

Haecceity

Proceedings of ANPA 32

Arleta D. Ford, *Editor*

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CONTENTS

<i>Arleta D. Ford</i> Editorial	1
<i>Ted Bastin et al.</i> Self-organization and the notion of level. A summary of three discussions	3
<i>John Amson</i> 'MoHoCH'. A Monster Holographic Combinatorial Hierarchy	24
<i>Anthony Blake</i> The language of will	52
<i>Dino Buzzetti</i> Neoplatonism, text and logic	89
<i>Geoffrey Constable</i> Ethereal thoughts	105
<i>Grenville J. Croll</i> Experimental observation of Kondo's Hyper Phi Meson and the Hyper Delta Hadron	129
<i>Barbara Gabrys, Julian Daizan Skinner</i> Zen meditation and mindfulness for modern times	135
<i>Nicola Graves Gregory</i> Human mathematics - new beginning from the Pythagoras Theano impulse	156

<i>Louis H. Kauffman</i> Non-Commutative worlds and classical constraints	166
<i>Garnet N. Ord</i> What happens between the ticks of a clock?	208
<i>Peter Rowlands</i> Nonlocality	227
<i>N C Thomas</i> A brief introduction to counterspace	251
ANPA Proceedings Editorial Policy	272
Guidelines for Preparation of Manuscripts	273
ANPA Statement of Purpose and Organization	274

EDITORIAL

Ted Bastn passed away in October last year. Instead of a photograph on the cover, I am enclosing here Angela's impression of a meeting with him¹:

It had been a long drive. We'd been in the car for over six hours, having had to drive across Wales to pick up Ted from his abode. Then we drove all the way to ANPA; there had been floods in Stroud so we had to go the long way, and long it was. Especially since the only music we could all agree on was classical, in the car. We finally got there, and then got into some confusion just for a little bit, until we finally parked up. Tired and hungry, we found our rooms, after having agreed to go for something that resembled food, in town. Within fifteen minutes of arriving we were walking down the street into Cambridge town centre; it seemed as if we had gone over some weird time loop having spent so long in the car together. We were weary, hungry, and together. The town was busy, being the height of Summer, and students cavorting all over; bikes, people visiting, and us in a somewhat overwhelmed state, we wandered around, realising that so many of the places we could go to were too far away; and we wanted to eat NOW, and with food intolerances, or intolerances to certain places to eat ('wink'), it was fast becoming a thing! Then, out the corner of our eye, we spied a pub; from the outside it felt 'ugh', but we'd come to the end of what energy we had to spare on looking, and so we reluctantly agreed, "Let's just go there". Inside it was busy, but soon we were attended to, and we were ushered downstairs into a cellar room that only had one other family in it. Surprised at this we found a corner and settled into a hearty meal of steak, mash, and veg. We ate in sporadic conversation, none of us really that bothered about talking, yet all agreeing and supporting how we had managed to get a good table in a restaurant that had turned out to be just what we needed.

After our meal we felt adjusted more to being in Cambridge and, our balance returning, we walked back to ANPA, and Ted, who had shared with us about the Chapel, took us in to give us the tour of it. It was dusk out, but we had a look inside and, lighted, chatted about the place, then, when finished, we stood outside, none of us really that keen on heading to the rooms. We decided to sit on the wall just outside the Chapel. The air around us changed as we felt something shift, and then Ted asked us the question that we had felt him stewing over for a while, but had never expected that he would ask. He asked us about death and dying? We talked about how we felt people get to choose, and when they are ready then they can let go of here. He said he felt it was hard because people here have a hard time. He asked us if we were frightened of dying; we both said no, but acknowledged that some of what happens is people have a fear of it and it is used to hold onto us. He said he felt that hold, and didn't want to hurt anybody, but that he was tired. He talked for a long time about his work, explaining to us that it was coming to an end. I felt he was

¹ Angela's text is slightly edited by K. Bowden

running out of reasons to be here, he was certainly trying to find more. We talked about people, and how they adapt, and finally he seemed to feel some resistance to what others were feeling about him, and from that he released some of his worry about other people; it wasn't that he was afraid, he just didn't want to leave others, yet didn't know how to admit to them that he'd had enough. It was the most open, fluid and real conversation I'd ever had with a person outside of my relationship with Jim. He asked us if we felt that dying was the end of us, or just the start of something else, and somehow during the conversation we all managed to sit on the same page of the world together, and I felt such love and peace with myself, and the two men I shared that moment with.

I loved Ted, he was a force of nature, and an institution in his own right, and I feel that he has flown to his heart's desire, wish you well my friend.

(Angela Honey, June 2012)

The second paper in the proceedings is a Mill discussion paper², never published before, rendered kindly from a manuscript to a readable PDF format by John Amson who also provided a Supplement of contributors and affiliations (see the last page of the paper). Let me quote John Amson:

The importance of this 1960 Working Paper is that it reports the first significant discussion of the innovative notion of a 'Hierarchy of Information Preserving Structures' with at least three 'Levels' — the precursor of what eventually, after much further detailed research, became the Bastin, Parker-Rhodes, Amson, Kilmister 'Combinatorial Hierarchy'. (*see the Mill paper in the proceeding*)

About the proceedings' title. Usually the title refers to a common subject of all the contributing papers. As I could not see any, I rendered the situation by the word: HAECCEITY suggesting by it what seems to be a common concern of the authors.

Wishing everybody great 2012 meeting!

Arleta D Ford

² We are very grateful to Suzanne Bastin and Yorick Wilkes who is Margaret Masterman's literary executor, for allowing us to publish the paper in this year ANPA proceedings. Every reasonable endeavour has been made by John Amson and myself to secure the permission of any unacknowledged Copyright Holder of the paper.

SELF-ORGANIZATION
AND THE NOTION OF
LEVEL

A SUMMARY OF THREE DISCUSSIONS
HELD IN
THE TOWER MILL
BURNHAM OVERY STAITHE, NORFOLK
AND IN CAMBRIDGE
SEPTEMBER – OCTOBER 1960

.....

First produced, in 1960, as
a Working Paper of the Information Structures Unit
Cambridge language Rsearch Unit
20 Millington Road, Cambridge, England

Transcribed into LaTeX and minimally edited
by JOHN AMSON

Anstruther, Fife, Scotland : john.amson@uwclub.net
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The importance of this 1960 Working Paper is that it reports the first significant discussion of the innovative notion of a ‘Hierarchy of Information Preserving Structures’ with at least three ‘Levels’ — the precursor of what eventually, after much further detailed research, became the Bastin, Parker-Rhodes, Amson, Kilmister ‘Combinatorial Hierarchy’.

Summary of Discussions held at Overy Staithe Mill September 26-27th 1960

Present: Richard Braithwaite, Fred Rigby, Margaret Masterman, Mary Hesse, Ted Bastin, Joan Miller, Tim Eiloart and Frances Ellis. The points raised were later discussed with Robert Thouless and Frederick Parker-Rhodes.

Self-Organizing Systems (SUMMARY BY MARY HESSE AND TED BASTIN)

The first discussions were regarded as clearing the ground for more detailed discussions later in the week. Initially an attempt was made to state the essential characteristics of a self-organizing system. This attempt aimed at specifying the machine, and must be set against the contrasting definitions (largely arising from the contributions of C.Bailey and generally felt to be much more satisfactory) produced at the end of the week, which looked to our changing state of knowledge of the structure of the machine to justify the use of the term "self-organizing". However, although preliminary discussions have to be seen against the background of a later radical change in viewpoint, it must be remembered that even under the later viewpoint some machines will be more suitable to exhibit the self-organizing property than others, and the preliminary discussion can be imagined as an attempt to compile a list of the attributes these "suitable" machines are usually found to have. Indeed Rigby, who was a major contributor, stressed that he could offer no general principles to explain the set of attributes he expected in a self-organizing machine; he was merely working with an intuitive idea of what self-organizing systems are, and generalizing from the attributes he had always found such machines to possess.

Three criteria were put forward for a self-organizing machine.

- a) It has at least one goal at any time.
- b) It can change its structure as well as its state in producing its output.
- c) The output from a given stimulus is less complex than the changes in the machine resulting in that output.

To these criteria, Bastin wished to add a fourth —

d) A self-organizing system must have a scan. Most generally this means that the system must be divided into at least two parts such that the state of one part as a whole, at a given instant, can be used to modify the {{p.1}} behaviour of the other part. The activity of the system must therefore be divisible into "time-slices" during any one of which the state of the one part may be assumed constant so that it can be scanned by the other part.

* * * * *

Discussion on Levels

Bastin said that a system with levels might be used to represent the discriminatory levels which arose in various sciences. He particularly mentioned the levels of degrees of discrimination defined in physics by the available "probing energy": (This is more fully described in Part II of this report).

Masterman said that any attempt to define level theoretically gave a picture unlike anything she had actually come across in any experimental field. (Bastin had assumed that his levels could be defined in the machine, by taking a level to be a separable or quasi-separable part of a machine, that has input to a level above it, and receives input from a level below it). What the experimentally discoverable levels seemed to have in common was some sort of mapping from one level on to another. Such mappings should reproduce structure from one level into another by a many-one (or sometimes a many-many) relation. Such a scheme of levels with structure-preserving mappings, leads naturally to the idea that in a succession of levels each represents an increase in discriminator potentiality; in the machine a simple structure is preserved and added to each change of level, and in the experimental field an experimental situation, already perceived in broad outline, has its detailed structure progressively filled in as levels of greater discrimination are reached.

Masterman gave the following examples of discriminatory levels in the literature:

1) 'Levels of Will', Margaret Masterman, in a paper presented to the Aristotelian Society, 1950.

2) 'Evolutionary Levels', J.N. Woodger (D'Arcy Thomsom volume). This is the locus classicus for the analysis of the concept of levels in biology.

3) 'Biochemical levels and hormone systems', J. Needham.

4) Bartlett and Head have written of many-levelled psychological schemata; material enters at a low level and works up to higher levels.

5) Many-levelled language systems. (Masterman does not believe in these as yet). {{p.2}}

Rigby was critical of the identification of levels with steps in discrimination. He instanced a railroad system in which one could think in terms of units of 1000 square miles of railroad, or of a whole state, or of the area covered by one signal box. It might be useful to make such a breakdown, but it would remain a mere convenience.

Masterman and Eiloart observed that if the unit of discrimination were taken small enough for it to matter that the flow of goods had to take place in discrete units (trains or even trucks) then the size of the unit would depend upon the experimental material, and would therefore cease to be arbitrary. They considered moreover that it was essential to systems of discriminatory levels that new kinds of phenomena should emerge at each level of greater discrimination.

Rigby¹ briefly described an idea he had had for representing an evolutionary system by a machine. A large set of vectors in a digital computer represent the organisms. Each has a genetic constitution represented as a set of parameters for a constant program. Each also has a "strength". Each is in an environment of random numbers. A scan is introduced to make the organisms evolve collaboratively. They evolve by changing their genetic constitution. They can also reproduce asexually if very strong. How much variance would the individuals stand ?

Bastin suggested a way of developing the mathematical treatment of energy-discrimination in physics. A basic "scale" number is assigned to each level. This number is, according to Eddington, to be identified with the electron / electromagnetic field coupling constant of current quantum field theory, and its value is to be derived from a particular partition of a general "tensor" of 256 components. In the CASPAR scheme this tensor is the 256-component level. Coupling constants of newer particle/field combinations are too near

¹Note: Rigby's suggestion aroused general interest *per se*, but it does not, actually, utilize any idea of "level".

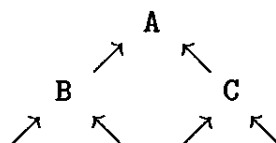
unity in value for the current type of calculation to be easily possible, and in this situation level theory might help in the following way: we might regard the electromagnetic case as one where there is a very exact experimental delineation of levels, corresponding to the large value of $1/\alpha$. In the other $\{\{p.3\}\}$ cases the epistemological element in the separation of levels is greater (they are really much more arbitrary) and the greater arbitrariness is represented by the smaller values of the coupling constants. The problem then is:- how to utilize the system of levels to cope mathematically with a situation where our mental requirements impose levels that are arbitrary in varying degrees.

Eiloart discussed levels in an administration system. He regarded an administrative system as a unit sufficiently clear-cut to illustrate some fundamental points concerning levels.

- 1) The complexity increases at each level.
- 2) The scope increases:

B C pass information A;

A chooses (scans) the information before acting.



A general problem is how to build the top levels in an experimental field (like language) where only the bottom level is understood.

Rigby was critical of the applicability of this example.

Eiloart described a self-organizing system consisting of a crawling mechanism on a plane, set to crawl along the edge of a plane figure like a rectangle, and such that it always turns a right angle when it reaches an edge. The right/left choice is made with probability that varies according to the past experience of the crawler, but such that neither probability is ever zero.

At the end of this discussion, the nature of the problem of determining what a scientific level was, and the great difficulties which lay in the way of employing the notion at all in any scientific field, were clearer to the contributors than its solution.

It was generally agreed that the thing to do next was to pursue a single example of an application to a machine of a system containing levels. This

system must be taken from a field in which, whatever else was unclear, the actual levels could be reliably distinguished from one another.

It was agreed that only Bastin's physical example (called by Rigby, to the surprise of some of the other contributors, "the simple physical case") fulfilled this criterion.

It was therefore agreed that Bastin should initiate the next discussion.
{{p.4}}

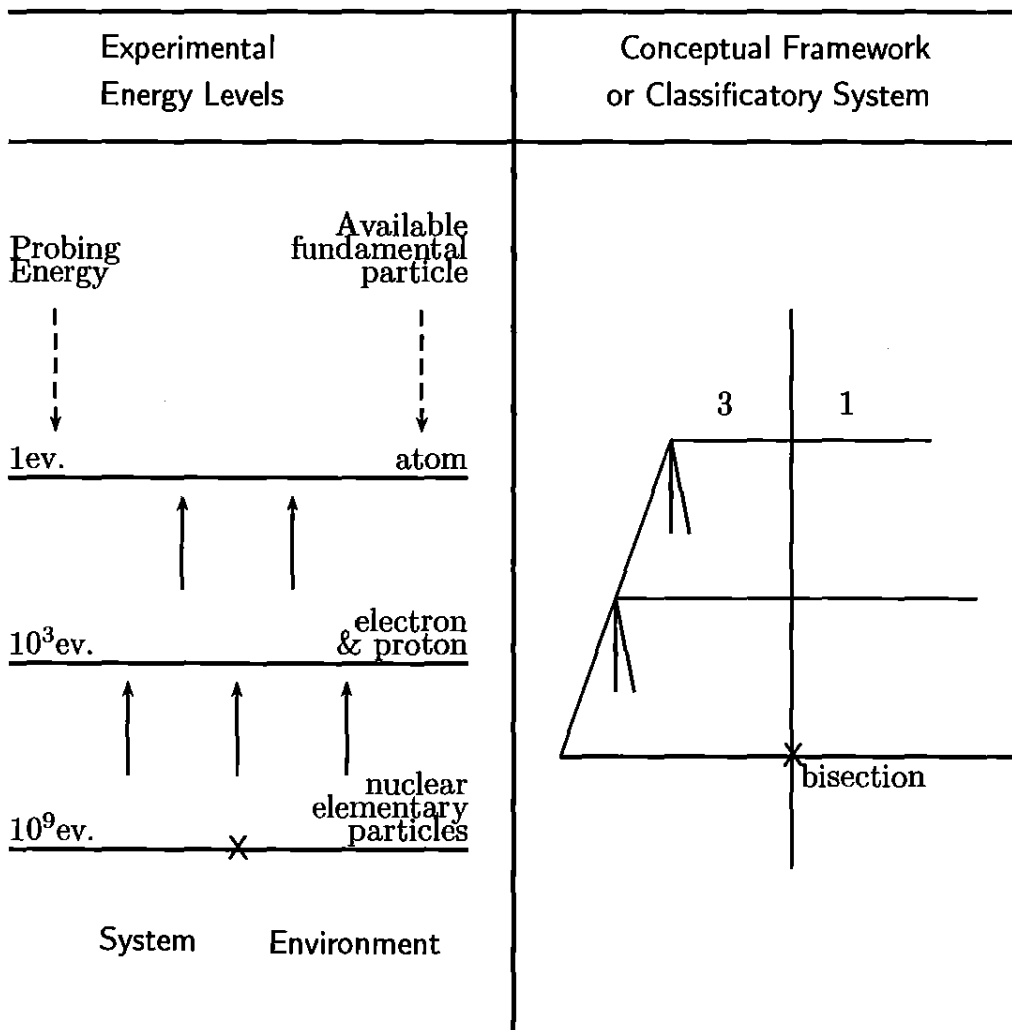
Summary of Discussions held at Overy Staithe Mill October 1st. 1960

Present: Mary Hoskyns, Richard Braithwaite, Christopher Bailey, Phyllis Bailey, Margaret Masterman, John Coales, Mary Hesse, Ted Bastin, Joan Miller, Elizabeth Pask, Gordon Pask, Frances Ellis, Sheila McKinnon Wood, Robin McKinnon Wood and Martin Kay.

Summary of Discussions (BY MARY HESSE AND TED BASTIN)

Bastin described two independent appearances of a hierarchical scheme of levels in physics:

...continued overleaf



{{p.5}}

1. Left-hand Diagram

This is a scheme of energy-levels defined in terms of the amount of "probing energy" that the physicist has at his disposal in order to detect his particles. The concept of an "elementary particle" is relative to this probing energy; thus, as shown in the figure, the only particle we can detect in Level I is the atom. With more energy it is possible to discriminate further detail. The hierarchy of levels thus defined continues in principle indefinitely downwards.

2. Right-hand Diagram

This represents a conceptual framework or classificatory system in which there is a definable homomorphic mapping from each level to the next level above it. Thus any simple structure defined at the top level is preserved in all the lower levels. The basic structure thus preserved throughout the levels is that of the 3 + 1 dimensionality of physical theory, and at the top level no other structure is discriminated. This 3+1 bisection at the top level causes a

corresponding bisection at each lower level. Bastin's theory of physical structure depends upon a device being available for superimposing these two schemes of levels, and he imagines the conceptual framework, characteristic of physical theory, in terms of this structure because it and no other makes the superimposition possible. To show this, replace the L.H. experimental levels by a model which consists of a finite hierarchical self-organizing system in which each level provides input to the level above it. Imagine furthermore that these inputs have some tangible physical counterpart in the model. The environment of the self-organizing system is the indefinitely continued set of lower levels. The only way in which this indefinitely continued set can be represented in the model is as a sub-set of the elements of the (arbitrary) bottom-level, thus defining a bisection of this level. This sub-set then represents input into the model from the environment and the order of the sub-set is the first introduction of a number into the system. This number arises out of the distinction between a measured system and the system by means of which measuring is performed. It is therefore reasonable, in the Eddingtonian manner, to identify such a constant with a $\{p.6\}$ dimensionless ratio of atomic constants which, physically speaking, do provide the scale for measurement.

The bisection in the bottom level of the model can now be identified with the bisection in the bottom level of the conceptual framework.

Summary (provided by Masterman)

Bastin appears to have:

- 1) A clear-cut physical interpretation of level in terms of the amount of energy used to discover particles.
- 2) A device for making this series of levels finite. At the bottom level (arbitrarily selected) some components are separated to represent the infinite regress of levels.
- 3) A way of introducing numbers into the system by superimposing a classification system with homomorphic mapping, and the resulting partitions, onto a self-organizing system with levels. The rule for superimposition is to identify the bisection point in the partition in the classificatory scheme with the point separating the components in the (arbitrary) bottom level from the rest in the self-organizing system.

4) A reason, depending on the use of a self-organizing system, for adopting Eddington's view that the fine-structure constant is to be identified with one of the numbers described in (3) above.

But Bastin has as yet no clear conception of the actual compound system thus derived (i.e. no physical picture of the particles). He has only a series of hunches for connecting his model with current physics.

In the discussion that followed, Bailey suggested that if the contributors present were to be of any help to Bastin, the question of the nature of a self-organizing system must be re-opened in the widest possible way.

He then described three increasing degrees of "self-organization":

- 1) A simple physical object with a source of energy (a rocket, say).
- 2) A machine with input and output.
- 3) A self-organizing machine (in a sense to be defined below).

Bailey wished to call a machine self-organizing if it is decided to say about the relation between its input and its output that not everything is known about this relation. But he started by saying that a system is self-organizing {{p.7}} if the relation between input and output "depends greatly on the past history of the system". To clarify this term, "greatly", he tried to re-distinguish (2) and (3), as given above, as follows: many networks which we should not wish to call "self-organizing" would fall into class (2), where the input-output relation depends to some extent on their past history; but a class (3) machine will have this property to a marked extent. For example, gyroscopes would not count as self-organizing, even though their motion might depend on a torque given up to, say, 3/4 hours ago.

A more illuminating way of looking at the distinction is in terms of the extent of our knowledge of the internal structure and states of a system; a self-organizing machine will then be a system with inputs and outputs, about the relations between which we shall say that not everything is known.

To develop the distinction further, consider two examples of self-organizing systems:

- i) Uttley's machine.

This starts as a *tabula rasa*, with no connections yet made between inputs and outputs (for all we know - a hollow box). In the end, it has wires con-

necting inputs to outputs; that is, the connections between inputs and outputs are completely known. But what is it like in the meantime? In the meantime it is self-organizing, that is, not everything is known about it; or, perhaps what is known (in terms of hardware) is ignored.

ii) Uttley's machine, so adapted that it never comes to the final state.

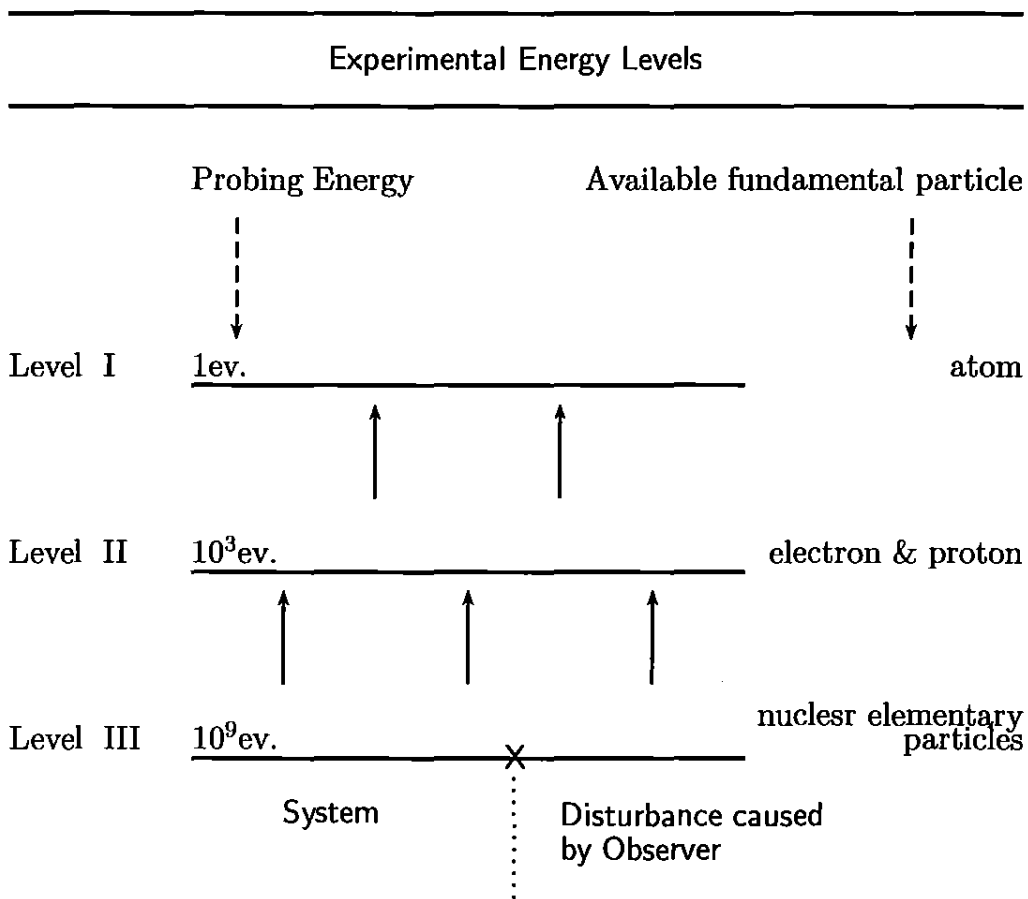
After a large number of trials, such a self-organizing system must either degenerate into an active or a passive network (in which case it "dies"), or else go on indefinitely absorbing its environment into its structure. If a self-organizing system is to model part of the world, therefore, one must ask whether that part of the world does in fact absorb its environment without limit. In this case, also, it will never be convenient to assume that everything is known about the machine; that is, it will always be self-organizing.

Bastin: I think my system is an instance of Bailey's last case, since I do indeed think of my partitioned bottom level as the simplest case of absorption of the environment. Because if I extend my model by adding another level, then those elements that were partitioned off become active parts of the model
{{p.8}}

Braithwaite: What is Bastin's environment? Bailey's description of a self-organizing system is one that is only characteristic of living organisms.

Bailey: No, I think I can see what Bastin is getting at here. Bastin's environment is the universe; and there is feed-back between observer and the universe. Braithwaite has to remember that Heisenberg has shown that we must consider the effect of the observer when considering the universe². The disturbance caused by the observer is modelled, in Bastin's schema, by the section to the right side of the partition in Level III, as shown below:

²This effect will be referred to below as "Heisenberg uncertainty"



Bastin: That is true, but I regard the Heisenberg position as an undesirable one from which we can escape with the help of the self-organizing model. For the use of this model takes us into a sphere about which we can make some positive assertions.

Pask: If we are considering the relations of Bastin's system to Heisenberg's Uncertainty Principle, then I wish also to consider its relation to Ashby's system. The Heisenberg uncertainty, after all, is uncertainty in making a measurement; the Ashby "uncertainty" is an "uncertainty" in determining the type of attributes which are to be used to describe the system. And Bastin's system, and the way in which he uses "measurement" seems to me far more like the Ashby "uncertainty" than like the Heisenberg uncertainty; and so I want to carry a comparison between Bastin and Ashby a little further. Ashby is only prepared to apply his systems (here called "Ashbean systems") in circumstances in which it is already agreed what to take as the $\{p.9\}$ effective states and transformations. Discussion of whether these states and transformations are the best ones for describing the world is excluded. Hence we shall call the uncertainty in the answer to this question "Ashbean uncertainty". Thus the Ashbean uncertainty

is Bailey's 'black box' uncertainty: that is to say, it is uncertainty of the same kind as Uttley's machine exhibits in its middle stages.

If you were trying to find out what Uttley's machine in its middle stages was doing, you would construct a mental model with a set of imaginary attributes which were variables in the model. Thus in terms of the model, the states of the system modelled — in this case Uttley's machine — could be defined by vectors in a phase space that had these attribute-variables as degrees of freedom, or coordinates. Experiments may then show that the set of attributes so far assumed is inadequate to define the system uniquely. Then it will be necessary to extend the set. (Such a set of attributes can be called a reference frame.)

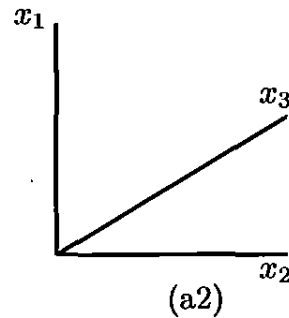
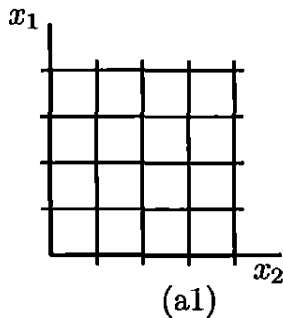
This extension may be made in two ways:

- a) By using a finer descriptive mesh, or
- b) by taking a larger system.

(a) again divides into two possibilities, namely

- al) the subdivision of the phase space more finely, or
- a2) the inclusion of a new descriptive property, as shown in the figures

below:



There is then uncertainty about what would be the best variables to add to the set at every stage in the process of discovering what Uttley's machine was doing; right up to the end point when complete knowledge is available. Remember, too, that the Uttley machine we are talking about has all this time `{{p.10}}` been reacting to stimuli from some external source, so there is a sense in which we have been finding out something of the external world in discovering the nature of the behaviour of the machine. In this sense, Uttley's

machine resembles the hierarchical structure Bastin wants to setup, but there seems at first to be a sharp distinction between the Ashbean and the Heisenberg uncertainty.

Bastin: I have already pointed out that I regard my hierarchy as a very simple case where elements from the partitioned set representing the environment can be incorporated into the system. This process of incorporation could clearly be regarded as itself a very simple representation of what happens when in Ashby's type of theory new variables or attributes are selected and incorporated within the system. Now, if I show, as I think I can, though I have not attempted to do so in this discussion, and even though I am not at all clear how the details of the demonstration would go, that the same device accounts for the existence in physics of the Planck uncertainty constant \hbar ; then surely I shall have shown that Heisenberg uncertainty is a special case of Ashbean uncertainty. That, at any rate, is what I meant to show. Verybriefly my demonstration is this:

The partition in my bottom level provides me with a number that has got, in some way, to be associated with a physical system. That is to say, it has to be a physically measurable number. The only way this is possible is for it to be identified with one of the well-known dimensional constants of physical theory. All of these numbers are currently thought of as derived by forming ratios of fundamental dimensional constants. The only one whose factors do not include any cosmologically based dimensional constant is the fine-structure constant $\alpha = e^2/hc$. This constant contains the factor \hbar , and if we accept for a moment that there are atoms (and hence an atomic unit of charge e) and that light has finite velocity, then we can regard our theory as defining a finite Heisenberg uncertainty, \hbar . Thus my demonstration deduces h from an uncertainty of the Ashbean type.

Pask: Yes, but we had already agreed that in Ashby's approach, states and transformations of the machine are exactly defined: though the interpretation of a given machine state in terms of an experimental system of which the machine is meant to be a model may change, nevertheless at every $\{\{p.11\}\}$ moment an unambiguous interpretation exists. The point here is that what we have agreed to call "Ashbean uncertainty" arises through a re-definition of states

and transformations. You are wanting to bring the uncertainty into the system and that in the Ashby approach you never can do. Your extension of Ashby's system would therefore have to be at the expense of clearness of definition of the states and hence of their interpretation.

Masterman: Actually the only way to escape from an Ashby system is to insist that the interpretation or identification of the self-organizing system is itself part of the scientist's problem.

Bastin: Ashby never allows that it can be.

Coales: Well, Ashby escapes from a discussion of interpretation by introducing the concept "variety". Bastin puts the "variety" below his bottom level. In so doing he causes the relevant question to become not (as with Ashby), how do you cope with the variety?, but how do you make use of the self-organizing character of the machine? And this boils down to, how do you introduce feedback, and why?

Bastin: My immediate concern was to deduce physical consequences from the fact that something had to be done to represent the introduction of variety into the system. I derived consequences from the fact that the variety represented by the indefinite sequence of levels had to be compensated for, that is, by an artifice. In other words I was arguing, illegitimately if you wish, from the fact that I could use a machine to represent a physical system, to certain consequences. I was not justifying my assumption by showing what such a use would be like. I have to take a feed-back loop, however. What form could it take?

Braithwaite: I do not see how any kind of temporal process can represent a logical structure or logical organization. It seems to me that the only thing that Bastin's machine can be is a machine whose activity in some way represents the process of extension of the body of physical knowledge.

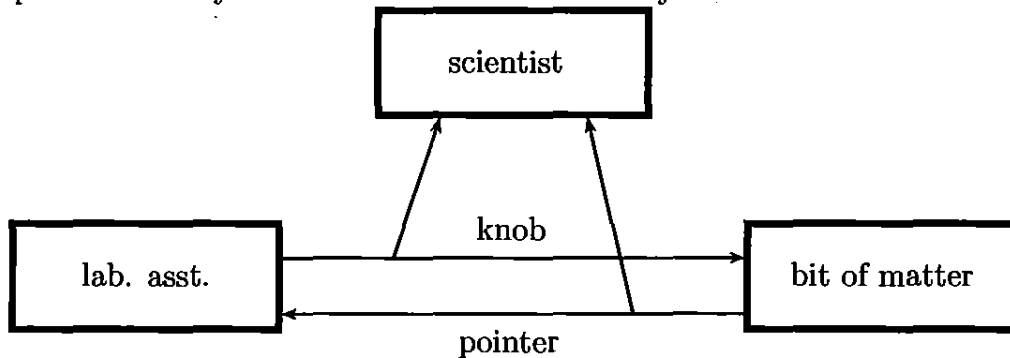
Masterman: It is essential to realize that Bastin is studying a field where the way the theory developed, regarding the theory for the moment as a classificatory system which is certainly one of the functions of any theory, is in principle relevant to the form the theory takes. I think Bastin should take {{p.12}} very seriously the suggestion that I think Hesse made at one moment.

She said one should think of the elementary particles as being like clouds.

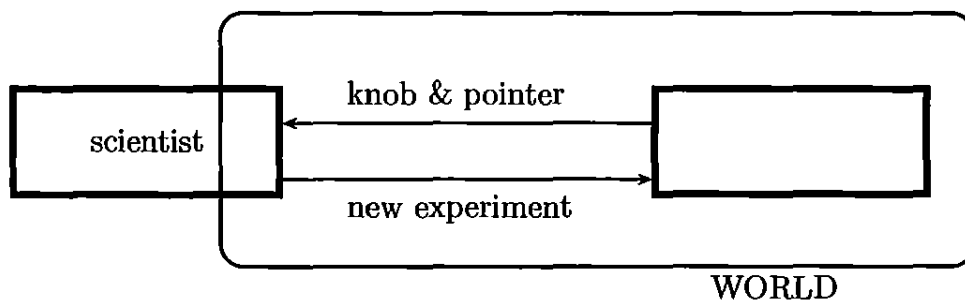
The individuality of clouds depends on the whole classification imposed on the weather by the meteorologist, but having been given this strange kind of individuality, they are capable of putting further experimental evidence into the total description. Bastin needs a pictorial analogy to replace the billiard ball, and modern quantum theory does not provide it. I find it helpful to say,

“Bastin’s particles are evanescent; they form and reform again; they are not in any ordinary sense entities; yet they can be classified; they are a great deal more like clouds than they are like billiard balls”.

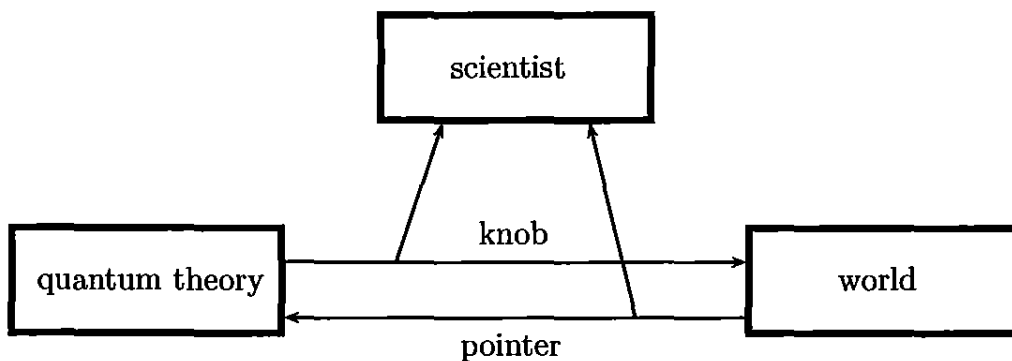
Bailey: I suggest a simple description of the classificatory problem raised by quantum theory which I summarize in this way:



As a result of finding himself in this situation, the scientist thinks of a new experiment, and this we represent as follows:

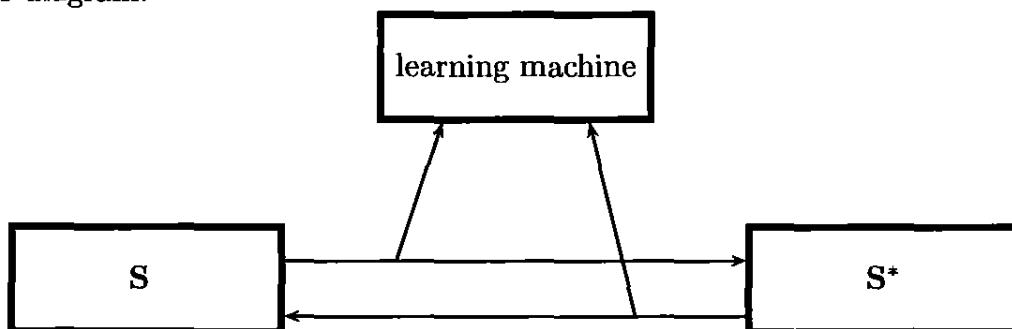


Here the scientist cannot extricate himself from his world. The classical physicist forgot this inconvenient fact, but the microscopic physicist can't and so he invents a type of theory in which this fact is taken care of explicitly. You then get the following diagram:



It seems

to me {{p.13}} that Bastin then tries to represent the whole situation as in this diagram:



Here **S** is Bastin’s model, and **S*** is the real world as represented by data about elementary particles. The moral to be drawn is that Bastin’s system has got to include a “learning machine” part. It has to be composite.

Bastin: Yes, but we seem to have gone very much back to the beginning again. You see what you say about microscopic physics would apply equally well to macroscopic physics. A sophisticated scientist in the middle of the nineteenth century might have argued just so. I mean we seem to have lost sight of the position we were at such pains to establish earlier: namely that it is Heisenberg uncertainty we expect to come out of our study, and not just a metatheory of a quite general kind about the process of making up theories.

Braithwaite, however, said that he felt that Bailey’s interpretation of Bastin’s position was probably right.

At this point the discussion broke for lunch.

* * * * *

The discussion resumed in the afternoon.

Hesse: I can see one way in which the hierarchies of Bailey and Bastin might be reconciled. We could say that Bailey’s “observer” or “scientist” in

the second stage corresponds to investigation of the electron-proton level in Bastin's hierarchy, since, at least in the Copengaen interpretation, there is an observe-experiment relation similar to that described by Bailey. The third stage of observer-experiment would then correspond to nuclear theory, in which quantum theory is still presupposed, and the relation between nuclear theory and quantum theory is here similar to that between quantum theory and classical physics on the second level. Bailey's "lab. assistant" then corresponds to a scientist using classical physics. {{p.14}}

Bastin: I see the argument in general. Of course my levels were levels of increasing discrimination and this progression in terms of discrimination is not to be identified with the historical progression :

classical theory → quantum theory → nuclear theory

Pask: I'm not very clear about Bastin's levels, and it may be worthwhile considering other appearances of levels in self-organizing systems. The one I am familiar with arises in a Turing machine which can make statements about its own state. In terms of these self-referential statements we can construct a hierarchy rather like Russell's types. The levels in the hierarchy are ordered in terms of the number of quantifiers needed to specify them. On the other hand, I don't think a Turing machine is self-organizing.

Anyhow, I don't think the "generalization" type of level which covers all the ways of getting levels out of the meta-language idea as well as those in a Turing machine is sufficient to represent levels of observation in Bastin's sense.

Bailey: Now that we have got onto levels, I'd like to return to some previous remarks of mine about the observer-world situation as a learning situation. It was because I wanted to introduce the topic of levels in a sufficiently fundamental way for us to understand how they arise that I thought it was worth going right back to the beginning and re-opening the whole question of a possible relation between the learning of a theory and the structure of the same theory in its widest generality.

You see, I think we have something like a theorem in this subject to the effect that when and only when a machine consists of at least one self-organized system being "studied" or scanned by a second self-organizing system, only then is the whole system going to be valuable in application to experimental data of

any kind. If a system is not going to be treated as composite in this sort of way,, then we might as well regard the machine as determinate. But now, any total system consisting of at least one self-organizing system scanning another is basically a system with at least two levels.

Pask: Let us consider the representation of the states of models by means of arrays of transition probabilities. There seem to be three possibilities {{p.15}} in ascending order of complexity:

1. Machines with fixed transition probabilities.
2. Machines with variable transition probabilities.
3. Systems where we don't know enough even to have a probability measure.

I should be very surprised if Bastin could do anything without going to type (3).

Bailey: Broadly, I agree with Pask, but I think something might be done in special circumstances with types (1) or (2).

Coales: I think type (1) should be adequate and should be investigated.

After this discussion, which was felt on all sides to have been a notable one, many of the participants felt that Bailey had produced a considerable clarification in the subject by his definition of a self-organizing system, and that he should write this up for publication.

Coales, however, felt that other schemata had been substituted by the participants for Bastin's, which was the one he (Coales) was interested in developing, and that this substitution had made the discussion go "up in the air".

A further discussion was therefore held between Coales, Bastin and Masterman to discuss the next step to be taken to develop the Bastin schema.

(A summary of this discussion is attached as an Appendix to this report).
{{p.16}}

Appendix
—
Summary of a Meeting
Held at 280 Hills Road, Cambridge
on
October 27th, 1960

Present: John Coales, Margaret Masterman, and Ted Bastin.

Summary by Margaret Masterman.

Coales said that he had asked for this further meeting in order to advise that an experimental approach to the self organizing- systems-in-physics work should be put in hand at once. He thought the large Associated Electronics PACE analogue computer equipment at Brussels could be used over Christmas to start experimentation, and that the form dictated by the computer to the experiments would itself stimulate further progress.

In the subsequent discussion, the fundamental principles which must govern the experimentation were determined. At first sight these principles appear to be independent of the theoretical constructions described by Bastin in the foregoing discussions and to have validity independently of the choice of particular sets of interconnections set out in the diagram [XXX] in Part II. Actually this independence is illusory however.

Additional Interpretative Principles:

1) Particle masses, which represent the inertia of atomic systems, must appear as the times taken to pass from one equilibrium state of the machine to another, there being one equilibrium state for each particle.

2) The quantum numbers that characterize elementary particles are to be represented as sets of initial transition probabilities in a square array.

3) A set of "correct" values or fundamental pattern of transition probabilities in one array must be sought as the main immediate object of the research. This fundamental pattern, if found, will generate the set of known elementary particles, as steady states of the whole machine and one will know $\{p_{17}\}$ which steady state represents which particle by finding the values that the transition probabilities have taken up in the steady states and then interpreting them as quantum numbers. The basic pattern to be found by this type of ex-

periment thus becomes the mainspring of the physical world. Such an idea is not altogether unfamiliar, for this array, which, if treated algebraically, would be a matrix, resembles the S-matrix proposed by Heisenberg. Bastin said that the pressure to experiment applied by Coales was entirely congruent with the experimental intuitions he himself was having. The novelty that had arisen out of this discussion was two-fold:

a) The stochastic approach to it, investigated mechanically, was to have priority over the logico-algebraic approach. (Bastin had hitherto concentrated, in his thinking, on the latter, even though the machine-stochastic approach had been described in very general terms in one of his publications). In any case, the essential features of the algebraic approach cannot be ignored in the approach now being advocated, for levels, defined by a mapping function, would have to be built in as interconnections in the actual analogue machine. Bastin therefore agreed that the stochastic method was the obvious next experimental step.

b) Coales, having located at Brussels an analogue machine of sufficient size to be set up in 3 levels, and having got Bastin a dated invitation to set it up, had made it possible for the first time for the technique which had been suggested to be carried out. {{p.18}}

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...continued overleaf

Supplement : Contributors

added by John Amson, 2012

(with some known Affiliations, c.1960

- 1 Ted Bastin ° ISU, CLRU
- 2 Richard Braithwaite ° CU Knightbridge Prof. Moral Philosophy
- 3 Christopher Bailey
- 4 Phyllis Bailey
- 5 John Coales ° CU Control and Systems Group, Dept.Eng.
- 6 Tim Eiloart ° founder, Cambridge Consultants Ltd (CCL) °
- 7 Frances Ellis
- 8 Mary Hesse ° CU Dept. History and Philosophy of Science
- 9 Mary Hoskyns
- 10 Martin Kay ° CLRU
- 11 Margaret Masterman ° CLRU
- 12 Joan Miller Philosopher
- 13 Frederick Parker-Rhodes ° CLRU
- 14 Elizabeth Pask
- 15 Gordon Pask ° Cybernetician; co-founder, 'System Research Ltd'

[see also : <http://turnbull.mcs.st-and.ac.uk/history/Biographies/Pask.html>]

- 16 Fred Rigby
- 17 Robert Thouless ° CU Educational Psychology
- 18 Robin McKinnon Wood ° CLRU, co-founder, 'System Research Ltd'
- 19 Sheila McKinnon Wood

[° : has a Wikipedia / Google entry.]

CLRU = Cambridge Language Research Unit

CU = Cambridge University, ISU=Information Systems Unit

oo

' M O H O C H '

A MONSTER HOLOGRAPHIC COMBINATORIAL HIERARCHY

'WHEREIN EVERYWHERE IS EVERYWHERE'

. . . A Report on Work in Progress . . .

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ABSTRACT

The formal process construction of **PuCH** (the Purely 'Combinatorial' Combinatorial Hierarchy) is investigated more deeply. It transpires that there is an intrinsic self-replicating 'breeding process' in **PuCH** which gives to **PuCH** a **MONSTER** tree-like structure with a remarkable **HOLOGRAPHIC PROPERTY**. This new system is given the name **MoHoCH**, and some provisional combinatoric properties and implications are tentatively explored. It is suggested that **MoHoCH** could be the first Set Theoretic model framework for Bohm's Implicate Order.

Introduction

This paper is a continuation of my papers ([A1], [A2]) :

' P U C H '

A PURELY 'COMBINATORIAL' COMBINATORIAL HIERARCHY

'ONLY WHAT WE CAN COUNT IS ACCOUNTABLE'

and

' G O L D B A C H , P U C H AND C H '

YET ANOTHER NEW COMBINATORIAL HIERARCHY

'WITHIN EVERYTHING IS THE SEED OF EVERYTHING'

both of which appeared in ANPA PROCEEDINGS No 31 Year 2011.

The section Numberings, *etc.*, continue on from that second paper.

7 Intrinsic ‘breeding chains’ in PuCH.

7.1 Informal introduction.

In §2 of [A1], I showed how the PuCH hierarchy comes into existence, using just four axioms from Naive Set Theory, the notion (as an additional axiom) of the Power Set *Power Set* $\mathcal{P}(\mathbf{S})$ (the set of all subset of a set \mathbf{S}) and its related ‘punctured’ power set $\mathcal{P}^*(\mathbf{S})$ (the power set with the Empty Set excluded).

We generate PuCH beginning with the Empty Set ‘Nothing’ \emptyset , and the next most primitive set : the ‘set containing the Empty Set’ $\{\emptyset\}$, which I labelled \mathbf{S}_0 . It is a set with 1 member.

We then form its Power Set $\mathcal{P}(\mathbf{S}_0)$. It has 2 members : the Empty Set \emptyset (every set contains the Empty Set) and the set of all its members, *i.e.* the subset containing its only member \emptyset , *viz.* $\{\emptyset\}$. It is a new set, the second most primitive set, which I labelled

$$\mathbf{S}_1 = \{ \emptyset, \{\emptyset\} \} = \mathcal{P}(\mathbf{S}_0) \quad (1)$$

Remark 7.1. Unlike PuCH, the initial ‘Ur-set’ in the classical CH was taken directly to be the equivalent of \mathbf{S}_1 , namely a set $\{0, 1\}$ with two distinct but undefined elements labelled 0 resp.1 and identified with the binary integers 0 resp.1, subsequent members of the generated Levels being identified with ‘bit-strings’ (binary/boolean lists) or, equivalently, with larger binary integers. \square

We then form \mathbf{S}_1 ’s set of subsets (but now excluding the \emptyset), which I labelled

$$\mathbf{S}_2 = \{ {}_1\{\emptyset\}, {}_2\{\{\emptyset\}\}, {}_3\{\emptyset, \{\emptyset\}\} \} = \mathcal{P}^*(\mathbf{S}_1); \quad (2)$$

— here I am using temporary prefix numbers to emphasise the counting.

Again we form its set of subsets (excluding the \emptyset), which I labelled

$$\begin{aligned} \mathbf{S}_3 &= \{ {}_1\{\{\emptyset\}\}, {}_2\{\{\{\emptyset\}\}\}, {}_3\{\{\emptyset, \{\emptyset\}\}\} \\ &\quad {}_4\{\{\emptyset\}, \{\{\emptyset\}\}\}, {}_5\{\{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}, {}_6\{\{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}, \\ &\quad {}_7\{\{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\} \} \\ &= \mathcal{P}^*(\mathbf{S}_2). \end{aligned} \quad (3)$$

Continuing this process of ‘forming the set of all subsets’ of a previous subset in the Hierarchy delivers the PuCH in its entirety.

Remark 7.2. Note that for **PuCH**, once we had reached S_1 , then for the next step to form S_2 , and all subsequent steps (‘Levels’ in the older **CH**-speak), we always excluded the ‘Empty Set’ subset from a previous subset before forming its Power Set. This was done to make **PuCH** match the counting sequence 3, 7, 127, ... $\sim 10^{38}$ so characteristic of the original (Bastin, Parker-Rhodes, Amson, Kilmister) Combinatorial Hierarchy **CH** (*cf.* all ‘discriminately closed subsets’ in **CH** intrinsically have the ‘zero’ excluded, thus incidentally identifying them with finite Projective Geometries [A4]).

Had we not done that (although we were unaware of this at that time in the 1960s) then the hierarchy we would have generated would have had been wholly identical to an earlier hierarchy known as a *Hereditarily Finite Set*, **HFS**, and almost the same as a ‘*The Cumulative Hierarchy*’ = ‘*The von Neumann Universe*’ **V**, both introduced by von Neumann (c.1955). The latter, in turn, is a less constrained version of the ‘*The Constructible Universe*’ introduced even earlier by Gödel (1940). These pre-dated our original **CH** (1965). [See §5 in [A1].] Easy introductions can be found, for example, in [W1] and [W2]. For a formal account see ‘*The Hierarchy of Constructible Sets*’ in Ch.13 in [Je]. For a more readily accessible account see [D1] and for a more comprehensive account in the general context of Set Theory see Ch.5, ‘*The Axiom of Constructibility*’, in [D2]. As a matter of historical record the original Gödel monograph on Constructibility appeared in [Gö]. □

Remark 7.3. As in **PuCH** and **MoHoCH**, in a ‘Hereditarily Finite Set’ each Level after the initial one is distinct from all preceding ones. Whereas in a ‘von Neumann Universe’ each level is always the Union of itself and all the preceding Levels. The first part of this paper deals only with the ‘Hereditarily Finite Set’ formulation of **PuCH** and **MoHoCH**. In a subsequent paper in this present series I shall introduce a ‘von Neumann Universe’ formulation by forming Unions and will examine the differences. □

We next observe that the Set of all Subsets of any given set **S** always includes the set **S** itself. So let us rewrite the descriptions (2) and (3) like this (using

' S_1 ' everywhere instead of ' $\{\emptyset, \{\emptyset\}$ ') :

$$S_2 = \left\{ {}_1\{\emptyset\}, {}_2\{\{\emptyset\}\}, {}_3S_1 \right\} = \mathcal{P}^*(S_1). \quad (4)$$

$$\begin{aligned} \text{and } S_3 = \left\{ {}_1\{\{\emptyset\}\}, {}_2\{\{\{\emptyset\}\}\}, {}_3\{S_1\} \right. \\ {}_4\{ \{\emptyset\}, \{\{\emptyset\}\} \}, {}_5\{ \{\emptyset\}, S_1 \}, {}_6\{ \{\{\emptyset\}\}, S_1 \}, \\ \left. {}_7\{ \{\emptyset\}, \{\{\emptyset\}\}, S_1 \} \right\} = \mathcal{P}^*(S_2). \quad (5) \end{aligned}$$

We see that the special '*breeding trigger*' set S_1 — from which the rest of PuCH is sequentially constructed by the action of forming successive 'Punctured Power Sets' (Power Sets with the Empty Set excluded) — is a '*node*' that is re-appearing more and more profusely at each successive construction.

To make it even easier to read, let us change to an even simpler notation, with $\{\emptyset\} \leftrightarrow a$ and $\{\{\emptyset\}\} \leftrightarrow b$, thus :

$$S_2 = \left\{ a, b, S_1 \right\} \quad (6)$$

$$S_3 = \left\{ \{a\}, \{b\}, \{S_1\}, \{a, b\}, \{a, S_1\}, \{b, S_1\}, \{a, b, S_1\} \right\} \quad (7)$$

whilst

$$\begin{aligned} S_4 = \left\{ \{ \{a\} \}, \{ \{b\} \}, \{ \{S_1\} \}, \{ \{a, b\} \}, \{ \{a, S_1\} \}, \{ \{b, S_1\} \}, \{ \{a, b, S_1\} \}, \right. \\ \{ \{a\}, \{b\} \}, \{ \{a\}, \{S_1\} \}, \dots \{ \{S_1\}, \{a, S_1\} \}, \dots \text{etc.} \dots \\ \{ \{a\}, \{b\}, \{S_1\} \}, \dots \{ \{S_1\}, \{a, S_1\}, \{b, S_1\} \}, \dots \text{etc.} \\ \left. \{ \{S_1\}, \{a, S_1\}, \{b, S_1\}, \{a, b, S_1\} \}, \dots \text{etc. etc. etc.} \right\} \quad (8) \end{aligned}$$

where the 1st row lists the singleton subsets, the 2nd row lists the subsets with 2 members, the 3rd row lists the subsets with 3 members, the 4th row lists the subsets with 4 members, and so on . . .

Remark 7.4. Counting successive appearances of S_1 , we see that :

there is 1 re-appearance in S_2 (out of a total of $2^2 - 1 = 3$ members);

there are 4 in S_3 (out of a total of $2^3 - 1 = 7$ members);

and a detailed survey (see Table 1 in the next Section §7.2) shows that

there are $256 = 2^8$ re-appearances in S_4 (out of a total of $2^7 - 1 = 127$ members).

Note that though 256 is greater than the 127 members in S_4 (the Punctured Power Set $\mathcal{P}^*(S_3)$) this is because many of the individual elements in S_4 are of the form having more than one re-appearance S_1 . For example $\{\{S_1\}, \{a, S_1\}\}$ in (8) has 2 re-appearances, whilst some others have as many as 3 or even all 4 re-appearances in a single such term in (8). \square

It should be theoretically possible to count the number of re-appearances in S_5 (out of a total of $2^{127} - 1 \approx 10^{38}$ members) (see TABLE 3 in §7.2), or indeed in any S_n , but there appear to be practical obstacles at the present moment.

However, at this stage of the investigation it is sufficient to observe that

- (i) there are very many re-appearances throughout all the stages of the sequential process of constructing **PuCH**.
- (ii) And each such re-appearance a separate (secondary) construction (breeding) process of **PuCH** is triggered.
- (iii) And in each of these (secondary) trigger-created copies of **PuCH** the same triggered breeding process is inherent, so that each of these carries a (tertiary) breeding property of the same kind — endlessly iteratively — so creating a MONSTER tree-like structure. The diagram on the following page suggests (very imperfectly) this tree-like breeding process . . .

Remark 7.5. We observe that there is a re-appearance of the ‘breeding trigger’ node S_1 as soon as the construction process has delivered the Punctured Power Set S_2 . We further observe that there are (potentially infinitely) many re-appearances of S_2 throughout the spreading tree-like construction as **PuCH** is generated. \square

Remark 7.6. Consequently, **PuCH** can be ‘broken’ into two halves anywhere one chooses at any position along any branch past the first appearance of a (copy of) S_2 , and each ‘half’ of the broken **PuCH** will contain (infinitely) many copies of **PuCH**. This is a remarkable property of the construction process outlined above. Any such ‘broken half’ of **PuCH** can again be broken anywhere one chooses into two halves (subject to the same initial restriction about S_2) and the same phenomenon is present — endlessly.

In effect, the totality of the MONSTER tree-like structure that is **PuCH** has a HOLOGRAPHIC PROPERTY. \square

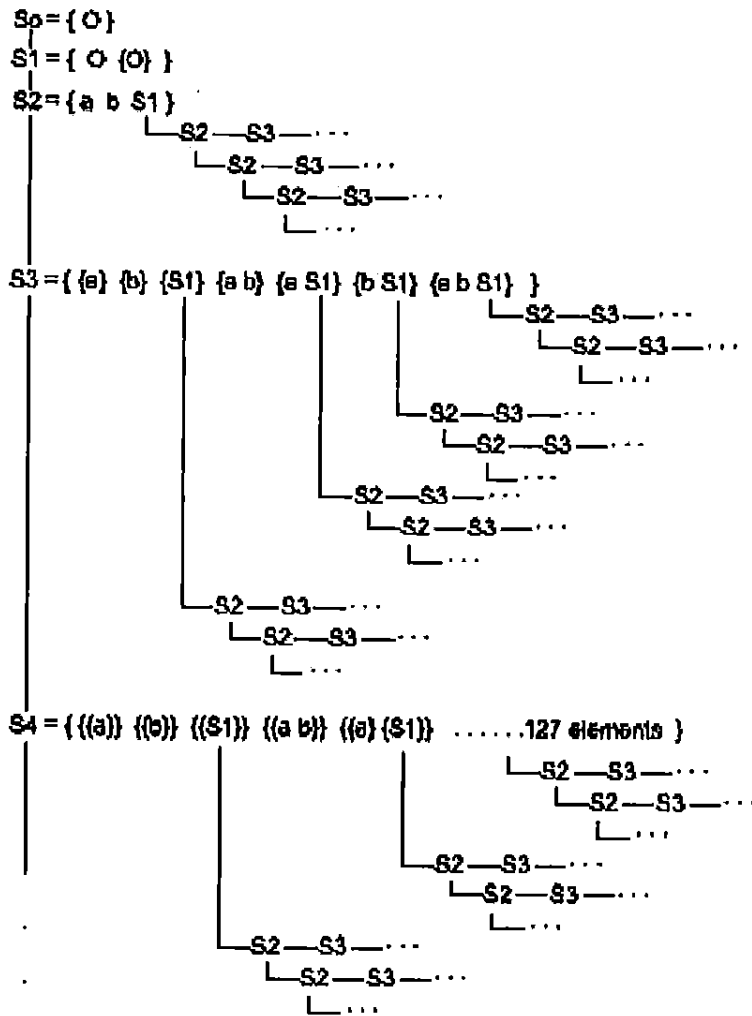


Figure 1: THE TREE-LIKE 'BREEDING PROCESS' INTRINSIC IN PuCH AND GIVING RISE TO MoHoCH

Note that only 3 'branches' of the 'tree' are shown growing from the 4th Level S₄; there are actually 256 such 'branches' of the 'tree' starting to grow there (see §7.2 below).

7.2 Combinatorics in MoHoCH.

The 'Combinatorial Hierarchical' properties (*cf.* the original CH philosophy) are challenging — and a full investigation is possibly awesome. However, some important initial studies are obtainable through a mix of empiricism and

analysis. The element-count sizes of the first few Levels are $|\mathbf{S}_1| = 2$, and

$$\begin{aligned} |\mathbf{S}_2| &= 2^2 - 1 = 3; & |\mathbf{S}_3| &= 2^3 - 1 = 7; \\ |\mathbf{S}_4| &= 2^7 - 1 = 127; & |\mathbf{S}_5| &= 2^{127} - 1 \approx 10^{38}; \end{aligned} \quad (9)$$

7.2.1 Combinatorics in Level \mathbf{S}_4

As a first step let us assess the first few successive appearances of the 'breeding trigger' node \mathbf{S}_1 . as indicated in Remark (7.4) and the expressions (6), (7) and (8).

We shall display, and count, these successive appearances in each class of subsets organised by subset-size. For visual simplicity we shall

(a) let the ordinal numbers 1 2 3 ... stand for the Level set elements ordered by increasing complexity,

$$\begin{aligned} \text{viz. } \mathbf{S}_2 &= \{ a, b, \mathbf{S}_1 \} \equiv \{ 1, 2, 3 \}; \\ \text{and } \mathbf{S}_3 &= \left\{ \{a\}, \{b\}, \{\mathbf{S}_1\}, \{a,b\}, \{a, \mathbf{S}_1\}, \{b, \mathbf{S}_1\}, \{a, b, \mathbf{S}_1\} \right\} \\ &\equiv \{ 1, 2, 3, 4, 5, 6, 7 \}; \end{aligned}$$

(b) highlight any digit representing a member in which a \mathbf{S}_1 appears by inserting an over-hat *e.g.* $\hat{3}$;

$$\text{thus } \mathbf{S}_3 \equiv \{ 1, 2, \hat{3}, 4, \hat{5}, \hat{6}, \hat{7} \};$$

(c) eliminate all visually intrusive 'set braces' $\{ \}$;

$$\text{thus } \mathbf{S}_3 \equiv 1, 2, \hat{3}, 4, \hat{5}, \hat{6}, \hat{7};$$

(d) collect subsets of same counting-size into separate rows, appropriately labelled, thus

$$\begin{array}{lll} \mathbf{S}_3 \equiv & & \\ \mathbf{S}_{(3,1)} = & 1, 2, \hat{3} & K_{(3,1)} = 1 \quad C_{(3,1)} = 3 \\ \mathbf{S}_{(3,2)} = & 4, \hat{5}, \hat{6}, & K_{(3,2)} = 2 \quad C_{(3,2)} = 3 \\ \mathbf{S}_{(3,3)} = & \hat{7} & K_{(3,3)} = 1 \quad C_{(3,3)} = 1 \end{array}$$

where $\mathbf{S}_{(n,m)}$ denotes the sub-collection of level \mathbf{S}_n containing subsets with m members;

and $K_{(n,m)}$ denotes the number of successive appearances of the 'breeding trigger' node \mathbf{S}_1 in $\mathbf{S}_{(n,m)}$;

and $C_{(n,m)}$ denotes the number of subsets with m members in $\mathbf{S}_{(n,m)}$ (simple binomial coefficients).

We are now in a position where we can display and count the appearances of the S_1 in each sub-collection $C_{(4,m)}$ of S_4 .

