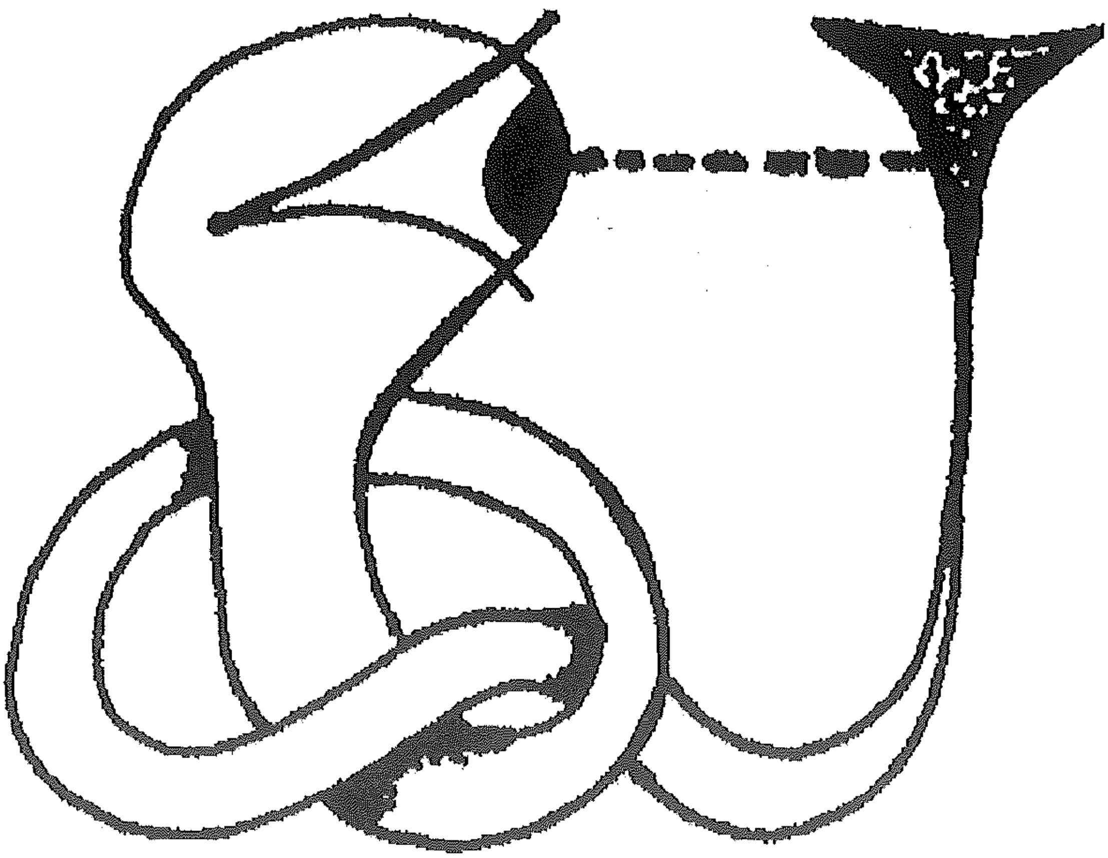
Reflexivity

Proceedings of ANPA 30

Arleta D. Ford, *Editor*



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THE BASIC TWO-ITY

Ted Bastin

There has been a good deal written (ANPA) on the possible application of the hierarchy algebra to the structure of physical theory and in particular to particles. One thing in this stands out as different from the conventional picture. In usual practice, theoretical innovation is a direct attempt to improve the current mathematical understanding of the physical world as expressed in laws represented by equations. Our algebraic approach is not an attempt to improve the way this has currently been done: it is something different. The hierarchy algebra is not an attempt to extend and improve existing writing on the physical world. It is something different. It is a *precondition* for imagining the physical world and stands separate from the formulae in which that imagination has always been expressed ordinary physics. It is vital to understand that these things are separate. There may well be influences from one side to the other, but the separation is what is always pre-eminent and it is the position from which we are constrained by our general approach to work. We call this separation "the basic two-ity".

An extremely important consequence or illustration of this principle appears through the astonishing calculation by Clive Kilmister of the precise value of the fine-structure constant which agrees exactly with experiment apart from being very gratifying, and very convincing of the general correctness of our whole approach.

One basic aspect of this calculation must be fully realized. The calculation comes in its entirety and in its detail through just taking the principles coming from the discrimination principle and following them. The fact that this works exactly is astonishing since nothing else may be added in.

The prevailing difference called here "the basic two-ity" needs further analysis to which I now turn. All began from the belief expressed that the world was in the first place a combinatorial structure to which perceived elements came progressively to be added by a process called *discrimination*. The question of what or who did the perceiving was left to be answered later on as the theory developed, but it was presupposed that perception or as we might better call it *recognition,* was inherent from the word go, and no explicit philosophy of perception was needed in those beginnings.

Can we present this position as part of the conventional view of scientific endeavour as *empirical?* Can we claim to be empiricist? And if so, what kind of empiricism?

Consider the profound and rigorous calculation by Kilmister of the exact value of the fine-structure constant. Kilmister set out with the intention of eliciting everything that was entailed when we adopted the statistical method. His calculation depended entirely on further elaboration of the principles that underlay the hierarchy algebra right from the critical point at which the number 137 appeared. His calculation agrees with its experimentally known value with complete accuracy. That is to say that is a calculated number whose values contain the experimental values so far as the latter are known sufficiently accurately for comparison to be possible, and it can be expanded indefinitely to predict experimental values that are not yet known This accuracy led us to think that the calculation must have represented the world as it really is.

The accuracy is of course remarkable and very satisfactory. However there would remain a more general question even if this degree of accuracy had not been obtained. We might have come up with one of a range of proffered values dotted around the desirable one. However, more critical appraisal makes us realise that our whole approach requires us to give an explanation of approximation itself, and by no means to be content to say that approximation must be accepted because it is found in the everyday character of calculation of theoretical quantities. To obtain a correct value like the exact value obtained by Kilmister is one thing, but to understand how there can be a range of values that are accessible to theoretical understanding is something that requires a new principle to be formulated.

So far we have been content to follow current ideas which see nothing wrong with the idea of a range of possible values, but now we have to be more demanding. If such calculations can once be completely right how can they ever be wrong? The answer lies deep in our entire approach. We have been behaving as though there was a physical world mapped out before us and as though our job was to pick up *just from that* the principles by which we should analyse it. In fact those principles have been furnished by the formulation of the hierarchy algebra before we began. In short, as we always knew, there is an inevitable jump that has always to be allowed for, and that we never *can* go from the hierarchy principles to their application as though they were all part of a whole. It is just at this point that we intervene in current thinking with something new. There must be a deliberate break in the construction process at which the hierarchy and its intended application are juxtaposed, and this juxtaposition cannot be incorporated into either. This break and its inevitability is what this paper is all about. It is entirely indispensible and therefore has to be very explicitly recognized.

Inevitably then we start from two ends, and these we have to find a way to reconcile. Since the combinatorial end is laid down when we begin, our problem lies in saying how that starting point interacts with the detailed physics. Can we perhaps suppose that at each point in the reconciliation process the physical detailed picture can say either "I can fit in with the presented combinatorial structure", or else "I cannot". There is no need to quibble about whether there can be a decision without a conscious observer: sufficient that there has to be a decision, and that decision has to reflect the present state of the detailed picture.

So what can we say about the effect that the hierarchy construction will have on the detailed structure? Evidently some analyses of the detailed background will make changes in the received picture, some more and some less surprising or unconventional. Let us contemplate a range of degrees of reliability or plausibility in applications of the hierarchy to the detailed structure in our attempts to understand what goes on between hierarchy and detailed picture. That sounds to be letting in a great deal of subjectivity, yet have we an option? Inevitably there is a judgment somehow to be made about the rightness of any detailed picture on the basis of whatever interpretation of the hierarchy picture that we accept. There is an *iteration* - namely a two-way interaction between what we have referred to as the two ends, and taking these together gave us our picture of the universe (physical world plus hierarchy structure).

We find we have introduced a new key concept - that of *iteration*. We are all the time making changes at one of our two ends, and then following that up by looking at the situation from the other end and considering that change. Thus we get a continuing process of successive reconsiderations which we may call iteration. This new concept answers the question of how the hierarchy affects the detailed structure. One at first asks how there can be this two-way interaction if the hierarchy end is fixed? The answer is that though it is fixed yet it has to be understood in a physical context which means that there has to be flexibility in the language in which it is expressed.

We may summarize our position thus. Theory construction, or the formulation of any complete picture is necessarily in two stages our "basic two-ity" and the separation is due to our starting from our discrimination logic.

THE STATUS OF THE COMBINATORIAL HIERARCHY

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I shall try to flesh out in some particular cases the procedure that Ted Bastin has sketched above and which I shall now call Precondition. That is, I want to see how the CH serves as a precondition for imagining the physical world. I found Ted's remarks very stimulating but somewhat Delphic. There is one clear application, to the calculation of the fine-structure constant. Our calculation of this agrees to one part in 10^7 with the experimental. To see that as a quantitative application of Precondition one could put it in this form: certain theories of electromagnetism and quantum mechanics can satisfy Precondition only if α has this value. Now I will try to unpick the significance of Precondition in two particular contexts: the construction of space, and quantum mechanics. You may well think: "Shouldn't this have been done forty years ago?" Yes, of course but (except unconsciously) it wasn't.

First as to space, which is not assumed a priori in the hierarchy. We base our whole discussion on the notion of process. New entities come into consideration. This process brings in order - it is premature to call it time as this stage. The first hierarchy level has 3 elements, symmetrically involved. In Combinatorial Physics [1] we somewhat resisted the guess that this showed the three-dimensionality of space. This was a mistake on our part. In our new version of the hierarchy these three elements are replaced by six in three pairs and the elements of a pair are related very much like positive and negative directions. This new version increases the pressure to see this three as the dimensionality of space. What more is needed to substantiate this in the form of Precondition?

Ask first, why should anyone be skeptical about the three-ness of the hierarchy being the three-dimensionality of space? I think the answer must be, in part, that just to give three directions leaves out "what is in between". One way of tackling this is to see what else is needed in order to construct something like space from the hierarchy framework. The clue comes from going back to the remark that process brings in order, that is, sequence of elements a_1, a_2, a_3, ... What is lacking as justification to call the order "time" is duration. By this I mean that one needs to say of two entities coming into consideration, not only "a_7 is later than a_4" but how much later. We carried out a careful investigation of what duration in this sense (which bears little relation to Bergson's duree) adds to the hierarchy scheme. I don't need to go into details. The first step is the need to interpolate between elements. If the integers are used to label elements, then interpolation requires the rationals instead of the integers. Not the real numbers – I have argued before here, with Weyl's help [2], against the continuum as being in some ways too much for a combinatorial theory and (as Weyl says) too little to capture the intuitive perception of time. Then we carry out a considerable generalisation of the Einstein-Milne argument, called the k-calculus by Bondi. The nature of the generalisation is that we discuss relations between sets of hierarchy elements (discriminately closed subsets, of course) rather than talking about sending light-signals between observers. I remember that when Milne's book [3J appeared, my supervisor, George McVittie wrote a witty review in which he said he had bought a torch and gone outside on a dark night to send light signals - but none came back. The result of our generalisation is to get a (rational approximation) to three-dimensional special relativity, I will just make four remarks:

1. This avoids the very weak argument usually advanced by the k-calculus people, who do it in one dimension and then generalise to three by little more than hand-waving.
2. The construction reaches three-space via space-time instead of the usual approach, which starts by assuming three-space.
3. No appeals here to light-signals - it is a pity that Viv Pope could not be here today. It was always, as Milne was acutely aware, puzzling that a discussion of something so basic as spatial relations had to be preceded by something depending on the physical properties of light.
4. The Precondition works here by showing that both geometry and special relativity can more or less reply "I can fit in".

Turning now to quantum mechanics, I want to argue that the situation is quite different. I want to persuade you that although quantum mechanics is a very efficient calculating tool for quite a large set of problems, it is devoid of explanatory power because it is irremediably inconsistent. That is why the most skilled practitioners say "No one understands it". I begin with the inconsistency; it is easiest to see this from history. You all know that Planck in 1900 justified his 1899 interpolation between Rayleigh's and Wien's formulae for black-body radiation by postulating a discrete nature for energy but putting it into a totally continuous theory. That is the basic inconsistency. The later history is like packing an overfull suitcase - you find something sticking out on one side, so you push it in only to find that something else pops out on the other. Heisenberg seemed to have solved the problem with matrix mechanics in 1925. The idea is that the continuum theory is dropped or at least not mentioned. Dirac tidied the whole thing up in 1926. But there is a snag: how do you know what physical system you are dealing with? You have the Hamiltonian, and this acts as a naming procedure but this Hamiltonian, even though written down in terms of Dirac's q-numbers, looks very like the continuum one. This makes one wonder whether the inconsistency is lurking. Indeed it is; Heisenberg found energy levels as the elements of the Hamiltonian when it was diagonalised. That did not go over into Dirac's q-number structure so it had to be replaced by an eigenvalue condition, $Hs = Es$, where E is a real number and s is the "state" of the system. This state-observable philosophy then exhibits the old inconsistency in new forms: collapse of the wave-function, the measurement problem, Schrödinger's cat and so on.

Later in 1926 came Schrödinger's six papers. They are exactly in the tradition I mentioned. They formulate an efficient calculating tool by replacing Heisenberg's infinite matrices and Dirac's non-commutative algebra by differential equations, which physicists were used to. But they go no further in removing the inconsistencies. I won't go on. In some ways quantum mechanics is like trying to explain the motion of the planets and the Sun by means of epicycles round the stationary Earth. Instead I turn to the lack of explanatory power. I could just cite the Bohr atom in the Old Quantum Theory; why is the angular momentum quantised? But instead I draw your attention to the more recent "wave-function scarring". There is an interesting discussion by Alisa Bokulich [3] in the case of "stadium billiards". This is a classical system consisting of a free, perfectly elastic particle moving on a perfectly smooth horizontal table with a cushion around in the form of a stadium; that is, the shape is one with two parallel straight sides joined at the ends by two semi-circles This looks very like a chaotic system, and so it proves. This means that a typical classical orbit fills up the whole enclosure an infinite number of times. But there are also periodic orbits which are easy to see. For example, a path coming parallel to the straight side and distant $\frac{1}{2}a(2-\sqrt{2}) = 0.2929\ldots\, a$ from it (where a is the radius of the end circles) will have the form of a rectangle. There is an infinite set (of measure zero) of such orbits.

Now quantise the system. Naive correspondence principle arguments say that wave functions will just show a uniform mess over the area [4]. But numerical solutions [5] showed many wave functions with a kind of ghostly trace of the periodic orbits. This is called scarring because it is an anomalous enhancement of the intensity along the unstable periodic orbits. And there is a "more or less" experimental confirmation of this [6]. What is interesting is the "explanation" of this by those in the trade. It is on these lines the wave function "corresponds" in some sense with the classical periodic solution. That is, the burden of the explanation rests on the classical mechanics solution. Quantum mechanics shows no explanatory power.

What then of Precondition? I think quantum mechanics, because of its basic inconsistency, has to answer "No, I cannot fit in". That should be no surprise because all this complication was just to put discreteness into a continuum theory. That problem is not ours, since we put in the discreteness from the beginning. So the optimistic gloss on that is to say

> And, like the baseless fabric of this vision,
> The cloud-capp'd towers, the gorgeous palaces,
> The solemn temples, the great globe itself,
> Yea, all which it inherit, shall dissolve,
> And, like this insubstantial pageant faded,
> Leave not a rack behind.

The corresponding pessimistic gloss is to point out that that we need to provide some kind of substitute for quantum mechanics from a discrete beginning. First, we need to find out just what it has really done We have not done this - I need a PhD student! If you again say "You ought to have done this forty years ago" I again agree. But we didn't. This time, however, I can say that I see great hope in some of the developments Lou Kauffman has shown us in the last few ANPA Proceedings with his commutator algebra.

REFERENCES

[1] Ted Bastin & Clive Kilmister, *Combinatorial Physics*, World Scientific 1995.

[2] H. Weyl, *Das Kontinuum*, 1917.

[3] A. Bokulich, *Brit. Jour. Phil.Sci.* 59, 217-235, 2008.

[4] M. Berry in R.H.G.Hellemn & R. Stora (eds.) *Les Houches Lecture Series Session XXXVI*, North Holland, 171-271, 1983.

[5] E. Heller, *Phys. Rev. Letters* 53, 1515, 1984.

[6] S. Sridhar, *Phys. Rev. Letters* 67, 785, 1991.

Riemann Fever

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Abstract. The confluence of computer science and quantum mechanics has inspired a proof of the Riemann Hypothesis for the discrete Clifford algebras $Cl(n,0)$ over $\mathbb{Z}_3 = \{0,1,-1\}$, as expected from the Weil Conjectures. The symptoms of Riemann Fever are clearly evident here - long periods of apathy punctuated by febrile investigations lasting from minutes to (here, two) weeks, accompanied by alternating explosions of euphoric insight into cosmic truths and insight becalmed in a sea of profundity; and as well a weakness for sweeping conclusions of great import. I recount the course of the affliction in my own case of this recurrent virulent ailment, which often leads to addiction to further attacks, and other complications.

Foreword

My first contact with the Riemann Fever virus (though I of course didn't realize it at the time) was in high school via Dantzig's book *Number - the Language of Science.* It was here that I first (I think) encountered the concept of the prime numbers and the mysteries surrounding them. The exposure was brief, but the virus nevertheless took hold. It was strengthened intermittently over the next ten years, which interval ended with me a computer scientist, software division.

There things stood for some 20 years, whence my interest in concurrent systems eventually led me to Clifford algebras. Quite unbidden, a very early thought here was, and I quote the sober little voice in my head, *There's something about the primes here.* Ach!! This was the early 1990's. But once again the outbreak was minor, as also with a 1996 item in Science magazine [1] about a connection between the primes and particle physics, recounted below, that one might otherwise expect to have resulted in a major attack.

Another ten or so years passed, and finally the toll extracted by the many lesser encounters over the years culminated in my acquiring a copy of John Derbyshire's excellent book *Prime Obsession* [2]. Hereafter, an incubation period of a couple of years resulted in the full-blown incident of Riemann Fever that is recounted in the following. It all began innocently enough, so read further at your own risk... [1]

[1] I have done only minor clean-up and organization of the record, plus added references.

Introduction

In 1996 Science magazine [1] reported a connection between analytic number theory and quantum physics whose core is the Riemann zeta function $\zeta(s)$, a statement intimately connected to the distribution of the primes. This result attracts attention because it is quite unexpected - or perhaps entirely *right* - that a purely number-theoretic property, primality, should have anything to do with the basic structure of reality. However, the probablistic nature of both the prime number distribution and the quantum formalism itself unfortunately shrouds the exact nature of the connection.

To this situation we bring the *synchronizational* model of quantum mechanics [5,6], which by being directly computational, provides a concrete and detailed *mechanism* for quantum processes. By "directly computational" is meant that the algebra functions *(1)* as a program specification language, *and (2)* as a language that allows the symbolic manipulation of this same program "code" according to the usual mathematical operations, and *and (3)* as a language describing physical reality, all simultaneously. This is a considerable extension of semantic power, and the underlying computational power of the model exceeds that of classical Turing machines. The statistical aspect of contemporary physical theory is, in the present view, the result of the combinatorics of the various algebraic possibilities.

Understand as well that the model describes and applies to *any* distributed system, including, especially, self-organizing ones.

More precisely, we use the discrete Clifford algebra $Cl(n,0)$ over $\mathbb{Z}_3 =$

$\{0,1,-1\}$ for this purpose, where the ± 1 captures presence/absence and spin/charge, at the price of compacting relationships based on multiplicity; and synchronization is encoded by idem- and nil-potent elements. Clifford algebras, being inherently hierarchical and well-suited to expressing and tracking symmetries and their transformations, have turned out to be very apt vehicles for the description of both physics and computation; see [7,8,9; 5,6].

Notation. Let x_i be a unit 1-vector, so $|x_i| = (x_i)^2 = +1$; and specify that $x_i x_j = -x_j x_i$ for $i \neq j$. Then the set $X = \{x_1, x_2, ..., x_n\}$ provides the generators for the Clifford algebra $Cl(n,0)$. Both addition and multiplication are associative, and multiplication distributes over addition as usual; the result of a multiplication can be understood as a rotation of the one factor by the other. Note that the Lie bracket $[A,B] = AB - BA$ is naturally defined in the Clifford algebras. For readability we will often write $a, b, c, ...$ for the x_i. Since multiplication is in general not commutative, our convention is "operate on the left on the operand to the right".

The hierarchical aspect of the Clifford algebras is realized via the graded dimensionality of its mutually orthogonal primitive elements 1, $\{x_i\}$, $\{x_i x_j\}$, $\{x_i x_j x_k\}, ..., x_1 x_2 ... x_n$, and together these form a basis for the $\sum \binom{n}{m} = 2^n$-dimensional combinatorial space of *same-different* distinctions that we will be working in. Sums in the algebra represent the simultaneous existence of said elements (eg. *states*), and a product AB is understood to be element A operating on element B. In [5,6] we use the co-boundary operator δ to construct higher-grade elements from lower grades[2] and so

[2]Via the criterion *if* $\partial_Y X = YX \cong Y'$ *then* $X = \delta Y$ *and* $\partial_{Y'} X = Y$. So, for

the algebra's graded hierarchy

$$\{1, a, b, c, d..., ab, ac, ad, ..., bc, bd, ..., ..., abcd, ...\}$$

should be seen in this constructive, structural light.

Calculating Frequencies

Since the elements of the algebra represent, in general, concrete physical entities, it is reasonable to require of them that they can be treated as having a wave-like aspect. For example, what is the frequency [spectrum] and oscillatory behavior of an arbitrary element of the algebra, eg. $a+b+ab$. To do this, view the fundamental *boundary* of the hierarchy as the primitive sensor vector $x_1+x_2+x_3+\ldots$, and assume that each such sensor x_i oscillates between the *discrete* values $+1$ and -1 (and never zero).[3]

Then x_i's oscillatory behavior is characterized by the discrete scalar function $sin(\phi+\omega_i t)$, where ω is the angular velocity, t is a monotonically increasing time counter, and ϕ a phase displacement (choose $\phi=0$). Thus ωt is a length, and taking $t=1$ lets ω carry the concept of wavelength (and thus frequency) without running into temporal and measurement issues.

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example, a principal boundary of the charge carrier $X=abc$ is the quark-form $Y=a+bc$, since $YX=(a+bc)(abc)=-a+bc$ and $a+bc\cong -a+bc$.

[3]That is, think of the hierarchy as being a reactive computational entity embedded in a surround that it senses via these one bit sensors: $x=+1$ denotes that whatever x senses is currently present in the surround, and $x=-1$ means conversely that whatever x senses is currently *not* present in the surround. The x_i are then combined via δ, recursively, to generate the hierarchy.

We wish to calculate $\sum sin(x_i)$ - the oscillatory behavior itself - and $\sum f_i$, the entity's frequency spectrum. Recall that frequency f and wavelength λ are inversely related: $\lambda = \frac{1}{f}$.

One might think that $\sum sin(x_i)$ can be calculated via the identity

$$sin(A) + sin(B) = 2\,sin(\tfrac{A+B}{2})\,cos(\tfrac{A-B}{2})$$

Unfortunately, it's unobvious how expand this to $sin(A)+sin(B)+sin(C)+ \ldots$ in any uniform way, although *if* it can be done, associativity guarantees that we will ultimately get to the same (top) node, the various paths - associatively speaking - are the same as the co-boundary relations and will yield the specific behaviours of each corresponding node.

Calculating $\sum f_i$, or rather $\Sigma\lambda_i$, is more fruitful:

$$\sum \lambda_i = \lambda_1 + \lambda_2 + \ldots = \frac{1}{f_1} + \frac{1}{f_2} + \ldots$$

Frequencies add as

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{f_1+f_2}{f_1 f_2} \quad \text{and} \quad \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} = \frac{f_1 f_2 + f_1 f_3 + f_2 f_3}{f_1 f_2 f_3}$$

and in general, letting $F = \{f_1, f_2, \ldots, f_n\}$ and abusing combinatorial notation, write

$$\sum \lambda_i = \lambda_1 + \lambda_2 + \ldots + \lambda_n = \frac{1}{f_1} + \frac{1}{f_2} + \ldots + \frac{1}{f_n} = \frac{\Sigma\begin{pmatrix} F \\ n-1 \end{pmatrix}}{\begin{pmatrix} F \\ n \end{pmatrix}}$$

The frequencies f are a *scalar* property of the 1-vector sensors that form the boundary of a system. The simplest case is when the f_i all have the same frequency, denoted by "1" in the following table of hierarchically constructed frequencies:[4]

Level Transition δ	$\lambda = \omega_1 + \omega_2 = \frac{1}{f_1} + \frac{1}{f_2} = \{f_1, f_2\}$	f	λ
a	1	1	1
$a+b \to ab$	$\{1,1\}$	2	.5
$a+bc \to abc$	$\{1,\{1,1\}\}$	$\frac{3}{2}$	.66
$a+bcd \to abcd$	$\{1,\{1,\{1,1\}\}\}$	$\frac{5}{3}$	.6
$a+bcde \to abcde$	$\{\{1\},\{1,\{1,\{1,1\}\}\}\}$	$\frac{8}{5}$	.625
$ab+cde \to abcde$	$\{\{1,1\},\{\{1,\{1,1\}\}\}$	$\frac{7}{6}$	.86
$ab+cd \to abcd$	$\{\{1,1\},\{1,1\}\}$	1	1
$abc+def \to abcdef$	$\{\{1,\{1,1\}\},\{1,\{1,1\}\}\}$	$\frac{4}{3}$	.75
$abcd+efgh \to abcdefgh$	$\{\{\{1,1\},\{1,1\}\},\{\{1,1\},\{1,1\}\}\}$	2	.5

Note that we are creating *under*tones, not *over*tones, so the wavelengths *grow* by the ratios shown, and the frequencies *fall* similarly. This reflects the fact that the higher the level of the hierarchy, the more global and "longer view" it reflects of the system's interaction with its surround.

[In Xanadu ...

[4]From top to bottom, the intervals are root, octave, fifth, major sixth, minor sixth, minor third, octave as root, fourth, and octave's octave. It's interesting that the pure-ratio musical intervals show up, although the occurrence here of small-number ratios in general is to be expected. Still missing: major 2nd (9/8 or 10/9, 8/7), minor 2nd (16/15), major 3rd (5/4) and minor 7th (16/9,9/5,7/4). Cf. Harry Partch, *Genesis of a Music*, p. 68.

The physical connection & Riemann

The energy difference E_{gap} between the first and second frequency levels in the above table is $1 - 0.5 = \frac{1}{2}$, and as well the second-to-third frequency level increment can be any one of $.66 - 0.5 = \frac{1}{6}$, $2 \times \frac{1}{6} = \frac{1}{3}$, or $3 \times \frac{1}{6} = \frac{1}{2}$, depending on how many of the co-occurrences $a + bc, b + ac, c + ab$ are present. Since in this $\mathbb{Z}_3$ algebra, $h = c = 1$, this gap corresponds to $0.5h$, where h is Planck's constant. Given the hierarchical context, this energy would then constitute the inherent rest-energy (a stand-in for mass at this stage of the hierarchical construction) of particles.

Howsoever, since Planck's constant h introduces a fundamental physical discreteness, we can reason that the shortest wavelength is the Planck length $\tilde{h}$, so $\lambda_1 = \tilde{h}$, and thus that $\lambda_m = m\tilde{h}$, whence $f_1 = \frac{1}{\tilde{h}}$ and $f_m = \frac{1}{m\tilde{h}}$. Substituting this into the sum, we get

$$\sum \lambda_i = \lambda_1 + \lambda_2 + \ldots = \frac{1}{f_1} + \frac{1}{f_2} + \ldots = \frac{1}{\tilde{h}} \sum_1^n \frac{1}{m}$$

which thus yields the (divergent) *harmonic series*: $1 + \frac{1}{2} + \frac{1}{3} + \ldots$. In the interests of generality [cf. Riemann], rewrite this as

$$\zeta(s) = \sum_{m=1}^{n} \frac{1}{m^s} = 1 + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s} + \ldots$$

where s is complex and the left-most Greek letter is *zeta*; the series converges for all $s \neq 1$. Following Euler's classic manipulation [2], multiply both sides of the above equation by $\frac{1}{2^s}$ and subtract the result from the original to get

$$(1 - \frac{1}{2^s})\zeta(s) = \sum_{m=1}^{n} \frac{1}{m^s} = 1 + \frac{1}{3^s} + \frac{1}{5^s} + \frac{1}{7^s} + \frac{1}{9^s} \ldots$$

Note that *all* terms containing a multiple of two have disappeared. Now iterate this process with $\frac{1}{3^s}, \frac{1}{5^s}, \ldots$, and divide to get

$$\zeta(s) = \prod_p (1 - \frac{1}{p^s})^{-1} = (\frac{1}{1-\frac{1}{2^s}})(\frac{1}{1-\frac{1}{3^s}})(\frac{1}{1-\frac{1}{5^s}})(\frac{1}{1-\frac{1}{7^s}})(\frac{1}{1-\frac{1}{11^s}})(\frac{1}{1-\frac{1}{13^s}})\ldots$$

where p ranges over the primes. It now follows that

$$\sum_m \frac{1}{m^s} = \zeta(s) = \prod_p (1 - \frac{1}{p^s})^{-1}$$

Thus an infinite sum over the integers equals an infinite product over the primes! It is therefore not surprising that this *Riemann zeta function* $\zeta(s)$ is intimately related to fundamental issues in prime number theory. Interpreting this physically, summing $\zeta(s)$ [in its Σ-form] over all s is some sort of universal wave function.

The last variation on this theme is the Möbius function, μ, the reciprocal of ζ:

$$\mu(s) = \frac{1}{\zeta(s)} = \sum_{m=1} \frac{\mu(m)}{m^s}$$

$$= \ldots(1 - \frac{1}{11^s})(1 - \frac{1}{7^s})(1 - \frac{1}{5^s})(1 - \frac{1}{3^s})(1 - \frac{1}{2^s})$$

$$\mu = 1 - \frac{1}{2^s} - \frac{1}{3^s} - \frac{1}{5^s} + \frac{1}{6^s} - \frac{1}{7^s} + \frac{1}{10^s} - \frac{1}{11^s} - \frac{1}{13^s} + \frac{1}{14^s} + \frac{1}{15^s} + \ldots$$

The terms of μ follow the rules:

$\mu(m)$:

- $\mu(1) = 1$
- $\mu(m) = 0$ if m has a square factor (which eliminates all m^s, $s > 1$)
- $\mu(m) = -1$ if m is a prime or the product of an *odd* number of different primes
- $\mu(m) = +1$ if m is the product of an *even* number of different primes

Recalling the path we have followed, we began in thc multi-vector world, asking the question, What is the frequency [spectrum] of (say) $a+b+ab$? This led us to the harmonic series, which led us to ζ and μ, both of which are, however, creatures of the *scalar* world. Thus, dimensionally speaking, we have *descended* from 2^n- dimensionality to the single dimension of the scalars $= Cl(0,0)$.

[did Kubla Khan ...

From physics to computation

So the question arises, is it possible to translate these scalar relationships *back* into vector language? *Yes* - by re-doing Euler's derivation, but this time in $Cl(n,0)$ over $Z_3 = \{0,1,-1\}$. Anticipating what is to come, we can drop the complex s because we have the imaginaries automatically with the Clifford algebra, since for $i \neq j$, $(x_i x_j)^2 = -1$; furthermore, we can drop s itself because the idempotent property we will employ flattens all exponents to 1. Nevertheless, the key *factor* properties remain, and it will become clear that it is idempotents that correspond to the scalar primes.

We begin with an initial "state vector" $S_0 = 1 + q_2 + q_3 + ...$ and require that $q^2 = q$, ie. q is idempotent. Now manipulate, just as before, to get $S_1 = S_0 - q_2 S_0 = (1-q_2)S_0$. In general,

$$S_i = S_{i-1} - q_{i+1}S_{i-1} = (1-q_{i+1})(1-q_i)...(1-q_3)(1-q_2)S_0 = PS_0$$

where P is the product $(1-q_{i+1})(1-q_i)...(1-q_2)$. The q's *must* be idempotent because if (eg.) $q^2 = \pm 1$, the fact that q is a factor of P is quickly lost, and with it the subtractive cancellation of q's multiples. The sum S_0 *cannot* have finite length because each multiplication generates terms that only match terms further out, so S_0 must always be longer than $q_{i+1}S_{i-1}$ if the desired subtractive cancellations are to occur.[5]

Suppose, now, that $q_i = -1+u_i$, where $u_i^2 = 1$, and further, associate the integer i with the vector q_i so that $q_{jk} = q_j q_k$, for example $q_6 = q_2 q_3$. Then

$$S_0 = 1+(-1+u_2)+(-1+u_3)+(-1+u_2)^2+(-1+u_5)+(-1+u_2)(-1+u_3)+(-1+u_7)$$
$$+ \quad (-1+u_2)^3+(-1+u_3)^2+(-1+u_2)(-1+u_5)+\ldots$$

corresponds to the sequence $1,2,3,4,\ldots$. Now multiply through by "2" $= q_2 = (-1+u_2)$ to get

$$(-1+u_2)S_0 =$$
$$(-1+u_2)+(-1+u_2)^2+(-1+u_2)(-1+u_3)+(-1+u_2)^3+$$
$$+ \quad (-1+u_2)(-1+u_5)+(-1+u_2)^2(-1+u_3)+(-1+u_2)(-1+u_7)+\ldots$$

and subtract this from S_0 to get

$$S_1 = (-1-u_2)S_0 = 1+(-1+u_3)+(-1+u_5)+(-1+u_7)+\ldots = \bar{q}_2 S_0$$

where $\bar{q} = -1-u$ is the conjugate of $q = -1+u$. Clearly, all the "even" terms - those with the factor $(-1+u_2)$ - have disappeared from the

[5] Although there may be a practical work-around, eg. require $q^r = 1$, whence r puts a ceiling on meaningful i. The situation reminds one of the "renormalization" issue in quantum field theory.

rhs and only the "odd"-numbered terms remain. Now repeat the process with q_3 to get

$$S_2 = (-1-u_3)(-1-u_2)S_0 = 1+(-1+u_5)+(-1+u_7)+\ldots = \bar{q}_3\bar{q}_2S_0$$

which rhs is missing all terms having "3" = q_3 as a factor. Clearly, if we continue this process, we will arrive at

$$S_k = P_{k+1}S_0 = \bar{q}_{k+1}\bar{q}_k\ldots\bar{q}_2S_0 = 1+(-1+q_{k+2})+(-1+q_{k+3})+\ldots$$

At this point, Euler argues [in the scalar case] that the remaining terms on the rhs approach zero, and hence that one could [analogously] write $P_{k+1}S_0 = 1$; but this is not possible here because $P_{k+1} = \bar{q}_{k+1}\bar{q}_k\ldots\bar{q}_2$ is not a reversible process [5] and hence *cannot* have S_0 as an inverse.[6,7] Rather, one must argue that the frequency of the remaining terms (after some point) is so low as to be insignificant.

Even with this, however, we *still* cannot divide through by P_{k+1} to get $\zeta = \frac{1}{P_{k+1}}$ because ignoring small terms doesn't change the fact that the very concept P_{k+1}^{-1} is meaningless when P_{k+1} by definition *has* no inverse. Rather, we must be satisfied with ζ's inverse, namely $\mu = P_{k+1}S_0$. Despite this, however, the zeroes of μ are in the same places (ie. arise from the same factors $\bar{q}$) as the zeroes of ζ, since it's the same P; we will return to this later.

At this point, rather, the question is, what does P_{k+1} look like when multiplied out? This polynomial can be constructed combinatorially:

[6]Theorem: For all $X \in Cl$, X has no inverse *iff* X has an idempotent factor.

[7]Notice, by the way, the fascinating interplay between the concepts of constructiveness and of a limit at ∞ versus physical ir/reversibility - only in the (by definition *unrealizable*) limit is the 2nd Law of Thermodynamics ultimately violated.

define the set $U = \{\bar{u}_2, \bar{u}_3, \bar{u}_5, \ldots\}$, let m be the grade of the m-vector, and as before, abuse combinatorial notation to write

$$\mu = 1 + \sum \mu_j = 1 + \sum_{n=1} (-1)^m \begin{pmatrix} U \\ n \end{pmatrix}$$

$$= 1 - (u_2 + u_3 + \ldots) + (u_2 u_3 + u_2 u_5 + \ldots) - (u_2 u_3 u_5 + u_2 u_3 u_7 + \ldots) \pm \ldots^{8}$$

Note the following properties of this sum:

1. If there are an odd number of u-factors, then the sign of the m-vector term will be negative because each u has negative sign.

2. If there are an even number of u-factors, then the sign of the m-vector term will be positive.

3. There are no multiples of any q_i present.

These are precisely the conditions that define the scalar version of μ.

Also, it should be noted that the conjugate forms that are an implicit part of this story follow naturally from the different possible orderings in the "non-prime" $q-$terms in S_0. Alternatively, the order in which the q_i are accumulated into P will also generate the conjugates: one could have either $(-1+u)(-1+v) = 1 - u - v + uv$ or $(-1+v)(-1+u) = 1 - u - v - uv$.

Thus $(-1 \pm u)(-1 \pm v)(-1 \pm w)$ has actually six different orderings, and

[8] Lest the reader worry that this Clifford μ doesn't show fractions as does the original scalar μ, notice that the m-vector terms are all self-inverse, squaring namely to ± 1, whence $\mu_i^{-1} = \pm \mu_i$. So this equation could as well be written as $1 - \frac{1}{u_2} - \frac{1}{u_3} \ldots$ (except, of course, that Clifford algebras don't do "division"). The inverse relationship between wavelength and frequency is thus maintained quite literally.

all of these are members of the same (irreversible) conjugate set of order $2^n n!$. [9]

[A stately pleasure dome decree ...

The Riemann Hypothesis for $Cl(n,0)$

This form of the μ-function (ie. with $q_i = -1 + u_i$, where $u_i^2 = 1$) produces a polynomial each of whose terms is a unique *solitary* m-vector $u_i ... u_j$, each of which has magnitude $|u_i ... u_j| = 1$. Furthermore, all these m-vectors are mutually perpendicular, and so a sum of such terms is the diagonal of the corresponding hypercube and therefore must satisfy the Pythagorean relation

$$|\mu_1| + |\mu_2| + ... + |\mu_k| = \left[\,|\mu_1|^2 + |\mu_2|^2 + ... + |\mu_k|^2\,\right]^{\frac{1}{2}}$$

Let ℓ count the number of q-factors currently making up μ in its product form. Then, at any such stage ℓ of μ's growth, sum the 1's of $\mu^{(\ell)}$'s sum-form terms:

$$M(k) \;=\; \sum_{j=1}^{k} |\mu_j| \;\leq\; \left[\sum_{j=1}^{k} |\mu_j|^2\right]^{\frac{1}{2}} \;=\; \mathcal{O}(k^{\frac{1}{2}})$$

where k is the number of terms. $M(k)$ is called Merton's function, and by showing that $M(k) = \mathcal{O}(k^{\frac{1}{2}})$, we have proven the Riemann Hypothesis for the spaces covered by $Cl(n,0)$. This is the expected result, due to the Weil Conjectures, which cover a wide range of spaces [3,4]; see the Appendix.

[9] Which for $n = 3$ yields 48; $Cl(3,0)$ is isomorphic to the Pauli algebra.

It is also worth pointing out that $\bar{q} = -1 - u$ is written in $\mathbb{Z}_3$, whereas in the real numbers $\mathbb{R}$, the $\mathbb{Z}_3$ minus one becomes $-1 = \frac{1}{-1} \mapsto \frac{1}{2}$, that is, all of the roots of μ, and therefore of ζ, lie on the line $x = \frac{1}{2}$.

[And all should cry, Beware, Beware!

Applications

We have thus arrived at the relationships

$$\mu = P_{k+1}S_0 = (\bar{q}_{k+1}...\bar{q}_5\bar{q}_3\bar{q}_2)(1+q_2+q_3+q_4...) = 1+q_{k+2}+q_{k+3}+...$$

$$= 1-(u_2+u_3+...)+(u_2u_3+u_2u_5+...)-(u_2u_3u_5+u_2u_3u_7+...)\pm ...\pm u_2u_3...u_{k+1}$$

We will henceforth refer to P_{k+1} generically as P. In the following, we try to tease some useful insights out of these.

[It is well at this point to recall that the components $q_i = -1 + u_i$ of the state S_0 possess implicitly *both* a discrete particulate aspect - in that the u_i are unitary - *and* a wave-like aspect - being finite, their magnitude can only oscillate.]

The Process Concept

Thinking back to how P was created, the original sum $S_0 = \sum q_i$ with $q^2 = q$, and particularly $q_{jk} = q_j q_k$, in effect specifies all possible products of the individual idempotent q's. Since these products are not commutative, there is an ambiguity as to their ordering; we allow any ordering. There is a second ambiguity in the order in which the various q's are chosen to become the factors of P, and again, we allow any ordering. As noted earlier, these various orderings are Cl's way of generating conjugate forms. Howsoever, with all possible orderings and the fact that the q's are irreversible, we have *all possible time-like processes.*

Looking at this computationally, and recalling that the mathematical expression ***is*** the literal mechanism of the computation, the rhs $1 + q_{k+1} + q_{k+2} + \ldots$ is *the state arrived at* after the application of the process/operator P to an initial state S_0. Indeed, aside from the fact that S_0 is not unitary, the form $PS_0 = 1 + q_{k+1} + q_{k+2} + \ldots$ describes a measurement P made on a state S_0 that yields the result "Yes [= the +1] , the state S_0 was found in the environment, which by the way is now equal to $q_{k+1} + q_{k+2} + \ldots$".

A Clifford algebra's multiplicative associativity masks the necessarily absolute sequentiality of the physical process, cf. the group E_8, which is namely *not* associative. The lack of associativity can namely enforce (eg.) a sequential right-to-left application of P's factors q_i to the current state. The price of so directly tracking this aspect of physicality is the loss of both manipulative and conceptual flexibility, the latter because it welds the use (and the physics) into a sequential view of what is in truth a distributed concurrent system that *cannot*, even in principle, be so captured (see the *Coin Demonstration* in [6]).

Put differently, by willy-nilly enforcement of sequentiality via non-associativity, one loses the natural (and very useful) *non*-specification of order-of-evaluation of a Clifford algebra. Of course, one can explicitly specify the useful exceptions in either case, but at least in the present computational case - where one is namely mostly interested *all* orderings - associativity is the more useful.

Synchronization & Causality

We can say that P is a *sequential computational process* because it is established in [5] that each of P's component actions, the idempotent operators q_i, is semantically equivalent to the computational *synchronization* operator ***Signal***(u_i). The *Signal* operation is paired with ***Wait***(u_i), which is *nilpotent*, and the pair correspond to the fermion/boson distinction in particle physics. Together they constitute the necessary and sufficient operators for any time-like, irreversible, computation - including "parallel" and "distributed" ditto, and as well the construction of memory and *if-then-else* [5].

Being nilpotent, ***Wait*** has no inverse. So, via the idempotence-*iff*-no-inverse theorem footnoted earlier, it follows that every nilpotent is grounded in a matching idempotent. For example, the nilpotent $-a+b+c$, a photon, is related to the electron $ab+ac$ via $a(-a+b+c)$. One does the following to get a nilpotent out of an idempotent.

The key is two unitaries that anti-commute: if U,V are two such unitaries, then all sign-variants of $U+UV$ are nilpotent. Since $U^2=V^2=1$ and $UV=-VU$, the operate-on-the-left sequence

$$(-1+V)(-1+U)$$

can, using the identities

$$(-1+V)=(-1+V)(-V) \quad \text{and} \quad (-1+U)^2=(-1+U)$$

be converted into

$$(-1+V)(-V)(-1+U)(-1+U)$$

$$= (-1+V)(V-VU)(-1+U)$$

where $W = V - VU$ is the desired nilpotent, playing the role of ***Wait***(U). The corresponding computational process is ***Signal***(U); ***Wait***(U); ***Signal***(V), where we now read left-to-right in computer program order:

First, *signal* U ...

"whilst in some other process"

... *wait* for U to occur, and only then *signal* V.

Thus, for the event ***Signal***(V) to be causally connected to the (preceding) event ***Signal***(U), there must exist the mediating nilpotent entity W. Indeed, if such a W doesn't exist, then the two events $-1+U$ and $-1+V$ aren't causally connectible.[10,11]

So, to cover all possible causal sequences in the present case, we must have at our disposal all the "prime" idempotents $\overline{q}_i = -1 - u_i$ and all the u_j that anti-commute with each u_i. Then the sequence $(-1-u_i)(-u_i - u_i v_j)(-1-v_j)$ exists and is a causal sequence for all u_i, u_j.

[10]One can think of such connecting nilpotents in a very concrete computational way, namely that each corresponds to a trip around the hardware interpretive *Ifetch* loop, which causally connects two successive instructions (= the idempotents). Alternatively, one can think of an idempotent $-1+U$ as being the *self*-boundary of its associated unitary U: $\partial_{-1+U} U \cong -1+U$, which can only be connected to another such self-boundary $-1+V$ by an entity that namely *has* no self-boundary, namely a nilpotent, since $\partial_W W = WW = 0$.

[11]Let ";" mean causally connected and ":" mean *not* so connected; and let A, B, C, D be idempotent. Then the sequence $(A;B) : (C;D)$ can be rewritten as $(AB) + (CD)$. Similarly, the sequence $A;(B:C);D$ could become any of $A(B+C)D$, $A(BD+C)$, or $A(B+CD)$, depending. The algebraic notation is more precise and flexible.

Arbitrary Idempotents

Notice that *any* choice of the idempotents $q = (-1+u)$ will generate the Riemannian result.

Example. The 1-vector product $(-1+u)(-1+v)(-1+w)...$ generates a complete polynomial, ie. one having all 2^n possible m-vectors. This set of idempotents satisfies all the steps of the above proof, but so do the similar products of 4-vector idempotents $-1+wxyz$, of 5-vector idempotents $-1+vwxyz$, etc. So perhaps the appearance of the scalar prime number statistics in the physical context is the result of the structure of the argument itself, rather than an appearance of the scalar primes *per se*.

Howsoever, this reveals (or perhaps merely systematizes) a way to model, at a strictly higher level, things like atoms and molecules, both of which clearly exceed the expressive power of $Cl(3,0)$. In a software specification and development context, it specifies how to structure a distributed system and verify its functionality one level at a time.

Example. The unitary u_i's have been written in lower case to encourage the interpretation that the u_i are *solitary* m-vectors $x_i...x_j$, but this need not be the case, as this property was never invoked in the above. Rather, the u_i can also be multi-vectors like $U_i = u+v+uv$ or $U_j = uv+uw$, which are also unitary. This means that the products, like U_iU_j, in the sum-form of μ can generate multiple terms, but these new μ_i do not change the Pythagorean result and the conclusion that $\Sigma|\mu| \leq \mathcal{O}(k^{\frac{1}{2}})$.

In a software context, the coalescence of such *compound* unitaries out of the sea of solitary unitaries is an *implicit* and *unavoidable* implication of the μ-hierarchy, *and* they themselves can be the subjects of causal sequences ...$(-1+V)(-1+U)$.... This in turn prompts the question: are these desirable?? Are they an opportunity to expand the object concept, and to build a meta-Ψ-hierarchy, corresponding to the hierarchy of the elements, molecules, meta-materials, ... ?

Or, like undocumented logic, are compound unitaries a threat to privacy and security? Note that as long as a given U_i is never actually assigned a name, it cannot be explicitly addressed, and it also both appears and dissolves automatically. Verification of system functionality, ie. the *proof* that it does or does not do X, is simplest when compound unitaries are denied objective existence, since then the system has a very tractable wave-like mathematical structure. Reversibility greatly improves system stability. Once assigned a name, however, a compound unitary requires that the system now *ensure* its persistence and accessibility, the chance of deadlock escalates, while at the same time its very existence encourages the creation of ever more irreversible processes.

Universal Hierarchy

Notice that the terms of the sum-form of P express all $\begin{pmatrix} n \\ m \end{pmatrix} = 2^n$ m-vectors, and hence is a basis of the space spanned by $Cl(n,0)$. Computationally, this space is the space of all possible distinctions.[12]

With this in mind, take the μ-form of our relationships and suppress the detail by defining various Q's and U_{P_k}:

$$\mu = P_k S_0 = (\bar{q}_k \cdots \bar{q}_5 \bar{q}_3 \bar{q}_2)(1 + q_2 + q_3 + q_4 \ldots) \;\; =$$

[12]Every m-vector, $m > 1$, calculates (scalar) exclusive-or = *same/different*, eg. A excludes B *vs.* A co-occurs with B.

$$1+q_{k+1}+q_{k+2}+\ldots=1+Q_{k+1,\ldots}$$

$$=$$

$$(1-(u_2+u_3+\ldots)+(u_2u_3+u_2u_5+\ldots)-(u_2u_3u_5+u_2u_3u_7+\ldots)\pm\ldots\pm u_2u_3\ldots u_k)S_0$$

$$=(1+U_{p_k})(1+Q_{2,3,4,\ldots})$$

$$=1+U_{p_k}+Q_{2,3,4,\ldots}+U_{p_k}Q_{2,3,4,\ldots}=1+Q_{k+1,\ldots}$$

The one's cancel, and re-arranging we get

$$U_{p_k}Q_{2,3,4,\ldots}=-\left(U_{p_k}+Q_{2,3,4,\ldots,k}\right)$$

wherein we see that the application of U_{p_k} to the total initial system state $Q_{2,3,4,\ldots}$ will invert U_{p_k} and $Q_{2,3,4,\ldots,k}$, which is exactly what should happen (since the q's by nature invert their object), *but* there are no q's (only u's) in U_{p_k}, so this formulation of the relationships shows that the inversions [can validly be seen to] occur via the rotations of the Q_i by the U_i rather than by multiplication by -1 in the $\overline{q}_i$ version. That is, both the wave and the particle views are simultaneously valid. Also, depending on the specific situation, the transition can be either reversible or irreversible.

Physically, U_{P_k} is the so-called quantum potential Ψ. As noted earlier, the $m>1$ elements of this space can be constructed using the coboundary operator, so (eg.) $\delta(u+v)=uv$ and $\delta(u+vw)=uvw$. Physically, this construction takes place dynamically and continually, and defines in its very reversibility the resonant structure of Ψ. But the interpretation of this structure as "the *quantum* potential" is restrictive, since, as we have seen, *any* set of unitary entities can be the elements of such a structure: call it the ***causal potential.***

The Object Concept

The individual U_i are the computational *objects* in this model, that is, entities having a persistent - though not necessarily permanent - existence. This persistence is signified by the fact of their unitarity: $U_i^2 = 1$, meaning that U_i encompasses its own inverse, and thus can change without changing, so to speak. It does this by possessing a persistent *internal state* that is different from its *name*: *ab* is the name, while $+ab$ and $-ab$ encode the internal state. [We discuss *naming* later.]

As a software structure, a given U_i has two modes of operation, one reversible and (hence) *space-like*, and the other irreversible and (hence) *time-like*. The latter is the prominent aspect of the objects definable by contemporary programming languages (eg. Java - they're all the same), since a given Java object is only active when it is entered via one of its functional ports, and once that function has been carried out, the object is once again utterly passive. This scenario corresponds to U_i being operated upon by the idempotent $-1+U_i$. It is strings of such activations, from U_i to U_j to U_k, that constitute the processes defined by *Wait* and *Signal* (and procedure calls in general).

The *space-like* aspect of a given U_i derives from its place in a surround of other similar objects, as defined by the sum-form of μ. The changes in the surround sensed by the primitive 1-vectors x_i, x_j are combined pair-wise (via δ) to produce the 2-vector object $x_i x_j$, which in turn can be combined with x_k to produce the 3-vector object $x_i x_j x_k$, etc. Thus every m-vector, $1 < m < n$, is both the collector of impulses from "below" (ie. from its constituent boundary entities) and the distributor of its own (consequent) state to the higher-level objects of which it is itself a boundary entity. This upward ascent δ is close kin to the calculus operation of integration $\int$, and similarly, the reversed, downward flow of the manifestation of the hierarchy's potential corresponds to differentiation [both denoted by ∂].

Hierarchical (ie. structural) relationships and hierarchy traversal are similarly space-like, since $\partial\delta = 1 = \delta\partial$ if the Cauchy-Riemann conditions $XU = YV$ and $YU = -XV$ hold, which they do if X, Y, U, V are reversible, which they are in the present case.

The upshot is that as software objects, the U_i are *always active* (at least conceptually). This trait, when combined with the reversibility conferred by unitarity, means that the U_i , seen from without, *oscillate*, and hence the entire hierarchy can be seen as a complex wave-like structure. The upward flow (and steadily decreasing frequency) performs a Fourier-like decomposition of the input vector, the primitive boundary $x_1 + x_2 + \ldots$, and the downward flow is a complex (but literal) reflection of that input. Notice that from an external point of view, the discrete particle-like aspect of the individual U_i has entirely disappeared from view ... one sees only waves and constructive and destructive interference - the *wave function* of the system as a whole!

From a programming point of view, all of the U_i are instances of the *same* abstract object, namely one that δ-combines two other objects into one, and itself is δ-combinable into another such instance. The general conceptual view is "inside looking out" rather than the usual "outside looking at" and thoroughly Heraclitean. The communication regime is *broadcast-listen* rather than the ubiquitous (and inherently time-like) *request-reply* of virtually all programming languages.

Finally, the possession of a wave-function U_P, a system's space-like aspect, constitutes a definitive criterion, otherwise lacking, of what it takes for a system to be "distributed". The dependence of the state of *every* locale on the state of *every other* locale, is both the ideal of a "distributed system" and *exactly* what a wave-based structure provides. Think of the waves in a bathtub - the height of every point on the surface is dependent on the heights of all the other points, and understand that this is a mere scalar version of the conceptual complexity that is organized in this fashion. Remote procedure call, ie. request-reply, is the wrong primitive for building truly distributed systems![13]

Discrete Physics

The recent book *The Origin of Discrete Particles* by Bastin and Kilmister [10] calculates the value of the inverse fine structure constant α^{-1} to one part in 10^7 on a closely reasoned and purely combinatorial basis.[14] The combinatorics of the present graded Clifford hierarchy match those of their Combinatorial Hierarchy (which is over $\mathbb{Z}_2 = \{0, 1\}$), which forms the structural basis for their calculation. Kilmister and the present author

[13]The *right* primitive is Co[A, B], which blocks until states A and B co-occur. *JavaSpaces* is the right kind of platform for this. Co[A, B], and it's complement, NotCo[A, B], are the author's extensions to the *Linda* distributed programming paradigm, from which *JavaSpaces* derives. See [4]/Linda, and [6].

[14]Yielding 137.036011393... versus the latest empirical value, 137.035999710(96)....

agree that there is most probably an isomorphism, though its exact form has never been investigated. Both structures are based on exclusive-or.

Bastin and Kilmister place the *process of constructing knowledge* at the center of their analysis:

The idea which underlies combinatorial physics is that of process. The most fundamental knowledge that we can have is of step-by-step unfolding of things; so in a sequence. This is the kind of knowledge we have of quantum processes, and that fact becomes specially evident in the experimental techniques of high-energy physics. Such a process is necessarily combinatorial but not conversely. [p. 3]

The fundamental act in Bastin and Kilmister's analysis, the *empirical* act, is that of finding an entity in the otherwise entirely unknown surround and determining whether or not it is novel. They prefigure the derivation of $\mu = P_k S_0$, which similarly specifies "take an element of the otherwise unknown universe [ie. pick a u_k from S_0 and form $\bar{q}_k = -1 - u_k$] and compare it to what is already known [ie. form $\bar{q}_k P_{k-1} = (-1 - u_k)(1 + \sum_{m=1}^{k-1} \begin{pmatrix} U \\ m \end{pmatrix}$ and note if any $u_k U_j = \pm 1$], and if it is novel [ie. for no element of $\begin{pmatrix} U \\ m \end{pmatrix}$ is $u_k U_j = \pm 1$] , adjoin u_k to what is already known [ie. add u_k to $\begin{pmatrix} U \\ m \end{pmatrix}$, the P hierarchy in its sum-form]".

Note that the acquisition of this *explicit* knowledge is shown, via q's idempotence, to be an irreversible process.

Naming

There are still many loose ends in the frequency calculation [eg. when the frequencies f_i differ. Nevertheless it seems apparent that the appearance of the prime integers in physics is connected to the individual measurement process - that is, particular prime numbers are associated with particular individual processes. The product P defines these associations.

This said, one can make an argument for why the x_i, the 1-vector generators of the algebra, should be assigned, *literally*, prime-number frequency values. This follows from computer science, in particular from algorithms for achieving mutual exclusion between computational processes. As noted above, the computational primitives for accomplishing this are called, generically, ***Wait***(e) and ***Signal***(e) where e is some event/state. However, there are many ways to do this and such algorithms have a long history of interest and research.[15]

Relevant here is a mutual exclusion algorithm for distributed contexts called the *Bakery algorithm*, inspired by the take-a-number systems often found where there is a queueing problem. The nice technical feature of this algorithm is that it separates (the mechanism of) the determination of *access sequence* from (the mechanism of) *granting access* per se, and is thus well-suited to today's logically and physically distributed systems. The feature of interest in the present context is that the algorithm demonstrates the intimate relationship between the integers and the fundamental concept of the discreteness of events and their ordering

[15]See the writings of C.A. Petri, E.W. Dijkstra, L.A. Lamport, and many others; and/or any good textbook on operating systems.

into processes.

The key point is that the integers *also* provide *unique names* for the participating entities, and the issue of naming is, like mutual exclusion, a central concern in computer science, both practically (a name is a *de facto* address) and theoretically (granting a name opens the door for the *object concept* and the type and attributes of said objects). Thus the Bakery algorithm inspires the thought that the *same* numbering strategy could be used simultaneously for both sequencing and naming in an elegant marriage of functionalities.

Returning now to ζ and μ, we indeed see how an *integer*-based numbering in the sum-form of ζ is turned into a *prime*-based [and nearly Gödel] numbering of the hierarchical m-vectors in the sum-form of P. Indeed, we see that the [integer-based] ordering ambiguities mentioned earlier are automatically swept aside to produce *unique* m-vector names that are namely *independent* of the ordering of the changes they connect *via* their co-boundary relationship (eg. in ab, flipping a and flipping b). It is particularly telling, in this context, that *every* possible combination over the u_i is automatically constructed.

Finally, returning to the physics, having shown that the *mechanism* for accomplishing the phenomena uses (indeed, *needs*) the primes to construct the *literal names* of the unitary entities constituting the causal potential's structure, it is but a short and natural step to hypothesize that Nature herself uses this naming scheme "to keep track of things", and that, therefore, the 1-vector unitaries x_i physically *too* are identified by being *prime* multiples of Planck's constant $\hbar$.

... Weave a circle round him thrice, And close your eyes with holy dread

For he on honeydew hath fed, And drunk the milk of Paradise.]

And then, quite suddenly, the Fever broke.

Those wishing to avoid further infection should contemplate the definitions and truths regarding ζ as expressed by the Weil Conjectures in the *Appendix*, expressed with the admirable and exquisitely inscrutable clarity and precision that we all expect, and usually get, from our mathematical colleagues.

If you do not understand this appendix, you will likely be spared further trauma. On the other hand, a prophylactic application of these conjectures *can* limit the scope of further attacks, as indicated by the present case. And certainly, an ignorance of whatever-the-f an Etale cohomology is will go a long ways too.[16]

If however you succeed in decoding the conjectures, there is a very good chance that you will instead be drawn into the very maw of the Riemann Hypothesis, namely over $\mathbb{R}$, the worst and most incurable form of Riemann Fever. Only a proof of the Hypothesis itself can cure it ... and forget the fame and the $\$10^6$ prizes - they'll be too late.

Few mathematical problems can lay claim to such a powerful combination of elementary nature, breadth of applications, and depth of theory inspired in the search for a proof. [3]

[16]Reader exercise: Exactly why, according to the conjectures, is the proof presented not news?

References

1. Cipra, Barry. "Prime Formula Weds Number Theory and Quantum Physics". Science v. 274, 20 December 1996, p. 2014-5.

2. Derbyshire, John. *Prime Obsession - Bernhard Riemann and the Greatest Unsolved Problem in Mathematics.* Pub. Joseph Henry Press (National Academy of Sciences) 2003. ISBN 0-309-08549-7.

3. Osserman, Brian. "The Weil Conjectures".www.math.ucdavis.edu /~osserman/classes/256B/notes/sem-weil.pdf

4. Wikipedia. The Weil Conjectures.

5. Manthey, Michael. "Synchronization - the Font of Physical Structure". Proc. ANPA 2007; and International Journal of Anticipatory Systems, Daniel Dubois, Ed. Issn 1373 - 5411, v. 20-21-22, IJCAS/BCS Symposium, Liege, Belgium. August 6-11, 2007. Available from the author.

6. Manthey, Michael. "A Combinatorial Bit-Bang Leading to Quaternions". Proc. ANPA 1997; and Proc. 1997 Helsinki Conference on Emergence, Hierarchy, and Organization (ECHO III). arXiv.org > quant-ph/9809033v1.

7. Hestenes, David. *New Foundations for Classical Physics*, 2nd edition. Kluwer, 1999. ISBN 0-7923-5514-8.

8. Lounesto, Pertti. *Clifford Algebras and Spinors.* London Mathematical Society Lecture Note Series #286, 2nd edition. Cambridge University Press, 2001. ISBN 0-521-00551-5.

9. Hestenes, David. *Clifford Algebra to Geometric Calculus - A Unified*

Language for Mathematics and Physics. Reidel/Kluwer, 1987. ISBN 90-277-2561-6.

10. Bastin, Ted and Kilmister, Clive. *The Origin of Discrete Particles.* World Scientific, Series on Knots and Everything, L. Kauffman, Ed. ; v. 42, 2009. ISBN-13: 978-981-4261-67-8.

I dedicate this paper, with profound gratitude, to Clive W. Kilmister (1924-2010).

Appendix - The Weil Conjectures [4]

Suppose that X is a non-singular n-dimensional projective algebraic variety over the field F_q with q elements. The zeta function $\zeta(X,s)$ of X is by definition

$$\zeta(X,s) = \exp\left(\sum_{m=1}^{\infty} \frac{N_m}{m}(q^{-s})^m\right)$$

where N_m is the number of points of X defined over the degree m extension $F_q m$ of F_q.

The conjectures state:

1. *Rationality.* $\zeta(X,s)$ is a rational function of T = q^{-s}. More precisely, $\zeta(X,s)$ can be written as a finite alternating product

$$\prod_{i=0}^{2n} P_i(q^{-s})^{(-1)^{i+1}} = \frac{P_1(T)\cdots P_{2n-1}(T)}{P_0(T)\cdots P_{2n}(T)}$$

where each $P_i(T)$ is an integral polynomial. Furthermore,

$$P_0(T) = 1 - T,\ P_{2n}(T) = 1 - q^n T \ ;$$

and for $1 \leq i \leq 2n-1$, $P_i(T)$ factors over C as $\prod_j (1 - \alpha_{ij} T)$ for some numbers α_{ij}... .

2. *Functional equation and Poincaré duality.*

$$\zeta(X, n-s) = \pm q^{\frac{nE}{2} - Es} \zeta(X,s)$$

or equivalently

$$\zeta(X, \tfrac{1}{q^n T}) = \pm q^{\frac{nE}{2}} T^E \zeta(X,T)$$

where E is the Euler characteristic of X. In particular, for each i, the numbers $\alpha_{2n-i,1}, \alpha_{2n-i,2}, \ldots$ equal the numbers $\frac{q^n}{\alpha_{i,1}}, \frac{q^n}{\alpha_{i,2}}, \ldots$ in some order.

3. *Riemann hypothesis.* $|\alpha_{i,j}| = q^{\frac{i}{2}}$ for all $1 \leq i \leq 2n-1$ and all j. This implies that all zeros of $P_k(T)$ lie on the "critical line" of complex numbers s with real part $\frac{k}{2}$.

4. *Comparison.* If X is a (good) "reduction mod p" of a non-singular projective variety Y defined over a number field embedded in the field of complex numbers, then the degree of P_i is the ith Betti number of the space of complex points of Y.

The Weil Conjectures were proven to be true by Deligne in 1973.

A Digital Universe.

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It is proposed, as a hypothesis, that the universe is a digital system. The Implications of this hypothesis are compared, where possible, with experiment. The probability of the hypothesis is assessed, and future tests are suggested.

1) Background.

Present science describes a variety of variables. Some are quantised into equal increments, for example angular momentum. Some, such as energy, are quantised into increments that vary in size according to some simple law (in this case Planck's Law). Mass appears to be incremental, but differs in that the masses of particles vary from one to another without (so far) any apparent rhyme or reason. Some, such as length and time, are held to vary continuously with no quantisation at all.

Such complexity seems improbable. Scientific principles usually turn out to be elegant rather than awkward and simple rather than complicated. The variety of variables described above is both awkward and complicated and is, therefore, questionable.

Richard Feynman, in his book 'The Character of Physical Law' makes the following observation. 'It always bothers me that, according to the laws as we understand them today, it takes a computing machine an infinite number of logical operations to figure out what goes on in no matter how tiny a region of space, and no matter how tiny a region of time. How can all that be going on in that tiny space? Why should it take an infinite amount of logic to figure out what one tiny piece of space/time is going to do? So I have often made the hypothesis that ultimately physics will not require a mathematical statement and that in the end the machinery will be revealed, and the laws will turn out to be simple, like the chequer board with all its apparent complexities.'

To extend the argument further, consider the following 'thought experiment'. Imagine a newly-appointed deity charged with creating a new universe. Such a deity might well conclude that, before getting on with the act of creating, thought should be given to the control laws that define the processes inherent in a universe. Soon a crucial question emerges, namely 'Should such laws be analogue or digital?' This decision might go either way – though a digital route might be judged to be the more likely. An entirely analogue universe would have no chemistry and hence no life, and consequently be somewhat constrained.

However, one conclusion can be drawn. If the creation of a universe is intelligence-based as proposed above, it is most unlikely that the resulting control system would be hybrid, some laws being analogue and others digital. Given a clean sheet of paper, no competent systems engineer – and we can assume the deity envisaged is one such – would start by creating an analogue/digital mix.

Transferring this thinking to our present universe, we might ask, 'Is it possible that all our present control laws and variables are analogue in nature?' The answer to this is 'No'. The experimental evidence that at least some variables are quantised is conclusive.

Alternatively, we might ask, 'Is it possible that all our present control laws and variables are digital?' The answer to this is, 'Yes'. If those variables that are presently assumed to be continuous are, in fact, quantised but so finely that we lack the means to detect and measure the increments concerned, then it is possible that all variables and, hence, all physical laws are digital in nature.

Thus, in the extreme, if only one of our variables can be proved to be quantised, it is likely that all our variables are quantised in one way or another. Our present assumption that some variables are continuous and analogue could turn out to be false.

The purpose of this paper, therefore, is to propose the hypothesis that the universe is a system that is entirely digital in nature, test the implications, assess the likelihood of validity, and suggest further tests that might prove decisive.

2) Historical Perspective.

The thought that everything is granular (i.e. digital) in nature is ancient in origin. For example, the Greek philosopher Epicurus (341 – 270 BC) proposed such a state of affairs, based principally on his observations of nature. His, and others', thoughts were promoted by the Roman poet Lucretius (96 – 55 BC) through a long work in Latin verse – a prime promotional medium in those days although one largely ignored by today's publicists.

A further thought was expressed in the poem, namely that each increment of nature was characterised by a quantity described and translated as 'swerve'. Perhaps a more accurate term – but less poetic - would have been 'angular momentum'. On the other hand, 'swerve' is possibly a more accurate term than 'spin'.

Much thought was given in ancient times to the existence of arithmetical relationships between the heavenly bodies, some of which continues to the present. Such material, for example astrological matters or 'the music of the spheres' is beyond the scope of the proposed hypothesis, although the possibility of useful input emerging from such sources is not to be ignored.

However, several modern scholars have been aware of the possibility of a digital universe. For example, in 1954 Einstein wrote the following in a letter to Besso. 'I consider it quite possible that physics cannot be based on the field model, i.e., on continuous structures. In that case nothing remains of my entire castle in the air, gravitation theory included (and of) the rest of modern physics.'

Some years earlier Dirac published a paper that noted the frequent occurrence of a ratio between natural forces of some 10^{42} and Eddington calculated there were 10^{79} particles in the universe.

More recently, serious work has been conducted under the auspices of the 'Alternative Natural Philosophy Association' (ANPA) into the scientific relevance of the series known as the 'Combinatorial Heirarchy' (CH). This series has just four terms:

$$2^2 - 1 = 3;\ 2^3 - 1 = 7;\ 2^7 - 1 = 127;\ 2^{127} - 1 = 10^{38}$$

The implications of the CH are covered in a book by Professor Clive Kilmister (Kluwer Academic Publishers).

The thought that the universe is digital in nature has been take further by others who propose that nature is programme-based, similar to the workings of a digital computer. A prominent writer in this field is Professor Pierre Noyes who has published several papers on the concept of 'Programme Universe' and associated matters.

Thus, the essence of the hypothesis proposed in this paper is far from new. Nevertheless, it is hoped the following material may advance the argument and, thereby, make a contribution.

3) The Implications of a Digital Universe and their Testability.

3.1) Discovery of New Forms of Quantisation.

The skill of scientists, and the accuracy and scope of their experiments, has increased with the passage of time. Thus, if the universe is a digital system, there should be a steady stream of discoveries revealing new quantised phenomena of one sort or another. Such a stream has, in fact, been in evidence. Examples are as follows.

3.1.1) The Quantised Hall Effect.

