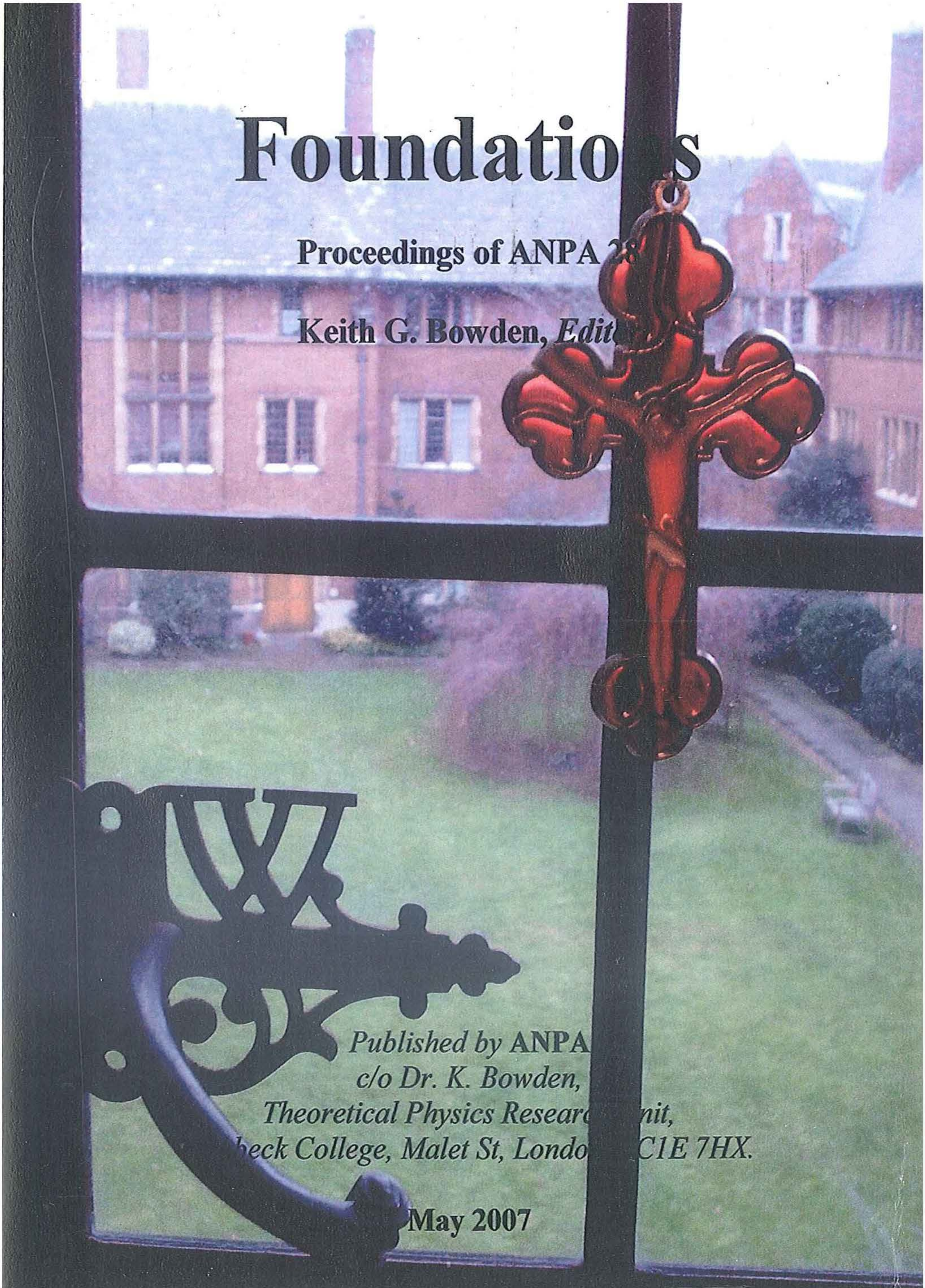

Foundatio s

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Editorial v2

Dear Louis,

Your implication that pulsars do not "flash" because of rotation has interesting consequences for the current attempts at detection of "gravity waves".

Indeed they imply that the gravity wave detectors (LIGO) will never detect ripples due to pulsars, because said pulsars are not emitting ripples.

This leads me to an ambiguity in the current language which I find extremely confusing and which is clearly (to me) confusing many discussions on the subject.

I wonder if anyone on this list can help clarify the situation.

First let me clarify something. LIGO (as I understand it) is testing two different independent things.

The first is whether we can detect a gravitational ripple from a pulsar AT ALL.

The second (and I believe the more important!) is whether or not the ripple travels at the speed of light (or other finite speed) or instantaneously.

Now I would propose that no-one in their right mind imagines (if the equipment were sensitive enough and the pulsar WERE rotating) that we would not detect a ripple at all. Assuming that a pulsar IS a rotating binary star then the gravitational field around it will be disturbed and if our equipment is sensitive enough we should be able to detect it in the form of a ripple at LIGO.

The real question is whether said ripple occurs simultaneously (is in phase with) with the visible "flash" at the Earth. The frequency of the pulsar in question is 10-4Hz (eight hours - about 1000 times faster than the Earth's rotation around the Sun!) so phase can be measured very accurately. There are obvious complications here but they do not affect my main question below.

So my question is about language.

I personally would define a gravity wave as a gravitational ripple traveling at the speed of light (or other finite speed). If such a thing were detected with the appropriate phase delay (zero compared with light amplitude at Earth) I would say "Gravity Waves exist."

If it were found that the phase delay was such as to reliably indicate that the gravitational ripple had "traveled" at infinite speed then I would say "Gravity Waves don't exist." The effect of mass movements is transmitted instantaneously.

Note here that by "ripple" I mean something more like a Mexican wave than like a light or sound wave. The oscillation in the ripple from the pulsar is due (disregarding harmonics due to nonlinearity) to the rotation of the pulsar, which is a "forcing function" and not due to natural oscillations in the medium. Gravitational ripples need not be oscillatory.

So what do the experts - the workers in the field - mean by "Gravity Wave"?

Do they mean a gravitational ripple moving exclusively at finite speed, probably the speed of light?

Or do they simply mean (as most laymen seem to interpret the phrase) a ripple detected at LIGO regardless of whether it got there instantaneously or not?

Looking forward to seeing you all in Cambs.

Best,
Keith

ANPA Proceedings Editorial Policy [28]

ANPA has been criticised in the past - in particular by members of its own Advisory Board - for having no formal editorial policy for its Proceedings. This has been balanced by a feeling within ANPA that we should keep ourselves open to all viewpoints. In the last few years as editor I have tried to tighten things up in such a way as I felt would satisfy our critics whilst not compromising our own position. This has been partially successful although for some time I have felt that it is time that there was a formally stated policy. The following has been approved by the Executive Council, although it is open to feedback from all. By "the editor" is meant the Editor or (an) appropriate nominated Referee(s) (note the capital R!)

1. The paper should make a new and original contribution to the fields of ANPA's interest. Survey papers are acceptable.

2. The default use of language for submitted papers in Physics {and Philosophy of Physics}* should be the common language of Physics as usually understood by Physicists {and, in particular, by Philosophers of Physics}*. Any other use of language should be carefully explained at the start of the paper and all appropriate definitions included there.
{* added by KGB}

3. The editor should be satisfied that the paper is ***presented*** in such a way that the majority of the readership will understand the author's intentions. In particular ***it should be clear*** that the author has a correct understanding of the subject matter. Please avoid forward references.

4. "Verbatim" reports will be accepted subject to the above three conditions only, regardless of whether the final draft is an accurate rendition of what was originally said.

5. Theories of any nature are acceptable material, provided they are compatible with the known facts, and provided they are deemed to be of interest to the readership. Theories of alternative, imaginary worlds are also acceptable, provided their nature is made clear.

ANPA Proceedings Notes for Authors

I would like to try to continue conformity of *style* for future issues of the Proceedings. Ideally I would like contributions to be submitted in International Journal of General Systems format (I have some copies of their Notes for Authors) or similar.

Times Roman, 14 point, is compulsory. **10 point is TOO SMALL to be reduced to A5.** If you are sending hard copy please send two copies single sided. Main heading 20 point capitalised and centred, other headings 16 point capitalised to the left. Author's name(s) capitalised and centred. Address italicised and centred. No underlining. At least a one inch bottom margin for footers; page numbers NOT top centre. *Only copy in good English will be considered, and remember, this is a formal Proceedings.* **Remember also to include your name (surprising how many people omit this!), affiliation and full address, email address and the version number (even if it is 1.0) or date of the draft ON THE COPY, centred below the main heading, or in 8 point font (tiny!) after it.** I often get sent more than one version of a paper and invariably mix them up! Send copy **to *KEITH BOWDEN, 139 SANDRINGHAM RD, BARKING, ESSEX IG11 9AH.***

The copy date for the ANPA2007 Proceedings is April 1st 2008. The issue will go to print on June 1st 2008. This will (really will) be adhered to rigidly this year!

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FOR EDITORIAL ADDRESS SEE THE PENULTIMATE PARAGRAPH OF THE TEXT

"NEW FOUNDATIONS" FOR THE COMBINATORIAL HIERARCHY

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I am going to talk about the basis for my calculation of the fine-structure constant as I have come to change it during the year. My paper overlaps Ted's but we have not had time to coordinate the two, so in hearing both you will have some idea of how it is we work together. The inverted commas in the title are to recall to me (at any rate) my youthful hero, Willard Quine. I was very excited by his lectures showing that logic while describing the whole world really describes nothing but arithmetic (because of Gödel's arithmetisation of syntax). In later life Quine published "Mathematical Logic" , a system which turned out to be inconsistent. This was followed by his "New Foundations" which is believed to be consistent.

In the early fifties of the last century Dirac said that he believed that his "Quantum Mechanics" was essentially correct but that, as a book, it lacked a Chapter 0 which would give the real foundations. It is in something of that spirit that I am working on the CH. Over the last eighteen months I have been (slowly and painfully) "woken from my dogmatic slumbers" recently by Keith's insistence but really my John Locke has been Ted. Don't worry if you can't remember what Kant's dogmatic slumbers were - the analogy doesn't go so far. Mine were the mathematician's usual vice: "If something works once, do it again, and again.." I look forward to Keith's paper this afternoon, which will have the opposite approach.

Historically I would describe my activities as "trying to understand the Parker-Rhodes construction". I took this to mean"to derive, from physically reasonable beginnings, his whole algebra". Now I see that this is not necessary. Rather, I want to derive a stripped-down version which is still adequate for the alpha calculation. If more is needed for later developments, it can be discovered when it is needed.

We postulate a universe that is constructed progressively, and so we speak of process. In the process elements are constructed in a sequence. It is convenient to speak of "elements coming into play". It will turn out that from the idea of being in play there follows a stripped-down version of the mathematics without a spurious appeal to Conway. To be a sequence implies that the process determines whether a new element is the same as one already in play or not. The determination of whether b is the same as a (call it ab) is a further element in play (call it c). The binary operation ab = c is called discrimination and we have to find out what this is.

One of the first things I was taught as a research student was: if you define something, then give an example to show that the concept is not empty. So here are two:-a binary operation suggests a group but this will be an example only if the set of group elements that can arise as products of two different elements is disjoint from the set of squares. In the Cayley table the diagonal elements can never occur in the rest of the table. But each line of the table has all the group elements in it so there can be only one element in the diagonal, that is, the identity.So every element is of order two and it is well-known that the group is then a direct product of cyclic groups of order two. The identity is a signal that the elements being discriminated are equal. Another way of writing this group is as a set of bit-strings (vectors over the field Z_2) with the binary operation now being addition and the signal for equality being the zero string. We all know what that example was; here is another.Instead of a group, begin with the quaternion group and replace all the diagonal elements in the Cayley table with the identity (which will then be the signal for identity). I have now been told by Arleta that this system is of respectable antiquity, known since the beginning of the last century as "hyperbolic quaternions". Here there is again a bit-string version with strings of length three but life is a little more complicated. You could number the bit-strings off by treating them as numbers in the scale of two:

100,	010,	110,	001,	101,	110,	111
1	2	3	4	5	6	7

Then define ab = a + b + H(a, b) where H is called the complexion of the pair a, b and is defined by

$$H(1, 2) = H(2, 3) = H(3, 1) = H(u, u) = 0,$$

$$H(u, v) + H(v, u) = 7, \quad H(u + 7, v) = H(u, v).$$

The system is not a group but it is a loop. We call it Q*.

I will not labour here the importance of level change, when certain subsets, those closed under discrimination, become elements at the "next level" because Ted has dealt with it but I will say a word about the idea of "level" because both Ted and Keith have asked me just what it means. I think the following is right, though it differs a little from what I told Keith recently. A level of type r is a set G of r elements, called generators of the level, together with the members of the level which are the elements of the discriminate closure of G and all those dcss which are discriminate closures of subsets of

G including G itself.

Now to put some detail on these ideas. This falls into two parts: the basic algebra, and the mechanism of level change. I begin by asking how the process determines whether two elements at the same level are the same or not. If a, b are in fact the same then no new element can arise in the discrimination. This is the first use of the "elements in play" idea. Hence either aa = a or else aa is not an element. The first has been suggested by Lou Kauffman but it will not do because the next step would have to be to check whether the element on the right-hand side was a or not: that is, exactly the same problem again. Hence aa is a non-element, which must be unique so that its appearance removes the need for further discrimination. I call this non-element z (to recall Parker-Rhodes's zero) and I shall prefer the word signal for non-element.

In order that z may signal equality I shall require that ab $\neq$ z whenever a is not the same as b, and I will call a structure with such a signal a discrimination system. When a, b are different, ab is then a third element c. That is, c is the discrimination of a putative new element b against one, a, already in play. This differs from the discrimination ba of a new element a against a b already in play so ba = d and d $\neq$ c. However, for obvious physical reasons c and d are closely related so I call this relation duality, say that c, d are duals and write d = c*, c = d*. This notion of duality is the critical new concept which, together with that of being in play allows me to find these new foundations.

Two elements may be (i) the same, (ii) duals,

(iii) essentially different. I have defined duality here for c and d but the structure of the system will turn out to be such that every element can be expressed (in many ways) as a discrimination, so that a* and b* exist as well. To determine whether two elements are duals or not will require a signal, just as equality did and I use y for this signal, so that uu* = y. The advent of this second signal requires a change in the definition of a dcs. Recall that a dcs is defined as a set of elements, so that signals must be excluded. This is achieved in the original definition by requiring that the discrimination of any two different elements of the set is in the set. Now we should require that the discrimination of any two essentially different elements should lie in the set.

Here are some more uses of "in play" and duality to show how much can be squeezed out of them. Suppose that ab = c. Consider ab*; this cannot be c since that is ab but the only elements in play are a, b, c, and their duals so it must be the case that ab* = c*. By a similar argument a*b = c* so in all ab* = a*b = (ab)* and a*b* = c = ab. Now consider bc; here again the only elements in play are a, b, c and their duals and so either bc = a or bc = a*. But bc = a* is ruled out because if bc = a* were to follow from ab = c, then ca* = b* would follow from bc = a* and applying this again a*b* = c* which contradicts a*b* = ab. Hence bc = a and ca = b. The triadic relation Rabc which expresses ab = c has the property that if R holds for any three elements it also holds for any cyclic permutation of them. Notice here a tension between the logical account and the historical one given by Ted. In his account there is also a triadic relation R but the symmetry of it is for any permutation not only the cyclic ones. These two will be reconciled below. Here is a table of these results:

	a	b	c	c*	b*	a*
a	z	c	b*			y
b	c*	z	a		y	
c	b	a*	z	y		
c*			y			
b*		y				
a*	y					

The rest of the table has not been filled in because it follows at once from uv* = u*v = (uv)*. It is clear that this is the structure called Q* at the beginning of the paper. It is a level of type 2, the set G being [a, b] and the members being (i) a, b, c = ab, a*, b*, c*, and (ii) the dcss [a, a*], [b, b*] and [ALL], the last one being the dcs with all the six elements in (i).

The table holds for any two elements a, b. If there were more generators than two and so a larger table, then any two generators would generate a sub-system with the same table, But note the subjunctive, for in this stripped-down version there is no direct step to more generators. Such a larger system can arise only at higher levels. The reason for this is of some importance. Suppose Q* has been completed and a putative new element u comes into play. The process cannot determine whether u is one of the elements of Q* by trying it against each element of Q* in turn because in each successive discrimination it would first be necessary to determine whether the comparison element was one which had already been used and so an infinite regress arises.There are only two ways ahead for the process; either u is simply incorporated as a new element independently of the existing Q* or else u must in some way be discriminated against Q* as a whole. I shall investigate this second course, which leads to level change, leaving the other one for the future. To be a little more general suppose the dcs to be T (not necessarily Q*) so that the triadic relation will have the form RTuv, where v is the result of the discrimination. To find the form that R must have consider first the case when u is in fact a member of T. Then no new element can come into play so that either RTuu or else a new signal must be introduced. In this case there is no need to reject the first possibility since the process has already incorporated discrimination between individual elements and so no infinite regress is involved. We can again put R in the form of an operation and use T, without fear of ambiguity to denote both the dcs and its operator. If u is in T, then Tu = u and correspondingly if u is not in T then Tu = w where w is different from u.

The physical importance of dcss imposes a further restriction on T for the results of discriminating the elements of a dcs against T must again be a dcs. If u, v are discriminated against T to give w, x then when uv is discriminated against T the result must be wx; that is, T(uv) = Tu.Tv. Moreover if $u \neq v$ then $Tu \neq Tv$ for it not, then T(uv) would be the signal z, contrary to the definition of Tu as an element. That is, T is an automorphism of the discrimination system and therefore preserves duality, Tu* = (Tu)*. It is straightforward to prove that for any dcs T there is an automorphism characterising it. The increase in complexity required by level change is provided by the discrimination between such automorphisms. This discrimination must be consistent with that between the original elements. To see how this happens note that the two automorphisms T, U will have to be the same if and only if Tp = Up for all the elements p in play.The discrimination between T, U, which we write as TU to be consistent with the original discrimination, must be such that (TU)p = z if and only if T = U , that is, TpUp = z for all p in play. So the discrimination between T and U must be that familiar in mathematics as the induced operation: (TU)p = TpUp for all p in play. When the process is discriminating between dcss in this way it has moved up a level. With this definition of discrimination between automorphisms the new level is again a discrimination system.

I must emphasise that the system as now set out is both non-commutative and non-associative so it is a far cry from the original Parker-Rhodes matrix algebra. But there is help at hand in navigating these strange waters. This help is not part of the process but is closely connected with it. It is a deliberate simplification of the structure which yet has enough detail left to give useful guidance on the way the algebra is going, rather as homology groups do in algebraic topology. Consider the two dcss [a, a*], [b, b*]. Define a new discrimination operation directly between the dcss, not by the extra structure introduced above by the general rule:

[u, v, w, ...][u', v', w', ...] =

[uu', uv', .. vu', vv', ...].

Then in the case of [a,a*], [b, b*], this becomes:

[a, a*][b, b*] = [ab, ab*, a*b, a*b*] = [c, c*].

Moreover [a, a*][a, a*] = [z, y] and [z, y][a, a*] = [a, a*]. So the three elements [a, a*], [b, b*], [c, c*] are those of the quadratic group S and [z, y] is the identity element. I call this simpler hierarchy the skeleton of the actual one. You can imagine that in my work with Ted it is very useful because he is happier with the quadratic group than with Q*. The skeleton has a more important feature. It combines simplicity with being inside, as it were, the actual hierarchy. This throws light on my calculation of $1/\alpha$. In [1] the calculation in the CH, or as I now call it the skeleton, gave $1/\alpha = 137{\cdot}0353...$ from two "corrections" to Frederick Parker-Rhodes's value of 137. One, inspired by David McGoveran, corrects Frederick's original idea. The second follows from the process view. Then a "third correction" derives 137·036012.... This is simply the result of going from the skeleton back to the actual hierarchy. In [2] I corrected some arithmetical errors to derive 137·036011. I have nothing to modify in the details of these calculations. NOTE ADDED IN JANUARY 2007. Following ANPA 28 Ron Weeden generously offered to check all my calculations and with his help we have a definitive value of 137·036011393... . This is within one part in 10^7 of the recent very accurate experimental determination by G Gabrielse, from the Lande g-factor for the electron, of 137·035999710... .

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PHYSICAL UNDERSTANDING COMES FIRST

TED BASTIN
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There is a physical picture to be understood that underlies Kilmister's 'process' even though it is an unfamiliar picture

I. THE HIERARCHY IS NOT AN UNINTERPRETED CALCULUS

What is the case for formalism, positivism, the uninterpreted calculus. We have some numbers! Why can we not use their similarity to numbers from physics to say we have explained those empirical numbers because we should then have given the calculus a successful interpretation? After a long period working with what was called 'programme universe' Noyes came to the conclusion that there was nothing in the calculus to show why this interpretation ought to be accepted. Noyes'view was quite sweeping: it amounted to saying that pure numbers could never be the subject of physical explanation. To put it another way, the calculus can never tell you what numbers from the world it should be compared with. We agree with Noyes' rejection if the view of measurement that he assumes is the only possible one.

A lot of thinking of a positivistic sort has been done that is inspired by the idea of an uninterpreted calculus. This idea has been prominent in the philosophy of science. According to it the mathematics employed in physical theory had its own intrinsic meaning and, as the final outcome of its development, produced conclusions (that could be numerical) that had in some way a correspondence with the physical world. The main body of the calculus did not initially have to have any such correspondence or physical meaning. However the metaphor of a zip fastener has been used to suggest how meaning can be infused into the physics from that existing in the calculus. The correspondence is supposed to establish the bottom of the zip, and then you just zip it up. To assume that anything in the calculus, let alone the bottom rung (to mix the metaphor) could have an evident physical meaning will sound warning bells, but then the whole approach is from my position as wrong as it could be.

Now Parker-Rhodes had come up with a binary algebraic scheme which gave some physically important numbers and it seemed plausible to treat that as an uninterpreted calculus. Then, so the argument goes, we need only study the ideas that this mathematics generates: that enquiry alone could be relied on to throw up all the 'meaning' of the physics. In our different view we expect our physical understanding to generate the mathematics and continual reference back to that is essential as we build mathematics and continual reference back to that is essential as we build the mathematical picture. By contrast some people worry away at the algebraic details of the hierarchy, thinking that they need to be got straight before they are contaminated with interpretation; and this way of working seems influenced by the uninterpreted calculus view.

I shall argue at some length that this is not the right approach but that reference back to the physical reasons why any particular mathematical scheme was adopted has to be undertaken at every stage to illuminate and guide the thinking. To start the ball rolling I mention one recent discussion with Mike Manthey to illustrate how approaches differ..

He asked how to fix group representations of second level entities. I recalled that for us new entities arose from the closure of some subsets that were associated somehow, and that one had to choose an algebra to secure the closure. Matrix transforms seemed the right way to introduce this idea. He said that it is desirable that the elements of the transform be the elements that we began with, and that since the algebra based on the quadratic group that we are using does not do that we should be better off using his Clifford group which would secure that advantage. My adherence to the quadratic group was because can say that it is contains the absolutely fundamental symmetry of the three elements with the minimum of algebraic frills, so that one can avoid giving physical significance to the frills.

We have avoided writing out the particular dcss that we use because the choice of generator pairs for any dcs is arbitrary and therefore any conclusions that you draw from the constitution of a particular dcs will be false

Mike again: at the risk of being tiresome, I again ask which of the three level-3 "dcss's" below are the ones your construction generates:

1. This is the one I got originally from Pierre, but which I no longer believe is a candidate:

{a,}	*{b},*	*{c},*
{a,b,ab},	*{a,c,bc},*	*{b,c,bc},*
{a,b,c,ab,ac,bc,abc}		*Total: 7*

2. This one seems to be the one you find:

{a,b,ab},	*{a,c,bc},*	*{b,c,bc},*
{a,bc,abc},	*{b,ac,abc},*	*{c,ab,abc}*
{a,b,c,ab,ac,bc,abc}		*Total: 7*

3. This one I'd very much like your reading on:

{a,bc,abc},	{b,ac,abc},	{c,ab,abc}
{a,b,bc,ac},	{a,c,ab,bc},	{b,c,ab,ac}
{a,b,c,ab,ac,bc,abc}	Total: 7	

I have talked about the uninterpreted calculus because it seems to go along with some attempts that are made to follow out the mathematical possibilities that the hierarchy algebra allows with out putting the physical understanding first, and that is not the attitude that I am advocating.

II. PARTICLES AND THE VOID

The view I have been putting is assumed by Kilmister in his *process* development which gives his accurate value for the fine-structure constant. I shall argue that physical understanding is necessary at every stage in this development..

Before there is any geometry there are elements that are presented sequentially in the form of elementary interactions. If there is to be a physical world some structure must arise out of this sequential presentation, and we call this the *process*. Sets of elements are constructed through the interactions so as to constitute manageable and separable units. Kilmister's process picture is restricted to what can come out of the study of sequentiality. Crudely speaking it is what you can get without a spatial background. It is essential to realise that the whole of the identification with particle level interactions can be reached this way, but we should be wrong to infer that we had particles as they are usually understood in a spatial context. Nevertheless these elements and units generate the particles of physics and we suppose that the process is essentially what we discover about particles in high energy interactions.

I start my gloss on the ideas behind Clive's process treatment. At any given stage in the process there are entities that are *in play*, and a new stage appears when an entity from outside this 'in play' set interacts with one of them. To make a binary system possible there must be a rule to express this interaction. We call this *discrimination* because it is simple to see it as a decision whether or not the entity from outside is yet in play. If it is not, a new entity comes into being and is added to them. Of course this picture contains the seeds of the idea that the 'outside' means *spatially* outside and that therefore the entities in play are in a restricted location. But we are not nearly there yet. It is true that one may imagine that some particles are more likely than others to present themselves for discrimination, and this idea might be used to introduce the next idea of some being spatially nearer than others, and to this extent introducing a spatial relationship. We should proceed by getting as far as we can without importing any such ideas.

We seek the physical ideas that underlie the algebraic representation of discrimination and all that follows from that in generating numbers of entities in a sequence of levels. Queries about the details of the algebra have to be referred ultimately back to these ideas. Thus the algebra is not self-supporting in the sense that it will of itself suggest to the mathematician what to do and what to devise at each decision point. It never was an uninterpreted calculus.

One may raise doubts whether it is proper to speak of physical understanding in view of the very bare ideas that have so far appeared. We speak of particles but what is physical about them? We have to answer this reasonable query by going back to the beginnings of Kilmister's development where there are at each stage in the process a number of entities that are 'in play' –having already been created. So there are things that are available and by implication things that are not, or not yet, available. We speak of these latter as the *background.* We argue that this difference is already enough to justify our talking of the entities as particles because we shall then have provided them with their most essential attribute, even though they have as yet no other.

We can look in a slightly different way at the understanding that constitutes a physical picture. It has to have the quality that is familiar from classical physics that events can be analysed into sequences of mechanical contacts so that an answer can always be given to the question "what caused this?". "Mechanical" seems to beg the question because we may think it commits us too much to classical ideas. We can try to avoid this by saying we have to be able to *imagine* the connexion of events in the sequence. This way of speaking has also the advantage that it permits the appeal to randomization since we can imagine repeated trials without involving ourselves in the controversy on randomization familiar from the quantum theory. It may seem to let a subjective element in through the back door. In truth it seems impossible to avoid placing some restrictions on what we shall accept as explanatory in specifying the connections of events, and to say they must be imaginable is near to saying that they must be the sort of thing you might expect particles to do, given of course that changes can be reduced back to all-or-none: discrete in fact.

III. SEQUENTIALITY AND EXTENSION

This section is the centre of my story: the lack of a sequential basis for the quadratic group

We are so far deliberately restricting ourselves to sequentiality, leaving non-sequentiality and so space or extension still to find a place. It is vital that the couplings that give the scale numbers come from pure sequentiality. It is relevant that they are pure numbers. In its very nature the process development deals entirely in sequentiality. We could say, though less precisely, that it all happens in time. There is no corresponding introduction of space or extension. Moreover the structure of discrete levels that appears relates only to the time-like aspect of things and there is therefore still a complementary aspect that has yet to be brought into the open. It might seem at first sight that some basic error has been committed since most discussion of the hierarchy has been on the expectation of a time/space interpretation. For us however it is important that the sequentiality aspect appear first, so that the introduction of anything that can represent spatial extension breaking into the process can be made quite deliberately and separately.

Several mathematical devices that were introduced to carry through the 'process' method, but which were difficult to justify from that alone, have been found to become redundant by Clive. He achieves that by starting from the place of the quadratic group as it appears in what he calls the 'skeleton'. That name refers to a simpler structure that is embedded in the whole theory but that gives, as an approximation, the bare values of the level scheme as Parker-Rhodes had it. The skeleton really puts things in terms of matrices.

I find that in more ways than one the skeleton is important in its own right and cannot be written off as just a detail of presentation. The original motivation for introducing levels was a physical understanding that is closely connected to the matrices and therefore to the skeleton. I shall deal with that later. In a recent considerable clarification of the process account Clive has been able to eliminate several pieces of mathematical argument that were physically difficult to justify. To do this he had to appeal to an already existing scheme of levels and this conflicted with the attempt to get a step-by-step logical development from process. It was recursive if not circular. My view is that a new appeal to something outside the purely sequential development is inevitable. We do indeed have to go back to look at the quadratic group, and to the very vision that brought it into action. This was the unique symmetry of the a, b, c. The triadic relation Rabc has the property Ruvw $\longrightarrow$ Rvwu $\longrightarrow$ Rwuv: namely cyclic permutation invariance. We said that since you cannot impose any order, we have departed from time and we must therefore be dealing with something new. Call it *space*.

Thus I think we have to go back to the original motivation of the hierarchy. We found the origin of dimensionality and of the 3-structure that is attributed to space in the quadratic group insofar as it presented the special symmetry of the three elements. Their order cannot be defined sequentially, so that the instructions for their employment have to be imposed. In fact they .have to be injected from outside the sequential process development. This is where spatiality appears. This unorderability characterizes space and is the origin of the space/time dichotomy. It was the origin of the hierarchy algebra, yet it has to be artificially imposed on the strictly sequential story. I do not say that other ways of getting over this logical break cannot be found: only that in this way we can advance to Kilmister's all-important numerical corrections

IV. BACK TO PARKER-RHODES

In Parker-Rhodes' original construction the bit-strings of length 2 generate 3 dcs. The automorphisms (non-singular 2x2 matrices) characterizing them are unique. These three matrices, as bit-strings of length 4, generate 7 dcss, and the 4x4 matrices characterizing them are not unique. The 127 dcss would require for their characterization $2^{127} - 1 \approx 10^{38.2}$ 256x256 matrices. Such sets cannot be independent; Parker-Rhodes' choice becomes impossible and the construction halts. The cumulative numbers of dcss when the hierarchy changes level are then 3, 10, 137 and 10^{38}. These four scale-constants are direct consequences of the Parker-Rhodes construction. The evident relation of the last two to the reciprocals of the coupling constants of the electromagnetic field and the gravitational field suggest that the first two are similarly related to nuclear forces (and that there is no "fifth field" in nature). Here however we confine ourselves to he value $\alpha = 1/137$ since only in this case is the physical constant known very accurately (to seven significant figures).

If the construction were constrained in some way to remain at the first three levels then 1/137 would be a probability, viz. the probability that a random pair of dcss were the same. We call 1/137 the 'bare value' of α. Such a constraint is not possible in the process construction. Instead we calculate the probability that the construction remains at the first three levels. For this purpose we assume a generalized ergodic hypothesis –that when there are several possibilities in the process each will occur with equal probability. At the first level a dcs may be one of the three at that level or none of them (because it is at a higher level) so each has probability ¼. Similarly at the next levels there are probabilities1/8 and 1/128, so long as Parker-Rhodes' choice of a normal set of matrices is made. The probability of being at none of the levels is therefore e where $1/e = 4.8.128 = 4096$, and so the probability that the construction condition happens to be fulfilled is $1 - e$. The probability 1/137 becomes $(1 - e)/137$ giving an improved value $1/\alpha = 137.033$.

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We now turn to levels. Do levels arise necessarily in

V. LEVELS, THE PROCESS.

It looks as though there is more to understand about them and that we need the skeleton to put our finger on that because it would not be evident from process alone. Levels provide the basic connexion of the process with physics via the particles. They are measures of the interaction strengths provided by particles. Parker-Rhodes set up his binary algebra deliberately to formalize the levels, and the levels were thought of as the way to increase the possibilities of description. Hence the most vital part of Kilmister's process story is to see precisely how it forces the appearance of the levels rather than our having to build them in deliberately.

All we can say about levels from process alone

The *discriminate closure* of any set of elements is the smallest dcs containing them.

A set of *generators* of a dcs is any set for which it is the discriminate closure.

A level of type r, L_r say, is a set G of r elements, called *generators* of the level, together with the *members of the level* which are

(a) the elements of the discriminate closure of G,

(b) all those dcss which are discriminate closures of subsets of G (including G itself).

The multiplicity of L_r is the number of different dcss which are members of L_r.

When all members of L_r have come into play the process has filled L_r.

If further elements come into play the level is inadequate to describe the further ranges of experiment and an increase in structure is required. Such an increase is provided by:-

The superior level to L_r is that defined by a set of generators, one for each of the dcss of L_r

For this increase of structure the process *goes up a level* to the superior level to L_r. The elements of this superior level are the dcss of the original level (together with their dcss of course).

In t\he process development progressively larger sets of elements are generated and these are to be used descriptively. A great deal depends on the sudden change that we call a change of level. This happens when a new set of generators is required. We now ask how the sudden increase in the size of elements corresponds with physics. All the physics happens at one level change at which electromagnetism comes into play. In the older picture we used to think of generating this new level by mapping two simpler ones on each other, and this idea could be given a meaning through the change from mechanics to electromagnetism. It seems that we cannot do without this kind of construction though it seems to demand insights from outside the strict process.

The principle of restarting with generators was foreshadowed by the prevalent idea that new descriptive power was to be obtained by mapping the completed structure at the current level onto itself, and there was thought otbe good theoretical reasons for doing that.

The 3-structure represented by the quadratic group that is left over unexplained by the process account is the origin of the vectors in their level jump. We do not explain it from space-time vectors, nor advance to a spatially understood electromagnetism at the level jump, but we are content to see how those ways of talking appeared. In truth we have to go back long before Parker Rhodes because we were aware that the quadratic structure had to be replicated or 'squared' to increase descriptive power, and that the step into electromagnetism in abstraction from a *metrical* dimensional structure would result.

In former accounts levels had their origin in the need to gain descriptive power by associating one level with a replica of it, and it seems that such an idea cannot be dispensed with. It was tied up with the matrix representation being a product of two vectors, and the idea was that one could see the essentials of the electromagnetic field arising from the combination of two accelerations of a test particle. All this talk may seem verbose, repetitive and very old hat, however somehow we have to answer the question how the process theory issues in a set of dynamical concepts, and all this is some sort of answer. We should see it in the context of the primary appearance of space from the quadratic group that has been discussed.

VI. DYNAMICAL CONCEPTS OF PARTICLES FROM PROCESS

From now on I shall be describing the growth of dynamical concepts from the space-like step that was left out of the sequential or process development.

We start physics from dimensionless numbers. Discrete particles will appear before there is a space to put them in. We argue that the jump from a combinatoric view of particles to ordinary dynamical concepts requires the adoption of the half-integral spin of the leptons as the central step. The disintegration that produces two photons of opposite spin resulted (Einstein, Rosen Podolski: Aspect) in the amazing discovery that whatever spin is detected in either determines what must be discovered as the spin of the other. This discovery is now usually said to show that the relationship between the participants in the reaction is *non-local.* It transcends the limitations of spatial connectivity. I propose to invert the argument and say that far from room having to be found for the non-local connectivity, it is that relationship that we start from. It is the first step, paradoxically, to building a space –which will of course be a relational space. Spatially, the separated particles are one event, but it is the recognition that *two* are required that opens the way to creating a multiplicity of events with spatial differentiation.

Clive thinks that the primitive space part of this idea may be right but that the connection with the half factor in spin is obscure. Perhaps the trouble is that the half-integral spin language already entails too much dynamics, and what I need is a combinatorial concept. The Pauli matrices are all-too-much that kind of thing. In the hierarchy algebra we have matrices creating a fourfold array and it has always been obvious that this is saying the same kind of thing as the Pauli matrices if we could see the two in the right way. At this point in the argument I am using the four-fold structure in the hierarchy level as deductive and as serving to introduce the Pauli matrices for the first time. The difficulty is in how to handle the spatial language that comes with the Pauli matrices as usually presented, and it is this that my non-locality idea should be able to see us through because it provides a way of placing the whole thing *progressively* in space step by step. First I look to see how the half-integral spin for leptons arises out of the Pauli matrices. There is all that stuff about having to do two rotations to get back to one change in the spin wave-function. We ignore that for the moment since it is probably only a way of putting into geometrical language something that has to be there anyway.

We know that the different interaction strengths appear in the most basic consideration of particles, but we need to look into the question of what physical significance to give to the constructive stages before the e.m. level is all there. It hasn't physical reality in the same way. We associate it with the quarks and consider it a success that the quarks do not have spatial existence.

V. CHARGE, SPIN, MASS – TOWARDS THE CLASSICAL CONCEPTS. PRIMITIVE COMBINATORICS -QUARKS

Charge. The combinatorial structure that emerges from high-energy experimentation is that of quarks. How do we identify these in the hierarchy algebra? The quarks are in the first place tripartite. However some insight may be gained, as is often the case, by going back to their early appearance. One needed two entities of opposite charge to provide three possibilities, and these became the quarks. You could have two together to give the neutral neutron. You could have two like that and then add a third to give the charged proton if the third addition was the positive one. (The possibility that you could get a negative particle chargewise symmetrical with the proton and therefore presumably the electron does not seem to have been used, presumably because people thought that the fact that they were dealing with heavy nuclei precluded that. The possibility of bringing in the anti-proton was not available at that date.) The vital thing that emerges from this bit of history is that the appearance of a concept identifiable with charge was needed to get the quark combinatorics off the ground. Naturally we look for an equivalence between these combinatorics and our first level.

The descriptors that characterize the quarks are normally expressed in classical language which entails spatial distribution. Since we have not introduced spatial separation we have to rely on the structure of the closed subsets at level 2. The only properties of these subsets that may be used in identification are the numbers of the four places in each that are ever occupied in the operations that go on. The cases are occupation 1, 2 and 3. The case of 4 leaves no room for operations on the occupancy. Hence there are three quarks. This multiplicity or 'dimensionality' by classical analogy leads us to associate the numbers with charge, spin, and magnetic moment respectively, and this identification merely brings in the inherent complexity of those concepts. Likewise we look forward to bringing in the detailed language now available for designating the quarks (colour and so on) but it is important to show that the bare form can be discussed separately as the basis for the more complex argument.

It may sound sensationally ambitious to find these germs of charge and spin in our combinatoric scheme but we have no alternative since we cannot take them as common-sense ideas that emerge from classical continuum physics. We have to have some picture rather than none. The quarks arise from juggling the experimental knowledge of combinations of particles. They are precursors of the particles, but it is going in the backwards direction to think of them as things of some material sort banging about inside a nucleus. We do not need to ask how many quarks there are REALLY? as though there were some way of getting behind the combinatoric specification. Current writers seem to find it unhelpful of the quarks to be so shy of exhibiting themselves in experiment, and even try to find explanations for their being that way. If there could be such an explanation then that would show our approach was wrong. The experimental circumstances that quarks would need to manifest themselves in the usual ways do not exist at our present stage of construction.

Spin

The other germ is spin. A careful scrutiny of the history has led us to reject the common view that spin cannot be discussed experimentally but has to be regarded as an abstract algebraic construct. On the contrary it has to be experimentally immediate. The two-valued character of spin in experiments of the kind first discussed by Einstein, Rosen and Podolsky is absolutely repeatable and comprehensible as an experimental fact that stands clear of any theoretical discussion whatever including the origins of quantum mechanics. When two fermions have a common origin as though they originate in the disintegration of a boson then we find that if one is found to have one value of spin then the other always is found to have the opposite one. This dual appearance is absolute and totally reliable experimentally: the only knowledge the viewer has to have is how to set up the experiments, and therefore this splitting has to be the first thing in our description of the world. It does not require any theory of any sort.

We combine two two-vectors each of one value of *combinatoria*l spin to get the 2x2 matrix that provides a representation of angular momentum, and therefore introduces the angular momentum concept to "spin" which it otherwise lacks so that it now becomes geometrical spin. Of course one still has the factor 137 to transform the spin angular momentum into orbital angular momentum and that can be handled separately.

It would seem to follow from all this that one has to go to the leptons before the concept of spin really applies at all, and that the bosons including the photons do not really count. Is it that in asserting that a boson has spin 1 one is really still looking to the lepton to provide a measure? Thus is it that interactions with matter by bosons always involve spin-half particles because they really are charged whereas the charge of bosons is sort of conventional?

To sum up, I look to the completely combinatorial picture and am confronted by the appearance of non-locality that we have to assume because of the way that two particles (or two photons, considering polarization) can be known to have opposite spins so that despite spatial considerations they must be taken as a combined system. I guess this is the proper starting point for us and that the combination of two particles in this way leads us to assign them the half-integralness that is taken over into the spin matrices.

If we do that then we have to decide how the spatial language gets a foot in the door. To answer that I speculate that the appearance of non-locality should be, paradoxically, the first step. That is to say that when we first confront the non-locality in the opposite spin or polarization situation that is when we start to build up distance or length in a world in which previously it has had no existence. In my musings we have been here before. We got the first spatial step away from sequentiality or temporal structure by going back to similarity of position and to the existence of the three-term structure long ago. I have been proposing that we need a conceptual fundamental innovation. The part played by the fractional charge is important. When we are dealing with protons and neutrons we have only the discrete value. The usual theorists naturally give this the full meaning as one value out of a potential (continuous or discontinuous) range. However for us it is just there to provide a convenient way to represent combination of two things in the right way. It may indeed be the first appearance of additive quantities, but that is something that has to be constructed very deliberately and no way can it be imported simply because *classical* charge is something that we are familiar with and we know that it can be added up.

. We have the kick-off point for charge from the quarks: how does spin fit in? The new bit of thinking is that it arises from the need to work at two levels with both existing at the same time. It is this level-complementarity that is the vital characteristic of our method and we have to see what change in our concepts it produces. It produces spin. Somehow there is a doubling as we go from the 4-vector level up a stage. In the past we played about with the idea that the factor 2 appeared because there were changes at the 4-level that required two successive operations to effect, but that could be provided at the next more complex level by one operation. We hoped that this would explain why any measurements would be changed by this factor, and hence the half-integral spin. This argument ran into difficulties. It is actually trying to do too much because it introduces a metric for measurement, even though that step has to be taken in due course. If we think about charge again we got our simple result by refusing the metrical interpretation and concentrating on the combinatorics of having a quantity (charge) which had two existence and non-existence states that together gave the required multiplicity. Here we use the same dodge with one vital change. Our level change has committed us to the richer electromagnetic language, even though we do not know how to use that yet.

This is the central bit, and is the most tricky. We have to imagine something very different from our usual ideas. The process has to contain not only increases (or decreases) in the occupancy of each level but, as well, the steps that take it continually between the levels. To the universe (or should I say sub specie aeternitatis or in the sight of God?) they all look the same. We might say that this requirement entails a switching backwards and forwards between levels except that there are no levels. The old puzzle emerges "What do we mean when we say that we are working at some one level?" We have to have recourse to a statistical principle here and say that for some reason that we forbear to go into further there is an a cessation of changes long enough to give the required degree of stability Anyway we have these elements that specify the level swap, but they only exist at the second level. The result is that when one of these elements appears it causes what we see as the two levels being successively in play as a single action, and therefore the one-and-two particle is born and that is what we call spin.

Evidently the quarks must generate two particles and this makes the difference leading into the electromagnetic language. They will have to appear with a physical difference labelling and this labelling has to use the new magnetic field or moment.

Of course the long path to our giving any particle like the electron an angular momentum like a cricket ball that can be added onto other angular momenta remains to be travelled –as also does the deriving of the classical notion of any angular momentum in terms of classical electromagnetism. We have to do it all of course, but the essential thing to remember is that all that is not already there really, as current theory leads us tacitly to assume.

We now have the two stories of the complementary attributes of charge and spin for the case of the stable particles. There are two long paths to tread to go further, but this is a convenient resting place.

Mass

Attributes of particles, which are called quantum numbers in low energy quantum physics, have a new status when we deal with particles at high energies. We shall call them *descriptors*. Each represents the possession, or the absence of a property. Ideally each takes the values 0, 1, (There are fractional values for some descriptors, such as charge, but it is clearest to think of these as combinations of all-or-none descriptors.) The whole impulse of high energy physics is to characterize each descriptor by an algebraic structure and to play down its definition within classical theory, though it is always tacitly assumed that there will always be a natural progression from the one to the other.

There is one glaring exception to all this -*mass*. A very large body of experimental knowledge relates to the masses of the different particles, and particles differ in the amount of mass that each possesses even though particles of a given sort have the same mass. Particle mass is thus like the classical mass in that it can take a range of numerical values, though it is different in not being able to move continuously between these values. Additivity, however, has been introduced. This difference between mass and the rest of the descriptors is a very good reason why high energy physicists feel a compulsion to understand the nature of mass and how it is generated. However this understanding is hard to come by and has led into some strange enquiries. The hunt for the Higgs boson was inspired by attempts to provide this understanding by Higgs and others, and a brief examination of their ideas may help.

The basic form of the Higgs picture is a bootstrap process whereby particles build each other up by their being present together. In the nature of the case one of them has to start the ball rolling, and that is the particle postulated by Higgs. That particle is bare of all attributes except mass, simply because one has to start somewhere and one has to have a word for the attribute with which one starts. To make this process work there must be a finite number of particles for there to be a top one. Analogies are sometimes made with ions in solutions where one of them acquires an 'effective mass' through its interactions with the ions surrounding it which is many times greater than its free mass. Unfortunately analogies like this may lead readers on to impute a similar sort of reality to the interactions in the boson case, and some of these models are quite bizarre. The whole point of the bootstrap is that it be abstracted from classical-type mechanisms and models, because the mass is the first additive concept to be introduced. The physical models become crude when they already presume its existence.

Perhaps the unclearness of the status of the bootstrap argument has been partly responsible for the popular insistence on *finding* the Higgs boson. People probably think that they could feel more sure about the bootstrap mechanism if such a particle could be found experimentally. There would then seem to be a clear 'yes' or 'no' to be got from the experimental search that would take the weight off the need for conceptual innovation. However, in our view the origin of mass – a problem that Newton famously declined to tackle, is inescapably posed by particle physics and should be tackled independently of the question of the experimental reality of the Higgs boson.

There is thus need for a sort of basic axiomatization of what is involved in getting something out of the mere act of putting particles together. These logical foundations should follow from our process build-up of the hierarchical algebra. At level 2 there are electrons in an abstract form since we have the electromagnetic coupling. We may say the particle of opposite charge –the proton- is also implicit if we are to talk about charge at all. Now we must look at the substructures at the lower levels, using the assumption that discriminate closure will be the guiding principle since it has led to the essential particularity of the electron (or electron-proton). The variety of such closed sets has –we think- to relate to the quarks. The most overriding argument for this identification is the fact the quarks –while having to be physically real- do not interact experimentally with anything: at least not in a spatially specifiable way. This strange fact is something that current theorists have to provide special mechanisms to explain; but for us it follows from the fundamental step that we take in going across the level 2 to 3 boundary. We can construct interactions between particles but we have not the conceptual apparatus to represent them spatially.

Our basic fourfold algebra should afford a point of view on CPT invariance. This invariance is normally assumed to be a group-theoretic structure but understanding of it in group-theoretic terms is usually inhibited by the fact that the time mentioned is obviously nothing to do with time as time is usually understood in physics, but is something that can be reversed. One is not told what it is though.

We described how the only theories of the origin of mass now available postulate a 'bootstrap' sort of build-up. A related idea emerges from the hierarchy theory. It must come from the construction that leads to the fine-structure constant. Mass appears in that constant through the Planck constant, h. However in order to get the dimensions right we cannot imagine either m or h being ratcheted up step by step *separately* to the integral value 137, since to do that would be to neglect the fact that both e^2 and m are dimensional. The way out of the dilemma was suggested to us by Rowlands who considers a combination of m and the square of the charge, e , which undergoes this process. Using square brackets to denote dimensionality, we have

$$[e^2 /hc] = [e .ML/T] = [e/m \times c^{2]} .$$

Level Change in the Combinatorial Hierarchy v2

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I am going to talk about the draft paper by Clive and I that was published in last year's Proceedings. One thing Arleta always accuses me of doing, when I talk, is spending all my time talking about what I am going to talk about. Just to be consistent I am going to do the same thing today. So before I start I would like to remind you a little about Finite Groups and Groupoids. This is a typical Group Q=({1, i, j, k, -1, -i, -j, -k}, *), the Quaternions. It consists of a set of eight objects and an infix operator * and for each pair of objects a and b in the set we assign an object ab=a*b which must also be in the set. This gives rise to a "multiplication" (or "addition") (or "Cayley") table. (Demonstrates.) Groups in particular have some extra properties as well as just closure. Clive (see paper in this volume) was largely talking about Groupoids which have no extra properties. Groupoids with associativity such that a(bc)=(ab)c for any a,b,c are called Semigroups. Semigroups with a (two sided) unit 1 (such that a*1=1*a=a) are called Monoids. Monoids such that every element a has a (two sided) inverse a^{-1} such that $aa^{-1}=a^{-1}a=1$ are called Groups. Commutative Groups such that ab=ba for any a,b are called Abelian. Some people consistently write Abelian Groups additively and NonAbelian groups multiplicatively. I do not. Once again, every set we describe in this paper is FINITE.

Thus we have a hierarchy of systems of structured finite sets. Category Theorists talk about the Category of Sets, the Category of Groupoids the Categories of Semigroups, Monoids, Groups and Abelian Groups etc and map between Categories using Functors, in this case the Forgetful Functor which forgets Commutativity, Inverses, Units, Associativity and even Operators respectively as we go back down the hierarchy. One idea that I would like to put across is that the Combinatorial Hierarchy (NB different hierarchy!) can be described at any of these levels – or at least that there are different Combinatorial Hierarchies that exist in each of these levels. (Where there is any ambiguity we will capitalise the Combinatorial Hierarchy and not the other hierarchy.) This is even true at the bottom level of just sets and subsets, albeit perhaps with a little bit of extra help to start it off.

Just a comment about finite groups before we go on. There are in fact a very finite number of finite groups of any particular size. I think there are for instance only three groups with eight elements. Some very peculiar things happen as we allow the number of elements to increase. It is increasingly hard to calculate the number of groups of any particular size. Further all the known finite groups can be classified into about a dozen different categories... except for a small number of unclassifiable ones called the "Sporadic Groups". Most of these are quite small except for two known as the Baby Monster and the Monster, which are huge and enormous, respectively. Things are not as predictable as one might think.

There has been a lot of talk about generators and I need a little bit more clarity about generators to say the things that I am going to say. Remember that the (minimal set of) generators of a group are a (smallest) set that will generate the whole group by taking successive products with the group operator. It is worth reminding ourselves for instance what the generators of Q are. At first sight it might be thought that i, j, k and -1 might be appropriate, but, in fact, a moment's reflection shows that i and j are sufficient, as i*j=k and i*i=-1. It should be remembered that these symbols i, j and k and particularly -1, -i etc are just that, nothing but symbols, that are convenient for representing the group Q. And also that {i, j} is not the ONLY minimal set of generators, because for example j and k are just as good, but that i and -1 are NOT.

It is interesting to look at the case where we can take a subset of the elements in a group, and that subset happens to be a group itself; we call this subset a subgroup. Conversely if we take a subset of the generators of a group it is not difficult to see that such a subset will always generate a subgroup of the group. Now, in the traditional terminology of the Combinatorial Hierarchy, subgroups are referred to as DCS's or Discriminately Closed Subsets, for obvious reasons. However we would like to emphasise here that there are DCS's which are NOT generated by subsets of the generators. For instance take the DCS {1,-1} of Q which is generated by {1,-1} itself. Now note that there is NO (minimal) set of generators of Q of which {1,-1} is a subset. We will call subgroups which ARE generated by a subset of the generators of a group, Primitively Generated Subsets or PGS's. Note that when we say "set of generators" we always mean minimal set unless stated otherwise.

So there are DCS's which are NOT PGS's. We think historically that these PGS's are the entities that Frederick originally emphasised as being fundamental to the CH. However note the paucity of references to such subgroups generated by a subset of the generators in the mass of literature on the Combinatorial Hierarchy which has accumulated over the last forty five years. Clive managed to find three or four such references. And yet without this concept we do NOT get the remarkably accurate calculation of the fine structure constant that Clive has obtained recently and we do not get historical consistency with Frederick's original work. Note also that Clive's example of a non-PGS DCS given in our formal paper relied on nonassociativity. This example in Q does not. Further everything we have said here about PGS's and DCS's applies equally to the more general cases such as Groupoids, as well as Groups, so we talk about Primitively Generated Subgroupoids, Primitively Generated Submonoids, etc and refer to them all as PGS's.

So why is this important, and why should we want to generalise it in this rather extreme way? Thinking about the title of our original paper, what we have been looking for is the *essence* of the Frederick construction, and this is the point of generalising. We have some special cases, and we want to think what the essence of these special cases is, so we look for more general cases which exhibit this essence. One of my motivations for doing this was the difficulty that I and others had found in grasping what the essence of the construction was from anything in the literature, or from talking with the main protagonists. Whenever I tried to do this and thought that I was getting somewhere it seemed that the thing would jump away from me. I think that now it is time to tie it down. Another motivation was that although the general case seemed to me to be vague, the most important special cases had become crystal clear. I was terribly impressed with what Clive calls "What Ted and I Do Now", which is described in the first paper in this volume, and I wanted to see how *far* we could generalise this very specific and very beautiful construction. The latest calculation of alpha, and Rossiter and Heather's work on Categories (and the existence of earlier work by Mike Wright), provided further impetus.

I would like to talk for a few minutes about the physical meaning of the Combinatorial Hierarchy. In particular I could title this section "Physical Understanding Comes Last" in contradistinction to Ted's title (later in this volume) because I want to show that ***out of*** the mathematics comes a picture that remarkably resembles some Physics that we are already familiar with. There have traditionally been two ways of thinking about this aspect of the Hierarchy. In the first Clive and Ted have often talked about entities moving from a set of the unknown to a set of the known and they have looked at this in the past as a rather ontological kind of picture except that the question has been raised of who is the knower? On the other hand there is a sort of epistemological way of looking at things in which I look at the world through rose tinted spectacles, as a result of which the world looks pink. I look at the world through whatever processes are involved in the process of observation, so it seems equally natural that the way I see the world will reflect the (mathematics of) the structure of the process of observation itself. For instance it is then not too surprising if the number of particle pairs which are stable within a Compton radius is 137 etc. These have been our ways of connecting the Hierarchy with the physical world in the past.

What we end up with in the paper is a picture of the Hierarchy as a sequence of mappings between levels. (I am not thinking about stop rules here although Arleta points out to me that I should really bring those back in, but I'm trying to liberalise the whole thing and see how it might feel if we don't know about stop rules.) Between each adjacent pair of levels is a mapping which we call the Level Change Operator, L. We note that a certain noncommutative picture of levels can be changed in a very natural way to a commutative picture of levels by a process A known as Abelianisation at each level. In order for this to be true it must also be true that AL=LA. The Level Change Operator must commute with Abelianisation. (Draws ladder diagram.) This picture is very familiar. It looks like something from Algebraic Topology. Mike Manthey has pointed us in this direction before.

Clive comments here that in the material he had hoped to present (but didn't quite get to) the case he was dealing with had an Abelianisation operator (taking the skeleton), and he had showed explicitly that this commuted with the level change operator (see his paper with Ted in this volume). Such explicit commutativity had also been shown in my work with John Amson for hoops and loops.

In Algebraic Topology the Level Change Operators would be called Boundary Operators, and in particular it would also be true that LL=0 (ie the kernel of a level change operator must contain the image of the next one) always. Indeed Wheeler has pointed out that this concept, that the boundary of a boundary is zero, subsumes all of Physics. Certainly Hodge Theory, differential forms, exterior derivatives, Clifford algebras, quaternions, Gibbs vector calculus, Maxwell's equations etc, etc all live here. So the question is left hanging, what would it mean for the Level Change Operator to be nilpotent? Well certainly, if it were, we would have structures called chain complexes, which immediately leads to homology and all the Physics that Wheeler was referring to.

On the other hand we would have a rather more special class of operators than is described mathematically in the formal paper. These operators would have to obey all the rules in the paper *plus* they would have to be nilpotent. This gives rise to the question, what is it that is being mapped to zero by the operators? Well, one view in the paper is that we are mapping elements in sets. However as John Amson has pointed out what we are really trying to do is describe "information preserving hierarchies" and it seems likely to me that what we should REALLY be looking at are mappings between levels of *information* rather than entities in sets and that when we do we will see chain complex structures and the Physics that goes with them. This is what I am looking at currently.

Now back to the paper. We present a rather strong claim that PGS's and not DCS's are the fundamental entities of the Hierarchy. This has a built in implication that much of what has been written in the past is at least flawed in that it doesn't recognise this. Well, what I want to show now is that both structures can be said to be fundamental depending upon context. Firstly, remember that all PGS's are DCS's, just not the other way around. Secondly, consider the question, "What is the (a) Combinatorial Hierarchy?" Well, the Combinatorial Hierarchy is a set of levels (grades) and I would like to remind you of another question that Ted rather famously asked Clive, "What is a level?" to which Clive responded, "Well... a level is a Group (or Groupoid etc) together with a specific set of generators." So for instance the Quaternions Q alone, do not have enough structure to constitute a level, whereas the pair (Q, {i, j}) of the Group Q together with the generators {i, j} explicitly stated does.

I would like to change this definition and define a Level (now a mathematical structure (and capitalised) like a Group) as being, not a Group(oid etc.), or specific Categorical Object with a specific set of generators, but a specific set of generators together with an operator on those generators. At other times I might say that a Level is a specific set of generators plus a specific set of relations, which has a particular symmetry that I find satisfying; but a specific set of generators with an operator on those generators is the definition that helps with the things that we are currently thinking about. Added in proof: NOTE that we have NOT specified which specific Category we are working in – Groups, Monoids or Groupoids etc. It is very satisfactory that we do not have to do so at this stage, so that a Level is an object more like a Categorical Object than a Group! Note also that to avoid ambiguity we (in the paper) use the word "grade" for levels in the Hierarchy. This nicely echoes my comments on topology earlier.

It is certainly true that if we specify the generators in (any of) these ways the fundamental entities of this particular Combinatorial Hierarchy *are* the PGS's, because the DCS's *in general* don't have a certain property that is needed to build the Hierarchy. This property is that for any DCS we need to be able to specify an automorphism that's derived from the DCS (Amson's fixers) and we only believe that we can do this for the PGS's for certain. Remember – a fixer (or *phixer*) maps every element in the DCS to itself, and every element outside the DCS to something other than itself. However this restriction of the fundamental entities of the Hierarchy from DCS's to PGS's depends upon the explicit statement of the set of generators for each level. In other words the set of PGS's is context dependent. In general the PGS's are chosen (in this way) from the pool of DCS's, so it is also a perfectly reasonable statement to say that the *DCS's* are the fundamental entities of the Combinatorial Hierarchy. (To make the confusion worse in the *classical linear binary* Combinatorial Hierarchy all the DCS's are PGS's, which is one reason the distinction got forgotten about.) Understanding of the Hierarchy today entails understanding of the meaning of this situation.

One of the things that is emphasise in the formal paper over and over again in various different forms as we develop the story is the actual form of the level (grade!) change operator. Fundamentally the level change operator consists of two things

1. a mapping from the generator set (Level) at level i to the new generator set at level i+1. The new generator set consists of a set of independent mappings that phix subsets of the old (lower level) generator set. They map (fix) the generators (and therefore the elements) of the subset to themselves

and they rotate (strictly permute) the generators in (not of!) the complement of the subset, for any

x in DCS, A(x)=x (fixing)

x not in DCS, A(x)<>x (unfixing)

2. a mapping from the discrimination operator at level i to the induced operator at level i+1,

(A+B)(_)=A(_)+B(_).

We wish to find an automorphism A which phixes the DCS (fixes the DCS and unfixes its complement). For any automorphism A we have a subgroup DCS which is so phixed, what we want this to do is make this work the other way round. However we find that we can only guarantee to find an automorphism that fixes the DCS if the DCS is a PGS.

So the story has not changed significantly from the past. The only change is that we are replacing Group(oid)s by Levels and so restricting the DCS's to be PGS's on the grounds that we can always find an automorphism that phixes a PGS. For instance in the notation above if x is not in the DCS we (apparently) simply rotate the generators in the complement of the DCS around (strictly permute them) and use the structure rules to give an A(x) which is not equal to x for any x in the complement. Unfortunately, there are problems with this.

Firstly what happens if there is only one generator in the complement of the PGS? Then we cannot strictly permute the complement. We will argue elsewhere that this instance can never arise.

Secondly consider the case of two generators x and y in the complement. Then we can write we can write our tentative automorphism (phixer)

A(x)=y and A(y)=x

and generate the rest of the complement using the structure rules.

But then A(xy)=A(x)A(y)=yx

which is fine for noncommutative groups but for commutative groups gives A(xy)=xy which means that xy must be in the PGS itself and not in its complement, which in turn means that it must be possible to generate xy from the generators of the PGS alone, which is a contradiction. And so on for A(xyz) etc.