TABLE 1
SOME FIRST COMBINATORICS IN S_4

$S_4 \equiv$			
$S_{(4,1)} =$	1, 2, $\hat{3}$ 4, $\hat{5}$, $\hat{6}$, $\hat{7}$: 4	7
		$K_{(4,1)} = 4$	$C_{(3,1)} = 7$
$S_{(4,2)} =$	12, $1\hat{3}$ 14, $1\hat{5}$, $1\hat{6}$, $1\hat{7}$: 4	6
	$2\hat{3}$ 24, $2\hat{5}$, $2\hat{6}$, $2\hat{7}$: 4	5
	$\hat{3}4$, $\hat{3}\hat{5}$, $\hat{3}\hat{6}$, $\hat{3}\hat{7}$: 7	4
	$4\hat{5}$, $4\hat{6}$, $4\hat{7}$: 3	3
	$\hat{5}\hat{6}$, $\hat{5}\hat{7}$: 4	2
	$\hat{6}\hat{7}$: 2	1
		$K_{(4,2)} = 24$	$C_{(4,2)} = 21$
$S_{(4,3)} =$	$12\hat{3}$, 124, $12\hat{5}$, $12\hat{6}$, $12\hat{7}$: 4	
	$1\hat{3}4$, $1\hat{3}\hat{5}$, $1\hat{3}\hat{6}$, $1\hat{3}\hat{7}$: 7	
	1 $4\hat{5}$, 1 $4\hat{6}$, $14\hat{7}$: 3	
	$1\hat{5}\hat{6}$, $1\hat{5}\hat{7}$: 4	
	$1\hat{6}\hat{7}$: 2	15
	$2\hat{3}4$, $2\hat{3}\hat{5}$, $2\hat{3}\hat{6}$, $2\hat{3}\hat{7}$: 7	
	$24\hat{5}$, $24\hat{6}$, $24\hat{7}$: 3	
	$2\hat{5}\hat{6}$, $2\hat{5}\hat{7}$: 4	
	$2\hat{6}\hat{7}$: 2	10
	$\hat{3}4\hat{5}$, $\hat{3}4\hat{6}$, $\hat{3}4\hat{7}$: 6	
	$\hat{3}\hat{5}\hat{6}$, $\hat{3}\hat{5}\hat{7}$: 6	
	$\hat{3}\hat{6}\hat{7}$: 3	6
	$4\hat{5}\hat{6}$, $4\hat{5}\hat{7}$: 4	
	$4\hat{6}\hat{7}$: 2	3
	$\hat{5}\hat{6}\hat{7}$: 3	1
		$K_{(4,3)} = 60$	$C_{(4,3)} = 35$

continued overleaf

$S_{(4,4)} =$	12 $\hat{3}$ 4, 12 $\hat{3}$ $\hat{5}$, 12 $\hat{3}$ $\hat{6}$, 12 $\hat{3}$ $\hat{7}$: 7		
	124 $\hat{5}$, 124 $\hat{6}$, 124 $\hat{7}$: 3		
	125 $\hat{6}$, 125 $\hat{7}$: 4		
	126 $\hat{7}$: 2	10	
	1 $\hat{3}$ 4 $\hat{5}$, 1 $\hat{3}$ 4 $\hat{6}$, 1 $\hat{3}$ 4 $\hat{7}$: 6		
	1 $\hat{3}$ 5 $\hat{6}$, 1 $\hat{3}$ 5 $\hat{7}$: 6		
	1 $\hat{3}$ 6 $\hat{7}$: 3	6	
	145 $\hat{6}$, 145 $\hat{7}$: 4		
	146 $\hat{7}$: 2	3	
	156 $\hat{7}$: 3	1	20
	2 $\hat{3}$ 4 $\hat{5}$, 2 $\hat{3}$ 4 $\hat{6}$, 2 $\hat{3}$ 4 $\hat{7}$: 6		
	2 $\hat{3}$ 5 $\hat{6}$, 2 $\hat{3}$ 5 $\hat{7}$: 6		
	2 $\hat{3}$ 6 $\hat{7}$: 3	6	
	245 $\hat{6}$, 245 $\hat{7}$: 4		
	246 $\hat{7}$: 2	3	
	256 $\hat{7}$: 3	1	10
	$\hat{3}$ 45 $\hat{6}$, $\hat{3}$ 45 $\hat{7}$: 6		
	$\hat{3}$ 46 $\hat{7}$: 3	3	
	$\hat{3}$ 56 $\hat{7}$: 4	1	4
	456 $\hat{7}$: 3	1	1
	$K_{(4,4)} = 80$	$C_{(4,4)} = 35$		
$S_{(4,5)} =$	12 $\hat{3}$ 4 $\hat{5}$, 12 $\hat{3}$ 4 $\hat{6}$, 12 $\hat{3}$ 4 $\hat{7}$: 6		
	12 $\hat{3}$ 5 $\hat{6}$, 12 $\hat{3}$ 5 $\hat{7}$: 6		
	12 $\hat{3}$ 6 $\hat{7}$: 3	6	
	1245 $\hat{6}$, 1245 $\hat{7}$: 4		
	1246 $\hat{7}$: 2	3	
	1256 $\hat{7}$: 3	1	
	1 $\hat{3}$ 45 $\hat{6}$, 1 $\hat{3}$ 45 $\hat{7}$: 6		
	1 $\hat{3}$ 46 $\hat{7}$: 3	3	
	1 $\hat{3}$ 56 $\hat{7}$: 4	1	
	1256 $\hat{7}$: 3	1	15
	2 $\hat{3}$ 45 $\hat{6}$, 2 $\hat{3}$ 45 $\hat{7}$: 6		
	2 $\hat{3}$ 46 $\hat{7}$: 3	3	
	2 $\hat{3}$ 56 $\hat{7}$: 4	1	4
2456 $\hat{7}$: 3	1	1	
$\hat{3}$ 456 $\hat{7}$: 4	1	1	
	$K_{(4,5)} = 60$	$C_{(4,5)} = 21$		
$S_{(4,6)} =$	12 $\hat{3}$ 45 $\hat{6}$, 12 $\hat{3}$ 45 $\hat{7}$, 12 $\hat{3}$ 46 $\hat{7}$: 9		
	12 $\hat{3}$ 56 $\hat{7}$, 12456 $\hat{7}$: 7		
	1 $\hat{3}$ 456 $\hat{7}$: 4	6	6
	2 $\hat{3}$ 456 $\hat{7}$: 4	1	1
	$K_{(4,6)} = 24$	$C_{(4,6)} = 7$		
$S_{(4,7)} =$	12 $\hat{3}$ 456 $\hat{7}$: 4	1	1
		$K_{(4,7)} = 4$	$C_{(4,7)} = 1$	

First we note that the total number, $K_{tot(n)}$, of the occurrences of the 'breeding trigger' node S_1 in the first three Levels are

$$K_{tot(1)} = 0, \quad K_{tot(2)} = 1, \quad K_{tot(3)} = 4. \quad (10)$$

Then we have the following summary of the occurrences of the 'breeding trigger' node S_1 in each sub-collection $C_{(4,m)}$ of different sizes in S_4 .

TABLE 2
THE OCCURRENCES $K_{(4,m)}$ OF S_1
IN EACH SUB-COLLECTION OF SIZE m IN LEVEL S_4 IN MoHoCH
AND THE BINOMIAL NUMBER $C_{(7,m)} = \binom{7}{m}$ OF THESE SUB-COLLECTIONS.

subset size : m	$K_{(4,m)}$	$C_{(7,m)}$	(ratio) $k \div c$
1	4	7	$\frac{4}{7} = \phi_4 \times 1$
2	24	21	$\frac{8}{7} = \phi_4 \times 2$
3	60	35	$\frac{12}{7} = \phi_4 \times 3$
4	80	35	$\frac{16}{7} = \phi_4 \times 4$
5	60	21	$\frac{20}{7} = \phi_4 \times 5$
6	24	7	$\frac{24}{7} = \phi_4 \times 6$
7	4	1	$\frac{28}{7} = \phi_4 \times 7$
totals	256	127	$\frac{256}{127} = \frac{2^8}{2^7-1}$

where the proportionality constant $\phi_4 = \frac{2^2}{2^3 - 1} = \frac{4}{7}$.

In Table 2 the ratios of the counts $K_{(4,m)}$ of the 'Breeding Trigger Nodes' S_1 to the subset sizes $C_{(7,m)}$ in Level S_4 in MoHoCH are seen to be in exact linear proportion to the subset size m . It is described by the combinatorial formula :

$$\frac{K_{(4,m)}}{C_{(7,m)}} = \phi_4 \times m = \frac{4}{7} \times m \quad (m = 1, 2, 3, 4, 5, 6, 7) \quad (11)$$

This observation is confirmed by the following Theorem :

Theorem 7.7.

(1) *In MoHoCH the number of occurrences of the Breeding Trigger Node S_1 in each subset collection of size m ($m = 1, 2, 3, 4, 5, 6, 7$) in Level S_4 is given by*

$$K_{(4,m)} = \frac{K_{tot(3)}}{|S_3|} \times C_{(7,m)} \times m \equiv \frac{4}{7} \times C_{(7,m)} \times m \quad (12)$$

(2) The total number $K_{\text{tot}(4)}$ of occurrences of the Breeding Trigger Node S_1 in Level S_4 is given by the recurrence relation

$$K_{\text{tot}(4)} = K_{\text{tot}(3)} \times 2^{(|S_3|-1)} = 4 \times 64 = 256 \quad (13)$$

Proof. (1) Using the display in the large Table 1 we readily see that the number of occurrences of the element labelled “1” in the subcollection $S_{(4,1)}$ is $1 = \binom{6}{0}$; in the subcollection $S_{(4,2)}$ it is $6 = \binom{6}{1}$; and in general it is $\binom{6}{m-1}$. Since we may permute any element into this 1st position (without materially changing the outcome), we see that the number of occurrences of each element in each subcollection $S_{(4,m)}$ is also $= \binom{6}{m-1}$. Since there are $4 = 2^2 = 2^{|S_1|}$ occurrences of the breeding trigger element S_1 in S_3 , there is a total of $4 \times \binom{6}{m-1}$ occurrences of the breeding trigger element S_1 in each subcollection $S_{(4,m-1)}$. But $\binom{6}{m-1} = \binom{7}{m} \times \frac{m}{7} = C_{(7,m)} \times m / 7$. Hence, in each subcollection $S_{(4,m)}$, the total of breeding trigger elements S_1 is $\frac{4}{7} \times C_{(7,m)} \times m$,

i.e. $\frac{K_{\text{tot}(3)}}{|S_3|} \times C_{(7,m)} \times m$, by (10), as asserted.

(2) We employ the well-known identity : $\sum_{m=0}^n \binom{n}{m} \times m = n \times 2^{n-1}$.

$$\begin{aligned} K_{\text{tot}(4)} &= \sum_{m=1}^7 K_{(4,m)} = \frac{4}{7} \times \sum_{m=1}^7 C_{(7,m)} \times m \\ &= \frac{4}{7} \times \left(\sum_{m=1}^7 \binom{7}{m} \times m \right) = \frac{4}{7} \times (7 \times 2^6) = 2^2 \times 2^6 = 4 \times 64 \\ &= K_{\text{tot}(3)} \times 2^{(|S_3|-1)} = 256, \end{aligned}$$

as asserted. □

7.2.2 Combinatorics in Level S_5

A table similar to TABLE 2 can be computed [A5] — albeit only partially — to count the occurrences of the ‘breeding trigger’ node S_1 in the first few sub-collection $C_{(5,m)}$ of different sizes in the huge Level S_5 .

TABLE 3
THE OCCURRENCES $K_{(5,m)}$ OF S_1
IN THE FIRST FEW SUB-COLLECTION OF SIZE m IN LEVEL S_5 IN MoHoCH
AND THE BINOMIAL NUMBER $C_{(127,m)} = \binom{127}{m}$ OF THESE SUB-COLLECTIONS.

continued overleaf

subset size : m	$K_{(5, m)}$	$C_{(127, m)}$	(ratio) $k \div c$
1	256	127	$\frac{256}{127} = \phi_5 \times 1$
2	32,256	8,001	$\frac{512}{127} = \phi_5 \times 2$
3	2,016,000	333,375	$\frac{768}{127} = \phi_5 \times 3$
4	83,328,000	10,334,625	$\frac{1024}{127} = \phi_5 \times 4$
5	2,562,336,000	254,231,775	$\frac{1280}{127} = \phi_5 \times 5$
6	$k?$	5,169,379,425	$\frac{k?}{127} = \phi_5 \times 6$
7	$k?$	89,356,415,775	$\frac{k?}{127} = \phi_5 \times 7$

Here we see that the proportionality constant $\phi_5 = \frac{2^8}{2^7 - 1} = \frac{256}{127} \approx 2.015748$ and the ratio of 'Breeding Trigger Node' counts in Level S_5 in **MoHoCH** to the subset size m in Table 3 is empirically seen (in the first few computable cases here) to be described by the combinatorial formula :

$$\frac{K_{(5, m)}}{C_{(127, m)}} = \phi_5 \times m = \frac{256}{127} \times m \quad (m = 1, 2, 3, 4, 5, \dots, 127) \quad (14)$$

This observation is confirmed by the following Theorem :

Theorem 7.8.

(1) *In MoHoCH the number of occurrences of the 'Breeding Trigger Node' S_1 in each subset collection of size m ($m = 1, 2, 3, \dots, 127$) in Level S_5 is given by*

$$\begin{aligned} K_{(5, m)} &= \frac{K_{tot(4)}}{|S_4|} \times C_{(|S_4|, m)} \times m \\ &\equiv \frac{256}{127} \times C_{(127, m)} \times m \end{aligned} \quad (15)$$

(2) *The total number $K_{tot(5)}$ of occurrences of the Breeding Trigger Node S_1 in Level S_4 is given by the recurrence relation*

$$\begin{aligned} K_{tot(5)} &= K_{tot(4)} \times 2^{|S_4| - 1} \\ &= 256 \times 2^{127 - 1} = 2^{134} \approx 2.1778 \times 10^{40} \end{aligned} \quad (16)$$

(3) *In full numerical precision (via MAPLE), the total number $K_{tot(5)}$ of occurrences of the Breeding Trigger Node S_1 in Level S_4 is*

$$\begin{aligned} K_{tot(5)} &= 128 \times 170, 141183, 460469, 231731, 687303, 715884, 105727 \\ &= 21778, 071482, 940061, 661655, 974875, 633165, 533184 \end{aligned} \quad (17)$$

Proof. (1) We use an analogous argument to that in the proof of the previous Theorem 7.7. The number of occurrences of the element labelled “1” in the subcollection $\mathbf{S}_{(5,1)}$ is $1 = \binom{126}{0}$; in the subcollection $\mathbf{S}_{(5,2)}$ it is $126 = \binom{126}{1}$; and in general it is $\binom{126}{m-1}$. Since we may permute any element into this 1st position (without materially changing the outcome), we see that the number of occurrences of each element in each subcollection $\mathbf{S}_{(5,m)}$ is also $= \binom{126}{m-1}$. Since there are $256 = 2^8 = 2^{(|\mathbf{S}_2|+1)} = \text{Ktot}_{(4)}$ occurrences of the breeding trigger element \mathbf{S}_1 in \mathbf{S}_4 / (see Theorem 7.7), there is a total of $256 \times \binom{126}{m-1}$ occurrences of the breeding trigger element \mathbf{S}_1 in each subcollection $\mathbf{S}_{(5,m-1)}$. But $\binom{126}{m-1} = \binom{127}{m} \times \frac{m}{127}$. Hence, in each subcollection $\mathbf{S}_{(5,m)}$, the total of breeding trigger elements \mathbf{S}_1 is $\frac{256}{127} \times \mathbf{C}_{(127,m)} \times m$, i.e. $\frac{\text{Ktot}_{(4)}}{127} \times \mathbf{C}_{(127,m)} \times m$, as asserted.

(2) We again employ the identity : $\sum_{m=0}^n \binom{n}{m} \times m = n \times 2^{n-1}$.

$$\begin{aligned}
\text{Ktot}_{(5)} &= \sum_{m=1}^{127} \text{K}_{(5,m)} = \frac{256}{127} \times \sum_{m=1}^{127} \mathbf{C}_{(127,m)} \times m \\
&= \frac{256}{127} \times \left(\sum_{m=1}^{127} \binom{127}{m} \times m \right) = \frac{256}{127} \times (127 \times 2^{126}) \\
&= 2^8 \times 2^{126} = 2^{134} \\
&= \text{Ktot}_{(4)} \times 2^{(|\mathbf{S}_4|-1)}
\end{aligned}$$

as asserted. □

From the previous Theorem (7.8) we infer this generalised combinatoric :

Theorem 7.9.

(1) *In MoHoCH the number of occurrences of the ‘Breeding Trigger Node’ \mathbf{S}_1 in each subset collection of size m ($m = 1, 2, 3, \dots$) in Level \mathbf{S}_N , ($N = 3, 4, 5, 6, \dots$) is given iteratively by*

$$\text{K}_{(N,m)} = \frac{\text{Ktot}_{(N-1)}}{|\mathbf{S}_{(N-1)}|} \times \mathbf{C}_{(|\mathbf{S}_{(N-1)}|, m)} \times m \quad (18)$$

(2) *The total number $\text{Ktot}_{(N)}$ of occurrences of the Breeding Trigger Node \mathbf{S}_1 in Level \mathbf{S}_N , ($N = 3, 4, 5, 6, \dots$) is given iteratively by*

$$\text{Ktot}_{(N)} = \text{Ktot}_{((N-1))} \times 2^{(|\mathbf{S}_{(N-1)}|-1)} \quad (19)$$

Using the closed formula (15) we can now compute (*via* MAPLE) the “full version” of Table 3. Each line of this table counts the number of complete copies of MoHoCH that originate from each of the 127 sub-collections m in question, each one of which contains a complete copy of this same table, and so on, *ad infinitum*

TABLE 4
THE OCCURRENCES $K_{(5,m)}$ OF S_1
IN ALL THE SUB-COLLECTIONS OF SIZE m IN LEVEL S_5 IN MoHoCH.

size of subset : m	$K_{(5,m)}$
1	256
2	32256
3	2,016000
4	83,328000
5	2562,336000
6	62520,998400
7	1,260840,134400
8	21,614402,304000
9	321,514234,272000
10	4215,408849,344000
11	49320,283537,324800
12	520104,808211,788800
13	4,984337,745362,976000
14	43,708807,920875,328000
15	352,792521,075636,576000
16	2634,184157,364753,100800
17	18274,652591,717974,636800
18	118247,752064,057482,944000
19	716055,831943,459202,272000
20	4,070212,097362,820728,704000
21	21,775634,720891,090898,566400
22	109,915108,591164,554059,430400
23	524,594836,457830,826192,736000
24	2372,080999,635408,953219,328000
25	10180,180956,768630,090899,616000
26	41535,138303,616010,770870,433280
27	161348,037256,354503,379150,529280
28	597585,323171,683345,848705,664000
29	2,112890,964071,308972,822209,312000

continued overleaf

20	4, 070212, 097362, 820728, 704000
21	21, 775634, 720891, 090898, 566400
22	109, 915108, 591164, 554059, 430400
23	524, 594836, 457830, 826192, 736000
24	2372, 080999, 635408, 953219, 328000
25	10180, 180956, 768630, 090899, 616000
26	41535, 138303, 616010, 770870, 433280
27	161348, 037256, 354503, 379150, 529280
28	597585, 323171, 683345, 848705, 664000
29	2, 112890, 964071, 308972, 822209, 312000
30	7, 140114, 292378, 906184, 019879, 744000
31	23, 086369, 545358, 463328, 330944, 505600
32	71, 493273, 430787, 499339, 347441, 049600
33	212, 245655, 497650, 388663, 687715, 616000
34	604, 578533, 841792, 016193, 534705, 088000
35	1653, 700107, 273136, 985470, 550810, 976000
36	4346, 868853, 403674, 361808, 304988, 851200
37	10987, 918490, 548176, 859015, 437610, 707200
38	26727, 369301, 333403, 170578, 091485, 504000
39	62598, 312311, 017707, 425827, 635321, 312000
40	141247, 473932, 552775, 730072, 613032, 704000
41	307213, 255803, 302287, 212907, 933346, 131200
42	644398, 536563, 024309, 763660, 543116, 275200
43	1, 304139, 895425, 168245, 950265, 384878, 176000
44	2, 547622, 121295, 677503, 716797, 496041, 088000
45	4, 805741, 728807, 755291, 102140, 731168, 416000
46	8, 757129, 372494, 131863, 786123, 110129, 113600
47	15, 420162, 590696, 188716, 666868, 954792, 569600
48	26, 247085, 260759, 470156, 028713, 114540, 544000
49	43, 198327, 824999, 961298, 463923, 667681, 312000
50	68, 764685, 109183, 611862, 860939, 715900, 864000
51	105, 897615, 068142, 762268, 805847, 162487, 330560
52	157, 808210, 689781, 371224, 102831, 065667, 394560
53	227, 607996, 187184, 670034, 763698, 652404, 896000
54	317, 792296, 563238, 973256, 085164, 156187, 968000
55	429, 608104, 613267, 500883, 226240, 433365, 216000
56	562, 396064, 221004, 728428, 950714, 749132, 646400
57	713, 037867, 137345, 280686, 705370, 485507, 462400
58	875, 660538, 589722, 274527, 532911, 122553, 024000
59	1041, 734089, 011910, 981765, 513290, 818209, 632000
60	1200, 642678, 861185, 538306, 015318, 231156, 864000
61	1340, 717658, 061657, 184441, 717105, 358125, 164800
62	1450, 612548, 066711, 052018, 907032, 026823, 948800
63	1520, 803477, 811874, 490019, 821888, 415218, 656000

Rows 64–127 ARE A MIRROR-IMAGE OF THE ABOVE ROWS 1–63

Recall that in §4 of my **PuCH** paper [A1] I asked the question :

Question : “Can we now productively ‘data-mine’ my new purely combinatorial **PuCH** for these and other other numbers ? Other ratios ? Using Combinatorial Tools and only Combinatorial Tools” ?

The same question applies here in **MoHoCH**, and the answer is again affirmative. Indeed, a similar observation applies here to the number of breeding trigger nodes S_1 in Level S_5 and hence to the number of complete copies of **MoHoCH**—growing out from Level S_5 . We can compute a catalogue of the dimensionless ratios of those counts as listed in Table 4, utilising the formula (16). Each ratio is given by the formula :

$$\begin{aligned}
 \text{Rat}(r, s) &= \left[\frac{256}{127} \times C_{(127, r)} \times r \right] \div \left[\frac{256}{127} \times C_{(127, s)} \times s \right] \\
 &= [C_{(127, r)}] \div [C_{(127, s)}] \times [r \div s] \\
 &= \frac{127!}{(127 - r)! r!} \times \frac{(127 - s)! s!}{127!} \times \frac{r}{s} \\
 &= \frac{(127 - s)! (s - 1)!}{(127 - r)! (r - 1)!} \tag{20}
 \end{aligned}$$

for each $r, s = 1, 2, \dots, 127$.

There are 8001 entries in the catalogue of these ratios. Figure 3 illustrates the distribution of the (logarithms) of these ratio values.

[X-axis : $\log_{10}(\text{Rat}(r, s))$, ($\forall r, s$), Y-axis : contents of the ‘X-(X+1)’-bin]

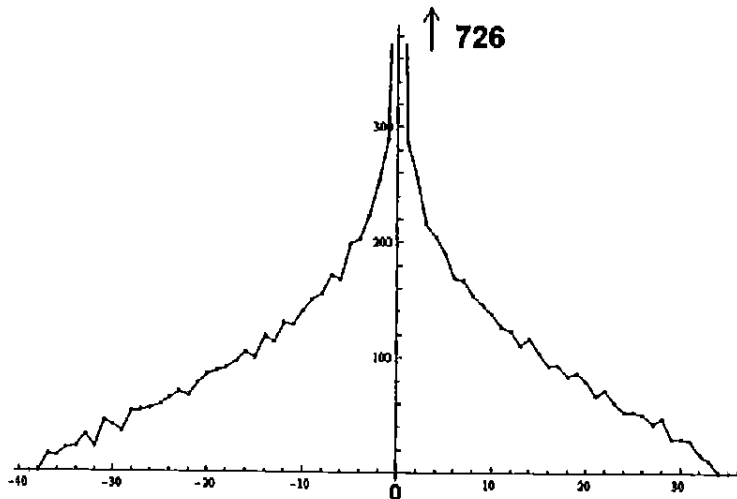


Figure 2: DISTRIBUTION OF (LOG)RATIOS OF THE COUNTS OF THE BREEDING TRIGGER NODE S_1 IN S_5 IN **MoHoCH**

Here is the (X,Y) data for Figure 3 :

```
(-39, 0), (-38, 0), (-37, 15), (-36, 14), (-35, 21),
(-34, 22), (-33, 33), (-32, 22), (-31, 45), (-30, 41),
(-29, 36), (-28, 53), (-27, 54), (-26, 56), (-25, 59),
(-24, 65), (-23, 70), (-22, 67), (-21, 78), (-20, 85),
(-19, 89), (-18, 91), (-17, 97), (-16, 105), (-15, 100),
(-14, 119), (-13, 114), (-12, 130), (-11, 129), (-10, 141),
(-9, 151), (-8, 156), (-7, 172), (-6, 168), (-5, 199),
(-4, 203), (-3, 223), (-2, 253), (-1, 290), *(0, 726),
( 1, 286), ( 2, 256), ( 3, 215), ( 4, 205), ( 5, 191),
( 6, 169), ( 7, 167), ( 8, 154), ( 9, 146), (10, 138),
(11, 127), (12, 123), (13, 111), (14, 117), (15, 104),
(16, 93), (17, 94), (18, 84), (19, 87), (20, 80),
(21, 67), (22, 72), (23, 61), (24, 53), (25, 53),
(26, 51), (27, 43), (28, 47), (29, 30), (30, 30),
(31, 28), (32, 16), (33, 11), (34, 0), (35, 0).
```

which clearly shows the exceptional concentration in the ******(X=0) bin — there are generally far more ratios of the order of unity than of the more extreme ratios with the order of 10^{-37} and 10^{+33} .

A sorted text file of the catalogue ([A6]) is available in 2-column 66-page (242kB) PDF format on request.

7.3 Implications — Questions

7.3.1 Rate of breeding in MoHoCH

Imagine we are ‘Boot-strapping’ **MoHoCH** into existence from some otherwise undefined ‘UR-moment’ in a discrete;y ticking virtual ‘time’. It may be helpful to look at Fig.1 again here.

At tick *No.0* of this Boot-strap “Big-Bang” there is only Nothing : the Empty-set \emptyset .

At tick *No.1* of the Boot-strap the first non-trivial set appears : the ‘set containing the Empty Set’ $\{ \emptyset \}$ — Level S_0 in **MoHoCH**.

At Boot-strap tick *No.2* there appears the first Power set and its punctured companion set : $\{ \emptyset, \{ \emptyset \} \}$ — Level 1 in **MoHoCH**. This happens to be the ‘breeding trigger node’ S_1 .

At Boot-strap tick *No.3* this breeding trigger node S_1 emerges as a proper subset of the next Level 2 in **MoHoCH**, $\{ \{ \emptyset \}, \{ \{ \emptyset \} \}, \{ \emptyset, \{ \emptyset \} \} \}$; which we can call the ‘critical breeding node’ S_2 (see Fig.1).

At Boot-strap tick *No.4* this ‘critical breeding node’ S_2 begins to generate the first of the infinitely-many branch-copies of the entire **MoHoCH** system. And at the same tick *No.4* the third Level S_3 also emerges together with the first four copies of the ‘breeding trigger node’ S_1 .

At Boot-strap tick *No.5* all four of the recently created copies of the breeding trigger node S_1 now begin to create their own critical breeding nodes S_2 . And the earlier critical breeding node S_2 in Level 2 has also begun to generate its own infinitely many copies of itself and S_2 which all contribute to generating their own infinitely-many copies of the entire **MoHoCH** system — each of which include every step that we have just described.

Remark 7.10. The question naturally arises :

“How rapidly do copies of the breeding trigger node S_1 appear at each successive Boot-strap tick ? ”

The iterative combinatorial formula (19) provides us with a step-by-step indication. Even so, as we examine the vast number of subsets that occur in the later Levels S_N we run into a progressively difficult computational problem.

We know (see (9)) that the number $(2^{127} - 1)$ of elements in S_5 is

$$170,141183,460469,231731,687303,715884,105727 \approx 10^{38}$$

which we can nickname ‘**BIG**’. Note that though this is a ‘big’ number it is still nowhere near as ‘big’ as my suggested ‘biggest possible counting number’ **ULTIMOL** (see §3 in [A1]). That was subjectively based on the number of things that could possibly be counted in say a Julian Year at the maximum possible rate of one count every Planck Second ($5.3912427 \times 10^{-44}$ SI seconds). By contrast, **NPSU**, the number of Planck Seconds that have elapsed in the Universe of Age 433.6×10^{15} years, is $\text{NPSU} = \frac{433.6 \times 10^{15}}{5.3912427 \times 10^{-44}} \approx 8.04267 \times 10^{60}$.

The count of subsets of size m ($m = 1, 2, 3, \dots, \text{BIG}$) in S_5 is the binomial number

$$\binom{\text{BIG}}{m} = \frac{\text{BIG} \times (\text{BIG} - 1) \times \dots \times (\text{BIG} - m + 1)}{1 \times 2 \times \dots \times m} \quad (21)$$

The numbers that arise here are so large as to be practically useless for any precise arithmetical processing for almost all m greater than a very conservative value of say 3 or 4 . . .

For example we have, for $m = 4$ and $m = 48$ and $m = 127$, (via MAPLE) :

$$\begin{aligned}
C_{(170141183460469231731687303715884105727, 4)} &\approx C_{(\approx 10^{38}, 4)} \\
&\approx 3.491616, 648422, 551328 \times 10^{151} \\
C_{(170141183460469231731687303715884105727, 48)} &\approx C_{(\approx 10^{38}, 48)} \\
&\approx 9.659280, 535744, 462651 \times 10^{1773} \\
C_{(170141183460469231731687303715884105727, 127)} &\approx C_{(\approx 10^{38}, 127)} \\
&\approx 6.821029, 698843, 875674 \times 10^{4641}
\end{aligned}$$

But we have to remember that the size of S_5 is of the order of ‘only’ 10^{38} , and there are ‘only’ about 10^{76} atoms in the whole Physical Universe which is ‘only’ as old as about 10^{61} Planck Seconds.

With these as benchmark numbers in mind, since the number of breeding trigger subset nodes S_1 in Level S_6 is proportional (see (18)) to these values of $C_{(\approx 10^{38}, m)}$ then these numbers of breeding trigger subset nodes become effectively unimaginable and meaningless beyond Level S_5 . Their numbers increase everywhere at an unimaginably rapid rate. And each one is the trigger for the creation of another distinct copy of the whole entire **MoHoCH** system — ‘WHEREIN EVERYWHERE IS EVERYWHERE’

7.3.2 What if MoHoCH has a Finite Stopping Rule ?

In my presentation of **PuCH** [A1] I introduced Louis Gidney’s perceptive suggestion that there might be a *New PuCH Stopping Rule : continue constructing new Levels from the old until the newest candidate fails to have a prime number of members.* If this were applied to **MoHoCH** and if perhaps one-day it was shown that such a test delivered a positive result, then the situation arises in which perhaps only the first Levels $S_0, S_1, S_2, S_3, S_4, S_5$ would be admitted. This would re-invoke the original **CH** Parker-Rhodes ‘STOP’ on the sequential construction and deliver a different curtailed version : ‘**MoHoCH***’. This would still be (a) essentially infinite in extent, and (b) would still possess a holographic property. The successive counts of the appearances of the breeding trigger nodes S_1 would eventually change and the Combinatorial

Properties would significantly alter. But there would be one very great difference : starting with each of the zillions of ‘trigger nodes’, each distinct branch of the evolving system would terminate with its own distinct copy of the Top Level S_5 . There would no longer be a single unique Top Level as in **PuCH** or the **CH**. Such a possibility deserves a much more detailed investigation — even if the testability of the primeness of the potentially last Level S_5 remains strictly out-of-reach of any foreseeable finite-time algorithm.

Moreover, if this prime-number Gidney ‘STOP’ did terminate the sequential construction, then this curtailed **MoHoCH*** construction would naturally admit an even more frighteningly complicated truly ‘fractal holographic’ infinite system :

the **Multi-Ouroboros-MoHoCH* Combinatorial Hierarchy**.

Recall that in my earlier [A3] recursive construction of the Bi-Ouroborosian Combinatorial Hierarchy we had two ‘dual’ Combinatorial Hierarchies, one formed first by uniquely identifying the elements of each member of each Level with binary numbers such as (00011), and those in the dual system with their dual binary numbers such as (11100) (resp.). The basic elements ‘0’ and ‘1’ — which are intrinsically “undefined” but “different” (see Remark (7.1)) — were then deliberately identified with the two dual Top Levels : Level IV and Level IV*. Which were then instantly ‘recursively fed-back’ into the Common Bottom Level Level O as the basic elements 0 and 1; thus creating the endlessly recursive Bi-Ouroborosian system.

Precisely the same kind of endlessly recursive construction of the Bi-Ouroborosian type can be applied to this new, curtailed, Combinatorial Hierarchy **MoHoCH*** by identifying each one of the zillions of copies of the two dual Top Levels **MoHoCh-Level IV** and **MoHoCh-Level IV*** with the basic elements $0 \approx \emptyset$, and $1 \approx \{\emptyset\}$ in the single Common Bottom Level S_1 . But now each feed-back loop from any Top Level can cycle up through the five Levels following any branch from any Trigger Node anywhere in the system. The Ouroborosian Cycles have all become ‘zillion-*Hydra*-headed’ on an incomprehensively complicated scale. The combinatoric analysis of this incredibly more complex system is obviously fraught with major difficulties. Its investigations, and its implications, will simply have to be postponed for the time being.

7.3.3 Does the ‘Goldbach Triple Partitioning’ in **GoCH** apply also to **MoHoCH** ?

My previous paper [A2] emphasised the appearance of Prime Numbers in **PuCH**, and especially the Mersenne Primes. As I explained there,

[...] use is made of Goldbach’s Conjecture (valid for numbers far larger than any appearing in the subsets of **PuCH**) to introduce the novel idea of the ‘**PuCH** Goldbach Triple Partition Principle’: ‘Every Odd Subset of every Level of **PuCH** higher than Level S_1 can be partitioned into the disjoint union of three Prime Subsets’ — and an ‘Iterative Goldbach Partitioning’ structure was introduced into **PuCH** together with the dual notion of an ‘Iterative Goldbach Spreading’. The new structure was re-named **GoCH**. The fact that any Prime Subset in **GoCH** is identifiable with a ‘Necklace’ (*alias* ‘Hoop’) as a cyclic prime group with a prime number of ‘beads’ suggests that **GoCH** is identifiable with a vast dendritic-like structure of chains of ‘Triple-Prime-Necklaces’.

It was suggested that unlike the classical **CH** (whose applications were apparently restricted solely to particle physics) this more-simply constructed set and number theoretic structure **GoCH** could also have far-reaching applications in biology and information-communication.

Quite plainly, the same considerations do strongly apply — with even greater consequences — to the vastly more infolded, **ultra-dendritic** and holographic structure in **MoHoCH**. And its vast dendritic structure of chains of ‘Triple-Prime-Necklaces’.

The first point of issue will, of course, be the unusual and great difficulty in trying to comprehend the nature and the implications of this new-fangled structure.

7.3.4 Of what kind of Set Theory is **MoHoCH** a model ?

Chapter 13 in [Je] begins by saying that Constructible sets were introduced by Gödel in his proof of consistency of the Axiom of Choice and of the Generalized Continuum Hypothesis. The class of all constructible sets (the Constructible Universe) is a transitive model of ZFC (Zermelo-Fraenkel Set Theory with the Axiom of Choice), and is the smallest transitive model of ZF (Zermelo-Fraenkel Set Theory) that contains all ordinal numbers.

I remarked earlier (Remark 7.2) that had we not deliberately followed Parker-Rhodes methodology in deleting the empty set \emptyset from all subsets in Level S_1 and

in all subsequent Levels, our process of forming **PuCH** would have delivered Gödels Constructible Universe instead of **PuCH**.

Two as yet unanswered questions arise from this observation :

(a) (A less provocative one) : Of what kind of Set Theory is **PuCH** a model, and does that model contain all or only some of the ordinal numbers ?

(And if only 'some', then 'which' ?)

(b) (A more provocative one) : Of what kind of Set Theory is **MoHoCH** a model, and what relationship has that model to the Continuum Hypothesis ?

There is one last question (even more provocative, perhaps) :

7.3.5 Is **MoHoCH** the first model for Bohm's Implicate Order ?

David Bohm [B2] has proposed "The hologram as an analogy for the implicate order". Let us recall (very briefly) what Bohm has implied (see [B1]) :

In a holographic reconstruction, each region contains a whole and undivided image. In Bohm's words : "There is the germ of a new notion of order here. This order is not to be understood solely in terms of a regular arrangement of objects (eg., in rows) or as a regular arrangement of events (e.g. in a series). Rather, a total order is contained, in some implicit sense, in each region of space and time. Now, the word 'implicit' is based on the verb 'to implicate'. This means 'to fold inward' ... so we may be led to explore the notion that in some sense each region contains a total structure 'enfolded' within it". Bohm noted that although the hologram conveys undivided wholeness, it is nevertheless static. In this view of order, laws represent invariant relationships between explicate entities and structures, and thus Bohm maintained that in physics, the explicate order generally reveals itself within well-constructed experimental contexts as, for example, in the sensibly observable results of instruments. With respect to implicate order, however, Bohm asked us to consider the possibility instead "that physical law should refer primarily to an order of undivided wholeness of the content of description similar to that indicated by the hologram rather than to an order of analysis of such content into separate parts . . ."

It is sometimes held that the Implicate Order should be regarded as a new form of descriptive order in physics, replacing what Bohm called the Cartesian Order (coordinates and well-defined points in space), perhaps an ontological

statement about the world and reality — about the way the world actually is.

Perhaps **MoHoCH** might provide a purely Set Theoretic and Combinatorial framework for the foundations of such an ontological enquiry ?

.....
TO BE CONTINUED
.....

8 Appendix : An algorithm for counting the S_1 in S_5 .

Here are the essential (pseudo-code) details of the algorithms I use in my Pascal program ([A5]) to count the occurrences $K_{(5,m)}$ of S_1 in the sub-collection of size m in Level S_5 in **MoHoCH**, as displayed in Table 3 above. (The internal loop 'addition' details can obviously be optimised further, in practice.)

The binomial numbers $C_{(127,m)} = \binom{127}{m}$ of these sub-collections are computed iteratively as usual (or by table look-up).

```
{ STEP-1 : Load the counts of S1 in S3 from observation }  
ES3[1] := 0;  
ES3[2] := 0;  
ES3[3] := 1;  
ES3[4] := 0;  
ES3[5] := 1;  
ES3[6] := 1;  
ES3[7] := 1;  
}
```

continued overleaf

```
{ STEP-2 : compute the counts of S1 in S4, by summing the
different collections of subsets in S4, ready for use in
the next stage for S5 }
```

```
GG := 0; global index for ES4 array
N := 1; { subcollections with 1 element at a time }
suum := 0;
for M1 := 1 to 7 do begin
    GG := GG+1;
    ES4[GG] := ES3[M1];
    suum := suum + ES4[GG];
end;
KS3[N] := suum;

N := 2; { subcollections with 2 elements at a time }
suum := 0;
for M1 := 1 to 7 do begin
for M2 := M1+1 to 7 do begin
    GG := GG+1;
    ES4[GG] := ES3[M1] + ES3[M2];
    suum := suum + ES4[GG];
end;end;
KS3[N] := suum;

N := 3; { subcollections with 3 elements at a time }
suum := 0;
for M1 := 1 to 7 do begin
for M2 := M1+1 to 7 do begin
for M3 := M2+1 to 7 do begin
    GG := GG+1;
    ES4[GG] := ES3[M1] + ES3[M2] + ES3[M3];
    suum := suum + ES4[GG];
end;end;end;
KS3[N] := suum;

N := 4; { subcollections with 4 elements at a time }
suum := 0;
for M1 := 1 to 7 do begin
for M2 := M1+1 to 7 do begin
for M3 := M2+1 to 7 do begin
for M4 := M3+1 to 7 do begin
    GG := GG+1;
    ES4[GG] := ES3[M1] + ES3[M2] + ES3[M3] + ES3[M4];
    suum := suum + ES4[GG];
end;end;end;end;
KS3[N] := suum;
```

continued overleaf

```

N := 5; { subcollections with 5 elements at a time }
suum := 0;
for M1 := 1 to 7 do begin
for M2 := M1+1 to 7 do begin
for M3 := M2+1 to 7 do begin
for M4 := M3+1 to 7 do begin
for M5 := M4+1 to 7 do begin
    GG := GG+1;
    ES4[GG] := ES3[M1] + ES3[M2] + ES3[M3] + ES3[M4]
              + ES3[M5];
    suum := suum + ES4[GG];
end;end;end;end;end;
    KS3[N] := suum;

N := 6; { subcollections with 6 elements at a time }
suum := 0;
for M1 := 1 to 7 do begin
for M2 := M1+1 to 7 do begin
for M3 := M2+1 to 7 do begin
for M4 := M3+1 to 7 do begin
for M5 := M4+1 to 7 do begin
for M6 := M5+1 to 7 do begin
    GG := GG+1;
    ES4[GG] := ES3[M1] + ES3[M2] + ES3[M3] + ES3[M4]
              + ES3[M5] + ES3[M6];
    suum := suum + ES4[GG];
end;end;end;end;end;end;
    KS3[N] := suum;

N := 7; { subcollections with 7 elements at a time }
suum := 0;
for M1 := 1 to 7 do begin
for M2 := M1+1 to 7 do begin
for M3 := M2+1 to 7 do begin
for M4 := M3+1 to 7 do begin
for M5 := M4+1 to 7 do begin
for M6 := M5+1 to 7 do begin
for M7 := M6+1 to 7 do begin
    GG := GG+1;
    ES4[GG] := ES3[M1] + ES3[M2] + ES3[M3] + ES3[M4]
              + ES3[M5] + ES3[M6] + ES3[M7];
    suum := suum + ES4[GG];
end;end;end;end;end;end;end;end;
    KS3[N] := suum;

```

{=====end-of-STEP-2=====}

continued overleaf

```
{ STEP-3 : compute the counts of S1 in S5, by summing the
different collections of subsets in S5, using the 127
different counts in ES4 which we have just computed }
```

```
HH := 0: global index for ES5 array
```

```
N := 1; { subcollections with 1 element at a time }
suum := 0:
```

```
for M1 := 1 to 127 do begin
```

```
    HH := HH+1;
```

```
    ES5[HH] := ES4[M1];
```

```
    suum := suum + ES5[HH];
```

```
end;
```

```
    KS5[N] := suum;
```

```
        KS5[1] is 256
```

```
N := 2; { subcollections with 2 element at a time }
```

```
suum := 0:
```

```
for M1 := 1 to 127 do begin
```

```
for M2 := M1+1 to 127 do begin
```

```
    HH := HH+1;
```

```
    tempES5 := ES4[M1] + ES4[M2];
```

```
    suum := suum + tempES5;
```

```
od:od:
```

```
    KS5[N] := suum;
```

```
        KS5[2] is 32,256
```

```
N := 3; { subcollections with 3 element at a time }
```

```
suum := 0:
```

```
for M1 := 1 to 127 do begin
```

```
for M2 := M1+1 to 127 do begin
```

```
for M3 := M2+1 to 127 do begin
```

```
    HH := HH+1;
```

```
    tempES5 := ES4[M1] + ES4[M2] + ES4[M3];
```

```
    suum := suum + tempES5;
```

```
end;end;end;
```

```
    KS5[N] := suum;
```

```
        KS5[3] is 2,016,000
```

continued overleaf

```

N := 4; { subcollections with 4 element at a time }
suum := 0:
for M1 := 1 to 127 do begin
for M2 := M1+1 to 127 do begin
for M3 := M2+1 to 127 do begin
for M4 := M3+1 to 127 do begin
    HH := HH+1;
    tempES5 := ES4[M1] + ES4[M2] + ES4[M3] + ES4[M4];
    suum := suum + tempES5;
end;end;end;end;
    KS5[N] := suum;
    KS5[4] is 83,328,000

```

```

N := 5; { subcollections with 5 element at a time }
suum := 0:
for M1 := 1 to 127 do begin
for M2 := M1+1 to 127 do begin
for M3 := M2+1 to 127 do begin
for M4 := M3+1 to 127 do begin
for M5 := M4+1 to 127 do begin
    HH := HH+1;
    tempES5 := ES4[M1] + ES4[M2] + ES4[M3]
                + ES4[M4] + ES4[M5];
    suum := suum + tempES5;
end;end;end;end;end;
    KS5[N] := suum;

```

```

KS5[5] is 2,562,336,000
HH      is 264,907,903 {total passes through all HH loops }
{ Total compute time ≈ 4 minutes : 1.103-microsecs per loop.}
{ Not continuing with next 122 stages with N=6,7,8,...,127,
  because of the impractical, long computation-times . . . }

{=====END-of-STEP-3, and END ALL=====}

```

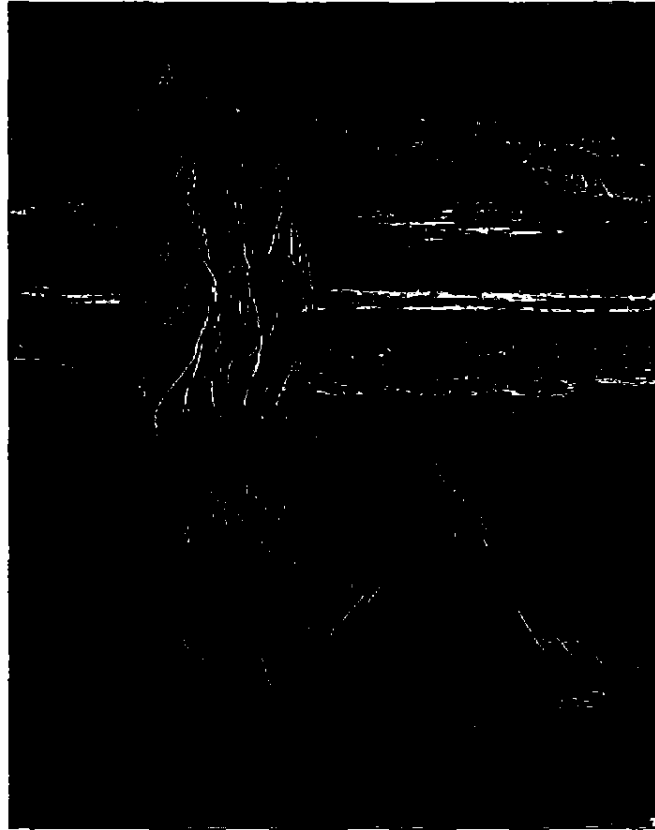
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-

THE LANGUAGE OF WILL

an essay by two brains in threefold unity



Anthony Blake

Illustration: *Abraham Entertains Three Strangers* by Gustave Dore

Man has in general two kinds of mentation: mentation by thought, in which words, always possessing a relative sense, are employed; and the other kind, which is proper to all animals as well as to man, mentation by form. *All and Everything* p. 15, G. I. Gurdjieff

This paper has been an attempt to develop a dialogue between the left and right sides of the brain/mind in order to express an idea of 'the language of will'. It is hoped that its use or misuse of certain formalisms will not cause offence and that it may actually give rise to some pleasure of contemplation.

THE MOTHER GODDESS

From women's eyes this doctrine I derive:
They sparkle still the right Promethean fire;
They are the books, the arts, the academes,
That show, contain, and nourish all the world. *Loves Labours Lost*,
William Shakespeare

I start from myth, admitting right away that speaking of myth involves making a further order of myth; we take from what we know of past beliefs and images elements that we then recombine in ourselves



according to our own spirit of myth making. It is to weave a story about stories, to dream about dreams. I have to start with this proviso because I am going to speak of the evolution of human mind in terms of a struggle and mutuality of potentialities in our brains and also portray this struggle and evolution through images of the Great Mother. Palaeolithic art seems to abound in portrayals of this figure, so much so that it was thought that She was the primordial deity of early humanity, only in historical times superseded by the God of male patriarchy and the hero. Very recently, as one could actually predict, the idea that

there was a universal original cult of the Mother is under attack. But this may be part of the same ongoing story to do with the arising of mind.

The story is like this. First was the Mother, the Mother of all, but she gave birth to a son who then turned against her, usurped her position and, in some stories then killed or dismembered her. Instead of the Mother being the 'source of all' the male hero became the One Source, or God. In Egypt, after all, we have Atum masturbating on the primordial hill to inseminate himself with the gods (the other side to the virgin birth!). In Sumeria, in the great epic *Enuma Elish* ('when on high') the Mother as Tiamat (the bitter waters of the ocean, whose consort was Apsu, the sweet waters of the abyss) gets cut in two to form the upper and lower bounds of the cosmos by one her sons Ea (later Marduk) because She threatens to annihilate them. I mention

this story because it vividly illustrates the idea that the 'son' cannot actually create anything only *divide what is*. In some African myths, the son actually devours his own mother.

Modern science fiction myths build on Sumerian stories, in particular the story of the *nam-shub* of Enki from which the Jews obtained their story of the Tower of Babel. A *nam-shub* is similar to the magic words *abracadabra*, which translates as 'create as I say'. In science fiction writer Neal Stephenson's fantasy *Snow Crash* the *nam-shub* was an incantation by which men lost their capacity to speak in a universal language but, at the same time, created in them in the capacity for individual conscious thought. The arising of individual thought marked the specialisation and divergence of languages - including the idea that each of us has our own idiolect - both making mutual understanding difficult and also leading to the creation of highly organized and conscious languages such as mathematics. The supposed original tongue of all mankind is said to be accessed by shamans in trance. It can be called the 'mother tongue' and the languages that replaced it the rebellious 'sons'. It is important to link with twentieth century phenomena such as the Viennese school which wanted to restrict language to what was 'logical' or meaningful in a very restricted sense - to 'kill the mother' in my mythic description..

The son of the mother becomes the *hero*. This leads to the cult of the individual as well as to many other things. The hero has *to discover something and bring it back home*. That is to say, he has to go into the unconscious and bring back an insight accessible to the conscious mind. We might think of it as a return to the womb: the imagery agrees with the idea of going through some dismantling of the conscious mind and a submergence in deeper levels of pre-conscious thought which, nowadays at least, we associate with creativity. One of the most interesting images I know of the subtle relation between the aspect I am associating with the 'Mother' and the other aspect of the 'son' is a painting of St. George and the Dragon by Uccello (1397-1475). The great warrior is driving his spear into the dragon but the young maiden holds the dragon by a silken thread.

THE MAN WITH TWO BRAINS

Like any mother, the Goddess loves and indulges her son. The imagery goes through complex transformations and includes of course the Christian icons of Mary and the Christ child which many believe have their roots in the Egyptian depictions of Isis and Horus. These depict the relationship as of the deepest love. I want simply to suggest that the relation of the Great Mother and the Hero Son is not only a recapitulation of family life but a semi-historical account of changes in human consciousness, and in the structure of the functioning of the brain. Out of some primitive matrix came a separated off structure capable of what we call individual conscious thought. This had to draw on the 'food' of the original kind of mind but would come to repudiate its origins and regard itself as self-sufficient and sufficient cause of its own agency. The process brought us to having two kinds of mind: one where we see ourselves as in control and another in which we do not. Awareness of the old 'hidden' mind surfaced in the twentieth century through such people as Freud but it was often regarded as sub-rational and a threat to 'civilization'.

Since around 1850, when it was first noticed, brain research has discovered that our brains seem to exhibit two kinds of mind each associated with a hemisphere. The left side of the brain tends to be logical and linear while the right tends to be pictorial and holistic. But there are no simple distinctions of function between the two sides.

This is a complex braid of connection which allows for both information exchange and *inhibition*. The two sides both co-operate and compete and in particular they compete for energy. The brain is said to use 20% of the body's energy and recent calculations suggest that it is unlikely that organisms like ours will ever be able to command significantly larger amounts, so our intelligence must be inherently limited in so far as it relies on information processing within single brains. The degree of synergy between the two sides is hence of great significance. It may be the case that 'technological' developments such as writing enhance one or other of the two sides. Recent technology is paying attention to images and sound rather than words and notation and hence may release more of the right brain.

It is interesting to reflect that Einstein was well aware that his 'thinking' was primarily in terms of feelings and images (right brain)

that only subsequently involved words and mathematics (left brain); while he later went on to spend considerable energy trying to refute the non-locality inherent in quantum theory. The bitterness and savagery exhibited by some scientists in defence of their beliefs is quite extraordinary especially when one realises that the resources of the right brain are purloined by the left brain in denial of the 'truth' of the right. Locality, causality, linearity and so on are all features of left brain mentation. It divides processes into steps, devises algorithms, works from axioms and produces the feeling of separation from the phenomenal world. One of the main features of the right hand brain is that it gives us *new experiences*, whereas all the left hand brain has is memories and representations. The left hand brain is alienated from the world while the right hand brain is in and of the world.

The right hand brain does not calculate; it *patterns*. For this mentation, everything 'resembles' everything else. There is no working things out from first principles because there are no first principles, only things as they are together. The left brain is abstract while the right is phenomenological in spirit. I say 'in spirit' because each side has its share of passion. The lateral distinction of the brain is not confined to humans. In birds, the right eye (linked to left brain) is involved in pecking for food whereas the left (linked to right brain) is generally aware of the environment and alert to dangers.

Now, when we have two elements or viewpoints that contrast with each other but are linked in some meaningful way the two sides regard or feel their situation also in a distinctive way. In such a *dyad* one side will tend to *exclude* and negate the other while this other side will tend to *include* and embrace the other. The idea here is that the two elements are not just like objects but also like *subjects* or minds and therefore capable of 'having an opinion' about their combination. There is a *meaning space* to be partitioned and the nature of each partition will depend on how many partitions there are, producing that number of *kinds* of meaning. It can also be said of ourselves that we are very much 'of two minds' and that this ambiguity of contradiction and complementarity is essential for our intelligence.

I do not want to lay out tables with lists of contrasting properties proposing to enumerate the differences between left and right brains

because such tables obscure the mutual relevance of the two sides which weave together in extraordinary ways as well as inhibiting each other. I think one has to rather *feel* their difference. However, a strong contender for marking the difference is the property of conscious thought. I simply mean by this something like, 'Well here I am thinking about this problem' where the critical factor is '*I think*'. This plays an enormous role in our lives not least giving the basis for the very concept that thoughts can be *owned* by people. In contrast there has long been discussed - at least in psychoanalysis - the idea of 'thoughts waiting to be thought'. Group Analyst David Armstrong proposed two kinds of thinking:

Thinking 1

Produced by a thinker and 'belonging' to her
They can be true or false
They are capable of being taught
Need to be explained, justified, etc.

Thinking 2

Precede any thinker
They just are
They can be learned from but not taught
Require nothing else but themselves

The occult philosopher Gurdjieff has a nice turn of phrase when he has one of his characters speak of thoughts 'a-thinking in me'. The contrast of thinking 1 and 2 has radical implications because thinking 1 has become by and large the very essence of our sense of and belief in our identity as free beings with will. Surely, we must in some sense 'be' the source of our own thoughts? Yet there is more and more evidence and discussion that points to the fact that the source of thoughts, including significant and creative ones, is in a part or kind of mind that is not accessible to what we identify as our conscious self. This was brought to a focus by experiments that showed that the conscious thought of having made a choice arises significantly *later* than neural events that presage it. In other words, decisions and choices are made before we are aware of 'making' them. This does not necessarily support the usual reductionist conclusion that we are just machines and our consciousness is just a 'user delusion'. I am talking about it here in terms of another kind of mind than that developed in our so-called conscious mentation. To put it in a crude but perhaps helpful way, on one side we have 'I

think' while on the other we have 'think I'. Instead of some imaginary isolated spirit or 'I' on the left we have the 'world' on the right, often seen of course as imaginary from the left-hand perspective. What is pretty certain is that the unconscious mind is capable of the most extraordinary acts of construction which have a logic that transcends what the conscious mind is capable of. As for example Spencer Brown intimated when he remarked that Ramanujan had a perception beyond the scope of existing mathematics. We should remember here that Ramanujan explicitly stated that his insights came from a manifestation of the goddess Lakshmi. It is as if all the good ideas are generated on the right, and then passed over to the left. The left has the job of rendering the ideas *useful in space and time* and in performing this task it assumes ownership of them and 'forgets' where they came from. The consciousness of the left brain involves it in treating many things as unconscious.

The right hand brain has to find ways of getting ideas across to the left through the barriers this side has put up to defend its identity. To speak like that is to be utterly anthropomorphic and I am reverting to a very mythical style of language. But, after all, these two minds I am speaking of are an essential feature of our humanity and intelligence. As an 'explanation' this view radically differs from any mechanistic one such as the naive neurophysiology of a Crick or even from the Turing machine model of intelligence. The Turing machine represents every possible machine in terms of a one-dimensional tape that goes back and forth under a reading/writing head. This is actually, I would say, a model of the left brain and does not begin to address the nature of the right side (note that the 'reader/writer head' looks very much like a model of an 'I'). It is still fairly true to say that the right brain is better appreciated in the world of psychoanalysis which has long grappled with the practical difficulties of gaining conscious access to this intelligence through, for example, dreams. In his book on the subject Matte-Blanco is forced to speak of the unconscious mind as consisting of 'infinite sets' which can never be fully translated into the finite sets and linear logic of the conscious mind. That is why material coming through as it were from the 'other side' can appear nonsensical.

It is speculated that the linear step by step sequential processing of the left hand brain developed from a function used in tracking animals, which makes some kind of sense. It became amplified with

the invention of writing and later of the alphabet; later still of our modern number system and equations. It is extraordinary how much has been achieved through this linear skill. By eliminating the richness and spaciousness of our living experience we have created spells - like abracadabra! - the word 'spell' meaning both magic and the use of an alphabet and abracadabra meaning (in Aramaic) 'create as I say'.

But I want to draw back a little from describing the two minds of the sides of the brain and ask, what sort of mind is speaking and who the author of this discourse? Only one of the sides may be speaking which, by obvious implication, would mean a serious inadequacy. And which brain is best equipped to know both sides? If both are speaking then is there some third bringing them into unison or co-operation? Or is the co-operation itself the third, rather like 'the third' in psychoanalysis?

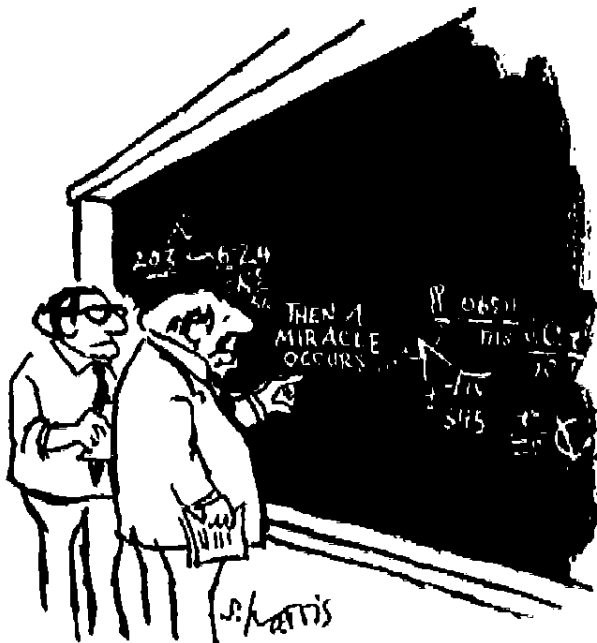
The two sides are not like things interacting, they are more akin to proto-persons, modes of mind (the simple two hemisphere division does not preclude yet more complex structure with several elements). I am reminded of the theology of Jesus as both God and man: two natures in one person. Most people these days regard Christian theology as arcane, archaic and irrelevant to them but I believe the theology managed to capture and reflect a new power of mind being born and leave us a message in its complexities and mysteries. If I map God onto the right brain and man on to the left, this is not an empirical claim but a matter of mythical or symbolic resemblance, to be seen within the context of the last two thousand years in which *man has become God* leading to the death of God as announced by Nietzsche.

One of the tortuous and anguished discussions of the period was around the question: how could Jesus know he was Christ, son of God and hence the same as God, if he did not know what was going to happen to him, as he could not have had if He were to die as a man? Our own dilemma resembles this. 'I' in my left brain does not have the kind of consciousness my right brain is supposed to have and so 'I' am not aware of what it sees. That is how we can surprise and deceive ourselves. Later I will talk about this 'I-ness' as distinct from any consciousness.

The theology of the early Church was developed after the emergence of modern mathematics that was distinctively marked by *The Elements* of Euclid (c. 300 BC) and flourished in Alexandria. *The Elements* is the next most widely read book after the Bible. The axiomatic method of Euclid marked a radical step away from earlier mathematics and its influence lasted for more than two millennia right into the nineteenth century. [When Euclid's fifth axiom was questioned God died!] It empowered abstraction. It released the possibility of modern mathematical physics and the astonishing concept of purely rational *proof* which allows a certainty entirely divorced from any particular actual experience.

The left brain was given an unparalleled way of advancing itself. It went on to transform the world out of all recognition. It was a magic to surpass all previous magic. I think of Dirac sitting in his room three years dreaming up the equation that would predict anti-matter. I am amazed at how few people are amazed at this logical, mathematical, abstract genius that has entered human life, while many are of course appalled at its consequences for the welfare or otherwise of human and indeed all life.

DIMENSIONS OF REPRESENTATION



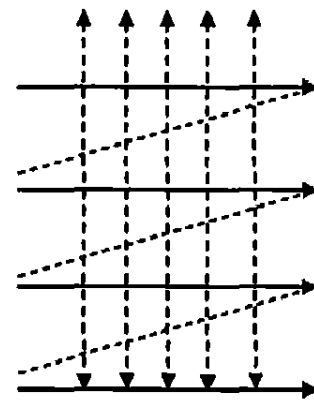
"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."

One of the most striking things I heard from Spencer Brown was that mathematics consisted of making marks on a plane surface. It was the first time I had come across a reference to the *physicality* of mathematical notation. What struck me later was that, in addition, mathematical script was like any other and written in a strip. This is so embedded in our culture it is barely noticed. Its significance is that only events, processes, actions, operations, etc. that are linear can be notated. Non-linear relations can be derived from equations but usually require diagrams

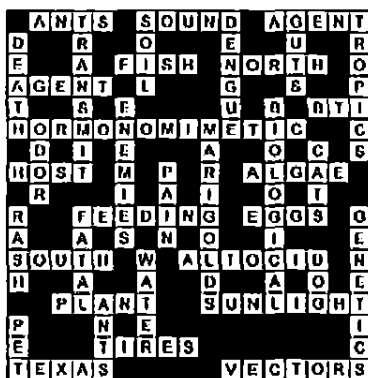
and images to depict them. Proponents of so-called 'visual mathematics' argue that images can show what equations cannot: we have to see what the equations might mean. The well-known example of the Mandelbrot set. has become part of modern art. Such imagery restores us to the sensory world. A fascinating instance of the significance of representational media is that of the *Hyperbolic Crochet Coral Reef* in which philosopher of science Margaret Wertham amongst others started crocheting coral reefs as part of an ecological project but then discovered that the forms they produced matched the needs of hyperbolic geometry.

Linear order allows us to very precise and to be able to check every particular step, because we actually go through the symbols one by one (as illustrated in this wonderful New Yorker cartoon). To allow for a writing/reading that operates in two dimensions must seem absurd. For one thing, we might not be able to say where the 'text' begins or ends. However, scholars have suggested that there are two kinds of reading, diachronic (historical) and synchronic (descriptive) which I relate to the sequential and the synoptic respectively.

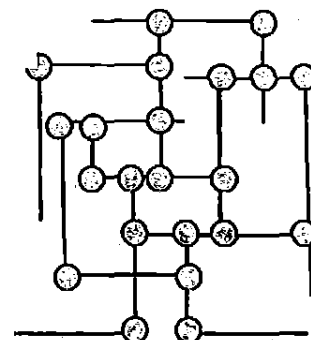
I made this diagram to attempt to depict what I had in mind as 'two-dimensional reading'. The vertical reading is not of the same kind as the horizontal as I crudely indicate by using double instead of single headed arrows; though such things as crosswords are an exception. Think of poetry where one comes to be aware of a meaning resonance between the lines. Alternatively, we might see a resemblance with the complex plane of representation of complex numbers. Here are some of such 'resemblances'.

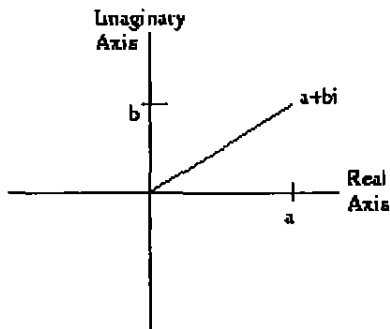


Here are some of such 'resemblances'.



IS OF THE FORM

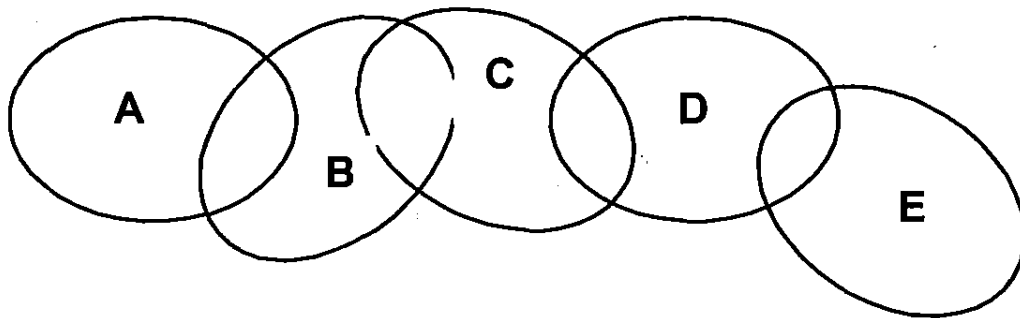




COMPLEX PLANE

As shown with the crossword illustration, there could be a number of points of intersection of vertical and horizontal, places where the two forms of writing/reading coincide. In the complex plane these are the (0,0) points in a constellation or order of zeroes. This reminds us of the well-known instance of Mendeleev who worked for three days solid on arranging the elements in the combined orders of their chemical properties and atomic weights then had a dream that revealed a solution that became the basis for the Periodic table.

When I first met David Bohm he was searching for a new understanding of space that avoided or was the precursor to Euclidean-Cartesian representation. His approach was 'ontological', thuswise:



The regions or topoi are concrete and A is somewhat 'inside' B, and B is somewhat 'inside' C and so on such that A is somewhat 'slightly inside' E. Gradations of insideness - or participating in the being of - corresponded to proximity and thence to distance. Bohm's feeling for this 'inside' approach found expression in his appreciation for the paintings of Rouault. In a letter to Charles Biederman he tells how his initial aversion to Rouault's painting gave way "to a remarkable

new steady vision which I can best describe as seeing in a new dimension". He once explained to me that he saw the colours in the painting merge into or act on each other in such a way that space *emerged* from them. Thus the colours were not 'in' space at all but prior to space.

Colour was a prime example of the phenomenology of Goethe. The narrow left brain view is that colour is 'subjective' and exists 'only in the brain'. There is a remarkable book called *Color Codes* which makes it clear that the kind of *order* associated with the *experience* of colour is definitely non-linear and follows a multitude of 'logics'. To take one example, in Hegel's view of colour the combination or harmony of all the colours is not white but the 'flesh-tone' used by painters.

The right brain has a window into the world of living complexity and does not have to 'understand' it as the left brain tries to. An image of this complexity is called 'Indra's Net' and portrays an infinite set of jewels such that every jewel reflects the radiance of every other jewel. A more homely example is that of a room, once one becomes aware of the transfinite possibilities of the arrangements of what is in it. *Arrangement* is an extraordinary feature of the world and, if the continuum is aleph one then it belongs to aleph two.