In 1985 Klaus von Klitzing was awarded the Nobel prize for work on the Quantised Hall Effect. This effect occurs when a current flows along a conducting sheet in the presence of a magnetic field perpendicular to the sheet. In these circumstances, a voltage is developed across the sheet. The ratio between the voltage and the current is known as the Hall Resistance. At low temperatures and high magnetic fields, this resistance is quantised and has values of h/e^2 divided by a series of simple integers (*h* being Planck's Constant and e the charge of the electron.) – see fig 1. At higher resistances the quantisation is different, equivalent to

h/e^2 multiplied by a similar series of integers. The factor h/e^2 is equivalent to approximately 25,000 ohms.
At still lower temperatures, the fractional quantised Hall Effect occurs where values of resistances of h/e^2 multiplied by simple fractions – e.g. 2/3 or 2/5 – can be observed. The striking fact, however, is that quantised Hall resistances do not depend upon the materials or samples used and are highly accurate and repeatable. In fact, the Hall Effect is now used for the standardisation of the unit of resistance, the Ohm.

Similar quantised changes in resistance/conductance have been detected with mono-crystalline metal fibres, stretched nano-wires, and the migration of silver ions in an electrolyte.

Sometimes the constant h/e^2 is doubled. This is attributed to electrons forming so-called Cooper pairs and behaving as a single particle of double charge.

3.1.2) The Josephson Effect.

This effect occurs at low temperatures when two super conductors are separated by an insulating gap – usually a very thin film. Electrons can cross the gap through the tunnel effect and, in certain configurations of junction, are highly sensitive to magnetic fields.

If a voltage is applied across a Josephson junction, an alternating current flows through the junction. The current varies at a microwave frequency related to the voltage by the equation

$$\nu = {}^{2e}/_{h}\, V$$

where v is the frequency of the current, e is the charge of the electron, h is Planck's constant, and V is the applied voltage.

It is a property of a superconductor that the associated magnetic flux is quantised such that

$$\Phi_0 = h/2e$$

where Φ_0 is the quantum of flux. (This matter is dealt with in more detail later in the paper).

The reciprocal of this term is the Josephson Constant which, due to the high accuracy and repeatability of experimental measurements, is now used to standardise the measurement of the Volt. The high accuracy of the Hall and Josephson effects enables them, in combination, to provide the most accurate means known of measuring the value of Planck's constant.

3.1.3) More recent Reports of Quantisation.

Several new areas of quantisation have been reported relatively recently. For example, there have been many studies into the quantisation of astronomical red-shifts. Such quantisation falls into two groups: a series of regularly-spaced preferred values of galactic red-shifts derived from a large database of observations of galaxies both near and far and located in any direction; and irregularly-spaced preferred values of red-shifts from distant quasars, particularly those associated with adjacent galaxies. Both forms of quantisation are discussed more fully later in this paper.

Very recently, there has been some speculation that gravitational potential is quantised, and evidence has been published by a group in Annecy (France) of the quantisation of the gravitation of 'cold' neutrons.

Apparently, when very low energy neutrons are 'bounced' in a potential well, tracking the motion of individual neutrons reveals quantisation of motion.

To summarise, therefore, more and more forms of quantisation are being reported as science progresses, a state of affairs that is consistent with the hypothesis proposed in this paper.

3.2) The maximum Value of Variables.

If, as hypothesised, the universe is a digital system, it is reasonable to suggest that it has system limits, such limits defining maximum values of variables.

Such thinking accords with the big-bang theory for the creation of the universe, now granted 'standard theory' status. Central to the Big-bang concept is the belief that the universe had a beginning and thus an age – T_0. Given this, it can be postulated that the universe has a boundary at cT_0 that contains within it the mass of the universe. (Such an observation may have to take into account the possibility that, due to a stretching of space, the speed of light denoted by 'c' may increase as this boundary is approached).

In short, therefore, nothing can be older, larger, or more massive within our universe than the universe itself, and maximum values of time, length and mass – such as may be required for calculations that relate to our universe – are thereby defined.

Such a concept is not entirely new. In the 1930s Milne calculated that the mass of the universe- i.e. the maximum mass that can exist within the universe – is given by

$$c^3T_0 / G$$

where c is the speed of light and G is the gravitational constant.

Recent observations from the Hubble telescope enable a precise prediction of the value of T_0 to be made – 13.72 billion years – and it is now widely accepted that the ultimate value is likely to be close to 14 billion years.

Thus, it can be argued that at least some variables are constrained to maximum values and that, by extension, most – if not all – variables are similarly constrained. One test of such an outcome will be whether any singularity problems that arise from the use of pure mathematics and the concept of infinity can be resolved by use of the maximum values described above.

Another test arises as follows. The concept of a maximum time interval leads to a minimum frequency and, hence, a minimum energy and mass. As will be shown later, such quantisation of mass is consistent with our hypothesis.

3.3) The Arithmetic of Quantised Variables.

The hypothesis requires that all variables are quantised in one way or another. Some of the implications that arise when such variables are added, subtracted, multiplied or divided are considered as follows.

3.3.1) The Addition and Subtraction of Quantised Variables.

Consider a variable that has a minimum size i.e. consists of quanta each of size 'a'. The value of such a variable will be $n \times a$, where n is some integer. If two quantities, say na and ma, of this variable are added, we have

$$ma + na = (n + m)a$$

which is unexceptional and displays unchanged quantisation. Similar arguments obtain for subtraction.

3.3.2) The Multiplication of Quantised Variables.

Imagine one quantised variable of value na that is to be multiplied by another of value mb to create a new variable. The result is given by

$$na \times mb = nm \times ab$$

Here we have generated a new quantum, ab, appropriate to the new variable, multiplied by a new integer nm.

It is not easy to find a variable that is derived by a simple multiplication involving mass, length or time. However, moment of inertia - mr^2 – might serve as an example. If the mass in question is ma and the length is nb, the moment of inertia is

$$ma \times (nb)^2 = mn^2 \times ab^2$$

Thus we have a new quantum ab^2 multiplied by a new integer mn^2.

It might be possible to devise an experiment to confirm – or otherwise – the quantisation of moment of inertia, and the existence of such quantisation would support the hypothesis being proposed.

As a refinement, unless there is the unlikely situation where either m or n are unity, the term mn^2 cannot be a prime number. The absence of prime numbers when quantisation of moment of inertia is detected would, therefore be a powerful indicator that the hypothesis is well-founded.

(A more useful experiment might be to look for quantisation in electrical power. If both voltage and current are quantised – see later in the paper – detectable quantisation of the power output should result).

3.3.3) The Division of Quantised Variables.

If one quantised variable is divided by another, we form a new variable as follows

$$\frac{na}{mb} = \frac{n}{m} \times \frac{a}{b}$$

The term a/b is a new form of quantisation, appropriate to the new variable. The term n/m is the quotient of two integers and can take several forms. If n and m are both unity, the new variable will be at its one-to-one value, i.e. a/b, a single quantum.

If n remains at unity but m is two or above, the new variable will have a series of values corresponding to a/b(1/2, 1/3, 1/4 ....), a form that might be termed 'reciprocal quantisation'.

If m remains at unity while n increases, the new variable will have a series of values corresponding to a/b(2, 3, 4, ...), i.e. conventional quantisation.

If neither n nor m are unity, the new variable will have a series of values corresponding to a/b multiplied by one of a series of fractions probably simple in character, i.e. fractional quantisation.

The Relevance of the Quantised Hall Effect.

Remarkably, the forms of quantisation described in (3.3.3) are precisely those exhibited by the quantised Hall Effect and in other effects that exhibit quantised resistance/conductance – see (3.1.1). In general, conventional and reciprocal quantisation are fairly simple to demonstrate while more effort is needed – e.g. the use of purer materials and lower temperatures – for the fractional quantisation to be observed.

The Hall resistance results from dividing a voltage by a current – as in Ohm's Law. Although the Hall Effect already has a well-established explanation based on solid-state physics, we proceed to test the supposition that this effect can also be attributed to dividing quantised voltage by quantised current – i.e. to dividing one quantised variable by another.

To do this we must derive an expression for the quantum of voltage and another for the quantum of current. If dividing one by the other yields the one-to-one value, i.e. the Hall resistance, the supposition above will be supported.

The minimum voltage will be given by the following expression

$$V_{\min} \times e = E_{\min}$$

where V_{min} is the quantum of voltage, e is the charge of the electron (the minimum charge), and E_{min} is the minimum quantum of energy.

The age of the universe is T_0 so the minimum frequency is given by

$$1/2\pi T_0$$

i.e. the number of cycles per second if one travelled round the circumference of the universe at speed c.

By Planck's law,

$$E_{min} = h\nu_{min} = h/2\pi T_0$$

so

$$V_{min} = h/2\pi e T_0$$

The quantum of current is calculated by considering an electron travelling around the circumference of the universe at speed c. The current thereby generated is given by

$$I_{min} = e/2\pi T_0$$

Thus the Hall resistance is given by

$$V_{min}/I_{min} = h/e^2$$

identical to the Hall resistance which is fundamental to the quantised Hall Effect.

That the hypothesis makes a prediction of the quantised Hall Effect, including the size of the Hall resistance, is encouraging. This is reinforced by the extreme accuracy of the Hall effect increments, which has lead to the observation by others that the effect is 'close to the bedrock of science', an observation that would indeed be in accordance with the explanation afforded by our hypothesis.

The Relevance of the Josephson Effect.

As described in (3.1.2), the Josephson Effect involves a voltage that produces an alternating current according to the equation

$$\nu = \frac{2e}{h}V$$

or

$$\frac{\nu}{V} = \frac{2e}{h}$$

where ν is the frequency of the current and V is the applied voltage.

Again, the implications of our hypothesis make such a relationship almost obvious.

As derived above,

$$\nu_{min} = \frac{1}{2\pi T_0}$$

and $$V_{min} = h/2\pi e T_0$$

It follows that $$\nu_{min}/V_{min} = e/h$$

The missing factor of 2 is, again, usually accounted for by the existence at the temperatures involved of Cooper Pairs of electrons – i.e. in effect a doubling of electronic charge.

The quotient

$$2e/h$$

is known as the Josephson constant, and the reciprocal is the quantum of magnetic flux.

Although the Josephson Effect, as the Quantised Hall Effect, has an accepted explanation derived from solid state physics, the analysis above indicates that it can also be accounted for by the implications of our hypothesis, and thereby supports the hypothesis.

So far, magnetic flux has been observed to vary only in constant steps i.e. conventional quantisation, without any evidence of reciprocal quantisation. However, as a further test of the hypothesis, an implication of the hypothesis is that such quantisation should exist. If experimental evidence of magnetic flux levels of

$$h/2e\,(1/2, 1/3, 1/4 \ldots)$$

can be obtained, the hypothesis would receive considerable additional support.

The Quantisation of Speed.

The examples cited above relate to voltage that is divided by current – in one case – and by frequency – in the other. If our hypothesis is valid, quantisations of the types described should also result when two variables from an entirely different field are used to form a quotient.

Speed is the result of dividing distance by time. If we make the significant assumption that both distance and time are primary variables and are not the result of other divisions or multiplications, speed should possess a one-to-one value, plus conventional quantisation at higher values and reciprocal quantisation at lower values. Is there any evidence of such quantisation?

At first, posing such a question seems less than promising. We are well accustomed to speed behaving as a well-behaved, continuous variable. For example our cars can travel at any speed chosen by the driver and do not accelerate in a series of sharp jerks from one speed to another.

But perhaps the speed of the average car is far removed from the one-to-one point of quantisation, with the result that any quantised steps are far too small to be observed. To make a guess, let us assume that the ratio between a quantum of distance and a quantum of time is 'c', the speed of light. In other words, let us look for quantisation at speeds given by a series of fractions such as 2c/3, c/2, c/3 etc.

One location where speeds of this magnitude are common and observable is at the outer reaches of the universe. Red-shifts in this region are assumed to result from high recessional speeds that cause emitted light to be red-shifted through the Doppler Effect.

If speed is quantised as described, we would expect such red-shifts to be similarly quantised. Is there any evidence of such quantisation?

The answer is ,'Yes'. Quasars are bright, usually distant, objects that have high red-shifts. During the last three decades there have been many reports that such red-shifts have preferred values and are quantised. However, the intervals between such values are irregular and, therefore, have been difficult to explain.

The existence of quantised red-shifts is sometimes disputed by astronomers, not only because the claimed effect appears to have no explanation or rational meaning, but also because the effect seems to be shown by only a small minority of quasars – those that appear to be closely associated with 'parent' galaxies.

The red-shifts of more than 5,000 quasars have been measured. A histogram showing the distribution of these red-shifts is shown as fig 2. It is obvious from even a brief glance that there is little or no evidence of any preferred red-shift values.

However, fig 3 shows the red-shift distribution of a set of quasars each associated with a parent galaxy. Here the picture is quite different and the existence of a series of preferred values is plainly apparent. The explanation offered by quantised red-shift enthusiasts is that such quasars have recently spawned from parent galaxies and are thus pristine, unsoiled by an extended life in space.

This subject has been explored in some detail in a recent paper (2000) by two eminent astronomers – G Burbidge and W M Napier. They conclude, using the evidence shown in fig 3, and evidence from other data sets, that significant peaks occur at preferred red-shifts of 0.06, 0.30, 0.60, 0.96, 1.41, 1.96, 2.63, 3.45, and 4.47. We now compare such values with shifts that can be attributed to the speed of light multiplied by a series of simple fractions.

Present-day thinking is that the relativistic Doppler formula has to be used to compute red-shifts when recessional speeds approach that of light – as is the case with the red-shift peaks cited above.

The relativistic Doppler formula is

$$Z + 1 = \sqrt{1 + \frac{v}{c}} \Big/ \sqrt{1 - \frac{v}{c}}$$

where Z is the red-shift, v is the speed of recession and c is the speed of light.

In Table 1 (column 1) a series of recessional speeds – all simple fractions of c – have been used to compute corresponding red-shifts (column 4), which are compared with the observed red-shift peaks (column5).

Table 1

v/c	$1+v/c$	$1-v/c$	$\sqrt{\frac{1+v/c}{1-v/c}}-1$ Predicted peak	Observed peak
1/4	1.25	0.75	0.291	0.30
1/3	1.33	0.667	0.41	0.60
1/2	1.5	0.5	0.732	0.96
2/3	1.667	0.333	1.237	1.41
3/4	1.75	0.25	1.646	
4/5	1.8	0.2	2.0	1.96
5/6	1.833	0.1667	2.316	
6/7	1.857	0.1429	2.605	2.63

Although there are some good matches, there are no matches for important fractions such as 1/3, 1/2, 2/3, 3/4, and 5/6. This test is regarded, therefore, as unconvincing.

However, views have been expressed recently concerning the validity of applying the relativistic Doppler formula as stated to high red-shifts. For example, Burbidge and Napier quote the formula

$$Z + 1 = (1 + Z_c)(1 + Z_d)(1 + Z_i)$$

where Z_c, Z_d, and Z_i are respectively red-shift components due to the expansion of the universe, random velocities (sometimes called peculiar velocities), and intrinsic properties (associated with the physics of objects).

Dr Ching-Chuan Su of Tsinghua University, Taiwan, is more specific and challenges the tacitly-made assumption that recessional speed does not change the resonant frequency of an emission or absorption line, and that Doppler shift is purely a kinematic property.

On the basis that moving clocks run slow and that an atom or molecule works like a clock when emitting or absorbing, it seems reasonable to meet the points raised above by modifying the relativistic Doppler formula by a factor of

$$\frac{1}{\sqrt{1 - v^2/c^2}}$$

Thus,

$$Z + 1 = \sqrt{\frac{1 + v/c}{1 - v/c}} \times \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$= \frac{1}{1 - \frac{v}{c}}$$

On this basis we construct Table 2, where predicted red-shift peaks are shown in column 2, and observed peaks are shown in column 3.

Table 2

$\frac{v}{c}$	$\frac{1}{1-\frac{v}{c}} - 1 = Z$	Observed red-shift peak	100Log(1+Z)
1/16	0.066	0.61	2.775
1/6	0.2		7.92
1/5	0.25		9.69
1/4	0.333	0.30	12.49
1/3	0.5	0.6	17.61
2/5	0.6667	0.6	22.19
1/2	1.0	0.96	30.1
3/5	1.5	1.41	39.79
2/3	2.0	1.96	47.71

Subject to the following explanatory notes, Table 2 shows an improved fit between the predicted and observed peaks of red-shift.

Note 1. Fig 3 shows one of the quasar red-shift distributions used by Burbidge and Napier. The distribution includes 78 quasars each being a radio source at 178 MHz. Peaks in the distribution are clearly discernable and align broadly with those predicted. The predicted peaks are marked in fig 4 to ease comparison.

Note 2. Burbidge and Napier report a strong peak at Z = 0.06. This is not shown in fig 3 as the sources concerned were optical rather than radio. Nevertheless, v/c = 1/16 yields a fair match between observation and prediction.

Note 3. At the prediction level of v/c = 1/6, the predicted red-shift aligns with a small peak – see fig 4. This peak is not listed by Burbidge and Napier.

Note 4. At v/c = 1/5, there is no peak in fig 4. This may be because a peak is present but is too weak to show in the small sample used. Alternatively, there may be theoretical reasons why 1/5 does not appear.

Note 5. Two peaks are predicted, one at v/c = 1/3 and the other at v/c = 2/5. Although Burbidge and Napier report only one peak in this area, the existence of a double peak is confirmed to some extent by fig 4.

Note 6. At v/c = 3/5 there is a significant difference between the observed peak (1.41) and the predicted peak (1/5). Examining the source data in fig 4 suggests that the 1.5 prediction is the better fit.

Note 7. Although higher observed peaks are reported by Burbidge and Napier, these are based upon relatively small amounts of data and do not appear in fig 4 at all.

Suggested further tests on the quantisation of speed.

The facts and arguments presented above support the hypothesis but are far from incontestable. Further tests to confirm - or otherwise - the quantisation of speed might include the following.

Particle accelerators propel particles to speeds close to that of light. No evidence has been reported thus far of preferred values of particle speed close to those predicted above. However, scientists have had no reason in the past to seek such values. Some research on these lines would now be welcome. If no quantisation is observed at normal temperatures, perhaps the effects of cooling to much lower temperatures might be explored.

The quantisation of red-shifts described above deals only with the reciprocal and fractional quantisation of light. If the hypothesis is correct, the speed of light should also be quantised in a conventional manner, showing preferred values at c(2, 3, 4, ...).

The idea that the speed of light can be exceeded is contrary to present-day science teaching, but a little - known event suggests that such speeds might after all exist. A certain Professor Nimtz is on record for using his allotted speaking time at a conference to reproduce a recording of Mozart's Jupiter Symphony. The unusual feature was that the signal was transmitted very rapidly - at the speed of light multiplied by a factor of approximately 5.3. This produced much argument among the scientific community of the day, such discussion centring around concepts such as group and phase velocities, and the idea that the speed of light could be exceeded was somewhat discredited, as was the scientific reputation of the professor.

As scientific attitudes are now, perhaps, more liberal, it is suggested the findings of Professor Nimtz, and others, be reviewed.

3.3.4) The Calculus Problem.

If a variable is quantised, by definition a small increment of that variable cannot 'tend to zero'. Calculus relies 'in the limit' upon reducing such an increment to zero. The implication is that the use of calculus is appropriate for variables that are continuous, not quantised. If the hypothesis that the universe is a digital system proves to be true, the use of calculus will lead to findings that are in error. In some cases, the errors may not be large, or even noticeable. In other cases, they may be significant and seriously misleading.

The corollary to the above implication is this: if the use of calculus can be shown to produce wrong or insufficient answers that are corrected when non-calculus means of calculation are employed, the hypothesis concerning the digital nature of the universe will have received a measure of support.

An attempt to derive such a measure of support is made as follows.

The electron is an enigmatic particle. We know it has an angular momentum of $h/2$ and that, if it were a spinning homogeneous sphere with such angular momentum, it would have to have a radius of at least 10^{-10} cm. (If the radius were any smaller, surface speeds of more than the speed of light would be required).

Conversely, scattering experiments show that the electron is very small in size and has a radius of less than some 10^{-16} cm, some six orders of magnitude less than the radius indicated above. Thus, the model of a spinning sphere doesn't fit the facts. A model that does is that of a point-like particle that orbits about a mean point and, irrespective of orbit, conserves its angular momentum.

Schrödinger's equation has been widely successful in describing the behaviour of sub-atomic particles so its use in this case seems reasonable. What is postulated is a free electron in space, not subject to any potentials but possessing kinetic orbiting energy arising from its angular momentum.

Under these conditions, Schrödinger's equation in polar coordinates becomes

$$\mathbf{h}^2/_{2M}\frac{\partial^2 u}{\partial r^2} + Eu = 0$$

where E is the kinetic energy of the orbiting electron, M is its mass and r is the radius of its orbit.

Irrespective of orbit, the angular momentum remains constant at

$$\mathbf{h}/_{2} = Mrv$$

where v is the tangential velocity of the electron. The kinetic energy is calculated as

$$E = 1/_{2}Mv^2 = \frac{\mathbf{h}^2}{8Mr^2}$$

Substituting for E in Schrödinger's equation yields

$$\partial^2 u/_{\partial r^2} + u/_{4r^2} = 0 \qquad (1)$$

One solution to this equation is $u = A\sqrt{r}$

As has been described in earlier ANPA papers, this result leads to the quantised structure model for the electron and thence to an explanation of the laws of electro-statics, electro-magnetics and gravity.

However, such development is subject to reservation due to the use of calculus in relation to what is hypothesised as a digital universe. We investigate, therefore, whether a digital version of equation (1) reveals information not exposed by this equation in its analogue form, as given above.

In particular, it is noted that equation (1) makes no reference to particle mass. As such, this equation can be expected to describe both the properties of the electron and those of other particles of semi-integral 'spin'. Such particles include the Muon (mass = 105.658389, σ = 0.00000015 MeV) and Tauon (mass = 1784.1, σ = 3 MeV). In this regard, the conventional solution to equation (1) does not discriminate between such particles or inform us about them.

The quantised structure of the electron reveals quantised 'shells' within which the electron particle has a specified speed and energy. Such shells have a constant distance of separation Δr, so r has a series of quantised values equivalent to Δr multiplied by a series of integers such as n – 1, n, n + 1, etc.

It is well known that in such circumstances

$$\partial^2 u \Big/ \partial r^2 = \frac{u_{n+1} - 2u_n + u_{n-1}}{\Delta r^2}$$

where r = nΔr

Thus, equation (1) becomes

$$u_{n+1} - 2u_n + u_{n-1} + {}^{u_n}\!/_{4n^2} = 0 \qquad (2)$$

It is guessed that the solution to this equation will not differ greatly from the analogue solution u = A√(nΔr). We try this solution and intend to add further terms to cancel out successive discrepancies.

Using the substitution above, equation (2) becomes

$$\sqrt{n+1} - 2\sqrt{n} + \sqrt{n-1} + {}^{\sqrt{n}}\!/_{4n^2} = 0$$

which is not exact and has a discrepancy. Having removed this discrepancy, and successive discrepancies by introducing further terms to the proposed solution, the following solution is derived.

$${}^{u_n}\!/_{\Delta r} = \sqrt{n} + {}^{1}\!/_{n^{3/2}} \times {}^{5}\!/_{4\times 64} - {}^{1}\!/_{n^{7/2}} \times 0.00344 - {}^{1}\!/_{n^{11/2}} \times {}^{1}\!/_{144} \qquad (3)$$

plus higher order terms.

But what is the meaning of these extra terms? Are they there to introduce a modest change to the probability function that one would now normally proceed to calculate? Or do they represent a sequence of particles? The second question is unconventional but seems worth exploring and, in the light of the following extract, cannot be considered as being totally barred by the original derivation of Schrödinger's equation.

The standard text book ‘An introduction to quantum physics’ (French and Taylor MIT Introductory Physics Series) includes the following remarks on the origins of Schrödinger’s equation. ‘It was Erwin Schrödinger who, in 1925, discovered an appropriate form of wave equation, making some deep formal analogies between optics and classical particle mechanics that had been evolved in the nineteenth century by W R Hamilton and others. We shall not attempt to retrace here the development of these fundamental analogies, but will simply try to make the form of the equation plausible......Clearly, we have not been driven inexorably to Schrödinger’s equation any more than Schrödinger was in his argument from analogy.....Finally, there is all the accumulated evidence that the Schrödinger equations work; they provide the basis for a correct analysis for all kinds of molecular, atomic, and nuclear systems. Whatever questionable features there may be in the manner of their formulation are swept away by the evidence of their manifest success’.

Let us guess, therefore, that the four terms in equation (3) represent a set of particles. It is reasonable to guess that the first term represents the electron – as was the case when considering the analogue equation (1).

Further, it seems sensible to examine first the case when n = 2 as this is the first shell that can be occupied by a particle of semi-integral ‘spin’. (The n = 1 shell is impossible as the orbiting speed of the particle would have to be ‘c’, the speed of light).

If we compare the second term with the first, we find the ratio between them is

$$\frac{4\times 64}{5}\times 2^2 = 204.8$$

If this ratio is multiplied by the mass of the electron – 0.511 MeV – we reveal a particle of mass

$$204.8 \times 0.511 = 104.6 \text{ MeV}$$

remarkably close to that of the Muon – 105.6 MeV.

We now calculate the ratio between the third and first term as

$$\frac{1}{0.00344} \times 2^4 = 4651.2$$

If this number is multiplied by the mass of the electron we arrive at a particle of mass

$$4651 \times 0.511 = 2376.7 \text{ MeV}$$

This cannot be recognised as any existing particle. However, if this mass is multiplied by the simple fraction ¾, we have

$$2376.7 \times 3/4 = 1782.6 \text{ MeV}$$

instantly recognisable as the Tauon – mass 1784 MeV.

Where does the ¾ fraction come from? An educated guess is that, as with the specific heats of gasses, we are dealing with a number of available degrees of freedom – in this case only three from a possible four.

That such predictions can emerge from a digital equation derived from Schrödinger's equation, rather than from the equivalent analogue equation, is an encouraging confirmation of our hypothesis. However, there are some 'loose ends' that remain.

For example, is there a rigorous theoretical basis for the calculation of the Muon and Tauon masses as described? Why, in the case of the Tauon, do we have to propose degrees of freedom to derive an accurate result? It is known that the Muon has a magnetic moment that differs very slightly from that of the electron. Can this now be explained?

On the other hand, the fourth term in equation (3) allows us to make a prediction concerning the existence of a fourth lepton in the series. Using the same method as before, we would expect a fourth lepton to have a mass of

$$144 \times 2^6 \times 0.511 = 4709 \text{ MeV}$$

perhaps multiplied by a simple fraction such as 1/2 or 5/6 to take account of degrees of freedom. If such a particle were revealed by future research, it would greatly strengthen support for the hypothesis, regardless of the 'loose ends' cited above.

3.2.5) Digital Logic.

The hypothesis proposed in this paper is that the universe is a digital system, and the required digitisation (quantisation) of variables has been explored at some length above. However, the inclusion of the word 'system' in the hypothesis requires more than just the non-linearity of variables; it requires that there should be at least some evidence that our processes and laws are underpinned by digital logic.

But what sort of logic should we seek? To make a guess, we should start by seeking evidence of binary logic, first because it is the simplest form of logic we know and second because the universe displays many examples if binary choice such as matter and anti-matter, positive and negative charge, light and dark, or even presence and absence. (To rule out the role of other forms of logic might at this stage be premature. For example, tertiary logic might have to be invoked to explain the concept of 'indistinguishability ')

If the workings of the universe are determined, at least to some degree, by binary logic, we would expect to discover that various quantities are related by powers of two. Some examples are examined as follows.

The relationship between the minimum mass and that of the electron.

The mass of the electron has been measured with some accuracy and is 9.1093897 x 10^{-28} gm.

As was shown in (3.3.3.), the minimum energy is given by $\hbar/T_o$, T_0 being the age of the universe, presently measured as 13.72 billion years, or 4.32970000 x 10^{17}seconds.

It follows that the minimum mass is

$$\hbar/c^2T_0 \text{ gm}$$

where $\hbar$ is 1.054573 (cgs units), and c is 2.997924 x 10^{10} cm/sec. Given these data, the ratio between the mass of the electron and that of the minimum mass can be calculated and is

3.3611 x 10^{38}

This is close to 2^{128}, which is

3.40282 x 10^{38}

Some discrepancy between these two findings is not surprising due to some uncertainty in measuring the age of the universe. If it is accepted that the true ratio is 2^{128}, the age of the universe becomes

13.89 billion years.

Although such a ratio – 2^{128} - yields a feasible result, and is a reassuringly 'round' number, there is as yet no theoretical reason why it should be accepted as correct. However, the existence of similar ratios in other fields suggests that some rational explanation may exist.

Ratio between the wavelength of the Cosmic Background Radiation and the Maximum Distance cT_0.

Fig 5 shows the spectrum of the Cosmic background radiation as measured by the Cobe satellite. The spectrum is remarkable in that it accurately follows the profile expected of black-body radiation that has cooled during the long period since the 'Big-bang' and thus is a major item of evidence supporting the Big-bang theory. The peak radiation occurs at a wavelength of 0.19 cm.

The length cT_0 is 12.98011 x 10^{27} cm. If this is divided by a factor of 2^{96} we obtain a length of 0.1638 cm, similar to that of the peak Cosmic Background radiation. Furthermore, as T_0 increases with the passage of time, this calculated length will increase in step. So, too, will the wavelength of 0.19 cm, in accordance with the adiabatic cooling produced by an expanding universe. The position of a 0.1638cm wavelength is shown in fig 5 for comparison purposes.

Again, the factor of 2^{96} is simple and round.

Ratio between the Period of Global Warming and the Age of the Universe.

Fig 6 shows evidence of cycles of global warming as gained from the analysis of deep ice cores taken from the Antarctic ice sheet. The predominant cycle of some 100,000 years is obvious, although other cycles – for example the Malenkovic cycles caused by the precessional effects of other planets – can be detected as well.

The cause of the 100,000 year cycle is not obvious, although various theories have been advanced.

The age of the universe is, say, 1.389×10^{10} years. If this is divided by 2^{16}, we derive a period of 211,000 years, or roughly double the period of climate change.

There are several ways of accounting for this factor of two discrepancy. One, for example, is that the 100,000 cycle may represent a probability function. This may well – as with Schrödinger – have resulted from squaring a wave function, such a function having twice the period of the resulting probability function.

Again, the factor 2^{16} is simple and round.

Quantisation of Galactic Redshifts.

The subject of the quantisation of distant quasars has already been discussed at length – see (3.3.3). There is, however, another form of quantised red-shift which relates to galaxies of all types (not just to selected quasars) of any red-shift and of any orientation with regard to the observer.

Such red-shifts were claimed by several observers to have preferred values at intervals of some 72 km/sec. In 1995, through a well-known paper by Guthrie and Napier, an attempt was made to subject such claims to rigorous testing, taking full account of the motions of the earth and the entire solar system. (Such quantisation was thought inconvenient and irrational by most astronomers, and there would have been some relief had it been disproved). However, Guthrie and Napier concluded that quantisation did exist, preferred values occurring at intervals of 37.6 km/sec.

Analysis continued as more data became available and, for example, Professor Tifft (University of Arizona) found other levels of quantisation, at some 8km/sec and 2.667 km/sec.

Techniques for measuring red-shifts have improved considerably during recent years, due in large part to developments in radio astronomy, and it is now possible to measure red-shifts within individual galaxies, for example along the arms of spiral galaxies. Interestingly, jumps in red-shift between the arms of individual spiral galaxies were observed to be quantised at intervals of 72 Km/sec. Then later studies showed other intervals that were 1/2, 1/3, and 1/6 of the 72km/sec value.

Further studies examined orbiting pairs of galaxies, thereby removing the need to take account of absolute red-shifts. The results showed that the relative red-shifts between such galaxies were quantised into multiples of 72km/sec. Accurate measurements using the 21cm hydrogen line showed red-shift differences clustered around 72, 144, and 216km/sec. Such quantisation was confirmed when extending the observations to groups of more than two galaxies.

Another investigation concentrated on dwarf irregular galaxies (very simple structures, mainly gas, that should present few complications) on an all sky basis. Once account had been taken of the motion of the Earth and solar system, strong quantisation was observed at intervals of 24km/sec – one third of the 72km/sec value.

Next, the study was extended to galaxies in another class, those with large amounts of rotation and interval motion – i.e. as different as possible from the dwarfs referred to above. Once more strong quantisation of red-shift was observed, but this time at intervals of half the 72km/sec level.

Such a spread of quantisation may appear puzzling, but not to readers of this paper. It aligns exactly with what happens in the quantised Hall Effect described earlier. What we have is reciprocal quantisation below a one-to-one point (72km/sec) and conventional quantisation above. As such, these phenomena suggest strongly that one quantised variable is being divided by another.

Deciding which two variables are involved is not simple. It is not impossible that speed is a more complicated variable than previously imagined – i.e. that distance and time are not the primary variables we assumed - and thus speed has two points of reciprocal quantisation, not just one.

Another possibility is that we are looking at the quotient of a different pair of variables that yields a variable that is capable of causing red-shifts. A prime candidate here is gravitational potential, GM/r. If this were so, it would be most fortunate in that, as the mass quantum is already known, we would then be able to calculate the size of the distance quantum. There may be complications, however, should the Gravitational constant 'G' turn out to be a variable!

The main point of this section is that 72km/sec looks as though it is a speed that is somehow significant. Is it related to the speed of light by a simple power of 2?

The answer is, 'Yes'. If the speed of light is divided by 2^{12} the result is

$$\frac{299792.4}{4096} = 73.19\,\text{km/sec}$$

An associated fact is the already commented-on redshift at Z = 0.06 – see (3.3.3.) – that corresponds to a recessional velocity of c/16, or $c/2^4$. Both 2^{12} and 2^4 are numbers that are reassuringly simple and round.

We have revealed ratios of 2 to the powers of 128, 96, 16, 12 and 4. There may well be more such ratios to be revealed. The existence of such ratios is at least an indicator that binary logic is in some way fundamental to our universe and, as such, supports our hypothesis.

Conclusion

Do the arguments above amount to a proof that the universe is a digital system? The answer to this is probably 'No'. The results quoted above have inconsistencies and some rely upon further assumptions, and widespread confirmation is needed from other sources – perhaps from some of the further tests suggested – before the hypothesis can gain sufficient strength to be accepted fully.

Are the arguments above so flimsy and faulted that the hypothesis should be discarded as highly improbable and, thus, merit no further consideration? Here again, the answer is probably 'No'. The arguments referred to seem individually to have some weight, and collectively to give a measure of support to the hypothesis.

If this paper merely alerts the reader to the credibility of the hypothesis, it will have served its purpose. Some may even conclude that, given the choice, it is easier to believe in the hypothesis and its implications than in the situation as presented by present-day science.

Should the hypothesis turn out to be correct, the implications for science would, as Einstein observed, be significant. Additionally, some development of pure mathematics might be needed as well. This is no criticism of pure mathematics which has served humanity well over many centuries and will certainly continue to do so. However the mathematics needed to describe a universe as defined by the hypothesis may well require all variables to be quantised, infinity to be finite, arithmetic to be probabilistic, simple calculus to be abandoned, and counting to be to base 2. Something of a change!

Geoffrey Constable August 2008

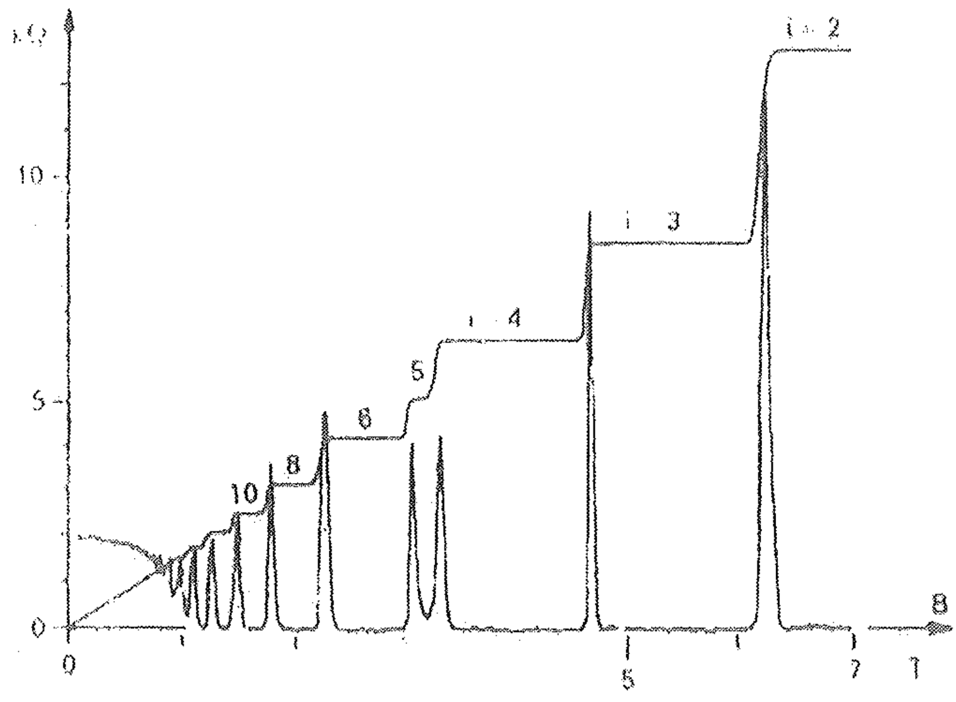

Fig 1 Plot of Quantised Hall resistance against magnetic flux

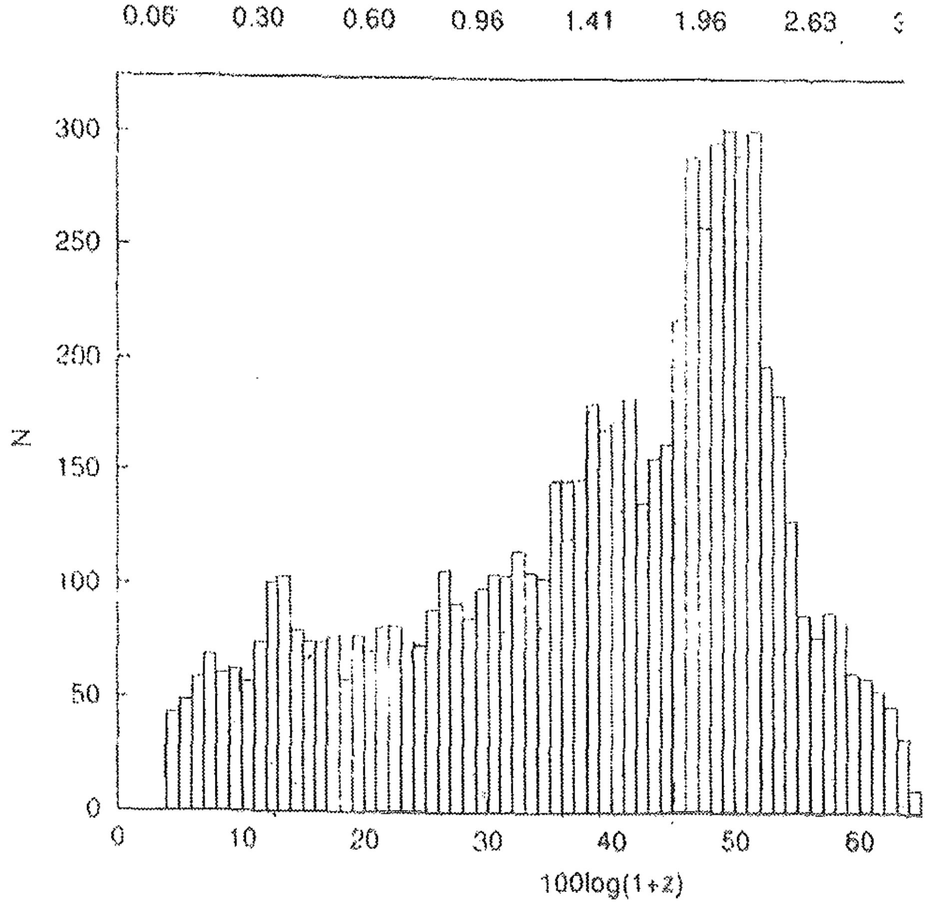

Fig 2 Red-shift Distribution of Full Population of Quasars

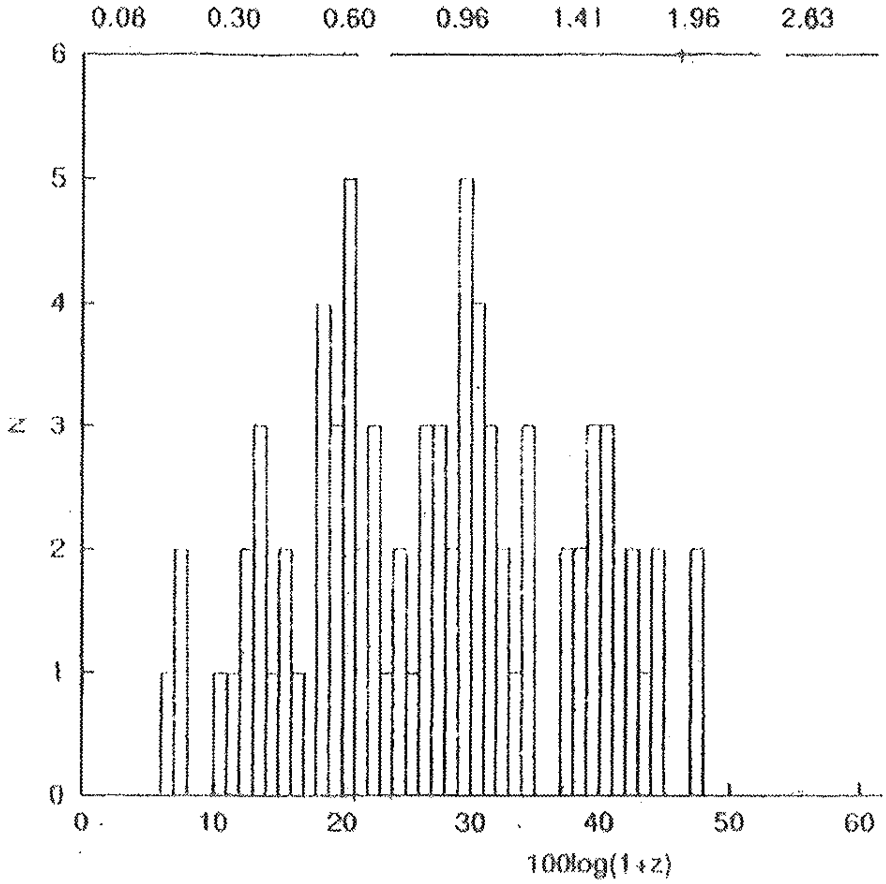

Fig 3 Red-shift Distribution of 78 Radio Quasars Closely Associated with Parent Galaxies

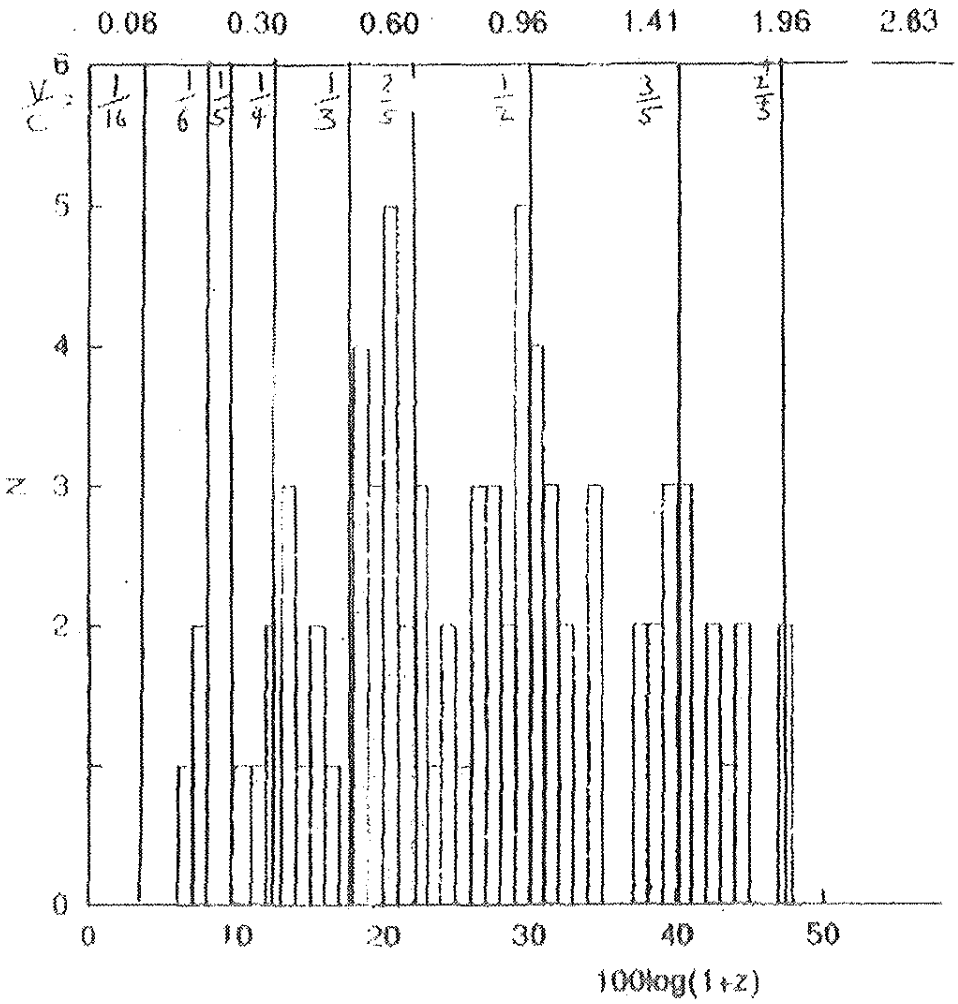

Fig 4 Red-shift Distribution of 78 Radio Quasars Showing Red-shift Peaks as Predicted by Hypothesis

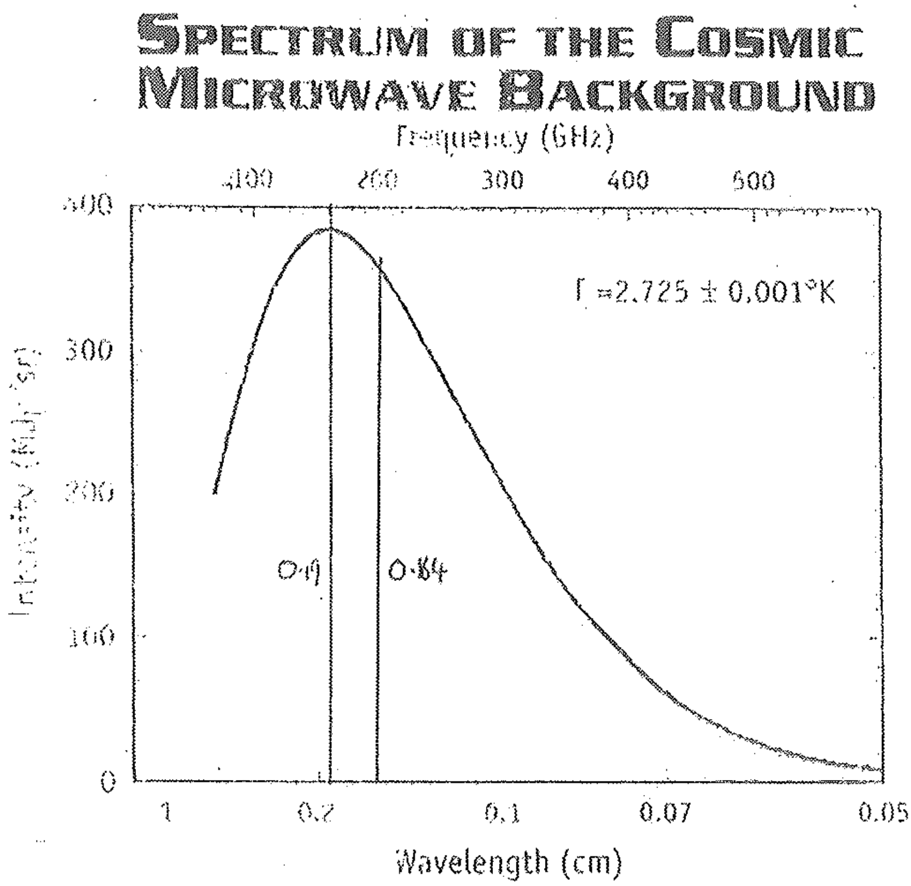
SPECTRUM OF THE COSMIC MICROWAVE BACKGROUND
Frequency (GHz)
300
400
500
T =2.725 ± 0.001°K
300
100
0
0.19
0.84
1
0.2
0.1
0.07
0.05
Wavelength (cm)

Fig 5

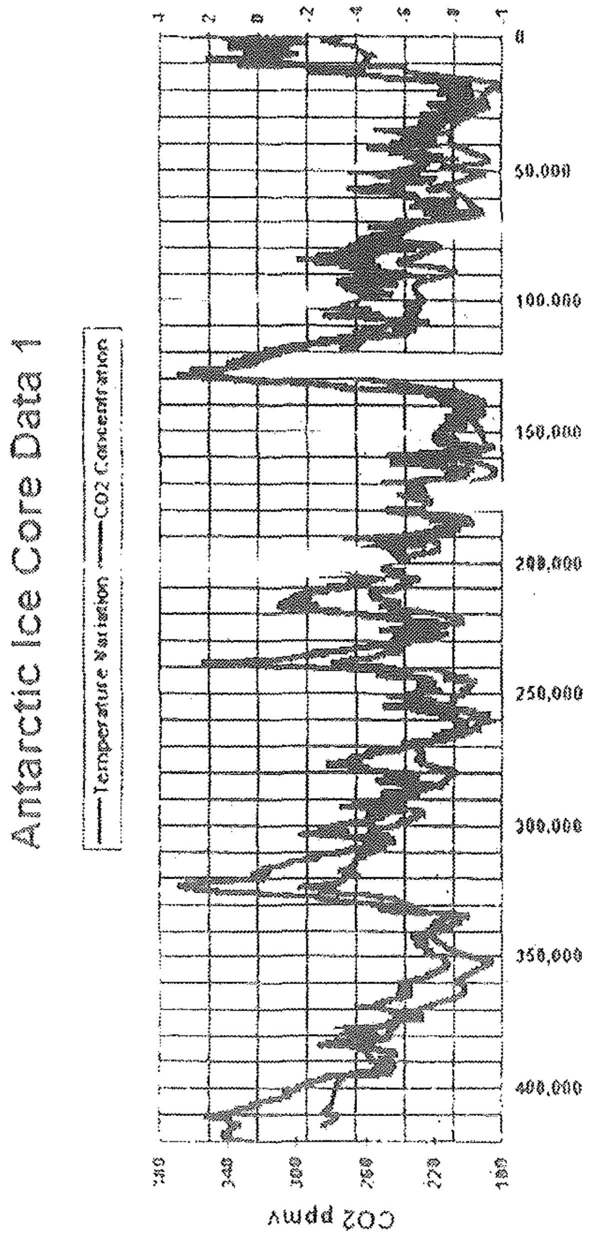
Antarctic Ice Core Data 1
Temperature Variation
CO2 Concentration
CO2 ppmv
0
50,000
100,000
150,000
200,000
250,000
300,000
350,000
400,000

Fig 6

What is Vacuum? A Nilpotent Solution

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Abstract. Vacuum is defined with exact mathematical precision as the state which remains when a fermion, with all its special characteristics, is created out of absolutely nothing. This definition leads to a form of relativistic quantum mechanics that requires only the construction of a fermionic creation operator. In this form of quantum mechanics, the characteristics of the weak, strong and electric interactions can be derived from the structure of the creation operator itself.

The nilpotent Dirac equation

Vacuum is defined as the state of minimum (but seemingly nonzero) energy in quantum mechanics, and it is an active component in quantum field theories. The main objective of projected unifying theories, for example, string theory, is to find the particular vacuum which makes their particle structures possible. It is not, however, a well-defined concept, and the reason why nature requires it at all is not at all clear. But it is now possible to show that vacuum has an exact, mathematically precise and logically satisfying meaning, and the discovery of that meaning is a very significant step in understanding the Standard Model of particle physics. It requires the most compact and powerful formulation of quantum mechanics available, and the one that leads most readily into a quantum field representation.

Fundamental physics is concerned only with fermions and their interactions via gauge bosons. In principle, the equation for fermions should solve everything. Clearly, it doesn't. The question is: can we make it do so? The fundamental equation for the fermion is the Dirac equation, conventionally written:

$$\left(\gamma^0 \frac{\partial}{\partial t} + \boldsymbol{\gamma}.\nabla + im\right)\psi = \left(\gamma^0 \frac{\partial}{\partial t} + \gamma^1 \frac{\partial}{\partial x} + \gamma^2 \frac{\partial}{\partial y} + \gamma^3 \frac{\partial}{\partial z} + im\right)\psi = 0 \quad (1)$$

where γ^0, γ^1, γ^2 and γ^3, are taken to be operators, which anticommute with each other, and with a fifth operator, $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, and where

$$(\gamma^0)^2 = (\gamma^5)^2 = 1 \qquad (\gamma^1)^2 = (\gamma^2)^2 = (\gamma^3)^2 = -1.$$

The first stage is to replace these with a tensor product of quaternions and vectors, an algebra with 8 base units:

i j k	quaternion units	**i j k**	multivariate vector units
1	scalar	*i*	pseudoscalar

The multivariate vector units are effectively a complexified quaternion system, which is commutative to ***i***, ***j***, ***k***. Multivariate vectors **a** and **b** follow the product rule: **ab** = **a.b** + *i* **a** × **b**. There are 64 possible products of the 8 basic units:

(±1, ± *i*)	4	units
(±1, ± *i*) × (**i**, **j**, **k**)	12	units
(±1, ± *i*) × (***i***, ***j***, ***k***)	12	units
(±1, ± *i*) × (**i**, **j**, **k**) × (***i***, ***j***, ***k***)	36	units

But this group of 64, needs only a *pentad* of 5 generators, for example:

ik **i***i* **j***i* **k***i* 1***j***

Such pentads can be seen as isomorphic to the γ algebra.

To create a pentad, we need, essentially, to take the 8 basic units and distribute the units of one 3-D structure among the rest, e.g.,

i	**i j k**	1	***i j k***
ik	**i***i* **j***i* **k***i*	1***j***	

The individual terms can then take either + or – values. 12 'gamma' pentads can be shown to exist simultaneously, all with the same overall structure. It is convenient for us to define 2, as mappings onto the gamma algebra:

$$\gamma^o = -i\boldsymbol{i} \quad \gamma^1 = \mathbf{i}\boldsymbol{k} \quad \gamma^2 = \mathbf{j}\boldsymbol{k} \quad \gamma^3 = \mathbf{k}\boldsymbol{k} \quad \gamma^5 = i\boldsymbol{j} \qquad \text{(A)}$$

$$\gamma^o = i\boldsymbol{k} \quad \gamma^1 = \mathbf{i}i \quad \gamma^2 = \mathbf{j}i \quad \gamma^3 = \mathbf{k}i \quad \gamma^5 = i \qquad \text{(B)}$$

Choosing mapping (A), we transform (1) into:

$$\left(-ii\frac{\partial}{\partial t}+k\mathrm{i}\frac{\partial}{\partial x}+k\mathrm{j}\frac{\partial}{\partial y}+k\mathrm{k}\frac{\partial}{\partial z}+im\right)\psi=0.$$

We now, 'trivially', multiply from the left by j, and the equation becomes:

$$\left(ik\frac{\partial}{\partial t}+i\mathrm{i}\frac{\partial}{\partial x}+i\mathrm{j}\frac{\partial}{\partial y}+i\mathrm{k}\frac{\partial}{\partial z}+ijm\right)\psi=0.$$

This switches the mapping to (B) at the same time as giving the fifth term a 'gamma' coefficient. We could, of course, have done this still using the gamma notation:

$$-i\gamma^5\left(\gamma^0\frac{\partial}{\partial t}+\gamma^1\frac{\partial}{\partial x}+\gamma^2\frac{\partial}{\partial y}+\gamma^3\frac{\partial}{\partial z}+im\right)\psi=0$$

$$\left(-i\gamma^5\gamma^0\frac{\partial}{\partial t}-i\gamma^5\gamma^1\frac{\partial}{\partial x}-i\gamma^5\gamma^2\frac{\partial}{\partial y}-i\gamma^5\gamma^3\frac{\partial}{\partial z}+\gamma^5 m\right)\psi=0$$

$$\left(\gamma^0\frac{\partial}{\partial t}+\gamma^1\frac{\partial}{\partial x}+\gamma^2\frac{\partial}{\partial y}+\gamma^3\frac{\partial}{\partial z}+\gamma^5 m\right)\psi=0.$$

So the process is not dependent on the algebraic symbolism.

In both formats, it is convenient to group the vector terms, as in:

$$\left(ik\frac{\partial}{\partial t}+i\nabla+ijm\right)\psi=0 \quad \text{or} \quad \left(\gamma^0\frac{\partial}{\partial t}+\boldsymbol{\gamma}.\nabla+\gamma^5 m\right)\psi=0.$$

For fairly obvious reasons, however, the quaternionic form is much more compact than the matrix form, and so this will always be preferred here. What we see immediately is that the transformed equation is now beautifully *symmetrical*:

$$\left(ik\frac{\partial}{\partial t}+i\nabla+ijm\right)\psi=0,$$

with the k, i, j and k units separating the energy, momentum and mass operators.

The real significance of the extra $j = -i\gamma^5$ becomes apparent when we try inserting a plane wave solution for ψ:

$$\psi = Ae^{-i(Et-\mathbf{p}.\mathbf{r})}.$$

Then applying the differential operator:

$$(kE + ii\mathbf{i}px + ii\mathbf{j}py + ii\mathbf{k}px + ijm)\, Ae^{-i(Et - \mathbf{p.r})} = 0$$

or

$$(kE + ii\mathbf{p} + ijm)\, Ae^{-i(Et - \mathbf{p.r})} = 0.$$

The multivariate nature of **p** allows us to write

$$\mathbf{pp} = (\boldsymbol{\sigma}.\mathbf{p})\,(\boldsymbol{\sigma}.\mathbf{p}) = pp = p^2.$$

So we can also use **σ.p** for **p** (or **σ.∇** for **∇**) in the Dirac equation, where **σ** is a pseudovector of magnitude –1.

In the Dirac equation, **σ.p** is the form taken by *helicity*. In effect, this means we need explicitly incorporate *spin* only where the vectors are not multivariate (e.g. using polar coordinates).

$$(kE + ii\mathbf{p} + ijm)\, Ae^{-i(Et - \mathbf{p.r})} = 0$$

is only valid if A is a multiple of $(kE + ii\mathbf{p} + ijm)$. That is, A is a *nilpotent* or square root of zero. This would have been true of ψ, *even if we had never multiplied from the left by* $\boldsymbol{j}$ *or substituted algebraic for matrix operators.* In principle, we don't have to say that the fermionic wavefunction *has* to be written in a nilpotent form. Only that it has to have a nilpotent character, whether this is explicitly recognised or not. And that this nilpotent character has a fundamental physical meaning. We will see that this is true whether the fermion is free or bound.

From wavefunction to 4-component spinor

The Dirac equation, of course, is not written for a single wavefunction, but a 4-component spinor. With the knowledge (from Hestenes, 1966) that a multivariate **p** or **∇** already incorporates fermionic spin, we can easily recognise the 4 variations required:

fermion / antifermion	$\pm E$
spin up / down	$\pm \mathbf{p}$

These options apply to both amplitude and phase, and to operators or eigenvalues, as well as free or bound states.

Leaving out the phase factors, this gives us amplitudes of the form:

$(\boldsymbol{k}E + \boldsymbol{i}i\mathbf{p} + \boldsymbol{i}\boldsymbol{j}m)$	fermion spin up
$(\boldsymbol{k}E - \boldsymbol{i}i\mathbf{p} + \boldsymbol{i}\boldsymbol{j}m)$	fermion spin down
$(-\boldsymbol{k}E + \boldsymbol{i}i\mathbf{p} + \boldsymbol{i}\boldsymbol{j}m)$	antifermion spin down
$(-\boldsymbol{k}E - \boldsymbol{i}i\mathbf{p} + \boldsymbol{i}\boldsymbol{j}m)$	antifermion spin up

where the sign conventions are, of course, arbitrary and purely conventional. It is, in fact, more convenient (for the physical meaning) to multiply throughout by i and reorganize the sign conventions, so that they become:

$(i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	fermion spin up
$(i\boldsymbol{k}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	fermion spin down
$(-i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	antifermion spin down
$(-i\boldsymbol{k}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$	antifermion spin up

With this convention, $\boldsymbol{i}\mathbf{p}$ / $i\boldsymbol{k}E$ represents the same helicity or handedness as $(-\boldsymbol{i}\mathbf{p})$ / $(-i\boldsymbol{k}E)$, but the opposite helicity to $(\boldsymbol{i}\mathbf{p})$ / $(-i\boldsymbol{k}E)$ or $(-\boldsymbol{i}\mathbf{p})$ / $(i\boldsymbol{k}E)$.

Also, while conventionally we would require 4 different phase factors as components of these amplitudes, and act upon them with a single differential operator, we can instead restructure the differential operator as a 4-component spinor, which acts on a single phase factor:

$\left(-\boldsymbol{k}\frac{\partial}{\partial t} - \boldsymbol{i}i\nabla + \boldsymbol{j}m\right)$	fermion spin up
$\left(-\boldsymbol{k}\frac{\partial}{\partial t} + \boldsymbol{i}i\nabla + \boldsymbol{j}m\right)$	fermion spin down
$\left(\boldsymbol{k}\frac{\partial}{\partial t} - \boldsymbol{i}i\nabla + \boldsymbol{j}m\right)$	antifermion spin down
$\left(\boldsymbol{k}\frac{\partial}{\partial t} + \boldsymbol{i}i\nabla + \boldsymbol{j}m\right)$	antifermion spin up

Most conveniently, we can group together the 4 operators and the 4 amplitudes in an abbreviated form of the equation as

$$\left(\mp \boldsymbol{k}\frac{\partial}{\partial t} \mp \boldsymbol{i}i\nabla + \boldsymbol{j}m\right)(\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)e^{-i(Et - \mathbf{p}.\mathbf{r})} = 0 .$$

We can also use the convention that E and $\mathbf{p}$ represent operators as well as amplitudes to express it as

$$(\pm ikE \pm \boldsymbol{i}\mathbf{p} + jm)(\pm ikE \pm \boldsymbol{i}\mathbf{p} + jm)e^{-i(Et-\mathbf{p}.\mathbf{r})} = 0 .$$

This suggests that we could derive the Dirac equation by simply taking the classical

$$E^2 - p^2 - m^2 = 0,$$

then factorizing using noncommuting algebraic operators to give,

$$(\pm ikE \pm \boldsymbol{i}\mathbf{p} + jm)\ (\pm ikE \pm \boldsymbol{i}\mathbf{p} + jm) = 0,$$

and, finally, applying a canonical quantization to the left-hand bracket.

Using a 'discrete' or anticommutative process of differentiation, as defined by Lou Kauffman (2004), where

$$\frac{\partial F}{\partial t} = [F, \mathcal{H}] = [F, E] \quad \text{and} \quad \frac{\partial F}{\partial X_i} = [F, P_i],$$

we can remove the phase factor from the amplitude and the mass term from the operator. Here, we can define a nilpotent amplitude

$$\psi = ikE + \boldsymbol{i}\mathbf{i}P_1 + \boldsymbol{i}\mathbf{j}P_2 + \boldsymbol{i}\mathbf{k}P_3 + jm$$

and an operator

$$\mathcal{D} = ik\frac{\partial}{\partial t} + \boldsymbol{i}\mathbf{i}\frac{\partial}{\partial X_1} + \boldsymbol{i}\mathbf{j}\frac{\partial}{\partial X_2} + \boldsymbol{i}\mathbf{k}\frac{\partial}{\partial X_3}$$

where $$\frac{\partial \psi}{\partial t} = [\psi, \mathcal{H}] = [\psi, E] \quad \text{and} \quad \frac{\partial \psi}{\partial X_i} = [\psi, P_i].$$

With some straightforward algebraic manipulation, we find that

$$-\mathcal{D}\psi = i\psi(ikE + \boldsymbol{i}\mathbf{i}P_1 + \boldsymbol{i}\mathbf{j}P_2 + \boldsymbol{i}\mathbf{k}P_3 + jm)$$
$$+ i(ikE + \boldsymbol{i}\mathbf{i}P_1 + \boldsymbol{i}\mathbf{j}P_2 + \boldsymbol{i}\mathbf{k}P_3 + jm)\psi - 2\,i(E^2 - P_1^{\,2} - P_2^{\,2} - P_3^{\,2} - m^2).$$

When is ψ nilpotent, then

$$\partial\!\!\!/ \psi = \left(i\boldsymbol{k}\frac{\partial}{\partial t} + \boldsymbol{i}\nabla \right)\psi = 0 .$$

This is a Dirac equation using discrete differentials. Generalising this to four states, with $\partial\!\!\!/$ and ψ represented as 4-spinors, then

$$\partial\!\!\!/ \psi = \left(\pm i\boldsymbol{k}\frac{\partial}{\partial t} \pm \boldsymbol{i}\nabla \right)(\pm i\boldsymbol{k}E \pm i\mathbf{i}P_1 \pm i\mathbf{j}P_2 \pm i\mathbf{k}P_3 + \boldsymbol{j}m) = 0$$

becomes the equivalent to the Dirac equation in this calculus.

Significantly we did not use i or $i\hbar$ in defining the differentials, though this is usually required in canonical quantization. We could, of course, have done so and obtained the same result. It would seem that the 'discreteness', by allowing us to eliminate the mass term, also allows us to use an operator that does not distinguish between quantum and classical contexts. It also allows us to use creation and annihilation operators that are exact negatives of each other, emphasizing the fact that the 'active' parts in the process are the space and time variations. If we convert the differentials to covariant derivatives, we can introduce distorted space-time without a mass term.

Crossing the Rubicon – quantum mechanics to quantum field theory

We have transformed

$$\left(-\boldsymbol{k}\frac{\partial}{\partial t} - i\boldsymbol{i}\nabla + \boldsymbol{j}m \right)(i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)e^{-i(Et - \mathbf{p}.\mathbf{r})}$$

$$\left(-\boldsymbol{k}\frac{\partial}{\partial t} - i\boldsymbol{i}\nabla + \boldsymbol{j}m \right)(i\boldsymbol{k}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)e^{-i(Et + \mathbf{p}.\mathbf{r})}$$

$$\left(-\boldsymbol{k}\frac{\partial}{\partial t} - i\boldsymbol{i}\nabla + \boldsymbol{j}m \right)(-i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)e^{i(Et + \mathbf{p}.\mathbf{r})}$$

$$\left(-\boldsymbol{k}\frac{\partial}{\partial t} - i\boldsymbol{i}\nabla + \boldsymbol{j}m \right)(-i\boldsymbol{k}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)e^{i(Et - \mathbf{p}.\mathbf{r})}$$

with 1 operator and 4 phase factors, into

$$\left(-k\frac{\partial}{\partial t}-ii\nabla+jm\right)(ikE+i\mathbf{p}+jm)e^{-i(Et-\mathbf{p}.\mathbf{r})}$$
$$\left(-k\frac{\partial}{\partial t}+ii\nabla+jm\right)(ikE-i\mathbf{p}+jm)e^{-i(Et-\mathbf{p}.\mathbf{r})}$$
$$\left(k\frac{\partial}{\partial t}-ii\nabla+jm\right)(-ikE+i\mathbf{p}+jm)e^{-i(Et-\mathbf{p}.\mathbf{r})}$$
$$\left(k\frac{\partial}{\partial t}+ii\nabla+jm\right)(-ikE-i\mathbf{p}+jm)e^{-i(Et-\mathbf{p}.\mathbf{r})}$$

with 4 operators and 1 phase factor.

The single phase gives the formalism enormously increased calculating power, as the phase factor is usually the first thing that has to be calculated. It also corresponds with Feynman's interpretation of negative energy states requiring reversed time. More fundamentally, it allows us to write

$$(\pm\, ikE \pm i\mathbf{p} + jm)\,(\pm\, ikE \pm i\mathbf{p} + jm) = 0$$

with many meanings.

The first is Pauli exclusion. A particle with a nilpotent wavefunction will be automatically Pauli exclusive, as the combination state with an identical particle $\psi_1 \psi_1$ will be zero. So far, we have only done this for free fermions. However, a massive change occurs when we use it to define Pauli exclusion in all fermionic states, whether free or bound. Here, we define E and $\mathbf{p}$ as operators, which may include any number of potentials or interaction terms. In the simplest case (the Coulomb interaction), E and $\mathbf{p}$ become

$$E = i\partial / \partial t \to i\partial / \partial t + e\phi + \dots$$
$$\mathbf{p} = -i\nabla \to -i\nabla + e\mathbf{A} + \dots .$$

but this would also apply for any other interaction, and any number of interactions. In principle, this means that we can incorporate QED and other quantum field theories, as the E and $\mathbf{p}$ terms can be redefined at every stage in the interaction process, so yielding, for example, Lamb-shifted versions of these operators.

The result of applying the new definitions of E and $\mathbf{p}$ as operators is to create a new expression for the single phase factor. It will no longer be the simple exponential of the free particle. However, the phase factor will still be defined uniquely for any given system, as it will be the unique expression required to give a resulting nilpotent amplitude after application of the operator.

$$(\text{operator acting on phase factor})^2 = \text{amplitude}^2 = 0.$$

It is important to note that this is *not* the Dirac equation.

This idea incorporates a massive change from conventional approaches, and is no longer simply equivalent to them. The whole apparatus of conventional relativistic quantum mechanics becomes redundant. We have eliminated the need for:

(1) An equation. The operator, once defined, specifies everything.