I had thought that this was all wrapped up and that we had a perfectly good theorem for generating phixers from PGS's. However, it seems that this is not true. In the last week before ANPA I have been throwing this back at Clive and asking him to have another go. In the last two minutes of this talk I am going to read out Clive's last letter to me. His letter is entitled "The Theorem". Clive handed this to me in person and I walked off with it thinking that it was done. Actually the first line of the letter is:

"I have been getting into a complex mess."

It goes on, "It's best if I explain why. It comes from there being two quite distinct ways of generalising Frederick. I have been guilty of jumping from one to the other without warning. To make it worse, they are inconsistent. In one way the theorem is true, indeed trivial, in the other I the other I judge it to be very hard."

Now, I think that Clive and I agree that there probably is a theorem, or at least that we can massage the context, such that there is a theorem. For instance Clive was fairly convinced that we couldn't do it for general groupoids at one point in time and that we would have to specialise to quasigroups, I believe it was. Hopefully we should be able to find a context in which the theorem is true, but it isn't clear how to do that at the moment. As the whole of this edifice hangs on the existence of this one theorem, I leave it as an open problem for the mathematicians.

The Internal Structure of the Level Change Theorem: I. Obstructions to Phixers

ANPA Working Paper 23/11/06 v2.3

Keith Bowden

A number of attempts have been made in the past to prove the following Theorem and special and general cases thereof:

Theorem 1 *Every DCS (or subgroup(oid)) of a group(oid) generates a (maybe non)unique automorphism which fixes the elements of that DCS and unfixes the elements of its complement. From now on we will only talk about groups but will attempt to do so in a general enough way to apply to groupoids and other structures.*

The earliest statement of this theorem that I have seen so far was by John Amson in the Appendix to PITCH [1]. It states the theorem for groups of the form C_2^N, although it does not give a proof but sketches some ideas in terms of Galois fields.

Such proofs usually will consist of two parts, firstly a construction of the automorphism and then a proof that it does what is required. It turns out that the easy part is the construction and the proof that it fixes the DCS. The hard part is the proof that it unfixes its complement (for an unspecified group).

Such a theorem was implicitly assumed by Griffor in [2] and others. More recently two more constructions were given in TEFC [3] without proof (or even comment on their relationship). This paper starts with a modified version of these two constructions, which stands a slightly better chance of being right in general, and discusses the problem in some detail.

We remind the reader that the purpose of this construction is to generate a set of ***independent*** *automorphisms to act as generators for the next level of the Combinatorial Hierarchy. Any other strategy will be equivalent. The importance of a proof of this theorem for Combinatorial Hierarchy research cannot be overemphasised.*

Language

We use a slightly different language than has been traditional

Definitions

A ***DCS (Discriminately Closed Subset)*** *of a Group is the underlying subset of a subgroup. By abuse of language we also use it to refer to the subgroup itself, the meaning being clear from the context. In a break with the past we always include the unit.*

A ***PGS (Primitively Generated Subset)*** *of a Group is the underlying subset of a subgroup generated by a subset of the generators of that Group. By abuse of language we also use it to refer to the subgroup itself, the meaning being clear from the context. In a break with the past we always include the unit.*

By (again abusing language) ***the generators of a Group, or a generator set, (or just generators)*** *we mean a minimal set of generators for that Group. It is assumed that these always have the same order (Arleta claims there is a theorem for this).*

A ***Level*** *is a Group together with a set of generators [CWK]. As there can be many sets of generators for a group we drop the usual meaning of level in the Combinatorial Hierarchy and instead use Grade for the latter purpose (and to spur us on to a connection with Topology). Everything we discuss in this paper will be assumed to take place in a fixed Grade, so that the meaning of level is always as given here.*

We may further abuse language by not differentiating between levels and generator sets, they are after all isomorphic. We also often assume that the reader can interpret meaning from context. Please report problems with such abuse.

When we talk about PGS's with no preamble (such as in the next paragraph below) we assume that we are speaking with respect to some fixed set of generators or Reference Level.

To ***fix*** *an element is to map it to itself thus* $\varphi(a)=a$. *To* ***unfix*** *an element is to map it to any other element thus* $\varphi(a)\neq a$. *To* ***phix*** *a subset (etc) is to fix its elements and to unfix those of its complement.*

If we talk about the "next" element of a set or group we just mean the next one in lexigraphical order (ie a new *one). This trick was taught to us by Clive.*

We usually take groups to be additive unless otherwise stated.

Theorem 2 *Every automorphism on a level (isomorphism between two levels) bijectively maps the generators of that level to a new set of generators, or simply rotates the generator set.*

Proof In what follows the reader is usually left to consider the effect of $\varphi(0)=0$. It is only necessary to prove that the image of the generator set under the automorphism is a new generator set. To see this we must look at the image of the process by which we construct the group from its generators and consider the action of the structure rules. For any sum of two generators a+b the automorphism constructs the image $\varphi(a)+\varphi(b)=\varphi(a+b)$, and so on to closure. Thus the image of the generator set under the automorphism is a new generator set.

Note that the new set of generators is not necessarily unique. It may be the rotation of another new set of generators. Similarly the identity automorphism just fixes the original set of generators and a set of equivalent automorphisms just rotate them. Thus levels define equivalence classes on automorphisms. We can say that the new set of generators is unique "up to (or mod) rotation". Note that such rotations themselves form a braided group.

Theorem 3 *Every bijection between two generator sets on the same group induces a unique automorphism on the underlying group.*

Proof The image of the generators under the automorphism is given by the bijection, this makes it unique. The image of the remaining elements is given by $\varphi(a+b)=\varphi(a)+\varphi(b)$ up to closure.

Unproven Theorem 4 *The following construction generates an automorphism that phixes (fixes and unfixes the complement of) any given PGS* P. *It is not obvious how to modify this for DCS's. (Impossible! K)*

We will talk about constructing an isomorphism from a level containing P *to a copy. The PGS contains its own generators. Map these to themselves in the copy. Now close the PGS under the group operation + and map each of its elements to themselves in the copy. That this fixes the PGS whilst preserving the structure rules is clear.*

Now choose the next unused nongenerator n *in the complement of the copy of the PGS and map the next unused generator* g *in the original to this. Close* $P \cup g$ *(forming a new PGS!) and use the structure rules to close its image under the automorphism, thus* n *becomes a new generator (under the potential automorphism) in the copy.*

Continue this process - choose the next unused nongenerator m *in the complement of the copy of the new PGS* Q *and map the next unused generator* f *in the original to this. Close* $Q \cup f$ *(forming a new PGS!) and use the structure rules to close its image under the automorphism, thus* m *becomes a new generator in the copy - until all the elements of the original group have been mapped bijectively to elements in the copy.*

That this bijective mapping is an automorphism is a consequence of using the structure rules to construct its image. That it fixes the PGS is a consequence of the choice of the identity mapping on the PGS. That it unfixes the complement of the PGS remains to be proven (or otherwise).

Discussion We know of three precedents to this construction. The first was discussed in private conversations with John Amson in which we suggested that the automorphism should fix the PGS (or DCS) and rotate the generators of its complement, the automorphism then being closed under the structure rules. (Note well that rather than the generators of the complement we should say the generators (of the group) *in* the complement (of the PGS). These generators do not generate the complement, indeed the complement is not necessarily a subgroup! We will, of course, continue to abuse language this way but this fact should always be kept in mind.)

Clive Kilmister later came up with a counterexample to this and the two alternative strategies given in TEFC. The first of these is to fix the PGS and then just to map the next element in the group to the next available element that it "can be" in the image and so on until all the elements are used up. Taking a very liberal interpretation of "can be" as "can be under the structure rules" this gives a similar algorithm to that given above. Fortunately the examples given all work whether we take this interpretation or not!

The second strategy is also similar to the ones given above in that it fixes the generators of the PGS and rotates the generators in the complement, except that it replaces *one* of the generators in the complement by the remaining generator in the original multiplied by any of the generators in the PGS, and rotates the rest *(ambiguous!)* The automorphism is then closed under the structure rules as usual.

This guarantees that the image of the generators under the automorphism is a different set of generators and not just the same set rotated. It is the fact that this strategy fixed the problem with Clive's counterexample that led us to believe that what is necessary is to map the set of generators in the complement not to a rotated copy of the set but to a new set of generators. Looking at the juggling with pseudopermutation matrices in the body of PITCH is also helpful.

Whether (in general) the set needs to be all new (as in our unproven theorem given here) or just partially new (as in Clive's strategy) is unknown as yet. As an introduction to further discussion we give two new and rather surprising theorems.

Theorem 5 *There exist Groups with DCS's which are never PGS's under* **any** *(minimal) set of generators.*

Proof It is only necessary to give an example and so we offer $C_2=(\{1,-1\}, \times)$ as a DCS of the Quaternions with the usual presentation $Q=(\{1, i, j, k, -i, -j, -k, -1\}, \times)$. It is easily seen that there is no level (eg, $\{i, j\}$) that can generate C_2 from a subset of its generators (because they all look like the closure of $(\{1,i\}, \times)=(\{1, i, -i, -1\}, \times)$).

Definitions

NGS nonprimitively generated subgroup – a DCS that is not a PGS

WNGS weakly nonprimitively generated subgroup – a DCS that is not a PGS for some level

SNGS strongly nonprimitively generated subgroup – a DCS that is not a PGS for any level

The Failure to Unfix

In order to look at obstructions to unfixing we will start by looking at automorphisms intended to unfix the complement of the trivial PGS, 0 (or 1 for Q, which we write multiplicatively by convention).

First let us look at rotating the generators in the complement of 1 in Q. We choose an automorphism generated by closing the bijection (i, j)→(j, i) where we have used parentheses to remind us that we are mapping ordered lists of generators. The automorphism generated is

a=(1, i, j, k, -i, -j, -k, -1)→(1, j, i, -k, -j, -i, k, -1)

which "wrongly" fixes -1. We note that the element that should be unfixed generates our special DCS C_2. So a phixes C_2 in Q. We may speculate:

Proposition Automorphisms that rotate generator sets fix SNGS's.
Corollary Automorphisms that fix SNGS's rotate generator sets.

There are 24 automorphisms of Q of which 4 are inner, one of which is the identity.

The other three inner automorphisms are all of the form ixi^{-1} *(j, k respectively)*. These phix the closures of ({1,i}, ×)=({1, i, -i, -1}, ×)) *(j, k, respectively)*. Indeed for nonabelian groups this is a consistent way of generating automorphisms phixing PGS's which are the closure of a single generator.

By construction none of these phix the trivial subgroup 1. So we have another surprising theorem.

Theorem 6 *(a) There exist groups for which there* **are no** *automorphisms that phix the trivial subgroup 1=({1}, ×). (b) Conversely there exist groups for which there* **are** *automorphisms that phix the trivial subgroup.*

Proof (a) We offer 1 in Q, by construction none of the 24 automorphisms work. (b) There is always a permutation that phixes 1 so we offer any group for which all the permutations are automorphisms, for instance C_4.

Again let us look at (essentially) Clive's original counterexample. He notes that the permutation matrix

1 0 0
0 0 1
0 1 0

which rotates (swaps) the generators 010 and 001 of C_2^3, and leaves 100 fixed should only fix the PGS generated by 100, but "wrongly" fixes 011 (think of the bit strings as being column vectors under premultiplication by the permutation matrix).

Now note that 011 generates the DCS {000, 011} which cannot be generated by a subset of the generators {100, 010, 001} and so is not a PGS, although it is a PGS of {101, 011, 110}.

So in what precise way are the special DCS's (NPGS's) related to the obstruction to unfixing? More soon!

Proof of Unfixing As we have mentioned above it is much harder to prove that a particular construction unfixes the appropriate elements of an unspecified group than it is to prove that it fixes the rest. This section will discuss this situation.

Independence

By restricting the DCS's to PGS's we have forced the subgroups to be independent in a certain sense. We assume that the automorphisms inherit this independence in some sense. This section will discuss this situation.

Appendix I The automorphisms of Q

We will generate the automorphisms of Q simply by closing the maps from the generator set {i, j} into Q. Using the notation
(j, i) ⇒ (1, j, i, -k, -j, -i, k, -1) ⇒ {1, -1}
meaning "the generator mapping (i, j)→(j, i) under the structure rules generates the automorphism (1, i, j, k, -i, -j, -k, -1)→(1, j, i, -k, -j, -i, k, -1) which phixes the DCS {1, -1}" we have the 24 automorphisms

1. (i, j) ⇒ (1, i, j, k, -i, -j, -k, -1) ⇒ {1, i, j, k, -i, -j, -k, -1}
2. (j, i) ⇒ (1, j, i, -k, -j, -i, k, -1) ⇒ {1, -1} (SNGS)
3. (i, -j) ⇒ (1, i, -j, -k, -i, j, k, -1)=i_i^{-1}=-i_i ⇒ {1, i, -i, -1} (inner)
4. (-i, j) ⇒ (1, -i, j, -k, i, -j, k, -1)=j_j^{-1}=-j_j ⇒ {1, j, -j, -1} (inner)
5. (-i, -j) ⇒ (1, -i, -j, k, i, j, -k, -1)=k_k^{-1}=-k_k ⇒ {1, k, -k, -1} (inner)
6. (j, -i) ⇒ (1, j, -i, k, -j, i, -k, -1) ⇒ {1, k, -k, -1}
7. (-j, i) ⇒ (1, -j, i, k, j, -i, -k, -1) ⇒ {1, k, -k, -1}
8. (-j, -i) ⇒ (1, -j, -i, -k, j, i, k, -1) ⇒ {1, -1}

9. (i, k) ⇒ (1, i, k, -j, -i, -k, j, -1) ⇒ {1, i, -i, -1}
10. (k, i) ⇒ (1, k, i, j, -k, -i, -j, -1) ⇒ {1, -1}
11. (i, -k) ⇒ (1, i, -k, j, -i, k, -j, -1) ⇒ {1, i, -i, -1}
12. (-i, k) ⇒ (1, -i, k, j, i, -k, -j, -1) ⇒ {1, -1}
13. (-i, -k) ⇒ (1, -i, -k, -j, i, k, j, -1) ⇒ {1, -1}
14. (k, -i) ⇒ (1, k, -i, -j, -k, i, j, -1) ⇒ {1, -1}
15. (-k, i) ⇒ (1, -k, i, -j, k, -i, j, -1) ⇒ {1, -1}
16. (-k, -i) ⇒ (1, -k, -i, j, k, i, -j, -1) ⇒ {1, -1}

17. (k, j) ⇒ (1, k, j, -i, -k, -j, i, -1) ⇒ {1, j, -j, -1}
18. (j, k) ⇒ (1, j, k, i, -j, -k, -i, -1) ⇒ {1, -1}
19. (k, -j) ⇒ (1, k, -j, i, -k, j, -i, -1) ⇒ {1, -1}
20. (-k, j) ⇒ (1, -k, j, i, k, -j, -i, -1) ⇒ {1, j, -j, -1}
21. (-k, -j) ⇒ (1, -k, -j, -i, k, j, i, -1) ⇒ {1, -1}
22. (j, -k) ⇒ (1, j, -k, -i, -j, k, i, -1) ⇒ {1, -1}
23. (-j, k) ⇒ (1, -j, k, -i, j, -k, i, -1) ⇒ {1, -1}
24. (-j, -k) ⇒ (1, -j, -k, i, j, k, -i, -1) ⇒ {1, -1}

Appendix 1 v1.0

Red Tiles Cottage, 31-12-06

Dear Keith,

I expect you are wanting feedback on your workpaper and I'm sorry that Christmas festivities and other things have delayed me. Better late than never, however. In what follows, for shortness, all praise or agreement is omitted:

You say:

Theorem 1 *Every DCS (or subgroup(oid)) generates a (maybe non)unique automorphism which fixes the elements of that DCS and unfixes the elements of its complement. From now on we will only talk about groups but will attempt to do so in a general enough way to apply to groupoids and other structures.*

Comment: uniqueness is very rare even in [the] simple case of [the] CH. (I am particularly sensitive to this as the non-uniqueness is the basis my [calculation of] $1/\alpha$.) *[Of course. K]*

More generally, your paper seems to suggest a spurious generality – I mean I doubt if it apples to any groupoid. *[See below. K]* This thought recurs here:

We remind the reader that the purpose of this construction is to generate a set of independent automorphisms to act as generators for the next level of the Combinatorial Hierarchy. Any other strategy will be equivalent. The importance of this proof for Combinatorial Hierarchy research cannot be overemphasised.

Because, if the generators are independent, then only some groups can be generated. For example, the symmetric group of order 6, ie the symmetry group of the triangle, can be generated by two generators a, b say, which are the operations of flipping over keeping a median fixed, because then ab and ba are rotations through $\pi/3$ one way or the other. But to generate the group needs the defining relations $a^2=b^2=e$ and $bab=aba$ (or something equivalent like $(ab)^2=ba$). And "defining relations" for the group theorist can be lack of independence for you. So you need to decide what structures you are dealing with.

["Independent generators" is an expression culled from the historical literature. I have always taken it to mean independent before the group relations are (or operator is) applied, and this is what I meant here. More clarity is clearly needed! ☺ K]

Next comment is just [on] a lack of clarity:

A Level [I always capitalise this now. K] is a Group together with a set of generators [CWK]. As there can be many sets of generators for a group we drop the usual meaning of "level" in the Combinatorial Hierarchy and instead use Grade for the latter purpose (and to spur us on to a connection with Topology). Everything we discuss in this paper will be assumed to take place in a fixed Grade, so that the meaning of Level is always as given here.

I mean: which is which? I guess that either a level is a collection of possible grades or vice versa? Which way?

We usually take groups to be additive unless otherwise stated.

Theorem 2 *Every automorphism on a Level (isomorphism between two Levels) bijectively maps the generators of that level to a new set of generators, or simply rotates the generator set.*

Just before the above is a very good definition of phix. I wholly approve. *[Praise?! ☺ K]*

Now, in the above, is the single sentence to mean "We usually take groups to be Abelian unless otherwise stated and write them additively"? *[No, no, it is not my habit to associate "additive" with Abelian. I happily write nonabelian groups additively. Sorry, I need to make this clearer. K]*

Here I find a word uncongenial:

Note that the new set of generators is not necessarily unique. It may be the rotation of another set of generators. Similarly the identity automorphism just fixes the original set of generators and a set of equivalent automorphisms just rotate them. Thus the Levels define equivalence classes of automorphisms. We can say that the new set of generators is unique "up to (or mod) rotation". Note that such rotations themselves form a braided group.

The word is "rotation" which seems to be a substitute for "permutation". *[Yes, agreed, this should be changed. It is a language we have been using for some years in conversations with John Amson. K]*

Now some comments at length on **Unproven Theorem 4.** This looks to be on the right lines but it is not yet correct. If it were to work out at all it would surely do so in the old CH so I look at an example: The group is $C_2 \times C_2 \times C_2 \times C_2$ and the subgroup is $C_2 \times C_2$. So using my old notation, the members of $C_2 \times C_2$ are generated by 1, 2, and so are {1, 2, 12}. (I take 0 for granted) and so the complement contains the generators 3, 4 and is {3, 13, 23, 123, 4, 14, 24, 124, 34, 134, 234, 1234} in (reverse) lexicographical order. So your rule for φ does this:

	P	Q				R					
x	1 2 12	3	13	23	123	4	12	24	124	34	...
φx	1 2 12	13	3	123	23	14	4	124	24	34	...

and I break off at 34 being fixed. The root of the trouble is in the circle, of course. *Any* other non-used non-generator e.g. 24 would have done. I can see a way out of this, but before going on to it, work out how to do C_2^2 as a subgroup of C_2^5. You start the same way but after having defined φ3=13, φ4=24 you find that φ5 cannot be 125 (which would be "next"). Once 13, 24 have come up using 1 and 2 from the original generators, then not just 1 and 2 but the PGS they generate won't do.

So I suggest that, after

image under the automorphism, thus n becomes a new generator (under the potential automorphism) in the copy.

you insert:

Now n is a combination of (one or more) of the generators of P with other elements. Call the PGS generated by these generators "sterilised" and call any non-generator containing these generators sterilised. To continue the process – choose the next unused and unsterilised non-generator m in the

complement of the copy of the new PGS Q and map the next unused generator f in the original to this. Close Q Uf (forming a new PGS!) and use *the structure rules to close its image under the automorphism, thus m becomes a new generator in the copy*, updating the sterilised PGS as well. Then continue,

Then go on as you have. *[Thank you for this excellent suggestion. K]*

To say in the discussion that the complement is not necessarily a subgroup is surely confusing? It is never a subgroup? *[But this is supposed to apply to subgroupoids and other objects as well, so perhaps I was being careful? It is confusing. K]*

Now all I have done here is doctor your rule to work for the CH, so the general theorem is still unproven.

The rest is work in progress for you. I haven't any further comment.

Best wishes for the New Year,
Yours,
Clive

Non-Commutative Worlds - Classical Constraints, Relativity and the Bianchi Identity

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1 Introduction to Non-Commutative Worlds

Aspects of gauge theory, Hamiltonian mechanics and quantum mechanics arise naturally in the mathematics of a non-commutative framework for calculus and differential geometry. In this paper we first give a quick review of our previous results about non-commutative worlds. Then follow two sections on new material. Section 2 discusses constraints on non-commutative worlds that are imposed by asking for correspondences between forms of classical differentiation and the derivatives represented by commutators in a correpsondent non-commutative world. This discussion of constraints parallels work of Tony Deakin [3] and will be taken up in joint work of the author, Deakin and Clive Kilmister. Section 3 is a very condensed review of the relationship of the Bianchi identity in differential geometry with the Einstein equations for general relativity. We then observe that every derivation in a non-commutative world comes equipped with its own Bianchi identity. This observation suggests that it will be fruitful to investigate general relativity in the non-commutative context.

This approach begins in an algebraic framework that naturally contains the formalism of the calculus, but not its notions of limits or constructions of spaces with specific locations, points and trajectories. Many patterns of physical law fit well into such an abstract framework. This fit is indicative of the secondary nature of point sets, topologies and classical differential geometries in physics. In this viewpoint one dispenses with spacetime and replaces it by algebraic structure. Behind that structure, space stands ready to be constructed, by discrete derivatives and patterns of steps, or by starting with a discrete pattern in the form of a diagram, a network, a lattice, a knot, or a simplicial complex, and elaborating that structure until the specificity of spatio-temporal locations appear.

Poisson brackets allow one to connect classical notions of location with the non-commutative algebra used herein. Below the level of the Poisson brackets is a treatment of processes and operators as though they were variables in the same context as the variables in the classical calculus. In different degrees one lets go of the notion of classical variables and yet retains their form, as one makes a descent into the discrete. The discrete world of non-commutative operators is a world linked to our familiar world of continuous and commutative variables. This linkage is traditionally exploited in quantum mechanics to make the transition from the classical to the quantum. One can make the journey in the other direction, from the discrete and non-commutative to the "classical" and commutative, but that journey requires powers of invention and ingenuity that are the subject of this exploration. It is our conviction that the world is basically simple. To find simplicity in the complex requires special attention and care.

In starting from a discrete point of view one thinks of a sequence of states of the world $S, S', S'', S''', \cdots$ where S' denotes the state succeeding S in discrete time. It is natural to suppose that there is some measure of difference $DS^{(n)} = S^{(n+1)} - S^{(n)}$, and some way that states S and T might be combined to form a new state ST. We can thus think of world-states as operators in a non-commutative algebra with a temoporal derivative $DS = S' - S$. At this bare level of the formalism the derivative does not satisfy the Leibniz rule. In fact it is easy to verify that $D(ST) = D(S)T + S'D(T)$. Remarkably, the Leibniz rule, and hence the formalisms of Newtonian calculus can be restored with the addition of one more operator J. In this instance J is a temporal shift operator with the property that $SJ = JS'$ for any state S. We then see that if $\nabla S = JD(S) = J(S' - S)$. then $\nabla(ST) = \nabla(S)T + S\nabla(T)$ for any states S and T. In fact $\nabla(S) = JS' - JS = SJ - JS = [S, J]$, so that this adjusted derivative is a commutator in the general calculus of states. This, in a nutshell, is our approach to non-commutative worlds. We begin with a very general framework that is a non-numerical calculus of states and operators. It is then fascinating and a topic of research to see how physics and mathematics fit into the frameworks so constructed.

Constructions are performed in a Lie algebra $\mathcal{A}$. One may take $\mathcal{A}$ to be a specific matrix Lie algebra, or abstract Lie algebra. If $\mathcal{A}$ is taken to be an abstract Lie algebra, then it is convenient to use the universal enveloping algebra so that the Lie product can be expressed as a commutator. In making general constructions of operators satisfying certain relations, it is understood that one can always begin with a free algebra and make a quotient algebra where the relations are satisfied.

On $\mathcal{A}$, a variant of calculus is built by defining derivations as commutators (or more generally as Lie products). For a fixed N in $\mathcal{A}$ one defines

$$\nabla_N : \mathcal{A} \longrightarrow \mathcal{A}$$

by the formula

$$\nabla_N F = [F, N] = FN - NF.$$

∇_N is a derivation satisfying the Leibniz rule.

$$\nabla_N(FG) = \nabla_N(F)G + F\nabla_N(G).$$

Discrete Derivatives are Replaced by Commutators. There are many motivations for replacing derivatives by commutators. If $f(x)$ denotes (say) a function of a real variable x, and $\tilde{f}(x) = f(x + h)$ for a fixed increment h, define the *discrete derivative* Df by the formula $Df = (\tilde{f} - f)/h$, and find that the Leibniz rule is not satisfied. One has the basic formula for the discrete derivative of a product:

$$D(fg) = D(f)g + \tilde{f}D(g).$$

Correct this deviation from the Leibniz rule by introducing a new non-commutative operator J with the property that

$$fJ = J\tilde{f}.$$

Define a new discrete derivative in an extended non-commutative algebra by the formula

$$\nabla(f) = JD(f).$$

It follows at once that

$$\nabla(fg) = JD(f)g + J\tilde{f}D(g) = JD(f)g + fJD(g) = \nabla(f)g + f\nabla($$

Note that

$$\nabla(f) = (J\tilde{f} - Jf)/h = (fJ - Jf)/h = [f, J/h].$$

In the extended algebra, discrete derivatives are represented by commutators, and satisfy the Leibniz rule. One can regard discrete calculus as a subset of non-commutative calculus based on commutators.

Advanced Calculus Looks Like Hamiltonian Mechanics or Quantum Mechanics in a Non-Commutative World. In $\mathcal{A}$ there are as many derivations as there are elements of the algebra, and these derivations behave quite wildly with respect to one another. If one takes the concept of *curvature* as the non-commutation of derivations, then $\mathcal{A}$ is a highly curved world indeed. Within $\mathcal{A}$ one can build a tame world of derivations that mimics the behaviour of flat coordinates in Euclidean space. The description of the structure of $\mathcal{A}$ with respect to these flat coordinates contains many of the equations and patterns of mathematical physics.

The flat coordinates X_i satisfy the equations below with the P_j chosen to represent differentiation with respect to X_j.:

$$[X_i, X_j] = 0$$
$$[P_i, P_j] = 0$$
$$[X_i, P_j] = \delta_{ij}.$$

Derivatives are represented by commutators.

$$\partial_i F = \partial F / \partial X_i = [F, P_i],$$
$$\hat{\partial}_i F = \partial F / \partial P_i = [X_i, F].$$

Temporal derivative is represented by commutation with a special (Hamiltonian) element H of the algebra:

$$dF/dt = [F, H].$$

(For quantum mechanics, take $i\hbar dA/dt = [A, H]$.) These non-commutative coordinates are the simplest flat set of coordinates for description of temporal phenomena in a non-commutative world.

Hamilton's Equations are Part of the Mathematical Structure of Non-Commutative Advanced Calculus.

$$dP_i/dt = [P_i, H] = -[H, P_i] = -\partial H/\partial X_i$$

$$dX_i/dt = [X_i, H] = \partial H/\partial P_i.$$

These are exactly Hamilton's equations of motion. The pattern of Hamilton's equations is built into the system.

The Simplest Time Series Leads to the Diffusion Constant. Consider a time series $\{X, X', X'', \ldots\}$ with commuting scalar values. Let

$$\dot{X} = \nabla X = JDX = J(X' - X)/\tau$$

where τ is an elementary time step (If X denotes a times series value at time t, then X' denotes the value of the series at time $t + \tau$.). The shift operator J is defined by the equation $XJ = JX'$ where this refers to any point in the time series so that $X^{(n)}J = JX^{(n+1)}$ for any non-negative integer n. Moving J across a variable from left to right, corresponds to one tick of the clock. This discrete, non-commutative time derivative satisfies the Leibniz rule.

This derivative ∇ also fits a significant pattern of discrete observation. Consider the act of observing X at a given time and the act of observing (or obtaining) DX at a given time. Since X and X' are ingredients in computing $(X' - X)/\tau$, the numerical value associated with DX, it is necessary to let the clock tick once, Thus, if one first observe X and then obtains DX, the result is different (for the X measurement) if one first obtains DX, and then observes X. In the second case, one finds the value X' instead of the value X, due to the tick of the clock.

1. Let $\dot{X}X$ denote the sequence: observe X, then obtain $\dot{X}$.
2. Let $X\dot{X}$ denote the sequence: obtain $\dot{X}$, then observe X.

The commutator $[X, \dot{X}]$ expresses the difference between these two orders of discrete measurement. In the simplest case, where the elements of the time series are commuting scalars, one has

$$[X, \dot{X}] = X\dot{X} - \dot{X}X = J(X' - X)^2/\tau.$$

Thus one can interpret the equation

$$[X, \dot{X}] = Jk$$

(k a constant scalar) as

$$(X' - X)^2/\tau = k.$$

This means that the process is a walk with spatial step

$$\Delta = \pm\sqrt{k\tau}$$

where k is a constant. In other words, one has the equation

$$k = \Delta^2/\tau.$$

This is the diffusion constant for a Brownian walk. A walk with spatial step size Δ and time step τ will satisfy the commutator equation above exactly when the square of the spatial step divided by the time step remains constant. This shows that the diffusion constant of a Brownian process is a structural property of that process, independent of considerations of probability and continuum limits.

Schroedinger's Equation is Discrete. Here is how the Heisenberg form of Schroedinger's equation fits in this context. Let $J = (1 + i\hbar H\Delta t)$. Then $\nabla\psi = [\psi, J/\Delta t]$, and we calculate

$$\nabla\psi = \psi[(1+i\hbar H\Delta t)/\Delta t] - [(1+i\hbar H\Delta t)/\Delta t]\psi = i\hbar[\psi, H].$$

This is exactly the form of the Heisenberg equation.

Dynamical Equations Generalize Gauge Theory and Curvature. One can take the general dynamical equation in the form

$$dX_i/dt = \mathcal{G}_i$$

where $\{\mathcal{G}_1, \cdots, \mathcal{G}_d\}$ is a collection of elements of $\mathcal{A}$. Write $\mathcal{G}_i$ relative to the flat coordinates via $\mathcal{G}_i = P_i - A_i$. This is a definition of A_i and $\partial F/\partial X_i = [F, P_i]$. The formalism of gauge theory appears naturally. In particular, if

$$\nabla_i(F) = [F, \mathcal{G}_i],$$

then one has the curvature

$$[\nabla_i, \nabla_j]F = [R_{ij}, F]$$

and

$$R_{ij} = \partial_i A_j - \partial_j A_i + [A_i, A_j].$$

This is the well-known formula for the curvature of a gauge connection. Aspects of geometry arise naturally in this context, including the Levi-Civita connection (which is seen as a consequence of the Jacobi identity in an appropriate non-commutative world).

One can consider the consequences of the commutator $[X_i, \dot{X}_j] = g_{ij}$, deriving that

$$\ddot{X}_r = G_r + F_{rs}\dot{X}^s + \Gamma_{rst}\dot{X}^s\dot{X}^t,$$

where G_r is the analogue of a scalar field, F_{rs} is the analogue of a gauge field and Γ_{rst} is the Levi-Civita connection associated with g_{ij}. This decompositon of the acceleration is uniquely determined by the given framework.

Non-commutative Electromagnetism and Gauge Theory. One can use this context to revisit the Feynman-Dyson derivation of electromagnetism from commutator equations, showing that most of the derivation is independent of any choice of commutators, but highly dependent upon the choice of definitions of the derivatives involved. Without any assumptions about initial commutator equations, but taking the right (in some sense simplest) definitions of the derivatives one obtains a significant generalization of the result of Feynman-Dyson.

Theorem With the appropriate [see below] definitions of the operators, and taking

$\nabla^2 = \partial_1^2 + \partial_2^2 + \partial_3^2$, $H = \dot{X} \times \dot{X}$ and $E = \partial_t \dot{X}$, one has

1. $\ddot{X} = E + \dot{X} \times H$
2. $\nabla \bullet H = 0$
3. $\partial_t H + \nabla \times E = H \times H$
4. $\partial_t E - \nabla \times H = (\partial_t^2 - \nabla^2)\dot{X}$

The key to the proof of this Theorem is the definition of the time derivative. This definition is as follows

$$\partial_t F = \dot{F} - \Sigma_i \dot{X}_i \partial_i(F) = \dot{F} - \Sigma_i \dot{X}_i [F, \dot{X}_i]$$

for all elements or vectors of elements F. The definition creates a distinction between space and time in the non-commutative world. A calculation reveals that

$$\ddot{X} = \partial_t \dot{X} + \dot{X} \times (\dot{X} \times \dot{X}).$$

This suggests taking $E = \partial_t \dot{X}$ as the electric field, and $B = \dot{X} \times \dot{X}$ as the magnetic field so that the Lorentz force law

$$\ddot{X} = E + \dot{X} \times B$$

is satisfied.

This result is applied to produce many discrete models of the Theorem. These models show that, just as the commutator $[X, \dot{X}] = Jk$ describes Brownian motion in one dimension, a generalization of electromagnetism describes the interaction of triples of time series in three dimensions.

Remark. While there is a large literature on non-commutative geometry, emanating from the idea of replacing a space by its ring of functions, work discussed herein is not written in that tradition. Non-commutative geometry does occur here, in the sense of geometry occuring in the context of non-commutative algebra. Derivations are represented by commutators. There are relationships between the present work and the traditional non-commutative geometry, but that is a subject for further exploration. In no way is this paper intended to be an introduction to that subject. The present summary is based on [8, 9, 10, 11, 12, 13, 14, 15, 16, 17] and the references cited therein.

The following references in relation to non-commutative calculus are useful in comparing with the present approach [2, 4, 6, 19]. Much of the present work is the fruit of a long series of discussions with Pierre Noyes. paper [18] also works with minimal coupling for the Feynman-Dyson derivation. The first remark about the minimal coupling occurs in the original paper by Dyson [1], in the context of Poisson brackets. The paper [7] is worth reading as a companion to Dyson. It is the purpose of this summary to indicate how non-commutative calculus can be used in foundations.

2 Constraints

The program here is to investigate restrictions in a non-commutative world that are imposed by asking for a specific correspondence between classical variables acting in the usual context of continuum calculus, and non-commutative operators corresponding to these classical variables. If, for example, we let x and y be classical variables and X and Y the corresponding non-commutative operators, then we ask that x^n correspond to X^n and that y^n correspond to Y^n for positive integers n. We further ask that linear combinations of classical variables correspond to linear combinations of the corresponding operators. These restrictions tell us what happens to products. For example, we have classically that $(x+y)^2 = x^2 + 2xy + y^2$. This, in turn must correspond to $(X+Y)^2 = X^2+XY+YX+Y^2$. From this it follows that

$2xy$ corresponds to $XY + YX$. Hence xy corresponds to

$$\{XY\} = (XY + YX)/2.$$

By a similar calculation, if $x_1, x_2, \cdots, x_n$ are classical variables, then the product $x_1 x_2 \cdots x_n$ corresponds to

$$\{X_1 X_2 \cdots X_n\} = (1/n!)\Sigma_{\sigma \in S_n} X_{\sigma_1} X_{\sigma_2} \cdots X_{\sigma_n}.$$

where S_n denotes all permutations of $1, 2, \cdots, n$. Note that we use curly brackets for these symmetrizers and square brackets for commutators as in $[A, B] = AB - BA$.

We can formulate constraints in the non-commutative world by asking for a correspondence between familiar differentiation formulas in continuum calculus and the corresponding formulas in the non-commutative calculus, where all derivatives are expressed via commutators. We will detail how this constraint algebra works in the first few cases. Exploration of these constraints has been pioneered by Tony Deakin [3]. The author of this paper, Tony Deakin and Clive Kilmister are planning a comprehensive paper on the consequences of these contraints in the interface between classical and quantum mechanics.

Recall that the temporal derivative in a non-commutative world is represented by commutator with an operator H

that can be intrepreted as the Hamiltonian operator in certain contexts.

$$\dot{\Theta} = [\Theta, H].$$

For this discussion, we shall take a collection $Q^1, Q^2, \cdots, Q^n$ of operators to represent spatial coordinates $q^1, q^2, \cdots, q^n$. The Q^i commute with one another, and the derivatives with respect to Q^i are represented by operators P^i so that

$$\partial\Theta/\partial Q^i = \Theta_i = [\Theta, P^i].$$

We also write

$$\partial\Theta/\partial P^i = \Theta^i = [Q^i, \Theta].$$

To this purpose, we assume that $[Q^i, P^j] = \delta^{ij}$ and that the P^j commute with one another (so that mixed partial derivatives with respect to the Q^i are independent of order of differentiation).

Note that

$$\dot{Q}^i = [Q^i, H] = H^i.$$

It will be convenient for us to write H^i in place of $\dot{Q}^i$ in the calculations to follow.

The First Constraint. The *first constraint* is the equation

$$\dot{\Theta} = \{\dot{Q}^i\Theta_i\} = \{H^i\Theta_i\}.$$

This equation expresses the symmetrized version of the usual calculus formula $\dot{\theta} = \dot{q}^i\theta_i$. It is worth noting that the first constraint is satisfied by the quadratic Hamiltonian

$$H = \frac{1}{4}(g_{ij}P^iP^j + P^iP^jg_{ij})$$

where $g_{ij} = g_{ji}$ and the g_{ij} commute with the Q^k. We leave the verification of this point to the reader, and note that the fact that the quadratic Hamiltonian does satisfy the first constraint shows how the constraints bind properties of classical physics (in this case Hamiltonian mechanics) to the non-commutative world.

The Second Constraint. The *second constraint* is the symmetrized analog of the second temporal derivative:

$$\ddot{\Theta} = \{\dot{H}^i\Theta_i\} + \{H^iH^j\Theta_{ij}\}.$$

However, by differentiating the first constraint we have

$$\ddot{\Theta} = \{\dot{H}^i\Theta_i\} + \{H^i\{H^j\Theta_{ij}\}\}$$

Thus the second constraint is equivalent to the equation

$$\{H^i\{H^j\Theta_{ij}\}\} = \{H^iH^j\Theta_{ij}\}.$$

We now reformulate this version of the constraint in the following theorem.

Theorem. The second constraint in the form $\{H^i\{H^j\Theta_{ij}\}\} = \{H^iH^j\Theta_{ij}\}$ is equivalent to the equation

$$[[\Theta_{ij}, H^j], H^i] = 0.$$

Proof. We can shortcut the calculations involved in proving this Theorem by looking at the properties of symbols A, B, C such that $AB = BA$, $ACB = BCA$. Formally these mimic the behaviour of $A = H^i, B = H^j, C = \Theta_{ij}$ in the expressions $H^iH^j\Theta_{ij}$ and $H^i\Theta_{ij}H^j$ since $\Theta_{ij} = \Theta_{ji}$, and the Einstein summation convention is in place. Then

$$\{A\{BC\}\} = \frac{1}{4}(A(BC + CB) + (BC + CB)A)$$

$$= \frac{1}{4}(ABC + ACB + BCA + CBA),$$

$$\{ABC\} = \frac{1}{6}(ABC+ACB+BAC+BCA+CAB+CBA).$$

So

$$\{ABC\}-\{A\{BC\}\} = \frac{1}{12}(-ABC-ACB+2BAC-BCA+2CAB-($$

$$= \frac{1}{12}(ABC - 2ACB + CAB)$$

$$= \frac{1}{12}(ABC - 2BCA + CBA)$$

$$= \frac{1}{12}(A(BC - CB) + (CB - BC)A)$$
$$= \frac{1}{12}(A[B, C] - [B, C]A)$$
$$= \frac{1}{12}[A, [B, C]].$$

Thus the second constraint is equivalent to the equation

$$[H^i, [H^j, \Theta_{ij}]] = 0.$$

This in turn is equivalent to the equation

$$[[\Theta_{ij}, H^j], H^i] = 0,$$

completing the proof of the Theorem.

Remark. If we define

$$\nabla^i(\Theta) = [\Theta, H^i] = [\Theta, \dot{Q}^i]$$

then this is the natural covariant derivative that was described in the introduction to this paper. Thus the second order constraint is

$$\nabla^i(\nabla^j(\Theta_{ij}) = 0.$$

If we use the quadratic Hamiltonian $H = \frac{1}{4}(g_{ij}P^iP^j + P^iP^jg_{ij})$ as above, then with $\Theta = g^{lm}$ the second constraint becomes the equation

$$g^{uv}(g^{jk}g^{lm}_{,jku})_v = 0.$$

Deakin and Kilmister observe that this last equation specializes to a fourth order version of Einstein's field equation for vacuum general relativity. This will be the subject of a paper of the author with Deakin and Kilmister.

The algebra of the higher order constraints is under investigation at this time.

3 Einstein's Equations and the Bianchi Identity

The purpose of this section is to show how the Bianchi identity (see below for its definition) appears in the context of non-commutative worlds. The Bianchi identity is a crucial mathematical ingredient in general relativity. We shall begin with a quick review of the mathematical structure of general relativity (see for example [5]) and then turn to the context of non-commutative worlds.

The basic tensor in Einstein's theory of general relativity is

$$G^{ab} = R^{ab} - \frac{1}{2}Rg^{ab}$$

where R^{ab} is the Ricci tensor and R the scalar curvature. The Ricci tensor and the scalar curvature are both obtained by contraction from the Riemann curvature tensor

R^a_{bcd} with $R_{ab} = R^c_{abc}$, $R^{ab} = g^{ai}g^{bj}R_{ij}$, and $R = g^{ij}R_{ij}$. Because the Einstein tensor G^{ab} has vanishing divergence, it is a prime candidate to be proportional to the energy momentum tensor $T^{\mu\nu}$. The Einstein field equations are

$$R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu} = \kappa T^{\mu\nu}.$$

The reader may wish to recall that the Riemann tensor is obtained from the commutator of a covariant derivative ∇_k, associated with the Levi-Civita connection $\Gamma^i_{jk} = (\Gamma_k)^i_j$ (built from the space-time metric g_{ij}). One has

$$\lambda_{a:b} = \nabla_b\lambda_a = \partial_b\lambda_a - \Gamma^d_{ab}\lambda_d$$

or

$$\lambda_{:b} = \nabla_b\lambda = \partial_b\lambda - \Gamma_b\lambda$$

for a vector field λ. With

$$R_{ij} = [\nabla_i, \nabla_j] = \partial_j\Gamma_i - \partial_i\Gamma_j + [\Gamma_i, \Gamma_j],$$

one has

$$R^a_{bcd} = (R_{cd})^a_b.$$

(Here R_{cd} is *not* the Ricci tensor. It is the Riemann tensor with two internal indices hidden from sight.)

One way to understand the mathematical source of the Einstein tensor, and the vanishing of its divergence, is to see it as a contraction of the Bianchi identity for the Riemann tensor. The Bianchi identity states

$$R^a_{bcd:e} + R^a_{bde:c} + R^a_{bec:d} = 0$$

where the index after the colon indicates the covariant derivative. Note also that this can be written in the form

$$(R_{cd:e})^a_b + (R_{de:c})^a_b + (R_{ec:d})^a_b = 0.$$

The Bianchi identity is a consequence of local properties of the Levi-Civita connection and consequent symmetries of the Riemann tensor. One relevant symmetry of the Riemann tensor is the equation $R^a_{bcd} = -R^a_{bdc}$.

We will not give a classical derivation of the Bianchi identity here, but it is instructive to see how its contraction leads to the Einstein tensor. To this end, note that we can contract the Bianchi identity to

$$R^a_{bca:e} + R^a_{bae:c} + R^a_{bec:a} = 0$$

which, in the light of the above definition of the Ricci tensor and the symmetries of the Riemann tensor is the same as

$$R_{bc:e} - R_{be:c} + R^a_{bec:a} = 0.$$

Contract this tensor equation once more to obtain

$$R_{bc:b} - R_{bb:c} + R^a_{bbc:a} = 0,$$

and raise indices

$$R^{b}_{c:b} - R_{:c} + R^{ab}_{bc:a} = 0.$$

Further symmetry gives

$$R^{ab}_{bc:a} = R^{ba}_{cb:a} = R^{a}_{c:a} = R^{b}_{c:b}.$$

Hence we have

$$2R^{b}_{c:b} - R_{:c} = 0,$$

which is equivalent to the equation

$$(R^{b}_{c} - \frac{1}{2}R\delta^{b}_{c})_{:b} = G^{b}_{c:b} = 0.$$

From this we conclude that $G^{bc}_{:b} = 0$. The Einstein tensor has appeared on the stage with vanishing divergence, courtesy of the Bianchi identity!

Bianchi Identity and Jacobi Identity. Now lets turn to the context of non-commutative worlds. We have infinitely many possible convariant derivatives, all of the form

$$F_{:a} = \nabla_{a}F = [F, N_{a}]$$

for some N_a elements in the non-commutative world. Choose any such covariant derivative. Then, as in the introduction to this paper, we have the curvature

$$R_{ij} = [N_i, N_j]$$

that represents the commutator of the covariant derivative with itself in the sense that $[\nabla_i, \nabla_j]F = [[N_i, N_j], F]$. Note that R_{ij} is not a Ricci tensor, but rather the indication of the external structure of the curvature without any particular choice of linear representation (as is given in the classical case as described above). We then have the Jacobi identity

$$[[N_a, N_b], N_c] + [[N_c, N_a], N_b] + [[N_b, N_c], N_a] = 0.$$

Writing the Jacobi identity in terms of curvature and covariant differention we have

$$R_{ab:c} + R_{ca:b} + R_{bc:a}.$$

Thus in a non-commutative world, every covariant derivative satisfies its own Bianchi identity. This gives an impetus to study general relativity in non-commutative worlds by looking for covariant derivatives that satisfy the symmetries of the Riemann tensor and link with a metric in an appropriate way. We have only begun this aspect of the investigation. The point of this section has been to show the intimate relationship between the Bianchi idenity and the Jacobi identity that is revealed in the context of non-commutative worlds.

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Combinatorial Calculations Using Relativistic Few Particle Dynamics

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Abstract

Quantum physics provides probably the most fundamental description of the nature of physical phenomena. Yet, most of our macroscopic observations appear to involve de-coherent, classical processes. Subtle connections between the microscopic realm and macroscopic phenomena remain intriguing to this day. In some instances, glimpses into the quantum realm are given by macroscopic phenomena and classical detections. This presentation will explore some of the subtle manifestations of quantum coherence in few particle systems. Calculations that provide connections between descriptions of physical phenomena using possibly combinatorial insights with those that connect to the popular models that accurately describe various experiments will be presented. A summary of a formulation for microphysical interactions that incorporates many of the ideals of modeling natural philosophy using parameters and concepts that are meaningful at every stage of any given calculation will be given.

I. Introduction

The insights presented will have roots in relativistic few-particle dynamics. We will therefore begin with a brief introduction into the formal structure of scattering theory. Scattering theory represents interacting quantum systems in terms of a complete set of well-understood basis states. One normally chooses a basis for which one has some intuitive ideas of how those states connect to the interacting system, either in terms of weakly or non-interacting states, or a manageable set of strongly interacting states that can directly be related to the system being described.

If the basis states do not fully describe the correct boundary behavior of the scattering, renormalization is often required. This is necessary because the descriptions of the scattering involve incoming and outgoing detections of physical states, which must be described in terms of any self-interactions or asymptotic bound state behaviors. Since bound states are kinematically and analytically distinct from scattering states, many approximation techniques that are appropriate for describing systems in one regime are not appropriate for describing the same system in another. In addition, any description of interacting states in terms of non-interacting basis states clearly requires a *re-dressing* of boundary states to account for legitimate self interactions that modify the masses (self-energies) of these asymptotic incoming and outgoing systems. Often this involves the canceling of calculated infinities in the scattering behaviors due to infinities from persistent self-interactions, considerably complicating higher order calculations for weakly interacting systems, and all calculations for strongly interacting systems.

Our approach will be to always use properly normalized boundary states parameterized by physical masses, charges, and measured quantum numbers. The problem then becomes one of developing analytically relevant scattering functionals that are both calculable and intuitive enough to allow meaningful connections to the phenomena being described. If one can develop analytic forms for the basic interactions, these forms can be consistently embedded into more complex processes in a manner consistent with both relativity and classical correspondence. We will explicitly develop a calculation that demonstrates how this is done.

The use of scattering theoretic techniques allows a direct examination of how fundamental measured parameters, such as coupling constants, enter into the models of a scattering process. Few particle scattering theory inherently incorporates the complicated kinematics of the scattering process independently of the specific dynamics of the interaction. Combinatorial calculations of the dynamical coupling constants can be directly incorporated into a structure that consistently represents the correct space-time kinematics of special relativity. This allows parallel paths of the exploration of the fundamentals of physical processes. One path involves understanding the hierarchy and nature of the fundamental interactions. The other involves understanding the nature of the arena upon which the actions and interactions are staged. In what follows we will demonstrate a formalism that incorporates the proper kinematics of the arena. We will also demonstrate how the dynamics of Coulomb interactions and Compton scattering can be incorporated into this formalism. The coupling constant associated with Coulombic processes is directly represented as an input parameter. We then demonstrate that we are able to directly calculate the fundamental process of Compton scattering using this formalism, giving results consistent with those calculated using perturbative quantum electrodynamics (QED). We show how one incorporates particle-antiparticle symmetries and pair creation within a finite particle number scattering formulation.

II. Fixed particle number relativistic scattering theory

In what follows, we will always focus on the properties of the particles. One should recognize that in any physically realized experiment, there will **always** be a finite number of particles in initial and final states.

However, high energy phenomenology demonstrates the possibility of pair creation (particle-antiparticle production) and annihilation, so that the actual number of particle states in the final state can be different from the number in the initial state. Single *quantum* states can be emitted from an excited or pseudo-stable system (like excited atoms and radioactive nuclei), but *particle* states require that lepton and baryon number conservation be satisfied for observed phenomenology. This behavior is represented in Figure 1. In the figure, the dashed lines represent the antiparticle states associated with the nearest solid line on the diagram.

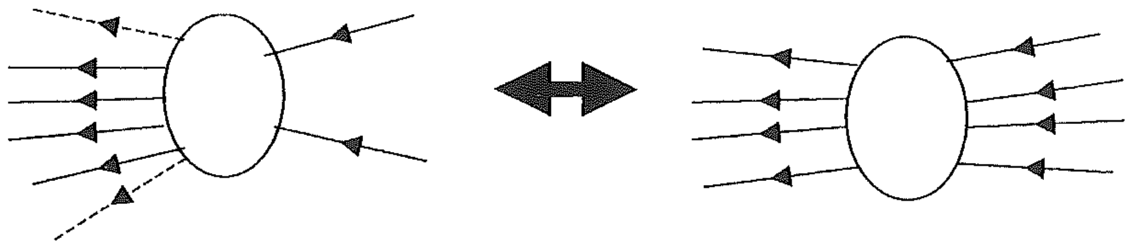


Figure 1 Pair creation

The diagram demonstrates that in all physically realizable cases, the asymptotic particle states can be represented in terms of a fixed particle number scattering process that has incoming/outgoing particle states transformed into outgoing/incoming antiparticle states. Once a procedure has been established for mapping these related amplitudes, one only needs to calculate the fixed particle number scattering amplitude related to any given particle scattering process.

For many problems in electrodynamics, the weakness of the electromagnetic coupling associated with charge allows one to use perturbative approaches in successively approximating the overall behavior of a scattering system of charges. However, there are some inherent flaws associated with naïve perturbative approaches. Dyson[1] showed that the renormalized perturbation series in the usual approach of quantum electrodynamics is *not* uniformly convergent when extended beyond 137 terms. Also, there is no known way to derive a unique, static non-relativistic ``potential'' which can be used unambiguously in a non-relativistic quantum mechanical Hamiltonian from *any* relativistic quantum field theory. This makes it difficult for one to make correspondences between the perturbative approaches in relativistic quantum field theory with those procedures developed in introductory courses in non-relativistic quantum mechanics. Finally, perturbative quantum field theory does not describe low lying bound states, since one cannot apply perturbative methods near the singular points of an amplitude. This problem is further compounded in quantum chromodynamics (QCD), since the basic asymptotic mass parameters of the quarks cannot be directly measured because of confinement. Similarly, an understanding of the phenomena of superconductivity required an appropriate description for the ground state (boundary state) basis of the system which could not be developed in a perturbative weak coupling limit because of an essential singularity at zero coupling in analytic forms involving the electron-electron coupling. A clear understanding of many fundamentals often requires that a non-perturbative approach be developed.

A. Finite particle number scattering theory

A finite particle number formulation for quantum processes has several advantages over the perturbative methods that utilize non-interacting basis states to describe the processes. These advantages include:

1. The formulation will have well defined and straightforward non-relativistic limits and classical correspondences.
2. The formulation produces analytic forms in cases for which perturbative series expansions would be invalid.
3. Bound state extraction is straightforward. Amplitudes directly exhibit the proper structure of the primary singularities.

However there are several difficulties with the whole approach. These include:

1. One expects extremely complicated functional forms for kinematic dependencies.

2. There is a need to construct exactly unitary relativistic amplitudes.

3. Cluster decomposability is **extremely** complicated for those systems requiring relativistic kinematics.

4. The inclusion of the creation and annihilation of particles is not trivial

The primary complication in relativistic finite particle number scattering formalism is in developing a mechanism that properly embeds dynamical clusters in the global space-time. We will assume that the discrete momentum basis of the finite volume space-time normally used in scattering theory has been made adequately consistent with combinatorial approaches. This means that we will not be examining the second path of understanding the *combinatorial* fundamentals of the *arena* of the scattering processes as discussed in the introduction. The present discussion will presume space-time kinematics consistent with special relativity. However, we *will* explore the dynamical implications of combinatorial couplings between quantum systems embedded in this space-time.

B. Unitarity

The conservation of probability fluxes and persistence of internal particle quantum numbers necessitates that any incoming particle types must ultimately wind up as corresponding outgoing particle types. This is known as unitarity, and it preserves the normalization of the state vectors used to describe the system, and sets constraints on the form of the scattering operators: The scattering amplitudes S and T are diagrammatically represented in Figure 2.

$$\langle \Psi_f | \Psi_o \rangle = \langle \Phi_f | S | \Phi_o \rangle \quad , \quad S^+ S = 1$$
$$S = 1 + iT \quad , \quad T - T^+ = iT^+ T$$

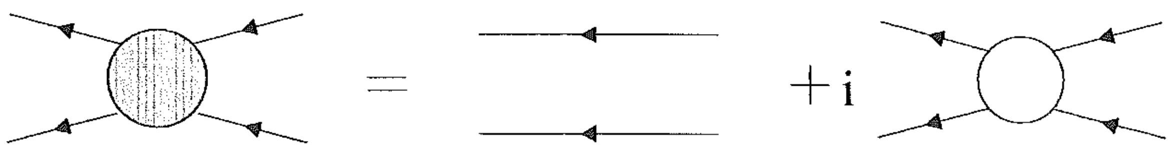


Figure 2 The Scattering and Transition Amplitudes

The identity operation represents no interaction between the particle states, indicated by the parallel lines on the right hand side of the equation in Figure 2. Unitarity is expressed in terms of amplitudes τ developed from basis state expectations of the transition operator T using

$$\tau_{(a)}\left(M_{(a)},\hat{q}_{(a)}\mid M'_{(a)},\hat{q}'_{(a)};\zeta_1\right)-\tau_{(a)}\left(M_{(a)},\hat{q}_{(a)}\mid M'_{(a)},\hat{q}'_{(a)};\zeta_2\right)=$$

$$(\zeta_2-\zeta_1)\iint dM''d^2\hat{q}''\tau_{(a)}\left(M_{(a)},\hat{q}_{(a)}\mid M'',\hat{q}'';\zeta_1\right)\left(\frac{1}{M''-\zeta_1}\right)\left(\frac{1}{M''-\zeta_2}\right)\tau_{(a)}\left(M'',\hat{q}''\mid M'_{(a)},\hat{q}'_{(a)};\zeta_2\right)$$

The variables involved in this description are the invariant energies, orientations, and off-shell parameters of the system.

C. Combinatorial input to dynamics

The question then becomes one of how combinatorial ideas can contribute to descriptions of the dynamics represented by a given scattering process. One possibility suggested by H. Pierre Noyes is that the dynamics of the fundamental interactions are to first order due to the purely combinatorial counting of a discrete set of states, which define the coupling constants. We refer to this description of dynamics as the Noyes Combinatorial Conjecture, which can be stated as follows:

•**Noyes Combinatorial Conjecture-** *Once a quantum interaction vertex forms, the likelihood of it re-opening into the specific final state with conserved quantum numbers is inversely proportional to the number of combinatorial states on that level of interaction.*

For example, consider the 137 level of the combinatorial hierarchy developed by the founding members of ANPA (excluding kinematic factors). Figure 3 demonstrates that only one of the 137 possible final state outcomes satisfies the constraint of this conjecture, approximately describing the appropriate coupling due to an electromagnetic interaction.

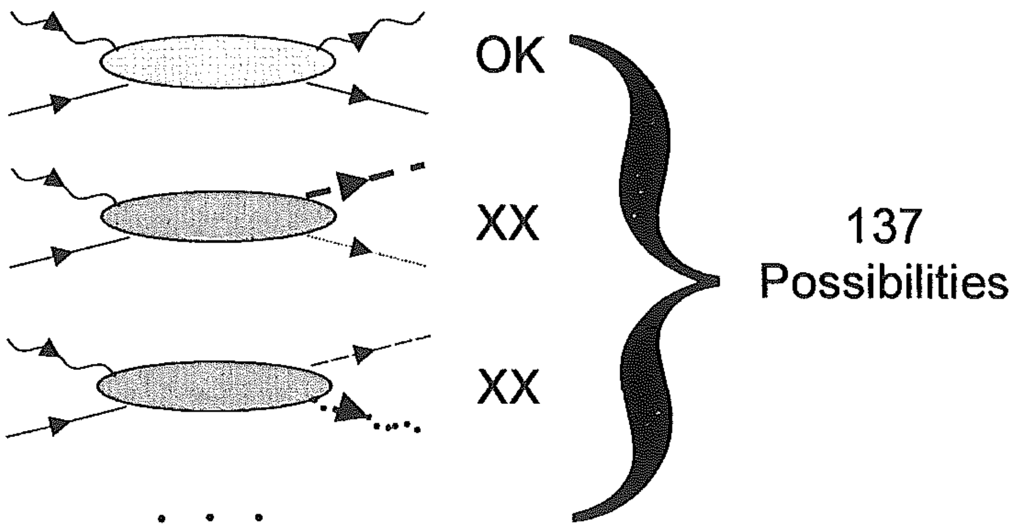


Figure 3 The Noyes Combinatorial Conjecture

This conjecture gives a possibility of a direct input of dynamical parameters calculated using combinatorial techniques into relativistic scattering predictions.

D. Dynamics of transition operator

The most direct incorporation of these couplings into amplitudes that can be used in scattering calculations will be through the non-relativistic electromagnetically bound states of atomic physics. Using scattering theoretic techniques, bound states are extracted by means of the off-energy-shell behaviors of the scattering amplitudes. From the Lippman Schwinger equation[5] for the transition operator

$$T(Z) = V - V \frac{1}{H - Z} V$$

any bound states associated with the energy operator H are seen to be reflected in the analytic behavior of the transition operator with respect to the parameter Z. The discrete spectrum of system Hamiltonian corresponds to poles in the off energy shell parameter Z. We will develop a transition amplitude that consistently incorporates any pre-calculated electromagnetic coupling constant into a Coulomb scattering process.

E. Coulomb scattering

The problem of quantum scattering of non-relativistic charged particles was solved by Mott and Massey[6] more than a half century ago. The form of the wave function associated with an incoming charge with wave vector k in the z-direction, charge Z_1e, and reduced mass μ scattering from charge Z_2e is given by

$$\psi_k(r,\vartheta) = N_k e^{ikz} \; {}_1F_1(-iZ_1Z_2\alpha\frac{\mu}{k};1;ik(r-z))$$

where the confluent hypergeometric function has an integral representation of the form

$${}_1F_1(a,b;w) = \frac{(b-1)!}{2\pi i}\int_\gamma \left(1-\frac{w}{t}\right)^{-a} e^t t^{-b}\,dt,$$

and α is the electromagnetic coupling constant. The contour γ in the integral is shown in Figure 4.