What I call vertical reading, colour, arrangement and so on are all facets of the right hand brain. I once took a Jungian analyst friend of mine to see Stonehenge. While I was feebly waxing on about astronomical alignments, she drew my attention to the colour of the lichen on the stones. The contrast was startling and also comic (humour may be in fact the most interesting example of the meeting of the two sides of the brain!).

BEING IN THE WORLD

....there is no inner man, man is in the world, and only in the world does he know himself. *Phenomenology of Perception p. xi*, Merleau-Ponty

It is more than curious that the thinking of the classical age of physics assumed that there were *primary* properties of material objects, all others being subjective or derivative. The mass and volume of an object were primary and real while its smell and colour

were not. The right brain appreciates how things taste, smell and look, all the sensory qualities or *qualia* of our experience. I would like to suggest that 'other kind of information' are located in or are part of the sensory richness of the world, the richness that has been steadfastly ignored or even disparaged in classical physics and recent philosophy. This is not to deny that another kind of richness has become available: just as the ascetic who denies his sensuality can release a possibly 'higher' range of sensations.

The right brain does not 'encode' the world as the left brain does or believes that it does. It does not make 'models of external reality'. It is *part and parcel* of the world it inhabits just as in Bohm's picture. Its mode of experience is *participation* while that of the left brain is *observation*. The left brain wears a white coat and goggles while the right dances in the meadow in bare feet.

It is more than a metaphor to say that the left brain *looks* while the right brain *listens* (or, as Heidegger would say, 'harkens'). In some cultures intelligence is located in the ears! My late friend Edward Matchett who studied and taught genius used a method he called 'neural education' in which music was used as a source of meaning-substance that could be assimilated directly into a problem and resolve it. Music in contrast with language gives us simultaneous streams of information. I am reminded that Elgar was reported to be able to hear whole symphonies on sight reading their scores.

I have used the term 'information' in a loose sense and not the now standard digital one. It is important to acknowledge the possibility that information in analogue form is richer than in digital (as some connoisseurs of music insist is true and value vinyl recordings over compact disks). The digital form of information belongs to the left brain and corresponds to its nature of treating everything as the same, as exemplified in Descartes' reduction of all materiality to extension alone. [I am surprised no one has observed that this was tantamount to adopting just the first rung of the *form* of the Scholastics. Matter in itself had no qualities and had to 'receive' them from the world of Form. The first of these was *quantity* but there were then many others reaching up through sentience and beyond.

Something of the character of right brain information can be *felt* perhaps in contemplating Chinese painting, especially of the Taoist

schools. This remarkable painting by Tao-chi brings the viewer inside it as the person in the hut enfolded by the mountains and he is not an observer.



Landscape by Tao-chi late 17th century

It is easy to find resemblances in many fields to the contrast between left and right brains we have been discussing. Speaking of China, there is the contrast between Confucianism and Taoism. In modern philosophy there is as one example the contrast between logical positivism and phenomenology. In Peter Rowlands' scheme the fermion is left and the structured vacuum is right - and *in combination are nothing*, which I will come back to later. We can even speak of the contrast between typical images of 'the west' and 'the east'. Interestingly enough - at least from the perspective of my metaphorical and mythical approach - history tells us of a shift of higher knowledge from the East to the West, from China and India, through the Middle East, to Europe and then North America.

HISTORY OF WILL

All other things have a portion of everything, but [WILL] is infinite and self-ruled, and is mixed with nothing but is all alone by itself. - Anaxagoras from *Commentary on Aristotle's Physics* by Simplicius [with the word 'mind' replaced by 'will'].

The duality of brain is essential for our creativity and I find it refreshing to contemplate that our very essence is rooted in contradiction. As I shall attempt to explain later, duality is the condition for will and without contradiction will can have no meaning. Also, will brings in something new and, as such, it would not be surprising to find - or find ourselves inventing - an historical view of will as somehow progressing.

The meaning of words/concepts does not stay still and it is questionable to assume that the same word was given the same meaning in different periods. 'Will' has its roots in Indo-European languages as Greek *elpis* 'hope', Latin *velle* 'wish, desire', German *wollen*, including the Sanskrit *vrnoti* 'chooses', the Avestan *verenav*. and the Old Saxon *willio*. It can designate both a verb and a noun. It has strong associations with desire and the term 'well' as in 'it goes well' means that things are going as one wishes. There is a more complex meaning to do with *freedom*. This meaning or complex of meaning has to do with the possibilities of freedom from desire (or oneself) and our relation to the Will of God. The rise of the monotheistic religions created the paradox of separate wills within one absolute single will. The aspect of desire finds its expression particularly in sex, as Schopenhauer was to aver, or will as a *blind urge*. But in the religious meaning it is allied with *conscience* or 'truly knowing' and *faith*. This might also be a kind of blindness, but in the sense of going beyond reason.

It is curious that Eastern religions such as Buddhism seem to have no equivalent for 'will' even though the phenomenological reality of it is evident in e.g. the Buddha's teachings. As a possible comparison we might think of the fact that the concept and term *energy* did not emerge in science until the nineteenth century.

It was St Augustine who first placed the issue of will firmly in the Christian vision. In a major departure from Greek tradition, he shifted the centre of gravity of the search for truth from Intellect to Will. In his

psychology, man has a trinity of faculties: Memory, Intellect and Will and Will was free to ignore or choose otherwise than intellect dictated. These days we might associate his 'will' with our modern sense of 'feeling'. What has remained over the last almost two thousand years is the intuition of a human being as a trinity of some sort. Yet, as we shall see, thinking in threes is hardly logical - as was known from the early days of the Church.

Will appears in the paradox of being able to do what one wants but also being able to *not* do what one wants. This was expressed by St Augustine as: "The mind commands the body and it obeys. The mind orders itself and meets resistance." This is also sometimes expressed by using the two terms 'will' and 'nill'. Psychologically, the operation of *nill* is supremely important for intelligence since it is needed for the 'suspension' of automatic responses, allowing for more conscious or thoughtful ones. In the twentieth century this became enshrined in the foundations of phenomenology as *epoche* or 'putting into brackets', which we will come back to later.

St Augustine was an acute observer of his own psychological process and was a major pioneer in thinking of man as *divided against himself*: we can 'know' what to do but 'choose' not to do it. The common naive view of today that we have a single will is questionable at best. A man can find himself at variance with the world, with his fellow men (as Sartre was to dictate, "The Other first arises in my world as a Not"), with God (as in the Christian doctrine of sin and the pangs of conscience) and even with himself. The first and last of these taken together gives us the common contemporary picture of man as a neurological machine for whom freedom is an illusion. In Islam the greater *jihad* is in the war with oneself (or one's *nafs*) and the lesser in the war against the *kafir* or infidels.

The scholasticism of the Middle Ages was largely based on Aristotle's ideas and, in particular, his scheme of Form and Matter. Matter was just stuff, inherently inchoate, needing to be shaped and made intelligible by Form. This, to my mind, is eerily similar to the relation of the right to the left sides of the brain which the left believes. The world of forms was hierarchical (even patriarchal, one might say); it largely disappeared from sight around the seventeenth century but is beginning to emerge again, centuries later, in such guises as Bohm's *active information*. There were different levels or

classes of existence. What was problematic was the *individual*. An individual was a member, say, of a genus or species but where did its particularity come from? The usual answer was that this was entirely due to the particular *matter* involved. The forms were always seen in *abstraction* from diversity and hence could not be responsible for individuality.

In the eleventh century the Jewish scholar and poet Solomon Ibn Gabirol, known to the Christian community as Avicebron, proposed that the very binding of form and matter required *will*. The two streams of Greek and Jewish thought were and remained divided against each other and here surfaced their division with a vengeance. Will appears closely associated with the ideas of monotheism. We can point out a resemblance between Augustinian psychology and Avicebron's cosmology, with a reference to Duns Scotus also:

Augustine	Avicebron	Scholastic
Will	Will	Haecceity
Intellect	Form	Quiddity
Memory	Matter	Hyle

Avicebron's position implied that every individual was an act of creation. Two centuries later, Duns Scotus became the champion of individuality in his doctrine of *haecceity* or 'thisness'. As Hannah Ardent claimed, Scotus was one of the first and greatest to explore the implications of the notion of will. He opposed Aquinas's view that freedom comes from 'intellectual appetite' as opposed to 'sensual appetite'. First of all he argued that freedom comes from having multiple options at one and the same time and, secondly, that there were two 'affections' of the will rather on the lines we discussed earlier - one for what is advantageous to us (desirable) and the other for what is *right* which may have nothing whatsoever to do with our happiness. Further, what was 'right' was entirely due to the Will of God.

The Will of God in monotheism maintained the element of 'caprice' or arbitrariness which we suppose was commonly ascribed to the gods in ancient times. It was strongly felt that God could not be restrained by reason. Remarks made by modern scientists such as Einstein and Hawkins refer to the *mind* of God and not His *Will* and come out

of the stream of Greek thought, not the Jewish. It is important to note that in Islam natural laws are regarded as 'the habits of Allah'. The philosophical *a priori* is somehow haunted by the unknown Will of God.

The 'capriciousness' of will appears again much later in Schopenhauer's *The World as Will and Idea* as *blind* impulse. Will cannot be explained. Since it is always what is original, it can only be known after the fact of its operation. Will can only be equated to will and cannot be derived from anything else. We are not even conscious of will: the intentionality of consciousness as 'consciousness of' in this case has no object. It is entirely subject.

The word 'subject' means 'thrown-under' or dominated but in the adjectival form 'subjective' became almost synonymous with illusory or opposed to the 'objective', derived from 'object' meaning 'thrown-against' or fact. At one time however 'subjective' meant *real* only gradually becoming its opposite. The nature and plight of the subject became a major issue in recent philosophy, starting perhaps with Soren Kierkegaard. A key issue was freedom to choose. No matter what the objective forces and evidence dictated, the subject was free to *choose otherwise*. Kierkegaard contrasted the aesthetic (intellectual) with the ethical (volitional) and called us to make the 'leap into faith'. Later, with the 'death of God', this was transposed down into for example the 'bad faith' and 'futile passion' of Sartre.

What is at stake with will is not merely difference or distinction but opposition and conflict. To summarise some of our earlier remarks, the will is involved in 'strife' with four elements:

GOD

OTHERS SELF

WORLD

Note that this form contains two complementary oppositions and can be read to imply that if they collapse into one (as below) we have the condition called *egoism*.

OTHERS-WORLD ↔ SELF-GOD

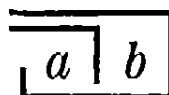
It is fairly clear that the concept or the possibility of will tends to throw a spanner in the works of reason. It may even be the case that it is finding another revelation in the discovery of the irrational, undecidable, unproveable, incomputable, etc. content in the heart of mathematics. In place of the simplistic classical view that truth has to be non-contradictory it is possible that something quite different is needed.

I have used the Heraclitean term 'strife' from his *dike eris* 'strife is justice': "We must know that war (*polemos*) is common to all and strife is justice, and that all things come into being through strife necessarily." But I also want to bring sex into the picture (though not quite in the same way as Schopenhauer does). Sex requires both opposition and union (as in the French *vivre la difference*) and is important because of its implications of generation because will may be understood as *essentially* dividing and uniting to create what is new. Understanding or enlightenment, according to Spencer Brown amounts to seeing the 'laws of creation'.

FORMALISM

To arrive at the simplest truth, as Newton knew and practiced, requires years of contemplation. Not activity. Not reasoning. Not calculating. Not busy behaviour of any kind. Not reading. Not talking. Not making an effort. Not thinking. Simply bearing in mind what it is one needs to know. And yet those with the courage to tread this path to real discovery are not only offered practically no guidance on how to do so, they are actively discouraged and have to set about it in secret, pretending meanwhile to be diligently engaged in the frantic diversions and to conform with the deadening personal opinions which are continually being thrust upon them. - G. Spencer- Brown

I want to play with some mathematical formalism you may be familiar with. I made much fuss about the linearity of language and mathematics. It is interesting that Spencer Brown actually violates this linearity though in a subtle way and not only in the way he expresses non-linear or second order equations by means of what is more or less a diagram:



He writes his two fundamental laws as

$$\neg \neg = \neg$$

$$\neg \neg =$$

I suggest that the second law of crossing should actually be written in a vertical way:

$$\begin{array}{c} \neg \\ \neg \\ \parallel \end{array}$$

An interesting feature of the vertical is that it oscillates between two states while the horizontal can have only one. In this guise, it is akin to the treatment of the square root of minus one as an oscillation between +1 and -1.

My distinction of the two ways of combining two operators or marked states in the calculus is an example of the possible plurality of meaning of 'distinction'. This is to bring into play the spirit of the right brain with a vengeance. It is not without support. In T. J. McFarlane's paper *Distinction and the Foundations of Arithmetic* he defines no less than *three* kinds of distinction (to allow for the full range of commutativity and associativity). Spencer Brown himself introduced a second operator 'score' to deal with arithmetic and there is yet another reading in which there are four operators or kinds of distinction.

Thus distinction is fine as a starting point but turns out to be variable of sorts; not a continuous one but subject to or correlative with *the number of distinct states allowed in the given calculus*.

Now I turn to an eigen form of reflexivity that Louis Kaufmann has shown us exemplified by the proposition of Heinz von Foerster; "I am the observer of I observing myself" . I write this in the following way:

$$I = (I (I))$$

'Moses equation'

This is the equation God gave to Moses when he said, 'I Am that I Am'. If we write this in a Spencer Brown way then the right hand side can be:

$$\overline{I \overline{I}} =$$

$$\overline{\neg} =$$

and just removing the I's comes to the same thing since So the I on the left of the 'Moses equation' can be interpreted as *nothing*: the ultimate reductionist view of human identity as vacuous or illusory.

In the latest edition of *The Laws of Form* Spencer Brown writes:

Thus we cannot escape the fact that the world we know is constructed in order (and I thus in such a way as to be able) to see itself.

This is indeed amazing.

Not so much in view of what it sees, although this may appear fantastic enough, but in respect of the fact that it *can see at all*.

But in order to do so, evidently it must first cut itself up into at least one state that sees, and at least one other state that is seen. In this severed and mutilated condition, Whatever it sees is *only partially* itself. We may take it that the world undoubtedly is itself (i.e. is indistinct from itself), but, in any attempt to see itself as an object, it must, equally undoubtedly, act* so as to make itself distinct from, and therefore false to, itself. In this condition it will always partially elude itself. [*He relates this word to the Greek for 'actor' and 'antagonist']

To anticipate myself a little I point out at this juncture that the only possible result of a *repetition* of the severance or distinction in the dyadic calculus that produces an observation of myself is to render me 'not-observing' or *unconscious*. If there is some other result then something new must come in and the second act of distinction be non-identical with the first. In this context, the repetition of a

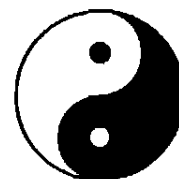
distinction changes the distinction (one cannot step into the same river twice - Heraclitus again).

Let me represent the first act of distinction by a vertical stroke | and the second act by a horizontal stroke ---. Then I write this form:



The *Tau* form expresses the right hand brain view of *how* two elements can 'meet' and/or 'be together'. It detaches from the usual sense of direction in reading. If we have two elements such as A B we can look for what is *between* them and also for what *encompasses* them, which gives a different 'reading' from scanning in the sequence A first and then B. The more holistic kind of reading was exactly what the Kabbalist practised in his contemplation of the letters in words. We will need to look more deeply at the meaning of the blank page later.

The Tau symbol is pretty obviously related to the (later) form of the Taoist duality of Yin and Yang:



But I could also use the *vertical* order of arrangement and write:



which suggests an hierarchical order or order of orders. The tripartite form exemplified above in the diagram is embedded in western theology and philosophy. In Hegel it became the 'negation of the negation' a phrase he used to express a *triadic* understanding in our basically *dyadic* language. Hegel drew on Christian mysticism and it is easy to find the recurrent idea of understanding the Unknown God by *going beyond the negation of what is known or thought*. I propose that there is an understanding of self-observation that is of the same form and implies an operation that *goes beyond consciousness*. The twentieth century mystic Gurdjieff made this remark:

"Not one of you has noticed the most important thing that I have pointed out to you," he said. "That is to say, not one of you has noticed that you do not

remember yourselves.” (He gave particular emphasis to these words.) “You do not feel yourselves; you are not conscious of yourselves. With you, ‘it observes’ just as ‘it speaks,’ ‘it thinks,’ ‘it laughs.’ You do not feel: I observe, I notice, I see. Everything still ‘is noticed,’ ‘is seen.’ ... In order really to observe oneself one must first of all remember oneself.” (He again emphasized these words.) “Try to remember yourselves when you observe yourselves and later on tell me the results. Only those results will have any value that are accompanied by self-remembering. Otherwise you yourselves do not exist in your observations. In which case what are all your observations worth?”

This passage exhibits an important ambiguity in its use of terms relating to awareness. Gurdjieff is implying that there is a usual kind of awareness - 'it observes' - and another kind where I 'remember myself'. The phrase 'self-remembering' which became emblematic of Gurdjieff's teaching is quite precise in indicating a kind of repetition that is yet utterly new. In common usage we have the slightly sardonic phrase 'once more with feeling' but there is indeed a sense of entering into the same again in such a way as to *really get it*. It is interesting that people who attempt to follow such indications always insist that this second order awareness cannot come into operation without intention (yet is only possible from something *given*).

I am going to connect together the exploration of the meaning of will with self-remembering. This means that it is not enough to be aware of oneself but one has *to realise* that one is aware of oneself. The word 'realise' is used advisedly in that the act is just that of 'making real': of truly owning what is revealed, taking responsibility for it just as it is. Whereas ordinary observation produces a separation due to which one part of ourselves can criticise or react to another part the second order observation takes it all in. One might even say that to self-remember it is *necessary to love* oneself. The observer and observed of the first order of distinction are realised as not-different in the second order. This is of the same form as 'overcoming one's own will' in a spiritual path.

The second order observation is not observation in the sense of the

first order. The result may feel like a Spencer Brownian 'cross-cross equals blank' precisely because will is unobservable and cannot be known. We do not know our own will so mistake who we are because we revert into treating ourselves as things *or observers of things*.

Gurdjieff called the state of collapse into objects, thoughts, etc. *identification*. The next, second order of identity gives awareness but not freedom. The third - self-remembering - gives freedom but what this freedom means is only then accessible. Another way of putting it is to say that in the first level one is an object whereas in the third one is - rather *enacts* - a *relationship with oneself*, which I like to call the 'true subject' in that this 'I' is *subject to itself*. What is between the first and third condition is usually called 'consciousness' and many people desire 'more' consciousness; however, in an important sense, the third order is not more conscious at all and can even appear as unconsciousness.

The latter idea is found in Hinduism where it represents levels of identity in the order: Waking, Dreaming, Dreamless Sleep; which seems to totally contradict our usual evaluation of levels in terms of more and more consciousness. Dreamless sleep is actually akin to *samadhi* - the identity of object and subject. In practical terms it is often said that 'from the perspective of the ordinary mind the higher state is unconscious'. This is of course precisely what we have with a reduction to the law of crossing:

$$\neg \neg =$$

The 'awareness of awareness' seems like non-awareness.

I want to distinguish the vertical from the horizontal ordering further by suggesting that it is in the horizontal that we see appear open-ended series such as:

$$I \quad I(I(I(I(I)))) \quad)$$

Which can expand or contract. Let us now make the jump to portray vertical and horizontal ordering together:

$$\begin{array}{c} (I) \\ (I(I)) \\ (I(I(I))) \end{array}$$

and so on. This *resembles* the combinatorial hierarchy based on the empty set \varnothing

$\{\varnothing\}$

$\{\varnothing \{\varnothing\}\}$

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It is shown here in four levels to remind us of the Pythagorean *tetraktys*. The simplistic series shown is not of course the same as the series of levels in the actual hierarchy. It shows only that there is a *progression in depth* and does not take account of the detail of the combinations. The interesting thing here is that we can see on the left a repetition of the 'same' element while on the extreme right there is a movement into depth as shown by the number of brackets. This gives a series of elements horizontally corresponding to the vertical levels reached. The aesthetic principle of this might be illustrated by a remark made by Ezra Pound:

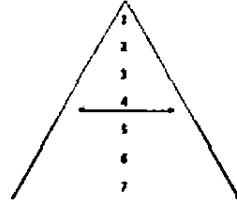
Genius ... is the capacity to see ten things where the ordinary man sees one, and where the man of talent sees two or three, plus the ability to register that multiple perception in the material of his art.

NUMBER AS AN ORDER OF ORDERS

I can think of no better expression to characterize these similarities than "**family resemblances**"; for the various resemblances between members of a family: build, features, colour of eyes, gait, temperament, etc. etc. overlap and criss-cross in the same way.- And I shall say: "games" form a family. . . .

for instance the kinds of number form a family in the same way. Why do we call something a "number"? Well, perhaps because it has a direct relationship with several things that have hitherto been called number; and this can be said to give it an indirect relationship to other things we call the same name. And we extend our concept of number as in spinning a thread we twist fibre

on fibre. And the strength of the thread does not reside in the fact that some one fibre runs through its whole length, but in the overlapping of many fibres.
Philosophical Investigations (67) Wittgenstein



The diagram shows an arrangement of elements that increase in depth and complexity according to the simple sequence of the natural numbers. The arrangement has *not been strictly derived* from some axioms or out of a single concept such as that of distinction. Right from the beginning it has presumed what it purports to show, which is intrinsic to *this kind of understanding*. The idea of a progressive sequence of number archetypes is ancient and can easily be dismissed as mythological. My suggestion is that, like the poor, it is always with us.

To focus a little more closely on what is involved in making a step from one level to another, we take the case of the movement from Two to Three or, as some interpret it, their 'warfare' or strife. A paradigmatic appearance of this transition is encountered in our visual perception of depth. This perception arises from the right side of the brain. The classical perspective of science will call this 'subjective' since what we actually receive as data consists of impacts (or 'marks') on the two-dimensional surfaces of the retinas. The sense and feeling of depth is something *projected* out of us. It then resembles what has been called 'quantum touching' (by Peter Marcer and others) that in fact our perceptions are not some kind of models constructed 'in here' inside the brain but really exist 'out there' just where objects 'are' (just as Plato proposed in his theory of vision). The impact of the third dimension can be keenly felt in playing with autostereograms which become three-dimensional if one looks at it in a certain way.

Holograms are also suggestive. I particularly remember one of a Victorian stereoscopic device that contained two photographs. The device existed in mid-air as it were and, by putting one's eyes to its

lenses, one could not only see the photographs but also combine them into another order of three-dimensional image.

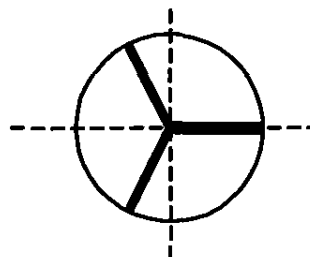
Depth is similar to colour in moving us away from a simple objective-subjective duality. A possibility by the way is that colour can count as a fourth level manifestation even though, again, what are presented to us, apparently, are only surfaces.

The 'war' or clash between Two and Three is expressed in what are called 'impossible objects' such as this triangular form.



Of course, the effect comes out of our automatic tendency to project into three dimensions. But it evokes the sense and feeling of *relationships*. If the viewer can somehow get *inside* the image the 'impossibility' dissolves. One has to go from looking at the image to *being inside it*. Such illustrations are but faint indications of a reality of Three that cannot be composed out of Two. This was something that Peirce was very keen on and he openly declared his allegiance to trichotomy. However, as he pointed out, people differ in their numerical allegiances. I want to suggest that *any* natural number can be taken as primary or as ultimate.

We can postulate *meaning spaces* resembling Hilbert space with N vectors or terms such that the 'direction' of any one of the N vectors *is in the context* of the others. The idea can be represented in the following diagram using the complex plane.



The example shows a threefold structure. The three terms (vectors) are given by the cube roots of 1 and in general for any N will be the Nth roots of 1. In the case of N = 3, the roots are:

$$1 \qquad \frac{1}{2} (-1 + i\sqrt{3}) \qquad \frac{1}{2} (-1 - i\sqrt{3})$$

The even N have simpler roots of course such as when $N = 4$ they are 1, i, -1, -i. The N distribute themselves equitably through the meaning space. Some elements repeat such as the obvious +1. But the meaning space itself changes with N. If we say it does not then we are arguing that the deeper spaces or levels are simply combinations of the shallower ones. In the progressive view, each new space is the same as the one before but creatively different. Such a view resembles the view of our identity which is along similar lines. The novelist William S. Burroughs invented the term 'identity transparency' to refer to just this aspect of our possible experience.

From the vertical order we derive the progression of 'worlds' but it is as if each level then has its own democratic or even 'political' task of sorting itself out through negotiation. There is the saying, 'Two's company but three's a crowd' which points out that even going from Two to Three is a challenge and the rules of the game must change. The worlds are so to say 'created' but are then left to sort themselves out as best they can. This means that the relationships of their terms are mutable. The number of the world, N, is set but the relationships, the mutualities, of the N terms have to be tweaked in every concrete instance. God is transcendent vertically but immanent horizontally.

It is also interesting that in this scheme we have a resemblance to Plato's account of the created world as an 'image' of the ideal world. The world of Ideas is vertical while the created world is horizontal.

THE CARTESIAN GENIUS

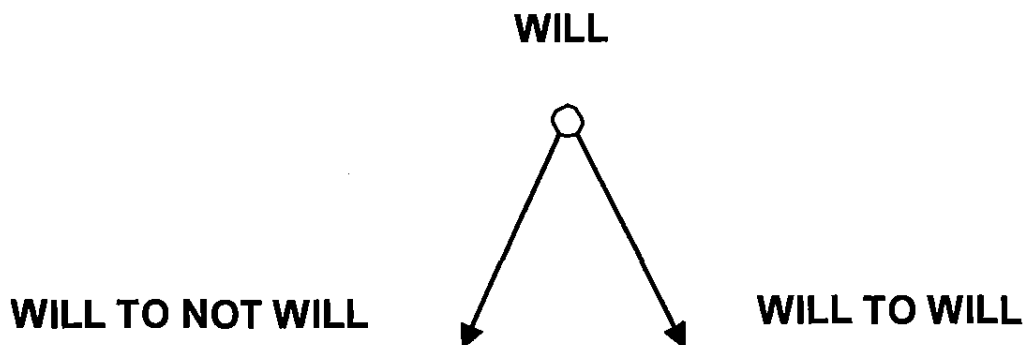
If you would be a real seeker after truth, it is necessary that at least once in your life you doubt, as far as possible, all things. Rene Descartes

The distinction Descartes made between *res cogitans* and *res extensa* - thinking substance and extended substance - arose from his *act of doubt*. More accurately he discovered that he could not doubt his doubt. As suggested earlier, his category of extension was a take up from the Scholastic doctrine of Forms, being the lowest form that matter could take while still being intelligible. At the other extreme, he has the Form of reason, the rational soul, one of the highest that matter could take. In essence then, his primary

distinction can be seen as just the two ends of the scholastic spectrum of Form. He simply left out all that came between. This proved supremely important because what was mainly left out was *life*. This was beginning of a world view in which everything was emptied out and seen as a blank canvas with only isolated points of meaning or souls to animate the canvas.

His cogitas can be more intelligibly understood as *will*. Referring forward again to future developments we can see here the rudiments of Schopenhauer's *World as Will and Idea*. The cogitas is revealed by the act of doubt. It is a kind of naked singularity. As such it had then to be sustained by a *higher act of will*, that of God.

In most combinatorial hierarchies, we have the primary substance or condition as some unknown at a zeroth level. The first level marks a beginning. In the case of our interpretation of Descartes, God is at the zeroth level while His Will is at level one. At level two this will divides into two contrasting modes.



Let us say that on the left hand side there is will as natural, mechanical law. As is well known, the laws of physics are largely negations. Now, how could we have will becoming unfreedom or determinism? The answer is strange: *only will is able to deny itself*. It then produces what are sometimes called 'framework conditions', including the conditions of time and space. On the left hand side this denial is akin to Cartesian doubt and embraces what we now tend to call 'reflexivity'. On the right hand side we have the appearance of freedom with the idea that 'man is made in the image of God', i.e. that he is free to choose. A short hand way of talking about what happens at level two is to say that it is made of negation. A struggle ensues which echoes in mythology as the 'war in heaven'. Thus we begin to see an approach to understanding the intertwining of such elements as the will of God, natural law, individuality and so on.

What might open up between the two directions and appear at deeper or 'more evolved' levels? An immediate answer would be - *life* - or autonomous existence. Eventually we will end up with the 'selfish gene'! What might also manifest is that we humans are not so much over to the right hand side and there is a question of *realising* the possibilities of freedom that are not guaranteed under conditions of organic existence.

The overall picture is one of asymmetrical symmetry. All the elements on a given level are equally true but in distinctive ways. The elements of the different levels have a transitive relation of implicate and explicate, each element of a given level capable of unfolding new possibilities at a deeper one. So everything is just the Will of God but in an ever-unfolding distinctive number of ways. As has been increasingly noticed, what we encounter in our mathematics for example is not so much some Platonic world of Forms but the *substance of our own acts*. This is simply indicated by saying we set up some rules and follow through the consequences. It is to engage in a reality we make by participating in that reality.

In this regard, we would aver that the underlying substance or given unknown which is variously called 'nothingness' or 'process' and so forth can be identified with will. Will is the emptiness of the blank page. Of course, to say this is to indulge in the right hand brain. The left would say that it merely 'symbolises' will simply because unknown = unknown.

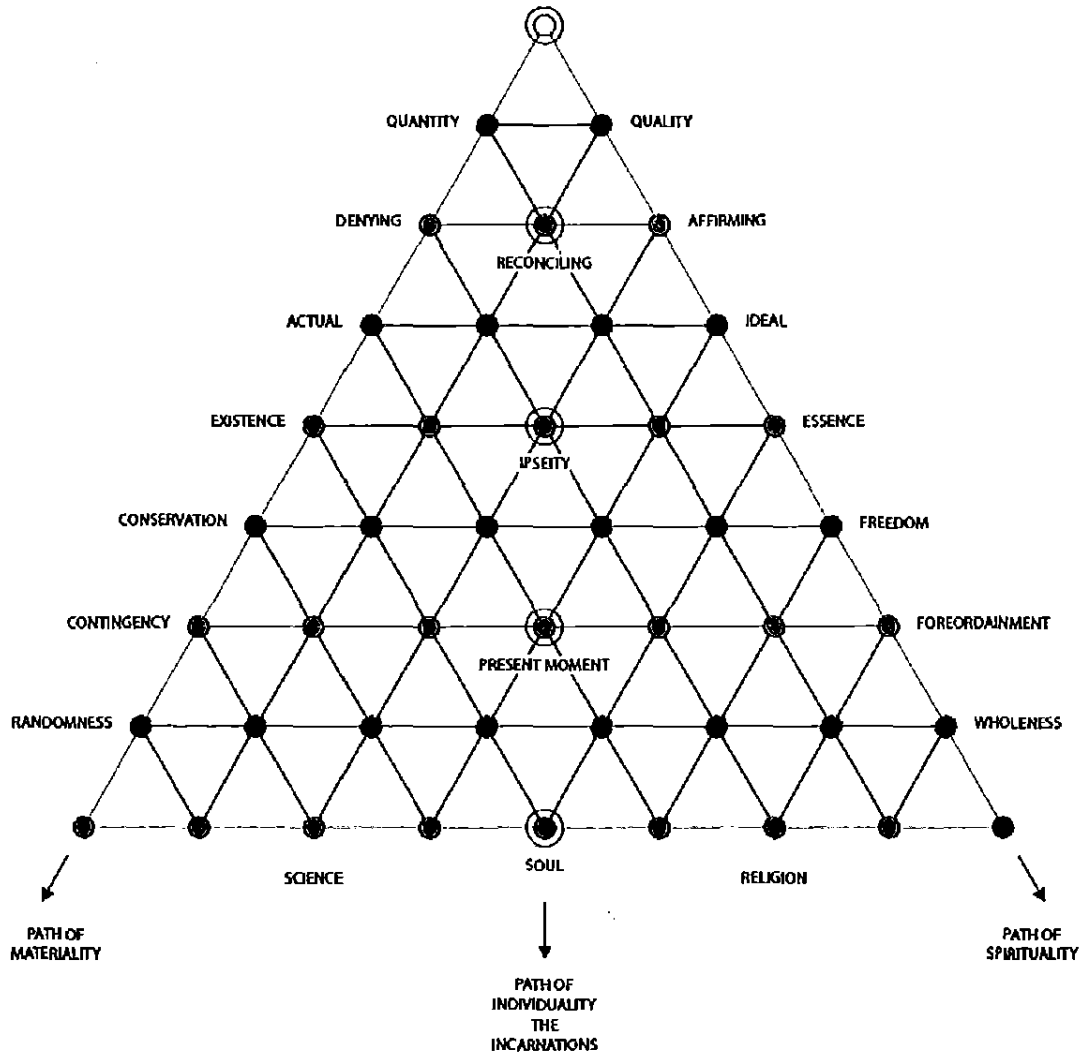
We mentioned that Descartes took over the first level of the Scholastic forms, i.e. quantity (extension). In general, the picture given by the kind of arrangement of levels and terms we have been playing with can be read as: towards the left, the quantitative; towards the right, the qualitative. Thus the numbers themselves can be seen as both numerical items and as archetypal qualities. However, like our two brains, the two views may be in opposition and even seek to suppress each other.

Returning to Descartes' *cogitas*, we can remark that thinking begins from contradiction. In this it is possible to glimpse how it is that thinking can be linked to the will in a special way. Thinking aims to remove doubt by taking it so far it becomes a certainty. It is the will affirming itself.

A NEVER ENDING STORY

Beauty is the translucence, through the material phenomenon, of the eternal splendour of the 'one'. - Plotinus cited by Heisenberg in his essay 'Science and Beauty'.

The unfolding or progression of levels looks something like the following *lattice*, where we have inserted labels at various points by way of illustration of what these positions or *topoi* might mean in a



general (therefore deceptive) sense:

The particular terms used are not so important because they cannot be determined by any authority or system but arise only in regard to some individual or individuals with purposes in a specific context. But we can specify the top point in terms of orientation, such as taking it as the will-to-know in the case of science or the will-to-be in the case of religious practice or the will-to-do in the case of technology, and so on. Alternatively, we can take *any* point within the nexus as a

starting point. *There is no absolute starting point.* 'God's will' is ubiquitous. Every term is a new beginning.

In traditional schemes, such a hierarchy - called an *emanation* - represented a declension of power as it unfolded as if some primordial energy runs down. In modern cultures, infused with vistas of evolution, the opposite interpretation can be made. We can see how it is possible to regard the left hand side as entropic with everything running down while the right hand side is syntropic or 'running up'.

The key act is that of 'creating contradiction' which is almost to say that rebellion (or Cartesian doubt) is built into the fabric of the universe. Humans are perhaps just a nodal point of the field of contradictions and not the sole exemplars of it. Of course, a traditional view was that humankind was created just so as to bring the godly characteristics into the creation, on the shop floor as it were. This has led to the prospect of highly evolved creatures becoming 'cosmic individuals' that is, capable of shaping the evolution of the universe. This was the ancient view of *shamans*. It goes beyond the Neoplatonic worldview as expressed by Plotinus:

So from this, the One Intellectual Principle, and the Reason-Form emanating from it, our Universe rises and develops part, and inevitably are formed groups concordant and helpful in contrast with groups discordant and combative; sometimes of choice and sometimes incidentally, the parts maltreat each other; engendering proceeds by destruction. Yet: Amid all that they effect and accept, the divine Realm imposes the one harmonious act; each utters its own voice, but all is brought into accord, into an ordered system, for the universal purpose, by the ruling Reason-Principle.
Ennead (3)

Another interpretation of the diagram is that it resembles generations and kinship. Every term derives from something and gives rise to something. The picture of the lattice suggests ripples of meaning being transmitted in all possible directions. Each term or *logotopos* - to coin a word from meaning and place - can be strong or weak but each has its role to play in a continuing mutual negotiation. Using the word 'system' for any particular level of the scheme - that is of a

particular value of N, the order or number of terms - we can sketch out some provisional rules. These are some of them:

RULE 1: *Consequence.* Every operation generated at any stage functions in all succeeding stages.

RULE 2: *Generation.* Middle terms of a system level are the children of the previous level's terms.

RULE 3: *Creativity.* The co-operation of the first two rules leads to new rules.

RULE 4: *Similarity.* Terms appearing along any line that can be drawn in the lattice are similar in nature but differ progressively.

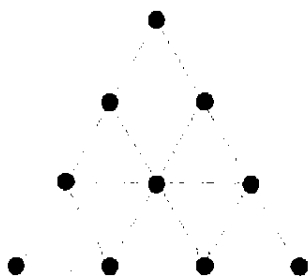
RULE 5: *Structure.* The terms of a system are not all equally connected.

RULE 6: *Depth.* Terms of a system are connected at different depths.

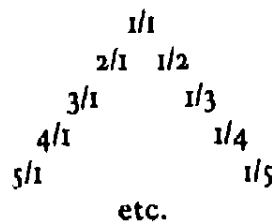
The term 'rule' is a translation of the Greek *nomos*, but this word originally meant a kind of melody in three, five or seven parts. It is useful to bear in mind that there are so-called *nomie games* in which the task is to generate new rules from a given set. Our lattice is not a candidate for Leibnitz's 'universal calculus' in that it does not have fixed rules and, instead of calculation, we have only the pull of creative interpretation in which rules are discovered *after* their emergence. Stabilisation of the meaning of terms is only possible through mutual agreement between people in shared circumstances.

GREAT NATURE

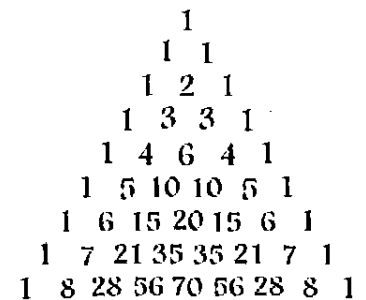
The Lattice form shows resemblances to many well-known forms such as:



Pythagorean Tetraktys



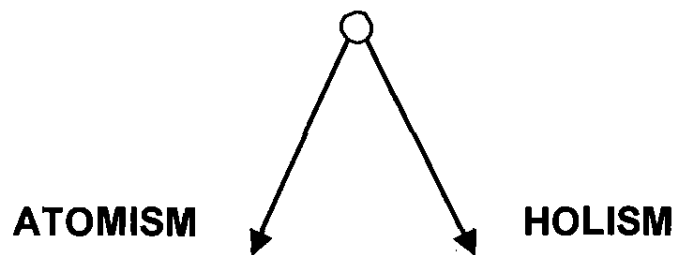
Lambda



Pascal's Triangle

The lambda form of undertones and overtones, that is of fractions and multiples suggests an interesting interpretation of the two directions of the lattice (reversed from those of the lambda figure above) as *atomic* and *holistic*.

WILL TO UNDERSTAND



The left side seeks after the fundamental least particle from which everything else can be constructed by combination. The right side seeks for what is most all-inclusive. Let us cite Anaxagoras again:

Neither is there a smallest part of what is small, but there is always a smaller (for it is impossible that what is should cease to be). Likewise there is always something larger than what is large.

The left has the attitude of separation and favours *locality* while the right has that attitude of togetherness and favours *non-locality*. [Perhaps in Peter Rowland's terms, the left side is the fermion and the right the vacuum.] This is strongly reminiscent of where we began with the theory of the divide of mind between the two hemispheres of the brain. One implication is that the left hand brain/mind identifies with its own will and separateness and *sees the world* just in these terms. People who are centred in the left cannot see any meaning in wholeness or seek to reduce it to some mechanism. Those centred on the right are usually inarticulate about what wholeness means and are unable to provide any practical guides to doing anything about it

The essential duality of reality is not produced by the contingent structure of our brains. Or, at least, that would be only a left brain explanation; just as the right hemisphere would expect duality to arise because *it* sees it as in the nature of all things everywhere. The further depths beyond duality are also then not a simple product of

duality but genuinely new creations. We can say in a colloquial way that these depths arise out of the dialogue between the two sides but that is to fix the two sides as 'objects' when their very meaning, including their structure in various forms, is being 're-calibrated' at every step. To repeat, this is like allowing for not only an iteration of operations according to a given rule or rules but to the operation of new rules. It is a little like playing music in different keys or modes or, rather, different scales.

So also for a presentation (not representation) of these ideas the very form of the lattice can be inverted and rotated, giving different perspectives, and we can go on to see it in three dimensions as a cone, which leads us into more new thoughts. The right brain will cry: 'There is no need for a model just 'eyes to see'' but the left will say: 'I cannot begin to understand unless I create it in my own terms.'

There is a progression of concreteness from equation to natural perception of the world, a higher order version of the sequence point, line, plane and solid that reaches into the mystical perception of the world as the presence of God - or of Matter, as this quote from the 'Mother' who was companion to Sri Aurobindo says:

There is a continuous perception, rendered by the vision, of a multicoloured light, consisting of all colours -- of all colours not in layers, but as if it were an association by dots of all colours. Two years ago...when I met with the Tantrics and got in touch with them, I started seeing this light and I thought it was a "Tantric light", the Tantric way of perceiving the material world. But now I see it constantly, in connection with everything, and it seems to be something that one might call "a perception of real Matter". All possible colours are mutually associated without being mixed, associated in luminous dots. Everything consists of it. And it seems to be the true way of being. I am not sure yet, but it is anyway a much more conscious manner of being. (The Mother speaking to Satprem in 1967)

And since the portions of the great and the small are equal in number, so too all things would be in everything. Nor is it possible that they should exist apart, but all things have a portion of everything. Anaxagoras

All the results issuing from all the cosmic sources great and small, taken together, were also then named by the cherubim the common-

cosmic Ansanbaluiazar; present day objective science also has the formula: Everything issuing from everything and again entering into everything. Gurdjieff

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Neoplatonism, Text and Logic

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o. The purpose of this paper is to expound a possible approach to finding a mathematical model for the the relationship between syntax and semantics. Its method consists in employing the ‘indicative shift,’ an operation on names introduced by Louis Kauffman,¹ to formalize the emanation of ‘Nous,’ or ‘Intellect,’ from the Neoplatonic ‘One,’ and in applying a logic of self-reference to the intrinsic formal structure of the Neoplatonic ‘Nous.’ A mathematical model of the conceptual relationships occurring in Neoplatonic metaphysics can provide a solution to our problem, for it would imply indetermination and codependency, just as is required by a proper analysis of the relationship between syntax and semantics, as we shall see here in more detail.

I. Neoplatonism

1. Neoplatonism is a philosophical doctrine that can be described as a comprehensive outlook of the whole of existing reality. It was originated by Plotinus (ca. 204/5 - 270 CE), who understood it as a reinterpretation of Plato’s thought.² A significant and ‘decisive step’ of Plotinus’ original and enlivening interpretation of Platonic doctrines was his ‘identification of metaphysical realities with states of consciousness.’³ Therefore ‘we may distinguish three aspects of Neoplatonism, which we may term respectively its metaphysical, exegetic and religious or experiential aspects.’⁽²⁾ It is worth noticing, however, that the underlying conceptual structure of the Neoplatonic system remains inherently the same and that its internal relations can therefore be equally and formally deployed in each one of its different discursive domains – namely the metaphysical, hermeneutical, psychological and religious one.

To our purposes, a significant contention, in this respect, is the statement found in an anonymous 6th-century manual of Neoplatonic philosophy, describing, from an exegetical point of view, Plato’s preferred literary form:

the dialogue is a cosmos and the cosmos a dialogue.⁴

What this statement tells us is that the structure of the universe is the same as the structure of a text and that we can legitimately transpose a metaphysical conceptual relationship into a textual context. But more interestingly, this assertion calls into play the relationship between the observed reality and the observer’s point of view as a codependent, self-reflexive and holistic relationship, which recalls John Archibald Wheeler’s notion of a self-observing universe, such as depicted in the famous Wheeler’s

¹ Cf. L. H. Kauffman, ‘Categorical Pairs and the Indicative Shift,’ in *Applied Mathematics and Computation*, 218 : 16 (2012), 7989-8004.

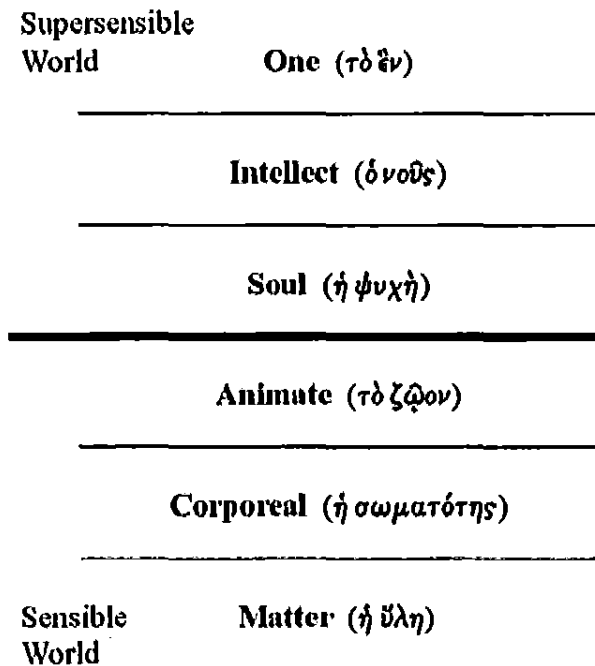
² Plotinus, *Enneades*, V.1.8, 10-14 Henry-Schwyzler: *Καὶ εἶναι τοὺς λόγους τούσδε μὴ καινοὺς μηδὲ νῦν, ἀλλὰ πάλαι μὲν εἰρησθαι μὴ ἀναπεπταμένως, τοὺς δὲ νῦν λόγους ἐξηγητὰς ἐκείνων γεγονέναι μαρτυροῖς πιστωσαμένους τὰς δόξας ταύτας παλαιὰς εἶναι τοῖς αὐτοῦ τοῦ Πλάτωνος γράμμασιν* (And these statements of ours are not new; they do not belong to the present time, but were made long ago, albeit not explicitly, and what we have said in this discussion has been an interpretation of them, relying on Plato’s own writings for evidence that these news are ancient).

³ R. T. Wallis, *Neoplatonism*, London, Duckworth, 1972, p. 5.

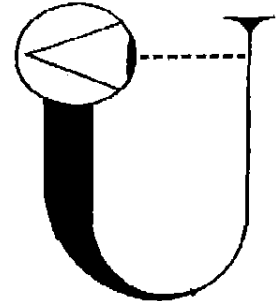
⁴ [Elias ?], *Prolegomena philosophiae Platonicae*, 16.3, p. 31 Westerink.

Eye (Fig. 1). Even if we would not be prepared, with sir Arthur Eddington, 'to accept the view that the substratum of everything is of mental character,'⁵ it should not be so hard to admit of a codependent relationship between the observer and the observed reality.

2. The structure of the whole of reality in the Neoplatonic system can be schematized as in Fig. 2. The chief division is between sensible and supersensible reality, each of which is further subdivided into three hypostases, or essential principles – the One, Intellect, and Soul, for transcendent reality; the Animate, the Corporeal, and Prime Matter, for physical reality. Hypostases are not altogether separate levels of



– Fig. 2 –



– Fig. 1 –

being. On the contrary, 'that the summit of each hypostasis overlaps the hypostasis above' is 'a feature [...] of Neoplatonism.'⁶ The hypostases proceed by emanation one from the other, all originating ultimately from the One. For 'the One is the potency of all things' and 'only from It originate all things that exist.' But 'it has to be said that all that originates from It comes into being without any movement in It,'⁷ and so without any intentional act of creation or exertion of the will. Lower hypostases proceed and detach themselves as an image of the upper neighbouring one by contemplating themselves in it as in their cause.⁸ Hypostases can then be seen in relation to each other as severing themselves from their original unity into two distinctly subsisting entities or, vice

versa, as two distinctly subsisting entities partially overlapping that unite and, from a certain point of view, can be seen as identical in as much as they share a common 'mean term,' or form.⁹ The former relation is a dynamic one, and describes the *procession* of an hypostasis from another, or the production and independent subsistence (*ὑπαρξίς*) of an effect potentially contained in its prior existing cause (Fig. 3). The latter is a static one, and describes the *participation* of two distinct hypostases into a common form, or 'mean term,' that makes them overlap and, with respect to it, in some way identical (Fig. 4). In

⁵ A. S. Eddington, *The Nature of the Physical World*, Cambridge, The University Press, 1928, p. 281.

⁶ A.C. Lloyd, *The Anatomy of Neoplatonism*, Oxford, Clarendon Press, 1990, p. 169.

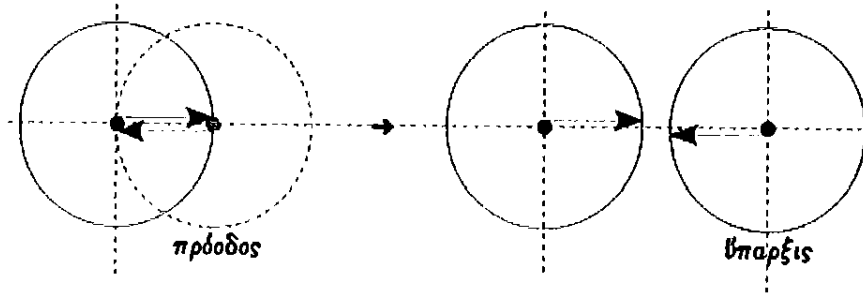
⁷ Plotinus, *Enn.*, V.1.7, 9-10, 22-23 Henry-Schwyzler: τὸ ἐν δύναμις πάντων [...] ἐξ αὐτοῦ δὲ πάντα <ἐν τοῖς οὐσίαις ἀν ἡν>. Ibid., V.1.6, 22-23: Τὸ οὖν γινόμενον ἐκεῖθεν οὐ κινήθειςτος φατέον γίγνεσθαι.

⁸ Ibid., V.1.6, 46-48: Ἀλλὰ ψυχῆς μὲν ἀμυδρὸς ὁ λόγος — ὡς γὰρ εἶδωλον νοῦ — ταύτη καὶ εἰς νοῦν βλέπειν δεῖ νοῦς δὲ ὡσαύτως πρὸς ἐκεῖνον [ἐν], ἵνα ἡ νοῦς (But the thought of the Soul is dim, and being an image of the Intellect, the Soul must look at it, just as the Intellect must look at the One, to become the Intellect).

⁹ 'Iamblichus' Law of Mean Terms' is, according to R. T. Wallis, 'a principle of great importance' in Neoplatonism: 'the essence of the law in question is that two dissimilar terms must be linked by an intermediary having something in common with each of them' (*Neoplatonism*, cit., p. 131).

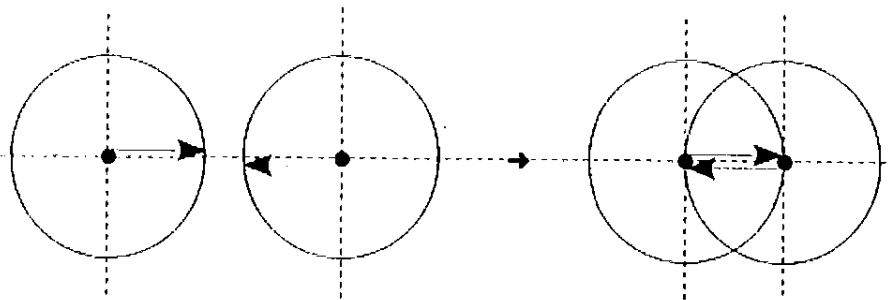
these two figures, we have used the spatial metaphor of the 'overlap' between the hypostases to obtain the diagrams that represent the two cardinal relations that connect

Procession



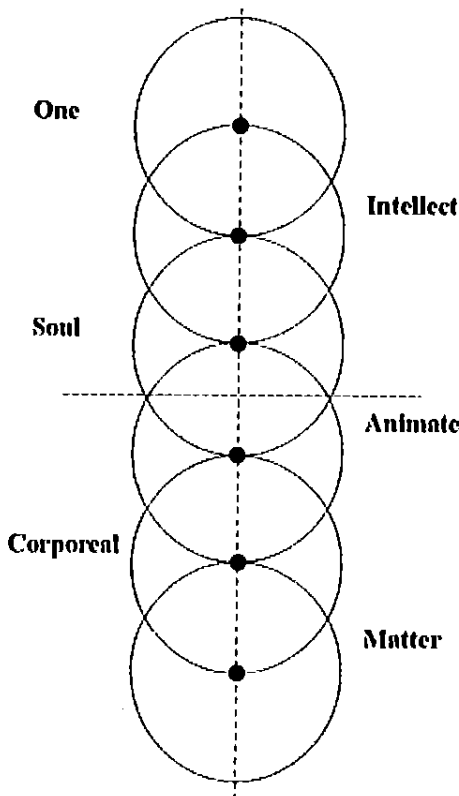
- Fig. 3 -

Participation



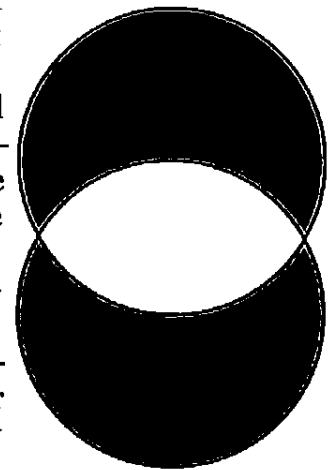
- Fig. 4 -

them together. The same kind of diagram can be used to represent the overall structure of the Neoplatonic metaphysical system, as shown in Fig. 5 below.



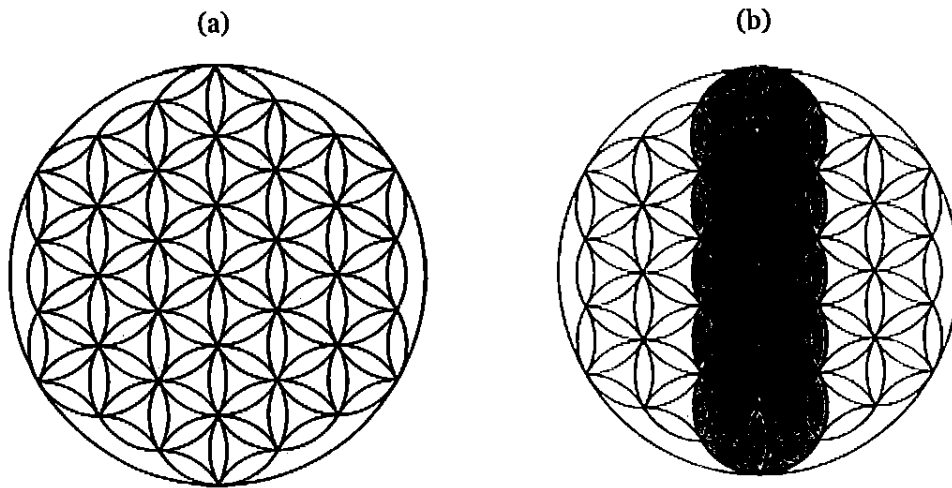
- Fig. 5 -

3. Such a diagram of the metaphysical Neoplatonic system has interesting properties of its own. The shape of the overlapping area is that of the *vesica piscis*, or *mandorla* (Fig. 6), which has since long been made the object of mystical speculation and, among scientists, of persisting beliefs in the geometric underpinnings of the cosmos. The *vesica piscis* is one of the simplest forms of the so-called 'sacred geometry' and one of its elemental symbols. It concurs in the creation of the 'flower of life,' in itself another highly symbolic geometrical construction. Our diagram of the Neoplatonic metaphysics can be easily superimposed upon it, as it can be seen in Fig. 7, below. The following illustrations (Fig. 8) show another remarkable case of arresting superposition of the Neoplatonic scheme with the Kabbalistic 'tree of life,' a

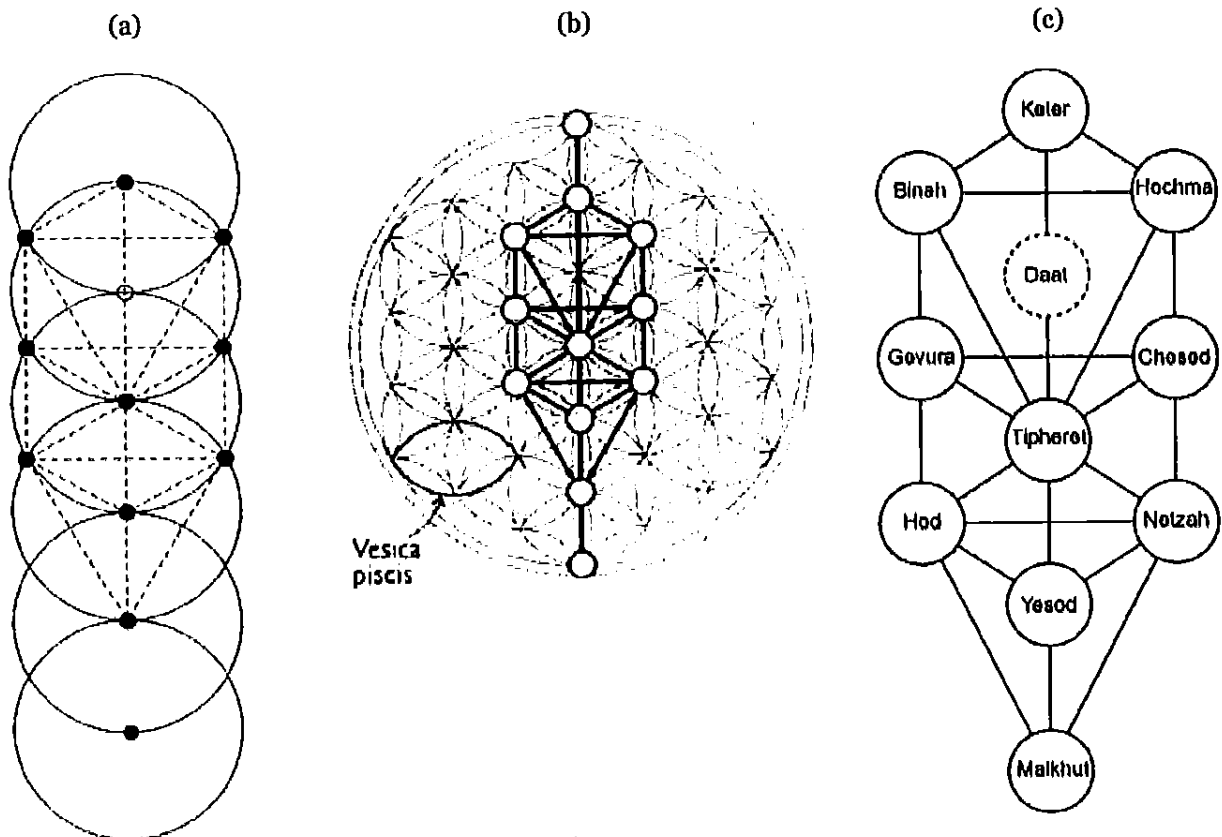


- Fig. 6 -

mystical symbol interpreted as a map of creation and a full model of reality. "The affinity between the Kabbalistic views [...] and the Neoplatonic theology was emphasized by Gershom Scholem, who understood some of its expressions in purely Neoplatonic terms."¹⁰ Some reservations, though, have actually been raised about his 'explicit Platonic proclivity.'¹¹



- Fig. 7 -



- Fig. 8 -

¹⁰ M. Idel, 'Jewish Kabbalah and Platonism in the Middle Ages and Renaissance,' in L. G. Goodman, *Neoplatonism and Jewish Thought*, Albany NY, SUNY Press, 1992, 319-51, pp. 338-39.

¹¹ Id., 'Some Concepts of Time and History in Kabbalah,' in E. Carlebach et al. (eds), *Jewish History and Jewish Memory: Essays in honor of Yosef Hayim Yerushalmi*, Hanover NH, University Press of New England [for] Brandeis University Press, 1998, p. 184.

4. The basic structure of Neoplatonic metaphysics can be further analysed in its constituent elements and relations. The more precise conceptualization of the formal relationships between the hypostases is due to Iamblichus, whose specialized notions and terminology were whole-heartedly accepted by his successors. The essential structure of Iamblichus' metaphysics has been schematized in the following way:¹²

THE BASIC STRUCTURE OF IAMBlichUS' METAPHYSICS

Arrows represent procession or irradiation, > participation by

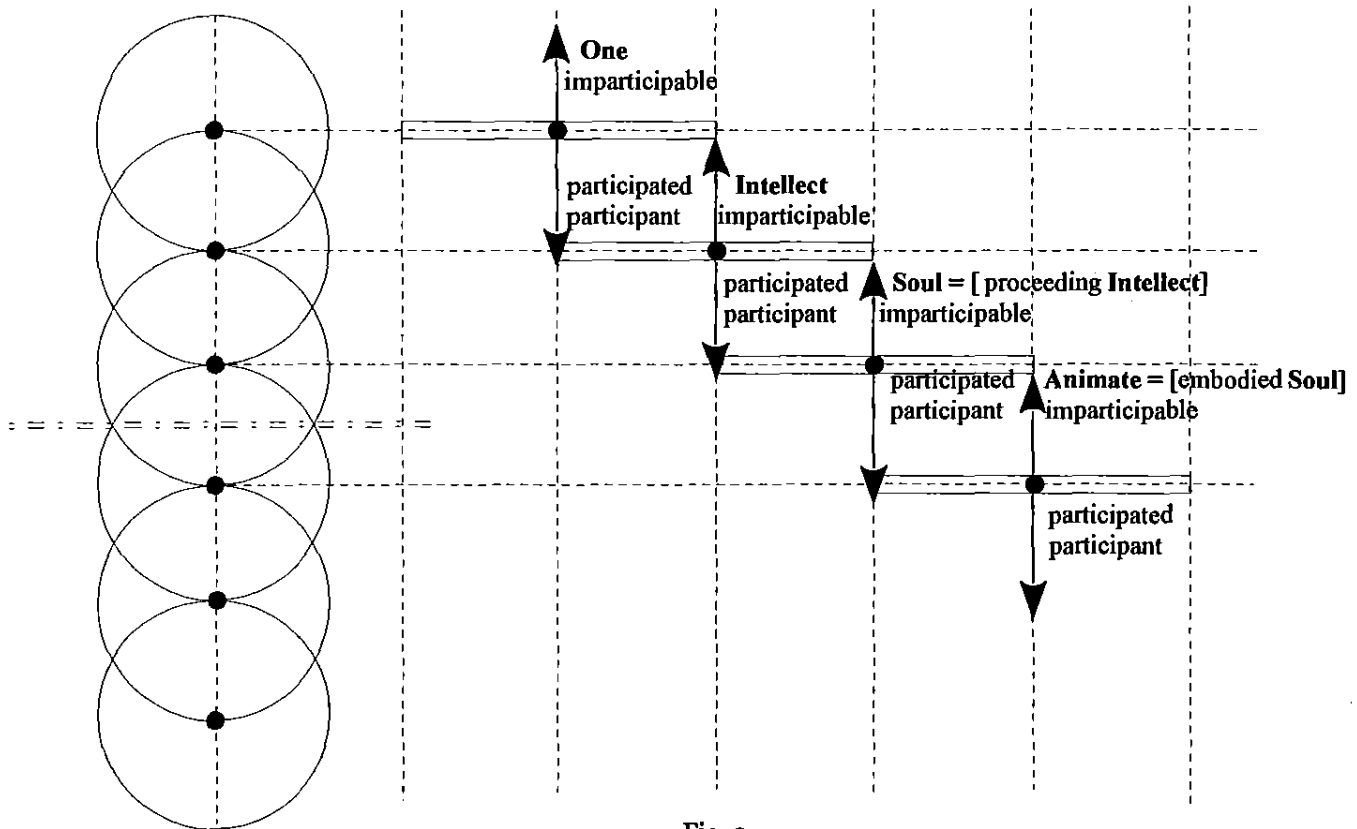
Imparticipable intellect

↓
 ((Participated intellect > Imparticipable soul) = Participant Intellect) → Proceeding intellect

↓
 ((Participated soul > body) = Participant soul) → Proceeding soul

NOTE: This omits the procession of Soul as a hypostasis from Intellect as a hypostasis, which is a complementary point of view representing what Plotinus (VI 2 [43] 23, 26-8) called the *external* activity of Intellect. Here the proceeding intellect is the *intellectus in habitu* and *passibilis* (Simplicius, *In De an.* 311, 29), the proceeding soul the non-rational soul.

According to Iamblichus, hypostases have a non-overlapping, or *imparticipable* part and an overlapping, or *participated* part. In this scheme, the overlapping part of the Intellect is participated by the imparticipable Soul and can be described as *participant Intellect*.