(2) Wavefunction, amplitude and phase factor as independent entities. These are completely determined by the operator.

(3) Spinor structure. This still exists but is completely fixed by predetermined sign changes. The 3 additional terms in the fermionic spinor are simply 'drones' of the lead term.

This reduction of the whole amount of input information to an operator does not reduce the calculating power of the method – on the contrary, it massively increases it. It also effects an almost seemless transition from quantum mechanics to quantum field theory without the need for the cumbersome apparatus of quantum field integrals, etc.

Fermions, specified in this way, can only be understood in relation to the entire universe (i.e. the entire quantum field). Renormalization and the hierarchy problem can be eliminated, though couplings still change, as expected, with the energy scale. No formal process of second quantization or additional mathematical formalism is required to specify the quantum field nature of the fermion creation operator. We can retain the simpler structures of quantum mechanics while specifying the action of the complete quantum field.

The phase factor is now simply an expression of all the possible variations in space and time which are encoded in the creation operator. This is uniquely defined once the operator is specified. A fermion is thus specified as a set of space and time variations. The mass term, as we see from the 'discrete differentiation' process, is purely passive, and is convenient, rather than necessary information.

There is, however, a payback. We can no longer define an isolated system. Defining a fermion 'disturbs the universe'. The energy conservation involved in $E^2 - p^2 - m^2 = 0$ only works over the entire universe, since the E and $\mathbf{p}$ terms contain the entire range of the fermion's interactions. The laws of thermodynamics become a necessary consequence, and the fermion becomes an *intrinsically dissipative* system.

The formalism drives us towards an understanding of the universe that cannot be separated from the way we define a fermion. And now, at last we begin to understand the question: what is vacuum? Suppose we want to create a fermion *ab initio*, that is, from entirely nothing, and that we want it to be created with certain interaction potentials embedded in its operator. Let us give this fermion a 'wavefunction' ψ_f. Then the 'hole' that its creation leaves in 'nothing' may be described as $-\psi_f$. The superposition of the states of fermion + 'hole' or $\psi_f - \psi_f$ equals 0, as does the product state $-\psi_f \psi_f$ if the wavefunction is nilpotent. (The effective equivalence of 'doubling' and 'squaring' here appears to have a fundamental origin (Rowlands (2007).))

The nilpotent structure and Pauli exclusion now become comprehensible if the entire universe adds up to nothing, and 'vacuum' becomes the 'hole' in nothing left by creating the fermion. However, vacuum itself is *not* nothing. To create the fermionic state ψ_f with the interaction potentials that we assigned to it means to create an 'environment' with makes this possible (i.e. a system of other fermions which create those potentials), but whose total effect adds up to $-\psi_f$. So, vacuum, for any fermionic state, becomes *the rest of the universe* for that state. This is what the fermionic operator is actually acting on. And that 'rest of the universe' must be so constructed as to appear in total as the mirror image of the fermion state.

The same condition must additionally be true for each of the other fermions which collectively create the vacuum state for the first. So, we could express the Pauli exclusion principle in the statement that no two fermions share the same vacuum (or the same phase factor). A zero condition for the entire universe is logically satisfying because it is necessarily *incapable* of further explanation. It is also a powerful route to understanding fundamental physical concepts because vacuum now becomes an active component of the theory. And nilpotency becomes a statement of a *physical* principle, rather than a purely mathematical operation.

Some consequences of the nilpotent formalism

Conventional quantum mechanics seemingly uses idempotent, not nilpotent wavefunctions, but the idempotent and nilpotent equations are exactly the same.

IDEMPOTENT

$$[(ik\partial / \partial t + \boldsymbol{i}\nabla + jm) j]\ [j(ikE + \boldsymbol{i}\mathbf{p} + jm)\ e^{-i(Et - \mathbf{p}.\mathbf{r})}] = 0.$$

operator *wavefunction*

NILPOTENT

$$[(ik\partial / \partial t + \boldsymbol{i}\nabla + jm) jj]\ [(ikE + \boldsymbol{i}\mathbf{p} + jm)\ e^{-i(Et - \mathbf{p}.\mathbf{r})}] = 0.$$

operator *wavefunction*

The equation thus contains both idempotent and nilpotent information about the wavefunction. The idempotent information has its significance, for vacuum, as we will see, but it is the nilpotent information that tells us about the relationship between the fermion and the rest of the universe. The choice of formalism used in quantum mechanics is not a neutral one. Different mathematical structures reveal non-equivalent levels of physical information.

The nilpotent formalism reveals that a fermion 'constructs' its own vacuum, or the entire 'universe' in which it operates. We can consider the vacuum to be 'delocalised' to the extent that the fermion is 'localised'. The 'local' can be defined as whatever happens inside the nilpotent structure ($\pm\ ikE \pm \boldsymbol{i}\mathbf{p} + jm$), and the 'nonlocal' as whatever happens outside it.

A Wheeler-type 'one fermion' theory of the universe (Feynman, 1972) is a serious possibility. However, a single fermion cannot be considered isolated. It must be interacting. In effect, it must construct a 'space', so that its vacuum is not localised on itself. If a fermion is point-like, its vacuum must be dispersed. In this sense, a single (noninteracting) fermion cannot exist. It can only be defined if we also define its vacuum.

Pauli exclusion is automatic with nilpotent wavefunctions. So, it is important to show that such wavefunctions are also Pauli exclusive in the conventional sense of being automatically antisymmetric ($\psi_1\psi_2 - \psi_2\psi_1$):

$$\begin{aligned}&(\pm\ ikE_1 \pm \boldsymbol{i}\mathbf{p}_1 + jm_1)\ (\pm\ ikE_2 \pm \boldsymbol{i}\mathbf{p}_2 + jm_2)\\ &\ - (\pm\ ikE_2 \pm \boldsymbol{i}\mathbf{p}_2 + jm_2)\ (\pm\ ikE_1 \pm \boldsymbol{i}\mathbf{p}_1 + jm_1)\\ &= 4\mathbf{p}_1\mathbf{p}_2 - 4\mathbf{p}_2\mathbf{p}_1 = 8\ i\ \mathbf{p}_1 \times \mathbf{p}_2.\end{aligned}$$

We see immediately that

$$\psi_1\psi_2 - \psi_2\psi_1 = -(\psi_2\psi_1 - \psi_1\psi_2).$$

The result is actually quite remarkable. It implies that, instantaneously, any nilpotent wavefunction must have a **p** vector in spin space (a kind of spin 'phase') at a *different orientation* to any other. The wavefunctions of all nilpotent fermions might then instantaneously correlate because the planes of their **p** vector directions must all intersect, and the intersections actually create the *meaning* of Euclidean space, with an intrinsic spherical symmetry generated by the fermions themselves.

At the same time, the equation could also be interpreted as suggesting that each nilpotent also has a unique direction in a *quaternionic phase space*, in which E, **p** and m values are arranged along orthogonal axes. We may suppose here that the mass shell or real particle condition requires the coincidence between the directions in these two spaces. In addition, the **p** vector carries *all the information* available to a fermionic state, its direction also determining its E and **p** values uniquely.

Fermionic spin and helicity

The nilpotent operator $(\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ immediately presents us with the Hamiltonian $\mathcal{H} = (\boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$. Using this Hamiltonian, we can see that fermionic spin is simply a result of the multivariate nature of **p**. If we *mathematically* define a pseudovector quantity $\boldsymbol{\sigma} = -\mathbf{1}$, then

$$\begin{aligned}[\boldsymbol{\sigma}, \mathcal{H}] &= [-\mathbf{1}, \boldsymbol{i}\,(\mathbf{i}p_1 + \mathbf{j}p_2 + \mathbf{k}p_3) + \boldsymbol{j}m] = [-\mathbf{1}, \boldsymbol{i}(\mathbf{i}p_1 + \mathbf{j}p_2 + \mathbf{k}p_3)] \\ &= -2\boldsymbol{i}\,(\mathbf{ij}p_2 + \mathbf{ik}p_3 + \mathbf{ji}p1 + \mathbf{j}\,p_3 + \mathbf{ki}p_1 + \mathbf{kj}p_2) \\ &= -2i\boldsymbol{i}\,(\mathbf{k}(p_2 - p_1) + \mathbf{j}(p_1 - p_3) + \mathbf{i}(p_3 - p_2)) \\ &= -2i\boldsymbol{i}\mathbf{1} \times \mathbf{p}.\end{aligned}$$

If **L** is the orbital angular momentum $\mathbf{r} \times \mathbf{p}$, then

$$\begin{aligned}[\mathbf{L}, \mathcal{H}] &= [\mathbf{r} \times \mathbf{p}, \boldsymbol{i}\,(\mathbf{i}p_1 + \mathbf{j}p_2 + \mathbf{k}p_3) + km] \\ &= [\mathbf{r} \times \mathbf{p}, \boldsymbol{i}\,(\mathbf{i}p_1 + \mathbf{j}p_2 + \mathbf{k}p_3)] \\ &= \boldsymbol{i}\,[\mathbf{r}, (\mathbf{i}p_1 + \mathbf{j}p_2 + \mathbf{k}p_3)] \times \mathbf{p}\end{aligned}$$

But $$[\mathbf{r}, (\mathbf{i}p_1 + \mathbf{j}p_2 + \mathbf{k}p_3)]\,\psi = i\mathbf{1}\,\psi.$$

Hence $$[\mathbf{L}, \mathcal{H}] = ii\,\mathbf{1} \times \mathbf{p},$$

and $\mathbf{L} + \boldsymbol{\sigma} / 2$ is a constant of the motion, because

$$[\mathbf{L} + \boldsymbol{\sigma} / 2, \mathcal{H}] = 0.$$

The spin ½ term characteristic of fermionic states has often been considered a rather strange property in seemingly requiring a fermion to undergo a 4π, rather than 2π, rotation to return to its starting point. However, if we regard a fermion as only being created simultaneously with its mirror image vacuum state, then we can regard the spin ½ term as an indication that taking the fermion alone only gives us half of the knowledge we require to specify the system.

Helicity ($\boldsymbol{\sigma}.\mathbf{p}$) is another constant of the motion because

$$[\boldsymbol{\sigma}.\mathbf{p}, \mathcal{H}] = [-p, \boldsymbol{i}\,(\mathbf{i}p_1 + \mathbf{j}p_2 + \mathbf{k}p_3) + ijm] = 0.$$

For a hypothetical fermion / antifermion with zero mass,

$$(kE + ii\,\boldsymbol{\sigma}.\mathbf{p} + ijm) \rightarrow (kE - iip)$$
$$(-kE + ii\,\boldsymbol{\sigma}.\mathbf{p} + ijm) \rightarrow (-kE - iip)$$

Each of these is associated with a single sign of helicity, $(kE + iip)$ and $(- kE + iip)$ being excluded, if we choose the same sign conventions for $\mathbf{p}$. Because we were required to choose $\boldsymbol{\sigma} = -\mathbf{1}$ in deriving spin for states with positive energy, the allowed spin direction for these states must be antiparallel, and so require left-handed helicity, while the helicity of the negative energy states becomes right-handed. Numerically, $\pm E = p$, so we can express the allowed states as

$$\pm E(k - ii)$$

Multiplication from the left by the projection operator

$$(1 - ij) / 2 \equiv (1 - \gamma^5) / 2$$

leaves the allowed states unchanged while zeroing the excluded ones.

Using a multivariate vector **p** or ∇ removes the need for an explicit spin (or total angular momentum) term, but this is not true where we are using ordinary vector terms, say with polar coordinates. Here, however, we can use a version of Dirac's prescription (1958) for expressing the momentum operator (with explicit spin term) in polar coordinates:

$$\nabla \rightarrow \left(\frac{\partial}{\partial r} + \frac{1}{r}\right) \pm i\frac{j+½}{r} .$$

Another significant aspect of spin arises when, using the original Dirac sign convention (and explicit universal constants), we write the nilpotent Hamiltonian in the form

$$\mathcal{H} = -ijc\boldsymbol{\sigma}.\mathbf{p} - iimc^2 = -ijc\mathbf{1}\,\mathbf{p} - iimc^2 = \boldsymbol{\alpha}c\mathbf{p} - iimc^2,$$

Since we have four separate spin states in the system, we may take $\boldsymbol{\alpha} = -ij\mathbf{1}$ as a dynamical variable, and define a velocity operator, as a dynamical variable, and $c\boldsymbol{\alpha} = -ij\mathbf{1}c$ as a velocity operator, which, for a free particle, becomes

$$\mathbf{v} = \dot{\mathbf{r}} = \frac{d\mathbf{r}}{dt} = \frac{1}{i\hbar}\,[\mathbf{r}, \mathcal{H}] = -ij\mathbf{1}c = c\boldsymbol{\alpha} .$$

The equation of motion for this operator then becomes:

$$\frac{d\boldsymbol{\alpha}}{dt} = \frac{1}{i\hbar}\,[\boldsymbol{\alpha}, \mathcal{H}] = \frac{2}{i\hbar}(c\mathbf{p} - \mathcal{H}\boldsymbol{\alpha}).$$

This is, of course, a standard result, and the solution, giving the equation of motion for the fermion, was first obtained by Schrödinger:

$$\mathbf{r}(t) = \mathbf{r}(0) + \frac{c^2\mathbf{p}}{\mathcal{H}}\,t + \frac{\hbar c}{2i\mathcal{H}}\,[\boldsymbol{\alpha}(0) - c\mathcal{H}^{-1}\mathbf{p}](exp\,(2i\mathcal{H}t / h) - 1).$$

The third term has no classical analogue, and predicts a violent oscillatory motion or high-frequency vibration of the fermion at $\approx 2mc^2/\hbar$, and amplitude $\hbar/2mc$, directly determined by the particle's rest mass.

Since it is derived from a velocity operator, defined as $c\boldsymbol{\alpha} = -ij\mathbf{1}$, the *zitterbewegung* has always been interpreted as a switching between the fermion's four spin states. It is certainly a vacuum effect. Its representation of the continual re-enactment of fermion creation is the most definite statement of the vacuum's existence. In relation to the nilpotent formalism, it seems to provide a physical mechanism for accommodating the instantaneous spin phase we have defined. (It can also be used to explain the Lamb shift.)

This is, additionally, possible if we imagine an alternative representation of nilpotency as representing a unique direction on a set of axes defined by the values of E, **p** and m. Half of the possibilities on one axis (those with $-m$) would be eliminated automatically (as being in the same direction as those with m), as would all those with zero m (since the directions would all be along the line $E = p$); such hypothetical massless particles would be impossible, in addition, for fermions and antifermions with the same helicity, as E, p has the same direction as $-E, -p$.

The nilpotent structure and the Coulomb interaction

We have seen that multiple physical meanings are encoded within the symbols of the nilpotent condition. Thus

$$(\pm\, ikE \pm \mathbf{i}\mathbf{p} + jm)\,(\pm\, ikE \pm \mathbf{i}\mathbf{p} + jm) \rightarrow 0$$

has at least *five* independent meanings.

classical	special relativity
operator × operator	Klein-Gordon equation
operator × wavefunction	Dirac equation
wavefunction × wavefunction	Pauli exclusion
fermion × vacuum	nonequilibrium thermodynamics

One of the most important aspects of the nilpotent structure (with its pseudoscalar, vector and scalar components) is that it *already incorporates the fundamental interactions.* Simply defining a nilpotent fermion by this mathematical formalism means that it is *necessarily* acting according to some or all of these interactions. They arise solely from its internal structure. Coulomb terms, for example, are simply the result of spherical symmetry of point sources.

To define a fermion, for example, with intrinsic spherical symmetry relative to a point source, we may use polar coordinates for the **p** term, according to the standard prescription previously described, to produce an operator of the form:

$$(\boldsymbol{ikE} - \boldsymbol{ii}\nabla + \boldsymbol{j}m) \rightarrow \left(\boldsymbol{ik}E - \boldsymbol{ii}\left(\frac{\partial}{\partial r} + \frac{1}{r} \pm i\frac{j+½}{r} \right) + \boldsymbol{j}m \right).$$

Now, whatever phase factor we apply this to, we will find that we will not get a nilpotent solution unless the $1/r$ term with coefficient $\boldsymbol{i}$ is matched by a similar $1/r$ term with coefficient $\boldsymbol{k}$. So, simply requiring *spherical symmetry* for a point particle, requires a term of the form A/r to be added to E. Deriving the solution for this case provides a model for all other cases. If all point particles are spherically symmetric sources, then the minimum nilpotent operator will be of the form

$$\left(\pm \boldsymbol{ik}\left(E - \frac{A}{r} \right) \mp \boldsymbol{ii}\left(\frac{\partial}{\partial r} + \frac{1}{r} \pm i\frac{j+½}{r} \right) + \boldsymbol{j}m \right).$$

To establish that this is a nilpotent, we must now find the phase to which this must apply to create a nilpotent amplitude. This is a convenient example for showing how an operator fixes the phase factor and quite quickly produces the characteristic solution for the Coulomb force (hydrogen atom, etc.). The solution is relatively straightforward. The ease of calculation is due to the fact that the structure provides dual information about both fermion and vacuum. We apply the specified operator to the phase factor

$$\phi = e^{-ar_{\nu}^{\gamma}} \sum_{\nu=0} a_\nu r^\nu$$

to find the amplitude (derived, as in the conventional solution, by inspired guesswork or trial and error), and equate the squared amplitude to zero.

$$4\left(E - \frac{A}{r} \right)^2 = -2\left(-a + \frac{\gamma}{r} + \frac{\nu}{r} + \dots \frac{1}{r} + i\frac{j+½}{r} \right)^2$$
$$-2\left(-a + \frac{\gamma}{r} + \frac{\nu}{r} + \dots \frac{1}{r} - i\frac{j+½}{r} \right)^2 + 4m^2.$$

Equating constant terms leads to

$$a = \sqrt{m^2 - E^2}.$$

Equating terms in $1/r^2$, following standard procedure, with $\nu = 0$, we obtain:

$$\left(\frac{A}{r}\right)^2 = -\left(\frac{\gamma+1}{r}\right)^2 + \left(\frac{j+\frac{1}{2}}{r}\right)^2 .$$

Assuming the power series terminates at n', and equating coefficients of $1/r$ for $\nu = n'$,

$$2EA = -2\sqrt{m^2 - E^2}\,(\gamma + 1 + n'),$$

and
$$\frac{E}{m} = \frac{1}{\sqrt{1 + \dfrac{A^2}{(\gamma + 1 + n')^2}}} = \frac{1}{\sqrt{1 + \dfrac{A^2}{\left(\sqrt{(j+\frac{1}{2})^2 - A^2} + n'\right)^2}}} .$$

When $A = Ze^2$ we have the 'hydrogen atom' solution in just 6 lines!

It is particularly significant here that the result emerges only from defining fermions as (discrete) point sources with spherical symmetry. All interactions involving fermions (electric, strong, weak, gravitational) have a Coulomb term (with $U(1)$ symmetry) which emerges in this way, and which turns out to be a purely scalar term, depending on the scalar values of the terms (iE, **p**, m) in the nilpotent operator. This manifests itself in the coupling constants associated with the interactions. It is interesting that Kauffman's discrete differentiation process results in an automatic curvature term with $U(1)$ symmetry.

Bosonic states and the weak interaction

The three quaternion units in the nilpotent operator have multiple, but connected, meanings. One of these is as operators for fundamental symmetry transformations, by pre- and post-multiplication of the nilpotent operator.

P	$\mathbf{i}\,(\mathbf{k}E + \mathbf{i}\mathbf{p} + \mathbf{j}m)\,\mathbf{i} = (\mathbf{k}E - \mathbf{i}\mathbf{p} + \mathbf{j}m)$
T	$\mathbf{k}\,(\mathbf{k}E + \mathbf{i}\mathbf{p} + \mathbf{j}m)\,\mathbf{k} = (-\mathbf{k}E + \mathbf{i}\mathbf{p} + \mathbf{j}m)$
C	$-\mathbf{j}\,(\mathbf{k}E + \mathbf{i}\mathbf{p} + \mathbf{j}m)\,\mathbf{j} = (-\mathbf{k}E - \mathbf{i}\mathbf{p} + \mathbf{j}m)$

It is easy to show that $CPT \equiv$ identity, etc. It is also apparent that C is effectively defined in terms of P and T, rather than being an independent operation, because only space and time are active elements. The variation in space and time is the coded information that solely determines the phase factor and the entire nature of the fermion state. The mass term (which connects with *C*) is a passive element, which can even be excluded from the operator without loss of information. The construction of a nilpotent amplitude effectively requires the loss of a sign degree of freedom in one component, *E*, **p** or *m*, and that the passivity of mass makes it the term to which this will apply.

The terms in the nilpotent 4-spinor, other than the lead term which determines the nature of the 'real' particle state, are effectively, the *P*-, *T*- and *C*-transformed versions of this state, the states into which it could transform without changing the magnitude of its energy or momentum. We can see them as vacuum 'reflections' of the real particle state, arising from vacuum operations that can be mathematically defined. Although a fermion cannot form a combination state with itself, we can imagine it forming a combination state with each of these vacuum 'reflections', and, if the 'reflection' exists or materialises as a 'real' state, then the combined state can form one of the three classes of bosons or boson-like objects.

So the three possible transformations also lead to the production of three types of bosonic state, which, when summed up over 4 terms, yield products which are scalars:

Spin 1 boson:

$$(ikE + \mathbf{i}\mathbf{p} + jm)\,(-\,ikE + \mathbf{i}\mathbf{p} + jm) \qquad T$$

Spin 0 boson:

$$(ikE + \mathbf{i}\mathbf{p} + jm)\,(-\,ikE - \mathbf{i}\mathbf{p} + jm) \qquad C$$

Fermion-fermion – Bose-Einstein condensate / Berry phase, etc.:

$$(ikE + \mathbf{i}\mathbf{p} + jm)\,(ikE - \mathbf{i}\mathbf{p} + jm) \qquad P$$

To demonstrate the scalar nature of the result of one of these combinations, over the 4 terms in the spinor, let us take the fermion-fermion combination:

$$(ikE + \mathbf{i}\mathbf{p} + jm)\,(ikE - \mathbf{i}\mathbf{p} + jm) = (ikE + \mathbf{i}\mathbf{p} + jm)\,(-2\mathbf{i}\mathbf{p})$$
$$(ikE - \mathbf{i}\mathbf{p} + jm)\,(ikE + \mathbf{i}\mathbf{p} + jm) = (ikE - \mathbf{i}\mathbf{p} + jm)\,(2\mathbf{i}\mathbf{p})$$
$$(-\,ikE + \mathbf{i}\mathbf{p} + jm)\,(-\,ikE - \mathbf{i}\mathbf{p} + jm) = (-\,ikE + \mathbf{i}\mathbf{p} + jm)\,(-2\mathbf{i}\mathbf{p})$$
$$(-\,ikE - \mathbf{i}\mathbf{p} + jm)\,(-\,ikE + \mathbf{i}\mathbf{p} + jm) = (-\,ikE - \mathbf{i}\mathbf{p} + jm)\,(2\mathbf{i}\mathbf{p})$$

Clearly, the terms on the RHS in ikE and jm total to 0, leaving only $4 \times (-\boldsymbol{i}\mathbf{p})(2\boldsymbol{i}\mathbf{p}) = 8p^2$, which is a scalar (normalizable to 1).

A spin 1 boson can be massless because

$$(ikE + \boldsymbol{i}\mathbf{p})\,(-\,ikE + \boldsymbol{i}\mathbf{p}) \neq 0$$

but a spin 0 boson (e.g. Goldstone / Higgs) cannot because

$$(ikE + \boldsymbol{i}\mathbf{p})\,(-\,ikE - \boldsymbol{i}\mathbf{p}) = 0$$

This result also shows that fermion and antifermion cannot have the same handedness. However, the fermion-fermion state can, again, be effectively massless, as in Cooper pairs, because

$$(ikE + \boldsymbol{i}\mathbf{p})\,(ikE - \boldsymbol{i}\mathbf{p}) \neq 0$$

The weak interaction can be considered as one in which fermions and antifermions are annihilated while bosons are created, or bosons are annihilated while fermions and antifermions are created. This is the action of a harmonic oscillator. Bosons, considered as created at fermion-antifermion vertices, are the products of weak interactions, or actions with a weak amplitude. Fundamentally, the sources of weak interactions are always at least dipolar, and so, in addition to the inverse linear (Coulomb) potential required for the scalar aspect, there is always a dipole or multipole potential (say, $\propto 1 / r^{-n}$, where $n \geq 2$). Such a combined potential (of virtually any algebraic form), when applied to a nilpotent operator, produces the energy levels of a harmonic oscillator, exactly as required. The interaction also has an $SU(2)$ symmetry, which seems, ultimately, to be related to the ± sign ambiguity attributable to the pseudoscalar term iE, as is the *zitterbewegung*, which can only proceed through a switch in the helicity state.

If we consider the nilpotent-nilpotent structures as defining the vertices for boson production via the weak interaction, then it appears that the pure weak interaction requires left-handed fermions and right-handed antifermions. In other words, it requires both a C violation and a simultaneous P or T violation. Since *zitterbewegung* is a transition that produces intermediate bosons, and is fundamental to the existence of the fermionic state, then a fermion, by its *very existence as a 4-spinor*, is always acting weakly, even if only with vacuum. In effect, the *zitterbewgung* ensures that a fermion is always a weak dipole in relation to its vacuum states, and the single-handedness of the weak interaction can be regarded as the result of a weak dipole moment connected with fermionic ½-integral spin.

Baryonic structure and the strong interaction

We have seen that Coulomb and weak interactions are fundamental to the structure of the nilpotent fermionic state, as (a) point-like spherically symmetric and (b) constructed as a 4-spinor with *zitterbewegung*. The first comes from the scalar structure within the nilpotent, the second from the pseudoscalar aspect involved with the iE term. (The electric term, which is scalar only, is closely associated with the scalar only term m.) A final interaction type, again from the *nilpotent's internal structure*, comes from the vector nature of the **p** term.

If we are to take the vector nature of **p** seriously, there must be some meaning to it having 3-components. But it is not obvious that a 3-component state vector will work, since

$$(ikE + \boldsymbol{i}\mathbf{p} + jm)\,(ikE + \boldsymbol{i}\mathbf{p} + jm)\,(ikE + \boldsymbol{i}\mathbf{p} + jm) = 0$$

However, we can write down terms of the form:

$$(ikE + \boldsymbol{i}\mathbf{p} + jm)\,(ikE + jm)\,(ikE + jm) \rightarrow (ikE + \boldsymbol{i}\mathbf{p} + jm)$$
$$(ikE + jm)\,(ikE + \boldsymbol{i}\mathbf{p} + jm)\,(ikE + jm) \rightarrow (ikE - \boldsymbol{i}\mathbf{p} + jm)$$
$$(ikE + jm)\,(ikE + jm)\,(ikE + \boldsymbol{i}\mathbf{p} + jm) \rightarrow (ikE + \boldsymbol{i}\mathbf{p} + jm)$$

It is then possible to have a nonzero 3-component state vector if we use the vector properties of **p** and the arbitrary nature of its sign (+ or –).

A state vector of the form,

$$(ikE \pm \boldsymbol{i}\,\mathrm{i}p_x + \boldsymbol{j}\,m)\,(ikE \pm \boldsymbol{i}\,\mathrm{j}p_y + \boldsymbol{j}\,m)\,(ikE \pm \boldsymbol{i}\,\mathrm{k}p_z + \boldsymbol{j}\,m)$$

privileging the **p** components, has six independent allowed phases, i.e. when

$$\mathbf{p} = \pm\,\mathbf{i}\,p_x\,,\ \mathbf{p} = \pm\,\mathbf{j}\,p_y\,,\ \mathbf{p} = \pm\,\mathbf{k}\,p_z$$

But these must be *gauge invariant*, i.e. indistinguishable, or all present at once. Also, we must interpret the E, **p**, m symbols as belonging to a totally entangled state, rather than the subcomponents.

The 6 possible phases are related by an $SU(3)$ symmetry, with 8 generators, exactly like that attributed to coloured 'quarks':

$$
\begin{array}{ll}
(ikE + \boldsymbol{i}\,\mathrm{i}p_x + \boldsymbol{j}\,m)\,(ikE + \;...\; + \boldsymbol{j}\,m)\,(ikE + \;...\; + \boldsymbol{j}\,m) & +RGB \\
(ikE - \boldsymbol{i}\,\mathrm{i}p_x + \boldsymbol{j}\,m)\,(ikE - \;...\; + \boldsymbol{j}\,m)\,(ikE - \;...\; + \boldsymbol{j}\,m) & -RBG \\
(ikE + \;...\; + \boldsymbol{j}\,m)\,(ikE + \boldsymbol{i}\,\mathrm{j}p_y + \boldsymbol{j}\,m)\,(ikE + \;...\; + \boldsymbol{j}\,m) & +BRG \\
(ikE - \;...\; + \boldsymbol{j}\,m)\,(ikE - \boldsymbol{i}\,\mathrm{j}p_y + \boldsymbol{j}\,m)\,(ikE - \;...\; + \boldsymbol{j}\,m) & -GRB \\
(ikE + \;...\; + \boldsymbol{j}\,m)\,(ikE + \;...\; + \boldsymbol{j}\,m)\,(ikE + \boldsymbol{i}\,\mathrm{k}p_z + \boldsymbol{j}\,m) & +GBR \\
(ikE - \;...\; + \boldsymbol{j}\,m)\,(ikE - \;...\; + \boldsymbol{j}\,m)\,(ikE - \boldsymbol{i}\,\mathrm{k}p_z + \boldsymbol{j}\,m) & -BGR
\end{array}
$$

The duality of the $\pm$ **p** or helicity states in these structures is a clear indication that the composite state described does not have zero mass.

The mediators of the transitions between the six component states will be six bosons of the form:

$$(ikE - \boldsymbol{i}\mathrm{i}p_x)\,(-ikE - \boldsymbol{i}\mathrm{j}p_y)$$

and two combinations of the three bosons of the form:

$$(ikE - \boldsymbol{i}\mathrm{i}p_x)\,(-ikE - \boldsymbol{i}\mathrm{i}p_x)$$

These structures are, of course, identical to an equivalent set in which both brackets undergo a complete sign reversal.

This $SU(3)$ symmetry or strong interaction is entirely nonlocal. That is, the exchange of momentum **p** involved is entirely independent of any spatial position of the 3 components of the baryon. We can suppose that the rate of change of momentum (or 'force') is constant with respect to spatial positioning or separation. A constant force is equivalent to a potential which is linear with distance, exactly as is required for the conventional strong interaction. Application to the nilpotent of the strong linear potential together with the Coulomb potential for the scalar aspect gives analytic solutions which have the well-known strong interaction characteristics of infrared slavery and asymptotic freedom.

Thus all the well-known interaction types involved with fermions can be seen to be characteristic consequences of aspects of their nilpotent structure alone. It is significant that the linear potential of the strong interaction is the only one that is optional, the nilpotency not being dependent directly on the vector nature of **p**. (Though electric charge is 'optional' in fermionic states, because it represents a scalar term *only*, there is still a scalar term and a Coulomb potential provided by the other interactions, even in the absence of electric charge. Strong charge is optional because nilpotency doesn't depend on p being a vector. Weak charge is the only one that is necessarily present in all fermionic states.)

In principle, the interactions intrinsic to a fermion are a product of the three types of quantity (pseudoscalar, multivariate vector and scalar) which the nilpotent representation contains. Ultimately, and more subtly, these are reflections of the need for a discrete (point) source to preserve spherical symmetry and hence to conserve angular momentum. We can, in fact, identify the 'interactions' and their associated symmetries as being connected with the three separately conserved aspects of angular momentum: magnitude (scalar, $U(1)$, spherical symmetry does not depend on the length of the radius vector); direction (vector, $SU(3)$, spherical symmetry does not depend on the choice of axes); and handedness (pseudoscalar, $SU(2)$, spherical symmetry does not depend on whether the rotation is left- or right-handed).

Partitioning the vacuum

We have seen that it is possible to define the entire structure of quantum mechanics by defining the creation operator for a single fermion ($\pm ikE \pm \boldsymbol{i}\mathbf{p} + jm$) as a nilpotent. There are four creation (or annihilation) operators here (or two of each):

$(ikE + \boldsymbol{i}\mathbf{p} + jm)$	fermion spin up
$(ikE - \boldsymbol{i}\mathbf{p} + jm)$	fermion spin down
$(-ikE + \boldsymbol{i}\mathbf{p} + jm)$	antifermion spin down
$(-ikE - \boldsymbol{i}\mathbf{p} + jm)$	antifermion spin up

But only the first term defines the real fermionic state. The others are vacuum 'reflections', or the states into which it could transform If we regard the four terms as operators, whose mass terms are passive (and eliminate when we use discrete differentials), the sum total of 'real' fermion plus vacuum reflections is zero, just as we would expect.

There is another way to look at this. If we take ($ikE + \boldsymbol{i}\mathbf{p} + jm$) and postmultiply it by

$$\boldsymbol{k}(ikE + \boldsymbol{i}\mathbf{p} + jm)$$

or

$$\boldsymbol{i}(ikE + \boldsymbol{i}\mathbf{p} + jm)$$

or

$$\boldsymbol{j}(ikE + \boldsymbol{i}\mathbf{p} + jm)$$

the result is $(ikE + i\mathbf{p} + jm)$, multiplied by a scalar. This can be done an indefinite number of times.

So these three *idempotent* terms behave as a vacuum operators. We can also see the three vacuum coefficients k, i, j as originating in (or being responsible for) the concept of discrete (point-like) charge. In effect, the operators, k, i and j perform the functions of weak, strong and electric charges, acting to partition the *continuous* (gravitational) vacuum represented by $-(ikE + i\mathbf{p} + jm)$, and responsible for zero-point energy, into discrete components, whose special characteristics are determined by the respective pseudoscalar, vector and scalar natures of their associated terms iE, $\mathbf{p}$ and m.

In this sense, they are related to 'real' weak, strong and electric localized charges, though they are delocalized. We can describe the partitions as strong, weak and electric 'vacua', and assign to them particular roles within existing physics:

$k\,(ikE + i\mathbf{p} + jm)$	weak vacuum	fermion creation
$i\,(ikE + i\mathbf{p} + jm)$	strong vacuum	gluon plasma
$j\,(ikE + i\mathbf{p} + jm)$	electric vacuum	isospin / hypercharge

(The electric vacuum – empty or filled – can be seen as responsible for the transition between weak isospin up and down states.)

We can see how the 3 bosonic states are related to vacua produced by the 3 quaternionic operators:

weak   *spin 1*

$$(ikE + i\mathbf{p} + jm)\, k\, (ikE + i\mathbf{p} + jm)\, k\, (ikE + i\mathbf{p} + jm)\, k\, (ikE + i\mathbf{p} + jm) \ldots$$
$$(ikE + i\mathbf{p} + jm)\, (-ikE + i\mathbf{p} + jm)\, (ikE + i\mathbf{p} + jm)\, (-ikE + i\mathbf{p} + jm) \ldots$$

electric   *spin 0*

$$(ikE + i\mathbf{p} + jm)\, j\, (ikE + i\mathbf{p} + jm)\, j\, (ikE + i\mathbf{p} + jm)\, j\, (ikE + i\mathbf{p} + jm) \ldots$$
$$(ikE + i\mathbf{p} + jm)\, (-ikE - i\mathbf{p} + jm)\, (ikE + i\mathbf{p} + jm)\, (-ikE - i\mathbf{p} + jm) \ldots$$

strong   *paired fermion state*

$$(ikE + i\mathbf{p} + jm)\, i\, (ikE + i\mathbf{p} + jm)\, i\, (ikE + i\mathbf{p} + jm)\, i\, (ikE + i\mathbf{p} + jm) \ldots$$
$$(ikE + i\mathbf{p} + jm)\, (ikE - i\mathbf{p} + jm)\, (ikE + i\mathbf{p} + jm)\, (ikE - i\mathbf{p} + jm) \ldots$$

The nilpotent structure of the lead term combined with the idempotent structure of the vacuum operator ensures that all real fermions have exactly supersymmetric boson partners (with the same E, $\mathbf{p}$, m) and vice versa. Ultimately, this means that we don't need renormalization (as we can show mathematically) or extra supersymmetric particles; and there is no hierarchy problem. The fermion and boson loops should cancel automatically.

The total vacuum $-1(\pm\, \boldsymbol{i}\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$, which is partitioned by the $\boldsymbol{k}$, $\boldsymbol{i}$, $\boldsymbol{j}$ operators, can be thought of as the continuous gravitational vacuum (with negative energy), which supplies the mechanism for the instantaneous transmission of quantum correlation, and which ensures that the nilpotent mechanism ensures the operation of both quantum holography, where E and m become the phase and reference phase, and the holographic principle, where the E and $\mathbf{p}$ terms create the effective 'bounding area' (Rowlands, 2007). The holographic aspects are dual to the nilpotent aspects, and can be observed directly by reversing the roles of vectors (connected to space) and quaternions (connected to charge) in the nilpotent structure. The holographic information will then determine the nature of the system, including connected information about its inertial mass and charge structure. For example, just as the electric charge determines the inertia of the electron, via the holographic principle (Rowlands, 2007), so the strong charge determines the inertia of the bare quarks at about 3 to 6 MeV, and the weak charge seemingly determines the inertia of the neutrinos at something like 0.13 eV.

The nilpotent operator $(\pm\, \boldsymbol{i}\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ can, finally, be regarded as a 10-D object (embedded in Hilbert space): 5 for iE, $\mathbf{p}$, m and 5 for $\boldsymbol{k}$, $\boldsymbol{i}$, $\boldsymbol{j}$; and six of these (all but iE and $\mathbf{p}$) are compactified. A classic prescription for a perfect string theory is one in which 'self-duality in phase space determines vacuum selection'. The nilpotent certainly fulfils this criterion and it is also a mass-shell system and incorporates the right groups. Though we have no need for a model-dependent theory to incorporate the interactions, it is important to be able to satisfy all the conditions that appeared to make string theory, *or a more fundamental abstract theory, of which the model-dependent theories are approximations*, seemingly necessary. It is significant that the nilpotent formalism achieves this through solving the problem of vacuum.

Conclusion

Vacuum has been used in quantum field theory without a complete understanding of what it is, why it is necessary, and how it should be described mathematically. Answering these questions gives us major leads into many significant aspects of quantum mechanics and particle physics (Rowlands, 2007). But the structure it reveals is also a generic one, not confined to fundamental physics, and has applications to all systems governed by holistic principles.

REFERENCES

Dirac, P. A. M., 1958 *The Principles of Quantum Mechanics*, fourth edition. Clarendon Press.

Feynman, R. P., 1972, The development of the space-time view of quantum electrodynamics, in *Nobel Lectures: Physics 1963-70*. Elsevier Publishing Company, Amsterdam.

Hestenes, D., 1966, *Space-Time Algebra*. Gordon and Breach, New York.

Kauffman, L. H., 2004, Non-commutative worlds, *New Journal of Physics*., **6**, 2-46.

Rowlands, P., 2007, *Zero to Infinity*, World Scientific, Singapore.

How General is Nilpotency?

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Abstract. Evidence is presented for the generality, criticality and importance of nilpotence and the associated criteria of Pauli exclusion, quantum phase factor and quantum holographic signal processing in relation to calculation, problem solving and optimum control of Heisenberg uncertainty in Quantum Interaction.

Nilpotent quantum mechanics

The nilpotent formulation of relativistic quantum mechanics / quantum field theory (Rowlands, 2007, 2008) is exceptionally compact and minimalistic, and for that reason particularly powerful. It is also unique in generating significant physical interpretations, not available to more restricted formalisms. In the nilpotent formalism, the only requirement for defining the entire quantum mechanical apparatus relating to a fermion and its interactions is to specify its creation operator in the form (ikE + i**p** + jm), where k, i, j are quaternion units and **p** takes the form of a multivariate or quaternion-like vector. Wavefunction, phase factor, amplitude, spinor structure, vacuum, and quantum mechanical equation are then automatic consequences of the initial definition and do not represent independent information.

While the operators E and **p** typically represent the derivatives $i\partial / \partial t$ and $-i\nabla$, they can also be supposed to incorporate field terms or be covariant derivatives, so that E could be, say, $i\partial / \partial t + e\phi + \ldots$, and **p** could be, say, $-i\nabla + e\mathbf{A} + \ldots$, with any number of field terms (whether electric, strong, weak or gravitational) added. The spinor structure is automatically provided by the four possible sign variations of E and **p**, so that the full specification of the operator becomes, in abbreviated form, ($\pm$ ikE $\pm$ i**p** + jm), with the sign variations representing the terms in a 4-component row vector. Once specified, this operator must determine a *unique* phase factor on which it acts to produce an amplitude or wavefunction which is nilpotent or squares to zero, and the amplitude must be a 4-component column vector, again of the form ($\pm$ ikE $\pm$ i**p** + jm), but this time multiplied by the phase factor, with the E and **p** representing eigenvalues (though not necessarily constant ones) rather than derivatives. The zeroing of this object by squaring then becomes the only quantum mechanical equation needed.

Physically, the nilpotency or squaring to zero may be interpreted as Pauli exclusion, as the combination state of two identical fermions cannot exist, but the operation also indicates the reason for Pauli exclusion, as being the need to maintain a zero totality universe. In effect, in creating a fermion from absolutely nothing in the form $(\pm ikE \pm i\mathbf{p} + jm)$, we also create the *vacuum* or rest of the universe to which this fermion relates in the form $-(\pm ikE \pm i\mathbf{p} + jm)$, and the combination state of fermion and vacuum or rest of the universe must be precisely $-(\pm ikE \pm i\mathbf{p} + jm)(\pm ikE \pm i\mathbf{p} + jm)$, or exactly zero, in the same way as the superposition. This is true whether the fermion is free or subject to any number of interaction potentials; in the latter case, vacuum or the 'rest of the universe' will be 'constructed' in such a way that the existence of a fermion in that state becomes possible. In this sense, every fermion contributes to the rest of the universe seen by every other fermion, and the 'universe' is defined by the state which makes all the nilpotent conditions possible simultaneously.

A nilpotent formulation of the fermion state means that the fermion can only be defined with respect to the entire universe, or, in another way of describing it, with respect to the entire quantum field. When we define energy conservation by writing $(\pm ikE \pm i\mathbf{p} + jm)(\pm ikE \pm i\mathbf{p} + jm) = 0$, we are saying that a closed system involving a fermion is impossible, that the conservation principle applies only over the whole universe, though, in being intrinsically relativistic, it is also local. Thermodynamics, as constructed in the first two laws, becomes a necessary consequence; the fermion is a fundamentally dissipative system, and this is required by its nilpotent structure.

Local conservation over the whole universe, which is effectively defined for each fermion from within the bracket $(\pm ikE \pm i\mathbf{p} + jm)$, is an important principle. However, the Pauli exclusion aspect of nilpotency, which is defined from *outside* the bracket, is equally significant, and is nonlocal. Here, every fermion must be constructed in such a way that its bracketed $(\pm ikE \pm i\mathbf{p} + jm)$ is different from any other, so that their product state is nonzero, and this must happen instantaneously, as there is no localization condition, as there is inside the bracket. So the state of every fermion in the universe must be subject to two universal conditions, one local and one nonlocal.

Conventionally, of course, Pauli exclusion between ψ_1 and ψ_2 is defined in terms of the antisymmetric wavefunction:

$$\psi_1\psi_2 - \psi_2\psi_1 = -(\psi_2\psi_1 - \psi_1\psi_2).$$

Applying the nilpotent condition to this leads to a truly remarkable result:

$$(\pm\, \boldsymbol{ik}E_1 \pm \boldsymbol{i}\mathbf{p}_1 + \boldsymbol{j}m_1)\,(\pm\, \boldsymbol{ik}E_2 \pm \boldsymbol{i}\mathbf{p}_2 + \boldsymbol{j}m_2)$$
$$-\,(\pm\, \boldsymbol{ik}E_2 \pm \boldsymbol{i}\mathbf{p}_2 + \boldsymbol{j}m_2)\,(\pm\, \boldsymbol{ik}E_1 \pm \boldsymbol{i}\mathbf{p}_1 + \boldsymbol{j}m_1)$$
$$= 4\mathbf{p}_1\mathbf{p}_2 - 4\mathbf{p}_2\mathbf{p}_1 = 8\, i\, \mathbf{p}_1 \times \mathbf{p}_2.$$

This result immediately implies that only the instantaneous direction of **p** ensures that the two nilpotent wavefunctions remain distinct, and that all the significant information about the state is contained in this term. Significantly, decoherence in combinations of more than one state is ultimately due to the vector nature of the **p** term.

Ultimately, the key significance of the nilpotent fermion state ($\pm\, \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m$) is that it necessarily constructs its vacuum, environment, or 'rest of the universe, as its own reversed image, $-(\pm\, \boldsymbol{ik}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$. The quantum mechanics, and the logic to which it relates, is fundamentally holistic. We have a simultaneous description, at all times, of the system and its environment. In addition, because we have derived it from a more fundamental information processing system, privileging zero totality, it is probable that that information processing system has applications beyond the narrow confines of quantum mechanics.

The nilpotent supersymmetry

Nilpotency is a particularly powerful idea because it introduces a natural holism into the definition of any conservative system. Clearly, some kind of holistic principle is necessary to make sense of the way that nature behaves, in structuring its operations as though all events had a universal and nonlocally determined unique birthordering. Richard Feynman once wrote: 'It always bothers me that, according to the laws as we understand them today, it takes a computing machine an infinite number of logical operations to figure out what goes on in no matter how tiny a region of space, and no matter how tiny a region of time. How can all that be going on in that tiny space? Why should it take an infinite amount of logic to figure out what one tiny piece of spacetime is going to do?' (Feynman, 1965) In fact, neither the brain nor any other natural system works like this. There is always a holistic process.

Nilpotent logic rather than digital logic reflects this by making the universal automatically the mirror image of the particular because the universe is constrained to have zero totality. This clearly operates in the case of quantum mechanics. The question that then emerges is how much can any system (e.g. life, consciousness, galactic formation, chemistry), which has a strong degree of self-organization manage to achieve this by being modeled on a nilpotent structure. The work of Hill and Rowlands (2007), and Marcer and Rowlands (2007), suggests that this is possible in a wide variety of contexts. The reason is that the nilpotency does not stem from quantum mechanics initially, but from fundamental conditions of optimal information processing which are prior to physics, chemistry and biology, and even to mathematics.

The work of Vitiello (2008) suggests a striking confirmation outside the authors' own work (Marcer and Rowlands, 2007). Vitiello has developed a quantum field model of the brain in which dissipation doubles the degrees of freedom of the system because the environment acts like the system's time-reversed mirror image. Essentially energy $-E$ dissipated by the system is balanced by energy E absorbed by the environment, and the two are structurally identical, though opposite in sign. This is an almost exact description of what happens in a nilpotent system. He also says that: 'Consciousness appears to be rooted in the permanent dialog of the subject with its Double.' Nilpotency provides a mechanism by which this can be achieved.

Nilpotence is the unique criterion of the universal computational rewrite system (NUCRS) set out in *Zero to Infinity* (Rowlands, 2007). It specifies a novel evolutionary worldview in terms of a staircase of matter of increasing complexity. This nilpotent staircase, obtained from the spontaneous symmetry breaking of the empty state taken as the worldview's initial nilpotent boundary condition (essential to the proper solution of any physical problem), begins by predicting the quantizations of Standard Model elementary particle matter as experimentally validated. It describes these (particles) as the sources and sinks of the 3 + 1 relativistic space time field, in such a way that the field and its sources and sinks operationally constitute the quantum computational 'machine order code' for all further (universal rewrite) computation corresponding to the NUCRS infinite universal alphabet and grammar. A field, Rowlands (2008, 2009) shows, concerns relativistic quantum mechanics derived from a single operator.

In principle, therefore, this nilpotent computational worldview is able to rewrite physical law at each level or stair in terms of the laws at the previous levels beginning from that of the initial 'machine order code' law. For example. it has already been shown (Hill and Rowlands, 2008) that the DNA / RNA genetic code is almost certainly a complete nilpotent rewrite of the NUCRS of the Standard Model of the elementary particles, rewritten at a molecular level, where the nilpotent worldview predicts the molecular biological structures and their molecular biology as they are actually known. And it generalizes molecular biology because the genetic code is now not only a unique, universal computational code but also a semantic one, as the NUCRS shows, of in principle the relativistic 3D geometry of its organisms in quantum holographic encoded form.

The symmetries of 3D space as a central feature of the NUCRS nilpotent QM machine order code and grammar

Consideration of the set of radial vectors to the surface of a unit sphere centre P, as defining each neighbourhood in 3D space, shows that any such vector through the centre *P* of the sphere and the plane perpendicular to it, are unique and correspond to the symmetries, *U*(1), *SU*(2), in relation to the symmetry of the sphere *SU*(3). Each *P* thus defines a ray space (Fig. 1), where the fundamental quantum mechanical spectral theorem of Hilbert and Von Neumann applies, such that *U*(1), *SU*(2) and *SU*(3) are respectively the symmetry groups, now known to determine the quantizations of the elementary particles of the electromagnetic, weak and strong forces, and *U*(1) × *SU*(2) is the symmetry of the electroweak force (the photon with its vector bosons) combined in just the right way. Thus (subject to the proviso that every *P* is indeed a Lie topological neighbourhood, where the symmetries are those of Lie groups together with their Lie algebras, as defined by their smooth tangent spaces of lines and planes) the NUCRS grammatical sub-alphabet of *P*, which must differentiate lexicographically between the three axes of space in order to describe three distinct orthogonal spatial axes, may be labelled 'electron', 'muon' and 'tau', in good accord with the Pauli exclusion principle for spin ½ (fermion) anti-commutative states, which applies to each and every unique unit vector and its two perpendicular plane axes at *P*.

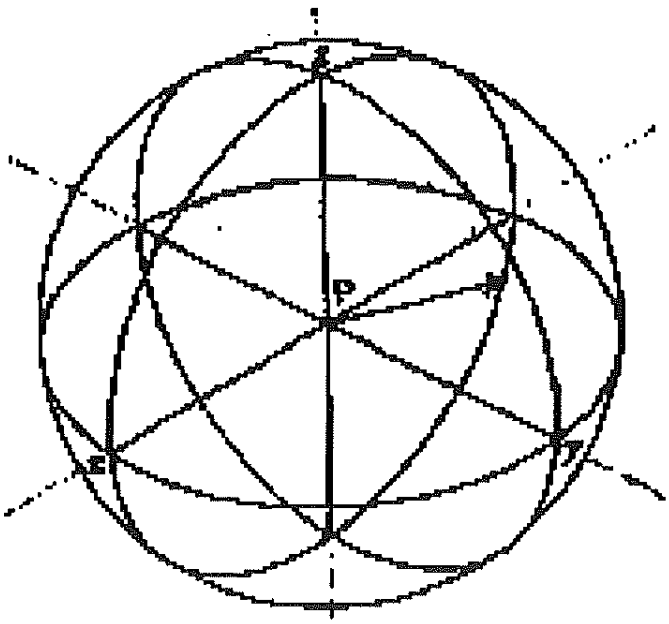

Fig. 1. 3D ray space at point P

In other words, in the NUCRS, where the elementary particles predicted are exclusively those of the Standard Model, and 3D space corresponds, as described above, to a quantum field, the particles with spin ½, known as the neutrinos take on, in a new testable hypothesis of their behaviour, an unexplored role in respect to the axes of 3D space. And in respect of this '3D spatial quantum field', there exists a Lie symmetry, that of the 3D Heisenberg Lie Group G, which in the form of its nilpotent Lie algebra g, as was known to Weyl in 1928, defines the Heisenberg uncertainty, as a Robertson relation,

$$\Delta U_\nu(P).\Delta U_\nu(Q) \geq \tfrac{1}{2}\,|U_\nu(Z)|$$

expressed in terms of standard root mean square deviations of the operators $\Delta U_\nu(P)$ and $\Delta U_\nu(Q)$, where $\{P, Q, Z\}$, defined below, are the canonical basis of g, and U_ν frequency ν, are, up to a unitary isomorphism, unique infinite dimensional irreducible unitary linear representations of G of the Schrodinger type in the standard Hilbert space $L^2(R)$ (Schempp, 1992). However as both g and G have Lie duals, there exist corresponding (Lie) exponential differentiable mappings with differentiable inverses, so that 'Heisenberg uncertainty' can, in this case, be used as the actual means to compute geometrically. As, for example, the description of $U(1, C)$ signal processing in the form of quantum holographic encoding / decoding in Magnetic Resonance Imaging (MRI) machines proves (Schempp, 1998).

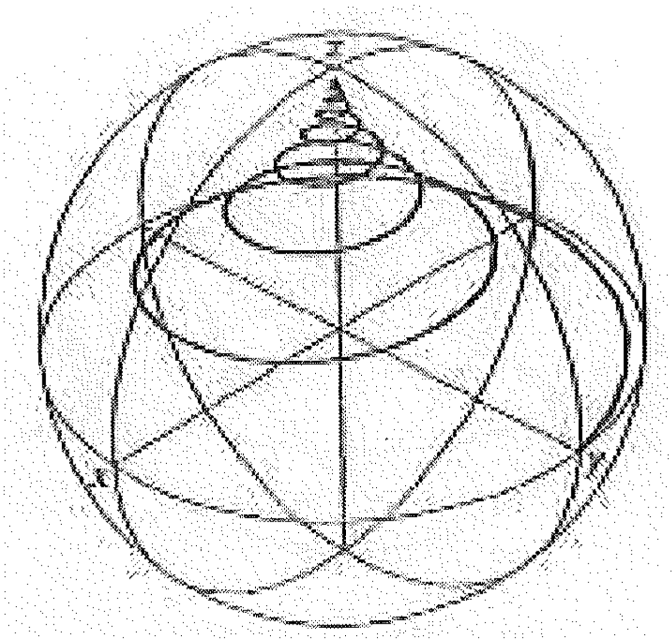

Fig. 2. Quantum wave collapse and re-expansion

Fig. 2 illustrates how in actual 3D space, the encoding / decoding Fourier transform action (in accord with the Heisenberg uncertainty principle defined by g the Lie algebra of G) actual happens in MRI. It shows the 'frequency induced signal' $U(1,C)$ described by the Heisenberg helix of G off resonance losing amplitude (z axis), i.e. thermodynamically decaying due to a transverse relaxation effect, but, remarkably, simultaneously regaining energy due to longitudinal relaxation, so as to embed the $U(1)$ signal in the complex plane $C = (x + iy)$, as a phase difference. For, with respect to G and g, G represented as the matrix

$$\begin{pmatrix} 1 & x & z \\ 0 & 1 & y \\ 0 & 0 & 1 \end{pmatrix}$$

is such that (x – path difference, y – phase difference) form a Fourier duality encoding pair $(x, y) = (x + iy)$ so embedding the complex plane C in G, and the infinitesimal Lie generators $\{P, Q, Z\}$ of g, where $(P, Q) = Z$; $(P, Z) = 0$; $(Q, Z) = 0$, are represented by the (3 line) matrices

$$P=\begin{pmatrix}0&1&0\\0&0&0\\0&0&0\end{pmatrix};Q=\begin{pmatrix}0&0&0\\0&1&0\\0&0&0\end{pmatrix};Z=\begin{pmatrix}0&0&1\\0&0&0\\0&0&0\end{pmatrix}$$

having Lie exponential diffeomorphisms

$$\exp P=\begin{pmatrix}1&1&0\\0&1&0\\0&0&1\end{pmatrix};\exp Q=\begin{pmatrix}1&0&0\\1&1&0\\0&0&1\end{pmatrix};\exp Z=\begin{pmatrix}1&0&1\\0&1&0\\0&0&1\end{pmatrix}$$

are the components of the automorphism of G, able to describe the inverse Fourier transform action of decoding, i.e. they give rise to the identity $\exp(xP + yQ + (z - ½xy)Z) = G$, where the conformal complex mode coordinates $T = ½(P + iQ)$ and $T^* = ½(P - iQ)$ permit through the linear Schrodinger representation U of G, a quantum mechanical description at the photon level in terms of the creation / annihilation operators $a = U(T)$ and $a^* = U(T^*)$ of an emitter/absorber model. Thus, for example, optimal control of quantum interactions in real time becomes possible as already experimental demonstrated in chemical reactions, equivalent to the solution of the Schrodinger equation for the chemistry (Dahleh et al, 1990, Judson and Rablitz, 1992, Rice, 1992).

Support for nilpotence from the Standard Model

The evidence from MRI presented in diagram B as specified above therefore explains the dichotomy between the facts of Heisenberg uncertainty and that quantum mechanical theory allows no loss of quantum mechanical information as is in principle the case in the quantum holographic process. at the nilpotent point where the collapse of the wave function, its annihilation turns into a re-expansion, a creation. However Diaz and Rowlands (2006), have shown that the NUCRS infinite alphabet determined by the nilpotents $X_n^2 = 0$, $X_n \neq 0$ corresponds to the infinite square roots of -1, $n = 1, 2, \ldots$. This is in line with Berry's hypothesis (1986), that the imaginary parts of the non trivial zeros of the Riemann zeta function are the eigenvalues of some still unknown self adjoint Hamiltonian operator with time reversal asymmetry (a property of the NUCRS) of which phase space trajectories are chaotic.

It is in agreement with the fact the each letter of the NUCRS infinite alphabet has the same 'nilpotent' formulation $X_n^2 = 0$ at all levels of its rewrite structure and so is self similar of fractal dimension 2 and therefore embeddable in the complex plane *C*; i.e. can be mapped in a conformal fashion onto the open unit disc (which is the geometry of $i = \sqrt{-1}$). It thus corresponds to the universal fractal attractor of the Golden number (Fig. 3) and relates to the wave behaviour seen at the boundary of the Mandelbrot set.

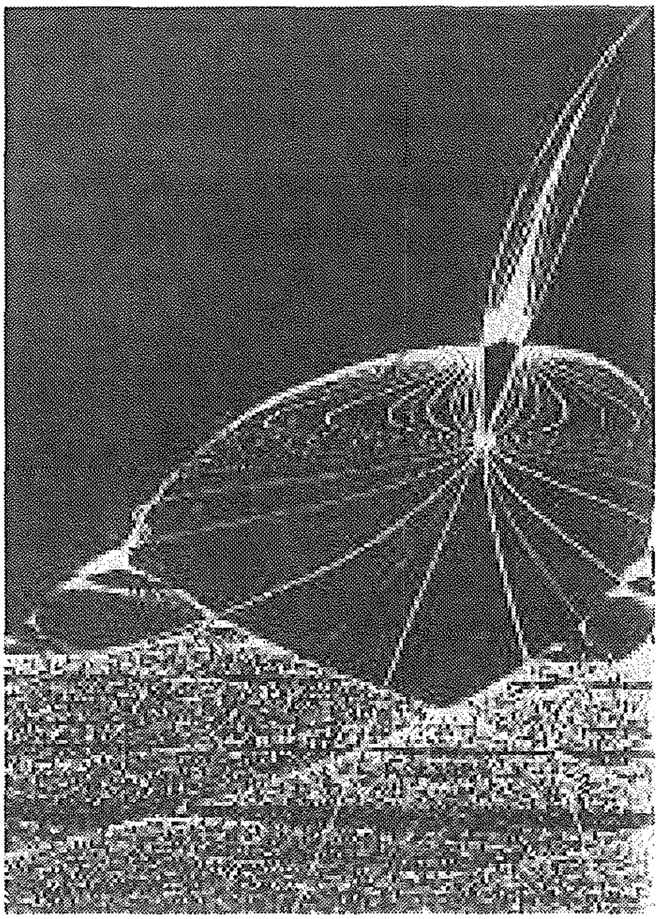

Fig. 3. The universal Golden attractor

The harmonic analysis on the 3D Heisenberg Lie group *G*, discovered and applied by Schempp (1986) to the description and control of MRI and Synthetic Aperture Radars, proves is indeed be the case. So also do the field symmetry $U(1, C)$ in terms of the polarization wave property of electromagnetic signals in relation, for example, to the quantum mechanical Aharonov Bohm effect; the helices of RNA / DNA (Hill and Rowlands, 2008) and the representation the Riemann sphere of the complex plane by stereographic projection, where the pole is the point at infinity called in perspective the 'vanishing point', which in art is a very late discovery of the nature of human perception (Fig. 4).

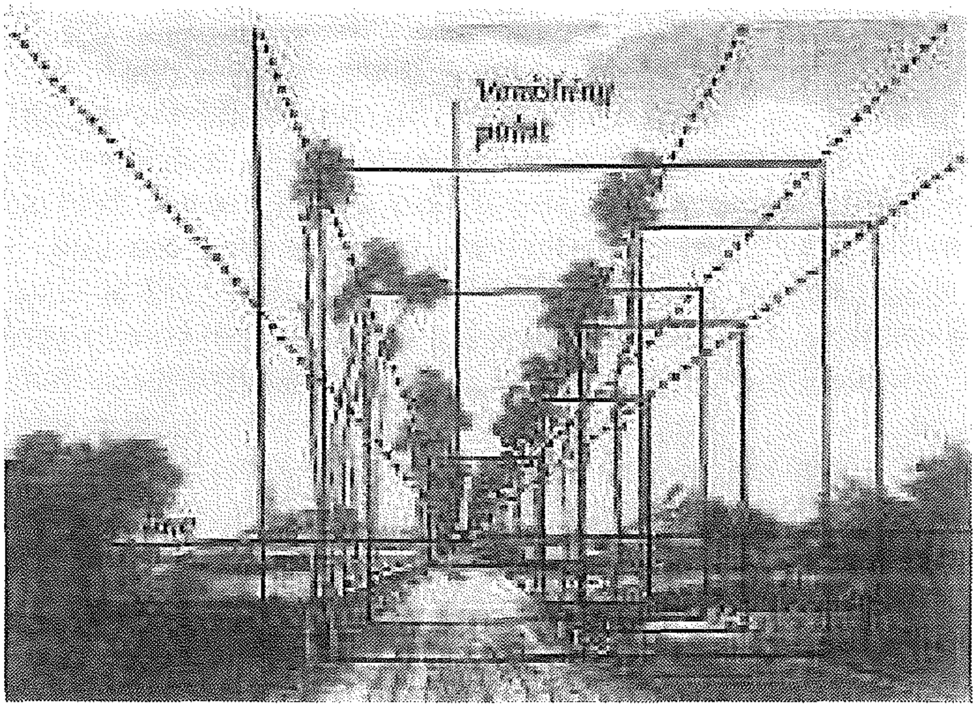

Fig. 4. Perspective with vanishing point / pole

In further support of this NUCRS hypothesis, Trell (2008) uses the 'cubit' symmetry structure of the eight cubes about the point P of the neighbourhood with its eightfold way cubit metric scaling of signature +++, –++, +–+, ++–, +––, –+–, ––+, ––– to calculate the relative masses of all the families of the elementary particles, in good accord with their experimentally determined values, and their quantizations as previously calculated by Rowlands. Mathematically these properties are, of course, those to be expected from Lie transformational theory, where bi-linearisation takes place via the calculation of invariants. A fact that would be explained if these invariants are all properties of 3D universal standing wave or a soliton.

The very highly detailed paper by Hill and Rowlands (2008) in relation to DNA / RNA biological systems and their geometric structures, Nature's Code fully underpins and extends that from the earlier complementary perspective of the nilpotent Heisenberg Lie group *G* (Marcer and Schempp, 1986, Gariaev et al, 2001, 2002), which predict, for example, the structure of DNA as a semantic wave bio-computer where the two Heisenberg helices of *G* and its dual *G*' and their base-pairing SU(2) hologram planes are the basis for the U(1, C) holographic helical signal processing taking place and the semiotics of the genetic code. This very comprehensive treatment fully substantiates the NUCRS hypothesis that the DNA / RNA genetic code is indeed a rewrite at a higher level of molecular complexity further extending the NUCRS universal alphabet. And where the DNA is known to include the encoding of the human brain at a yet higher level (Marcer and Rowlands, 2007). To summarize therefore. 3D space should no longer be considered as just a plenum within which matter moves, but as a quantum field such as Einstein describes in general relativity, where, it has been said, 'matter bends space and space shapes matter'. An example would be changes in scaling resulting from nearby massive objects that can be expected from Trell's research to subject $U(1)$ signals to lensing effects. Indeed as shown in *Zero to Infinity* (Rowlands, 2007), this is, in fact, the actual means by which the dichotomy between nilpotent quantum mechanics and general relativity is resolved thermodynamically.

Further support, at a higher level of complexity, comes from the three spin ½ particles, the electron, muon and tau, and the composite (nuclear) matter with spin – these matter symmetries would (in addition to the Trell metric signature of 'cubit' scaling) reflect those of $SU(3)$, which concern the quark patterns, where these are, rather than the doublets of $SU(2)$, expressed in the form of triplets conventionally labelled the red, yellow and blue colour forces interactions produced by gluons (colour bosons) between the quarks.

It is also clear that in relation to the 3D spatial field, that there must also exist $SU(2)$ processes able to change the properties of $SU(3)$ matter interior to any nuclear composite. These processes can thus be hypothesized to define, for example, the two very well known phenomena of nuclear fusion and nuclear fission (as the two NUCRS create, conserve productions at this level) where the interactions involve the neutrinos and are recognised to produce the composite nuclear matter/elements of the periodic table in the stars, which are seen here to be 'super' quantum mechanical ray spaces composed from neighbourhoods P.

The NUCRS / nilpotent quantum mechanics (NQM) scaling phenomena and relativity – special and general – are essentially one and the same and so relate to the fact that the electro-magnetic vector symmetry $U(1)$ and 'gravity' / scaling, can be expected in the first approximation to obey the same inverse square Coulomb law as is indeed in the case. For in the case of gravity masses replace charges, but due to their scalar rather than vector nature only attract one another.

Conclusion

Evidence from phenomena at various levels of complexity seems to suggest that a nilpotent structure providing an automatic determination of the environment simultaneously with the system is the most efficient and most ubiquitous information processing system in Nature, and acts, in effect, as Nature's machine order code.

REFERENCES

Berry, M. V., 1986, Riemann's Zeta Function: a model of quantum chaos, *Springer Lecture Notes in Physics*, **263**, 1-17, *Quantum Chaos and statistical nuclear physics*, editors Seligman T.H. and Nishioka H., Springer, Berlin.

Dahleh, M., Pierce, A.P. and Rabitz, H., 1990, Optimal Control of Uncertain Quantum Systems, *Phys. Rev. A*, 42, 3, 1065-1079.

Diaz, B. and Rowlands, P., 2006, The infinite square roots of –1, *International Journal of Computing Anticipatory Systems*, **19**, 229-235.

Feynman, R. P., 1965, *The Character of Physical Law*, BBC Books, London.

Gariaev, P., Birstein, B., Iarochenko, A., Leonova, K. A., Marcer, P., Kaempf, U. and Tertishy, G., 2001, The DNA-wave Biocomputer, *International Journal of Computing Anticipatory Systems*, **10**, 290-310.

Gariaev, P., Birstein, B., Iarochenko, A., Leonova, K. A., Marcer, P., Kaempf, U. and Tertishy, G., 2002, Fractal Structure in DNA Code and Human Language: Towards a Semiotic of Biogenetic Information, *International Journal of Computing Anticpatory Systems*, **12**, 255-273.

Hill, V. and Rowlands, P., 2008, Nature's Code, *AIP Conference Proceedings*, **1051**, 117-126.

Judson R.S. and Rabitz H., 1992, Teaching Lasers to Control Molecules, *Phys. Rev. Lett.*, **68**, 10, 500-1503.

Marcer, P. and Rowlands, P., 2007, How Intelligence Evolved? *Quantum Interactions. QI-2007 Papers from the AAAI Spring Symposium. Technical report SS-07-8*, 46-51.

Marcer, P. and Schempp, W., 1986, A Mathematically Specified Template for DNA and the Genetic Code, in terms of the physically realizable Processes of Quantum Holography Fedorec, A. and Marcer, P. (eds.), *Proceeding of Greenwich (University) Symposium on Living Computers*, 45-62.

Rice S. A., 1992, New Ideas for Guiding the Evolution of a Quantum System, *Science*, **258**, 412-413.

Rowlands, P., 2007, *Zero to Infinity*, World Scientific, Singapore.

Rowlands, P., 2008, What is vacuum?, arXiv:0810.0224.

Rowlands P., 2009, Relativistic quantum mechanics derived from a single operator, *International Journal of Computing Anticipatory Systems*, forthcoming.

Schempp W., 1986, *Harmonic Analysis on the Heisenberg Group with Applications in signal theory. Pitman Notes in Mathematics*, Longman Scientific and Technical, London.

Schempp, W., 1992, Quantum Holography and Neuro-computer Architectures, J*ournal of Mathematical Imaging and Vision*, 2, 279-326.

Schempp, W., 1998, *Magnetic Resonance Imaging, Mathematical Foundations and Applications*, John Wiley, New York.

Trell, E., 2008, Elementary Particle Spectroscopy in Regular Solid Rewrite, *AIP Conference Proceedings*, **1051**, 127-141.

Vitiello, G., 2008, Quantum Dissipation, classicality and brain dynamics, *Proceedings 2nd QI-2008 Symposium*, 60-67.

Nature's Code III

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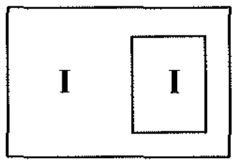
Abstract. A variety of mathematical structures is explored, which relate to the algebra and geometry of a nilpotent universal rewrite system. The structures are shown to encode significant biological and physical information.