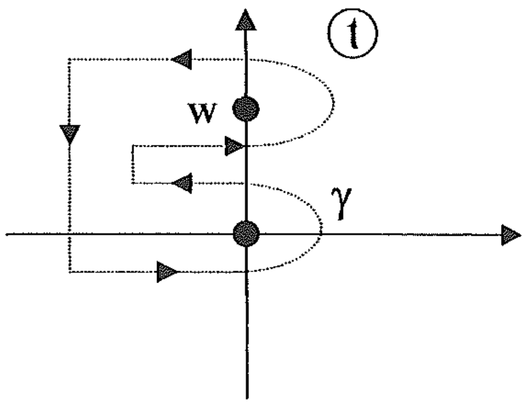


Figure 4 Contour for Components of Scattering Solution

The calculation allows the confluent hypergeometric function to be decomposed into incoming and outgoing components:

$${}_1F_1(a,b;w) = W_1(a,b;w) + W_2(a,b;w)\,.$$

The outgoing part of the wave form has an asymptotic behavior given by

$$W_2(-iZ_1Z_2\alpha\frac{\mu}{k};1;ik(r-z))\xrightarrow[r\to\infty]{}\frac{e^{ik(r-z)}\left[ik(r-z)\right]^{-1-iZ_1Z_2\alpha\frac{\mu}{k}}}{\Gamma(-iZ_1Z_2\alpha\frac{\mu}{k})}\left(1+\frac{\left(1+iZ_1Z_2\alpha\frac{\mu}{k}\right)}{ik(r-z)}+\cdots\right).$$

In the Born limit, we should expect to recover the Rutherford scattering result for the scattering phase shift Δ_0, in the form $\sin\Delta_o(q)\underset{\substack{BornLimit\\ m_Q\to 0}}{\Longrightarrow}\frac{1}{2}Z_1Z_2\alpha\frac{\mu}{q}$,

where the coupling constant $\alpha\approx 1/137$ at low energies. The appropriate correspondences with finite quantum mass scattering has been demonstrated in the references[7]. In order to recover the Coulomb phase factor, we define the parameter $Q(Z_1Z_2\alpha,q,\mu,m_Q)\underset{m_Q\to 0}{\Longrightarrow}Z_1Z_2\alpha\frac{\mu}{q}$

and identify a general form for the phase shift as

$$\Delta(q,\vartheta\quad)=Q\log\left(\frac{m_Q^2+4q^2}{m_Q^2-M_X^2}\right)+2\eta(Q)+\nu(q,Q),$$

where the phase factor η is given in terms of gamma functions by

$$e^{2i\eta(a)}\equiv\frac{\Gamma(ia)}{\Gamma(-ia)}$$

and the factor ν is given by

$$\nu(q,Q)=\frac{1}{2}Q\left[(1+4\gamma)-2\log\left(\frac{m_Q^2+4q^2}{m_Q^2}\right)\right].$$

The bound state spectrum associated with hydrogenic atoms is properly described by the poles in the gamma functions from the expression for η. A straightforward simplification identifies the phase shift and scattering amplitude as

$$\Delta(q,\vartheta)=\frac{1}{2}Q\left[\left(1+4\gamma-2\pi\right)+2\log\left(\frac{m_Q^2}{m_Q^2-M_X^2}\right)\right]+2\eta(Q)$$

$$f_q(\vartheta)=\frac{\sin\Delta(q,\vartheta=0)}{q}\left(\frac{m_Q^2+4q^2}{m_Q^2-M_X^2}\right)e^{i\Delta(q,\vartheta)}.$$

The outgoing waveform then satisfies the expected form of an outgoing spherical wave

$$\psi_k(r,\vartheta)\xrightarrow[r\to\infty]{}\frac{e^{ikr}}{r}f_k(\vartheta).$$

Therefore, the derived form gives an example of an analytic form for relativistic two particle scattering of charges due to a finite mass quantum m_Q (which can be taken to zero) and coupling constant α.

F. Kinematic factors

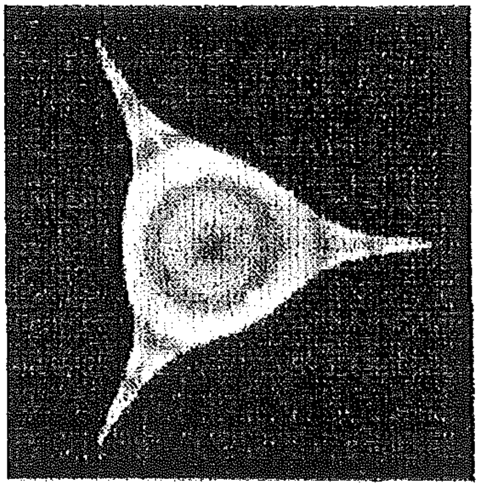
We next develop the kinematic descriptions that can properly embed the appropriate dynamics as developed in the previous example for a relativistic generalization of Coulomb scattering. Faddeev[8] developed a non-relativistic few particle formalism that properly incorporated unitarity and cluster decomposability. The Faddeev formulation has the following characteristics:

- The theory is a multichannel formulation of few body scattering theory using two body scattering amplitudes (including bound states) as inputs, and utilizing only *physical* parameters (like masses and charges).
- Full off-shell unitarity for the few body system is guaranteed if the two body input is unitary. There is no need for renormalizability concerns.
- Disconnected scattering processes are properly handled, resulting in non-singular kernels.
- Cluster decomposability is explicitly demonstrated for non-relativistic kinematics.

•Bound state extraction is straightforward due to the channel decomposition that explicitly exhibits them as primary singularities

•Multiparticle bound states are directly calculable

One of the most profound implications of three particle scattering was discovered independently by Efimov (the Efimov effect[9]) and Noyes (the Eternal Triangle effect). The effect predicts the existence of long range 3-body bound states, despite the absence of pair-wise bound states. This occurs if the phase shift has significant low energy modifications in the two-body interactions, allowing the two body long-range stationary states to generate three-body bound states near threshold energies. This effect has been recently observed[10] in bosonic cesium atoms cooled to 10^{o}nK. A magnetic field is then used to fine-tune the scattering length (which is inversely related to the pairwise energy scale) between pairs. The team observed an Efimov resonance at a scattering length of $850a_{o}$, where a_o is Bohr radius, indicating the long-range nature of the two-body interactions. The resultant three-body probability distribution is indicated in the following image:



The reader is invited to examine reference 10 for a further discussion of this experiment.

The three body Faddeev equations for the transition amplitude involves decomposing this amplitude into channels associated with the possible clusters of the system. For three particles, the possible clusters are given by the three possible pair-plus-spectator combinations. The transition operator for the three particle system is driven by the two particle transition operators $T_{(a)}$

$$T_{ab}(Z) = \delta_{ab}T_{(a)}(Z) - \sum_{d} \bar{\delta}_{ad}T_{(a)}(Z)R_o(Z)T_{db}(Z)$$

$$\bar{\delta}_{ad} \equiv 1 - \delta_{ad} \quad , \quad T \equiv \sum_{a,b} T_{ab}$$

which can be diagrammatically represented as

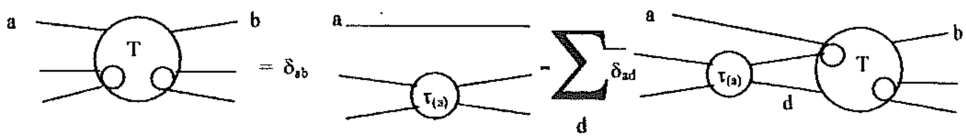


Figure 5 Three particle transition operator.

The first term on the right hand side of Figure 5 is disconnected due to the non-interaction of the spectator labeled "a". The fully connected Faddeev amplitude W can be defined by

$$W_{ab}(Z) \equiv T_{ab}(Z) - \delta_{ab} T_{(a)}(Z).$$

This amplitude satisfies the fully connected Faddeev equation

$$W_{ab}(Z) = -\bar{\delta}_{ab} T_{(a)}(Z) R_o(Z) T_{(b)}(Z) - \sum_d \bar{\delta}_{ad} T_{(a)}(Z) R_o(Z) W_{db}(Z)$$

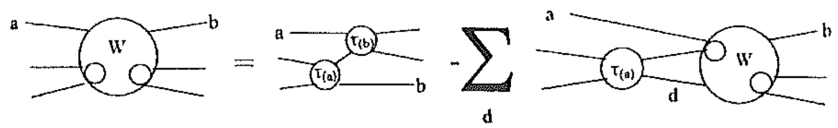


Figure 6 Connected Faddeev equations.

Since there are no disconnected terms in this equation, there are no self-energy bubbles, or singularities due to disconnected clusters. Also, all particle legs represent physical particles. Several of the infinities associated with perturbative approaches to quantum scattering (such as self-energy bubbles/mass renormalization, charge renormalization at non-physical vertices, etc.) are absent in this approach. However, the full complication of the internal kinematics and dynamics are present in the intermediate states parameterized by channels "d" in the sum.

Unitarity of the scattering amplitudes guarantees conservation of probability fluxes. The unitarity of the overall few particle amplitude is guaranteed by the exact unitarity of the input amplitudes $T_{(a)}$, given by

$$T_{(a)}(Z_1) - T_{(a)}(Z_2) = T_{(a)}(Z_1)[R_o(Z_2) - R_o(Z_1)]T_{(a)}(Z_2).$$

The full amplitude T must also satisfy the appropriate unitarity constraint

$$T(Z_1) - T(Z_2) = T(Z_1)[R_o(Z_2) - R_o(Z_1)]T(Z_2)$$
$$T = \sum_{ab} T_{ab}$$

The two-particle input amplitudes T(a) must therefore be embedded into the three-particle dynamics in a way that preserves this unitarity along with cluster decomposability.

Cluster decomposability is essentially the requirement that the kinematics of distant, non-interacting systems should not in any way influence the dynamics of an interacting cluster. This is represented in Figure 7:

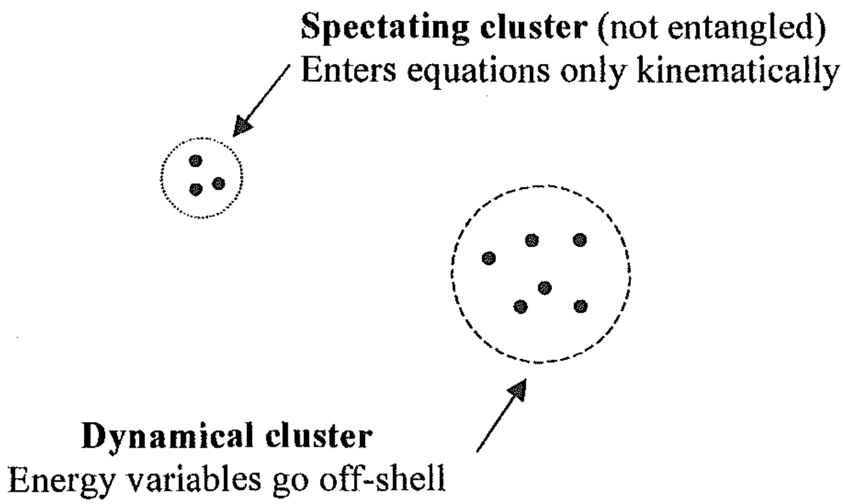


Figure 7 Cluster Decomposition

Cluster Decomposition (which is related to quantum dis-entanglement or decoherence) is **necessary** for the realization of the classical limit! Classical physics requires that systems behave in a manner without quantum entanglement, and therefore, without cluster decomposability, one cannot have correspondence with classical physics.

There are considerable problems with incorporating cluster decomposability in a system whose kinematics is consistent with special relativity. As previously mentioned, the kinematics of the overall system must include the (non-linear) relativistic kinematics of the spectating cluster in a way that does not change the dynamical spectra of the dynamical cluster. Since relativistic kinematics involves non-trivial square root connections between energies and momenta, it is difficult to construct systems that are additive in the kinematic parameters. To do this, one needs to acknowledge the difference between the off-shell ($Z \neq E_{physical}$) and off-diagonal ($M \neq M'$) behavior in order to correctly include the bound states. $\langle M, \hat{q} | T(Z) | M', \hat{q}' \rangle$. The non-trivial accomplishment of our efforts has been to properly embed two-particle scattering into three-particle space in a manner that insures unitarity, relativistic kinematics, and cluster decomposability. The kinematic parameters needed to describe two particle scattering must also include the Lorentz boost velocity that connects the pair rest frame to that of the total system. This parameterization is demonstrated in Figure 8 and Figure 9.

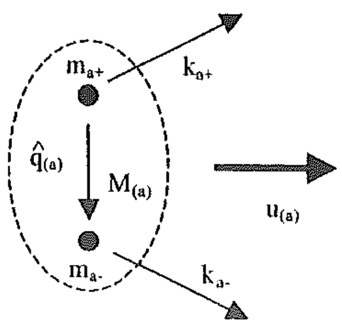


Figure 8 Kinematic description of two-particle state

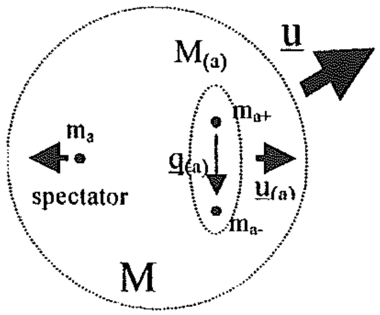


Figure 9 Kinematic parameters of three-particle state

The precise form of the scattering amplitude in the two-particle space is given by

$$\left\langle \underline{k}_{a+}\underline{k}_{a-}\left|t_{(a)}(\zeta)\right|\underline{k}_{a+0}\underline{k}_{a-0}\right\rangle \equiv \left(u^0_{(a)}\right)\delta^3\left(\underline{u}_{(a)}-\underline{u}'_{(a)}\right)\Theta\left(M_{(a)}-(m_{a+}+m_{a-})\right)\Theta\left(M_{(a)0}-(m_{a+}+m_{a-})\right)\otimes \frac{\tau_{(a)}\left(M_{(a)},\hat{q}_{(a)}\mid M_{(a)0},\hat{q}_{(a)0};\zeta\right)}{\sqrt{\rho^{(2)}_{(a)}(M_{(a)})}\sqrt{\rho^{(2)}_{(a)}(M_{(a)0})}}$$

and its embedding into the three particle space has the form

$$\left\langle \underline{k}_{a}\underline{k}_{a+}\underline{k}_{a-}\left|T_{(a)}(Z)\right|\underline{k}'_{a}\underline{k}'_{a+}\underline{k}'_{a-}\right\rangle = \left(u^0\right)\delta^3\left(\underline{u}-\underline{u}'\right)\left(u^0_{(a)}\right)\delta^3\left(\underline{u}_{(a)}-\underline{u}'_{(a)}\right)\frac{\tau_{(a)}\left(\omega_{(a)},\hat{q}_{(a)}\mid \omega'_{(a)},\hat{q}'_{(a)};\zeta_{(a)}\right)}{\sqrt{\left[\rho^{(3)}_{(a)}\left(M,u^0_{(a)}\right)\rho^{(3)}_{(a)}\left(M',u^0_{(a)}\right)\right]}}$$

The Jacobian factors ρ in the denominators involve the variable change from a momentum description of the parameters to a description in terms of the invariant energies, boost velocities, and orientations. The key elements of the solution are as follows:

•Velocity conservation instead of momentum conservation.

The off-energy-diagonal behavior of relativistic systems implies that if momentum is conserved, then the Lorentz frame of reference associated with the different energies must be different. This can be seen in terms of the generators of the Poincare' group $[K_j, P_k] = i\,\delta_{jk} H$. If one chooses a unique frame of reference, one cannot have off-diagonal energies and on-diagonal momenta:

$$\underline{v} = \frac{\underline{P}}{E} \neq \frac{\underline{P}'}{E'} = \underline{v}' \quad unless \quad \underline{P} \neq \underline{P}'.$$

•Spectating cluster kinematics enter only parametrically. If the kinematics of the spectating cluster directly enters calculations involving off-diagonal intermediate states, then its kinematics would directly affect the dynamics of the dynamical cluster. This would violate the requirement of cluster decomposability. The off-energy-shell parameter ζ and the off-energy-diagonal parameter ω that properly describe the two-particle system embedded in the three particle space are given by

$$e_a(M_o, u^0_{(a)}) \equiv u^0_{(a)} \sqrt{m_a^2 + M_o^2 u^2_{(a)}} - M_o u^2_{(a)}$$

$$\zeta_{(a)} = \frac{Z - e_a}{u^0_{(a)}} \quad , \quad \omega_{(a)} = \frac{M - e_a}{u^0_{(a)}}$$

•Natural parameters M, $\underline{u}$, $\underline{u}_{(a)}$, This prevents parametric entanglement of off-diagonal energy parameters with orientation angles and relative Lorentz frames of reference.

The Jacobian factor represented in Figure 10.

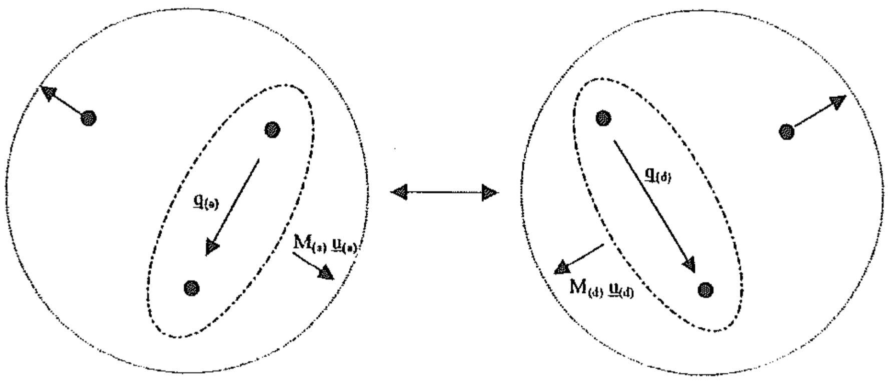


Figure 10 Jacobian of intermediate states

As a demonstration of the complicated kinematics associated with this non-perturbative approach, this Jacobian is expressed below:

$$F_{(ad)}\left(u_{(a)}, u'_{(d)}, \hat{u}_{(a)} \cdot \hat{u}'_{(d)}\right) = \left[\frac{\varepsilon_a^{(a)}\varepsilon_d^{(d)}}{q'_{(a)} q'_{(d)}}\right]^{\frac{1}{2}} \frac{M'^2_{(a)} M'^2_{(d)}}{\varepsilon'_d \varepsilon_a^{(d)} \varepsilon_{d-}^{(d)} u^0_{(a)}} \left\{1 + \left(-\frac{\underline{q}'_{(d)} \cdot \underline{u}'_{(d)}}{\varepsilon_a^{(d)}} + \frac{u'^2_{(d)}}{u^{0'}_{(d)} + 1}\right)\right.$$

$$- \frac{1}{u^0_{(a)}} \left(\frac{M'^2_{(d)} u'^2_{(d)} \underline{q}'_{(d)} \cdot \underline{u}'_{(d)}}{\varepsilon_d \varepsilon_a^{(d)} \varepsilon_{d-}^{(d)}} + \frac{u^{0'}_{(d)} \underline{u}_{(a)} \cdot \underline{q}'_{(d)}}{\varepsilon_{d-}^{(d)}} + \underline{u}_{(a)} \cdot \underline{u}'_{(d)}\right) +$$

$$\left.\frac{\left(\underline{u}'_{(d)} \times \underline{u}_{(a)}\right)}{u^0_{(a)}} \cdot \left(\frac{\underline{q}'_{(d)} \times \underline{u}'_{(d)}}{\varepsilon_a^{(d)}} + \frac{\underline{q}'_{(d)} \times \underline{u}'_{(d)}}{u^{0'}_{(d)} + 1} \left[\frac{M'^2_{(d)} u'^2_{(d)}}{\varepsilon_d \varepsilon_a^{(d)} \varepsilon_{d-}^{(d)}} + \frac{u^{0\,\prime}_{(d)}}{\varepsilon_{d-}^{(d)}}\right]\right)\right\}^{-1}$$

The salient characteristics of the solution are that:

1. The formalism is fully Lorentz invariant, and its on-shell form generates full four-momentum conservation;
2. Unitarity is assured for any scattering amplitude as long as the input interactions are unitary;
3. The amplitudes generated have unique fully off-shell forms, which are appropriate generalizations of the non-relativistic scattering formalism;
4. The formalism satisfies proper cluster decomposability;
5. All amplitudes and parameters involved in calculations have well defined non-relativistic limits, and the equations uniquely go to the Faddeev equations in this limit.

III. Inclusion of particle-antiparticle symmetries

We next turn from fixed particle number scattering theory to one that includes antiparticles and the potential of pair creation. One lesson learned from the fixed number formalism is that the correct choice of parameters and variables is a significant part of the solution. The natural parameters to describe a particle-particle scattering process can be expressed in terms of the 4-momenta in Figure 11:

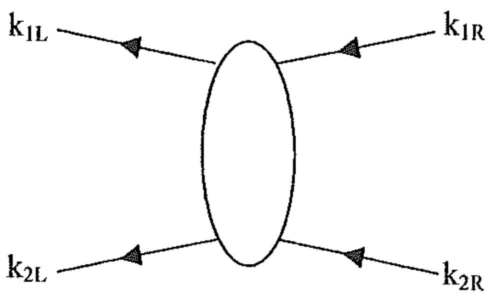


Figure 11 Particle-particle scattering

the Mendelstaam variables given by

$$s_{\frac{L}{R}} \equiv (\vec{k}_{1_{\frac{L}{R}}} + \vec{k}_{2_{\frac{L}{R}}})^2$$
$$t_a \equiv (\vec{k}_{aR} - \vec{k}_{aL})^2$$
$$u_a \equiv (\vec{k}_{aR} - \vec{k}_{\neg aL})^2$$

and the invariant masses and relative orientation parameters:

$$m_a = \sqrt{\vec{k}_a \cdot \vec{k}_a} \qquad -m_a = \sqrt{-\vec{k}_a \cdot -\vec{k}_a}$$
$$\xi_{t_a} \equiv \hat{k}_{aR} \cdot \hat{k}_{aL} \qquad \xi_{u_a} \equiv -\hat{k}_{aR} \cdot \hat{k}_{\neg aL}$$

The four-momentum of a given particle state in the 2-particle center-or-momentum frame can be expressed

$$\vec{k}_a = \left(\varepsilon(M, m_a, m_{\neg a}), q(M^2, m_a, m_{\neg a})\hat{k}_a\right)$$

where the invariant energy and momentum are given by

$$\varepsilon(M, m_1, m_2) \equiv \frac{1}{2M}(M^2 + m_1^2 - m_2^2)$$
$$q^2(M^2, m_1, m_2) \equiv \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}.$$

The distinguishable particle-antiparticle scattering amplitude is parameterized in terms of the particle-particle amplitude in Figure 11 by reversing the signs of the second particle's four-momentum and interchanging incoming and outgoing particle states, as indicated in the particle-antiparticle (bar) channel Figure 12:

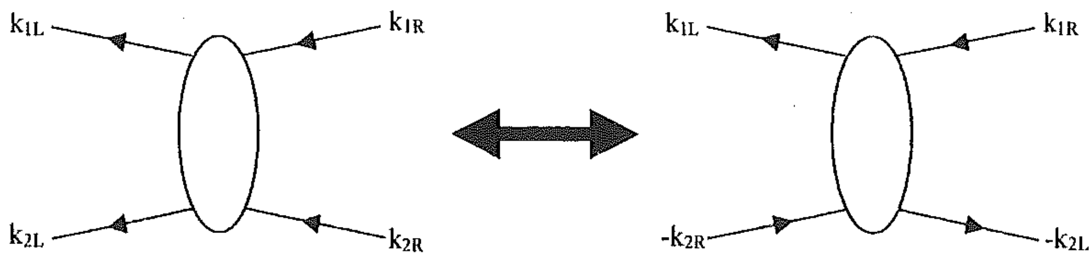


Figure 12 Distinguishable particle-antiparticle scattering

The invariant energies and momenta satisfy

$$(\varepsilon_{1R} - \varepsilon_{1L})^2 - (q_R \hat{k}_{1R} - q_L \hat{k}_{1L})^2 = (\bar{\varepsilon}_{1R} - \bar{\varepsilon}_{1L})^2 - (\bar{q}_R \hat{\bar{k}}_{1R} - \bar{q}_L \hat{\bar{k}}_{1L})^2$$
$$\overline{M}^2_{\substack{L \\ R}} = (\varepsilon_{1^L_R} - \varepsilon_{2^R_L})^2 - (q_{\substack{L \\ R}} \hat{k}_{1^L_R} - q_{\substack{R \\ L}} \hat{k}_{2^R_L})^2 = (\vec{k}_{1^L_R} - \vec{k}_{2^R_L})^2$$

which have on-diagonal forms given by

$$-2q^2(M^2, m_1, m_2)(1 - \xi) = -2q^2(\overline{M}^2, m_1, -m_2)(1 - \bar{\xi})$$
$$\overline{M}^2(M^2, \xi) = \left(\frac{m_1^2 - m_2^2}{M}\right)^2 - 2q^2(M^2, m_1, m_2)(1 + \xi).$$

The corresponding Mendelstaam parameters then satisfy

$$\begin{aligned}
\bar{s} &= \bar{M}^2 \\
\bar{t} &= -2q^2(\bar{M}^2, m_1, -m_2)(1-\bar{\xi}) \\
\bar{u} &= 2(m_1^2+m_2^2) - \bar{M}^2 - \bar{t} \\
\bar{\xi} &= 1 + \frac{2(m_1^2+m_2^2) - \bar{s} + (\bar{t}-\bar{u})}{4q^2(\bar{s}, m_1, -m_2)} = \frac{(m_1^2-m_2^2)^2}{4\bar{s}q^2(\bar{s}, m_1, -m_2)} + \frac{\bar{t}-\bar{u}}{4q^2(\bar{s}, m_1, -m_2)}
\end{aligned}$$

Finally, the identical particle-antiparticle annihilation/pair creation amplitude is parameterized in terms of the particle-particle amplitude in Figure 11 by reversing the signs of the first particle's outgoing four-momentum (changing it to an incoming antiparticle) and reversing the second particle's incoming four-momentum (changing it to an outgoing antiparticle), as indicated in the particle-antiparticle (bar) channel Figure 13:

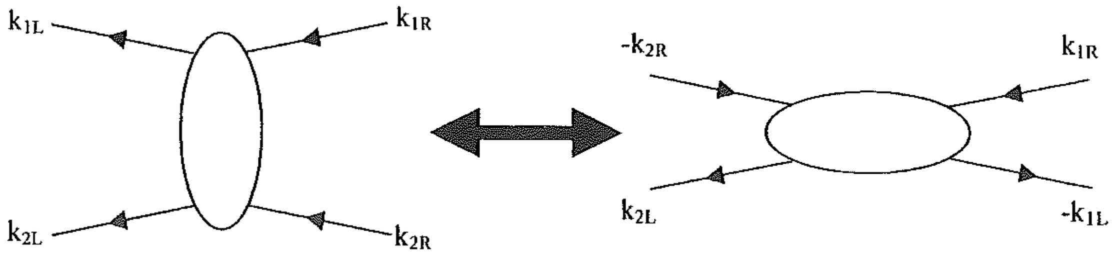


Figure 13 Annihilation/creation channel scattering

The invariant energies and momenta satisfy

$$\begin{aligned}
(\varepsilon_{1R} - \varepsilon_{2L})^2 - (q_R\hat{k}_{1R} + q_L\hat{k}_{1L})^2 &= (\varepsilon_{1RX} - \varepsilon_{2LX})^2 - (q_{RX}\hat{k}_{1RX} + q_{LX}\hat{k}_{1LX})^2 \\
M_{aX}^2 = (\varepsilon_{1aR} - \varepsilon_{aL})^2 - (q_R\hat{k}_{aR} - q_L\hat{k}_{aL})^2 &= (\vec{k}_{aR} - \vec{k}_{aL})^2
\end{aligned}$$

which have on-diagonal forms given by

$$\left(\frac{m_1^2-m_2^2}{M}\right)^2-2q^2(M^2,m_1,m_2)(1+\xi)=\left(\frac{m_1^2-m_2^2}{M_X}\right)^2-\left(q_{1X}^2+q_{2X}^2+2q_{1X}q_{2X}\xi_X\right)$$
$$M_X^2(M^2,\xi)=-2q^2(M^2,m_1,m_2)(1-\xi)$$

The corresponding Mendelstaam parameters then satisfy

$$s_X=M_X^2$$
$$u_X=\left(\frac{m_1^2-m_2^2}{M_X}\right)^2-\left(q_{1X}^2+q_{2X}^2+2q_{1X}q_{2X}\xi_X\right)$$
$$t_X=2(m_1^2+m_2^2)-M_X^2-u_X$$
$$\xi_X=\frac{(m_1^2-m_2^2)^2}{2q(s_X,m_1,-m_1)q(s_X,m_2,-m_2)M_X^2}+\frac{t_X-u_X}{4q(s_X,m_1,-m_1)q(s_X,m_2,-m_2)}$$

Therefore, once an analytic form has been established for the particle-particle scattering amplitudes in a fixed-particle-number scattering formalism, antiparticle scattering, annihilation, and pair creation amplitudes can be generated in a straightforward manner.

IV. Generation of non-perturbative amplitudes

A direct calculation of a physical scattering process will be demonstrated to illustrate the usefulness of this formulation. One possible choice that can only be calculated using this type of formulation is demonstrated in Figure 14.

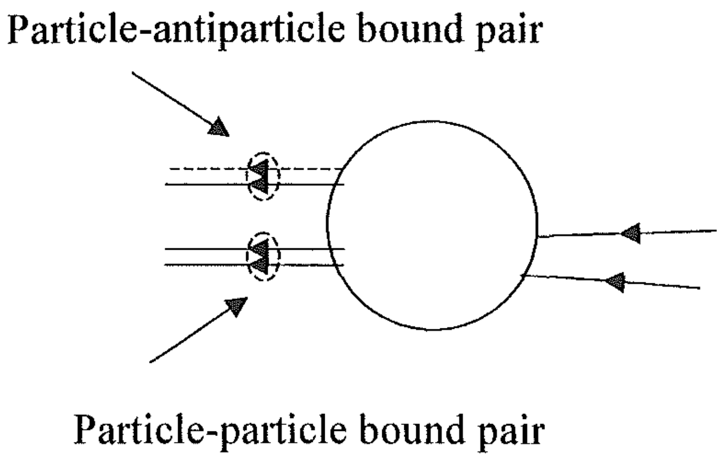


Figure 14 Pair creation from particle-particle scattering

A three-particle scattering amplitude could, in principle, be folded into a two particle scattering into bound-pair/bound particle-antiparticle final state, requiring an analytic form from which bound state extraction in the final state could be done. However, a more direct calculation that illustrates our formulation is demonstrated in Figure 15

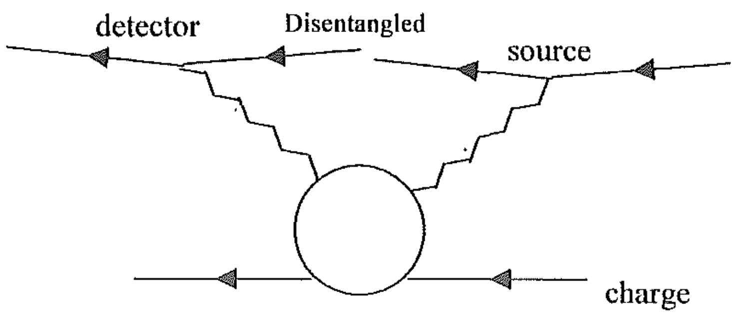


Figure 15 Compton Scattering and the Quantum-Classical Transition

This figure represents a three-particle form for standard Compton scattering of a photon from a charged particle. In the classical limit, the source and the detector must interact only through the photon scattering with the charged particle. Clearly, cluster decomposability must be a property with regards to direct source-detector interactions. The cluster decomposable and unitary form should result in an amplitude which satisfies the optical theorem. In this calculation, we do *not* use analyticity as a defining characteristic of the amplitudes. This means that we are not "fishing" amongst the various analytic properties of the amplitude to determine one that satisfies unitarity and symmetry properties. Instead, the relative properties of particles and antiparticles are defined *only* through the transformation properties of the amplitudes (i.e. through their interactions). New particle types will not be produced by the transformation (i.e., left handed massless neutrino is associated only with right handed anti-neutrino)

V. Compton scattering

We now calculate the amplitude associated with the emission of a photon from an excited source, the subsequent distant scattering of that photon from a charged particle, and its subsequent asymptotic absorption to place a detector in an excited state. This scattering is represented in Figure 16:

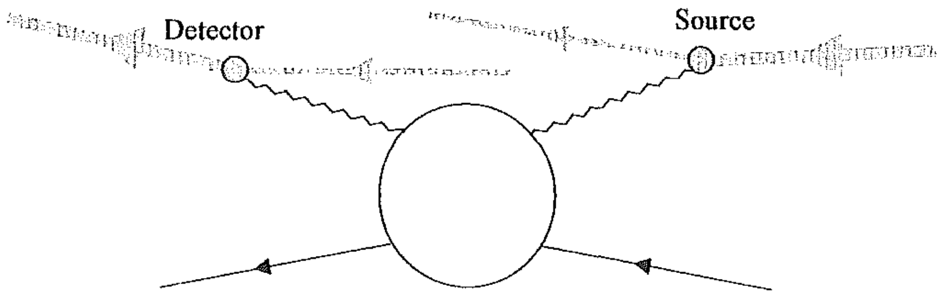


Figure 16 Compton Scattering

The fully connected Faddeev amplitude that corresponds to this scattering involves only the driving term demonstrated in Figure 17

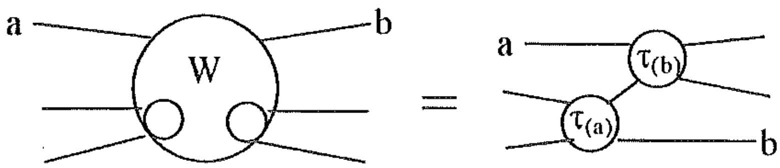


Figure 17 Connected Faddeev Compton amplitude

The various clusters involved in Compton scattering have features that guide the calculation. The source cluster interacts with the particle via a two-body unitary scattering amplitude from which one can extract the quantum in question. In order for this process to include the kinematic possibility of describing the emission of a quantum that can be interpreted as a boundary state, this scattering must be *anelastic* in the sense that the source changes mass when the quantum is emitted. Similarly, the detector must engage in a unitary *anelastic* two-body scattering with the particle.

A. Quantum entanglement

The specific postulate needed to connect Compton scattering to a three-particle scattering process is that the source and detector be disentangled from each other, except through the interaction with the particle. This then means that the underlying philosophy used here is the *Wheeler-Feynmann* point of view that everything that can be accomplished by treating photons as particles can equivalently be accomplished by treating them as implicit characteristics of particle-particle interactions. The apparent particulate behavior of the photons must then be extracted from their interaction with the source/detector.

B. Extracting photons

Asymptotic particulate photon states are indistinguishable from asymptotically emitted and absorbed electromagnetic interactions. In order to construct a viable three-particle representation of Compton scattering, the kinematics of photons as asymptotic states should be extracted from the source/detector behavior. This equivalence is illustrated in Figure 18:

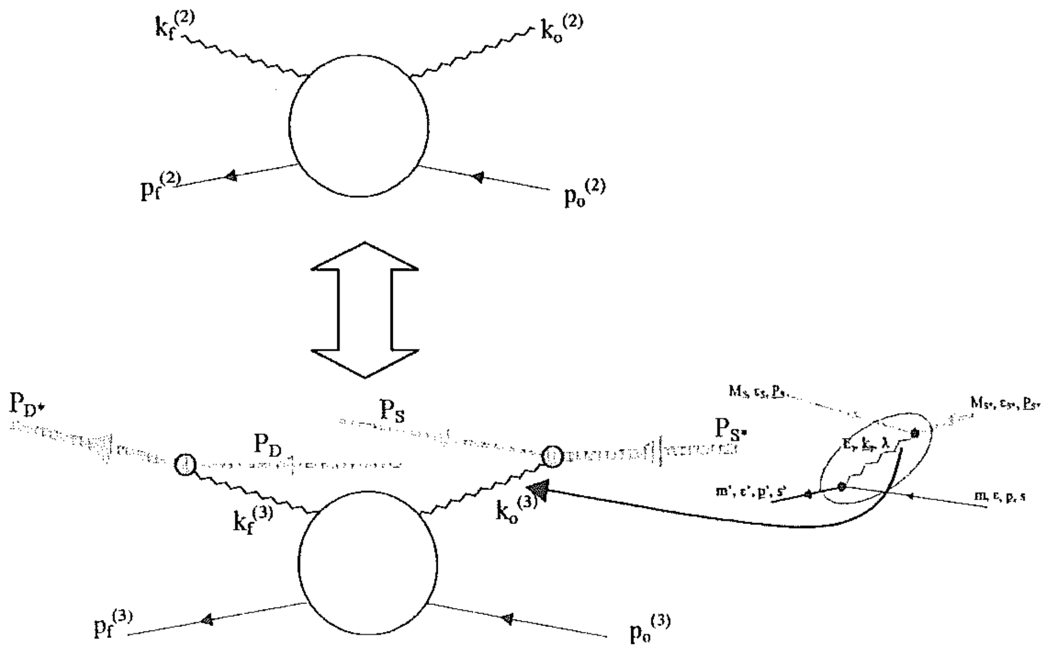


Figure 18 Photons as particles and quanta

The explicit diagrams representing photon scattering from a charged particle are given in Figure 19

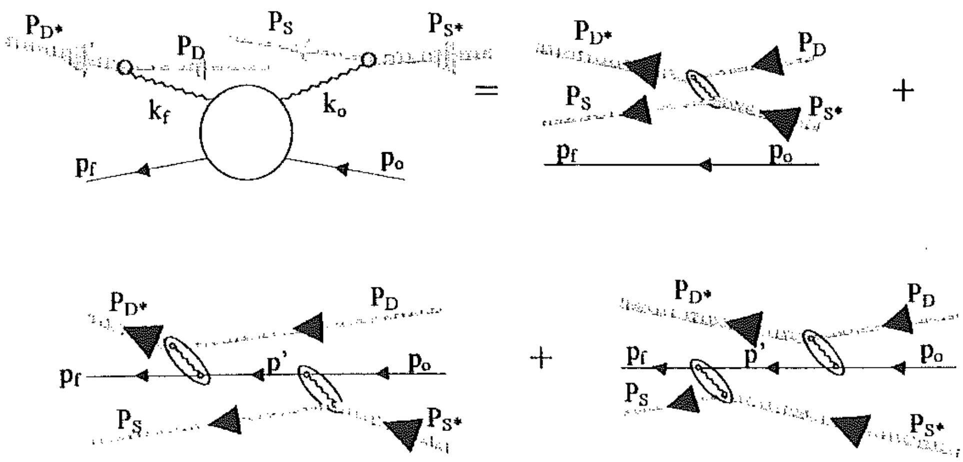


Figure 19 Compton scattering diagrams

The kinematic parameters labeling excited states are labeled with an asterisk. The classical dis-entanglement of the source and detector requires that there be no direct interaction between these components during the scattering process in Figure 20.

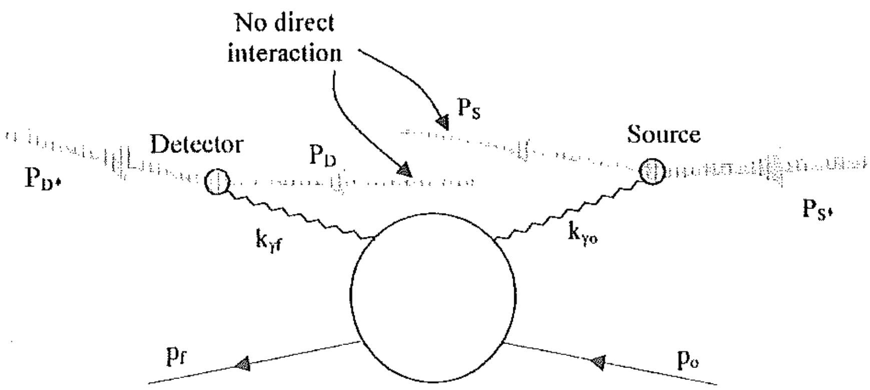


Figure 20 Dis-entanglement of source and detector

The Compton driving terms should incorporate a relativistic propagator that properly incorporates the causal contributions of particles and antiparticles as illustrated in Figure 21.

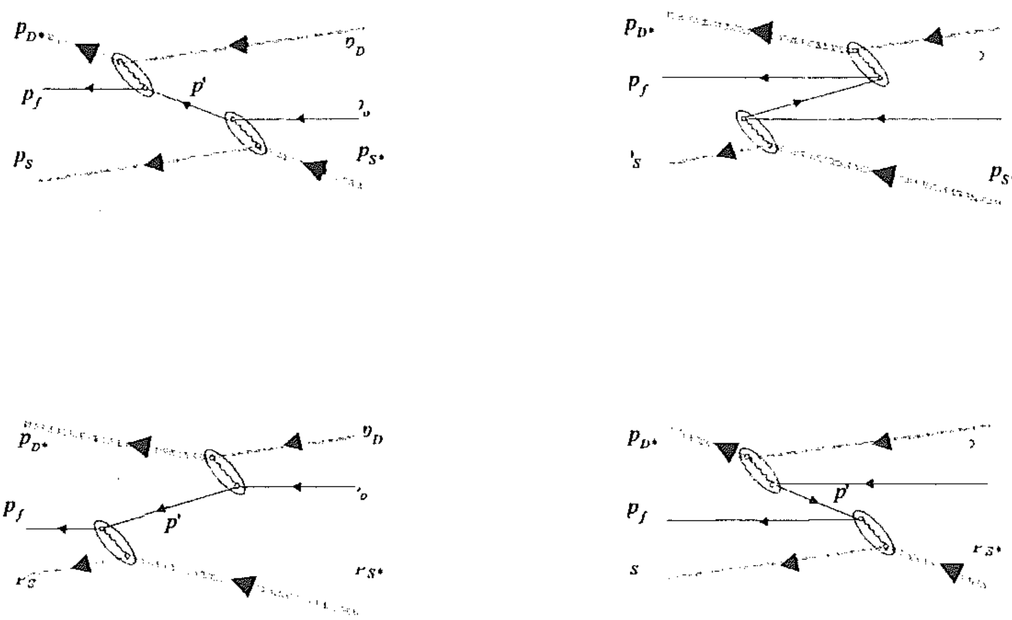


Figure 21 Relativistic propagators for Compton driving term

This form allows us to use the conventional interpretation of negative energy particles as positive energy antiparticles in the construction of the usual causal propagator for the problem at hand. The S-matrix for the scattering is constructed using the usual identification S=1+iT. The unit term in the S-matrix for the process computed here corresponds physically to the self consistent calibration of the source and detector with the flux normalization of the transition amplitude.

Figure 21

$$A(k_f,\lambda_f;p_f,s_f \mid k_o,\lambda_o;p_o,s_o) =$$
$$\sum_{s'}\left(\frac{1}{2\pi i}\right)^2 \left(\Gamma^{\mu^*}(p_f,s_f;p_o+k_o,s')e_\mu^*(k_f,\lambda_f)\frac{2m\wp_+(p_o+k_o)}{m^2-(p_o+k_o)^2}e_\nu(k_o,\lambda_o)\Gamma^\nu(p_o,s_o;p_o+k_o,s') \right.$$
$$\left. +\Gamma^\nu(p_o,s_o;p_o-k_f,s')e_\nu(k_o)\frac{2m\wp_+(p_o-k_f)}{m^2-(p_o-k_f)^2}e_\mu^*(k_f)\Gamma^{\mu^*}(p_f,s_f;p_o-k_f,s') \right).$$

The couplings Γ can be directly calculated to first order in the electromagnetic charges. The result, taken from the reference [11], has the form

$$A(k_f,\lambda_f;p_f,s_f \mid k_o,\lambda_o;p_o,s_o) \approx$$
$$\left(\frac{1}{2\pi i}\right)^2 \left(\bar{\mathbf{u}}(p_f,s_f)\frac{q}{c}\gamma^\mu e_\mu^*(k_f,\lambda_f)\frac{\{m\mathbf{1}+\not{p}_o+\not{k}_o\}}{m^2-(p_o+k_o)^2-i0^+}e_\nu(k_o,\lambda_o)\frac{q}{c}\gamma^\nu\mathbf{u}(p_o,s_o)+\right.$$
$$\left.\bar{\mathbf{u}}(p_f,s_f)\frac{q}{c}\gamma^\nu e_\nu(k_o,\lambda_o)\frac{\{m\mathbf{1}+\not{p}_o-\not{k}_f\}}{m^2-(p_o-k_f)^2-i0^+}e_\mu^*(k_f,\lambda_f)\frac{q}{c}\gamma^\mu\mathbf{u}(p_o,s_o)\right)=$$
$$\alpha\left(\frac{1}{2\pi i}\right)^2 \left(\bar{\mathbf{u}}(p_f,s_f)\gamma^\mu e_\mu^*(k_f,\lambda_f)\frac{\{m\mathbf{1}+\not{p}_o+\not{k}_o\}}{m^2-(p_o+k_o)^2-i0^+}e_\nu(k_o,\lambda_o)\gamma^\nu\mathbf{u}(p_o,s_o)+\right.$$
$$\left.\bar{\mathbf{u}}(p_f,s_f)\gamma^\nu e_\nu(k_o,\lambda_o)\frac{\{m\mathbf{1}+\not{p}_o-\not{k}_f\}}{m^2-(p_o-k_f)^2-i0^+}e_\mu^*(k_f,\lambda_f)\,\gamma^\mu\mathbf{u}(p_o,s_o)\right),$$

where the electromagnetic coupling constant α (which must be input from other considerations) is explicitly demonstrated. This solution exactly corresponds to that obtained using second order perturbative quantum electrodynamics. The inclusion of dynamical electromagnetic couplings into a fixed particle number scattering formulation with relativistic kinematics and physically consistent cluster decomposability, classical, and non-relativistic correspondence, has been demonstrated. The parameters in the prior non-perturbative result (or its approximate form) can be immediately related to the corresponding pair annihilation process in precisely the same manner as is done using perturbative quantum electrodynamics. We therefore claim that this is the first fixed particle number scattering formulation that successfully includes pair creation as well as the potential for multi-quantum extractions as attributes of its formalism.

VI. **Conclusions and discussion**

We have demonstrated a successful calculation of a verified experimental result using calculational methods consistent with those utilized by ANPA approaches to natural philosophy. The motivation of this particular presentation has been to address a specific question posed by founding members of ANPA. The problem posed to us was how might combinatorial results be incorporated into physically testable models or formulations.

Our approach has been to clearly separate the analytic forms needed to describe the dynamics (which vary amongst the hierarchy of fundamental interactions) from the analytic behavior needed to embed that dynamics into the space-time kinematics associated with multiple frames of reference. We have demonstrated two calculations (relativistic Coulomb and Compton scattering) where combinatorial couplings (in the demonstrated cases, the electromagnetic coupling α) can be incorporated into physically meaningful processes. The space-time kinematics need only be consistent with discrete energies and momenta that satisfy the principles of special relativity on the usual coarse scales associated with experiment.

We have circumvented any need to develop complicated renormalization methodologies, at the expense of the necessity of developing complicated kinematic descriptions of intermediate quantum coherent states needed to describe interacting systems in terms of physical particle boundary states. However, in the process we have developed well defined methods that allow correspondence of the amplitudes calculated in our formulation with those that can be obtained using suitably renormalized perturbative techniques.

Acknowledgements

The author gratefully acknowledges many useful and insightful discussions with Walter Lamb, Marcus Alfred, Petero Kwizera, Alex Markevich, Valodya Karmanov, and Richard Blankenbecler. Discussions with Viv Pope on interpretations of the asymptotic state of photons have also been helpful. This presentation has been motivated by helpful discussions and questions expertly posed by Ted Bastin and Clive Kilmister. The body of work presented summarizes the successful efforts of several decades of conceptual and technical formulations of physical processes as espoused by H.P. Noyes, whose mentorship and guidance has been crucial in its completion.

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CLASSICAL MECHANICS FROM QUANTUM MECHANICS VIA COMMUTATORS draft 5

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ABSTRACT

Begin with a classical model of structureless point particles having coordinates $\underline{q}$ that are functions of time; specify neither the connectivity nor the dimension of the continuous space in which the particles move. Write an hierarchy of identities for $\dot{\theta},\ddot{\theta},...$ where $\theta(\underline{q})$ is any function characteristic of the model. Quantize these. The quantizations are not identities. They limit the forms permitted for θ and the QM Hamiltonian H. If θ is arbitrary H is either linear (particle physics) or quadratic (classical physics). Choosing quadratic H, determine the condition under which θ satisfies the first two quantizations; this is thought to be a predictivity condition. It turns out that θ must satisfy a fourth order PDE. Reverse-quantize H to determine the physical meaning of the coefficient operators. Identify θ with one or more of these; the PDEs become new field equations.

Quantization restricts to flat coordinates and a flat space. Nevertheless the PDEs, derived for the most general quadratic H, are believed to lead to a new tensor field equation for GR. This equation is discussed.

INTRODUCTION

There follows an outline of the project described in the Abstract. I believe that this project has reached a degree of maturity that now merits our attention. It may be that the results are nonsense; the argument has considerable scope for error. But, if there is nonsense, it is deep, intriguing nonsense and still deserves our attention! On the other hand, if the results are *bone fide* then, we have the beginnings of a new way to understand macrophysics. The Correspondence Principle has been eschewed.

The argument is abstract and there is a great deal of mathematics. Some of the mathematics is of intrinsic interest. But much is tedious calculation. I have therefore chosen to base this outline in a series of flow charts. The rules for interpreting these diagrams are simple: text in oval boxes expresses human input (hypotheses, choices, assignments); text in sharp-cornered boxes indicates well defined calculation; text in round-cornered boxes outlines results; arrows indicate the flow of either inference or data.

The mathematics is the standard operator algebra of Quantum Mechanics (QM). Operators, having Hermitian symmetry and real spectra, represent real observables. The spectra, which may be either discrete or continuous or partly both, are the sets from which measured values are drawn. The operators may or may not commute (and therein lies the richness of QM).

Constant scalars (c-numbers) represent either numerical or physical constants. The time is represented by a scalar t that commutes with all operators. Time in QM, therefore, cannot be measured directly. Rather clocks are devices that measure a coordinate q_0; and they are designed so that q_0 keeps roughly in step with t. Errors, however, are unavoidable in principle [1], [2].

Most of the rules and formulae required have been gleaned from the standard literature of QM; but I have organised and generalised them. In particular the commutators, that naturally appear in QM, are intimately connected with the calculus; and some of the patterns of CM (such as Hamilton's equations expressed in Cartesian coordinates) are merely the consequences of the algebra of commutators.

These results, and many others, are independently arrived at in the papers of Louis Kauffman [3]. But Louis' work covers a much wider canvas. He is concerned with operators (and their commutators) that are the elements of free algebras. Their spectra may not be real; and, in many of the contexts that he examines, this fact is of no consequence. He shows, by examining various frameworks of definition, that commutators obey and, indeed, explain many of the patterns of mathematics and theoretical physics.

OVERVIEW- Fig. 1

Fig. 1 presents an overview of a recipe by which new field equations are derived from the formalism of QM. This formalism is applied to a rudimentary classical model; see Fig. 2.

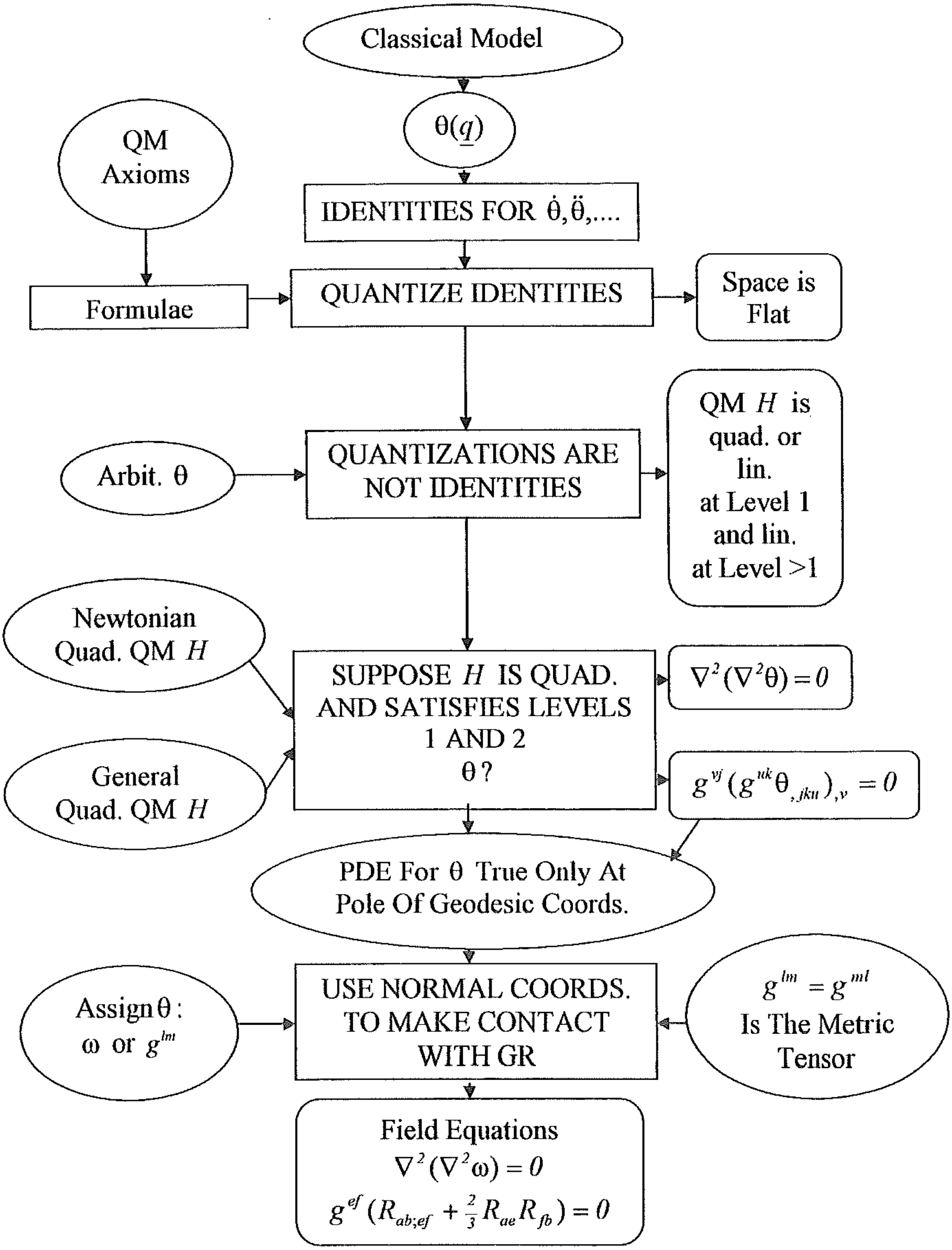
Structureless point-particles move in a differentiable manifold. Nothing is known about either the number and interaction of the particles or the dimension and connectivity of the space. The coordinates of the particles $\underline{q}$ are continuous functions of a continuous time measure t. This time is that (of the consciousness and of the clock) belonging to a single observer who monitors the movement of the particles (by radar or some such). As yet we do not insist that t is identical with the quantum time. Indeed, if we talk about consciousness, clocks and observation then, it is more consistent to identify t as the clock coordinate q_0. Notice that because, in this model, the coordinates are functions of time we may choose any one of them to equal t.

Associated with the system of particles is a function of their coordinates $\theta(\underline{q})$ that has physical significance. This function might be, for example, a scalar potential, an element of a vector potential, a field strength or an element of a metric tensor. It is possible to write an hierarchy of differential identities that express $\dot{\theta} \equiv d\theta / dt, \ddot{\theta}, \dddot{\theta}, \ldots$ in terms of the partial derivatives $\theta_{,j} \equiv \partial\theta / \partial q^j, \theta_{,jk}, \theta_{,jkl}, \ldots$ and the rates $\dot{\underline{q}}, \ddot{\underline{q}}, \dddot{\underline{q}} \ldots$; see Fig. 2.

We require to quantize the identities; and for this we need Axioms and consequent Formulae; see Figs. 3 and 4. In the process we introduce coordinate operators $\underline{Q}$, their conjugates $\underline{P}$, an operator Θ representing $\theta(\underline{q})$ and an operator H that drives the dynamics of the quantized system; this last plays the same role as the classical Hamiltonian. A limitation imposed by the Axioms is that the coordinates must be flat (e.g., Cartesian or Galilean) and, in consequence, the space is likewise.

We find that the quantizations (operator equations) are not algebraic identities. That is they are not satisfied by arbitrary pairs of the operators

Fig 1- OVERVIEW



$\Theta(\underline{Q})$ and $H(\underline{P},\underline{Q})$. Thus the quantizations are Constraints (Kauffman); and, therefore, they imply law. It can be shown that if θ is arbitrary then the first level quantization (see Fig. 5) is satisfied only by H that is a polynomial in the $\underline{P}$ of order either 1 (linear) or 2 (quadratic); the coefficients in the quadratic polynomial commute with the $\underline{Q}$. The second and higher level quantizations are satisfied only by H that is linear in the $\underline{P}$.

Macrophysics is characterised by quadratic Hamiltonians; microphysics, in so far as it is amenable to the Hamiltonian method, is characterised by linear Hamiltonians (Dirac). So here, given our objective of explaining the structure of CM using QM, we confine discussion to quadratic Hamiltonians. We also consider only the first two levels of quantization. This is partly because the higher levels involve very complex expressions; but also, it turns out, the higher levels are less important to our objective.

There are reasons to believe that a quadratic Hamiltonian, that satisfies *both* quantizations 1 and 2, provides a more accurate description of the dynamics than an arbitrary quadratic Hamiltonian (that satisfies only quantization 1); see Fig. 6. But, given such an Hamiltonian, θ must be restricted so that it also satisfies quantization 2.

With the 'Newtonian' Hamiltonian (Cartesian coordinates)

(1) $$H \equiv \tfrac{1}{2m}\sum_{j=1}^{3} P_j^2 + \Omega(\underline{Q}); \quad \underline{Q} \equiv \underline{q}I; \quad \Omega(\underline{Q}) \equiv \omega(\underline{q})I;$$ see Fig. 6

this restriction can be expressed as

(2) $$\nabla^2(\nabla^2\theta) = 0$$

Now (1) represents the motion in E3 (Euclidean 3-space) of a single particle of mass m under a scalar potential $\omega(\underline{q})$. Notice, however, that the scalar potential ω does not appear in (2). Given that quantization 2 is satisfied, variation of H implies variation of θ; so $\theta(\underline{q})$, which is a function characteristic of the system, must then appear as a term in H. The only function of the coordinates that appears in the Hamiltonian (1) is the scalar potential ω. Therefore we assign

(3) $\quad \theta \equiv \omega \Rightarrow \nabla^2(\nabla^2\omega) = 0$

Any solution of the familiar

(4) $\quad \nabla^2\omega = 0$

is a solution of (3). It follows that the Hamiltonian (1) given (3) is suitable to describe the low-speed motion of a single particle, in E3, under either scalar Newtonian gravity or an electrostatic potential.

But the solutions of (3) involve terms additional to the solutions of (4). To have evaded observation these additional terms must be either very small or zero. For example the spherically symmetric (S-S) solution of (3) is ($k_1,\ldots,k_4$ are constants of integration)

(5) $\quad \omega = \dfrac{k_1}{r} + k_2 r^2 + k_3 r + k_4$

whereas the S-S solution of (4) is

(6) $\quad \omega = \dfrac{k_1}{r} + k_4$

These solutions are *both* suitable to the field of a point source at the origin.

The most general quadratic Hamiltonian (see Figs.5,6 and 7), that has the necessary Hermitian symmetry and coefficients that commute, is

$$H \equiv \frac{K}{2}[G^{uv}(\underline{Q})P_u P_v + P_u P_v G^{uv}(\underline{Q})]$$

(7) $$+\frac{1}{2}[F^j(\underline{Q})P_j + P_j F^j(\underline{Q})] + V(\underline{Q});$$

$$G^{uv} \equiv g^{uv}(\underline{q})I; \quad g^{uv} = g^{vu}; \quad F^j \equiv f^j(\underline{q})I; \quad V \equiv v(\underline{q})I$$

where K is a scalar constant and the suffices run over the coordinates of all the particles. The Einstein summation convention in force. When we enquire as to the restriction on θ, needed to ensure that quantization 2 is satisfied given (7), we find

(8) $g^{vj}(g^{uk}\theta_{,jku})_{,v} = 0$; Einstein summation convention in force

Notice that the functions $f^j(\underline{q})$ and $v(\underline{q})$ have cancelled out. Any one of the functions $g^{uv} = g^{vu}$, f^j or v are candidates for θ. Concentrating on the g^{uv}, the single partial differential equation (PDE) (8) then yields the set

(9) $g^{vj}(g^{uk}g^{lm}{}_{,jku})_{,v} = 0$; see Fig. 7

The functions g^{uv} can be regarded as elements of the metric tensor of a Riemannian space of dimension n_c; the matrix g^{uv} must, of course, have an inverse g_{jk}. There is reason to believe that this Riemannian space has physical significance. In general its dimension n_c is not the dimension n of the space of the classical model; see Figs 1 and 2. That is, unless there is but one particle in the model, $n_c > n$. In what follows we refer to the Riemannian space as the Coordinate Space (dimension n_c) and to the space containing the particles, in the classical model, as the Particle Space (dimension n).

The coordinates of a point $\underline{q}$, in the coordinate space, are subject to the Axioms; see Fig 3. So they are flat as is the space. Further, even assuming that g^{uv} is a metric tensor, (9) is not a tensor equation unless the space is flat. A way out of this impasse is to assume that (9) is true only at the pole of geodesic coordinates in the Riemannian coordinate space. Geodesic coordinates can be chosen to be flat in the neighbourhood of the pole; and, in that neighbourhood, the space is effectively flat because

(10) $g^{uv}_{,k} = 0$

at the pole. Elsewhere the space can be curved, the functions g^{uv} more general and we have freedom to use appropriate coordinates.