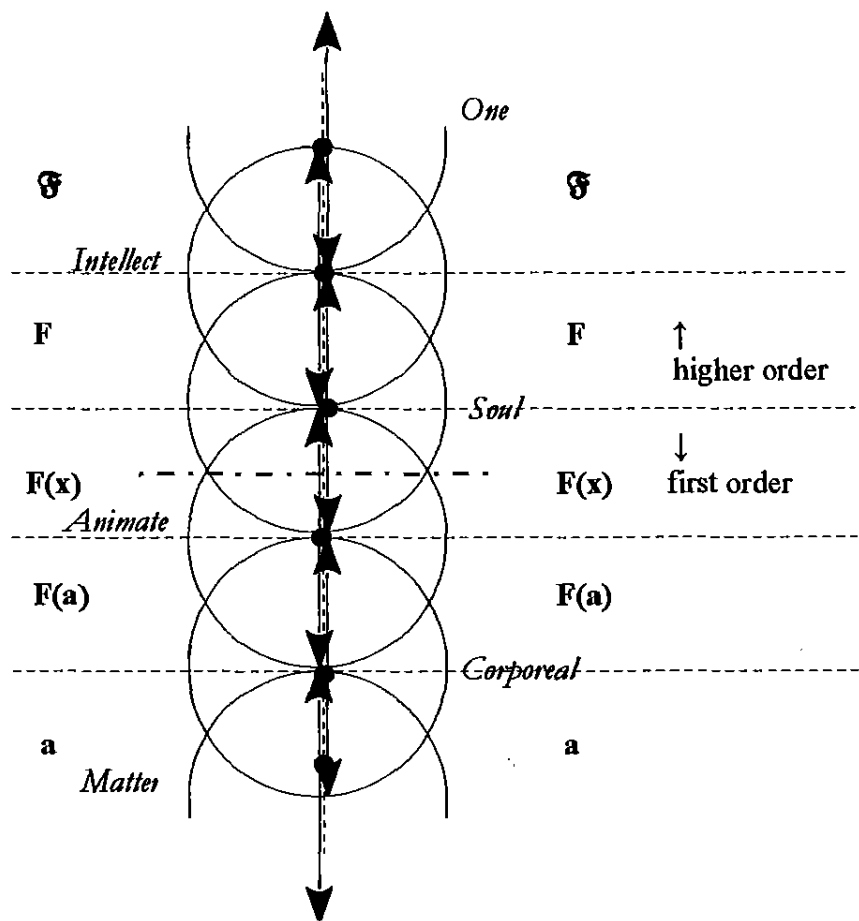


- Fig. 9 -

¹²A. C. Lloyd, 'The Later Neoplatonists,' in *The Cambridge History of Later Greek and Early Medieval Philosophy*, Cambridge, Cambridge University Press, 1967, p. 301.

The imparticipable Intellect proceeds to the participant Intellect, which in turn can be described as *proceeding* Intellect. The participated and participant intellect can be seen as staying, respectively, in a passive or active participation relationship with the Soul. The imparticipable Soul proceeds from the proceeding Intellect, and so on to the lower hypostases. All these relations can be visualised, using our diagrams, as in Fig. 9 .

5. The several hypostases of this hierarchical comprehensive structure of reality requires different logical forms to be described. From a metaphysical point of view, each hypostasis is the subject, or substrate, of ontologically different forms. These forms are properties of their respective hypostases, and can be talked about as their attributes and expressed as predicates of their names. As we have a metaphysical hierarchy of hypostases and their respective forms, we also have a corresponding hierarchy of expressions to describe and talk about them. The appropriate language to describe this metaphysical structure is a



- Fig. 10 -

hierarchical language comprising different logical types. In Fig. 10 we exemplify the various expressions that can be used to symbolize the different semantic categories required to designate each specific ontological entity. As forms proceed from the higher hypostases and can be irradiated throughout to the lower ones, we equally have corresponding expressions used at different levels of predication to designate their matching ontological forms. We can use the standard formal language of first-order logic to designate entities and forms at the level of corporeal things and animated bodies. We need a higher-order language and a second-order form of predication to talk about forms in the imparticipable Soul, and still an upper-order language to express forms and

relations in the Intellect. Transfinite notions seem appropriate to designate forms in the Intellect.

Different logical types are then seemingly required for the expression of forms at different ontological levels. But the hierarchy of types is not open: it ends up at the level of the imparticipable Intellect, or *nous*. We may notice that it works to talk about the lower hypostatic levels and to express discursive thought, but that it does not work to talk about higher hypostatic levels and to express non-discursive thought, such as is required to expose how the imparticipable *nous* actually thinks. So, why is the hierarchy of types not open? Why does it go up only to the third degree, and why does it get barred when it reaches the One?

6. A possible answer can be found by making use of the *indicative shift* operator introduced by Louis Kauffman. This operator is 'a construction for indirect self-reference' that 'formalizes an operation on names that can also be regarded as an *expansion* of a name in the sense that if "A" is the name of A then the expansion E"A" refers to A"A", the result of appending the contents of the name to the name.' If we 'adopt the symbol # for the shift,' and *a* is the name referring to *b*, we have:

$$a \rightarrow b$$

and

$$\# a \rightarrow b a .$$

If we assume that 'A' is the name of A and that the name points to its content, as in

$$'A' \rightarrow A$$

then,

$$\# A \rightarrow A 'A'$$

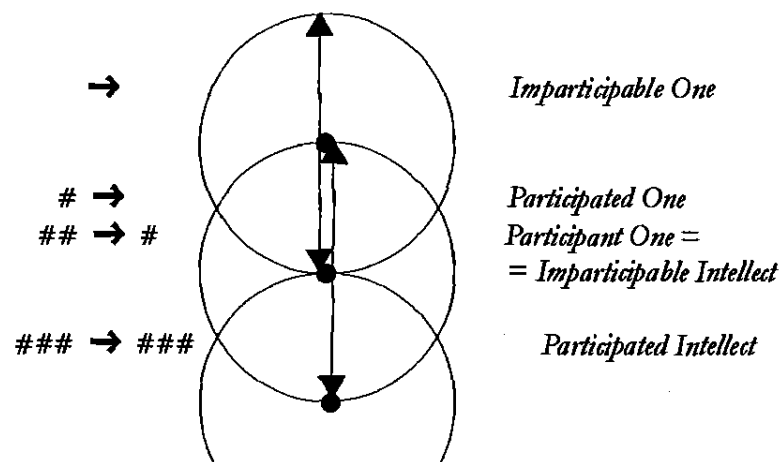
and 'self-reference results when one expands the name of the expansion operator':¹³

$$\# '\#' \rightarrow \# '\#' .$$

We can then use the indicative shift to represent the transition from the void to self-reference, as shown here below:

$$\begin{aligned} & \rightarrow \\ \# & \rightarrow \\ \# \# & \rightarrow \# \\ \# \# \# & \rightarrow \# \# \# . \end{aligned}$$

Now, we can use this derivation to formalize the procession of the Intellect from the One, as in Fig. 11. Here we start with a formula expressing the void, being itself nameless or referred to by no name. This



- Fig. 11 -

formula can be easily interpreted as the Neoplatonic One, utterly unspeakable, referring to itself. The One, conceived of as imparticipable is absolutely transcendent and beyond any kind of ontological or conceptual distinction. In this sense it can be seen as void and deprived of any feature and characterization whatsoever. The next formula can be seen as referring to a *henad*, or to a participated aspect of the One. A henad can be thought of as a

¹³ Kauffman, 'Categorical Pairs,' cit., p. 7989-91.

'mean term,' or an ontological property that allows participation between the One and the Intellect. As a mean term, ontological or conceptual, a henad can be seen as belonging to the One, i.e. as *participated* One, or as belonging to the Intellect, i.e. as Intellect *imparticipable* by the immediately lower hypostasis, the Soul. It is to this double aspect of the henad conceived of, on the one hand, as a property or a predicate of the One, and on the other, as a property or a predicate of the Intellect, that we can respectively refer the second and the third formula, "# → " and "# # → # ", of our formal derivation. As belonging to the One, the henad is 'ineffable,' while on the contrary, as belonging to the Intellect, the henad is altogether 'intelligible.'¹⁴ The double aspect of the henad constitutes the ontological structure that explains the 'overlap' of the two hypostases, the One and the Intellect. In its turn, the last formula refers to the Intellect *participated* by the Soul and expresses its self-referential nature, for the Intellect is at once 'what is thinking (*νοοῦν*) and what is being thought (*νοούμενον*).' The Intellect is a unity that consists of an intrinsic duality, it is 'one and the same, because it is one with itself,' and it 'is two things,' because it is 'thinking' and 'thought'; but these two things 'are simultaneous and exist together,' for they imply each other by being mutually related and by being the same, one referring to the other, and in fact the same thing referring to itself.¹⁵

7. The activity of the Intellect is thought, and it is thought thinking of itself. Thus the Intellect thinks everything at once: 'it thinks not by seeking, but by having,' for 'it has everything' in itself and 'in eternity'; it has no past, nor future, 'it only is, and its "is" is for ever'; it is not in time, like the Soul, and it does not think 'one thing after another,' dwelling on them in time, thus 'letting some things go and attending to others.'¹⁶ Accordingly, the activity of the Intellect can be described as a kind of 'non-discursive' thought.¹⁷

Non-discursive thought, such as the kind of thought Plotinus is concerned with, has been described by Antony Lloyd as 'a type of thought which would be *simple*, that is, contain no complexity.' And from its simplicity 'three properties' are to be deduced: non-discursive thought involves (a) 'no transition from concept to concept'; (b) 'no distinction between the thinker or the thinking on one side and the object of his thinking or the thought on the other side';¹⁸ and (c) 'thinking of everything at once.'⁽²⁶⁷⁾ The lack of distinction between the act and the object of thought, makes it a self-referential kind of thought. But this lack

¹⁴ Proclus, *Institutio theologica*, 162, 1-3 (p. 141, 28-30) Dodds: Πᾶν τὸ καταλάμπων τὸ ὄντως ὄν πλήθος τῶν ἐνάδων κρύφιον καὶ νοητὸν ἐστὶ κρύφιον μὲν ὡς τῷ ἐνὶ συνημμένον, νοητὸν δὲ ὡς ὑπὸ τοῦ ὄντος μετεχόμενον (All those henads which illuminate true Being are secret and intelligible: secret as conjoined with the One, intelligible as participated by Being).

¹⁵ Plotinus, *Enn.*, V.1.4, Henry-Schwyzzer 30-32, 40: Ἄμα μὲν γὰρ ἐκεῖνα καὶ συνυπάρχει καὶ οὐκ ἀπολείπει ἄλληλα, ἀλλὰ δύο ὄντα τοῦτο τὸ ἐν ὁμοῦ [...] νοοῦν καὶ νοούμενον. [...] Ταῦτόν δέ, ἐπεὶ ἐν ἑαυτῷ (For they are simultaneous and exist together and one does not abandon the other, but this one is two things at once [...] what is thinking and what is being thought. [...] But also one and the same, because it is one with itself).

¹⁶ Ibid., V.1.4, Henry-Schwyzzer 16, 17-19, 21-22: νοεῖ δὲ οὐ ζητῶν, ἀλλ' ἔχων. [...] ἐν αἰῶνι πάντα, καὶ ὁ ὄντως αἰὼν [...] τὰ μὲν παριείς, τοῖς δὲ ἐπιβάλλων. Καὶ γὰρ ἄλλα καὶ ἄλλα [...] Ἐχει οὖν <ἐν τῷ αὐτῷ> πάντα [...] καὶ ἔστι μόνον, καὶ τὸ "ἔστιν" αἰεί (it thinks not by seeking, but by having. [...] all things are in eternity, and the true eternity [...] letting some things go and attending to others. And in fact one after another [...] It has therefore everything <in itself> [...] and it only is, and its "is" is for ever).

¹⁷ Cf. A. C. Lloyd, 'Non-Discursive Thought: An enigma of Greek philosophy,' in *Proceedings of the Aristotelian Society*, New Series, 70 (1969-1970), 261-274; and Id., 'Non-propositional Thought in Plotinus,' in *Phronesis*, 31:3 (1986), 258-65.

¹⁸ Id., 'Non-Discursive Thought,' cit., p. 263.

of distinction should not be understood as an absolute identity. It can be conceived of as a *distinctio formalis a parte rei*, a special kind of distinction introduced by Duns Scotus:¹⁹ due to the particular ontological nature of the Intellect, a formal distinction implies the identity of the Intellect with each one of the 'objective forms or formalities' that belong to it and that can be thought of, at the same time, severally and distinctly from it. The kind of identity that holds between the Intellect and its formalities, though, does not apply to other entities ontologically different and non-self-referential, such as perceivable and sensible objects.

The reason for the identity of the Intellect and its forms can be traced back to the circumstance that the object of thought 'is an intentional object—however we wish to analyse that notion.' And the peculiarity of an intentional object is that, contrary to objects that exist in the realm of sensible perception, their essence or act of being is the same as their representative nature or act of representing. An intentional object cannot be identified with 'what is being referred to' by a name 'in the logician's technical sense of "refer";' and 'it is not possible there to say,' dealing with an intentional object, 'what is being thought of, unless we say that it is identical with what is being thought.'²⁰ The act and the object of thought are here identical, and self-reference is implied throughout. But self-reference can be dealt with formally by means of the indicative shift, as Louis Kauffman has convincingly shown: "The completion of the naming process for the process of naming is self-referential. When we refer to ourselves in language we refer to our own ability to make and complete the act of naming."²¹ So the behaviour of an intentional object does not seem so elusive as Antony Lloyd purports it to be and, as he acknowledges too, what we ought to say about it 'must also be distinguished from the alternative of merely denying that there is' in actual fact 'an object' of this kind. Moreover, as Kauffman invites us to do, we can further argue that the notion of the Intellect thinking of itself may be adequately expressed by rephrasing a statement of Heinz von Foerster²² in the following way: 'The Intellect thinking of itself is the thought of relation between the Intellect and its thinking of itself.' Is then non-discursive thought such an 'enigma,' as professor Lloyd described it in the very title of his paper?

This analogy between the Neoplatonic Intellect and von Foerster's self-observing systems is not far-fetched, for as von Foerster reflects on self-observation, Plotinus reminds us that 'we know ourselves [...] by our becoming identical with the Intelligence (*νοῦς*),'²³ by thinking as the Intellect thinks of itself. And

to become *νοῦς* is to acquire self-knowledge, for *νοῦς* not only has self-knowledge, he is self-knowledge [...] This, then, is true self-knowledge: to become *νοῦς χωριστός*, in whom there is no

¹⁹ 'While the *distinctio realis* exists between two really different things, and the *distinctio rationis* multiplies our concepts of one and the same thing, to enable us to consider it from different (*d. rationes cum fundamento in re*) or identical (*d. rationes sine fundamento in re*) standpoints, the *distinctio formalis a parte rei* points, in one and the same individual substance, to the objective forms or formalities that are realized in it, and really in it, *independently of any intellectual act of ours*. Having once established this *distinctio formalis a parte rei*, Scotus makes extensive use of it in his metaphysics' (M. De Wulf, *History of Medieval Philosophy*, 3rd. edn., London, Longmans, 1909, pp. 372-73).

²⁰ Lloyd, 'Non-Discursive Thought,' cit., p. 271.

²¹ Cf. Kauffman, 'Categorical Pairs,' cit.

²² 'I am the observed relation between myself and observing myself' (H. von Foerster, *Understanding Understanding: Essays on cybernetics and cognition*, New York, NY, Springer, 2003, p. 257); and Kauffman comments: 'We encourage the reader to expand further on these themes' ('Categorical pairs,' cit., p. 8003).

²³ Plotinus, *Enn.*, V.3.4, 4-7 Henry-Schwyzzer: *γινώσκωμεν δὲ αὐτοὺς [...] ἐκεῖνο [νοῦς] γινόμενοι.*

difference between knower, object known, and the act of knowledge. [...] When I become that which is self-knowledge I know myself.²⁴

8. But what are the most engaging formal properties of a non-discursive, self-referential form of thought? A first and foremost character is the collapse of the distinction between object language and metalanguage, or between different semantic categories or logical types within a many-tiered object language allowing higher-order predication. Non-discursive thought does not admit of a theory of types. But the collapse of the hierarchy of logical types has a peculiar character. Usually, defenders of first-order logic as the only genuine kind of logic reject higher-order predication and reduce the expressive power of the object language to a single level of assertion, the lowest one. They admit of a metalanguage totally separate from the object language. The semantic and ontological counterparts of this position are an exclusively referential semantics and the rejection of abstract or metaphysical entities. The world is made only of concrete objects and everything else is the result of linguistic constructions, or purely nominal definitions, whose meaning has to be resolved in terms of primarily referring expressions.

The collapse of logical types in non-discursive thought is of a totally different nature. It is a reduction to the top level of the hierarchy, beyond which there is only the totally ineffable and undefinable. The semantic and ontological consequences are completely different. In this language we can only express what we 'think' and not what we 'think of or about' something, unless we identify 'act and object of thought.'²⁵ Objects of thought can be conceived of as 'intentional objects' that cannot be referred to in the ordinary denotational sense, but only conveyed as the meaning or sense – as opposed to reference – of the corresponding expressions. Ontologically, they can be thought of as ideas or metaphysical supersensual entities. We have a sort of collapse to the top level of the logical and ontological hierarchy instead of a collapse to the bottom level, assumed as unique, of existing reality.

The Neoplatonic system, then, admits of two kinds of thought and logic, a discursive and a non-discursive one. The discursive kind of logic, in the Neoplatonic system, comprises a theory of logical types, adequate to the expression of ontological hypostases and forms up to the level of the participated Intellect. The non-discursive kind of logic is suited to the expression of forms and relations at the level of the imparticipable Intellect. And whereas by collapsing to standard first-order logic we can only admit of a purely denotative and referential semantics, a kind of semantics that in a somewhat derogatory way has been labelled the "Fido"-Fido' theory of meaning,²⁶ by adopting Neoplatonic discursive and non-discursive logic we can apply a fully-fledged and comprehensive range of semantical categories.

As we have seen, Plotinian non-discursive thought can be dealt with formally by a logic of self-reference. The discursive *vs* non-discursive opposition seems to vanish, but for the specific formal and operational properties of the respective systems. Just to mention one of these features, we may refer to an aspect of Spencer-Brown's 'calculus of indication,' a

²⁴ Ph. Merlan, *Monopsychism, Mysticism, Metaconsciousness: Problems of the soul in the Neoaristotelian and Neoplatonic tradition*, The Hague, Martinus Nijhoff, 1963, pp. 80-81.

²⁵ Lloyd, 'Non-Discursive Thought,' cit., p. 271.

²⁶ Cf. G. Ryle, 'The Theory of Meaning,' in C.A. Mace (ed), *British Philosophy in Mid-Century: A Cambridge symposium*, London, Allen and Unwin, 1957, pp. 239-64.

'formalism to represent the act of *distinction*.'²⁷ As Louis Kauffman put it, 'self-reference and the idea of distinction are inseparable (hence conceptually identical).'²⁸ In this context, a self-referential process can be conceived of 'as self-indication,' (123) or as 'a form that *reenters* its indicational space,' a form in other words 'that informs itself.' Invoking a geometrical analogy, we may compare self-indication to a Klein bottle, a sort of self-reentering bottle 'where inside and outside become hopelessly confused.' (122) In this calculus, every indication is expressed by 'the same name or token,' a name that can be understood in two ways: either as an operation or 'act of distinction,' or as the 'value' of that very operation. Consequently, 'the only explicit symbol of the calculus,' the mark of distinction, 'acquires a double sense,' (111) and this means that in the calculus of indications 'operators and operands are interchangeable.' This formal property of the calculus leads to a collapse of logical type distinctions. We enter, thus, the 'enchanted land' of 'self-referential forms'²⁹ and, relying on them, we can try to formalize that kind of non-discursive, self-referential thought, that is the proper distinctive character of the Neoplatonic Intellect.

In a few words, it seems as if we could make use of a system endowed with such formal properties to find an adequate way of dealing with the relationship between syntax and semantics as embodied in a text.

II. Text, syntax and semantics

1. According to John Haugeland, a defender of GOFAI (Good Old-Fashioned Artificial Intelligence),³⁰ and an advocate of 'the *sufficiency* of physical symbol systems for producing intelligence,'³¹ there is no difficulty whatsoever in connecting syntactic structures to semantic ones. From this point of view, what is assumed as the guiding principle of a formalist analysis of language is a memorable 'phrase' with a 'memorable name,' the so-called 'Formalists' Motto': 'If you take care of the syntax, the semantics will take care of itself.'³² But the awareness of a more problematic relationship between syntax and semantics comes to the fore just as we deal, quite surprisingly, with the digital representation of a text.

From a computational point of view, text is defined as 'information coded as characters or sequences of characters.'³³ The basic type of data to represent a text thus consists in a string of coded characters. The means to assign a structure to the string of characters representing a text is provided by markup, a technique that can be defined as 'the use of embedded codes, known as tags, to describe a document's structure.' In other words, markup is 'the denotation of specific positions' in a string of characters 'with some

²⁷ F. J. Varela, *Principles of Biological Autonomy*, New York/Oxford, North Holland, 1979, p. 106-07; cf. G. Spencer Brown, *Laws of form*, London, Allen & Unwin, 1969.

²⁸ L. H. Kauffman, 'Self-reference and Recursive Forms,' in *Journal of Social and Biological Structures*, 10:1 (1987), 53-72, p. 53.

²⁹ Id. and F. J. Varela, 'Form Dynamics,' in *Journal of Social and Biological Structures*, 3:2 (1980), 176-206, p. 200.

³⁰ 'What I shall call Good Old-Fashioned Artificial Intelligence—GOFAI for short [...] rests on a particular theory of intelligence and thought—essentially Hobbes idea that ratiocination is *computation*' (J. Haugeland, *Artificial Intelligence: The very idea* [1985], 1st pbk. ed., Cambridge MA, MIT Press, 1989, p. 112.

³¹ A. Newell and H. A. Simon, 'Computer Science as Empirical Inquiry: Symbols and search,' in *Communications of the ACM*, 19:3 (1976), 113-126, p. 118.

³² Haugeland, *Artificial Intelligence*, cit., p. 106.

³³ A. C. Day, *Text Processing*, Cambridge, Cambridge University Press, 1984, p. 1.

assigned tokens.³⁴ Now, recalling a distinction introduced by the Danish linguist Louis Hjelmslev, we may say that the string of characters constitutes the *expression* of a digital text and that the information it conveys constitutes its *content*.³⁵ Accordingly, the syntactic structure of a digital text consists in the structure that the markup assigns to a string of characters by denoting in it some specific positions, whereas its semantic structure 'is not always reducible to character positions' within the text. Thus, the 'formalists' motto, that rests on the assumption of a one-to-one correspondence between syntactic and semantic structure, clearly breaks down. So, the digital representation of the text forces us to give up this very simplistic assumption about the relationship between syntax and semantics and to ponder more heedfully on a general fact that characterizes this relation.

2. Language, in general, admits of synonymous and polysemic expressions. Synonymy can be defined as 'more than one form having the same meaning,' and polysemy as 'the same form having more than one meaning.'³⁶ The same meaning can be expressed in more than one way, just as the same expression can be interpreted differently and can be assigned more than one meaning. If you take a decision about the syntax and single out an expression, its meaning remains indeterminate and the choice among a number of possible interpretations is left completely open. On the other hand, if you take a decision about the semantics and fix the import of what you want to say, the way it can be expressed remains indeterminate and the choice among a number of possible expressions is left equally open. The result of a choice is an act of distinction, either about the syntax or about the semantics, and as soon as it is done, the process goes on on the other side. So, the relation between the expression and the content or, for that matter, between syntax and semantics, is a codependent indetermination relationship, and in order to relate them formally, a suitable model of this kind of relationship has to be found.

Now, as it happens, the markup can be exploited precisely to analyse this indetermination relationship. Markup expressions behave as diacritics³⁷ and as diacritical expressions can exert an inherent self-referential function, simply by acting as a discriminating mark, that exhibits self-reflexively what can only be shown and cannot be said in the object language. In their self-referential capacity, markup expressions are ambiguous and ambivalent. They are in themselves particular expressions of the text, and belong to it, but at the same time they are endowed with a kind of self-describing metalinguistic force, that relates some semiotic elements of the text to other semiotic elements of the same text. They can thus be considered as an instance of 'reentrant forms,' capable of being considered as a value, or a form of expression, on the one hand, and as a 'prescription,'³⁸ or a rule, acting on these very values or expressions on the other: taken as values, or forms of expression, they belong to the text; on the other hand, taken as operators, they can be understood as rules, expressed in the object language, to draw inferences about specific elements of the expression, or the content of the text.

³⁴ D. R. Raymond et al., 'Markup Reconsidered,' Paper presented at the First International Workshop on Principles of Document Processing, Washington, D.C., October 22-23, 1992, <http://www.darrellraymond.com/markup.pdf> (accessed 2 May 2012).

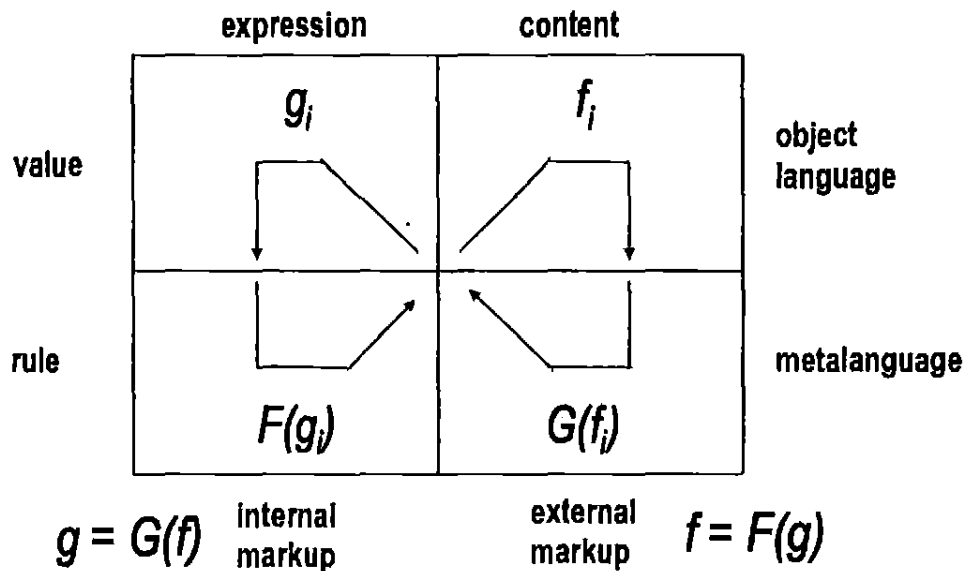
³⁵ See L. Hjelmslev, *Prolegomena to a Theory of Language*, tr. F. J. Whitfield, rev. Engl. ed., Madison WI, The University of Wisconsin Press, 1961.

³⁶ G. N. Leech, *Semantics: The Study of Meaning*, Harmondsworth, Penguin Books, 1974, pp. 101-102.

³⁷ Cf. D. Buzzetti, *Diacritical Ambiguity and Markup*, in Id., G. Pancaldi, and H. Short (eds), *Augmenting Comprehension: Digital Tools and the History of Ideas*, London-Oxford, Office for Humanities Communication, 2004, pp. 175-18; and Id., *Digital Editions and Text Processing*, in M. Deegan and K. Sutherland (eds), *Text Editing, Print, and the Digital World*, Aldershot, Ashgate, 2009, pp. 45-62.

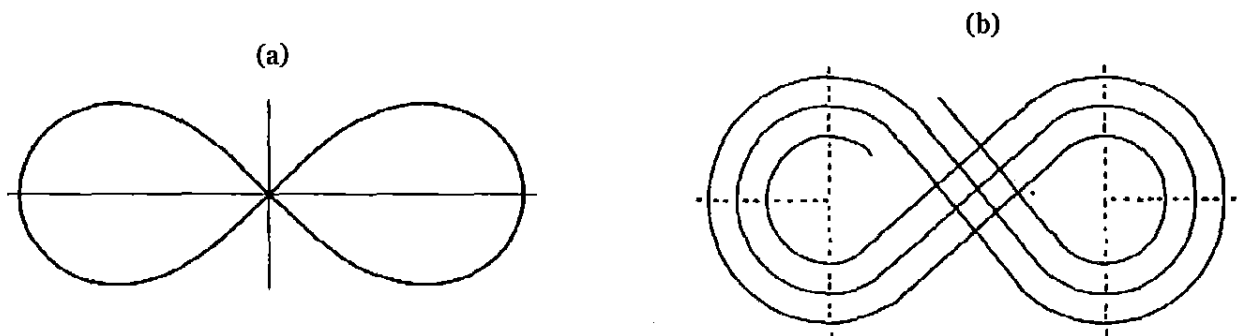
³⁸ Varela, *Principles of Biological Autonomy*, cit., p. 124.

Based on these observations, a 'dynamic model' of the relationship between syntactic and semantic structures of a text can be devised. A first attempt to obtain such a model can be described by means of a diagram (Fig. 12):³⁹



- Fig. 12 -

This diagram can be thought of as a kind of multidimensional matrix, whose elements are connected by a series of operations. The process resulting from this series of operations is a kind of loop, or continuing spiral, as shown in Fig. 13.



- Fig. 13 -

3. But this model is not complete and it can be improved by taking into account some essential Neoplatonic insights. As we have seen, the basic formal structure of the Neoplatonic system holds in each of its specific discursive domains. Plotinus himself is aware of this basic aspect of his theoretical construction. Referring to the three primary hypostases – One, Intellect, and Soul – he affirms explicitly: 'And just as in nature there are these three degrees, so we ought to think that they are present also within ourselves.'⁴⁰ There is, then, an essential homology between the metaphysical and the psychological

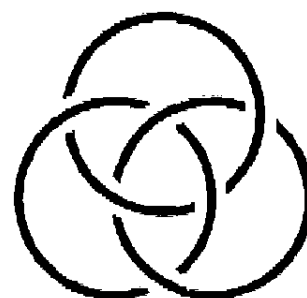
³⁹ For a more detailed description, see D. Buzzetti, *Digital Text Representation: Expression and Content*, in A. D. Ford (ed), *Contexts: Proceedings of ANPA 31*, [London], ANPA, 2011, 124-145, pp. 134-36.

⁴⁰ Plotinus, *Enn.*, V.1.10, 5-6 Henry-Schwyzler: "Ὡσπερ δὲ ἐν τῇ φύσει τριτὰ ταῦτά ἐστι τὰ εἰρημένα, οὕτω χρὴ νομίζειν καὶ παρ' ἡμῖν ταῦτα εἶναι."

structure. And also in the religious and hermeneutic domains we can find the same structural arrangement. The overall formal structure of the system is holistic and all its structural components are codependent. The same kind of codependency holds, therefore, among the specific elements and notions of all its different domains. The very same structural homology can be extended to textuality: the hermeneutic domain is clearly textual, and the isomorphism between the metaphysical and the textual realms is expressly stated in the maxim ‘The dialogue is a cosmos and the cosmos a dialogue.’⁴¹ Apparently, then, we can rely on a model gathered from the formal structure of the Neoplatonic system to analyse the indetermination relationship that holds between textual syntactic and semantic structures.

Now, the relationship between syntax and semantics cannot be properly understood without taking into account, as Peirce reminds us, a ‘triadic relation’⁴² between a sign, the object it refers to, and an interpreter. According to Peirce, ‘a sign endeavours to represent, in part at least, an Object [...] but to say that it represents its Object implies that it affects a mind’⁴³ – or a mind-like interpreting system anyway. This means that a ‘sign’ or, more generally, what he calls a ‘*representamen*,’ is ‘a subject of a triadic relation to a second, called its *object*, for a third, called its *interpretant*.’ In this sense, the meaning of ‘the word *representation*’ is confined to ‘the operation of a sign,’ or to ‘its *relation* to the object for the interpreter of the representation.’⁴⁴ An adequate model of the relationship between syntax and semantics should therefore take into account Peirce’s ‘thirdness,’ which ‘is what it is, owing to things between which it mediates and which it brings into relation to each other.’⁴⁵ A third, or an *interpretant*, then, ‘has a mode of being which consists in the Secondness,’ or the *object*, ‘that it determines’ for a *representamen* or a *sign*.⁴⁶ It is worth noticing, though, that a Peircean interpretant is not an interpreter, but rather the sense made of the sign. So, with a more familiar terminology, we may speak of the *sign*, or the information carrier, the *sense* made of the sign, and the *reference* the sign stands for, as the three necessary elements of a semiotic relation.

The sign, the object, and the interpretant, as conceived by Peirce, are strictly connected and interlaced, just as the ‘symbolic,’ the ‘real,’ and the ‘imaginary’ orders defined by Lacan,⁴⁷ with which they can be easily aligned. The close interconnection ‘of the real, the symbolic and the imaginary,’ as exposed by Lacan ‘in their basic circularity,’ is ‘presented by means of the Borromean knot,’ which ‘defines itself as the way in which we *imagine* the *real* effect of the *symbolic*.’⁴⁸ (Fig. 14). The Borromean ‘knot,’ though, is not properly a knot, but rather an interlacing of three separate rings. Actually, a better representation of this interconnection can be provided by a trefoil knot, as shown in Fig. 15.



– Fig. 14 –

We can try to interpret the trefoil knot, that can be assumed as an adequate model of the threefold relationship between sign, sense, and reference, from a Neoplatonic point of

⁴¹ See above, note 4.

⁴² Ch. S. Peirce, ‘A Syllabus of Certain Topics of Logic,’ EP 2:272-3, 1903.

⁴³ Id., ‘Some Amazing Mazes, Fourth Curiosity,’ CP 6.347, c. 1909.

⁴⁴ Id., *Lowell Lectures*, CP 1.540-542, 1903.

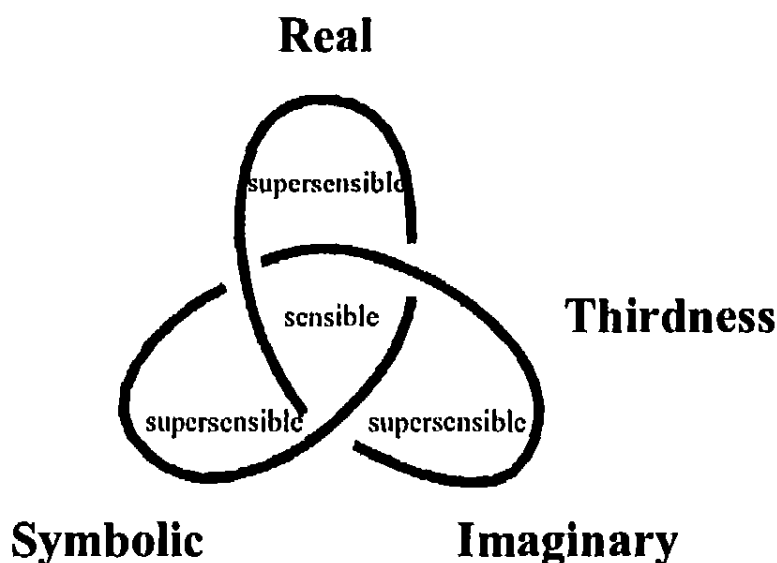
⁴⁵ Id., ‘A Guess at the Riddle,’ CP 1.356, c. 1890.

⁴⁶ Id., *Lowell Lectures*, CP 1.536-537, 1903.

⁴⁷ Cf. J. Lacan, *Les quatre concepts fondamentaux de la psychanalyse: 1964*, Paris, Éditions du Seuil, 1973.

⁴⁸ Ph. Julien, *Pour Lire Jacques Lacan*, 2.e éd., Paris, E.P.E.L., 1990, pp. 212, 221.

view. In this diagram each loop can be understood as a specific discursive domain that represents, respectively, the symbolic, the real and the imaginary order – that correspond,



– Fig. 15 –

in turn, to Peirce's notions of sign, object and interpretant. Each loop encloses two distinct areas, one of them overlapping with the corresponding area of the other loops, and another one kept apart from them. The overlapping area can be thought of respectively as the sensible component of the sign, the object, or the interpretant. The sensible component of a sign is its physical nature; that of an object is its body, or the substrate where a formal or metaphysical object inheres as a form; and, for the interpretant, usually the cerebral state of the person who refers a sign to an object by means of its sense. The other and distinct area can be understood as the abstract or supersensual component of the sign, of the object, or of the interpretant respectively. These components can be conceived of as the specific and distinctive abstract forms that the sign, the object and the interpretant must possess to be put in the triadic relation that characterizes them as the three distinct elements of a semiotic relation. The advantage of the Neoplatonic perspective is that each of these three semiotic elements can be dealt with as a functional part of a formal structure endowed with highly expedient expressive and representational properties capable of representing in a suitable way the interconnection of the three semiotic elements and the indetermination of their relationship.

The traditional Neoplatonic semiotics matches this model. Its fundamental elements are *words* (*φωναί, λέξεις, ὀνόματα*), *things* (*πράγματα*), and *concepts* or *thoughts* (*ἔννοιαι*, as existing in the Soul, and *νοήματα*, as existing in the Intellect). As it can be seen, these elements correspond respectively to the three basic orders of our model, namely the symbolic, the real and the imaginary, or the sign, the object and the interpretant, or sense. The interconnection of these three orders is explicitly stated by Simplicius, who insists on the unifying function of the Intellect:

For neither are significant expressions (*λέξεις*) wholly separate from the nature of beings, nor are beings detached from the names (*ὀνόματα*) which are naturally suited to signify them. Nor, finally, are intellectual concepts (*νοήματα*) extraneous to the nature of the other two; for these three things were previously one, and became differentiated later. For Intellect (*νοῦς*), being

identical with realities and with intellection (*νόησις*), possesses as one both beings and the intellectual concepts of them, by virtue of its undifferentiated unity (*ἀδιάκριτος ἕνωσις*), and there [sc. in the intelligible world] there is no need for language.⁴⁹

Moreover, Neoplatonism provides a consistent hermeneutical theory based on a fundamental holistic principle stated by Proclus in this metaphysical form: 'All things are in all things, but in each according to its proper nature.'⁵⁰ Transferred in the hermeneutic domain, this principle lays the foundations of Iamblichus' theory of *skopos* (*σκοπός*), according to which all exegesis is governed by the question: 'What is the 'aim' of a work, its intention? (*skopos*)' Being a reflection of the whole cosmos, a text speaks of everything, but in relation to what it wants to deal with, and thus constitutes its particular aim, or *skopos*. So, in a text, or in an expression of 'the human language proceeding from incarnated souls,' the 'words' (*λέξεις* or *φωναί*) signify, 'through mediation of simple and universal "notions" (*ἔννοιαι*) that are in the Soul and that coincide with the *signifieds* of these words,' the intelligible 'realities' (*πράγματα*)⁵¹ that exist eternally and unchangeably in the Intellect, and are identical with their own intellectual apprehension. An interpretation practice based on this primary hermeneutic principle proceeds in absolute conformity with a codependent and holistic point of view.

A model of the textual relation between syntax and semantics based on these principles can account for a consistent set of codependent, holistic and self-referential structural relations. Could an appropriate logic of self-reference provide a viable means to formalize a model of this kind?

⁴⁹ Simplicius, *In Categorías*, 12, 13-19 Kalbfleisch: οὔτε τῶν σημαντικῶν λέξεων πάντη κεχωρισμένων τῆς τῶν ὄντων φύσεως οὔτε τῶν ὄντων ἀπηρητημένων τῶν σημαίνειν αὐτὰ πεφυκότων ὀνομάτων, ἀλλ' οὐδὲ τῶν νοημάτων ἔξω τῆς ἀμφοῖν ὄντων φύσεως· ἐν γὰρ ὄντα πρότερον τὰ τρία ταῦτα διεκρίθησαν ὕστερον. ὁ μὲν γὰρ νοῦς αὐτὰ τὰ πράγματα ὧν καὶ αὐτὴ ἡ νόησις ταῦτ' ἔχει τὰ τε ὄντα καὶ τὰ τῶν ὄντων νοήματα διὰ τὴν ἀδιάκριτον ἕνωσιν, καὶ φωνῆς ἐκεῖ οὐδὲν χρεῖα.

⁵⁰ Proclus, *Institutio theologica*, 103.1 (p. 92, 13) Dodds: Πάντα ἐν πᾶσιν, οἰκείως δὲ ἐν ἑκάστῳ.

⁵¹ Ph. Hoffmann, 'What Was Commentary in Late Antiquity? The Example of the Neoplatonic Commentators,' in M. L. Gill and P. Pellegrin, *A Companion to Ancient Philosophy*, Oxford/Malden MA, Wiley-Blackwell, 2009, p. 611.

Ethereal Thoughts

Geoffrey Constable

26 January 2012

Abstract

It is argued in this paper that a 'quantum soup' of charged virtual particles possesses mechanical properties, and should more accurately be referred to as 'quantum jelly'. As many virtual particles possess mass, it is argued further that quantum jelly should be capable of transmitting waves, that these might be electromagnetic (EM) waves, and that quantum jelly might be the medium (i.e. the ether) that permits such waves to exist and propagate.

It is also proposed that quantum jelly should be capable of transmitting both transverse and longitudinal EM waves. Using the assumptions that quantum jelly is perfectly compressible and that the transverse waves are electromagnetic and travel at speed 'c', such longitudinal EM waves should travel faster at speed $c\sqrt{2}$.

If there is no evidence of longitudinal EM waves (and hence no knowledge of their speed of propagation), Maxwell's Equations remain unchallenged. If longitudinal EM waves do exist, and travel at $c\sqrt{2}$, Maxwell's equations as presently stated may have to be revisited.

It is concluded that the existence of longitudinal EM waves is 'highly probable' and that quantum jelly might be the ether in which EM waves exist.

Introduction

This paper is part of an extended series, presented to ANPA over a period of some 20 years, aimed at reconsidering some of the fundamental assumptions on which science relies. One such assumption is that, while some variables are known to be quantised and, therefore, discontinuous, others are continuous and thus infinitely variable. The hypothesis that the latter is not so and that all variables are quantised in one way or another - the universe behaving as a digital (not an analogue) system - was presented at ANPA 30 through the paper 'A Digital Universe'.

Background

This hypothesis has far-reaching implications. One is that some particles with half-integral spin have extended, probabilistic, structures. An electron, for example, is not always at its exact mean position – a view most physicists would accept. It may occasionally be very distant, the probability of the electron's presence at a particular point being inversely proportional to the distance between that point and its mean position.

This outcome leads to a further implication, namely that the forces of electrostatics and magnetism (and, incidentally, gravity as well) are explained by interaction between particles with extended structures (ANPA 18 'An Investigation, Arising from the Theory of Quantised Variables, into Inter-particle Forces').

A consequence of this implication is that the concept of a 'field' is no longer needed to explain 'action at a distance'. Fields are models that enable certain forces to be calculated with accuracy but do not explain their existence.

This view immediately encounters a difficulty. It is well known that electromagnetic (EM) waves can be transmitted and that such a phenomenon is

explained by Maxwell's equations. These equations depend upon the existence of electrostatic and electromagnetic fields and their derivatives and, if the concept of a field is flawed, why do such waves exist? This question was a stimulus for the arguments that follow.

Quantum Soup

The concept of 'virtual particles' is now an accepted part of modern physics. Such particles flash in and out of existence and, in particular, are present almost universally, even in a high vacuum. Such particles are often described collectively as 'Quantum Soup'.

As said above, many particles with half-integral spin have uncertainty concerning position. Thus, when a point distant from a mean particle position is viewed, the particle in question appears to flash in and out of existence and can, therefore, be considered at that point to be virtual. As there are very many, say, electrons in the universe, any element of space will contain a large number of virtual electrons.

Interestingly, the largest contribution to such virtual particles will arise from electrons that are the most distant because, while the probability of the presence of such a virtual particle varies inversely with the distance to its mean position, the area of the radiating shell (that contains such mean positions) varies directly as the square of this distance.

It seems reasonable to assume, therefore, that virtual particles from the very distant parts of the universe are the predominant contributors to Quantum Soup, which is consistent with the fact that the quality of Quantum Soup varies little with the proximity of massive bodies such as planets or stars.

(Such an argument might help to explain a puzzling phenomenon which so far remains a mystery. It is well-known that linear velocity is relative, not absolute. However, angular velocity is absolute – the datum for measuring absolute zero rotation being provided, apparently, by the most distant fixed stars of the universe. As rotation can be measured instantly, any form of communication (to measure rotation) between a rotating body and these fixed stars seems unlikely. Hence, some manifestation of the most distant stars must, in one way or another, be present everywhere where rotation is possible in order to provide a universal and immediate means of measuring rotation. The proposed quantum soup of virtual particles could provide such a means.)

Quantum Jelly

Virtual electrons repel each other so, when contained as in quantum soup, can be expected to form a lattice-type structure. Displacement of one virtual electron will give rise to an appropriate restoring force and cause a disturbance to its neighbours. Thus, the lattice will behave as a solid with mechanical properties – including the possession of an equivalent to Young's Modulus. Quantum Soup may therefore be a misnomer – Quantum Jelly being a more appropriate term.

As virtual electrons have mass, such a jelly should be able to transmit waves. Because this jelly should be homogeneous and isotropic, both transverse and longitudinal waves should be possible. But what are these waves, and are they already known to science? The obvious first guess is that these are electromagnetic waves – already very well-known to science – and that quantum jelly is the long-sought-after ether that permits such waves to propagate.

The Michelson-Morley Experiment

This famous experiment was conducted in 1887 and was aimed at detecting the presence of an ether. Monochromatic light from a single source was split into two beams at right angles to each other. Using pairs of mirrors, each beam travelled several meters before being recombined with the other to produce interference fringes. All this was mounted on a rotatable table, so that the entire apparatus could be turned about a vertical axis.

As Earth travels through space, it was reasoned, the presence of an ether would create an 'ether wind'. Rotating the table would cause a change in the effective distance travelled through the ether by one beam as compared with the other, and would cause a shift in the observed interference pattern. During many tests no such shift was observed. It was concluded there was no ether wind and, therefore, no ether – a view that has continued until the present day. Plainly, such a result has to be accounted for if the idea of linking quantum jelly to the existence of an ether is to progress.

A Velocity-insensitive Ether

The Michelson-Morley finding that no ether wind could be detected is firm and cannot be challenged. But, does that mean for certain that there is no ether? What if the ether is of a form that is insensitive to relative velocity? (Such a suggestion may not be quite as strange as it sounds. After all, special relativity depends on the premise that the observed speed of light remains constant irrespective of relative velocity between different frames of reference. Furthermore, that linear velocity cannot be measured relative to an absolute datum – for example, an ether – is a further indication that a velocity-insensitive ether may exist).

At this point, two examples from nature may prove helpful. For the first, visualise a skier on a mountain slope. There is a blizzard so the skier can see only two meters, say, in any direction. The skier stops but, in doing so, starts an avalanche. Looking down, the skier sees he is stationary with respect to the snow. However, he knows he is moving because the snowflakes that earlier were drifting past his nose are now whistling past his ears.

If in this example the snowflakes represent an ether, movement through it can be detected because the flow of snowflakes (the elements of the ether) can be observed.

Second, however, visualise a midnight river trip in the tropics to enjoy the display of fireflies. The night is dark (no moon or stars) and the river is wide (banks invisible). Drifting down stream, no motion can be detected relative to the water. The swarm of fireflies puts on a dazzling display, each insect emitting sporadic, bright, pulses (100msec) of bioluminescence. In this case the observer (unaided by sophisticated measuring equipment) cannot detect motion through the swarm of fireflies – the flashes of light are far too brief and the motion of the fireflies too swift and erratic.

If, in this case, the swarm of fireflies represents the elements of an ether, movement through it cannot be observed for the reasons given above.

In this respect, the flashing fireflies are similar to virtual particles that flash in and out of existence. However, virtual particles flash very much more quickly – so quickly, in fact, that the flash is probably much too brief, and motion of the particles too uncertain, for motion relative to a group of virtual particles to be detected by experiments such as that made by Michelson/Morley.

Thus, the concept of an ether that transmits EM waves, but does not yield an ether wind, is not one to be casually dismissed.

Longitudinal EM Waves

At this point in the argument, it may help to consider a third analogy from nature. It is well known that seismic waves, as produced by earthquakes, have two forms: transverse and longitudinal. The transverse form involves side to side displacement of the medium (i.e. at right angles to the direction of propagation of the wave) and the longitudinal form involves displacement of the medium parallel to the direction of propagation. For this reason, longitudinal waves are often referred to as 'pressure waves'.

In an earthquake, for example, the longitudinal waves arrive first and rock 'fore and aft' while the transverse waves arrive second and rock 'port to starboard'.

The speeds of these two waves are given by the following equations

$$C_L^2 = \frac{E(1-\nu)}{\rho_0(1+\nu)(1-2\nu)} \quad (1)$$

$$C_S^2 = \frac{E}{\rho_0(1+\nu)} \frac{1}{2} \quad (2)$$

So,

$$\frac{C_L}{C_S} = \sqrt{\frac{(2-2\nu)}{(1-2\nu)}} \quad (3)$$

Where

C_L is the speed of the longitudinal wave

C_S is the speed of the transverse wave

E is Young's Modulus

ρ_0 is the mass density

ν is Poisson's Ratio

Taking the case of a solid composed of virtual particles, it seems reasonable to guess that such a solid is 'perfectly compressible' and, therefore, that Poisson's Ratio is zero. Using equation (3),

$$C_L = \sqrt{2}C_S$$

If the waves being considered are electro-magnetic, C_S (the transverse wave) will correspond to 'c', the speed of light. By simple deduction, therefore, any longitudinal EM wave can be expected to travel faster, at speed $c\sqrt{2}$.

Thus, there are two simple tests of whether the ideas put forward so far have any substance. Do longitudinal EM waves exist and, if so, do they travel faster than light by a factor of the square root of two?

If experimental evidence indicates that the answers in both cases are 'No', Maxwell's equations remain unchallenged.

If experimental evidence indicates that the answers are 'Yes', the validity of Maxwell's equations as at present stated may have to be reconsidered.

Nikola Tesla

Any literature search into longitudinal EM waves will swiftly arrive at references to this remarkable American scientist, electrical engineer, and genius. Born in Serbia, Tesla acquired a great interest in electricity, and soon moved to New York to work for Edison. After a brief period, he set himself up as an independent consultant and advised Westinghouse on how to overcome the limitations of DC power transmission by the introduction of poly-phase (in practice three phase) AC transmission systems – an achievement for which he

is probably best remembered. (See 'The Complete Patents of Nikola Tesla' edited by Jim Glenn 1994 ISBN 1 – 56619 – 266 – 8).

Tesla then became interested in the radiation that arose from the repeated switching of high voltage DC e.g. that produced by a high voltage dynamo, a small capacitor storage bank and a hand operated knife blade shorting switch. This radiation seemed to differ from that created by oscillating currents (e.g. radio waves) and had the following characteristics.

- 1) The switch operator was struck by a pressure blast plus sensations of needle-like penetrations.
- 2) The shorting wire vaporised explosively with an impulse resembling lightning.
- 3) When protected by a glass plate, the switch operator still experienced the needle-like effects – i.e. effects not due to emitted particles.
- 4) These effects were not produced by shorting AC voltages, hence effects due to impulses, not alternations.
- 5) No explosion took place when capacitor shorting was via longer, more resistive leads, but pressure blast plus needle-like effects persisted, hence effects not due to explosion.
- 6) When insulation to earth was improved for all parts of the apparatus, the effects were unchanged, hence effects not due to any shorting to earth.

This experiment was repeated with the manually-operated knife blade switch replaced by a remotely-controlled rotary spark switch, thereby enabling these effects to be examined at greater distances. The following results were noted.

- 1) The effect intensified – due to repeated pulses – and did not diminish with distance from observer to apparatus.

- 2) Radiation appeared to travel in straight lines.
- 3) Shock effects were felt far beyond the termination of visible sparks. The dynamo generated just 15,000 volts but the extent of shock effects indicated far higher voltages. Hence input voltage was being transformed into a higher voltage.
- 4) When a large copper shield was constructed to protect the observer, effects were undiminished. Hence, the observer is not protected by a 'Faraday Cage'.
- 5) Input leads were immersed in mineral oil – effects were undiminished. Hence, effects do not depend upon the presence of air.
- 6) One lead of a galvanometer was connected to a copper plate and the other was grounded. The effects caused a direct current to flow from the plate to ground.
- 7) The rays neither diminished with the square of the distance nor the inverse of the distance from their source. They seemed to spread out in a progressive shock shell without any apparent loss.

At this point Tesla concluded the following.

'The rays are new, light-like in that they travel in straight lines, but are sound-like and thus longitudinal.' (Presumably, the observation as to the rays being 'sound-like' arose from the observed pressure blast; given this, the observation as to the rays being longitudinal waves follows automatically.)

Further experiments were made when the rotary park switch was replaced by a 'heavy duty magnetic arc switch', immersed in mineral oil. This switch was introduced to facilitate high repetition pulse rates and measures were taken to

eliminate any oscillations after switching. This change enabled the following observations to be made.

- 1) The needle effect was not now present although the pressure effect remained.
- 2) At shorter pulse lengths, the pressure effect disappeared.
- 3) Metal objects showed a great variety of 'white fluidic discharges' and sometimes displayed actual motion.
- 4) Copper cylinders produced remarkable volumes of electrical discharges and, if cylinder dimensions were carefully selected and tuned, very strong discharges were noted.
- 5) Discharges from a cylinder with many radial 'cuts' showed a significant further increase in discharge.
- 6) Can even larger amplification be achieved with spirally wound coils? Tests with such coils showed 'tremendous white brushes leaping from their free ends'.
- 7) The electrostatic voltage over one inch of winding reached 10,000 volts so a single 10 inch coil could yield 100,000 electrostatic volts.
- 8) Such a coil was introduced into the discharge strap, i.e. as a secondary to the primary strap. Further improvement was noted. 'Strap and coil are not inductors in this instance; they are resistive capacitors. (Transformer action is electrostatic induction)'.
9) For maximum efficiency of operation, the copper mass of the primary has to equal the copper mass of the secondary, no matter how thin the coil windings are. (Tesla remarked that the coil capacitors were then in 'electrostatic resonance'. Using such principles, Tesla was able to produce a device that produced well over 1,000,000 volts but was 'no higher than a child'.

- 10) Discharges were of 'an intense white colouration' and could 'permeate all matter', sometimes with a slight heating effect but more often with a cooling effect.
- 11) Tesla's 'impulse transformers' greatly magnified the power supplied to them and their radiant effects were equally magnified. 'Electrostatic power' could be projected to very great distances, lighting special lamps to full candle power and driving electric motors at many hundreds of feet.

Although the invention of polyphase power had brought about major improvements to distribution systems, Tesla now foresaw that even greater improvements might be possible using the techniques described above. He undertook field trials that produced the following outcomes.

- 1) To reduce losses due to spark-like discharges, Tesla installed large copper spheres to surround the active terminals – see the well-known 'Wardenclyffe Tower' for example.
- 2) For maximum effect, both receivers and transmitters had to be grounded and the copper in the receiver had to match that in the transmitter.
- 3) Both low pressure gaseous and vacuum tubes could replace copper. Spheres filled with low pressure argon were found to be the most effective in reducing losses and produced 'tremendous radiance'.

Tesla's Patents

Tesla was cited as inventor in a remarkable number of patents – at least more than 300. Some related to the inventions described above.

US patent No 645,576 (1900) describes the wireless transmission of huge electrical power over enormous distances with minimal losses. Both transmitter

and receiver consisted of spirally wound coils, the former fed with pulsed high voltage DC. In the words of the patent,

‘I conclude that with electromotive impulses not greatly exceeding fifteen or twenty millions of volts, the energy of many thousands of horsepower may be transmitted over vast distances, measured by many hundreds or even thousands of miles’.

In US patent 787,412 (1905) Tesla describes an invention for creating standing electro-magnetic waves throughout the Earth and, thereby, achieving enormous power amplification. The transmitter was, again, a spirally wound coil fed with pulsed high voltage DC. The receiver was a mechanical device synchronised with the standing waves. In the words of the patent,

‘I have found it practicable to produce in this manner an electrical movement thousands of times greater than the initial – that is the one impressed upon the secondary by the primary.’

The Pierce Arrow

Following various misunderstandings between Tesla and his sponsors, work on power transmission did not proceed. However, Tesla continued his interest in the opportunities provided by ‘Longitudinal EM Waves’ particularly in relation to power amplification.

He was quoted in the New York Herald Tribune (11 September 1932) as follows.

‘Electric power is present everywhere in unlimited quantities and can drive the world’s machinery without the need of coal, gas, or any of the other common

fuels..... I showed that the universal medium is a gaseous body in which only longitudinal pulses can be propagated, involving alternating compressions and expansions similar to those produced by sound waves in air.'

This statement was preceded by a remarkable episode in 1931, witnessed in detail by Tesla's nephew. Aged 70, Tesla modified a standard production car, a Pierce Arrow, to run on a 'mysterious radiation from the ether'.

The internal combustion engine was replaced by an 80 horsepower, fan cooled, electric motor. Power was supplied by a dashboard mounted 'power receiver', equipped with 12 'special tubes' and two 3 inch rods to be pushed in to achieve power.

A six foot rod aerial was mounted in the rear of the car.

The car was subjected to a week's intensive testing in and around Buffalo, NY, and ran well, achieving 90 mph 'with ease' and requiring no conventional fuel.

On the return rail journey to New York, Tesla remarked to his nephew that he intended to interest the shipping world in this invention as a means of providing low-cost marine power.

Although coverage of the Pierce Arrow tests appeared in the local and non-local press, there are no reports of this invention going further with respect to either automotive or marine propulsion.

Longitudinal EM Wave Weapons

During his remaining years, Tesla pursued his interest in the military use of longitudinal EM waves. There is relatively little information on this period that

has passed to the public domain. It is known that some of Tesla's patent applications were seen to be of strategic value and thus were treated as official secrets. It is also known that, at Tesla's death (in a cheap hotel room – his home at that time), officials sequestered all his documents for official use, though some were subsequently released to the Tesla Museum in Belgrade.

The most that can be said with certainty concerning this period is that Tesla's ideas for weapons and the concept of EM longitudinal waves were obviously thought to be of considerable importance by those in authority, and it is suspected that significant research into this field has been continued by several nations, but behind closed doors.

Professor Konstantin Meyl (Germany)

There has been a recent renewal of interest in Tesla's achievements. Several workers in different countries (e.g. USA, Germany, Russia and Mexico) are trying to replicate his various experiments and findings. A prominent worker is Professor Konstantin Meyl who has taken the two Tesla patents referred to earlier as a point of departure, and has used modern electronics to produce a small-scale Tesla-type apparatus to test some of Tesla's claims.

The apparatus in question has a transmitter and a receiver. Both utilise the conducting sphere principle pioneered by Tesla, the transmitter receiving a modest power input from a generator of variable MHz frequency. The frequency is tuned for resonance to optimise receiver output, peaks being typically at 7 MHz (high power) and 4.8MHz (somewhat lower power). Simple tests show that the EM coupling at 4.8MHz from transmitter to receiver is via conventional transverse waves, 'Tesla Radiation' (i.e. longitudinal EM waves) accounting for the 7MHz peak. (Why Meyl knows that the 7MHz radiation is

longitudinal in nature is not absolutely clear. However, that is his clearly stated belief.)

As the distance from transmitter to receiver does not change during the experiment, it is believed there is no change in resonant wavelength and, thus, that the speed of propagation at 7MHz exceeds the speed of light 'c' by a factor of

$$\frac{7}{4.8} = 1.46$$

Meyl assumes the correct factor to be 1.5 which, if nothing else, provides some indication of the expected experimental accuracy.

Within such accuracy, a factor of $\sqrt{2}$ would seem to be equally reasonable. However, this work has yet to be replicated by other workers or, if already replicated, has yet to be published.

Incidentally, Meyl states that his apparatus confirms Tesla's claims concerning power amplification, amplification factors (with no additional power source) of two to three being commonly observed, and sometimes as high as 10. Such amplification is explained by Meyl as the apparatus taking power from the 'environmental field energy'.

Meyl concludes with some vector algebra to argue that the existence of longitudinal EM waves is compatible with Maxwell's Equations.

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There is a widespread view amongst academics, professional physicists and engineers that Maxwell's equations are sound, as these have robustly withstood the all the tests of time. Such a view is often augmented with the comment that, as Maxwell's Equations exclude the possibility of longitudinal EM waves, it is impossible for such waves to exist.

One example of such a view is given by Dr Bruhn, who examined in detail the vector algebra analysis of Meyl, pointed to flaws in the Meyl argument, and concluded with the bald statement that 'Solutions of the Maxwell Equations are always transversal. **Longitudinal waves are impossible**'.

Koen van Vlaenderen (Holland)

Sometimes the view expressed above is presented in a somewhat less forceful manner. For example, van Vlaenderen concedes that, if extra (scalar) terms are added to the equations of Maxwell, such terms representing electromagnetic potentials, then longitudinal wave solutions become possible. Moreover, such solutions start to predict the effects, such as electrostatic power phenomena, originally described by Tesla.

However, to what degree is it permissible arbitrarily to introduce such scalar terms? That these yield solutions that align with previous observations may turn out to be highly significant. Alternatively, as with Einstein's arbitrarily introduced 'Cosmological Constant', they may later have to be accepted as a 'mistake'.

(The present writer cannot resist using this opportunity to remind the reader that arguments such as these become redundant when considering the possibility that EM waves are transmitted, not by fields, but by vibrations in an omnipresent

quantum jelly. In such a scenario, not only are both transverse and longitudinal waves possible, they are inevitable.)

S B Karavashkin (Ukraine)

There is some evidence that longitudinal EM waves have been widely researched in Eastern Europe. One important piece of information is the book 'On longitudinal Electromagnetic Waves' by Karavashkin, the first chapter of which has been published by SELF Transactions (VOL 1 1994). The abstract of this chapter reads as follows.

'This is the initial version of an introduction chapter of a monograph devoted to the theoretical and experimental proof of the longitudinal electromagnetic wave's existence. The chapter proves that the known Maxwell divergence equation works correctly only in stationary fields. Its form for dynamical fields is derived. Some typical inexactitudes having led scientists to the conclusion that energy does not propagate in the near field are shown and the contradiction between the Ampere Law and Lorentz equation for dynamical magnetic fields acting on a charge are considered as well. As the supplement to this paper, the author published the review to the primary experiment on radiation and reception of the longitudinal EM wave as demonstrated by S B Karavashkin.'

Anatoly A Denisov D E (Professor, Leningrad Polytechnical Institute, Russia)

This Institute is located in the city now known as St Petersburg and is large (18,000 students) with 1,500 teaching staff. Prof Denisov was an electrical engineer of high standing and reputation, and even served as a member of the Supreme Soviet.

On September 20, 1990 in Moscow, in the presence of Prof Denisov and Dr Marienko, Karavashkin gave a demonstration of the transmission and reception of longitudinal EM waves. Prof Denisov subsequently wrote a testimonial to describe this event, summarised as follows.

The demonstration was given in a fairly small room and consisted of a transmitter, a receiver, and an oscilloscope (H313) to display the receiver output. The transmitter and receiver (both hand-made) were tuned to a frequency of 30kHz with a band pass of 5 – 6 kHz. The distance between transmitter and receiver was ca 3m.

In Prof Denisov's own words (as translated)

'When turning the radiator about any edge, the signal amplitude at the oscillograph reduced in approximately 5 times. The radiation direction was approximately 180 degrees. The author (Karavashkin) said the residual radiation was caused by a very limited space in the room, and this was corroborated by the following experiment.

When approaching the hand to the radiating side of the radiator, the signal amplitude at the oscillograph increased a few times. The touching of the other planes of the radiator gave no effect.

If the distance between the radiator and receiver varied while the hand kept approached, then the signal amplitude varied as this distance.

When approaching the hand to the receiver, even without touching, signal amplitude increased abruptly. When I stood between the radiator and receiver, the signal amplitude increased.

When the radiator turning about relatively to the receiver, then the approaching my hand to the radiating surface caused abrupt increase of the signal at the oscillograph.....

On the whole, basing on the above experiments, I can conclude the following. At this experiment was generated a pure longitudinal EM wave having the

properties that cannot be effected for transverse waves on the mentioned frequency band and by so portable device. First after the Arago and Fresnel experiments of 1816 was there demonstrated the possibility to generate and to receive the basically new type of EM field. It opens a huge, unknown before, branch of physics of phenomena and processes. It abruptly broadens both research and engineering scope of mankind. This is the more interesting that already the primary experiment revealed the counteraction of this radiation with the biological object.'

This statement seems to establish the following.

- 1) The waves generated were not transverse EM waves. (Biological matter is transparent to such waves at this frequency.)
- 2) These waves were instantly recognised by Prof Denisov as longitudinal EM waves.
- 3) For such recognition to be possible, Prof Denisov would have needed to possess some prior knowledge of the properties of longitudinal EM waves.

Tzontchev, Rivera-Juarez, Onoochin, Espinoza (University of Zacatecas, Mexico) 2003

The team, two members of which have Russian origins, published a paper with the title, 'On the Penetration of the Longitudinal Component of EM fields into Metals'.