Introduction

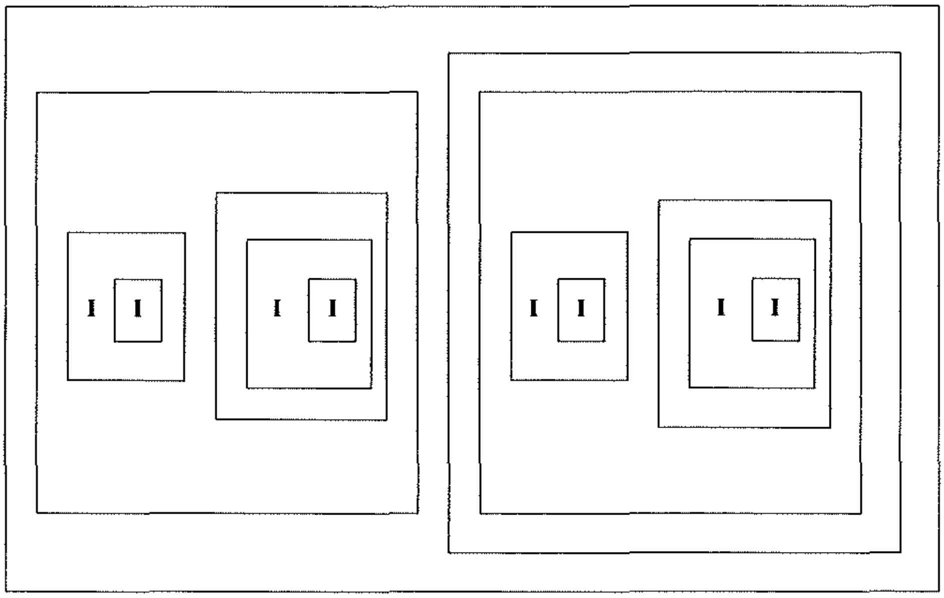
The question we have been concerned with in our previous presentations (Hill and Rowlands, 2007, 2008a, 2008b, Rowlands, 2007, joint presentation in chapter 19, Rowlands and Hill, 2006) is: how can the highly ordered replicating state which we call 'life' form within a universe where the tendency of natural processes is towards a state of increasing disorder? Life could not possibly emerge from a purely random arrangement of physical and chemical interactions. A simple calculation will show that the probability of this happening is zero. There must therefore be some driving process, and, to overcome the tendency to increased entropy and loss of information, it must be essentially a process which maximises information. It would seem that nature has an intrinsic information processing or machine order code, which probably determines the behaviour of all ordered systems, physical and chemical as well as biological, large scale as well as small scale. The existence of such a code seems to be suggested by the generation of a *universal rewrite system*, with its own mathematical structure, from the single assumption of a zero totality universe; and it would appear that the successive stages which this system automatically generates correspond with the algebraic and geometrical structures which are fundamental in physics and biology in particular.

The universal rewrite system has a number of significant aspects. One is that it is a series based on *cardinality*, rather than ordinality, as required by Georg Wikman (2008), i.e. a system in which change generates a new whole incorporating each previous cardinal (whole) state into a new cardinality – though, here, we have a cardinality of zeros, rather than of infinities. Each stage or 'alphabet' in the rewrite system is a totality, completely incorporating and extending the previous totality. Another is that each stage is a *bifurcation*, splitting the previous zero totality 'universe' into two. Finally, a key stage is reached at order 64, where the complete 64-part algebra can be produced by a combination of just 5 of the terms used as generators, and where scalar values can be applied to make the combination of 5 terms or pentad into a *nilpotent*, squaring itself to zero, and so providing the required zeroing with the maximum efficiency. It would seem that this structure can emerge at more than one level, and the 5-fold nature suggests a connection with the Fibonacci sequence, which is ubiquitous in nature, and the Platonic solids leading up to the dodecahedron and icosahedron.

In relation to *cardinality*, Lou Kauffman (2008) has given an analysis of Heinz von Foerster's statement, 'I am the observed link between myself and observing myself', in terms of the Fibonacci sequence. If we represent 'myself' by 'I' and 'observing myself' by 'I' surrounded by a box, then the *observed* link in 'I am the observed link between myself and observing myself' requires surrounding the two symbols by another box.



Redefining this symbol as the new 'I', leads to an infinite series of successively more complicated symbols, such as:



We can see from the diagrams that they are capable of a representation in terms of cardinality, in this sense parallel to the structure of the universal rewrite system. It is only a representation, of course, not a replacement, because it depends on the dimensionality which the universal rewrite system generates. In addition, if we start outside the box and take every path inward that involves crossing one line at each stage, then the total number of lines crossed at each stage forms a Fibonacci sequence: 1, 1, 2, 3, 5, 8 … . Clearly the universal rewrite system incorporates the Fibonacci series in this way – as well as others. Significantly, perhaps, the fact that a Fibonacci sequence has been produced is only apparent at the fifth level. 5 viewpoints in this representation are necessary to achieve a full state of observation – and to recognise that a Fibonacci sequence has occurred. Within the DNA helix we also see this cycle of 5 realised, where 5 dodecahedrons in a ring align with 1 cycle of double-stranded DNA and each of 2 dodecahedral faces aligns with the dNTPs of the X-ray crystallography scattergraph. It appears that the 5 again 'completes cycles'. This is also seen in the geometry where we have 5 star tetrahedras. Here, each star tetrahedron self-constructs an internal octahedron and the points of the star transcribe a cube. If we now take 5 such star tetrahedra the result will be the construction of an internal icosahedrons, transcribing of an icosidodecahedron and an external dodecahedron. The vertices of the 5 octahedra produce the icosidodecahedron and those of the 5 cubes (or 5 star tetrahedra) produce the dodecahedron, hence completion of a new cycle and the expression of the Golden section.

Double 3-dimensionality

The algebra that emerges from the rewrite system can be conveniently represented in the following table:

TABLE 1. The Rewrite Algebra.

Group	**Algebraic Units**	**3-D**
Order 2	$(1, -1)$	$0 \times$ 3-D
Order 4	$(1, -1) \times (1, i_1)$	$0.5 \times$ 3-D
Order 8	$(1, -1) \times (1, i_1) \times (1, j_1)$	$1 \times$ 3-D
Order 16	$(1, -1) \times (1, i_1) \times (1, j_1) \times (1, i_2)$	$1.5 \times$ 3-D
Order 32	$(1, -1) \times (1, i_1) \times (1, j_1) \times (1, i_2) \times (1, j_2)$	$2 \times$ 3-D
Order 64	$(1, -1) \times (1, i_1) \times (1, j_1) \times (1, i_2) \times (1, j_2) \times (1, i_3)$	$2.5 \times$ 3-D

The algebra requires nested 3-dimensional or anticommutative systems (i_1, j_1, and by implication $i_1j_1 = k_1$), (i_2, j_2, and by implication $i_2j_2 = k_2$), (i_3 ...), which are independent of each other (commutative). In principle, the 3-D systems continue to infinity, but, at order 64, physical systems allow the truncation of the series and zeroing of further terms by creating a nilpotent structure, squaring to zero.

Now, physics is structured on 4 fundamental parameters, but, because 2 of them are dimensional, the 4 parameters require specification by 8 algebraic units:

	Time	Space	Mass	Charge
	↓	↓	↓	↓
units	1	3	1	3
	i	**i j k**	1	i j k

Biology has much the same structure. This time, there are 4 bases, which, in the bonding between two strands, become equivalent to 8 algebraic units:

	T	A	C	G
	↓	↓	↓	↓
units	1	3	1	3

The algebra allows 64 possible combinations of the units:

$\pm 1, \pm i, \pm j, \pm k,$
$\pm \mathbf{i}, \pm \mathbf{i}i, \pm \mathbf{i}j, \pm \mathbf{i}k, \pm \mathbf{j}, \pm \mathbf{j}i, \pm \mathbf{j}j, \pm \mathbf{j}k, \pm \mathbf{k}, \pm \mathbf{k}i, \pm \mathbf{k}j, \pm kk,$
$\pm i, \pm ii, \pm ij, \pm ik,$
$\pm i\mathbf{i}, \pm i\mathbf{i}i, \pm i\mathbf{i}j, \pm i\mathbf{i}k, \pm i\mathbf{j}, \pm i\mathbf{j}i, \pm i\mathbf{j}j, \pm i\mathbf{j}k, \pm i\mathbf{k}, \pm i\mathbf{k}i, \pm i\mathbf{k}j, \pm i\mathbf{k}k$

5 combined units (pentad) are the simplest possible generators for the 64:

$$ik \qquad i\mathbf{i} \;\; i\mathbf{j} \;\; i\mathbf{k} \qquad j$$

Physically, these become the new quantities: energy (E), momentum (**p**, 3 units, representing 3 spatial dimensions) and mass (m). Similarly, within the DNA translation process we see the 64 triplet codons generated from the 4 bases A, T, G and C (4^3). This process yields the 20 amino acids that are the constituents of proteins and here we see the 5-fold broken symmetry nature as well as that seen manifested within the icosahedral / dodecahedral symmetry of the DNA helix itself.

In historical terms, physics required 2 independent observable 3-D systems to understand physical process involving change. To accommodate relativity (i.e. the fact that any 3-D system cannot be defined without an accompanying scalar) they each become '4-D' = 3 (+ 1), e.g. space (+ time). This pattern has been retained in later developments, and can be seen to be essential to the structuring of all fundamental information processing, whether physical, biological, or more abstract, and whether small-scale or large-scale. There are always at least 2 × 3-D systems required to specify a system in this way, one of which is nonconserved, and one conserved, and the combination – the variation of the nonconserved quantity against the background of the conserved one – gives us a description of the process of change.

Now, historically (in classical physics) the 3-D (or 4-D) systems were space (+ time), which was nonconserved, and momentum (+ energy), which was conserved, and the combination of space + momentum, which totally specified the dynamics of a system, was called 'phase space'. Both dimensional systems were in some sense observable. Quantum physics, however, shows that space and momentum are not truly independent; instead, the genuinely independent dimensional systems are space (+ time), which is nonconserved, and charge (+ mass), which is conserved, and only the first is observable. It was by combining the two in a 5-fold broken symmetry, that we could construct a second observable system, corresponding to the momentum (+ energy) that, historically, we had used to describe change.

It is because the second system, as used in measurement, is constructed partly from the first that the two 3D systems are not truly independent. In quantum mechanics, we say that the space (**r**) and momentum (**p**), and energy (**H**) and time (**t**) operators, acting on the wavefunction (ψ) (an object varying in space and time) are not commutative. That is each does not act as though the other did not exist. Mathematically, the operations $(\mathbf{rp} - \mathbf{pr})\psi$ and $(\mathbf{Ht} - \mathbf{tH})\,\psi$ are not zero (but $i\hbar\,\psi$), and we cannot know both position and momentum, or both energy and time, accurately at the same moment (the well-known Heisenberg uncertainty). On the large scale, with the combination of many quantum systems, the quantum coherence disappears, and the space (+ time) and momentum (+ energy) become independent; we then generate classical physics, which is the historical, but not logical starting point.

We may note that our two initial '4-D' systems would not be completely independent if they had the same scalar value, so one of the systems is separated from the other by being multiplied throughout by the 'pseudoscalar' *i*, which is like the first term in yet *another* 3-D system never completed. This extra term, through its incompleteness, seems to represent variation, and, in particular, variation with time. The passage of time seems to act like a 4-D system, in which structures are doubled to represent the old and the new as complete structures simultaneously in a higher space. This is the classic bifurcation or doubling process associated with the universal rewrite system, and in quantum mechanics, is something like the 'many worlds' interpretation in which the 'universe' bifurcates into two every time a measurement is made. There, however, it is assumed that the bifurcation is into two conserved 'universes'; here, by contrast, one is conserved, and one is not. In the universal rewrite system there is only a new cardinality or a new conserved state; the old one is lost.

The emergence of the icosahedral / dodecahedral symmetry

We have seen, previously, that the structure of DNA and translation of its code (via triplet codons) into proteins suggests an icosahedral / dodecahedral symmetry. The transformation of the icosahedron to dodecahedron (and vice versa) is effectively a transformation of the space, from privileging one 3-D to another 3-D, such as space to charge, or space to momentum space, lattice to reciprocal lattice, or inside space to outside space. The dodecahedron / icosahedron duality thus corresponds to that of the phase space of Heisenberg uncertainty, and needs a double 3-dimensionality; the structure can only be created through transforming one 3-D to another and this process can be seen to go via other, *Archimedean*, solids (which are composed, unlike the Platonic solids, of two or more types of regular polygons), e.g. the icosidodecahedron which has 32 faces. Ultimately, we need two 3-Ds to get uniqueness and variation; significantly, thermodynamics, which is a signature of a unique birth-ordering of events, comes from the vector nature of the **p** term in quantum mechanics, and the decoherence which results from this; quantum mechanics is impossible without it.

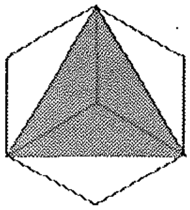
The symmetry which incorporates 2 separate 3-D structures is necessarily always a 5-fold one, with a broken symmetry, and this broken symmetry is manifested in many ways across a wide range of mathematics, for example, in the insolubility of the quintic equation, which relates directly to the symmetries that are important in biology and physics. The symmetries of regular 2-D n-gons, where n is not prime, are constructed from those of the p_i-gons, where the p_is are the primes from which n is constructed. In the nineteenth century, Galois linked this with the insolubility of the quintic equation (already established by Abel) by showing that the only algebraic equations that can be solved by a general method or formula are those whose underlying symmetry can be built up from such prime-sided shapes. Cubic equations require the symmetries of a triangle and quartic equations those of a tetrahedron; but quintic equations require the 60 rotational symmetries of a dodecahedron, which are indivisible and cannot be resolved into prime-sided shapes. The pentagonal faces of the dodecahedron cannot be rotated in combination with other shapes to produce these symmetries – a fact which is connected with the impossibility of tiling space with a 5-fold repeating structure, and the fact that 5-fold symmetry produces a breaking of symmetries structured purely from powers of 2. The 60 rotational and 60 reflectional symmetries of the dodecahedron, in fact, emerge from the possible 5! = 120 ways of ordering 5 objects, just as the 12 rotational and 12 reflectional symmetries of the tetrahedron emerge from the 4! = 24 ways of ordering 4 objects. These numbers are significant in both biology and physics.

Within the universal rewrite geometry (Figure 1), we have a single tetrahedron at order 4; 2 tetrahedra then produce a star tetrahedron which automatically constructs an octahedron internally, with the outer star points making the vertices of a cube (order 8); while 4 tetrahedra = 2 star tetrahedra plus 2 internal octahedra (order 16). The next stage is 4 star tetrahedrons with 4 internal octahedra (order 32). At this point, two paths can be chosen; either these 4 star tetrahedrons can undergo a phase change to a single 3rd order tetrahedron (order 32) and combine with a second 3rd order tetrahedron to form a 3rd order star tetrahedron (order 64); or the system can enter a broken symmetry state. Here a 5th star tetrahedron is borrowed from another set of 4 (say the –32 group) and after a readjustment to a position where all the star points are equidistant, will yield the dodecahedral symmetries. For the dodecahedral symmetries (broken symmetry), the order becomes 5 star tetrahedra / 5 cubes (the cubes being defined by the outer points of the stars) / 5 octahedra (internally created within each star tetrahedron) which now, together, produce an internally constructed icosahedron and the icosidodecahedron (produced by the vertices of the 5 octahedra) and the outer points of the star tetrahedra produce the vertices of an external dodecahedron.

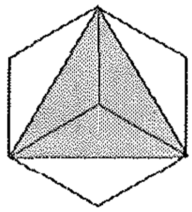
So we now have 5 objects, i.e. star tetrahedra, and these can be ordered in 120 different ways to produce 120 different dodecahedral forms, which could each represent a different particle. If we now consider the other chiral dodecahedrons, these may represent the vacuum states and would produce another 120 to give a total of 240 particles.

This 5th star tetrahedron may perhaps be borrowed from the –32 set (4 star tetrahedral) required to complete order 64 and vice versa similar to the *zitterwebegung*. Of course this leaves 3 remaining star tetrahedra within our original –32 set, i.e. there is a total of 8 star tetrahedra within this process of the creation of a broken symmetry state and there is a possibility of different shuffling states here, perhaps resulting in different particles. This process is also reminiscent of the hydrogen molecule and the sharing of an electron.

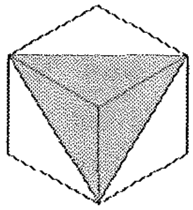
Order 2: 2 base types
purine + pyrimidine
[DNA:Binary coding
0,1 (2^1)]
2
MASS



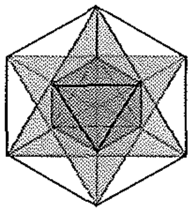
Order 4: 4 DNA bases
2 pur.,2 pyr.
[DNA:Binary coding, reading 2bp
(2^2 or 4^1 central base)]
4
TIME



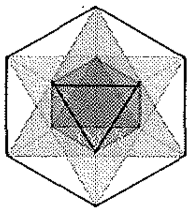
+ sense ss DNA synth
A,T,G,C
+ 4



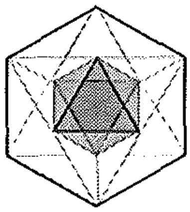
- sense ss DNA synth
T,A,C,G
- 4



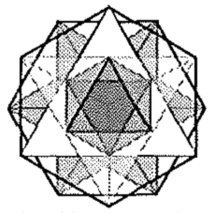
Order 8: ds DNA
A T G C
T A C G
[DNA:Binary coding,reading
2bp (2^3=8 family codons)]
8
CHARGE



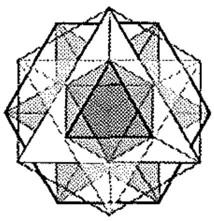
RNA strand synthesis
+ sense mRNA (U,A,C,G)
(copy of - sense DNA)
+8



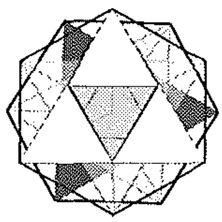
Unzipped DNA
-sense DNA exposed
and copied (mRNA)
-8



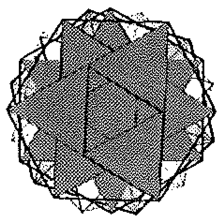
Order 16: Transcription
-sense DNA, + sense mRNA
[DNA:Binary coding, reading 3bp
4^2 codons with reverse codons
giving same aa reverse translation]
16
SPACE



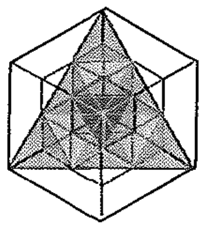
- sense m RNA
+ sense ribosomal
RNA positioning
+16



+ sense tRNAs
- sense ribosomal
RNA positioning
+16

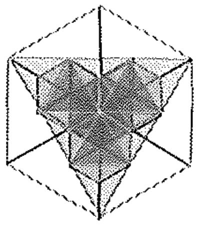


Order 32:Translation
mRNA/ribosome, tRNA/
ribosome, tRNA/a.a.,
tRNA/mRNA
32

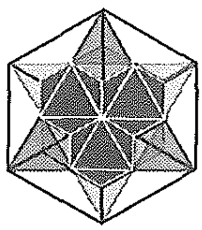


+ sense tRNAs
- sense mRNA
(1 U/C 3) Pyrimidine
[4^2+pyr as 3rd base]
+ 32

SYMMETRY STARTS

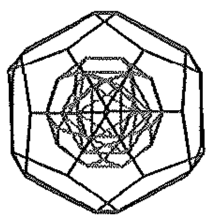


+ sense tRNAs
- sense mRNA
(1 A/G 3) Purine
[4^2+pur as 3rd base]
- 32

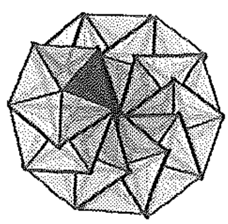


Order 64: Translation
all triplet codons
64 [4^3 DNA triplet code]

TO BREAK



Protein created
prodn. of protein chain
of amino acids (20)



Helix formation
developing protein
chain folds into
helices

BROKEN SYMMETRY

Figure 1. The geometric universal rewrite system relating (URS) to physics and biology. The process begins with the URS order 2 producing mass and the two DNA base types (purine and pyrimidine). Subsequent rounds of doubling produces firstly the tetrahedron (order 4), time and the four DNA bases; order 8 presents the star tetrahedron (ST), the emergence of charge plus the formation of classical double stranded DNA; Order 16, 2 STs, space and the initiation of the process of DNA transcription (mRNA copy of one strand of DNA is made); Order 32, 4 STs presents the mechanism of translation of the RNA code into amino acids within the ribosome, a geometric phase change into a 3rd order tetrahedron. At this point symmetry starts to break. Order 64 can be represented by two 3rd order tetrahedrons forming a 3rd order ST and represents the full translation of all 64 triplet codons into 20 amino acids. Broken symmetry allows the formation of a 5-fold geometry which results from the addition of an extra 5th ST to produce the dodecahedron, the icosidodecahedron and the icosahedron representative of the formation of an amino acid chain (a protein) that can then form helical structures (alpha helices (5-fold nature) and beta pleated sheets 8-fold nature). The recently proposed binary system of DNA evolution (Wilhelm and Nikolajewa, 2004) is shown in grey within brackets.

Breaking Symmetry

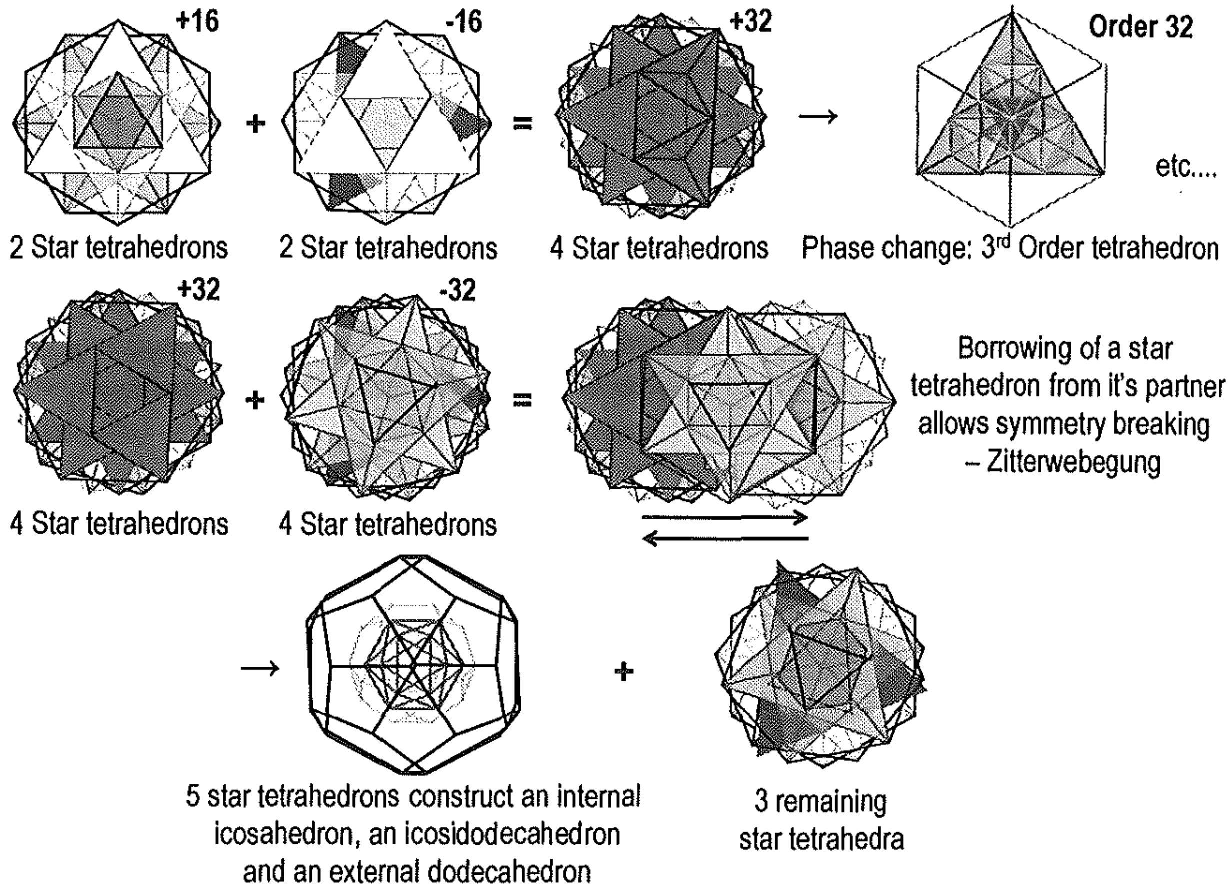


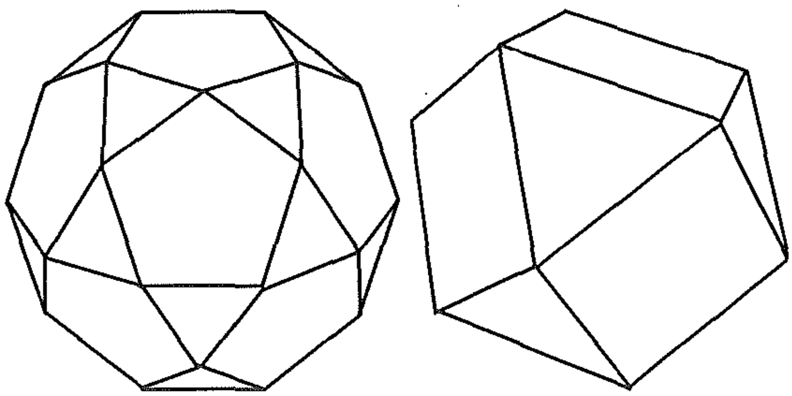
Figure 2. Breaking symmetry. The doubling of order 16 (2 star tetrahedrons (STs)) produces a complete set of 4 STs (order 32). At this stage there are two possible geometric pathways; Firstly, a phase change which allows a 3rd order tetrahedron (20 tetrahedrons representing 20 amino acids and 32 triplet faces upon 4 octahedra representing 32 of the 64 triplet codons of DNA); Secondly an extra star tetrahedron (perhaps borrowed from another order 32 set) allows the symmetry to break into a 5 fold system. Here we see the 5 STs held within a sphere with star points equidistant to each. This automatically self constructs an internal icosahedron, an internal icosidodecahedron and an external dodecahedron (see Figure 4). There are 3 remaining STs left over within the second set of STs, i.e. there are 8 STs in total with one group of 5 and one of 3 STs.

The edge transitive polyhedra

The 5 Platonic solids (2 dual pairs and the self-dual tetrahedron) group with 4 of the 13 Archimedean solids (2 dual pairs) to form the 9 edge-transitive polyhedra. These 2 Archimedean dual pairs include the cuboctahedron and its reciprocal the rhombic dodecahedron plus the icosidodecahedron and its reciprocal the rhombic triacontahedron.

1. The cuboctahedron

The cuboctahedron has 14 faces (8 triangular and 6 square), 24 edges and 12 vertices. Within our geometric rewrite system, order 4 is a star tetrahedron with an internally constructed octahedron while the star points describe an external cube. Moving from the octahedron, up through to the cube we pass through the cuboctahedron. Interestingly, our Uni-Phi colleagues, Nassim Haramein and Elizabeth Rauscher, have seen a fundamental significance in the cuboctahedron, in connection with the structure of physical space. This, we observe, has a dual pair with the rhombic dodecahedron, which can be used to tessellate space.



Icosidodecahedron **Cuboctahedron**

Figure 3. Two Archimedian solids. The icosidodecahedron and the cuboctahedron.

2. The icosidodecahedron

The icosidodecahedron has 30 vertices in total, 20 triangular faces and 12 pentagonal faces, making a total of 32 faces; it also has 60 edges; 6 great circle decagons; and 5 hexagonal planes of same side length as the pentagonal planes. The rhombic triacontahedron is its dual and any 2 vertices plus the centre make Golden triangles. Its symmetry group is the icosahedral group A_5. Within the previously described geometric construction of 5 equidistant star tetrahedra (concentrically placed within an imaginary sphere) we can see the emergence of a number of Archimedean solids by a progression from the internal icosahedron upwards through the icosidodecahedron, towards the dodecahedron.

Our work on Nature's code has shown that the icosidodecahedron, whose dual is the rhombic triacontahedron, is fundamental to the rewrite geometry and each has special significance within biology. This solid is interesting in that it has 32 faces reminiscent of order 32 – half the total number of DNA's triplet codons; 60 edges – the number of codons that actually code for an amino acid, 20 triangular faces – the 20 amino acids that make up proteins; and 12 pentagonal faces each representing 5 triplet codons ($12 \times 5 = 60$ coding triplet codons).

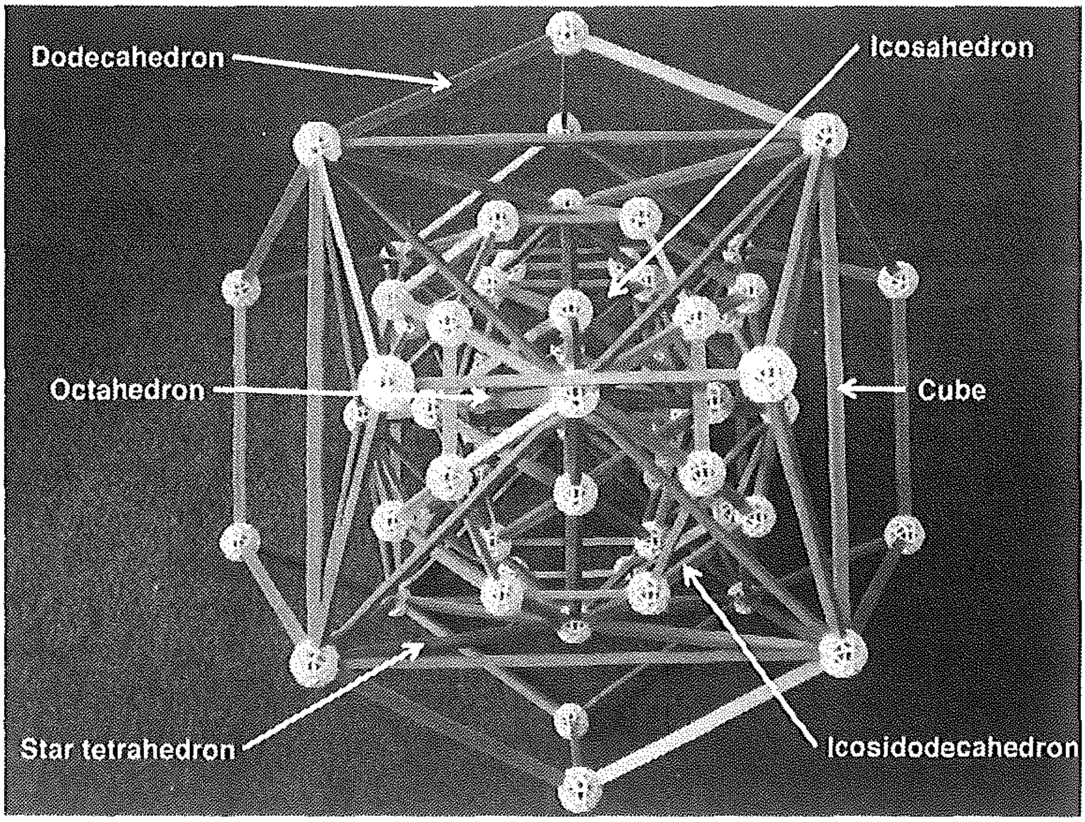


Figure 4. A Zome tool model showing part of the broken symmetry-state geometry that realizes the 5 Platonic solids and the icosidodecahedron. The 5 star tetrahedra (ST) held within an imaginary sphere, with the star points equidistant to each other, produce the vertices of a dodecahedron (only one ST is shown here), the points of each ST define a cube and 5 cubes also define a dodecahedron. Each ST contains a self constructed, internal octahedron and the vertices of 5 such octahedrons construct the icosidodecahedron. These 5 octahedrons also construct an internal icosahedron.

The reciprocal of the icosidodecahedron, the rhombic triacontahedron, in a 2-D projection gives the same outline as half of the pentagonal outline shown within an X-ray crystallographic scattergraph of the DNA double helix. Presumably, placing the second chiral form would complete the picture, indicate the reverse strand of the DNA helix and present a second set of 32 faces giving a total of 64 (total number of triplet codons). Interestingly, the outline of the spiral of pentagonal discs and the ring of dodecahedra, described in earlier work conforms exactly to that of a 3D projection of a 120 cell (4D equivalent of the dodecahedron) in a specific perspective (see Fig 5).

Our previous work has suggested that the icosidodecahedron is the shape that corresponds, in principle, to the all-important order 64, plus 32 being one chiral form and minus 32 giving us the chiral partner. If we have an icosidodecahedron, combining an icosahedron inside and a dodecahedron outside (or vice versa), then each (because of the intrinsic 5-fold symmetry) is a representation of a double 3-D (or, including the timelike variation between them, to a double 3-D, with the extra pseudoscalar). If we have a cuboctahedron, combining a cube outside and octahedron inside (or vice versa), each of its component shapes represents a single 3-D (with a timelike variation providing an extra pseudoscalar). The two aspects could represent 2 3-Ds, but not simultaneously. There is no instantaneous double 3-D and no 5-fold symmetry.

We can, incidentally, reduce the 5-D nilpotent $ikE + i\mathbf{p} + jm$ to a 3-D nilpotent $ikE + ip + jm$ by removing the 3-dimensionality of **p**, so that it becomes scalar p. (Here, if a cube represents space, then its reciprocal octahedron represents momentum, or $E / p / m$.). Effectively, one fermion on its own behaves this way, but, in relation to other fermions, it constructs a 3-D space and a 3-D momentum, or 'physical space'. We see something like this in the geometrical diagrams related to the universal rewrite system where we can see the 6-sided cube repeating itself with increased inner structure.

Now, an icosidodecahedron incorporates all states at once, as do the icosahedron and the dodecahedron taken separately. However, unless there is variation between them, then we don't have a full picture, because we don't see the extra pseudoscalar structure. The icosidodecahedron incorporates all possible states of itself at once, including the extremes of icosahedron and dodecahedron (we could perhaps think about these as like the **r**, **t** and **p**, **H** extremes of the physics), and all possible states in between – an infinite number of phases. We can imagine switching between phases or cycling through them, or, in quantum mechanical terms, as imagining that all happen at once.

We should think of this as a property of 'physical space', not of physics or biology, as such. In this context, when we talk about fundamental physical particles, we should realise that the 'charge space', which tells us how many physical particles we should have, is exactly dual with the momentum (+ energy) space, which constitutes quantum mechanics. The first tells us how space (+ time) combine with charge (+ mass) to transform them; the second tells us how charge (+ mass) combine with space (+ time) to transform them. The first is the conserved picture; the second the nonconserved.

When something happens, i.e. when some interaction occurs, one particular phase is selected, but only at that moment. In quantum mechanics, all possible states are equivalent (degenerate) until one is selected over all the others by making an observation (as with Schrödinger's cat). Gauge invariance (i.e. phase invariance) means all possible states are equal. Fixing means one particular state is selected. Famously in physics, the weak interaction fixes the gauge of the weak vacuum, privileging the real fermion state in all observable situations over the other three (vacuum states) in the Dirac wavefunction, and also the left-handed helicity over the right-handed. The predominance of matter over antimatter is a property of 'physical space', not a result of 'conditions during the big bang'! So also is the left- over right-handed chirality. It is a result of the production of the 5-fold state reducing the number of sign degrees of freedom.

The fermion, as structured in the Dirac wavefunction, always has four possible states – particle / antiparticle, right- / left-handed – one is 'real', and the other three are vacua. The free particle behaves as though it is continually switching between them, by a 'jittery motion' in and out of vacuum (*zitterbewegung*), and the mass and velocity of the particle are determined by the switching rate. When we observe the particle, of course, its behaviour is determined by which of the four terms represents its 'real' (observable) state. The icosahedron / dodecahedron switching is like *zitterbewegung*. We can see it as 'particle' / vacuum or space / momentum space, etc. Of course, there is also chirality of the shapes – left- and right-handed versions. The in-built chirality of biology is due to the chirality of physical space.

We can perhaps think of a real biological system, e.g. DNA / RNA, virus, protein, as having all the possibilities at once, or as near that as possible, and interacting when the possibility is selected which matches what is required by the interacting system. This must be true of chemical systems in general, as chemical systems are quantum mechanical, and biological systems, of course, are also interacting as chemical systems.

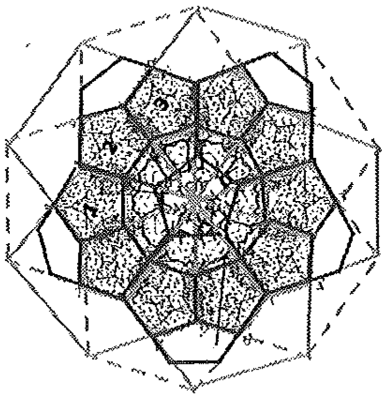
It appears that biological systems are very near chaotic. This would make sense in terms of the need to produce as many possibilities as possible to find the one which 'works'. Chaos occurs when the continual bifurcation of the system goes at too fast a rate. The defining of new possibilities in the universal rewrite system can be seen as a continual bifurcation, and the rate which just stays inside chaos will be the most efficient possible that maintains structural order.

The icosidodecahedral phase

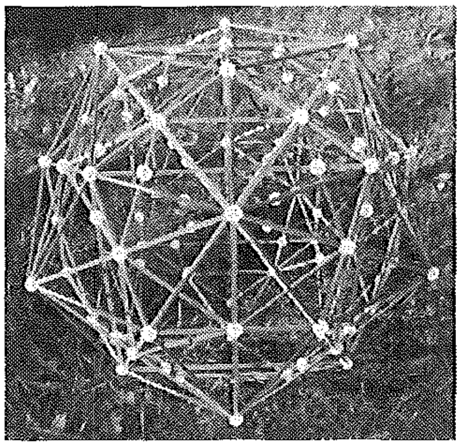
We have previously seen the uniqueness of each DNA profile as probably being in some way analogous to Pauli exclusion. Now Pauli exclusion in nilpotent theory has two forms. First, if $\psi_1 = (ikE_1 + i\mathbf{p}_1 + jm_1)$ squares to 0, then the combination state with $\psi_2 = (ikE_2 + i\mathbf{p}_2 + jm_2)$ would be 0 unless ψ_1 is different from ψ_2. But, conventionally, Pauli exclusion is done by defining the combination state as $(\psi_1\psi_2 - \psi_2\psi_1)$, and then saying that the wavefunctions ψ_1 and ψ_2 are antisymmetric, which means that, if we exchange ψ_1 and ψ_2, then the combination state is the negative of what it was before. So $(\psi_1\psi_2 - \psi_2\psi_1) = -(\psi_2\psi_1 - \psi_1\psi_2)$. Now, if you apply this definition of combination state, i.e. $(\psi_1\psi_2 - \psi_2\psi_1)$, to ψ_1 and ψ_2, using nilpotents, you find that after all the subtractions, only one thing is left, and that is $8i\mathbf{p}_1 \times \mathbf{p}_2$. This is certainly antisymmetric, as required, but it also has a deep significance. $\mathbf{p}_1$ cross $\mathbf{p}_2$ only has a nonzero value when $\mathbf{p}_1$ and $\mathbf{p}_2$ are in different directions – it doesn't matter about their scalar magnitudes. In other words, we can only tell if ψ_1 and ψ_2 are different by the instantaneous relative directions in 3-D of their momentum operators $\mathbf{p}$. So, each is uniquely specified by this alone. We can also imagine projection of the magnitudes of E, $\mathbf{p}$ and m along three axes being unique for each ψ, and the spin (momentum) vector carrying the information about E (handedness), $\mathbf{p}$ (direction), m (magnitude). This would link in with the first way of seeing Pauli exclusion. Now, the only things that specify an Archimedean solid are the magnitude, handedness / chirality, and instantaneous direction in 3-D determined against some arbitrary fixed axis. So if the identity of a living organism's DNA is uniquely defined by the relative 'position' of its defining solid (presumably uniquely determined by its sequence of bases) or direction of its defining axis with respect to some fixed reference, then this (plus size and handedness) is the same set of information as we use in *E*-**p**-*m*, and we could backwards project the solid onto *E*-**p**-*m* to characterize it. (Significantly, if we look at the particle states, they are in principle all connected as though they were versions of each other.)

We can imagine the area between the inner icosahedron and the outer dodecahedron as in constant flux, with the icosahedron, in effect going up and down (and jumping about inside). The solid would then be expanding and contracting up and down the 3 axes at once, migrating through states that are actually Archimedean solids, i.e. passing through each 'phase' up to the dodecahedron. This process starts with the icosahedral triangular faces and, as the solid increases in size up the 3 axes, it transmutes into other forms as the pentagonal faces start to appear and the triangular ones start to span out and reduce in size until we end up with the flat dodecahedral pentagonal faces alone. The previously described geometric progression shows the construction of an icosidodecahedron (where we can also realise a reciprocal form of the rhombic tricontrahedron with relevance to the top-down view of a scattergraph of the DNA helix). If you view from outside this fluctuating object from a random view point – say a vertex of an icosahedron and its 5 associated triangles – you will see the pentagonal unit of 5 triangles effectively start to flatten and spread out and the pentagonal faces start to appear as the object increases in size from the icosahedron. This pentagonal area will effectively appear to the observer, to turn and expand in a spiral on its way up to the pentagonal face of the outer dodecahedron. The direction this turns will depend on the chirality of the final dodecahedron.

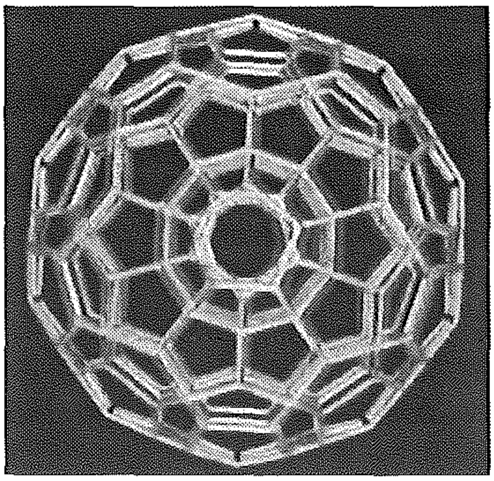
Each living entity can be considered as being in one of an infinite number of phase states between the icosahedron and dodecahedron, with each state representing a different codon usage determined by what faces are effectively formed at a specific point. Hence we can have the extremes of the say thermophiles, which have 70% GC content and therefore make heavy use of GC biased codons for an amino acid, to the other extreme of AT rich organisms. In addition, everything again becomes unique within a species as there are always tiny mutations introduced at every round of DNA replication. As long as these mutations are within the boundaries set by the phase, no new species will be formed. Perhaps stable species in evolutionary terms are those which can be represented by Archimedean solids and have greater symmetry characteristics. We can start to consider whether the actual Archimedean solids are states that are more stable positions and that things in between tend towards one or other of them and a species hovers near these more stable forms. This would be something like resonance in 3D. An interaction of one microorganism or living system with another in close proximity could allow a new combined resonance state to be set up which would shift codon usage in each, with a subsequent mutual information exchange occurring to allow deeper and deeper interaction. This may be observed initially, as a symbiosis and eventually as a full integration, as proposed for mitochondria within eukaryotic cells.



A



B



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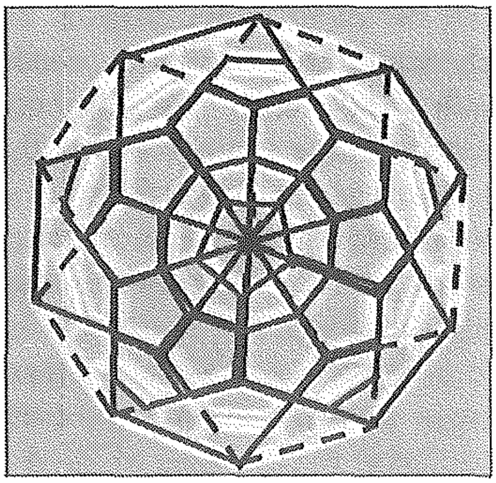

D

Figure 5. The geometry of the DNA helix. A; The outline of the spiral of pentameric discs (5 tetrahedrons/disc) and dodecahedra overlaid upon

a computer generated scattergraph of DNA. B; the archimedian solid the rhombic tricontrahedron the reciprocal of the icosidodecahedron. C; 120 cell projection into 3D. D; The outline of a spiral of pentameric discs and dodecahedra overlaid onto the 120 cell projection into 3D.

The rewrite algebra in physics and biology

Of the total of 64 generators in the rewrite algebra, 60 can be arranged into 12 nilpotent pentads, with the remaining 4 being the nondimensional units 1, –1, *i*, – *i*. One way of arranging the 12 pentads is as follows:

$i\mathbf{i}$ $i\mathbf{j}$ $i\mathbf{k}$ ik j	$-i\mathbf{i}$ $-i\mathbf{j}$ $-i\mathbf{k}$ $-ik$ $-j$
$i\mathbf{i}i$ $i\mathbf{i}j$ $i\mathbf{i}k$ $i\mathbf{k}$ $\mathbf{j}$	$-i\mathbf{i}i$ $-i\mathbf{i}j$ $-i\mathbf{i}k$ $-i\mathbf{k}$ $-\mathbf{j}$
$j\mathbf{i}$ $j\mathbf{j}$ $j\mathbf{k}$ ii k	$-j\mathbf{i}$ $-j\mathbf{j}$ $-j\mathbf{k}$ $-ii$ $-k$
$i\mathbf{j}i$ $i\mathbf{j}j$ $i\mathbf{j}k$ $i\mathbf{i}$ $\mathbf{k}$	$-i\mathbf{j}i$ $-i\mathbf{j}j$ $-i\mathbf{j}k$ $-i\mathbf{i}$ $-\mathbf{k}$
$k\mathbf{i}$ $k\mathbf{j}$ $k\mathbf{k}$ ij i	$-k\mathbf{i}$ $-k\mathbf{j}$ $-k\mathbf{k}$ $-ij$ $-i$
$i\mathbf{k}i$ $i\mathbf{k}j$ $i\mathbf{k}k$ $i\mathbf{j}$ $\mathbf{i}$	$-i\mathbf{k}i$ $-i\mathbf{k}j$ $-i\mathbf{k}k$ $-i\mathbf{j}$ $-\mathbf{i}$

Each pentad here is a fermion, with the first term representing the energy operator, the next 3 the momentum operator, and the last term the mass operator. So these structures (if we treat them symbolically) can be seen as representing 12 fermions, say, 6 quarks and 6 leptons, or 6 quarks / leptons and 6 antiquarks / antileptons. The total becomes 2 × 12 = 24 if we include left- and right-handed states (the parity *P* duality); and 2 × 2 × 12 = 48 if we include fermion and antifermion states (the charge conjugation *C* duality), in addition to quarks and leptons.

generation		isospin	
1	electron neutrino	up	$i\mathbf{i}$ $i\mathbf{j}$ $i\mathbf{k}$ ik j
	electron	down	$i\mathbf{i}i$ $i\mathbf{i}j$ $i\mathbf{i}k$ $i\mathbf{k}$ $\mathbf{j}$
2	muon neutrino	up	$j\mathbf{i}$ $j\mathbf{j}$ $j\mathbf{k}$ ii k
	muon	down	$i\mathbf{j}i$ $i\mathbf{j}j$ $i\mathbf{j}k$ $i\mathbf{i}$ $\mathbf{k}$
3	tau neutrino	up	$k\mathbf{i}$ $k\mathbf{j}$ $k\mathbf{k}$ ij i
	tau	down	$i\mathbf{k}i$ $i\mathbf{k}j$ $i\mathbf{k}k$ $i\mathbf{j}$ $\mathbf{i}$

generation		isospin	
1	antielectron-neutrino	up	$-i\mathbf{i}$ $-i\mathbf{j}$ $-i\mathbf{k}$ $-ik$ $-j$
	antielectron	down	$-i\mathbf{i}i$ $-i\mathbf{i}j$ $-i\mathbf{i}k$ $-i\mathbf{k}$ $-\mathbf{j}$
2	antimuon-neutrino	up	$-j\mathbf{i}$ $-j\mathbf{j}$ $-j\mathbf{k}$ $-ii$ $-k$
	antimuon	down	$-i\mathbf{j}i$ $-i\mathbf{j}j$ $-i\mathbf{j}k$ $-i\mathbf{i}$ $-\mathbf{k}$
3	antitau-neutrino	up	$-k\mathbf{i}$ $-k\mathbf{j}$ $-k\mathbf{k}$ $-ij$ $-i$
	antitau	down	$-i\mathbf{k}i$ $-i\mathbf{k}j$ $-i\mathbf{k}k$ $-i\mathbf{j}$ $-\mathbf{i}$

generation		isospin	
1	up quark	up	$i\mathbf{i}$ $i\mathbf{j}$ $i\mathbf{k}$ ik j
	down quark	down	$i\mathbf{i}i$ $i\mathbf{i}j$ $i\mathbf{i}k$ $i\mathbf{k}$ $\mathbf{j}$
2	charmed quark	up	$j\mathbf{i}$ $j\mathbf{j}$ $j\mathbf{k}$ ii k
	strange quark	down	$i\mathbf{j}i$ $i\mathbf{j}j$ $i\mathbf{j}k$ $i\mathbf{i}$ $\mathbf{k}$
3	top quark	up	$k\mathbf{i}$ $k\mathbf{j}$ $k\mathbf{k}$ ij i
	bottom quark	down	$i\mathbf{k}i$ $i\mathbf{k}j$ $i\mathbf{k}k$ $i\mathbf{j}$ $\mathbf{i}$

generation		isospin	
1	antiup-quark	up	$-i\mathbf{i}$ $-i\mathbf{j}$ $-i\mathbf{k}$ $-ik$ $-j$
	antidown-quark	down	$-i\mathbf{i}i$ $-i\mathbf{i}j$ $-i\mathbf{i}k$ $-i\mathbf{k}$ $-\mathbf{j}$
2	anticharmed-quark	up	$-j\mathbf{i}$ $-j\mathbf{j}$ $-j\mathbf{k}$ $-ii$ $-k$
	antistrange-quark	down	$-i\mathbf{j}i$ $-i\mathbf{j}j$ $-i\mathbf{j}k$ $-i\mathbf{i}$ $-\mathbf{k}$
3	antitop-quark	up	$-k\mathbf{i}$ $-k\mathbf{j}$ $-k\mathbf{k}$ $-ij$ $-i$
	antibottom-quark	down	$-i\mathbf{k}i$ $-i\mathbf{k}j$ $-i\mathbf{k}k$ $-i\mathbf{j}$ $-\mathbf{i}$

These particle states can be represented either by an icosahedron, with triangular faces (4 × 5 × 3), or a dodecahedron, with pentagonal faces (4 × 3 × 5). Another alternatative, is the icosidodecahedron with 32 faces, 20 of which are triangular (4 × 5 × 3) and 12 pentangular (4 × 3 × 5).

The biological system becomes algebraically equivalent to the physical one through the A-T and C-G pairing becoming algebraically equivalent to space-time and charge-mass. Also, both the four parameters of physics and the four bases of genetics can be represented by a tetrahedron. (In the case of physics, this leads to a representation of *C*, *P* and *T* symmetries.)

The tetrahedron has 12 rotation symmetries, corresponding to the 12 pentads. In biology, each face of the tetrahedron is a triplet codon. Each pentad is a unit on a DNA strand that highlights a triplet and 10 defines a double helix of DNA and aids in the placement of the dodecahedra – 3 pentagonal faces (1.5 dodecahedra) give placement of 3 dNTPs or a triplet, and back faces would represent the opposite DNA strand 3 dNTPs. The total can become $2 \times 12 = 24$ if we consider the opposite chiral form of the dNTPs and DNA and hence a mirror form of the dodecahedron.

64 possible combinations are manifested in the triplet code ($4 \times 4 \times 4$), and the triplet code is created because of the *need* to have the 64 combinations. 60 of these actually code – the others being STOP mechanisms, etc. This separation of the number of algebraic units into 60 + 4, and the further creation of 48 units within the 60, can be accomplished by some such algebraic classification of the codons as:

48	2 first bases different
12	2 first bases the same
4	3 bases the same

(There are further ways of dividing up the 60 into combinations such as 36 + 12 + 12.). The third order tetrahedron includes all 64. The total doubles again for physical particles if we include the *3-dimensionality* of the quarks (as we did originally for space and charge), or 3 + 1-dimensionality for quark-fermion (as with space-time and charge-mass). Then we have 3 colours of quark and $2 = 1 + 1$ becomes $4 = 1 + 3$, as previously, meaning that we have $2 \times 2 \times 2 \times 12 = 96$ possible fermionic / antifermionic states – these are the ones known to exist.

In the corresponding genetics, the 60 translated codons can be represented by the 12 pentagonal faces of a dodecahedron; the dodecahedron, with 12 pentagonal faces, can be taken as equivalent to 12 pentads, each representing 5 triplets. A dodecahedron incorporates 5 positive and 5 negative tetrahedral within (that constitute the 5 star tetrahedra), each of which can rotate 12 times, again dividing the 60 into 12 pentads (× 2, 1 positive the other negative). 48 + 12 would further emerge from 4 dodecahedra + a fifth, completing a cycle of 5. It is possible that the amino acids can be grouped into 5 yet to be determined groups represented by one face of a dodecahedron in a dodecahedral arrangement of codons (5 per face) and also a star tetrahedral arrangement – 64 triplets (8 octahedra – each face representing a triplet) plus 20 tetrahedra (representing 20 amino acids). This 60 is also reiterated again in the system of the pentameric disks of 5 tetrahedra that create the double helix and the highlighting of a triplet codon. The 12 pentads in genetics can be seen in the dodecahedral arrangement of the codons (5 triplets per face) on 12 faces (minus stop codons), as shown in Figure 5A.

Now, to each 4 fermions / antifermions, there is a gauge boson, or quantum of interaction. There are 24 (spin 1) gauge bosons, or particles mediating interactions between fermions: 12 are definitely known to exist (8 gluons for the quarks, 2 W bosons, 1 Z boson and 1 photon), but Grand Unification theories, which must be true in principle, and which can be made to work perfectly, also predict that another 12 exist – 6 X bosons and 6 Y bosons. It would seem that these are *already* manifested in a subtle way. In addition, any single gauge (spin 1) boson state requires a fermion / antifermion combination, one being right-handed and the other left-handed. So a single boson state defines 4 possible fermion / antifermion components. These are, of course, the 4 states in the Dirac wavefunction for a fermion, with $\pm iE$, $\pm$ **p**.

At this point, we invert the derivation of the 12 from a 5-unit pentad, and map the fermions and bosons onto a new pentad structure, of which the pseudoscalar component (the iE term) is 24 leptons / antileptons, and the vector component (the **p** term) 72 quarks / antiquarks. Bosons are scalar particles, and scalars are the squared products of pseudoscalars and vectors, just as bosons are the squared products of fermions / antifermions. So if the 24 bosons occupy the *scalar* part of the pentad (the m term), then we can use nilpotency to group the 96 fermions (24 leptons and 72 quarks) with the 24 bosons into a single structure with 120 fermions plus bosons.

But each 4 of the real fermions also contributes to a vacuum boson (never seen, but still mathematically necessary), and each of the real bosons requires 4 vacuum fermions / antifermions (again never seen, but still necessary). So the total of fermions plus bosons, including vacuum states, is 240, which, as we will see, is the one required by a group E_8 representation. (The vacuum doubling is equivalent to the T duality, which, because of nilpotency, is not independent of CP.)

Mathematically, in fact, the only way of including bosons and fermions in the same representation is through the exceptional groups E_6, E_7, E_8, and these would seem to be represented by the stages 48 + 12 = 60, 96 + 24 = 120, 192 + 48 = 240. So, in a nilpotent system, we have a physical as well as mathematical reason for combining fermions and bosons in the same representation, and, in the nilpotent structure, a fermion is necessarily, in some sense, its own vacuum boson, and vice versa. The 12 bosons we would put with the 48 are the 8 gluons, the 2 W and 1 Z boson, and the photon, that is, all except X and Y.

In terms of Platonic sold geometry, we can say that the 12 rotational symmetries of the tetrahedron represent only 1 set of vertices on a dodecahedron. This can be doubled because of the minor twin (star tetrahedron), leading to 24 'scalars' (bosons) – the 'identity' set. Choosing another vertex set leads to 4 more, in a kaleidoscopic pattern, or a total of 120. So, we have 1 'identity' and 4 additional reflections. Then taking left-hand and right-hand rotations, we double again to 240. The dodecahedron itself has 60 rotation symmetries. It can be constructed from 5 cubes, each with a star tetrahedron. So there are 10 tetrahedra in total. Then, we have $12 \times 10 \rightarrow 120$ or $24 \times 5 \rightarrow 120$. Taking the two chiral dodecahedral forms gives us $2 \times 12 \times 10 \rightarrow 240$. (As we can see below, this can be related to the 24-cell, 120-cell and 600-cell.) Applying the same reasoning to genetics, we can suppose that doublings occur for left- and right-handed strands and codon / anticodon, or 2-stranded DNA + mRNA + tRNA, to produce $2 \times 2 \times 60 = 240$ total units.

One final point is that the icosahedral number 20, just like the pentad, and the number 60, has a double significance in this system. First of all, 4 pentads (fermion / antifermion, right- and left-handed) (or $\pm iE$, $\pm \mathbf{p}$) are needed to make a boson, so this is $4 \times 5 = 20$ units. Secondly, the particle themselves require a 20 for a quark / lepton / equivalent boson generation. The original 6 quarks / leptons are in three generations, each divided into two states of weak isospin (up / down). To write down a single line of this, say the first, we need 16 fermions – (3 colours of quark + 1 lepton) × 2 (for left- / right-handed) × 2 (for particle antiparticle) – plus 4 gauge bosons (in this case, 2 colourless gluons + Z^0 + photon). In a sense, everything else is a repetition of this pattern, multiplied by 6, or even 3 and a series of 2s.

4-dimensional representations

Though space is intrinsically 3-dimensional, the representation of a physical or biological system by a double set of coordinates, one of which is fixed and the other moving, or one conserved and the other nonconserved, can be structured using a 4-dimensional space. In some ways, this is close to the idea of time as a fourth dimension (and, mathematically, the time term doubles the algebraic units by adding imaginary versions to all the real terms), but a more exact way of thinking about it would be to say that the simultaneous visualisation of two places relating to an object in 3-dimensions gives the sensation of the passage of time. There are seventeen separate centres in the brain involved with visual perception, and it is specially significant that the perception of movement requires a totally different centre to that for the perception of an object as a body in 3-dimensional space, and that an individual can possess one facility without the other. There are also other ways of seeing 4-dimensionality as being intrinsic to physics and biology, particularly where the dimensionality is not necessarily directly spatial. Physics, for example, has a 4-dimensionality associated with the parameters space, time, mass and charge, and another (ultimately related) one involving the fundamental particles, while biology has a natural 4-dimensional 'space' associated with the 4 bases of DNA. Each also has a natural 8-dimensionality, with an intrinsic 6-dimensional component. The use of higher-dimensional 'spaces' brings in new aspects to the idea of Platonic solids and also connects them to other fundamental mathematical concepts, such as groups and the kissing number.

The kissing number is a way of describing the most efficient packing of space using identical spheres, or, in dimensions greater than 3, hyperspheres. In any dimensionality of space, it is the number of equally-sized spheres or hyperspheres that can simultaneously touch any central one. In one dimension, it would be 2, in 2 dimensions it is 6 (as in graphite, graphene, and the benzene and pyrimidine rings). In 3 dimensions it is 12, placed at the vertices of an icosahedron, and in 4 dimensions 24. Most of the exact kissing numbers for higher dimensions are not known, but for 8 dimensions it is 240, and for 5, 6 and 7, the respective numbers are believed to be 40, 72 and 126. The corresponding lattices for the kissing numbers from 1 to 8 are those of the Lie groups A_1 (= *Z*), A_3 (= D_3), D_4, D_5, E_6, E_7 and E_8. These numbers also bring into play another significant mathematical number, the Riemann zeta function, $\zeta(n)$ (see Marcer and Rowlands, 2009), for, according to the Minkowski-Hlawaka theorem, lattices must exist in n dimensions with hypersphere packing densities greater than or equal to $\zeta(n) / 2^{n-1}$. However, if we define $\zeta(n)$, for complex $n = a + bi$, by the summation,

$$\zeta(n) = \sum_{t=1}^{\infty} \frac{1}{r^n}$$

projects an infinite number of values of 0 for $\zeta(n)$, when $a = ½$, and b seemingly random, which is suggestive of fermions with spin ½ and effectively random energy and momentum. Perhaps this means that, to take space to the limit of fermionic point-singularity (the only physical thing that can conceivably be defined in this way), with the complexity introduced by fermion spin, and zero minimum packing, we need to generate an infinite number of possible values of $\zeta(n) = 0$ by generating an infinite number of almost random possible states of energy and momentum.

The kissing numbers and groups for these dimensions are also closely related to the Platonic solids which they allow. Only in 3 and 4 dimensions are there more than three Platonic solids (though in 2 dimensions there are an infinite number of regular flat polygons, each of which is its own dual). Only in these dimensions are there solids with the 5-fold symmetries of the dodecahedron and icosahedron. Other dimensionalities have only equivalents of the tetrahedron, cube and octahedron. The 4-dimensional solids are a particularly interesting case, because here there are *six* Platonic solids – higher than in any other dimension – and one solid, the 24-cell, that is unique, and not even present in 3 dimensions although there are some indications of a relationship with the cuboctahedron.

The self-dual pentatope (or pentachoron), the 4-dimensional analogue of the tetrahedron, has the numbers associated with the algebra of compacted fermionic structures. It is made up of 5 tetrahedral cells, with 10 faces, 10 edges and 5 vertices. Projection of the pentatope onto 3 dimensions shows 5 tetrahedra making one pentagonal disc (minus 7°12'). We have shown that 10 such discs complete one cycle of the DNA helix and when considered to be parts (top and bottom sections) of icosahedra with reciprocal internal dodecahedra, pinpoints the nucleotide positions viewed along the axis of DNA.

The tesseract, the 4-dimensional analogue of the cube (which can be simulated from the combination of a cube with another upon each face and connecting their vertices), and its dual, the 16-cell, the analogue of the octahedron, have the numbers associated with the uncompacted algebra. The tesseract has 8 cells, 24 faces, 32 edges and 16 vertices, while the 16-cell has 16 cells, 32 faces, 24 edges and 8 vertices. The 16 vertices may be represented by the 24 possible unit values of a quaternion of the form $a + bi + cj + dk$, where one of the terms a, b, c, d is either 1 or –1 and the others 0; they form a group under multiplication, always producing a product from inside the group.) The representation of the tesseract as a projection of this 4dimensional cube into 3dimensions and put into motion via rotation, with one cube seemingly 'emerging' from the other, can be used to give a representation of 4 dimensions emerging from a double coordinate system.

The self-dual 24-cell, with no corresponding 3-dimensional figure, has the numbers associated with the possible types of fermions and bosons. This is important because the three colour-types of quark and the single type of lepton, combined, can be considered to make up a kind of 4-

dimensional 'space'. The 24-cell has 24 octahedral cells (6 meeting at each edge), 96 triangular faces, 96 edges, and 24 vertices. (The 24 vertices again form a group under multiplication, being represented by the 24 possible unit values of a quaternion of the form $a + bi + cj + dk$, where the terms a, b, c, d are either all integers or all integers plus ½.) Now, there are 96 possible fermion states, made up of (3 colours of quark + 1 lepton) × (2 states of weak isospin in each generation: up / down) × (3 generations) × (2 for fermion / antifermion) × (2 for left- and right-handed), and 24 bosons to carry their interactions ('gauge bosons') (8 gluons, photon, W^-, W^+, Z^0, plus 6 X- and 6 Y-bosons for the strong-electroweak combination). (The same is also true for the vacuum partners, which in effect double the total number of states: 4 real fermion states (fermion / antifermion, left- and right-handed) contribute to each vacuum boson state; and each real boson state requires 4 vacuum fermion states. So we can imagine the 96 fermions as being represented along, say, the edges and meeting at the vertices, whose number is also the kissing number in 4 dimensions. It may be that we could simultaneously see the 24 cells and the 96 faces as the vacuum partners. It is also significant that 72 of the fermions are quarks or antiquarks, and 24 are leptons or antileptons; and 72 is, appropriately, the kissing number for a 6-dimensional space, just as 24 is that for 4 dimensions.

The 120-cell, which is the 4-dimensional analogue of the dodecahedron, and its dual, the 600 cell, which is the 4-dimensional analogue of the icosahedron, have the numbers associated with taking all the possible set of fermion and boson states together on a single footing. They also incorporate all the significant numbers of the 3 previous structures, pentatope, tesseract and 24-cell. It is at this level that the 5-fold dodecahedral / icosahedral symmetry first emerges. The 120-cell is made up of 120 dodecahedral cells, with 4 at each vertex, the vertex figure being a tetrahedron. It has 720 pentagonal faces, 1200 edges and 600 vertices; 5 copies of the dual 600-cell can be inscribed in these vertices. The dual 600-cell is made up of 600 tetrahedral cells, 20 meeting at each vertex; there are 1200 triangular faces, 720 edges and 120 vertices, the vertex figure being an icosahedron. The edges of the vertices have length $1 / \phi$, and yet again form a group under quaternionic multiplication. (This time the members are of the form $a + bi + cj + dk$, where the terms a, b, c, d are all constructed from the so-called 'golden field', $x + \sqrt{5}\, y$, and x and y are rational numbers.) 24 are vertices of a 24-cell (16 being vertices of a tesseract and 8 of a 16-cell) and 96 are vertices of a snub 24-cell (which is made up of 120 regular tetrahedra and 24 icosahedra). The 96:24 or 4:1 fermion / boson ratio also reflects the division between the terms *i*i, *i*j, *i*k, *ik*, and the 'redundant' *j* in the nilpotent algebra; significantly, the *j* is attached to a purely scalar value, exactly as required for bosons. In DNA, the 120 can represent the 60 codons used for producing amino acids present on the sense strand and the 60 corresponding codons on the antisense strand. The total process always requires doubling. (On the basis of the third base often being in effect redundant, we can divide the 120 codons / anticodons into those with first two bases nonidentical and those with first two bases identical, which divide in the ratio 96:24. It might even be possible to divide the 96 into 72 and 24, as in the analogous physics application, by, say, separating out the codons with two final bases identical.) The 120-cell, projected onto 3 dimensions also shows remarkable similarity to a computer-generated scatttergraph of DNA viewed along its axis, showing the complete stacking of the 10 nucleotides as pentagonal discs.

A dodecahedral universe has been proposed, in the form of a hypersphere surrounded by 120 modified dodecahedra. This would be bounded by reversed images of itself, each rotated by 36°. We might imagine this in a particle space, with the fermion reflecting itself in vacuum.

Higher dimensional representations

Although 4 dimensions has a special role in producing the 'double 3-dimensionality' needed in both physics and biology, even higher dimensionalities have significance in representing aspects of the resulting structures. The significance of 5 dimensions is obvious from the generating algebraic units *i*i, *i*j, *i*k, *ik*, *j*, and the projected kissing number of 40 has significance, for example, in respect of the dual process needed in the production of 20 amino acids.

In $n = 5$ dimensions and higher, the 'n-simplex' or hypertetrahedron has $(n + 1)$ faces, each of which is an $(n - 1)$-simplex; the 'n-cube' or hypercube has $2n$ faces, each of which is an $(n - 1)$-cube; and the 'n-dimensional cross-polytope' (equivalent to a hyperoctahedron) has 2^n faces, each of which is an $(n - 1)$-simplex. While the n-simplex is self-dual, the n-cube and n-dimensional cross-polytope are duals of each other. There are no other Platonic solids in these dimensions. The special nature of 4 dimensions results from the fact that the unit sphere is a group, as it also is in 1 and 2, and no other dimensionality. The unit sphere in 1 dimension can be thought of as the two points represented by the real numbers –1 and 1, whereas, in 2 dimensions, it becomes the sphere of the unit complex numbers, $e^{i\theta}$. In 4 dimensions the unit sphere becomes the group of unit quaternions or $SU(2)$, which is also the *double cover* of the rotation group in 3 dimensions, with 2 elements of the group corresponding to each 3-dimensional rotation. Because of this relationship, we can show that, in 4-dimensions, the 24-cell becomes the group of symmetries of the tetrahedron, while the 600-cell becomes the group of symmetries of the icosahedron. We can also see the 'double cover' aspect emerging in physical and biological applications, and as being ultimately generated in the 'doubling' process of the universal rewrite system.

8-dimensionality is a particularly interesting case because it is in many respects, and certainly in geometrical algebra, the end of the line before repetition. The kissing number for an 8-dimensional space is 240, and we already have an 8-dimensional aspect to the fundamental algebra, with its 8 fundamental units, and in the concept of fermion space-time plus vacuum space-time, or 4 fermions plus 4 antifermions. 8 is also the dimension of the highest division algebra, or octonions, the next (and last) stage up from quaternions; and the 8 fundamental units have many octonion-like properties. (They are not octonions, because they preserve associativity, but they can be mapped onto octonions in a way which avoids this.) In 8 dimensions, we have the Gosset polytope, which has 240 vertices, and which can be conveniently visualized by being projected onto a 3-dimensional space. This has 240 vertices (which could represent particle states plus vacuum states), and these can also be associated with the 240 root vectors of the rank 8 exceptional group E_8 (which have an icosahedral / dodecahedral structure). This is the highest group generated from the symmetry of the octonions, and has long been considered as a possible candidate for the group that would unify all the particles and interactions. Its subgroups include the other exceptional groups, derived from the octonions, E_7, E_6, F_4 and G_2, in addition to $O(16)$, $SU(8)$, and the groups considered to be important in particle physics, $SU(5)$, $SU(3)$, $SU(2)$ and $U(1)$. Because it is an exceptional group, it can incorporate both fermions and bosons in a single representation, and, in addition, because it has 8 additional dimensions, in addition to the 240 root vectors, it can also incorporate mass and gravity. The total process of transcription and translation in DNA also involves 240 units, 60 for codons on the sense strand of the DNA and 60 for the codons on the antisense strands, in addition to 60 for mRNA and 60 for tRNA. Here, we note that four individual units are needed for each codon / anticodon combination, in the same way as four fermion / antifermion units are needed for each boson. In addition, if we include the four extra (mostly STOP) codons in DNA, we produce the number 248 associated with the E_8 *group*, rather than the root vector lattice.