Any point in the space can be chosen as the pole of geodesic coordinates; and, within certain requirements of continuity and differentiability, a tensor equation is true at all points in the space and using any coordinate system. The question therefore arises: what tensor equation reduces to (9) at the pole of geodesic coordinates in a Riemannian space? This question has been answered by Clive Kilmister. Using normal coordinates [4], [5] as geodesics he gets

(11) $g^{ef}(R_{ab;ef} + \frac{2}{3}R_{ae}R_{fb}) = 0$; R_{ab} is the covariant Ricci tensor

By assuming that there is but a single particle ($n_p = 1$) and setting the dimension of the particle space to four ($n = 4$) we can then make contact with GR; there are, however, additional complications. See Figs. 6,7 and 8.

If we take the original classical model literally, the space between the point particles is empty; (11) then applies only to empty space. Notice that solutions of

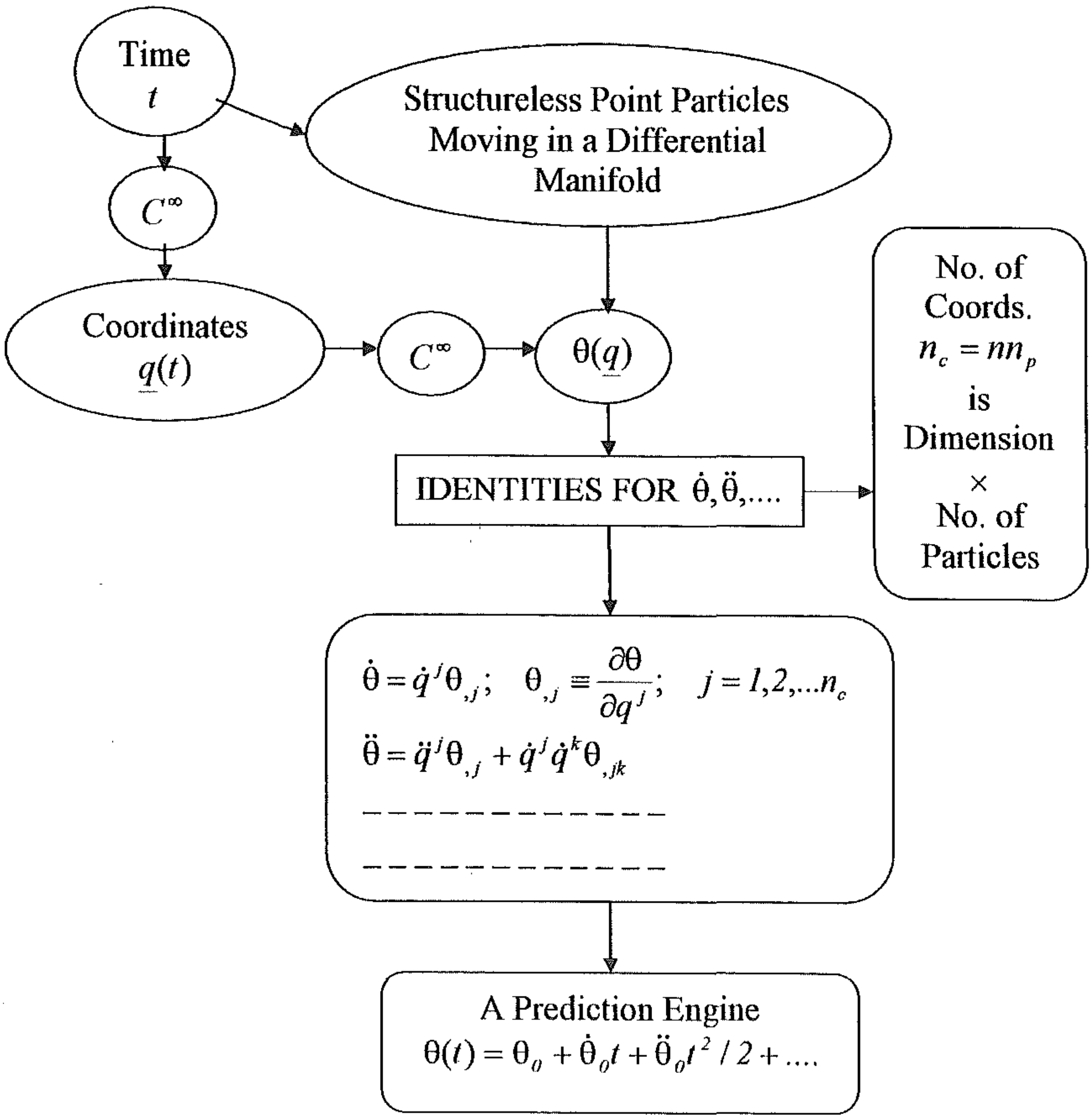
(12) $R_{ab} = 0$

are always solutions of (11). The tensor equation (12) is Einstein's original field equation for gravity in empty space. The relation between (11) and (12) thus echoes the relationship between (3) and (4).

MODEL AND IDENTITIES- Fig. 2

Fig. 2 assumes that θ and the $\underline{q}(t)$ are indefinitely differentiable. Subsequent results suggest that it is sufficient for the present purpose if these functions are class C^4. The figure quotes the first two identities. Notice that these formulae use the Einstein summation convention. The Taylor's expansion shows that the set of identities can be used to predict θ as a function of t in terms of initial values of the coordinates and their rates $\underline{q}, \dot{\underline{q}}, \ddot{\underline{q}}, \dddot{\underline{q}}$.... It is, of course, necessary to know the function $\theta(\underline{q})$ to carry this out.

Fig 2- MODEL & IDENTITIES



AXIOMS- Fig. 3

With a single exception the axioms required for the quantization of the identities are the same as the axioms used by Schrödinger to set up his famous wave equation for the hydrogen atom. The extra axiom leads to a standard definition for rate operators. We have merely assumed that these axioms apply in general. The consequences are remarkable.

Schrödinger begins with a classical expression for the total energy as the sum of the kinetic and the potential energies. Notice that he uses Cartesian coordinates. There is the apparent assumption that the proton does not move. In fact the electron mass m can be corrected for the motion of the proton leaving the appearance of the expression for the energy unchanged. The formula given in the figure denotes the potential energy as a general function of the coordinates $v(\underline{q})$ rather than the specific electron-proton electrostatic potential $-\varepsilon^2/r$ used by Schrödinger.

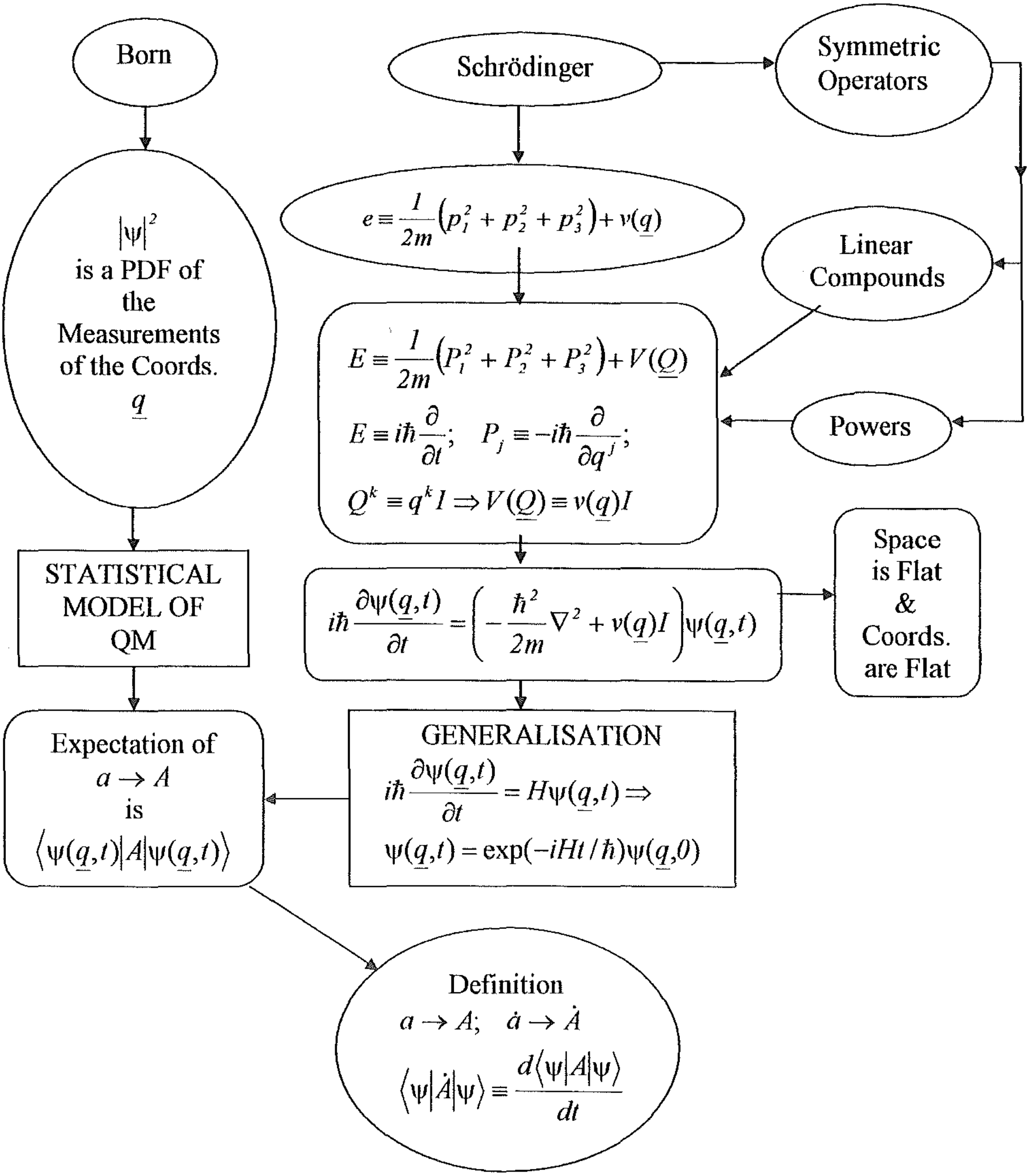
The coordinates and functions of the coordinates are replaced by themselves multiplied by the unit operator; the result is always self-adjoint (Hermitian symmetric with a continuous, real spectrum). The energy and momenta are replaced by self-adjoint differential operators; the structure is otherwise unchanged. The self-adjointness property is intimately linked with the fact that the range of t and the spectra of the operators $\underline{P}$ and $\underline{Q}$ occupy the whole of $[-\infty,\infty]$. Each side of the resulting operator equation is made to act on a function $\psi(\underline{q},t)$; thus the wave equation is complete.

Notice that because the structure is unchanged, when scalars are replaced by operators, we can infer that a weighted sum (linear combination) goes to a weighted sum; and powers go to powers. Thus

(13) $\alpha a+\beta b \rightarrow \alpha A+\beta B$ and $a^n \rightarrow A^n$

where α,β are c-numbers, a,b are scalar observables, A,B their representative operators and n is a positive integer. Because the kinetic energy operator does not commute with the potential energy operator we can infer that the first axiom (13) holds whether or not A and B commute.

Fig. 3- AXIOMS


Born
$|\psi|^2$ is a PDF of the Measurements of the Coords. $\underline{q}$
STATISTICAL MODEL OF QM
Expectation of $a \to A$ is $\langle \psi(\underline{q},t) | A | \psi(\underline{q},t) \rangle$
Schrödinger
Symmetric Operators
$e \equiv \frac{1}{2m}(p_1^2 + p_2^2 + p_3^2) + v(\underline{q})$
Linear Compounds
$E \equiv \frac{1}{2m}(P_1^2 + P_2^2 + P_3^2) + V(\underline{Q})$
$E \equiv i\hbar \frac{\partial}{\partial t}; \quad P_j \equiv -i\hbar \frac{\partial}{\partial q^j};$
$Q^k \equiv q^k I \Rightarrow V(\underline{Q}) \equiv v(\underline{q}) I$
Powers
$i\hbar \frac{\partial \psi(\underline{q},t)}{\partial t} = \left(-\frac{\hbar^2}{2m} \nabla^2 + v(\underline{q}) I \right) \psi(\underline{q},t)$
Space is Flat & Coords. are Flat
GENERALISATION
$i\hbar \frac{\partial \psi(\underline{q},t)}{\partial t} = H \psi(\underline{q},t) \Rightarrow$
$\psi(\underline{q},t) = \exp(-iHt/\hbar) \psi(\underline{q},0)$
Definition
$a \to A; \quad \dot{a} \to \dot{A}$
$\langle \psi | \dot{A} | \psi \rangle \equiv \frac{d \langle \psi | A | \psi \rangle}{dt}$

We have one remaining axiom to establish. If $a \to A$ and $\dot{a} \to \dot{A}$ how do we define $\dot{A}$? Heisenberg gave a definition that leads to the same formula as the definition given in the figure; the later is based on Born's statistical interpretation of ψ. Both definitions are standard.

Note that, as a consequence of the axioms, t *is* the quantum time [1], [2]. Further, with the exception of the unitary operator $\exp(-iHt/\hbar)$, all the operators are independent of t. This is the Schrödinger Representation.

FORMULAE- Fig. 4

The formula quoted in the first box of the figure is perhaps the most famous result in QM; it follows from the representations used by Schrödinger; see Fig. 3. We also conclude, from these representations, that all coordinates mutually commute, all momenta mutually commute and only coordinates and their conjugate momenta do not commute.

A consequence of these facts is that: commutators with P_j behave like partial derivatives with respect to q^j; and commutators with Q_k behave like partial derivatives with respect to p_k. Louis Kauffman calls commutators like this Derivations. It follows from the formulae

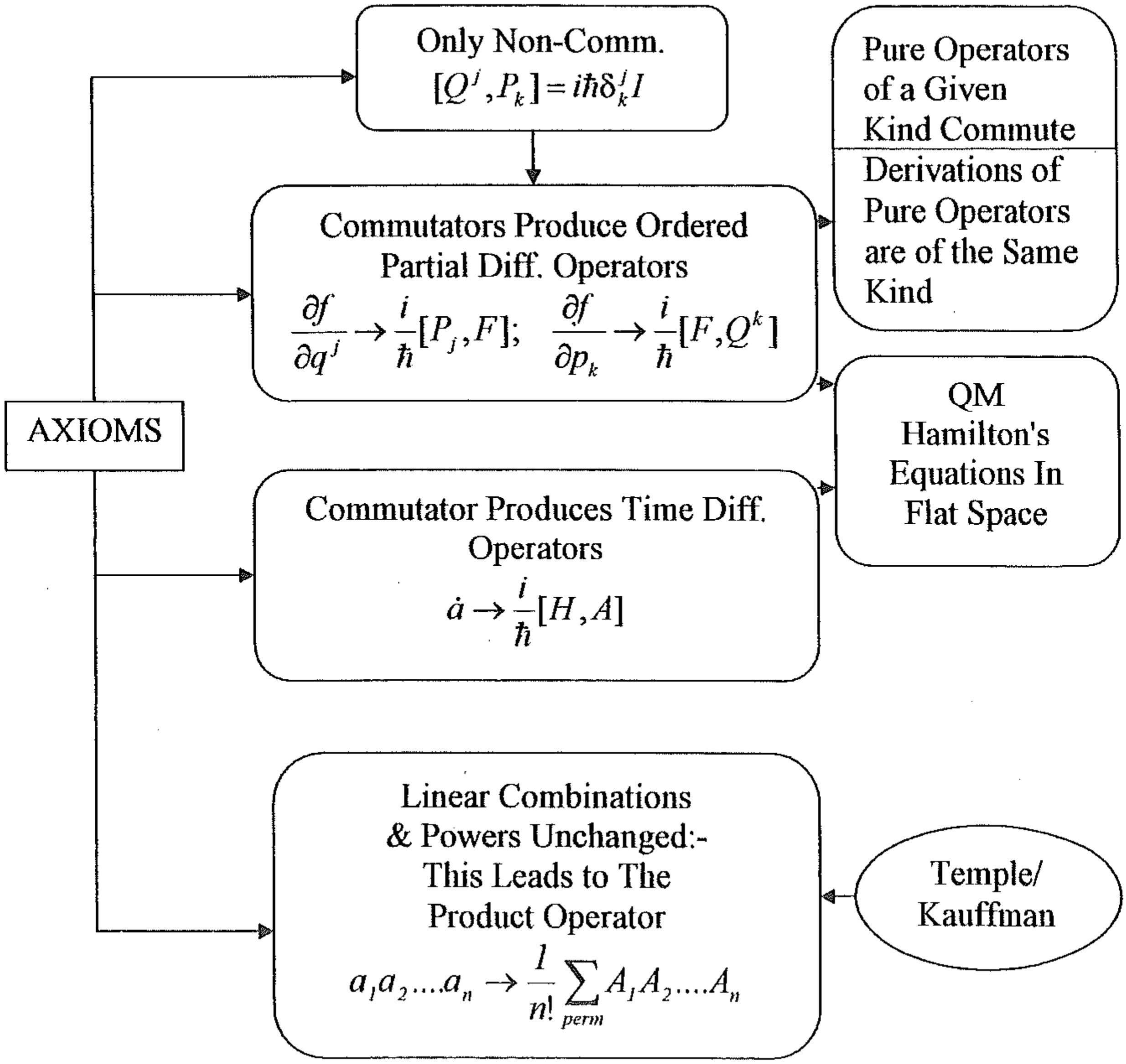
$$(14a)\quad \frac{\partial f}{\partial q^j} \to \frac{i}{\hbar}[P_j, F]; \quad \frac{\partial f}{\partial p_k} \to \frac{i}{\hbar}[F, Q^k]$$

and

$$(14b)\quad \dot{a} \to \dot{A} \equiv \frac{i}{\hbar}[H, A]$$

that, in QM, Hamilton's equations are identities. But recall that both the coordinates and the space are flat; so the QM Hamilton's equations are much more restricted than in the classical case.

Fig. 4- FORMULAE



An operator which depends only on the $\underline{Q}$ and c-numbers is said to be Pure in the $\underline{Q}$. An operator which depends only on the $\underline{P}$ and c-numbers is said to be Pure in the $\underline{P}$. Derivations of pure operators are of the same kind. All pure operators of a given kind commute with operators of the same kind.

A consequence of the axioms concerning linear combinations and powers is the formula given for the operator that represents a product. Temple deduces the formula for a product of two terms [6]

(15) $ab \to (AB + BA)/2$

Louis Kauffman has deduced the formula for the general case. This is a vital result.

QUANTIZE IDENTITIES- Fig. 5

The figure quotes the level 1 identity and its quantization. The notation is

(16) $\Theta_{,j} \equiv \frac{i}{\hbar}[P_j, \Theta]; \quad H^{:k} \equiv \frac{i}{\hbar}[H, Q^k]$

and, as usual, the Einstein summation convention is in force.

The reverse quantization of (7) yields the classical Hamiltonian quoted. Hamilton's (classical) equations yield equations of motion which allow us to interpret the physical meaning of the terms in (7).

CONDITIONS ON θ - Fig. 6

The meaning of the hypothesis about H_n becomes clear when we quantize the expansion given in Fig. 2

(17a) $\theta(t) = \theta_0 + \dot{\theta}_0 t + \ddot{\theta}_0 t^2/2 + ... \to \Theta(t) = \Theta + \dot{\Theta}t + \ddot{\Theta}t^2/2 + ...$

The operators $\dot{\Theta}, \ddot{\Theta}$,.... are generated according to the rule

(17b) $\left.\frac{d^s\theta}{dt^s}\right|_{t=0} \rightarrow \overset{s}{\Theta} \equiv \frac{i}{\hbar}[H, \overset{s-1}{\Theta}]; \quad \overset{0}{\Theta} \equiv \Theta$; see Fig. 4

and so

(18) $\Theta(t) = \exp(iHt/\hbar)\Theta\exp(-iHt/\hbar)$

But (18) is the definition of the Heisenberg representation of θ (Θ being the Schrödinger representation). Thus, if $H = H_n$ then, the RHS of the first n quantizations can be substituted for the $\overset{s}{\Theta}$, $s \leq n$ in (17a). The Heisenberg representation is thereby approximated to $n + 1$ terms. Each term is an operator expression involving the derivations of H and Θ. In other words this is a quantization of the prediction engine of Fig. 2; and the hypothesis claims that *more terms make the prediction better.*

The remainder of the figure gives a graphical illustration of how (3) is derived from (1) and (9) is derived from (7).

GRAVITY FIELD EQUATION- Fig. 7

This figure makes reference to the last three boxes of Fig. 5 in the case of a truncated classical Hamiltonian (i.e., one without linear terms). It turns out that the equations of motion have the appearance of geodesic equations where the $g^{\mu\nu}$ are the metric coefficients of a Riemannian space (the coordinate space) of dimension n_c. But the space is flat as are the coordinates; and it follows that the motion is uniform in a straight line!

The remainder of the figure is a reminder to the reader of the argument, given under the heading Overview above, which allows the tensor field equation (11) to be derived to make possible contact with GR. The essence of this argument is that, if the $g^{\mu\nu}$ are the metrical coefficients of a *curved* Riemannian coordinate space then, (9) can be true only at the pole of geodesic coordinates. By assuming normal coordinates Kilmister deduces that the only tensor equation that reduces to (9), at the pole, is (11). If we assume that there is but a single particle and we set

(19) $n_p = 1; \quad n_c = n = 4$; see Fig. 2

Fig. 5- QUANTIZE IDENTITIES

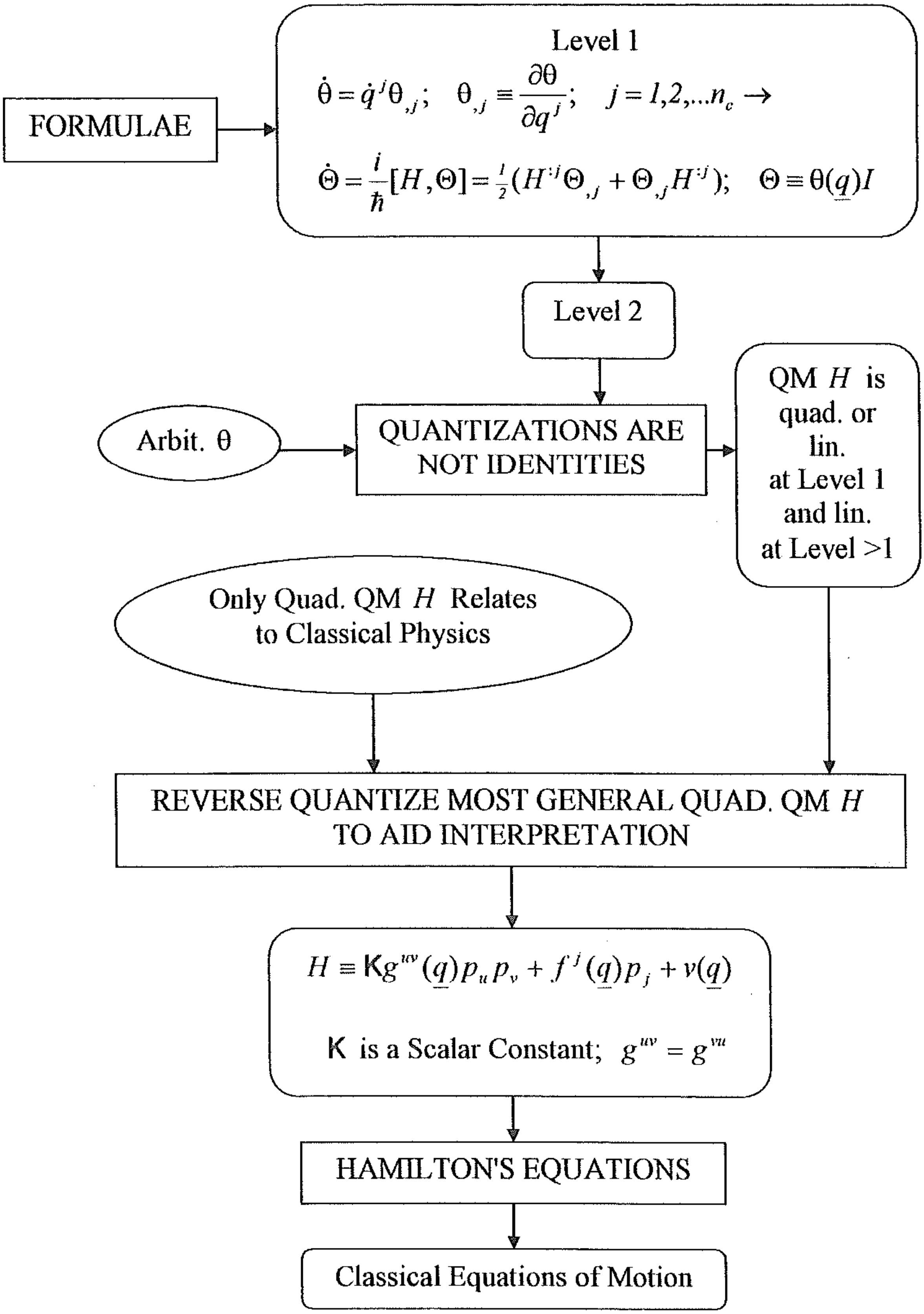


Fig. 6- CONDITIONS ON θ

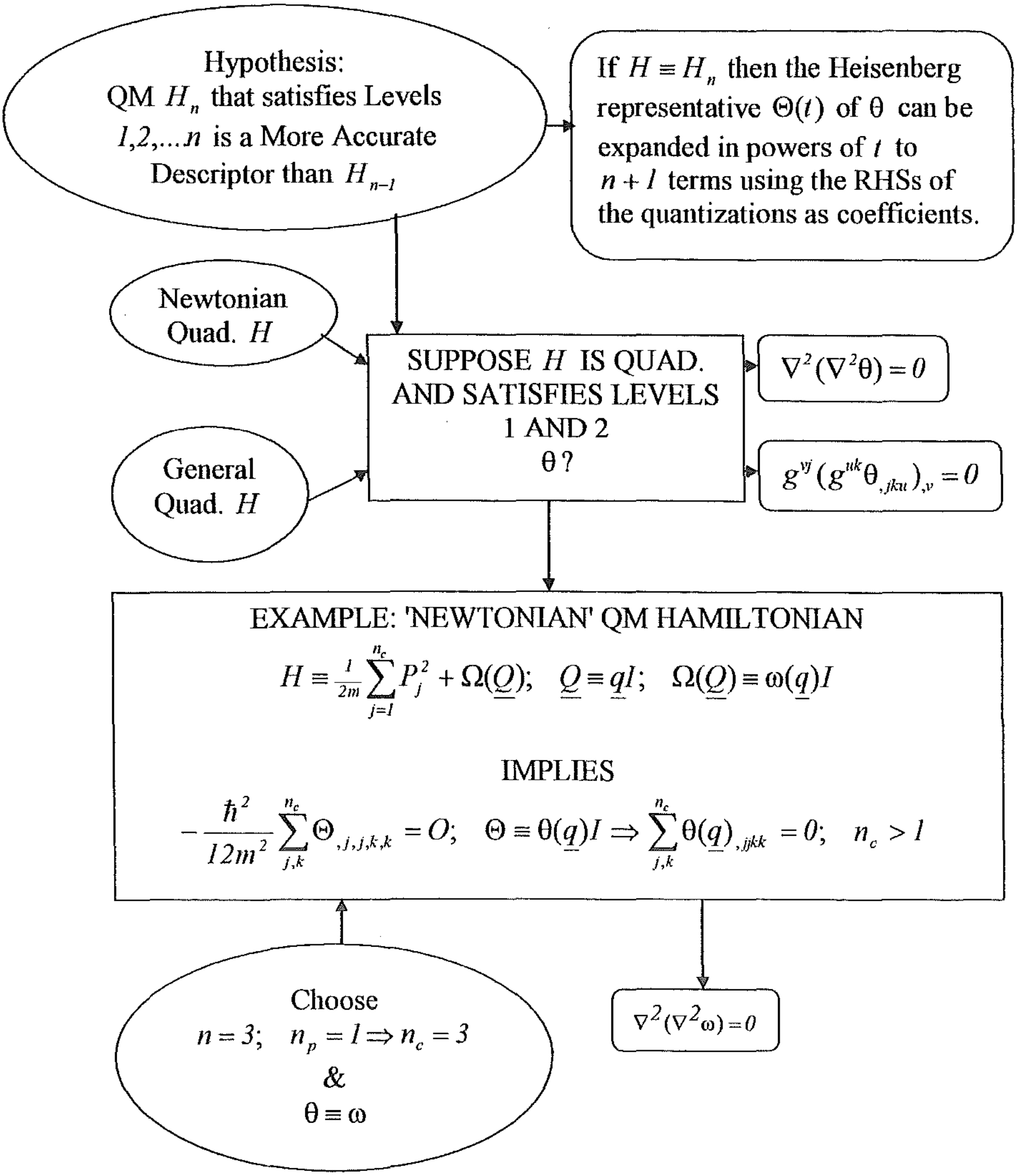
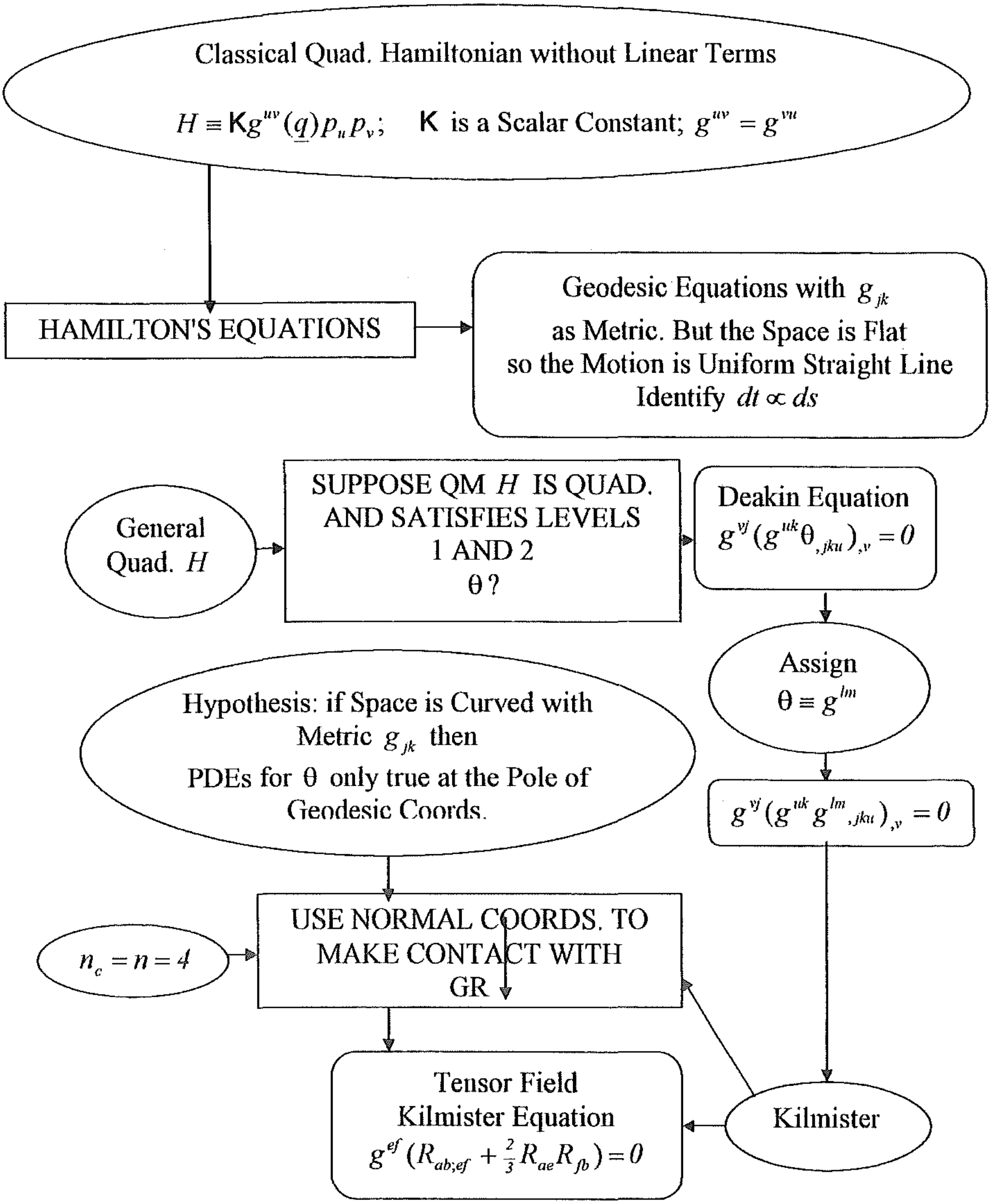


Fig 7- GRAVITY FIELD EQUATION



we can make contact with GR. The complications alluded to above are that to make contact with GR:

a) the metric must have the correct signature.

b) the time coordinate is not, in general, t but a function of t.

c) the classical Hamilton's equations of Fig. 7 use t as time; so the resulting equations of motion are geodesics only if

(19a) $dt \propto ds$

where ds is an infinitesimal increment of the metrical distance in the coordinate space. This matter requires more study.

The dimensionality $n = n_c = 4$ has a special significance in relation to the law (12). This tensor equation expands to

(20) $n(n+1)/2$

unique algebraic equations. But (12) is the contraction of the tensor equation

(21) $R^d_{abc} = 0$

where R^d_{abc} is the Riemann-Christoffel or curvature tensor. The condition (21) *defines* the space as flat [5], [7]; and it comprises

(22) $n^2(n^2-1)/12$

unique algebraic equations [7]. We deduce that solutions of the tensor equation (12) cannot imply curvature if

(23) $n(n+1)/2 \geq n^2(n^2-1)/12 \Rightarrow n < 4$

because, under condition (23), (12) will imply (21). The same is likely to be true for the solutions of (11).

RELATION OF KILMISTER EQUATION 'K' TO GR- Fig. 8

Strictly, (11) makes contact with GR only when (19) holds. In the language of GR the single particle is a test particle; we ask not what is causing the curvature, if any, of the coordinate space. But, by applying (11) to GR, we imply that the coordinate space of dimension $n_c = n = 4$ has a *physical significance*.

In the classical model there is no matter between the point-particles in the particle 4-space. So it is reasonable to assume that, in the coordinate 4-space also, there is no matter in the immediate vicinity of the test particle. But we may be obliged to accept that the coordinate space contains *energy*. Einstein's later law, for gravity in empty space, is

(24) $R_{ab} = \Lambda g_{ab}$

where Λ is a constant. Substitute (24) into (11) and we find that

(25) $\Lambda = 0$

Accept this result and we have

(26) $G_{ab} = -\chi T_{ab} \Rightarrow R^{u}_{v} \equiv G^{u}_{v} - \frac{1}{2} G \delta^{u}_{v} = -\chi (T^{u}_{v} - \frac{1}{2} T \delta^{u}_{v})$

which can be substituted into (11) to give a tensor equation for T_{ab}. If we believe that the coordinate space is truly empty this equation is a null identity. But, otherwise, it gives *possible* distributions of mass-energy-momentum-stress.

To illustrate this idea consider the weak gravity/ low speed equation (3) which has an S-S solution (5). The appropriate version of the latter is

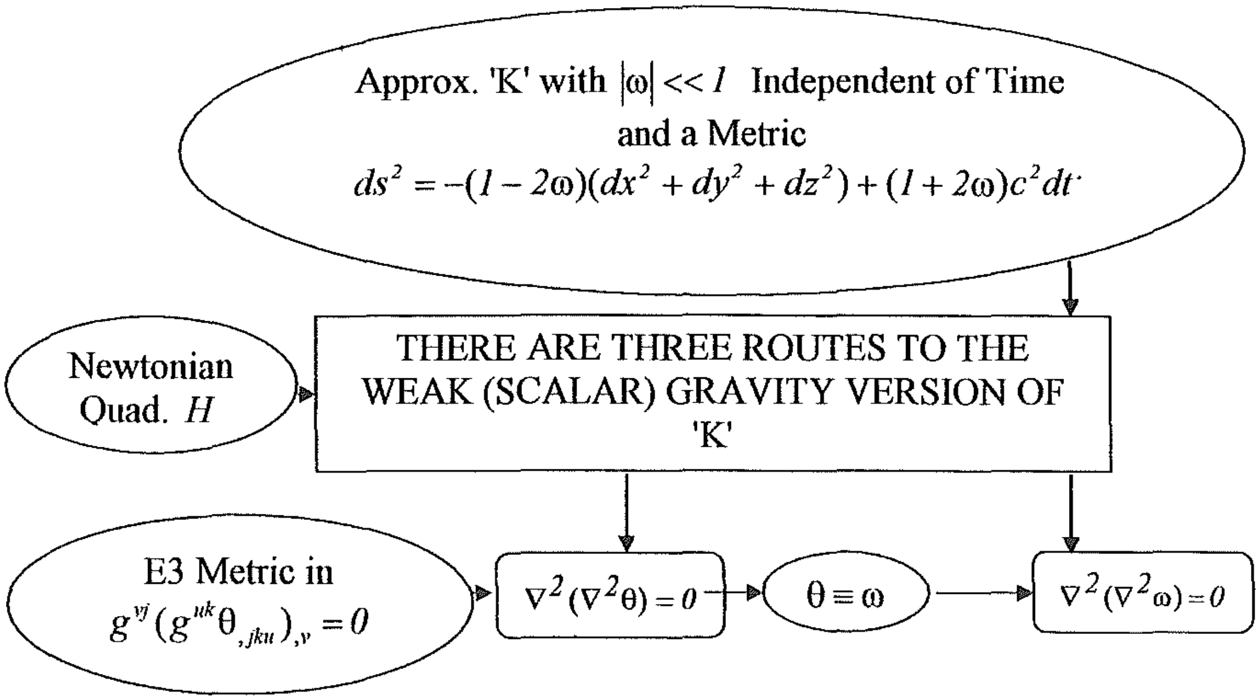
(27) $\omega \approx \dfrac{k_1}{r} + k_2 r^2$

where $k_4 = 0$ to ensure $|\omega| << 1$ and the condition $k_3 = 0$ is required by an alternative (Schwarzschild [7]) linearised solution of (11). Define the density

Fig. 8- RELATION OF KILMISTER EQUATION 'K' TO GR

$$G^u_v \equiv R^u_v - \tfrac{1}{2} R \delta^u_v; \quad G_{ab} + \Lambda g_{ab} + \chi T_{ab} = 0; \quad \chi \equiv 8\pi \mathrm{G}/c^4; \quad g^{ef}(R_{ab;ef} + \tfrac{2}{3} R_{ae} R_{fb}) = 0$$

'K' applies to empty space. If $T_{ab} = 0$, i.e., the space is truly empty then, $G_{ab} = -\Lambda g_{ab} \Rightarrow R_{ab} = \Lambda g_{ab}$. But 'K' requires that constant $\Lambda = 0$. All solutions of $R_{ab} = 0$ (the Einstein law for Empty Space) satisfy 'K' but not vice versa. Merely to set up the 'K', using the Schwarzschild Method, requires a computer.



S-S solution of $\nabla^2(\nabla^2\omega) = 0$, with a single particle of mass m at the origin, gives the Inverse Square Force. But it can also allow accelerated expansion (Dark Energy), beyond a sufficient distance, provided that Space is pervaded with a Negative Energy

$$\rho_{energy} \equiv 2\nabla^2\omega/\chi = 12k_2/\chi; \quad k_2 < 0.$$

On the hypothesis that the total energy of the Universe is zero the parameters of this motion can be evaluated

$$c^2\rho + \rho_{energy} = 0; \quad \omega \approx \frac{k_1}{r} + k_2 r^2; \quad \ddot{r} \approx -\omega'; \quad k_1 \equiv -\frac{\mathrm{G}m}{c^2}; \quad k_2 \equiv -\frac{2\pi \mathrm{G}}{3c^2}\rho$$

where $\rho = 4 \times 10^{-30}\, gm\, cm^{-3}$ is the mean mass-density of the Universe. We find

$$(-k_2)^{-1/2} \approx 4 \times 10^{28}\, cm = 4.2 \times 10^{10}\, ly$$

(28) $\rho \equiv 2\nabla^2\omega/\chi$; Poisson's equation

and (3) becomes

(29) $\nabla^2\rho = 0$

which gives a *possible* continuous distribution of matter-energy responsible for the potential ω. The case (27) requires

(30) $\rho = 12k_2/\chi$

and , in order to produce *positive acceleration* at sufficient distance, $k_2 < 0$. So, if $k_2 \neq 0$, space must be pervaded with a constant density of *negative energy* (in order to exhibit the observed Dark Energy positive acceleration). In the figure I have evaluated k_2 on the hypothesis that the total energy of the universe is zero. This calculation is merely suggestive; it is based on the approximation (3) and models a single gravitating particle at the origin.

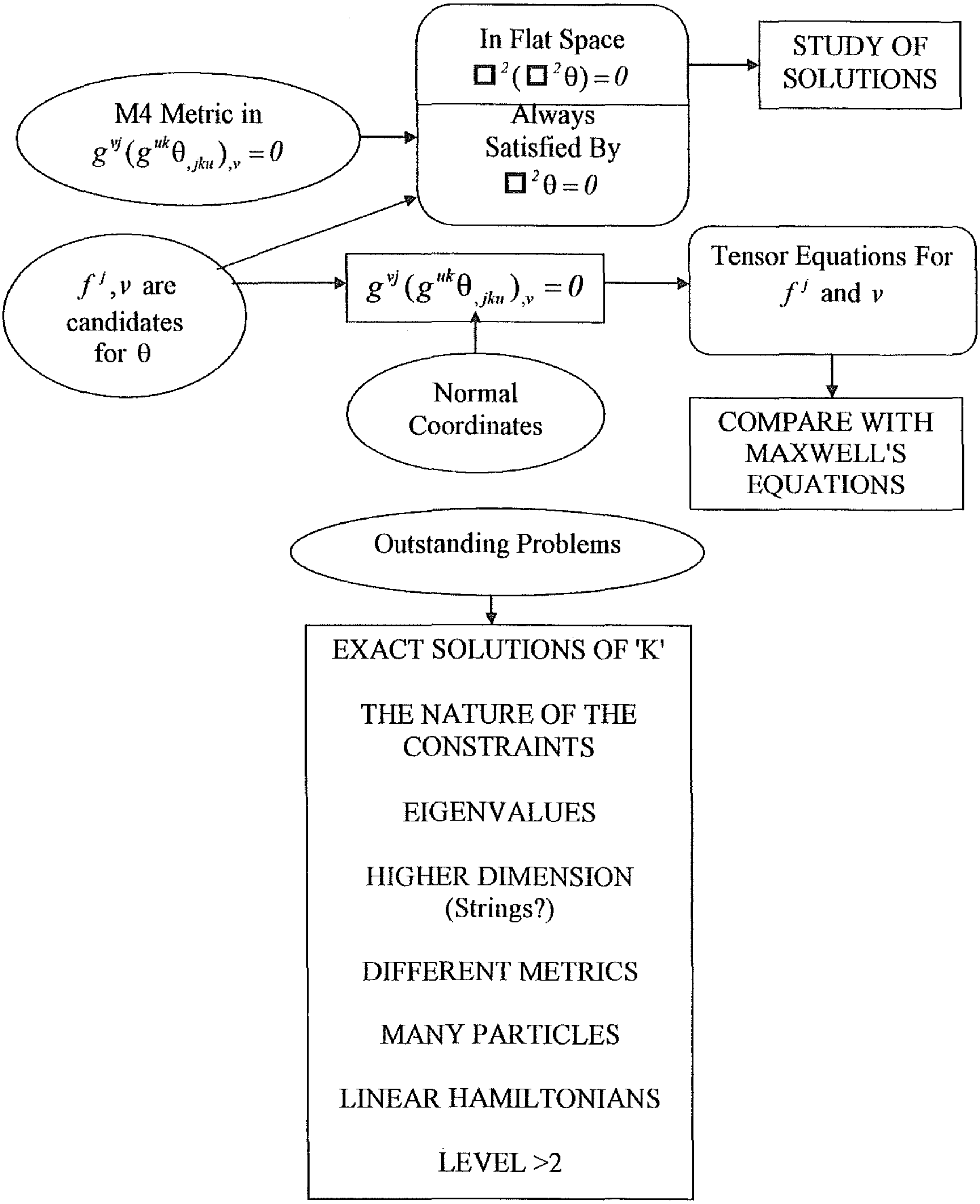
Note that the acceleration associated with $k_2 < 0$ is distinct from the Hubble red-shift; the latter is a property of Einstein's usual equations combined with an appropriate choice of metric [8].

THE FUTURE- Fig. 9

The figure is self-explanatory. But it is, perhaps, desirable to illustrate a possible solution to one of the outstanding problems: how does the new formalism deal with two or more particles? We consider the simplest version of this problem: the motion of two particles in E3, under a scalar potential, with the Newtonian approximation. We have $n_p = 2, n = 3$ and so $n_c = 6$.

The appropriate Hamiltonian operator is an adaptation of (1)

Fig. 9- THE FUTURE



(31) $$H \equiv \frac{1}{2}\left(\sum_{j=1}^{3} P_j^2 / m_1 + \sum_{j=4}^{6} P_j^2 / m_2\right) + \Omega(\underline{Q}); \quad \underline{Q} \equiv \underline{q}I; \quad \Omega(\underline{Q}) \equiv \omega(\underline{q})I$$

Particle 1 has mass m_1 and Cartesian coordinates q^1, q^2, q^3. Particle 2 has mass m_2 and Cartesian coordinates q^4, q^5, q^6. The metrical coefficients of the flat coordinate space are therefore

(32) $$g^{uu} = \frac{1}{2m_1}; \quad u = 1,2,3; \quad g^{vv} = \frac{1}{2m_2}; \quad v = 4,5,6; \quad g^{jk} = 0; \quad j \neq k$$

The general equation (8) then becomes, with an appropriate choice for θ,

(33) $$\left(\frac{\nabla_1^2}{m_1} + \frac{\nabla_2^2}{m_2}\right)^2 \theta = 0; \quad \theta \equiv \omega; \text{ valid at all points in coordinate space}$$

According to Einstein the expression of physical law must be independent of the choice of coordinates. In this case $\omega(\underline{q})$ must be an invariant independent of rotation and translation of the coordinate axes. The only such invariants, associated with the classical model, are the distance between the two particles

(34) $$l = +[(q_1 - q_4)^2 + (q_2 - q_5)^2 + (q_3 - q_6)^2]^{1/2}$$

and functions of same. So ω is a function of l. Therefore

(35) $$\nabla_1^2\omega = \nabla_2^2\omega = \left(\frac{d^2}{dl^2} + \frac{2}{l}\frac{d}{dl}\right)\omega$$

and (33) becomes (after division by $(1/m_1 + 1/m_2)^2$)

(36) $$\left(\frac{d^2}{dl^2} + \frac{2}{l}\frac{d}{dl}\right)^2 \omega = 0$$

The general solution of this ODE is ($c_1, \ldots, c_4$ are constants of integration)

(37) $\omega = \frac{c_1}{l} + c_2 l^2 + c_3 l + c_4$; compare with (5)

If l is small enough, we may neglect the terms $c_2 l^2 + c_3 l$. Then ω provides an inverse square force along the line joining the particles. In this sense, the new field equation (8) is successful. The formalism is suitable for an electrostatic force or Newtonian gravity. In view of the placing of ω in the Hamiltonian (31), and the fact that the coordinate space is flat, we favour the former choice. Gravity must surely involve curvature of the coordinate space governed by (11).

CONCLUSIONS

It seems that the recipe summarised in the Abstract and in Fig.1 contributes to the unravelling of two puzzles. First, it forges a link between QM and CM which, ostensibly, has nothing to do with the Correspondence Principle. Second, it provides an explanation of the geometrisation of physics. See [9] for an interesting alternative.

The theory provides an automatic link between the coefficients (field terms that inform the equations of motion) in the quadratic Hamiltonians of CM and the new field equations. The later include equations for gauge fields as well as for gravity.

The new field equations are expressed in an abstract Riemannian space (whether curved or flat) with dimension $n_c = n$ (dimension of the particle space)$\times n_p$(the number of particles); see Fig. 2. This offers the possibility of high dimension (i.e., greater than 3, 4 or 5) and consequent complexity. Nevertheless the simplest two-particle problem ($n_c = 6$) gives encouraging results.

The fact that the new field equations (8) turn out to be fourth order is another source of complexity; but it may presage a further enrichment of physics! Certainly the new gravitational equations (11) are vastly more complex than the conventional equations (12). Nevertheless the solutions of (12) are also solutions of (11). We likewise expect solutions of the conventional EM equations to be special cases of the solutions of new gauge field equations (not yet studied in detail).

A notable feature is that (11) gives *possible* distributions of mass-energy-momentum-stress. Because of the high order, however, these distributions are very flexible. The tensor equation (11) requires that the cosmological constant is zero.

A linearised version of (11) gives extra terms (by virtue of its higher order); see (3) and (4). These might be interpreted as accounting for the observed Dark Energy acceleration; however this conclusion is highly speculative.

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COMMENTS ON TONY DEAKIN'S CLASSICAL MECHANICS FROM QUANTUM MECHANICS VIA COMMUTATORS.

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At the ANPA meeting I read and commented on Tony Deakin's paper because he was unwell. There is no point in reproducing that but it seems worthwhile to make a few remarks on the printed paper. Over nearly forty years Tony has been doggedly worrying away at a problem. I would put the problem like this: Read any good book on quantum mechanics (e.g. Dirac), and really believe what is said at the beginning, rather than what is then done to solve the problems. Earlier Proceedings show various stages in this. Now Tony has got far enough to feel he can put it all together. It is most unfortunate that he is not able to come and talk about it this year.

Tony's model consists of point particles in a differentiable manifold. The coordinates of the particles q are continuous functions of a continuous time measure t. Then he remarks that because the coordinates are functions of time we may choose any one of them to equal t. I have two remarks about that. Firstly, it is necessary for the coordinate in question to be a monotonic function of t. Secondly, in a situation with n particles such a choice seems to privilege some particles over others. I shall return to this comment later. Next Tony considers a function $\theta(q)$ of the coordinates, which is supposed to have some physical significance. Then one can write down a sequence of identities for the time derivatives of θ in terms of the partial derivatives of θ and the time derivatives of q. Now

Tony quantises these identities by the rules that the books give. In his earlier work, as I remember it, he derived the results by a detailed examination of measurements. Whichever way you go about it the quantised identities are no longer identities and so their satisfaction engenders constraints. For example, take the first derivatives:

$$\dot{\theta} = \dot{q}^r\theta_{,r} = \frac{\partial H}{\partial p_r}\frac{\partial \theta}{\partial q^r} .$$

If everything becomes an "operator" as the books suggest, and taking $\hbar = 1$ you get

$$i[\theta, H] = \tfrac{1}{2}i(H^r\theta_{,r} + \theta_{,r}H^r)$$

which it is easy to see is a limitation on H. In fact it requires H to be only linear or quadratic in the p's. The second identity allows only the linear case.

Now Tony says that macrophysics is characterised by quadratic Hamiltonians and microphysics by linear ones. Given the object of the paper, he confines attention to the quadratic case. I have a slight qualification; the linear Hamiltonian to which he refers is Dirac's and so has non-commuting coefficients. Perhaps this does not matter; but more seriously it is a Hamiltonian for special relativity and this seems odd in the context of the way this paper starts. None the less, what comes next is sufficiently exciting to make one ignore such petty quibbles. He takes a general quadratic Hamiltonian satisfying the first two constraints and looks at the effect on θ when the coefficients in the quadratic Hamiltonian are taken as g^{ij} for curvilinear coordinates and θ is taken as one of them. The constraints do not come out to be of tensor form - a familiar enough situation in quantising - but they can be seen to be equivalent in canonical coordinates to the tensor equations:

$$g^{ef}(R_{ab;ef} + \tfrac{2}{3}R_{ae}R_{fb}) = 0.$$

Some problems begin to arise in my mind when I read on, for Tony says, reasonably enough, that the finctions g^{ij} can be regarded as the elements of a metric tensor in a Riemannian space (subject to the matrix g being non-singular).He suggests that this space has physical significance; my first reaction is to say: of course, this is just like the formulation given by Hertz in his lovely book on mechanics.Then Tony goes on to point out that the dimension of this Riemannian space is greater than that of the classical model except in the case of one particle. So this exception makes one think that no extra time dimension has been slipped in here, that we are still safely in a classical (Newtonian) model. But then later on he suggests taking the single particle case and setting the dimension of the particle space to 4 so as to make contact with general relativity. Or rather with another form of gravitation theory in which the field equations are the tensor equations above. My worry is:- how can four dimentions arise? And if the response is, you put in time, then I would say: how can this be extended to the case of two particles? Is the dimension of the coordinate space then 8 or is there some constraint to make it 7? (Physically, is there a universal time or does each particle have its own?.)

I may well have been unduly hard on Tony's outstanding paper. It calls to mind a comment told to me by someone who attended Hermann Bondi's Part 3 lectures called general relativity shortly before Bondi left Cambridge. The comment was "He leaves these young men with a puzzle why anyone should study a theory so completely beset with difficulties." So:- go on to read Tony's paper, not mine!

The Creature

Jimmy Honey *v1.0ed KGB*

"They've caught one, come and see it." Susan looked at the intrusion that was in the form of the engineer Mike fflowers as she asked, "Where did they catch it?" He was leaning into her room by holding onto the door frame, obviously wanting to leave, as he replied, "Section E56 - are you coming or what?" She closed her wrist-com and got to her feet saying that she certainly was, then she followed Mike along the narrow corridors to the Science section where she found most of the staff had already gathered. She moved as close to the plasti-steel cage as she could and saw through the huddle of excited workers a creature that looked to her like a giant yellow slug with six legs, two eyes and a large mouth.

"So this is the critter that is causing us so much extra work is it?" This was said by Gerald, the irrigation operator and was answered by the doctor, who was the closest thing they had to a Science Officer, "This is one of the creatures that has been attacking our feed and supply lines in spite of the robo guards that are supposed to be protecting them."

"It looks so small and slug like."

"It is small and extremely slug like but this is the creature that has been interfering with our work and keeping us well behind our schedule."

"So what happens now?" Susan asked looking around at the tired faces until the doctor replied, "The task falls to me to try to find out how we can repel it and if not then how to eradicate it." Susan moved closer to him as she said, "Why eradicate it when all that is needed is that it leaves our cables alone?" The doctor shrugged, saying that he must find a way to stop it or they would be here for years and years, then he shrugged again and went back to studying the creature. Susan and the rest of the crew of Land Creation Section four o seven, all started to offer ways to stop the attacks and some of them started to moan about how far behind schedule they had become due to the ugly slug things. Susan went back to her office wondering how long it would be until they could leave this planet and get back home. She thought of her mother and father and smiled as she turned on her wrist comp and started to tap in numbers.

Over the next few days the main topic of conversation around the base was how the doc was getting on with finding a solution to their pest problem. At every opportunity one of the crew would buttonhole him for information until he finally had enough and started to write up daily sheets on what he had tried and the failed results of each test. Susan read these with enthusiasm, brought on mainly from boredom and a desire to leave this place as soon as she could. She read that the doctor had first tried to use various levels of electricity but because the creature was made up from a mixture of liquids, it could manipulate itself making its body into a conductor and then just keep on grounding the source. He had tried sounds at different volumes and from different sources and all without any apparent effect. He had moved on to repellents and finally poisons with the same results.

She studied the latest report and nodded to herself, then went to look for the doctor and, when she found him in his office staring into space, asked if she could help him with his research. She saw that he was surprised and happy to have someone else to talk to about his problem, and what else could he try instead of just complaining that he had not succeeded? He asked her why she was offering and she replied that she was fed up with her mind pod and wanted to leave sooner rather than later. He smiled understanding how easily that occurred and agreed to allow her total access to the creature.

So she began her study of the creature. At first she just watched it as the doctor tried putting different scents into its cage. The organism rose up on its back limbs and waved its face at them and then fell back with a wet sloshing noise that made her flinch. When the doctor had repeated this with different odours he shrugged, scratched his head and left telling her to do what she wanted with it as long as she did not harm it then he left her alone.

She sat away from the cage just watching it, putting together what the doctor had found out about it. It had a spinal cord with complicated nerve endings that connected to its six limbs that worked independently to one another. It breathed the heavy atmosphere but could take large amounts of many other chemicals including sulphur without any harm. It devoured every form of vegetation and was not bothered by rain nor the storms that happened on a weekly basis. It did not react to loud noises but seemed to be able to hear, or maybe it was that it felt vibrations under its fat wet belly. She knew that it and its kind had attacked many of their cables that they used to control the machinery and robots, and that without these cables, which carried electricity, water, and recom juice, they could not do their job. The robo vids showed them chewing on the cables and pulling on them, until the roboguards came along, and then they dismantled them.

How they managed this was beyond her comprehension for its feet/hands were primitive with no opposable thumbs but they did it somehow for she had seen what was left of them. The next day the doctor was called away to help with an accident on a cargo ship a few light years away, so it was left to her to look after the thing and to do whatever she might.

The first thing that she did was to watch it eat its latest meal of vegetables, which it ate using its top limbs, only taking the food to its large wet mouth. She sat watching it until it had finished its meal and then she put the water bottle against the side of the cage. The creature obviously had problems sucking from it, so she looked around for some container that she could use for a water dish. She finally found some saucers that had not been used for a very long time judging from the layers of dust on them. She gently blew away the dust, wiped one against her overall then filled it with water. She put it on the circular floor of the cage and gently turned it until the water and dish were inside. She idly wondered if the creature would show some form of emotion at having water that it was used to drinking on the ground, from a tube.

The creature immediately attacked the container showing for the first time its sharp pointed teeth as it tore the dish apart. Susan gasped from amazement; the thing had suddenly changed into a devil from a slug. She slumped back in her chair studying the creature that now pushed what was left of the dish to one corner, the corner she noticed that had the circular floor, then it lapped up the water from the floor. She realised that it was the dish that it had attacked and not what it contained but she had no idea as to why it had acted so. She went and got another saucer and repeated the experiment but this time the creature ignored both the dish and the water that it contained. She waited until it finally went and sipped the water.

Susan looked at the dish wondering if because it had been recoded that it was somehow more acceptable to the slug.

Suddenly she noticed that this dish was a light blue colour while the other one had been a deep blue.

Over the next hour she experimented with different colours and discovered that the creature reacted to all of the lighter colours and none of the darker ones.

This was the solution to their problems, all they had to do was to color all of their feed pipes dark colors and the robo workers and guards and the creatures left them alone.

She became the hero of the hour and tales of her expertise with the slug creatures preceded her and continued to be a talking point. She tried to shrug it off explaining that it had been an accident rather than good research. At her next interview the incident was brought up and she later found out that it was because of this that she got the job that she knew she was not experienced enough for.

She immediately went out and commissioned a small necklace pendant in the form of the creature and it became her lucky charm.

Relational Existing, Internal Relations and Sentience

v.1.08

Louis Gidney, Strontian, May 2007

Abstract

In some ways the thought-experiment described here harks back to the refutation by G. E. Moore and Russell of Objective Idealism - a monism where so-called "external relations" characteristic of Materialism were held to be impossible. Here I regard some relations as internal to their relata under one description but not under another; in terms of physics: to one observer (or another physical process) but not to another. That is important here, but the key idea in what is presented is "relational existing" posited as a characteristic of elemental physical interactions, such that within interactions something fleetingly "exists to" something else, but not objectively and would not have come into being at all if the interaction had not taken place. So in the absolute sense it never existed. The 'shape' of a particular incarnation of Schroedinger's wave equation might be seen in this way, such that its values represent changing "degrees of existing" rather than probabilities, thus making existence a complex variable instead of an all-or-nothing quantity. "Relational existing" is seen here as a primitive analogue of private experience, which it is its purpose to 'place' within physical processes. It is proposed that conscious entities are systems of "merged" primitive acts within which interactions are mostly "internal"; nothing "exists to" anything else within them. The "relational existings" within such systems lose their individuality; making such systems, in some sense, "wholes". The existence of something *to* such systems is conceived to be a consequence of "external interactions" in the sense that the participants in the interactions could have been other than they happened to be. To tie these various strands together a non-standard ontology is introduced based on acts and "relational existing".

Consciousness aside, this scheme provides a new way of seeing what we already know about physical, chemical, and cellular interactions in an entirely new light, and opens new possibilities for research, such as the close study of the "externality", "internality" of interactions and the transition from one to the other within structures such as molecules and other systems. It should help us understand why "binding particles/forces" and other scatter fragments are observed as a result of high energy collisions, but not otherwise.

Introduction

At the outset, I should say that this is not a theory of consciousness, because one or two people who looked at the draft asked questions typically related to theories of consciousness. But it is not physics either. I will call it a "topography", analogous to the way this word is applied to landscape, to mean an overview of the salient elements to be included in a conceptual scheme and how they are envisaged as related to one another. This is the only word I have yet thought of that does not invoke misleading associations. It is also suitably vague! --but capable of being brought into sharper focus later. A blurred picture can sometimes give a better preliminary idea of the whole.

I've set myself the modest aim of focusing on one question only, namely: what a conceptual basis of physics might be like if it allowed a place for consciousness. I will try to show that it is possible to place the concept of "relational existing" or "existing to", which are part of a non-standard ontology and in a sense a kind of "proto-consciousness", in a relation to the basic concepts of physical processes which, I believe, matches our intuitive sense of both.

Topographies

What sorts of 'things' do we know about - and in what ways can we think of them as related? The chemical elements can be thought of as variants of the same sort of 'thing' in the naive sense that they are more like each other than they are like, say, rotations, frequencies, sounds or colours. When I was an Art student in the fifties, nearby was a library of Imperial College (which later became attached to the Science Museum). Its main staircase was lined with a row of sealed glass jars which supposedly contained specimens of the chemical elements. I assume that if a new chemical element had been isolated, a new jar would have been added to the line.