The abstract reads as follows. 'The penetration of EM waves, with frequency 14 MHz in which the E vector is directed along the wave vector, through metallic shields of different design, in the near field of the antenna, is investigated. Despite the fact that in a normally incident transverse wave the theory predicts a decay of the signal of some 40 db, for shields used in the experiments the decay of the measured signal was less than 1.2 db, i.e. an anomalously large

penetration of the signal through the metallic layer was detected. This effect is due to the type of penetrating EM wave, i.e. its longitudinality.'

The apparatus used was very similar, in principle, to that used in the early investigations of Tesla. Two van de Graaf generators were used, one active and one passive. Each generator was connected to its own copper sphere, radius 10cm, and each sphere was connected to a spherical copper antenna, radius 9.5cm. On discharge of one sphere to the other, the circuit resonated at 14 MHz.

The potential between the two antenna was recorded using a Tectronix oscilloscope. When one antenna was shielded by a conducting film (which according to well-established principles should have attenuated the strength of the measured signal by 40db) it was found that the attenuation was merely 1.2 db. Thus the shield had been penetrated far more than expected – a phenomenon observed and recorded by Tesla ca 100 years earlier.

However, Tesla offered no explanation as to why such penetration took place. The team in Mexico designed the apparatus so that the E vector is along the direction of propagation with result that the H vector is always parallel to the surface of the shield and so cannot produce eddy the currents that, as produced by a transverse EM wave, would block penetration.

This paper is interesting, first, because it corroborates Tesla's experiments and, second because the production of the 'Longitudinal Component' is reported in an entirely 'matter-of-fact manner' as though, by this time, such a phenomenon was regarded by the team as ordinary and no longer novel.

Reports of weapons derived from Longitudinal EM waves

The literature contains reports by several authors describing military research into (and the subsequent deployment and use of) such weapons. The authors are plainly knowledgeable and competent, and the drafting is lucid. However, the content is slightly alarmist in nature and the credulity of the reader is tested by references to the use of such weapons being linked to unidentified flying objects (UFOs) and crop circles.

Much of this material dates from the late sixties/seventies/early eighties when cold war tensions led to widespread apprehension concerning the possession by unfriendly nations of terrifying weapons. Indeed, in those times, there were good grounds for such apprehension. Furthermore, some of Tesla's later work was in the public domain as was the occurrence of official interest in it. Seen in this light, some published speculation into where such work might lead is hardly surprising.

Nevertheless, during several decades there has been no specific statement from any Government that such weapons exist or that they have been deployed and used; can official secrets be kept so well by several nations for so long?

Recently, there have been official reports that EM weapons have been developed and will be deployed. However, it has been stressed that such weapons use convention techniques and components (such as Transverse waves and magnetrons) and are designed solely to disable enemy equipment and computer systems – quite different from the hugely powerful weapons allegedly enabled by longitudinal EM waves.

Conclusions

As can be seen from the above, there is a considerable amount of information in the public domain concerning the existence and properties of longitudinal EM waves. However, the quality of such information is variable.

By way of example, a peer-reviewed article in a learned journal can probably be trusted, as can a granted and published patent (which, after all, is a formal legal declaration, and would not be granted or published without the full approval of the examiner).

An article published in a heavy-weight national newspaper is less formal but, as serious journalists check their sources, can provide useful and trustworthy information.

A piece posted on the internet, particularly if it is aimed at promoting a particular product or point of view, has to be treated with caution. The information it gives may be interesting but is it balanced and complete?

Faced with such a mixture, firm conclusions as to the existence of longitudinal EM waves cannot be drawn. But that does not imply that no conclusions can be offered. It merely means that, as the information itself is qualified by probabilities, similar qualifications have to be applied to the conclusions, which are listed as follows.

- 1) That longitudinal EM waves exist, in one form or another, seems to be highly probable. This view relies heavily upon the written evidence arising from Eastern Europe and elsewhere, and the patents and corroborated experimental work of Tesla.

- 2) Whether such waves travel at the speed of light factored by the square root of two is uncertain. The only supporting evidence is provided by Professor Meyl, with no reworking or corroboration by others. It is thought, therefore, to be too soon to rely upon it alone. (Further information on this issue may emerge in future but, for the moment, is likely to be classified.)
- 3) In the light of (1), it is probable that Maxwell's equations, as presently stated, need to be revisited.
- 4) It is also probable that classified research has been undertaken by several nations into the strategic uses of longitudinal EM waves. Whether such weapons have resulted, and have been deployed and used is uncertain.
- 5) Whether endless power can be derived from the environment, as described as part of the Pierce Arrow project, is a tantalising and important question. Such derivation has only been corroborated by Meyl – though others have worked in this field, but not knowingly with longitudinal EM waves. Without further corroboration, such a phenomenon has to be rated as 'hard to believe', but (in the light of Tesla's patents and the eye-witness accounts given by Tesla's nephew) not totally improbable. The economic benefit should Tesla's work in this regard prove to be capable of replication would be enormous. Further investigation would surely be sensible.
- 6) In the light of (1) and (3), the thought that quantum jelly provides the medium (the ether) for the propagation of all forms of EM waves is an idea that survives – at least for the time-being!

Experimental Observation of Kondo's Hyper Phi Meson and the Hyper Delta Hadron

V1.05

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Kazuo Kondo left an *a priori* particle theory predicting a cascade of massive hyper mesons and massive hyper hadrons. The CDF collaboration at the Tevatron appears to have recently provided evidence supporting the existence of the Hyper Phi Meson. This new observation combined with the existing observation that the mass of the Top Quark coincides with the mass of the Hyper Delta Hadron suggest that Kondo's epistemological model merits further attention.

1. Introduction

In amongst Kazuo Kondo's four to five million words of scientific output [Croll, 2006] are a set of specific *a priori* predictions [Croll, 2008] regarding the existence of a cascade of high mass Mesons and Hadrons isomorphic to those presently observed. Hyper Mesons and Hyper Hadrons are described in terms of Akitsugu Kawaguchi's geometry of higher order spaces [Kawaguchi, A., 1931, 1932, 1933] [Kawaguchi, M., 1962, 1968],

and are based upon a series of mass quanta related by the fine structure constant. The Kondo Model (KM) is by definition locked into the masses and properties of the electron and the muon of the Standard Model (SM).

2. Cascade of Mass Quanta

KM first principles suggest that, for two mass quanta θ and Θ :

$$m_e = 2\theta, \quad m_\mu = 2\theta + 3\Theta$$

which, when compared with the experimental masses

$$m_e = 0.511 \text{ MeV}, \quad m_\mu = 105 \text{ MeV}$$

gives

$$\theta = 0.2555 \text{ MeV}$$

and

$$\Theta = 35 \text{ MeV}$$

from which follows, approximately, the familiar ratio:

$$\alpha = \theta / \Theta \approx 1 / 137$$

Thus, an open ended cascade of mass quanta are proposed:

θ'	$= \theta \alpha$	$=$	0.015	MeV
θ	$= \Theta \alpha$	$=$	0.255	MeV
Θ	$= \theta / \alpha$	$=$	35	MeV
Ψ	$= \Theta / \alpha,$	$=$	4.795	GeV
Ψ'	$= \Psi / \alpha$	$=$	656	GeV

The use of the paragraph symbol was probably due to the fact that it was the only unused symbol left available on Kondo's typewriter.

3. Hyper Mesons and Hyper Hadrons

We enumerate the KM structure and masses of the Mesons and Hyper Mesons in Table 1 and the structure and masses of the Hadrons and Hyper Hadrons in Table 2.

Table 1 – Mesons and Hyper Mesons

	Name	KM Structure	#Mass Quanta	Meson Quantum = Θ (MeV/c ²)	Hyper Meson Quantum = Ψ (GeV/c ²)
a)	η	$\left[\begin{array}{c ccc} 1 & 2 & 3 & 2 & 1 \\ \hline 1 & 2 & 3 & 2 & 1 \end{array} \right]$	16	560	76.72
b)	ρ or ω	$\left[\begin{array}{c ccc} 1 & 2 & 3 & 3 & 2 & 1 \\ \hline 1 & 2 & 3 & 3 & 2 & 1 \end{array} \right]$	22	770	105.49
c)	K	$\left[\begin{array}{c cc} 1 & 2 & 3 & 2 & 0 \\ \hline 1 & 2 & 3 & 2 & 0 \end{array} \right]$	14	490	67.13
d)	K^*	$\left[\begin{array}{c ccc} 1 & 2 & 3 & 3 & 2 & 1 \\ \hline 1 & 2 & 3 & 4 & 3 & 2 & 1 \end{array} \right]$	26	910	124.67
e)	η'	$\left[\begin{array}{c ccc} 1 & 2 & 3 & 2 & 3 & 2 & 1 \\ \hline 1 & 2 & 3 & 4 & 3 & 2 & 1 \end{array} \right]$	28	980	134.26
f)	ϕ	$\left[\begin{array}{c ccc} 1 & 2 & 3 & 4 & 3 & 2 & 1 \\ \hline 1 & 2 & 3 & 4 & 3 & 2 & 1 \end{array} \right]$	30	1050	143.85
g)	π	$\left[\begin{array}{c c} 1 & 2 \\ \hline 1 & 2 \end{array} \right]$	4	140	19.18

Table 2 – Hadrons and Hyper Hadrons

	Name	KM Structure	#Mass Quanta	Hadron Quantum= Θ (MeV/c ²)	HyperHadron Quantum= \Uparrow (GeV/c ²)
a)	N:	$\begin{bmatrix} 1 & 2 & 3 & . & . & 2 & 1 \\ 1 & 2 & 3 & . & . & 2 & 1 \\ 1 & 2 & 3 & . & . & 2 & 1 \end{bmatrix}$	27	945	258.93
b)	Σ :	$\begin{bmatrix} 1 & 2 & 3 & . & . & 2 & 1 \\ 1 & 2 & 3 & . & . & 2 & 1 \\ 1 & 2 & 3 & 4 & 3 & 2 & 1 \end{bmatrix}$	34	1190	163.03
c)	Ξ :	$\begin{bmatrix} 1 & 2 & 3 & . & . & 2 & 1 \\ 1 & 2 & 3 & 4 & . & 2 & 1 \\ 1 & 2 & 3 & 4 & 3 & 2 & 1 \end{bmatrix}$	38	1330	182.21
a')	Δ :	$\begin{bmatrix} 1 & 2 & 3 & . & 3 & 2 & 1 \\ 1 & 2 & 3 & . & 3 & 2 & 1 \\ 1 & 2 & 3 & . & 3 & 2 & 1 \end{bmatrix}$	36	1260	172.62
b')	Y^* :	$\begin{bmatrix} 1 & 2 & 3 & . & 3 & 2 & 1 \\ 1 & 2 & 3 & . & 3 & 2 & 1 \\ 1 & 2 & 3 & 4 & 3 & 2 & 1 \end{bmatrix}$	40	1400	191.80
c')	Ξ^* :	$\begin{bmatrix} 1 & 2 & 3 & . & 3 & 2 & 1 \\ 1 & 2 & 3 & 4 & 3 & 2 & 1 \\ 1 & 2 & 3 & 4 & 3 & 2 & 1 \end{bmatrix}$	44	1540	210.98
d')	Ω :	$\begin{bmatrix} 1 & 2 & 3 & 4 & 3 & 2 & 1 \\ 1 & 2 & 3 & 4 & 3 & 2 & 1 \\ 1 & 2 & 3 & 4 & 3 & 2 & 1 \end{bmatrix}$	48	1680	230.16
e)	Λ :	$\begin{bmatrix} 1 & 2 & . & 3 & . & 2 & 1 \\ 1 & 2 & . & 3 & . & 2 & 1 \\ 1 & 2 & 3 & 2 & 3 & 2 & 1 \end{bmatrix}$	32	1120	153.44

3. Experiment

The CDF collaboration [Aaltonen et al, 2011] have recently observed an excess mass distribution in the 120-160 GeV/c² range which is not described by Standard Model processes. The theoretical mass of Kondo's Hyper Phi Meson at 143.85 GeV coincides very closely with the Mean of the Gaussian component of this observed excess at 144 ± 5 GeV. The theoretical mass of Kondo's Hyper Delta Hadron at 172.62 GeV corresponds very closely with the observed mass of the Top Quark (M_{top}) which is 172.5 ± 1.7 (*stat* + *JES*) ± 1.2 (*syst*) GeV/c² [Margaroli, 2011].

4. Summary

We have briefly outlined the a priori KM structure and masses of Mesons, Hadrons, Hyper Mesons and Hyper Hadrons. We cite experimental results which suggest that the mass excess seen recently at the Tevatron is very close to the mass of the KM Hyper Phi Meson, and that the mass of the Top Quark is very close to the mass of the KM Hyper Delta Hadron.

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Zen meditation and mindfulness for modern times

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Julian Daizan Skinner²

Introduction

Zen is perhaps the most talked about and the most misunderstood philosophy. While it eludes definitions, words of D.T. Suzuki convey its meaning well:

“Zen in its essence is the art of seeing into the nature of one’s own being, and it points the way from bondage to freedom. By making us drink right from the fountain of life it liberates us from all the yokes under which we finite beings are usually suffering in this world.” (Suzuki and Humphreys 1969)

The documented practice of meditation and mindfulness goes back at least two and half thousand years. Zen teachings and practice originated in India at the times of Shakyamuni Buddha (The Awakened One of the Shakya clan), around 500 BC. In time it has spread to China then Japan, and in 20th century came to the West, to Europe and America. In the course of these wanderings it has adapted to local and cultural circumstances. However the direct transmission from masters to worthy successors assured that Zen is alive and that the core of teaching remains pure.

For us Zen is a way of life, and though some would argue it is non-religious, its roots are firmly planted in Buddhism. Zen practice is not monolithic – some

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types lead to enlightenment, the ultimate aim, some aim to enhance physical well-being and life satisfaction. The five main divisions of Zen were classified by Fifth Ancestor of the Chinese Huayan School, Kuei-feng Tsung-mi as follows: *bompu* (ordinary), *gedo* (an outside way), *shojo* (small vehicle), *daijo* (large vehicle) and *saijojo* (the highest vehicle) (Kapleau 1980). This classification reflects the increasing insight into our True Nature – all within very similar forms of practice.

Bompu or *ordinary* Zen is non-religious and its aim is an improvement of the physical and mental health. Philosophically content-free it is good for everybody: if practiced regularly it will improve the health, increase well-being and help to achieve concentration and clarity of mind. The key benefit is learning how to keep your mind where you put it – a skill necessary to achieve anything in life. It is noteworthy that *bompu* Zen underlies all higher forms of practice as to have a healthy body and a disciplined mind provides a vehicle for deepening insight.

In contrast the fourth type of Zen, *daijo* or Great Vehicle (Mahayana) has an ultimate goal of achieving *kensho* - seeing into one's True Nature - and becoming the Way in daily life. This is the true Buddhist Zen as Buddhism is a religion of Enlightenment; intended for people with burning desire to see the things as they really are and so experience absolute Reality first hand. The practice of *zazen* – a sitting meditation – embodies the innate Buddha-nature that we all are endowed with. There are differences in approach to enlightenment between two main Zen schools, Rinzai and Soto: Rinzai strives to awakening while Soto claims that since *zazen* actualises our pure True Nature one day it will become obvious to a sincere practitioner.

Every Zen master faces a challenge how to implement Zen Buddhist teaching in daily life for the variety of practitioners. One of us (JDS) has received Dharma Transmission in both Rinzai and Soto traditions, and his teaching emphasises meditation, mindfulness and everyday practice. He focuses on the practical application of Eastern philosophy to the stresses and strains of modern Western life.

In 2010 JDS developed two eight weeks long courses, *Zen meditation and mindfulness for health and well-being* and *Zen meditation and mindfulness for insight*. The former embodies *bompu* type of Zen whereas the latter addresses *daijo* type of Zen. He has trained since several meditation and mindfulness teachers to deliver these courses to healthy subjects. BJK delivers them twice a year at Oxford University (health and well-being) and annually at Oxford Zen Society (insight).

Meditation tradition

In this paragraph we take a brief tour of concepts that are pertinent to our practice.

- *Meditation* is a practice which broadly speaking has either narrow focus on concentration (using a mantra or image) or on insight which is wider, more investigative which leads to clearly seeing things how they really are. The former is called shamatha (calm abiding), the latter prajna (wisdom or clear seeing). *Four types of meditation* are sitting, walking, standing and lying down.
- *Mindfulness* is simply paying attention, with an internal focus, e.g. mindfulness of the body or with an external focus on an activity such as flower arrangement or Japanese tea ceremony. *Four foundations of*

mindfulness are mindfulness of the body, mindfulness of sensations, mindfulness of the mind and mindfulness of mind objects.

The Zen approach is that meditation is a simple and natural thing, nothing special to believe in. A distinctive feature of Zen Rinzai tradition to which we belong is teaching meditation for wellbeing in and outside monastic setting.

Meditation and health

A cursory search on Amazon (March 2012) revealed 35 top guided mindfulness meditation books. The number of research papers published annually with 'meditation' or 'mindfulness' in the title or content goes into hundreds. Why is that so?

In the second half of the twentieth century there was a breakthrough in the medical science in America. Dr Herbert Benson, a father of modern mind-body medicine, has pioneered the use of meditation in medical science. He studied physiological effects of stress and demonstrated that the *relaxation response* can relieve stress as opposed to the *fight-or-flight reaction*. Factors such as metabolic rate, heart rate and blood pressure are increased by stress and decreased by meditation. The importance of his research was to show the direct connection between relaxation response and meditation (Redwood 2008).

This theme was picked up subsequently by John Kabat-Zinn (Kabat-Zinn, Lipworth et al. 1985) who introduced meditation (carefully stripped of any religious meaning) as a major factor in treating psychiatric disorders and patients with chronic pain. The efficiency of eight week courses – with group meetings every week and participants practicing around an hour daily in their

own homes – has been tested and proved many times over (Kabat-Zinn 2001). This strand has been further developed in Britain by Prof Mark Williams, Oxford Mindfulness Centre (Williams 2010). In contrast our courses are aimed at healthy participants.

These courses are based on a premise that the human brain has a property of *neuroplasticity*. Both the brain and the nervous system can change structurally and functionally under the influence of the signals from the environment. Learning and memory are examples of plasticity. Implicitly it follows meditators are able to change some neural paths in the brain through meditation, and this can result in a changed outlook on life events. This profound result of sustained practice, known for many centuries to meditation practitioners, has been confirmed by scientific studies in the last two decades (Chiesa and Serretti 2010).

There are many practices which are classified under an umbrella of ‘mindfulness meditation’; while they have roots in contemplative or religious practices – especially Hinduism and Buddhism – their foundation is non-religious (Lutz, Dunne et al. 2007). This causes a problem for neuroscience investigation as there are many subtle and less so parameters essential for conducting a well-defined experiment. For example, what paths change in the brain if a meditator practices with open eyes, and when eyes are closed? Hence a strict comparison of the effects of different techniques on meditators has to be defined in broader terms, for example looking at psychological factors such as increased life satisfaction.

One of the definitions of meditation is *not-doing* as opposed to action (Lutz, Dunne et al. 2007). The former is an anathema to a busy Western person. We are

supposed to manage our time in the most efficient way; some time management trainings advise people to schedule every five minutes. Keeping track of time is the ultimate achievement; not having time is interpreted as that we have 'made it'. Curiously though, the outcome of not-doing helps us with doing if translated into daily activities. This is something we want to learn and implement in daily practice. And we pose a practical question: what can we get out of not-doing? How is it going to influence us, help us to cope with life's stresses, can it preserve and improve our mental and physical health?

Zen for health and well-being course addresses illnesses of modern times, as well as illustrates the benefits of meditation and mindfulness. Three examples are given below:

Stress - especially emotional

Stress is an unavoidable feature of modern life, and low-ish level of it can be beneficial. It is *the non-specific response of the organism to any pressure or demand* (Hans Selye). However prolonged exposure to stress leads to impaired performance and reduces immunity to diseases. Handling stressors, whether external (e.g. work) or internal (thoughts, feelings...) involves development of outer and inner resources for dealing with them. Meditation and mindfulness helps to develop your inner resources in two main ways:

- providing an oasis of calm and relaxation in which we can recuperate
- perceiving clearly what we are going through. When we encounter stress, we don't act out and we don't suppress it – we are simply aware.

Mindfulness centred on the body helps to identify and alleviate stress and works on many deep levels. Hence we can learn to change our *reaction* to some

stressors such as deadlines, relationship problems or public performance to a *response*:

- **The classical stress reaction** - three stages: i. fight or flight (mobilization of the body for immediate reaction), ii. a slower resistance action iii. exhaustion
- **Response to stress** – the *ultimate effect on our health of the total psychological stress we experience depends in large measure on how we come to perceive change itself in all its various forms, and how skilful we are in adapting to continual change while maintaining our own inner balance and sense of coherence* (Jon Kabat-Zin).

In an interview Herbert Benson stated that his team has found psychological benefits of meditation as well (Redwood 2008). The relaxation response was found to allay anxiety, mild and moderate depression, and to moderate excessive anger and hostility. He also compares health and well-being to a three-legged stool: one leg is medication; the second leg is surgery and other medical procedures. For the balance there has to be a third leg and “*that leg is self-care*”. Self-care includes relaxation response and other factors, such as proper nutrition, the belief system of the patient, social environment etc. Therefore one has to bear in mind that meditation is not cure for all: especially in mental disorders it is often used simultaneously with traditional medical treatments.

Anxiety and depression

Rather than escaping emotional pain we deal with it through AA (awareness and acceptance). Emotional responses are often rooted in the animal level of our being; hence one of the most effective ways of dealing with them is through mindfulness of the body. The key attitude is that of being present with emotional pain, neither suppressing nor acting on it. This approach is also valid for dealing

with very strong states such as fear or depression; practising by simply coming face to face with the moment-to-moment reality of our symptoms. The results of scientific research (Kuyken, Byford et al. 2008) show that mindfulness and meditation (often combined with Cognitive Behavioural Therapy as done at Oxford Mindfulness Centre) is as effective as maintenance anti-depressants in preventing a relapse into depression, and is more effective in enhancing peoples' quality of life (Teasdale, Segal et al. 2000).

Elevated function

After meditating I feel calm, centred and ready to compete – but, more importantly, the technique allows me to 'just play' and enjoy the game without worrying about the result. Jonathan Rowson, the three times British chess champion

In addition to preventing ill-health on mental and physical levels and to stress-proofing meditation can boost or elevate your function. This is widely accepted in the East; in Japan many companies will send their employees for short meditation training in a Zen temple. Meditation improves attention, concentration and clarity of the mind; enhances creativity and problem – solving skills; induces general psychological well-being; encourages positive personality growth with the length of the meditation practice; increases self- esteem; increases empathy; delays the aging process; heightens perception; may even boost IQ. Public speaking becomes easier; personal performance improves, physiological age is 'younger' than chronological age (Wallace, Dillbeck et al. 1982). Another level of elevated functions – metaphysical – is not 'proved' conclusively by scientific methods.

To sum up, the key wellbeing benefits of meditation and mindfulness are:

- **training the mind to focus and concentrate** – daily practice, with obvious benefits
- **enhancing mental, physiological and psychological planes**
- **improving general psychological well-being** – typically increasing with the length of practice – meditators are more confident, relaxed, satisfied, conscientious and less anxious than their less experienced colleagues
- **a tool to enhance youthfulness and longevity** – regularly practising meditation for five years brings benefit in being physiologically 12 years younger than non-meditators .

Practices taught in this course

All Rinzai Zen masters of today have Hakuin Zenji as their ancestor. Hakuin (1686-1769) revived Zen teachings in Japan, and was a prolific writer. Nowadays he is remembered in the lay world for his book on health *Yasenkanna*³ (Yampolsky 1971).

Here we concentrate only on the health aspect which is the primary and underlying aspect of Zen practice. There are people who meditate zealously but yet their health suffers – from what became colloquially known as ‘Zen sickness’. This can be overcome by a practice of introspection, roughly by

³ *Yasenkanna* (“Chat on a boat in the evening”, English translation RDM Shaw and W Schiffer, *Monumenta Niponica* XIII, no. ½ (1956), pp 101-127.)

More about meditation and mindfulness teacher training: <http://www.zenways.org/>. *Zen meditation and mindfulness for health and well-being* course is a part of a ‘Building a successful career in the sciences’ programme at Oxford University: <http://www.mpls.ox.ac.uk/skills/courses/meditation-and-mindfulness>. More about *Zen meditation and mindfulness for insight* course: <http://www.oxfordzensociety.org/>

allowing the inward spirit and vital force to penetrate into the space below the navel.

What is so important about this space below the navel? This is *hara* – the vital centre of man. There is a book of this title by Karlfried Graf von Dürckheim. He asserts that ‘...anchorage in the vital centre which is Hara guarantees man enjoyment of a power which enables him to master life in a new and different way. It is a mysteriously sustaining, ever renewing, ordering and forming power, as well as liberating and integrating one.’ (Dürckheim, Kospoth et al. 1977)

With some simplification there are three essential physical components to the Zen training: posture, breathing and balance between tension and relaxation. On the mental level there is attention and concentration. All these are linked to *hara*; a well-developed *hara* is essential for ‘being centered’, for being focused, for keeping our attention fully on any activity or accessing meditative states.

We teach four fundamental meditation and mindfulness techniques for health and well-being: bodyscan (a simplification of a traditional Zen practice called *nanso no ho* or soft ointment meditation), counting breath, following breath and *fusho* (the practice of presence or unborn mind). They serve as an anchor for attention to which we revert when distracted, and their relation to foundations of mindfulness is summarised below:

Attaching attention

- Bodyscan → Strong physical anchor
- Counting breath → Abstract and physical anchor
- Following breath → Physical anchor
- Fusho → No anchor

Spiritual development

Mindfulness and meditation are traditional, age-old Eastern ways of developing and deepening insight. Mindfulness means being aware of what's going on in the present moment; being one with any activity we choose; not day-dreaming. Zen places particular emphasis on developing awareness of the body which is the vehicle for enlightenment. That's where mindfulness plays vital role as we have the body-mind: what's going on in the body affects the mind and vice versa.

Zen meditation and mindfulness for insight eight week course introduces Zen meditation and mindfulness practices aimed at developing insight into how the reality is and who we really are. The ultimate focus is on *kensho* – literally seeing or experiencing your true nature – ‘...a blissful realization where a person's inner nature, the originally pure mind, is directly know as illuminating emptiness, a thusness which is dynamic and immanent in the world’ (Peter Harvey).

Mental level

In addition to documented effects of meditation and mindfulness on health there is their influence on the mental level. From the ability to concentrate stems an increased mental clarity, heightened ability to learn, mental strength and being grounded, increased empathy (with some detachment). There are several current studies on the influence of meditation on mental tasks, e.g. improvement of the ability to prioritise and manage tasks, the ability to voluntarily focus on specific information and the ability to stay alert to the environment (three sub-components of attention) (Jha, Krompinger et al. 2007). All these effects are

linked to neuroplasticity of the human brain. Both the brain and the nervous system can change structurally and functionally under the influence of the signals from the environment, thus our experiences and our thoughts.

Mental effects of meditation can be summarised as follows:

- **the skill of concentration:** ability to concentrate can be learned and improved; impact on all areas of life when we are able to stay focused
- **increased mental clarity, heightened ability to learn**
- **concentration leading to relaxation** – concentration narrows our focus of attention thus ‘squeezing out’ distractions
- **overcoming principal barrier to mindfulness** – lack of willingness to see things as they really are. If we stop fooling ourselves and have courage to see things as they are, we can overcome this barrier. We allow things to arise and pass, to resolve, letting them go, thus healing and gaining mental strength and groundedness.

Spiritual level

A central feature of this course is outlining the road to *kensho*, enlightenment, through practice. We recall that the key aim of mindfulness training is to keep the mind where we put it. How this work changes, expands and influences mind – and ultimately our Being - is traditionally expressed in Zen through *Ten oxherding pictures*. Similar stages in spiritual practice are found in other cultures as well. For example, in Christianity there is St John of the Cross’ “The Dark Night of the Soul”. On one level the Tibetan Buddhist classics *The Tibetan Book of the Dead* describes how to prepare for death and desirable rebirth; a modern interpretation is of passage through several psychological states.

The Tibetan tradition emphasizes the path as being *between* confusion and enlightenment, and urges the practitioner to choose the latter. Of course the final goal is enlightenment but at every stage, in every realm, there is a clear choice between soft and alluring light (wrong!) and brilliant intimidating luminosity (correct!). The path can be cyclical – there are six realms where one can be reborn: the realm of hell, the hungry ghost realm, the human realm, the realm of the jealous gods, the realm of the gods. These realms are thematically tied to living in *samsara* (life in ignorance and suffering) and serve as a springboard to entering *dharmakaya* (the absolute Buddha-nature). In all realms the liberation happens via seeing through and not falling for projections of one's mind but instead finding the courage to face and enter luminosity (the naked mind).

Psychologically in the realm of hell, the most intense of all, a practitioner is fighting with hatred and aggression (hot hell) or pride (cold hell). In the hungry ghosts realm one suffers insatiable hunger of possessing all what is to be possessed so there is no joy of new acquisitions. In the animal realm everything is sane, solid and humourless – so anything unpredictable threatens the base of living. The human realm is a sort of mixture between animal and hungry ghost realms: seeking predictability while striving constantly for something (research and development, enriching life). In addition there is passion and intrigue, and impermanence.

We argue that the Zen way leads *from* confusion to enlightenment. Traditionally the path has ten stages as described below. It is universally recognized that this is not a one-off process but it is cyclical; it reaches deeper and deeper cleansing and healing levels resulting in deepening understanding of and connection to our true nature. It does not however describe the emotional landscape through which

this path is weaving. To a great extent it seems to be unimportant; a typical response of a Zen master would be: ‘cut off’!, ‘more zazen’!, ‘let go!’.

There are reasons for that, and one is that a Zen practitioner has to let go of any experience, even of kensho as not to stop the possibility of infinite spiritual development.

Stages of insight

The ten oxherding pictures: Zen road map of the journey of spiritual insight

The ten pictures illustrate stages of insight, and are often accompanied by poems⁴. Over the centuries prominent Zen masters added their commentaries, sometimes in a very oblique way. Here we present a modern version: verses by Ryōsen (BJG) and commentaries by JDS from teacher’s manual as taught to meditation and mindfulness teachers.

1. The search for the bull



Transmutation

Breath by breath
I open my chest
Breath by breath
I let the dragon soar
Breath by breath
I let the light in
Breath by breath
Chest opens
Breath by breath
Dragon soars
Breath by breath
Light shines
Breath by breath
Dew drop on the blade of grass

⁴ For example, see traditional poems by Kakuan, a 12th century Chinese Zen master. A good translation is by Nyogen Senzaki and Paul Reps, as presented in a book **Zen Flesh, Zen Bones**. Illustrations for *Ten oxherding pictures*: Ryōan-ji temple, Kyoto, Japan

From Teacher's Manual: We're doing our practice and there is an engagement with it, a commitment to find what we are looking for. In this stage we are studying the self, our mind and body and we realize that ultimately we are not our mind and body.

2. Discovering the footprints



Three worlds

I have given you:
 A shadow of the past
 A glimpse of the future
 A body of the present -
 A sightless man, you held this body tight

From Teacher's Manual: The footprints are the effects – previous actions are the cause. We realize clearly that things, being made up of causes are inherently changeable and have a degree of instability about them that can never fully and permanently satisfy us.

3. Perceiving the bull



University Park

A ginkgo tree silently stands
 bridging heaven and earth.
 Its naked crown supports the sky,
 its leaves carpet the ground.
 A late hedgehog rustles the leaves
 into a winter house,
 and rainy clouds fleetingly rest
 on dormant nets of branches.
 Tenderly, a kiss of a smooth bark
 falls on an open palm –
 the heart beats thanks for such
 generosity of this tree.

From Teacher's Manual: We get a clear view on things now, our meditation takes on a life of its own. Reality begins to be seen as networks of very fine vibrations. We can feel like we've made it – we've seen the grandeur and beauty of reality.

4. Catching the bull



Tyrtej

Like a resinous splinter, you keep shedding
Alighted rags whirling all around you
Burning, you're ignorant of becoming free
Or if all what is yours will be lost?
What will be left: just chaos and ashes
Storm-driven into an abyss? – or
Among the cinders star-begotten diamond will remain
The eternal victory dawn...

Cyprian Kamil Norwid (free translation by Ryōsen)

From Teacher's Manual: At this stage we come face to face with all the suffering inherent in our previous deluded or asleep lives. There are specific obstacles that arise in this stage, that vary for each person. It's very beneficial to have a teacher or a reference point who has gone through it before you.

5. Taming the bull



The night watch

The full moon hangs in the raindrops
Even the black clouds
Cannot obscure his luminosity

From Teacher's Manual: We finally reach a stage of equanimity. We are sufficiently de-identified from our stuff that it can arise and pass away quite freely. With it can come a quality of relief.

6. Riding the bull home



Just so dance

Twirling with the leaves
Flirting with the breeze
Weaving a web of joy
Of an Indian summer

From Teacher's Manual: Things can become quite pleasant, we can feel like we can ride the vibrations of rise and fall without any particular effort. Our mind is now trained. We are good meditators.

7. The bull transcended

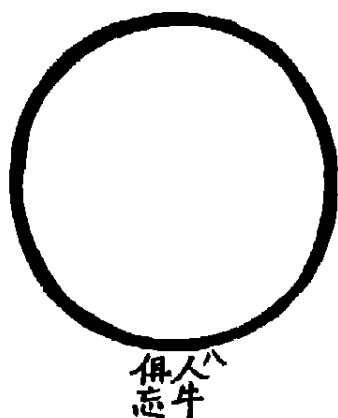


Form is emptiness, emptiness is form –
the moon in his belly,
his chest the clouded sky

From Teacher's Manual: We don't have anything, we're not identified with anything in particular. The famous Zen text, the Mumonkan says: "All the

illusory ideas and delusive thoughts accumulated up to the present will be exterminated, and when the time comes, internal and external will be spontaneously united. You will know this, but for yourself only, like a dumb man who has had a dream”.

8. Both Bull and Self transcended



[left blank on purpose]

From Teacher's Manual: Then we come to the stage of the famous Zen empty circle. In this stage, reality disappears for a moment and so do you. Zen calls it 'the great death'. We let go of everything and here is no time, space or anything else. There is really nothing we can say about this place, it is a condition of complete discontinuity.

9. Reaching the source



An early spring
The newborn catkins –
Their silvery down wet still –
Tickle the sulky sky

From Teacher's Manual: In this place we're often touched by joy and relief. We know we're home and paradoxically we can know that we always were home. Everything is as it is, and has a quality of perfection about it.

10. Returning to the world



昭和辛卯夏
西荻利權
金入十
手鬱

On the way from the market

Armful of fresh roses
Carried through the streets
Gladdens the hearts of passers-by

From Teacher's Manual: With all of this spiritual work it's not enough to have realization, we have to live our understanding, and in doing this, we naturally help others. How we live and what we do become vastly important at this stage.

Summary

In this paper we have described the implementation of Zen meditation and mindfulness practices as two eight week long courses. There is a degree of overlap between them but they have different aims. The basic, non-religious course aims for improvement of health and well-being. The advanced course aids spiritual development as leading to enlightenment. We can summarise the main points of both courses as follows:

- **Key concept – meditation as a journey:** Zen practice is a life-long process, and regular practice is essential in order to develop deep insight which then changes your life. It is an adventure with many surprises and insights along the way. But Zen places emphasis in finding the Truth in your ordinary life.
- **Happiness as a way of caring for others:** the more we forget about ourselves, the happier we become, and eventually serve with our life.
- **Changing your life:** outwardly, life may be the same but there are changes inside – a true Zen practitioner is happy regardless of external circumstances.
- **Longevity:** a side-effect of meditation. Zen masters are renowned for their longevity – for example, Joshu Zenji lived to 120 years.
- **Beyond daily practice:** achieving mastery in your profession through inner work and outer experience.

How we did it

Each of these courses lasts eight weeks, and comprises 1.5 – 2 hrs long meeting with a themed talk and guided meditation, backed up by a CD. Participants are expected to practice on their own about half an hour daily in between the meetings. Typical homework for a week: to continue sitting meditation and update a meditation diary. Take some time for mindful stretching. Continue practicing and noting daily life mindfulness activities. In addition, do a three-minute mini-version of current practice daily and note the effects this has in the meditation diary.

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Human mathematics – new beginnings from the Pythagoras Theano impulse

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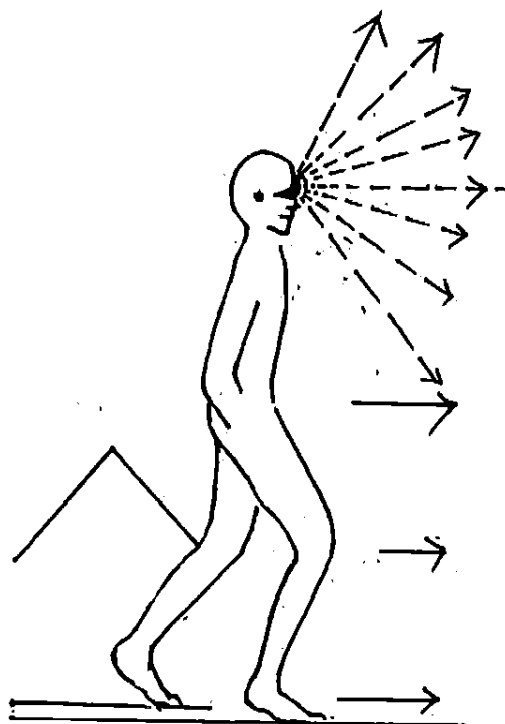
It was Pythagoras who coined the term, ‘mathematics’ - ‘*ta mathemata*’, literally, ‘those things which have been learned’. This was not just sums and times tables etc, but holistic learning, i.e. both the beginnings of mathematics as we think of it now and spiritual development, at the same time⁽¹⁾.

Nowadays ‘mathematics’ is defined not by intension but solely by extension, as a sprawling body of powerful, symbolic knowledge⁽²⁾, mainly the expansion of the ancient Greek ‘*logistike*’, the study of calculation. I am using ‘symbolic knowledge’ in the traditional way here, i.e. knowledge written in symbolic form. It would be more appropriate, given mathematical knowledge’s claim to precision, to describe it as being written in sign form. The difference between signs and symbols is that signs are in 1:1 correspondence with their meanings (as in road signs), whereas the relationship between the meanings of a symbol and the symbol form is many to one. Of course, mathematical signs are also symbols: one powerful example is ‘0’ for ‘zero’⁽²⁾.

Logistike was only one part of mathematics; the neglected parts include not only significant, purely abstract mathematical investigation, but also the other side of knowledge, the human knowing, the living essences of learning.

Pythagoras was primarily a philosopher (etymologically, ‘lover of wisdom’ (or ‘wise man of love’?)). His mathematics was embedded in an ontology where the natural numbers, ‘*arithmoi*’, were distinct entities from lines or magnitudes, ‘*megethoi*’, the measure in geometry. This distinction and the concern for a philosophical basis for mathematics began to crumble in the 3rd century AD when Diophantus included fractions as numbers. By the Renaissance, the notion of number had been so far stretched that irrationals, negatives, even imaginaries were accepted, as Girard said, ‘for their utility’. Pragmatism had come to stay.

Current CONVENTIONAL
 MATHEMATICS is a
 CONSTRUCTION of mainly LEFT
 BRAIN, QUANTITATIVE thought,
 based on VISUAL measure.
 Materialist science ANALYSES
 EXTERNAL realities and uses this
 mathematics with its STATIC,
 SOLID LOGIC to make machines
 that work EFFICIENTLY.



We HUMAN beings
 live QUALITATIVE
 MATHEMATICS.
 Our RIGHT BRAIN,
 HEART thinking
 perceives WHOLENESS
 We relate through
 our MULTISENSORY
 organisms to HETERO-
 GENEUS realities.
 Witnessing ourselves
 learning, we CHANGE
 and grow.



These overlap and are complementary

Mathematics is generally thought of as being separate from, even above ideology, but all our knowledge is human knowledge, subject to human motivations, and part of human belief systems. Euclid's 'Elements', the archetype of mathematical demonstration, begins with definitions, postulates and common notions that we agree to agree on, because words have to begin somewhere. These underlying ideas are beliefs which are subject to change; one example is the parallel postulate on the emergence of the non-Euclidean geometries in the late 19th century.

Pragmatism is a short term philosophy: it looks neither very deep, nor very far ahead. It is the philosophy of the sorcerer's apprentice: "Look! I can do it. It works!", then "Oh dear, I can't turn it off!" The lack of self-awareness in the pragmatism of materialist science has resulted in the self-deception that it is value-free when it is, in fact, founded on a belief which is not explicitly stated and for which there is no evidence, namely that matter precedes consciousness.

The first drawing shows the unquestioned, linear progress of materialist science, underpinned by left-brain, quantitative mathematics. This has given us today's sophisticated technology. We have a lot to thank it for. We also live with the unforeseen negative side effects.

Real mathematical development is qualitative: the emergence of new ideas upon which quantification may subsequently be imposed. Two crucial new mathematical ideas of the 17th century, the Cartesian number grid and subsequently the differential calculus, were simultaneously qualitative and quantitative. They created a global, quantitative structure for space, time and change leading, in the following 3 centuries, to the drive to quantify all ensuing new qualitative mathematical concepts wherever possible

It was the pragmatism of the extension of the concept of number which allowed the unexamined conflation of numbers and lines in the immensely powerful Cartesian concept of the number line.⁽⁴⁾ This spatial concept of measure, deriving from the sense of vision, is imposed upon time which has its own inherent very different kind of measure, related more naturally to hearing and music.

Rilke expresses it beautifully in his Sonnet to Orpheus (Part One, no.xii),

*“Heil dem Geist, der uns verbinden mag;
denn wir leben wahrhaft in Figuren.
Und mit kleinen Schritten gehn die Uhren
neben unserm eigentlichen Tag.*

Bless the spirit who loves to connect us;
for truly we live in shapes.
And the clocks in tiny steps
walk alongside our actual day.” (my translation)

Clock time, a homogeneous visual concept, is imposed on heterogeneous human reality. As an official in the National Institute of Standards and Technology in Boulder, USA, neatly said, “Our clocks do not measure time. Time is defined to be what our clocks measure”⁽⁵⁾ The desire to measure comes from the human desire to control, which is reasonable within reason, but is now irrational.

Vision not only gives us the beauty of the rose and the perfect solids, it is also the sense of control par excellence. Visual concepts dominate mathematics, as image dominates our cultural life. Seeing and hearing are different in nature: when we listen, we tune in; when we see, we see only surfaces. Humanity, hypnotised by its own image, is now alienated from its inner life.

The confused concept of the number line, and the ubiquitous dominance of vision over the inherent mathematica of other senses, have had a profound impact on the human psyche which has not been realised. One example is the spurious idea of linear progress, based on quantitative comparatives: in the mechanical world ‘better’ may be ‘bigger’ or ‘smaller’, but is always ‘faster’.

The differential calculus built not only on the Cartesian grid but also on the radically new algebraic idea of a variable rather than an unknown⁽⁶⁾. It is formidable but vital for us to attempt to imagine what mathematics was like before these core 17th century developments and even before them, the world-changing introduction of a sign for zero.

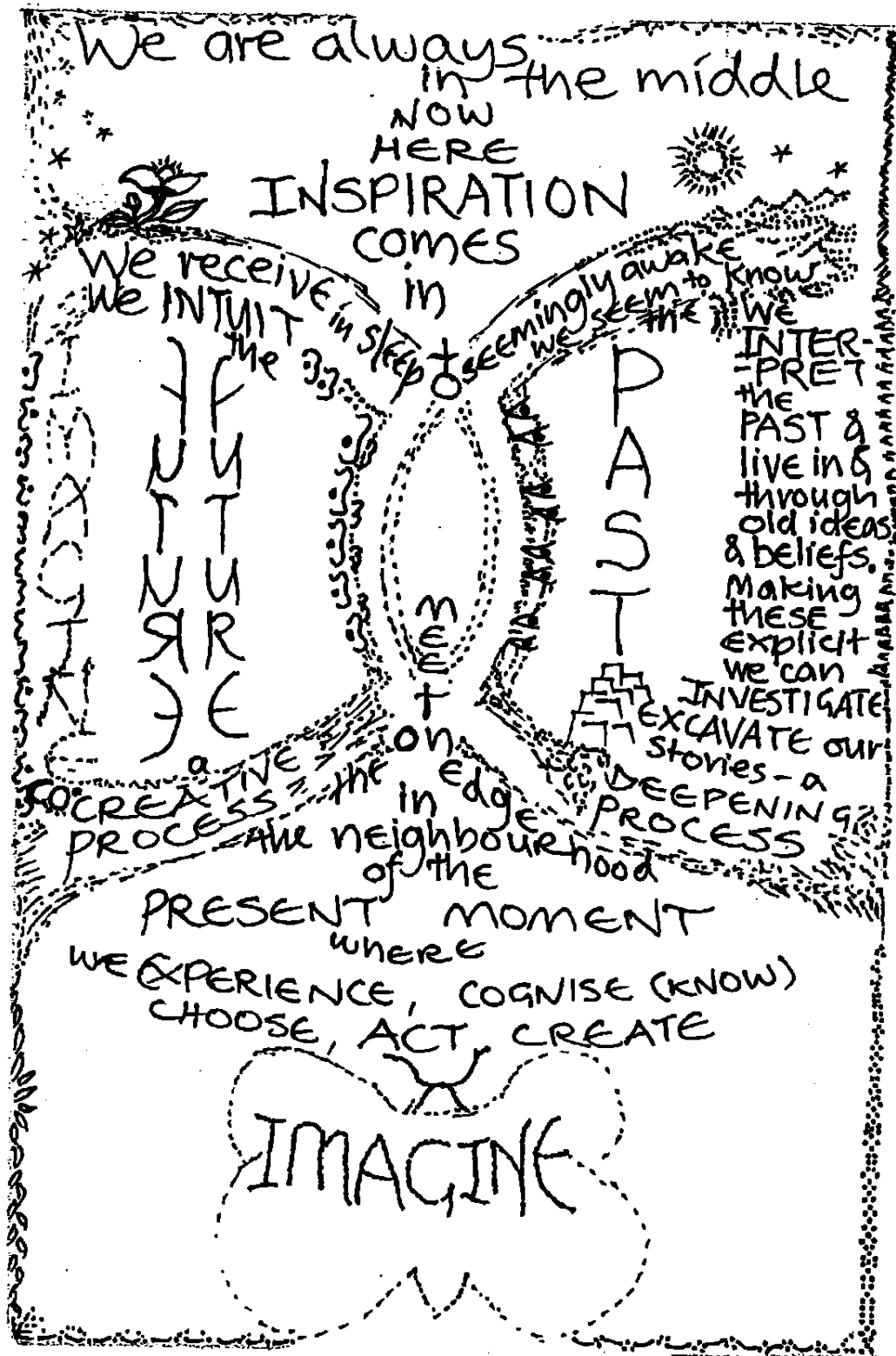
Much of the qualitative development in the following centuries has been algebraic. Qualitative, spatial mathematical developments only started to emerge with the non-Euclidean geometries. Before that Cartesian coordinate geometry overshadowed Desargues’ projective geometry.

Interestingly non-commutative algebraic structures share with real, experienced time the characteristic that order matters. These algebraic advances have often been absorbed into quantitative mathematical forms.

Quantitative number ('*hule*') was the subject of *logistike*. Its counterpart was *Arithmetike*, the study of the qualities or forms ('*eide*') of the integers, triangular, square, perfect etc. These descriptions of numbers' internal structures derive from *logistike*. There were also human associations which are alien to us today, 5 being the number of marriage etc.

Today's number theory continues the abstract mathematics of *arithmetike*. Yet the qualities of numbers investigated, remains those of the Greek period, primarily the primes. This work has yielded wonderful fruit bridging many other mathematical disciplines. Nevertheless we have not broadened our notion of the qualities of numbers to take into account the extended *logistike* now available. The particularities of the complex plane (for example, the infinite differentiability of holomorphic functions) are an aspect of 2-ness. The amount of material available, for example from research into higher dimensions of so many mathematical topics, is daunting but not impossible. We might even extend this enquiry to consider the different natures of the different number domains, integers, rationals, imaginaries, surreals etc.⁽⁷⁾

For centuries the other aspect of *arithmetike*, the human symbolic import of the integers, was relegated to numerology. Around the end of the 19th century more serious interest in this area arose again: C.S.Peirce, A.N.Whitehead, C.G.Jung, W.Pauli, M-L.von Franz, J.Bennett and others concerned themselves with the 'vast though vague ideas' (Peirce's words) contained in the natural numbers. A.Blake in 'Meaning of Numbers' (ANPA Proc. 31) gives a very useful overview of some of these approaches, and suggests various directions in which research might continue.



What might it mean to revive Pythagoras' and Theano's impulse now? On the one hand this would include the revival of *arithmetike* in symbolic mathematics, i.e. extending our present number theory to consider new qualities of the integers, and qualities of different number domains. The

other, human, spiritual, experiential aspect I am calling Human mathematics and trying out different definitions. At the moment I favour 'the human experience of symbolic mathematics and the mathematics inherent in human living'.

Note I am using 'symbolic' in the traditional way, since extant mathematics is symbolic, even though the symbols are normally understood only as signs. Conventional mathematics investigates the remarkable entities these signs indicate, as well as their interrelations, which are legion, and new entities continue to emerge. Human mathematics investigates the symbolic aspects, the human meanings of these entities and their relations.

Note also that mathematics is not necessarily symbolic. Maybe symbolic forms are a later mathematical development. Much of mathematics up until the Renaissance was written in words or abbreviations. For example, the 'equals' sign, '=', was only invented by Robert Recorde in 1557. Perhaps given that different areas of learning have different natures, the essences of learning, the appropriate mathematica, may take other forms. It remains to be seen.

Following Euclid's example I am explicating my guiding beliefs, my *koina*. (They are probably not as clear as I hope they will become; some may well be redundant. This is a work in progress.)

1. Human life is a great gift and opportunity.
2. We human beings are spiritual (conscious), multisensory (polyaesthetic) beings with an ability to change.
3. We do not understand our own nature, our own wholeness.
4. We are part of a greater wholeness.
5. We share this life, this universe with other beings whose nature and consciousness we do not understand.
6. Our self-awareness gives us the possibility to choose our actions. This gives us the responsibility to choose wisely.
7. We can only choose as wisely as our understanding allows.
8. Therefore we have a responsibility to increase and deepen our understanding of ourselves and our universe and the relationships between them.

I am also choosing to explicate the telos of Human mathematics, namely, to develop essences of understanding of this beautiful, living world in which we find ourselves, of ourselves and our place in this world, of other beings here and our relationships with them, and other

interrelationships, in order to create harmony in alignment with that of the wholeness of which we are part⁽⁸⁾.

Euclidean mathematics was embedded in a Platonic static world view. For this reason the telos did not need to be stated; it was to discover the mathematics of the eternal reality underlying the phenomenal world. Human mathematics begins from common sense, consensus reality, the world of time, change and choice, hence the necessity of explicating the telos.

The philosophy corresponding to this stance is phenomenology. Drawing (2) shows the phenomenological starting place (not 'point'), the discrete moment of awareness, symbolised here as the mandorla or vesica piscis formed by the two intersecting curves demarcating the past and future.

So what might all these fine words mean practically? Here is a list of some of the possible areas to be developed and researched. My ANPA 2010 paper, 'God be in my Heart and in my Thinking: towards self-awareness in mathematics and science' gives more details.

1 Explicit emphasis on the mathematical experience⁽⁹⁾ which complements the emphasis on results. (Process as well as product)

1.1 Self-awareness: the intention to develop self-awareness in the act of doing mathematics.

1.1.1 This includes witness awareness:

1.1.1.1 Awareness that we are part of the wholeness of life.

1.1.1.2 Awareness of our motivations, explicating our beliefs (from the past) and our directions and desires (towards the future)

1.1.2 It also implies whole human thinking, not just abstract head thinking, but also heart thinking, moral, visceral, multisensory, intuitive, imaginative thinking.

1.2 Ongoing clarification of the telos of human mathematics, including refining our senses and revisiting Platonic values.

1.2.1 Beauty: mathematics evokes a sense of wonder.

1.2.2 Truth: demonstration (*deiknymi*) clarifies and deepens our understanding; it helps us refine our sense of truth and reasoning.

1.2.3 Good: we refine our sense of the harmonies and wholenesses in life and develop abilities to create harmony as appropriate in different situations and different ways.

2 Development of mathematics from other senses than vision, for example:

2.1 Mathematics of gesture (kinaesthetic sense): eurythmy, Laban and Indian dance, embryology, social interactions etc.

2.2 Mathematics of hearing: music, rhythm, harmony, Vedic sounding etc.

2.3 mathematics of the interrelations of heterogeneous senses in the oneness of the self.

3 Development of mathematics from different areas of the external world, for example:

3.1 Ethnomathematics, the arts and crafts, knitting, weaving, pottery, architecture etc.

3.2 Studies of the natural world, biology, geology etc.

4 Investigating symbolic mathematics for human meaning, sacred geometry and music, meaning of numbers etc

5 Investigating mathematical language, for example:

5.1 Grammar: 3rd and 2nd person (for example Pogacnik cosmograms), prepositions, verbs, such as \geq , $=$, \rightarrow , functions etc.

5.2 Signs and symbols, logic and archetypes

These are not all separate and the list is not complete. A journey of a thousand miles begins with a single step. Clearly many people (only a few of whom I have mentioned) have already taken many steps on the path that I am calling here Human mathematics.

Notes

1. Interestingly 'spiritual' in German is 'geistig', which also means 'intellectual' (in Greek also, 'psyche' means 'mind' and 'soul'). We may speculate as to how mathematics and our understanding of it might have developed if Leibniz had received his due recognition (it is, of course, his calculus notation that we use, not Newton's) and if German had become the global embedding language for mathematics.
2. Some people cannot numb themselves to the power of the symbols and see them as mere signs; their situation is generally not recognised and they are effectively condemned to mathematical illiteracy.
3. See 'Who Carved Up the Integers? They Never Died', NGG , *Mathemagica* 1976, pp 13-26, for a more detailed history of the development of the concept of number, the reasons and some of the results.
4. Ibid, pp 21-23 for some of the deep, complex philosophical developments, mostly not explicit, involved in the emergence of the concept of the number line. See also 'God be in my Heart and in my Thinking: towards self-awareness in mathematics and science, ANPA Proc 2010 p.16.
5. Reported by Tim Folger, *Discover* magazine, June 2007, online http://discovermagazine.com/2007/jun/in-no-time/article_view?b_start:int=1
6. See 'Greek Mathematical Thought and the Origin of Algebra', J.Klein, Dover,1992, for the fascinating story of the development of algebraic concepts. See 'Who Carved...' pp.20-23 for an initial exploration of the intricate interrelationships of geometric and algebraic notions and the ideological changes implied in these developments.
7. Gerhard Kowol has begun this type of enquiry in 'Complex numbers – the harmonising element in mathematics', *Phlogiston Journal of the History of Science*, 1998, pp. 201-218.
8. Propositions are topological and significant in Human mathematics. The propositions underlined denote belonging and cooperation. Problems have arisen to the extent that Francis Bacon's dictum, 'Knowledge is power' has been interpreted as power over nature, rather than power to do. Human mathematics seeks to cooperate and collaborate.
9. P.J.Davis' and R.Hersh's thesis in their excellent 'The Mathematical Experience', Mariner Books, 1999, is precisely that living mathematics is not just dry results. I am saying that this needs to be explicit in our understanding of mathematics and this then leads further and deeper into mathematics both as powerful, symbolic knowledge and as a human developmental path.

Non-Commutative Worlds and Classical Constraints

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1 Introduction

Aspects of gauge theory, Hamiltonian mechanics, relativity and quantum mechanics arise naturally in the mathematics of a non-commutative framework for calculus and differential geometry. In this paper, we first give a review of our previous results about discrete physics and non-commutative worlds. The simplest discrete system corresponds directly to the square root of minus one, seen as an oscillation between one and minus one. This way thinking about i as an *iterant* is explained below. By starting with a discrete time series of positions, one has immediately a non-commutativity of observations since the measurement of velocity involves the tick of the clock and the measurement of position does not demand the tick of the clock. Commutators that arise from discrete observation suggest a non-commutative calculus, and this calculus leads to a generalization of standard advanced calculus in terms of a non-commutative world. In a non-commutative world, all derivatives are represented by commutators. We then give our version of Feynman-Dyson derivation of the formalism of electromagnetic gauge theory. The rest of the paper investigates algebraic constraints that bind the commutative and non-commutative worlds.

Section 2 is a self-contained miniature version of the whole story in this paper, starting with the square root of minus one seen as a discrete oscillation, a clock. We proceed from there and analyze the position of the square root of minus one in relation to discrete systems and quantum mechanics. We end this section by fitting together these observations into the structure of the Heisenberg commutator

$$[p, q] = i\hbar.$$

Section 3 is a review of the context of non-commutative worlds with a preview of part of the Feynman-Dyson derivation. This section generalizes the concepts in Section 2 and places them in the wider context of non-commutative worlds. The key to this generalization is our method of embedding discrete calculus into non-commutative calculus. Section 4 is a discussion of iterants

and matrix algebra. We show how matrix algebra in any dimension can be regarded as describing the pattern of acts of observation (time shifting operators corresponding to permutations) on periodic time series. Section 5 gives a complete treatment of our generalization of the Feynman-Dyson derivation of Maxwell's equations in a non-commutative framework. This section is the first foray into the consequences of constraints. This version of the Feynman-Dyson derivation depends entirely on the postulation of a full time derivative in the non-commutative world that matches the corresponding formula in ordinary commutative advanced calculus. Section 6 discusses constraints on non-commutative worlds that are imposed by asking for correspondences between forms of classical differentiation and the derivatives represented by commutators in a correspondent non-commutative world. This discussion of constraints parallels work of Tony Deakin [3, 4] and will be continued in joint work of the author and Deakin. At the level of the second constraint we encounter issues related to general relativity. Section 7 continues the constraints discussion in Section 5, showing how to generalize to higher-order constraints and obtains a commutator formula for the third order constraint. The first appendix, Section 8, is a very condensed review of the relationship of the Bianchi identity in differential geometry and the Einstein equations for general relativity. We then observe that every derivation in a non-commutative world comes equipped with its own Bianchi identity. This observation suggests one way to investigate general relativity in the non-commutative context. Section 9 is a philosophical appendix.

2 Quantum Mechanics - The Square Root of Minus One is a Clock

The purpose of this section is to place i , the square root of minus one, and its algebra in a context of discrete recursive systems. We begin by starting with a simple periodic process that is associated directly with the classical attempt to solve for i as a solution to a quadratic equation. We take the point of view that solving $x^2 = ax + b$ is the same (when $x \neq 0$) as solving

$$x = a + b/x,$$

and hence is a matter of finding a fixed point. In the case of i we have

$$x^2 = -1$$

and so desire a fixed point

$$x = -1/x.$$

There are no real numbers that are fixed points for this operator and so we consider the oscillatory process generated by

$$R(x) = -1/x.$$

The fixed point would satisfy

$$i = -1/i$$

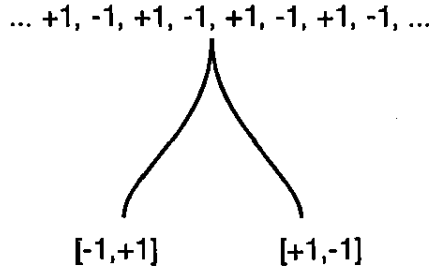


Figure 1: A Basic Oscillation

and multiplying, we get that

$$ii = -1.$$

On the other hand the iteration of R yields

$$1, R(1) = -1, R(R(1)) = +1, R(R(R(1))) = -1, +1, -1, +1, -1, \dots$$

The square root of minus one is a perfect example of an eigenform that occurs in a new and wider domain than the original context in which its recursive process arose. The process has no fixed point in the original domain.

Looking at the oscillation between +1 and -1, we see that there are naturally two phase-shifted viewpoints. We denote these two views of the oscillation by [+1, -1] and [-1, +1]. These viewpoints correspond to whether one regards the oscillation at time zero as starting with +1 or with -1. See Figure 1.

We shall let $I\{+1, -1\}$ stand for an undisclosed alternation or ambiguity between +1 and -1 and call $I\{+1, -1\}$ an iterant. There are two iterant views: [+1, -1] and [-1, +1].

Given an iterant $[a, b]$, we can think of $[b, a]$ as the same process with a shift of one time step. These two iterant views, seen as points of view of an alternating process, will become the square roots of negative unity, i and $-i$.

We introduce a temporal shift operator η such that

$$[a, b]\eta = \eta[b, a]$$

and

$$\eta\eta = 1$$

for any iterant $[a, b]$, so that concatenated observations can include a time step of one-half period of the process

$$\dots abababab \dots$$

We combine iterant views term-by-term as in

$$[a, b][c, d] = [ac, bd].$$

We now define i by the equation

$$i = [1, -1]\eta.$$

This makes i both a value and an operator that takes into account a step in time.

We calculate

$$ii = [1, -1]\eta[1, -1]\eta = [1, -1][-1, 1]\eta\eta = [-1, -1] = -1.$$

Thus we have constructed the square root of minus one by using an iterant viewpoint. In this view i represents a discrete oscillating temporal process and it is an eigenform for $R(x) = -1/x$, participating in the algebraic structure of the complex numbers. In fact the corresponding algebra structure of linear combinations $[a, b] + [c, d]\eta$ is isomorphic with 2×2 matrix algebra and iterants can be used to construct $n \times n$ matrix algebra. We treat this generalization elsewhere [?, ?, ?].

The Temporal Nexus. *We take as a matter of principle that the usual real variable t for time is better represented as it so that time is seen to be a process, an observation and a magnitude all at once.* This principle of “imaginary time” is justified by the eigenform approach to the structure of time and the structure of the square root of minus one.

As an example of the use of the Temporal Nexus, consider the expression $x^2 + y^2 + z^2 + t^2$, the square of the Euclidean distance of a point (x, y, z, t) from the origin in Euclidean four-dimensional space. Now replace t by it , and find

$$x^2 + y^2 + z^2 + (it)^2 = x^2 + y^2 + z^2 - t^2,$$

the squared distance in hyperbolic metric for special relativity. By replacing t by its process operator value it we make the transition to the physical mathematics of special relativity.

2.1 Quantum Physics, Eigenvalue and Eigenform

In quantum modeling, the state of a physical system is represented by a vector in a Hilbert space. As time goes on the vector undergoes a unitary evolution in the Hilbert space. Observable quantities correspond to Hermitian operators H and vectors v that have the property that the application of H to v results in a new vector that is a multiple of v by a real factor λ . Thus

$$Hv = \lambda v.$$

One says that v is an eigenvector for the operator H , and that λ is the eigenvalue.

2.2 The Wave Function in Quantum Mechanics and The Square Root of Minus One

One can regard a wave function such as $\psi(x, t) = \exp(i(kx - wt))$ as containing a micro-oscillatory system with the special synchronizations of the iterant view $i = [+1, -1]\eta$. It is these synchronizations that make the big eigenform of the exponential work correctly with respect to

differentiation, allowing it to create the appearance of rotational behaviour, wave behaviour and the semblance of the continuum. In other words, we are suggesting that once can take a temporal view of the well-known equation of Euler:

$$e^{i\theta} = \cos(\theta) + i\sin(\theta)$$

by regarding the i in this equation as an iterant, as discrete oscillation between -1 and $+1$. One can blend the classical geometrical view of the complex numbers with the iterant view by thinking of a point that orbits the origin of the complex plane, intersecting the real axis periodically and producing, in the real axis, a periodic oscillation in relation to its orbital movement in the two dimensional space. The special synchronization is the algebra of the time shift embodied in

$$\eta\eta = 1$$

and

$$[a, b]\eta = \eta[b, a]$$

that makes the algebra of $i = [1, -1]\eta$ imply that $i^2 = -1$. This interpretation does not change the formalism of these complex-valued functions, but it does change one's point of view and we now show how the properties of i as a discrete dynamical system are found in any such system.

2.3 Time Series and Discrete Physics

We have just reformulated the complex numbers and expanded the context of matrix algebra to an interpretation of i as an oscillatory process and matrix elements as combined spatial and temporal oscillatory processes (in the sense that $[a, b]$ is not affected in its order by a time step, while $[a, b]\eta$ includes the time dynamic in its interactive capability, and 2×2 matrix algebra is the algebra of iterant views $[a, b] + [c, d]\eta$).

We now consider elementary discrete physics in one dimension. Consider a time series of positions

$$x(t) : t = 0, \Delta t, 2\Delta t, 3\Delta t, \dots$$

We can define the velocity $v(t)$ by the formula

$$v(t) = (x(t + \Delta t) - x(t))/\Delta t = Dx(t)$$

where D denotes this discrete derivative. In order to obtain $v(t)$ we need at least one tick Δt of the discrete clock. Just as in the iterant algebra, we need a time-shift operator to handle the fact that once we have observed $v(t)$, the time has moved up by one tick.