It is significant, that, in 8 dimensions, there is no need for the icosahedron / dodecahedron structure and for 5-fold symmetry breaking, because all 8 dimensions can be seen at once. The 5-fold symmetry breaking is only apparent in 3D. The nonperiodic icosahedral or 5-fold structure of quasicrystals is removed when the spatially observed data is plotted onto a 6-D space, with axes drawn so as to connect opposite vertices of the icosahedron.

Conclusion

It would seem that many different branches of mathematics – geometrical algebra, the Platonic solids and edge-transitive polyhedra, the Fibonacci series, exceptional groups, higher dimensional geometry, the quintic equation – can be integrated into a coherent pattern with the universal rewrite system, and shown to be fundamental to the information processing that drives many of the structures and processes in both biological and physical worlds. Specific applications may be expected to follow, in addition to those outlined in this and in earlier papers.

Appendix: Experimental confirmation of Nature's code

It is interesting to note that, since our initial publications, there have been a number of experimental findings which could be interpreted as providing confirmation of our theoretical predictions. Tystovich et al (2007) have now shown that 'Complex plasmas may naturally self-organize themselves into stable interacting helical structures that exhibit features normally attributed to organic living matter', with 'thermodynamic and evolutionary features thought to be peculiar only to living matter such as bifurcations that serve as 'memory marks', self-duplication, metabolic rates in a thermodynamically open system, and non-Hamiltonian dynamics'. This is in addition to the discovery of a double helical 'DNA-type' nebula, about 80 light years long, and indicative of 'a high degree of order', about 300 light-years from the centre of the Milky Way (Morris et al, 2006). Another significant finding is the observation of 'remarkably long-lived electronic quantum coherence' in a photosynthetic complex (Engel et al, 2007). Here, the authors state (in line with our general predictions about the most efficient processing system) that: 'This wavelike characteristic of the energy transfer within the photosynthetic complex can explain its extreme efficiency, in that it allows the complexes to sample vast areas of phase space to find the most efficient path.' In connection with the emphasis by Marcer et al (2005) on the quantum Carnot engine principle (operating at a single temperature) in self-organized systems governed by the universal rewrite system (whether physical or biological), it is notable that biological 'machines' of size less than 10 μm are necessarily isothermal.

A new classification scheme has been proposed (Wilhelm and Nikolajewa, 2004) for the genetic code that is based upon a binary representation of the purines and pyrimidines. This new scheme has led to new ideas about the evolution of the genetic code. The hypothesis is that it started with a binary doublet code and developed via a quaternary doublet code into the contemporary triplet code (Figure 1). This scheme supports the work presented here regarding the theory of Nature's code.

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References

Engel, G. S. Calhoun, T. R., Read, E. L., Ahn, T.-K., Mančal, T., Cheng, Y.-C., Blankenship, R. E., and Fleming, G. R. 2007, Evidence for wavelike energy transfer through quantum coherence in photosynthetic systems *Nature* 446, 782-786.

Hill, V. and Rowlands, P., 2007. Nature's code, in K. Bowden (ed.), *Foundations* (*Proceedings of XXVIII ANPA Conference*, Cambridge, August 2006), 282-300.

Hill, V. and Rowlands, P. 2008a. Nature's code II, in K. Bowden and A. D. Ford (ed.), *Exactness* (*Proceedings of XXIX ANPA Conference*, Cambridge, August 2007), 160-177.

Hill, V. and Rowlands, P. 2008a. Nature's code, to be published in *AIP Conference Proceedings* (presented at CASYS, Liège, August 2007).

Kauffman, L. H., 2008. Fibonacci form and the quantized mark, in K. Bowden and A. D. Ford (ed.), *Exactness*, 2008, 183-218.

Marcer, P., Mitchell, E., Rowlands, P. and Schempp, W. 2005. Zenergy: The 'phaseonium' of dark energy that fuels the natural structures of the

Universe, *International Journal of Computing Anticipatory Systems*, 16, 189-202.
Marcer, P. and Rowlands, P., 2009. Riemann hypothesis, to be published in *IJCAS* (presented at CASYS, Liège, August 2007).
Morris, M., Uchida, K. and Do, Tuan. 2006. A magnetic torsional wave near the Galactic Centre traced by a 'double helix' nebula, *Nature*, 440, 308.
Rowlands, P., 2007. *Zero to Infinity The Foundations of Physics*, World Scientific.
Rowlands, P. and Hill, V., 2006. Fundamental structures applied to physics and biology, in K. Bowden (ed.), *Conceptions* (*Proceedings of XXVII ANPA Conference*, Cambridge, August 2005), 324-60.
Tsytovich, V. N., Morfill, G. E., Fortov, V. E., Gusein-Zade, N. G., Klumov, B. A. and Vladimirov, S. V. 2007. From plasma crystals and helical structures towards inorganic living matter, *New Journal of Physics* 9, 263-73.
Wikman, G., 2008. Different but not Distinct, in K. Bowden and A. D. Ford (ed.), *Exactness*, 267-334.
Wilhelm, T. and Nikolajewa, S., 2004. A new classification scheme of the genetic code. J. Mol. Evol. 59, 598-605.

REFLEXIVITY, EIGENFORM AND FOUNDATIONS OF PHYSICS

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Abstract: This essay is a discussion of the concept of reflexivity and its relationships with self-reference, re-entry, eigenform and the foundations of physics.

I. Introduction

Reflexive is a term that refers to the presence of a relationship between an entity and itself. One can be aware of one's own thoughts. An organism produces itself through its own action and its own productions. A market or a system of finance is composed of actions and individuals, and the actions of those individuals influence the market just as the global information from the market influences the actions of the individuals. Here it is the self-relations of the market through its own structure and the structure of its individuals that moves its evolution forward. Nowhere is there a way to effectively cut an individual participant from the market and make him into an objective observer. His action in the market is concomitant to his being reflexively linked with that market. Just so for theorists of the market for their theories, if communicated become part of the action and decision-making of the market. Social systems partake of this same reflexivity, and so does apparently objective science and mathematics. In order to see the reflexivity of the practice of physical science or mathematics, one must leave the idea of an objective domain of investigation in brackets and see the enterprise as a large conversation among a group of investigators. Then, at once, the process is seen to be a reflexive interaction among the members of this group. Mathematical results, like all technical inventions, have a certain stability over time that gives them an air of permanence, but the process that produces these novelties is every bit as fraught with circularity and mutual influence as any other conversation or social interaction.

How then, shall we describe a reflexive domain? It is the purpose of this paper to give a very abstract definition that nevertheless captures, what I believe to be the main conceptual feature of reflexivity. We then immediately prove that eigenforms, fixed points of transformations, are present for all transformations of the reflexive domain. This will encourage us and it will give us pause.

The existence of eigenforms will encourage us, for we have previously studied them with the notion that "objects are tokens for eigenbehaviour". Eigenforms are the natural emergence of those tokens by way of recursion. So to find the eigenforms dictated by a larger concept is pleasing. But we shall also need to pause.

For the existence of fixed points for arbitrary transformations will show us that the domain we have postulated is indeed very wide.

It is not an objectively existing domain. It is a clearing in which structures can arise and new structures can arise. A reflexive domain is not an already-existing structure. Not at all. To be what it claims to be, a reflexive domain must be a combination of existing structure and an invitation to create new structure and new concepts. The new will become platforms from which further flights of creativity can be made. Thus in the course of examining the concept of reflexivity we will find that the essence of the matter is an opening into creativity, and that will become the actual theme of this paper.

We are particularly interested in the way these concepts of reflexivity affect fundamentals of topology and fundamentals of physics. The last parts of this essay are a reformulation of elementary mathematics of matrices, complex numbers and exponentials in terms of process, reflexivity and eigenform.

We then show how quantum mechanics and discrete physics acquire a new point of view in the light of these interpretations. The reader may wish to skip directly to Section XII to see how this part of the argument proceeds.

Our essay begins with explication of the notion of eigenform as pioneered by Heinz von Foerster in his papers [4, 5, 6, 7] and explored in papers of the author [11, 12]. In [5] The familiar objects of our existence can be seen to be nothing more than tokens for the behaviors of the organism, creating apparently stable forms. Such an attitude toward objects makes it impossible to discriminate between the object as an element of a world and the object as a token or symbol that is simultaneously a process.

The notion of an eigenform is inextricably linked with second order cybernetics. One starts on the road to such a concept as soon as one begins to consider a pattern of patterns, the form of form or the cybernetics of cybernetics. Such concepts appear to loop around upon themselves, and at the same time they lead outward to new points of view. Such circularities suggest a possibility of transcending the boundaries of a system within. When the circular concept is called into being, the boundaries turn inside out.

An object , in itself , is a symbolic entity, participating in a network of interactions, taking on its apparent solidity and stability from these interactions. We ourselves are such objects, we as human beings are "signs for ourselves", a concept originally due to the American philosopher C. S. Peirce [10]. Eigenforms are mathematical companions to Peirce's work.

In an observing system, what is observed is not distinct from the system itself, nor can one make a separation between the observer and the observed. The observer and the observed stand together in a coalescence of perception. From the stance of the observing system all objects are non-local, depending upon the presence of the system as a whole. It is within that paradigm that these models begin to live, act and enter into conversation with us.

After this journey into objects and eigenforms, we take a wider stance and consider the structure of spaces and domains that partake of the reflexivity of object and process. We make a definition of a *reflexive domain* (compare [1] and [18]). Our definition populates a space (domain) with entities that could be construed as objects, and we assume that each object acts as a transformation on the space. Essentially this means that given entities A and B, then there is a new entity C that is the result of A and B acting together in the order AB (so that one can say that "A acts on B" for AB and one can say "B acts on A" for BA). This means that the reflexive space is endowed with a non-commutative and non-associative algebraic structure. The reflexive space is expandable in the sense that whenever we define a process, using entities that have already been constructed or defined, then that process can take a name, becoming a new entity/transformation of a space that is expanded to include itself.. Reflexive spaces are open to evolution in time, as new processes are invented and new forms emerge from their interaction.

Remarkably, reflexive spaces always have eigenforms for every element/transformation/entity in the space! The proof is simple but requires discussion.

Given F in a reflexive domain, define G by Gx = F(xx).
Then GG = F(GG) and so GG is an eigenform for F.

Just as promised, in a reflexive domain, every entity has an eigenform. From this standpoint, one should start with the concept of reflexivity and see that from it emerge eigenforms. Are we satisfied with this approach? We are not satisfied. For in order to start with reflexivity, we need to posit objects and processes. As we have already argued in this essay, objects are tokens for eigenbehaviours. And a correct or natural beginning is process where objects are seen as tokens of processes.

By now the reader begins to see that the story we have to tell is a circular one. We give a way to understand this circularity with Section X where we discuss creativity in recursive process and the emergence of novelty.

The reader will see that we have woven a tale the goes back and forth between recursion and idealized eigenforms. This means that we are sometimes considering abstractions such as reflexive domains and their algebraic properties and we are sometimes looking at the particulars of recursions directly related to automata or to specific complex numbers. Here follows a précis of the paper from the point of view of both the algebras and the physics.

This paper explores the analogies of fixed points, observations and observables, eigenvectors and recursive processes in relation to foundations of physics. In particular we shall re-open the books on the complex numbers and view them in terms of recursion and reflexivity, finding new and natural ways to think about their roles in physical theory (Section XIII).

To give a hint, think of the oscillatory process generated by $\mathbf{R(x) = -1/x}$. The fixed point is $\mathbf{i}$ with $\mathbf{i^2 = -1}$, but the processes generated over the real numbers must be directly related to the idealized i. We shall let $\mathbf{I\{+1,-1\}}$ stand for an undisclosed alternation or ambiguity between $\mathbf{+1}$ and $\mathbf{-1}$ and call $\mathbf{I\{+1,-1\}}$ an *iterant.* There are two *iterant views*: **[+1,-1]** and **[-1,+1]**. These, seen as points of view of alternating process will become the square roots of negative unity. We introduce a temporal shift operator η such that

$$[a,b]\eta = \eta[b,a] \text{ and } \eta\eta = 1$$

so that concatenated observations can include a time step of one-half period of the process **...abababab...** . We combine iterant views term-by-term as in **[a,b][c,d]** = **[ac,bd]**. Then we have, with i = **[1,-1]**η (**i** is view/operator),

$$ii = [1,-1]\eta\ [1,-1]\eta = [1,-1][-1,1]\eta\ \eta = [-1,-1] = -1.$$

This gives rise to a new process-oriented construction of the complex numbers, quaternions, and in fact of all of matrix algebra.

We relate this point of view to thinking about the role of complex numbers in quantum mechanics and the role of temporal shift operators in discrete physics, that begins with the understanding that temporal shift operators allow discrete calculus to be represented in a non-commutative (Lie algebraic) context where all derivatives are represented by commutators. (Section XIV.)

We also relate these ideas of reflexivity and fixed points to left or right distributive non-associative algebras and their relationships with knot theory in Section VI. We relate this with approaches to wholeness in physics and philosophy such as the work of Barbara Piechosinska [16]. A *magma* is a non-associative algebra with a single binary operation that is left-associative:

$$a*(b*c) = (a*b)*(a*c).$$

Note that this axiom says that every element A of the magma is a structure preserving mapping of the magma to itself:

$$A(x*y) = (A*x)*(A*y).$$

The notion of a magma is another view of what should be a self-reflexive domain. We raise questions about the relationship of magmas and reflexive domains and, in Section VI, illustrate the remarkable and deep relationships among magmas and knots and braids.

II. Objects as Tokens for Eigenbehaviours

In his paper "Objects as Tokens for Eigenbehaviours" [5] von Foerster suggests that we think seriously about the mathematical structure behind the constructivist doctrine that *perceived worlds are worlds created by the observer.* At first glance such a statement appears to be nothing more than solipsism. At second glance, the statement appears to be a tautology, for who else can create the rich subjectivity of the immediate impression of the senses? At third glance, something more is needed. In that paper he suggests that the familiar objects of our experience are the fixed points of operators. These operators *are* the structure of our perception. To the extent that the operators are shared, there is no solipsism in this point of view. It is the beginning of a mathematics of second order cybernetics.

Consider the relationship between an observer O and an "object" A. The key point about the observer and the object is that "the object remains in constant form with respect to the observer". This constancy of form does not preclude motion or change of shape. Form is more malleable than the geometry of Euclid. In fact, ultimately the form of an "object" is the form of the distinction that "it" makes in the space of our perception. In any attempt to speak absolutely about the nature of form we take the form of distinction for the form. (paraphrasing Spencer-Brown [3]). It is the form of distinction that remains constant and produces an apparent object for the observer. How can you write an equation for this? The simplest route is to write

$$\mathbf{O(A) = A.}$$

The object **A** is a fixed point for the observer **O**. The object is an eigenform. We must emphasize that this is the most schematic possible description of the condition of the observer in relation to an object **A**. We only record that the observer as an actor (operator) manages through his acting to leave the (form of) the object unchanged. This can be a recognition of the symmetry of the object but it also can be a description of how the observer, searching for an object, makes that object up (like a good fairy tale) from the very ingredients that are the observer herself. This is the situation that Heinz von Foerster has been most interested in studying. As he puts it, if you give a person an undecideable problem, then the answer that he gives you is a description of himself. And so, by working on hard and undecideable problems we go deeply into the discovery of who we really are. All this is symbolized in the little equation **O(A) = A.**

And what about this matter of the object as a token for eigenbehaviour? This is the crucial step. We forget about the object and focus on the observer. We attempt to "solve" the equation **O(A) = A** with **A** as the unknown. Not only do we admit that the "inner" structure of the object is unknown, we adhere to whatever knowledge we have of the observer and attempt to find what such an observer could observe based upon that structure.

We can start anew from the dictum that the perceiver and the perceived arise together in the condition of observation. This is a stance that insists on mutuality (neither perceiver nor the perceived causes the other). A distinction has emerged and with it a world with an observer and an observed. The distinction is itself an eigenform.

III. Compresence and Coalescence

We, identify the world in terms of how we shape it. We shape the world in response to how it changes us. We change the world and the world changes us. Objects arise as tokens of a behavior that leads to seemingly unchanging forms. Forms are seen to be unchanging through their invariance under our attempts to change, to shape them.

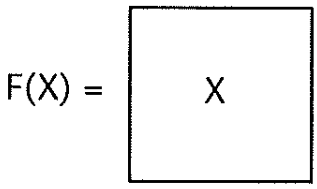
For an observer there are two primary modes of perception -- *compresence* and *coalesence*. Compresence connotes the coexistence of separate entities together in one including space. Coalesence connotes the one space holding, in perception, the observer and the observed, inseparable in an unbroken wholeness. Coalesence is the constant condition of our awareness. Coalesence is the world taken in simplicity. Compresence is the world taken in apparent multiplicity.

This distinction of compresence and coalesence, drawn by Henri Bortoft [2], can act as a compass in traversing the domains of object and reference. *Eigenform is a first step towards a mathematical description of coalesence.* In the world of eigenform the observer and the observed are one in a process that recursively gives rise to each.

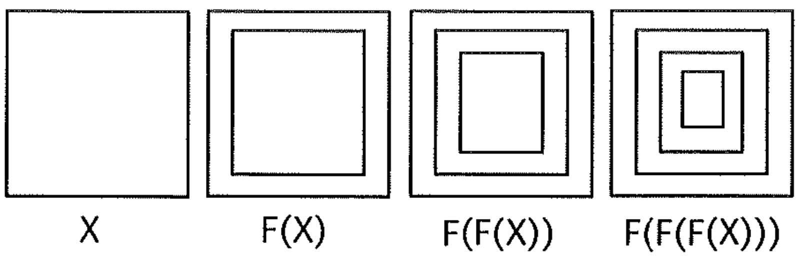
IV. The Eigenform Model

We have seen how the concept of an object has evolved to make what we call objects (and the objective world) processes that are interdependent with the actions of observers. The notion of a fixed object has become a notion of a process that produces the apparent stability of the object. This process can be simplified in a model to become a recursive process where a rule or rules are applied time and time again. The resulting object of such a process is the *eigenform* of the process, and the process itself is the *eigenbehaviour*.

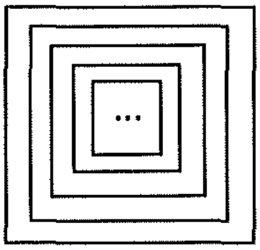
In this way we have a model for thinking about object as token for eigenbehaviour. This model examines the result of a simple recursive process carried to its limit. For example, suppose that

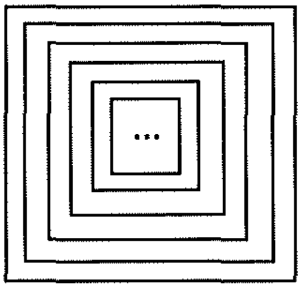


That is, each step in the process encloses the results of the previous step within a box. Here is an illustration of the first few steps of the process applied to an empty box X:



If we continue this process, then successive nests of boxes resemble one another, and in the limit of infinitely many boxes, we find that

X=F(F(F(...)))= 

F(X) =  = X

the infinite nest of boxes is invariant under the addition of one more surrounding box. Hence this infinite nest of boxes is a fixed point for the recursion. In other words, if X denotes the infinite nest of boxes, then

$$\mathbf{X = F(X).}$$

This equation is a description of a state of affairs. The form of an infinite nest of boxes is invariant under the operation of adding one more surrounding box. The infinite nest of boxes is one of the simplest eigenforms.

In the process of observation, we interact with ourselves and with the world to produce stabilities that become the objects of our perception. These objects, like the infinite nest of boxes, may go beyond the specific properties of the world in which we operate. They attain their stability through the limiting process that goes outside the immediate world of individual actions. We make an imaginative leap to complete such objects to become tokens for eigenbehaviours. It is impossible to make an infinite nest of boxes. We do not make it. We *imagine* it. And in imagining that infinite nest of boxes, we arrive at the eigenform.

The leap of imagination to the infinite eigenform is a model of the human ability to create signs and symbols. In the case of the eigenform X with X = **F(X)**, X can be regarded as the name of the process itself or as the name of the limit process. Note that if you are told that

$$\mathbf{X = F(X),}$$

then substituting **F(X)** for **X**, you can write

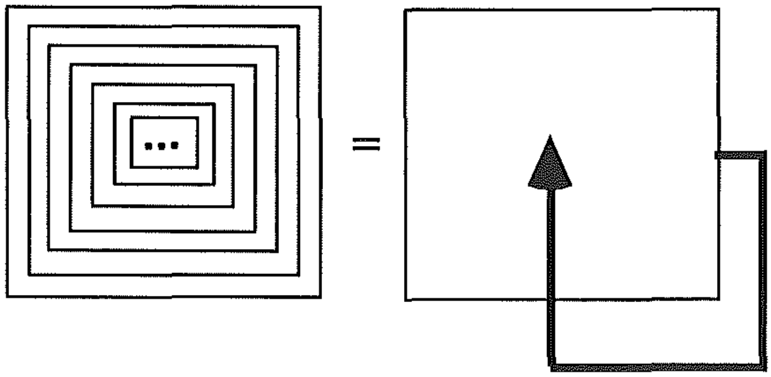
$$\mathbf{X = F(F(X)).}$$

Substituting again and again, you have

$$X = F(F(F(X))) = F(F(F(F(X)))) = F(F(F(F(F(X))))) = \ldots$$

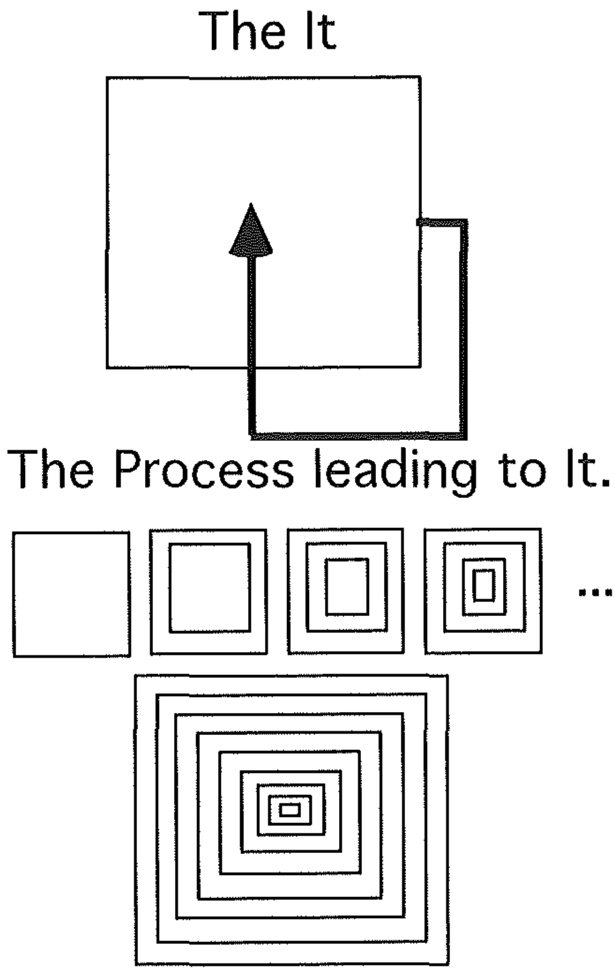
The process arises from the symbolic expression of its eigenform. In this view *the eigenform is an implicate order for the process that generates it.*

Sometimes one stylizes the structure by indicating where the eigenform **X** reenters its own indicational space by an arrow or other graphical device. See the picture below for the case of the nested boxes.



Does the infinite nest of boxes exist? Certainly it does not exist in this page or anywhere in the physical world with which we are familiar. The infinite nest of boxes exists in the imagination. It is a symbolic entity.

Eigenform is the imagined boundary in the reciprocal relationship of the object (the "It") and the process leading to the object (the process leading to "It"). In the diagram below we have indicated these relationships with respect to the eigenform of nested boxes. Note that the "It" is illustrated as a finite approximation (to the infinite limit) that is sufficient to allow an observer to infer/perceive the generating process that underlies it.



Just so, an object in the world (cognitive, physical, ideal,...) provides a conceptual center for the exploration of a skein of relationships related to its context and to the processes that generate it. An object can have varying degrees of reality just as does an eigenform. If we take the suggestion to heart that objects are tokens for eigenbehaviours, then an object in itself is an entity, participating in a network of interactions, taking on its apparent solidity and stability from these interactions.

An object is an amphibian between the symbolic and imaginary world of the mind and the complex world of personal experience. The object, when viewed as process, is a dialogue between these worlds. The object when seen as a sign for itself, or in and of itself, is imaginary.

Why are objects apparently solid? Of course you cannot walk through a brick wall even if you think about it differently. I do not mean apparent in the sense of thought alone. I mean apparent in the sense of appearance. The wall appears solid to me because of the actions that I can perform. The wall is quite transparent to a neutrino, and will not even be an eigenform for that neutrino.

This example shows quite sharply how the nature of an object is entailed in the properties of its observer.

The eigenform model can be expressed in quite abstract and general terms. Suppose that we are given a recursion (not necessarily numerical) with the equation

$$\mathbf{X(t+1) = F(X(t))}.$$

Here **X(t)** denotes the condition of observation at time **t.** **X(t)** could be as simple as a set of nested boxes, or as complex as the entire configuration of your body in relation to the known universe at time **t.** Then **F(X(t))** denotes the result of applying the operations symbolized by **F** to the condition at time **t**. You could, for simplicity, assume that **F** is independent of time. Time independence of the recursion **F** will give us simple answers and we can later discuss what will happen if the actions depend upon the time. In the time independent case we can write

$$\mathbf{J = F(F(F(...)))}$$

the infinite concatenation of F upon itself. Then

$$\mathbf{F(J) = J}$$

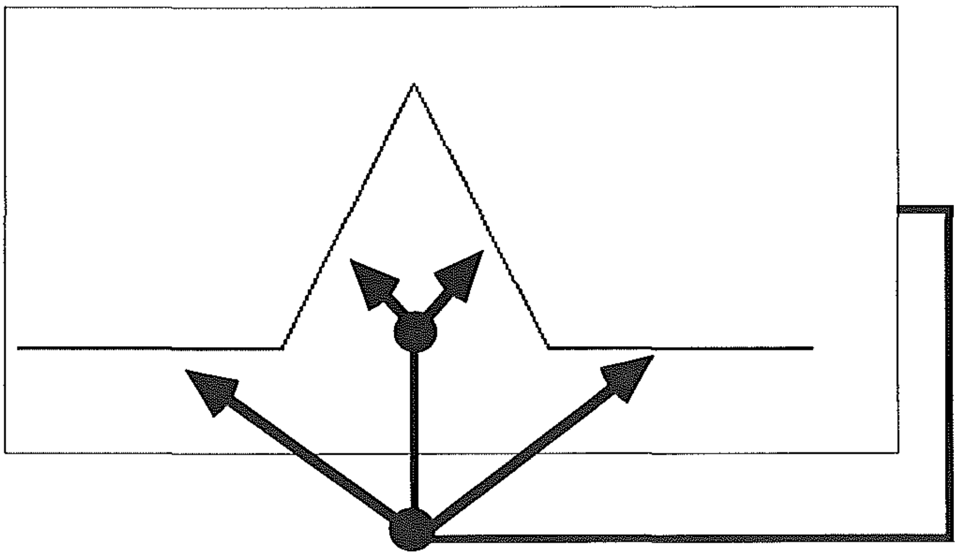
since adding one more **F** to the concatenation changes nothing.

Thus **J,** the infinite concatenation of the operation upon itself leads to a fixed point for **F. J** is said to be the eigenform for the recursion **F.** We see that every recursion has an eigenform. Every recursion has an (imaginary) fixed point.

We end this section with one more example. This is the eigenform of the Koch fractal [14]. In this case one can write symbolically the eigenform equation

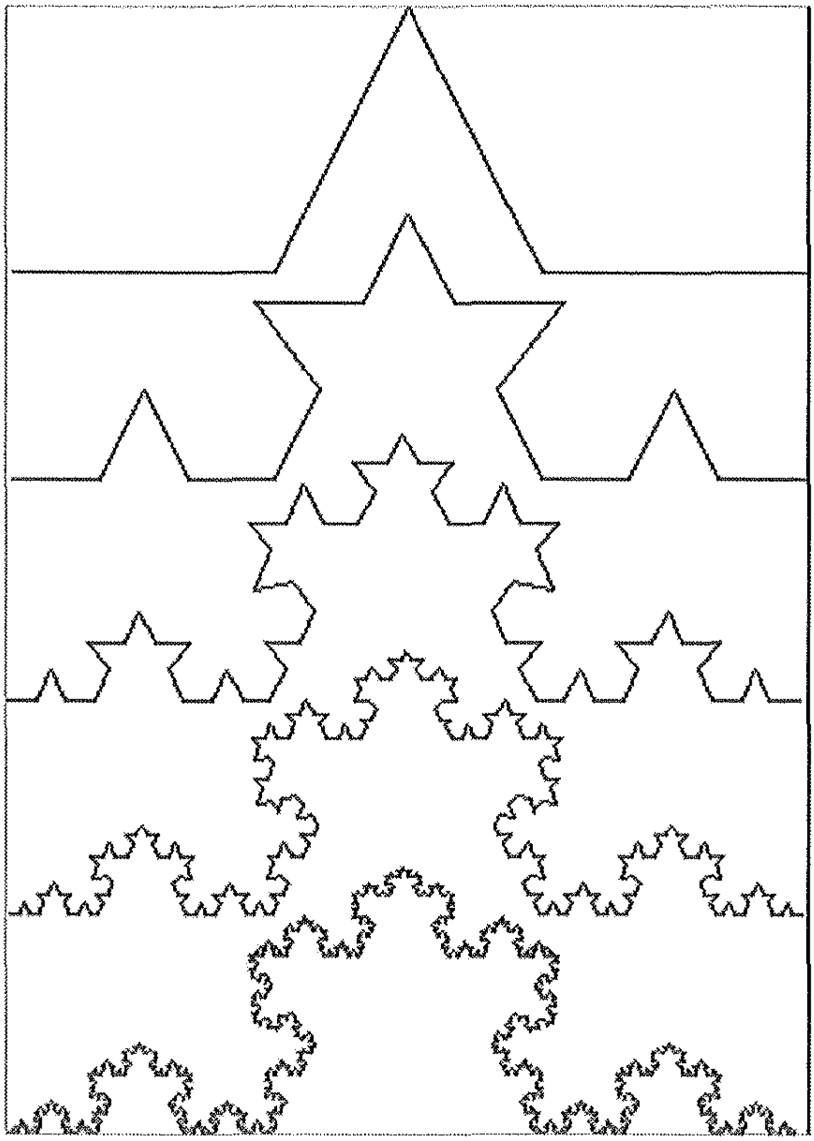
$$K = K\{\ K\ K\ \}K$$

to indicate that the Koch Fractal reenters its own indicational space four times (that is, it is made up of four copies of itself, each one-third the size of the original. The curly brackets in the center of this equation refer to the fact that the two middle copies within the fractal are inclined with respect to one another and with respect to the two outer copies. In the figure below we show the geometric configuration of the reentry.



$$K = K \{ K K \} K$$

In the geometric recursion, each line segment at a given stage is replaced by four line segments of one third its length, arranged according to the pattern of reentry as shown in the figure above. The recursion corresponding to the Koch eigenform is illustrated in the next figure. Here we see the sequence of approximations leading to the infinite self-reflecting eigenform that is known as the Koch snowflake fractal.



Five stages of recursion are shown. To the eye, the last stage vividly illustrates how the ideal fractal form contains four copies of itself, each one-third the size of the whole. The abstract schema

$$K = K \{ K K \} K$$

for this fractal can itself be iterated to produce a "skeleton" of the geometric recursion:

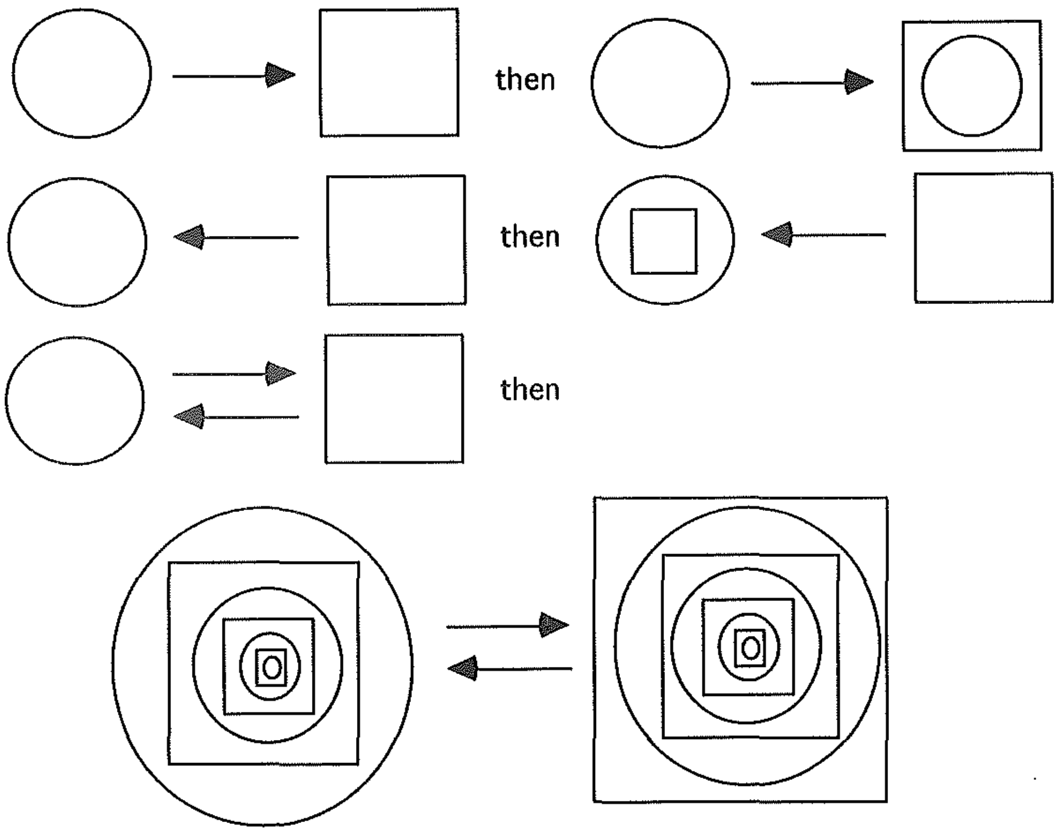
$$\begin{aligned} K &= K \{ K K \} K \\ &= K \{ K K \} K \ \{ \ K \{ K K \} K \ \ K \{ K K \} K \ \} K \{ K K \} K \\ &= \ldots \end{aligned}$$

We have only performed one line of this skeletal recursion. There are sixteen **K**'s in this second expression just as there are sixteen line segments in the second stage of the geometric recursion. Comparison with this symbolic recursion shows how geometry aids the intuition. The interaction of eigenforms with the geometry of physical, mental, symbolic and spiritual landscapes is an entire subject that is in need of deep exploration.

It is usually thought that the miracle of recognition of an object arises in some simple way from the assumed existence of the object and the action of our perceiving systems. This is a fine tuning to the point where the action of the perceiver and the perception of the object are indistinguishable. Such tuning requires an intermixing of the perceiver and the perceived that goes beyond description. Yet in the mathematical levels, such as number or fractal pattern, part of the process is slowed down to the point where we can begin to apprehend the process. There is a stability in the comparison, in the correspondence that is a process happening at once in the present time. The closed loop of perception occurs in the eternity of present individual time. Each such process depends upon linked and ongoing eigenbehaviours and yet is seen as simple by the perceiving mind. The perceiving mind is itself an eigenform.

Mirror-Mirror

In the next figure we illustrate how an eigenform can arise from a process of mutual reflection. The figure shows a circle with a an arrow pointing to a rectangle and a rectangle with an arrow pointing toward a circle. For this example, we take the rule that an arrow between two entities (P -----> Q) means that the second entity will create an internal image of the first entity (Q will make an image of P). If P -----> Q and Q ------> P, then each entity makes an image of the other. A recursion will ensue. Each of P and Q generates eigenforms in this mutuality.



In this example we can denote the initial forms by C (for circle) and B (for box). We have C -----> B and B -----> C. The rule of imaging is (symbolically):

If P -----> Q then P -----> QP.
If P < ------ Q, then PQ <------ Q.

We start with the mutual reference C < -----> B. This condition of mutual mirroring can be described by two operators C and B:

C(P)= CP corresponds to C -----> P.
B(Q) = BQ corresponds to Q < ----- B.

We are solving the eigenform equations

$$C(Y) = X,$$
$$B(X) = Y.$$

We have the mirror-mirror solution

$$X = BCBCBCBC...,$$
$$Y = CBCBCBCB...,$$

just as in the Figure.

We are quite familiar with this form of mutual mirroring in the physical realm where one can have two facing mirrors, and in the realm of human relations where the complexity of exchange (mutual mirroring) between two individuals leads to the eigenform of their relationship.

V. Reflexive Domains

A reflexive domain D is an arena where actions and processes that transform the domain can also be seen as the elements that compose the domain. Every element of the domain can be seen as a transformation of the domain to itself.

In actual practice an element of a domain may be a person or company (collective of persons) or a physical object or mechanism that is seen to be in action. In actual practice we must note that what are regarded as objects or entities depends upon the way in which observers inside or outside the domain divide their worlds.

It is very difficult to make a detailed mathematical model of such situations. Each actor is an actor in more than one play. His actions undergo separate but related interpretations, depending upon the others with whom he interacts. Mutual feedback of a multiplicity of ongoing processes is not easily described in the Platonic terms of pure mathematics.

Nevertheless, we take as a general principle for a mathematical model that D is a certain set (possibly evolving in time), and we let [D,D] denote a selected collection of mappings from D to D. An element F of [D,D] is a mapping F:D ----->D.

We shall assume that there is a 1-1 correspondence I:D -----> [D,D]. This is the assumption of reflexivity. Every element of the reflexive domain is a transformation of that domain. Each denizen of the reflexive domain has a dual role of actor and actant.

Given an element g in D, I(g):D ----> D is a mapping from D to D, and for every mapping F:D ----> D, there is an element g in D such that I(g) = F. The reflexive domain embodies a perfect correspondence between actions, and entities that are the recipients of these actions.

An important precursor to this notion of reflexive domain in mathematics is the notion of Goedel numbering of texts. One chooses a method to encode a text as a specific natural number (a certain product of prime powers). Then texts that speak about numbers can, in principle speak about other texts and even about themselves. If a text is seen as a transformation on the field of numbers, then that text is itself a number (its Goedelian code) and so can be transforming itself. The precision of this idea enabled Goedel to construct mathematical systems that could talk about their own properties without contradiction and he showed that all sufficiently rich mathematical systems have this property. In this way, these systems become self-limiting due to the possibility of statements whose coded meaning becomes "This statement has no proof in the system of mathematics in which it is written," while the surface meaning of the same statement is a discussion of the properties of certain numerical relations. The domain of numerical relations appears innocuous, and yet it sows the seeds of its own limitations through this ability to reflect itself through the mirror of the Goedel coding.

The Goedelian example is not just a piece of mathematics. It is a reflection with mathematical precision of the condition of our language, thought and action. We are always equipped to comment on our own doings and in so doing to create new language about our old language and new language about our worlds. All our apparent well-thought-out and directed actions in worlds that seem to extend outward from us in an objective way are fraught with the circularity not just of our meta-comments, but also with the circular return of the consequences of those actions and the influence of our very theories of the world on the properties of that world itself.

We now prove a fundamental theorem about reflexive domains. We show that every mapping F:D -----> D has a fixed point p, an element p in D such that F(p) = p. What does this mean? It means that there is another way, in a reflexive domain, to associate a point to a transformation. The point can be seen as the fixed point of a transformation and in that way, the points of the domain disappear into the self-referential nature of the transformations.

Let me tender persuasions. Suppose that p = F(p). Then we can regard this equation as an expression of p in terms of F and itself and write

$$
\begin{aligned}
p &= F(p) \\
&= F(F(p) \\
&= F(F(F(p))) \\
&= F(F(F(F(p))))
\end{aligned}
$$

and continue in this fashion until the appearance of p on the right hand side is lost in the depths of the composition of F upon itself.

$$p = F(F(F(F(F(F(F(F(F(F(F(F(F(F(F(F(F(F(F(F(\ldots)))))))))))))))))))).$$

The infinite composition of F upon itself is invariant under one more composition with F and so F(p) = p is consistent with this process.

To show that an entity p is a fixed point for a process F is to show that p can be confused with the infinite concatenation of F upon itself. This is an image of the way objects become tokens for eigenbehaviours in the language of Heinz von Foerster [ ]. Later in this paper we will discuss the production of many examples of such eigenforms, fixed points of repeated transformations. For now, here is the proof of the fixed point theorem for reflexive domains.

Fixed Point Theorem. Let D be a reflexive domain with actor/actant correspondence F:D -----> [D,D]. Then every F in [D,D] has fixed point. That is, there exists a p in D such that F(p) = p.

Proof. Define G:D ----> D by the equation
Gx = F(I(x)x)for each x in D.
Since I:D -----> [D,D] is a 1-1 correspondence,
we know that G = I(g) for some g in D.
Hence Gx = I(g)x = F(I(x)x) for all x in D.
Therefore, letting x = g,
I(g)g = F(I(g)g) and so p = I(g)g is a fixed point for F.
Q.E.D.

We shall discuss this proof and its meaning right now in a series of remarks, and later in the paper in regard to examples that will be constructed.

Remark1.
Suppose that we reduce the notational complexity of our description of the reflexive domain by simply saying that for any two entities g and x in the domain there is a new entity gx that is the result of the interaction of g and x. (We think of gx as I(g)x = I(g) applied to x.) In mathematical terms, we define

$$gx = I(g)x.$$

Then the proof of the fixed point theorem appears in a simpler form: We define Gx = F(xx) and note that GG = F(GG). Thus GG is the fixed point for F!

I like to call G " F's Gremlin".

According to Webster [Webster's New Collegiate Dictionary , G. C. Merriam Publishers (1956)] a gremlin is "One of the impish foot-high gnomes whimsically blamed by airmen for interfering with motors, instruments, machine guns, etc.; hence any like disruptive elf."

This is an apt description of our G. At first G looks quite harmless. Applying G to any A we just apply A to itself and apply F to the result. GA = F(AA). The dangerous mixture is comes when it is possible to apply G to itself! Then GG = F((GG) and GG is sitting right in there surrounded by F and you cannot stop the action. Off goes the recursion

GG = F(GG)
= F(F(GG))
= F(F(F(F(GG))))
= F(F(F(F(F(F(F(F(GG))))))))

The diabolical nature of the Gremlin is that he represents a process that once started, is hard to stop. Such are the processes by which we make the world into a field of tokens and symbols and forget the behaviours and processes and reflexive spaces from which they came. Fixed points and self-references are the unavoidable fruits of reflexivity, and reflexivity is the natural condition in a universe where there is no complete separation of part from the whole.

Remark 2.
A reflexive domain is a place where actions and events coincide. An action as a mapping of the whole space, because there is no intrinsic separation of the local and the global. Feedback is an attempt to handle the lack of separation of part and whole by describing their mutual influence.

When we define a new element g of D via gx = F(x) for any mapping F:D---->D, and we have a notion of combination of elements of D: a,b -----> ab, then we can define gx = F(xx) and so get gg = F(gg). Here we have not made a big separation between the elements of D and the mappings, since each element g of D gives the mapping I(g)x = gx. But in fact, we could define ab = I(a)b in a reflexive domain.

Whenever anyone comes up with a transformation, we make that transformation into an element of the domain by the definition gx = F(x). We transmute verbs to nouns. The reflexive domain evolves.

The space is not given a priori. The space evolves in relation to actions and definitions. The road unfolds before us as we travel.

Remark 3.

We create languages for evolving concepts. The outer reaches of set theory (and category theory) lead to clear concepts, but these concepts are not themselves sets or categories. A good example is the famous Russellian concept of sets that are not members of themselves. Russell's concept is not a set. Another example is the concept of set itself. There is no set that is the set of all sets. This very limitation on the notion of set is its opening. It shows us that set theory is an evolving language.

Language and concepts expand in time.

Here is a transformation on sets: F(X) = {X}. The transform of a set X is the singleton set whose member is X. If X is not a member of itself, then F(X) is also not a member of itself. But a fixed point of the transformation F is an entity U such that {U} = U. We have shown that within the domain of sets that are not members of themselves, there is no fixed point for the transformation X -----> {X}. This fragment of set theory (sets that are not members of themselves) is not yet a reflexive domain. We shall at least allow sets that are members of themselves if we wish to have a set theory with reflexivity.

Remark 4.

Transcendence

The leap to infinity via self-reference.
The production of the finity of a new level of infinity.
The completion of an incompletion.
The emergence of eternity from the world of time.

How then is observation different from action? If observation is a form of recursion coupled with the production of the finity of the limiting form, then observation is a transcendence to a new level. The model of observation as simple eigen-vector must be shifted to observation as the production of eigenform. It is not enough to produce eigenform. The fixed point is itself an active element and can itself engage in transformation

In the creation of spaces of conversation for human beings, we partake of a reflexivity of action and apparent object, where it is seen that every local manifestation of process, every seemingly fixed entity in a moving world is an indicator of global transformation. The local and the global intertwine in a reflexive and cybernetic unity.

Retuning (returning/tuning/retuning) to thoughts of reflexivity.
One creates by going outside oneself, but the creation returns in the form of a conversation with one's self. There is a feedback loop between the person/designer and the world that she makes. Each acts in the creation of the other. Priorities may be assigned, but it is the loop that interests us, and the possibility of stability (or at least temporal persistence) of what is created in that loop.

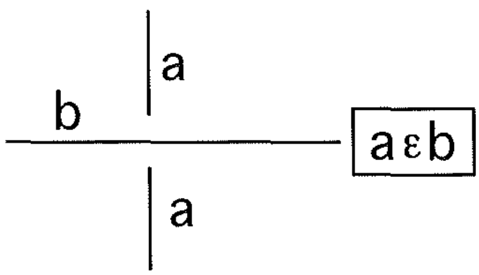
VI. Knot Sets, Topological Eigenforms, Quandles and Right and Left Distributivity

We shall use knot and link diagrams to represent sets. More about this point of view can be found in the author's paper "Knot Logic" [9]. In this notation the eigenset Ω satisfying the equation

$$\Omega = \{\Omega\}$$

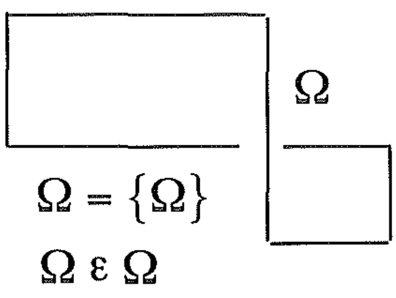
is a topological curl. If you travel along the curl you can start as a member and find that after a while you have become the container. Further travel takes you back to being a member in an infinite round. In the topological realm Ω does not have any associated paradox. This section is intended as an introduction to the idea of *topological eigenforms*, a subject that we shall develop more fully elsewhere.

Set theory is about an asymmetric relation called *membership.* We write **a** ε **S** to say that **a** is a member of the set **S.** In this section we shall diagram the membership relation as follows:

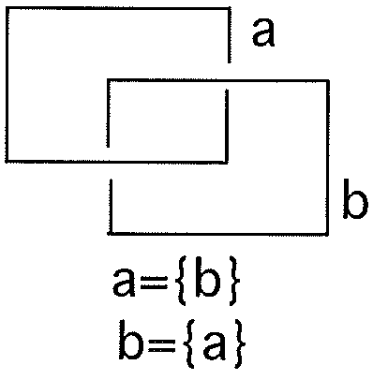


This is *knot-set notation.* In this notation, if b goes once under a, we write a={b}. If b goes twice under a, we write a={b,b}. This means that the "sets" are multi-sets, allowing more than one appearance of a member. For a deeper analysis of the knot-set structure see [KL].

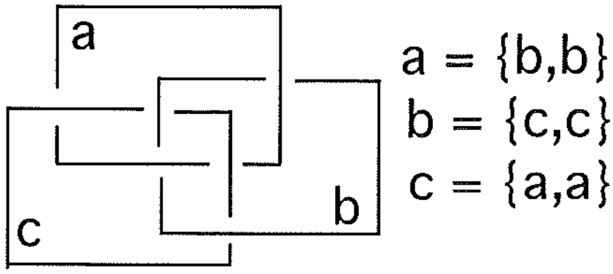
This knot-set notation allows us to have sets that are members of themselves,



and sets can be members of each other.



Here a mutual relationship of **a** and **b** is diagrammed as topological linking.

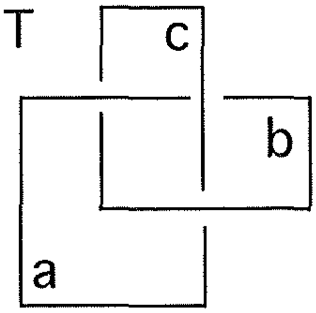


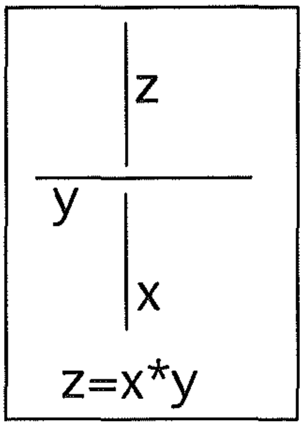
Here are the *Borromean Rings.* The Rings have the property that if you remove any one of them, then the other two are topologically unlinked. They form a topological tripartite relation. Their knot-set is described by the three equations in the diagram. Thus we see that this representative knot-set is a "scissors-paper-stone" pattern. Each component of the Rings lies over one other component, in a cyclic pattern.

To go beyond this first level of knot set theory we need to examine the formal structure of the relationships among the arcs on a link diagram.

Quandles and Colorings of Knot Diagrams

There is an approach to studying knots and links that is very close to our knot sets, but starts from a rather different premise. In this approach each arc of the diagram receives a label or "color". An arc of the diagram is a continuous curve in the diagram that starts at one undercrossing and ends at another undercrossing. For example, the trefoil diagram below has three arcs.





$$b = a*c$$
$$c = b*a$$
$$a = c*b$$

Each arc corresponds to an element of a "Trefoil Color Algebra" **IQ(T)** where **T** denotes the trefoil knot. We have that **IQ(T)** is generated by colors **a**,**b** and **c** with the relations

$$\begin{gathered} \mathbf{a*a = a}, \\ \mathbf{b*b = b}, \\ \mathbf{c*c = c}, \\ \mathbf{a*b = b*a = c}, \\ \mathbf{b*c = c*b = a}, \\ \mathbf{a*c = c*a = b}. \end{gathered}$$

Each of these relations in the diagram above is a description of one of the crossings in T. The full set of relations describes the coloring rules for an algebra that contains these relations and allows any two elements to be combined to a third element. This three-element algebra is particularly simple. If two colors are different, they combine to form the remaining third color. If two colors are the same, they combine to form the same color.

When we take an algebra of this sort, we want its coloring structure to be invariant under the Reidemeister moves (illustrated below). This means that when you make a new diagram from the old diagram by a topological move, the resulting new diagram inherits a unique coloring from the old diagram. Then one can see from this that the trefoil must be knotted since all diagrams topologically equivalent to it will carry three colors, while an unknotted diagram can carry only one color.

As the next diagram shows, invariance of the coloring rules under the Reidemeister moves implies the following *global relations on the algebra:*

$$\begin{gathered} \mathbf{x*x = x} \\ \mathbf{(x*y)*y = x} \\ \mathbf{(x*y)*z = (x*z)*(y*z)} \end{gathered}$$

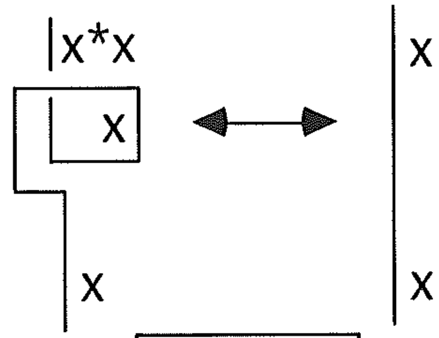
for any **x**, **y** and **z** in the algebra (set of colors) **IQ(T)**.

An algebra that satisfies these rules is called an *Involutory Quandle* [9], hence the initials **IQ**. Perhaps the most remarkable property of the quandle is its right-distributive law corresponding to the third Reidemeister move, as illustrated below. The reader will be interested to observe that in a multiplicative group **G**, the following operation satisfies all the axioms for the quandle: $g*h = hg^{-1}h$.

In an additive and commutative version of this axiom we can write $a*b = 2b - a$. Here the models that are most useful to the knot theorist are to take **a** and **b** to be elements of the integers **Z** or elements of the modular number system $Z/dZ = Z_d$ for some appropriate modulus **d**. The knot being analyzed restricts the modular possibilities. In the case of the trefoil knot the only possibility is $d = 3$, and in the case of the Figure Eight knot (shown after the Reidemeister moves below) the only possibility is $d = 5$.

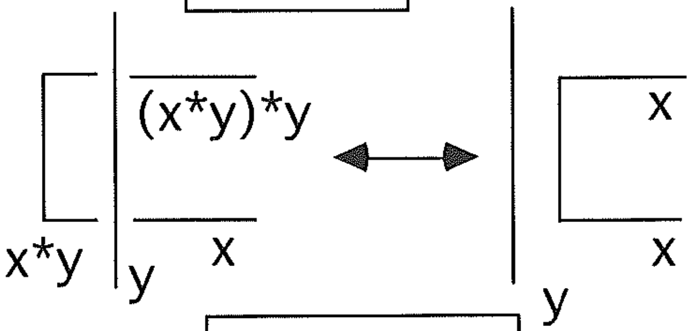
This analysis then shows that there cannot be any sequence of Reidemeister moves connecting the Trefoil and the Figure Eight. They are distinct knot types.

I.



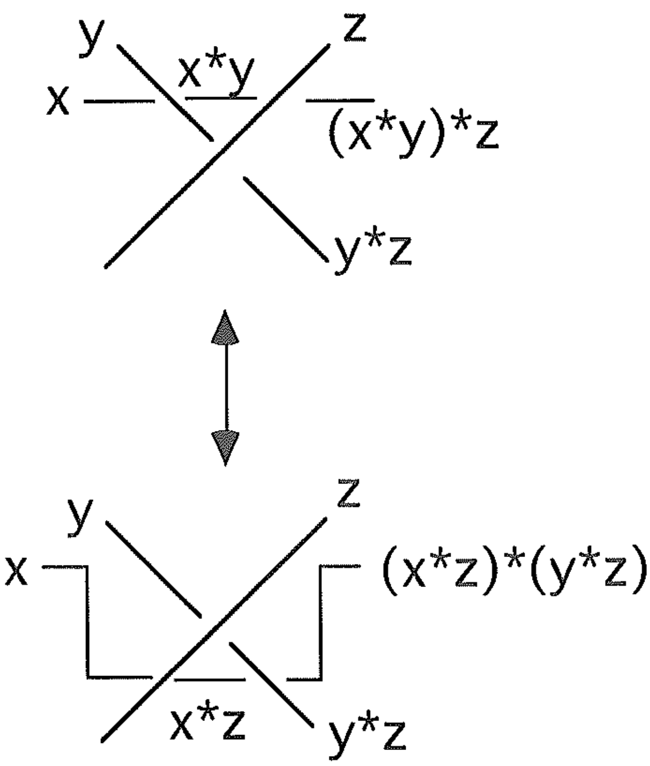
$$x*x=x$$

II.



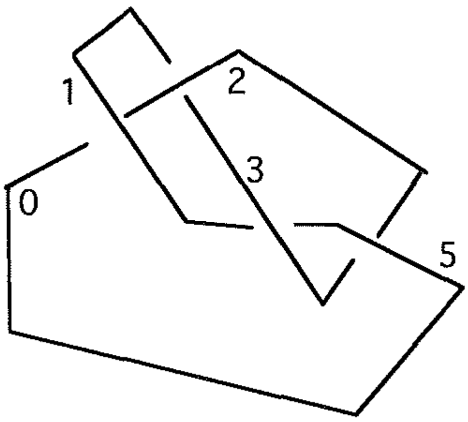
$$(x*y)*y = x$$

III.



$$(x*y)*z = (x*z)*(y*z)$$

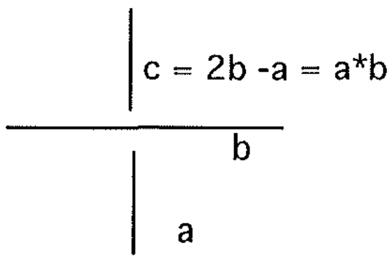
Here is the example for the Figure Eight Knot.



2 x 1 -0 = 1
2 x 2 -1 = 3
2 x 3 - 1 = 5
-> 0 = 5

Z/5Z = {0,1,2,3,4} with 0 = 5.

We have shown how an attempt to label the arcs of the knot according to the quandle rule



a*b = 2b -a, leads to a labeling of the Figure Eight knot in **Z/5Z.** In our illustration we have shown that there is a compatible coloring using four out of the five elements of **Z/5Z.** If you apply Reidemeister moves to the diagram for the Figure Eight knot you will see that other versions of the knot require all five colors. It is interesting to prove that there is no diagram of the Figure Eight knot that can be colored in less than four colors.

It should be noted that the knot diagrams give a remarkable picture of non-associative algebra structure and that each arc-label **a** in a diagram is both an element of the algebra and a transformation of the algebra to itself via the mapping $O_a(x) = x*a$. Note that the right distributivity of this operation has the equation

$$O_a(x*y) = (x*y)*a = (x*a)*(y*a) = O_a(x)*O_a(y)$$

That is, we have

$$O_a(x*y) = O_a(x)*O_a(y).$$

The right distributive law tells us that *each quandle operation is a quandle homomorphism*. That is, each quandle operation is a structure preserving mapping of the quandle to itself. This is an underlying algebraic meaning of the third Reidemeister move. Since the mappings O_a are invertible, we see that any quandle **Q** is in 1-1 correspondence with a certain collection of automorphisms of itself. In this sense a quandle is a reflexive domain with a limitation on the allowable collection of self-mappings. In fact we have a very simple fixed point theorem for quandles since

$$\mathbf{O_a(a) = a*a = a.}$$

Every element of the quandle is fixed by its own automorphism. Since we take **[Q,Q]** to be the set of mappings of **Q** to itself of the form $\mathbf{O_a(x) = x*a}$, we see that any quandle is a reflexive domain of a restricted sort. (Not every set theoretic mapping of **Q** to **Q** is realized in the above manner.)

How far is the quandle from being a reflexive space in the full sense of the word? Lets look at the fixed point construction. We define **G(x) = (x*x)*F** for a given element F of the quandle . Is it then the case that **(x*x)*F = x*g** for some g in the quandle? The answer is yes, but for very simple reason: We have **x*x = x** so that **(x*x)*F = x*F** and consequently **(F*F)*F = F*F**. In fact **F*F = F** so **F** is already its own fixed point. We see therefore that in a quandle the fixed point theorem is satisfied automatically due to the axiom **x*x = x** for all **x.**

On the other hand if **F:Q ---->Q** is an arbitrary mapping from **Q** to **Q,** then we cannot expect that **F** will have a fixed point. Suppose, for example, we define **F(x) = (x*(a*x)** and use the Trefoil quandle. Then

$$\mathbf{F(a) = (a*(a*a)) = a,}$$
$$\mathbf{F(b) = (b*(a*b)) = b*c = a,}$$
$$\mathbf{F(c) = (c*(a*c)) = c*b = a.}$$

Thus F has no fixed point, verifying that the Trefoil quandle is not a full reflexive domain.

Left Distributivity
We have written the quandle as a right-distributive structure with inveritible elements. It is mathematically equivalent to use the formalism of a left distributive operation. In left distributive formalism we have **A*(b*c) = (A*b)*(A*c).** This corresponds exactly to the interpretation that each element **A** in **Q** is a mapping of **Q** to **Q** where the mapping **A[x] = A*x** is a structure preserving mapping from **Q** to **Q.**

$$\mathbf{A[b*c] = A[b]*A[c].}$$

We can ask of a domain that *every element of the domain is itself a structure preserving mapping of that domain.* This is very similar to the requirement of reflexivity and, as we have seen in the case of quandles, can often be realized for small structures such as the Trefoil quandle.

We call a domain M with an operation * that is left distributive a *magma.* Magmas are more general than the link diagrammatic quandles. We take only the analog of the third Reidemeister move and do not assume any other axioms. Even so there is much structure here. A magma with no other relations than left-distributivity is called a *free magma.*

The search for structure preserving mappings can occur in rarefied contexts. See for example the work of Laver and Dehornoy [9] who studied mappings of set theory to itself that would preserve all definable structure in the theory. Dehornoy realized that many of the problems he studied in relation to set theory were accessible in more concrete ways via the use of knots and braids. Thus the knots and braids become a language for understanding for formal properties of self-embedded structure.

Structure preserving mappings of set theory must begin as the identity mapping since the relations of sets are quite rigid at the beginning. (You would not be able to map an empty set to a set that was not empty for example, and so the empty set would have to go to itself.) The existence of non-trivial structure preserving mappings of set theory questions the boundaries of definability and involves the postulation of sets of very large size. See [16] for a good exposition of the philosophical issues about such embeddings and for an approach to wholeness in physics that is based on these ideas.

It is worth making a remark here about sets. Consider the collection Aleph of all sets whose members are themselves sets and such that any investigation into membership will just reveal more sets as members. Typical elements of Aleph are the empty set { }, the set whose member is the empty set { { } } and of course various curious constructs that have infinitely many members such as

{ { } , {{ }}, {{{ }}}, {{{{ }}}}, ... }

and we may even consider sets that are members of themselves (eigen-sets!) such as

{ { { { { ... } } } } }.

The key thing to understand about Aleph as a class of sets is that any member of Aleph is, by definition, a subset of Aleph. And any subset of Aleph is by definition a member of Aleph. This is a beautiful property of the class Aleph, and it is a paradoxical property if we imagine that Aleph is a set! For if Aleph is a set, then we have just shown that Aleph is in 1-1 correspondence with the set of subsets P(Aleph) of Aleph. If X is any set then we denote the set of subsets of X by P(X). Cantor's Theorem (proved here in Section VIII and related in that section o the fixed point theory of reflexive domains) tells us that *for any set X, P(X) is larger than X.*

This means that there cannot be a 1-1 correspondence between Aleph and P(Aleph) if Aleph is a set.

We can only conclude that Aleph is not a set. It is a class, to give it a name. It is an unbroken wholeness whose particularities we can always consider, but whose totality will always elude us. The way that the totality of Aleph eludes us is right before our eyes. Any particular element of Aleph is a set and it is a collection of sets as well. But we cannot complete Aleph. Any attempt to approximate Aleph as a set will always have some subsets that have not been tallied inside itself and so the set of subsets of the approximation will grow beyond that approximation to a new and larger domain of sets. Philosophically, this observation of the unreachability of Aleph, the set of all sets, as a set itself is very interesting and important. We see here how a perfectly clear mathematical concept may always remain outside the bounds of the formalities to which it refers and yet that concept is indeed composed of these formalities. It is the leading presence of the ultimately huge and unattainable Aleph that leads us to consider exceeding large sets in the pursuit of a flexibility in self-embeddings of set theory. At the end of Section VIII we take an alternative view of Aleph and consider what would have to change if Aleph were admitted to be a set.

Enough said about the abstract reaches of the magma. We should not expect that any given structure is a reflexive space. But it is possible to create languages that can expand indefinitely and thus partake of the ideal of reflexivity.

VII. Church and Curry

In this section we point out how the notion of a reflexive domain first appeared in the work of Alonzo Church and Haskell Curry [1] in the 1930's. This method is commonly called the "lambda calculus". The key to lambda calculus is the construction of a self-reflexive language, a language that can refer and operate upon itself. In this way eigenforms can be woven into the context of languages that are their own metalanguages, hence into the context of natural language and observing systems.

In the Church-Curry language (the lambda calculus), there are two basic rules:

1. Naming. *If you have an expression in the symbols in lambda calculus then there is always a single word in the language that encodes this expression.* The application of this word has the same effect as the application of the expression itself.

2. Reflexivity. *Given any two words A and B in the lambda calculus, there is permission to form their concatenation AB, with the interpretation that A operates upon or qualifies B.* In this way, every word in the lambda calculus is both an operator and an operand. The calculus is inherently self-reflexive.

Here is an example. Let **GA** denote the process that creates two copies of **A** and puts them in a box.

$$GA = \boxed{AA}$$

In lambda calculus we are allowed to apply **G** to itself. The result is two copies of **G** next to one another, inside the box.

$$GG = \boxed{GG}$$

This equation about **GG** exhibits **GG** directly as a solution to the eigenform equation

$$X = \boxed{X}$$

thus producing the eigenform without an infinite limiting process.

More generally, we wish to find the eigenform for a process **F**. We want to find a **J** so that **F(J) = J.** We create an operator **G** with the property that

GX = F(XX)

for any **X**. When **G** operates on **X, G** makes a duplicate of **X** and allows **X** to act on its duplicate. Now comes the kicker. Let **G** act on herself and look!

GG = F(GG)

So **GG** is a fixed point for **F**.

We have solved the eigenform problem without the excursion to infinity. If you reflect on this magic trick of Church and Curry you will see that it has come directly from the postulates of **Naming** and **Reflexivity** that we have discussed above. These notions, *that there should be a name for everything,* and that *words can be applied to the description and production of other words*, allow the language to refer to itself and to produce itself from itself. The Church-Curry construction was devised for mathematical logic, but it is fundamental to the logic of logic, the linguistics of linguistics and the cybernetics of cybernetics.

I like to call the construction of the intermediate operator **G**, the "gremlin" (See [10].) Gremlins seem innocent. They just duplicate entities that they meet, and set up an operation of the duplicate on the duplicand. But when you let a gremlin meet a gremlin then strange things can happen. It is a bit like the story of the sorcerer's apprentice. A recursion may happen whether you like it or not.

An eigenform must be placed in a context in order for it to have human meaning. The struggle on the mathematical side is to control recursions, bending them to desired ends. The struggle on the human side is to cognize a world sensibly and communicate well and effectively with others. For each of us, there is a continual manufacture of eigenforms (tokens for eigenbehaviour). Such tokens will not pass as the currency of communication unless we achieve mutuality as well. Mutuality itself is a higher eigenform. As with all eigenforms, the abstract version exists. Realization happens in the course of time.

VIII. Cantor's Diagonal Argument and Russell's Paradox

Let **AB** mean that *B is a member of A.*

Cantor's Theorem. Let S be any set (S can be finite or infinite).
Let **P(S)** be the set of subsets of S. Then **P(S)** is bigger than S in the sense that for any mapping **F: S -----> P(S)** there will be subsets C of S (hence elements of **F(S)**) that are not of the form **F(a)** for any **a** in S. In short ,the power set **P(S)** of any set S is larger than S.

Proof. Suppose that you were given a way to associate to each element **x** of a set **S** a subset **F(x)** of S. Then we can ask whether **x** is a member of **F(x)**. Either it is or it isn't. So lets form the set of all **x** such that x is not a member of **F(x)**. Call this new set **C.** We have the defining equation for **C** :

Cx = ~F(x)x.

Is **C = F(a)** for some a in S?
If C = **F(a)** then for all **x** we have
F(a)x = ~F(x)x.
Take **x = a.** Then
F(a)a = ~F(a)a.

This says that a is a member of **F(a)** if and only if **a** is not a member of **F(a)**. This shows that indeed **C** cannot be of the form **F(a)**, and we have proved that the set of subsets of a set is always larger than the set itself.

Note that in the usual language,

C = { x in X | x is not a member of F(x) }.

Note the problem that the assumption that **C = F(a)** gave us. If **C = F(a)**, then **F(a)a = ~F(a)a.** We would have a fixed point for negation. But there is no fixed point for negation in classical logic! If we had enlarged the truth set to

{T, F, I}

where **~I = I** is an eigenform for negation, then **F(a)a** would have value **I.** What does this mean? It means that the index **a** of the set **F(a)** corresponding to it would have an oscillating membership value. The element **a** would be like Groucho Marx who declared that he would not join any club that would have him as a member. We would be propelled into sets that vary in time.

Note that our proof of Cantor's Theorem has exactly the same form as our earlier proof of the existence of fixed points for a reflexive space.

The mapping **F:X -----> P(X)** takes the role of the 1-1 correspondence between **D** and **[D,D]**. The reader will enjoy thinking about this analogy. In the Cantor Theorem we have used the non-existence of a fixed point for negation to deduce a difference between and set **X** and its powerset **P(X)**. In the study of a reflexive domain we have shown the existence of fixed points, but we have seen that such domains must be open to new elements and new transformations.

There are many points of view about Cantor's Theorem. Lets start again by considering the assemblage (we shall not call it a set) **Aleph** of all sets whose members are sets that are members of **Aleph**. That is, a set **S** is a member of **Aleph** if every member of **S** is a set and when you look at the members of the members, they too are sets, and this process of finding sets continues to all depths. We allow the possibility of infinite depth of membership and hence the possibility of self-membership for sets in **Aleph**. Note that **Aleph** is a natural concept - the concept of sets that are made up from sets. But by definition, any set **S** that is a member of **Aleph** is also a subset of **Aleph**. And by definition, any subset of **Aleph** is a member of **Aleph!** Thus **Aleph** is *identical* with **P(Aleph)**. *According to Cantor's Theorem, Aleph is not a set.*

What is the contradiction that Cantor's Theorem produces for **Aleph?** Cantor forms *C = {x in Aleph| x is not a member of x }* since we can take **F:Aleph -----> P(Aleph)** to be the identity mapping. But is this a contradiction?! It would be a contradiction if we knew that **C** is a set. Then **C** would be a member of itself if and only if it was not a member of itself.