Those jars evoked the enticing idea of an encyclopaedic collection of everything in the world. Early scientists seem to like storing specimens in jars. Not everything readily lends itself to being put into a jar, but in 1746 in the city of Leyden, Pieter van Musschenbroek, even made a special jar that could store static electricity --for a few hours at least. As far as I know no one tried to make a jar for storing magnetism. But there are certain 'things' you could never put into a jar. The spin of a child's top for instance. You can put the child's top in a jar, but how do you put in just its spin?

Heat was once thought to be a mysterious 'effluvium'. But at the time of Joule and his contemporaries, the warmth of a body came to be thought of as the vibrational motion of its constituent parts. The production of heat by motion, via friction, had been known from time immemorial, but this new conception led to a quantitative equivalence of heat to mechanical work, which made possible an understanding of the reverse process in heat-engines. It was a notable step at the time, even though we now know it was not the whole story. That was long before electromagnetic radiation had been discovered, so radiant heat was envisaged as the mechanical agitation of some invisible "aether".

At that time gravitational force was understood as acting along a straight line between two bodies. This conception also worked for electrostatically charged bodies. But the idea could not be usefully extended to "electromagnetism" as it gradually came to be understood to be one phenomenon with three aspects: mechanical motion and two distinct kinds of force operating mutually at right angles. The concept of a closed loop (of electric current or of lines of magnetic force) was another concept that formed part of this understanding.

You can see that this was a significantly different "topography" from the idea of a force acting along a straight line (and from stuff that can be stored in jars), and it illustrates well, the way I will use the word "topography".

Electromagnetism was gradually seen to make more sense when flowing electric currents rather than static charges were considered. It made even more sense when all three aspects were changing simultaneously, particularly when alternating rather than constant currents were employed. Although physical apparatus was required for the study of electromagnetism, as a phenomenon it was essentially immaterial. In fact it could become entirely independent of any apparatus. For example it could be launched as waves in an "aether", now thought of as electromagnetic rather than mechanical in character. And these waves could continue to propagate even if the apparatus that launched them were switched off, or even destroyed.

I hope the above paragraphs illustrate what I mean by a "topography"; a combination of the various conceptions to be included in a conceptual scheme, and the ways in which they are thought to be related to one another. I have already mentioned some typical relations. It is enough for now, to keep in mind just a few unsophisticated kinds of relation, such as: "being of the same or a different kind"; "being contained in"; "being perpendicular to"; "being in relative motion to"; etc.

Consciousness

Why do I think consciousness is important or even relevant to the conceptual basis of physics? My simple-minded answer is that consciousness, and our experience of its intimate involvement with the physicality of our bodies, is the most evident of all facts. So any theory of the physical that aspires to be comprehensive, but does not leave room for theories of embodied consciousness to be built, cannot be wholly adequate. It could be that it would not be wholly adequate for pure physics either, even for physicists who do not believe consciousness is relevant to their work.

Now we come to the main topic: what sort of a "topography" can encompass both consciousness and physical processes --and offer a plausible relationship between them? We learn from the history of physics, that when things once thought to be incompatible later came to be understood as related, it was typically as a result of a change in the conception of one or both that made it possible. For instance it would have been difficult to relate heat to mechanical work before it had been conceived as a form of motion, and one can think of many other examples. I have already mentioned the topographical change that was needed to understand electromagnetism. Clearly, if we started out thinking of 'mind' as a woolly cloud, and 'matter' as clockwork, we can see straight away that it is unlikely we would have much success.

So we must try to address the thorny question: what is the essence of consciousness? There is no need to review the rich variety of states of human consciousness and the different kinds of experience we can have. We are familiar with that already. It is complicated. It may have no limits in its variety. And it does not belong to physics.

I will approach the question in the opposite direction; by stripping away the aspects of human experience that do not seem to be essential to consciousness; the "add-ons" to its essential core, so to speak. For instance, blind people can be just as conscious, have as much humour, empathy with others, and so on, as sighted people. So clearly, eyesight is not an essential feature of consciousness. The same is true even for most deaf-blind mutes. I think it is reasonable to assume that intelligence, artistic talents, and so on, are not essential either. All that can be absent and there can still be consciousness, and rapport with others.

Is there a limit to this stripping process? Can we finally arrive at a point beyond which it is impossible to go without losing what can still be considered to be a rudimentary form of consciousness? If so, what would it be like? It seems reasonable to suppose that whatever this something is, if it does not retain, in some sense, the fact of there being something that could be called experiencing, then we would have gone beyond the point where the essential feature of everyday consciousness is present.

What if we memory were absent? Presumably any possibility of reflecting on past experiences, or recognising something that had been experienced before would be lost. However, I believe it is possible that even then there can still be experience, even if it be restricted to the immediate present moment. Clearly there must come a point at which most of what we usually associate with consciousness could be lacking, while at the same time, experiencing of the surroundings, still occurs.

When we get this far in the stripping away process, we are treading on delicate ground where we touch upon questions of human suffering and values, and it becomes difficult to continue speaking in terms of human consciousness. Also, it seems reasonable to make provision for the possibility of non-human modes of consciousness. At this point I begin to feel the need for a more precise definition of experiencing. Is consciousness still the correct word? I had thought of using the word "sentience" to mean something more primitive than human consciousness but which still possesses that essential feature, namely that *to* the sentient organism there is still "something there", even if a full awareness of it is lacking; even if there is "nobody home". This may seem an odd thing to say, for how could an organism (or anything else) retain any supposed "essential feature of consciousness" if it lacked so much; if it lacked a sense of self-hood for instance?

In order to take this abstracting process as far as possible I will now stop using words like consciousness, experience, sentience, and so on --which can never be defined in a way that suits everybody in any case, with the neutral phrase "exists to". This seems to be adequate all the way down the scale. It works at the everyday scale. The customary meaning of "John sees, (feels, touches or in some other way experiences) a table" is not unduly distorted if instead we say: "the table *exists to* John". In fact this way of saying it removes preconceptions about how the experiencing is accomplished. It also has the advantage that it retains a definite meaning if we wish contemplate the possibility of rudimentary modes of 'experiencing' (or perhaps "proto-experiencing") by worms, insects, micro-organisms, or even molecules and atoms, if we should happen to find "relational existing" useful in physics at that level.

Comments on Method

Having come this far, we may stand back for a moment to muse upon the possibility that this mode of "relational existing" (which differs from absolute objective existing analogously to the way private experience differs from "objective" fact) might be a universal feature of all physical interactions. This is an attractive idea, for it holds out the possibility that the sophisticated kinds of consciousness may evolve by the "merging" of rudimentary forms of relational "existing to". We shall see later that such merging can be understood in terms of interactions that correspond to external relations, changing to interactions corresponding to internal relations.

Let's hold it there for a moment. What I have suggested so far, could be dismissed as a kind of naive, reductionist "atomism of consciousness" which employs fragments of experience instead of Democritus' atoms; that I have done nothing more than say that if you think of consciousness being stripped as bare as is conceivably possible, then abstract what is left and call it "relational existing", you can then reverse the process and declare that to be an explanation of embodied consciousness.

But is that a criticism? There is much truth in it. But there is also some truth in the idea that if you don't put what you want into the foundations of a conceptual scheme, you may never get it out later. For example there was an attempt to reduce electromagnetism to mass, length and time. But later, an electrical dimensional quantity was introduced. However I hope to show that in the case of consciousness, there is no need to introduce anything new, such as what some thinkers have called a "psi factor", but that it can be understood simply as existence; as *physical interactions giving rise to relational "existing to".*

So it is a "nothing added" scheme, in the sense that relational "existing to" does not interfere with any established way of understanding physical processes. It is simply the recognition that what we are looking for is already there, but that we have not seen its "place" in an overall "topography", and have never left room for it.

Thus far, what I have proposed is no more than a different way of looking at what we already know in physics. However, further thought brings one to the realisation that it cannot be made to hang together properly without revising our ontological ideas, and when we do, it looks as if it might have implications for physics. So before I end my "likely tale", I must introduce what could be called a "non-standard ontology". It does not have much to do with the existence of anything in the classical sense, but since it has to do with the "ground" of the world, ontology seems an appropriate label.

A recent news item (April 2007) ran under the headline "Quantum physics says goodbye to reality", and continued "...physicists from Austria claim to have performed an experiment that... [suggests] the uneasy consequence that reality does not exist when we are not observing it." This expresses exactly the "topography" I am developing. It also exposes the endless confusion caused by the conflation of the meanings of "real" and "exist" in common parlance.

In the non-standard ontology I am proposing, in order to fit the bits together, "exist" (in the relational sense of "exists to" already described) gets reserved for the everyday life-world (which we can think of as different for organisms with different sense organs and different experiencing speeds), while the physical happenings beneath experienced appearances come to be regarded as in essence pure activity.

Happenings, acts, and so on, I insist should not be thought of as existing (nor as not existing) since what they do instead is "happen" or "act". So the "uneasiness" of the journalist quoted above is no more than a consequence of a commonly assumed version of "Realism" which assumes that for any actions or changes, to occur there must first exist something that acts or changes.

In my thought experiment it is proposed that "pure actions" (of some kind, speaking speculatively, perhaps binding forces) are real; and are the ultimate "ground" that holds the world together; but that it is an error to say that they exist. However, I propose that they be considered as having a property such that within every act, "relational existing" occurs. That is my proposed non-standard ontology in a nutshell. It could be summed up even more briefly (but badly) by saying that it proposes a real physicality that acts but does not exist, and an everyday experience-world which exists but only relationally --and not forever.

Mathematical Serving Suggestions

Louis Kauffman [1, 2], following Spencer-Brown's Laws of Form [3] has shown that logical paradoxes can be dealt with in terms analogous to complex numbers in algebra. The most remarkable thing about that work is that out of a "static" logic, time or change (typically as frequencies) emerges.

Recall that to make common algebra "closed" (in the sense that all its operations could be reversed and combined in every possible way without "jumping out" of the field and still give a meaning solution) it was necessary to extend the number field to the negative, to the irrational and to the complex numbers. Then look at, say, the propositional logic. It can construct, but it cannot cope with paradoxes, so it is still imperfect compared to algebra. Kauffman and others have shown how this shortcoming can be remedied.

It seems to me their work has not received the attention it deserves, for if the process be reversed, then starting from time and change one can derive the "static" true/false as a special case. If we then map Logic to Ontology, such that True becomes Exist. False becomes not-Exist (in the sense of "relational existing" I have already described), then and the complex values arising from paradoxes map to "pure action", thus giving precisely the alternative ontology I have proposed above. This suggests that perhaps paradoxes are more general and more fundamental than the simpler True/False logic. Could this be what Zen masters have been trying to tell us for centuries? Furthermore, we arrive at a very natural relation between action and relative existing.

[According to G. Spencer Brown [4], Bertrand Russell told him that he and Whitehead used his Theory of Types as a temporary fix to deal with paradoxes in their Principia. Interestingly “types” is a higher level notion. I suppose something similar could be done in algebra (if its historical development were included within it) and some operator concocted as a kind of "expander" or “level changer” to extend the field when some operation produced a result that stepped outside the field it started in, and thus create some kind of hierarchical structure. But this is hardly relevant here.]

Einstein’s famous formula tells us that mass is equivalent to a certain amount of energy which in turn can be expressed in terms of action times a frequency. This seems to imply that entities exhibiting mass-like properties are literally stable structures of pure energy. But (so far as I know) an understanding of how and why energy "condenses", so to speak, into the specific stable structures that it does, exhibiting mass-like properties, is still lacking. It is only in the last five or ten years, since neutrons and protons have been split and their fragments found to be short-lived, that it has become plausible to even think in such a way. So far as I know, what a stable energy structure of “pure activity” might be like, it is not yet possible to say until the fine details of quarks, etc, have been worked out.

Even so, I think we can already say that the last vestiges of classical materialism have now evaporated, leaving the possibility of physical theories based on the Process ideas of Alfred North Whitehead and others. According to those ideas the ultimate ‘stuff of the world’ is not an existing ‘stuff’ at all, but pure (disembodied) process or activity (not to be confused with the activity of objectively pre-existing ‘stuff’) -- as described above. This is year and next could be a very important moment in history for physics. In November of this year (2007), the enormously powerful LHC at CERN will be switched on for the first time, and next year it should operate at full power. Also, the Voyagers I & II have now verified the"Pioneer anomaly", so we have much new data to look forward to. Hopefully, it will shed more light on these matters.

Relational Existing, Activity, and Sensing

We are not yet quite done with "relational existing"; something *existing to* something else *within* interactions. As conceived here it is a state of affairs that has duration. The recognition that continual 'activity' in one form or another, is common to both physical and mental processes including sensing is, I suppose, self-evident. (For studies of the latter see for example J. J. Gibson [5] and much other work since.). The philosophical significance of that work in physiology is that we now know that experiencing ("relational existing" or *existing to*) is an active process; that sensation is not simply the passive intake of information. We also now know that "matter" is a hive of activity; that even in the most ancient lump of rock, besides thermal agitation, all kinds of interactions are holding the molecules together at the electronic scale; that various nuclear processes are holding neucleons and quarks together as energy in a persisting structure. Neither of these facts was known to the Ancients, who regarded matter as something passive that could only move or change when acted upon by something external to itself.

At the outset of modern philosophy Descartes assumed (or laid it down as a matter of principle) that what he called "mind" and "extending things" had no feature in common. Even today people still debate the "connection" of "mind" and "matter", quite often with little knowledge of how "immaterial" the concept of matter has become in physics, or on the other side: not knowing that sensing is impossible without action (such as eye tremor, etc) on the part of sense organs. So here we have a common feature, namely activity. To have some sort of image of an elemental "pure act", one might imagine a photon emitted by one atom and absorbed by another, then delete both atoms, and then try to envisage that within that 'act' something corresponding to "relational existing" takes place. This is only a rough "visual aid", because no one knows what goes on at the vastly smaller nuclear scale.

The Possibility of Conscious Machines

Perhaps it is worth adding a comment about how the possibility of conscious machines appears within the scheme I have outlined. My personal suspicion is that consciousness can only be achieved by live organisms, but other possibilities cannot be ruled out. The key idea is that for a machine (or anything else) to be conscious it would have to capitalise on "relational existing" from the 'bottom up' in such a way that what *exists to* the whole is more, in some sense, than what had *existed to* the parts, ie- such that the transient, fragmentary "relational existings" possessed by the parts by virtue of their activity, become "merged" in some sense (very loosely analogous to the way in which, if we have two eyes we don't 'see double', but rather we have a unified but enhanced view, with a better appreciation of distance). Presumably, if sentience can indeed be properly understood as the merging of the separate "relational existings" of the parts of a conscious entity, then it seems to me obvious that this "merging" must take place at all organisational levels for the elemental "relational existings" to become full blown human consciousness.

If so, then anything less would be like trying to build an imitation electric motor powered by an imitation battery. If the real thing ("relational existing" in this case, rather than electromagnetism and chemistry) is regarded as a feature of nature, it cannot be imitated by emulations that start at a larger size scale than the genuine article. (For instance at the scale of electronic components, physical processes and their corresponding "relational existings" are not merged or organised at all, but are still randomly orientated and undirected amongst the constituents of the materials used. In other words, the constituents are still externally related.) If that could be achieved, I see no obvious reason why it should be impossible using non-organic structures.

Is Relational Existing Necessarily Intermittent ?

This is a philosophical question, which perhaps someone else can do something with. There seems to be "circumstantial evidence" suggesting that it may be built into the nature of the world that "relational existing" at all levels, including human experience *necessarily* has an on/off or intermittent character, mediated by actions. This conjecture could be summed up by saying that without change, "relational existing" cannot occur. To take a couple of examples from physiology: It is possible to effect the disappearance of everything in one's visual field if (without moving) we can manage to stare at a fixed point for a couple of minutes in a room in which nothing is moving. This is slightly tricky because our eyes have a tendency to move involuntarily, but most people can do it. Another example: It can happen, if we awake in a comfortable bed, that we do not know exactly where our legs are until we move them. In other words: until we act. All this and more has been studied by experimental psychologists of course. In association with this (but not connected) here are a two more examples: first the signals in nerves are pulses and not analogue signals; second, in physics we find that emissions of energy are quantised. There is no obvious connection between these assorted facts except that they are vaguely suggestive of a conjecture that just might be important, or might suggest something else that is. So it could be useful to ponder these facts from time to time. For the purposes of my thought experiment I have defined "relational existing" as occurring within actions (not *caused* by action, but loosely analogous to the fact that the appearing of a magnetic field is an accompaniment of the displacement of an electrical charge) --and things in our experience as being in essence the result of many momentary primitive acts of sensing summed together. If we include bodily movements as actions, then what is meant by the existence of something we are not currently experiencing, is that by performing certain actions we can experience it, ie, make it *exist to* us repeatably whenever we want. Here ends my "likely tale".

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3. G. Spencer Brown, Laws of Form, London, Allen and Unwin, 1969.

4. *ibid* p.xiv (Preface to the First American Edition, Julian Press, 1972)

5. J. J. Gibson, The Senses Considered as Perceptual Systems, Houghton Mifflin, Boston, 1966.

NAMING THE MYSTERY: FAITH, TRUTH AND THEOLOGY v.1.0

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The purpose of this paper is to give an overview of modern theology in the hope that the general and philosophical issues raised may suggest parallels and points of convergence to an audience whose main concern is with the physical sciences. Where do we start? As we shall see, contemporary theology believes that context is important. What I will say, what I regard as important, has been affected by the fact that my initial training was as a physicist, or 'natural philosopher' as Oxford still calls the degree. Near to the start of courses I teach for students I usually introduce two different sets of issues:

- Richard Dawkins once wrote to a national newspaper in horror at the gift of a large amount of money to endow an academic post at Cambridge for the study of science and religion: 'Theologians don't do anything, don't affect anything, don't achieve anything, don't even mean anything.'[1] We can't avoid noticing what other disciplines are up to; what their practitioners think about us.

- But, at the same time we try to take seriously immediate and internal concerns. I often begin my teaching with a number of 'church door' questions to which the students are asked to respond, and from which we begin to piece together the theological and doctrinal implications of such encounters, and the resources from the tradition which may help to explicate them: 'Surely there is no harm in my student grand-daughter being rebaptized?' 'Wasn't it good that our Muslim colleague took communion with us today?'

Theology has a number of concerns which must be taken seriously, and which point our attention in varying directions:

Housekeeping
Listening to the children

[1] *The Independent*, 19 March 1993.

Talking with the neighbours
An adult conversation

HOUSEKEEPING

How much do we have to clean up? Where are the boundaries of our responsibilities?

> I had set out on a journey, with no other purpose than that of exploring a certain province of natural knowledge; I strayed no hair's breadth from the course which it was my right and my duty to pursue; and yet I found that, whatever route I took, before long, I came to a tall and formidable-looking fence. Confident as I might be in the existence of an ancient and indefeasible right of way, before me stood the thorny barrier with its comminatory notice-board - "No Thoroughfare. By order. Moses." There seemed to be no way over; nor did the prospect of creeping round, as I saw some do, attract me. True there was no longer any cause to fear the spring guns and man-traps set by former lords of the manor; but one is apt to get very dirty going on all-fours. The only alternatives were either to give up my journey - which I was not minded to do - or to break the fence down and go through it.[2]

Since the end of the nineteenth century this has been more of a problem for theology than science. Owen Chadwick makes the point with his customary clarity and economy:

> Science versus Religion - the antithesis conjures two hypostatized entities of the later nineteenth century: Huxley St. George slaying Samuel smoothest of dragons; a mysterious undefined ghost called Science against a mysterious indefinable ghost called Religion; until by 1900 schoolboys decided not to have faith because Science, whatever that was, disproved Religion, whatever that was.[3]

What is the subject matter of theology? Who are the conversation partners? A traditionally Methodist way of looking at theology has been to imagine a quadrilateral of influences: Scripture, tradition, experience reason.[4] Do other disciplines operate in this

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[2] Thomas Henry Huxley, *Collected Essays*, Macmillan, 1893-4, Volume V, p. vii - viii.
[3] Owen Chadwick, *The Secularisation of the European Mind in the Nineteenth Century*, Cambridge University Press, 1975, p. 161.
[4] See, W. Stephen Gunter, *Wesley and the quadrilateral*, Nashville, Abingdon Press, 1997.

way? Modern studies of science suggest that the links between experiment, data, theory and context are more important than many scientists have previously allowed.[5]

LISTENING TO THE CHILDREN

That sort of model can be a recipe for highly academic (in both the good and bad senses of the word) theology. In the twentieth century there was a shift to something new. It was probably brought about both by disaster – war, hunger, poverty – and also by a greater awareness of these things. After all as Jesus notes 'you will always have the poor with you.' But now faster reports brought them into our living rooms, and television made the victims stare accusingly at us.

Dietrich Bonhoeffer wrote from his prison cell:

> We have for once learnt to see the great events of world history from below, from the perspective of the outcaste, the suspects, the maltreated, the powerless, the oppressed, the reviled - in short, from the perspective of those who suffer.[6]

Famously, Gustavo Gutiérrez (*A Theology of Liberation*[7]) and many others have claimed that theology is *the second step* behind action, to free the oppressed, to change the conditions of the poor. The Theologies of Liberation, and there are now many, claim to be theology done in the streets, and hovels and at the barricades. From being at the centre, at the high point of theological endeavour, we (the theologically literate) become the technicians, following and trying to understand the really important work that takes place elsewhere. Has the modern (Enlightenment) search for certainly, a fixed point, been replaced by Post-modern relativism? As Sallie McFague writes: 'The "view from the body" is always a view from somewhere versus the view from above, from nowhere…'[8]

The state of the world as a whole becomes a theological issue. The poor are trapped by sin into a subhuman way of life. Poverty is not the role model for a holy life; or in itself the guarantee of salvation, but poverty is that state from which God brings liberation. God has a preferential option for the poor:

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[5] For an introduction see, (Ed) Jay Labinger and Harry Collins *the one culture? A Conversation about Science*, Chicago, University of Chicago Press, 2001.
[6] Dietrich Bonhoeffer, *Letters and Papers from Prison*, London, SCM Press, 1971.
[7] Gustavo Gutiérrez, *A Theology of Liberation*, London, SCM Press, 1974.
[8] Sallie McFague, *The Body of God - An Ecological Theology*, SCM 1993, p. 95.

> The theology of liberation challenges a privatized conception of salvation, where redemption is exclusively connected with the relationship of the individual to God. Messianic liberation embraces the whole of life.[9]

Theology also is seen not just as the writing of academic books, but also in the worship of the Church. Compare Robert Robinson's (1735 -1790) hymn:

> Here I raise my Ebenezer;
> Hither by Thy help I'm come;
> And I hope, by Thy good pleasure,
> Safely to arrive at home.
> Jesus sought me when a stranger,
> Wandering from the fold of God;
> He, to rescue me from danger,
> Interposed His precious blood.

with Brian Wren's hymn (1989):

> Are you the gambler-God, spinning the wheel of creation,
> giving it randomness, willing to be surprised,
> taking a million chances, hopeful, agonized,
> greeting our stumbling faith with celebration?
> If hope will listen, love will show and tell,
> and all shall be well, all manner of things be well.[10]

> Worship cares for us. Inappropriate liturgy can strip us of our basic sense of worth and dignity... *Liturgy, which is the vehicle through which worship is expressed, creates an environment in which human beings confront those sides of themselves which under normal circumstances they dare not face.*

The above is a quotation from an early example of a book which combines serious observation of the Christian community at worship with a study of how people's worshipping life affects their well-being:

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[9] Theo Witvliet *A Place in the Sun*, London, SCM Press, 1985, p. 40.
[10] For Robertson: Methodist Church, *Hymns and Psalms*, London, Methodist Publishing House, 1988, no. 517; for Wren: Brian Wren, *Bring Many Names*, Oxford, Oxford University Press, 1989.

> [I]t is the Christian claim that each human being can only be completed beyond her or himself... [Liturgy] creates a safe environment to push our questions hard and face perplexity. People come to church to be with God and to hide from God, to scream at God and to embrace God, to be with others and to be with themselves... Liturgy is an activity through which a community celebrates its values, passes on its norms and recreates a sense of its own identity through memory and forgiveness... It is both the preservation and expectation of hope.[11]

This shift of emphasis is something of a return to the early centuries when the first theologians were simply the pastors and bishops of the Church, and their theology consisted simply of their commentaries on the books of the Bible, and their sermons. One contemporary claim is that theology is done everywhere, recognized or not. Thus Ann Tomlinson's monograph on pastoral theology is called *Training God's Spies*:

> The Church has a much wider and more glorious calling, namely to be a sign and a sacrament to all humanity of God's redemptive work in creation. Every member of the body has a God-given ministry to point to God's ultimacy and sovereignty, to articulate a sense of the sacred in a relative world, to mediate His presence and action. Ministry is the task of communicating the truth, of offering people a "transcendent reference" for their lives, of expanding their awareness; it is a vocation to mystagogy, a calling to act as guardian and guide of insight and mystery, awakening others to the reality of God.[12]

TALKING WITH THE NEIGHBOURS

A few years ago at a conference I was mugged by a group of Afro-American feminist theologians. They had presented their reworking of the doctrine of the Trinity, and I had suggested in a question that we might all benefit from a dialogue between their picture and the traditional doctrine. No, I was heard as saying that the WASPs want to keep control.

Mary Daly writes: 'It might be interesting to speculate on the probable length of a "depatriarchalized Bible."' A very short pamphlet, she suggests. Daly believes that the problems lie with *the central core symbolism of Christianity*. Why should the past have a priority? For her the women's movement is an 'exodus community.'[13] There is

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[11] Robin Green, *Only Connect*, London, 1987, p. 5-17.
[12] Ann Tomlinson, *Training God's Spies*, Contact Monograph, 2001 Number 11, p. 8.
[13] Mary Daly, *Beyond God the Father*, Boston, Beacon Press, 1974 and *Gyn/Ecology*, London, Women's Press, 1991.

considerable diversity here, see for example, the insights of 'Womanist Theology' (e.g., Jacquelyn Grant, *White Women's Christ, Black Women's Jesus*) which explores whether there are links between the experience of poor women and other theologies of liberation. Is the sisterhood strong enough to encompass the differences between white middle class affluent speculation and Black songs of consolation in poverty?

I deliberately introduced some of the concepts of 'Liberation Theology' in the earlier section, which had a patronizing title. Theology has now moved from 'Listening to the Children' to 'Talking with the Neighbours.' What started as a conversation *within* the Church has now broadened out. Gustavo Gutiérrez ground breaking book from the 1970s, *A Theology of Liberation*, now feels very dated. Most of the book consists of a dialogue between a Roman Catholic priest working in the slums of Peru and his superiors back in Rome. Things have now moved on:

> It [the church in Asia] must be humble enough to be baptised in the Jordan of Asian religiosity and bold enough to be baptized on the cross of Asian poverty... The theology of power-denomination and instrumentalization must give way to a theology of humility, immersion, and participation.
>
> Aloysius Pieris- Sri Lanka
>
> Stanley Samartha: "It is not necessary for Indians to become Jews first before they can catch a glimpse of the fullness of Christ."[14]

It is instructive to look at the different way these issues are treated in successive editions of the best book surveying the area, Ford and Muers, *The Modern Theologians*[15]:

The Modern Theologians I
- *New Challenges in Theology*
  - *Latin American Liberation Theology*
  - *Black Theology*
  - *Asian Theology*
  - *Feminist Theology*

The Modern Theologians III
- *Particularizing Theology*
  - *Feminism, Gender, and Theology*

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[14] For more details, see the various editions of *The Modern Theologians*, below.

[15] David Ford, *The Modern Theologians*, Oxford, Blackwell, 1st edition, 1989; 3rd edition (with Rachel Muers), 2005.

Black Theology of Liberation
Latin American Liberation Theology
African Theology
Theologies of South Asia
Contextual Theology in East Asia
Postcolonial Biblical Interpretation

I *Evangelical and Orthodox Theologies*
Theology and Religious Diversity
Ecumenical Theology
Theology of Religions

III *Global Engagements*

Theology between Faiths

Theology in Many Media

Let's take one example. What is 'Postcolonial Biblical Interpretation' trying to achieve? If you know your way around the Hebrew Bible you will remember the story of the rise and fall of many different empires, so perhaps we should try to situate colonialism at the centre of the Bible and biblical interpretation. Postcolonial Biblical Interpretation 'endeavors to revive and reclaim silenced voices, sidelined issues, and lost causes.' To 'scrutinize biblical interpretation and expose the ideological content hidden behind its apparent claim to neutrality.' To reread the Bible in the light of postcolonial concerns and conditions. The vital questions are now those of 'plurality, hybridity, multiculturalism, nationalism, diaspora, refugees, and asylum seeking.'

> After September 11, 2001 the West, especially the USA, has seized its chance to push through an imperialist agenda accompanied by the noble rhetoric of planting the seeds of democracy, liberty, and human rights in nations ruled by despots. This imperial intervention in public discourse is often redefined as humanitarian assistance and as a liberal enterprise serving both moral and strategic purposes. There is a thin line between humanitarian assistance and the imposition of one's

> values on other peoples… As Aimé Césaire put it long ago: "No one colonizes innocently."[16]

The fundamental claim here is that changed circumstances mean changed theologies. Remember the overall title that Ford and Muers give to this section of their book, *Particularizing Theology*. The 'hermeneutic of suspicion' is dominant here. Who benefits?

Let me sound a note of caution by going back a hundred years. Oliver Lodge was one of those who attempted another reconstruction of religion, this time in the light of the newly prominent place of science at the beginning of the twentieth century. His dismissal of Original Sin catches all the disdain and misunderstanding common to those who think new facts mean that theology must change: 'As a matter of fact it is non-existent, and no one but a monk could have invented it.'

> If this is Christianity, Lodge has assuredly the honour of being the first Christian and it is not improbable that he will have the additional honour of being the last.[17]

Admittedly, Lodge's additional interest in psychical research did not make it any easier for either rationalist scientists or orthodox Christians to accept his views. 'Sir Oliver's theism is of a kind which would make the hair of a Christian stand on end.'[18]

Much more recently Wolfhart Pannenberg, one of the dominant theologians of our time, sought to develop his idea that the Spirit of God can be described helpfully in terms of field theory: 'Wolfhart Pannenberg seems to think that modern field theory offers a way of thinking about spirit, though to a physicist a field is about as spiritual as a tenuous gas!'[19]

'We have had more than enough proposals for the radical revision of a half-understood Christianity on the basis of a half-understood sociology.' Sighs Alister McGrath.[20]

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[16] RS Sugirtharajah, in Ford and Muers, p. 537-8 and 550-1.
[17] Joseph McCabe, *The Religion of Sir Oliver Lodge*, London, The Rationalist Press Association, Watts and Co. 1914, p.45.
[18] W. H. Mallock, 'Oliver Lodge on Religion and Science', *Fortnightly Review*, New Series, LXXVIII, 1905, p. 839.
[19] John Polkinghorne, *Scientists as Theologians*, p. 8; for a detailed discussion see, Polkinghorne, *Reason and Reality*, p. 92 - 94. Pannenberg's thesis is developed in *Toward a Theology of Nature* (Lousville: Westminster/John Knox Press, 1993) and in his *Systematic Theology* (Edinburgh, T&T Clark, 1991-8).
[20] McGrath, Alister, *A Scientific Theology, Volume II: Reality*, London: T&T Clark, 2002, p. 109.

Trying to take seriously the claims of other and new voices, always carries with it the danger of fragmentation. So it isn't surprising that Cambridge recently gave birth to 'Critical Orthodoxy': a call to return to the old sources as the solution not just for theology but for all the troubles of the modern world.[21]

AN ADULT CONVERSATION

Like many other churches the Methodist Church currently faces a set of problems concerning what standards are acceptable for members of the Church. I'll risk a caricature by saying that on the one side are those who see a set of rules given in scripture, and interpreted through the tradition, which mark out God's people as holy. These rules set a positive, unchanging, example about how to live contentedly before the whole world. On the other hand are those who claim the Bible teaches of a God of Love, who made and loves his world. Therefore it is not surprising that new insights arise from God's world which can help the Church better to understand God's will for it.

That *is* a caricature. But the important point is that there is enormous diversity on this issue. So the Methodist *Faith and Order* committee was asked to write a report on *Living with Contradictory Convictions in the Church.* The report set out a history of difference and noted that the analogy you use will very often decide the answer you get. Removing the Church's discrimination against homosexuality is like abolishing slavery, or alternately, the liberal tolerance of this and other heresies is like tolerance of the heresy of apartheid.

> Scripture's witness to diversity is in itself diverse... the 'inclusive' texts of Ruth and Jonah are set against the more 'exclusive' emphases of Ezra, while the notorious differences of Paul and James on faith and works are held together within the canonical New Testament. There are also four Gospels... More commonly, however, dissenting voices are clearly stigmatized and placed outside the community of faith.

Inevitably the live issues for today are about sex. After you have debated it for years in church conferences, even sex becomes boring, but the underlying issues are crucial. As *Contradictory Convictions* asks:

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[21] John Milbank, *Theology and Social Theory: Beyond Secular Reason*, Oxford, Blackwell, 2nd edition, 2006, Catherine Pickstock, *After Writing: on the liturgical consummation of philosophy*, Oxford, Blackwell, 1997.

> [Are] we now facing a question where some kind of diversity can or must be lived with, or whether unity 'at any price' is not a price worth paying. Is what we are examining an acceptable or unacceptable form of diversity?[22]

J.W. Van Huyssteen is a philosophical theologian. His *Gifford Lectures* are called *Alone in the World?* Van Huyssteen sets out to ask the question what makes human being distinctive, if indeed they are. I won't go into the main argument of the book, simply to say that van Huyssteen is clear from the start that his enterprise can only be attempted by listening to different voices. The philosophical theories of 'foundationalism' tried to prove that there were firm foundations from which to build theories of knowledge. There is general agreement now that the work of philosophy since the end of the nineteenth century has shown that foundationalism is a failed dream. So:

> Crucial to a post-foundationalist theory of rationality, then, is a theory of experience that will enable us to reason adequately about the various facets of our human experience... In fact, post-foundationalist theology, like science, relies on a community, a community that converses with itself but also seeks to engage in dialogue across the disciplines because of the rational resources we share... A post-foundationalist notion of rationality thus creates a safe space where our different discourses and actions are seen at times to link up with one another and at other times to contrast or conflict with one another. It is precisely in the hard struggle for interpersonal and interdisciplinary communication that the many faces of human rationality are revealed.[23]

Safe spaces, community, conversation. These are recurrent themes in modern theology, for example in the work of David Ford, Daniel Hardy and the philosopher Mary Midgley. Since Thomas Huxley they have been viewed with some suspicion by scientists, who claim the right to judge what is meaningful. My central contention here is that this will not do. Midgley rightly claims that there are many kind of causes, and of explanations, so that 'the diversity of questions is a central topic...'[24] Science has worn blinkers and majored on reductionism in order to solve problems; sometimes in order to be allowed to solve problems by those in authority – both religious and political. Theology likes to do the same. It is natural to like firm ground under one's feet. The demands of community and conversation sometimes mean that the ground moves, and as Galileo discovered, that reality can be dangerous. But the earth does move.

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[22] *Methodist Conference Agenda 2006*, p. 237-250.
[23] J.W. Van Huyssteen, *Alone in the World? Human Uniqueness in Science and Theology. The Gifford Lectures 2004*, Grand Rapids, Michigan, Eerdmans 2006, p. 13-23.
[24] Mary Midgley, *Science and Poetry*, London, Routledge, 2001, p.119.

Theologians are not called to proclaim a flat earth, or for that matter an unevolving creation, but rather to reach out to touch and listen to the variety of voices which sing God's story. We begin by acknowledging that we stand in one particular place but from there we can at least see, if not necessarily always comprehend, the world.

Some *A Priori* Torah Decryption Principles[1]

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The author proposes, a priori, a simple set of principles that can be developed into a range of algorithms by which means the Torah might be decoded. It is assumed that the Torah is some form of transposition cipher with the unusual property that the plain text of the Torah may also be the cipher text of one or more other documents written in Biblical Hebrew. The decryption principles are based upon the use of Equidistant Letter Sequences (ELS's) and the notions of Message Length, Dimensionality, Euclidean Dimension, Topology, Read Direction, Skip Distance and Offset. The principles can be applied recursively and define numerous large subsets of the 304,807! theoretically possible permutations of the characters of the Torah.

1. Introduction

As a result of the controversy generated by the "Bible Code" book by Michael Drosnin [8] and the resultant publicity given to the work of Witztum, Rips & Rosenberg [1] (WRR), the author was motivated to develop this set of a priori Torah decryption principles in late 1997. They are described with some additional commentary and provide the background for other more recent work completed by the author.

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[1] Presented at the 2nd conference of the Int. Torah Codes Society, Jerusalem, Israel, 5th June 2000

Controversy aside, the author maintains a working hypothesis that the Torah is a cryptogram and that it is intended that it be decoded. More specifically, the author's view is that the Torah is some form of transposition cipher[2] and that ELS's are the basis for any possible decryption. The author takes the view that the Torah may have an unusual property in that the plain text of the Torah may also be the cipher text for at least one other lengthy document written in Biblical Hebrew.

Though the above suggestion may seem ludicrous, the existence of anagrams is unremarkable and exemplify the principle of readable text being readily concealed within other readable text. Of course, the length of a traditional anagram is very short in comparison with the proposition being made with regard to the Torah. Long transposition ciphers with a lucid cipher text are exceedingly difficult to create [9] using conventional computation and it is possible that this difficulty may have been overcome in respect of the Torah by the use of some form of Quantum Computer[3].

Note that despite the extreme difficulty of creation of long transposition ciphers (using conventional finite state automata), it is entirely possible that decryption (using conventional finite state automata) may be relatively easy. In this respect there may turn out to be a direct analogy between the RSA cipher[4] and the Torah. RSA uses the relative difficulty of factorisation of large integers as its Trap Door function. Likewise, the Torah may have some transposition algorithm and a long key as the basis for its own one way barrier function.

The author is aware of the status of the Torah and ignores the obvious theological and philosophical problems that work in this area poses, save to note that should it turn out to be the case that the Torah is a large scale (possibly multiple) transposition cipher, then there may be severe limitations as to speculation with regard to its origin.

Since the method of any encoding and decoding of the Torah are unknown to us, perhaps we should approach the matter in some sort of logical sequence.

2. Use of ELS's

2.1 Background

Much has been written regarding ELS based codes in the Torah and other secular and religious material in a wide variety of languages. Though attempts have been made to establish rigorous proofs that there are or are not [11] ELS based "codes" in the Torah, the issue is not resolved. For the proponents there remains a series of stunning coincidences which defy statistical analysis due to their uniqueness. For the sceptic, the Torah remains an ordinary document that appears entirely random when subjected to a variety of statistical analyses [10] [12]. The author has results that fit squarely in both camps.

Common factors in most forms of this work are the use of ELS's, per WRR's original definition, though some work has been done on Gradually Increasing/Decreasing Letter Sequences. A further and more imposing common factor is that most work so far has focussed upon the use of ELS's and the attempted detection of proximate conceptually related word pairs. Few attempts have been made to widen the field to embrace other potential properties of the Torah and other textual material.

In this paper, though ELS's are used as the basis for the various algorithms, there is almost no commonality with other previous work in this area.

2.1 Algorithm One

It is apparent that if we skip through texts whose length L is prime, using a skip distance D where $1 < D < L$, we can transpose an original text into L-2 different derivative texts very easily:

Let T[*i*] be the *i*th character of an array containing the Torah, or some other section of the Hebrew Bible or a control and P[*i*] be the *i*th character of an array containing the permuted document. If the lengths of T and P are L and L is prime and D is the ELS skip distance, then we can permute the input document using the simple 'C' routine:

/* Algorithm One */

```
for (i=0;i<L;i++)
          {
P [i] = T [D*i % L];   /* % is the 'C' modulus operator */
          };
```

For example, the text string "MARY HAD A LITTLE LAMB!" which is 23 characters long including spaces, can be transposed into the nonsensical string "MR A ITELM!AYHDALTL AB" by considering it as a one dimensional ring of characters and using a skip distance (or "key") of two.

There are 23 factorial permutations of a meaningful 23 character string and it is extremely unlikely that any of the 21 permutations defined above should result in a readable derivative text. For longer strings, the probability of finding a meaningful sequence in one of the derivative permutations tends to zero at the exponential rate of 1/(N-1)!

Though vanishingly small, the possibility cannot be excluded that there exists an algorithm (perhaps based upon Algorithm One) which might permute the characters of the Torah into some other document as a result of wilful and very clever design.

By way of further example, Table 1 contains a list of English words that can be permuted into other English words using Algorithm One and a skip distance of two. Table 1 was compiled by manually searching the pages of a pocket dictionary. Readers might like to attempt the creation of longer, meaningful word strings such that they permute into other meaningful word strings by the simple application of Algorithm One.

Table 1

Feast	Fates
Fount	Futon
Green	Genre
Point	Piton
Treat	Tetra
Moans	Mason
Pearl	Paler
Perry	Pryer
Prise	Piers
Taint	Titan
Weird	Wider

Note that Algorithm One significantly reduces the number of anagram candidates. Many five character words are anagrammatical. However, of the 120 possible permutations of the characters of a five character word, Algorithm One only defines five of them. This is important when considering longer strings and more complex algorithms because the number of possible permutations becomes effectively infinite (in fact uncountably finite). Algorithm One, its derivations and variations address the computable [5] permutations of the characters of the Torah.

4. Establishing Message Length

4.1 Data Communications fundamentals

One of the fundamental issues in the field of Data Communications is the means by which the length of a transmitted section of text is communicated between sender and recipient. Techniques include the use of blocks of a pre-determined size and variable length blocks with a pre-specified delimiter. It is not unreasonable to assume that message texts from unknown sources might have a length which was prime. Note that the 3 Terawatt SETI message transmitted from the Arecibo radio telescope in 1974 consisted of 1679 bits in a 73 * 23 B&W binary image.

Other potentially relevant data communications techniques include the use of error detection and correction methods and the use of data compression techniques to reduce the data volume and transmission time. Basic techniques include parity checking, Cyclic Redundancy Checks, Hamming codes and so on. Which, if any, of these techniques might be of relevance in analysing the Torah remains to be seen.

In the development of the algorithms that follow, no assumption is made about the data that is being analysed. Once a text and a message length has been chosen, the various approaches to further processing it naturally follow.

4.1 Torah Message Length

There are numerous versions of the Torah, whose content and length vary by a small amount. The traditions surrounding the historic copying and distribution of the Torah are strict and have led to the near letter perfect preservation of the various versions of the Torah for in excess of a thousand years. WRR and the author base their work upon the digital version of the Koren edition [6] of the Torah. The Koren edition is itself based upon the ancient Masoretic Textus Receptus. The author assumes that the particular features of the Koren edition of the Torah are there by design and various inferences are drawn. There is no reason however why differing inferences should not be drawn or that these methods should not be applied to any other version of the Torah (eg the Leningrad Codex), or any other document in whatever language.

The Koren edition of the Torah is 304,807 characters long including two inverted letter nuns. 304,807 is prime.

4.2 The Inverted Letter Nuns

There are 22 basic characters in the Hebrew alphabet and in addition there are two instances where the letter nun, which is the 14th character in the Hebrew alphabet, is written upside down in the Torah. The inverted letter nuns bracket the short section of text comprising Numbers 10 verses 35 & 36:

> 35 When the Ark would journey, Moses said "Arise HASHEM, and let Your foes be scattered, let those who hate You flee from before you"
>
> 36 And when it rested, he would say, "Reside tranquilly, O HASHEM, among the myriad thousands of Israel". [7]

Nobody knows why the inverted letter nuns are there. One theory has it that the short section of text between the inverted nuns is a book in its own right.

Due to the unusual nature of the inverted nuns (especially their resemblance to a pair of ASCII square brackets), we make an assumption that their presence is intended to draw attention to the fact that there are three distinct textual sections within the Torah. We assume that their presence and absence, together with the presence and absence of the section of text between the inverted nuns and the inverted nuns themselves is related to the issue of establishing message length.

Table 2 sets out some of the possible combinations of text sections and gives the prime factors of the lengths of each of the various combinations of the sections of text. We refer to the three major text sections as T1, T2 and T3 and the letter nuns as N1 and N2.

By inspection, the length of the text between the inverted letter nuns in the Koren edition is 85 characters, thus text section T2 has length 85. The lengths of sections T1 and T3 are 206,588 and 98,132 respectively.

Table 2

Text	**Length**	**Number Of Factors**	**Prime Factors**
T1,N1,T2,N2,T3	304807	1	304807
T1,T2,T3	304805	2	5, 60961
T2	85	2	5, 17
T1	206588	3	2, 2, 51647
T3	98132	3	2, 2, 24533

Other combinations of text section can be examined (including or not including inverted nuns)

5. Dimensionality & Euclidean Dimension

Given the factorisation characteristics of the various text sections as given in Table 2 we could consider the resultant text as either a linear string, a two dimensional array, a three dimensional cuboid or some other shape in M dimensions.

Clearly, with a regular, M-dimensional layout of text, the process of skipping through the text (along whichever dimension) to generate further derivative texts using variations of Algorithm One is made much simpler when there are no jagged edges.

Note that WRR's analysis assumes that the text of the book of Genesis is laid out in two dimensions, in rectangular arrays of varying size with an incomplete last row. The irregularity of these shapes would not permit the efficient creation of large derivative texts using ELS's and a simple algorithm.

6. Linear permutation

For a text of 304,807 characters there are 304,805 permutations of this text using ELS's and Algorithm One as described in section 3. Automatic evaluation of these 304,805 derivative versions of the Torah using software designed to either detect meaningful Hebrew text or measure randomness is not difficult.

Examination of a string of length L where L is prime using Algorithm One is referred to as a Linear Permutation. Use of **Algorithm Two** (the Maxquad function) to measure the randomness of the Linear Permutations is reported separately.

7. Rectangular permutation

7.1 Methods of arranging text

If we now turn our attention to the second row of Table 2 where T1, T2 & T3 are concatenated, we note that the length of the document is 304,805, which factorises into 5 * 60,961, potentially implying that the text should be laid out in a two dimensional array comprising 5 rows of 60961 columns. We have no information however about how we should arrange this text within the confines of a two dimensional array. Is the text to be treated as a five long strings, each of length 60961, or is it to be treated as 60,961 strings each of length 5?. Do we arrange the text in each row right to left or left to right? Is the text to be treated as a 5 * 60961 ring, or is it to be treated as a 5 * 60,961 Mobius ring? Or is the topology more complicated still?

7.2 Textual Topology & Read Direction

Thus the establishment of the Topology and text Read Direction of any encoding in two, three and more dimensions is crucial if we are to fully investigate permutations of the Torah.

In considering the topology as being either a ring or a Mobius ring, we note that these are two specific variations of the 5! different ways of joining up the ends of a 5 * 60,961 array. In fact there are 120 different topologies obtained by placing the five rows in each of 120 different combinations.

Of course, we do not know if we are to arrange the text left-to-right or right-to-left within each row. Since there are 5 rows, there are 2^5 or 32 different ways we can arrange the rows, and each of these should be considered.

7.3 Skip Distance

Having arranged the text in rows and columns in two dimensions using all the various combinations of topology and read direction, we can then use a simple variation of Algorithm One to skip through the complete two dimensional array every D characters, starting at the first character and omitting values for D which are not co-prime to 5 and 60,961 (or to each of the vertex lengths if the dimensionality is not two).

7.4 Offset

Of course, starting Algorithm One at the first character is restrictive - there is no reason why Algorithm One could not be started at some other character position - the Offset position. The same character sequence is generated, but the resultant string is byte shifted by the Offset amount. Offset only becomes significant when considering recursion as otherwise the same sequences are generated, rotationally shifted by one or more bytes.

7.5 Tractability

Note that in the above discussion, we assume that the text is being laid out in 5 long strings each of length 60,961 which gives rise to the possibility of there being 5! differing topologies and 2^5 different row read directions. Of course, we could just as easily assume that there are 60,961 strings of length 5 to be placed in 60,961! different topologies with 2^60,961 row read directions. Clearly, the former is tractable, whereas the latter is not (yet). For this reason, and in all further analysis, text is laid out in a smaller number of longer rows and ELS skipping takes place along the long direction only.

7.6 Recursion

Examination of a string of length L where L has two prime factors using the method outlined above is referred to as Rectangular Permutation. The resultant algorithm is known as **Algorithm Three**. Algorithm Three is recursive and recursion can take place to any number of Levels, though at higher levels of recursion the issue of computability becomes relevant. Algorithm Three is specified more exactly in Appendix A.

7.7 Number of Permutations

For Torah text T1T2T3 and T2 alone, the total number of permutations of the text using Algorithm Three to various levels of recursion is:

	T1T2T3	T2
Topologies	120	120
Read Directions	32	32
ELS Skips	243839	32
Offsets	304805	85
Recursion Level 1	2.85E+14	1.04E+07
Recursion Level 2	8.15E+28	1.09E+14
Recursion Level 3	2.32E+43	1.14E+21
Recursion Level 4	6.63E+57	1.19E+28
Recursion Level 5	1.89E+72	1.24E+35

Note that T1T2T3 and T2 are similar in that they both have 5 rows. There is therefore a possibility that T2 may be the index for the whole Torah. Any Algorithm Three permutations of T2 that are lucid may be indicative of the possibility that application of the same Algorithm Three parameter sequence to T1T2T3 may result in the derivation of a New Torah.

Based upon estimates of currently available technology a single large device could be built to achieve exhaustive evaluation of T2 to Recursion Level 3.

8 Some A priori short cuts

Given the substantial number of permutations of even T2 that can be evaluated, there exists the possibility that any encoder may have built in some clues as to the correct topology and read direction in the design of the message itself. This might avoid exhaustive evaluation of every possible combination.

8. 1 The Topological Interlock

The author proposes (a priori) that if the text of the Torah is laid out as a 5*60,961 array (or a 5*17 array as in the case of T2) and is examined using ELS sequences in the long and short directions with the same key, for one topology (being the correct topology) the two sub components generated during this process may be the same, similar or complementary. The author denotes this test as the Topological Interlock test. The two components being the horizontal and vertical Topological Interlock components.

For example, assume a semi-nonsense 21 character plain text "GOSSIERMISNOMEREXODUS" laid out on a 7 by 3 array. The rows are numbered 1 to 3 and the columns labelled A to G, in standard spreadsheet format. Using a skip of 5, the Horizontal and Vertical Topological Interlock Components are calculated as follows:

	A	B	C	D	E	F	G	Vertical Component						
1	G	O	S	S	I	E	R	A1	B3	D2	F1	G3	B2	D1
2	M	I	S	N	O	M	E	E3	G2	B1	C3	E2	G1	A3
3	R	E	X	O	D	U	S	C2	E1	F3	A2	C1	D3	F2

Horizontal Component

A1	F1	D2	B3	G3	E1	C2
A2	F3	D1	B2	G2	E3	C1
A2	F2	D3	B1	G1	E2	C3

The actual text of the two interlock components is therefore:

	A	B	C	D	E	F	G	Vertical Component						
1	G	O	S	S	I	E	R	**G**	**E**	**N**	**E**	**S**	**I**	**S**
2	M	I	S	N	O	M	E	D	E	O	X	O	R	R
3	R	E	X	O	D	U	S	S	I	U	M	S	O	M

Horizontal Component

G	**E**	**N**	**E**	**S**	**I**	**S**
R	U	S	I	E	D	S
M	M	O	O	R	O	X

One row of the interlock components match exactly and indeed spell an interesting word "GENESIS". We may therefore be drawn to the conclusion that the 7 by 3 layout with traditional English left-right top-bottom reading is the correct topology for further investigation of this text.

8. 2 The Directional Interlock

Similarly, in considering text laid out in a 5 * 60,961 array in what may have been determined to be the correct topology, there are four directions in which text can then be read starting from the first character. These are denoted North, South, East and West. The author proposes that read direction may also be indicated by a Directional Interlock test, where the correct read direction is indicated by one of the four Directional Interlock components being distinctly different from the other three, when each component is created using the same ELS key.

Let us again assume that we are examining some text again laid out on a 7 * 3 array. Using spreadsheet notation and a key of 5, we can derive the North, South, East and West components of the Directional Interlock.

North

```
A1 C2 E3 F1 A2 C3 D1
F2 A3 B1 D2 F3 G1 B2
D3 E1 G2 B3 C1 E2 G3
```

West

```
A1 C2 E3 G1 B1 D2 F3
A3 C1 E2 G3 B3 D1 F2
A2 C3 E1 G2 B2 D3 F1
```

Torah

	A	B	C	D	E	F	G
1	T		E	O	L	E	S
2		A	H	D	C	N	E
3	R	I	P	S	I	I	T

East

```
A1 F1 D2 B3 G3 E1 C2
A3 F3 D1 B2 G2 E3 C1
A2 F2 D3 B1 G1 E2 C3
```

South

```
A1 B3 D2 F1 G3 B2 D1
E3 G2 B1 C3 E2 G1 A3
C2 E1 F3 A2 C1 D3 F2
```

The texts of the four directional interlock components are therefore:

North

```
T H I E   P O
N R   D I S A
S L E I E C T
```

West	**Torah**	**East**
T H I S D I	T E O L E S	T E D I T L H
R E C T I O N	A H D C N E	R I O A E I E
P L E A S E	R I P S I I T	N S S C P

South

```
T I D E T A O
I E   P C S R
H L I   E S N
```

In the above example, there is ample evidence to suggest that West is the correct read direction.

8. Cubic Permutation

T1 and T3 each have three prime factors and can be permuted by algorithms similar to those above, with the additional factor being that there are eight choices of corner from where the ELS skip through could commence from.

9. Summary & Conclusion

Having given due consideration to the establishment of Message Length, Dimensionality, Euclidean Dimension, Topology, Read Direction, Skip Distance and Offset, the Torah and various parts of it can be recursively permuted into myriad further long strings using a simple algorithm. The order in which these various steps can be applied to the Torah can be varied in a relatively limited way.

Given the extreme difficulty of creating long transposition ciphers where the cipher text is also lucid, it is incredibly unlikely that even the short section of text comprising T2 should be capable of permutation into a readable sequence by any variation of the algorithms described above (or any other algorithms), except by virtue of intentional design.

The task of evaluating the above algorithms and any output will be very substantial. However, Algorithm Three is suitable for execution on highly parallel architectures and also in specialist PLA type hardware where sequence generation (and some evaluation) can take place at clock speed rates.

To exhaustively evaluate T2 to recursion Level 3 will require, for example, the development of specialist hardware in substantial volume operating within a large networked configuration. Alternatively, if sufficient publicity can be generated, a significant proportion of the machines connected to the internet operating in unison could be just as effective.

If it is thought that T2 may be the index for T1T2T3, then the parameters from T2 sequences that pass a priori readability or lucidity thresholds can be applied automatically to T1T2T3.

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Appendix A Algorithm Three

Here is some indicative 'C' code for Algorithm Three:

```
int t2[85], loop_t2[85], rp_t2[85];                    // T2, TEMP T2, PERMUTED T2

int topology [120] [5] = {                             // TOPOLOGY TABLE
                    { 0, 1, 2, 3, 4},
                    { 0, 1, 2, 4, 3},
                      etc                              // 120 Rows Long
                    { 4, 3, 2, 0, 1},
                    { 4, 3, 2, 1, 0},
                    };

int top, row, offset, skip;
int r0, r1, r2, r3, r4;                                // Row Flip Binary Counters
int i, j, k, m, u;                                     // Loop Counters etc

for (top=0;top<120;top++)                              // 120 Topologies
{ i=0;
 for (u=0;u<5;u++)                                     // Load the 5 rows in order
  {for(m=0;m<17;m++)                                          // Each of 17 Characters
   {loop_t2[i++]=t2[(17*topology[top][u])+m];};        // Depending upon which Topology
  };
 for(r0=0;r0<2;r0++)
 { reverse_row(loop_t2,0);                             // Left Right Reverse The First Row
   for(r1=0;r1<2;r1++)
   { reverse_row(loop_t2,1);                           // Left Right Reverse The Second Row
     for(r2=0;r2<2;r2++)
     { reverse_row(loop_t2,2);                         // Left Right Reverse The Third Row
      for(r3=0;r3<2;r3++)
      {reverse_row(loop_t2,3);                         // Left Right Reverse The Fourth Row
        for(r4=0;r4<2;r4++)
```

```
{reverse_row(loop_t2,4);                         // Left Right Reverse The Fifth Row
 for(offset=0;offset<85;offset++)                // For 85 Offsets
{for(skip=0;skip<85;skip++)
 {if(skip % 5 !=0 && skip % 17 !=0)              // For (net) 32 Skips
  {for(m=0;m<85;m++)
      {rp_t2[m]=loop_t2[(m*skip+offset)%85];;}   // Permute T2
   analyze(rp_t2);                               // Analyse or pass to next Recursion Level
   };
   };
  };
  };
 };
 };
};
};
};
                                        // Don't expect this to compile!
```

Torah, Kondo & the Combinatorial Hierarchy

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1.05

A few ideas from three large but very different bodies of knowledge are introduced, compared and contrasted in order to gain further insight into the underlying nature of reality. We conclude with the *a priori* hypothesis that the underlying data structures behind the electron and the positron must be of tetrahedral form.

1.0 Introduction

The Torah is the original Hebrew version of the first five books of the Bible. Following a suggestion that the Torah might be some sort of cryptogram, and having learnt that the length of the popular Koren Edition of the Torah was prime, a formal *a priori* decryption methodology was conceived. A subsequent investigation established that the methodology was effective. Indeed, it was relatively easy to algorithmically permute a small but well defined part of the Torah into multiple alternate readable (but not yet meaningful) forms through the use of simple but effective statistical and deterministic filtering algorithms. During the course of this work, two potentially significant coincidences emerged which serve to support the thesis that the Torah may be more than it immediately appears to be in a manner that may be relevant to the physical sciences. The results of this project were reported at a small conference in Jerusalem, Israel in June 2000 and finally published, albeit in the wrong order, in 2006 & 2007 in the proceedings of the Alternative Natural Philosophy Association (ANPA) [Croll, G.J., 2007] [Croll, G.J., 2006].

Whilst researching the subject of cryptography at the British Library, the author discovered, less than three metres away, a large number of mathematical monographs on the physical, biological and other sciences mostly written by Kazuo Kondo, late Emeritus Professor of Mathematics at Tokyo University, Japan. Following lengthy examination of the several hundred monographs in this body of work and related extensive previous material, it became clear that this body of knowledge may be of significance in the quest for a unified theory. An introduction to Kondo's work was written [Croll, G.J., 2006] and distributed to Kondo's wife and approximately ten now retired physicists and mathematicians who had worked with Professor Kondo. As a result, Professor Tomoaki Kawaguchi generously furnished the author with an original and near complete set of Kondo's post RAAG monographs. Sadly, Mrs Reiko Kondo is recently deceased. Professor Laxmi Chandra Jain has generously provided copies of his Post RAAG work [K321 et al], together with further work on his Functional Non-Symmetric Field Theory [Jain, L.C. & Jain, Smt. M., 2005] and on other work in the Jainist religious tradition. In addition, Professor Jain provided the author with copies of his 40 year written correspondence with Professor Kondo which add some significant personal insights.

By 2004 the author was aware of the existence of ANPA West through its now defunct website, however he had been unable to make contact. Through a series of virtuous coincidences, Professor Brian Josephson kindly indicated that ANPA 26 was in progress in Cambridge. The author became involved and was thus exposed to a third large body of knowledge, within which the Combinatorial Hierarchy was an important focus.

It gradually became apparent to the author that there were connections between these three subjects. The purpose of this paper therefore is to introduce new material on the first two of these subjects, to highlight key features of the third subject, to discuss the three pairs of subjects and to draw any resulting inferences and conclusions.

2.0 Torah

The Torah is an ancient document that has been preserved in letter perfect form for at least a thousand years. There are just a few versions, the author's researches having been performed on the Koren edition which is available digitally. There is a strict tradition regarding the copying and use of the Torah. There are exhortations to preserve its integrity in all three Abrahamic traditions [Kaplan, A., 1997]. The length of the Torah, including two inverted letter nuns, is both a prime and an emirp (304,807 & 708,403) which immediately raises the interest of a cryptographer, particularly in view of the 1974 SETI transmissions from the radio telescope at Arecibo which were encoded on a 73 by 23 binary matrix.

The Torah describes in the first verses of Genesis an *ex-nihilo* creation by a divinity whose most common name יהוה (JHVH) occurs 1839 times. Coincidentally, 1839 is the Neutron/Electron rest mass ratio to four significant digits. 1839 is also exactly thrice 613, where 613 is an important and well known number in the Jewish faith, as this is the number of Mitvot of the Jewish faith. The Mitzvot are the positive and negative obligations which are written in the surface text of the Torah. Note that 613 is prime. The Mitzvot were established and codified by the Jewish sage Maimonides approximately 800 years ago. Note also that there are two actual and one hidden occurrences of JHVH in the 85 character section of text between the inverted nuns, perhaps hinting at 1836, or Mp/Me to a good approximation. See figures 1 & 2.

ויהיבנסעהארנויאמר	0
משהקומהיהוהויפצוא	1
יביכוינסומשנאיכמפ	2
ניכובנחהיאמרשובהי	3
הוהרבבותאלפיישראל	4

Figure 1 - Original text of section T2 of the Torah (Numbers 10, verses 35-36), with characters comprising two JHVH's underlined

*

הוהרבבותאלפיישראל	4
ניכובנחהיאמרשובהי	3
ויהיבנסעהארנויאמר	0
יביכוינסומשנאיכמפ	2
משהקומהיהוהויפצוא	1

Figure 2 - T2 with rows switched to reveal AJHVH in the central, asterisked, "pivot" column. Note the word "April" in column 1 on the left. Coincidentally this diagram was first drawn in April 1999.