We adjust the discrete derivative. We shall add an operator J that in this context accomplishes the time shift:

$$x(t)J = Jx(t + \Delta t).$$

We then redefine the derivative to include this shift:

$$Dx(t) = J(x(t + \Delta t) - x(t))/\Delta t.$$

This readjustment of the derivative rewrites it so that the temporal properties of successive observations are handled automatically.

Discrete observations do not commute. Let A and B denote quantities that we wish to observe in the discrete system. Let AB denote the result of first observing B and then observing A . The result of this definition is that a successive observation of the form $x(Dx)$ is distinct from an observation of the form $(Dx)x$. In the first case, we first observe the velocity at time t , and then x is measured at $t + \Delta t$. In the second case, we measure x at t and then measure the velocity.

We measure the difference between these two results by taking a commutator

$$[A, B] = AB - BA$$

and we get the following computations where we write $\Delta x = x(t + \Delta t) - x(t)$.

$$x(Dx) = x(t)J(x(t + \Delta t) - x(t)) = Jx(t + \Delta t)(x(t + \Delta t) - x(t)).$$

$$(Dx)x = J(x(t + \Delta t) - x(t))x(t).$$

$$[x, Dx] = x(Dx) - (Dx)x = (J/\Delta t)(x(t + \Delta t) - x(t))^2 = J(\Delta x)^2/\Delta t$$

This final result is worth recording:

$$[x, Dx] = J(\Delta x)^2/\Delta t.$$

From this result we see that the commutator of x and Dx will be constant if $(\Delta x)^2/\Delta t = K$ is a constant. For a given time-step, this means that

$$(\Delta x)^2 = K\Delta t$$

so that

$$\Delta x = \pm\sqrt{(K\Delta t)}$$

This is a Brownian process with diffusion constant equal to K .

Thus we arrive at the result that any discrete process viewed in this framework of discrete observation has the basic commutator

$$[x, Dx] = J(\Delta x)^2/\Delta t,$$

generalizing a Brownian process and containing the factor $(\Delta x)^2/\Delta t$ that corresponds to the classical diffusion constant. It is worth noting that the adjustment that we have made to the discrete derivative makes it into a commutator as follows:

$$Dx(t) = J(x(t + \Delta t) - x(t))/\Delta t = (x(t)J - Jx(t))\Delta t = [x(t), J]/\Delta t.$$

By replacing discrete derivatives by commutators we can express discrete physics in many variables in a context of non-commutative algebra. We enter this generalization in the next section of the paper.

We now use the temporal nexus (the square root of minus one as a clock) and rewrite these commutators to match quantum mechanics.

2.4 Simplicity and the Heisenberg Commutator

Finally, we arrive at the simplest place. Time and the square root of minus one are inseparable in the temporal nexus. The square root of minus one is a symbol and algebraic operator for the simplest oscillatory process. As a symbolic form, i is an eigenform satisfying the equation

$$i = -1/i.$$

One does not have an increment of time all alone as in classical t . One has it , a combination of an interval and the elemental dynamic that is time. With this understanding, we can return to the commutator for a discrete process and use it for the temporal increment.

We found that discrete observation led to the commutator equation

$$[x, Dx] = J(\Delta x)^2/\Delta t$$

which we will simplify to

$$[q, p/m] = (\Delta x)^2/\Delta t.$$

taking q for the position x and p/m for velocity, the time derivative of position and ignoring the time shifting operator on the right hand side of the equation.

Understanding that Δt should be replaced by $i\Delta t$, and that, by comparison with the physics of a process at the Planck scale one can take

$$(\Delta x)^2/\Delta t = \hbar/m,$$

we have

$$[q, p/m] = (\Delta x)^2/i\Delta t = -i\hbar/m,$$

whence

$$[p, q] = i\hbar,$$

and we have arrived at Heisenberg's fundamental relationship between position and momentum. This mode of arrival is predicated on the recognition that only it represents a true interval of time. In the notion of time there is an inherent clock or an inherent shift of phase that is making a synchrony in our ability to observe, a precise dynamic beneath the apparent dynamic of the observed process. Once this substitution is made, once the correct imaginary value is placed in the temporal circuit, the patterns of quantum mechanics appear. In this way, quantum mechanics can be seen to emerge from the discrete.

The problem that we have examined in this section is the problem to understand the nature of quantum mechanics. In fact, we hope that the problem is seen to disappear the more we enter into the present viewpoint. A viewpoint is only on the periphery. The iterant from which the viewpoint emerges is in a superposition of indistinguishables, and can only be approached by varying the viewpoint until one is released from the particularities that a point of view contains.

3 Review of Non-Commutative Worlds

Now we begin the introduction to non-commutative worlds and a general discrete calculus. Our approach begins in an algebraic framework that naturally contains the formalism of the calculus, but not its notions of limits or constructions of spaces with specific locations, points and trajectories. Many patterns of physical law fit well into such an abstract framework. In this viewpoint one dispenses with continuum spacetime and replaces it by algebraic structure. Behind that structure, space stands ready to be constructed, by discrete derivatives and patterns of steps, or by starting with a discrete pattern in the form of a diagram, a network, a lattice, a knot, or a simplicial complex, and elaborating that structure until the specificity of spatio-temporal locations appear.

Poisson brackets allow one to connect classical notions of location with the non-commutative algebra used herein. Below the level of the Poisson brackets is a treatment of processes and operators as though they were variables in the same context as the variables in the classical calculus. In different degrees one lets go of the notion of classical variables and yet retains their form, as one makes a descent into the discrete. The discrete world of non-commutative operators is a world linked to our familiar world of continuous and commutative variables. This linkage is traditionally exploited in quantum mechanics to make the transition from the classical to the quantum. One can make the journey in the other direction, from the discrete and non-commutative to the "classical" and commutative, but that journey requires powers of invention and ingenuity that are the subject of this exploration. It is our conviction that the world is basically simple. To find simplicity in the complex requires special attention and care.

In starting from a discrete point of view one thinks of a sequence of states of the world S, S', S'', S''', \dots where S' denotes the state succeeding S in discrete time. It is natural to suppose that there is some measure of difference $DS^{(n)} = S^{(n+1)} - S^{(n)}$, and some way that states S and T might be combined to form a new state ST . We can thus think of world-states as operators in a non-commutative algebra with a temporal derivative $DS = S' - S$. At this bare level of the formalism the derivative does not satisfy the Leibniz rule. In fact it is easy to verify that $D(ST) = D(S)T + S'D(T)$. Remarkably, the Leibniz rule, and hence the formalisms of Newtonian calculus can be restored with the addition of one more operator J . In this instance J is a temporal shift operator with the property that $SJ = JS'$ for any state S . We then see that if $\nabla S = JD(S) = J(S' - S)$, then $\nabla(ST) = \nabla(S)T + S\nabla(T)$ for any states S and T . In fact $\nabla(S) = JS' - JS = SJ - JS = [S, J]$, so that this adjusted derivative is a commutator in the general calculus of states. This, in a nutshell, is our approach to non-commutative worlds. We begin with a very general framework that is a non-numerical calculus of states and operators. It is then fascinating and a topic of research to see how physics and mathematics fit into the frameworks so constructed.

A simplest and fundamental instance of these ideas is seen in the structure of $i = \sqrt{-1}$. We view i as an *iterant* (see Section 4), a discrete elementary dynamical system repeating in time the values $\{\dots -1, +1, -1, +1, \dots\}$. One can think of this system as resulting from the attempt to solve $i^2 = -1$ in the form $i = -1/i$. Then one iterates the transformation $x \rightarrow -1/x$ and

finds the oscillation from a starting value of $+1$ or -1 . In this sense i is identical in concept to a *primordial time*. Furthermore the algebraic structure of the complex numbers emerges from two conjugate views of this discrete series as $[-1, +1]$ and $[+1, -1]$. We introduce a temporal shift operator η such that $\eta[-1, +1] = [+1, -1]\eta$ and $\eta^2 = 1$ (sufficient to this purpose). Then we can define $i = [-1, +1]\eta$, endowing it with one view of the discrete oscillation and the sensitivity to shift the clock when interacting with itself or with another operator. See Sections 2 and 4 for the details of this reconstruction of the complex numbers. The point of the reconstruction for our purposes is that i becomes inextricably identified with elemental time, and so the physical substitution of it for t (Wick rotation) becomes, in this epistemology, an act of recognition of the nature of time.

Constructions are performed in a Lie algebra \mathcal{A} . One may take \mathcal{A} to be a specific matrix Lie algebra, or abstract Lie algebra. If \mathcal{A} is taken to be an abstract Lie algebra, then it is convenient to use the universal enveloping algebra so that the Lie product can be expressed as a commutator. In making general constructions of operators satisfying certain relations, it is understood that one can always begin with a free algebra and make a quotient algebra where the relations are satisfied.

On \mathcal{A} , a variant of calculus is built by defining derivations as commutators (or more generally as Lie products). For a fixed N in \mathcal{A} one defines

$$\nabla_N : \mathcal{A} \longrightarrow \mathcal{A}$$

by the formula

$$\nabla_N F = [F, N] = FN - NF.$$

∇_N is a derivation satisfying the Leibniz rule.

$$\nabla_N(FG) = \nabla_N(F)G + F\nabla_N(G).$$

Discrete Derivatives are Replaced by Commutators. There are many motivations for replacing derivatives by commutators. If $f(x)$ denotes (say) a function of a real variable x , and $\tilde{f}(x) = f(x+h)$ for a fixed increment h , define the *discrete derivative* Df by the formula $Df = (\tilde{f} - f)/h$, and find that the Leibniz rule is not satisfied. One has the basic formula for the discrete derivative of a product:

$$D(fg) = D(f)g + \tilde{f}D(g).$$

Correct this deviation from the Leibniz rule by introducing a new non-commutative operator J with the property that

$$fJ = J\tilde{f}.$$

Define a new discrete derivative in an extended non-commutative algebra by the formula

$$\nabla(f) = JD(f).$$

It follows at once that

$$\nabla(fg) = JD(f)g + J\tilde{f}D(g) = JD(f)g + fJD(g) = \nabla(f)g + f\nabla(g).$$

Note that

$$\nabla(f) = (J\tilde{f} - Jf)/h = (fJ - Jf)/h = [f, J/h].$$

In the extended algebra, discrete derivatives are represented by commutators, and satisfy the Leibniz rule. One can regard discrete calculus as a subset of non-commutative calculus based on commutators.

Advanced Calculus and Hamiltonian Mechanics or Quantum Mechanics in a Non-Commutative World. In \mathcal{A} there are as many derivations as there are elements of the algebra, and these derivations behave quite wildly with respect to one another. If one takes the concept of *curvature* as the non-commutation of derivations, then \mathcal{A} is a highly curved world indeed. Within \mathcal{A} one can build a tame world of derivations that mimics the behaviour of flat coordinates in Euclidean space. The description of the structure of \mathcal{A} with respect to these flat coordinates contains many of the equations and patterns of mathematical physics.

The flat coordinates Q^i satisfy the equations below with the P_j chosen to represent differentiation with respect to Q^j :

$$\begin{aligned} [Q^i, Q^j] &= 0 \\ [P_i, P_j] &= 0 \\ [Q^i, P_j] &= \delta_{ij}. \end{aligned}$$

Here δ_{ij} is the Kronecker delta, equal to 1 when $i = j$ and equal to 0 otherwise. Derivatives are represented by commutators.

$$\begin{aligned} \partial_i F &= \partial F / \partial Q^i = [F, P_i], \\ \hat{\partial}_i F &= \partial F / \partial P_i = [Q^i, F]. \end{aligned}$$

Our choice of commutators guarantees that the derivative of a variable with respect to itself is one and that the derivative of a variable with respect to a distinct variable is zero. Furthermore, the commuting of the variables with one another guarantees that mixed partial derivatives are independent of the order of differentiation. This is a flat non-commutative world.

Temporal derivative is represented by commutation with a special (Hamiltonian) element H of the algebra:

$$dF/dt = [F, H].$$

(For quantum mechanics, take $i\hbar dA/dt = [A, H]$.) These non-commutative coordinates are the simplest flat set of coordinates for description of temporal phenomena in a non-commutative world.

Hamilton's Equations are Part of the Mathematical Structure of Non-Commutative Advanced Calculus.

$$\begin{aligned} dP_i/dt &= [P_i, H] = -[H, P_i] = -\partial H / \partial Q^i \\ dQ^i/dt &= [Q^i, H] = \partial H / \partial P_i. \end{aligned}$$

These are exactly Hamilton's equations of motion. The pattern of Hamilton's equations is built into the system.

The Simplest Time Series Leads to the Diffusion Constant and Heisenberg's Commuator.
 Consider a time series $\{Q, Q', Q'', \dots\}$ with commuting scalar values. Let

$$\dot{Q} = \nabla Q = JDQ = J(Q' - Q)/\tau$$

where τ is an elementary time step (If Q denotes a times series value at time t , then Q' denotes the value of the series at time $t + \tau$). The shift operator J is defined by the equation $QJ = JQ'$ where this refers to any point in the time series so that $Q^{(n)}J = JQ^{(n+1)}$ for any non-negative integer n . Moving J across a variable from left to right, corresponds to one tick of the clock. This discrete, non-commutative time derivative satisfies the Leibniz rule.

This derivative ∇ also fits a significant pattern of discrete observation. Consider the act of observing Q at a given time and the act of observing (or obtaining) DQ at a given time. Since Q and Q' are ingredients in computing $(Q' - Q)/\tau$, the numerical value associated with DQ , it is necessary to let the clock tick once, Thus, if one first observe Q and then obtains DQ , the result is different (for the Q measurement) if one first obtains DQ , and then observes Q . In the second case, one finds the value Q' instead of the value Q , due to the tick of the clock.

1. Let $\dot{Q}Q$ denote the sequence: observe Q , then obtain \dot{Q} .
2. Let $Q\dot{Q}$ denote the sequence: obtain \dot{Q} , then observe Q .

The commutator $[Q, \dot{Q}]$ expresses the difference between these two orders of discrete measurement. In the simplest case, where the elements of the time series are commuting scalars, one has

$$[Q, \dot{Q}] = Q\dot{Q} - \dot{Q}Q = J(Q' - Q)^2/\tau.$$

Thus one can interpret the equation

$$[Q, \dot{Q}] = Jk$$

(k a constant scalar) as

$$(Q' - Q)^2/\tau = k.$$

This means that the process is a walk with spatial step

$$\Delta = \pm\sqrt{k\tau}$$

where k is a constant. In other words, one has the equation

$$k = \Delta^2/\tau.$$

This is the diffusion constant for a Brownian walk. A walk with spatial step size Δ and time step τ will satisfy the commutator equation above exactly when the square of the spatial step divided by the time step remains constant. This shows that the diffusion constant of a Brownian process is a structural property of that process, independent of considerations of probability and continuum limits.

Thus we can write (ignoring the timeshift operator J)

$$[Q, \dot{Q}] = (\Delta Q)^2/\tau.$$

If we work with physics at the Planck scale, then we can take τ as the Planck time and ΔQ as the Planck length. Then

$$(\Delta Q)^2/\tau = \hbar/m$$

where m is the Planck mass. However, we shall also Wick rotate the time from τ to $i\tau$ justifying $i\tau$ on the principle (described above) that τ should be multiplied by i to bring time into coincidence with an elemental time that is both a temporal operator (i) and a value (t). With this we obtain

$$[Q, \dot{Q}] = -i\hbar/m$$

or

$$[m\dot{Q}, Q] = i\hbar,$$

and taking $P = m\dot{Q}$, we have finally

$$[P, Q] = i\hbar.$$

Heisenberg's commutator for quantum mechanics is seen in the nexus of discrete physics and imaginary time.

Schroedinger's Equation is Discrete. Here is how the Heisenberg form of Schroedinger's equation fits in this context. Let $J = (1 + i\hbar H\Delta t)$. Then $\nabla\psi = [\psi, J/\Delta t]$, and we calculate

$$\nabla\psi = \psi[(1 + i\hbar H\Delta t)/\Delta t] - [(1 + i\hbar H\Delta t)/\Delta t]\psi = i\hbar[\psi, H].$$

This is exactly the form of the Heisenberg equation.

Dynamical Equations Generalize Gauge Theory and Curvature. One can take the general dynamical equation in the form

$$dQ^i/dt = \mathcal{G}_i$$

where $\{\mathcal{G}_1, \dots, \mathcal{G}_d\}$ is a collection of elements of \mathcal{A} . Write \mathcal{G}_i relative to the flat coordinates via $\mathcal{G}_i = P_i - A_i$. This is a definition of A_i and $\partial F/\partial Q^i = [F, P_i]$. The formalism of gauge theory appears naturally. In particular, if

$$\nabla_i(F) = [F, \mathcal{G}_i],$$

then one has the curvature

$$[\nabla_i, \nabla_j]F = [R_{ij}, F]$$

and

$$R_{ij} = \partial_i A_j - \partial_j A_i + [A_i, A_j].$$

This is the well-known formula for the curvature of a gauge connection. Aspects of geometry arise naturally in this context, including the Levi-Civita connection (which is seen as a consequence of the Jacobi identity in an appropriate non-commutative world).

One can consider the consequences of the commutator $[Q^i, \dot{Q}^j] = g_{ij}$, deriving that

$$\ddot{Q}^r = G_r + F_{rs}\dot{Q}^s + \Gamma_{rst}\dot{Q}^s\dot{Q}^t,$$

where G_r is the analogue of a scalar field, F_{rs} is the analogue of a gauge field and Γ_{rst} is the Levi-Civita connection associated with g_{ij} . This decomposition of the acceleration is uniquely determined by the given framework.

Non-commutative Electromagnetism and Gauge Theory. One can use this context to revisit the Feynman-Dyson derivation of electromagnetism from commutator equations, showing that most of the derivation is independent of any choice of commutators, but highly dependent upon the choice of definitions of the derivatives involved. Without any assumptions about initial commutator equations, but taking the right (in some sense simplest) definitions of the derivatives one obtains a significant generalization of the result of Feynman-Dyson. We give this derivation in Section 5 of the present paper, using diagrammatic algebra to clarify the structure. In this derivation we use X to denote the position vector rather than Q , as above.

Theorem With the appropriate [see below] definitions of the operators, and taking

$$\nabla^2 = \partial_1^2 + \partial_2^2 + \partial_3^2, \quad H = \dot{X} \times \dot{X} \quad \text{and} \quad E = \partial_t \dot{X}, \quad \text{one has}$$

1. $\ddot{X} = E + \dot{X} \times H$
2. $\nabla \cdot H = 0$
3. $\partial_t H + \nabla \times E = H \times H$
4. $\partial_t E - \nabla \times H = (\partial_t^2 - \nabla^2)\dot{X}$

The key to the proof of this Theorem is the definition of the time derivative. This definition is as follows

$$\partial_t F = \dot{F} - \sum_i \dot{X}_i \partial_i (F) = \dot{F} - \sum_i \dot{X}_i [F, \dot{X}_i]$$

for all elements or vectors of elements F . The definition creates a distinction between space and time in the non-commutative world. A calculation reveals that

$$\ddot{X} = \partial_t \dot{X} + \dot{X} \times (\dot{X} \times \dot{X}).$$

This suggests taking $E = \partial_t \dot{X}$ as the electric field, and $B = \dot{X} \times \dot{X}$ as the magnetic field so that the Lorentz force law

$$\ddot{X} = E + \dot{X} \times B$$

is satisfied.

This result is applied to produce many discrete models of the Theorem. These models show that, just as the commutator $[X, \dot{X}] = Jk$ describes Brownian motion in one dimension, a generalization of electromagnetism describes the interaction of triples of time series in three dimensions.

Taking $\partial_t F = \dot{F} - \sum_i \dot{X}_i \partial_i(F) = \dot{F} - \sum_i \dot{X}_i [F, \dot{X}_i]$ as a definition of the partial derivative with respect to time is a natural move in this context because there is *no time variable* t in this non-commutative world. A formal move of this kind, matching a pattern from the commutative world to the mathematics of the non-commutative world is the theme of the Section 6 of this paper. In that section we consider the well known way to associate an operator to a product of commutative variables by taking a sum over all permutations of products of the operators corresponding to the individual variables. This provides a way to associate operator expressions with expressions in the commutative algebra, and hence to let a classical world correspond or map to a non-commutative world. To bind these worlds more closely, we can ask that the formulas for taking derivatives in the commutative world should have symmetrized operator product correspondences in the non-commutative world. In Section 6 we show how the resulting constraints are related to having a quadratic Hamiltonian (first order constraint) and to having a version of general relativity [3, 4] (second order constraint). Such constraints can be carried to all orders of derivatives, but the algebra of such constraints is, at the present time, in a very primitive state. We discuss some of the complexities of the constraint algebra in the Appendix to this paper.

In Section 7 we discuss the relationship of the Bianchi identity in non-commutative worlds and its role in the classical derivation of Einstein's equations. This suggests other avenues of relationship between general relativity and non-commutative worlds. The reader may well ask at this point if we propose quantum gravity via this framework. In our judgement, it is too early to tell.

Remark. While there is a large literature on non-commutative geometry, emanating from the idea of replacing a space by its ring of functions, work discussed herein is not written in that tradition. Non-commutative geometry does occur here, in the sense of geometry occurring in the context of non-commutative algebra. Derivations are represented by commutators. There are relationships between the present work and the traditional non-commutative geometry, but that is a subject for further exploration. In no way is this paper intended to be an introduction to that subject. The present summary is based on [15, 17, 18, 19, 20, 21, 22, 23, 24, 25] and the references cited therein.

The following references in relation to non-commutative calculus are useful in comparing with the present approach [2, 5, 7, 29]. Much of the present work is the fruit of a long series of discussions with Pierre Noyes. paper [27] also works with minimal coupling for the Feynman-Dyson derivation. The first remark about the minimal coupling occurs in the original paper by Dyson [1], in the context of Poisson brackets. The paper [8] is worth reading as a companion to Dyson. It is the purpose of this summary to indicate how non-commutative calculus can be used in foundations.

4 Iterants, Discrete Processes and Matrix Algebra

The primitive idea behind an iterant is a periodic time series or “waveform”

$$\dots abababababab \dots$$

The elements of the waveform can be any mathematically or empirically well-defined objects. We can regard the ordered pairs $[a, b]$ and $[b, a]$ as abbreviations for the waveform or as two points of view about the waveform (a first or b first). Call $[a, b]$ an *iterant*. One has the collection of transformations of the form $T[a, b] = [ka, k^{-1}b]$ leaving the product ab invariant. This tiny model contains the seeds of special relativity, and the iterants contain the seeds of general matrix algebra! Since this paper has been a combination of discussions of non-commutativity and time series, we include this appendix on iterants. A more complete discussion will appear elsewhere. For related discussion see [9, 10, 11, 12, 13, 14, 16, 28].

Define products and sums of iterants as follows

$$[a, b][c, d] = [ac, bd]$$

and

$$[a, b] + [c, d] = [a + c, b + d].$$

The operation of juxtaposition is multiplication while $+$ denotes ordinary addition in a category appropriate to these entities. These operations are natural with respect to the structural juxtaposition of iterants:

$$\dots abababababab \dots$$

$$\dots cdcdcdcdcdcd \dots$$

Structures combine at the points where they correspond. Waveforms combine at the times where they correspond. Iterants combine in juxtaposition.

If \bullet denotes any form of binary composition for the ingredients (a, b, \dots) of iterants, then we can extend \bullet to the iterants themselves by the definition $[a, b] \bullet [c, d] = [a \bullet c, b \bullet d]$. In this section we shall first apply this idea to Lorentz transformations, and then generalize it to other contexts.

So, to work: We have

$$[t - x, t + x] = [t, t] + [-x, x] = t[1, 1] + x[-1, 1].$$

Since $[1, 1][a, b] = [1a, 1b] = [a, b]$ and $[0, 0][a, b] = [0, 0]$, we shall write

$$1 = [1, 1]$$

and

$$0 = [0, 0].$$

Let

$$\sigma = [-1, 1].$$

σ is a significant iterant that we shall refer to as a *polarity*. Note that

$$\sigma\sigma = 1.$$

Note also that

$$[t - x, t + x] = t + x\sigma.$$

Thus the points of spacetime form an algebra analogous to the complex numbers whose elements are of the form $t + x\sigma$ with $\sigma\sigma = 1$ so that

$$(t + x\sigma)(t' + x'\sigma) = tt' + xx' + (tx' + xt')\sigma.$$

In the case of the Lorentz transformation it is easy to see the elements of the form $[k, k^{-1}]$ translate into elements of the form

$$T(v) = [(1 + v)/\sqrt{(1 - v^2)}, (1 - v)/\sqrt{(1 - v^2)}] = [k, k^{-1}].$$

Further analysis shows that v is the relative velocity of the two reference frames in the physical context. Multiplication now yields the usual form of the Lorentz transform

$$\begin{aligned} T_k(t + x\sigma) &= T(v)(t + x\sigma) \\ &= (1/\sqrt{(1 - v^2)} - v\sigma/\sqrt{(1 - v^2)})(t + x\sigma) \\ &= (t - xv)/\sqrt{(1 - v^2)} + (x - vt)\sigma/\sqrt{(1 - v^2)} \\ &= t' + x'\sigma. \end{aligned}$$

The algebra that underlies this iterant presentation of special relativity is a relative of the complex numbers with a special element σ of square one rather than minus one ($i^2 = -1$ in the complex numbers).

The appearance of a square root of minus one unfolds naturally from iterant considerations. Define the "shift" operator η on iterants by the equation

$$\eta[a, b] = [b, a]\eta$$

with $\eta^2 = 1$. Sometimes it is convenient to think of η as a delay operator, since it shifts the waveform ...*ababab*... by one internal time step. Now define

$$i = [-1, 1]\eta$$

We see at once that

$$ii = [-1, 1]\eta[-1, 1]\eta = [-1, 1][1, -1]\eta^2 = [-1, 1][1, -1] = [-1, -1] = -1.$$

Thus

$$ii = -1.$$

Here we have described i in a *new* way as the superposition of the waveform $\epsilon = [-1, 1]$ and the temporal shift operator η . By writing $i = \epsilon\eta$ we recognize an active version of the waveform that shifts temporally when it is observed. This theme of including the result of time in observations of a discrete system occurs at the foundation of our construction.

4.1 MATRIX ALGEBRA VIA ITERANTS

Matrix algebra has some strange wisdom built into its very bones. Consider a two dimensional periodic pattern or “waveform.”

.....
 ...*abababababababab*...
 ...*cdcdcdcdcdcdcdcd*...
 ...*abababababababab*...
 ...*cdcdcdcdcdcdcdcd*...
 ...*abababababababab*...

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}, \begin{pmatrix} b & a \\ d & c \end{pmatrix}, \begin{pmatrix} c & d \\ a & b \end{pmatrix}, \begin{pmatrix} d & c \\ b & a \end{pmatrix}$$

Above are some of the matrices apparent in this array. Compare the matrix with the “two dimensional waveform” shown above. A given matrix freezes out a way to view the infinite waveform. In order to keep track of this patterning, lets write

$$[a, d] + [b, c]\eta = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

where

$$[x, y] = \begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix}.$$

and

$$\eta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

The four matrices that can be framed in the two-dimensional wave form are all obtained from the two iterants $[a, d]$ and $[b, c]$ via the shift operation $\eta[x, y] = [y, x]\eta$ which we shall denote by an overbar as shown below

$$\overline{[x, y]} = [y, x].$$

Letting $A = [a, d]$ and $B = [b, c]$, we see that the four matrices seen in the grid are

$$A + B\eta, B + A\eta, \bar{B} + \bar{A}\eta, \bar{A} + \bar{B}\eta.$$

The operator η has the effect of rotating an iterant by ninety degrees in the formal plane. Ordinary matrix multiplication can be written in a concise form using the following rules:

$$\begin{aligned}\eta\eta &= 1 \\ \eta Q &= \bar{Q}\eta\end{aligned}$$

where Q is any two element iterant.

For example, let $\epsilon = [-1, 1]$ so that $\bar{\epsilon} = -\epsilon$ and $\epsilon\epsilon = [1, 1] = 1$. Let

$$i = \epsilon\eta.$$

Then

$$ii = \epsilon\eta\epsilon\eta = \bar{\epsilon}\bar{\epsilon}\eta\eta = \epsilon(-\epsilon) = -\epsilon\epsilon = -1.$$

We have reconstructed the square root of minus one in the form of the matrix

$$i = \epsilon\eta = [-1, 1]\eta = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

More generally, we see that

$$(A + B\eta)(C + D\eta) = (AC + B\bar{D}) + (AD + B\bar{C})\eta$$

writing the 2×2 matrix algebra as a system of hypercomplex numbers. Note that

$$(A + B\eta)(\bar{A} - B\eta) = A\bar{A} - B\bar{B}$$

The formula on the right corresponds to the determinant of the matrix. Thus we define the *conjugate* of $A + B\eta$ by the formula

$$\overline{A + B\eta} = \bar{A} - B\eta.$$

These patterns generalize to higher dimensional matrix algebra.

It is worth pointing out the first precursor to the quaternions: This precursor is the system

$$\{\pm 1, \pm\epsilon, \pm\eta, \pm i\}.$$

Here $\epsilon\epsilon = 1 = \eta\eta$ while $i = \epsilon\eta$ so that $ii = -1$. The basic operations in this algebra are those of epsilon and eta. Eta is the delay shift operator that reverses the components of the iterant. Epsilon negates one of the components, and leaves the order unchanged. The quaternions arise directly from these two operations once we construct an extra square root of minus one that commutes with them. Call this extra root of minus one $\sqrt{-1}$. Then the quaternions are generated by

$$\{i = \epsilon\eta, j = \sqrt{-1}\bar{\epsilon}, k = \sqrt{-1}\eta\}$$

with

$$i^2 = j^2 = k^2 = ijk = -1.$$

The “right” way to generate the quaternions is to start at the bottom iterant level with boolean values of 0 and 1 and the operation EXOR (exclusive or). Build iterants on this, and matrix algebra from these iterants. This gives the square root of negation. Now take pairs of values from this new algebra and build 2×2 matrices again. The coefficients include square roots of negation that commute with constructions at the next level and so quaternions appear in the third level of this hierarchy.

4.2 Matrix Algebra in General

Construction of matrix algebra in general proceeds as follows. Let M be an $n \times n$ matrix over a ring R . Let $M = (m_{ij})$ denote the matrix entries. Let π be an element of the symmetric group S_n so that $\pi_1, \pi_2, \dots, \pi_n$ is a permutation of $1, 2, \dots, n$. Let $v = (v_1, v_2, \dots, v_n)$ denote a vector with these components. Let $\Delta(v)$ denote the diagonal matrix whose i -th diagonal entry is v_i . Let $v^\pi = (v_{\pi_1}, \dots, v_{\pi_n})$. Let $\Delta^\pi(v) = \Delta(v^\pi)$. Let Δ denote any diagonal matrix and Δ^π denote the corresponding permuted diagonal matrix as just described. Let $[\pi]$ denote the permutation matrix obtained by taking the i -th row of $[\pi]$ to be the π_i -th row of the identity matrix. Note that $[\pi]\Delta = \Delta^\pi[\pi]$. For each element π of S_n define the vector $v(M, \pi) = (m_{1\pi_1}, \dots, m_{n\pi_n})$ and the diagonal matrix $\Delta[M]_\pi = \Delta(v(M, \pi))$.

Theorem. $M = (1/(n-1)!) \sum_{\pi \in S_n} \Delta[M]_\pi [\pi]$.

The proof of this theorem is omitted here. Note that the theorem expresses any square matrix as a sum of products of diagonal matrices and permutation matrices. Diagonal matrices add and multiply by adding and multiplying their corresponding entries. They are acted upon by permutations as described above. This means that any matrix algebra can be embedded in an algebra that has the structure of a group ring of the permutation group with coefficients Δ in an algebra (here the diagonal matrices) that are acted upon by the permutation group, and following the rule $[\pi]\Delta = \Delta^\pi[\pi]$. This is a full generalization of the case $n = 2$ described in the last section.

For example, we have the following expansion of a 3×3 matrix:

$$\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & k \end{pmatrix} = \frac{1}{2!} \left[\begin{pmatrix} a & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & k \end{pmatrix} + \begin{pmatrix} 0 & b & 0 \\ 0 & 0 & f \\ g & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & c \\ d & 0 & 0 \\ 0 & h & 0 \end{pmatrix} + \right. \\ \left. \begin{pmatrix} 0 & 0 & c \\ 0 & e & 0 \\ g & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & b & 0 \\ d & 0 & 0 \\ 0 & 0 & k \end{pmatrix} + \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & f \\ 0 & h & 0 \end{pmatrix} \right].$$

Here, each term factors as a diagonal matrix multiplied by a permutation matrix as in

$$\begin{pmatrix} a & 0 & 0 \\ 0 & 0 & f \\ 0 & h & 0 \end{pmatrix} = \begin{pmatrix} a & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & h \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

It is amusing to note that this theorem tells us that up to the factor of $1/(n - 1)!$ a unitary matrix that has unit complex numbers as its entries is a sum of simpler unitary transformations factored into diagonal and permutation matrices. In quantum computing parlance, such a unitary matrix is a sum of products of phase gates and products of swap gates (since each permutation is a product of transpositions).

A reason for discussing these formulations of matrix algebra in the present context is that one sees that matrix algebra is generated by the simple operations of juxtaposed addition and multiplication, and by the use of permutations as operators. These are unavoidable discrete elements, and so the operations of matrix algebra can be motivated on the basis of discrete physical ideas and non-commutativity. The richness of continuum formulations, infinite matrix algebra, and symmetry grows naturally out of finite matrix algebra and hence out of the discrete.

5 Generalized Feynman Dyson Derivation

In this section we assume that specific time-varying coordinate elements X_1, X_2, X_3 of the algebra \mathcal{A} are given. *We do not assume any commutation relations about X_1, X_2, X_3 .*

In this section we no longer avail ourselves of the commutation relations that are in back of the original Feynman-Dyson derivation. We do take the definitions of the derivations from that previous context. Surprisingly, the result is very similar to the one of Feynman and Dyson, as we shall see.

Here $A \times B$ is the non-commutative vector cross product:

$$(A \times B)_k = \sum_{i,j=1}^3 \epsilon_{ijk} A_i B_j.$$

(We will drop this summation sign for vector cross products from now on.) Then, with $B = \dot{X} \times \dot{X}$, we have

$$B_k = \epsilon_{ijk} \dot{X}_i \dot{X}_j = (1/2) \epsilon_{ijk} [\dot{X}_i, \dot{X}_j].$$

The epsilon tensor ϵ_{ijk} is defined for the indices $\{i, j, k\}$ ranging from 1 to 3, and is equal to 0 if there is a repeated index and is otherwise equal to the sign of the permutation of 123 given by ijk . We represent dot products and cross products in diagrammatic tensor notation as indicated in Figure 2 and Figure 3. In Figure 2 we indicate the epsilon tensor by a trivalent vertex. The indices of the tensor correspond to labels for the three edges that impinge on the vertex. The diagram is drawn in the plane, and is well-defined since the epsilon tensor is invariant under cyclic permutation of its indices.

We will define the fields E and B by the equations

$$B = \dot{X} \times \dot{X} \text{ and } E = \partial_t \dot{X}.$$

We will see that E and B obey a generalization of the Maxwell Equations, and that this generalization describes specific discrete models. The reader should note that this means that a significant part of the *form* of electromagnetism is the consequence of choosing three coordinates of space, and the definitions of spatial and temporal derivatives with respect to them. The background process that is being described is otherwise arbitrary, and yet appears to obey physical laws once these choices are made.

In this section we will use diagrammatic matrix methods to carry out the mathematics. In general, in a diagram for matrix or tensor composition, we sum over all indices labeling any edge in the diagram that has no free ends. Thus matrix multiplication corresponds to the connecting of edges between diagrams, and to the summation over common indices. With this interpretation of compositions, view the first identity in Figure 2. This is a fundamental identity about the epsilon, and corresponds to the following lemma.

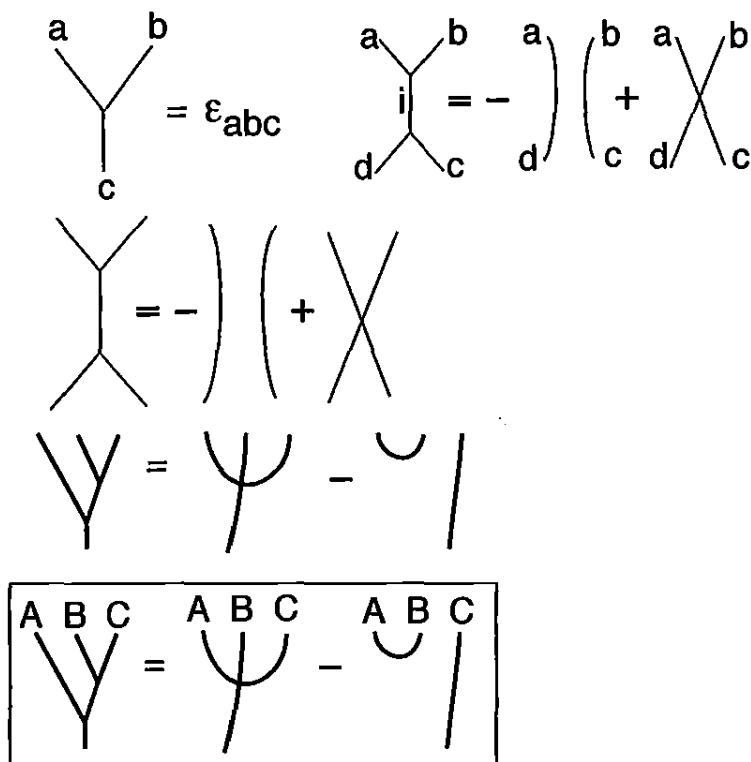


Figure 2: Epsilon Identity

Lemma. (View Figure 2) Let ϵ_{ijk} be the epsilon tensor taking values 0, 1 and -1 as follows: When ijk is a permutation of 123, then ϵ_{ijk} is equal to the sign of the permutation. When ijk

contains a repetition from $\{1, 2, 3\}$, then the value of epsilon is zero. Then ϵ satisfies the labeled identity in Figure 2 in terms of the Kronecker delta.

$$\sum_i \epsilon_{abi} \epsilon_{cdi} = -\delta_{ad} \delta_{bc} + \delta_{ac} \delta_{bd}.$$

The proof of this identity is left to the reader. The identity itself will be referred to as the *epsilon identity*. The epsilon identity is a key structure in the work of this section, and indeed in all formulas involving the vector cross product.

The reader should compare the formula in this Lemma with the diagrams in Figure 2. The first two diagrams are two versions of the Lemma. In the third diagram the labels are capitalized and refer to vectors A, B and C . We then see that the epsilon identity becomes the formula

$$A \times (B \times C) = (A \cdot C)B - (A \cdot B)C$$

for vectors in three-dimensional space (with commuting coordinates, and a generalization of this identity to our non-commutative context. Refer to Figure 3 for the diagrammatic definitions of dot and cross product of vectors. We take these definitions (with implicit order of multiplication) in the non-commutative context.

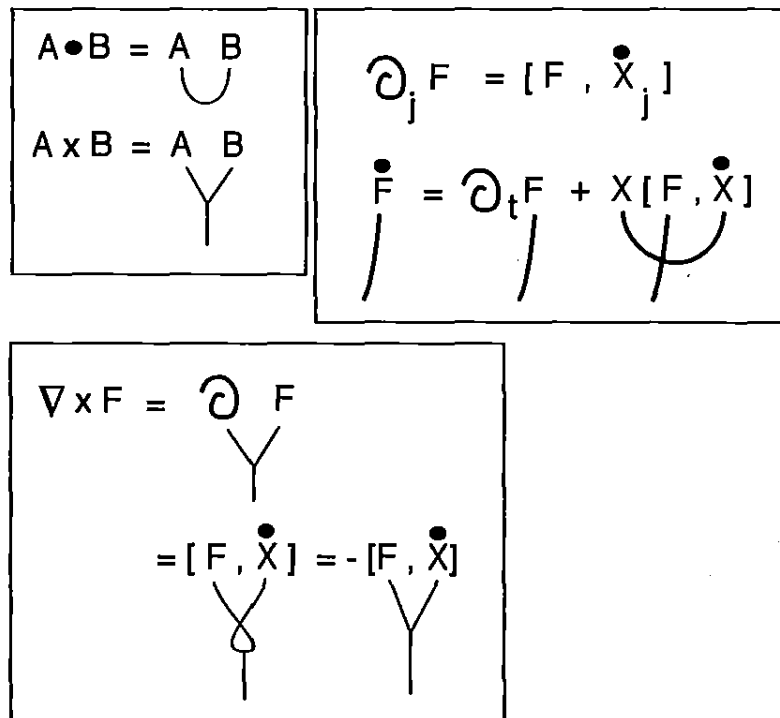


Figure 3: Defining Derivatives

Remarks on the Derivatives.

1. Since we do not assume that $[X_i, \dot{X}_j] = \delta_{ij}$, nor do we assume $[X_i, X_j] = 0$, it will not follow that E and B commute with the X_i .

2. We define

$$\partial_i(F) = [F, \dot{X}_i],$$

and the reader should note that, these spatial derivations are no longer flat in the sense of section 1 (nor were they in the original Feynman-Dyson derivation). See Figure 3 for the diagrammatic version of this definition.

3. We define $\partial_t = \partial/\partial t$ by the equation

$$\partial_t F = \dot{F} - \sum_i \dot{X}_i \partial_i(F) = \dot{F} - \sum_i \dot{X}_i [F, \dot{X}_i]$$

for all elements or vectors of elements F . We take this equation as the global definition of the temporal partial derivative, even for elements that are not commuting with the X_i . This notion of temporal partial derivative ∂_t is a least relation that we can write to describe the temporal relationship of an arbitrary non-commutative vector F and the non-commutative coordinate vector X . See Figure 3 for the diagrammatic version of this definition.

4. In defining

$$\partial_t F = \dot{F} - \sum_i \dot{X}_i \partial_i(F),$$

we are using the definition itself to obtain a notion of the variation of F with respect to time. The definition itself creates a distinction between space and time in the non-commutative world.

5. The reader will have no difficulty verifying the following formula:

$$\partial_t(FG) = \partial_t(F)G + F\partial_t(G) + \sum_i \partial_i(F)\partial_i(G).$$

This formula shows that ∂_t does not satisfy the Leibniz rule in our non-commutative context. This is true for the original Feynman-Dyson context, and for our generalization of it. All derivations in this theory that are defined directly as commutators do satisfy the Leibniz rule. Thus ∂_t is an operator in our theory that does not have a representation as a commutator.

6. We define divergence and curl by the equations

$$\nabla \bullet B = \sum_{i=1}^3 \partial_i(B_i)$$

and

$$(\nabla \times E)_k = \epsilon_{ijk} \partial_i(E_j).$$

See Figure 3 and Figure 5 for the diagrammatic versions of curl and divergence.

Now view Figure 4. We see from this Figure that it follows directly from the definition of the time derivatives (as discussed above) that

$$\ddot{X} = \partial_t \dot{X} + \dot{X} \times (\dot{X} \times \dot{X}).$$

This is our motivation for defining

$$E = \partial_t \dot{X}$$

and

$$B = \dot{X} \times \dot{X}.$$

With these definition in place we have

$$\ddot{X} = E + \dot{X} \times B,$$

giving an analog of the Lorentz force law for this theory.

Just for the record, look at the following algebraic calculation for this derivative:

$$\begin{aligned} \dot{F} &= \partial_t F + \Sigma_i \dot{X}_i [F, \dot{X}_i] \\ &= \partial_t F + \Sigma_i (\dot{X}_i F \dot{X}_i - \dot{X}_i \dot{X}_i F) \\ &= \partial_t F + \Sigma_i (\dot{X}_i F \dot{X}_i - \dot{X}_i F_i \dot{X}) + \dot{X}_i F_i \dot{X} - \dot{X}_i \dot{X}_i F \end{aligned}$$

Hence

$$\dot{F} = \partial_t F + \dot{X} \times F + (\dot{X} \bullet F) \dot{X} - (\dot{X} \bullet \dot{X}) F$$

(using the epsilon identity). Thus we have

$$\ddot{X} = \partial_t \dot{X} + \dot{X} \times (\dot{X} \times \dot{X}) + (\dot{X} \bullet \dot{X}) \dot{X} - (\dot{X} \bullet \dot{X}) \dot{X},$$

whence

$$\ddot{X} = \partial_t \dot{X} + \dot{X} \times (\dot{X} \times \dot{X}).$$

In Figure 5, we give the derivation that B has zero divergence.

Figures 6 and 7 compute derivatives of B and the Curl of E , culminating in the formula

$$\partial_t B + \nabla \times E = B \times B.$$

In classical electromagnetism, there is no term $B \times B$. This term is an artifact of our non-commutative context. In discrete models, as we shall see at the end of this section, there is no escaping the effects of this term.

Finally, Figure 8 gives the diagrammatic proof that

$$\partial_t E - \nabla \times B = (\partial_t^2 - \nabla^2) \dot{X}.$$

This completes the proof of the Theorem below.

$$\begin{aligned}
\dot{F} &= \partial_t F + X[F, X] \\
\dot{F} &= \partial_t F + XFX - XXF \\
\ddot{X} &= \partial_t \dot{X} + \dot{X}\dot{X}\dot{X} - \dot{X}\dot{X}\dot{X} \\
&= \partial_t \dot{X} + \dot{X} \times \dot{X} \\
\boxed{\ddot{X} = \partial_t \dot{X} + \dot{X} \times (\dot{X} \times \dot{X})}
\end{aligned}$$

Figure 4: The Formula for Acceleration

Electromagnetic Theorem With the above definitions of the operators, and taking

$$\nabla^2 = \partial_1^2 + \partial_2^2 + \partial_3^2, \quad B = \dot{X} \times \dot{X} \quad \text{and} \quad E = \partial_t \dot{X} \quad \text{we have}$$

1. $\ddot{X} = E + \dot{X} \times B$
2. $\nabla \cdot B = 0$
3. $\partial_t B + \nabla \times E = B \times B$
4. $\partial_t E - \nabla \times B = (\partial_t^2 - \nabla^2)\dot{X}$

Remark. Note that this Theorem is a non-trivial generalization of the Feynman-Dyson derivation of electromagnetic equations. In the Feynman-Dyson case, one assumes that the commutation relations

$$[X_i, X_j] = 0$$

and

$$[X_i, \dot{X}_j] = \delta_{ij}$$

are given, *and* that the principle of commutativity is assumed, so that if A and B commute with the X_i then A and B commute with each other. One then can interpret ∂_i as a standard derivative with $\partial_i(X_j) = \delta_{ij}$. Furthermore, one can verify that E_j and B_j both commute with the X_i . From

$$E = \partial_t \dot{X} \quad B = \dot{X} \times \dot{X}$$

$$\dot{X} = E + \dot{X} \times B$$

$$\begin{aligned} \nabla \cdot B &= [B, \dot{X}] \\ &= B \dot{X} - \dot{X} B = \dot{X} \dot{X} \dot{X} - \dot{X} \dot{X} \dot{X} = 0 \\ \nabla \cdot B &= 0 \end{aligned}$$

Figure 5: Divergence of B

this it follows that $\partial_t(E)$ and $\partial_t(B)$ have standard interpretations and that $B \times B = 0$. The above formulation of the Theorem adds the description of E as $\partial_t(\dot{X})$, a non-standard use of ∂_t in the original context of Feynman-Dyson, where ∂_t would only be defined for those A that commute with X_i . In the same vein, the last formula $\partial_t E - \nabla \times B = (\partial_t^2 - \nabla^2)\dot{X}$ gives a way to express the remaining Maxwell Equation in the Feynman-Dyson context.

Remark. Note the role played by the epsilon tensor ϵ_{ijk} throughout the construction of generalized electromagnetism in this section. The epsilon tensor is the structure constant for the Lie algebra of the rotation group $SO(3)$. If we replace the epsilon tensor by a structure constant f_{ijk} for a Lie algebra \mathcal{G} of dimension d such that the tensor is invariant under cyclic permutation ($f_{ijk} = f_{kij}$), then most of the work in this section will go over to that context. We would then have d operator/variables X_1, \dots, X_d and a generalized cross product defined on vectors of length d by the equation

$$(A \times B)_k = f_{ijk} A_i B_j.$$

The Jacobi identity for the Lie algebra \mathcal{G} implies that this cross product will satisfy

$$A \times (B \times C) = (A \times B) \times C + [B \times (A) \times C]$$

where

$$([B \times (A) \times C])_r = f_{ktr} f_{ijk} A_i B_k C_j.$$

This extension of the Jacobi identity holds as well for the case of non-commutative cross product defined by the epsilon tensor. It is therefore of interest to explore the structure of generalized non-commutative electromagnetism over other Lie algebras (in the above sense). This will be the subject of another paper.

$$\mathcal{D}_t B = \dot{B} + \dot{X} [\dot{X}, B]$$

$$\begin{aligned} \dot{B} &= (1/2)[\dot{X}, \dot{X}] = [\dot{X}, \dot{X}] \\ &= [E, \dot{X}] + [\dot{X} \times B, \dot{X}] \\ &= -\nabla \times E + [\dot{X} B, \dot{X}] \end{aligned}$$

Figure 6: Computing \dot{B}

5.1 Discrete Thoughts

In the hypotheses of the Electromagnetic Theorem, we are free to take any non-commutative world, and the Electromagnetic Theorem will be satisfied in that world. For example, we can take each X_i to be an arbitrary time series of real or complex numbers, or bitstrings of zeroes and ones. The global time derivative is defined by

$$\dot{F} = J(F' - F) = [F, J],$$

where $FJ = JF'$. This is the non-commutative discrete context discussed in sections 2 and 3. We will write

$$\dot{F} = J\Delta(F)$$

where $\Delta(F)$ denotes the classical discrete derivative

$$\Delta(F) = F' - F.$$

With this interpretation X is a vector with three real or complex coordinates at each time, and

$$B = \dot{X} \times \dot{X} = J^2 \Delta(X') \times \Delta(X)$$

while

$$E = \ddot{X} - \dot{X} \times (\dot{X} \times \dot{X}) = J^2 \Delta^2(X) - J^3 \Delta(X'') \times (\Delta(X') \times \Delta(X)).$$

Note how the non-commutative vector cross products are composed through time shifts in this context of temporal sequences of scalars. The advantage of the generalization now becomes apparent. We can create very simple models of generalized electromagnetism with only the simplest of discrete materials. In the case of the model in terms of triples of time series, the

$$\begin{aligned}
\partial_t B + \nabla \times E &= \dot{X} [\dot{X}, B] + [\dot{X} B, \dot{X}] \\
&= \dot{X} [\dot{X}, B] + [\dot{X} B, \dot{X}] + [\dot{X} B, \dot{X}] \\
&= -\dot{X} \dot{X} B + \dot{X} \dot{X} B \quad (\text{Note that } \dot{X} B = B \dot{X}) \\
&= \dot{X} \dot{X} B = B \times B \\
\boxed{\partial_t B + \nabla \times E = B \times B}
\end{aligned}$$

Figure 7: Curl of E

generalized electromagnetic theory is a theory of measurements of the time series whose key quantities are

$$\Delta(X') \times \Delta(X)$$

and

$$\Delta(X'') \times (\Delta(X') \times \Delta(X)).$$

It is worth noting the forms of the basic derivations in this model. We have, assuming that F is a commuting scalar (or vector of scalars) and taking $\Delta_i = X'_i - X_i$,

$$\partial_i(F) = [F, \dot{X}_i] = [F, J\Delta_i] = FJ\Delta_i - J\Delta_i F = J(F'\Delta_i - \Delta_i F) = \dot{F}\Delta_i$$

and for the temporal derivative we have

$$\partial_t F = J[1 - J\Delta' \bullet \Delta]\Delta(F)$$

where $\Delta = (\Delta_1, \Delta_2, \Delta_3)$.

6 Constraints - Classical Physics and General Relativity

The program here is to investigate restrictions in a non-commutative world that are imposed by asking for a specific correspondence between classical variables acting in the usual context of continuum calculus, and non-commutative operators corresponding to these classical variables.

$E = \partial_t \dot{X} \longrightarrow \partial_t E = \partial_t^2 \dot{X}$
$\nabla \times B = \partial \dot{X} \dot{X}$ $= -\partial \dot{X} \dot{X} + \partial \dot{X} \dot{X}$ $= \partial [\dot{X}, \dot{X}] = (\partial \partial) \dot{X} = \nabla^2 \dot{X}$
$\partial_t E - \nabla \times B = (\partial_t^2 - \nabla^2) \dot{X}$

Figure 8: Curl of B

By asking for the simplest constraints we find the need for a quadratic Hamiltonian and a remarkable relationship with Einstein's equations for general relativity [3, 4]. There is a hierarchy of constraints of which we only analyze the first two levels. An appendix to this paper indicates a direction for exploring the algebra of the higher constraints.

If, for example, we let x and y be classical variables and X and Y the corresponding non-commutative operators, then we ask that x^n correspond to X^n and that y^n correspond to Y^n for positive integers n . We further ask that linear combinations of classical variables correspond to linear combinations of the corresponding operators. These restrictions tell us what happens to products. For example, we have classically that $(x + y)^2 = x^2 + 2xy + y^2$. This, in turn must correspond to $(X + Y)^2 = X^2 + XY + YX + Y^2$. From this it follows that $2xy$ corresponds to $XY + YX$. Hence xy corresponds to

$$\{XY\} = (XY + YX)/2.$$

By a similar calculation, if x_1, x_2, \dots, x_n are classical variables, then the product $x_1 x_2 \dots x_n$ corresponds to

$$\{X_1 X_2 \dots X_n\} = (1/n!) \sum_{\sigma \in S_n} X_{\sigma_1} X_{\sigma_2} \dots X_{\sigma_n}.$$

where S_n denotes all permutations of $1, 2, \dots, n$. Note that we use curly brackets for these symmetrizers and square brackets for commutators as in $[A, B] = AB - BA$.

We can formulate constraints in the non-commutative world by asking for a correspondence between familiar differentiation formulas in continuum calculus and the corresponding formulas in the non-commutative calculus, where all derivatives are expressed via commutators. We will detail how this constraint algebra works in the first few cases. Exploration of these constraints

has been pioneered by Tony Deakin [3, 4]. The author of this paper and Tony Deakin are planning a comprehensive paper on the consequences of these constraints in the interface between classical and quantum mechanics.

Recall that the temporal derivative in a non-commutative world is represented by commutator with an operator H that can be interpreted as the Hamiltonian operator in certain contexts.

$$\dot{\Theta} = [\Theta, H].$$

For this discussion, we shall take a collection Q^1, Q^2, \dots, Q^n of operators to represent spatial coordinates q^1, q^2, \dots, q^n . The Q^i commute with one another, and the derivatives with respect to Q^i are represented by operators P^i so that

$$\partial\Theta/\partial Q^i = \Theta_i = [\Theta, P^i].$$

We also write

$$\partial\Theta/\partial P^i = \Theta^i = [Q^i, \Theta].$$

To this purpose, we assume that $[Q^i, P^j] = \delta^{ij}$ and that the P^j commute with one another (so that mixed partial derivatives with respect to the Q^i are independent of order of differentiation).

Note that

$$\dot{Q}^i = [Q^i, H] = H^i.$$

It will be convenient for us to write H^i in place of \dot{Q}^i in the calculations to follow.

The First Constraint. The *first constraint* is the equation

$$\dot{\Theta} = \{\dot{Q}^i \Theta_i\} = \{H^i \Theta_i\}.$$

This equation expresses the symmetrized version of the usual calculus formula $\dot{\theta} = \dot{q}^i \theta_i$. It is worth noting that the first constraint is satisfied by the quadratic Hamiltonian

$$H = \frac{1}{4}(g_{ij} P^i P^j + P^i P^j g_{ij})$$

where $g_{ij} = g_{ji}$ and the g_{ij} commute with the Q^k . We leave the verification of this point to the reader, and note that the fact that the quadratic Hamiltonian does satisfy the first constraint shows how the constraints bind properties of classical physics (in this case Hamiltonian mechanics) to the non-commutative world.

The Second Constraint. The *second constraint* is the symmetrized analog of the second temporal derivative:

$$\ddot{\Theta} = \{\dot{H}^i \Theta_i\} + \{H^i H^j \Theta_{ij}\}.$$

However, by differentiating the first constraint we have

$$\ddot{\Theta} = \{\dot{H}^i \Theta_i\} + \{H^i \{H^j \Theta_{ij}\}\}$$

Thus the second constraint is equivalent to the equation

$$\{H^i \{H^j \Theta_{ij}\}\} = \{H^i H^j \Theta_{ij}\}.$$

We now reformulate this version of the constraint in the following theorem.

Theorem. The second constraint in the form $\{H^i\{H^j\Theta_{ij}\}\} = \{H^iH^j\Theta_{ij}\}$ is equivalent to the equation

$$[[\Theta_{ij}, H^j], H^i] = 0.$$

Proof. We can shortcut the calculations involved in proving this Theorem by looking at the properties of symbols A, B, C such that $AB = BA, ACB = BCA$. Formally these mimic the behaviour of $A = H^i, B = H^j, C = \Theta_{ij}$ in the expressions $H^iH^j\Theta_{ij}$ and $H^i\Theta_{ij}H^j$ since $\Theta_{ij} = \Theta_{ji}$, and the Einstein summation convention is in place. Then

$$\begin{aligned}\{A\{BC\}\} &= \frac{1}{4}(A(BC + CB) + (BC + CB)A) \\ &= \frac{1}{4}(ABC + ACB + BCA + CBA),\end{aligned}$$

$$\{ABC\} = \frac{1}{6}(ABC + ACB + BAC + BCA + CAB + CBA).$$

So

$$\begin{aligned}\{ABC\} - \{A\{BC\}\} &= \frac{1}{12}(-ABC - ACB + 2BAC - BCA + 2CAB - CBA) \\ &= \frac{1}{12}(ABC - 2ACB + CAB) \\ &= \frac{1}{12}(ABC - 2BCA + CBA) \\ &= \frac{1}{12}(A(BC - CB) + (CB - BC)A) \\ &= \frac{1}{12}(A[B, C] - [B, C]A) \\ &= \frac{1}{12}[A, [B, C]].\end{aligned}$$

Thus the second constraint is equivalent to the equation

$$[H^i, [H^j, \Theta_{ij}]] = 0.$$

This in turn is equivalent to the equation

$$[[\Theta_{ij}, H^j], H^i] = 0,$$

completing the proof of the Theorem.

Remark. If we define

$$\nabla^i(\Theta) = [\Theta, H^i] = [\Theta, \dot{Q}^i]$$

then this is the natural covariant derivative that was described in the introduction to this paper. Thus the second order constraint is

$$\nabla^i(\nabla^j(\Theta_{ij})) = 0.$$

Note that

$$\begin{aligned} \nabla^i(\nabla^j(\Theta_{ij})) &= [[\Theta_{ij}, H^j], H^i] \\ &= -[[H^i, \Theta_{ij}], H^j] - [[H^j, H^i], \Theta_{ij}] \\ &= \nabla^j(\nabla^i(\Theta_{ij})) + [[H^i, H^j], \Theta_{ij}] \\ &= \nabla^i(\nabla^j(\Theta_{ij})) + [[H^i, H^j], \Theta_{ij}]. \end{aligned}$$

Hence the second order constraint is equivalent to the equation

$$[[H^i, H^j], \Theta_{ij}] = 0.$$

This equation weaves the curvature of ∇ with the flat derivatives of Θ .

A Relationship with General Relativity. Again, if we define

$$\nabla^i(\Theta) = [\Theta, H^i] = [\Theta, \dot{Q}^i]$$

then this is the natural covariant derivative that was described in the introduction to this paper. Thus the second order constraint is

$$\nabla^i(\nabla^j(\Theta_{ij})) = 0.$$

If we use the quadratic Hamiltonian $H = \frac{1}{4}(g_{ij}P^iP^j + P^iP^jg_{ij})$ as above, then with $\Theta = g^{lm}$ the second constraint becomes the equation

$$g^{uv}(g^{jk}g_{jku})_v = 0.$$

Deakin and Kilmister [4] observe that this last equation specializes to a fourth order version of Einstein's field equation for vacuum general relativity:

$$K_{ab} = g^{ef}(R_{ab;ef} + \frac{2}{3}R_{ae}R_{fb}) = 0$$

where $a, b, e, f = 1, 2, \dots, n$ and R is the curvature tensor corresponding to the metric g_{ab} . This equation has been studied by Deakin in [4]. It remains to be seen what the consequences for general relativity are in relation to this formulation, and it remains to be seen what the further consequences of higher order constraints will be.

The algebra of the higher order constraints is under investigation at this time.

7 On the Algebra of Constraints

We have the usual advanced calculus formula $\dot{\theta} = \dot{q}^i \theta_i$. We shall define $h^j = \dot{q}^j$ so that we can write $\dot{\theta} = h^i \theta_i$. We can then calculate successive derivatives with $\theta^{(n)}$ denoting the n -th temporal derivative of θ .

$$\begin{aligned}\theta^{(1)} &= h^i \theta_i \\ \theta^{(2)} &= h^{i(1)} \theta_i + h^i h^j \theta_{ij} \\ \theta^{(3)} &= h^{i(2)} \theta_i + 3h^{i(1)} h^j \theta_{ij} + h^i h^j h^k \theta_{ijk}\end{aligned}$$

The equality of mixed partial derivatives in these calculations makes it evident that one can use a formalism that hides all the superscripts and subscripts (i, j, k, \dots). In that simplified formalism, we can write

$$\begin{aligned}\theta^{(1)} &= h\theta \\ \theta^{(2)} &= h^{(1)}\theta + h^2\theta \\ \theta^{(3)} &= h^{(2)}\theta + 3h^{(1)}h\theta + h^3\theta \\ \theta^{(4)} &= h^4\theta + 6h^2\theta h^{(1)} + 3\theta h^{(1)2} + 4h\theta h^{(2)} + \theta h^{(3)}\end{aligned}$$

Each successive row is obtained from the previous row by applying the identity $\theta^{(1)} = h\theta$ in conjunction with the product rule for the derivative.

This procedure can be automated so that one can obtain the formulas for higher order derivatives as far as one desires. These can then be converted into the non-commutative constraint algebra and the consequences examined. Further analysis of this kind will be done in a sequel to this paper.