But **C** is not a set! **C** is itself a contradiction. **C** is the Russell paradox. We have that **C** is a member of **C** if and only if **C** is not a member of **C.** Cantor's process applied to **Aleph** produces a set that is supposed to be a new subset of **Aleph**, but in fact it is a paradoxical set. We could take the point of view that this shows that there are cases where the Cantor definition *C = {x in X| x is not in F(x)}* leads to an undefined set, a set for which one cannot actually decide on the membership of certain elements. In that viewpoint, **Aleph** may be considered an example of a set to which Cantor's Theorem does not apply.

We say, how did this happen? Isn't it always clear whether or not x is in **F(x)**? You would think so. But in the case at hand we have **F(x) = x** and the question becomes: *does x belong to x?* And then we see that as far as **C** itself is concerned this question creates an iterant, an oscillation, a paradox. By applying Cantor's argument to **Aleph**, *we have found iterants and imaginary values at the very heart of set theory.*

The notion that we can always specify a set by a definition in the form **S = {x | P(x) }** where **P(x)** is a logical proposition is *naive.* The propositional statement provides a criterion of distinction, but it is possible that this criterion will be circular or undecideable. So we have to keep attending to what we define, and find out when it makes the sense. Why should such things be automatic?

IX. The Secret

What is the simplest language that is capable of self-reference? We are all familiar with the abilities of natural language to refer to itself. Why this very sentence is an example of self-referentiality. The American dollar bill declares "This bill is legal tender.". The sentence that you are now reading declares that you, the reader, are complicit in its own act of reference. But what is the simplest language that can refer to itself?

The simplest language would have a simple alphabet. Let us say it has only the letter R. The words in this language will be all strings of R's. Call the language LS. The words in LS are the following:

R,
RR,
RRR,
RRRR,
and so on.

Two words are equal if they have the same number of letter R's. E*ach word makes a meaningful statement of reference via the rule:*

If X is a word in LS, then RX refers to XX.
RX refers to XX, the repetition of X.

Thus RRR refers to RRRR (not to itself), and R refers to the empty word.

There is a word in LS that refers to itself. Can you find it?

Lets see.
RX refers to XX.
So we need XX = RX if RX would refer to RX.
If XX = RX, then X = R.
So we need X = R.
And RR refers to itself.

The little language LS looks like a pedantic triviality, but it is actually at the root of reflexivity, Godel's incompleteness Theorem, recursion theory, Russell's paradox and the notion of self-observing and self-referring systems. It seems paradoxical that what looks like a trick of repeating a symbol can be so important. The trick is more than just a trick.

Just to show you how this works, consider Russell's paradox again. Russell asks us to consider the set of all sets that are not members of themselves. Lets call this set B for "Bertrand Russell". Lets write YX to mean "X is a member of Y". And write ~YX to mean "X is not a member of Y". OK?

Then Russell's set is defined by the equation

$$BX = \sim XX.$$

(= means "if and only if" in a logical context) Read it out loud:

"X is a member of B if and only if X is not a member of X". Exactly. What about B? Is B a member of B? Try it. Let X = B. Then

$$BB = \sim BB.$$

"B is a member of B if and only if B is not a member of B."

This is the Russell paradox. You see that in the form BX = ~XX the Russell paradox is an instance (in a slightly more complex language) of exactly our LS trick of self-reference.

The Russell paradox continues to act as a mystery at the center of our attempts to relate syntax and semantics. In that center is a little trick of syntactical repetition. I would like to think that when we eventually discover the true secret of the universe it will turn out to be this simple.

The snake bites its tail. The Universe is constructed in such a way that it can refer to itself. In so doing, the Universe must divide itself into a part that refers and part to which it refers, a part that sees and a part that is seen.

Let us say that R is the part that refers and U is the referent. The divided universe is RX and RX = U and RX refers to U (itself). Our solution suggests that the Universe divides itself into two identical parts each of which refers to the universe as a whole. This is

$$RR.$$

In other words, the universe can pretend that it is two and then let itself refer to the two, and find that it has in the process referred only to the one, that is itself.

The Universe plays hide and seek with herself, pretending to divide herself into two when she is really only one. And that is the secret of the Universe and that is the universal source of our trick of self-reference.

X. The World of Recursive Emergence and Creativity

We have repeatedly insisted that a formal fixed point or eigenform is associated with any transformation T in any domain where infinite composition of transformations is possible. Thus we make E = T(T(T(T(T(...))))) and find that E = T(E). This is the symbolic fixed point that sometimes corresponds to a stability in the original domain of the recursion. We have also seen that one can take a seed z for the recursion and repeatedly form

$$z, T(z), T(T(z)), T(T(T(z))), \ldots$$

in a temporal sequence or recursive process. Then the finite products of this process can exhibit similarity to the infinite eigenform, and they can also exhibit novelty and emergence structure in ways that are most surprising. It is this appearance of creativity and novelty in recursive process that makes reflexivity more than abstract mathematics and more than a philosophical idea.

The purpose of this last section is to exhibit an example involving cellular automata that illustrates these ideas and gives us a platform for thought. In this example, we are using an algorithm that I call 7-Life. It is a variant of the Life automaton of John H. Conway. Conway's automaton is governed by the rule B3/S23 which means that a white square in the grid is born (B) when it has 3 neighbors and it survives (S) when it has exactly 2 or 3 neighbors. Life has the property that there are many intriguing formations and processes, but statistically most configurations dies out to a collection of isolated static patterns (still lifes) and oscillating patterns that do not grow or interact outside themselves.

7-Life has the rule B37/S23 and has many of the properties of Life, plus the phenomenon that many starting configurations grow, self-interact and produce streams of *gliders*. The gliders are five-square formations (occurring in Life as well) that occur spontaneously and regenerate themselves, appearing to move along diagonal directions in the process. The most striking property of 7-Life is the long term persistence of such self-interacting configurations, growing slowly in complexity over time.

In the Figures 1,2, and 3 we indicate the result of applying the 7-Life algorithm to a simple and not-quite symmetrical starting configuration shown in Figure 1. In Figure 2 we see the result of 33911 iterations of the process. We now have a galaxy of complex interactions. The small entities radiating away from the galaxy are gliders, as described above, and if a reader were to watch the process using a computer program, he or she would see a teeming, seemingly random mass of activity. Then in Figure 3 we see that after 49281 iterations something new has emerged. It seems that a highly patterned *dragon* is emerging from the chaos of the complex process. The tip of this dragon moves forward relentlessly. The body of the dragon interacts with the glider radiation and begins to roil in the chaotic process. So far, the growing tip of the dragon has not interacted with any gliders.

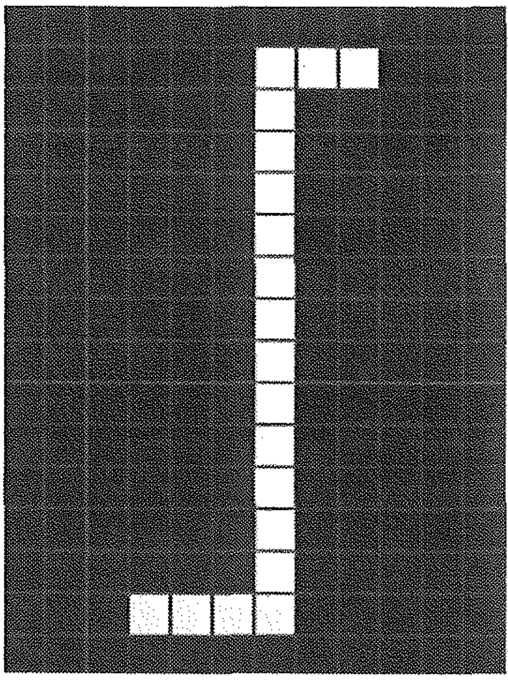


Figure 1. The Starting Configuration

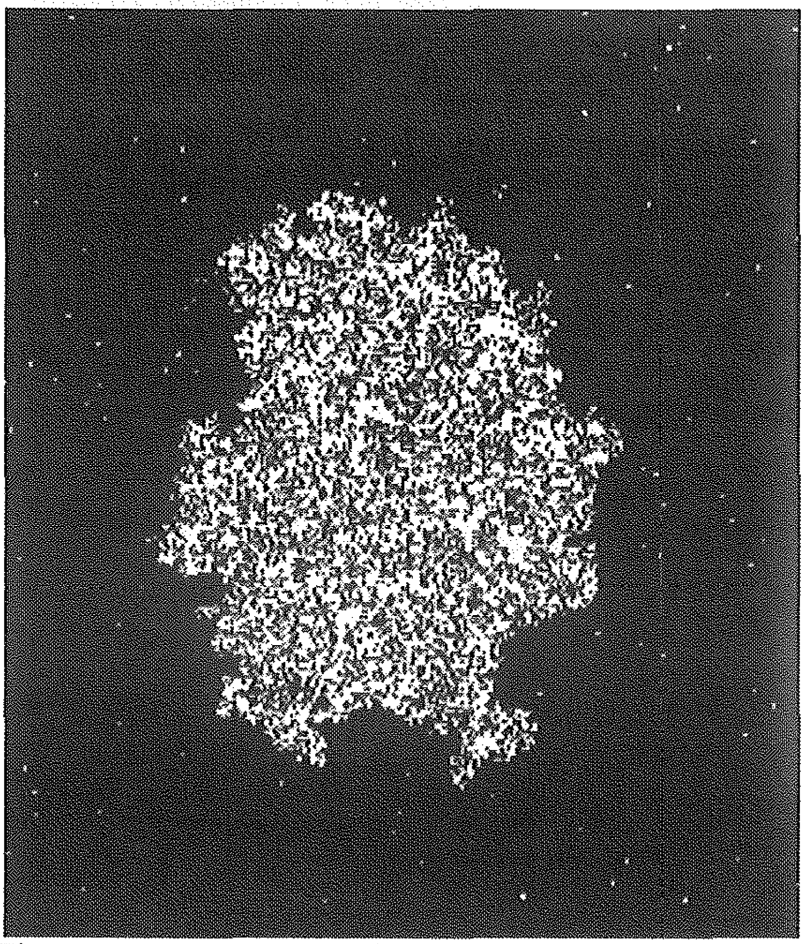


Figure 2. After 33911 Iterations

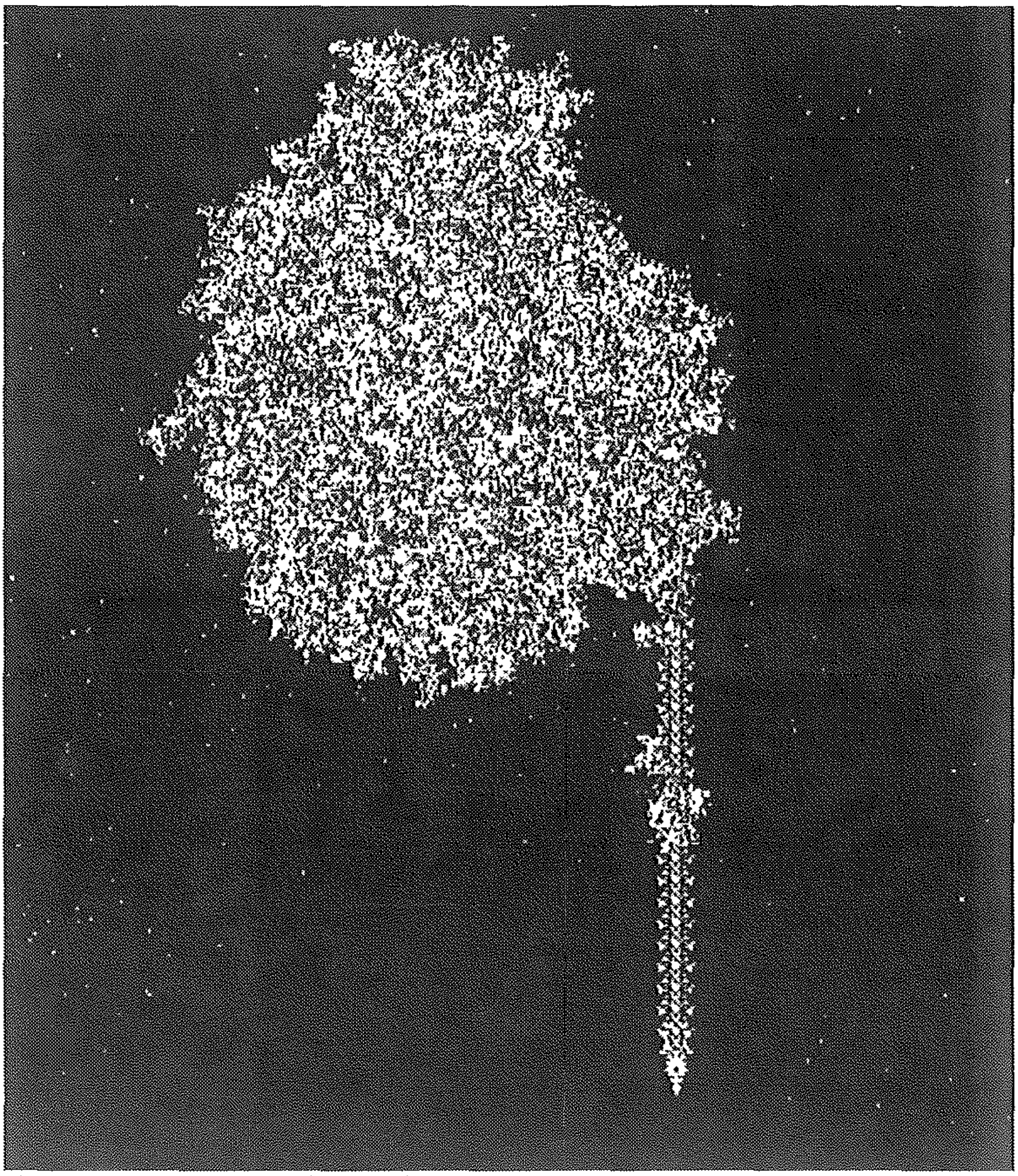


Figure 3. After 49281 Iterations

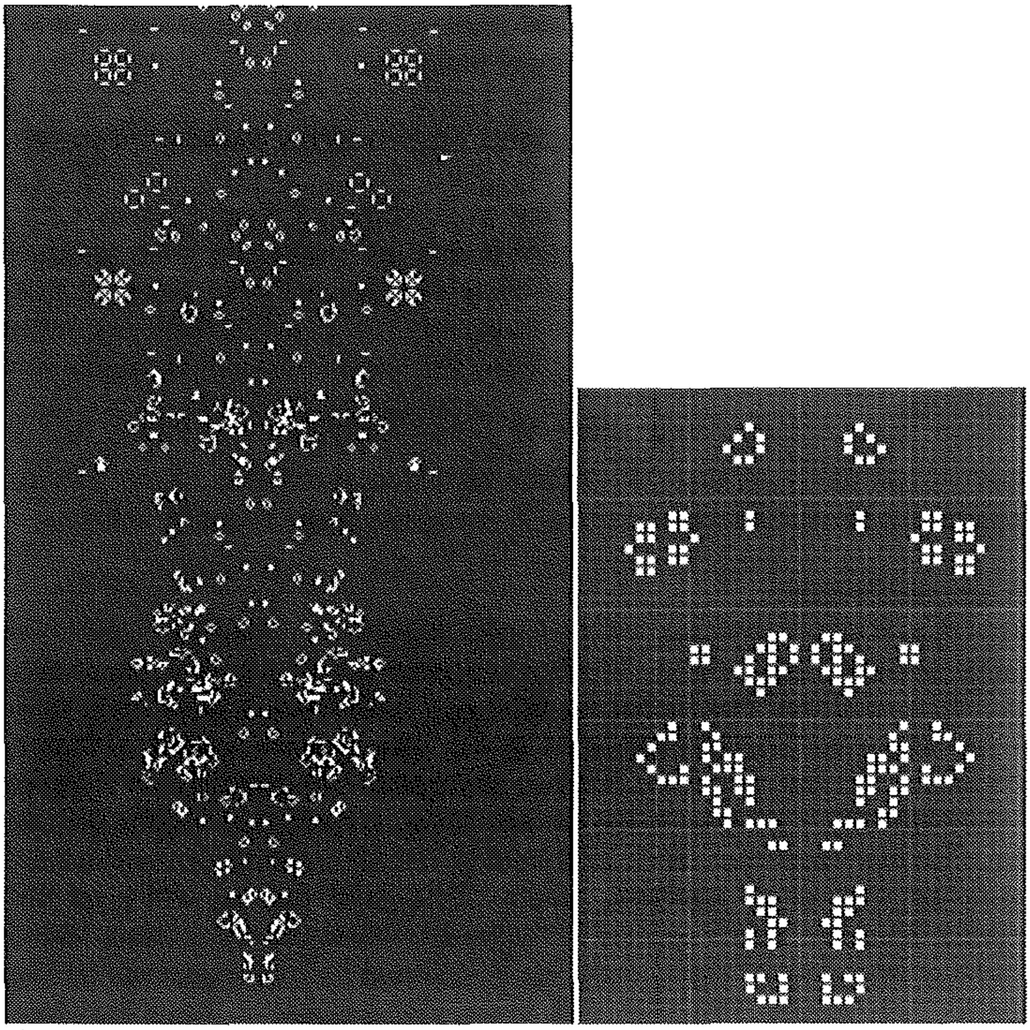


Figure 4. The Growing Tip

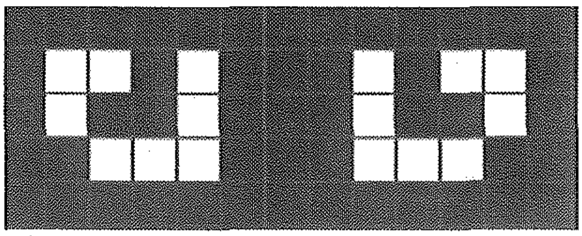


Figure 5. The Generating Tip GG

Figures 4 gives closeups of the tip of the dragon and Figure 5 isolates the generator, GG, of the dragon itself. This configuration GG of 16 squares in mirror symmetry, when placed on an otherwise blank lattice will generate the dragon in the 7-Life algorithm.

What has happened is that this 16-square generator GG has appeared the course of the complex interactions, and it has had enough room to move forward in its own pattern -- forming the dragon behind it and periodically regenerating itself. The generator of the dragon, GG, is not our invention. GG is a natural consequence of the complex process of 7-Life. GG emerges, but with much lower probability than the gliders. The result is an appearance of novelty and creativity in the complex process as it happens over time. We can only speculate what more complex entities would eventually emerge in 7-Life over many more iterations.

Just so does DNA emerge from the complex process of the world of the earth and sun.

We see from this example that eigenforms that are processes, such as the self-generating GG, *can and will emerge of their own accord* from complex systems based on recursion. In this sense, such systems begin to generate their own reflexive spaces. The novel and self-reproducing forms that emerge from them can be seen in a similar light.

All these observations are made by an observer. The observer is clever only in the distinctions that he or she makes, and that is enough to found an entire universe.

XI. In Zermelo's Bar

The section is a multi-logue about the attempts to solve the equation of the observer in relation to his/her observation. We first encounter Mr. D, who has solved his own equation in such a way that he has no head and instead has a great open space of possibility where his head was supposed to be. This requires a drink to ingest and we go to Zermelo's Bar, where we find two mathematicians arguing over the solution to an equation whose solution is the Golden Ratio, a proportion well known to the Greeks. The mathematicians are a little hard to follow, but their discussion turns on all the essential issues of recursion, reality and infinity that we will need for this adventure. Then Dr. Von F appears in the bar (we think you can guess who this is) and explains the nature of eigenforms. He is followed by a character named Charlie and Dr. CC, a linguist and logician, then by Dr. HM, a biologist. Later there appears a physicist, Dr. JB and finally Dr. R himself, the source of the self-referential paradox. We hope that you will join in on this discussion yourself.

Infinite Recursion and Its Relatives

Our problem is to solve the equation

$$\mathbf{O(A) = A}$$

for **A** in terms of **O**.

For example, suppose that the observer **O** is Mr. D, a man who insists that he has no head. We interview him. Well Mr. D, why do you say that you have no head? Mr. D. replies. Oh it is so simple, you will see at once what I mean. In fact, consider what you yourself see. Look directly around. Do you see your head? No. You see and feel a great open space of perception where your head is supposed to be, and a flow of thoughts and feelings. But no head! The body comes in. Shoulders, arms, legs, shoes and the world. But no head. Instead of a head there is a great teeming void of perception. Once I realized this, I knew that the relationship of a self to reality was indeed deep and mysterious.

As we can see, Mr. D has discovered that what is constant for his visual observer is a body without a head. He has solved the problem of finding himself as a solution of the equation of himself in terms of himself. Perhaps we need a drink.

We walk into Zermelo's Bar and two mathematicians appear on the scene. One says to other: How do you solve this equation? I want a positive real solution.

$$1 + 1/A = A.$$

The second one says: Nothing to it, we multiply both sides by the unknown **A** and rewrite as

$$A + 1 = A^2.$$

Then, solving the quadratic equation, we find that

$$A = (1 + \sqrt{5})/2.$$

The first mathematician says: Nice tricks you have there, but I prefer infinite reentry of the equation into itself. Look here:

If $A = 1 + 1/A$, then

$$\begin{gathered} A = \\ 1 + 1/A = \\ 1+1/(1+1/A) = \\ 1+1/(1+1/(1+1/A)) = \\ 1+1/(1+1/(1+1/(1+1/A))) \end{gathered}$$

and I will take this reentry process to infinity and obtain the form

$$A = 1+ 1/(1+1/(1+1/(1+1/(1+ 1/(1+1/(1+ \ldots)))))).$$

The second mathematician then says: Well I like your method. We can combine our answers and write a beautiful formula!

$$(1+\sqrt{5})/2 =$$

$$1+ 1/(1+1/(1+1/(1+1/(1+ 1/(1+1/(1+ \ldots))))))$$

Why do you like this formula? says the second guy. Well, says the first guy, the left hand side is a definite irrational number and it is easy to see by squaring it that it satisfies the equation $A^2 = A + 1$ as we wanted it. But irrational numbers have a curiously tenuous existence unless you know a way to calculate approximations for them. On the other hand, your right hand side can be regarded as the limit of the fractions

$$1 = 1/1$$

$$1+1/1 = 2/1 = 2$$
$$1+1/(1+1/1) = 3/2$$
$$1+1/(1+1/(1+1/1)) = 5/3$$
$$1+1/(1+1/(1+1/(1+1/1))) = 8/5$$
$$1+1/(1+1/(1+1/(1+1/(1+1/1)))) = 13/8$$
$$1+1/(1+1/(1+1/(1+1/(1+1/(1+1/1))))) = 21/13$$

with the first few terms of this limit being

$$(1+\sqrt{5})/2 = 1.618...$$

On top of this your infinite formula actually does reenter itself as an infinite expression it really is of the form

$$A = 1 + 1/A.$$

The first guy comes back with: Well it sounds to me like you really believe in the "actual" infinity of the terms on the right-hand side. I also like to imagine that they are all there existing together in space with no time.

Right ! says the second guy. We know that this is an idealization, but it lets us actually reason to correct answers and to put them in an aesthetically pleasing form.

The bartender is listening to all of this, and he leans over and says: You guys have to meet a couple of others on this score. There is Dr. Von F and Dr. CC. They both have some ideas very similar to yours. Hey, here is Dr. Von F now. Dr. Von F, could you tell these fellows about your eigenforms?

Jah! Of course! It is all very simple. We just combine this notion of recursion with the most general possible situation. Suppose we have any observer **O** and we wish to find a fixed point for her. Well then we just let the observer act without limit as in

$$A = O(O(O(O(O(O(O(O(...))))))))).$$

After infinity, one more application of **O** does not change the result and we have

$$O(A) = A.$$

This is very simple, no? And it shows how we make objects. These objects are the tokens of our repeated behaviors in shaping a form from nothing but our own operations. As I have said before, the human identity is precisely the fixed point of such a recursion. "I am the observed link between myself and observing myself." [2]

The first mathematician makes a comment: What you are doing is a precise generalization of my infinite continued fraction! If I had defined

$$O(A) = 1 + 1/A$$

then we would have

$$O(O(O(...))) = 1+ 1/(1+1/(1+1/(1+...))).$$

But I am puzzled by your approach, for it would seem that you are willing that your solution A will have no relation with how the process starts, and also it may not related to the original domain in which it was constructed! For example, in my mathematics, I could consider the operator

$$O(A) = -1/A$$

and this operator does not have a fixed point in the real numbers, but if we take **A=i** where $i^2 = -1$ (the simplest imaginary number), then **O(i) = i.** Are you suggesting that

$$i = -1/-1/-1/... \quad ?$$

Dr. Von F replies: Jah, Jah! This is very important! The fixed point can be a construction that breaks ground into an entirely new domain! Actually, I am mainly interested in those fixed points that do break new ground. We are looking for the places where new structures emerge. In your mathematics you have illustrated this in two ways. In the first recursion, the values converge to an irrational number (the golden ratio). All the finite approximations are rational fractions (ratios of Fibonacci numbers) but in the limit of the infinite eigenform, you arrive at this beautiful new irrational number! And in your second example all the finite approximations oscillate like a buzzer, or a paradox, between positive unity and negative unity, but the eigenform is a true representative of the imaginary square root on minus one! And don't forget that this "imaginary" quantity is fundamental to both logic and physics. The fully general eigenforms are fundamental to the ontology of the world.

Suddenly the door to Zermelo's Bar opens and in walks a character that everyone calls "Charlie." Charlie! say the barkeep, where have you been? We have a good discussion on signs going here. You have to hear this stuff. Charlie says, Well I heard just about everything Dr. Von F said as I admit here to a bit of eavesdropping on the other side of the door! These eigenforms of Von F are quite familiar to me as I have thought continuously along these lines for many years. You see, any sign once you look at it in the context of its reference and the continuous expansion of its interpretant becomes a growing complex of signs referring to other signs, growing until the references close on themselves and, as Dr. Von F correctly describes, these closures are the eigenforms, the tokens for apparently stable behaviors. As the complex of signs grows, the complex itself is a sign and as the closures occur, that sign becomes a sign for itself. We humans are in our very nature such signs for ourselves.

Dr. Von F says: Well I always say that I am the observed link between myself and observing myself. I am a sign for myself!

At this point Dr. CC chimes in: But Dr. Von F and Charlie, this excursion to recursion and infinity seems quite excessive! It is all right for mathematicians to imagine such a thing, but we humans exist in language and the finiteness of expressions. Surely you do not suggest that this profligate composition of the operator and expansion of sign complexes actually happens!

Well, Dr. CC, says Von F, I am really a physicist and well aware of the speed of physical process in relation to the very slow pace of our verbal thought. Surely you have stood between two facing mirrors and seen the near-instantaneous tunnel of reflections created by light bouncing back and forth between the mirrors. Yes, I am seriously suggesting that the self-composition of the observer is carried to high orders. These orders are sufficiently large and accomplished with such a high speed that they appear infinite in the eyes of the observer. Now you may detect the beginning of a paradoxical flight here. The very observer who is too slow to detect the difference between a large number and infinity is yet so quick and subtle that he/she can produce this flight to infinity. But I beg your pardon, this is still a matter of the interaction of slow thought and fast action. Wave your arm back and forth rapidly in front of your eyes. For all practical purposes the arm appears to be in two places at the same time! You do not deny that it is "you" that moves the arm, and it is "you" that perceives it.

I simply go further and suggest that every perception is based on such an illusion of permanency, based on the self composition of your self. You do it all and you are surprised at the result. You do it all, but you can not perceive all that you do!

Charlie adds: I agree but do not have to rest on physics. Our shortsighted view of our own nature arises from the difficulty in reckoning that our true nature is as signs for ourselves. It is only at the limit of eigenbehaviours that such signs appear simple. We partake of the complexity of the universe.

Dr CC replies: Ah Charlie and Dr. Von F, I have been working in the linguistic and logical realm and you will see that our points of view are mutually supporting. For I imagine the structure of the observer as a big network of communicating entities. These entities have so much interrelation among themselves that their identities begin to merge into one identity and that is the apparent identity of the self.

Charlie interrupts with: Yes! That is the essence of continuity.

Dr. CC continues. I agree! The infinity in my view is not with any one of them, but with the aggregate of them that has become so large as to begin to merge into a continuity.

But let me explain: If A and B are entities in my "community of the self", then they can interact with each other and with themselves. These processes of interaction produce new entitles who exist at the same level as the original entities. Can you imagine this? Of course you can, you are such an entity. For example, I suggest to you that you are the self that thinks kindly of others, that you satisfy the equation **SX** = **KX** where **S** is "you" and **KX** is the being "thinks kindly of **X**". *Then that entity* ***S*** *exists.* In the world of language, *every definable entity exists.* The consequence is that **S** might even think kindly of herself as in **SS = KS**. That **S** can think kindly of herself is, in this linguistic world, dependent on the condition that the kindly thinking observer is an observer at the same level as any other observer. Now there are many such entities. Watch this magic trick. Let

$$\mathbf{GX = O(XX)}.$$

The gentility **G** is the observer who observes an entity observing herself. What happens when G observes herself? Then **G** observes herself observing herself and we have a fixed point, an eigenform!

$$\mathbf{GG = O(GG)}.$$

I have constructed the eigenform without the infinite composition of the observer upon herself. Of course once this self-reflexive construction comes into the being of language then it runs automatically to the level of practical infinity and produces your recursion.

$$\mathbf{GG = O(GG) = O(O(GG)) = O(O(O(GG))) = \ldots}$$

I believe my linguistic construction provides the context for your observer's self interaction. The true infinity in my world is a distributed infinity of beings each coming into being as a name for a process of observation. This continues without end and is the basis of the coincidence of the language and the metalanguage in this world.

At this point Dr. HM, a biologist, walks into the room. He remarks: I see that you have been discussing the stability of perceptions from physical and linguistic principles. Let me tell you how I see these matters in my domain. The beings you talk about are biological, not just logical. They exist in the evolutionary flow of coordinations of coordinations that give rise to the mutual patternings that you call "language" and "thought". It is not at all surprising that each such being, coordinated with the others in the deep flow of its history in biological time will appear layered like an onion with the actions of each on each. The long time history of mutual interaction and coordination will generate the appearance of the eigenforms. But there is no "disembodied observer" who generates these forms from some abstract place. In biology there is no problem of mind (abstract observer) and body. They are one. Mind and observer both refer to the conversational domain that arises in the construction of the coordination of coordinations that is language. The disembodied observer is a fantasy that is convenient for the mathematician or the physicist. In the biological realm all forms are generated through time in an organic way.

And finally, Dr. JB enters the room, a very theoretical physicist. He says: Ah it is not surprising, but you all have the business of objects and eigenforms quite wrong. Let me start with the views of the biologist Dr. HM. You see, there is no time. None. Time is an illusion. Of course in order to tell you about this insight I shall have to use words that appear to describe states in time. That is my fate to be so projected into language. You must forgive me.

Each moment of being is eternal, beyond time. I prefer to call such moments "time capsules." Each moment contains that possibility that it can be interpreted in terms of a "history", a story of events leading up to the "present moment" that constitutes the time capsule as a whole. But this history is a pattern in eternity. That the history can be told with some coherence and that we manage to tell the story of "past events" leads us to believe that these past events "actually happened". But in fact what has happened is happening now and only now in the eternity of the time capsule whose richness derives from the superposition of its quantum states.

At this point the bartender chimes in: I'll drink to that. Time is a grand illusion and a wee scotch from my bar will convince ye o' that in less time than it takes to wink an eye!

All well and good, says Dr. R, who just walked into the bar, but as I was telling my friend Frege, if there is one thing that will give us trouble it is this notion of eternity and the non-existence of time. For as I told Gottlob just the other day, you have only to imagine the timeless reality of the set of all sets that are not members of themselves and you will have to leave logic behind! I gave up long ago my travails on this issue with Professor Whitehead. We tried to make logic go first and it was a disaster. Now I let logic run along behind and there is no problem at all. As far as fixed points are concerned my favorite is Omega, the set whose only member is Omega herself. You see that the act of set formation is nothing but an act of reflection. Omega finds herself in reflecting on herself.

Dr. CC retorts: Well, Russell, I hardly expected you to capitulate your position on logic. Your Type is hardly likely to just slip away. I prefer to make a specimen of your famous set in the following way. I let **AB** mean that "B is a member of A". Then I define your set of all sets that are not members of themselves" by the equation

$$\mathbf{Rx} = \sim\mathbf{xx}.$$

Then we can pin the specimen to the board by substituting **R** for **x** as in

$$\mathbf{RR} = \sim\mathbf{RR}.$$

This **RR** is a fixed point for negation. It is neither true nor false. I do not leave logic behind. I imagine new states of logical discourse that are beyond the true and the false. Your set performs this transition to imaginary Boolean values.

Now Dr. HM says: Well I see you fellows are beginning to foment an argument. I feel that I must point out to you that logical paradox occurs only in the domain of language. There is no such matter as the paradox of the Russell set in the natural domain. In the natural domain, all apparent contradictions are only antimonies in the eyes of some observer. Nature herself runs in the single valued logic of the evolutionary flow. This is why I emphasize that it is only in the linguistic domain of coordinations of coordinations that the eigenforms arise. At the biological level there are processes that can be seen as recursions, but this seeing is already at the level of the coordinations. There is no mystery in this, but it is necessary to round out the mathematical models with the prolific play and dynamics of the underlying biology. In this sense biology is prior to physics as well as cognition.

At this point a tremor shakes the bar and the lights go out. I am sorry folks, the bartender says from the darkness, but this is another one of our natural events in the single valued logical flow of biological time -- a small earthquake. I will have to ask you to leave now for your own safety. And so the discussion ended, unfinished but perhaps that was for the best.

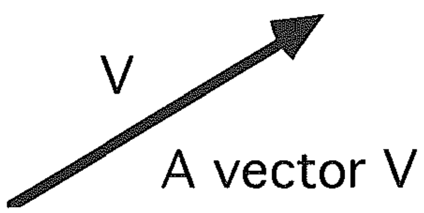
A Remark

The story in this section presents a number of different points of view about the cybernetics of fixed points. Fixed points can be produced by infinite recursion, by direct self-reference, through the linguistics of lambda calculus, and by approximation to infinites. Mr. D is a fictionalized version of Douglas Harding the man who indeed realized that he did not have a head, and had the courage to write about it. The good Drs. at the bar represent these points of view and are thinly disguised representatives of the viewpoints of Heinz von Foerster, Alonzo Church and Haskell Curry (Dr. CC), Humberto Maturana and the physicist Julian Barbour. Charlie represents the American mathematical philosopher Charles Sanders Peirce. All this is only the beginning. The most famous fixed point of them all is the Universe herself, acted here by the bartender.

XII. Quantum Physics, Eigenvalue and Eigenform

There are two reasons for including a discussion of quantum mechanics in this essay. On the one hand the quantum mechanics has been a powerful force in asking us to rethink our notions of objects and causality. On the other hand, von Foerster's notion of eigenform is an outgrowth of his background as a quantum physicist. We should ask what eigenforms might have to do with quantum theory and with the quantum world.

In this section we meet the concurrence of the view of object as token for eigenbehaviour and the observation postulate of quantum mechanics. In quantum mechanics observation is modeled not by eigenform but by its mathematical relative the eigenvector. The reader should recall that a *vector* is a quantity with magnitude and direction, often pictured as an arrow in the plane or in three dimensional space.



In quantum physics [11], the state of a physical system is modeled by a vector in a high-dimensional space, called a Hilbert space. As time goes on the vector rotates in this high dimensional space. Observable quantities correspond to (linear) operators **H** on these vectors **v** that have the property that the application of **H** to **v** results in a new vector that is a multiple of **v** by a real factor **λ**. (An operator is said to be linear if H(av +w) = aH(v) + H(w) for vectors v and w, and any number a. Linearity is usually a simplifying assumption in mathematical models, but it is an essential feature of quantum mechanics.)

In symbols this has the form

$$\mathbf{Hv} = \lambda\mathbf{v}.$$

One says that **v** is an *eigenvector* for the operator **H**, and that **λ** is the *eigenvalue.* The constant **λ** is the quantity that is observed (for example the energy of an electron). These are particular properties of the mathematical context of quantum mechanics. The **λ** can be eliminated by replacing **H** by **G** = **H/λ** (when **λ** is non zero) so that

$$\mathbf{Gv} = (\mathbf{H}/\lambda)\mathbf{v} = (\mathbf{Hv})/\lambda = \lambda\mathbf{v}/\mathbf{k} = \mathbf{v}.$$

Thus

$$\mathbf{Gv} = \mathbf{v}.$$

In quantum mechanics observation is founded on the production of eigenvectors v with **Gv = v** *where v is a vector in a Hilbert space and* ***G*** *is a linear operator on that space.*

Many of the strange and fascinating properties of quantum mechanics emanate directly from this model of observation. In order to observe a quantum state, its vector is projected into an eigenvector for that particular mode of observation. By projecting the vector into that mode and not another, one manages to make the observation, but at the cost of losing information about the other possibilities inherent in the vector. This is the source, in the mathematical model, of the complementarities that allow exact determination of the position of a particle at the expense of nearly complete uncertainty about its momentum (or vice versa the determination of momentum at the expense of knowledge of the position).

Observation and quantum evolution (the determinate rotation of the state vector in the high dimensional Hilbert space) are interlocked. Each observation discontinuously projects the state vector to an eigenvector. The intervals between observations allow the continuous evolution of the state vector. This tapestry of interaction of the continuous and the discrete is the basis for the quantum mechanical description of the world.

The theory of eigenforms is a sweeping generalization of quantum mechanics that creates a context for understanding the remarkable effectiveness of that theory. If indeed the world of objects is a world of tokens for eigenbehaviours, and if physics demands forms of observations that give numerical results, then a simplest example of such observation is the observable in the quantum mechanical model.

Is the quantum model, in its details, a consequence of general principles about systems? This is an exploration that needs to be made. We can only ask the question here. But the mysteries of the interpretation of quantum mechanics all hinge on an assumption of a world external to the quantum language. Thinking in terms of eigenform we can begin to look at how the physics of objects emerges from the model itself.

Where are the eigenforms in quantum physics? They are in the mathematics itself. For example, we have the simplest wave-function

$$\varphi(x,t) = e^{i(kx - \omega t)}.$$

Since we know that the function $E(x) = e^x$ is an eigenform for operation of differentiation with respect to x, $\varphi(x,t)$ is a special multiple eigenform from which the energy can be extracted by temporal differentiation, and the momentum can be extracted by spatial differentiation. We see in $\varphi(x,t)$ the complexity of an individual who presents many possible sides to the world. $\varphi(x,t)$ is an eigenform for more than one operator. It is this internal complexity that is mirrored in the uncertainty relations of Heisenberg and the complementarily of Bohr. The eigenforms themselves, as wave-functions, are *inside* the mathematical model, on the *other side* of that which can be observed by the physicist.

We have seen eigenforms as the constructs of the observer, and in that sense they are on the side of the observer, even if the process that generates them is outside the realm of his perception. This suggests that we think again about the nature of the wave function in quantum mechanics. Is it also a construct of the observer? To see quantum mechanics and the world in terms of eigenforms requires a turning around, a shift of perception where indeed we shall find that the distinction between model and reality has disappeared into the world of appearance.

This is a reversal of epistemology, a complete turning of the world upside down. Eigenform has tricked us into considering the world of our experience and finding that it is our world, generated by our actions. It has become objective through the self-generated stabilities of those actions.

A Quick Review of Quantum Mechanics

DeBroglie hypothesized two fundamental relationships: between energy and frequency, and between momentum and wave number. These relationships are summarized in the equations

$$E = \hbar w,$$
$$P = \hbar k,$$

where **E** denotes the energy associated with a wave and **p** denotes the momentum associated with the wave. Here $\hbar = h/2\pi$ where **h** is Planck's constant.

Schrödinger answered the question: *Where is the wave equation for DeBroglie's waves?* Writing an elementary wave in complex form

$$\psi = \psi(x,t) = \exp(i(kx - wt)),$$

we see that we can extract DeBroglie's energy and momentum by differentiating:

$$i\hbar\partial\psi/\partial t = E\psi \quad \text{and} \quad -i\hbar\partial\psi/\partial x = p\psi.$$

This led Schrödinger to postulate *the identification of dynamical variables with operators* so that the first equation ,

$$i\hbar\partial\Psi/\partial t = E\Psi,$$

is promoted to the status of an equation of motion while the second equation becomes the definition of momentum as an operator:

$$p = -i\hbar\partial/\partial x .$$

Once **p** is identified as an operator, the numerical value of momentum is associated with an eigenvalue of this operator, just as in the example above. In our example $p\psi = \hbar k\psi$.

In this formulation, the position operator is just multiplication by **x** itself. Once we have fixed specific operators for position and momentum, the operators for other physical quantities can be expressed in terms of them. We obtain the energy operator by substitution of the momentum operator in the classical formula for the energy:

$$E = (1/2)mv^2 + V$$
$$E = p^2/2m + V$$
$$E = -(\hbar^2/2m)\partial^2/\partial x^2 + V.$$

Here **V** is the potential energy, and its corresponding operator depends upon the details of the application.

With this operator identification for **E**, Schrödinger's equation

$$i\hbar\partial\psi/\partial t = -(\hbar^2/2m)\partial^2\psi/\partial x^2 + V\psi$$

is an equation in the first derivatives of time and in second derivatives of space. In this form of the theory one considers general solutions to the differential equation and this in turn leads to excellent results in a myriad of applications.

In quantum theory, observation is modeled by the concept of eigenvalues for corresponding operators. *The quantum model of an observation is a projection of the wave function into an eigenstate.*

An energy spectrum **{Ek}** corresponds to wave functions ψ satisfying the Schrödinger equation, such that there are constants Ek with $\mathbf{E}\psi = Ek\psi$. An *observable* (such as energy) **E** is a Hermitian operator on a Hilbert space of wavefunctions. Since Hermitian operators have real eigenvalues, this provides the link with measurement for the quantum theory.

It is important to notice that there is no mechanism postulated in this theory for how a wave function is "sent" into an eigenstate by an observable. Just as mathematical logic need not demand causality behind an implication between propositions, the logic of quantum mechanics does not demand a specified cause behind an observation. This absence of an assumption of causality in logic does not obviate the possibility of causality in the world. Similarly, the absence of causality in quantum observation does not obviate causality in the physical world. Nevertheless, the debate over the interpretation of quantum theory has often led its participants into asserting that causality has been demolished in physics.

Note that the operators for position and momentum satisfy the equation $\mathbf{xp - px = \hbar i}$. This corresponds directly to the equation obtained by Heisenberg, on other grounds, that dynamical variables can no longer necessarily commute with one another. In this way, the points of view of DeBroglie, Schrödinger and Heisenberg came together, and quantum mechanics was born. In the course of this development, interpretations varied widely. Eventually, physicists came to regard the wave function not as a generalized wave packet, but as a carrier of information about possible observations. In this way of thinking $\psi*\psi$ ($\psi*$ denotes the complex conjugate of ψ) represents the probability of finding the "particle" (A particle is an observable with local spatial characteristics.) at a given point in spacetime. Strictly speaking, it is the spatial integral of $\psi*\psi$ that is interpreted as a total probability with $\psi*\psi$ the probability density. This way of thinking is supported by the fact that the total spatial integral is time-invariant as a consequence of Schrodinger's equation!

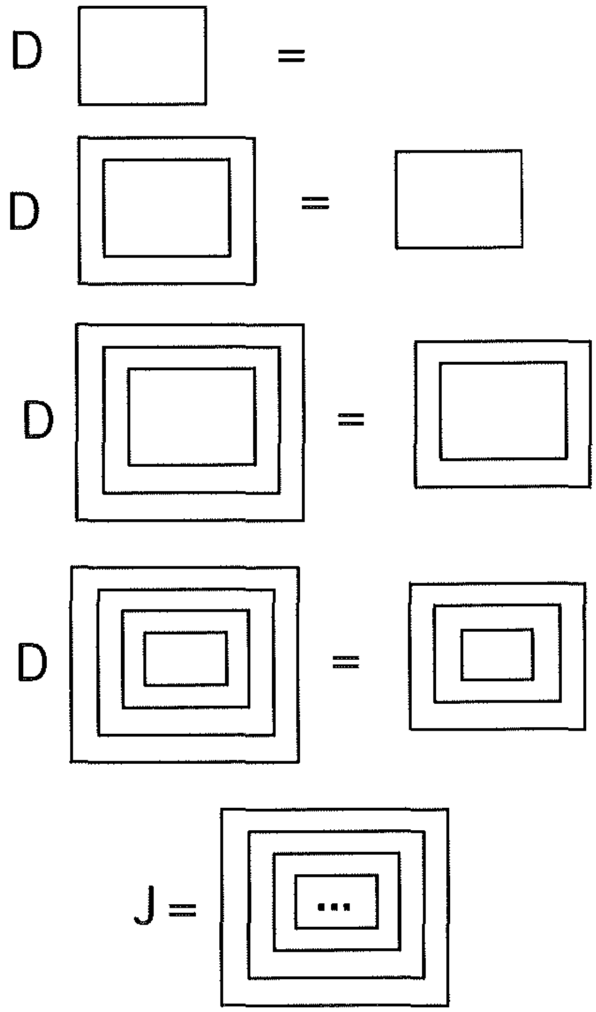
XIII. Iterants, Complex Numbers and Quantum Mechanics

We have seen that there are indeed eigenforms in quantum mechanics.

The eigenforms in quantum mechanics are the mathematical functions such as e^x *that are invariant under operators such as* $\mathbf{D} = d/dx$.

But we wish to examine the possibly deep relationship between recursion, reflexive spaces and the properties of the quantum world. The hint we have received from the theory of the quantum is that we should begin with the mathematics which is replete with eigenforms. In fact, this hint seems very rich when we consider that **i**, the square root of minus one, is a key eigenform in our panoply of eigenforms and it is a key ingredient in quantum mechanics.

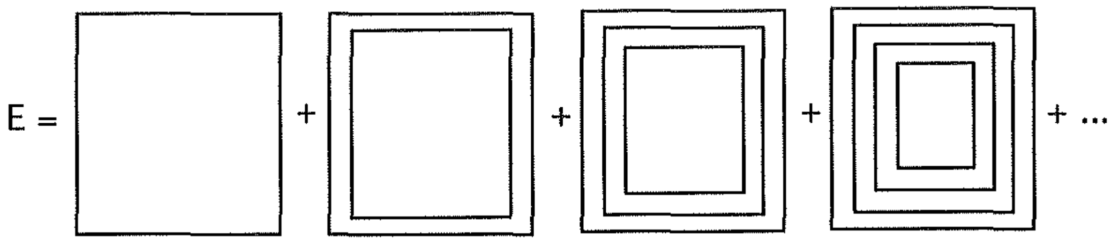
Lets begin by looking at the simpler case of differentiation. Consider an operator **D** that *removes* a box from around **X**.



DJ = J

Our familiar infinite nest of boxes is an eigenform for the "differentiation" operator **D**.

But we can go further. Consider an infinite series E of nested boxes as shown below.



Then extending **D** formally so that **D(X + Y) = D(X) + D(Y)**, we see that **D(E) = E** since **D** shifts the first box to void, the second box to the first box, the third box to the second box and so on.

Calculus and the Mathematics of Eigenforms

The exponential function is invariant under differentiation. Thus it is an eigenform for the operator **D=d/dt:**

$$D(\exp(t)) = \exp(t) \text{ where } D=d/dt.$$

In fact,

$$\exp(t) = 1 + t/1! + t^2/2! + t^3/3! + \ldots$$

where

$$D1 = 0,$$

$$Dt^{(n+1)}/(n+1)! = t^n/n!$$

from which it follows that

$$D(\exp(t)) = \exp(t).$$

If we think of the exponential function as a nest of boxes, each of which corresponds to one of the terms $t^n/n!$, then we see that the invariance of the nest of boxes **E** (above) under the formal differentiation operator has exactly the form of the invariance of **exp(t)** under differentiation in the calculus.

Another simple example of this sort is the series

$$S = 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + \ldots$$

Here we can write

$$S = 1 + x(1 + x + x^2 + x^3 + x^4 + x^5 + \ldots).$$

Thus

$$S = 1 + x S$$

and so **S** is an eigenform for the operator

$$T(A) = 1 + xA.$$

Now **i** is a close relative of this operator. If we define

$$R(A) = -1/A$$

then **R(i) = i** since $i^2 = -1$ is equivalent to **i = -1/i.**

Using the infinite recursion we would then write (see the discussion in Zermelo's Bar)

$$i = -1/-1/-1/-1/-1/\ldots,$$

making **i** an infinite reentry form for the operator **R.** Lets choose a notation for abbreviating such forms. We will write **i = [-1/*]** where * denotes the reentry of the whole form into that place in the right-hand part of the expression. Ok?

Similarly, if **F(x) = 1 + 1/x,** then the eigenform would be **[1 + 1/*]** and we could write

$$(1+\sqrt{5})/2 = [1 + 1/*].$$

With this in place we can now consider wave functions in quantum mechanics such as

$$\psi(x,t) = \exp(i(kx - wt)) = \exp([-1/*]\,(kx - wt))$$

and we can consider classical formulas in mathematics such as Euler's formula

$$\exp([-1/*]\varphi) = \cos(\varphi) + [-1/*]\,\sin(\varphi)$$

in this light. Really, we must start here with Euler's formula, for this formula is the key relation between complex numbers, **i** and waves and periodicity.

We have to return to the finite nature of **[-1/*]**. This eigenform is an oscillator between **-1** and **+1**. It is only **i** in its idealization or in its appropriate synchronization that it has the property that **i = -1/i.** As a real oscillator, the equation **R(i) = -1/i** tells us that when **i** is **1**, then i is transformed to **-1** and when **i** is **-1** then **i** is transformed to **+1**. There is no fixed point in the real domain. The eigenform is achieved by leaving the real domain for a new and larger domain. We know that this larger domain can be conceptualized as the plane with Euclidean rotational geometry, but we want to here explore the larger domain in terms of eigenforms.

We are now going to do this exploration, but we have to warn the reader: *We find that **i** itself is a fundamental example of a discrete physical process, and it is in the "microworld" of such discrete physical processes that not only quantum mechanics, but also classical mechanics is born.*

Iterants and Iterant Views

In order to think about i, consider an infinite oscillation between **+1** and **-1**:

$$\ldots -1,+1,-1,+1,-1,+1,-1,+1,\ldots$$

This oscillation can be seen in two distinct ways. It can we seen as **[-1,+1]** (a repetition in this order) or as **[+1,-1]** (a repetition in the opposite order). This suggests regarding an infinite alternation such as

... a,b,a,b,a,b,a,b,a,b,a,b,a,b,...

as an entity that can be seen in two possible ways, indicated by the ordered pairs **[a,b]** and **[c,d]**. We shall call the infinite alternation of **a** and **b** the *iterant* of **a** and **b** and denote it by **I{a,b}**. Just as with a set **{a,b}**, the iterant is independent of the order of **a** and **b**. We have **I{a,b} = I{b,a}**, but there are two distinct views of any iterant and these are denoted by **[a,b]** and **[b,a]**.

The key to iterants is that two representatives of an iterant can *by themselves* appear identical, but *taken together* are seen to be different. For example, consider

... a,b,a,b,a,b,a,b,a,b,a,b,a,b,...

and also consider

...b,a,b,a,b,a,b,a,b,a,b,a,b,a,...

There is no way to tell the difference between these two iterants except by a direct comparison as shown below

... a,b,a,b,a,b,a,b,a,b,a,b,a,b,...
... b,a,b,a,b,a,b,a,b,a,b,a,b,a,...

In the direct comparison we see that if one of them is **[a,b]**, then the other one should be **[b,a]**. Still, there is no reason to assign one of them to be **[a,b]** and the other **[b,a]**. It is a strictly relative matter. The two iterants are entangled (to borrow a term from quantum mechanics) and if one of them is observed to be **[a,b]**, then the other is necessarily observed to be **[b,a]**.

Lets go back to the square root of minus one as an oscillatory eigenform.

... -1,+1,-1,+1,-1,+1,-1,+1,...

What is the operation **R(x) = -1/x** in this case? We usually think of a starting value and then the new operation shifts everything by one value with **R(+1) = -1** and **R(-1) = +1**. Thus would suggest that

R(... -1,+1,-1,+1,-1,+1,-1,...) = ... +1,-1,+1,-1,+1,-1,+1,...

and these sequences will be different when we compare, them even though they are identical as individual iterants.

... -1,+1,-1,+1,-1,+1,-1,+1,...
... +1,-1,+1,-1,+1,-1,+1,-1,...

However, we would like to take the eigenform/iterant concept and make a more finite algebraic model by using the iterant views **[-1,+1]** and **[+1,-1]**. Certainly we should consider the transform **P[a,b] = [b,a]** and we take

-[a,b] = [-a, -b],

so that

-P[a,b] = [-b,-a].

Then

-P[1,-1] = [1,-1].

In this sense the operation **-P** has eigenforms **[1,-1]** and **[-1,1]**. You can think of **P** as the shift by one-half of a period in the process

...abababababab.... .

Then **[-1,1]** is an eigenform for the operator that combines negation and shift.

We will take a shorthand for the operator **P** via

P[a,b] = [a,b]' = [b,a].

If **x=[a,b]** then **x' = [b,a]**.

We can add and multiply iterant views by the combinations

[a,b][c,d] = [ac,bd],
[a,b] + [c,d] = [a+c, b+ d],
k[a,b] = [k,a,k,b] when k is a number.

We take **1 = [1,1]** and **-1 = [-1,-1]**. This is a natural algebra of iterant views, but note that **[-1,+1][-1,+1] = [1,1] = 1**, so we do not yet have the square root of minus one.

Consider **[a,b]** as representative of a process of observation of the iterant **I{a,b}**. **[a,b]** is an *iterant view.* We wish to combine **[a,b]** and **[c,d]** as *processes of observation.* Suppose that observing **I{a,b}** requires a step in time. That being the case, **[a,b]** will have shifted to **[b,a]** in the course of the single time step. We need an algebraic structure to handle the temporality. To this end, we introduce an operator η with the property that

$$[a,b]\eta = \eta[b,a] \text{ with } \eta^2 = \eta\eta = 1$$

where **1** means the identity operator. You can think of η as a *temporal shift operator* that can act on a sequence of individual observations. The algebra generated by iterant views and the operator η is taken to be associative.

Here the interpretation is that **XY** denotes "first observe **X**, then observe **Y**". Thus $X\eta Y\eta = XY'\eta\eta = XY'$ and we see that **Y** has been shifted by the presence of the operator η, just in accord with our temporal interpretation above.

We can now have a theory where **i** and its conjugate **-i** correspond to the two views of the iterant **I{-1,+1}**. Let **i** = [1,-1]η and **-i** = [-1,1]η . We get a square roots of minus one:

$$ii = [1,-1]\eta[1,-1]\eta = [1,-1][-1,1]\eta\eta = [-1,-1] = -[1,1] = -1.$$

The square roots of minus one are iterant views coupled with temporal shift operators. Not so simple, but not so complex either! If **e** = [1,-1] then **e'** = [-1,1] = -**e** and **ee** = [1,1] = 1 with **ee'** = -1.

$$i = e\eta$$
$$ii = e\eta e\eta = ee'\eta\eta = ee' = -1$$

With this definition of i, we have an algebraic interpretation of complex numbers that allows one to think of them as observations of discrete processes.

This algebra contains more than just the complex numbers. With **x** = [a,b] and **y** = [c,d], consider the products $(x\eta)(y\eta)$ and $(y\eta)(x\eta)$:

$$(x\eta)(y\eta) = [a,b]\eta[c,d]\eta = [a,b][d,c] = [ad,bc]$$

$$(y\eta)(x\eta) = [c,d]\eta[a,b]\eta = [c,d][b,a] = [cb,da].$$

Thus

$$(x\eta)(y\eta) - (y\eta)(x\eta)$$
$$= [ad-bc, -(ad-bc)]$$
$$= (ad - bc)[1,-1].$$

Thus

$$x\eta y\eta - y\eta x\eta = (ad - bc)i\eta.$$

We see that, with temporal shifts, the algebra of observations is non-commutative. Note that for these processes, represented by vectors [a,b], the commutator $x\eta y\eta - y\eta x\eta = (ad - bc)i\eta$ is given by the determinant of the matrix corresponding to two process vectors, and hence will be non-zero whenever the two process vectors are non-zero and represent different spatial rays in the plane.

There is more. *The full algebra of iterant views can be taken to be generated by elements of the form*

$$[a,b] + [c,d]\eta$$

and it is not hard to see that this is isomorphic with 2 x 2 matrix algebra with the correspondence given by the diagram below.

$$[a,b] + [c,d]\eta \longleftrightarrow \begin{bmatrix} a & c \\ d & b \end{bmatrix}$$

We see from this excursion that there is a full interpretation for the complex numbers (and indeed matrix algebra) as an observational system taking into account time shifts for underlying iterant processes.

Let **A = [a,b]** and **B = [c,d]** and let **C = [r,s], D = [t,u]**. With **A' = [b,a]**, we have

$$(A + B\eta)(C + D\eta) = (AC + BD') + (AD + BC')\eta .$$

This writes 2 x 2 matrix algebra in the form of a hypercomplex number system. From the point of view of iterants, the sum **[a,b]** + **[b,c]**η can be regarded as a superposition of two types of observation of the iterants **I{a,b}** and **I{c,d}**. The operator-view **[c,d]**η includes the shift that will move the viewpoint from **[c,d]** to **[d,c]**, while **[a,b]** does not contain this shift. Thus a shift of viewpoint on **[c,d]** in this superposition does not affect the values of **[a,b]**. One can think of the corresponding process as having the form shown below.

... a a a a a a a a a a a a a a a ...
... c d c d c d c d c d c d c d ...
... b b b b b b b b b b b b b b b ...

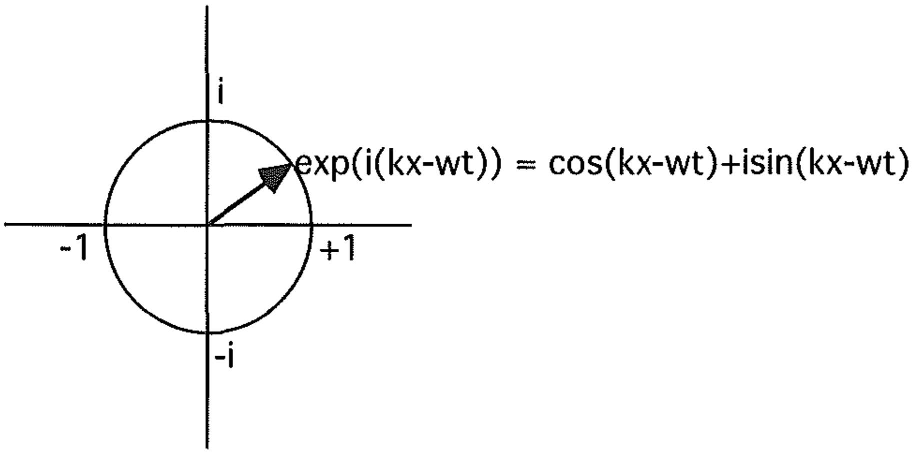
The snapshot **[c,d]** changes to **[d,c]** in the horizontal time-shift while the vertical snapshot **[a,b]** remains invariant under the shift. It is interesting to note that in the spatial explication of the process we can imagine the horizontal oscillation corresponding to **[c,d]**η as making a boundary (like a frieze pattern), while the vertical iterant parts a and b mark the two sides of that boundary.

Returning to Quantum Mechanics

You can regard **ψ(x,t) =exp(i(kx - wt))** as containing a micro-oscillatory system with the special synchronizations of the iterant view **i = [+1,-1]**η. It is these synchronizations that make the big eigenform of the exponential **ψ(x,t)** work correctly with respect to differentiation, allowing it to create the appearance of rotational behavior, wave behavior and the semblance of the continuum. Note that

$$\exp(i\varphi) = \cos(\varphi) + i\sin(\varphi)$$

in this way of thinking is an infinite series involving powers of **i**. The exponential is synchronized via **i** to separate out its classical trigonometric parts. In the parts we have **cos(φ) + i sin(φ) = [cos(φ), cos(φ)] + [sin(φ), -sin(φ)]**η, a superposition of the constant cosine iterant and the oscillating sine iterant. Euler's formula is the result of a synchronization of iterant process. One can blend the classical geometrical view of the complex numbers with the iterant view by thinking of a point that orbits the origin of the complex plane, intersecting the real axis periodically and producing, in the real axis, a periodic oscillation in relation to its orbital movement in the higher dimensional space.



The diagram above is the familiar depiction of a vector in the complex plane that represents the phase of a wave-function. I hope that the reader can now look at this picture in a new way, seeing **i** = **[+1,-1]**η as a discrete oscillation with built-in time shift and the exponential as a process oscillating between **cos(kx-wt) + sin(kx-wt)** and **cos(kx-wt)-sin(kx-wt).** The exponential function takes the simple oscillation between **+(kx-wt)** and **-(kx-wt)** and converts it by a complex of observations of this discrete process to the trigonometric wave-forms. All this goes on beneath the surface of the Schrodinger equation. This is the production of the eigenforms from which may be extracted the energy, position and momentum.

Higher Orders of Iterant Structure. What works for 2 x 2 matrices generalizes to n x n matrix algebra, but then the operations on a vector $[x_1,x_2,\ldots,x_n]$ constitute all permutations of n objects. That is a generating element of iterant algebra is now of the form $x\,\sigma = [x_1,x_2,\ldots,x_n]\sigma$ where σ is an element of the symmetric group S_n. The iterant algebra is the linear span of all elements $x\,\sigma$, and we take the rule of multiplication as

$$x\,\sigma\; y\,\tau = xy^{\sigma}\;\sigma\tau$$

where y^{σ} denotes the vector obtained from y by permuting its coordinates via σ ; **xy** is the vector whose k-th coordinate is the product of the k-th coordinate of x and the k-th coordinate of y ; $\sigma\tau$ is the composition of the two permutations σ and τ .

Hamilton's Quaternions

Here is an example. Hamilton's Quaternions are generated by the iterant views

$$I = [+1,-1,-1,+1]\sigma$$
$$J = [+1,+1,-1,-1]\lambda$$
$$K = [+1,-1,+1,-1]\tau$$

where

$$\sigma = (12)(34)$$
$$\lambda = (13)(24)$$
$$\tau = (14)(23).$$

Here we represent the permutations as products of transpositions. One can verify that

$$I^2 = J^2 = K^2 = IJK = -1.$$

For example,

$$\begin{aligned} I^2 &= [+1,-1,-1,+1]\sigma\ [+1,-1,-1,+1]\sigma \\ &= [+1,-1,-1,+1][-1,+1,+1,-1]\sigma\ \sigma \\ &= [-1,-1,-1,-1] \\ &= -1. \end{aligned}$$

and

$$\begin{aligned} IJ &= [+1,-1,-1,+1]\sigma\ [+1,+1,-1,-1]\lambda \\ &= [+1,-1,-1,+1][+1,+1,-1,-1]\ \sigma\ \lambda \\ &= [+1,-1,+1,-1]\ (12)(34)(13)(24) \\ &= [+1,-1,+1,-1]\ (14)(23) \\ &= [+1,-1,+1,-1]\ \tau. \end{aligned}$$

In a sequel to this paper, we will investigate this iterant approach to the Quaternions and other algebras related to fundamental physics. For now it suffices to point out that the quaternions of the form $\mathbf{a + bI + cJ + dK}$ with $\mathbf{a^2 + b^2 + c^2 + d^2 = 1}$ (a,b,c,d real numbers) constitute the group **SU(2)**, ubiquitous in physics and fundamental to quantum theory. Thus the formal structure of all processes in quantum mechanics can be represented as actions of iterant viewpoints.

Nevertheless, we must note that making an iterant interpretation of an entity like $\mathbf{I} = [+1,-1,-1,+1]\sigma$ is a conceptually natural departure from our original period two iterant notion. Now we are considering iterants such as **I{+1,-1,-1,+1}** where the iterant is a multi-set and the permutation group acts to produce all possible orderings of that multi-set. The iterant itself is not an oscillation. It represents an implicate form that can be seen in any of its possible orders. Once seen, these orders are subject to permutations that produce the possible views of the iterant. Algebraic structures such as the quaternions appear in the explication of such implicate forms.

The reader will also note that we have moved into a different conceptual domain from the original emphasis in this paper on eigenform in relation to recursion.

Indeed, each generating quaternion is an eigenform for the transformation **R(x) = -1/x.**

The richness of the quaternions arises from the closed algebra that arises with its infinity of eigenforms that satisfy this equation, all of the form **U = aI + bJ + cK** where $a^2 + b^2 + c^2 = 1$. This kind of significant extra structure in the eigenforms comes from paying attention to specific aspects of implicate and explicate structure, relationships with geometry and ideas and inputs from the perceptual, conceptual and physical worlds. Just as with our earlier examples (with cellular automata) of phenomena arising in the course of the recursion, we see the same phenomena here in the evolution of mathematical and theoretical physical structures in the course of the recursion that constitutes scientific conversation.

Quaternions and SU(2) Using Complex Number Iterants
Since complex numbers commute with one another, we could consider iterants whose values are in the complex numbers. This is just like considering matrices whose entries are complex numbers. For this purpose we shall allow given a version of **i** that commutes with the iterant shift operator η. Let this commuting **i** be denoted by ι (iota). Then we are assuming that

$$\iota^2 = -1$$
$$\eta\,\iota = \iota\,\eta$$
$$\eta^2 = +1.$$

We then consider iterant views of the form $[a + b\iota, c + d\iota]$ and $[a + b\iota\,, c + d\iota\,]\eta = \eta\,[c + d\iota\,, a + b\iota\,]$. In particular, we have $e = [1,-1]$, and $i = e\eta$ is quite distinct from ι . Note, as before, that $e\eta = -\eta e$ and that $e^2 = 1$. Now let

$$I = \iota e$$
$$J = e\eta$$
$$K = \iota\eta\ .$$

We have used the commuting version of the square root of minus one in these definitions, and indeed we find the Quaternions once more.

$$I^2 = \iota e\,\iota e = \iota\,\iota\ e\,e = (-1)(+1) = -1,$$
$$J^2 = e\eta\ e\eta = e\,(-e)\,\eta\eta = -1,$$
$$K^2 = \iota\eta\,\iota\eta = \iota\,\iota\,\eta\eta = -1,$$
$$IJK = \iota e\,e\eta\ \ \iota\eta = \iota\ 1\ \iota\,\eta\,\eta = \iota\,\iota = -1.$$

Thus

$$I^2 = J^2 = K^2 = IJK = -1.$$

This must look a bit cryptic at first glance, but the construction shows how the structure of the quaternions comes directly from the non-commutative structure of our period two iterants. In other, words, quaternions can be represented by 2 x 2 matrices. This is the way it has been presented in standard language. The group **SU(2)** of 2 x 2 unitary matrices of determinant one is isomorphic to the quaternions of length one.

$$\begin{pmatrix} z & w \\ -\overline{w} & \overline{z} \end{pmatrix} = [\, z \,,\, \overline{z}\,] + [\, w \,, -\overline{w}\,]\eta$$

In the equation above, we indicate the matrix form of an element of **SU(2)** and its corresponding complex valued iterant. You can easily verify that

1: z = 1, w = 0,
I: z = ι, w = 0,
J: z = 0, w = 1,
K: z = 0, w = ι.

This gives the generators of the quaternions as we have indicated them above and also as generators of **SU(2)**.

Similarly, **H = [a,b] + [c + di, c-di]η** represents a Hermitian 2 x 2 matrix and hence an observable for quantum processes mediated by **SU(2)**. Hermitian matrices have real eigenvalues. It is curious how certain key iterant combinations turn out to be essential for the relations with quantum observation.

XIV. Time Series and Discrete Physics

In this section we shall use the convention (outside of iterants) that successive observations, first A and then B will be denoted BA rather than AB. This is to follow previous conventions that we have used. We continue to interpret iterant observation sequences in the opposite order as in the previous section. This section is based on our work in [20] but takes a different interpretation of the meaning of the diffusion equation in relation to quantum mechanics.