These coincidences may be viewed as a method for determining the Torah's integrity. The numeric values of each character can be summed (encoding the 22 characters as 1234567891234567891234) to determine if there is any significance in the checksum or its factors. The checksum is 10495013 and it is interesting to note that there are 10 commandments, 49 is the year of Omer which occurs immediately before a Jubilee year, there are 50 years in a Jubilee and there are 13 obligations of the Jewish faith.

Within the Jewish tradition, the Torah is viewed as being "The Instrument of Creation". One of the principle works of Jewish Mysticism, the Zohar, states "God looked into the Torah and Created the World". Entire books have been written about the first words and verses of the Torah. Stan Tenen [Tenen, 2007] has spent over 30 years looking into the geometrical structures and properties contained in Genesis. It is long said that the first character of the Torah, Bet (ב)is there to indicate that a partition was made out of the void. The first character of the Hebrew alphabet is Aleph (א) which has no sound. Aleph was not the first character of the Torah, because although it has no sound, it is not nothing. Bet was therefore the first character, representing the establishment of a partition with both upper and lower bounds denoted by the horizontal arms of the character Bet. According to Jewish tradition, before the act of creation there was nothingness so devoid that not even number existed. Out of this void a finite partition was created.

The Torah text can be studied at various levels: simple; hint; conceptual; and hidden, the Hebrew letters for these levels forming the Hebrew word Pardes (פרדס) "paradise". These frequently occurring subtleties lying just below the surface text are lost in translation. Contemplation of the anagrams of the English word "Snipe" illustrates this point well.

2.1 Kabbalah

The written religious tradition of the Torah is made whole by the Talmud, which is a documented oral tradition. There is also a mystical tradition of the Kabbalah, most of which is not available in the west or in English and much of which is secret. There are hierarchies of access to information about the Torah and the Kabbalah. It is said that the Kabbalah should not be studied before the age of 40. From the age of 50 other rights and privileges are rumoured to pertain. Caution is recommended in the selection of books regarding the Kabbalah, Kaplan and Scholem being reliable.

One of the most important documents in the Kabbalistic tradition is the Sefer Yetzirah. Rabbi Aryeh Kaplan's authoritative commentary [Kaplan, A., 1997] on it begins:

"The Sefer Yetzirah is without question the oldest and most mysterious of all Kabbalistic texts. The first commentaries on this book were written in the 10th Century, and the text itself is quoted as early as the sixth. References to the work appear in the first century, while traditions regarding its use attest to its existence even in Biblical times. So ancient is this book that its origins are no longer accessible to historians. We are totally dependent on traditions with regard to its authorship".

The Sefer Yetzirah is a very small and concise book. In its Short version, it is only some 1300 words long, the Long version being 2500 words. The Gra version, preferred by Kaplan, begins:

With 32 mystical paths of Wisdom
Engraved Yah
The Lord of Hosts
The God of Israel
the living God
King of the universe
El Shaddai
Merciful and Gracious
High and Exalted
Dwelling in eternity
Whose name is Holy -
He is Lofty and Holy
And He created His universe
with three books (Sepharim)
with text (Sepher)
with number (Sephar)
and with communication (Sippur)

Kaplan then goes on to analyse the above verse word by word, taking 14 pages to do so. Regarding the last three lines he writes:

> *"These three books are said to be 'text, number and communication' The Hebrew word for 'text' here is 'Sepher', which literally means 'book'. 'Number' is 'Sephar' from which the English word 'cipher' is derived. 'Communication' is 'Sippur' which more literally is 'telling".*

Note the correspondence with the three distinct parts of the Torah defined by the two inverted letter nuns denoted T1, T2 and T3 in my earlier papers.

The Sefer Yetzirah continues in a manner that expresses mathematical, geometrical astronomical and universal constructs, the construction and labelling of parts of the human body, language, aspects of consciousness and spirituality.

For the mathematician, verse 4:16 is interesting:

Two stones build two houses
Three stones build 6 houses
Four stones build 24 Houses
Five stones build 120 houses
Six stones build 620 houses
Seven stones build 5040 houses
from her on go out and calculate
that which the mouth cannot speak
and the ear cannot hear

For we see the factorial sequence and learn Kaplan's comment:

"Thus from the permutations of the alphabet, a name can be formed for every star in the universe. This is in accordance with the teaching that every star has an individual name"

Interpretation of the Sefer Yetzirah is not merely literal, or mathematical: it can be physical as in verse 3:4:

Three Mothers, AMSh (אמש)
in the Universe are air, water, fire.
Heaven was created from fire
Earth was created from water
And air from Breath decides between them.

Kaplan trained and worked as physicist as we can see from his interpretation of the above verse:

"In the simplest physical terms, 'water' represents matter, 'fire' is energy, and 'air' is the space that allows the two to interact. On a somewhat deeper physical level, fire, water and air represent the three basic physical forces. 'Fire is the electromagnetic force, through which all matter interacts. The atomic nucleus, however consists of like positive charges, which would repel each other if only electromagnetism existed. There must therefore exist another force which can bind the nucleus together. This is the 'strong nuclear' or pionic force, which binds the nucleus together, represented by 'water'. If this nuclear force were to interact with all particles, however, all matter would be mutually attracted together, forming a solid lump denser than a neutron star. On the other hand, even within each elementary particle, there is a need for a cohesive force to counter the electromagnetic repulsion within the particle itself. This force can be

neither electromagnetic nor pionic. This is the 'air' which represents the 'weak nuclear' force, which 'decides between' the other two. It is this force that allows light particles (leptons) such as electrons to exist."

Thus we see the terminology of modern theoretical physics within a credible and authoritative commentary on the Sefer Yetzirah, an ancient and mystical document which itself can be viewed as a commentary or even a user manual for the Torah.

3.0 Kondo

Almost every month for three decades until his death aged 90 in December 2001, Professor Emeritus Kazuo Kondo, Dept. Applied Mathematics, Tokyo University, Japan, published a monograph developing his ideas on his mathematically based natural philosophy. There are 358 monographs in total, each monograph comprising 16-48pp (referenced hereon as K1-K358). It is surely one of the largest bodies of work completed by a single individual. More remarkable is the content and the fact that it has lain undiscovered and unappreciated (other than by his few remaining colleagues & students) for so long.

Kondo's mathematically based natural philosophy presupposes that reality consists of information elements, in myriad permutations and combinations and that they are indistinguishable, finite and countable.

That is to say, in Kondo's view, there is no underlying physical reality, merely number. Reality is perceived through the counting of number. Kondo gives only the merest clue regarding the storage location of number, however it is not difficult to gain an impression of where Kondo views it to be held. The front and back covers of the four volumes of his RAAG work [Kondo, 1955, 1958, 1962, 1968] and the last page of the last monograph in the post RAAG series have embossed or printed upon them a curious symbol containing the famous Platonic slogan in Greek "God ever Geometrises".

There is a mapping between Kondo's integral parameter space and the continuous three dimensional space we perceive through a mathematical construct known as the Kawaguchi space. The Kawaguchi space was first referred to as such by Synge in the early part of the twentieth century.

Kawaguchi spaces are a class of higher order space. The properties of such spaces have been studied widely. Miron [Miron, 1998] has recently produced a number of standard texts on the subject, though the bulk of the work on Kawaguchi spaces has been completed in Japan in the first half of the twentieth century [see Kawaguchi, M., 1962 & 1968].

A critically important result, due to Kawaguchi, Hombo and Kondo [K291, Page 21] proves that for Kawaguchi spaces of a higher order, the dimensionality of such spaces need not exceed three. Kondo's interpretation of this result, which he discusses widely, is that it is the mathematical reason why reality has the dimensionality that we observe.

The existence of pre-spaces as a model of reality has been previously postulated by Hiley [Hiley, B.J., 2000] and others. The pre-space model of reality is appealing because of the simplicity of representation of the complex structures within reality such as molecules, crystals and biological forms etc. The Kawaguchi transformation facilitates the mapping of just a few numeric parameters into complex spherical and other topological continuous forms.

The use of the Kawaguchi space is further appealing in that there is a natural mathematical axiom: particles are spaces and spaces are particle. This neatly and mathematically addresses the wave particle duality. Such duality does not exist within Kondo's work as the mathematics of Kawaguchi spaces naturally deals with both interpretations in a consistent manner.

The boundary between the macroscopic and microscopic is clearly defined by consideration of the dimensionality of the Kawaguchi spaces representing objects. Zero and positive dimensionality corresponds to macroscopically observable items. Negative dimensionality corresponds to the microscopic, unobservable or quantum mechanical world.

In the last recast of his wide ranging ideas, in his ninth decade, Kondo proposes an equivalence between number and infinity. Cogniscent of the universality of Gödel's incompleteness theorem and influenced by Kant's critique of pure reason, Kondo finally addresses the need to include incompleteness within a mathematically consistent model of reality.

Kondo's integral parametric pre-space, though mathematically consistent, is of necessity incomplete. Completeness is achieved by equating the pre-space with an infinite identity, highly reminiscent of the famous identity $e^{i\pi} = -1$. On the right hand side of the equation is rationality, arithmetic, finity and incompleteness. On the left are irrationality, infinity and completeness.

Kondo's work is a varied mix of abstract mathematics and extended discussion across a very wide variety of topics such as nuclear physics, relativity, chemistry, biology, optics, information theory, plant and animal taxonomy, physiology and linguistics. Running through all the work is the constant use of the Kawaguchi space as the mathematical vehicle underpinning the natural world.

Kondo does not ignore the contemporary view, including the standard model. Far from it. Very many of his discussions address the differences between his own ideas and those of contemporaries such as Einstein, Dirac, Bohr etc. The use of higher order geometry gives a richer and more applicable theory.

3.1 The Kawaguchi Space

The following is extracted from "Three Phases of Epistemological Penetration to Nature" [Kondo, K., 1997, page 2]:

...in the following exposition we shall first show that the fundamental constitution of epistemological recognition has to be put in terms of expoint co-ordinates. They are a systematical means of comparison of information built up in the domain of real numbers.

Expoints are expressed by sets of terms of co-ordinates:

$$x^i,\ i=1, 2, \ldots N$$

and parameters

$$\mu^\lambda,\ \lambda = 1, 2, \ldots L$$

by the expoint coordinates

$$x^i_{\lambda(r)} = \frac{\partial^r x^i}{\partial x^i_{\lambda 1} \ldots \partial x^i_{\lambda r}} \qquad r = 0,1, \ldots M$$

or, in the simpler case of L =1

$$x^{(r)i} = \frac{d^r x^i}{dt^r} \qquad r = 0, 1, \ldots M$$

The manifold of expoints is called a space. It is shown that the significant phases are restricted to the space configurations under the Zermelo-Géhéniau conditions in terms of $x^{i}_{\lambda(r)}$, which space we indicate by $K^{(M)}_{LN}$ [i.e. For M Derivatives, N Dimensions and L Parameters].

3.2 The Construction of Elementary Particles

The following is extracted from K293 of November 1995:

Definition I. A particle is carried on the Higher Order Space $K^{(M)}_{LN}$ One can say that the space is itself a particle and the particle is itself a space.

The constitution and the behaviour of the particle are, therefore, restricted by the Zermelo-Géhéniau conditions.

Definition II. A specific kind of particle carried on the line element of $K^{(M)}_{N}$ is called a **lepton**. The leptons are restricted by the Zermelo conditions.

Definition III Mesons are of the next kind of particle given for L = 2. A more complicated class given for L = 3 are **baryons**. Both of baryons and mesons are collectively called **hadrons.**

The lepton is under the Zermelo condition equation:

$$\Delta_1 F \equiv (x^{(1)i} \partial_{(1)i} + \kappa) F = F$$

where

$$\kappa = \sum_{r=2}^{M} r\, x^{(r)i} \partial_{(r)i}$$

represents the mass of the particle.

Following which Kondo then establishes, using the KH theorem, that there are only two types of quanta associated with the lepton. The mass quanta obtained from the equations of the lepton are denoted θ and Θ. θ obtaining from order 2 and Θ from orders 3, 4 or more.

The theoretical formulae for the mass construction of the electron and muon give

$$m_e = 2\theta, \quad m_u = 2\theta + 3\Theta$$

and compared with the experimental masses

$$m_e = 0.5011, \quad m_u = 105\text{Mev}$$

Giving

$$\theta = 0.25055 \text{ Mev}$$

and

$$\Theta = 35 \text{ Mev}$$

from which follows the ratio

$$\alpha = \theta / \Theta = 1 / 139.69$$

which is quite close to the experimentally determined value.

Skipping a discussion of quark confinement, K293 yields a mass construction of the Nucleon of

N:
$$\begin{bmatrix} 1\,2\,3\,2\,1 \\ 1\,2\,3\,2\,1 \\ 1\,2\,3\,2\,1 \end{bmatrix} \Theta$$

giving a mass of 27 Θ = 945 Mev, close to Mp of 938.272 Mev. The three rows correspond to three quarks.

Mesons having the masses 14 Θ and 16 Θ, i.e. the kaon (490 Mev) and the η particle(530 Mev) are then suggested, which are also close to experimentally determined values.

3.3 The Ultimate Microscopic Mass assembly

Following a detailed homo/cohomological argument given in K291, Kondo states in K293 that "..the ultimate microscopic mass assembly must be at any rate of three dimensional shape and of the simplest kind. It cannot but be a three-dimensional simplex having tetrahedral form". Based on its homological structure, the tetrahedron can carry the following mass quanta:

4 Vertices carry 4 unnamed lower quanta having mass θ_0

6 Edges carry 3 Nucleons having mass θ_1

4 Faces carry 4 Kaons having mass θ_2

1 Interior carries 1 further unnamed lower quanta having mass θ_3

$$\theta_2 \qquad \begin{bmatrix} 2\,3\,2 \\ 2\,3\,2 \end{bmatrix} \qquad = 14\,\Theta$$

$$\theta_1 \qquad \begin{bmatrix} 1\,2\,3\,2\,1 \\ 1\,2\,3\,2\,1 \\ 1\,2\,3\,2\,1 \end{bmatrix} \qquad = 27\,\Theta$$

At a first approximation, the tetrahedron has six edges to provide 3 pairs of supports for the 3 Nucleon equivalents to carry 3 * 27 Θ. It has four triangular faces providing supports for 4 Kaon equivalents to carry 4 * 14 Θ.

In all the tetrahedron carries

$$3 * 27\ \Theta + 4 * 14\ \Theta = 81\Theta + 56\ \Theta = 137\ \Theta = \P = 4795\ \text{Mev}$$

on this basic mass assembly unit. See figure 3.

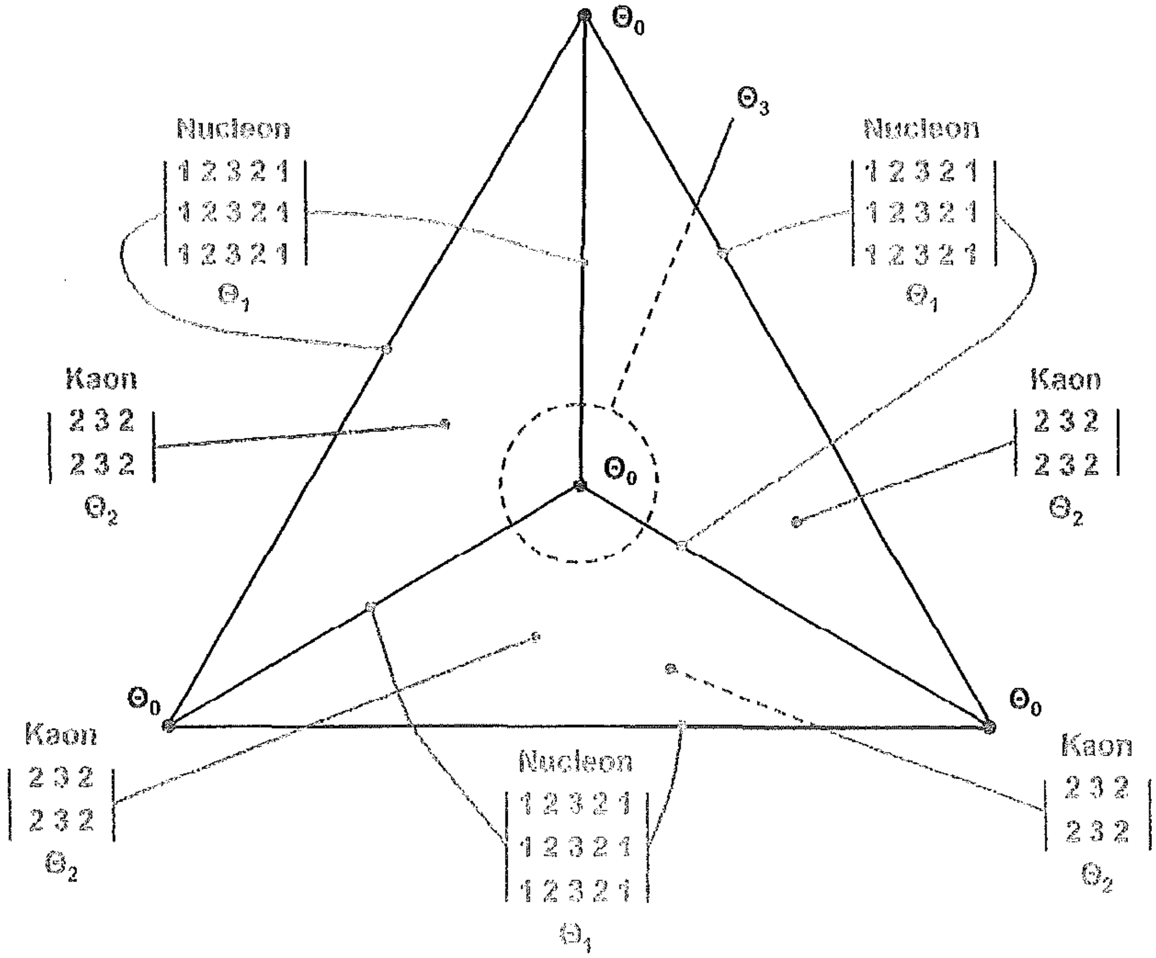

Figure 3 – Kondo's Tetrahedral Mass Assembly

Which leads to Kondo's Primary Fine Structure Constant Theorem that 137 smaller mass quanta are assembled into a unit of larger mass quantum.

Or the smaller and the larger mass quanta θ and Θ say, occur in the ratio

$$\alpha = \theta / \Theta = 1 / 137$$

Kondo envisages a series of mass quanta which can be amplified from θ and $\Theta = \theta / \alpha$ to

$$\text{...........}, \nu = \alpha\,\theta, \theta, \Theta, \P = \Theta / \alpha, \text{.........}$$

At a better approximation for the tetrahedron, the vertexes are equipped to carry 4 units of a certain lower kind of quanta θ_0 and the interior is equipped to carry another mass unit, say θ_3, giving a total mass carried of

$$\P = 4\,\theta_0 + 56\,\theta_2 + 81\,\theta_1 + \theta_3$$

Correcting Kondo's typos & simplifying his assumptions for clarity, Kondo postulates that

$$\theta_1 = \theta_2 = \Theta$$

and

$$\theta_0 = \theta_3 = \theta$$

In which case

$$\P = 5\,\theta + 137\,\Theta$$

substituting $\alpha = \theta / \Theta$, $\alpha = \Theta / \P$

$$1 = 5\,\alpha^2 + 137 \qquad \alpha$$

or we have an expression in terms of a continuous fraction

$$\alpha = 1/(137 + 5\,\alpha)$$

Solving the quadratic for α yields

$$1/\alpha = 137.036\ (4793)$$

Solving the third approximation

$$1/\alpha = 127 + 5\,\alpha - 8\,\alpha^2 - 32\,\alpha^3 = 137.0360(482)$$

Terminating after a finite number of steps is necessary because the fundamental recognition is finite, which brings us neatly to the next section.

4.0 The Combinatorial Hierarchy

Having come to it most recently, the author is less familiar with the Combinatorial Hierarchy and the several thousand pages of related ANPA material of the last 27 years. This section is therefore shorter.

Quoting Clive Kilmister [Kilmister, C., 2004]:

> *"Over forty years ago Frederick Parker-Rhodes devised a curious algebraic construction of a hierarchical system of four levels (graded group) of cumulative multiplicities 3, 10, 137 and 10^{38}. The process terminates at the fourth level.".*

Furthermore:

> *"Careful examination of the Parker-Rhodes Combinatorial Hierarchy (CH) and its elaborations yields a value of the reciprocal of the fine structure constant of $1/\alpha$ = 137.0360, completely agreeing with the seven significant figures of the experimental determinations".*

The fourth term in the CH has a correspondence with the magnitude of the gravitational coupling constant G.

The CH offers a distinct alternative to the standard model and provided a focus for the initial work of ANPA, from which a variety of mostly digital models of reality have emerged. The physical models posited by ANPA authors challenge almost every precept of conventional theoretical physics. A variety of philosophical viewpoints are accommodated within ANPA publications.

5.0 Torah & the Combinatorial Hierarchy

Cynthia Kolb-Whitney [Kolb-Whitney C., 2005] derives an approximation of Plank's constant:

$$h \approx \pi e^2/c\sqrt{(128\ Mp\ /\ 3\ Me)}$$

substituting Mp / Me = 1836 obtained from the Torah in section 2.0 above

$$h \approx \pi e^2/c\sqrt{(128 * 612)}$$

Turning the relation round, as suggested by the reviewer of Cynthia's paper

$\alpha = \sqrt{(1/(32 * 612))}$

i.e. $\alpha = 139.94$

substituting Mn for Mp, though this makes no sense in the hydrogen atom, we have

$\alpha = \sqrt{(1/(32 * 613))}$

ie $\alpha = 140.05$

Both values are about 2% in error, however it is the form of both equations which is of interest. There being 32 paths of wisdom in the first verse of the Sefer Yetzirah, 32 row flip operations in the operation of my Algorithm Three and the last and first characters of the Torah (לב) spelling 32 (or reading "heart"). There are 32 actions of creation in Genesis and 32 paths of wisdom (states of consciousness) from Kabbalistic sources. If we assume integer values for the information structures behind the Proton and the Neutron each of which might contain 1839 or 1836 information elements, we would expect that 613 or 612 information elements might relate to the 3 underlying quarks of QCD, or the 12321 in each row of Kondo's Nucleon structure.

Note that a branch of the Kabbalah is Gematria, where characters are assigned numeric values. The Hebrew root of Kabbalah is Kabal (לבק) "to receive", which has a value of 137.

The second stanza of the Sefer Yetzirah reads:

Ten Sefirot of Nothingness
And Twenty Two Foundation Letters:
Three Mothers
Seven Doubles

and twelve elementals

The three, seven and ten members of the first, second and first two levels of the CH could correspond with the three mothers, seven doubles and ten sefirot. Page 26 of Kaplan's book shows a simple diagram of 10 points linked by 22 lines. There are three horizontals, seven verticals and twelve diagonals. The form of the diagram is of three interlinked tetrahedra joined at two common vertices with four additional edges. See Figure 4.

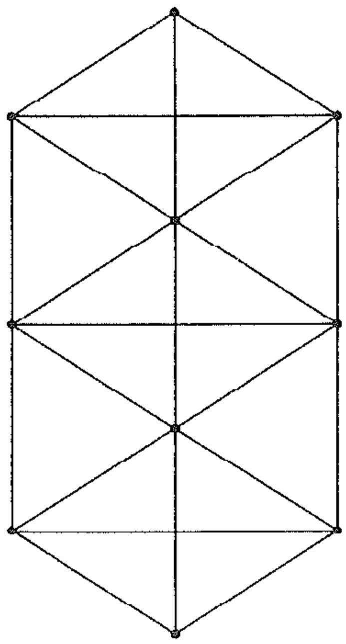

Figure 4 – The ten sefirot.

6.0 Torah and Kondo

There is a long tradition of permuting the Torah. Kabbalistic meditation occurs on permutations of the various divine names and other material. Within my earlier decryption paper I show how strings of length L where L is prime can be turned into L-2 derivative strings using Algorithm One, which simply extracts every Nth character in turn. The extraction of such permutations is merely the extraction of Galois fields from the set of all possible permutations of a string of length L.

In one of his later and most remarkable monographs [K316, Nov 1997], "Perception of Space, Matter and Bio-Psychological Information, Part I: Galois Field, Multidimensional Space and Differential Geometry", Kondo establishes a reality based upon a finite number of information elements which do not have individuality but can be counted. Consciousness is summed up in his Galois Field Recognition Lemma which states that

> *"The epistemological recognition by human being must primarily start with extraction of Galois fields"*

There is not enough space here to reproduce K316, however to re-emphasise the point, Kondo starts with countable information elements, describes the mechanism of consciousness as being merely the extraction of Galois fields then, in the same paper, leaps into mathematical musicology in order to give an example. K316 then continues in a most remarkable fashion. It is clear however that the worlds of Torah, Kabbalah and Kondo are linked by Combinatorics and Galois fields irrespective of whether such links are meaningful.

Finally, note that the four characters of the Tetragrammaton (JHVH) can label the vertices of Kondo's tetrahedral mass assembly (and the four components of a quaternion). Furthermore the 24 total or 12 unique permutations of the letters of the tetragrammaton provide a labelling for the rotational symmetries of the Tetrahedron [Sirag, S., 1989].

7.0 Kondo & the Combinatorial Hierarchy

There are two obvious overlaps between these two fields.

Firstly, Kondo frequently discusses an equivalence between the higher order space and quaternionic formulations, the latter occurring frequently in ANPA publications [Manthey, M., 1999] [Rowlands, P.R. & Cullerne, J.P., 1999] Although expressed differently, Kondo writes of both mathematical languages speaking of the same underlying structures. Although Kilmister [Bastin T. & Kilmister C., 2005] argues against re-expressing perfectly good theories in quaternionic form, there is good reason to do so given the vast use to which Kondo has put the geometry of higher order spaces. There are also huge practical and computational advantages if fundamental underlying structures can be expressed in quaternionic form.

The second overlap is that in Kondo's way of thinking, for each type of higher order space, there is an equivalent material entity. Whether that be a hadron, a halide, a horse or a human. This is reminiscent of the uniqueness of the members of each Discriminately Closed Subset (DCS) of the CH.

8.0 Torah, Kondo & the Combinatorial Hierarchy

All three bodies of knowledge suggest or deal with the three Quark model of QCD. In the case of Kondo's model of the Nucleon

$$N: \quad \begin{bmatrix} 1\,2\,3\,2\,1 \\ 1\,2\,3\,2\,1 \\ 1\,2\,3\,2\,1 \end{bmatrix} \Theta$$

it is clear to see where the threeness arises. The equivalence between the 9-ness of Kondo's quark and the 612-ness or 613-ness suggested by the Torah is a puzzle if it is anything at all.

All three bodies of knowledge deal with a structure that appears ex void - The Torah speaks of a creation *ex nihilo*, in the CH there is the first discrimination against void and in Kondo there is the simultaneous appearance ex void of *m* integers. There is a finiteness in each body also: the automatic termination at 10^{38} in the CH; the tradition of a finite partition in Torah and the axiomatic assumption of finiteness in Kondo. Finally, all three bodies of knowledge are essentially digital and combinatoric. This is explicit in Kondo and CH and implicit in the Torah.

9.0 Concluding Remarks

If the 1839 coincidence in the Torah is purposeful and we are indeed in possession of an instrument of creation and, perhaps, a deeply mystifying user manual, then we may be in a better position to deduce the underlying information infrastructures of reality.

Kondo and ANPA have done much in this direction, however much more remains to be done. Kondo's assertion that the ultimate microscopic mass assembly is tetrahedral is essentially supported in the Torah, except that the *ultimate* tetrahedron must be much smaller still, there being 1839 or so in each nucleon. In which case, the underlying data structures behind the electron and the positron must be of tetrahedral form.

Dedication & Thanks

I dedicate this paper to my mother and father on the occasion of my father's 80th birthday. I thank my gliding friends for their good humoured company during the writing of this paper and Kevin Moloney in particular for setting Figures 3 & 4.

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On Biology as an Emergent Science

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Abstract

Biology is considered here as an "emergent science" in the sense of Anderson and of Laughlin and Pines. It is demonstrated that a straightforward mathematical definition of "biological system" is useful in showing how biology differs in structure from the lower levels in Anderson's "More is Different" hierarchy. Using cells in a chemostat as a paradigmatic exemplar of a biological system, it is found that a coherent collection of metabolic pathways through a single cell in the chemostat also satisfies the proposed definition of a biological system. This provides a theoretical and mathematical underpinning for Young's fundamental model of biological organization and integration. Evidence for the therapeutic efficacy of Young's method of analysis is provided by preliminary results of clinical trials of a specific application of Young's model to the treatment of cancer cachexia.

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1 Why "Emergent Science"?

1.1 Anderson, and Laughlin and Pines on "emergence"

In a famous paper Anderson[1] has pointed out that there is a natural hierarchy of scientific ideas. He starts with the usual (reductionist) strategy of the search for the laws obeyed by the elementary entities of physics, but then points out that the possibility of reduction does *not* imply constructivity. Rather, if science Y underlies some science X: "The elementary entities of science X obey the laws of science Y". The $Y \rightarrow X$ hierarchy Anderson proposes is: elementary particle physics $\rightarrow$ solid state or many body physics; many body physics $\rightarrow$ chemistry; chemistry $\rightarrow$ molecular biology; molecular biology $\rightarrow$ cell biology; .... ; physiology $\rightarrow$ psychology; psychology $\rightarrow$ social sciences. Anderson goes on to state that "... this hierarchy does not imply that science X is 'just applied Y'. At each stage entirely new laws, concepts and generalizations are necessary, requiring inspiration and creativity to just as great a degree as in the previous one." I heartily agree!

Although my conventional scientific career (in elementary particle physics) started out with the conventional scientific (reductionist) assumption that the only way to solve a basic scientific problem was to find the elementary entities, the laws they obey, and then construct higher levels of science from that basis, I now realize that I was mistaken. I have become convinced that 21^{st} century science will be most exciting and fruitful if its basic problem is taken to be not only to find out if there are general hierarchy bridging laws that connect each level to the next and lead to novel types of complexity, but also if there are bridging laws which overarch the "elementary" bridging connection. This is one message I read into the paper on emergent science by Laughlin and Pines[2] who explicitly start from Anderson's analysis. Laughlin's book[3] is criticized by Leggett[4] under the title "Emergence Is in the Eye of the Beholder." However, I am still particularly impressed by the fact that the values of $\hbar c/2e$ and $e^2/\hbar$ obtained by electric measurements in complex systems can be obtained to much higher accuracy than the values which can be obtained by direct "elementary particle" measurements, despite the fact that the details of the theories used to understand the complex systems providing the data for these results have not reached consensus agreement.

1.2 The Organizing Principle of Darwinian Biology

Laughlin and Pines[2] also note that "For the biologist, evolution and emergence are part of daily life." As Fred Young remarked when I started discussing emergent science with him, "Everything I ever said at ANPA (cf.[5]) was in the direction of emergent science". This was sure to catch my attention because for several years Fred Young[6] has been trying to explain to me how his thesis work[7] is becoming more and more important for him as an explanatory tool of use in understanding how recent metabolic and physiological research all fits together. When James Lindesay joined these discussions of Fred's work, our joint understanding began to take shape as a paper[8]. Briefly, I saw that Young's results and Lindesay's mathematical deduction from them could be interpreted as the starting point for adding the links "cell biology $\leftrightarrow$ biological systems $\leftrightarrow$ ecological systems and evolutionary biology" to Anderson's proposed hierarchy in a specific way.

The careful reader will note that — in contrast to the earlier steps in Anderson's hierarchy — I have used the symbol "$\leftrightarrow$" for the connective between levels of the hierarchy once biology enters the picture. The symbol "$\rightarrow$" used by Anderson is an irreversible transition which replaces the "elementary particles" of the lower level by the laws they obey as the "elementary entities" of the new phenomena which occur at the more complex (higher) level, and which require the invention of new organizing principles, etc. which "emerge" from the careful study of this richer world of ideas. Biology arrived at its fundamental organizing principle by another route. Some biologists did not even believe that the phenomena they studied obeyed all of the laws of physics, in particular the second law of thermodynamics! Further, it was found useful to ask what *purpose* a particular aspect of these complex biological systems had "evolved" to satisfy. This kind of *teleological* explanation had been banished from physics after a very hard struggle, but its pragmatic usefulness in biology is hard to deny. That a "higher level" organizing principle can in fact lead to *deductive* and *demonstrable* conclusions when applied "top down" to a lower level *biological* entity is one of the points I wish to make below. This is why I replace "$\rightarrow$" by "$\leftrightarrow$" in the hierarchy once we enter the biological realm. Of course I must avoid the traps that lead to error when teleological reasoning is used carelessly. I hope the reader will reserve judgment as to whether I succeed in doing this until my methodology is presented clearly. In particular I believe that my methodology *also* avoids the traps pointed out by Anderson and by Laughlin and Pines, with whose basic conclusions I do agree.

Biology has sometimes been called a "Baconian" science in the sense that it started by amassing all kinds of details and facts assumed to be relevant to the subject and then induced general rules governing these facts. This methodology is to be contrasted with the tradition in the "mathematical" sciences which started from the astronomical practice of using numerical and geometrical models to make testable predictions. Skipping over vital historical details, this had the historical result that the "physical sciences" came to rely primarily on reductionism and hypothetical-deductive methodology for testing. Chemistry started out as a Baconian science, but began making the transition to a physical science in the nineteenth century thanks to electro-chemistry, thermodynamics and statistical mechanics. Quantum mechanics more or less allowed that transition to be completed; this transition has often been used as the leading example of the triumph of the reductive-hypothetical-deductive methodology.

Biology has not as yet made much *fundamental* use of mathematics. Its greatest nineteenth century success was the explanation of evolution via Malthus' observation that (in a stable environment and over a sufficient period of time) a persistent population **must** have birthrate = deathrate. Starting from that deduction and with observations (in particular Darwin's) of descent with modification, Darwin and independently Wallace came to the conclusion that "evolution by natural selection" is inevitable. This was the *non-quantitative* starting point for a scientific "evolutionary biology". Since then this has remained unshaken as the organizing principle of *scientific* biology. Clearly — if my description of the history is roughly correct — this is a very different route to a basic organizing principle than the routes followed in those "physical sciences" which now rest on hypothetical-deductive mathematical foundations.

One qualitative fact about biology which makes a methodological difference between it and the physical sciences is that "natural selection" inevitably presupposes the existence of some sort of *environment* within which the biological systems evolve, making it logically *impossible* to discuss biological systems without considering their interaction with that environment. I make explicit use of this fact in my definition of what is meant by a "biological system" in the sub-section which follows. A corollary of this point of view is that the paradigm of most importance in getting the study of biology off the ground is a persistent, evolved system. We will see that this provides a *reference state*, allowing fluctuations away from that reference state to be studied *quantitatively.*

1.3 A Proposed Mathematical Definition of a Biological system

I **define** a persistent *biological system* **B** as a finite, countable *population* of individual constituents C^B which in a suitable *environment* at constant temperature and pressure is a throughput (of molecules), steady state system satisfying the first and second laws of thermodynamics. The environment must supply food and fuel (F) at a rate sufficient to maintain the steady state. **B** acts catalytically to convert the food and fuel into product molecules P which are retained by the individual *living* constituents and waste molecules W which are disposed of by the environment. The environment must also remove the waste heat required by the second law in such a way as to maintain the postulated constant temperature and pressure. The environment must remove that selection of living individual constituents whose disposal will maintain within the system (on average) a *constant distribution* of living constituents over all the states which can occur during the *lifetime* of any of them. The environment must absorb all dead constituents. This implies that "dead constituents" become part of the environment "at death"and are no longer part of **B**. In our context the (average) number of (living) constituents satisfy the growth rate equation

$$\dot{C}^B = k_B C^B \tag{1}$$

and are said to be in a state of *stable population* (SP).

Note that C^B is a "counting number". As such, it is necessarily *dimensionless* in terms of a physicist's dimensional units of mass, length and time "M, L, T". Then $\dot{C}^B$ and k_B each have the dimension of inverse time (T^{-1}). By taking Eq. 1 as our defining equation for biology (in the context of a SP reference state), we emphasize in a different way the importance of the *environment* in our definition of biology. Lacking any evidence for a persistent *physical* environment, any biological *system* satisfying our definition — let alone its individual constituents — *must* have a finite lifetime.

2 The Young Model for the Organization and Integration of Biological Systems

2.1 The Chemostat as Paradigm for a Biological System

Our definition of product molecules P given in Sec. 1.3 allows us to specify the distribution of living constituents by the (average) number of product molecules they contain at any stage during the life cycle of each constituent. We illustrate how this can be done by narrowing the specific paradigm for a biological system used here to a group of cells in a chemostat maintained in a SP state. For the purposes of our theoretical analysis we assume that we can treat each cell in the chemostat as a coherent combination of its chemical constituents. Then we can use the symbol "C^B" to stand for a *molecule* in the chemist's sense†. This allows us to write chemical equations (conserving the numbers of each type of atom and the sum of their masses) connecting individual molecules to cells, which we take to be one of the (implicit) axioms of *biochemistry*.

We are now dealing with a population of *growing* cells inside the

†A chemist's "molecule" is a coherent structure which contains *one or more* chemical "atoms", while a physicist usually thinks of an "atom" as composed of still more elementary constituents, and of a "molecule" as composed of *two or more* atoms.

chemostat absorbing nutrient molecules F and producing product molecules P which are retained by the cell and waste molecules W which are excreted into the solution surrounding the cell. Since the cell eventually divides into two cells which — at our level of analysis — are indistinguishable, we index the growing cells by the number of product molecules n_P they have added in the range $N_P \leq n_P \leq 2N_P - 1$. Cell division is then the irreversible process

$$C^B(2N_P) \Rightarrow 2C^B(N_P) \tag{2}$$

The basic biochemical process in this context is

$$F + C^B(N_P + n_P) \Rightarrow C^B(N_P + n_P + 1) + W \tag{3}$$

Consequently the chemical equation describing the operation of the chemostat in this simplest case is

$$N_F F + \Sigma_{n_P=0}^{2N_P-1} C^B(N_P + n_P) \Rightarrow \Sigma_{n_P=1}^{2N_P-1} C^B(N_P + n_P) + 2C^B(N_P) + N_W W \tag{4}$$

which, by defining a *complete population* of cells (i.e. a population which, in the appropriate environmental context, when supplied with N_F nutrient molecules, can produce two clones by cell division) as $\Sigma C^B \equiv \Sigma_{n_P=0}^{2N_P-1} C^B(N_P + n_P)$, we can rewrite as

$$N_F F + \Sigma C^B \Rightarrow \Sigma C^B + C^B(N_P) + N_W W \tag{5}$$

or as

$$N_F F \underset{\Sigma C^B}{\Rightarrow} C^B(N_P) + N_W W \tag{6}$$

A growing cell has to add N_P product molecules to its structure before it can divide and start the process over again. One of those two copies (clones) must be removed at some subsequent time (in its life cycle or when it dies); this pruning is required to maintain the SP state. In this particulate description of the overall process, any *growing* cell will have to add each individual product molecule *sequentially.* We assume that the context in which the equations apply is a *through-put steady state* (SP state). Then the rate at which the molecules of F move into the growing cell, the rate at which the molecules of P join the growing cell, the rate at which the cell divides into two clones (beginning cells), and the rate at which one of these two growing cells is eventually pruned from the cell colony are all the same. That is

$$[\dot{C}^B] = k_B[C^B];\ [\dot{F}] = k_B[F];\ [\dot{P}] = k_B[P];\ [\dot{W}] = k_B[W] \tag{7}$$

Here the symbol $[X]$ ($X \in C^B, F, P, W$) means the *concentration* (i.e mass per unit volume) of the substance X, For small molecules (i.e. molecules whose atomic content and (if needed) molecular structure are known) this mass is most conveniently measured in terms of *moles* (i.e. *gram molecular weights*). These equations immediately suggest that it may be possible to treat concentrations of small molecules as biological systems in an appropriate context. We develop this idea in the next sub-section, in which we give precision to the concept of *metabolic pathway.*

The careful reader will have noted that we have use the symbol $\Rightarrow$ denoting the *irreversibility* of the chemical reaction not only for cell-division (Eq. 2) but also for the individual step (Eq. 3) in which the cellular environment *catalyzes* the transition from food molecule(s) to the product and waste molecules. We assume that this can only happen when the initial and final molecules are in the correct *stoichiometric ratios* (see next section). This is because we are interested in this paper only in the passage of molecules through the cell (or to their location within the cell) when this path does go through some catalytic site (which we will call an *enzyme*) that guarantees that we are talking about a throughput steady state which is *far from equilibrium* and **not** about the equilibrium states with which much of physical chemistry is concerned. Thus there are no two-way transitions at the basic level and the usual use of detailed balance rate constants is, from the start, inapplicable. This brings us to the discussion of (enzymatic) metabolic pathways in the next section.

2.2 A Coherent Collection of Metabolic Pathways as a Paradigm for a Biological System

The food/fuel molecule or molecules F that initiate the basic process (Eq. 3) could have entered the cell at many different places, and the waste molecule or molecules that complete the process can leave the cell at many different places, but (in our abstract model) the critical transition occurs at only one place along the path(s) connecting the input and output surface patches, namely where some enzyme $E_{F \Rightarrow PW}$ catalyzes the reaction $h_F F \Rightarrow h_P P + h_W W$. We call this "one dimensional" route through the cell a *metabolic pathway* and represent its action by the biochemical equation

$$
\begin{array}{ccccc}
 & & E \cdot h_F F & & \\
 & \nearrow & & \searrow & \\
h_F F + E & & & & E + h_P P + h_W W \\
 & \nwarrow & & \swarrow & \\
 & & E & &
\end{array}
\qquad (8)
$$

Eq. 8 represents the irreversible, catalytic action of a *single* enzyme molecule, which may dynamically change its shape during the process but automatically resumes its initial shape after the process is completed[‡]. Note that, for the biochemical processes used in our paradigm, this process *must* occupy a (3+1)-dimensional *space-time volume* and hence *must* be *nonlocal.* The numbers h_F, h_P and h_W ***must*** be integers because both the number of (chemical) atoms and the amount of (chemical) mass are conserved in the process. Their ratios $h_{X/Y} \equiv h_X/h_Y = (h_Y/h_X)^{-1} = h^{-1}_{Y/X}; X, Y \in F, P, W, ...$ are called *stoichiometric ratios.* If we measure the *concentration* $[X]$ [which has *physical* dimensions ML^{-3} (i.e mass per unit volume)] of any chemical

[‡]This restoration of the initial state of the enzyme provides one "feedback" control mechanism. *Some* feedback control loop in the information flow is *required* for any persistent, self-organizing complex system to exist.

substance X in *moles* (i.e. in gram-molecular weights per unit volume), then the stoichiometric ratios are identical to the concentration ratios. Then the equation also can be read as the number of moles of each substance which will react in this way when catalyzed by one mole of the enzyme. Note that we can **now** rigorously and quantitatively bridge the *small molecule* $\leftrightarrow$ *cell* mass magnitude gap by writing, as a corollary to Eq. 6

$$h_F N_F = h_P N_P + h_W N_W \tag{9}$$

Note that this is an *algebraic* equation connecting positive definite *integers* and **is not** a chemical equation.

A few comments are needed here. Note that the N_P apparently independent metabolic pathways implied by Eq. 4 — which are needed in order to allow Eq. 7 to be treated as defining the hierarchical nesting of a collection of biological systems — must act *coherently*, at least at the conceptual level; this assumption is also needed in order for the cell to be thought of as a coherent chemical molecule. The conceptual advantage of this step is to allow the very complicated process of cell growth and division to be made into the simple doubling of the starting cell via the sequence of steps (Eq. 4) that leads to cell division (Eq. 2). Then the rate k_B at which the transition occurs is a *quantitative* and experimentally measurable function of the concentrations of **small** molecules of known structure called here F, P and W, even if we do not know the molecular weight of the enzyme invoked by Eq. 8, or the details of how the catalytic result is achieved, let alone knowing the molecular weight of the cell!

Some such critical conceptual step is needed in order for the mathematical model we are constructing to be able to *explain* how chemostats can *determine* empirically what function of these concentrations the cell growth rate k_B is. That such functions are known is an empirical **fact**[9]. It is this fact which allows us to go from it to a simple mathematical formulation of Young's model. Explicitly we quote from Fred's thesis ([7], p.1)

> ...the value of k_B is a reproducible function of the medium composition[9] ...

which we write formally as

$$k_B = K_B([F_1], [F_2], ..., [F_j], ..., [F_J]) \qquad (10)$$

Here the nutrients F_j are distinguished from each other by the unique enzymes E_j which catalyze the *irreversible* reactions

$$h_{F_j} F_j \underset{E_j}{\Rightarrow} h_{P_j} P_j + h_{W_j} W_j \qquad (11)$$

that remove h_{F_j} molecules of F_j from the metabolic pathway and replaces them with product (P_j) and waste (W_j) molecules, conserving chemical mass and atom flux. J is the number of *types* of enzymes *and* the number of *types* of metabolic pathways we consider important in any particular analysis. K_B is *not* a function in the usual mathematical sense. For us, if the values of the parameters are known over the ranges of values and to the accuracy needed for our immediate purposes, a "table lookup" plus any well defined "interpolation procedure" suffice to make this framework into a *testable theory* in Popper's sense.

Accepting that Eq. 10 is a reproducible *empirical statement* based on table lookup has important consequences. In that context the inescapable **fact** is that all the numerical quantities (in this case k_B and each of the $[F_j]$) have an experimental range of uncertainty. I formalize this fact by assuming that *whenever* we assert Eq. 10, we are claiming that there are $2(J+1)$ numbers called $k_{min}, k_{max}, [F_j]_{min}, [F_j]_{max}$ such that for **any** choice of numerical values within these ranges, no matter how correlated, the asserted equality provides an acceptable representation of the data for our purposes. If this statement becomes suspicious, the careful experimenter will look for an explanation either in some source of systematic error, or some theoretical constraint or possibility that has been ignored. Note that in either case, these limits become testable hypotheses in Popper's sense, and new experiments can either reduce the experimental uncertainty or produce new empirical knowledge. This is standard procedure in physics.

With this understood, we can use some hypothesis that makes nutrient "j" the "most important" for the purposes of our analysis, and formally "invert" Eq.10 by defining

$$[F_j] = (K_B)_j^{-1}(k_B; [F_1], [F_2], ..., [F_{j-1}], [F_{j+1}], ..., [F_J]) \approx (K_B)_j^{-1}(k_B) \tag{12}$$

which means that, to the extent that the approximation is valid, we can ignore what is going on in the concentrations of the other nutrients and find some monotonically increasing function of k_B, $k_{min} < k_B < k_{max}$ to fit the observed values of the *correlated* variation of $[F_j]$, for $[F_j]_{min} < [F_j] < [F_j]_{max}$ (or *visa versa*), — i.e. $k_B = (K_B)_j([F_j])$. With this basic phenomenology understood we can make testable empirical hypotheses about and place reasonable theoretical constraints on the concept of "metabolic pathway" in the context of a stable population of bacterial cells in a chemostat.

2.3 Single Enzyme Control in a single metabolic pathway as an irreversible transition

The simplification of Eq. 8 for each emzyme/pathway j to Eq. 11 allows us to compare it to the detailed model for catalytic action in the irreversible reaction

$$2\ CO + O_2 \underset{Cat.}{\Rightarrow} 2\ CO_2 \tag{13}$$

as analyzed by Grinstein, et. al.[10]. As the authors note,

> Since the reverse reaction $CO_2 \rightarrow CO + O$ is not allowed, the system defined by the above rules cannot satisfy detailed balance for any underlying Hamiltonian.

which reinforces the remark already made at the end of sub-section 2.1 that the processes we are considering *cannot* be described by the rules of equilibrium physical chemistry. It also warns us (in our non-equilibrium context of irreversible, steady state, throughput processes) that we cannot expect the essential mathematics needed for theoretical biology to resemble the continuum mathematics used in classical theoretical physics. I fear this fact about biological systems is often ignored by biochemists analyzing enzyme reactions *in vitro.* The advantage Young has in basing his model on chemostat data is that these empirical studies are, in fact, *in vivo* experiments. They allow us to go *directly* from chemical measurements (concentrations of small molecules) to a parameter (k_B) that measures the (average) time it takes a *living* organism to replicate itself in an environmental *context* that allows a biological system composed of such organisms to achieve a persistent steady state (SP-state).

The rules the authors[10] refer to in the quote given above describe the way to parameterize the rates at which the incoming molecules attach to the catalytic surface, rearrange bonds to form the product molecules of the outgoing gas, and the rates at which the outgoing molecules detach. These details need not concern us here, nor do the numerical methods Grinstein, et. al. are forced to use because they lack a Hamiltonian model. What does concern us is that the catalyzed transition $2CO+O_2 \Rightarrow 2CO_2$ is a worked out example analogous to the way the F molecules come along the *incoming* part of the metabolic pathway to a specific enzyme and the P and W molecules leave on the distinctly different *outgoing* part of the same pathway. We *could* make a more detailed model of this process, but we *are not required* to do so in order to achieve our results. All we need abstract from the complicated process that goes on in the ill-defined space-time volume around the enzyme is the fact that this transition separates the metabolic pathway into an incoming and an outgoing part, and that it *fixes* the stoichiometric ratios of all the substances in this single metabolic pathway whose mass flow is continuous through this volume.

The reason we are not concerned with the geometrical details is that the basic equations (Eq. 7) are *space-scale invariant* **and** only depend on spacial averages (concentrations) as functions of time. To smooth these out in the complicated region where F attaches to the enzyme, the enzyme rearranges F into P and W and these leave, we assume this region has an average length L. We assume that an average flow velocity for molecules along this metabolic pathway through the cell can be defined by $v = k_B L$. Then within this region, we can measure distance along the pathway by a spatial coordinate $x = vt = Lk_B t$ when $0 < x < L, 0 < t < T_B = k_B^{-1}$. Assume that the pathway is in an SP-state (i.e. $v = const. = k_B L = L/T_B$). *Upstream* of this *transition region* (i.e $x < 0$) the concentration $[F_j]$ must have a constant *input* value which we call $[F_j]_I$. *Downstream* of the transition region the concentrations $[P_j]$ and $[W_j]$ must have constant *output* values which we call $[P_j]_O$ and $[W_j]_O$. We then know that the concentration $[F_j]$ must fall from its input value $[F_j]_I$ to zero as it passes through the transition region. This shows that we can *always* describe the steady state action of any enzyme which causes an irreversible phase transition by

$$[\dot{F_j}] = -k_B[F_j] \tag{14}$$

if we use the *algebraic sign* conventions a) that k_B is a positive definite constant and b) that the time rate of change of a concentration is *positive* when it is the same as the sign of the rate change of a *growing* cell. Similarly $[P_j]$ must start at zero at $x = 0$ and rise to the output value $[P_j]_O$ at $x = L$. Then a little thought tells us that if Eq. 8 is used to represent an *irreversible* transition, we must have that

$$if\ [Y_j] \in [E_j], [E_j \cdot F_j], [P_j], [W_j],\ then\ [\dot{Y_j}] = k_B[Y_j] \tag{15}$$

Now we must face up to the fact that,empirically, the chemostat data often exhibit a substantial range of values of k_B, as is implied by Eq. 10 and the quotation which it formalizes. Indeed, the question of *how* such a strictly correlated (by any set of values or range of values for k_B for which the quote and or Eq. 10 are a correct representation of the facts) can come about was the problem Fred Young's thesis[7] set out to solve.

Here the Darwinian organizing principle of natural selection comes to our aid. Any organism in a biological system will benefit by extending the range of the concentrations of nutrients which it can tolerate and continue to reproduce, and the speed with which it can make use of them in competition with other organisms or with genetically modified members of it own species. But the metabolic pathways *within* each organism (having the same genome) will gain collectively (for its genotype) if their action is tuned to make maximum use of the total supply which can be absorbed by the organism as a whole. On both counts we expect an organism-wide coordination to be selected for, and not just maximum range and efficiency of the action of the individual catalytic pathways. As is not surprising, from the point of view of the central dogma of molecular biology, this coordination is provided by the genetic control of the production of the enzymes themselves. Since this is more easily explained by using the control mechanism discovered by Fred Young in his thesis than by discussing a single enzyme pathway, we now turn to that explanation in the next sub-section.

2.4 Two Enzymes linked by a feedback loop in a single metabolic pathway

The basic feedback control loop for metabolic regulation which Fred Young[7] discovered, written as a chemical equation, is

$$\begin{array}{ccccc} & & h_U U & & \\ & \nearrow & & \searrow & \\ h_I I + E_I & & & & E_O + H_O O \\ & \nwarrow & & \swarrow & \\ & & h_D D & & \end{array} \qquad (16)$$

Note that this connects two irreversible enzyme-catalyzed transformations, namely

$$h_I I + h_D D \underset{E_I}{\Rightarrow} h_U U \underset{E_O}{\Rightarrow} h_D D + h_O O \tag{17}$$

Note that D — which is analogous to the the enzyme E in Eq. 8 (if we replace $E \cdot F$ by $D \cdot I = U$) — is conserved in the sense that it retains (in a SP-system) a constant concentration. Note also that (in a SP-system) the net effect is to transform I to O irreversibly, conserving mass and chemical atoms, at a rate

$$k_B = \frac{[\dot{O}]}{[O]} = -\frac{[\dot{I}]}{[I]} = \frac{[\dot{U}]}{[U]} = \frac{[\dot{D}]}{[D]} \tag{18}$$

This is, of course, the same conclusion we reached about the action of the individual enzymes (cf. Eq.'s 14 nd 15). This means that we can go on connecting nodes (representing enzymes which unequivocally direct mass flow in the direction defined by positive growth rate or, in feed-back links, unequivocally in the opposite direction) in a way that will never upset the SP character of the system *provided* the environment is stable and we can prove that the system is stable against "normal" fluctuations in the environment.

Further, Eq. 18 implies that $\frac{[\dot{O}]}{[\dot{I}]} = -h_{O/I}$, the negative of the stoichiometric ratio of the concentrations. Not only is the rate of decrease of I precisely equal to the rate of increase of O (a fact we could derive directly from chemical mass and atom conservation), but if we think of the cell as a factory for the production of O, ratios of the stoichiometric coefficients in Eq. 16 could serve as the set-points for some rate control system that optimizes the use of resources I to the rate at which they are provided. This is obvious to a chemical engineer. That natural selection has "engineered" such a system is a deduction from the Darwinian organizing principle.

Fred Young's approach is to "reverse-engineer" the data on the concentrations in steady state throughput experiments using what is known about the structure and working of the cell so as to tease out how the control system operates. One advantage of using his control cycle, rather than concentrating on the genes (and hence the enzymes) *directly*, is that his control loop allows this to be done using only the concentrations of the small molecules as the empirical starting point. As he notes ([7], p.8): "The interrelationships between cellular components that define the steady-state and illustrate the scope of regulation which is independent of specific [genetic] induction-repression mechanisms have been comprehensively tabulated."

One general mechanism for cell-wide rate control recognized by Young is the relation between protein synthesis and ribosome synthesis when both are thought of as a function of k_B (cf. [7], Fig. 7, p.37 and related text). For a stable population of growing cells, and a large range of values of k_B, the rate of protein synthesis (per genome equivalent of DNA) is constant, whereas the relative rate of ribosome synthesis is a rapidly increasing function of k_B. Clearly the value of k_B where these two curves cross is a "rate control set point" .

Why is this true? The proteins are manufactured by the ribosomes. The particular protein called for is coded on an "instruction tape" (messenger ribonucleic acid — mRNA). This tells the ribosome which of the 20 possible amino acids to attach next onto the growing protein (polypeptide) chain. An "expressed" DNA-gene uses (ignoring ambiguities in the code) one of 20 "three letter codons" (corresponding to the 20 amino acids which can be used to make a protein chain) to provide the information added sequentially to the mRNA instruction tape. The fact that the concentrations of ribosomes, ribosomal nucleic acid (rRNA), mRNA and tRNA are **all** proportional to k_B then tells us (accepting the one gene - one enzyme doctrine and still subtler approximations[§]) that we can expect the concentration of any enzyme (a "large molecule" made up of one or more polypeptide (protein) chains) produced by this machinery to also be proportional to k_B. The fact that the (relative) rate of protein synthesis as a function of k_B is constant (in the range where it crosses the rate of ribosome production) can be interpreted as due to the likelihood that the rate of transcription for any of the codons that specifies any amino acid is approximately the same as for any other codon.

The next step is to note that the amount of the enzyme synthesized is controlled by the expression of the gene and that this, in turn, is con-

[§][DNA is a shorthand for *deoxyribonucleic acid*, The transfer RNA (tRNA) — with 20 varieties — transfers *uniquely* the sequential information from an expressed mRNA transcript of a DNA "structural" gene one codon at a time by having one end which attaches by complementary base pairing to the RNA codon and picks up on the other end the cognate amino acid which is added to the growing polypeptide chain. If this apparatus worked perfectly there would be approximately one "constant" rate (with a fine structure of 2 or 20 or some number less than 64 rates) for this process. But the mRNA can itself get degraded at some rate between the time when the information is transferred to it physically and the time when it is read. Consequently information transfer and the transfer of the material coding of that information can have different average rates. Fortunately, for the purposes of this paper, we can ignore these complexities.

trolled by the operator-promoter region of the gene. These controls can be either positive (enzyme induction) or negative (enzyme repression) and can be effected by a change in the concentration of any appropriate small molecule or protein in the metabolic pathway upstream of the enzyme in question, even though it is not directly involved in the $I \Rightarrow O$ catalysis. [Such a change can even "turn off" the gene completely and hence form the starting point for a *concentration threshold*-controlled on-off "switch". We will mention such switches in the discussion of the cancer cachexia treatment, but not model them in this paper.]

Two applications of this control loop are discussed in [7]. The first is glucose metabolism. For it "I" is some phosphate in the food which is picked up by adenosine-diphosphate (ADP) [identified with "D"] to form adenosine-triphosphate (ATP) [identified with "U"], which then passes on the phosphate to some downstream product [identified with "O"] and returns ADP as the feedback which completes the cycle. Thus there is an internal "set-point" for the internal control loop — namely the stoichiometric ratio $h_{U/D}$ — and an external set-point, $h_{I/O}$. Both are under the genetic control of the two genes associated with the two enzymes E_I and E_O. The second application is the addition of a monomer as the next link in the chain of a growing polymer. The second example is illustrated in [7], Fig. 5, p.28, which is reproduced below, together with its figure caption as Figure 1. I trust that, after the above sketch of how the whole thing works, this captioned figure is self-explanatory.

What Fig. 1 does not point out is that the genetic control mechanism acts on a long time scale, appropriate to a secular change in the rate and/or amount at which food is flowing into the system from the environment, while the internal control loop acts on a shorter time scale which can smooth out short term fluctuations in the concentrations due to other causes. That this feedback is *stable* follows immediately from the irreversibility of the direction of flow at the two enzymes. These absorbing state phase transitions (from $M + X$ to $(X \cdot M)$ and from $(X \cdot M)$ to $X + P$ — with the return control loop path that takes X "back" to the initiating phase transition in place) also enforce the stoichiometric set points for both the interior and the exterior rate control. The rate controlled *range* of k_B comes from: a) the fact that the number of ribosomes per cell increases with k_B; b) the fact that the range of k_B is limited at the upper end by the maximum number of ribosomes which the cell can hold in a steady state; c) the fact that the range of k_B is limited at the lower end by the minimum amount of nutrient which will allow, at least, the minimum stable number of cells to maintain themselves in the chemostat at the flow rates for nutrient input and for the solvent carrier input.

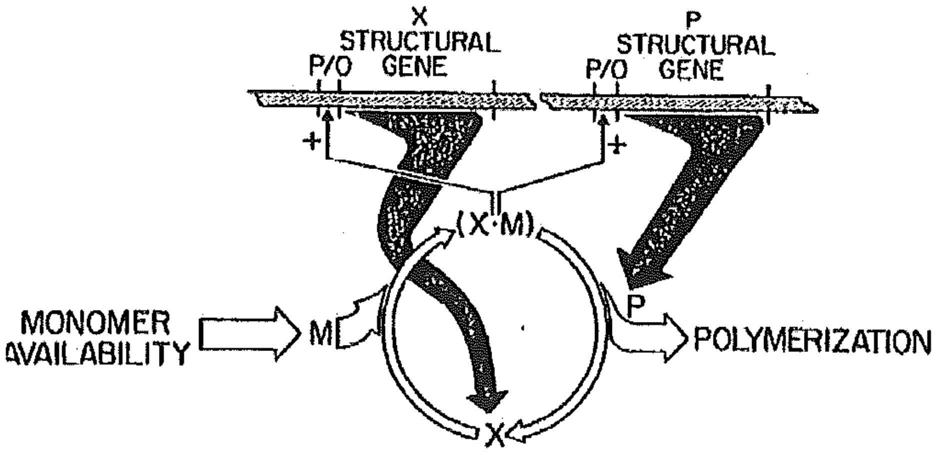

Figure 1: Representation of how monomer charge can be maintained at various steady-state growth rates through the use of rate effectors. When rates of monomer producing and utilizing reactions are balanced the concentration of $(X \cdot M)$ is proportional to the rate of synthesis of $(X \cdot M)$. $(X \cdot M)$ is a positive effector for synthesis of both X and P. In this way the rates of synthesis of P and X will be proportional to the rate of synthesis of $(X \cdot M)$. During non-steady-state conditions the concentration of $(X \cdot M)$ will no longer be proportional to the rate of synthesis of $(X \cdot M)$ and consequently rates of synthesis of P and X will no longer be equal to the rate of monomer availability. P/O is the operator promoter region of the structural gene. X represents a carrier of monomer units. (The thickness of the arrow is not meant to reflect the relative reaction rates.)