The interested reader may enjoy seeing how this formalism can be carried out. Below we illustrate a calculation using *Mathematica*TM, where the program already knows how to formally differentiate using the product rule and so only needs to be told that $\theta^{(1)} = h\theta$. This is said in the equation $T'[x] = H[x]T[x]$ where $T[x]$ stands for θ and $H[x]$ stands for h with x a dummy variable for the differentiation. Here $D[T[x], x]$ denotes the derivative of $T[x]$ with respect to x , as does $T'[x]$,

In the calculation below we have indicated five levels of derivative. The structure of the coefficients in this recursion is interesting and complex territory. For example, the coefficients of $H[x]^n T[x] H'[x] = h^n \theta h'$ are the triangular numbers $\{1, 3, 6, 10, 15, 21, \dots\}$ but the next series are the coefficients of $H[x]^n T[x] H'[x]^2 = h^n \theta h'^2$, and these form the series

$$\{1, 3, 15, 45, 105, 210, 378, 630, 990, 1485, 2145, \dots\}.$$

This series is eventually constant after four discrete differentiations. This is the next simplest series that occurs in this structure after the triangular numbers. To penetrate the full algebra of constraints we need to understand the structure of these derivatives and their corresponding non-commutative symmetrizations.

$$T'[x] := H[x]T[x]$$

$$\begin{aligned} & D[T[x], x] \\ & D[D[T[x], x], x] \\ & D[D[D[T[x], x], x], x] \\ & D[D[D[D[T[x], x], x], x], x] \\ & D[D[D[D[D[T[x], x], x], x], x], x] \\ & H[x]T[x] \end{aligned}$$

$$H[x]^2T[x] + T[x]H'[x]$$

$$H[x]^3T[x] + 3H[x]T[x]H'[x] + T[x]H''[x]$$

$$H[x]^4T[x] + 6H[x]^2T[x]H'[x] + 3T[x]H'[x]^2 + 4H[x]T[x]H''[x] + T[x]H^{(3)}[x]$$

$$H[x]^5T[x] + 10H[x]^3T[x]H'[x] + 15H[x]T[x]H'[x]^2 + 10H[x]^2T[x]H''[x] + 10T[x]H'[x]H''[x] + 5H[x]T[x]H^{(3)}[x] + T[x]H^{(4)}[x]$$

7.1 Algebra of Constraints

In this section we work with the hidden index conventions described before in the paper. In this form, the classical versions of the first two constraint equations are

1. $\dot{\theta} = \theta h$
2. $\ddot{\theta} = \theta h^2 + \theta \dot{h}$

In order to obtain the non-commutative versions of these equations, we replace h by H and θ by Θ where the capitalized versions are non-commuting operators. The first and second constraints then become

1. $\{\dot{\Theta}\} = \{\Theta H\} = \frac{1}{2}(\Theta H + H\Theta)$
2. $\{\ddot{\Theta}\} = \{\Theta H^2\} + \{\Theta \dot{H}\} = \frac{1}{3}(\Theta H^2 + H\Theta H + H^2\Theta) + \frac{1}{2}(\Theta \dot{H} + \dot{H}\Theta)$

Proposition. The Second Constraint is equivalent to the commutator equation

$$[[\Theta, H], H] = 0.$$

Proof. We identify

$$\{\dot{\Theta}\}^\bullet = \{\ddot{\Theta}\}$$

and

$$\{\dot{\Theta}\}^\bullet = \{[\Theta H]H\} + \{\Theta \dot{H}\}.$$

So we need

$$\{\Theta H^2\} = \{\{\Theta H\}H\}.$$

The explicit formula for $\{\{\Theta H\}H\}$ is

$$\{\{\Theta H\}H\} = \frac{1}{2}(\{\Theta H\}H + H\{\Theta H\}) = \frac{1}{4}(\Theta HH + H\Theta H + H\Theta H + HH\Theta).$$

Thus we require that

$$\frac{1}{3}(\Theta H^2 + H\Theta H + H^2\Theta) = \frac{1}{4}(\Theta HH + H\Theta H + H\Theta H + HH\Theta).$$

which is equivalent to

$$\Theta H^2 + H^2\Theta - 2H\Theta H = 0.$$

We then note that

$$[[\Theta, H], H] = (\Theta H - H\Theta)H - H(\Theta H - H\Theta) = \Theta H^2 + H^2\Theta - 2H\Theta H.$$

Thus the final form of the second constraint is the equation

$$[[\Theta, H], H] = 0.//$$

The Third Constraint. We now go on to an analysis of the third constraint. The third constraint consists in the the two equations

1. $\{\ddot{\Theta}\} = \{\Theta H^3\} + 3\{\Theta H\dot{H}\} + \{\Theta\ddot{H}\}$
2. $\{\ddot{\Theta}\} = \{\ddot{\Theta}\}^\bullet$ where

$$\{\ddot{\Theta}\}^\bullet = \{\{\Theta H\}H^2\} + 2\{\Theta H\dot{H}\} + \{\{\Theta H\}\dot{H}\} + \{\Theta\ddot{H}\}$$

Proposition. The Third Constraint is equivalent to the commutator equation

$$[H^2, [H, \Theta]] = [\dot{H}, [H, \Theta]] - 2[H, [\dot{H}, \Theta]].$$

Proof. We demand that $\{\ddot{\Theta}\} = \{\ddot{\Theta}\}^\bullet$ and this becomes the longer equation

$$\{\Theta H^3\} + 3\{\Theta H\dot{H}\} + \{\Theta\ddot{H}\} = \{\{\Theta H\}H^2\} + 2\{\Theta H\dot{H}\} + \{\{\Theta H\}\dot{H}\} + \{\Theta\ddot{H}\}$$

This is equivalent to the equation

$$\{\Theta H^3\} + \{\Theta H\dot{H}\} = \{\{\Theta H\}H^2\} + \{\{\Theta H\}\dot{H}\}$$

This, in turn is equivalent to

$$\{\Theta H^3\} - \{\{\Theta H\}H^2\} = \{\{\Theta H\}\dot{H}\} - \{\Theta H\dot{H}\}$$

This is equivalent to

$$(1/4)(H^3\Theta + H^2\Theta H + H\Theta H^2 + \Theta H^3) - (1/6)(H^2(H\Theta + \Theta H) + H(H\Theta + \Theta H)H + (H\Theta + \Theta H)H^2) \\ = (1/2)(\dot{H}(1/2)(H\Theta + \Theta H) + (1/2)(H\Theta + \Theta H)\dot{H}) - (1/6)(\dot{H}H\Theta + \dot{H}\Theta H + H\dot{H}\Theta + H\Theta\dot{H} + \Theta H\dot{H} + \Theta\dot{H}H)$$

This is equivalent to

$$3(H^3\Theta + H^2\Theta H + H\Theta H^2 + \Theta H^3) - 2(H^3\Theta + 2H^2\Theta H + 2H\Theta H^2 + \Theta H^3) \\ = 3(\dot{H}H\Theta + \dot{H}\Theta H + H\Theta\dot{H} + \Theta H\dot{H}) - 2(\dot{H}H\Theta + \dot{H}\Theta H + H\dot{H}\Theta + H\Theta\dot{H} + \Theta H\dot{H} + \Theta\dot{H}H)$$

This is equivalent to

$$H^3\Theta - H^2\Theta H - H\Theta H^2 + \Theta H^3 \\ = (\dot{H}H\Theta + \dot{H}\Theta H + H\Theta\dot{H} + \Theta H\dot{H}) - 2(H\dot{H}\Theta + \Theta\dot{H}H)$$

The reader can now easily verify that

$$[H^2, [\dot{H}, \Theta]] = H^3\Theta - H^2\Theta H - H\Theta H^2 + \Theta H^3$$

and that

$$[\dot{H}, [H, \Theta]] - 2[H, [\dot{H}, \Theta]] = (\dot{H}H\Theta + \dot{H}\Theta H + H\Theta\dot{H} + \Theta H\dot{H}) - 2(H\dot{H}\Theta + \Theta\dot{H}H)$$

Thus we have proved that the third constraint equations are equivalent to the commutator equation

$$[H^2, [H, \Theta]] = [\dot{H}, [H, \Theta]] - 2[H, [\dot{H}, \Theta]]$$

This completes the proof of the Proposition. //

Discussion. Each successive constraint involves the explicit formula for the higher derivatives of Θ coupled with the extra constraint that

$$\{\Theta^{(n)}\}^\bullet = \{\Theta^{(n+1)}\}.$$

We conjecture that each constraint can be expressed as a commutator equation in terms of Θ , H and the derivatives of H , in analogy to the formulas that we have found for the first three constraints. This project will continue with a deeper algebraic study of the constraints and their physical meanings.

8 Appendix – Einstein's Equations and the Bianchi Identity

The purpose of this section is to show how the Bianchi identity (see below for its definition) appears in the context of non-commutative worlds. The Bianchi identity is a crucial mathematical ingredient in general relativity. We shall begin with a quick review of the mathematical structure of general relativity (see for example [6]) and then turn to the context of non-commutative worlds.

The basic tensor in Einstein's theory of general relativity is

$$G^{ab} = R^{ab} - \frac{1}{2}Rg^{ab}$$

where R^{ab} is the Ricci tensor and R the scalar curvature. The Ricci tensor and the scalar curvature are both obtained by contraction from the Riemann curvature tensor R_{bcd}^a with $R_{ab} = R_{abc}^c$, $R^{ab} = g^{ai}g^{bj}R_{ij}$, and $R = g^{ij}R_{ij}$. Because the Einstein tensor G^{ab} has vanishing divergence, it is a prime candidate to be proportional to the energy momentum tensor $T^{\mu\nu}$. The Einstein field equations are

$$R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} = \kappa T^{\mu\nu}.$$

The reader may wish to recall that the Riemann tensor is obtained from the commutator of a covariant derivative ∇_k , associated with the Levi-Civita connection $\Gamma_{jk}^i = (\Gamma_k^i)_j$ (built from the space-time metric g_{ij}). One has

$$\lambda_{a;b} = \nabla_b \lambda_a = \partial_b \lambda_a - \Gamma_{ab}^d \lambda_d$$

or

$$\lambda_{;b} = \nabla_b \lambda = \partial_b \lambda - \Gamma_b \lambda$$

for a vector field λ . With

$$R_{ij} = [\nabla_i, \nabla_j] = \partial_j \Gamma_i - \partial_i \Gamma_j + [\Gamma_i, \Gamma_j],$$

one has

$$R_{bcd}^a = (R_{cd})_b^a.$$

(Here R_{cd} is *not* the Ricci tensor. It is the Riemann tensor with two internal indices hidden from sight.)

One way to understand the mathematical source of the Einstein tensor, and the vanishing of its divergence, is to see it as a contraction of the Bianchi identity for the Riemann tensor. The Bianchi identity states

$$R_{bcd:e}^a + R_{bde:c}^a + R_{bec:d}^a = 0$$

where the index after the colon indicates the covariant derivative. Note also that this can be written in the form

$$(R_{cd:e})_b^a + (R_{de:c})_b^a + (R_{ec:d})_b^a = 0.$$

The Bianchi identity is a consequence of local properties of the Levi-Civita connection and consequent symmetries of the Riemann tensor. One relevant symmetry of the Riemann tensor is the equation $R_{bcd}^a = -R_{bdc}^a$.

We will not give a classical derivation of the Bianchi identity here, but it is instructive to see how its contraction leads to the Einstein tensor. To this end, note that we can contract the Bianchi identity to

$$R_{bca:e}^a + R_{bae:c}^a + R_{bec:a}^a = 0$$

which, in the light of the above definition of the Ricci tensor and the symmetries of the Riemann tensor is the same as

$$R_{bc:e} - R_{be:c} + R_{bec:a}^a = 0.$$

Contract this tensor equation once more to obtain

$$R_{bc:b} - R_{bb:c} + R_{bbc:a}^a = 0,$$

and raise indices

$$R_{c:b}^b - R_{:c} + R_{bc:a}^ab = 0.$$

Further symmetry gives

$$R_{bc:a}^{ab} = R_{cb:a}^{ba} = R_{c:a}^a = R_{c:b}^b.$$

Hence we have

$$2R_{c:b}^b - R_{:c} = 0,$$

which is equivalent to the equation

$$(R_c^b - \frac{1}{2}R\delta_c^b)_{:b} = G_{c:b}^b = 0.$$

From this we conclude that $G_{:b}^{bc} = 0$. The Einstein tensor has appeared on the stage with vanishing divergence, courtesy of the Bianchi identity!

Bianchi Identity and Jacobi Identity. Now lets turn to the context of non-commutative worlds. We have infinitely many possible covariant derivatives, all of the form

$$F_{:a} = \nabla_a F = [F, N_a]$$

for some N_a elements in the non-commutative world. Choose any such covariant derivative. Then, as in the introduction to this paper, we have the curvature

$$R_{ij} = [N_i, N_j]$$

that represents the commutator of the covariant derivative with itself in the sense that $[\nabla_i, \nabla_j]F = [[N_i, N_j], F]$. Note that R_{ij} is not a Ricci tensor, but rather the indication of the external structure of the curvature without any particular choice of linear representation (as is given in the classical case as described above). We then have the Jacobi identity

$$[[N_a, N_b], N_c] + [[N_c, N_a], N_b] + [[N_b, N_c], N_a] = 0.$$

Writing the Jacobi identity in terms of curvature and covariant differentiation we have

$$R_{ab:c} + R_{ca:b} + R_{bc:a}.$$

Thus in a non-commutative world, every covariant derivative satisfies its own Bianchi identity. This gives an impetus to study general relativity in non-commutative worlds by looking for covariant derivatives that satisfy the symmetries of the Riemann tensor and link with a metric in an appropriate way. We have only begun this aspect of the investigation. The point of this section has been to show the intimate relationship between the Bianchi identity and the Jacobi identity that is revealed in the context of non-commutative worlds.

9 Philosophical Appendix

The purpose of this appendix is to point to a way of thinking about the relationship of mathematics, physics, persons, and observations that underlies the approach taken in this paper. We began constructions motivating non-commutativity by considering sequences of actions $\dots DCBA$ written from right to left so that they could be applied to an actant X in the order $\dots DCBAX = \dots (D(C(B(AX)))) \dots$. The sequence of events A, B, C, D, \dots was conceptualized as a temporal order, with the events themselves happening at levels or frames of successive "space". *There is no ambient coordinate space, nor is there any continuum of time.* All that is given is the possibility of structure at any given moment, and the possibility of distinguishing structures from one moment to the next. In this light the formula $DX = [X, J] = XJ - JX = J(X' - X)$ connotes a symbolic representation of the measurement of a difference across one time interval, nothing more. In other words DX represents a difference taken across a background difference (the time step). Once the Pandora's box of measuring such differences has been opened, we are subject to the multiplicities of forms of difference $\nabla_K X = [X, K]$, their non-commutativity among themselves, the notion of a flat background that has the formal appearance of quantum mechanics, the emergence of abstract curvature and formal gauge fields. All this occurs in these calculi of differences *prior* to the emergence of differential geometry or topology or even the notion of linear superposition of states (so important to quantum mechanics). Note that in this algebraic patterning each algebra element X is an actant (can be acted upon) and an actor (via the operator ∇_X). In Lie algebras, this is the relationship between the algebra and its adjoint representation that makes each element of the algebra into a representor for that algebra by exactly the formula $adj_A(X) = [A, X] = -\nabla_A(X)$ that we have identified as a formal difference or derivative, a generator for a calculus of differences.

The precursor and conceptual background of our particular formalism is therefore the concept of discrimination, the idea of a distinction. A key work in relation to that concept is the book "Laws of Form" by G. Spencer-Brown [28] in which is set out a calculus of distinction of maximal simplicity and generality. In that calculus a mark (denoted here by a bracket $\langle \rangle$) represents a distinction and is seen to be a distinction between inside and outside. In this elemental mathematics there is no distinction except the one that we draw between the mathematician and the operator in the formal system as sign/symbol/interpretant. This gives full responsibility to the mathematician to draw the boundaries between the formal system as physical interaction and the formal system as symbolic entity and the formal system as Platonic conceptual form. In making a mathematics of distinction, the mathematician tells a story to himself/herself about the creation of a world. Spencer-Brown's iconic mathematics can be extended to contact any mathematics, and when this happens that mathematics is transformed into a personal creation of the mathematician who uses it. In a similar (but to a mathematician) darker way, the physicist is intimately bound to the physical reality that he studies.

We could have begun this paper with the the Spencer-Brown mark as bracket: $\langle \rangle$. This empty bracket is seen to make a distinction between inside and outside. In order for that to occur the bracket has to become a process in the perception of someone. It has to leave whatever

objective existence or potentiality it has alone (all one) and become the locus or nexus of an idea in a perceiving mind. As such it is stabilized by that perception/creation and becomes really a solution to $\{ \langle \rangle \} = \langle \rangle$ where the curly bracket (the form of perception) is in the first place identical to the mark $\langle \rangle$, and then distinguished from it by the act of distinguishing world and perceiver. It is within this cleft of the infinite recursive and the finite

$$\langle \rangle = \{ \langle \rangle \} = \{ \{ \langle \rangle \} \} = \{ \{ \{ \langle \rangle \} \} \} = \dots = \{ \{ \{ \{ \{ \dots \} \} \} \} \}$$

that the objectivity of mathematics/physics (they are not different in the cleft) arises. All the rest of mathematics or calculus of brackets needs come forth for the observer in the same way. Through that interaction there is the possibility of a deep dialogue of many levels, a dialogue where it is seen that mathematics and physics develop in parallel, each describing the same boundary from opposite sides. That boundary is the imaginary boundary between the inner and outer worlds.

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What Happens Between the Ticks of a Clock?

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Abstract

The question in the title is sequentially asked of Newton, Minkowski and Dirac. Newton proposes a model of a digital clock that, in a suitable continuum limit provides a simple model for the absolute time of classical mechanics. Minkowski, noting the invariant speed of light under a change of inertial frames, takes Newton's clock and modifies it to be relativistically correct at events. He notices that this implicates spacetime algebra with its odd signature. It also assigns a geometric rest mass to the clock and suggests that the reference frames usually invoked in special relativity are implicitly of infinite mass. Dirac then notices that Minkowski's clock automatically uses a form of wave propagation and the differential equation for his clock is the Dirac equation. Minkowski's clock is however completely classical and the result appears paradoxical. The paradox is resolved by noticing two reasonable but quite different answers to the title question.

Key words: quantum mechanics, special relativity, stochastic processes

1. Introduction

If one measures the worth of a scientist's contribution by the length of time it takes to replace his theories with something better, Newton has to be holding a pretty unbreakable record. Classical mechanics is still with us after 300 years and although superseded in principle by relativity and quantum mechanics, both theories are approximated well by Newtonian mechanics at 'normal' scales. Furthermore, Newton's differential calculus still provides a basis for both relativity and quantum mechanics. His influence is felt throughout science in the almost universal acceptance of the differential equation as the language of science.

By comparison, relativity gets full marks for elegance, physical cogency and accuracy. In particular, Minkowski's concept of spacetime has come to be accepted as a physical entity as well as a mathematical framework for relativistic theories.

However, if empirical accuracy is the criterion by which we judge physics, quantum mechanics is arguably the most precise theory we have. Despite this, there

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continue to be aspects of the theory that many scientists find controversial[1, 2]. The practically universal agreement that *quantum mechanics provides a superb probability calculus* does not extend to questions involving the theory's interpretation. Opinions on interpretive issues cover a broad spectrum. At one extreme, the Copenhagen view suggests that the mathematics of quantum mechanics is a calculational tool that does not represent an external physical reality. In Bohr's words,

"There is no quantum world. There is only an abstract physical description. It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature." [3]

At the other extreme, the many-worlds interpretation postulates continually bifurcating universes in order to construct an external reality that reflects the mathematics. Even the deBroglie-Bohm picture, arguably the most 'physical' picture of all has a spectrum of interpretation for the reality of the wavefunction. This lack of consensus on the interpretation of quantum mechanics makes discussions about objects within the probability calculus problematic. For example, the only answer to the question "What is a wavefunction?" that would satisfy all interpretations would be a list of the required mathematical properties of wavefunctions and a mention of the Born postulate. There is simply no consensus on the origin or physical reality of the wavefunction.

While the interpretative problems are commonly thought to occur with the invocation of wavefunctions in quantum mechanics, it is interesting to note that both special relativity and quantum mechanics represent shifts away from the Newtonian concept of absolute time. In relativity, proper time is path dependent and in quantum mechanics the phase of a wavefunction is path dependent. Both path-dependent features are modifications to the Newtonian picture in which each point in physical space can be assigned a clock that ticks in a synchronized fashion with every other clock.

To illustrate the fact that relativity and quantum mechanics share a modified view of time we shall appeal to authority. We ask Newton to build a simple two-photon clock to provide a basis for his concept of absolute time. Minkowski then modifies the clock to make it relativistically correct and finally Dirac comments on the continuum limit that Minkowski uses to make spacetime continuous. In all cases we start out with digital clocks since, in any event, continuum limits are idealizations that cannot be checked physically. They are mathematical conveniences that are perhaps the most persistent inheritance from Newton.

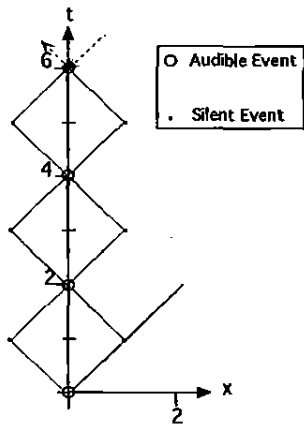


Figure 1: Two featureless Photons confined to a box. They cross at $x = 0$ every 2 units of time giving an audible event. They move with speed $c = 1$. Points where paths change direction are considered silent events. Silent events are part of the spacetime geometry of the paths but are not part of the audible world line event sequence.



2. Newton's Clock

Consider the two-photon clock sketched in Fig.(1). We imagine there exists some device that ticks when the two paths cross. Silent events occur where the photons reverse direction. The sequence of audible events is spaced two units apart in the rest frame of the clock.

From a moving frame, the Galilean transformation simply adds velocities and the time interval between ticks is invariant. The path that Fig.(2) illustrates shows the location of events in spacetime, but we need a digital clock that just ticks in response to the events. Consider the 'boundary path' of a clock illustrated in Fig.(3).

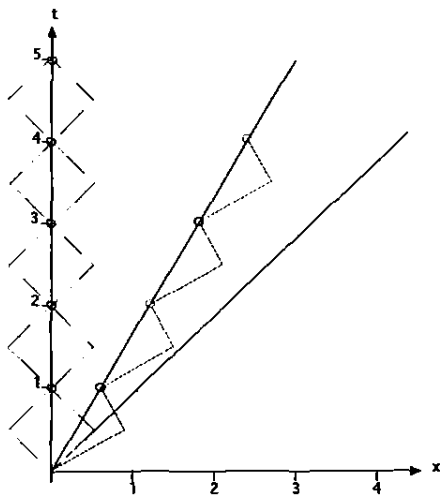


Figure 2:

A stationary and moving clock. Assuming a Galilean transformation, the tick rate of a moving clock does not depend on the velocity of the moving clock. The audible and silent events occur at the same times in both frames. In terms of counting events, we do not need to consider separate clocks for different frames.

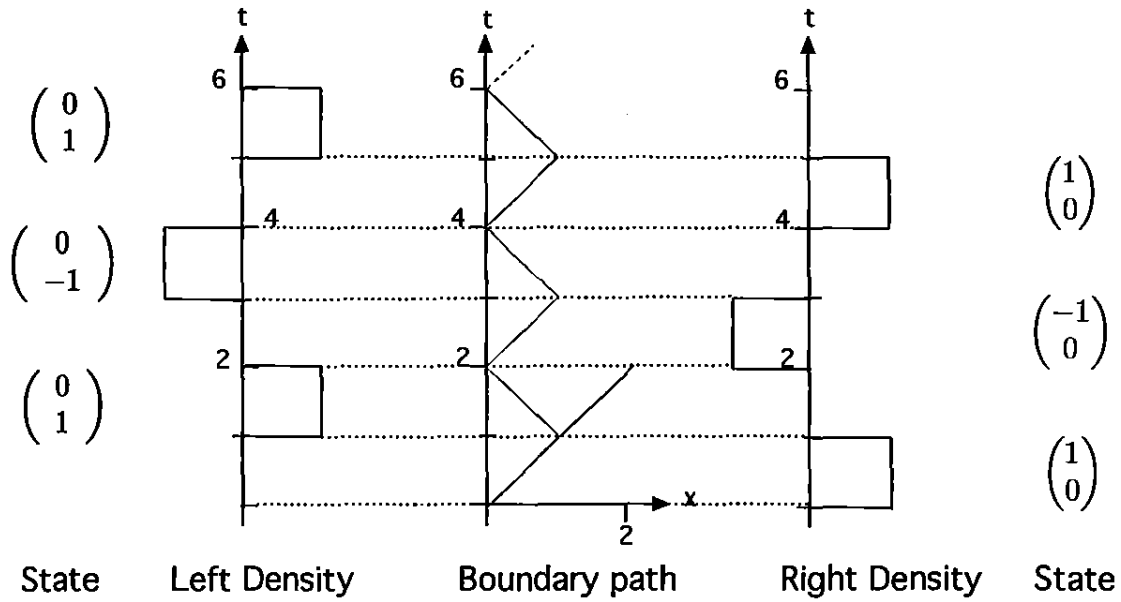


Figure 3: We use the right-hand boundary of the chain of rectangles to count events. A 2-component column vector cycles between the 4 possible states of the boundary. Each state gives rise to a density that changes sign with every two events.

This is just the right-hand boundary of the chain of areas that constitutes the clock. A boundary path segment can be in one of four states depending on its direction in spacetime and its orientation. A column vector with four states

$$s_k \in S = \left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix} \right\} \quad (1)$$

represents an indicator function that registers the presence or absence of an oriented boundary of a rectangle with the accompanying orientation. If the two photon clock is in state s_k at time k then the subsequent state, after the next event is

$$s_{k+1} = T s_k \quad (2)$$

where T is the transfer matrix:

$$T = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \quad (3)$$

T , operating on a state transfers us through the next event and T^2 takes us through two events, returning to the same 'direction' in the boundary but with opposite

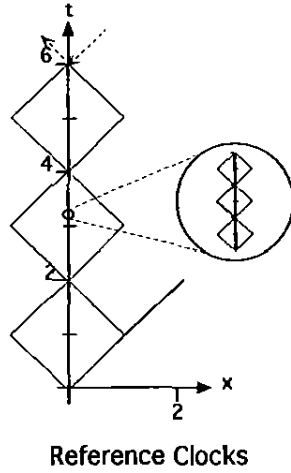


Figure 4:
A two-photon clock of higher frequency may be used as a reference frame clock to increase the precision of our period four clock.

orientation. The two-photon clock ticks cyclically through the set S . Thus powers of the transfer matrix give us the number of ticks of the clock.

If all we have is a single clock, we could use it to sequence events that happen over time scales much greater than 1 by counting time in unit intervals. We *cannot* use it as a reference frame clock to measure events happening on time scales much less than one, since multiple events could occur between ticks and not be sequenced. We can however imagine a reference 2-photon clock working in the same way as T but at a much higher frequency, say N , where N is an integer much greater than 1 Fig.(4). If T_ν is the transfer matrix for the high-frequency N -clock, and it is synchronized with T , we must have an equivalence between the N -th power of T_ν and T itself. Thus we can write

$$T_\nu^N = T \quad (4)$$

giving T_ν as an N th root of T or

$$T_\nu = \begin{pmatrix} \cos\left(\frac{\pi}{2N}\right) & -\sin\left(\frac{\pi}{2N}\right) \\ \sin\left(\frac{\pi}{2N}\right) & \cos\left(\frac{\pi}{2N}\right) \end{pmatrix} \quad (5)$$

Taking a continuum limit gives a transfer matrix representing a reference clock with arbitrarily high precision. That is we define T_R as

$$\begin{aligned} T_R(t) &= \lim_{N \rightarrow \infty} T_\nu^{Nt} = \begin{pmatrix} \cos\left(\frac{\pi t}{2}\right) & -\sin\left(\frac{\pi t}{2}\right) \\ \sin\left(\frac{\pi t}{2}\right) & \cos\left(\frac{\pi t}{2}\right) \end{pmatrix} \\ &= \cos\left(\frac{\pi t}{2}\right) I_2 + T \sin\left(\frac{\pi t}{2}\right) \end{aligned} \quad (6)$$

$$= \cos\left(\frac{\pi t}{2}\right) I_2 - i \sigma_y \sin\left(\frac{\pi t}{2}\right) \quad (7)$$

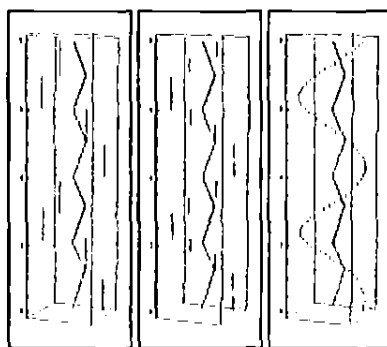


Figure 5: Increasing the frequency of events. The Transfer matrix can be used to increase the frequency of events. It does this by 'rotating' the indicator functions for the four states s_k . The first frame shows a single boundary of the two-photon clock and the associated indicator function as a two-component function of t (vertical lines). The path is in a single state at a given time so only one of the s_k is non-zero. The second frame shows the result of a two-photon clock running at double the frequency. The indicator function is still showing the path is always only in a single state but now the new events are shown rotated from the four states in S . The third frame indicates a reference clock running at 10 times the photon-clock frequency and one can see the emergence of one of a spiral that appears in the continuum limit.

where I_2 is the 2×2 identity matrix and σ_y is the second Pauli matrix. This clock agrees with our period four clock T at the integers but uses a rotation matrix to interpolate between events. That is $T_R(n) = T^n$ for integer n .

Note that when we use T_R as an operator on the states s_k , these no longer appear to behave as *discrete* characteristic functions that indicate the presence or absence of a boundary of a rectangle. Initially, for our two-photon clock, the s_k clearly indicate that a boundary is in one of four *mutually exclusive* states.

However, unless t is an integer, $T_R(t)s_k \notin S$. For example $T_R(1/2)s_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

It appears that our indicator function is in a 'superposition' state. Since we are only using the transfer matrix to count objects, the 'superposition state' must have a simple interpretation.

Indeed, notice that the original s_k are orthogonal because a boundary can only be in a single state. State 1 is orthogonal to states 2 and 4. State 2 is orthogonal to states 1 and 3. However the two dimensional space spanned by S is *not the coordinate space of the boundary* but an algebraically constructed space whose orientation has been chosen so that successive events 'rotate' the s_k through the elements of S making the role of the states as characteristic functions obvious. Using the transfer matrix to increase the frequency of events simply uses the extra degree of freedom in the choice of basis set to map the extra events onto a spiral. Observing Fig.(5) we see that the higher frequency clock T_ν in eqn(4) behaves

just as the two-photon clock except that it expresses the indicator function as a rotation of *the entire set S*. The orthogonality that is an expression of the fact that the clock must be in a single state is preserved in the rotated basis. This is illustrated in the second frame of Fig.(5). There the frequency is doubled and the indicator function for the path splits so that the second half of every inter-event interval is rotated by $\pi/4$. This is just an accommodation of the fact that at this resolution there are eight possible states and the refined clock visits each sequentially. The last frame of the figure shows a refinement of the clock by a factor of 10. At this resolution one can see the outline of a spiral limit while still seeing the digital aspect of the clock between ticks.

$T_R(t)$ is an infinite precision idealization of our two photon clock obtained from an ensemble of such clocks of higher frequency. It can be used for analog conversion since $T_R([t]) = T^{[t]}$ and the two clocks agree at the events at integer times. $T_R(t)$ is a prototype of a frame clock that gives us access to an arbitrarily large number of events between integers and justifies the use of the real parameter t to describe position along the t -axis of the two-photon clock.

One interesting feature of the analog form of our clock is that although we start with the planar path of Fig.(2) as an idealization of a clock mechanism restricted to a plane, the refinement of that clock to allow for higher precision *automatically* invokes a complex structure. The planar structure of a real two dimensional space-time is not big enough to support a description of synchronized clocks of higher precision than the one we started with, and *the result is an expansion in dimension through the use of complex numbers*.

This is, in part, illustrated by the fact that the analog approximation to our digital clock satisfies a differential equation. If $U(t)$ is a column vector, then using (6) we see that the analog of the difference equation (2) is

$$\frac{dU}{dt} = \left(\frac{\pi}{2}\right) T_R U \quad (8)$$

However, the shift matrix T is just a 2×2 version of the unit imaginary and we can write

$$T_R(t) = \exp[i\pi t/2] \quad (9)$$

and we see that our continuum clock is just a unimodular complex number whose phase is proportional to t . The Galilean transformation suggests that if we accept (9) as a continuum limit of a digital clock, we can use it as a reference for all points in space. We can imagine that all objects in the universe have access to this clock.

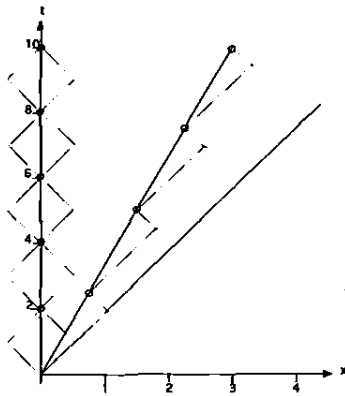


Figure 6: The moving clock path is a chain of oriented areas, however the boundaries of the areas remain parallel to the light cones and the enclosed areas stay fixed. As a result the distance between events of the moving clock in the stationary frame appears longer.



3. Minkowski's Clock

Newton's Clock nicely illustrates how one can go from a finite sequence of events of a digital clock to a countably infinite sequence of events in such a way that the continuum limit invites the use of Newton's differential calculus. It is also interesting to see how complex numbers are implicated. Newton's clock can be implemented through the simple replacement of t by it , suggesting that i is in a sense, a clock [4].

One place that the clock needs to be amended is in the assumption of the Galilean transformation. In changes of inertial frames, the speed of light is invariant and as a result the magnitude and sign of the oriented areas between the audible events of the two-photon clock are also invariant [20]. The paths appear as in Fig.(6). It is clear that time will not be *Absolute* as was the case for Newton's clock. The clock's tick-rate must be path dependent so a single universal clock, synchronized at each point in space, will not work.

We can however take a continuum limit from the two photon clock for either the lab or moving frame. Assume that $v < 1$, and in anticipation of the fact that a clock may tick at different rates in different frames, we shall need the extra precision of a high frequency 'frame' clock to detect differing frequency and phase. From Fig.(6) we can see that the column vector counting states stays in the upper component longer than the stationary case, with the lower component reducing residency time by the same factor. The transfer matrix will have to reflect this. It will have to modify the residency times in the two states so the phase of the silent event is shifted (Fig.(7)).

Consider the reference clock T_R at time $\epsilon \ll 1$. To lowest order in ϵ we have

$$\begin{aligned} T_R(\epsilon) &= \begin{pmatrix} 1 & -\frac{\pi\epsilon}{2} \\ \frac{\pi\epsilon}{2} & 1 \end{pmatrix} \\ &= I_2 + \left(\frac{\pi\epsilon}{2}\right) T \end{aligned} \quad (10)$$

Increasing the residency time in the upper state and decreasing the residency time in the lower state by v gives

$$T_M(\epsilon) = \begin{pmatrix} 1 + \frac{\pi}{2}v\epsilon & -\frac{\pi\epsilon}{2} \\ \frac{\pi\epsilon}{2} & 1 - \frac{\pi}{2}v\epsilon \end{pmatrix} = I_2 - \frac{\pi}{2}\epsilon (i\sigma_y + v\sigma_z) \quad (11)$$

If we take the limit of the transfer matrix raised to the power t/ϵ as ϵ goes to zero we get

$$\begin{aligned} T_M(t) &= \begin{pmatrix} \cos\left(\frac{\pi t}{2\gamma}\right) - v\gamma \sin\left(\frac{\pi t}{2\gamma}\right) & -\gamma \sin\left(\frac{\pi t}{2\gamma}\right) \\ \gamma \sin\left(\frac{\pi t}{2\gamma}\right) & \cos\left(\frac{\pi t}{2\gamma}\right) + v\gamma \sin\left(\frac{\pi t}{2\gamma}\right) \end{pmatrix} \\ &= \cos\left(\frac{\pi t}{2\gamma}\right) I_2 - \gamma \sin\left(\frac{\pi t}{2\gamma}\right) (v\sigma_z + i\sigma_y) \end{aligned} \quad (12)$$

Let us check this against the sketch in Fig.(6). According to that sketch, for $v = 0$ we should get events at $t \in \{0, 2, 4, \dots\}$ and we can see from eqn(12) that this is the case since for $v = 0$, $\gamma = 1$ and the odd part of T_M vanishes at the even integers. Furthermore, the frame clock with relative velocity $-v$ should give us events with inter-arrival times modified by the Lorentz factor γ , ie. $t \in \{0, 2\gamma, 4\gamma, \dots\}$. This also appears to be the case in eqn(12) since the odd component vanishes at precisely

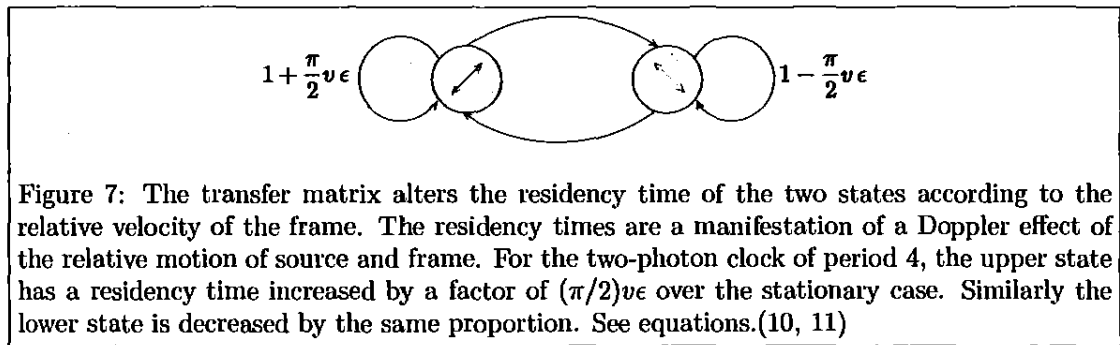


Figure 7: The transfer matrix alters the residency time of the two states according to the relative velocity of the frame. The residency times are a manifestation of a Doppler effect of the relative motion of source and frame. For the two-photon clock of period 4, the upper state has a residency time increased by a factor of $(\pi/2)v\epsilon$ over the stationary case. Similarly the lower state is decreased by the same proportion. See equations.(10, 11)

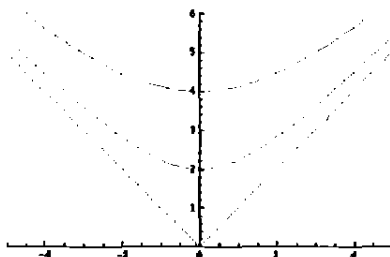


Figure 8: An ensemble of reference frames with different relative speeds leads to a locus of points of first and second events from the two-photon clock.

these points, ie.

$$\begin{aligned}
 T_M(2\gamma n) &= \begin{pmatrix} \cos\left(\frac{\pi(2\gamma n)}{2\gamma}\right) - v\gamma \sin\left(\frac{\pi(2\gamma n)}{2\gamma}\right) & -\gamma \sin\left(\frac{\pi(2\gamma n)}{2\gamma}\right) \\ \gamma \sin\left(\frac{\pi(2\gamma n)}{2\gamma}\right) & \cos\left(\frac{\pi(2\gamma n)}{2\gamma}\right) + v\gamma \sin\left(\frac{\pi(2\gamma n)}{2\gamma}\right) \end{pmatrix} \\
 &= \cos(n\pi) I_2
 \end{aligned} \tag{13}$$

The transfer matrix for our two-photon clock correctly gives us the locus of points corresponding to events in the frame-clock and it is interesting to see what happens at the silent events. When $v = 0$ the silent events occur at $t \in \{1, 3, 5, \dots\}$ and for non-zero v we would expect the silent events to occur at $t \in \{\gamma, 3\gamma, 5\gamma, \dots\}$ and indeed the even term in eqn(12) vanishes there, leaving the odd term

$$r = \pm\gamma(v\sigma_z + i\sigma_y). \tag{14}$$

Notice that if we take the matrix product of this expression with itself we get $-I_2$. If we think of $i\sigma_y$ as a unit vector representing time and σ_z as a unit vector representing space then r is a unit vector with γ representing a normalization factor. The ensemble of all vectors r over all possible values of v (ie. $\{v : |v| < 1\}$) gives the locus of points of unit distance from the origin Fig.(8). The region between the light cone and the hyperbola is the 'unit cell' for our two photon clock. Points within the unit cell correspond to events that our two-photon clock *cannot resolve*. Note that the two unit vectors $i\sigma_y$ and σ_z have norms respectively $(i\sigma_y)^2 = -I_2$ and $(\sigma_z)^2 = I_2$ and they are orthogonal under the symmetric product $A \cdot B = (AB + BA)/2$.

It is interesting to note that our transfer matrix T_R is very much like a conventional clock with a rotating hand marking time. In eqn(12), had the coefficients of both trigonometric functions been scalars, the clock would indeed have been a rotating vector. However note that while the even component is a scalar, the odd component is a vector. The clock that we subsequently call Minkowski's clock mixes a scalar and a vector Fig.(9).

The reason that Minkowski's clock mixes the two types of objects is that the clock has to accommodate the Lorentz transformation at events. We can see that the reference frame clock does this explicitly at all events. Noticing that r in equation(14) is the unit vector of the time axis of the two-photon clock in the moving frame, we should be able to obtain T_M by using a 'rotation' facilitated by the bivector $B = \sigma_z(i\sigma_y)$ [6]. In fact we should be able to obtain T_M as

$$T_M = \exp[-B\theta/2]T_R \exp[B\theta/2]. \quad (15)$$

Completing the matrix multiplication it can be seen that this is the case provided

$$\theta = \operatorname{arctanh}(v) \quad (16)$$

as might be expected. To add two more spatial dimensions to this we need only embed the process in higher dimensions.

Suppose the relative speed of the frame and the clock is

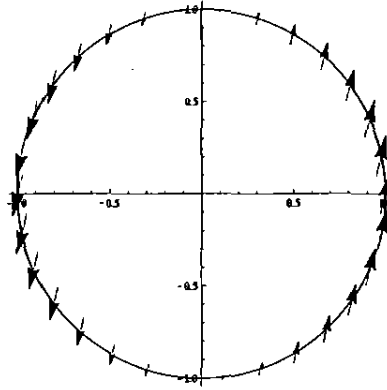


Figure 9: Minkowski's clock mixes scalar and vector. The vector portion points in the direction of the t -axis in the inertial frame of reference and its length is modulated through the clock cycle.

$$\mathbf{v} = v_x\sigma_x + v_y\sigma_y + v_z\sigma_z \quad (17)$$

where the σ_k are the Pauli matrices representing unit vectors. We then have the scalar

$$\mathbf{v}^2 = (v_x^2 + v_y^2 + v_z^2) I_2 \quad (18)$$

and the transfer matrix for an infinitesimal step, becomes

$$T_M(\epsilon) = \begin{pmatrix} I_2 + \frac{\pi}{2}\mathbf{v}\epsilon & -\frac{\pi}{2}\epsilon I_2 \\ \frac{\pi}{2}\epsilon I_2 & I_2 - \frac{\pi}{2}\mathbf{v}\epsilon \end{pmatrix} = I_4 - \frac{\pi}{2}\epsilon (\sigma_z \otimes \mathbf{v} + i\sigma_y \otimes I_2)$$

This transfer matrix functions in exactly the same way as the previous version, except it codes for a reference frame that moves at constant velocity in a three dimensional space. The (1,1) and (2,2) blocks code for the Doppler shifts of the clock, as did the (1, 1) and (2, 2) elements in the two dimensional version.

The continuum limit of this may be taken as in the previous case, using an eigenvalue expansion:

$$\lim_{\epsilon \rightarrow 0} T_M(\epsilon)^{t/\epsilon} = \cos\left(\frac{\pi t}{2\gamma}\right) I_4 - \sin\left(\frac{\pi t}{2\gamma}\right) \gamma(v_x\alpha_x + v_y\alpha_y + v_z\alpha_z + i\beta) \quad (19)$$

Where the α 's and β anticommute and have unit norm.

Notice here that the Dirac matrices α and β function as *orthonormal vectors* representing the 4 directions of spacetime. Of particular interest is the occurrence of the combination $i\beta$, a vector with negative norm. We can see from equation(19) and its predecessor (12) that the odd signature of spacetime is embedded in the continuum limit of our two-photon clock.

We can see that the negative norm of the time vector encodes *both the periodicity and the directional information* needed for a relativistic clock. The clock is completely classical, *we know exactly what it is counting.* (ie. no 'interpretation' necessary!)



4. Dirac's Clock

In Minkowski's clock, the continuum limit that takes us from the two-photon clock to the reference clock seems benign. In this limit we go from two silent ticks between two successive audible ticks (the silent pair being necessary to accommodate boosts, see Fig.(6)) to an indefinite number. The continuum limit itself is a direct inheritance from Newton but how this limit is taken distinguishes between classical special relativity and quantum propagation!

To see this notice that the two-photon clock, relativistic or not, appears completely classical. We know what it is counting and the 'digital' worldline it produces is not in need of elaborate interpretation. In the relativistic case, what happens between the ticks is relativistically correct and we see how a worldline in Minkowski space could be a result by simple linear interpolation. For example, if we think of the two photon clock with a frequency in conventional units of $\nu_e = \frac{m_e c^2}{h}$, the electron's Compton frequency, the time between audible ticks would be tiny on any scales that would be used in macroscopic physics. In fact it would be tiny even on scales of the quantum behaviour of non-relativistic electrons themselves! For all

practical purposes, a two photon clock running at the Compton frequency could serve as a reference frame clock for macroscopic physics in that it could sequence events down to fine scales and would respond appropriately to relativistic time dilation.

It is worth revisiting the reference clock equation(11) replacing the units with standard units for the electron. This gives the transfer matrix

$$T_M(t) = \cos\left(\frac{mc^2t}{\hbar\gamma}\right) I_2 - \gamma \sin\left(\frac{mc^2t}{\hbar\gamma}\right) \left(\frac{v}{c}\sigma_z + i\sigma_y\right) \quad (20)$$

where m is the particle mass. If we write $p = m\gamma v$ and $E = m\gamma c^2$ the above is

$$T_M(t) = \cos\left(\frac{mc^2t}{\hbar\gamma}\right) I_2 - \sin\left(\frac{mc^2t}{\hbar\gamma}\right) (pc\sigma_z + iE\sigma_y)/mc^2 \quad (21)$$

and the vector part of the transfer matrix eqn(20) becomes

$$\mathbf{r} = -\gamma(v\sigma_z/c + i\sigma_y) \rightarrow -(pc\sigma_z + iE\sigma_y)/mc^2 \quad (22)$$

so that the invariant squared length is:

$$\mathbf{r}^2 = (p^2c^2 - E^2)/m^2c^4 = -1 \Rightarrow E^2 = m^2c^4 + p^2c^2 \quad (23)$$

Notice here that the energy and momentum are components of an invariant vector that arises without the usual appeal to conservation principles through dynamics. Mass and energy here are not background attributes but manifestations of the clock mechanism that recognizes the fundamental frequency and the invariant spacetime area.

It is also worthwhile pointing out exactly *why*, according to this model, conventional treatments of special relativity have to put in the mass-energy relation by hand. When we extracted the frame clock T_R eqn. (12), we did this by taking a limit in which the frequency of events between two ticks of the two-photon clock became arbitrarily large. We see from (20) that this is an infinite mass limit! Discussions about the relations between reference frames are discussions about relations between infinite mass clocks. Finite mass clocks(particles) have a finite number of events in their paths where reference frame and particle clocks agree. Our reference frame has been assigned an infinite mass to interpolate between the ticks of the two-photon clock. However, if we compare two clocks with each other through a limit of increasing mass for both clocks, the invariant proper time relation eqn(14) remains as a relation for clocks of all frequencies. The Lorentz transformation then becomes a relation between frames that has to be augmented with the addition of a background mass whose behaviour is then deduced by dynamics.

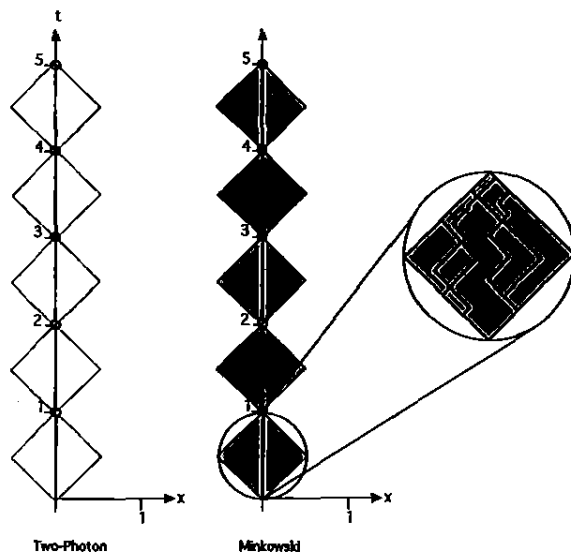


Figure 10: Minkowski's clock ticks at the audible events. In the continuum limit the number of silent ticks between audible ticks is indefinite. In terms of paths, many paths are possible but none are actually singled out by the audible events.

Finally, it is worth noting that Minkowski's clock, eqn(21) satisfies the Dirac equation. This may be seen either through differentiation, or through its relation to the Feynman chessboard model[8],[19].



5. What *really* happens between the ticks of a clock?

In the two-photon clock there are two silent events between each audible tick and the path of the two photons is fixed. In Minkowski's clock the continuum limit allows for an indefinite number of silent events and the path of the photons is not fixed. If we have no access to the inner machinery of a clock between audible ticks, then even classically we cannot say which clock is which. Mathematically this comes down to a familiar identity that is embedded in the continuum limit, namely

$$T = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \sum_{n=0}^{\infty} \frac{(T\pi/2)^n}{n!}. \quad (24)$$

Our tick matrix on the left moves the two-photon clock discretely between states and is associated with the two photon paths in the sketch in Fig.(10). Minkowski's clock recognizes that the right hand side of (24) is another representation of the Transfer matrix *that does not imply a specific two-photon path but does allow a continuum limit.* This brings the particle-wave paradox of clocks into sharp focus.

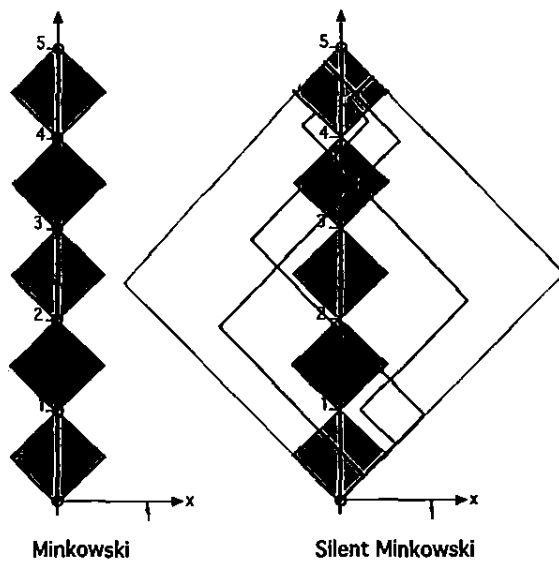


Figure 11: Minkowski's clock ticks at the audible events that forms a discrete version of a worldline, equivalent to the two-photon clock. If audible ticks are silenced and are not part of the discrete worldline then the paths that describe the clock cover a larger spacetime area and the wave propagation feature of the clock extends past the Compton scale.

On scales much greater than the Compton scale, if Minkowski's clock has audible ticks at the Compton scale then for all practical purposes the clock is classical. As Fig.(10) illustrates, we may not be able to distinguish what happens between audible ticks but this is largely irrelevant since the audible ticks are so close together the result is for all practical purposes a smooth worldline. From a quantum mechanical perspective, the clock wavefunction continually collapses on scales below the scale of interest and the clock exhibits the quantum Zeno effect. The classical two-photon clock and the paradoxical Minkowski clock are equivalent in this instance.

On scales less than the Compton scale, we are specifically probing what happens between the ticks of a clock and this depends on the clock mechanism. If Newton's calculus is applicable so that Minkowski's clock is a fair description of the internal machinery, we *do* have a form of wave propagation between audible ticks as witness the fact that the transfer matrix satisfies the Dirac equation. However the differential equation itself does not anticipate the next audible tick, it just describes the evolution between ticks. Quantum mechanics appears to result by removing the 'Classical' restriction on the worldline that the sequence of audible ticks be audible.

This is perhaps more easily seen in the path picture Fig.(11). If there is no mechanism to make the audible ticks of Minkowski's clock observed, the paths do not have to collapse to the audible ticks of the two-photon clock. By making audible ticks silent, clock paths may spread out over a much larger spacetime area and wave propagation is no longer confined to the Compton scale.

Table 1: Comparing Pictures. Conventional pictures of relativity and quantum mechanics start in the continuum and install physics with continuous time evolution assumed. The clock picture distinguishes time scales above and below the fundamental frequency of the clock. This leads to different views of the relationship between relativity and quantum mechanics.

Feature	Conventional	Minkowski's Clock
Spacetime Signature	Kinematics	Clock Feature
Lorentz Transformation	Kinematics	Infinite Mass Limit
Mass	Background	Clock Frequency
Dynamics	" Spacetime tells matter how to move."	" Spacetime is a <i>construct of matter</i> "
Quantum Mechanics	Field Overlay	Clock Synchronization

This picture of audible vs. silent events has historical precedence in 'old quantum theory'. To see this take Minkowski's clock as a model of an electron. ie. replace the smooth worldline of an electron with Minkowski's clock using the electron's Compton frequency $\nu_e = \frac{m_e c^2}{\hbar}$. Wrap this on a cylinder of radius r in such a way that a path around the cylinder loses exactly n events due to time dilation relative to the rest path (the 'twin paradox'). Do this in the non-relativistic limit and you get $m_e r v = n \hbar$. This is Bohr's quantization rule from old Quantum theory. It corresponds to making the 'audible' ticks at the Compton frequency around the cylinder *silent*. This also give $\lambda_B = \frac{\hbar}{m v}$ as the d'Broglie wavelength that is a manifestation of the scaling up of the wave propagation implied by special relativity.

The physics suggests that the rest mass of an object is a fundamental frequency of that object's worldline *as a clock*. Quantum mechanics then *happens* when all events are silent and synchronized between two observations!

Minkowski's clock thus suggests that even fundamental particles are clocks and that rest mass is just the fundamental frequency of an object's worldline. Table(1) compares the particle-as-clock picture with the conventional picture.

6. Conclusions

The relationship between classical physics and quantum mechanics has been a controversial topic since the inception of quantum mechanics. Articles by Bell[1, 2] point out many of the paradoxical characteristics of current formulations. A thoughtful treatment of the Bohm picture may be found in [5]. Original work on the Chessboard model that underlies the clock model may be found in [7, 8] and in previous ANPA proceedings by these authors. Other alternative pictures may

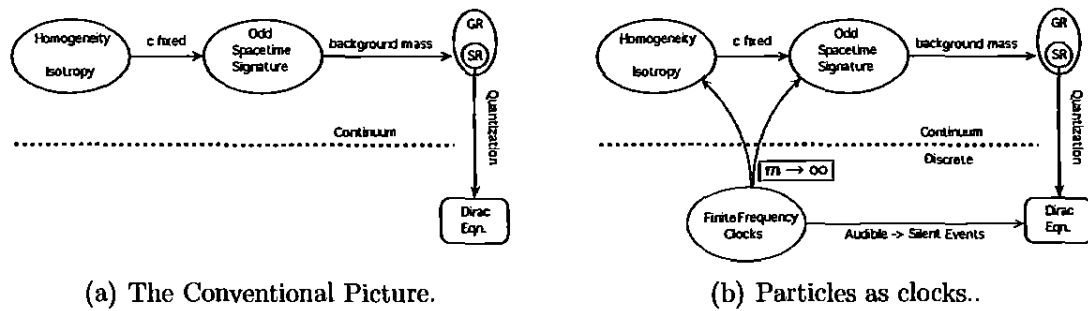


Figure 12: (a) In conventional pictures on the relation between relativity and quantum mechanics, homogeneity and isotropy are assumed at the start. Quantum mechanics is invoked by a canonical quantization procedure. (b) In this picture we start with discrete time and as a result, the reference frames of relativity are seen as infinite mass idealizations. Quantum mechanics is seen as a scaling up of the unobserved events between the ticks of a clock.

be found in references [9] to [15]. Antecedent work on this subject by the author may be found in [16] to [20]

Minkowski's clock is a model of a finite mass particle. It implicates the odd signature of spacetime as a consequence of the constant light-speed postulate implemented on finite frequency clocks. It suggests that 'rest mass' is the fundamental frequency of a particle's worldline, which itself acts as a clock. In this view the conventional inertial frames of special relativity are infinite mass approximations.

The clock picture suggests that quantum propagation is the synchronization of events into a single clock with silent events between initial and final states. The fact that this is not obvious from conventional formulations stems from the power and convenience of the differential calculus. If you start with a description in which time is continuous and events may be arbitrarily close together, physics is naturally built using differential equations. However, as in Fig.(12(b)), formulating relativity in the continuum removes the association of mass with frequency because the derivative presupposes arbitrarily closely spaced observable events. Mass then enters classical physics through dynamics and only reappears as a fundamental frequency at the Dirac equation.

In contrast, the alternative picture maintains a distinction between audible and silent events and classical and quantum behaviour result from different limiting processes that reflect the distinction between the two.

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Nonlocality

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Abstract. Though sometimes considered to be a ‘problem, for quantum physics, nonlocality is rather a fundamental aspect of the way *local* interactions actually operate. Locality and nonlocality can be seen as the two components of a dual system in which each determines the behaviour of the other. The defining characteristics of the weak, strong and electric interactions can be shown to be completely determined by the nonlocal vacuum structures with which they are associated, while gravity provides a nonlocal dual to the combined interactions, which has inertia as its local manifestation. An integrated description of local interactions and nonlocal vacuum structures is derived, based on nilpotent quantum mechanics and its unique algebraic structure.

Local and nonlocal

How do we account for the nonlocality or instantaneity of quantum correlations, when all direct physical measurement or observation must be local, or confined to information transmitted at the speed of light or less? Nonlocality is now demonstrated by the experiments of Aspect (1982), and many others, but is often still regarded as a problem. Nonlocality supposedly defies ‘common sense’ and our expectations. That ‘information’ cannot be transmitted faster than c is taken as ‘fundamental’.

In fact, the fundamental duality of nature, as evident from the nilpotent universal computational rewrite system (Rowlands and Diaz, 2002, Diaz and Rowlands, 2005, Rowlands, 2007) requires nonlocality as much as locality. The local cannot be separated from the nonlocal. Even the terms are misleading. ‘Local’ refers to the entire universe as much as nonlocal does. And they are not separate occurrences: local interactions ultimately stem from nonlocal processes.

Nilpotent quantum mechanics (NQM) shows there is no such thing as an isolated system, and so a complete local description, which originates in the individual particle, still require knowledge of the contents and disposition of the whole universe. Using nilpotent version of quantum mechanics, it is possible to show that both the local and nonlocal aspects

of the strong, weak and electric interactions contribute to their special characteristics.

In addition, 'information', in the computational sense of a code symbol equivalent to 1 or 0, *can* be transmitted faster than c through the spin correlations of entangled states. This is in partial contradiction to the use of the term in physics, where 'information' is usually restricted to the results of bosonic transfer. Really, however, we should define two types of information transfer, local (through bosonic transfer) and nonlocal (through quantum correlation), and both provide 'information' in the computational sense. (Here, one has to be sceptical of the results of the Opera experiment (Agafonova, 2010), which has apparently detected neutrinos travelling at speeds faster than light, though this involved fermion rather than boson transfer. The only conceivable explanation might be if the neutrino is a Majorana particle, allowing switching between $+E$ and $-E$ or $+t$ and $-t$, but this seems to be extremely unlikely.)

In principle, the difference between local and nonlocal is not in the phenomena they describe, but in the method of description, essentially whether we use an iterative or recursive computational paradigm. We should expect to see anticipatory features (Dubois, 1992) in many physical systems, especially in ones driven by gravity, which is the most holistic of all the physical interactions. Local interactions are determined by the collective nonlocal effect of the rest of the universe, and, in this sense, could not take place without anticipation. Gravity, however, as we shall argue, has a special anticipatory significance among the four known interactions in providing the most directly observable aspect of nonlocality.

Nilpotent quantum mechanics

Nilpotent quantum mechanics gives us a very precise definition of the boundary between local and nonlocal. It must be relativistic, as Lorentzian space-time or energy-momentum connection needed to define the local. Without a proper account of the local, we cannot specify what we mean by nonlocal. The simplest way to derive nilpotent quantum mechanics is to begin with Einstein's energy-momentum conservation equation (with the usual convention that $c = 1$):

$$E^2 - p^2 - m^2 = 0$$

We need an algebra to factorize this equation, and so we use a combined algebra made from the tensor product of four quaternion units ($1, i, j, k$)

and a set of four multivariate vector units (i, j, k) to which these are commutative.

The four quaternion units, $i, j, k, 1$, follow the standard multiplication rules:

$$\begin{aligned} i^2 = j^2 = k^2 = ijk &= -1 \\ ij = -ji &= k \\ jk = -kj &= i \\ ki = -ik &= j. \end{aligned}$$

The multivariate vector units, i, j, k, i , are effectively those of complexified quaternions ($ii = i$, $ij = j$, $ik = k$, $i1 = i$), and follow the multiplication rules of Pauli matrices:

$$\begin{aligned} i^2 = j^2 = k^2 = -iijk &= 1 \\ ij = -ji &= ik \\ jk = -kj &= ii \\ ki = -ik &= ij. \end{aligned}$$

In multivariate vector algebra, which is also described as the Clifford algebra of 3D space, two vectors \mathbf{a} and \mathbf{b} have a full product $\mathbf{ab} = \mathbf{a} \cdot \mathbf{b} + \mathbf{ia} \times \mathbf{b}$. Terms like ii , ij , ik are pseudovectors (e.g. area and angular momentum) and i is a pseudoscalar (e.g. volume). The eight base units ($1, i, j, k, i, i, j, k$) have a similar structure to Penrose's twistors, with four real or norm 1 components and four imaginary or norm -1 components. There is a significant difference, however, in that the connection between the units of space and time is a quantum rather than classically relativistic one.

In the combined algebra, each of the four units $1, i, j, k$, is commutative to each of i, j, k . This algebra, which is isomorphic to that of the gamma matrices of the Dirac equation, requires 64 units, which are + and - versions of:

i	j	k	ii	ij	ik	i	1
i	j	k	ii	ij	ik		
ii	ij	ik	iii	ijj	ikk		
ji	jj	jk	iji	ijj	ijk		
ki	kj	kk	kii	ikj	ikk		

However, the 64 can be generated from a subset of 5, which can be matched to the 5 gamma matrices in a number of ways, for example:

$$\gamma^0 = ik \quad \gamma^1 = ii \quad \gamma^2 = ji \quad \gamma^3 = ki \quad \gamma^5 = ij$$

The algebra now allows us to factorize

$$E^2 - p^2 - m^2 = 0$$

in the form

$$(ikE + iip_x + ijp_y + ikp_z + jm)(ikE + iip_x + ijp_y + ikp_z + jm) = 0$$

or

$$(ikE + ip + jm)(ikE + ip + jm) = 0.$$

If we now apply a canonical quantization procedure to the first bracket in these squared expressions, to replace the terms E and \mathbf{p} by the operators $E \rightarrow i\partial/\partial t$, $\mathbf{p} \rightarrow -i\nabla$ (this time equating \hbar to 1), and assume that the operators act on the phase factor for a free fermion, $e^{-i(Et - \mathbf{p}\cdot\mathbf{r})}$, we obtain the nilpotent Dirac equation for a free fermion:

$$\left(\mp k \frac{\partial}{\partial t} \mp i\nabla + jm \right) (\pm ikE \pm ip + jm) e^{-i(Et - \mathbf{p}\cdot\mathbf{r})} = 0 \quad (1)$$

If we use a multivariate vector for the \mathbf{p} or ∇ term it automatically includes spin (through the extra \times term in the full product) (Hestenes, 1966). So, here, \mathbf{p} is interchangeable with $\sigma\cdot\mathbf{p}$ and ∇ with $\sigma\cdot\nabla$. However, if we should revert to using ordinary vectors at any time, we would have to include an explicit spin or angular momentum term.

As usual, 4 simultaneous solutions are required for the wavefunction: 2 for fermion / antifermion \times 2 for spin up / spin down. Rather than a 4×4 matrix differential operator and a column vector wavefunction, we use a row vector operator and a column vector wavefunction, each of which may be represented in abbreviated form by $(\pm ikE \pm ip + jm)$. In the nilpotent formalism, the four solutions can be represented as, say:

$(ikE + ip + jm)$	fermion spin up
$(ikE - ip + jm)$	fermion spin down
$(-ikE + ip + jm)$	antifermion spin down
$(-ikE - ip + jm)$	antifermion spin up

The observed particle state is the first in the column, while the others are the accompanying vacuum states, or states into which the observed particle could transform by respective P , T and C transformations:

$$\begin{array}{l}
 P \quad \quad \quad i(ikE + i\mathbf{p} + jm) i = (ikE - i\mathbf{p} + jm) \\
 T \quad \quad \quad k(ikE + i\mathbf{p} + jm) k = (-ikE + i\mathbf{p} + jm) \\
 C \quad \quad \quad -j(ikE + i\mathbf{p} + jm) j = (-ikE - i\mathbf{p} + jm)
 \end{array}$$

Replacing the observed fermion state spin up with any of the others would simultaneously transform all four states by P , T or C . It is often convenient to specify just the first term, with the others assumed to be automatic consequences.

The dual status of the energy term $\pm ikE$ originates in the dual nature of the pseudoscalar term $\pm i$, which is purely mathematical in origin. As is evident from (1) the time coordinate (in $\partial / \partial t$) follows the same relative sign, so 'negative energy' particles go backwards in time. In the single observable direction of time allowed by the second law of thermodynamics, all material objects are observed to have positive energy.

Negative energy, as we have argued elsewhere, represents vacuum rather than the local quantized state, and the apparent disparity between matter and antimatter in the universe is really a result of the fact that, as in the four-component Dirac wavefunction, one set of states exists in observable real space, and the other in an unobservable 'vacuum space', as required by zero totality.

The dual nature of $\pm i$ is also a factor in creating existence of two states of helicity. Once we have decided on a sign convention for \mathbf{p} , the spin state of the particle (or, more conventionally, the helicity or handedness $\sigma \cdot \mathbf{p}$) is determined by the ratio of the signs of E and \mathbf{p} . So $i\mathbf{p} / ikE$ has the same helicity as $(-i\mathbf{p}) / (-ikE)$, but the opposite helicity to $i\mathbf{p} / (-ikE)$.

Nilpotent quantum mechanics produces all the standard results of conventional relativistic quantum mechanics, which can easily be obtained by replacing (1) with

$$-i\gamma^5 \left(\gamma^0 \frac{\partial}{\partial t} + \gamma^1 \frac{\partial}{\partial x} + \gamma^2 \frac{\partial}{\partial y} + \gamma^3 \frac{\partial}{\partial z} + im \right) \psi = 0$$

Standard classic results obtainable through nilpotent quantum mechanics (Rowlands, 2007) include spin $\frac{1}{2}$, one-handed helicity for weakly interacting states, and the *zitterbewegung* which emerges as an automatic switching process between the four states in the wavefunction, and which is interpreted as a mass-generating switching between the

fermion and its antifermion vacuum partner, and the two helicity states, which are already mixed in real fermions.