We have just reformulated the complex numbers and expanded the context of matrix algebra to an interpretation of i as an oscillatory process and matrix elements as combined spatial and temporal oscillatory processes (in the sense that **[a,b]** is not affected in its order by a time step, while **[a,b]η** includes the time dynamic in its interactive capability, and 2 x 2 matrix algebra is the algebra of iterant views **[a,b] + [c,d]η**). We now consider elementary discrete physics in one dimension. Consider a time series of positions **x(t), t = 0, Δt, 2Δt, 3Δt, ...** . We can define the velocity v(t) by the formula **v(t) = (v(t + Δ) - v(t))/Δt = Dx(t)** where **D** denotes this discrete derivative. In order to obtain v(t) we need at least one tick **Δt** of the discrete clock. Just as in the iterant algebra, we need a time-shift operator to handle the fact that once we have observed **v(t)**, the time has moved up by one tick. Thus we shall add an operator **J** that in this context accomplishes the time shift:

$$x(t)J = Jx(t+\Delta t).$$

We then *redefine* the derivative to include this shift:

$$Dx(t) = J(x(t+\Delta) - x(t))/\Delta t\ .$$

The result of this definition is that a successive observation of the form **x(Dx)** is distinct from an observation of the form **(Dx)x**. In the first case, we observe the velocity and then **x** is measured at $t + \Delta t$. In the second case, we measure **x** at **t** and then measure the velocity. Here are the two calculations:

$$\begin{aligned} x(Dx) &= x(t)\,(J(x(t+\Delta) - x(t))/\Delta t\,) \\ &= (J/\Delta)(x(t+\Delta t))(x(t+\Delta t) - x(t)) \\ &= (J/\Delta t)(x(t+\Delta t)^2 - x(t+\Delta t)x(t)). \end{aligned}$$

$$\begin{aligned} (Dx)x &= (J(x(t+\Delta t) - x(t))/\Delta t\,)x(t) \\ &= (J/\Delta t)(x(t+\Delta t)x(t) - x(t)^2). \end{aligned}$$

We measure the difference between these two results by taking a commutator **[A,B] = AB - BA** and we get the following formula where we write $\Delta x = x(t+\Delta t) - x(t)$.

$$\begin{aligned} [x,(Dx)] &= x(Dx) - (Dx)x \\ &= (J/\Delta t)(x(t+\Delta t) - x(t))^2 \\ &= J\,(\Delta x)^2/\Delta t \end{aligned}$$

This final result is worth marking:

$$[x,(Dx)] = J\,(\Delta x)^2/\Delta t.$$

From this result we see that the commutator of **x** and **Dx** will be constant if $(\Delta x)^2/\Delta t = K$ is a constant. For a given time-step, this means that $(\Delta x)^2 = K\,\Delta t$ so that $\Delta x = +\sqrt{(K\,\Delta t\,)}$ or $-\sqrt{(K\,\Delta t\,)}$. In other words,

$$x(t+\Delta t\,) = x(t) + \sqrt{(K\,\Delta t\,)} \quad \text{or} \quad x(t) - \sqrt{(K\,\Delta t\,)}.$$

This is a Brownian process with diffusion constant equal to **K.**

Digression on Browian Processes and the Diffusion Equation
Assume, for the purpose of discussion that in the above process, at each next time, it is equally likely to have + or - in the formulas

$$x(t + \Delta t) = x(t) + \sqrt{(K \Delta t)} \quad \text{or} \quad x(t) - \sqrt{(K \Delta t)}.$$

Let **P(x,t)** denote the probability of the particle being at the location x at time t in this process. Then we have

$$P(x, t + \Delta t) = (1/2)(P(x - \Delta x) + P(x + \Delta x)).$$

Hence

$$(P(x, t + \Delta t) - P(x,t))/\Delta t)$$
$$= ((\Delta x)^2/2\Delta t)(P(x - \Delta x) - 2P(x,t) + P(x + \Delta x,t))/(\Delta x)^2)$$
$$= (K/2)(P(x - \Delta x) - 2P(x,t) + P(x + \Delta x,t))/(\Delta x)^2).$$

Thus we see that **P(x,t)** satisfies the a discretization of the diffusion equation

$$\partial P/\partial t = (K/2)\partial^2 P/\partial x^2.$$

Of course, this demands comparison with the Schrodinger equation in the form (with zero potential) shown below.

$$i\hbar\partial\psi/\partial t = -(\hbar^2/2m)\partial^2\psi/\partial x^2$$

In the Schrodinger equation we see that we can rewrite it in the form

$$\partial\psi/\partial t = i(\hbar/2m)\partial^2\psi/\partial x^2$$

Thus, if we were to make a literal comparison with the diffusion equation we would take $K = i(\hbar/m)$ and we would identify

$$(\Delta x)^2/\Delta t = i(\hbar/m).$$

Whence

$$\Delta x = ((1+i)/\sqrt{2}) \sqrt{[(\hbar/m)\Delta t]}$$

and the corresponding Brownian process is

$$x(t + \Delta t) = x + \Delta x \text{ or } x - \Delta x.$$

The process is a step-process along a diagonal line in the complex plane. We are looking at a Brownian process with complex values! What can this possibly mean? Note that if we take this point of view, then **x** is a complex variable and the partial derivative with respect to **x** is taken with respect to this complex variable. In this view of a complexified version of the Schrodinger equation, the solutions for Δx as above are real probabilities. We shall have to move the **x** variation to real x to get the usual Schrodinger equation, and this will result in complex valued wave functions in its solutions.

In our context, the complex numbers are themselves oscillating and synchronized processes. We have $\mathbf{i} = [1,-1]\eta$ where η is a shifter satisfying the rules of the last section, and $[1,-1]$ is a view of the iterant that oscillates between plus and minus one. Thus we are now observing that solutions to the Schrodinger equation can be construed as Brownian paths in a more complicated discrete space that is populated by both probabilistic and synchronized oscillations. This demands further discussion, which we now undertake.

The first comment that needs to be made is that since in the iterant context Δx is an oscillatory quantity it does make sense to calculate the partial derivatives using the limits as Δx and Δt approach zero, but this means that the interpretation of the Schrodinger equation as a diffusion equation and the wave function as a probability is dependent on this generalization of the derivative. *If we take Δx to be real, then we will get complex solutions to Schrodinger's equation.* In fact we can write

$$\psi(x, t + \Delta t) = (1 - i)\psi(x, t) + (i/2)\psi(x - \Delta x) + (i/2)\psi(x + \Delta x)$$

and then we will have, in the limit,

$$\partial\psi/\partial t = i(\hbar/2m)\partial^2\psi/\partial x^2$$

if we take $(\Delta x)^2/\Delta t = (\hbar/m)$.

It is interesting to compare these two choices. In one case we took

$$(\Delta x)^2/\Delta t = i(\hbar/m)$$

and obtained *a Brownian process with imaginary steps.*

In the other case we took

$$(\Delta x)^2/\Delta t = (\hbar/m)$$

and obtained *a real valued process with imaginary probability weights.* These are complementary points of view about the same structure.

With $(\Delta x)^2/\Delta t = (\hbar/m)$, $\psi(x, t)$ is no longer the classical probability for a simple Brownian process. We can imagine that the coefficients $(1-i)$ and $(i/2)$ in the expansion of $\psi(x, t + \Delta t)$ are somehow analogous to probability weights, and that these weights would correspond to the generalized Brownian process where the real-valued particle can move left or right by Δx or just stay put. Note that we have

$$(1 - i) + (i/2) + (i/2) = 1,$$

signaling a direct analogy with probability where the probability values are imaginary. But this must be explored in the iterant epistemology!

Note that **1-i = [1,1] -[1,-1]η** and so at any given time represents either **[1,1] - [1,-1] = [0,2]** or **[1,1] - [-1,1] = [2,0]**. It is very peculiar to try to conceptualize this in terms of probability or amplitudes. Yet we know that in the standard interpretations of quantum mechanics one derives probability from the products of complex numbers and their conjugates. To this end it is worth seeing how the product of **a+bi** and **a-bi** works out:

$$
\begin{aligned}
(a + bi)(a - bi) &= aa + bia + a(-bi) + (bi)(-bi) \\
&= aa + abi - abi - bbii \\
&= aa - bb(-1) \\
&= aa + bb.
\end{aligned}
$$

It is really the rotational nature of **exp(it)** that comes in and makes this work. **exp(it)exp(-it) = exp(it - it) = exp(0) = 1** The structure is in the exponent. The additive combinatory properties of the complex numbers are all under the wing of the rotation group.

A fundamental symmetry is at work, and that symmetry is a property of the synchronization of the periodicities of underlying process. The fundamental iterant process of **i** disappears in the multiplication of a complex number by its conjugate. In its place is a pattern of apparent actuality. It is actual just to the extent that one regards **i** as only possibility. On making a reality of **i** itself we have removed the boundary between mathematics and the reality that "it" is supposed to describe. There is no such boundary.

XV. Epilogue

The problem that we have resolved in this paper is the problem to understand the nature of observation in quantum mechanics. In fact, we hope that the problem is seen to disappear the more we enter into the present viewpoint. A viewpoint is only on the periphery. The iterant from which the viewpoint emerges is in a superposition of indistinguishables, and can only be approached by varying the viewpoint until one is released from the particularities that each point of view contains.

It is not just the eigenvalues of Hermitian operators that are the structures of the observation, but rather the eigenforms that populate the mathematical models at all levels. These forms are the indicators of process. Mathematics, instead of being a descriptive symbol system for various algorithms, comes alive as an interrelated orchestration of processes. It is these processes that become the exemplary operators and elements of the mathematics that are put together to form the physical theory. We hope that the reader will be unable, ever again, to look at Schrodinger's equation the same way, after reading this argument.

References

1. H. P. Barendregt, "The Lambda Calculus - Its Syntax and Semantics," North Holland Pub. (1981,1985).

2. H. Bortoft, The Whole - Counterfeit and Authentic, Systematics, vol. 9, No. 2, Sept. (1971), 43-73.

3. G. Spencer-Brown, "Laws of Form," George Allen and Unwin Ltd. (1969).

4. Heinz von Foerster, "Observing Systems," The Systems Inquiry Series, Intersystems Publications (1981).

5. Heinz von Foerster, Objects: tokens for (eigen-) behaviors, in "Observing Systems," The Systems Inquiry Series, Intersystems Publications (1981), pp. 274 - 285.

6. Heinz von Foerster, Notes on an epistemology for living things, in "Observing Systems," The Systems Inquiry Series, Intersystems Publications (1981), pp. 258 - 271.

7. Heinz von Foerster, On constructing a reality, in "Observing Systems," The Systems Inquiry Series, Intersystems Publications (1981), pp. 288 - 309.

8. L. H. Kauffman, Self-reference and recursive forms, Journal of Social and Biological Structures (1987), 53-72.

9. L. H. Kauffman, Knot logic, in "Knots and Applications", ed. by L. H. Kauffman, World Scientific Pub. Co. (1995), pp. 1-110.

10. L. H. Kauffman, The mathematics of Charles Sanders Peirce, in Cybernetics and Human Knowing, Volume 8, No. 1-2, (2001), pp. 79-110.

11. Louis H. Kauffman, Eigenform, Kybernetes - The Intl J. of Systems and Cybernetics, Vol. 34, No. 1/2 (2005), Emerald Group Publishing Ltd, p. 129-150.

12. Louis H. Kauffman, Eigenforms - Objects as Tokens for Eigenbehaviors, Cybernetics and Human Knowing, Vol. 10, No. 3-4, 2003, pp. 73-90.

13. F.W. Lawvere, Introduction to "Toposes, Algebraic Geometry and Logic" Springer Lecture Notes on Mathematics Vol. 274 (1970) , pp. 1-12.

14. B. B. Mandelbrot, "The Fractal Geometry of Nature", W. H. Freeman and Company (1977, 1982).

15. W. S. McCulloch, What is a number that a man may know it, and a man, that he may know a number?, in "Embodiments of Mind", MIT Press (1965), pp. 1-18.

16. B. Piechocinska, Physics from Wholeness, PhD. Thesis, Uppsala Universitet (2005).

17. J. J. Sakurai, Modern Quantum Mechanics, Benjamin/Cummings Publishing Company, Inc. (1985).

18. D. Scott, Relating theories of the lambda calculus, in "To H. B. Curry: Essays on Combinatory Logic, Lambda Calculus and Formalism", (P. Seldin and R. Hindley eds.), Academic Press (1980), pp. 403-450.

19. L. Wittgenstein, "Tractatus Logicus Philosophicus", Routledge and Kegan Paul Ltd, London and New York (1922).

20. Louis H. Kauffman, Non-Commutative Worlds, New Journal of Physics, Vol. 6, (2004), 173 (47 pages).

ON THINKING

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Edited version of talk given at ANPA, August, 2008 with additions.

> "I *must*, before I die, find *some* way to say the essential thing that is in me, that I have never said yet – a thing that is not love or hate or pity or scorn, but the very breath of life, fierce and coming from very far away. Bringing into human life the vastness and the fearful passionless force of non-human things . . ."
> Bertrand Russell, letter to Constance Malleson, 1918

THE THINKER

Start with Odysseus in Homer's *Iliad* (c. 1,000 BC).

Odysseus was considered as strange (the 'cunning') and different from the other heroes. Instead of acting on impulse he could reflect and consider things in his mind. He could solve problems.

A speculation is that he marked a change in human consciousness – or at least within a certain domain or historical type of culture. This idea was developed by Julian Jaynes [1]. His thesis: at an earlier period information from the right brain appeared as coming to people from outside of them; then there was a change such that it was 'integrated' with the left brain to produce the modern type of mind where a person thinks.

The image at least of man changed in Greece so that by the time of the philosophers (by 4th century BC) it was very different. [2] In the early Homeric age thought was in the *lungs* – because it was identified with speech. The brain was considered to be *sexual* – possibly our 'creative'. It was identified with the *daimon* (see how Socrates talks about the daimon) and amongst the Romans with *genius*.

Not only a culture can change its view of man but different cultures embody different views. Kuriyama, for example, points out that what the 'pulse' is or means in western medicine is quite different from what it means in Chinese medicine. [3] Westerners have puzzled about this since the 17th century. Looking at images (and sculptures) we see the Greeks emphasising the *muscles* because they thought in terms of what we would now call 'will': they divided voluntary from involuntary movements and saw the muscles as organs of will (women were not shown in this way because they were regarded as without will).

THINKING IS INSIDE

Early cultures were very *physical* in their view of man. Example, in India the *Atman* was originally seen as a 'little man' inside the heart and only later became abstract and invisible.

St Augustine in his *Confessions* remarks on the strange phenomenon of people reading without even moving their lips. Originally, reading meant always reading aloud; then one moved the lips silently, but later even this ceased. This gave the impression of some *mental private process* that became linked with thinking.

A suggestion: that belief in thinking is an *inference* and that the belief in some totally *direct* knowledge of it is mistaken. Allied hint: we only know we are *conscious* through other people; the idea of a totally private world is an invention of some cultures.

SCIENCE AND CULTURE

It is a puzzle why science – as we tend to identify it today – arose in Europe. Consider the achievements of e.g. China and the Arab world earlier, as evidenced in Needham's masterwork on Chinese science. [4] Needham was a Marxist and in love with China (he married a Chinese). He could speak and read Chinese, Arabic, Hebrew, Greek, etc. In a talk he said that he had best concluded that the explanation of science arising in Europe was related to Christianity and Capitalism – very striking for a Marxist!

On a personal note: the arising of science in Europe puzzled me and led me from physics to the history and philosophy of science, which I studied in Cambridge. [5].

It appears that early European scientists such as Newton were much influenced by stories, legends, and myths about *ancient science.* They believed that a real science has existed in the ancient past and their task was to rediscover or re-create it again. For instance, there was the idea that

Enoch had studied astronomy under the British Druids. For instance, Kepler who considered the 'music of the spheres'; that Newton spoke of *seven* colours because of the ancients for whom seven was highly significant.

A speculation is that it took the evolution of a certain kind of culture to enable the emergence of modern science. An ancillary point is the historical fact that by 1500 'reason' was under the thumb of 'faith' in the Islamic world.

The way of science that emerged by the 17th century took a certain direction. According to Wolfgang Pauli, 'science took a wrong path' by eliminating e.g. alchemy. [6]. Pauli says science followed the *triad* as in mathematical physics and eschewed the *tetrad,* thereby missing out on *becoming* and modes of transformation of substance.

In Newton, 'hypotheses non fingo' meant not only the adoption of a new method of mathematical physics but the rejection of any sense that objects or anything in the universe besides man had any power of their own. This was the rejection of the 'occult'. It meant that everything was to be considered as a mechanism with no inwardness of its own. This stemmed from the Church which was fighting a battle with the occultists: consider the influence of John Dee at the court of Elizabeth. Christianity like Buddhism has an anti-nature stance (though it also has the other side as well). In China, there has always been an alternation of Confucianism with Taoism (Needham speculated about the possible future emergence of a *Taoist* science) that was reflected in the practice of Buddhism.

Margaret Wertheim documents the influence of the Church on physics and points out that this extended even to denying women any soul! (thanks to Henri Bortoft for telling me of her book). [7]

In the birth of modern physics there was incorporated an alienation of man from nature. Francis Bacon: put Nature on the rack to extract her secrets. Harvey and vivisection: animals were machines. This meant the emergence of a view of man as a separate special entity in whom the divine attributes of will, reason, etc. were present but nowhere else. Weinberg's 'the universe is found to be meaningless' was inevitable, because that was the inherent programme of modern science.

Side-bar: there has always been - and necessarily so - some thread of alternative or underground science [8] [9]. I had a correspondence with Heitler about his view that the universe was intelligent: he could only think in terms of the abstract principle of intelligence not of its concreteness.

SCIENTIFIC METHOD

In the nineteenth century William Whewell [10] established the word 'science' in place of 'natural philosophy' as in the sense 'to know'. He also set out what is known as the *hypothetico-deductive* method clearly: man invents pictures or ideas about the world that are then put to the test. He said that 'fancy' or imagination was necessary to generate some new idea but it was a scientific idea only if it could be tested and led to something not previously known. Since his time it has been the way to ignore or marginalise *how* a good hypothesis is created.

A side-bar on the strip cartoon *Hey BC!* In one, the main character becomes frustrated when his contemporaries go round declaiming their discoveries – of fire, the wheel, etc. – until he seizes hold of one of them and declaims 'I have discovered the discoverer!'

A speculation is that as scientists discovered strange new objects they were simultaneously revealing the 'thinker' - while still maintaining old models of the separate human mind. This led to such commonly expressed puzzles as 'why mathematics, the product of pure thought, can turn out to fit the real world so well'. Such a view derives from the assumption that mind is separate from the world in the first place. And this assumption derives from articulating a world entirely in terms of separate – and non-living non-aware – things, which reflected back into the view of minds as also separate (private) and thinking as abstract and divided from perception.

Some Sufi teachings aver that 'thinking is on the same level as material objects'. They both belong to what is called the *alam-i-ajsam* the world of objects (*alam* = 'world'). Sufism, as many other traditions do, postulates (or reports from experience) that there are other worlds. For example the *alam-i-arvah* translated variously as 'the world of spirits' or, more recently, as the 'world of *energies*'. In the modern western world something of this is reflected in *psycho-analysis.* Psycho-analysis suggests that thinking is not what it seems. This has become something deeper than a concern with curing mental ills and offers a new way in to understanding thought and the thinker or 'discoverer' as creative thinker.

The Shadow of the Object

A standard idea in psycho-analysis (Melanie Klein, Bion) is that thinking arises from frustration. Crudely put, in the womb there is instant gratification – food supply is continuous, etc. Once born, there appears a gap between desire and satisfaction: the baby cries in hunger but is not instantly fed. This is the source of what we coldly call 'explanation'. Such proto-explanation fills the gap and creates a new world for the infant. [11]

A side-bar: the original meaning of the word 'chaos' was *yawning gap* [12] and in ancient cosmologies it *precedes* the creation of the gods or the 'universe'. Also remember that the word 'universe' literally means 'turning into the one' and there was no suggestion that the universe *is* one.

The new world is full of imaginary beings – but we must caution ourselves that we do not understand what imagination is – including the beings who become 'other people' (but not yet). It is also able to provide a new kind of satisfaction, something that works through in time to become the satisfaction in explaining as such: we come to like to 'understand'. Another thing is that the 'objects' of the world take on characteristics that are *projections* from the psyche. In a way, the objects are endowed with what the psyche feels it lacks.

A side-bar: the word 'subjective' originally meant what was *real*. Do not imagine that the term 'projection' can be identified with inventing something unreal.

The psycho-analytic view makes knowing an active and creative thing and not simply some supposed 'accurate' reflection in the mind of what is 'out there'. It is a quite distinct view from the usual one of supposing we make 'models' or pictures that approximate to what is really going on.

The standard empiricist view – cf. Locke - of sense data coming into the brain and being processed to make sense of the world by some miraculous decoding derives from thinking-in-objects and, is not only the result of a Cartesian split, but contrary to recent research that shows that the brain/mind 'reaches out into the world' to make its perceptions. The common assumption of a stimulus-response structure is wrong.

The psycho-analytic view may be misunderstood as merely 'projection' or subjective in the modern sense, and there is the complementary side of the world coming to presence in us, the two sides undivided. *How* the world comes to presence in us concerns perception but also thinking.

The Russian psychologist Vygotsky, now very influential in the west in educational theory, says that we learn to think through participation in conversations. Education now has an archetypal form in considering conversations that involve agents who *can ask questions* that other participants cannot yet ask themselves. It is the interiorisations of such questioning that create thinking in children. To think means *to ask oneself questions one cannot yet answer.*

In general, as expressed especially by Foulkes the individual mind is seen as some sort of node or 'condensation' out of what he calls a *matrix*. This has been taken up in a practical way by various people such as Mary Abercrombie, who reported and discussed work she did with science students on their understanding of science, drawing on her experience of taking part in Foulkesian groups. [13] Gordon Lawrence developed the *Social Dreaming Matrix* [14]. In his method, dreams are used as a source of new thinking about a shared situation. There is nothing involved of the usual 'interpretation of dreams' as diagnostic of the individual who has the dream. There is not even any interpretation at all. Participants engage in 'free association' to explore links between the dreams that might provide insight.

Side-bar: the matrix could be seen as a quantum field and brains as particles, as in Bohm's approach [15] and extended in Sarfatti's view of consciousness as a 'back-action' on the field from brains. Brains are treated as quantum particles. This is related to Patrick de Mare's idea that 'mind is between brains not in brains'. [16] de Mare was Bohm's therapist and introduced him to his ideas of *dialogue*.

Psycho-analysis is linked to the idea of the *unconscious* and there is an increasing acknowledgement of what Lawrence calls *'unconscious intelligence'*. Matte-Blanco deals with the unconscious in terms of infinite sets and struggles with the phenomena of 'translation' between the world of the conscious and the unconscious. [17] Meanwhile, more and more people admit that the most powerful source of thinking is unconscious, which radically shifts from the classical view of building everything on the simplest, clearest and most distinct ideas. [18].

SCIENCE IS PART OF NATURE-NATURING

Johannes Scotus Eriugena (c. 815–877) speaks of (a) that which creates and is not created (b) that which is created and creates (nature-naturing) (c) that which is created and does not create (d) that which is neither created nor creates. [19] (a) is God as origin and (d) is God as realisation or perfecting; the alpha and omega. [See addendum]

The old model of private minds trying to know an objective world ignores most of what works and is interesting. First of all there is the fact that mind and thinking arise in societies. Empiricists and existentialists alike have to sneak in society by back-doors. There cannot be language arising in a child without a human community. Secondly, the arising of thinking comes with a more hidden process of ways of seeing that are rooted more in emotion than in thought. Emotion in its turn, as in contemporary discourse on what is called *affect*, is rooted in our biology and evolution.

Side-bar: more than one physicist has turned Hume on his head by suggesting that what is called 'physical intuition' comes out of the process of the physical body becoming partially conscious. This may be related to current interest in *shamanism.*

Scientists embrace or reject ideas according to their deep-rooted emotional attitudes, which have somehow become established in them in ways that must include the formatory influences of family and culture, even religion and language, as well as their biology (e.g. in terms of brain functions). In affect theory there are six elements: Hate, Fear, Anger, Disgust, Love and Curiosity. Scientists can become enraged by ideas contrary to their own internal stance or world view. As with most people, they believe they are being objective and are usually not conscious of the underlying forms of thought they bear in them. Instances in the history of science related to these ideas: Cantor driven into lunatic asylums, and Boltzmann to suicide; Schrödinger's equation called 'obscene' etc.

The psycho-analytic perspective looks at what is happening in the scientist as integral to what he does. Even though certain ideas win out in a Darwinian fashion as the survivors of some struggle to dominate, there remain aspects that are not dealt with or concluded at all. See, for instance: quantum theory works and what it means can be ignored but has continued to trouble (some) people for a hundred years now.

Bohm argued that every theory proposed had its own proper domain of truth. [20] David Deutsch proposed there could not be any one type of complete theory but there had to be *four*: [21]. Feyerabend asserted that there could not be any one method that fitted all things. [22] The suggestion may be that *what* we study is integrally linked with *how* we study it.

This extends to the prospect that how we can think about something depends on that something and there cannot be any kind of thinking that is universal.

Side-bar: I had conversation with an industrial scientist who told me that in his first job he had to find out why certain chemicals were decaying as they were stored and he approached the task by *imagining what it was like to be the chemicals.* Of course, he wrote up his work without mentioning anything of this. Years ago, Medewar exposed that we hardly ever get to hear of what scientists actually do to arrive at their results. [23]

There are two issues: the seduction of oneness and the nature of *participative consciousness* in contrast with 'observer consciousness'.

Oneness is a deeply rooted stance common in both religion and science. It is a top-down approach and links with hierarchical views and authority-based systems. Purveyors of this approach each claim to know the One that rules all. This is not a necessary stance for science to take. But it comes out a lot in the casual talk of 'knowing the mind of God' – as in Einstein and Hawking – however metaphorically we take such statements. What this does is to identify the self with God, or give it some absolute status.

Participation is relevant to finding an alternative to oneness. A background idea is that everything is deeply connected in such a way that any one phenomenon can illuminate any other. This is quite different from supposing that there is something behind them all that we can know apart from the diversity. *Unity is in diversity and diversity in unity.* They are inseparable. Separating out some oneness from diversity is an illusion. So is separating out the thinker from the world as if they were of two divided natures.

In the way of participation, the scientist *goes into* the phenomena and does not stand apart as an onlooker. This was exemplified in the *Biosphere 2* project in which eight men and women spent two years hermetically sealed in a specially created mini-biosphere – with an ocean, desert, rain forest, agriculture and even technosphere – and were subject to extreme conditions: the opposite to 'men in white coats doing experiments'. [24] The project had its roots in the ideas of the Russian biogeochemist Vernadsky [25] and the early experiments done based on his ideas in one of the hidden 'science cities' of Soviet Russia.

The humans were within the Biosphere 2, which was within Biosphere 1. Biosphere 2 was the instrument of study of Biosphere 1 by *being of the same nature.*

Side-bar: the whole idea of observation and experiment needs to be transformed here. *Observation is a profound human act that has arisen out of life and its evolution.* [26] Konrad Lorenz, the naturalist, was the last occupant of the chair of epistemology first held by Kant. Major scientists have had powers of observation of remarkable degree. Example: Leuwenhoek (a copy of his microscope is in the Whipple Science museum where I studied) who saw bacteria a hundred years before anyone else did. Example: as reported in Abercrombie, it takes a significant step for a student to be able to 'read' an X-ray and 'see' what it shows. Observation in this sense precedes or is besides the kind of 'observation' linked to taking numbers from a machine in an experiment.

Goethe did not divide thinking or 'conception' from perception as most people now do. What he did involved *transforming his perception* not just adding concepts onto it. He *saw* what he was thinking. His thinking was aimed at becoming akin to the natural process of 'nature-naturing' (see Scotus Eriugena cited earlier).

Observational biology is now marginalised but still alive. E. O. Wilson writes about the *naturalist's trance.*

"In a twist my mind came free and I was aware of the hard workings of the natural world beyond the periphery of ordinary attention, where passions lose their meaning and history is in another dimension, without people, and great events pass without record or judgement. . . I focused on a few centimetres of ground and vegetation. I willed animals to materialize, and they came erratically into view. "

and

"I came to a small glade that opened onto the sandy path. I narrowed the world down to the span of a few meters. Again I tried to compose the mental set - call it the naturalist's trance - by which biologists locate more elusive organisms." [27]

THE REAL OBJECTS ARE COLLECTIVES

Actual science is conducted through groups of people, institutions, languages, finance, apparatuses, as well as a variety of kinds of object, processes, including computation and kinds of experience. The actual complexity of science is vastly beyond the simplistic dyad of thinking and phenomena. Bertrand Latour speaks of 'collectives', in particular of humans and non-humans. [28] There is never any such thing as a single mind or a single object in science; nor of objects plus minds. All our experience and work comes out of a complex totality that is enjoying itself!

Vernadsky again: the idea of the *noosphere* as emergent from the biosphere and integral to it. The idea of the noosphere came from Le Roy, a student of Bergson and was acquired by Teilhard de Chardin who was at the Sorbonne at the same time as Vernadsky. Westerners believed the idea was de Chardin's partly because Vernadsky became lost to view in Stalinist Russia and the West took up Chardin's mystical view, in contrast with Vernadsky who treated it materially.

Bohm considered that mind and matter must arise from a common source and their separation is only a surface thing. The common source was called *hylé* or *ylem* in olden times. I would like to call it *meaning*. Bohm speaks of the triad Matter, Energy and Meaning.

The modern development is to incorporate evolution and cognitive science into our thinking as suggested by Deutsch.

In psycho-analysis – e.g. in Bion – thinking is seen as akin to the *digestive system*, hence as in a process of transformation. This has been explored by e.g. Gendlin who proposed a scheme whereby inchoate 'felt-sense' may be transformed into articulate new theory: hence, not starting from clear ideas at all though deeply involving language. Bion speaks of 'alpha-elements' that are experienced but not conceptualised turning into 'beta-elements' that can be thought about. There is some primordial process of rendering *experience* into elements of meaning, thus making new thoughts possible. Such a process is rooted in nature. New thinking comes from the deeper levels beyond habitual forms of consciousness (remember Wilson's 'elusive organisms' – these are not just animals and plants but qualities of life). The rest is calculation, or following some system of computation.

Side-bar: Turing's universal machine was an attempt to model intelligence as a mechanism, which is something that intelligence always does. Simulations of intelligence are not only in the form of current AI but more widely-based: consider what a novel is. This leads to the Gnostic speculation that we ourselves are just simulations of intelligence and not its source.

A 'collective' is a *world* and science takes place in a world (see above on the *alam* in Sufism). The diversity of life is to be loved, as in Wilson's *Biophilia*: 'you cannot understand anything unless you love life'. Complexity is not just to be 'solved' but to be embraced as a source of wisdom. Nature loves all Her children equally. Even as in the Bible: 'Not a sparrow falls but that the Lord thy God, knoweth it'.

The mystic-philosopher Simone Weil says that only love can lead the intellect. [29] Some scientists acknowledge this: you must love what you research else you turn into a killer of it. Science must be an expression of the biosphere, or "the very breath of life, fierce and coming from very far away" of Russell, that is, a cosmic phenomenon (as stated in Vernadsky). Science that does not 'explain' science is incomplete, but it is called beyond itself.

THE CALLING

Heidegger speaks of the word 'think' as allied to the word 'thank' and of our need to respond to the gods with thankfulness; the implication being that what we usually call thinking is only a mechanism, a technology and what he is after is not 'our' thinking at all, the thinking of Odysseus, but the 'miracle of thought' [30]

THE DESTROYER

I had thought of finishing my talk with reference to Shiva, the Destroyer, as a paradigm of thinking. Thinking must destroy, must make new spaces; it is of the nature of evolution. A typical move is to declare 'all this is illusion'. As Rilke put it: 'Is it possible that nothing has been discovered yet, nothing understood? – It is possible' [31]

ADDENDUM 1

I was going to mention my 'meaning equation' in the talk but there was not time. Reverting to the triadic mathematical style of modern science (see reference to Pauli) produces an expression which actually corresponds closely with those of many ANPA members. It is of the same nature as the Christian Trinity, Matchett's 3-M equation and Gurdjieff's triad, to cite some perhaps unusual examples.

In my book *The Supreme Art of Dialogue* I introduce the 'meaning equation'

$$M(1) + M(2) = M(3)$$

The + and = are like operators but there is pretension to mathematical precision here. However it is likely that the variant forms:

$$2.1 = 3$$
$$3 = 1.2$$
$$3 = 2.1$$

have meaning. If we qualitatively associate M(1) with what is 'material' and M(2) with what is 'mental' then M(3) is 'meaning'. The form 321 reads 'love giving rise to research', or simply 'science arises from meaning'.

The basic thing is that in order to understand something we have to *add* something of our own to it. What emerges is a co-creation. In the past this 'equation' has been seen as saying that mind comes after matter and makes something imaginary. But the quest for 'pure' knowledge is a chimera. If it were attained we would be in death.

One manifestation of the principle is that when we become aware [M(2)] of a natural process [M(1)] it becomes transformed [M(3)]. Reference Patrick de Mare's idea that we speak [M(1)] and then become aware of speaking [M(2)] to transform speaking into dialogue [M(3)]. This is totally different from the usual view that our conscious produces 'subjective' or imaginary entities. M(2) can be language.

There is re-iteration. In its turn, M(3) is operated with M(4) towards M(5) and so on. Or one can say that the M(3) takes the role of M(1). This movement is *evolutionary.* Such is the way of science. Knowing in the old sense is not the concern. M(3) changes us. What it means to perceive or think changes in the act of perceiving and thinking.

sensation + thinking = perception

In the psycho-analysis of Lacan there is the Real, the Imaginary and the Symbolic.

T. S. Eliot: "We must be still and still moving
Into another intensity
For a further union, a deeper communion . . ."

ADDENDUM 2

The tetradic form of Scotus Eurigena is now mostly known in terms of the *semiotic square* of Griemas. In the abstract:

A	-A
A & - A	neither A nor –A

Hence:

Creative	Created
Created creating	Neither created nor creating

The first pair is the most common and the second pair the most interesting. That which is neither created nor creating was allied to the Holy Ghost in the way of the *perfecting* of the universe.

END QUOTE

This fragment by Goethe (c. 1780) says it all so much better. It is interesting that it appeared on the cover of the first issue of the science journal *Nature* Nov 4, 1869 introduced by Thomas Huxley. [32]

> "NATURE! We are surrounded and embraced by her: powerless to separate ourselves from her, and powerless to penetrate beyond her.
>
> Without asking, or warning, she snatches us up into her circling dance, and whirls us on until we are tired, and drop from her arms.
>
> She is ever shaping new forms: what is, has never yet been; what has been, comes not again. Everything is new, and yet nought but the old.
>
> We live in her midst and know her not. She is incessantly speaking to us, but betrays not her secret. We constantly act upon her, and yet have no power over her.
>
> The one thing she seems to aim at is Individuality; yet she cares nothing for individuals. She is always building up and destroying; but her workshop is inaccessible.
>
> Her life is in her children; but where is the mother? She is the only artist; working-up the most uniform material into utter opposites; arriving, without a trace of effort, at perfection, at the most exact precision, though always veiled under a certain softness.

Each of her works has an essence of its own; each of her phenomena a special characterisation: and yet their diversity is in unity.

She performs a play; we know not whether she sees it herself, and yet she acts for us, the lookers-on.

Incessant life, development, and movement are in her, but she advances not. She changes for ever and ever, and rests not a moment. Quietude is inconceivable to her, and she has laid her curse upon rest. She is firm. Her steps are measured, her exceptions rare, her laws unchangeable.

She has always thought and always thinks; though not as a man, but as Nature. She broods over an all-comprehending idea, which no searching can find out.

Mankind dwell in her and she in them. With all men she plays a game for love, and rejoices the more they win. With many, her moves are so hidden, that the game is over before they know it.

That which is most unnatural is still Nature; the stupidest philistinism has a touch of her genius. Whoso cannot see her everywhere, sees her nowhere rightly.

She loves herself, and her innumberable eyes and affections are fixed upon herself. She has divided herself that she may be her own delight. She causes an endless succession of new capacities for enjoyment to spring up, that her insatiable sympathy may be assuaged.

She rejoices in illusion. Whoso destroys it in himself and others, him she punishes with the sternest tyranny. Whoso follows her in faith, him she takes as a child to her bosom.

Her children are numberless. To none is she altogether miserly; but she has her favourites, on whom she squanders much, and for whom she makes great sacrifices. Over greatness she spreads her shield.

She tosses her creatures out of nothingness, and tells them not whence they came, nor whither they go. It is their business to run, she knows the road.

Her mechanism has few springs — but they never wear out, are always active and manifold.

The spectacle of Nature is always new, for she is always renewing the spectators. Life is her most exquisite invention; and death is her expert contrivance to get plenty of life.

She wraps man in darkness, and makes him for ever long for light. She creates him dependent upon the earth, dull and heavy; and yet is always shaking him until he attempts to soar above it.

She creates needs because she loves action. Wondrous! that she produces all this action so easily. Every need is a benefit, swiftly satisfied, swiftly renewed.— Every fresh want is a new source of pleasure, but she soon reaches an equilibrium.

Every instant she commences an immense journey, and every instant she has reached her goal.

She is vanity of vanities; but not to us, to whom she has made herself of the greatest importance. She allows every child to play tricks with her; every fool to have judgment upon her; thousands to walk stupidly over her and see nothing; and takes her pleasure and finds her account in them all.

We obey her laws even when we rebel against them; we work with her even when we desire to work against her.

She makes every gift a benefit by causing us to want it. She delays, that we may desire her; she hastens, that we may not weary of her.

She has neither language nor discourse; but she creates tongues and hearts, by which she feels and speaks.

Her crown is love. Through love alone dare we come near her. She separates all existences, and all tend to intermingle. She has isolated all things in order that all may approach one another. She holds a couple of draughts from the cup of love to be fair payment for the pains of a lifetime.

She is all things. She rewards herself and punishes herself; is her own joy and her own misery. She is rough and tender, lovely and hateful, powerless and omnipotent. She is an eternal present. Past and future are unknown to her. The present is her eternity. She is beneficient. I praise her and all her works. She is silent and wise.

No explanation is wrung from her; no present won from her, which she does not give freely. She is cunning, but for good ends; and it is best not to notice her tricks.

She is complete, but never finished. As she works now, so can she always work. Everyone sees her in his own fashion. She hides under a thousand names and phrases, and is always the same. She has brought me here and will also lead me away. I trust her. She may scold me, but she will not hate her work. It was not I who spoke of her. No! What is false and what is true, she has spoken it all. The fault, the merit, is all hers."

REFERENCES

[1] Jaynes, J.(1976): *Origin of Consciousness in the breakdown of the Bicameral Mind*

[2] Onians, R. B.(1988): *The Origins of European Thought: About the Body, the Mind, the Soul, the World, Time and Fate*

[3] Kuriyama, S. (2002): *The Expressiveness of the Body and the divergence of Greek and Chinese Medicine*

[4] Needham, J.: (1956-) *Science and Civilisation in Ancient China*:

[5] Blake, A.: *A Critical Essay on the History of Science, Systematics Vol III, no. 2* at http://www.systematics.org/journal/vol3-1/HistoryOfScience.htm

[6] Laurikainen, K. V. (1989): *Beyond the Atom, the philosophical thought of Wolfgang Pauli*

[7] Wertheim, M. (1997): *Pythagoras's Trousers: God, Physics and the Gender Wars*

[8] See in Blake loc. Cit

[9] Foster, D. (1987): *The Philosophical Scientists*

[10] Whelwell, W. (1989): *Theory of Scientific Method* edited by Robert Butts

[11] Cf. Sartre, J. P. (1966): *Being and Nothingness,* 'The Other first arises as the Not'

[12] Cornford, F. M. (1957): *From Religion to Philosophy: A Study in the Origins of Western Speculation (Mythos)*

[13] Abercrombie, M. (1960): *The Anatomy of Judgment*

[14] Lawrence, G.(1998): *Social Dreaming @ Work*

[15] Bohm, D. & Hiley !993), B: *The Undivided Universe*

[16] Cited in Blake, A. (2008): *The Supreme Art of Dialogue*
[17] Matte-Blanco, I. (1988): *Thinking, Feeling and Being*
[18] Descartes, R.: *Discourse on Method*
[19] Johannes Scotus Eriugena: *De divisione naturae* (*Periphyseon*). See Carabine, D: *John Scottus Eriugena* and discussion in Laurikainen, K. V. (1989): *Beyond the Atom, the philosophical thought of Wolfgang Pauli*
[20] Bohm, D. (1957): *Causality and Chance in Modern Physics*
[21] Deutsch, D.(1997): *Fabric of Reality*
[22] Feyerabend,P. K. (1993): *Against Method*: Outline of an Anarchistic Theory of Knowledge
[23] Medewar, P. (1964): *Is the Scientific Paper a Fraud?* BBC talk
[24] Allen, J. (1991): (edited Blake) *Biosphere 2 – the human project.*
[25] Vernadsky, V.(1998): *The Biosphere: Complete Annotated Edition*
[26] Lorenz, K. (1977):*Behind the Mirror*
[27] Bortoft, H. (1996): *The Wholeness of Nature: Goethe's Way of Science*
[27] Wilson, E. O.(1984): *Biophilia*
[28] Latour, B.(1999): *Pandora's Hope: An Essay on the Reality of Science Studies*
[29] Weil, S. (1951): *Waiting on God*
[30] Heidegger, M.(1963): *What is Called Thinking?*
(1970) *Discourses on Thinking*
[31] Rilke, R. M. (1950): *The Notebooks of Malte Laurids Brigge*
[32] See http://www.nature.com/nature/about/first/aphorisms.html

Goethe and the Dynamics of Being

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Introduction

This talk is about the alternative natural philosophy developed by Goethe round about 1800. Two expectations seem to be aroused whenever this is mentioned. Firstly, because Goethe was a poet and dramatist, he must have developed a poetic form of science. In fact what he did is nothing like this, as we shall see. Secondly, Goethe is usually associated – quite wrongly in many ways – with Romanticism, and therefore it is thought that he must have developed some kind of romantic alternative to mainstream science. This idea is very attractive to some people, but it is very misleading. We can now recognize that what Goethe was really doing is the phenomenology (in the philosophical sense) of nature.

Whenever Goethe is mentioned in connection with science, it is usually in connection with his work on colour, and especially his disagreement with what he believed Newton said about it. So his approach to science is presented in the light of being controversial. "Goethe versus Newton" was appropriated after German unification for nationalistic purposes, when Goethe was hijacked to bolster up German self-confidence in the face of the call to modernize. Those who were unhappy with this found the self-identification of what it is to be 'German' in the philosophy of wholeness, as distinct from what is 'not German', which they labelled 'Anglo-Saxon' and identified with a more atomistic philosophy. The "Goethe versus Newton" controversy was seen as being singularly expressive of this putative difference. It was carried over to America by those fleeing from Nazi persecution, where it surfaced as part of the countercultural movement in the 1960s, and it continues to have an attraction today for those who are seeking a holistic "alternative" to what they see as the undesirable features of modern life.

Not only is this completely contrary to the spirit of Goethe, but it also obscures the fact that his own understanding of science developed over time, eventually evolving into a sophisticated philosophy of science which includes the possibility of different alternatives *within science* (1). Goethe's notion of 'modes of conception' *(vorstellungsarten)* in science looks forward to much later developments in the philosophy of science, and his growing awareness that "the history of science is science itself" now seems highly prescient in the light of the so-called "new" history of science, with its recognition of the *intrinsic historicity* of scientific knowledge. But whereas in the Anglo-American world we jump straight to Kuhn in the 1960s, in Continental Europe there is a long tradition before that, exemplified by the phenomenological theory of science, and especially Heidegger's existential conception of science, in which Goethe's mature understanding of science would find its place quite easily.

The tendency to focus on Goethe's "controversial" work on colour also has the unfortunate consequence of drawing attention away from his equally important work on the metamorphosis of plants. In its own day this was certainly surprising and even radical. Goethe himself was in no doubt as to the kind of difficulty it presented:

> "When we try to recognize the idea inherent in a phenomenon we are confused by the fact that it frequently – even normally – contradicts our senses.
>
> The Copernican system is based on an idea which was hard to grasp; even now it contradicts our senses every day.....
>
> The Metamorphosis of plants contradicts our senses in this way" (2).

What Goethe discovered here turns out to be what we now know to be a fact from electron microscopy. The question is, as one professor of genetics put it to me, how could Goethe have got it right all that time ago, without any of the tools of developmental genetics? The answer to this is in the *practice of seeing* which Goethe cultivated throughout his life. This is not merely seeing in the empirical sense, but *phenomenological seeing* in which the idea becomes phenomenal and appears 'there' in the phenomenon itself (3).

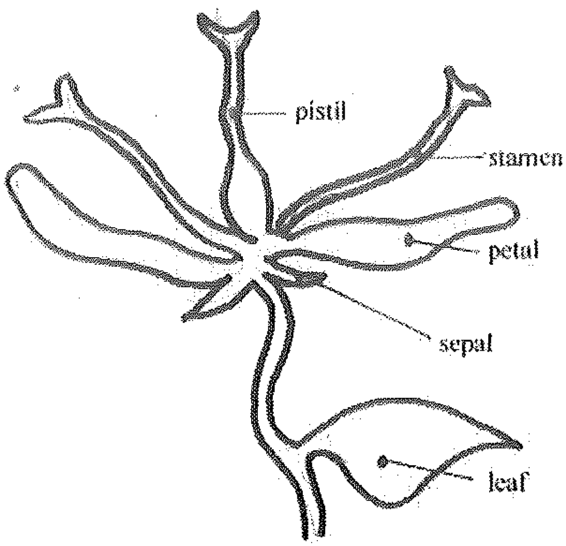
Whenever a new idea is introduced, there is an inevitable tendency to try to reduce the unfamiliarity by relating it to more familiar ways of thinking. This is certainly what happened in the case of Goethe. The result is that the *dynamic* quality of his thinking has all too often been overlooked, so that the remarkable transformation of the idea of "the one and the many" which this brings about has not been noticed. It is this transformation that I want to focus on.

The Idea of Metamorphosis

Goethe begins *The Metamorphosis of Plants* (1790) with the observation that

> "Anyone who observes even a little the growth of plants will easily discover that certain of their external parts sometimes undergo a change and assume, either entirely, or in a greater or lesser degree, the form of the parts adjacent to them" (4).

By 'external parts' he means the various organs growing from the stem of the plant. Firstly there are the vegetative leaves winding up the stem, and then the rings of organs comprising the flower: the sepals which contain the floral bud, and which open to reveal one or more rings of petals surrounding an inner ring(s) of stamens, all of which surround the central organs (pistil and ovary) at the end of the stem where reproduction takes place and seeds are formed.

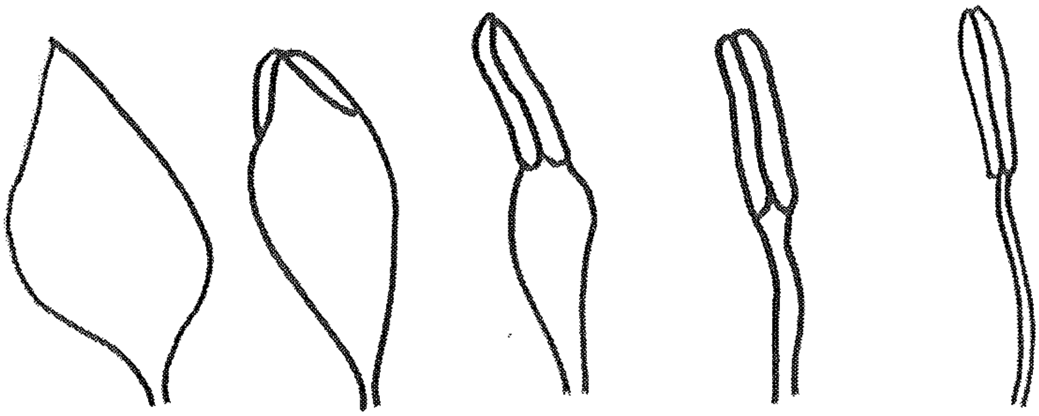


Goethe brings out what he means more clearly in his next observation:

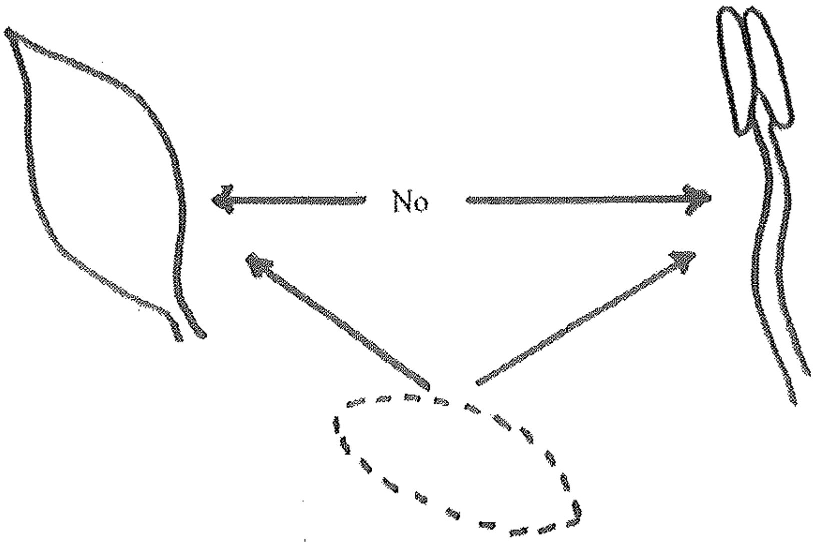
> "So the simple flower, for example, often changes into a double one, if petals develop in the place of stamens and anthers. These petals may either perfectly resemble the other petals of the corolla, both in form and colour, or they may still retain visible signs of their origin".

An example of this is provided by the difference between the wild and the cultivated rose. The wild rose has a widely open flower with a single ring of petals, within which there are several rings of stamens. The cultivated rose, on the other hand, has a closed flower consisting of several rings of petals, within which there is a single ring of stamens. The difference in appearance is striking: on the one hand a simple flower opens to view, and on the other an enclosed flower which hides itself and has become a symbol of beauty and mystery. The difference botanically is that rings of stamens in the wild rose have "metamorphosed" into rings of petals. So where stamens should be, now there are petals – an example of what Goethe calls retrogressive metamorphosis, because here the plant takes a backward step with respect to its normal developmental sequence.

When we notice the fact that petals sometimes appear in the place of stamens, we may have the intuition that there is some kind of inner connection between petals and stamens. Organs which appear at first to be distinct and separate, now seem to belong together. But are there instances in the normal developmental sequence of the plant where we can recognize this "secret affinity", as Goethe puts it, between petals and stamens? There are indeed. It is so evident in the white water lily, for example, that, when we recognize it, we could easily believe the idea had become "visible" and that we are seeing it with our eyes. In this plant we find several intermediate stages between petals and stamens. Here again there are several successive rings of organs, with each ring showing a distinct intermediate form on the way from petal to stamen (5):



Several developmental stages can be seen simultaneously here, so that when we look at a waterlily the overall effect is that we seem to "see" one organ turning gradually into another one. But this is not what is happening: a petal does not materially turn into a stamen. Rather, what we are seeing here is one organ manifesting in different forms, and not an organ turning into another one – i.e. no finished petal changes into a stamen. The metamorphosis is in the embryonic stage of plant growth and not at the adult stage:



Goethe expresses this as follows (referring specifically to the retrogressive case of petals in the place of stamens):

> "If we see that in this way it is possible for the plant to make a retrograde step and reverse the order of growth, we shall become all the more aware of the normal course of Nature, and shall learn to understand those laws of transformation by which she produces one part out of another and creates the most varied forms by the modification of one single organ."

Almost the first thing we notice here is the possible misunderstanding which this invites. Goethe begins by referring to the transformation that "produces one part out of another", which in the context could lead us to think that a stamen is produced out of a petal at the adult stage instead of in the embryonic growth form of the organ. But he then goes on immediately to say that nature "creates the most varied forms by the modification of one single organ", which expresses the idea very clearly. All the organs appended to the stem of the plant are to be seen in this way – from the stem leaves through to the reproductive heart of the flower (6). So in the next paragraph Goethe considers, not just adjacent organs like petals and stamens, but all the organs as "modifications of one single organ":

> "The secret affinity between the various parts of the plants such as leaves, calyx [sepals], corolla [petals], and stamens, which are developed one after the other and as it were one out of the other, has long been recognised in a general way by naturalists; indeed much attention has been given to the study of it. The process by which one and the same organ presents itself to us in manifold forms has been called *the metamorphosis of plants*".

Here again, the first thing we notice is the suggestion that the successive organs are developed "one out of the other", which could be misleading if we took it to mean that an organ at the adult stage transformed physically into another one. Goethe certainly does not mean to say this, as we can see from the fact that he adds "as it were" to qualify it. Immediately after this comes the clear statement that metamorphosis is "the process by which one and the same organ presents itself to us in manifold forms", which is completely different from the idea of one organ turning into another one. It is the ability of the vegetative shoot to develop into different forms which leads to the diversity of organs, and not some miraculous ability on the part of a finished organ to change its form into a different organ. The metamorphosis is in the earlier embryonic stage of the coming-into-being of the organs, and not at the later adult stage of organs that are already finished. Goethe's way of thinking is intrinsically dynamic: it goes back "upstream" into the coming-into-being of the organs, instead of beginning "downstream" with the organs that are already formed. Metamorphosis is only to be found in the coming-into-being, and the failure to realize this leads us to look in the wrong direction by trying to understand metamorphosis in a downstream way. This is the source of much of the misunderstanding of Goethe's work (7).

Protean Thinking

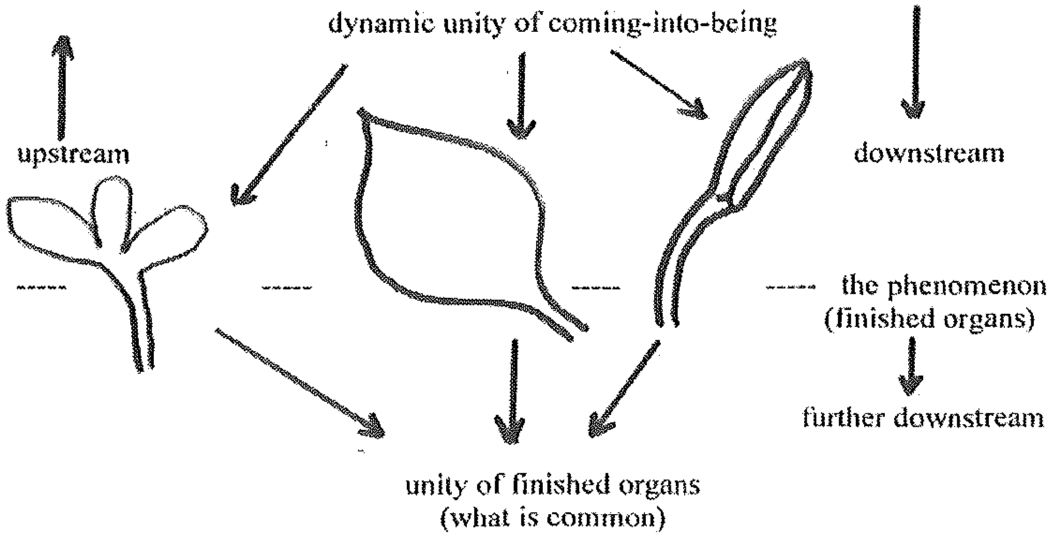
So far we have seen that in some sense the different organs of the plant are one organ. But what kind of 'one' is this? What kind of 'one' can present itself in manifold forms, and what is the relationship between this 'one' and the 'many' forms in which it manifests? We are now going to explore this question in some depth.

In his own time, and indeed ever since, an answer seems to have been given to this question which is based on an assumption that is in fact nowhere to be found in Goethe's work. This is the assumption that he was searching for what all the different plant organs have in common – their "lowest common denominator". By trying to find what is the same in all of them, i.e. in which there is no difference at all between them, it is supposed that Goethe discovered a unity in the diversity of the organs. The movement of thinking in this case evidently has the effect of *excluding* difference from unity. We can see this clearly in some of the statements that have been made about what Goethe was doing – e.g. that he "was transfixed by uniformities and commonalities in nature", and that he sought for "the general plan common to all organs" by trying to find "the simplest form of plant organ from which the anatomist's mind had stripped all the specializations required by the organs of real living plants". Statements such as these, which are typical, clearly do not portray nature in the way that Goethe expressed to Schiller, as "working and alive, striving out of the whole into the parts". On the contrary, they portray nature more as dead and finished.

When we read what Goethe says carefully, paying attention to the movement of thinking, we can see for ourselves that he was doing something radically different from just looking for what all the plant organs have in common. We have already seen that he says "nature creates the most varied forms by the modification of one single organ," and describes metamorphosis as " the process by which one and the same organ presents itself to us in manifold forms". Elsewhere, in letters and the diary of his Italian journey, he says that he is "becoming aware of the form with which again and again nature plays, and in playing brings forth manifold life", and that "the thought becomes more and more living that it may be possible out of one form to develop all plant forms". Notice that he does not say the form with which nature plays again and again is nature's model or ground plan of the plant, just as he does not say that he is trying to reduce all plant organs to one form (8). Yet again, on another occasion when referring to the organs of the plant, he says: "It had occurred to me that in the organ of the plant which we ordinarily designate as *leaf,* the true Proteus is hidden, who can conceal and reveal himself in all forms". Reading what Goethe says, it is difficult not to get the sense that he is doing the very opposite of searching for what all the different organs have in common. He is talking about the creation of difference within unity, not arriving at unity by the exclusion of difference. The direction of his thinking is the other way round.

The reference to Proteus gives us an indication of the direction Goethe's thinking takes. Proteus is the Greek God who can hide and reveal himself in any form he chooses. He can present himself in manifold forms, ever differently, and yet it is always Proteus. Now we would not try to understand Proteus by collecting together different manifestations and trying to see what they all have in common. Such a procedure would be far "too late". What is essential about Proteus is the coming-into-being, the *appearing,* and not the specific form in which he appears. The attempt to find a common identity based on the different appearances could only result in an "average Proteus" – which is an absurd notion that would only take us even further away from the ever-dynamic Proteus. So clearly, Goethe does not want us to look at the organs of the plant and find what they have in common, excluding all the ways in which they are different from one another and including only the ways in which they are the same, until at last we arrive at a kind of "average organ" which is the common plan according to which they are all formed. It takes only a moment's thought to realize that no real differences could ever be produced from such an "average organ," because it is reached by excluding all differences in the first place. It is a cul-de-sac.

So Goethe is not saying: begin with the finished organs as they are on the adult plant and then try to abstract a unity from them. If this were the case we could *only* end up with what they all have in common. For Goethe the organs in their finished state are already "downstream", and to abstract from them only the unity of what they have in common – a process which excludes the ways in which they are different - is to go even further downstream. But Goethe goes in the opposite direction and tries to catch nature "in the act" – i.e. as "working and alive, striving out of the whole into the parts". He goes back "upstream" from the organs in their finished state, so that he doesn't derive the unity *from* the diversity, instead he "brings the diversity back into the unity from which it originally went forth"(9). In this way the movement of his thinking can follow the coming-into-being of the organs and end with them in their finished state.



We always begin with the phenomenon, which is already downstream. The difference in whether we then go further downstream in our search for unity, or whether we go upstream into the coming-into-being of the phenomenon. If we go upstream we discover the dynamic unity of the emerging organs, so that we now come into the phenomenon from the unity instead of trying to come to the unity from the finished phenomenon. So we can begin to think in a Protean way that "creates the most varied forms by the modification of one single organ". If we don't recognize the difference between these two movements of thinking, then we easily fall into the error of trying to "reach the milk by way of the cheese" by projecting the unity abstracted from the finished organs back into the beginning, as if this unity of the dead end were the unity of the living origin.

Goethe and Schelling

Goethe came to a fuller understanding of the dynamical form of his own thinking partly as a result of his conversations with Schelling – the highly precocious philosopher who was appointed to the chair of philosophy at the University of Jena in 1798 as a result of Goethe's influence. It was through these discussions that Goethe came to realize that the direction of his own thinking took him upstream from seeing nature as product into seeing nature as producing. Schelling emphasized that to understand nature we must rise from nature as fact to nature as "the action itself in its acting". He says that "In the usual view, the original productivity of nature disappears behind the product. For us the product must disappear behind the productivity" (10). Of course, in emphasizing the reversal Schelling in no way intends to imply a separation between the productivity and the product – his thinking here is dynamical. We can now easily recognize that this is indeed what Goethe is doing by going back upstream into the coming into being of the phenomenon, instead of beginning downstream with the phenomenon in its finished state and trying to explain it from there – which can only take us even further downstream. The difference between nature as productivity and nature as product is often described in terms of the distinction between *natura naturans* ("nature naturing") and *natura naturata* ("nature natured"). It is the latter which we think of as the natural world, whereas for Schelling it is really the former that is implied by the idea of *Nature*. Goethe's remark that it is possible to present nature as "working and alive, striving out of the whole into the parts", clearly refers to nature in the sense of "nature naturing". The distinction between *natura naturans* and *natura naturata* is often attributed to Spinoza, whose work was read and admired by Goethe, Schelling, and others at the time. However, this distinction did not in fact originate with Spinoza, for whom it was certainly important, but will be found earlier in Renaissance nature philosophy, and goes back to Scotus Eriugena in the 10th century.

The Self-Differencing Organ

If one and the same organ presents itself to us in different forms, then each organ *is* that organ, but differently, and not *another* organ – Proteus is always one and the same Proteus, but differently, and not another Proteus. It is always the very same one and not another one, and yet it is always becoming different from itself. It becomes other without becoming another – the other of itself and not another one. Goethe's "one and the same organ" manifesting as different forms is a self-differencing organ producing differences of itself. So the different organs we see are the self-differences of one organ. What we discover here is the extraordinary idea of self-difference instead of self-sameness, the idea that something can become different from itself whilst remaining itself instead of becoming something else. When we go upstream into the coming-into-being we discover the self-differencing organ which appears downstream as several different organs – to borrow from Deleuze, we find "there is *other,* without there being *several"* (11). So we find that the unity of coming-into-being is the dynamical unity of self-differencing, in which difference is *intrinsic* to unity. Here the unity is in the very dynamics of self-differencing. There is no separation here (if we find it in our thinking, it is because we have fallen downstream without noticing): the self-differencing *is* the unity and concomitantly the unity *is* the self-differencing. This dynamical unity is evidently the very opposite of the unity of the finished products, which is the static unity of self-sameness that is reached by the *exclusion* of all difference.

Here it may be helpful to note that in English the word "being" is a participle, so that it is simultaneously a noun and a verb. We usually think of "being" as a noun, as *a* being, in which case it just seems to be an abstract general term denoting anything at all without reference to the specific characteristics which make it one particular kind of thing or another. So a table is a being, and so is a rock, and a fish, and a plant, - in fact we can think of anything as a being. It is the ultimate abstract generalization, and as such seems so empty of content that to say of something that it is a being is to say no more than that it exists as a thing. But there is another aspect to being, which is expressed by the fact that the word "being" is also a verb – it is be-ing as well as being. Thus in this less familiar perspective, being is already dynamic instead of static as it is when we think of being as a being. Being as be-ing is *intrinsically* dynamic (12). The two different modes of unity – the "upstream" and the "downstream" modes – are expressions of the difference between these two different perspectives of "being":

be-ing → unity of coming-into-being = dynamic unity of self-differencing

being → unity of finished products = static unity of self-sameness

Intensive and Extensive Distinctions

When one thing is different from another thing, the distinction is extensive; but when something is different from itself the distinction is intensive. What this means will become clearer with the example provided by the hologram. If we have a transmission hologram on a glass plate – let us say of a horse galloping towards us – what would we expect to find if we divided the plate physically into two halves? If we had a photographic plate instead of a hologram, we know what the answer would be: two halves of the plate with half the horse on one and the other half on the other. What is surprising about hologram division is that we would find the *whole* horse on each of the two halves of the plate (13). We can divide the photographic plate but not the hologram of the horse. The contrast with a photograph is striking: if we want another photograph we have to make a copy of the first one, and then there will be two photographs – one and another one. But there cannot be "two" holograms here – even though it looks like there are physically – because this does not take into account the optical indivisibility of the hologram, whereby the attempt to divide it results in the whole again instead of two halves. If we now ask how many there are, what can we say? We cannot say there are two, because this would be no different from the case of photographic reproduction, where there clearly are two (one and another one). In the case of the hologram it seems that each is the *same* one – one and the other of itself instead of one and another one. In a sense there is only one and not two, yet clearly not in a numerical sense because then we would be unable to distinguish this case from the original hologram before it was divided. The division of the hologram is *intensive* because it remains whole when divided, and consequently the distinction between the "two" which are one and the very same one (one and the other of itself) is an intensive distinction. We can call this "multiplicity in unity". The division of the photograph, on the other hand, is an *extensive* division because it results in two halves. Copying the photograph is also extensive, because the result is "one and another one" and not "one and the other of itself".

This difference can also be seen in the vegetative reproduction of plants. When a gardener propagates a plant by taking cuttings, what is happening *organically* is similar to what is happening *optically* when a hologram is divided in the manner described above. For example, if a leaf is taken from a fuchsia plant, and divided into several pieces, each of which is then planted separately, eventually these cuttings will grown into adult fuchsia plants. So where we had a single plant to begin with, there will now be several plants, which can be separated and moved to different locations as if they were simply physical objects. But *organically* there is only One plant here, because each is the very same one and not another one. Like the hologram, the plant is divisible and yet remains whole – so that it is really "indivisible" in a subtle sense. Organically each one is the very same plant, so although there appear to be many plants there is really One plant here. The plant has become multiple without becoming many plants – even though this is how it seems to us when we count the plants, because when we do we count them as physical objects. The difference is between the non-numerical multiplicity of "multiplicity in unity" and the numerical multiplicity of many ones. This *must* be an intensive multiplicity otherwise the unity would fragment.

So here we have a unity which includes multiplicity *within* it without being divided - and thereby ceasing to be unity. Extensively we can have either one or many – one only or many ones - but intensively we can have one and many at the very same time because the one *is* many. The difference here is that the one can be multiple intensively without being many extensively. Such a "multiplicity in unity" constitutes an intensive dimension of One, as distinct from the extensive dimension of many ones. In what follows we shall use an initial capital letter in this way to donate the 'one' which is intensively "multiplicity in unity", and a lower case letter to donate the 'one' which is extensively one of "many ones". Thus, in vegetative propagation there is One plant organically where we see many plants. So the unity of coming-into-being, which is the dynamical unity of self-differencing, produces "multiplicity in unity" which is an intensive dimension of One.

We can see this quite strikingly in other examples of vegetative reproduction, where what appears to us as many plants is in fact One plant be-ing itself multiply. Consider a strawberry bed in the garden, for example. We see this as many strawberry plants, whereas it is in fact One plant. The strawberry propagates by sending out a creeping runner along the ground, from the tip of which it puts out roots and a new strawberry plant shoots out. But organically this is the very same plant – the other of itself and not another one. This means that the entire strawberry bed is organically One plant, a "multiplicity in unity" instead of many separate plants, in "which there is *other* without there being *several*". Yet another very striking example of the difference between an intensive and an extensive distinction is provided by the growth of potatoes. John Seymour describes this:

> "The potato is not grown commercially from seed, but from sets, which are just potatoes, and so all the potatoes of one variety in the world are *one plant*. They are one individual that has just been divided and divided."

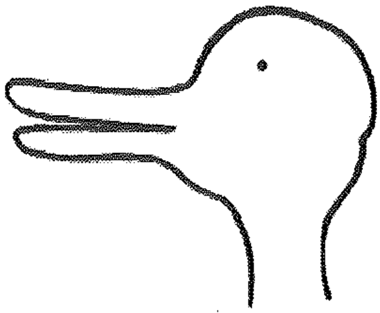
To produce a new variety it is first necessary to fertilize a plant with the pollen of another. After that:

> ".... The breeder arranges for the new variety to be multiplied by setting the actual potatoes from it – and if it proves a popular variety the original half dozen or so potatoes on the first-ever plant of that variety may turn – by division and subdivision – into billions and billions of potatoes – all actually parts of that first plant. It would be interesting to know how many billion tons that first King Edward plant has developed into during its life!"(14).

So the King Edward potato is dynamically One plant in space and throughout time. It is a non-numerical, organic "multiplicity in unity" which constitutes an intensive dimension of One, and not the numerical multiplicity of many ones we see when we are buying potatoes. We can of course see it both ways: as One which is intensively multiple, or as many ones which are extensively separate so that each one is another one. But in the latter mode we lose the organic "indivisibility" of the whole, and we see the potatoes as no more than physical bodies like a pile of bricks. Here again we can see the difference between following the coming-into-being and starting from the already finished products. Whereas "downstream" we see many plants, "upstream" we discover One plant be-ing itself multiply.

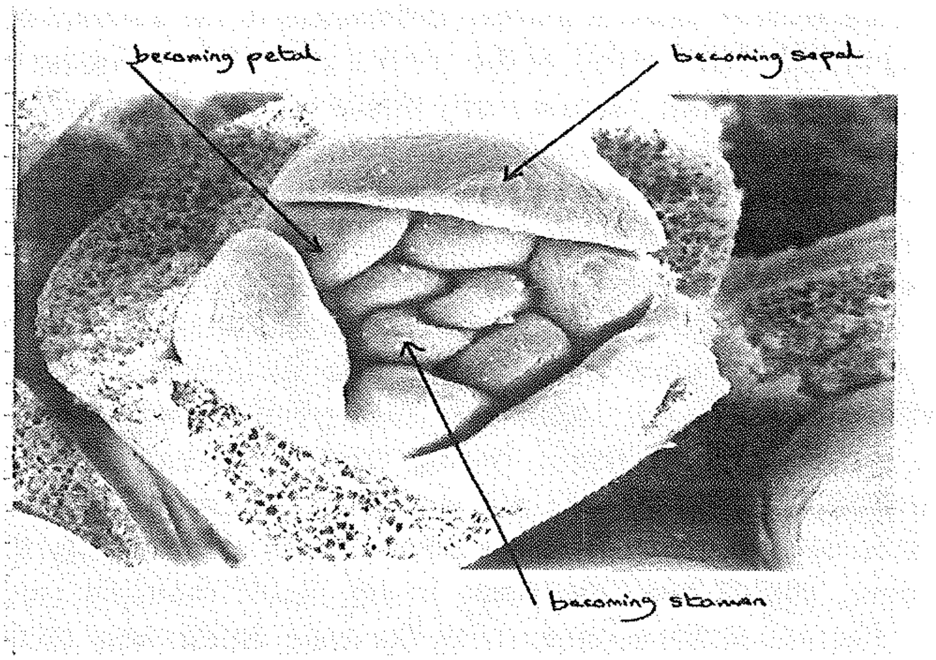
It is this idea of an *intensive* distinction which we need in order to see the transformation of the idea of "the one and the many" in Goethe's dynamical thinking. However, the examples we have given above to illustrate this kind of distinction only consider multiplicity and not genuine diversity. Hologram division, or plant cuttings, result in identical holograms, or plants, whereas the "one organ" Goethe is describing can present itself in various *different* forms – as vegetative leaf, sepal, petal, stamen. Nevertheless, the same form of thinking is needed here for genuine diversity as for simple multiplicity, i.e. the dynamical unity of *self-difference* is intensive. Here again it is possible to give examples which can function as "templates for thinking" (15). For instance, the hologram model can be extended to the case of the multiple hologram, in which several *different* images can be recorded on one and the same hologram without becoming confused – as they would be in the case of a photograph which had been multiply exposed. The important point here is that each different image is recorded on the entire hologram - not one image on one part of the hologram, and another on another part, and so on. If each exposure is taken at a slightly different angle, and then the angle at which the hologram is looked at is also changed slightly, what is seen is one image (e.g. a horse) turning into a different one (e.g. a cow) in the very same place. It is possible to produce such a multiple hologram with several different images. This illustrates how the dimension of wholeness can contain many within it in such a way that each one *is* the whole, but differently, because in this case each different image in the hologram is the whole hologram and not part of it in an extensive sense.