The general applicability of Fred Young's general in-out, up-down rate control loop feedback model, i.e.

$$\begin{array}{ccccc} & & U & & \\ & \nearrow & & \searrow & \\ \rightarrow I & & & & O \rightarrow \\ & \nwarrow & & \swarrow & \\ & & D & & \end{array} \qquad (19)$$

to most (possibly all) biological systems, and to many well-modeled physical systems that provide significant analogies for biological systems, may not be obvious. After all it was only first discovered in *E-coli* metabolism. But careful perusal of Young's thesis[7], should begin to remove doubt on that score. The thesis was deliberately undertaken, not to solve a specific problem but to find a mechanism that could account in a general way for how rate control of metabolism can lead to multiple rates of growth in stable populations and for growing systems exhibiting balanced exponential growth. Subsequent developments in biology and other fields provide ample evidence for the fact that Young's model is *ubiquitous* in its applicability. This topic will be discussed in[11], where the connected chain: non-equilibrium steady state $\rightarrow$ absorbing state phase transition $\rightarrow$ allometric scaling laws $\rightarrow$ fractal scaling $\rightarrow$ Kolmogorov scaling is developed. The abstract of an earlier talk by Young on this subject at an international meeting in Shanghai is presented here as Appendix 1.

Although the primary purpose of this paper was to prepare a mathematical and logical basis for the Young model, whose technical structure will be presented elsewhere[8], we wish to also take this occasion to point out that, once the biological principles are understood, the top-down analysis of metabolic pathways which Young's general model makes possible did not have to wait for mathematical development in order to be applied. A general computer program called HiNET, was developed by Young and collaborators, and specifically applied to the problem of cancer cachexia. Normal nutrition for our species and many others has a replete-hungry cycle with on-off switches changing the metabolic pathways between the two stable states. In certain shock states, there a great need for nutrition at any cost and the body in these states starts eating anything inside it, including its own structure. Normally this state turns off when the danger is past, but cancer and some cancer therapies can produce a shock state that does not return to normal; consequently the body wastes away even though ample nutrition is provided by injection into the veins. Using his analysis, Young found a way to treat the patient with combinations of FDA-approved drugs. They alter the concentrations of small molecules in the direction which returns the body to normal nutritional states and solves the problem. Preliminary clinical trials were so succesful that second stage trials were approved on a faster than usual time schedule. An older short report of this is given in Appendix 2.

In conclusion, I believe that Young's control loop feedback in-out: up-down cycle model for a throughput system is a good candidate to become an emergent fundamental law of biological systems going beyond the Darwinian organizing principle. Using such control loops as coupled nodes in a hierarchical model for top-down analysis of functional metabolic pathways, of which the first example is Young's HiNET model, bids fair to become a fruitful research tool for uncovering novel emergent biological organizing principles during the 21^{st} century.

3 Acknowledgments

This paper rests primarily on the decades of work by Fredric S. Young on his unique approach to the problem of the organization and integration of biological systems. I am most indebted to him for his invitation to participate in this research. I am also indebted to James V. Lindesay for his collaboration on clarifying the mathematical structure of the feedback system involved in the two-enzyme control loop, and to both of them for the three-way discussions we had during 2004-2006. I owe much to the the clarification of the logical and philosophical structure of what we are attempting to do provided by Walter R. Lamb while he was still with us.

4 Appendix 1: The Universal Modular Organization of Hierarchical Control Networks in Biology

Fredric S.Young
Vicus Biosciences

Fredric S.Young, Vicus Biosciences

Progress in the physical sciences have always involved conceptual and theoretical simplification and unification. Modern biology has resisted this tendency and has focused almost completely on the details. The sequencing of the human genome has not been translated into comprehensive models and has not led to new therapies. Using reverse engineering, we have abstracted a theoretical description of the universal modular organization of biological control systems which are modeled as the construction of a fractal representing the hierarchical control network or HiNet. Disease therapy becomes a problem of shifting the state of the HiNet to a configuration closer to normal homeostasis. This has enabled the rational and systematic developmet of combination therapies for clinical trials. An emphasis on energy and control manifolds connects this approach to catastrophe theory. The modularity of HiNet allows a hierarchical network decomposition and modeling of local processes on low dimensional control manifolds. Modeling of the integrated global organization of a biological system requires control spaces of many more dimensions than 3 as stated by Thom in *Structural Stability and Morphogenesis.* A HiNet model of allometric scaling supports the recent application by Ji-Huan He of El Naschie's ε^{∞} theory to biology. (Abstract of paper presented at the 2005 International Symposium on Non-Linear Dynamics: Celebration of M.S. El-Naschie's 60 Anniversary, December 20-21, Shanghai, China)

5 Appendix 2: The Obsolescence of Reductionist Biology: Systems Biology Modeling and Cancer Cachexia Therapy Development Based on Emergent Patterns of Organization Rather Than on Genes and Molecules

Dr. Fredric Young, Chief Scientist, Vicus Therapeutics, LLC

Vicus has developed a hierarchical network (HiNET) model of emergent patterns of organization based on principles of self-organized criticality, phase-transitions, integral control and reaction blocks. We will describe our HiNET model of cancer cachexia, a catastrophic wasting disorder secondary to advanced cancer, and its predicted EKG-based biomarkers and reaction-block drug targets. We will show data from our retrospective and prospective VT-122 clinical trials and contrast our clinical results with previous failed attempts targeting specific dysregulated pathways and proteins. (Abstract of paper presented at *Beyond the Genome 2006: Top Ten Opportunities in the Post-Genome Era*, June 19=21, 2006, Fairmont Hotel, San Francisco, California.)

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THE QUANTUM POTENTIAL AND THE EPIGENETIC LANDSCAPE

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1. SUMMARY

In this paper I compare the account of quantum measurement given in the Bohm interpretation with the account of biological development given by Waddington. This is a comparison of two pictures: Bohm's quantum potential and Waddington's epigenetic landscape.

Active information

Every cell in an adult organism has the same DNA. In any one cell, only part of the genetic information of the whole is active, the rest is inactive. To understand the differentiation of the embryo into all the cell types of the adult organism is to understand how information either remains active, or becomes inactivated.

In the Bohm interpretation, Q the quantum potential *informs* the particles of the whole experimental arrangement and coordinates their trajectories. Thus an interference pattern emerges when the two slits are open, but disappears when one is closed, according to the form of quantum potential. Rather than being a mechanical potential, Q is an information potential. Information is active according to the total situation in which the particle finds itself.

The Potentiality of Cells

A single cell is *totipotent* if it is able to divide and produce all the differentiated cells of an organism. In totipotential cells, the whole of the genetic information is potentially active. Totipotential cells specialise into the *pluripotent* cells, which, in humans, can develop into any of the three major tissue types: ectoderm, mesoderm or endoderm.

Pluripotent cells undergo further specialisation into *multipotent* cells. Blood stem cells give rise to red cells, white cells and platelets, each of which are considered to be *terminally differentiated.*

Differentiated cells usually lose the potential to give rise to new individuals: information is active in the cell's function, but the rest is inactive. Differentiation of a blood stem cell into platelets, white blood cells and red blood cells is a contraction of wholeness, a projection from a larger into a smaller space.

Bifurcation of Alternatives

Consider the example of a potential barrier. The quantum potential can thought of as lowering the height of the barrier, allowing particles to pass through. In this way, the quantum potential explains the mystery of how particles penetrate the barrier. Confronted with the potential barrier, the particle enters one of two distinct *channels*, either reflected or transmitted. Information in the channel that the particle enters is actually active, whereas in the empty channel, information is actually inactive. Information in the empty channel retains the potential to act on the particle, since the channels can be made to overlap again.

Consider a pluripotent cell that gives rise to ectodermal tissue. At this stage, the information for both skin and nerve cells is still potentially active. If the cell follows the skin channel, the nerve information has become inactive, where the skin information is active. Many plant cells remain totipotent. The actually inactive information may be reactivated to allow an entire plant to be regenerated from a single cell.

The epigenetic landscape is Waddington's image of the channels that guide cells in embryological development (see Figure 2).

Moving Boundaries

In the usual analysis, the form of the quantum potential is determined by fixed boundary conditions. In Waddington's epigenetic landscape, the fate of cells is not determined in the embryo, but at each bifurcation. A more exact analogy of epigenesis with the quantum potential would require that the boundary conditions change while the particle is moving through the system. In such a moving boundary problem, the quantum potential itself changes through time.

'[T]he distinctive quality of morphogenetic dynamics in living organisms appears to be the that shape is generated within and by a moving boundary, the dynamics changing the shape while changing the shape feeds back onto the dynamics, stablizing the modes that generate the form and creating the conditions for the next bifurcation' (Goodwin, 1994: 100).

2. ACTIVE INFORMATION

Every cell in an adult organism has the same DNA. Nevertheless, different cell types have different patterns of activity, where some genes are expressed and other are not. Only those stretches of DNA relevant to the cell's role are translated into proteins. In any one cell, only part of the genetic information of the whole is active, the rest is inactive. To understand the differentiation of the embryo into all the cell types of the adult organism is to understand how information either remains active, or becomes inactivated.

So there is the idea that information is present in a cell, which may be either active or inactive. This same idea, of active information, is presented by Bohm and Hiley (1993) to describe the wholeness of quantum processes.

Popular accounts of, say, the double slit experiment, talk of a particle passing through one of the slits 'knowing' that the other slit is open. In the Bohm interpretation, the quantum potential *informs* the particles of the whole experimental arrangement and coordinates their trajectories. Thus an interference pattern emerges when the two slits are open, but disappears when one is closed, according to the form of quantum potential.

The quantum potential does not produce a mechanical force, as its effects do not depend on its strength, only on its form. Rather than being a mechanical potential, it is an information potential. Information of the whole experimental arrangement is potentially active everywhere but only actually active where the particle is.

To bring out the idea of active information, Bohm and Hiley provide many illustrations: a ship guided by a radio signal, a person following a map, and a cell guided by its genetic information: '[I]t is only the form of the DNA molecule that counts, while the energy is supplied by the rest of the cell (and indeed ultimately by the environment as a whole). Moreover, at any moment, only a part of the DNA molecule is being 'read' and giving rise to activity. The rest is potentially active according to the total situation in which the cell finds itself' (Bohm and Hiley, 1993: 36).

3. UNDIVIDED WHOLENESS

Bohm and Hiley (1993: 135-137) describe an Einstein-Podolsky-Rosen-type experiment, where a molecule of total spin zero disintegrates into two atoms, each with spin one-half. If we measure the spin of atom A in a particular direction, then we can predict that the spin of B is opposite. The EPR criticism is that as atom B has not disturbed by the measurement of A, then its spin must have been an element of reality before the measurement of A.

We could have chosen any of the components of the spin of B and each, by the EPR argument, would have to have been an element of reality before the measurement of A. However, according to quantum theory, these components do not commute and cannot be defined together, so at most one could be an element of reality at any given time. If we accept that measuring A cannot affect the state of B, the quantum theory appears incomplete. However, if the state of B is nonlocally correlated with A, then the criticism fails.

Bohm (1980: 186-189) makes the following vivid analogy. A fish swims in a tank and is being filmed by two cameras 90 degrees to one another (*ibid.*, Figure 7.1). An observer of the images from the two cameras sees different pictures, but ones that are correlate with each other. When one fish moves, so does the other and in a predictable direction. A movement started by one will be completed by the other, etc. Movement in one image does not cause movement in the other. The two images are two dimensional projections from a higher dimensional reality that is their common ground, namely the fish.

The quantum system of the original molecule in the EPR experiment, though divided by the disintegration into two atoms, remains whole, and its parts, though no longer locally in contact are still correlated. Each atom is a three dimensional system, projected from a six dimensional system, which holds both projections within it.

Classical physics is based on what Bergson (1911) calls the order of solid bodies, bodies that are separate and external to one another and act on each other through mechanical forces. Quantum physics, on the other hand, reveals an order of organic connection, where parts separated over great distances may nonetheless act as a single whole. It is in this sense that Bohm (e.g. 1980) has spoken of quantum phenomena revealing a new kind of order. Parts that appear separate are so only relatively speaking and emerge from a background in which they are implicit. Parts are an explicate order brought forth from the wholeness of the implicate order. Different experimental arrangements bring forth different appearances, so that, for example, the position of a particle cannot be explicated at the same time as its momentum. Hiley (2001) has made this particularly clear recently by showing that the particle's position and momentum are within different shadow manifolds projected from a higher dimensional space.

There are many examples of a wholeness that remains even when divided among plants and animals. Plants can grow a whole new plant from cuttings, from a root, shoot or, more unusually, a leaf, or from vegetative structures, such as corms or tubers. Animals such as *Hydra* and *Planaria* can be cut in half and regenerate into two individuals.

Bortoft (1982), in attempting to bring out the character of quantum wholeness, draws on these examples from the living realm: '[N]o matter how many times we divide the fuchsia plant [to take a cutting] it remains whole ... when we divide the plant, it is always the original plant but never the same specimen' (Bortoft, 1982: 48[1]). 'The potato is not grown commercially from seed, but from *sets*, which are just potatoes, and so all the potatoes of one variety in the world are *one plant*. They are one individual that has just been divided and divided' (Seymour, 1977: 116). Bortoft takes the latter quote from a book quaintly named, *The Countryside Explained*.

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[1] I thank Professor Basil Hiley for kindly lending me a copy of Henri Bortoft's thesis.

An explanation of this living wholeness is to be framed within the implicate order and not in terms of the local mechanisms of the explicate order. Indeed, regeneration has defied mechanistic explanation, a fact testified, with some puzzlement, by mainstream authors: 'Given that Abraham Trembley's investigations of regeneration in *Hydra* (published in his *Mémoires* in 1744) launched the era of experimental biology, it is ironic that the problem of regeneration still awaits a satisfying mechanistic explanation' (Newmark and Alvarado, 2002). It does not occur to the authors that well-known biological phenomena might not fit onto their Procrustean bed!

4. INFINITE POTENTIAL

The idea of the implicate and explicate orders was inspired by the writings of the Renaissance theologian, Nicholas of Cusa. From the premise of the infinity of God, Nicholas was led away from scholastic philosophy to the vision that in the Divine, all opposites coincide and all distinctions are overcome. 'God is the "implicatio" of all opposites. But what in God is "implicatio" and "complicatio," becomes "explicatio" in the universe, which results from multiplicity, distinction, and opposition... God is as it were contracted in beings; He is the absolute quiddity [the essence, that which makes things what they are] of all the things in which He is contracted.' [2] In the explicate order, bodies are separate, distinct and different from one another. In the implicate order, this difference disappears and they are seen as manifestations of the same whole. They are projections or contractions from a higher dimension into a lower dimension[3].

Nicholas of Cusa also stresses that the world partakes in the infinity of the Godhead and thus has an 'infinite potential.'[4] Bohm (1984) considers the idea that 'nature may have in it an infinity of potentially or actually significant qualities' (p. 132) as an alternative to mechanism. It has been expressed to me that biology is marked out from physics by dealing with so

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[2] 'The Philosophy of Nicholas of Cusa,' Center for Applied Philosophy: The Radical Academy, http://radicalacademy.com/philcusa.htm.

[3] Bortoft (1982) draws on Abbott's (1952) story of *Flatland*. The effort required to see two explicate parts as an undivided whole is like that of a two-dimensional Flatlander trying to comprehend the sphere that passes through his world.

[4] *Infinite Potential* was the name chosen by David Peat for his biography of David Bohm.

many different entities and with so many different qualities. Physics, from the point of view of mechanism, aims to reduce the diversity of the world to the behaviour of a finite set of fundamental entities, whose qualities are fixed.

Bohm shows that the history of physics, in particular in the twentieth century, simply does not bear out the assumptions of mechanism, indeed it has contradicted them. The position of a qualitative infinity of nature is also not simply an alternative to mechanism, but includes it. We may always entertain that the finite set of entities and qualities with which mechanism explains a certain phenomenon is a relevant and useful contraction of nature's infinity for that context. To understand that to account for phenomenon requires such a contraction saves us from 'multiplying entities without necessity.' Lastly, mechanism, in its claim that certain features are fundamental and fixed, contradicts the spirit of the scientific method, which urges the scrutiny of every feature of a theory. At every stage in the history of science, that scrutiny has revealed contradictions and led to a better and richer understanding of the world.

Now let us turn to look at the infinite potential of the living state. A single cell is *totipotent* if it is able to divide and produce all the differentiated cells of an organism. The ability is found in, for example, the cells of the meristematic tissue of a plant cutting or tuber, or a human embryo. In totipotential cells, the whole of the genetic information is potentially active. They are responsible for the quality of living wholeness that we have described earlier.

Totipotential cells specialise into the *pluripotent* cells, which, in humans, can develop into any of the three major tissue types:

a. Endoderm – lining of the gut
b. Mesoderm – muscle, bone and blood
c. Ectoderm – nerves and skin

Pluripotent cells undergo further specialisation into *multipotent* cells. They are committed to give rise to a limited range of cell types, which serve a common function. Blood stem cells give rise to red cells, white cells and platelets, each of which are considered to be *terminally differentiated.*

Differentiated cells usually lose the potential to give rise to new individuals: information is active in the cell's function, but the rest is inactive. Along a pathway of differentiation, the information that is potentially active becomes less and the cell takes on a more definite form.

Differentiation is the contraction of the wholeness originally present in the totipotent embryo, by stages pluripotent, multipotent and terminally differentiated. Differentiation of a blood stem cell into platelets, white blood cells and red blood cells is a contraction of wholeness, a projection from a larger into a smaller space. To understand differentiation is thus to understand how the original wholeness contracts so as to be lost, actually and potentially.

5. BIFURCATION OF ALTERNATIVES

So how in quantum systems is the original wholeness lost and information inactivated? Let us consider the example of barrier penetration (Bohm and Hiley, 1993: §5.1).

The quantum potential can thought of as lowering the height of the barrier, allowing particles to pass through. In this way, the quantum potential explains the mystery of how particles penetrate the barrier. A particle of energy E will pass through if E is greater than $V + Q$, where V is the classical potential and Q the quantum potential. Also, the quantum potential does not take a constant value, but varies like a landscape, with plateaux and troughs. Only those particles at the front of the wave packet may enter the barrier, and most of these are reflected back. 'There is a critical trajectory, which divides the trajectories that go through from those that do not. This evidently resembles a bifurcation point of a kind typical of the non-linear equations describing unstable systems' (Bohm and Hiley, 1993: 75).

Confronted with the potential barrier[5], the particle enters one of two distinct *channels*, either reflected or transmitted. Information in the channel that the particle enters is actually active, whereas in the empty channel, information is actually inactive. Information in the empty channel retains the potential to act on the particle, since the channels can be made to overlap again. As time passes and the particles undergoes more and more interactions, it becomes more and more difficult to be able to reconstruct the original overlapping channel. Hence the measurement may be considered an irreversible process.

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[5] Hiley (2002) describes an equivalent situation for the Stern-Gerlach magnet.

As Beattie describes in his excellent review of Conrad Waddington's work[6], Waddington was the first to construct bifurcation diagrams of development, long before their emergence in the sciences of chaos and complexity. 'In embryonic development we are confronted with alternative modes of development, the choice between which is taken in reference to an external stimulus, or to an internal one ... In considering the effects of genes, we find alternatives, the choice between which may be taken in response to diffusible substances' (Waddington, 1956).

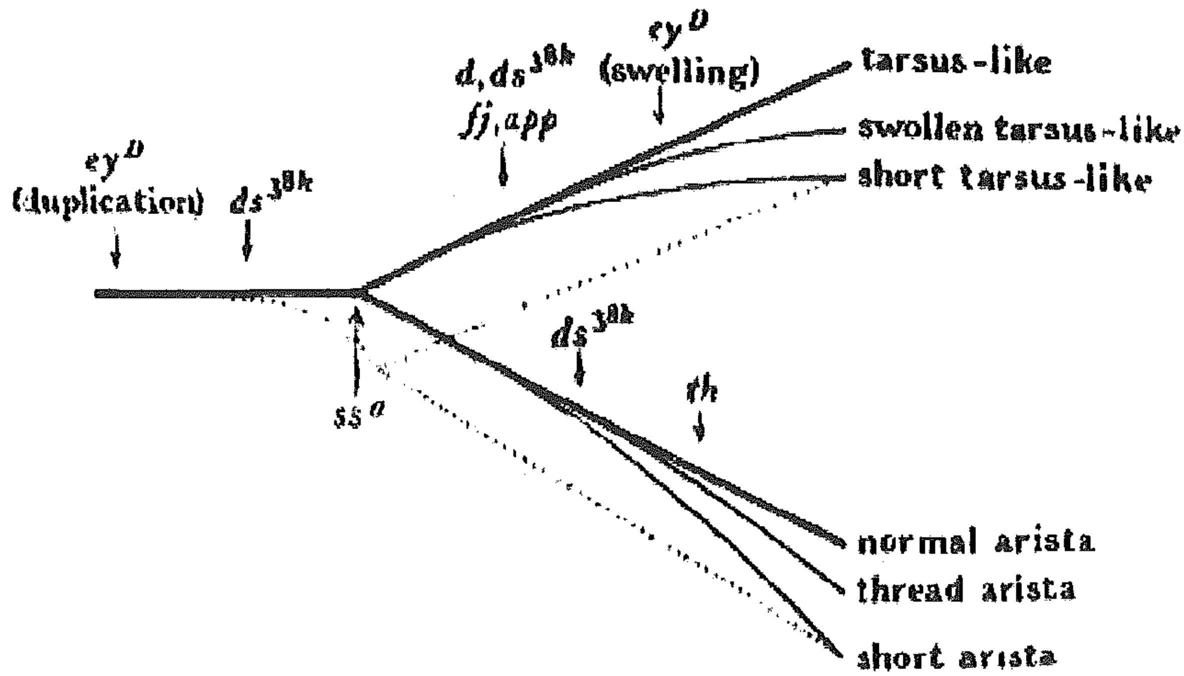

Figure 1. Bifurcation diagram showing the fate of an imaginal disc from the antennae of *Drosophila* (Waddington, 1940).

Consider a pluripotent cell that gives rise to ectodermal tissue. At this stage, the information for both skin and nerve cells is still potentially active. Yet there is a bifurcation along the pathway, where the cell will enter either the nerve channel or the skin channel. If the cell follows the skin channel, the

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[6] I thank Professor Alan Beattie for kindly allowing me to quote from his paper, 'Figures in an epigenetic landscape' (2004). http://www.lancs.ac.uk/ias/documents/complexity workshop/a b figures in an epigenetic landscape d2.doc

nerve information has become inactive, where the skin information is active. If it follows the nerve channel, the reverse is true.

Many plant cells remain totipotent. The empty channels that they have left behind during their differentiation still have an influence on their behaviour. The actually inactive information may be reactivated to allow an entire plant to be regenerated from a single cell. Mechanism fails to account for the phenomenon of regeneration because the approach concentrates on actually active information and ignores genetic information that may remain potentially active in a differentiated cell.

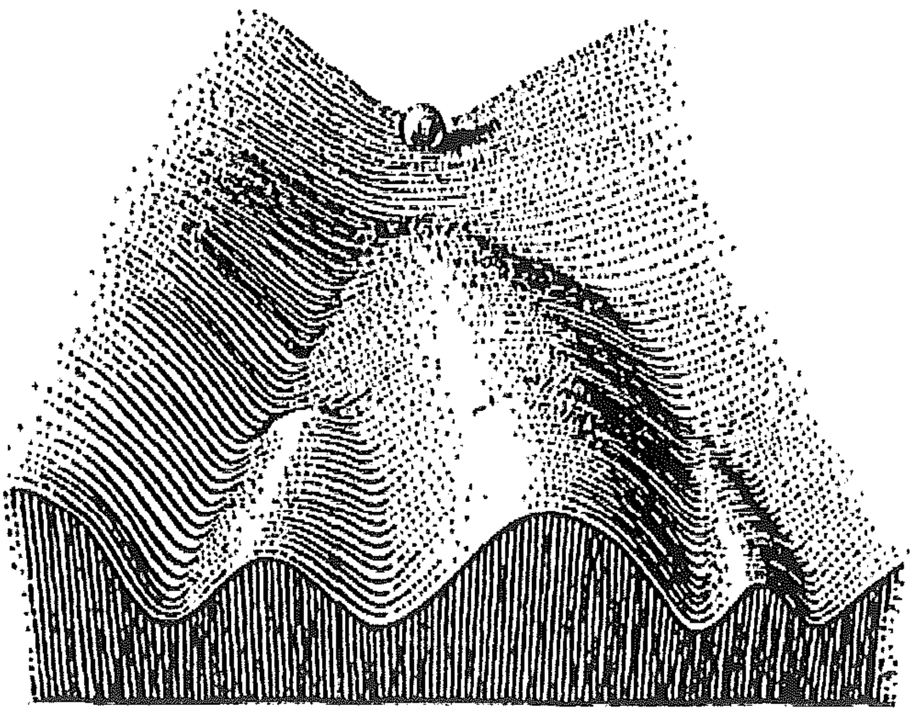

Figure 2. The epigenetic landscape is Waddington's image of the bifurcating channels that guide cells among alternative paths in embryological development (Waddington, 1957).

There is a difference between the quantum potential and the epigenetic landscape. In the quantum potential, the particles are on the plateaux and the valleys represent transitions between possible trajectories, where the particle receives a violent acceleration or deceleration. As Figure 2 shows, in Waddington's epigenetic landscape, the ball representing the cell, or developing organism as a whole, lies in the valley and the hills represent improbable transitions.

6. BROKEN SYMMETRY

Goodwin (1994) summarises his work on morphogenesis in *Acetabularia*. This giant single-celled alga has a rhizoid, by which it attaches to the rock, a long stalk and a parasol-like cap, by which it earns its vernacular name, the Mermaid's Cap. If the cap is cut off, then the alga regenerates through a particular sequence of changes of form, given below:

- A new cell wall forms a hemisphere over the cut end.
- A tip arises, pushed out by pressure from the vacuole.
- The tip grows into a stalk.
- The tip flattens into an annulus.
- A whorl emerges with laterals at intervals round the tip.

Goodwin and his co-workers have developed a model, based on interactions between calcium concentration and the elasticity of the cytoplasm, which captures this sequence. *Acetabularia* shares this pattern of changes with other members of its order, the Dasycladales. None, except the members of the recently evolved family Acetabulariaceae, share the final step of producing the beautiful cap.

The regenerating tip begins with spherical symmetry. The tip grows and the spherical symmetry breaks, arriving at the circular symmetry of the annulus. Finally, the circular symmetry breaks into the rotational symmetry of the laterals, placed at intervals round the tip. 'The technical term to describe the transition from a state of higher symmetry (lower complexity) to one of lower symmetry (higher complexity) is a *bifurcation*' (Goodwin, 1994: 89).

We can see the same pattern of broken symmetry in high speed photographs of a drop falling into a thin layer of milk (see Figure 3). The spherical symmetry of a drop transforms into the circular symmetry of the crater of liquid pushed up by the impact. Finally, the crater breaks into a crown of 24 points, possessing rotational symmetry.

D'Arcy Thompson was inspired by these photographs when they emerged at the turn of the twentieth century. A section of *On Growth and Form* is devoted to splashes and their analogies with other structures (1961: 67-70). The crater rising from the liquid surface is the pot thrown from the semi-liquid clay by the potter's hand. The crown of the milk splash is the fluted calyx of the hydroid *Campanularia* or the bell of the single celled *Vorticella*.

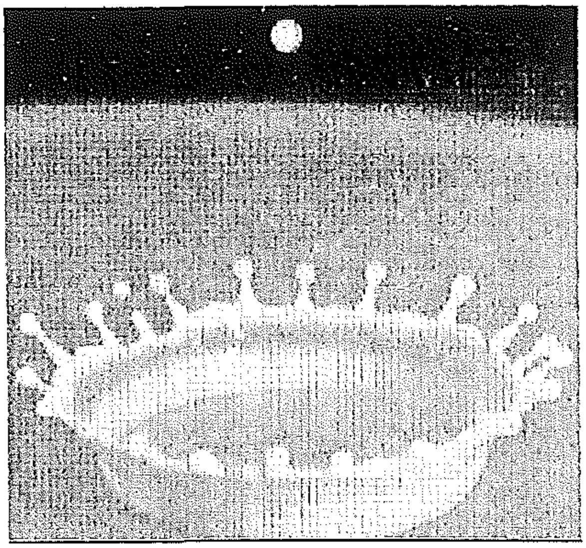

Figure 3. The crown formed by a milk drop falling into a thin layer of the liquid.

7. MOVING BOUNDARIES

In the usual analysis, the form of the quantum potential is determined by fixed boundary conditions. The fate of a particle depends on its position in the wave packet at the start. There is nonetheless a random assignment of positions to particles. Therefore, the fate is given by a combination of the initial and boundary conditions.

In Waddington's epigenetic landscape, the fate of cells is not determined in the embryo, but at each bifurcation. If the fate were determined in the embryo this would not be epigenesis, but preformation. Preformation is an ancient theory of development, where the egg and the sperm contain miniature adults, which develop by unrolling, 'evolution' in its original sense. A more exact analogy of epigenesis with the quantum potential would require that the boundary conditions change while the particle is moving through the system. In such a moving boundary problem, the quantum potential itself changes through time.

The morphogenesis of *Acetabularia* is a moving boundary problem, as the boundary of the morphogenetic field moves as the cell grows: 'the field dynamics generates a pattern that leads to a particular shape that the affects the dynamics, resulting in an unfolding of form through a sequence of changes. We can describe this as an implicate order in the dynamics being explicated in the shape, which then influences the implicate order'

(Goodwin, 1994: 96). The shape emerges as an expression of the whole dynamics, yet the shape serves to redefine the context of the dynamics. The growth of the regenerating tip sets up the conditions for the symmetry breaking that result in the annulus. The subsequent flattening of tip makes possible the bifurcation to the laterals of the whorl.

Goodwin concludes: 'the distinctive quality of morphogenetic dynamics in living organisms appears to be the that shape is generated within and by a moving boundary, the dynamics changing the shape while changing the shape feeds back onto the dynamics, stablizing the modes that generate the form and creating the conditions for the next bifurcation' (Goodwin, 1994: 100).

8. UNANSWERED QUESTIONS

This paper is based on a talk I gave to the TPRU at Birkbeck in July 2005. I thank Basil Hiley, Keith Bowden, Tony Booth and Ray Brummelhuis for the discussion that resulted. A number of questions emerged I list them here, firstly, to show possible applications of the ideas presented here, secondly to indicate future work and thirdly to invite the reader to join me on the same path.

- Why has no-one investigated the nonlocal propagation of changing the boundary of a quantum system?
- How can potentially active genetic information become actually active again? How can there be interference between an empty and a filled channel? A nerve or a skin cell becomes a stem cell again, regaining its ability to activate both sets of information. Is that seen? How could it occur?
- What are the theories concerning loss of totipotency? Why are animals so different from plants? Is there an advantage to terminal differentiation? Maybe it is good to have cells committed to their fate, compared to cancer where cells dedifferentiate and keep dividing.

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The geometry of DNA: a structural revision

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I would like to suggest a modification to the structure for the salt of deoxyribose nucleic acid (D.N.A.) that was proposed by Francis Crick and James Watson in April 1953.

In 1995 I began an investigation into the structure of DNA with the intention of producing a series of drawings and paintings of the double helix. My interest stemmed from certain features of my work as an artist, specifically my inquiries into the nature and depiction of space. In the manner of renaissance perspectival artists such as Uccello, I embarked on scale drawings of the helical structure using the standard textbook dimensions that derive from x-ray diffraction data. In the course of this work, discrepancies emerged, and it became clear to me that the Crick and Watson structure does not conform to geometric principles. Indeed my attempts to translate their theory from two into three dimensions ran into considerable topological problems.

Since the article in nature was published, their proposal would appear to have been fully vindicated by almost all-available empirical evidence. However, I have since discovered that a growing minority do recognise flaws in Crick and Watson's conclusions, specifically in terms of its topology and thereby the ability of the structure to replicate itself. Although some research has even gone so far as to question the very existence of the double helical structure itself, my proposal retains the double helix as its fundamental basis.

Without compromising the essence of their structure, I propose a resolution of the geometrical inconsistencies by means of a simple change in the position of alignment between the purines and pyrimidines. This realignment is founded entirely upon geometric principles and further investigation revealed a series of mathematical equations that describe a three-dimensional geometric helix that conforms to the known ratios of DNA. In this paper, I will demonstrate the mathematical basis for this re-reading of DNA and moreover the simplicity and purity it engenders, when applied to the molecular structure of the bases.

The Double Helix

The structure of the salt of deoxyribose nucleic acid is undoubtedly a double helix, with ten bases to each turn. Moreover, the approximate dimensions of a complete turn of DNA's helix are well known: the diameter of the helix is 20Å (angstroms), the base height is 3.4Å and thus the helix extension is 34Å. These data enabled a simplified and systematic perspective projection of the structure, see **fig1** below.

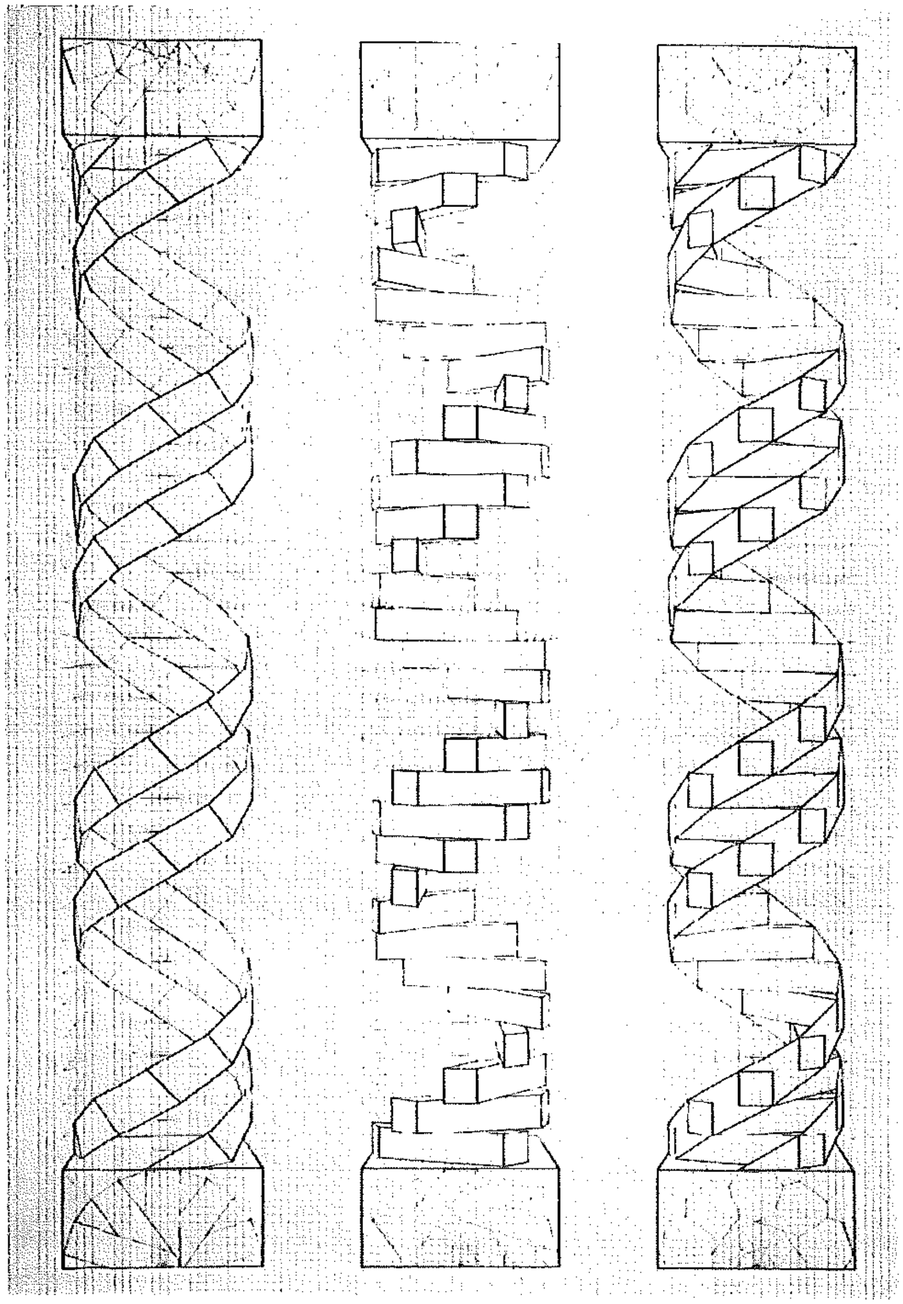


fig 1

The Mathematics

Although, theoretically, it is possible to construct a helix from almost any series of polygons, there happens to be only one polygonal formation that would fulfil all the necessary criteria: ten regular pentagons orientated about a decagon, see **fig 2** below.

When translated into three-dimensional space, these pentagons would become prisms with all lengths equal (**ii**).

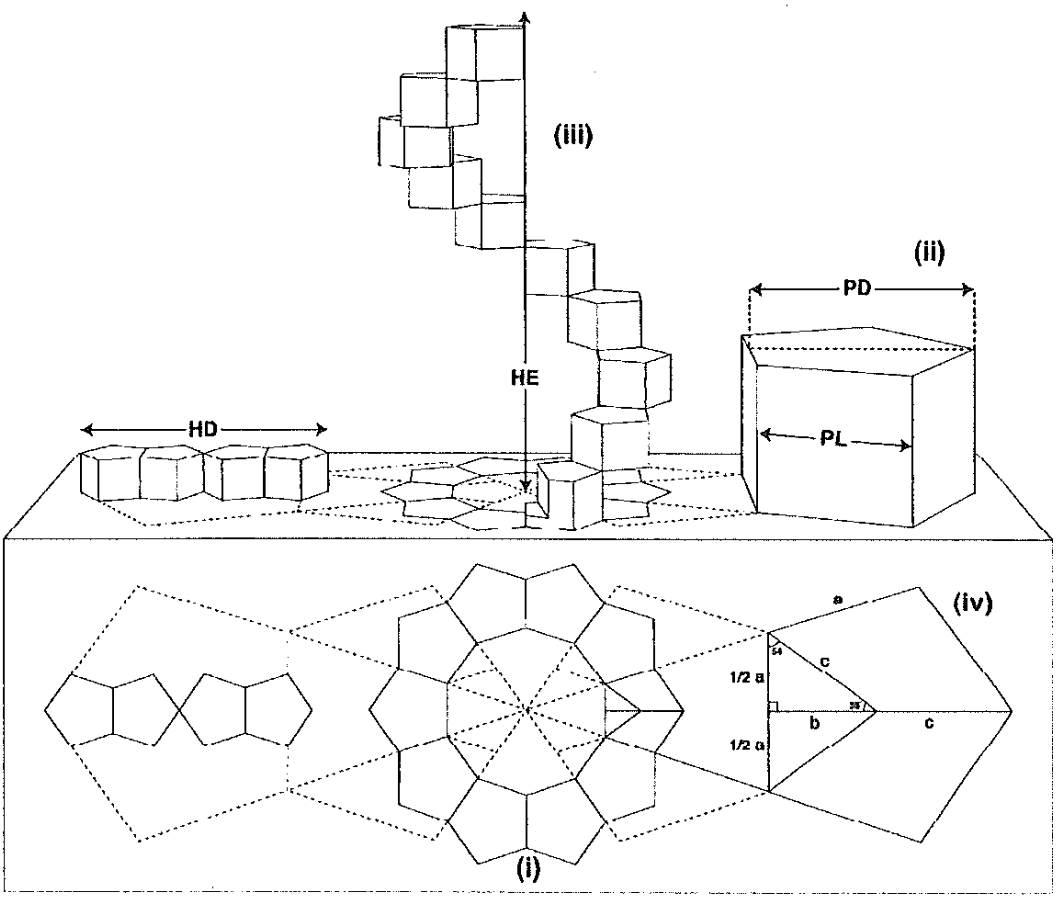


fig 2

A complete turn of the helix is formed by progressive rotation and extension of the ten regular prismatic pentagons (**iii**).

It follows, therefore, that the diameter of the helix (**HD**) and the height of the helix extension (**HE**) have a direct and constant proportional ratio to the pentagon length (**PL**) and pentagon diameter (**PD**).

In order to establish the precise nature of this ratio we must devise some trigonometrical equations.

The values of **a b c (iv)** are ascertained as follows:

$1/2a = c \times \sin 36°$
$b = c \times \cos 36°$
$c = 1/2a \div \sin 36°$ *or* $b \div \cos 36°$

The values of PL and PD may be determined as follows:

$PL = 2 \times 1/2a = a$
$PD = b + c$

It follows, therefore, that:

$HD = 4 \times (b + c)$
$HE = 10 \times a$

The above equations enable us to establish the mathematical constant ratios correct to ten decimal places:

PL = 1
PD = 1.5388417686
HD = 6.1553670744
HE = 10

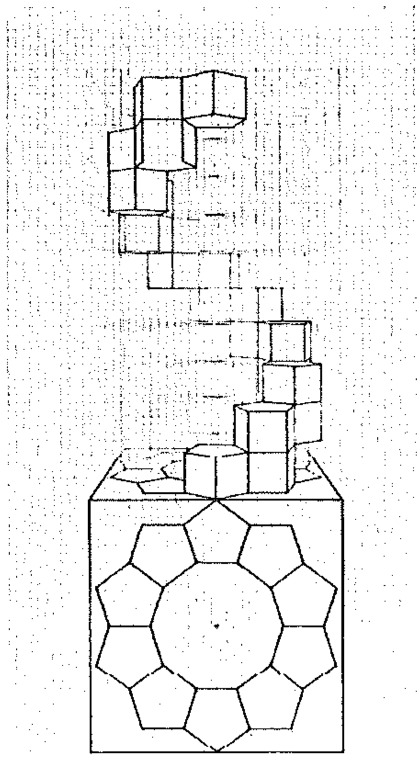
From these ratios, it is possible to come by a set of interrelated equations that would determine any of the measurements of a helix constructed from regular prismatic pentagons:

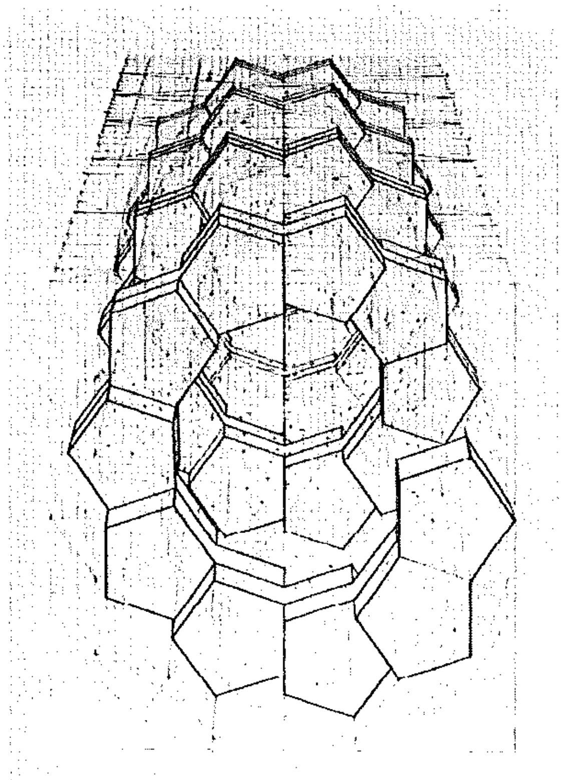
$PL = PD \div 1.5388417686$ *or* $HD \div 6.1553670744$ *or* $HE \div 10$
$PD = PL \times 1.5388417686$ *or* $HD \div 4$
$HD = PL \times 6.1553670744$ *or* $PD \times 4$
$HE = PL \times 10$

We now find ourselves in a position to apply any one of the known dimensions of DNA to the above equations. For example, if the PL – i.e. the base height – is known to be approximately 3.4 Å:

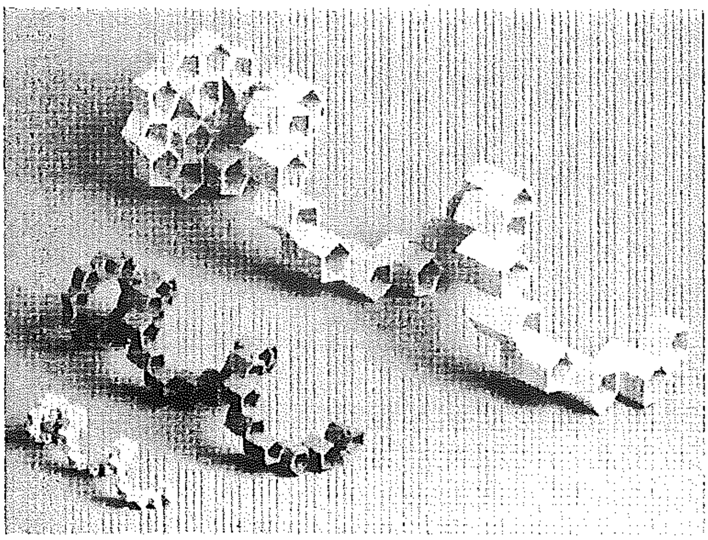
PD = 3.4 (PL) x 1.5388417686 = 5.23206201324 Å
HD = 3.4 (PL) x 6.1553670744 = 20.928248053 Å
HE = 3.4 (PL) x 10 = 34Å

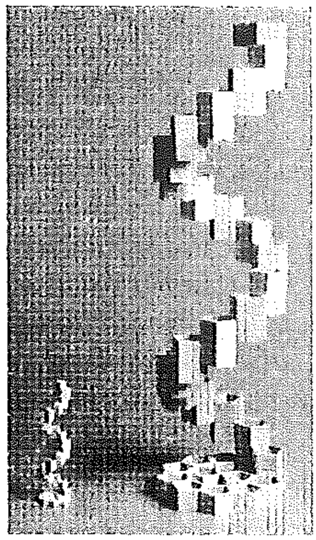
These figures would appear to cohere with the known dimensions of DNA, as revealed by the x-ray diffraction data and we now find ourselves in a position to construct an exact dimensional composite of the DNA double helix.





In **fig 2** I used single prismatic pentagons to illustrate the 3-dimensional helix. However, to ensure stability in the structure as it would exist in reality, it is necessary to place adjacent pentagons on each plane and then stack accordingly, thereby creating the double helix.





The application of geometry to the DNA molecular structure

The pairing of purines and pyrimidines in the Crick and Watson proposal locates the molecular pentagons that make up a part of the purines on the outside of the paired bases (**fig 4**). However, it has been demonstrated that a more fluent geometry would require adjacent pentagons at the heart of the base pairing. It is therefore necessary to re-orientate the established molecular pairing, so that the pentagons of adenine (A) and guanine (G) form hydrogen bonds to the hexagonal structures of both thymine (T) and cytosine (C) (**fig 5**). This spatial arrangement of the hydrogen bonds enables the necessary adjacent second pentagon to become viable. Moreover, it also accords with our understanding of molecular bonding and maintains the specificity of G with C and A with T.

Fig 4 Crick and Watson pairings

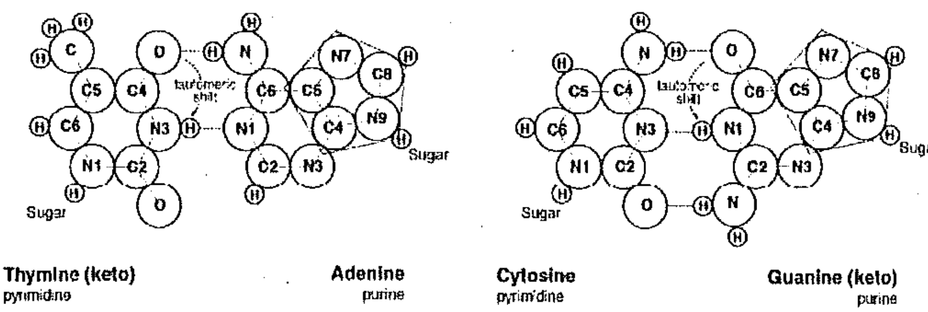
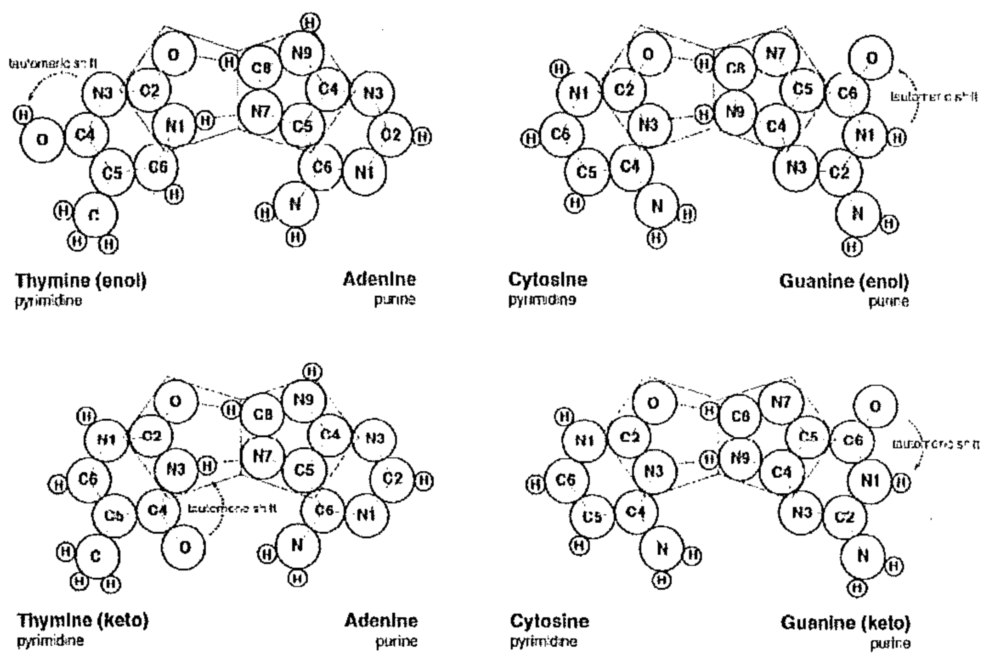
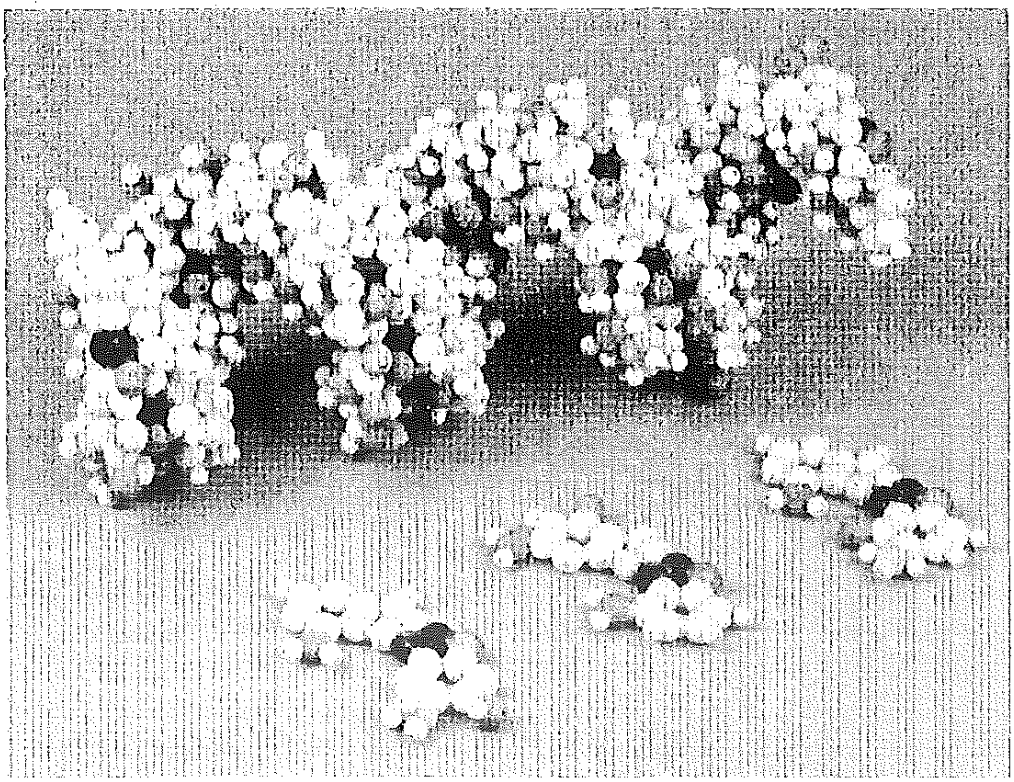


Fig 5 Proposed pentagonal pairings

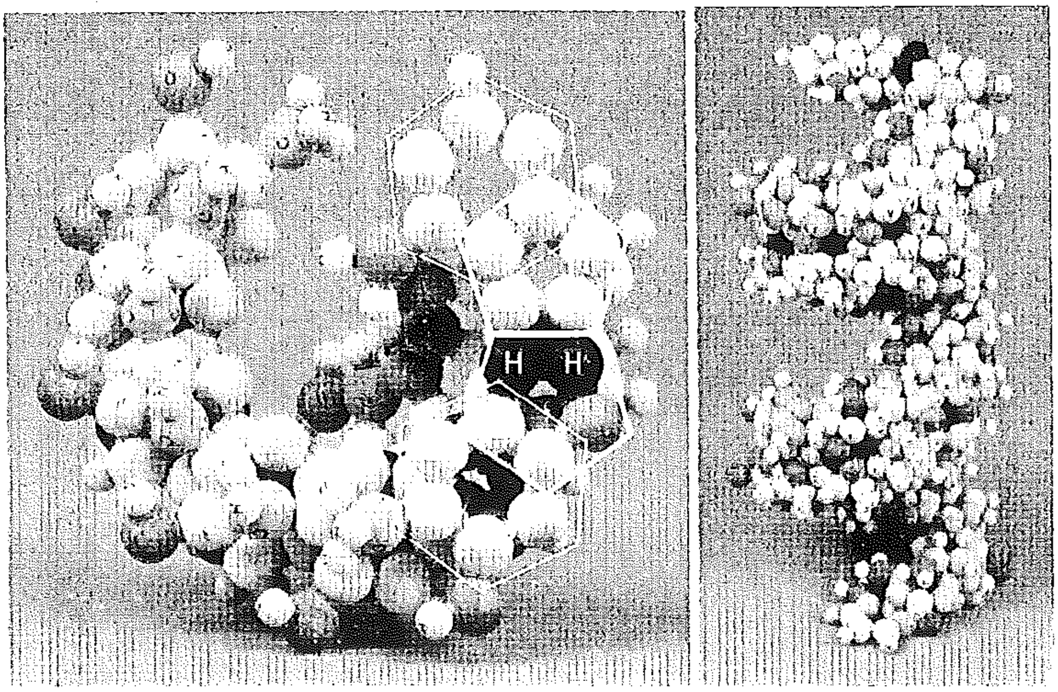


At this point, it would seem appropriate to refer to the different formations that the molecules of guanine and thymine can take, namely the enol and keto states. Guanine and thymine may exist in either enol or keto configurations and are believed to pass through tautomeric shifts from one to the other – in other words, an indiscernible and spontaneous leap by the hydrogen atom from a specific oxygen atom to a specific nitrogen atom or vice-versa. The Crick and Watson proposal requires the keto formation, whereas this alternate proposal would appear to be viable in either of the formations.

In addition, it should be noted that presently the sugar-phosphate chains are depicted attached to N(1) of the pyrimidines and N(9) of the purines by covalent (glycosidic) bonds. However, both prior to 1953 and even today, when the molecules are represented in isolation, hydrogen atoms are illustrated attached to these nitrogen atoms. For the purposes of this proposal, the four hydrogens are retained and one of them – guanine N(9) – now plays a practical role in the pairing of guanine with cytosine.



This alternative geometrical formulation reveals each base pairing to be an almost mirror image of the other, allowing a structure which, when fully assembled, exhibits both uniformity and stability. Most importantly perhaps, the alignment of the 'spurs' – that is to say, the NH_2's (G, C & A) or CH_3 (T) – on each base pair is consistent. If either base pair were inverted, these spurs would naturally project in the opposite direction without upsetting the main generative architecture of the helix. This ensures that the spurs attached to cytosine and adenine construct a constant sequence specific 'secondary helix' within the space generated by the primary helix, and that the spurs attached to guanine and thymine provide a sequence specific 'capping' of the cavities generated by the conformation of the primary helix. This sets in motion a simple and seemingly clockwork mechanism that may be utilised in the coding for amino acids and subsequent protein structures.

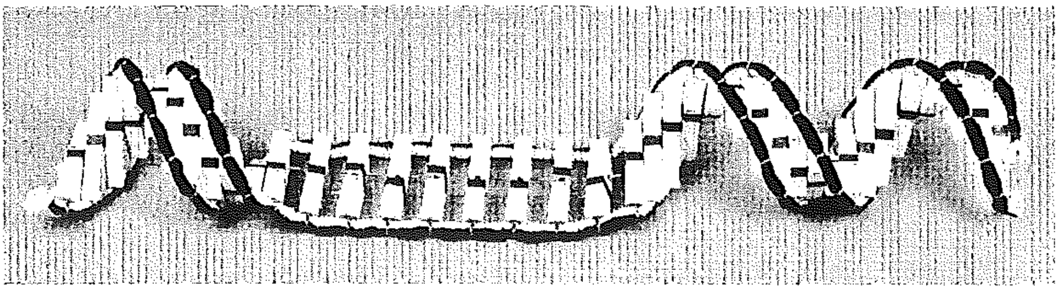


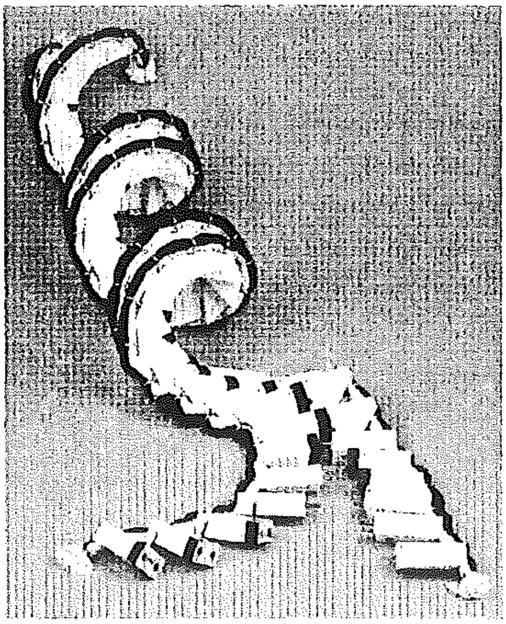
With respect to the sugar-phosphate chains, it has already been stated that at present, covalent bonds are used to attach them to the bases. The nature of the C/W proposal is such that it both requires and also relies in entirety on the sugar phosphate chains to corral the bases into the helix.

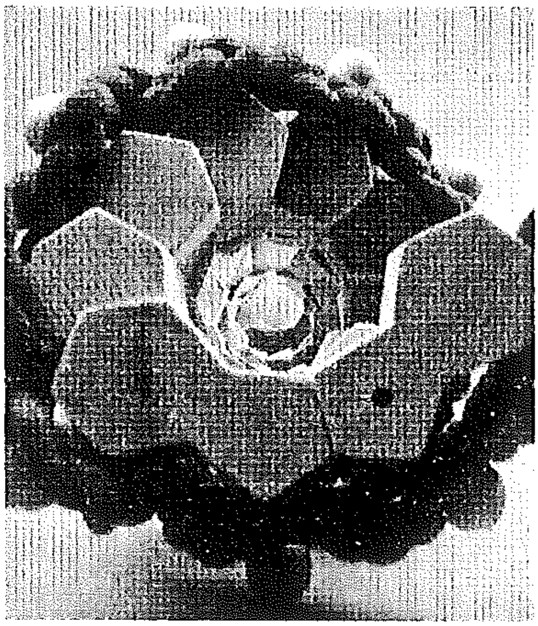
While it would be possible to incorporate covalent bonds into this proposal, I must say that, to my mind, the nature of this revised structure does suggest a strong possibility that the sugar phosphate chains may be secured by means of multiple hydrogen bonds. The sheer quantities of accessible hydrogen and oxygen on both the revised structure and also on the sugar-phosphate chains would perhaps tie in with such a suggestion. The first point of separation will inevitably continue to be the hydrogen bonds that hold base to base because they have considerably less resistance than the multiplicity of bonds that would hold the base-pair's to the sugar-phosphate chains.

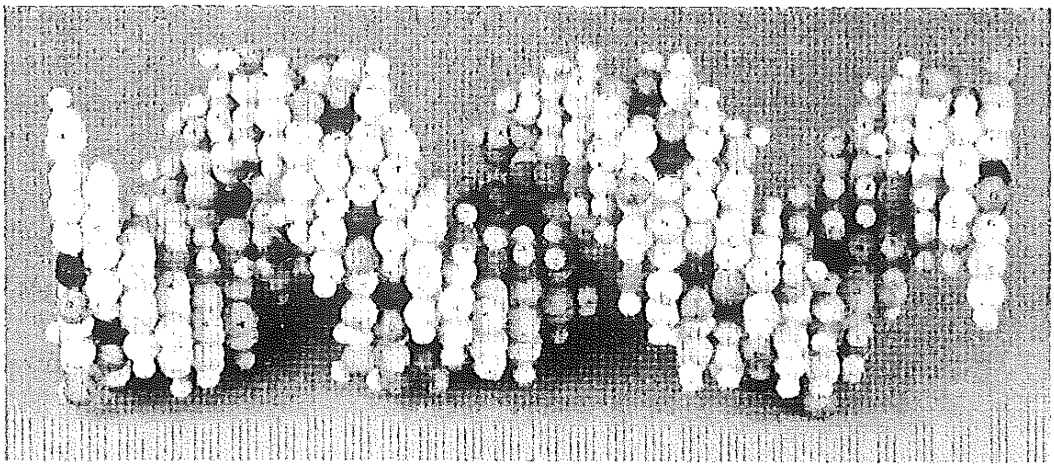
Conclusions

This proposed structure for DNA is wholly founded upon mathematical principles. Although the geometrical modification to the base pairings is relatively minor, the resulting double helix manifests a clarity altogether distinct from that offered by Crick and Watson and it would appear to shed light upon a number of areas of continuing uncertainty.