Nilpotent quantum mechanics also produces many new results. Among the most important are the descriptions of three different boson-type states, which are combinations of the fermion state which any of the P , T or C transformed ones, the result being a scalar wavefunction.

$$\begin{array}{ll}
 (\pm ikE \pm \mathbf{ip} + jm) (\mp kE \pm \mathbf{ip} + jm) & \text{spin 1 boson} \\
 (\pm ikE \pm \mathbf{ip} + jm) (\mp ikE \mp \mathbf{ip} + jm) & \text{spin 0 boson} \\
 (\pm ikE \pm \mathbf{ip} + jm) (\pm ikE \mp \mathbf{ip} + jm) & \text{fermion-fermion combination}
 \end{array}$$

One of the most significant aspects of this formalization is that a spin 1 boson can be massless, but a spin 0 boson cannot, as then $(\pm ikE \pm \mathbf{ip})$ $(\mp ikE \mp \mathbf{ip})$ would immediately zero: hence massless Goldstone bosons must become massive Higgs bosons in the Higgs mechanism

Vacuum and the separation of local and nonlocal

The key aspect of nilpotent quantum mechanics, is the fact that an operator of the form $(ikE + \mathbf{ip} + jm)$ automatically generates a phase term on which it operates to produce a nilpotent amplitude of the form $(ikE + \mathbf{ip} + jm)$, that is, one that squares to zero. We don't really need an equation. This is true whether or not the fermion is free, as we can incorporate field terms or covariant derivatives into the operator, with, for example,

$$E \rightarrow i\partial / \partial t + e\phi + \dots,$$

and

$$\mathbf{p} \rightarrow -i\nabla + e\mathbf{A} + \dots$$

We can still represent the operator as $(ikE + \mathbf{ip} + jm)$, but the phase term will no longer be $e^{-i(Et - \mathbf{p}\cdot\mathbf{r})}$. It will be whatever is needed to create an amplitude of the general form $(ikE + \mathbf{ip} + jm)$, which squares to zero, with the eigenvalues E and \mathbf{p} representing more complicated expressions that will result from the presence of the field terms

The nilpotent structure immediately gives us a *formal* way of separating the local from the nonlocal. The bracketed term representing the fermion creation operator or wavefunction determines how conservation of energy applies to that fermion, as squaring the wavefunction and equating to zero gives us back the energy-momentum equation, and, of course, it is local, as the required Lorentzian structure is intrinsic. However, the addition and multiplication of nilpotent

wavefunctions construct the nonlocal processes of superposition and combination, and these processes do not require a Lorentzian structure. In effect, anything inside the fermion bracket is local and anything outside it is nonlocal.

In NQM the total structure of the universe is exactly zero. Pauli exclusion, a fundamentally nonlocal phenomenon, is an immediate consequence. Imagine creating a fermion wavefunction of the form $\psi_f = (ikE + ip + jm)$ from *absolutely nothing*; then we must simultaneously create the dual term, $\psi_v = -(ikE + ip + jm)$, which negates it both in superposition and combination:

$$\begin{aligned}\psi_f + \psi_v &= (ikE + ip + jm) - (ikE + ip + jm) = 0 \\ \psi_f \psi_v &= -(ikE + ip + jm)(ikE + ip + jm) = 0\end{aligned}$$

We have identified the dual term as the vacuum appropriate to that fermion state, in principle, the rest of the universe needed to maintain a fermion in that particular state. Pauli exclusion then tells us that no two fermions can have the same quantum numbers because the combination state would be zero. It also implies that no two fermions can share the same vacuum. Vacuum is intrinsically nonlocal. Because the fermion is localized, then the rest of the universe is necessarily nonlocalized. If the fermion is a point, as experiments suggests that it may be, then the rest of the universe is defined as everything outside that point. So the nonlocal connection which makes Pauli exclusion possible can be thought to occur through the vacua for each fermion.

The nilpotent structure clearly demands a holistic approach to physics. When we write down an operator or amplitude in the form $(\pm ikE \pm ip + jm)$, the brackets may suggest that we have created a closed system, but in fact the E and p terms may contain an unlimited number of potentials. We have created a system but it is open. Closure or energy conservation is only maintained over the entire universe, and requires the second law of thermodynamics as well as the first. So, though the bracket may define locality, locality does not imply a closed system.

The creation of the fermion state is the creation of a local region in phase space, to which everything else becomes nonlocal; the creation of the two regions is simultaneous. Any subsequent change inside the bracket (a rewriting of the structures of E and p) also affects everything else outside it, and vice versa. For example, decoherence of a superposition state by imposition of a physical potential (the 'measuring

apparatus' of the Copenhagen interpretation) can be regarded as local determining nonlocal.

In the nilpotent formalism, the vacuum can be structured, directly reflecting that of matter. If we take $(\pm ikE \pm ip + jm)$ and post-multiply it by the idempotent $k(\pm ikE \pm ip + jm)$ any number of times, the only effect is to introduce a scalar multiple, which can be normalized away.

$$\begin{aligned}
 &(\pm ikE \pm ip + jm) k(\pm ikE \pm ip + jm) k(\pm ikE \pm ip + jm) \dots \\
 &\quad \rightarrow (\pm ikE \pm ip + jm)
 \end{aligned}$$

Similarly with $j(\pm ikE \pm ip + jm)$ or $i(\pm ikE \pm ip + jm)$.

The three idempotent terms have the mathematical characteristics of vacuum operators. They also correspond to the respective transformations of alternate brackets by T , C and P , or to the production of the spin 1, spin 0 and fermion-fermion bosonic states. The character of the original fermionic state remains unchanged, while vacuum versions of the three bosonic states are created.

In principle, the quaternionic operators k, j, i , like T, C and P , which are associated respectively with E, m and \mathbf{p} , split the continuous vacuum into discrete units, and we can to some extent regard these units as the respective 'charges' or sources of the weak, electric and strong interactions, acting to create the vacuum necessary for these forces to act.

The nilpotent structure and fundamental interactions

Locality and nonlocality are totally dual. Nonlocal aspects of the interactions are expressed in their symmetry groups for the interactions, and these can be derived directly from the nilpotent structure. The translation to the local effects comes about when we use the same structure to determine the potentials to be added to the operators E and \mathbf{p} in the fermionic nilpotent.

The simplest interaction, the Coulomb interaction, follows the $U(1)$ symmetry group and is part of the structure of each of the electric, strong and weak forces. In the case of the electric force it is the entire interaction; for the strong force it is the one gluon exchange; and for the weak force it provides the hypercharge and the B^0 gauge field, which creates the combination with the electric force in the Weinberg-Salam electroweak theory.

The $U(1)$ symmetry group comes from the characterization of a fermion as a point source with spherical symmetry: spherical symmetry is preserved and demonstrated by rotation whatever the length of the radius vector used. It is a purely scalar symmetry defined only by the magnitude of the charge, or source of the interaction, and is effectively equivalent to defining a coupling constant for the interaction, which maintains its value independent of the distance from the source. The nonlocal effect which is responsible for this symmetry can be identified as the one which is least specific and most general for all fermions: Pauli exclusion. In NQM, this comes from nilpotency itself.

A free fermion has no interactions. An interacting fermion is one that has been brought within the range of influence (or field) of another or others. If fermions are point particles and their influence is spherically symmetric, then it will be convenient to express the influence of one point-source on another by changing the coordinates of the 'receiving' particle from Cartesian to polar, with the point-particle source at the centre of the coordinate system also defined as the centre of physical influence.

Here, we use a version of Dirac's standard prescription (1958) for converting the differential operator to polar coordinates, with the explicit inclusion of a fermionic spin / angular momentum term:

$$\sigma \cdot \nabla = \left(\frac{\partial}{\partial r} + \frac{1}{r} \right) \pm i \frac{j + 1/2}{r}$$

The fundamental condition necessary to assign this operator to a fermion state is that it maintains Pauli exclusion and leads to a nilpotent solution when applied to a phase factor. This leads to the local manifestation of the $U(1)$ symmetry.

It can be seen, simply by inspection, that it will be impossible to obtain a nilpotent solution (i.e. a nilpotent amplitude) and Pauli exclusion with any phase factor unless the operator ikE includes potential energy term varying with $1 / r$ to cancel out the effect of that in the ii part of the operator.

$$(ikE - ii\nabla + jm) \rightarrow \left(ikE - ii \left(\frac{\partial}{\partial r} + \frac{1}{r} \pm i \frac{j + 1/2}{r} \right) + jm \right)$$

So, simply requiring *spherical symmetry* for a point particle, requires a term of the form A / r to be added to E .

Deriving the solution for this case provides a model for all other cases. If all point particles are spherically symmetric sources, then the minimum nilpotent operator will be of the form

$$\left(\pm ik \left(E - \frac{A}{r} \right) \mp i \left(\frac{\partial}{\partial r} + \frac{1}{r} \pm i \frac{j + \frac{1}{2}}{r} \right) + jm \right)$$

To establish that this is a nilpotent, we must now find the phase to which this must apply to create a nilpotent amplitude. This is a convenient example for showing how an operator fixes the phase factor and quite quickly produces the characteristic solution for the Coulomb force, as in the hydrogen atom.

The solution is relatively straightforward. The ease of calculation is due to the fact that the structure provides dual information about both fermion and vacuum. We apply the specified operator to the phase

$$\phi = e^{-ar} \sum_{\nu=0}^{\infty} a_{\nu} r^{\nu}$$

to find the amplitude (derived, as in the conventional solution, by inspired guesswork or trial and error), and equate the squared amplitude to zero.

$$4 \left(E - \frac{A}{r} \right)^2 = -2 \left(-a + \frac{\gamma}{r} + \frac{\nu}{r} + \dots \frac{1}{r} + i \frac{j + \frac{1}{2}}{r} \right)^2 - 2 \left(-a + \frac{\gamma}{r} + \frac{\nu}{r} + \dots \frac{1}{r} - i \frac{j + \frac{1}{2}}{r} \right)^2 + 4m^2.$$

Equating constant terms, we find:

$$a = \sqrt{m^2 - E^2}.$$

Equating terms in $1/r^2$, with $n = 0$:

$$\left(\frac{A}{r} \right)^2 = - \left(\frac{\gamma + 1}{r} \right)^2 + \left(\frac{j + \frac{1}{2}}{r} \right)^2.$$

Assuming the power series terminates at n' , and equating coefficients of $1/r$ for $n = n'$:

$$2EA = -2\sqrt{m^2 - E^2} (\gamma + 1 + n'),$$

and

$$\frac{E}{m} = \frac{1}{\sqrt{1 + \frac{A^2}{(\gamma + 1 + n')^2}}} = \frac{1}{\sqrt{1 + \frac{A^2}{\left(\sqrt{(j + \frac{1}{2})^2 - A^2} + n' \right)^2}}}$$

When $A = Ze^2$ we have the 'hydrogen atom' solution in just 6 lines.

The strong interaction

The strong interaction begins in a combination state, which reflects the *vector* nature of the \mathbf{p} term in the nilpotent wavefunction. Effectively, the vector aspect requires a source term and corresponding vacuum with

three components. A combination is outside the bracket and so is nonlocal. Nilpotency, of course, prevents us writing a combination of the form

$$(ikE + ip + jm) (ikE + ip + jm) (ikE + ip + jm) = 0$$

which would automatically zero.

However, if we separate out three components of \mathbf{p} in the form

$$(ikE \pm iip_x + jm) (ikE \pm ij p_y + jm) (ikE \pm ikp_z + jm)$$

then we have a nonzero combination. Spin is defined in only one direction at a time, so, at any given instant, the wavefunction will reduce (after normalization) to

$$(ikE + iip_x + jm) (ikE + jm) (ikE + jm) \rightarrow (ikE + iip_x + jm)$$

$$(ikE + jm) (ikE + ij p_y + jm) (ikE + jm) \rightarrow (ikE - ij p_y + jm)$$

$$(ikE + jm) (ikE + jm) (ikE + ikp_z + jm) \rightarrow (ikE + ikp_z + jm)$$

The change of sign in the second case is significant.

Since we need to maintain the symmetry between the three directions of momentum, there will be six possible outcomes, using both + and - values of momentum terms, that is, a superposition of six combination states:

$$(ikE + iip_x + jm) (ikE + jm) (ikE + jm) \rightarrow (ikE + iip_x + jm)$$

$$(ikE - iip_x + jm) (ikE + jm) (ikE + jm) \rightarrow (ikE - iip_x + jm)$$

$$(ikE + jm) (ikE + ij p_y + jm) (ikE + jm) \rightarrow (ikE - ij p_y + jm)$$

$$(ikE + jm) (ikE - ij p_y + jm) (ikE + jm) \rightarrow (ikE + ij p_y + jm)$$

$$(ikE + jm) (ikE + jm) (ikE + ikp_z + jm) \rightarrow (ikE + ikp_z + jm)$$

$$(ikE + jm) (ikE + jm) (ikE - ikp_z + jm) \rightarrow (ikE - ikp_z + jm)$$

The six 'phases' represented here must all be valid at the same time, as required by the rotation symmetry of space or of gauge invariance. The symmetry is recognisably that of the group $SU(3)$, with the same structure as the three symmetric and three antisymmetric 'colour' combinations in the standard representation:

$$\psi \sim (BGR - BRG + GRB - RGB + RBG - GBR)$$

Just as $U(1)$ establishes that spherical symmetry of a point source requires the rotation to be performed independently of the length of the radius vector, so $SU(3)$ requires the rotation to be independent of the coordinate system used. In terms of Noether's theorem, while $U(1)$ conserves the magnitude of angular momentum, $SU(3)$ conserves the direction. The exact symmetry of the symmetric and antisymmetric phases also explains why the strong interaction is nonchiral.

The $SU(3)$ symmetry represents a nonlocal interaction between the parts of the wavefunction or 'quarks' incorporating p_x , p_y , and p_z . This interaction is independent of the physical separation of the components

(and they must be spatially or temporally separated to have different local and nonlocal manifestations). The local effect of this is a transfer of \mathbf{p} between the three brackets of each wavefunction. As it is local, with \mathbf{p} changing within each bracket, this is not instantaneous, unlike the actual combinations and superpositions, and so there will be a nonzero local force or rate of change of momentum. However, because it does not depend on separation, this force will be constant. The local manifestation of a constant force is a potential that varies linearly with distance.

As the $U(1)$ condition for spherical symmetry / Pauli exclusion / nilpotence is also a requirement, then the minimal potential to describe this local strong interaction is a combination of terms that are linear and inverse linear with distance. It may be significant that the linear potential of the strong interaction is the only one that is optional to the fermion state, the nilpotency not being dependent directly on the vector nature of \mathbf{p} .

Now it is significant that, because we have both positive and negative \mathbf{p} terms for the same E term, the six phases incorporate a (maximal) superposition of left- and right-handed components. This means that the mass term m is necessarily nonzero, though the unbroken gauge invariance which results from no momentum direction being preferred, means that, in quantum field terms, the boson mediators managing the transitions between the six component states must be massless, as well as spin 1.

The so-called mass gap problem has been a significant one in particle physics for a number of decades, and is part of one of the mathematical problems for which the Clay Institute has offered prizes. It would seem that nilpotency suggests that the problem is now within our grasp. Significantly, also, the solution would appear to be that of the Higgs mechanism, which supplies mass to all the fermions. Many commentators have argued that the bulk of the mass of a proton comes from the boson or gluon exchange and not from the Higgs mechanism. Our analysis would suggest, however, that they are in fact the same process.

The duality of the $\pm \mathbf{p}$ or helicity states in these structures is a clear indication that the composite state described does not have zero mass. The mediators of the transitions between the six component states will be six bosons of the form:

$$(ikE - \ddot{i}p_x) (- ikE - \ddot{i}p_y)$$

and two combinations of the three bosons of the form:

$$(ikE - \ddot{i}p_x) (- ikE - \ddot{i}p_x)$$

These structures are, of course, identical to an equivalent set in which both brackets undergo a complete sign reversal. They are also equivalent, with normalization, to ones with a massless fermion acting on a 'strong' vacuum,

$$(\pm ikE \pm iip_x) (\pm ikE \mp ij p_y) = (\pm ikE \pm iip_x) i (\pm ikE \pm iip_y) i.$$

A 'colour' transition can be seen as either an exchange of the components of \mathbf{p} between the individual quarks, or a relative switching of quark positions. The colours, therefore, either move with the respective p_x, p_y, p_z components, or switch with them. The effect of either process is the same, and a sign reversal in \mathbf{p} , as with $i (\pm ikE \pm iip_y) i$, is an additional necessary result.

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To establish that the local interaction produced by the combination of direct and inverse linear potential results in a force with the characteristics of the strong interaction, we need to extend the Coulomb solution of for this new nilpotent operator. For either a quark-antiquark combination, or a combination of three quarks, we will produce an operator of the form:

$$\left(\pm k \left(E + \frac{A}{r} + Br \right) \mp i \left(\frac{\partial}{\partial r} + \frac{1}{r} \pm i \frac{j+1/2}{r} \right) + ijm \right)$$

Once again, we need to identify the phase factor to which this operator applies, but a relatively simple extension of the one for the Coulomb potential suggests that its form is:

$$\phi = \exp(-ar - br^2) r^\nu \sum_{\nu=0} a_\nu r^\nu$$

Applying the operator and the nilpotent condition, we obtain:

$$E^2 + 2AB + \frac{A^2}{r^2} + B^2 r^2 + \frac{2AE}{r} + 2BEr = m^2$$

$$- \left(\alpha^2 + \frac{(\gamma + \nu + \dots + 1)^2}{r^2} - \frac{(j+1/2)^2}{r^2} + 4b^2 r^2 + 4abr - 4b(\gamma + \nu + \dots + 1) - \frac{2a}{r}(\gamma + \nu + \dots + 1) \right)$$

Then, once again assuming a termination in the power series (as we did with the Coulomb solution), we can equate:

$$\begin{aligned} \text{coefficients of } r^2 \text{ to give:} & \quad B^2 = -4b^2 \\ \text{coefficients of } r \text{ to give:} & \quad 2BE = -4ab \\ \text{coefficients of } 1/r \text{ to give:} & \quad 2AE = 2a(\gamma + \nu + 1) \end{aligned}$$

The immediate consequences of these equations are

$$\begin{aligned} b &= \pm \frac{iB}{2} \\ a &= \mp iE \\ \gamma + \nu + 1 &= \mp iA. \end{aligned}$$

The ground state case (where $\nu = 0$) then requires a phase factor of the form:

$$\phi = \exp(\pm iEr \mp iBr^2/2) r^{\mp iqA-1}$$

We can identify the imaginary exponential terms in ϕ as representing asymptotic freedom; the $\exp(\mp iEr)$ is typical for a free fermion.

The complex $r^{\nu-1}$ term can be restructured as a component phase, $\chi(r) = \exp(\pm iqA \ln(r))$, which varies more slowly with r than the rest of ϕ . We can therefore express ϕ in the form:

$$\phi = \frac{\exp(kr + \chi(r))}{r}$$

where

$$k = \pm iE \mp iBr/2.$$

At high energies, and small r , the first term dominates, which approximates to a free fermion solution or asymptotic freedom. At low energies, however, when r is large, the second term, which incorporates the confining potential Br , dominates, leading to infrared slavery. Once again, a nonlocal symmetry, related to the nilpotent structure, determines the known characteristics of a local interaction.

The weak interaction

We have so far discussed nonlocal combination and superposition states that lead to the local Coulomb and strong interactions. There is, however, in all fermion states an additional superposition resulting from the spinor structure of the Dirac wavefunction, and the *zitterbewegung* that connects the superposed states. The fermion wavefunction has two fermion and two antifermion states, each representing two directions of spin or helicity.

While the co-existence of two spin states can be thought of as, in some sense, 'real', and accounted for by the presence of mass, the co-existence of two energy states can only be meaningfully understood in the context of the simultaneous existence of fermion and vacuum. A fermionic source cannot be separated from its vacuum partner. So a fermion or antifermion cannot be created or annihilated, even with an antifermionic or fermionic partner, unless its vacuum is simultaneously annihilated or created.

The nonlocal connection between fermion and antifermion, or fermion and vacuum, which ultimately emerges from the dual nature of the pseudoscalar aspect of the term ikE , can be seen as creating a kind of virtual dipole, which will necessarily have local consequences. In the case of the *zitterbewegung*, this involves a switching between fermion and antifermion states which is an aspect of the *weak* interaction, in which one state is annihilated and another created by a boson exchange, the only interaction in which such a process occurs.

So, even though the transitions between the two energy states, ikE and $-ikE$, may be virtual, the *zitterbewegung* would seem to require the production of an intermediate bosonic state at a vertex where one fermionic state is annihilated and another is created to replace by it. In fact, all weak interactions operate according to the same mechanism, and the mediator is a spin 1 boson $(\pm ikE \pm ip + jm) k(\pm ikE \pm ip + jm)k$, equivalent to the state produced by a fermion acting on a 'weak' vacuum. Fermions are the only particles which act as weak sources, and the fact that they are a kind of dipole under the weak interaction suggests that all weak sources act, in effect, as weak dipoles, and the immediate manifestation of this dipolar nature occurs with the fermion's $\frac{1}{2}$ -integral spin.

Though the superposition of fermion and antifermion states is nonlocal, we can expect it to have local consequences for a fermion nilpotent. Locally, a dipole manifests itself through a squaring of the distance dependence of the force. So inverse square forces become inverse fourth power dipoles and linear forces become squared distance dipoles. There is also the possibility that the existence of multiple dipoles will produce multipole forces. It would seem that the weak interaction, like the electric and strong interactions, is built into the structure of the nilpotent operator, and its nature is determined by that of the pseudoscalar iE operator, whose sign uniquely determines the helicity of a weakly interacting particle, or more specifically its weakly interacting component.

To accommodate a local interaction which emerges from this nonlocal superposition, we can investigate the nilpotent solutions that would

emerge from an operator incorporating a Coulomb potential together with any other potential which is a function of distance from a point source with spherical symmetry, apart from the linear potential which is characteristic of the strong interaction. So, let us assume a nilpotent operator with a form such as

$$\left(k \left(E - \frac{A}{r} - Cr^n \right) + i \left(\frac{\partial}{\partial r} + \frac{1}{r} \pm i \frac{j+1/2}{r} \right) + ijm \right)$$

where n is an integer greater than 1 or less than -1 .

As before, we will need to find a phase factor which will make the amplitude nilpotent. Again, we will seek to extend the Coulomb solution, using the information that polynomial potential terms which are multiples of r^n require the incorporation into the exponential of terms which are multiples of r^{n+1} . So, we may suppose that the phase factor is of the form:

$$\phi = \exp(-ar - br^{n+1}) r^\gamma \sum_{\nu=0}^{\infty} a_\nu r^\nu$$

As in the previous examples, we apply the operator and square to zero, with a termination in the series, to obtain:

$$4 \left(E - \frac{A}{r} - Cr^n \right)^2 = -2 \left(-a + (n+1)br^n + \frac{\gamma}{r} + \frac{\nu}{r} + \frac{1}{r} + i \frac{j+1/2}{r} \right)^2 - 2 \left(-a + (n+1)br^n + \frac{\gamma}{r} + \frac{\nu}{r} + \frac{1}{r} - i \frac{j+1/2}{r} \right)^2$$

Equating constant terms, we find

$$a = \sqrt{m^2 - E^2}$$

Equating terms in r^{2n} , with $\nu = 0$, gives

$$C^2 = -(n+1)^2 b^2$$

Equating coefficients of r , where $\nu = 0$:

$$AC = -(n+1)b(1+\gamma),$$

$$(1+\gamma) = \pm iA.$$

Equating coefficients of $1/r^2$ and coefficients of $1/r$, for a power series terminating in $\nu = n'$, leads to

$$A^2 = -(1+\gamma+n')^2 + (j+1/2)^2$$

and

$$-EA = a(1+\gamma+n').$$

Combining these results produces:

$$\left(\frac{m^2 - E^2}{E^2} \right) (1+\gamma+n')^2 = -(1+\gamma+n')^2 + (j+1/2)^2$$

$$E = -\frac{m}{j+1/2} (\pm iA + n').$$

The last equation has the form of a harmonic oscillator, with evenly spaced energy levels deriving from integral values of n' . The constant A has the dimensions of charge (q) squared, or interaction energy \times range, and if numerically equal to $\pm \frac{1}{2} \hbar c$ has exactly the value required by the uncertainty principle, if the range of an interaction mediated by the Z boson is calculated at $\hbar / 2M_Z c = 2.166 \times 10^{-18}$ m, as observed. Then we obtain the exact formula:

$$E = -\frac{m}{j + \frac{1}{2}} \left(\frac{1}{2} + n' \right).$$

In this interpretation, we can describe the *zitterbewegung* as a dipolar switching between fermion and vacuum antifermion states, with a weak dipole moment $(\hbar c / 2)^{3/2} / M_Z c^2$, of magnitude 8.965×10^{-18} em, and a weak magnetic moment, of order $4.64 \times 10^{-5} \times$ the magnetic moment of the electron. A (the phase term required for spherical symmetry) then has its origin in the random directionality of fermionic spin. The possible appearance of an imaginary factor i in A has an interesting relation to the fact that a complex potential or vacuum is required for CP violation in the pure weak interaction.

The 'weak' behaviour, like that of the other interactions, comes from the structure of the nilpotent fermionic state. The dual state $\pm ikE$ which produces the dipolar switching is also manifestly an $SU(2)$ symmetry, and this can be related to the fact that the spherical symmetry of the point source proceeds from the fact that it is independent of the handedness of the rotation, which, in terms of Noether's theorem, becomes the conservation of the handedness of angular momentum.

There are, however, two different $SU(2)$ symmetries involved, though they are related. The $SU(2)$ of spin describes two helicity states, left- and right-handed. However, there is another $SU(2)$ symmetry, weak isospin, which describes a fermionic weak interaction as being independent of whether or not an electric charge is present and generating its own contribution to mass.

In effect, weak isospin tells us that the weak interaction, though often occurring simultaneously and in combination with the electric interaction, has an independent origin. In the case of weak isospin, the $SU(2)$ is a switching between different ratios of left- and right-handedness, and so of mass, determined by the presence or absence of the electric charge in one of the two states. So a superposition of, say, $a_1(ikE_1 + ip_1 + jm_1) + a_2(ikE_1 - ip_1 + jm_1)$ might become $b_1(ikE_2 + ip_2 + jm_2) + b_2(ikE_2 - ip_2 + jm_2)$, with both spin 1 and spin 0 vertices, the additional spin 0 vertex being the one which changes the mass.

Significantly, while all the forces contribute to the production of the scalar mass term, the electric force is the only which contributes to nothing else. Overall, the calculation shows that an additional potential of the form Cr^n , where n is an integer greater than 1 or less than -1 , that is any potential emerging from a system in which there is complexity, aggregation, or a multiplicity of sources, leads to a harmonic oscillator solution for the nilpotent operator, irrespective of the value of n , and, in fact, any polynomial sum of potentials of this form would produce the same result.

Gravity

Gravity presents a different problem from the other three forces. It has the same $U(1)$ symmetry as all the other forces, but is not successfully quantized. Also the value of the cosmological constant predicted from a quantum-style argument is 10^{123} times higher than experimental data, based on 'dark energy', would suggest. If, as Seth Lloyd suggests on the basis of the holographic principle (Lloyd, 2002, Giovannetti, 2004), there are 10^{123} possible bits of data in the observable universe, then the prediction is as wrong as it could conceivably be, suggesting that we should look for an answer in a totally different direction. In fact, it suggests, that, rather than applying to a single quantum state (local), it actually applies to the whole universe (nonlocal). Other indications which point in this direction are gravity-gauge theory correspondence, which suggests that gravity is somehow a totality of all the other gauge forces. This has always seemed obvious in the context of the nilpotent structure, where the vacua of the weak, strong and electric forces seem to be derived by partitioning the general gravitational vacuum. The Higgs field, zero-point energy and the cosmic microwave background radiation further suggest continuity in mass-energy, even identity at every point.

A major problem with quantum gravity is that it is to all appearances irreconcilable with a strict understanding of locality, even though it is necessarily a local theory. Black holes, as understood in general relativity, are a specific result of assuming locality applies to gravity. Now, according to the rules of locality, the maximum entropy within any space is determined by the volume of the space, the total entropy being the sum of the entropies of any partitioning within it. In the case of a black hole, however, the maximum entropy, considered in quantum terms, is determined by the area of the event horizon.

Everything indicates that gravity is really a nonlocal property, a manifestation of the total vacuum, $-(\pm ikE \pm \mathbf{ip} + jm)$, even in its negative potential energy, and that it is not a local force. Gravity could be the carrier of all the information separately available from the other three forces, and, in fact, of all their local manifestations, but it would be their inverse rather than their summation, exactly as reflected in its negative energy. We could regard the attractive nature of the gravitational interaction between like particles as a *result* of its being a vacuum, rather than a local force, and therefore requiring negative energy.

The local manifestation associated with gravity may be the positive energy inertial local reaction, which is what we really observe and equate to the gravity of localized clumps of matter. For a fermion we could represent this by the nilpotent structure $(\pm ikE \pm \mathbf{ip} + jm)$ itself. The inertial force, however, is fictitious and repulsive, which means that it could be quantized, as a repulsive force requires a spin 1 gauge boson, rather than the spin 2 particle which creates the renormalization problems associated with quantum gravity.

We have shown previously that, if gravity is instantaneous and nonlocal, but physical *observation* is local, involving time-delayed luminal or subluminal interaction, we can no longer use the Lorentzian space-time of a local coordinate system in the description of gravity (Rowlands, 1994, 2007). To do so effectively creates a noninertial frame for the gravitational system, with the resulting appearance of fictitious inertial forces. Relativistic and 'gravitomagnetic' effects appear but are properties of the local coordinate system, and are not due to the gravitating source. One such effect is an acceleration-dependent inductive force analogous to that appearing in electromagnetic theory:

$$F = \frac{G}{c^2 r} m_1 m_2 \sin \theta \frac{dv}{dt}. \quad (2)$$

Sciama (1953) proposed using a real force of this kind to explain inertia. He assumed that the inertia of a body of mass $m = m_1$ could be derived from the action of the total mass $m_{\text{u}} = m_2$ within a Lorentzian event horizon of radius r_{u} , making the force in equation (2) equal to Kma , with K a constant. In our understanding, the continuous mass-field or vacuum which provides gravitational nonlocality must also define the standard by for a unit inertial mass for the entire universe, in the same way as the almost constant gravitational field g allows us to define a unit mass at the Earth's surface.

In this case, the mass m_{u} will define a radial inertial field of constant magnitude, from the centre of a given local coordinate system to the event

horizon defined by r_u , at the same time as the gravitational field (Gm_u / r_u^2), independently of the local coordinate system, defines a unit of gravitational mass within the same radius. Allowing isotropy to remove the angular dependence, we obtain:

$$\frac{Gm_u}{c^2 r} \frac{dv}{dt} = \frac{Gm_u}{r_u^2}$$

$$a = \frac{dv}{dt} = v \frac{dv}{dr} = \frac{c^2 r}{r_u^2}$$

For a nonlocal gravity, this acceleration would be a fictitious one, the result of using a local Lorentzian coordinate system to model the instantaneous interaction. It is a fundamental physical process, analogous to a centrifugal force, which determines the cosmology (however we interpret this) rather than being a result of it. Integration of the acceleration term with respect to r will also produce a velocity related to the Hubble constant, H_0 :

$$v = \frac{cr}{r_u} = H_0 r,$$

allowing the acceleration to be interpreted in terms of this constant as

$$a = \frac{v^2}{r_u} = H_0^2 r.$$

Combining the inertial acceleration with the gravitational force due to total mass m at any distance, allows us to calculate an equivalent vacuum density, ρ , from

$$F = \frac{Gm}{r^2} - H_0^2 r = \left(\frac{4}{3} \pi G \rho - H_0^2 \right) r$$

In terms of a Poisson-Laplace equation, this becomes:

$$\nabla^2 \phi = 4\pi G \left(\rho - \frac{3H_0^2}{4\pi G} \right) = 4\pi G \left(\rho + \frac{3P}{c^2} \right) = 4\pi G (\rho - 3\rho_{vac})$$

The vacuum density here,

$$\rho_{vac} = \frac{H_0^2}{4\pi G}$$

then becomes equivalent to a 'dark' energy density or negative pressure

$$-P = \frac{H_0^2 c^2}{4\pi G}$$

which can also be expressed through the cosmological constant

$$\lambda = 8\pi G \rho_{vac} = 2H_0^2$$

Defining the critical density for a 'flat' universe as

$$\rho_{crit} = \frac{3H_0^2}{8\pi G}$$

we obtain

$$\frac{\rho_{vac}}{\rho_{crit}} = \frac{2}{3}.$$

As stated previously (Rowlands, 1994, 2007), this value is within current observational limits, but was obtained twenty years before the earliest measurements from supernovas.

If inertia is local, it must be quantizable, even if nonlocal gravity is not. A discrete gravity theory by de Souza and Silveira (1998) has been based on a concept of *extended causality*, which is a development of the special relativistic 4-vector space-time, including the proper time, which in our notation would be represented by a nilpotent $(ikt + ir + jt)$, analogous to $(ikE + ip + jm)$. In this theory, a single object (either a particle or field) at two points in Minkowski space-time (s) must satisfy the causality constraint $\Delta t^2 + \Delta s^2$. In our interpretation this becomes $\Delta(ikt + ir + jt)^2 = 0$. 'Extended causality' applies when we shift t and x by infinitesimal steps dt and dx .

Applying a massless scalar gives us a discrete field equation, and a field source represented by a scalar charge to generate a 'graviton'-like object and a metric for a discrete gravitational field. In our terms, the field is a repulsive inertial one, fully quantized via the Dirac nilpotent, and the 'graviton'-like object is correspondingly identified as a spin 1 boson or photon-like pseudo-boson (Rowlands, 2007). The field can only be quantized in this way because it is inertial, with positive energy, not gravitational, with negative energy, as negative energy, in our understanding, represents nonlocal vacuum rather than the local quantized state. In effect, we have reversed the argument used for the other forces, working from vacuum to the quantized local state.

Local interactions result from nonlocal effects

We have, in effect, shown that there are only three local physical manifestations which result from the nonlocal combinations and superpositions associated with a nilpotent fermion, and these are the ones associated with the electric (or other pure Coulomb), strong and weak interactions. They are also aspects which can be derived from the *algebraic structure* of the three terms, jm , ip , and ikE , which make up the nilpotent operator or wavefunction.

The fundamental basis of the work in this paper has been a long-held conviction that physics ultimately requires just four parameters, space, charge, time, mass, which are symmetrical elements of a noncyclic group of order 4, and which are respectively characterized by multivariate

vector, quaternion, pseudoscalar and scalar algebras, or, in Cliffordian terms by vector, bivector, trivector and scalar algebras (Rowlands, 2007). The combination of the four algebras (with units $i, j, k; i, j, k, i; 1$) gives us the 64-part algebra of the fundamental physical unit, the fermionic state, with a minimal set of five generators, typically $ik; ii; jj; ki; ij$.

The process of creating these generators, however, by 'compactification' of the units, requires the creation of new physical quantities: quantized energy (E), quantized momentum (p) and rest mass (m). At the same time the symmetry of one of the two 3-dimensional quantities (space or charge) must be broken. In practice, it is charge, and the weak, strong and electric components of charge take on the respective characteristics of pseudoscalar, vector and scalar algebra. In effect, the modification of charge shows the nonlocal or vacuum side of the compactification process, while the compactification to energy, momentum and rest mass shows the local.

The algebraic characteristics acquired are manifested nonlocally through the vacua associated with the energy, momentum, and rest mass components of the nilpotent wavefunction. The different algebraic characteristics of the three components then ultimately determine the nature of the local interactions which result from the local symmetries. We have shown that it is possible to go all the way from the nonlocal vacua to the recognizable physical effects which are characteristic of the different forces.

At the same time, gravity presents a kind of inverse effect to that of all the other forces, as we can see from the negative energy involved. While we can do the calculations for weak, strong and electric forces from the perspective of the local structure of the fermion state, and work iteratively to determine the effect of the rest of the universe, with gravity we have to do the calculation universally or recursively to work back to the inertial effect on the quantized fermionic state.

Mass of the Higgs boson

Researchers at the LHC facility in CERN are actively attempting to find the Higgs boson. Though this particle has not yet been discovered and the limits of exclusion are as yet only at the $2s$ level, interest has been evoked by anomalies seen in the data at energies of approximately 120, 140 and 246 GeV. It is not obvious why the Higgs boson should have any particular mass, from within the Standard Model, but at some point theorists will need to address this problem, and it is important to try to

anticipate the result. My own previous speculations have variously suggested 121.5, 137.5, 181.5 and 227.5 GeV, though with a relatively low level of confidence (Rowlands, 2007).

A 246 GeV mass would be interesting as coinciding with the vacuum expectation value of the Higgs field. This would effectively merge the Higgs boson with the vacuum which produces it, which is a possibility I have previously considered. A mass of half this value (123 GeV) would be even more interesting because it would suggest that the particle has the maximal energy required for either its fermion or its antifermion component to have the 'wrong' handedness and so produce the spin 0 combination (Rowlands, 2011). Either of these masses would mean that 181.5 GeV would remain as the total mass previously derived from the 'structured' information available about particles, so representing the sum of the fermion contributions.

The spin 1 boson has the structure

$$\begin{matrix} (ikE + ip + jm) & (- ikE + ip + jm) \\ LH\ fermion & RH\ antifermion \end{matrix}$$

And can be massless

$$(ikE + ip) \quad (- ikE + ip)$$

The spin 0 boson has the structure

$$\begin{matrix} (ikE + ip + jm) & (- ikE - ip + jm) \\ LH\ fermion & LH\ antifermion \end{matrix}$$

or

$$\begin{matrix} (ikE - ip + jm) & (- ikE + ip + jm) \\ RH\ fermion & RH\ antifermion \end{matrix}$$

and cannot be massless. One half or the other (left-handed antifermion or right-handed fermion) has the 'wrong' handedness and is not weakly interacting. This forces it to acquire mass. If it is maximally for the Higgs boson, then we might expect this process to require half of the vacuum energy or Higgs field, i.e. 123 GeV, with the two possibilities giving us the combined value of the Higgs field (246 GeV).

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A Brief Introduction to Counterspace

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Introduction

Counterspace is a term applied to a discovery by Rudolf Steiner (1861-1925) that besides the kind of space we are accustomed to, there is another quite different kind of space which can only be inhabited by a different kind of consciousness. That space is quite distinct from ordinary space, actually exists he claimed, and concerns life and the subtler aspects of light, chemistry and heat. His colleague George Adams applied the then modern ideas of non-euclidean geometries to characterise counterspace mathematically. The author of this article has attempted to take the whole project further, applying it to various spheres of science. The essential idea is that objects may exist in both spaces at once, and when they do so strain arises due the conflict of their different invariants. This leads to stress that accounts for the forces we experience as well as other forces sometimes referred to as "subtle". The first successful application of the idea was to the force of gravity. A brief account, without formal proof, will be given of the basic geometry which can readily be substantiated in the literature, and a more detailed exposition of the author's work in the reference.

Space

Space is understood here to be a subject of physics, described mathematically by some kind of geometry. Space and geometry are of course distinct concepts. There are different kinds of possible geometry, starting with projective geometry which is the most general if we remain with geometry rather than topology. It has only one numerical invariant, cross-ratio, and is linear i.e. projective transformations obey the law of superposition, and straight lines transform again into straight lines, flat planes into flat planes. Strictly speaking there is no infinity in pure projective geometry (e.g. the model of two-dimensional projective geometry given by the lines and planes in a fixed point), but when projective geometry is applied to Euclidean space one of its planes becomes for the latter an unreachable "plane at infinity", which homogeneous coordinates can handle effectively. That plane is - for Euclidean geometry - an "ideal plane" composed of "ideal points". The first specialisation of projective geometry is to restrict transformations to those that leave a plane invariant (as a whole i.e. all its points transform into points also lying in it) which is then called the plane at infinity, yielding affine geometry. Since points and lines at infinity are transformed into points and lines also at infinity, parallelism is conserved in affine geometry. Division ratios are also invariant e.g. two parallel line segments will retain the ratio of their lengths invariant. Remarkably it is possible to impose a further restriction such that volume becomes invariant for so-called special-affine transformations, even though length and angle are not invariants. Finally if only those transformations are allowed which

transform spheres into spheres then metric Euclidean geometry is obtained in which length, angle, area and volume are all invariant. Our everyday space is Euclidean to a high degree of accuracy, only diverging from that when relative velocities comparable with that of light are involved. It is characterised by the invariants obeyed.

Arthur Cayley (and others) found a way of bringing a metric into projective geometry by imposing an invariant quadric surface S on it, for then two given points P and Q define a line intersecting S in two points I and J , say, which may be real or imaginary. The cross-ratio of those four points (PQ, IJ) is invariant under the specialised projective transformations that leave S invariant, so the length PQ may be associated with that cross-ratio. In the Euclidean case S is degenerately composed of an imaginary circle in the plane at infinity given by $x^2+y^2+z^2=0=w$ in homogeneous coordinates, where w is the fourth coordinate which when zero gives a point at infinity.

Duality and Polarity

In projective geometry a law of duality exists such that every theorem has a dual theorem where the roles of points and planes are interchanged e.g. three distinct non-collinear points define a unique plane, while dually three distinct non-collinear planes determine a unique point. In the two-dimensional case the theorems of Pascal and Brianchon, once considered distinct, were seen to be dual and therefore logically the same. This

duality is an inescapable fact of projective geometry, and the success in science of applying its point-based approach suggests that the application of the dual plane-based approach may be equally fruitful. In physics tensor analysis is applied to the definition of the metric of a space, given by

$$ds^2 = g_{ij} dx_i dx_j \quad (1)$$

where ds is the infinitesimal distance between two "adjacent points", dx_i and dx_j are their infinitesimal coordinates, and g_{ij} is the metric tensor composed of functions of the coordinates. The convention is that repeated suffices are independently summed (from 1 to 3 for space). For Euclidean space g_{ij} is simply the matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2)$$

giving Pythagoras' Theorem $ds^2 = dx_1^2 + dx_2^2 + dx_3^2$, but for a non-Euclidean space each term of the matrix may be replaced by a function of the coordinates. The above equation (1) is called a "tensor equation" because it remains true on change of coordinates. For a projective description homogeneous coordinates are used and so i and j range from 1 to 4 in (1).

In curved space or in a manifold the metric changes from point to point, and g_{ij} may be regarded as representing Cayley's absolute quadric which

varies across the manifold. The constancy of the "signature" stressed in Relativity theory simply means that the quadric remains of the same type across a curved space (not necessarily in a manifold) as only a projective transformation may change its signature to some degree, but not a Euclidean one. Even a projective transformation cannot interchange ellipses, ruled quadrics or imaginary quadrics.

Counterspace

Notice that equation (1) is defined in terms of the coordinates of points, and in the Theory of Relativity and in modern physics generally, that is always the case. However the principle of duality suggests it is also possible to define a metric as follows:

$$d\tau^2 = g_{ij} d\rho_i d\rho_j \quad (3)$$

where g_{ij} is a metric tensor, and $d\rho_i$ is an infinitesimal in plane coordinates. Consider the projective equation of a plane

$$\alpha_1 \cdot x_1 + \alpha_2 \cdot x_2 + \alpha_3 \cdot x_3 + \alpha_4 \cdot x_4 = 0$$

The constants α_i define the plane, and all points $\{x_1, x_2, x_3, x_4\}$ satisfying this equation lie in that plane. However the x_i may be fixed defining a fixed point, and the α_i may be varied to give all the planes containing that point. The equation is thus fully dual, and the α_i may then be regarded as

the coordinates of those planes. If $\alpha_2=\alpha_3=0$ and $\alpha_4=x_4=1$ then $x_1=-1/\alpha_1$, so that the x_i ($i=1$ to 3) are proportional to $-1/\alpha_i$. Thus these coordinates may be envisaged as the reciprocals of the negative intercepts of the plane with the coordinate axes. $d\rho_i$ above are the infinitesimals of such plane coordinates.

The Euclidean dual takes g_{ij} equal to the unit matrix I (similar to (2) but for four coordinates) giving the dual of an imaginary circle which is an imaginary cone (this was the insight of George Adams). The latter is Cayley's absolute quadric for a metric space dual to that of Euclid. In the Euclidean case the imaginary circle lies in the plane at infinity, so in the dual case the (real) vertex V of the imaginary cone plays the role of the infinite. By duality we expect that the dual of the plane at infinity will be a "point at infinity", and V is that point. Such an infinity is an "infinite inwardness" in contrast to the "infinite outwardness" of the plane at infinity. Instead of having a "point of view", by duality an entity in such a space has a "plane of view", which plays for it the role of the periphery from which it gazes infinitely inwards towards the unreachable point V . Such a space is referred to here as "counterspace". Its geometrical description is dual in every respect to that of ordinary Euclidean space. However the apparent simplicity of that is beguiling as we shall see. Rudolf Steiner claimed to have been able to enter a dual or polar consciousness such as this, and thus end up in a counterspace. People known to the author have experienced such a "turning inside out" of consciousness, or cosmic consciousness. It must be emphasised that the

notion of counterspace makes no sense without taking into account the possibility of a type of consciousness for which planes or orientations are basic instead of points or locations.

The invariants of counterspace are termed by the author "turn" and "shift", where turn is the variable τ in equation (3), measuring the separation of planes, while shift is the measure of the separation of points. By duality the equations governing turn are like those governing length in space, and so turn does not measure an angle, but a separation which may become infinite if one of the planes contains V . Shift on the other hand behaves like an angle, varying from 0 to 2π radians. It is measured by the angle between the lines through the points and V . Strictly speaking there are no angles in counterspace, but this Euclidean perspective is a useful "crutch" to envisage the meaning of shift. Note that in counterspace there are no parallel planes, but there are "parallel points", for two points on a line through V have zero shift, dual to two parallel planes in space which have a zero angle between them. We begin to see that although a "simple" duality is involved, the conceptual consequences are in fact interesting.

The duals of area and volume exist in a metric counterspace, and they are called polar-area and polar-volume, and can be quite non-intuitive. The canonical formulae for polar area and volume have the same form as for space but in terms of turn and shift rather than distance and angle e.g. the polar-volume of a sphere is

$$\frac{4}{3}\pi\rho^3$$

where ρ is the polar radius, which is the turn in from Euclidean infinity of any tangent plane to the sphere. The polar volume may be described by all the planes that do not intersect the sphere, just as the ordinary volume is described by all its interior points. Polar-area can be more misleading e.g. that of a cone is described by all the planes in its vertex that do not intersect the surface. Intuitively that seems like a volume, but it should be recalled that the planes only have two degrees of freedom, not three.

Linkages

It may be asked: what is the result of an object residing in two spaces at once i.e. in space as we know it and in a counterspace? It transpires that generally the invariants of the two spaces cannot be obeyed simultaneously. As an engineer, instead of abandoning the whole idea the author realised that this may be regarded as giving rise to a "strain" in the object. If a steel bar is compressed, for example, the strain is defined as the percentage change in length. It is a purely geometric concept. However we well know that stress accompanies the strain i.e. a field of stress pervades the bar, leading to strong forces on the supports, resisting the change in length. This makes a transition from geometry to physics, for stress is a physical concept. The hypothesis is that *an object residing in both space and counterspace may suffer strain when any movement occurs, leading to stress*. Such stress is the cause of the forces we study in

physics. The first application of the idea was to the force of gravity, and indeed Newton's law was obtained. The result is that gravity may be described as *the gradient of shift strain* (see the reference for a detailed derivation). An important case when no stress arises is when an object is rotated about an axis through V. This may explain the universal tendency for rotary motion to arise in liquids, gases and to a lesser degree solids. Water vortices are a beautiful example, assuming a counterspace infinity (like V) at the centre of the Earth.

When an object is in both spaces at once that constitutes a linkage made by the object between the spaces, which are otherwise disjoint. The linking may be via points, lines, planes, polar-area or polar-volume, and in the case of gravity point-linkages were analysed assuming points of the object were in both spaces at once, like "stitches" between the spaces. These linkages are governed by tensors and may be affine, special-affine or metric. The point-based linkages are seemingly abstract at first, but will be replaced by a more realistic interpretation later. The nature of the stress depends upon whether the invariants of space dominate, in which case the stress arises in counterspace but reacts upon space via the linkage, or whether the invariants of counterspace dominate in which case the stress arises directly in space. The former applies to gravity which is why it is such a mysterious force. It should be noted that the explanation of gravity given by General Relativity relies on a curvature of space, which is however purely geometric. Force only arises when an object is prevented from free-falling along a geodesic of space-time. Einstein had to introduce

the energy-momentum tensor to describe the force. However, that does not explain the force, it merely enumerates it. Another route has been followed here, so far for Euclidean-style metrics, which explains the force as well as enumerating it.

Counterspace and Chaos Theory

George Adams had the insight that there is a counterspace infinity at the growing-point of every plant. This was at first intimidating (anyway for the author) for it implies billions of counterspaces, not just one! The branch of chaos theory based on the so-called "chaos game", which generates fractals by combining several affine transformations of the same kind, helped to resolve this. There is a fundamental or primal space represented by a triangle, say, which is then "tiled" by images of itself. Those "tiles" represent similarity transformations of the primal space, and the fractal is the synergy of all the transformations. The fractally produced fern leaf is a well known example. The multitude of counterspace infinities could likewise be regarded as images of a primal counterspace, and those images are referred to as CSIs (counterspace images or counterspace infinities). A pointwise linkage referred to above for gravity is then seen to be a dynamic activity between the two spaces rather than a mathematical abstraction i.e. linkage takes place at a CSI. The derivation of Newton's law of gravity depends upon this fractal interpretation, as do the results for gases and liquids where pointwise linkages are also

involved. All of this only applies to point linkages which, like fractals, are scale invariant.

This does not mean that there is only one primal counterspace, however, and it emerges later that there is one for every chemical element, as well as one in the Sun and in the Earth. This remains an open area of investigation.

Scaling

If two planes are linked and their separation is increased by 1 degree, say, how large is the corresponding change in turn in counterspace? There is nothing fundamental to determine this, so the linkage must be *scaled*, and various physical constants turn out to be such scaling constants, most notably the so-called velocity of light. The latter is not in fact a velocity but the scaling constant between space and counterspace for light and other radiation. Now consider a physical copper cube. If it is heated it expands, suggesting that temperature relates to scaling. Very fruitful results follow from adopting this proposal, most notably the ideal gas law. Other examples of scaling constants are permeability, permittivity, Planck's constant and so on i.e. all the fundamental constants of physics. The search for a Theory of Everything would like to derive these constants from some over-arching law, but such an ultimate mechanisation is not adopted here and the constants are accepted as contingent facts.

Time

Two distinct lines of enquiry independently suggested that time is related to radial turn (see the reference). That means that if counterspace is described by spherical polar coordinates then the polar radius of a sphere is inversely proportional to time, so that the greater its physical radius, or the smaller its polar-radius, the further back it is in time. There has to be a sphere centred on the CSI for the present moment, which is called the *valence sphere* because of its intimate relationship to chemistry. All chemical action occurs now, not yesterday or tomorrow. The remaining two-dimensionality of counterspace concerns time-independent relationships, which are thus tangential, so two distinct planes tangential to a sphere (centred on the CSI) are separated by a turn which is not time-like, whereas any turn involving radial difference involves time. The polar-volume of the valence sphere contains the past, which is unchangeable, while its Euclidean inside relates to the future and is where change is possible. Time is something of an illusion, as the corresponding reality concerns whether change is or is not possible, the present being where change is occurring. Thus instead of adding another dimension to the three of space, as in Relativity, a radial dimension of counterspace is involved instead. The conventional four dimensions of space-time are supposed to be qualitatively indistinguishable, and Einstein suggested that it is our consciousness that singles out one as measuring time. Hence the need for imaginary numbers to relate it to the others. The view presented here is quite different, time being qualitatively quite different from space.

Linkage Types

Special-affine geometry, as pointed out earlier, preserves volumes without preserving lengths or angles. This seems surprising, and shows that although volumes are calculated using lengths and angles, they are nevertheless independent of them. Liquids at constant temperature exactly exhibit this geometry, and for example the brakes of a car depend critically upon the virtual incompressibility of the brake fluid. Gas on the other hand may expand or be compressed, and affine geometry better describes it, while solids at constant temperature and pressure exhibit the invariants of metric geometry. Heat relates to scaling and is not necessarily to be related to projective geometry, as might otherwise be supposed. The linkages are thus seen to be metric for solids, special-affine for liquids and affine for gases. This applies to point-linkages involving CSIs. Applying this enables Boyle's Law for ideal gases to be derived, and then taking into account the relation of temperature to scaling leads to the Ideal Gas Law (see the reference). A similar approach to liquids using special affine linkages yields many of the properties of liquids, but is still ripe for further work. The significance of this is that the laws can have a different non-materialistic basis, where forces may be explained in a different way using stress rather than collisions and rate of change of momentum. That opens up a wider approach to forces than is provided by collision.

Corresponding to the affine geometries there are polar-affine geometries suitable for counterspace linkages where counterspace dominates. These relate to light, tone, chemistry and life and will be explained below.

Light

For light another kind of linkage tensor is necessary. Planar linkages are unsuitable, and the polar approach to that for point linkages fails as turn is not scale-invariant. The linkage is assumed to be affine as light is expansive, in which case comparability is only possible in one counterspace "direction". That narrows down the choice to the polar area of cones, or turns between planes intersecting in a plane containing a CSI. The most suitable candidate is the polar area of cones. Now a cone may represent the dual of a bivector, and it has the three properties essential for photons: a magnitude (polar-area), a sense (it is projectively oriented) and a direction (it has an axis). A bivector is a simple tensor and thus for all these reasons is well suited to be the linkage tensor for light, appearing as a cone in space and the dual of a circle in counterspace. Such cones are referred to as photon cones, but suffer none of the disadvantages of conventional photons: there is no particle/wave duality problem as neither particles nor waves are involved, and the laws of reflection are derived very simply, in contrast to the complicated "sum of paths" explanation necessary in quantum physics. Consequently photon cones are adopted as the linkage tensor for light. It is shown in the reference that diffraction, reflection, absorption and Snell's law of refraction are all easily derived

from this proposal. Also the independence of its properties from the velocity of the observer as in Relativity, as well as the so-called "constancy of the velocity of light *in vacuo*" which follows from the fact that the scaling constant c is indeed constant.

The directional aspect requires the polar-areas to be "parallel" which means the cones compared must either have the same vertex (dual to areas lying in the same plane) or the vertices must lie on a line through a CSI to be "parallel". That meets the affine requirements and is of critical importance for explaining refraction, reflection etc..

The waves physics observes, associated with photons, only arise at the moment of absorption. Since light can only be studied experimentally by observing its interactions with matter, it has been assumed that it has wave properties, which is false except in such interactions. The solution of the second order differential equation involved for the tensor shows this, but is not in the reference as that work has been done since its publication.

Chemistry

The valence sphere introduced earlier governs chemistry, and changes in the surface of a sphere are studied that obey Laplace's equation $\nabla^2 \phi = 0$. This is necessary to ensure single-valued, finite and continuous action. Laplace's equation in the surface of a sphere involves surface spherical harmonics, and at this juncture the equations of quantum physics thus arise

as e.g. in the conventional study of a particle on a sphere for electrons. However in our case no particles need be involved, but instead the turns of planes and the polar areas of photon cones tangential to the valence sphere. As such changes occur "in" (tangential to) the sphere they are time-invariant, which is interesting as for us time-invariance is a given, whereas Schrödinger arbitrarily partitioned his wave-functions into time-invariant and progressive parts for convenience. The scaling tensor is a trivector in this case, and an analogue of the covalent bond is readily derived (see the reference). Quantisation arises for the same reason as in quantum physics. Wave like "tones" also arise from the surface spherical harmonics. Thus the most complete accord with quantum physics arises for chemistry. The "tones" are seen as the counterpart of vibrating strings in string theory, without the encumbrance of eleven dimensions, or the implausible energies involved.

Life

The scaling tensor for life is metric, and a two-component spinor turns out to be most apt. Spinors in any case cannot be affine. The reason they are applicable lies in the formal equation of an isotropic vector:

$$x_1^2 + x_2^2 + x_3^2 = 0$$

These are associated with spinors, and formally the coordinates may be point-coordinates or plane coordinates. Furthermore this is the projective

equation of the absolute imaginary circle at infinity for point coordinates - as we saw earlier - giving the Euclidean metric, and it is also the projective equation in plane coordinates for the absolute imaginary cone introduced for the counterspace metric.

It might be thought that a "solid" metric tensor is the least suitable for life! However it is applied to the membranes involved in all living organisms, governing the clearly metric actions involved in diffusion across membranes etc.. A human body contains membranes within membranes, working inwards from the skin. This characterises counterspace where there is no adjacency as in space, but "inside" and "outside" predominate.

If a membrane is regarded in counterspace with a CSI at its centre, then the "inside" of the membrane is its polar volume, and is what our Euclidean consciousness regards as the outside. For the plasma membrane of a cell, all other cells are "inside" it as viewed in counterspace i.e. an organism is "inside" all its cells! This gives a new insight into the nature of an organism, and the synergy of all its cells - every one containing the whole organism - governs its holistic properties in healing, nourishment etc.

Much work remains to be done by qualified biologists to take all this further.

Particle Physics

Professor Bohm pointed out forcefully that in a bubble-chamber experiment there is no particle "crashing its way" along a track producing bubbles as it goes. Rather there is a reasonably well-defined track and the wave-function collapses at intervals where bubbles occur due to interactions, at which points a particle fleetingly appears only to vanish again.

An electron poses great difficulties because it cannot have a finite volume, as if it did the electric charge would be spread out and then Coulomb's law would result in its disintegration. Therefore it must be a point particle. It is not "smeared out" by the wave-function as is sometimes claimed, but is a point particle whose location is stochastically described by the wave function in line with Born's interpretation. However a point particle with charge and mass has an infinite charge density together with an infinite mass density. No Standard-Model based attempts to remedy this have been successful, although string theory can do so at a huge price in the energies involved and the complications of the Calabi-Yau spaces involved. The counterspace approach solves the problem in quite a different manner. Recall that for an affine comparison of polar areas the comparison must be made at a point, or at polar-parallel points. The electron is proposed to have such a point, called the Affine Interaction Location or AIL, at which all affine comparisons are made for its interactions with other entities. However, the charge is distributed in

counterspace where no explosive repulsion occurs, and the mass arises from a new concept: *radial momentum*, which is also not contained in the AIL, but centred on it (farewell LHC?!). The AIL is like the post-box of a house, through which all correspondence is posted. It is not, however, a particle as conventionally envisaged. Other particles, where they arise, may be considered similarly.

It is not in accord with quantum physics to suppose particles exist as point particles all the time, as Bohm and others point out. This even applies to atoms. A helpful analogy is to imagine a Maltese cross on the surface of a strapped torch. Shine the torch on a wall and the cross appears. Its area, size and shape may be measured, and then the torch is turned off. Later it is turned on again, and *hey presto* all the measurements are the same as before! That does not prove the cross was on the wall all the time, and neither do repeatable experiments with particles or atoms prove they exist between the experiments. This is the view adopted here, and is fully in line with known physics. When an interaction conjures a particle into existence, it is conjured lawfully and consistently.

Many of the properties of the atom of a chemical element have recently been derived more simply than in conventional quantum physics. It is beyond the scope of this article (and accompanying lecture) to go much further with this. It is based on hybrid resonant cavities:-

Hybrid Resonant Cavities

A mathematical experiment was conducted. An attempt was made to equate the spatial version of Laplace's equation in spherical polar coordinates with a counterspace version. It is a standard technique to equate two dimensionless equations via some kind of scaling constant when solving second-order differential equations. When the counterspace side was related to spatial quantities via scaling and the resulting equation was solved, the standard spherical wave equation was obtained. This opens up the possibility of proposing the existence of *hybrid resonant cavities* between space and counterspace, and this has been applied to the physics of atoms, when they appear. They are regarded as heat-cavities, that with the fundamental frequency for the valence sphere being a hydrogen atom, other elements being related to higher harmonics. Protons and neutrons are interpreted not as particles but as modes unless detached by an interaction. The spinor introduced for the scaling tensor of life processes plays an important role, although it is augmented to a four-component spinor to take in the electric and magnetic properties. Electrons are only supposed to appear upon ionisation. This has yet to be published in detail, but a further book is being written. It is also somewhat provisional as new insights improve it, being now in its fifth incarnation. The aim is to find a way of treating nuclear waste as well as to re-found quantum physics on a new basis.

Heisenberg's uncertainty relation may be regarded as a consequence of the fact that polar quantities may co-exist in affine linkages but not when reduced to a metric linkage by a measurement.

Non-locality is implicit in the whole approach, as counterspace is inherently non-local.

Conclusion

This brief, aphoristic account accompanying the lecture is intended to introduce the approach to science envisaged by the application of linkages between space and counterspace, and to outline some of the results obtained. It is important that known facts are accommodated, not as a "proof" but as ensuring the enterprise is well founded ready for new predictions and applications. It is intended to widen the scope of science to accommodate more subtle aspects of the world including life, which is regarded by biologists as an unscientific concept acting as a shorthand for the complexities embraced. Non-locality is implicit and consciousness plays an important role in the distinction between space and counterspace. Much remains to be done!

Reference

Thomas, N.C., *Science Between Space and Counterspace*, New Science Books, London 1999, reprinted 2008 by Temple Lodge Press.

Web page: www.nct.anth.org.uk

ANPA PROCEEDINGS EDITORIAL POLICY

ANPA has been criticised in the past - in particular by members of its own Advisory Board - for having no formal editorial policy for its Proceedings. This has been balanced by a feeling within ANPA that we should keep ourselves open to all viewpoints. In the last few years as editor I [K.B.] have tried to tighten things up in such a way as I felt would satisfy our critics whilst not compromising our own position. This has been partially successful although for some time I have felt that it is time that there was a formally stated policy. The following has been approved by the Executive Council, although it is open to feedback from all. By "the editor" is meant the Editor or (an) appropriate nominated Referee(s) (note the capital R!)

1. The paper should make a new and original contribution to the fields of ANPA's interest. Survey papers are acceptable.
2. The default use of language for submitted papers in Physics and Philosophy of Physics should be the common language of Physics as usually understood by Physicists, and, in particular, by Philosophers of Physics. Any other use of language should be carefully explained at the start of the paper and all appropriate definitions included there.
3. The editor should be satisfied that the paper is *presented* in such a way that the majority of the readership will understand the author's intentions. In particular *it should be clear* that the author has a correct understanding of the subject matter. Please avoid forward references.
4. "Verbatim" reports will be accepted subject to the above three conditions only, regardless of whether the final draft is an accurate rendition of what was originally said.
5. Theories of any nature are acceptable material, provided they are compatible with the known facts, and provided they are deemed to be of interest to the readership. Theories of alternative, imaginary worlds are also acceptable, provided their nature is made clear.

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- **FORMAT:** Only electronic submission as a PDF file or a WORD file (as a second option) will be accepted
- **FONTS SIZE:** The body text should be in at least 14 point type - if submitted in A4 format, and 10 point type - if submitted in A5 format
- **EMBEDDED FONTS:** All fonts must be embedded
- **LINE SPACE:** The manuscript should be at least 1.5 spaced throughout
- **JUSTIFIED TEXT:** The text must be justified on both sides
- **MARGINS:** The left, right and the top margins should be the same: 20mm. The bottom margin should be 30mm
- **MAX NUMBER OF PAGES:** The length of the manuscript must not exceed 30 pages (please, contact the editor if your manuscript is longer than that)
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- **DEADLINE:** Contributions must be submitted latest 1 January following the ANPA meeting

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1. The primary purpose of the Association is to consider coherent models based on a minimal number of assumptions, so as to bring together major areas of thought and experience within a Natural Philosophy alternative to the prevailing scientific attitude. The Combinatorial Hierarchy, as such a model, will form an initial focus of our discussions.
2. This purpose will be pursued by research, publications and any other appropriate means including the foundation of subsidiary organisations and the support of individuals and groups with the same objective.
3. The Association will remain open to new ideas and modes of action, however suggested, which might serve the primary purpose.
4. The Association will seek ways to use its knowledge and facilities for the benefit of humanity and will try to prevent such knowledge and facilities being used to the detriment of humanity.

ORGANISATION

1. The Founder of the Association was Pierre Noyes. The Founder Members were Pierre, John Amson, Ted Bastin, Clive Kilmister and Frederick Parker-Rhodes. They will be known herein as the Founders. The Executive Council is the governing body of the Association. It consists of: (a) The Founders and all past Presidents of the Association, the President, the Co-ordinator and the Treasurer, (b) Ordinary members nominated by classes (a) and (b), who serve for three years, with the possibility of re-nomination.
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