A further illustration of the intensive dimension of self-difference is provided by the experience of seeing a multi-perspectival figure, such as the reversing cube or the duck/rabbit. Thus, in the case of the duck/rabbit, each different figure that we see – whether duck or rabbit – is the whole figure. So it is an instance of "multiplicity in unity". One figure does not occupy only part of the picture, while the other figure occupies the other part – it is duck/rabbit, not duck *and* rabbit:



There are no lines left over, unused, by either figure – and no extra lines need to be added in either case. Each figure is complete in itself, and yet it is not the only possibility. The duck and the rabbit are nested intensively in one another. Either can come into manifestation, but not both together, side by side, extensively – if we try to do this, the duck and the rabbit will each be a duck/rabbit. Each one is the very same One and not another one, but differently. So this illustrates the idea of the intensive unity of self-difference, which includes difference without fragmenting the unity. Here only two figures are included, and we do not know of multivalent figures which are more than bivalent. However, although we may not be able to draw such a possibility, we can certainly conceive it, and thereby consider the possibility in principle of a multivalent figure with three, four, five, ...figures nested intensively. Such a possibility would go some way towards illustrating the intensive unity of the self-differences of the One organ which manifests as leaf, sepal, petal, stamen, etc. – although its limitation is that it is not intrinsically dynamical.

It has already been mentioned that Goethe's phenomenological way of seeing the metamorphosis of the plant has been confirmed by modern research in electron microscopy and developmental genetics. The electron micrograph below shows the development of a floral bud at an early embryonic stage (16). Here we can see the self-differencing organ coming-into-being as sepal, petal, and stamen. This is Goethe's "diversely metamorphosed organ".



Becoming Other in Order to Remain Itself

Ron Brady, a philosopher who contributed much towards understanding Goethe's morphology, describes the intrinsically dynamic form of life as "becoming other in order to remain itself" (17). He says that:

> "The forms of life are not 'finished work' but always forms *becoming*, and their 'potency to be otherwise' is an immediate aspect of their internal constitution..... The *becoming* that belongs to this constitution is not a process that finishes when it reaches a certain goal but a condition of existence – a necessity to change in order to remain the same". (18).

Here we have a clear recognition of the self-differencing organ, ever changing into other modes of itself, so that what we see as the diversity of organs *is* the living unity of the plant.

So far we have considered only the organs of the plant, but we can now expand our horizon to consider the dynamics of "becoming other in order to remain itself" in the variety of different forms which an individual plant species can take. If we consider a single species of plant, we will see individual plants of this species taking on different forms according to the conditions of the environment in which they are growing. Changes in environmental factors such as soil conditions, weather patterns, the light, and so on, are seen to result in marked differences in the external form (the phenotype) of individual plants of the species (19). When Goethe travelled across the Alps into Italy, he saw many plants which were familiar to him in Southern Germany, but modified in accordance with the change in environment. Thus, in the Alps he noticed that, in general, branches and stems were more delicate, buds further apart, and leaves narrower, than they were in the same species in Germany. He recognized that in such cases he was seeing different manifestations of the same plant and not different plants. This phenotypic variety of a species is not extensively many different plants, but intensively One plant coming-into-being as different expressions of itself, be-ing itself differently in changing circumstances. The idea of "one and the many" is turned inside out here: Goethe does not see many different plants which are basically the same (downstream, static) but One plant be-ing itself multiply (upstream, dynamic). The one is not separate from the many in this way of thinking. On the contrary, what we find here is that, in the words of Deleuze: "Multiplicity is the inseparable manifestation, essential transformation and constant symptom of unity. Multiplicity is the affirmation of unity; becoming is the affirmation of being" (20).

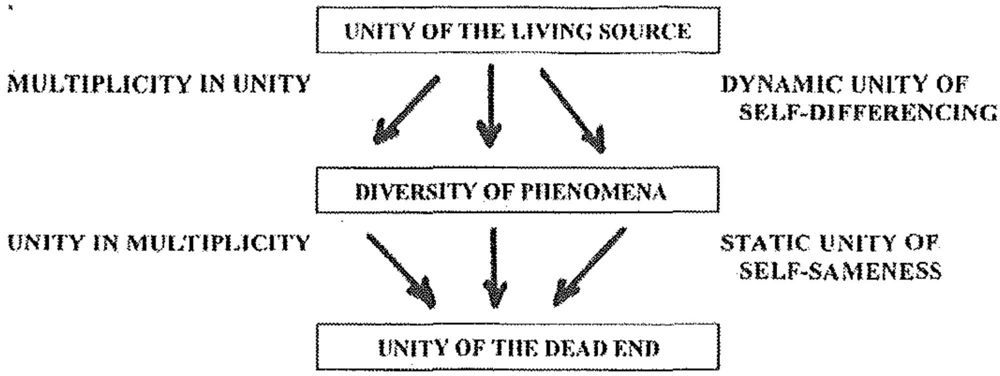
We must be careful when we are trying to conceive the plant in its environment in an organic way that we do not inadvertently fall into a way of thinking that is more appropriate to the inorganic realm. It is all too easy for our thinking to loose sight of the very quality of livingness which is the organism's own "potency to be otherwise", and for us to fall into thinking of the organism as if it were responding in a mechanical manner to the influences of the environment. The *living* organism does not just adapt to external circumstances in a passive manner, as it would do if it were only an inert body responding to external forces according to the laws of physics. The specific form which a plant takes in its surroundings is not the result of external conditions acting directly on the plant to cause the modification which we observe. The conditions clearly do influence the specific form which the particular plant takes, but they do not cause it. Such a way of thinking fails to take into account the living organisms own contribution to its specific form. Goethe spoke of the particular individual plant as being a "conversation" between the living organism and its environment. This metaphor draws our attention to the plant's *active* contribution to the form which it takes in specific conditions, emphasizing the fact that the individual expression of the plant which we see is the outcome of the active response of the organism to the "challenge" posed to it by the environment. This is stated very clearly by Steiner: "We must conceive at a deeper level than the influences of external conditions something which does not passively allow itself to be determined by these conditions but actively determines itself under their influence"(21). The living organism configures itself *actively*, instead of being conditioned passively, in response to the environment. The external conditions stimulate the plant but do not determine it. The plant responds actively out of its own "potency to be otherwise" to produce the form of itself which the environment evokes. Holdrege describes this as follows:

> "Imagine that you are holding a groundsel seed in your hands before planting it. Depending on how, when and where you plant the seed, a limitless variety of forms can arise. All these potential forms are not, of course, stored in the seed. The concrete forms are emergent characteristics that arise out of a germinal state and develop in the interplay between the plant's plasticity and the environmental conditions. In particular surroundings the potential of the plant is evoked, but what appears is only one manifestation of the myriad ways in which this plant could develop"(22).

The specific form which an individual plant takes is neither determined by the environment nor predetermined by the organism itself. As Holdrege indicates, we must avoid the trap of thinking in a "finished product" manner, as if the potential forms were there already in the organism like peas in a pod. This is the kind of thinking which tries to "get to the milk by way of the cheese", thereby eclipsing the dynamical quality of the organism be-ing itself differently according to the situation in which it is placed.

As well as the variety resulting from environmental factors, there is the much greater variety which can arise from the genetic variation taking place within the species. This is what interests the breeder. He or she is always on the look out for "interesting" variations which can then be propagated – the process of artificial selection which Darwin took as his model for the idea of natural selection (23). This is how the huge variety in any one species of plant arises. There are, for example, a thousand different varieties of Peony. Many of these are on display together on the same day at the Chelsea Flower Show in London. It is an astonishing variety to behold, and yet what we see before us extensively as many different plants is organically One plant which is intensively multiple – a "multiplicity in unity" which is an expression of the dynamic unity of self-differencing. It is One plant be-ing itself differently and not just many different plants of a common kind. Of course, we usually see more in the "downstream" mode of the latter than in the "upstream" mode of the *living* plant. But if we can shift our thinking upstream, we can recognize that the diversity of peonies we see *is* the living unity of the Peony. How different it would be if we looked for unity among the peonies by trying to find what they all have in common. If what is living is always "becoming other in order to remain itself," then we must learn to recognize diversity as the dynamic unity of life, so that we can see the unity concretely as being identical with the diversity of the phenomena. This is not what we would expect to find: that the unity is "hidden" right in front of us as the diversity.

As we have considered the varieties of a single plant species as a whole, so we can go on to consider the different species within a genus - and then different genera within a family - also as a whole. It is an enlivening experience to see a particular family of plants in the light of the idea of the dynamic unity of self-differencing. We begin to see the different kinds of plant within the family intensively as One plant be-ing itself multiply, instead of just seeing different plants extensively that have something in common. The extensive perspective, which remains on the outside of the phenomenon, tries to draw off the unity by abstracting what is the same, eliminating differences. The result is a lifeless "unity in multiplicity" which is a dead end. We can recognize that the movement of thinking in this case is the opposite of that which sees the diversity of the phenomenon unfolding as the living unity of its coming-into-being. Anyone can learn to practise this living way of seeing for themselves. For example, we can become familiar with the different members of the *Rosaceae* family – the rose, cherry, apple, blackberry, strawberry, etc. – and begin to see them as One plant in the form of "multiplicity in unity". Learning to see in this way has the consequence that we begin to see each member of the family reflected in all the others, so that the rose is seen in the apple, as the strawberry is seen in the rose, for example, without there being any sense whatsoever that one kind of organism somehow changes physically into another. What we are seeing in this way is the metamorphosis of One plant into different modes of itself, and not the external change of one plant into another. How different the *experience* of this is from that of looking for what these different plants have in common, i.e. from seeing the *Rosaceae* in the mode of the static unity of self-sameness. This latter way of seeing leads only to an abstract generalization – something like an "average plant" of the *Rosaceae* family – which functions as an organizational schema, or "blueprint", for all the plants of the family. Although such a concept has been used in biology at times - and has often been mistakenly identified with Goethe's approach - it is nevertheless so rigid and static that it completely lacks the flexible and dynamic quality which is characteristic of life. It is in fact no more than a lifeless counterfeit of living being, the very opposite of Goethe's living perception of nature for which he says "we must make ourselves as mobile and flexible as nature herself".



The *Urpflanze*

The dynamical unity which is the plant is to be found at every level: the organs of the plant, the varieties of a single species, the species within a family, and ultimately the One plant which is the whole plant kingdom - what Goethe called the *Urpflanze*. It is very important that we try to think dynamically when we are considering this notion, going "upstream" into the coming-into-being, and do not fall back into the habit of static thinking which begins "downstream" with the organisms in their finished state. Much of the misunderstanding which there has been about Goethe's *Urpflanze* is a result of failing to do this. Time and again we read that Goethe searched for the general plan common to all plants, looking for uniformity and commonality in the multiplicity of living organisms, trying to reduce the diversity of nature to unity, and so on. But this just gets what he was doing back-to-front. Only by thinking in a dynamical way can we come to appreciate Goethe's notion of the *Urpflanze,* and the way that it differs significantly from some of the misinterpretations to which it has frequently been subjected.

The name *Urpflanze* is usually translated into English as either 'primordial plant' or 'archetypal plant'. Both of these *could* be seen in the Goethean way – i.e. as referring to the intrinsically dynamic unity of coming-into-being which is the One plant be-ing itself differently (self-differencing) as "multiplicity in unity" – but almost invariably they are not seen in this way. The term 'primordial plant' is usually taken to refer to some supposed primitive ancestral plant from which all other plants have developed procreationally over time. The use of the adjective 'archetypal' immediately suggests an association with the philosophy of Platonism. In the standard established interpretation, this is a two-world philosophy and hence strongly dualistic. According to this interpretation, which has been held widely, Plato conceived the fundamental ontology of the world as being on two different levels. There is the level of what we see around us, the world of changing appearances that we experience through the senses, and the ontologically superior level of the world of Ideas or Forms which are eternally one and the self-same, and constitute the original "templates", or "models", according to which the multifarious appearances of the world of the senses are formed, and of which they are only imperfect copies (24). In the standard interpretation, these two different ontological levels are conceived as being not only distinct but also as being *separated* from one another – hence the two-world dualism. For all the *many* different instances of some particular kind of thing in the sensible world, there is *one* Form or Idea in the intelligible world which functions as the ideal archetype according to which all the particular instances are formed – this is referred to as "the one over many", where a *separation* is implied between the one and the many. It seems clear that this is a very external, spatial way of thinking: another world is imagined which seems to be "outside" the familiar world, and yet this second world is imagined in the image of the familiar world, which it therefore seems to duplicate in an idealized way. The difficulties to which this leads are well known, and are so intractable that we cannot help but wonder why Plato ever thought in this way in the first place. The answer may well be that he didn't. What we think of as Plato's philosophy may really be a misinterpretation of Plato. It is possible that Plato himself did not subscribe to the two-world theory implied by this common (mis)interpretation of the theory of Ideas or Forms – which Gadamer, who affirmed that "Plato was no Platonist", referred to as pseudo-Platonism, or vulgar Platonism. However, Plato himself may have unwittingly encouraged this misunderstanding by the way that he presented the Ideas, and their relation to the world of the senses, and later he went to considerable lengths to correct it (25).

We can see from all that has been said about Goethe's dynamic way of thinking, that what he means by the *Urpflanze* has nothing whatsoever to do with the notion of the archetype in the standard interpretation of Plato. For this reason, translating it as 'archetypal plant' is very misleading indeed, because it invites us to associate it with a notion which is completely out of tune with his whole way of thinking (26). What Goethe means by the *Urpflanze* is the dynamic unity of the coming-into-being of all plants as the self-differencing of One plant, which is therefore intensively multiple but appears to us extensively as all the many different plants. What this means is that each plant *is* the *Urpflanze* be-ing one possible mode of itself – the number of possibilities is indeterminate. Hence, paradoxically, it is everywhere visible and nowhere visible – although once we begin to think dynamically, this is no paradox at all. Instead of being *separate* from the many particular plants that we see, i.e. as "the one over many", Goethe's *Urpflanze* is One which comes into concrete manifestation simultaneously with the many - *with which it is identical* because the many are now the self-differences of One. This is very different indeed from the two-world theory which *separates* the One from the many. There is no such dualism in Goethe's thinking, for which, in his own words, "The universal and the particular coincide: the particular is the universal, appearing under different conditions".

We can also see this in the light of Hegel's conception of the *concrete* universal - as distinct from the more familiar abstract universal – which Hegel believed was the great advance he had made upon previous philosophers. Whereas the abstract universal is reached by *excluding* all differences, and so contains only what is common, the concrete universal is "the universal which differentiates or particularizes itself and yet is one with itself in its particularity" (27). This expresses very clearly what has been said about the *Urpflanze*, which we came to by following the *movement* of Goethe's thinking (28). We must be careful not to loose sight of the fact that the concrete universal in *intrinsically* dynamical – i.e. it "particularizes itself". Otherwise we will fall into the "downstream" trap of thinking of the concrete universal simply as an *inclusive* form which already contains all possible plant forms within itself. But the *Urpflanze* (the concrete universal) does not include all its possible manifestations, as if the indeterminate variety of the plant kingdom were somehow already contained within it, just waiting to emerge when the specific conditions for each kind of plant are right. If we were to perceive the diversity of plants in the manner of the concrete universal, we would have a purely dynamic experience of seeing different plant forms appearing one after another, as if each unfolded out of the other. Goethe describes such an experience:

> "When I closed my eyes and lowered my head, I could imagine a flower in the centre of my visual sense. Its original form never stayed for a moment; it unfolded, and from within it new flowers continuously developed with coloured petals or green leaves" (29).

We must read this intensively, as One plant be-ing itself differently, and not extensively as merely one plant after another. This is the dynamic unity of the concrete universal which "particularizes itself and yet is one with itself in its particularity".

Unfortunately, the concrete universal of Goethe's dynamical thinking is all to often mistaken for the abstract universal of a static generalization. We can see this confusion in the way that comparative morphology developed in Britain in the Victorian period, especially in the work of the famous anatomist Richard Owen, director of the Kensington Natural History Museum in London. In his major work on the vertebrates, *On the Archetype and Homologies of the Vertebrate Skeleton* (1848), Owen set out an idealized picture of the simplest vertebrate form. But this is not the unity of the living source of all potential variations. On the contrary, it is the most basic pattern common to all vertebrates, the least common denominator shared by all members of the vertebrate class. Such an abstract generalization is evidently at the opposite pole to the concrete universal Goethe had in mind. This reduction to the minimal commonality from which all the specialized organs required by actual living organisms have been excluded, is really the unity of the dead end. Owen and others believed that in this way it would be possible to discover the most general ground plan (the "blueprint") of the vertebrates – which is what he called the "archetype" of the vertebrates. There is, of course, nothing wrong with such an abstract generalization; what matters is the use to which it is put. In fact it can be very helpful in the process of discovery. For example, it was in this way that Owen came to the concept of homology, whereby different organs – e.g. the bones in the human hand, the wing of the bat, and the paddle of the porpoise – can all be recognized as instances of the same basic anatomical structure fulfilling different functions (30). There were many anatomists who just saw this kind of abstract unity of organization common to different organisms as no more that a geometrical abstraction –

it was even sometimes compared to a mathematical axiom. (31). But Owen was at the forefront of those who wanted to see more than this in it. He wanted the unity of the *abstract* universal to be a *transcendent* unity, as if it pre-existed as the ground plan of the vertebrate class at a "higher" ontological level than the actual organisms themselves. We can begin to recognize the two-world theory of Platonism here - which, as already mentioned, is really pseudo-Platonism. (32). This is very different from Goethe's idea of the science of morphology, which can only be understood in a thoroughly dynamical way and not in terms of static "Platonic" archetypes. Because his way of thinking is intrinsically dynamic, it is possible to describe the dynamics of being without introducing a false dualism which separates being and appearance. But to do this we have to go upstream into the coming-into-being, instead of beginning downstream with the finished organisms.

We can of course begin at the end by abstracting the unity of what the finished organisms have in common. As we have seen, there is nothing wrong in doing so. But we also need to keep our attention on the movement of thinking, otherwise we will make a fundamental mistake which has far-reaching consequences – which is the mistake that Owen and others made when they turned the abstract universal into a transcendental unity. Having formed this abstract unity, which can only come at the end, we then project it back into the origin, and imagine that it is there in the phenomena, or "behind" the phenomena, all the while. In other words, we assume that this downstream abstraction is ontologically fundamental. In which case we now have to try to understand how difference could emerge from an abstract unity from which all difference has been excluded. It is impossible. Since all difference has been excluded from this unity – in favour of what is common – then none can emerge from it. It is an ontological cul-de-sac. Yet this mistake has been made time and again. What this misses is the dynamic unity of the living source, the unity of coming-into-being, for which it substitutes the static unity of the dead end. The result is that we try to "reach the milk by way of the cheese," and so get everything the wrong way round. Goethe's understanding of the dynamics of being, on the other hand, goes upstream towards the unity of the living source, so that the movement of his thinking follows the coming-into-being of the phenomena to end where we usually begin. Consequently in Goethe's dynamical thinking of "the one and the many" there is no separation of the One from the many, and the two-world dualism of pseudo-Platonism simply doesn't arise.

Conclusion

Alfred North Whitehead famously said that the Western philosophical tradition consists of a series of footnotes to Plato. But the influence of Platonism extends far beyond philosophy. The impact it had on western Christianity led Nietzsche to declare (and Heidegger agreed) that the doctrine of the Church is "Platonism for the people" (33). There is also the influence which Platonism had on the transition to the heliocentric system of planetary astronomy, and the development of mathematical physics in the 17th century, which we can see exemplified in Copernicus, Kepler, Galileo, Newton, and others. This is the historical source of the idea that there are *transcendent* "laws of nature", which leads one philosopher to conclude that "Metaphysics [Platonism] is alive and well and lives on in modern physics" (34).

Wherever it occurs, Platonism is characterized by "... an opposition between the multifarious appearances involved in perpetual change and an immutable realm of existence, forever persisting in strictest self-identity" (35). Knowledge of this 'Being-as-it-is-in-itself' (*ontos on*) is considered to be the only true knowledge, whereas everything else is only belief and opinion about changing appearances. The characteristic of genuine knowledge is that:

> "Since it is concerned with Being-as-it-is-in-itself, it is free from all relativity with regard to subjects, their standpoints, and the vicissitudes of their lives. Because of the persistent self-identity of this Being, genuine knowledge is perpetually true, under all circumstances and for everyone".

Seeing this now, in the perspective of Goethe's dynamic way of thinking, we can begin to appreciate the limitation of this dualistic ontology. When movement (change) is excluded from being, we get the separation of being from all that is dynamical, so that everything falls apart into a realm of pure being without change and a realm of change without being – which is a realm of "mere appearance" having no reality itself. But if there is movement in being this dualism disappears. However, there cannot be extensive movement in being, whereby one thing changes into another thing. There can only be an intensive movement whereby something changes into another mode of itself and not into something else. This is the intensive movement of self-differencing. Consequently there can be difference in being (which must be self-difference), contrary to the traditional notion of Being-as-it-is-in-itself from which all difference is excluded – and therefore reduced to an inferior ontological status. If difference is excluded we are left with the unity of being as "the identical and self-same" (Plato, *Timaeus*, 28c). But if there is difference in being, because it is self-difference this *is* the unity of being - as we have seen with Goethe's phenomenonological description of the metamorphosis of the plant.

We cannot escape from the dichotomy of objectivism and relativism within the dualistic framework of Platonism because these are two sides of the same Platonic coin. If appearances are separated from being, and vice-versa, then either what we know must be "perpetually true, under all circumstances and for everyone", in which case it is objective, or else it is dependent on the circumstances and situation, in which case it is relative. But since in the dynamics of being there is no such opposition between a multiplicity of "mere" appearances and Being-as-it-is-in-itself, the either/or of the objectivism/relativism dichotomy simply doesn't arise. We can have ontological pluralism without epistemological relativism. We have explored this dynamical approach in the context of Goethe's phenomenology of the living plant, but we should expect to find the dynamics of being exemplified wherever we encounter the coming-into-being of something which is always one and the same and yet different. This is what we find, for example, in the philosophy of meaning and understanding (hermeneutics), where we see the same dynamic of self-differencing in understanding the meaning of a work that we have found in Goethe's science of life (36).

Notes and References

1. Dennis L. Sepper, *Goethe Contra Newton: Polemics and the project for a new science of colour* (Cambridge: Cambridge University Press 1988), pp. 91-99, 169-70, and chapter 5, especially pp. 179-195.
2. Douglas Miller, ed., *Goethe: Scientific Studies* (New York: Suhrkamp Publishers, 1988), p.308.
3. The subtlety of this intuition of the idea belonging to the phenomenon is dealt with very carefully in Ronald H. Brady, "The Idea in Nature: Rereading Goethe's Organics", in David Seamon and Arthur Zajonc, eds. *Goethe's Way of Science: A Phenomenology of*

Nature (Albany, N.Y.: State University of New York Press, 1998), pp. 83-111. See also Henri Bortoft, *The Wholeness of Nature: Goethe's Way of Science* (Edinburgh: Floris Books, 1996).

4. The English translation of *Die Metamorphose de Pflanze* that I am using is basically the one which appeared in the *Journal of Botany* in 1863. It is available from Biodynamic Farming and Gardening Association (Kimberton, PA., 1993). Another translation is given in Douglas Miller, ed., *Goethe: Scientific Studies* pp. 76-97. My choice of translation is simply determined by the fact that it is the one with which I am most familiar because I have used it in lectures and workshops.
5. Diagrammatic representation of intermediate stages between petal and stamens in the white water lily taken from Gerbert Grohmann, *The Plant, vol 1* (Kimberton, Pa: Biodynamic Literature, 1989), p. 43.
6. Although we will not go into this – because we are concerned here with getting the idea of metamorphosis, and not with the details of plant growth as such – the central organ of the pistil, which includes the ovary, is included in this transformation. For example, it can happen that there is a retrogressive step where stem leaves appear in the place of sepals (the leaves which form the outer casing enclosing the floral bud) - there is a photograph of a dandelion in which this can be seen in Grohmann, op. cit., p.63. But more striking than this is the case of retrogression which Goethe called a proliferous carnation (para. 105 of *Metamorphosis*). In this case he observed a carnation in which the seed capsules of the ovary were transformed back into sepals (notice how easy it is to convey the false idea that this is a physical transformation), and in place of the seed capsules a second flower grew out of the first.
7. We are now much more familiar with this dynamical thinking as a consequence of the way that biology has developed, so that it is much easier

   for us to understand the idea of metamorphosis in the right way than it might have been previously. For example, stem cell research has drawn our attention to cells in embryos (embryonic stem cells) which are unspecialized – they proliferate by cellular division without specializing - but which can also differentiate, in which case a stem cell can take on any one of a number of specialist functions, e.g. becoming a liver cell, or blood cell, or nerve cell, and so on. This is an astonishing discovery, and yet it is clearly in line with the idea of metamorphosis in the plant, where the vegetative cone of the plant remains in an embryonic state and can therefore develop into different organs according to circumstances. Similarly, a stem cell can develop into a specialized cell in different ways according to circumstances – but we would never expect to see an already formed blood cell turn into a liver cell, for example. The fundamental process of life which Goethe recognized in the metamorphosis of the plant, is dynamically similar to the embryonic development of specialized cells from the stem cells in the growth of organisms.

8. Goethe's work on metamorphosis extends beyond the individual plant, to include variations in a plant species, members of a family of plants, such as the *Rosaceae* for example, and ultimately to the plant kingdom as a whole. The last two quotations from Goethe refer to different plants, whereas so far we have considered only the organs of a single plant. However it will not be difficult to recognize that this is a natural extension to make, and we will be considering it below. In any case, it is easy to see how both these quotations apply to the different organs of a single plant equally as well as to different plants.
9. Rudolf Steiner, *Goethe's World View* (Spring Valley, N.Y.: Mercury Press, 1985), p.81.
10. F.W.J. Schelling, *First Outline of a System of the Philosophy of Nature* (Albany, N.Y.: State University of New York Press, 2004), p.15. First published in 1799, when Schelling was twenty-four, it built upon his earlier work, *Ideas for a Philosophy of Nature*, published in 1797 (and revised in 1803). An English translation of the latter was published by Cambridge University Press in 1988. These are the first English translations of these works to be published. The interaction between Goethe and Schelling and their mutual influence is described in detail in Robert Richards, *The Romantic Conception of Life: Science and Philosophy in the Age of Goethe* (Chicago: University of Chicago Press, 2002). However, it needs to be emphasized that, notwithstanding their fruitful interaction, Goethe's way of science is very different in practice from the approach to nature developed by Schelling and others (notably Hegel) at the time, which was called *Naturphilosophie* or "nature philosophy". This was a development of the post-Kantian philosophy which is usually, if misleadingly, called "German Idealism", which in this case takes the transcendental approach (in Kant's sense of the term) to find the "conditions for the possibility "

of nature. This is very different from Goethe's phenomenology of nature.

11. Gilles Deleuze, *Bergsonism* (New York: Zone Books, 1991), p.42. The use of the term 'differencing' may initially cause difficulty for some. A precedent for this may be the use of the term 'presencing', which is now commonplace in discussions of Heidegger's philosophy, although there are some who continue to find it objectionable. Normal language use often focuses on the noun and not the verb, the static instead of the dynamic. But when it is the latter which needs to be emphasized, then it may be useful to introduce an unfamiliar, and therefore at first awkward, term which is more dynamic. Thus 'presencing' says more than is conveyed by 'presence'. Similarly, 'differencing' conveys more than 'difference'. Nigel Topping, a student on the M.Sc. Holistic Science Course at Schumacher College, pointed out to me that, as we easily go from 'dance' to dancing', so we could just as easily go from 'difference' to 'differencing'.
12. It is now almost a commonplace that western philosophical thinking has been dominated for the most part by one of these perspectives, giving priority to being as a being. However, this is only "for the most part". Beginning with Hegel in the 19th century, and accelerating remarkably in the 20th century with philosophers as different as Heidegger and Merleau-Ponty, Bergson and Deleuze, a dynamic alternative to the mainstream western philosophical tradition had developed which emphasizes being as be-ing.
13. This will not work with the kind of holograms that can now be obtained commercially because they are manufactured by a different process, and anyone who tries it today will be both disappointed and annoyed. The division process could be done as described with transmission holograms when these were first developed.
14. John Seymour, *The Countryside Explained* (London: Faber and Faber, 1977), p.116
15. This expression was used by David Bohm.
16. This electron micrograph was very kindly given to me by Dr. Bruce Kirchoff when I visited the University of North Carolina (Greensboro) in November 1996, to give a talk on "Goethe's Science of the Wholeness of Nature".
17. Ronald H. Brady, "Form and Cause in Goethe's Morphology", in F. Amrine, F.J. Zucker, and H. Wheeler (eds.), *Goethe and the*

Sciences: A Re-Appraisal (Chicago: University of Chicago Press, 1987), p. 286.

18. Ibid., p.287

19. An excellent discussion of varieties, with carefully observed examples, is given in Craig Holdrege, *Genetics and the Manipulation of Life: The Forgotten Factor of Context* (Hudson, NY: Lindisfarne Press, 1996), chap.1.

20. Deleuze, quoted in Todd May, *Gilles Deleuze: An Introduction* (Cambridge: Cambridge University Press, 2005), p. 60. See Gilles Deleuze, *Nietzsche and Philosophy* (New York: Columbia University Press, 1983; first published 1962), p. 24.

21. Rudolf Steiner, *A Theory of Knowledge Based on Goethe's World Conception* (New York: Anthroposophic Press, 1968), p. 88.

22. Holdrege, op. cit., p.46.

23. Darwin was deeply impressed, overwhelmed even by the ubiquity of variation. Before he did his work with barnacles, Darwin had believed that variation is the exception in nature, occurring only in times of crisis. His barnacle work changed that. Here he found that there are no unvarying forms, and that barnacle species are, as he put it, "*eminently* variable". What made the work of classification so difficult was that "Every part 'of every species' was prone to change; the closer he looked, the more stability seemed an illusion [Adrian Desmond and James Moore, *Darwin* (London: Penguin Books, 1992), p. 373]. Barnacles, he told Hooker, are infinitely variable; and in the context of his theory of what he called 'the transmutation of species', he went further to see variations as incipient species. There is a switch in gestalt here, like the reversing cube: in one perspective the phenomenon appears as the variations of a species, whereas in another perspective the very same phenomenon appears as the initial stages of new species. Goethe and Darwin both encountered the organism's "potency to be otherwise" which is the self-differencing dynamic of life. But, whereas Goethe saw this unceasing variation phenomenologically, so that he *understood* it as the expression of life itself (Goethe's aim was to understand life from life itself), Darwin wanted to *explain* it (in this regard he thought more like a physicist). He eventually "found" an explanation in the key to the success of Victorian capitalism: the division of labour. Prompted by the idea of the 'physiological division of labour' put forward by the French zoologist Henri Milne-Edwards [Desmond and Moore, p.394 and p.241], and the considerable experience of his wife's family (the Wedgewoods) in assembly-line manufacture (of pottery), Darwin applied the metaphor of the division of labour to see Nature as a 'workshop' – Nature's 'manufactory of species' – in which variation produced greater functional diversity of species, so that overcrowding did not necessarily result in direct competition for food and other resources. Thus species with small functional differences could all be supported in the same area without open competition by occupying different niches for which they were each functionally adapted in their own specific way, with the result that: "Just as a crowded metropolis like London could accommodate all manner of skilled trades each working next to one another, yet without any direct competition, so species escaped the pressure by finding unoccupied niches in Nature's market place". [Desmond and Moore, p. 420]. The Malthusian problem of overpopulation and competition was solved in Nature, it seemed, in much the same way that it had been in 19th century industrial Britain.

24. The reason why the two different terms, Form and Idea, are often used in connection with Plato's philosophy, follows from the fact that in Greek two different words were used: *eidos* and *idea*. When these were translated into Latin they became *forma* and *idea,* from where they entered into English as 'form' and 'idea'. It is customary to write 'Idea', rather than 'idea', when dealing with Plato, to emphasize that in this context it does not mean an idea that has been mentally abstracted from experience, and which therefore belongs only to the subject, as in the philosophy of empiricism – e.g. Hume's "faded copies" of sense impressions.

25. The key dialogue here is the *Parmenides*. The conventional view is that this dialogue represents Plato facing up to the difficulties with the theory of Ideas and becoming self-critical, which leads him, if not to reject it, at least to a revision of the theory which presents it in a much diluted form. But Gadamer strongly disagrees with this conventional view of the *Parmenides.* He sees it as a mature work representing the culmination of Plato's thinking, and not as a rejection of his own earlier work. It is Plato's attempt to correct the mistaken interpretation of the Ideas, which he may himself have unwittingly encouraged but never intended. Gadamer also believes that the theory of Ideas is not the real core of Plato's philosophy, as usually believed, but that the true focus is the philosophy of "the one and the many" which is a necessary condition for understanding the

Ideas in the right way. A very clear discussion of Gadamer's understanding in this regard, and its application today, is given in Brice R. Wachterhauser, *Beyond Being: Gadamer's Post-Platonic Hermeneutic Ontology* (Evanston, Illinois: Northwestern University Press, 1999).

26. It may even be that Goethe's dynamical thinking of "the one and the many" could lead us nearer to what Plato was really trying to say.

27. Edward Caird, *Hegel* (London: Blackwood, 1883; Cambridge Scholars Press, 2002), p.135.

28. Hegel was also in Jena during the golden period of the first few years of the 19th century, at the same time as Schelling (with whom he worked closely before they disagreed), Hölderlin, Novalis (Friedrich von Hardenberg), Schleiermacher, the Schlegels, Schiller, and Goethe, among others, were giving birth to the movement of German Romanticism in poetry and art and the movement of German Idealism in philosophy. The outcome of this period for Hegel was the *Phenomenology of Spirit.* It is hardly conceivable that Hegel and Goethe would not have met, especially in view of the fact that Hegel was at the time collaborating philosophically with Schelling. Much later, in 1821, Hegel wrote to Goethe expressing his appreciation of the latter's notion of the *Urphaenomen* and its significance for philosophy [Rudolf Steiner, *Goethe The Scientist* (New York: Anthroposophic Press, 1950), p. 179]. He is reported to have said that what he did abstractly, Goethe did concretely.

29. Goethe described this experience in his review of Purkinje's*Sight from a Subjective Standpoint* (1824). See Douglas Miller, ed., *Goethe: Scientific Studies*, p.xix. The dynamical quality of Goethe's perception of organic nature is strongly emphasized in Rudolf Steiner, *A Theory of Knowledge Based on Goethe's World Conception* (New York: Anthroposophic Press, 1968), chapter 16, especially pp. 90-93. Goethe once suggested that the way in which multiple patterns can be produced with the kaleidoscope could be used as a metaphor for his dynamic experience of seeing the plant – and he was well aware of the danger of taking such a metaphor too seriously and turning it into a "model". Keeping this caution in mind, we can now offer the technologically updated metaphor of the multiple hologram for this experience of seeing the multiply unfolding plant.

30. Peter J. Bowler, *Evolution: The History of an Idea* (Berkeley, California: University of California Press, 1989), p. 132.

31. Adrian Desmond, *The Politics of Evolution: Morphology, Medicine and Reform in Radical London* (Chicago: University of Chicago Press, 1992), p.368.
32. Owen said that the general vertebrate "unity of plan", as it was often called, pointed towards a "predetermined pattern, answering to the 'idea' of the Archetypal World in the Platonic Cosmogony" (quoted in Adrian Desmond, op. cit., p.364). The Platonic influence in comparative anatomy in Britain at this time stemmed initially from the poet and philosopher Samuel Taylor Coleridge. He had been very much influenced by German *Naturphilosophie,* which he saw as a way to combat the prevailing mechanico-corpuscular philosophy in Britain that to him was the cornerstone of the materialism he abhorred so much. Coleridge attracted the surgeon Joseph Henry Green as his leading medical disciple, who had himself been educated partly in Germany, and had studied philosophy in Berlin. Green's protégé in turn was Richard Owen. These, among others, formed the influential group which John Stuart Mill called the "Germano-Coleridgeans", whose aim was to promote transcendental morphology as a science of Platonic "archetypes" existing in the Divine Mind. The morphological archetypes, therefore, became the Thoughts of God. So in this respect the science of morphology became similar to the science of mathematical physics, as this was developed in the 17th century by Galileo, Kepler, Descartes, and Newton. In the latter case the influence of the two-world theory led to the idea that there are mathematical laws of nature which are separate from, and on a higher ontological level than, the empirical phenomena encountered by the senses. In this case it is the mathematical laws – which function as the equivalent of the "archetypes" in biology – that are considered to be Thoughts in the Mind of God. The development of these ideas in 19th century biology is described in rich detail, and in a wonderfully readable way, in the work by Adrian Desmond referred to above (see especially chapter 8)
33. Martin Heidegger, *Introduction to Metaphysics,* trans. Gregory Fried and Richard Polt (New Haven and London: Yale University Press, 2000), p.111.
34. G.B. Madison, *The Hermeneutics of Postmodernity: Figures and Themes* (Bloomington: Indiana University Press, 1990), p. 130.

35. This and the following quotation are from Aron Gurwitsch, *Phenomenology and the Theory of Science* (Evanstan: Northwestern University Press, 1974), p. 51.
36. This is the subject of a book, *The Dynamics of Being: from Phenomenology through Goethe's Way of Seeing to Hermeneutics,* which I am writing.

What does guide our actions?
A wider context of S.R. Covey's philosophy

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Abstract. The fundamentals of S.R. Covey's practical philosophy are analysed and comparison is drawn with other philosophical and religious approaches.

I. Introduction

In the second half of the last century a peculiar phenomenon swept the Western world: a mass search for meaning of an individual life. In the proliferation of self-help and personal growth books the works of Stephen R. Covey stand out [in particular 1, 2] if only through the sales of millions of copies. His practical philosophy became an industry not only in the US but worldwide; several large corporations have sought his advice on running their business through a 'principle-centred' leadership. The cornerstone of this approach is a combination of ethical and spiritual development and **action** in a real world.

In order to understand the success of Covey's 'applied philosophy' it is necessary to examine the foundations on which it rests. This is done in the next section, with emphasis on ways to become independent. Section III briefly examines the practical side of this philosophy, and focuses on the implications of managing time with the aid of a *compass* and a *clock*. Section IV sets this analysis in a wider context, and touches on the spiritual dimension.

This paper comes with the health warning: if you think that philosophy should **not** appeal to the masses, stop reading now!

II. Covey's philosophy in a nutshell

Success in a technological society

Covey's thinking grew out of the analysis of what others considered to be a success and the route to achieve it. He surveyed the 'success literature' published in America in the last two centuries. From this he concluded that initially success should be achieved via the *Character Ethics* or internalisation of the principles of integrity, hard work, modesty, courage, fairness etc. However, in the first quarter of the last century there was a visible shift of focus to the *Personality Ethics*, with foundations in personality, attitudes and techniques; these are roughly summarised in public relations and positive mental attitude. As I see it, subscribing to the personality ethics results in an ego-centred view of the world (Figure 1).

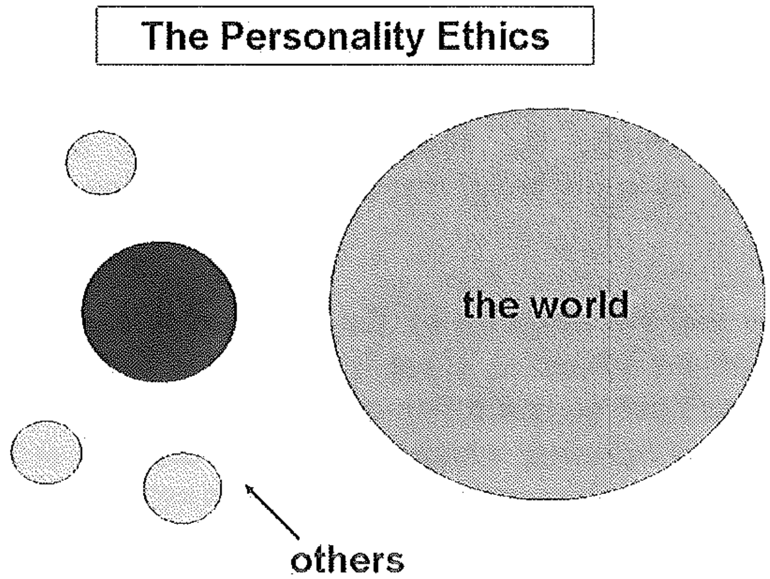

Figure 1 The self-centred view of the world, dominating the Western thought for the last century. A comic strip representation.

Many self-help and management books reinforce this view. However, while self-centred living can deliver short-term benefits, in the long run it will fall short of expectations. This way of life promises more than it can deliver, as it ignores principles of growth and change.

Principles or natural laws

The basis of Covey's "Character Ethics" (Figure 2 below) is an assumption that some "universal, timeless, self-evident *principles* common to every enduring, prospering society throughout history" [1] govern human effectiveness. Hence principles or *natural laws* are a part of these ethical, social philosophy or religious systems which withstood the test of time. Fundamental principles comprise fairness, integrity and honesty, human dignity, potential and growth. In short, principles are guidelines for our conduct, and correct actions and true values follow from them.

Covey does not claim to have invented the Character Ethics but simply to have "identified and organised these principles into a sequential framework". He states that humans possess self-awareness, understood as an ability to think about their own thought processes. This provides a route to an examination of principles, values and paradigms. While he very carefully refrains from a reference to any specific religion or ethics, the Judeo-Christian point of view occasionally surfaces, as in his belief that Man has dominion over all things in the world.

Since we act upon and are acted on by the world, it is essential to identify the limits of our influence. There is a *Circle of Concern*, i.e. events and actions we have no real influence but which still affect us. There is also a *Circle of Influence* which comprises events and actions we have some control over. The art is to focus our efforts in the latter. (The Circle of Concern should be inside the Circle of Influence for 'proactive people'). Working on things one can do something about, and enlarging this class is one of essential ingredients of success. His approach has attracted both acclaim and critique (see, for example, [3]) as it could create an impression that we can become completely independent of others. This has been rectified in his next, co-authored book [2] which I consider to be the most useful and applicable of his works.

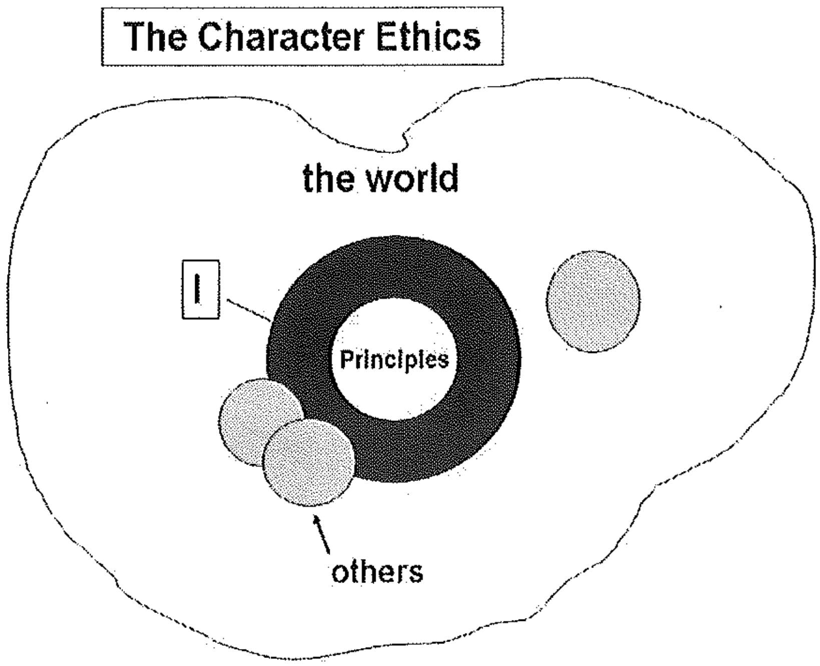

Figure 2 The Character Ethics in a comic strip representation. Actions and values are grounded in principles (universal laws).

Development of self-awareness
One has to recognise from the outset that Covey advocates action and fulfilling one's responsibilities in a technological society. This leaves aside a question of purely spiritual pursuits which by their nature are not 'productive' as commonly understood. He acknowledges though the need for spiritual development and for reflection – time put aside e.g. for reading scriptures, prayer and meditation or simply pondering ethical and moral questions.

How can we realise our human potential? According to Covey we need to "identify and apply the principle or natural law that governs the results you seek" [1]. This goes against the instant results and gratification mentality as it involves a lot of effort, self-questioning and patience needed for personal development. This approach differs greatly from that advocated by many popular self-help systems; it is the way of self-development in any worthy ethical or spiritual systems.

The question arises how can we change in order to become better – and better organised – human beings? This can be achieved through development of seven habits of highly effective people [1]. A habit is defined as an intersection of ***knowledge*** (what to do and why), ***skill*** (how to do) and ***desire*** (want to do). The path from dependence through independence to interdependence leads via cultivation and implementation of these habits (Figure 3).

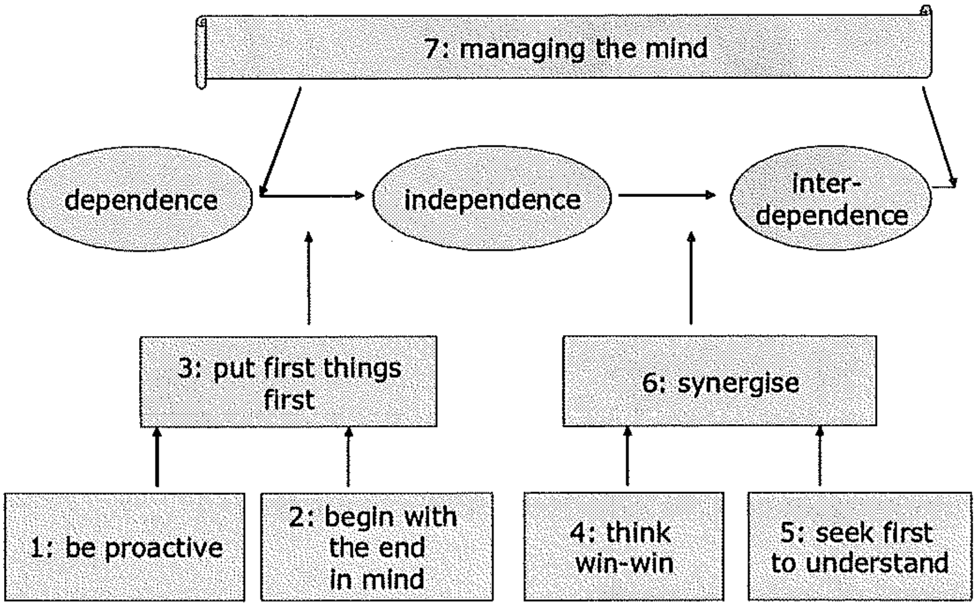

Figure 3 The seven habits according to Covey which foster self-development. His 7th habit 'sharpen the saw' is represented here by 'managing the mind' [4].

From dependence to independence

The first habit to cultivate is to be proactive, i.e. take responsibility for our actions and to be able to override our social and other conditioning through a measured response to a stimulus in the decision-making process. This freedom to choose is informed by our *self-awareness*, *imagination* (to go mentally beyond present reality), *conscience* (an innate awareness of right and wrong) and *independent will*. The second habit 'begin with the end in mind' means looking at our plans and actions in the frame of a grand picture: a clear understanding of our destination, perhaps even destiny. This embraces a principle that all things are created twice: firstly mentally, then physically. (E.g. in research a grant application requires projection of the final result of an enquiry and setting it out on paper; if the application is successful the research is then carried out. The details of results may vary from the expected ones, adding excitement; or a greater challenge of an unexpected can reward the researcher.) A practical way to nurture the second habit is to develop a personal mission statement or personal long, medium and short term objectives. This serves as a marker along the path of personal development **if** based on the correct principles.

The first two habits underpin the third one, that of putting first things first. It requires mastering of the first two habits in order to become principle centred and giving the priority, in every single moment, to the most important things which our vision encompasses. It necessitates discipline to honour promises, meet deadlines, and fulfil obligations as a daily practice. Further, the mastery of the third habit brings independence from being ruled by crises and wishes of others. If we fall back on principles and live our life's mission then we are not swept by events as leaves on a windy street but are able to achieve to what we have set to.

These three habits are encompassed and reinforced by a habit number seven – that of managing your mind or as Covey puts it, 'sharpening the saw'. This is recognition that in order to function well and live harmoniously we need to hone our physical, spiritual, mental and social/emotional dimensions. The physical well-being and health underlies the other three, and is achieved through proper nutrition and exercise. Spiritual dimension is arguably the most important one, and as most private it is left to us to choose the way to spiritual renewal. Mental recreation should be a worthy one and comprises lifelong learning. Finally the social/emotional dimension is of an utmost importance in dealing with others, and encompasses passage from independence to interdependence. On face of it, this type of advice is a very common one; parts of it can be traced back to ancient Greece. The maxim 'Know thyself' is attributed to at least six philosophers, and was inscribed in the forecourt of Temple of Apollo in Delphi, according to Pausanias [5]. The knowledge of oneself should lead ultimately to understanding others.

Harder to accept is the idea of having a personal 'mission' and 'vision', since so often these words are being abused. As a matter of fact each of us has a mission though we may rarely think or talk about it. For example, some children dream from an early age to be a doctor and to cure currently incurable diseases such as Aids. Others, like the Dalai Lama, are born into their mission as a religious leader. Politicians may claim a mission to improve their countries' economies thus bring welfare to their people. Scientists may aspire to reach the stars, find whether there is life on Mars or develop new sources of energy for the benefit of human kind.

III. Putting theory into practice

First things first

A diligent and regular application of the above habits will result in life and time management where the activities and tasks are organised and executed around priorities. This is referred to as an (emerging) time management of the fourth generation – where we need to manage ourselves, not time. It is *principle-centred*, *conscience-directed*, *defines our unique mission*, *helps us balance life by identifying roles* and gives *a context through weekly organising*. In practical terms, the focus is on relationship and results, with time being subjugated to them [2].

The novelty of Covey's time management methodology was a proposition that the most successful people are guided both by a *compass* and a *clock*. What is a compass? In analogy with walking, rather than wander aimlessly along the country roads leading from A to B we would use the compass to point you in the right direction. If we want to go to the Northern Pole, the compass is set to point to the north. Then while going we would consult it frequently to check, whether we're still going in the right direction; while dealing with obstacles in the direct path we would still keep our orientation right. The same reasoning applies to our ethical compass – it is informed by mission and vision, and should always point to 'true north'. And True North is defined by principles or natural laws.

Reputed to be the most popular book on time management ever, *First things first* [2] prescribes an integral way of life and work based on a few building blocks described above. It seemed to me that Covey's approach can be adapted to academic practice in the sciences [4]. This is because his philosophy stems from generic principles and the interpretation of what these natural laws are and where they come from is left to the reader. This leaves scientists a freedom to function within their particular frame of reference. However the description of practical details of such time management is beyond the scope of this article.

In my view the relationship between the *compass* and the *clock* is the most important factor of Covey' success: his practical philosophy provides a frame of reference to believers and non-believers alike by avoiding explicit religious message. It yet provides a moral guidance of which details are left to an individual. Covey addresses the yearning to make sense and live meaningfully in a modern world though some people may find his language and guru-like exposition not palatable. He attempts to place Man back in an ordered universe where things have purpose not always evident to us. In doing so he goes back to a couple of centuries (viz. [6] for a thorough analysis of the making of modern identity). Yet there is a power of choice between stimulus and response which resonates better with a modern man.

IV. Wider context

The horizons of a modern man

One way of looking at Covey's proposition is to see it as a conflict between the place of man in a hierarchical order and a modern individuality. Then, seemingly, Covey tries to turn back the clock by subjecting an individual to a rule of natural laws. However it is possible to perceive his efforts as reconciliation of individualism and some greater, perhaps cosmic, order. The latter interpretation justifies a mission as something essential to one's life, something worth dying for. This fits readily with philosophy of morality. For example, according to Taylor [6] the expression of morality as that of following a voice of nature within us is due to a philosopher Jean Jacque Rousseau. The essential condition of leading a contented and rightful life is to be in an intimate moral contact with our Self.

How can this intimate contact be achieved? In today's understanding the way is through spiritual practice which may take a form of prayer, meditation or ethical considerations. This is also what Covey means by asking a reader to 'deeply connect with your vision and mission'; meaning to examine the underlying principles.

How well does Covey address the needs of modern society? Taylor [7] identifies three malaises which trouble us: individualism, the disenchantment of the world or "instrumental reason" and the consequences for political life of these. Individualism at its worst became ideology of the 'me' generation where 'I' and 'my' needs are central (cf Covey's Personality Ethics). In order to overcome it one has to go beyond the current culture of authenticity. This way, the needs of others have their rightful place, the care of and sharing with others enriches (cf Covey's Character Ethics). Instrumental reason is the common tendency to measure everything in terms of cost and benefits and where the maximum efficiency is god [7]. Covey has his own understanding of efficiency and tries to redress the balance through bringing about synergy with others as one of critical ingredients of success. The success of any industry, as well that of families, is brought down to personal efforts of each member and their subscription to a common mission and vision [2]. He carefully avoids the murky waters of politics, as pointed out by his academic critics [3].

This leaves some essential questions open: what are natural laws? By whom or by what are they established? How do we know whether we are in harmony with them, or are trespassing?

Covey simply acknowledges the existence of *natural laws* or *principles* which underlie and ought to guide our actions, and wisely does not prescribe any explicit religious or political meaning. His personal interpretation is a Christian one – the source is God, and presumably the laws would be summed up as Ten Commandments. Consider other great teachings, for example Buddhism - the basic structure of human society relies on sense of responsibility, stemming from compassion and altruism. Success in life is due to determination, will and courage which flow out of altruism. Buddhism also teaches interdependence of all sentient and non-sentient beings thus widening the responsibility beyond human kind [8]. As everything is interrelated, our welfare ultimately depends on the welfare of others.

The source of action (and motivation) is mind which 'operates' on two levels of consciousness: the grosser level produced by the brain and the independent, ultimate, innermost subtle level of consciousness. Our influence and power to transform is over the former; the laws (Dharma) flow from the latter.

Graf von Dürckheim expresses the need for and the meaning of spiritual development through reconciliation of Western and Eastern thought [9]. For him, the three basic concerns of human kind are to live or survive, live meaningfully and be accepted by community. They make us tick and are central to our living and fulfilment. They are reflected in otherworldly Being as undivided plenitude, absolute order and all-embracing unity. Being manifests itself in existence, and its tri-unity is interpreted in Christianity as the power, wisdom and goodness of God the Father. Buddhism has three treasures – Buddha, Dharma (the law) and Sangha (the community of disciples) [9]. With regard to the three basic concerns, we can experience self-consciousness of three kinds - that is of our own strength, of our own value and of our links with others, respectively. On a deeper level, we can experience self-consciousness from our True Nature, which is going beyond one's ego and touching our own centre or Being. It is claimed that such phenomenon happens e g in near-death experiences [10].

For many of Covey's readers his practical philosophy would suffice – there is a way to live in accordance with ones' conscience and yet practical enough to generate income and contribute to the society. This is laudable, as put forward by Dürckheim:
"It is a truism that all work, all art and all professional activity require practice if they are to succeed. This we accept, and in order that we may establish ourselves in the world, it is obvious that we must be at pains in all our vocations, avocations and transactions to practise and assimilate experience." [11]

V. Conclusion

The central thesis of Covey's philosophy is the statement that man has power to make and break habits through self-awareness: observation,

reflection and a free will to choose response to a stimulus. His practical approach addresses needs of modern man and strikes a chord with ethical messages of philosophy of morality, Buddhism and Christianity.
In analogy to spiritual practice one can use the layers of Covey's philosophy simply for increased efficiency in daily life, as Zen can be practised just for improving health. However this would be only a little progress on 'recipe' type of time management approaches – the deeper meaning of putting first things first is then irrevocably lost.

Acknowledgment
I would like to thank Adam Parker Rhodes for pointing out the relevance of Charles Taylor's book 'Sources of the self' to this contribution.

Note
In the above text the term 'man' is used in general way and comprises men, women and trans-gender individuals.

Selected references

[1] *The 7 habits of highly effective people*, S.R. Covey, Simon & Schuster Ltd; Rev Ed edition (4 Jan 1999) ISBN-10: 0684858398
[2] *First things first,* S.R. Covey, A. R. Merrill, R.R. Merrill, Simon & Schuster UK Ltd., London 1999 ISBN 0-684-85840-1
[3] *Guru philosophy as adult education: Stephen Covey and the cult of educational effectiveness*, T. Fenwick and J. Parsons (1996) paper presented to the Ontario Theological Seminary/Institute Christian Studies "Heart and Mind Conference", Regina, Saskatchewan, May 18-20. http://www.ualberta.ca/~tfenwick/ext/pubs/print/covey.htm (last accessed 31/12/2008)
[4] *A preparation for the academic career in the sciences: coping with complexity,* B.J. Gabrys and J.A. Langdale, book in preparation
[5]http://en.wikipedia.org/wiki/Know_thyself
[6] *Sources of the self: The making of the modern identity*, C. Taylor (1989), Cambridge University Press, ISBN 978-0-521-42949-8
[7] *The ethics of authenticity*, C. Taylor (1991), Harvard University Press, ISBN 0-674-26863-6
[8] *The transformed mind: reflections on truth, love and happiness,*

His Holiness the Dalai Lama (2000), Mackays of Chatham plc, Chatham, Kent, ISBN 0-340-81937-5

[9] *The call for the master*, Karlfried Graf Von Dürckheim, Penguin Arkana, ISBN 0 14 019 345 6

[10] *Eindeloos bewustzijn: een wetenschappelijke visie op de bijna-dood ervaring*, Pim van Lommel, Ten Have, (7th edition) 2008, ISBN 978 90 259 5778 0

[11] *The way of transformation: daily life as spiritual exercise*, Karlfried Graf Von Dürckheim, (1985 ed.), George Allen and Unwin, London, ISBN 0-04-291020-X

A New Type of Jet Engine: Some *A Priori* Engineering Design Principles[1]

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This document outlines the basic design, fuel, materials and principles of operation of a new type of jet engine

1 Introduction

The author is fortunate to have been able to acquire through purchase and donation a near complete copy of the lifetime works of Kazuo Kondo, late Emeritus Professor of Mathematics, Tokyo University, Japan [Croll, 2006]. Careful study of this material, which relies largely upon Kawaguchi's geometry of higher order spaces [Kawaguchi, A., 1931, 1932, 1933] [Kawaguchi, M., 1962, 1968], enabled some predictions to be made [Croll, 2008] about particles expected to be produced by the world's largest machine, the Large Hadron Collider, at CERN on the Franco-Swiss border. Other information contained within these papers provides guidance about a theorised dimensionality reduction experienced by superfluids [K218] and alternative nuclear fusion processes [K219][K347]. This information, combined with an engineering design first conceived in 1996, provides a

[1] Presented at ANPA 2008, Wesley College, Cambridge, UK, 5th August 2008

basis for outlining these *a priori* engineering design principles for possible future use.

2 Basic Design

The new type of jet engine under consideration comprises a small (<10mm dia), flawless, cylindrical piece of Quartz, Diamond or other hard, transparent material. This cylindrical nozzle contains a central cylindrical channel of between 1 and 500 microns diameter, which rapidly opens out to an exhaust funnel at one end. Pressurised helium II, doped with a small amount of deuterium or other dopant, is introduced into the nozzle below the lambda temperature – somewhat below 2K. Immediately prior to exhaust through the funnel end of the nozzle, the superfluid helium II and its dopants are forced through a "pinch", engineered to reduce the diameter of the central channel somewhat further. To achieve ignition, a concentric laser pulse is focussed around the pinch to symmetrically & explosively compress the superfluid helium II. A small fraction of the deuterium dopants in the centre of the central channel are believed to combine together into helium releasing a significant amount of energy. The resulting plasma immediately exhausts the nozzle through the carefully shaped funnel, cooling as it expands. Further cooling is enabled by introducing coolant through concentric channels leading into the exhaust funnel. Device operation is pulsed via fuel pressure, with pulse timing matched to the design of a prismatic cylindrical collar which surrounds part of the nozzle, and the resonance of the entire system in operation. The prismatic collar feeds back part of the burn energy from the exhaust area to achieve continuous ignition without further use of the laser. Variable location of the prismatic collar around the nozzle, combined with the design of the central channel, provides a throttling mechanism.

3 Nozzle

The essence of the nozzle is the use of a hard, clear, crystalline material and a very high ratio between the diameter of the central channel and the diameter of the rearmost section of the exhaust funnel. The burn takes place in only the central part of the material in the central channel. This provides a sufficient buffer for the hot plasma to stay contained until it cools by expansion upon entering the rear part of the funnel where further pressurised concentric cooling is provided, preventing plasma contact with the nozzle surface.

4 Fuel

The fuel is a careful balance of helium and deuterium, with deuterium as a minor dopant. Helium II is the superfluid carrier & the amount of deuterium dopant is such that the superfluidity of the helium II carrier is not impaired under operating conditions. There must be sufficient deuterium available such that the probability of deuterium-deuterium combination is sufficiently high. Superfluidity permits a high fluid transfer velocity through the central channel.

5 Combustion

Combustion relies on causing the mixture of helium and deuterium superfluid to recombine into helium only as it instantaneously changes state from superfluid to plasma. There can be no memory of what the proportion of helium and deuterium was in the mixture prior to the instant of recombination. Given Kondo's notion of the underlying nature of matter, there is an element of Gödelian undecidability as to what actually happens at the moment of recombination. Recombination simply occurs, there being no achievable proof that it should or should not do so. There is a suggestion [K219] that recombination is spontaneous.

6 Exhaust

Choice of dopant will determine the nature of the exhaust. With pure deuterium, the exhaust should simply be a high temperature helium plasma with a zero net charge and no free neutrons. Exhaust energy may be fed back by a concentric prismatic collar which focuses energy on the pinch. Atmospheric operation my be viable, and may be a source of further coolant in the manner of a traditional bypass jet.

7 Ignition

Initial ignition occurs through the use of a relatively modest (<1Kw) industrial laser providing a concentric pulse focussed on the central channel around the pinch. It is possible that a simpler design may be achieved by abandoning the prismatic collar and using pulsed laser ignition on a continuous basis.

8 Operation

Fuel pressure is pulsed at the nozzle's natural resonant frequency, which is determined by the size, shape & length of the central channel. The theorised dimensionality reduction [K218] occurring in pressurised superfluids enables them to exhibit the properties of both liquids and solids: semi-solids. At the correct resonant frequency, temperature and pressure, some of the material in the central channel becomes temporarily semi-solid, preventing back-burn through earlier stages of the device. Pulsed operation assists in despatching hot exhaust prior to re-establishing nozzle temperature below the lambda point ready for the next pulse. A hope is continuous operation.

9 Thrust

Reactive force is provided by exhausting helium plasma or gas at very high velocity (1-10 km/s^{-1}) from the exhaust nozzle. Central channel fuel flow rates would be in the region 1-10 g/s^{-1}. Coolant flow rates would be in the region of 100-10,000 g/s^{-1}. It is expected that thrust would be in the region of 1-100 tonnes.

10 Cooling System

A particular engineering challenge is system design to minimise the amount of coolant required, whilst maximising production rate. Fuel and coolant has to run through the nozzle at < 2K whilst in close proximity to a hot plasma. Nozzle design should ensure that most energy is refracted to the rear.

References

Croll G.J. (2006), The Natural Philosophy of Kazuo Kondo. Proc ANPA, Cambridge, arXiv:0712.0641v1

Croll G.J. (2008), Predicting Large Hadron Collider Observations using Kazuo Kondo's Mass Quantum Cascade. Proc. ANPA, Cambridge, arXiv:0804.4299v1

Kawaguchi, A., (1931, 1932, 1933), Theory of Connections in the Generalised Finsler Manifold, I, II, III, Proceedings of the Imperial Academy, Tokyo, Vols 7, 8, 9. http://projecteuclid.org/DPubS/Repository/1.0/Disseminate?view=body&id=pdf_1&handle=euclid.pja/1195581142

Kawaguchi, M., (1962), An Introduction to the Theory of Higher Order Spaces I: The Theory of Kawaguchi Spaces, RAAG Memoirs, Vol III

Kawaguchi, M., (1968) An Introduction to the Theory of Higher Order Spaces II: Higher Order Spaces in Multiple Parameters, RAAG Memoirs, Vol IV

K218, Kondo, K., (1987) Post RAAG Report Number 218, A Priori Classification of Chemical Mass Quanta and Confusions in the Periodic Table with References to Magnetism and Superconductivity.

K219, Kondo, K., (1987) Post RAAG Report Number 219, On Radio Activities, Nuclear Fission and Nuclear Fusion

K347, Kondo, K., (2000) Post RAAG Report Number 347, Some Crucial Questions on Elementary Particles and Nuclei

ANPA Proceedings Editorial Policy

ANPA has been criticised in the past - in particular by members of its own Advisory Board - for having no formal editorial policy for its Proceedings. This has been balanced by a feeling within ANPA that we should keep ourselves open to all viewpoints. In the last few years as editor I [K.B.] have tried to tighten things up in such a way as I felt would satisfy our critics whilst not compromising our own position. This has been partially successful although for some time I have felt that it is time that there was a formally stated policy. The following has been approved by the Executive Council, although it is open to feedback from all. By "the editor" is meant the Editor or (an) appropriate nominated Referee(s) (note the capital R!)

1. The paper should make a new and original contribution to the fields of ANPA's interest. Survey papers are acceptable.

2. The default use of language for submitted papers in Physics and Philosophy of Physics should be the common language of Physics as usually understood by Physicists, and, in particular, by Philosophers of Physics. Any other use of language should be carefully explained at the start of the paper and all appropriate definitions included there.

3. The editor should be satisfied that the paper is ***presented*** in such a way that the majority of the readership will understand the author's intentions. In particular ***it should be clear*** that the author has a correct understanding of the subject matter. Please avoid forward references.

4. "Verbatim" reports will be accepted subject to the above three conditions only, regardless of whether the final draft is an accurate rendition of what was originally said.

5. Theories of any nature are acceptable material, provided they are compatible with the known facts, and provided they are deemed to be of interest to the readership. Theories of alternative, imaginary worlds are also acceptable, provided their nature is made clear.

GUIDELINES FOR PREPARATION OF MANUSCRIPTS FOR THE ANPA PROCEEDINGS

- FORMAT: Only electronic submission as a PDF file or a WORD file (as a second option) will be accepted
- FONTS SIZE: The body text should be in at least 14 point type.
- EMBEDDED FONTS: All fonts must be embedded
- LINE SPACE: The manuscript should be double-spaced (or at least 1.5 spaced) throughout
- JUSTIFIED TEXT: The text must be justified on both sides
- MARGINS: The left, right and the top margins should be the same: 30mm. The bottom margin should be 40mm
- MAX NUMBER OF PAGES: The length of the manuscript must not exceed 30 pages (please, contact the editor if your manuscript is longer than that)
- PAGE SIZE: The basic page size should be the standard European A4 format page (length: 297mm; width: 210mm)
- PAGE NUMBERS: The page number should NOT be typed
- DEADLINE: Contributions must be submitted latest 1 January following the ANPA meeting

Alternative Natural Philosophy Association
Statement of Purpose

1. The primary purpose of the Association is to consider coherent models based on a minimal number of assumptions, so as to bring together major areas of thought and experience within a Natural Philosophy alternative to the prevailing scientific attitude. The Combinatorial Hierarchy, as such a model, will form an initial focus of our discussions.

2. This purpose will be pursued by research, publications and any other appropriate means including the foundation of subsidiary organisations and the support of individuals and groups with the same objective.

3. The Association will remain open to new ideas and modes of action, however suggested, which might serve the primary purpose.

4. The Association will seek ways to use its knowledge and facilities for the benefit of humanity and will try to prevent such knowledge and facilities being used to the detriment of humanity.

Organisation

1. The Founder of the Association was Pierre Noyes. The Founder Members were Pierre, John Amson, Ted Bastin, Clive Kilmister and Frederick Parker-Rhodes. They will be known herein as the Founders. The Executive Council is the governing body of the Association. It consists of: (a) The Founders and all past Presidents of the Association, the President, the Co-ordinator and the Treasurer, (b) Ordinary members nominated by classes (a) and (b), who serve for three years, with the possibility of re-nomination.

2. The Members of the Association are (a) the members of the Executive Council and (b) others nominated by the Members and approved by the Executive Council.

3. The President has the responsibility for calling meetings of the Executive Council, at least annually, for the determination of overall policy.

4. The Treasurer is the responsible financial officer of the Association for the receipt and disbursement of funds and shall maintain and make available appropriate records, including annual accounts.

5. The President and the Co-ordinator services will include the organisation of meetings and the editing of the Proceedings of such meetings for publication, co-ordination of, and participation in, the research activities of the Association, preparation when appropriate of research reports and publication of such reports, and other such duties as may be assigned.

6. Members of the Executive Council may as appropriate receive funds for travel, expenses, etc.

7. The Executive Council has selected an independent Advisory Board. It may adopt its own rules for the operation and replacement of members. The Executive Council may nominate candidates to the Board. Any member of the Board, or the Board collectively, may make recommendations to the Executive Council, or directly to the Membership. Action taken on such recommendations must be promptly reported by the Executive Council to the Board in writing.

8. No one of the general public who has booked and paid the appropriate fees should be excluded from attending ANPA meetings except as decided by a majority of the Executive Council in a formal meeting. Notification of ANPA meetings will be sent to all members.

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