The implications of these findings are inevitably far-reaching and would potentially affect many areas of research. However, these lie beyond my own capacity to evaluate and I will restrict myself to reiterating the principal features of this proposal:

- Geometric equations predict the dimensions of DNA's structure. Not only does the pentagonal geometry predict the helical dimensions but it would also demonstrate 'principle causation'.

- The pentagonal geometry provides the dynamics required to build a consistent, stable and uniform helical structure and also establishes *why* there should be consistently ten bases contained within a single turn of the helix. Incidentally, when converted to the molecular dimension I would certainly predict degrees of variation, certainly between 9.5 and 10.5 bases per turn, but perhaps even more.

- Both the hollow centre and side-by-side structural formation ensure instant access at any point within the helix. This would permit the DNA (even circular) to open and close during its replication functions without entangling itself.

- The modification to the base pairing would appear to be able to exist in either the enol or keto formations.

- While the sugar-phosphate backbones will undoubtedly prove integral to the stability of the helical structure, it is the geometry of the base-pair molecules themselves that is ultimately responsible for the formation of the helix.

I have now set down a reinterpretation of DNA's structure as I see it. It should be remembered that, by necessity, I publish this paper in my capacity as an artist and in the knowledge that nothing I have set down has yet been subjected to scientific scrutiny. It should be read as such, allowing the visual material to complement the text and vice-versa. This, after all, was the trigger for my initial interest. Aware of the further implications, I offer it in the hope that it may stimulate wider research and debate.

Mark E. Curtis

Addendum

Since my first offering of this proposal in 1997 it has met, perhaps not altogether surprisingly, with a decidedly cool response. However, that said, I remain no less convinced that there is a case to answer and that more specific scientific research needs to be undertaken - I therefore continue to maintain what I have done so already with confidence, and would like to offer the following quote from Plato as some justification for my apparent dogmatism.

"We must in my opinion begin by distinguishing between that which always is and never becomes from that which is always becoming but never is. The one is apprehensible by intelligence with the aid of reasoning, being eternally the same, the other is the object of opinion and irrational sensation, coming to be and ceasing to be, but never fully real. In addition, everything that becomes or changes must do so owing to some cause; for nothing can come to be without a cause. Whenever, therefore, the maker of anything keeps his eye on the eternally unchanging and uses it as his pattern for the form and function of his product the result must be good; whenever he looks to something that has come to be and uses a model that has come to be, the result is not good."

Plato - Timaeus. (28)

Unfortunately, I have neither the access nor indeed the understanding to take these ideas any further and feel it both wise and prudent to leave any potential future areas of research to those people with a more specialist knowledge. In response to those whose minds appear fixed within the present paradigm and who would use its 'issue' to pick holes in some of the detail of this alternate proposal I can but quote Thomas Kuhn:

" ...the choice between competing paradigms regularly raises questions that cannot be resolved by the criteria of normal science. To the extent, as significant as it is incomplete, that two scientific schools disagree about what is a problem and what a solution, they will inevitably talk through each other when debating the relative merits of their respective paradigms. In the partially circular arguments that regularly result, each paradigm will be shown to satisfy more or less the criteria that it dictates for itself and to fall short of a few of those dictated by its opponent... The normal scientific tradition that emerges from a scientific revolution is not only incompatible but often actually incommensurable with that which has gone before... Because it has that character, the choice is not and cannot be determined merely by the evaluative procedures characteristic of normal science, for these depend in part upon a particular paradigm, and that paradigm is at issue."

Thomas Kuhn - The Structure of Scientific Revolutions. Ch IX & XII

Finally, I should like to mention that over these last 10 years I have both unearthed and also been pointed towards numerous papers, articles and books that I believe to be well worth revisiting in the light of this geometry. In particular I would draw attention to the following:

Papers

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Gamow, G. (1954) *Nature*, vol **173**, 318.
Furberg, S. (1949) *Nature*, vol **164**, 22.
Pauling, L. & Corey, R. B. (1953) *Proc. Natl. Acad. Sci,* **39**, 84-96
Hoogsteen, K. (1959) *Acta. Cryst*, vol **12**, 822.
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Bansal, M. (2003) *Current Science*, vol **85**, 1556-1563.

Books

Levene, P. A. & Bass, L. W. (1931) 'Nucleic Acids'.
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Angst... ou Ennui?

or The Paris Texas Hilton Interpretation of Quantum Mechanics v1.2

Jim Colvin

"Why does everybody get to mess around with the fabric of reality except me?" Lisa Simpson

It was raining. The party was over and there was little to do other than clear up the mess. Jerry Cornelius lay on the sofa, one hand on his needle gun and the other on Mitzi Beesley.

"There is no deep underlying physical reality. There are only macroscopic quantum phenomena. The rest is metaphysical baggage." Jerry Cornelius was in his element, bringing quotes out of a hat, albeit not always accurately. "Quantum physics teaches us to abandon the distinction between information and reality. If reality exists and if we will never be able to make an operational distinction between reality and information, the hypothesis suggests itself that reality and information are the same. We need a new concept which encompasses both."

"As is well known", said Jerry, carelessly waving the needle gun in the direction of Alphonsus Pi, "during euphoric experiences such as orgasm, the exercise zone and so on, the bloodstream contains analogs of chemicals such as opiates. In a similar way during religious, meditative and other spiritual experiences the bloodstream has been found to contain analogs of (so called) hallucinogens such as mescaline, psilocybin or lysergic acid diethylamide. Hence I do not distinguish between these experiences. (Others do.) To me they are all examples of 'direct contact with the void'".

"Since the void does not exist", responded Professor Pi, "one could not possibly have any contact with it. I suspect that drugs foster illusions. Of course the void itself (ahem) is an illusion. And existence is also an illusion." Jerry ignored him and continued, "I have speculated why a chemical structure such as lysergic acid should give one 'direct experience' of the void. My hypothesis (such as it is) it that such chemicals may have a molecular structure that, from an Everett point of view, gives direct quantum contact between the Many Worlds."

“I thought like this for a long time,” said Pi, “but no longer. I believe that the essence of the matter is the state of relaxation ... everyday ego-mind being a high (mental) arousal state anticipative of causal action to be taken; contemplative mind being a medium-to-low arousal state that performs (I believe) quantum computation on perceptions of its surround; and perception of the Void an even lower state, and sans any content - just pure awareness in a state of profound relaxation. So in this view, the presence or absence of any particular molecule is, at bottom, incidental.” Miss Brunner, who also appeared to be somewhere on the settee, groaned.

“I agree, but it’s a matter of causality. The molecules get you there”, replied Jerry irritably. “I do find it a little strange however when people describe such meditative states as "states of relaxation". It is true that in a sense one may be able to achieve them by relaxation, or that one is relaxed when one is in such a state, but to refer to them as states of relaxation always seems to me to be missing the point. Well, the main thing is to know where you are going - regardless of how you get there – and to know what to do when you get there.”

The washing machine repair man, who was now well into his new career, chipped in, “I (at least) feel no need to subscribe to the Many Worlds thing for Quantum Mechanics (or meditative experience) to make sense - I think it's a cop-out.” Jerry’s mind was wondering. For some reason he was picturing the day he took his mother to Southend-on-Sea. Mrs. Cornelius started the day by drinking twelve pints of lager. She then became hungry and followed this up with two large portions of fish and chips and an eel pie. Jerry had a vivid image of his mother vomiting noisily over the side of the pier. It was probably the eel pie that did it, thought Jerry.

Jerry recovered his composure. “Many Worlds is not a "copout", it is simply one of a variety of ways of looking at phenomena - or at least this is the general view amongst Theoretical Physicists today - and it was the viewpoint from which I was talking.” Mrs. Cornelius was very resilient. She had picked herself up, drunk some more beer, and proceeded to try every ride in the Coney Island style fun fair that took up most of the seafront at Southend. Her capacity for abusing herself seemed insatiable, thought Jerry, completely forgetting his English. What an upbringing! He wondered to what extent it had influenced his career choices.

"There are however some famous people," continued Jerry, "including Quantum Computer Scientists - David Deutsch in particular - and - strangely - Cosmologists, who *believe* (exclusively) in Many Worlds. (There are others who *believe* in the Quantum Potential in the same way.) I think that this was the point of view from which you were responding. In this case I agree with you - it is a copout (or, at least, a religion)." Jerry suspected that his telephone was being tapped again. Whenever he spoke like this on the phone the telltale clicks and buzzes of interceptor equipment were apparent. When he reverted to more innocuous subjects they disappeared again.

"But do not underestimate the usefulness of Many Worlds for understanding and learning new ideas. The way in which I was using it is, I believe, appropriate. At other times I will switch to Bohm, at others Copenhagen, etc. These interpretations are all mathematically equivalent. It does not matter which one you use to calculate, you will always get the same answers. But it does matter which one you use to model. Each will lead to different thinking about the same situation. Some will lead to new insights in one particular situation. Others will lead to different insights in different situations. That is the situation in Physics today, and I find it most satisfactory."

"Think about the LSD25 molecule from the Many Worlds point of view. If it is shown to have an exceptionally large degree of quantum coherence for its size – and if this turns out to be true of other similar chemicals – it cannot be denied that we should start to think more carefully about the nature of psychedelic experience." The sights and sounds of both Southend and of Mrs. Cornelius continued to drift unbidden through Jerry's consciousness.

"However... this is a hypothesis that remains to be tested. One plan is to persuade Anton Zeilinger", Jerry eyed the needle gun, "to repeat his two slit experiment with the LSD25 molecule. If my hypothesis is right it should show a considerably higher degree of coherence than anything that has been tried before." Sarah Bellectomice's lurchers were licking something, which once may have been edible, out of a cardboard carton on the floor. The Archbishop's daughter rose unsteadily from Jerry's clutch on the settee and staggered towards the door, kicking the dogs on the way past. The sound of an old Pink Floyd record drifted in through the door. It was "Animals". Jerry smiled. He never could play the rhythm to "Dogs".

"Unfortunately" Jerry continued, "lysergic acid diethylamide is not stable enough for Zeilinger's current approach and the only solution is to simulate the entire experiment." Nobody seemed to be listening to him. Jerry wondered if they had any idea of the consequences of his proposal. He had access to the most powerful computer in the University, and he had been racking his brains to find a use for it for some time. Jerry looked at his watch. The Multiverse was becoming very unstable. Jherek was up to his old tricks. Jerry sighed. It was time to adjust the chronoflow again.

Jimmy Colvin has been writing Science Fiction for some fifty years.
Many of his stories are about Jerry Cornelius.

Nature's Code

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Introduction

In our previous presentation on 'Fundamental structures applied to physics and biology' (Rowlands and Hill, 2006), we examined the mathematical structures apparently underlying different aspects of physics and biology in relation to their possible common origin and especially in relation to the universal rewrite system and the consequent nilpotent Dirac algebra. We now feel confident in proposing that this is part of a universal system of process applicable to Nature, which we may describe as 'Nature's code'. In this connection, we draw attention to our demonstration of the significance to fundamental processes of such concepts as 4 basic units, 64- and 20-unit structures, symmetry-breaking and 5-fold symmetry, chirality, double 3-dimensionality, the double helix, the Van der Waals force and the harmonic oscillator mechanism, and our explanation of how they necessarily lead to self-aggregation, complexity and emergence in higher-order systems. Biological concepts, such as translation, transcription, replication, the genetic code and the grouping of amino acids appear to be driven by fundamental processes of this kind, and it would seem that the Platonic solids, pentagonal symmetry and Fibonacci numbers have significant roles in organizing 'Nature's code', and, in particular, in organizing the structures and operations of DNA and RNA, and the processes that we describe as the genetic code.

Biological systems, though operating at the edge of chaos, are extremely ordered, whereas the tendency for nature is to become more disordered. Biology is, in effect, a race between order and entropy with the odds stacked in favour of entropy. So biological systems must create order, i.e. process information, with as much efficiency as possible. It would appear that the efficient processing of information requires certain algebraic and geometric structures, which are also found in systems organized at other scales, in particular, physics.

A diagram will summarise our previous findings on the unification of physics and biology:

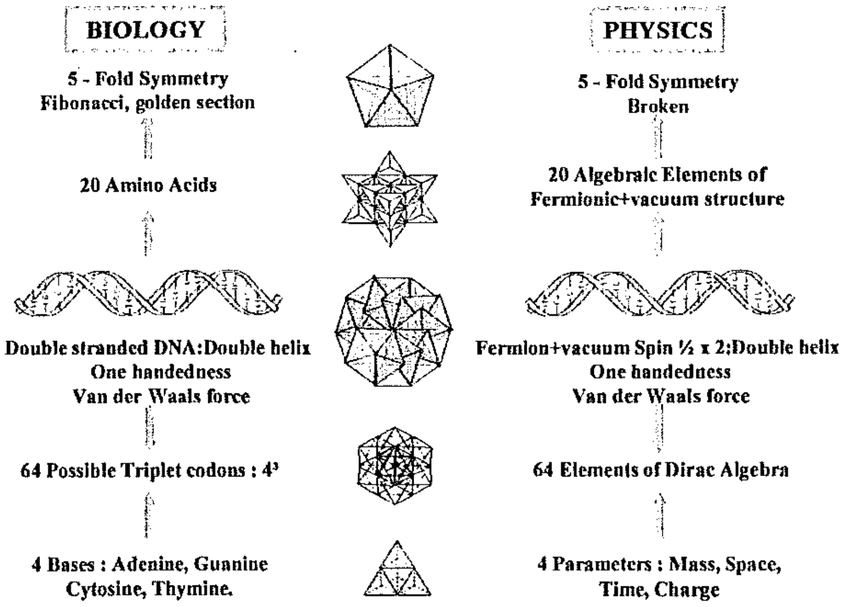


Fig 1. THE UNIFICATION OF PHYSICS AND BIOLOGY

Here, we see that the Dirac nilpotent fermion plus vacuum structure, with its four fundamental components (space, time, mass and charge), the 64 elements of its algebra, the double helical structure, chirality, and 5-fold broken symmetry (E, $\mathbf{p}$, m), corresponds closely to the structure of the DNA / RNA genetic code, with its four bases (adenine, guanine, cytosine and thymine), 64 triplet codons, double helical structure, chirality, and 5-fold axial symmetry.

Other structural connections may also be identified. Thus, it is even possible that some biological form of the Pauli exclusion principle may operate. It is a widely known fact that the enzymes responsible for DNA replication and repair within bacteria (the DNA polymerases) all have an error rate generally between 1×10^3 and 1×10^7. However, these error rates are values given for *in vitro* situations and cannot be directly applied *in vivo*. Different bacterial species have differing numbers of these polymerases each responsible for slightly different processes and each has an error rate that will be cumulative as a whole. These error rates increase when the system is under stress or within suboptimal conditions, and it may be that this error rate is part of a system for adaptive evolution. When the actual error rates are considered as a whole it is unlikely that any one bacterial cell is in fact a true identical clone of another – a situation that may remind us of the Pauli exclusion principle.

Triplet codons and amino acids

One particular significant connection between the two systems is that the 64 components of the full system are in some sense a 'hidden' structure. The Dirac nilpotent plus vacuum only requires 20 elements of the 64-component algebra, just as the 64 codons code for only 20 amino acids. Even the breakdown of the two sets of 64 show very similar mathematical patterns, along with the 20 units in each case, which are required:

AUG $1\boldsymbol{ki}$; AUA $1\mathbf{i}\boldsymbol{i}$; ACA $1\mathbf{j}\boldsymbol{i}$; AAA $1\mathbf{k}\boldsymbol{i}$; AAC $1\boldsymbol{j}$;
UGG $-1\boldsymbol{ki}$; UGC $-1\mathbf{i}\boldsymbol{i}$; UAC $-1\mathbf{j}\boldsymbol{i}$; UCC $-1\mathbf{k}\boldsymbol{i}$; UUU $-1\boldsymbol{j}$;
GAC $i\boldsymbol{k}1$; GAA $i\mathbf{i}\boldsymbol{i}$; GUA $i\mathbf{j}\boldsymbol{i}$; GCA $i\mathbf{k}\boldsymbol{i}$; GGG $i\boldsymbol{j}$;
CAC $-\,i\boldsymbol{k}1$; CAA $-\,i\mathbf{i}\boldsymbol{i}$; CGA $-\,i\mathbf{j}\boldsymbol{i}$; CUA $-\,i\mathbf{k}\boldsymbol{i}$; CCG $-\,i\boldsymbol{j}$.

Here, the codons are grouped into four pentads, with the first base determining whether the first coefficient is 1, as -1, i or $-i$. The second base in the three central codons of each pentad is represented by a vector term, corresponding to a different base in each; while the quaternion labels correspond to the final bases, which are different for the pseudoscalar, vector and scalar terms. In this representation, we might imagine the stop codons taking algebraic forms such as -1, i and $-i$, though a more systematic representation of the 20 units might privilege vectors rather than complexified quaternions in the third and fourth pentads:

AUG $1\boldsymbol{ki}$; AUA $1\mathbf{i}\boldsymbol{i}$; ACA $1\mathbf{j}\boldsymbol{i}$; AAA $1\mathbf{k}\boldsymbol{i}$; AAC $1\boldsymbol{j}$;
UGG $-1\boldsymbol{ki}$; UGC $-1\mathbf{i}\boldsymbol{i}$; UAC $-1\mathbf{j}\boldsymbol{i}$; UCC $-1\mathbf{k}\boldsymbol{i}$; UUU $-1\boldsymbol{j}$;
GAC $\mathbf{k}1$; GAA $i\boldsymbol{i}\mathbf{i}$; GUA $i\boldsymbol{j}\mathbf{i}$; GCA $i\boldsymbol{k}\mathbf{i}$; GGG $\mathbf{j}$;
CAC $-\,\mathbf{k}1$; CAA $-\,i\boldsymbol{i}\mathbf{i}$; CGA $-\,i\boldsymbol{j}\mathbf{i}$; CUA $-\,i\boldsymbol{k}\mathbf{i}$; CCG $-\,\mathbf{j}$.

For many amino acids, the third base in the codon is partially redundant. Nearly all amino acids are predominantly coded by the first two bases, which remain the same as well as unique to that acid – only serine, leucine and arginine, with 6 triplet codons each, are exceptions. In all these cases, one alternative has the complete range of options for the third base, while the other has a choice of two. In serine, the alternatives are UC and AG; in leucine they are UU and CU; and, in arginine, CG and AG. In all cases, where the first two bases make six bonds (in the conventional arrangement), the third base is entirely redundant, with all four options for the third base (A, U, G, C) being available. This is true also in three of the cases where they make five bonds; in nine other cases, there is a choice between two options for the third base, and in one case, there is just a single option. Where the first two bases make only four bonds, there are three options for the third base in one case, two options in five cases, and a single option in one case. There is a general tendency, therefore, for a decreasing number of options for the third base, where there are more bonds made by the first two bases, as we might expect. In those (seven) cases where two different amino acids share the same two first bases, the third base divides into U / C or A / G in nearly every case; only isoleucine (U / C / A) and methionine (G) provide a slight exception to the pattern. In the case of bacterial start codons, it is not at all unreasonable that the most significant bases are the *last* two, which are invariably UG, with the first base entirely redundant (A / U / G / C); while stop codons correspondingly *begin* with the same base (U) with limited options for the final two (AA, GA and AG).

An analysis of grouping of amino acids according to their specific triplet codons also yields interesting relationships. A standard method of grouping is shown in Figure 1 and is dependent upon such factors / properties as size, polarity, charge, hydrophobicity, etc. However, if we attempt to group the amino acids according to the positioning of the nucleotide type within the triplet codon a different picture arises. When we attempt amino acid grouping using the A, T / U, G, C in the first position of the triplet codon, a completely random result is obtained and a similar lack of grouping is given by the position of the base within the third position – usually termed as the 'redundant base'. This is not surprising, in that this third position placement allows for variation in coding for the same amino acid. However, when the group is defined by the middle base (i.e. as in the columns in Table 1) we find there is a definite pattern (Figure 2) which gives a similar group profile to the standard system of grouping by chemical properties and yet because there are four bases a fourth group is defined. Table 2 lists the amino acids of each new group.

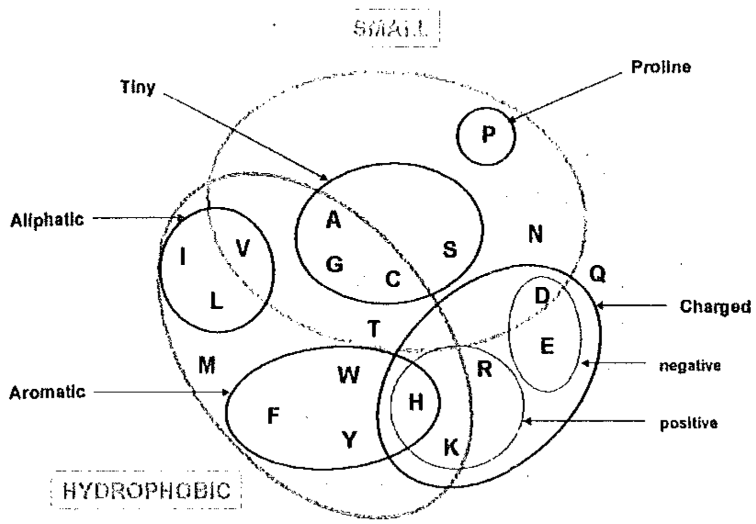


Fig 2. Standard 'Properties' Grouping of Amino Acids

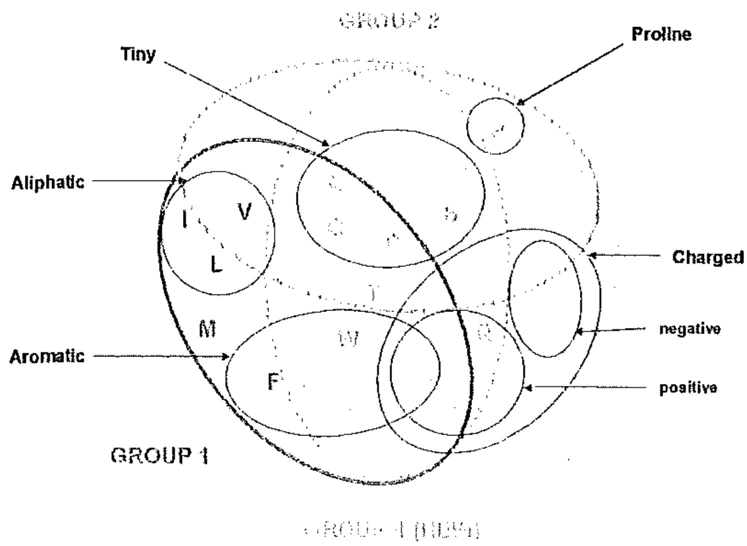


Fig 3. Grouping of Amino Acids Using The Second Base of Triplet: Group 1: T/U; Group 2: C; Group 3: A; Group 4: G.

When the properties of these four groups (Table 2) are looked at closer we see that group 1 now carries all the recognised bacterial start codons; ATG, TTG, CTG, GTG and group 4 contains the amino acids which display the 'extremes' of a certain property (Table 3); e.g. glycine is the smallest most flexible amino acid that acts as 'structure breaker' within proteins; tryptophan is the least used amino acid within proteins, it is the largest, is aromatic and absorbs UV light; arginine is the most basic with the most extensive delocalised charge, generally present in protein-nucleic acid interactions; cysteine is involved in di-sulfide bridge formation and is one of only two sulphur containing amino acids, the other being methionine with its triplet codon acting as the start codon.

GROUP 1	GROUP 2	GROUP 3	GROUP 4
Isoleucine: I	Proline: P	Glutamine: Q	Glycine: G
Leucine: L	Alanine: A	Asparagine:N	Cysteine: C
Valine: V	Serine: S	Tyrosine: Y	Tryptophan:W
Methionine: M	Threonine: T	Histidine: H	Arginine: R
Phenylalanine : F		Lysine: K	(Serine: S)
		Aspartate: D	
		Glutamate: E	

Table 1. The 4 New Amino Acid Groups

Glycine	Smallest, most flexible, structure breaker, achiral.
Tryptophan	Largest, aromatic, rarest, absorbs uv light.
Arginine	Most basic, extensive delocalised charge, present in protein-nucleic acid interactions.
Cysteine	Disulfide bridge formation, typically extracellular, 1 of 2 S containing amino acids (other = methionine, the start codon).

Table 2. Properties of Group 4 Amino Acids
Extremes : smallest, largest, most basic, S containing.

The algebraic structure can be considered as composed of 32 + terms and 32 – terms. It is interesting to see if such a split can be seen within the triplet codon set, and also to see if the full set of algebraic terms can be allocated to the 64 codons, in addition to the allocation of the 20 algebraic units to the corresponding 20 amino acids.

The system that revealed the previously described group structure within the chemical properties of the amino acids was by division into groups depending upon the bases A, T, G and C as the middle base of each triplet codon. The codon table (Table 1) can also be split into two groups dependent upon the type of middle base (pyrimidine or purine) within the triplet. The purines and pyrimidines hydrogen bond to each other upon opposite strands of the DNA helix and can be considered as opposites upon the + and – ve sense DNA strands. Splitting the triplet codons into these two groups dependent upon the middle base of the triplets does indeed reveal another level of order. The group with a pyrimidine base (U / T, C) as the middle codon (Group A) reveals a trend for triplet codons that code for amino acids of predominantly nonpolar / hydrophobic nature and those with a purine (A, G) as the middle base (Group B) as those coding for amino acids of a polar / hydrophilic nature (Figure 3).

If we divide each of these two groups of 32, further into 16 codons with their respective middle bases A, U / T, G and C (Figure 4), we see that the Group 1, containing U / T as the middle base code, for amino acids that are all distinctly hydrophobic, nonpolar and neutral; Group 2, containing C as the middle base, code for amino acids that are all neutral and have two amino acids that are hydrophilic and two that are hydrophobic. Group 3, containing A as the middle base, are all polar and 6 out of 7 are hydrophilic, 4 are charged (2 + and 2 –) and 2 neutral. Group 4, with G as the middle base of the triplet codons, gives the most mixed group of amino acids with 3 out of 5 being polar, 1 charged + ve and 4 neutral, 2 hydrophilic and 2 hydrophobic. It appears that Groups 1 and 3 have the most distinctive clustering of amino acid properties based upon nonpolarity / polarity, and Groups 2 and 4 those of less well defined grouped properties. It may be that, for the less well defined groups, there are other properties not yet considered that have greater applicability.

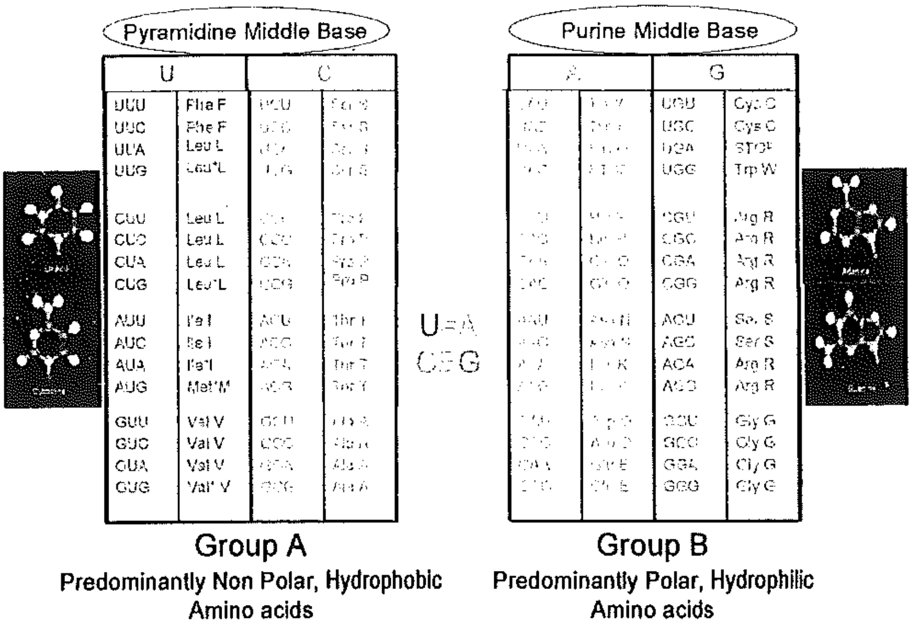


Fig 4. The 32+ and 32- Split of the 64 triplet Codons

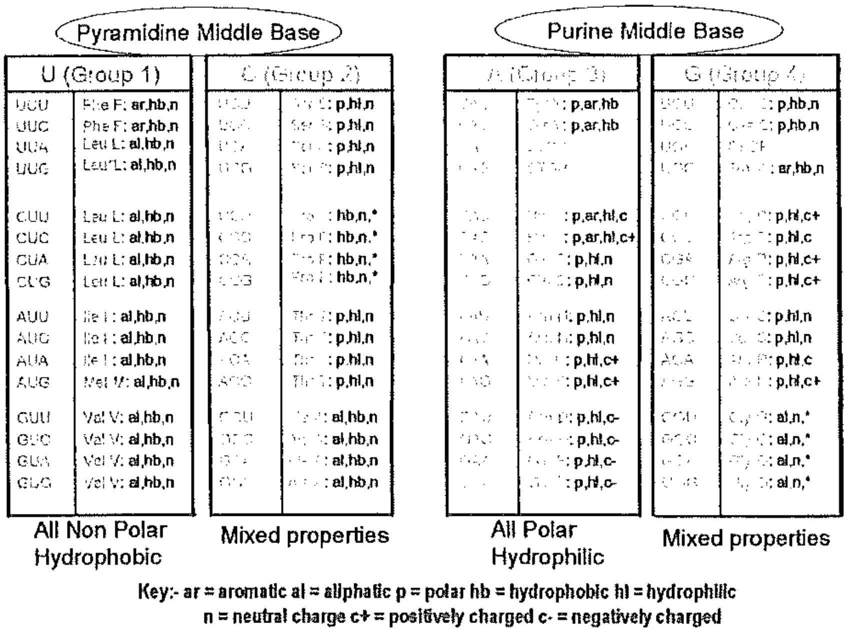


Fig 5. The Chemical properties of the 20 Amino Acids

As we have said previously (Rowlands and Hill, 2006), the 20 of the reciprocal icosahedron is clearly visible in physics in the structure of the nilpotent operator which has 4 groups of 5, and contains the dualities of mass-energy / charge, fermion / vacuum, space / phase space, localised / nonlocalised, etc. This icosahedral structure may also be applicable to the 20 amino acids in that there is some indication of amino acid grouping into 4 groups, dependent upon the middle base of the associated triplet codon. If a tetrahedron is placed upon each face of the icosahedron to give a tessellated form, with a total of 60 triangular faces, we can then allocate triplet codons to each tetrahedral, triangular face, that relate to the appropriate amino acid. It is interesting that here we have to lose 4 triplet codons of the 64 to give us the required 60 and we do have 3 known stop codons. Evolution may well have resulted in the loss of one stop codon and it is already known that there are variations of which codons code for specific amino acids in the process of 'codon capture' (Brooks *et al*, 2002). There are also known stop codon variations; for example, the codons that normally code for arginine, AGA and AGG, code for stops in vertebrate mitochondria (Ivanov *et al*, 2001) while the stop codon UGA, has been replaced by tryptophan in Mycoplasma species (Chaudhuri and Yeates, 2005), and UGA by selenocysteine and UAG by pyrrolysine in Archaea (recently found rare amino acids) in some archaebacter (Cobucci-Ponzano *et al*, 2005). In physics, the 64 fundamental algebraic units are made up of 12 (= 3 × 4) sets of 5 generators for the entire group (each with an in-built 3-D property) and the 4 units of ordinary complex algebra ($\pm 1, \pm i$), with no dimensionality.

The role of the Fibonacci numbers

Fibonacci numbers are an integer sequence in which each new term is defined simply as the sum of the two previous integers: 0 1 1 2 3 5 8 13 21 34 ...; the ratios of successive integers: 3/2, 5/3, 8/5, 13/8, 21/13, 34/21 ... converge towards the Golden Section value, $\Phi = (1 + \sqrt{5}) / 2 = 1.618$ ... , while the inverse ratios tend towards $\Phi' = (1 - \sqrt{5}) / 2 = -0.618$ ..., so that $\Phi + \Phi' = 1$ and $\Phi\Phi' = -1$, where the two numbers are the roots of the equation $x^2 - x - 1 = 0$.). The Fibonacci series is recognizable in quasicrystals with 5-fold symmetry, where adjacent short (S) and long (L) 'tiles' may follow the sequence S L LS LSL LSLLS LSLLSLSL LSLLSLSLLSLLS, with the L / S radio tending towards the 'Golden Section'. The crystals typically show long-range order but no periodicity. Of course, the Golden Ratio is not uniquely determined by the Fibonacci series, but by any series constructed according to the relationship $f(n + 1) = f(n) + f(n - 1)$, where $f(n)$ is an integer for all n. In the work described here, the Golden Ratio is the necessary result of the fundamental nature of a 5-fold broken symmetry, and the Fibonacci series becomes one means of expressing it mathematically.

A classic case of the Fibonacci sequence is the growth of a spiral shell ending in a point-singularity. D'Arcy Thompson (1917) observed that a shell grows in size without changing its shape, leading to growth in a logarithmic or equiangular spiral, with radius $r = a\ exp\ (b\theta)$ from the point source. This shape can be observed, typically, when the long side of a Golden Triangle, an isosceles triangle from within a star pentagon, with apex angle = $180^{o} / 5 = 36^{o}$ (the Golden attractor), is continually used as a base for a new Golden Triangle. Here, the successive bases have the ratios 1Φ, 1Φ + 1, 2Φ + 1, 3Φ + 2, 5Φ + 3, 8Φ + 5 … . Illert, whose two 3-D systems are constituted from a fixed reference system (ordinary 3-D space) and a set of moving 3-D coordinates representing growth, has used the analogy of the spiral watchspring acting as a classical harmonic oscillator, obeying Hooke's Law ($F = kx$). Significantly, quasicrystals with icosahedral symmetry show no periodicity in ordinary 3-D space, but become periodic, and lose their Fibonacci character when structured within a 6-D cube, constructed from a parallel real 3-D space, and a perpendicular imaginary 3-D space. It may be that the significant distinction between organic and inorganic forms, in this context, is brought about by the extra half-3-dimensionality representing variation with time (and leading to helicity). In physics, this is the role of the weak interaction, which ordered crystalline structure is designed to suppress (Illert, 1992, Illert and Santilli, 1996).

In fact, in the fundamental rewrite system with its 5-component nilpotent operator, we may see, at once, all the fundamental units of *natural process* that apply to biology as well as physics. Defining the operator simultaneously leads to the creation of point singularity and discreteness; compactification (from 8 units to 5) and chirality (as a result of the loss of some sign degrees of freedom in the compactification); symmetry-breaking (between the 5 units) and spontaneous symmetry-breaking (because of the chirality); (double) helicity and angular momentum (with its double 3-D nature); irreversibility (because of the chirality of the time and energy operators); 5-fold symmetry (cubical → spiral) and the Fibonacci sequence; and a harmonic oscillator-based tendency to aggregation and complexity (because of the pseudoscalar nature of the time / energy term needed for nilpotency).

In physics, the act of 'creation' is that of the fermionic state (and requires the simultaneous creation of relativity and quantum mechanics); but the structure can repeat at higher levels because it is a recurring pattern. An example from chemistry is the phenomenon of spontaneous chiral symmetry breaking in chiral autocatalytic systems in which continuous stirring of a solution of a salt such as $NaClO_3$ (a nonequilibirum system in thermodynamic terms) can cause it to go through a symmetry-breaking transition in which it crystallizes almost entirely into either laevo- or dextrorotatory forms (Kondepudi and Asakura, 2001). The significance of the creation of a dominant angular momentum state (through stirring) is apparent.

Pentagonal symmetry within DNA

In our previous work (Rowlands and Hill, 2006), we have discussed the pentagonal symmetry within DNA, especially in connection with the way in which 5 tetrahedrons can break symmetry by making one pentagonal disc.

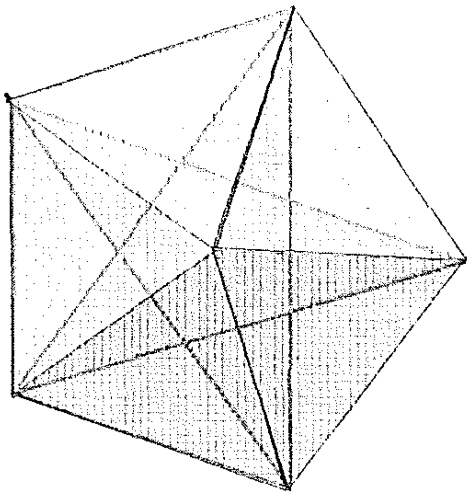
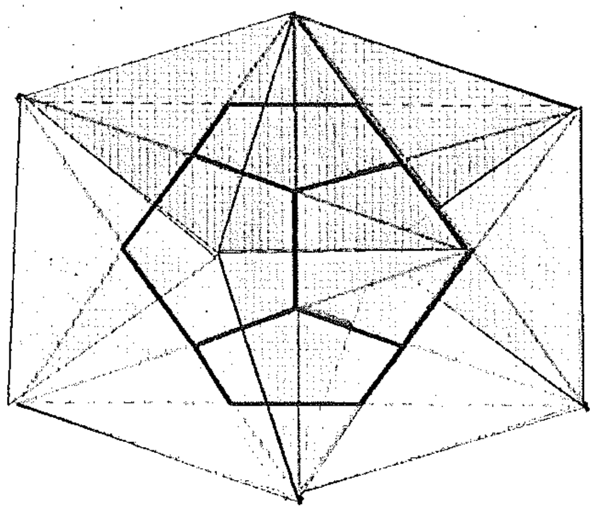


Fig 6. Breaking Symmetry: 5 Tetrahedrons Make One Pentagonal Disk

The structure revealed in Figure 20 can also be generated by a projection onto 3-space of the self-dual pentatope, which is the 4-dimensional analogue of the tetrahedron. The pentatope, with 5 vertices, 10 edges and 10 faces, is the simplest regular figure in 4D (http://mathworld.wolfram.com/Pentatope.html). The connection may well be an expression of the fact that space, though fundamentally 3-dimensional, is part of a larger structure of nested 3-dimensionalities, which, in the universal rewrite system, is created after the scalar mass, pseudoscalar time, and 3D conserved quaternion charge; and some of the profoundest insights into physical space's 3D structure may come from the properties of the larger structure within which it is embedded. In this sense, physical space can only exist as the 3D spatial projection of the higher dimensionality incorporated in the nilpotent structure, but, in view of the fact that the 'fifth' dimensional of the nilpotent structure (the proper time) is an invariant, and therefore essentially redundant, it is significant that the regular Platonic-type figures reach their maximal extent (6) in 4D; at higher dimensionalities they reduce to 3. The manifestation of the extra dimensionality in biology may then be related to a chaotic variation with respect to time.

If we now overlay two pentagonal disks a 3D representation of the icosahedron can again be visualised in 2D as shown in Fig. 21 (where we have highlighted the reciprocal internal dodecahedron).

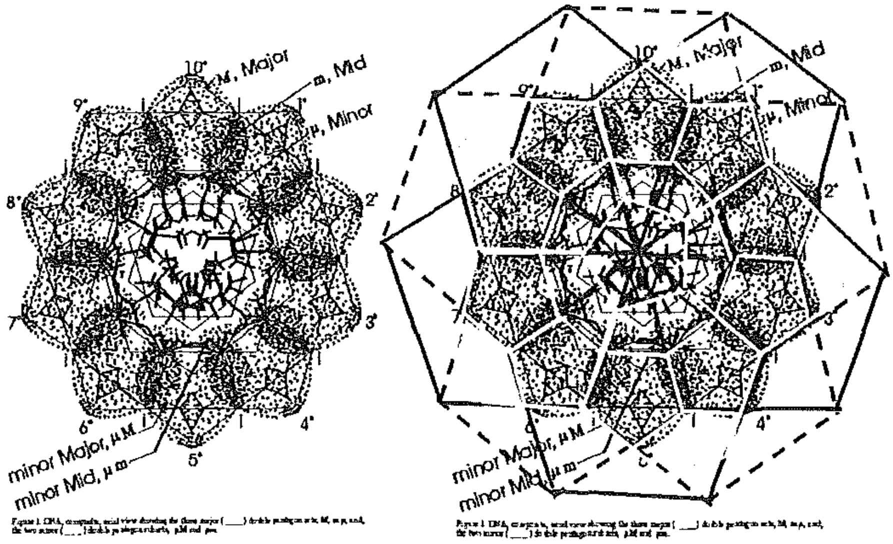


2 Pentagonal discs overlaid to produce an icosahedron (2D) plus internal dodecahedron.

Fig 7. Icosahedron Plus Reciprocal Dodecahedron Inside

This pentagonal scheme fits well with both the Curtis and the Watson and Crick models of DNA. In the Watson-Crick model it is the pentose sugar rings of the dNTPs that now relate to the pentagonal faces of the internal dodecahedra.

A computer generated scattergraph of DNA generated by Robert Langridge was analysed by Reginald Brookes (1998) who suggested a different pentagonal geometry composed of concentric double pentagons as shown in Fig. 29 but our proposed geometry also fits well upon this scattergraph (Fig. 29). Again, the scattergraph shows a remarkable similarity to a 3D projection of a regular solid in 4D. This time it is the 120-cell or hyperdodecahedron (http://mathworld.wolfram.com/120-Cell.html), and again this seems to make sense with relation to the higher dimensionality when the rewrite structure's most fundamental unit – incorporating a time-varying, even chaotic, sequence, which is not present in more ordered structures, such as those of inorganic crystalline materials – is projected onto a 3-dimensionality. Significantly, the dodecahedron and icosahedron, with their pentagonal symmetries, are represented only in 3D and 4D – other dimensionalities only have cubes, tetrahedra and octahedra as Platonic solids.



A. R.Brookes pentagonal geometry overlaid upon R.Langridge's scattergraph of DNA

B. New pentagonal geometry overlaid upon R.Langridge's scattergraph of DNA

Fig 8. Computer Generated Scattergraph of DNA: Pentagonal Geometry

Two further aspects of the double helix may be mentioned here, in connection with its appearance in both biological and physical contexts. One is the first observation of the structure on the large scale (80 light years = 7.6×10^{17} m) in a 'DNA-type' nebula about 300 light-years from the centre of the Milky Way, and indicative of 'a high degree of order' (Morris *et al*, 2006.) – an observation which may be significant, from the point of view of the universal applicability of the nilpotent rewrite system. Here, the requirement of a strong magnetic field acting on the rotating body suggests that the best analogy is with a fermion acting in a boson-like manner in a magnetic field.

The other is the phenomenon of spectrin repeats, found in several proteins involved in cytoskeletal structure, for example, spectrin, alpha-actinin and dystrophin. These repeats create a triple helical bundle, which we may liken to the DNA / RNA connection or to vacuum process of fermion becoming virtual boson and then virtual fermion again; or the real process by which all fermionic states absorb and emit gauge bosons (equivalent to a combination of fermion and antifermion) in all discrete interactions.

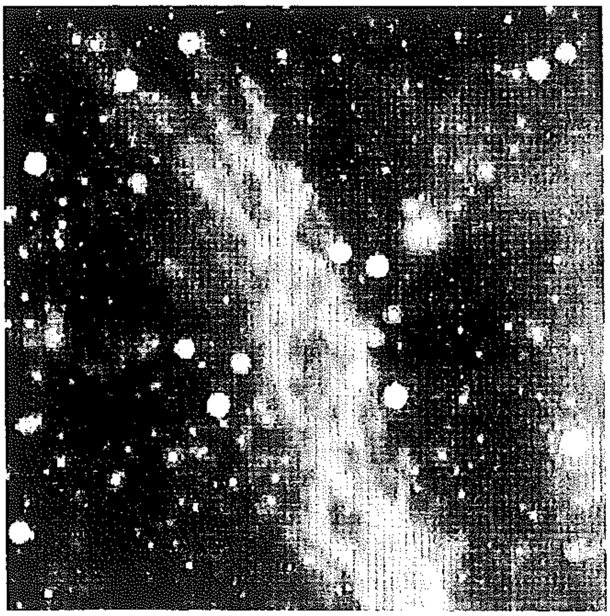


Fig 9. Double Helix-Shaped Nebula at The Center of Milky Way

Credit: NASA/JPL-Caltech/UCLA

The cube and the harmonic oscillator

As we have suggested, the packing of pentagonal disks may have relevance to the spiral which is so important in both DNA and the spin of the fermion with its vacuum. However, two disks can also be viewed in 2D as a cube. The switching between the two perspectives of the cube is interesting in that it can also be considered as a harmonic oscillator. One can visualize the cube spinning and the fermion and vacuum switching places (*zitterbewegung*) when the front flips to the back and the back face then holds precedence. The cycle can then complete when the front face flips forward again. A second element this relates to is the Klein-4 group and dual group (which can be seen as relating to particle and vacuum), where the complementary colours say red and cyan, used for the group / dual group, point to opposite corners of the cube and could be considered as switching. Rotation of the cube would, of course, produce the same effect.

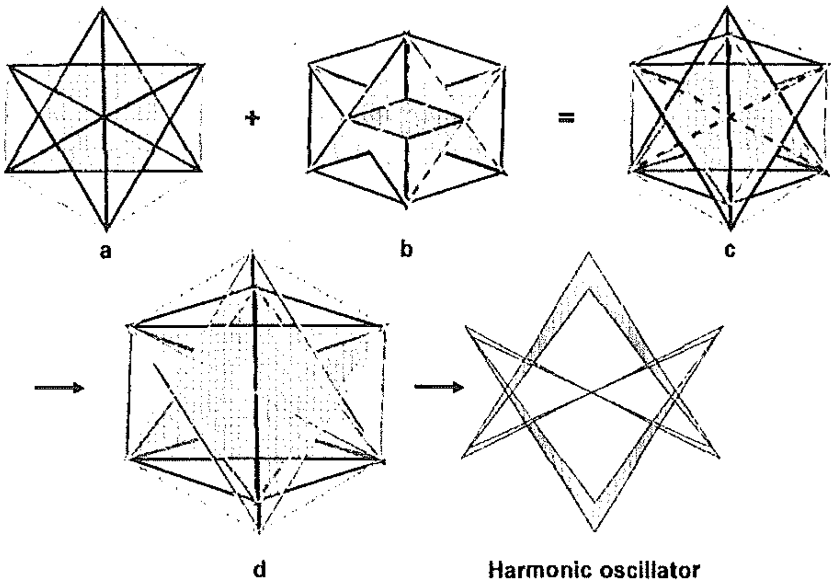


Fig 10. The Spinning Cube as The Harmonic Oscillator

Extending the dimensions, a 4D cube with an internal star tetrahedron in rotation can be seen creating and annihilating two discs of five tetrahedra as if it were going through a phase-switching transition, exactly as in *zitterbewegung*, the fourth dimension being identified with $\pm t$, $\pm E$ and $\pm w$, and being the 'vacuum space' in which the real 3D structures are created. The dodecahedron in 3D is composed of 5 cubes. If we take a 3D cube and consider each of the four axes through the corners, with a cube rotating upon each, there is a position that is created from the 5 cubes that constructs the dodecahedron. As we know we can construct the star tetrahedron within each cube and if we extend this for the constructed dodocahedron, we will have a total of 10 interlocking tetrahedra. Interestingly, these interlocking tetrahedra can be seen to produce an appearance of two vortices, one going one way and the other the other way in a manner analogous to a double helix.

A particular example of a cubical symmetry with a nilpotent physical, but not yet a biological, application (though it is likely that one is ready to be found), is provided by the Rubik cube structure, which is a regular cube, divided into 27 smaller cubes or cubelets, with each face of the large cube divided into 9 cubelet faces. The 54 cubelet faces are divided equally between 6 colours (or 3 colours and 3 anticolours), so that there are 9 of each colour, and each of the cubelets has the same relative ordering of colours on its faces. The structure, however, can be rotated internally, so that any colour shows on any of the cubelet faces of the larger cube. Corner cubelets have been viewed as giving the correct $\pm$ 1/3, $\pm$ 2/3 twists corresponding to the electric charge values observed for the quark components of mesons and baryons. The 3 colours (primary) and 3 anticolours (secondary), however, could also be seen as representing the three directions of the momentum operator in the nilpotent structure when the cube is in its most organized state $(ikE + i\mathbf{i}p_x + jm)$ $(ikE + i\mathbf{j}p_y + jm)$ $(ikE + i\mathbf{k}p_z + jm)$. The six faces then represent the six possible phases, i.e. when the whole momentum $\mathbf{p} = + \mathbf{i}p_x, - \mathbf{i}p_x, + \mathbf{j}p_y, - \mathbf{j}p_y, + \mathbf{k}p_z, - \mathbf{k}p_z$. Anything else then represents a degree of mixing between two or more phases. When each face has 3 colours and 3 anticolours, the mixing is perfect. Of course, the quantum mechanical picture of the proton requires perfect mixing between the 6 phases ($SU(3)$ symmetry). (This would require a 6 × 6 × 6 cube with faces containing equal numbers of each coloured cubelet.) Another way of looking at it is to rotate the perfectly organized cube (each face one colour), allowing each face to be seen by the viewer for the same amount of time. The 8 agents that produce all possible colour changes (combinations of colour and anticolour) represent the 8 gluons. There are only 8, not 9, because only 2 combinations of red / antired, green / antigreen, blue / antiblue are considered to be independent. (The standard 3 × 3 × 3 becomes relevant if we consider colour / anticolour as occupying the same cube and we consider 3 colours alone. A 4 × 4 × 4 cube would give us the convenient total of 64 cubelets.)

The rewrite process as Nature's code

If the existence of four bases reflects the basic structure of a universal rewrite system, then we would expect to find a similar set of dualities to those observed for space, time, mass and charge, and we would expect to find some natural process in which they could emerge in a relatively uncomplicated way. If the rewrite process gives us Nature's code, it is not obvious that, apart from existing at the most fundamental level, it will also reappear in similar form in particular structures higher up the natural hierarchy. Such reproductions of features of small-scale systems in larger ones, however, certainly exist in nature, and generally reflect some concept of *coherence* between the small-scale elements, organized by some powerful driving mechanism. The double helical structure and five-fold symmetry of DNA would suggest an application of 'Nature's code' by such a mechanism even if there were no other indications in the number and characteristics of the base elements, and the fact that the component structures are based on the relatively basic chemistry of only a few light elements suggests that the organizing principle is more significant to the process than the specific characteristics of the chemistry.

The nitrogenous bases found in nucleotides are, in fact, relatively simple structures, with a common feature: a heterocyclic ring derived from the parent compounds pyrimidine and purine. The ring structure is composed of both nitrogen (position 1 and 3) and carbon (positions 2, 4, 5 and 6). The pyrimidines T and C, and the purines A and G, are relatively minor variations on this pattern; and purine can be considered as a derivative of pyrimidine as it consists of a pyrimidine ring with an imidazole ring (a five-component ring, again with two nitrogen atoms) fused together. Here, we have one duality. Significantly, also, each of the purines has a pyrimidine *partner* (which makes it 'dual', in another sense), and the bonding atoms of these complementary atoms will line up oppositely on a single strand of DNA. So, we have O-N[-OH] and N-NH[-NH] bonds linking A-T reversing the order for the same bonds used in C-G. If we want a closer analogy with the physical case, we can say that the more structured A and G might be considered to correspond to the more structured physical parameters space and charge, which each have a specific partner in time and mass. Also, the A-T partnership might be considered the 'driver' or source of variation in the same way as the nonconserved space-time partnership is in physics. Thymine certainly initiates change with its replacement by uracil in RNA, while adenine triphosphate (ATP) is the main source of energy transfer in living systems. This suggests the approximate correspondence: A / space; T / time; C / mass; G / charge.

The question remains how such bases could emerge from some natural process which is not purely random (in the sense that, though the chemical structures may emerge randomly, they will be quickly 'selected' for their relevance to the overall scheme). Very possibly, special conditions (for example, high temperature, electricity, magnetism, liquid or gaseous environment) will be required to generate the structures in the right proportions – and so the generation of 'life' is not an *inevitable* result of chemical chaos; it is an inevitable consequence only of the conditions being available for the universal rewrite mechanism to operate on a scale higher than the most fundamental as a result of the creation of some necessary condition of coherence. Given the right conditions, however, positive feedback mechanisms might be expected to take place to enhance a process that would need to evolve only once. Fossil evidence from prokaryotes, which resemble present-day bacteria but may be as much as 3.3 billion years old, suggest that the genetic code developed at a single time at a relatively early date in the Earth's history. There was no multiple evolution.

Proteins are, of course, the essential basis of life, and include a number of enzymes which play a significant role in the transfer of genetic information; but proteins are made of twenty amino acids, selected only from those that follow the coded sequence presented by the four bases in DNA, and there is no other driving mechanism to link them in the seemingly random polypeptide chains, with their multiplicity of folded shapes. Though amino acids are relatively simple chemicals, which are easy to reproduce from basic elements under extreme conditions, there is no mechanism for linking them in protein structures which would then develop a coding system to reproduce them. It is virtually impossible to believe that the more complicated system logically preceded the simpler one, though one could see DNA, RNA and protein structures evolving together through a process in which the evolution of one aided and was aided by the evolution of the others. There are some arguments for believing that RNA preceded DNA, but DNA, with its double helix, is much the more stable structure, and it is certainly difficult to believe that the bases A / T and C / G should form their pairings by accident, rather than as part of the molecule's original design. Although tRNAs also form double stranded conformations they involve the inclusion of over 30 rare bases, with up to 10% of the total number of bases. It is apparent from this, that these are more complicated molecules than DNA. An argument for the temporal precedence of RNA would have to conclude that the extra bases were discarded when the more 'perfect' base pairings of A, T, C and G made the creation of DNA possible.

The structure of the bases as variations on a single basic design almost suggests a modification process to produce the appropriate levels of 'duality', and the creation of DNA through a combination of single types of the elements with each of the valences 1 to 5 (namely, H, O, N, C, P, three of which are gaseous in the free state) suggests something like a minimalist approach. Carbon is, of course, the only basis for creating the molecules that we call 'organic', because its valence 4 gives it the perfect bonding for coherently structured large molecules; and the hexagonal benzene ring is one of the most stable (though dynamic), simple organic structures, easily formed. (The 'dynamic' 2-D hexagonal (or flattened cube) symmetry of benzene reflects that of the more dynamically interesting parent graphite as opposed to the more perfectly ordered, and therefore more inert, 3-D tetrahedral structure of diamond.) The most efficient way of driving an evolving process involving small atoms or molecules to break the sterile uniformity produced by the benzene ring itself might be through interaction with a gaseous atmosphere or gas molecules dissolved in liquids (a kind of biochemical equivalent of the physical 'vacuum'); and, from the point of view of stability, bonding, and the kind of 'closure' produced by the base pairing, the two valence 3 nitrogen substitutions found on the rings in each of the bases would seem to provide definite advantages.

The significant role of ATP then suggests the potential importance of the combination of bases and triphosphate as a precursor of DNA. The pentavalence of the phosphorus atoms provides significant opportunities for bigger linkages. Here, we have to imagine the driving process as creating helical structure via a harmonic oscillator mechanism. To create an extended helical structure we need pentagonal symmetry. To create pentagonal symmetry, we need a combination of the two hexagonal structures produced by the A-T and C-G linkages (the hexagonal structure being a double tetrahedron in 2-D), and a pentose sugar (another small 5 ring structure with a gaseous element substitution) interposing between the base and the phosphate. Significantly, proteins, deriving from DNA coding, show elements of helical structure, but much more disrupted than that of DNA itself.

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Nilpotent Operators and Particle States

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Abstract. The nilpotent formulation of quantum mechanics is not only a powerfully streamlined method of calculation, with intrinsic supersymmetry and automatic removal of divergences, it also incorporates the electric, strong and weak interactions as direct consequences of the structure of the fermionic operator alone. They do not require any additional 'physical' assumption for their sources and structures, and there are strong indications also that the same is true for fundamental particle states.

An algebraic structure of quantum mechanics

The nilpotent formulation of quantum mechanics is the most streamlined version available (Rowlands, 2005, 2006). It is fully relativistic; it uses only (differential) operators; it doesn't need mysterious objects like wavefunctions and spinors; and all terms have the same format, based on the operators E, $\mathbf{p}$, m. These terms are compartmentalised using the quaternions $\boldsymbol{k}$, $\boldsymbol{i}$, $\boldsymbol{j}$, in a similar way to real and imaginary parts. The operators are also full quantum field operators – we don't need second quantization. They are, in addition, intrinsically supersymmetric. Interactions and particle states are immediately explained, while calculations are relatively easy, easier than nonrelativistic ones. Renormalization of free particles is not needed, while significant divergences simply disappear.

The formalism can be derived from the concept of zero using a universal rewrite system, which seems to have much more general applications. This derivation automatically includes quantization and special relativity as part of the abstract formal structure – it doesn't need to assume them. The algebraic structure can be shown to be derived from the algebras of the four fundamental parameters, space, time, mass and charge, and their mathematically symmetric relationships. However, though the derivation from the universal rewrite system is the only truly fundamental one, there are less profound ones from more conventional theories, e.g. by converting the gamma matrices of the conventional Dirac equation into algebraic operators (multivariate 4-vector quaternions or complex double quaternions). The most direct is from the classical relativistic equation:

$$E^2 = p^2 + m^2$$

but it should be remembered that, fundamentally, it is the classical equation that is derived, not the quantum mechanics.

So, let us take the equation in the form:

$$E^2 - p^2 - m^2 = 0$$

and factorize using noncommuting algebraic operators (multivariate 4-vector quaternions or complex double quaternions):

$$(\pm\, ikE \pm \mathbf{ip} + jm)\,(\pm\, ikE \pm \mathbf{ip} + jm) = 0.$$

To make quantize this canonically, we simply make E and $\mathbf{p}$ operators, say $i\,\partial / \partial t$ and $-i\nabla$, rather than numerical variables. So, we can, for example, make the first bracket an operator, operating on a phase term of some kind, say $e^{-i(Et\, -\, \mathbf{p.r})}$ for a free particle plane wave, and the second bracket the amplitude which results from this operation.

$$(\pm\, ikE \pm \mathbf{ip} + jm)\,(\pm\, ikE \pm \mathbf{ip} + jm) =$$
$$(\pm\, k\partial / \partial t \pm i\mathbf{i}\nabla + jm)\,(\pm\, ikE \pm \mathbf{ip} + jm)\, e^{-i(Et\, -\, \mathbf{p.r})} = 0$$

This now becomes equivalent to the Dirac equation for a free particle

The algebra we are using is a tensor product between two quaternion systems, one of which is complexified (or converted to 4-multivariate vectors). These two algebras remain commutative with respect to each other. The eight fundamental units required are:

i j k	quaternion units	**i j k**	multivariate vector units
1	scalar	*i*	pseudoscalar

The combined algebra is intriguingly close to twistor algebra, a complex 4-D space-time, which is now used in QCD, in that there are four units with norm 1 and four with norm −1, though, in this case, each of these sets of 4 is broken up into a 3 + 1 combination. The multiplication rules of the two subalgebras are the standard ones: $i^2 = j^2 = k^2 = ijk = -1$, $ij = -ji = k$, etc., for the quaternions, and $\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = -i\mathbf{ijk} = 1$, $\mathbf{ij} = -\mathbf{ji} = i\mathbf{k}$, etc., for the multivariate 4-vectors.

Multivariate vectors have a full product, combining the scalar and vector products: $\mathbf{ab} = \mathbf{a.b} + i\,\mathbf{a} \times \mathbf{b}$. So $\mathbf{aa} = a^2$, and, for a multivariate $\mathbf{p}$,

$$\mathbf{pp} = (\boldsymbol{\sigma}.\mathbf{p})\,(\boldsymbol{\sigma}.\mathbf{p}) = pp = p^2$$

This means that we can also use **σ.p** for **p** (or **σ.∇** for **∇**) in the quantum equation, where **σ.p** is helicity, and **s** is a pseudovector of magnitude –1. Also, Hestenes (1966) showed that a multivariate **p** or **∇** could accommodate spin, even in the Schrödinger equation, because of the equivalence of the i **a** × **b** term in **∇∇**.

$$\nabla\nabla\,\psi = \nabla.\nabla\,\psi + i\,\nabla \times \nabla\psi\,.$$

We will only need to explicitly incorporate spin where the vectors are not multivariate (e.g. when we are using polar coordinates).

The nilpotent operator

Now, if we take

$$(\pm\, k\partial / \partial t \pm \mathbf{i}i\nabla + \mathbf{j}m)\,(\pm\, ikE \pm \mathbf{i}\mathbf{p} + \mathbf{j}m)\, e^{-i(Et - \mathbf{p}.\mathbf{r})} = 0$$

as the Dirac equation for a free particle, we can interpret the four possible sign variations of $\partial / \partial t$ and **∇** in the first bracket as making up the four components of a row vector; and the four possible sign variations of E and **p** in the second bracket as making up the four components of a column vector. In principle, we have made both the operator and the amplitude into *essentially identical* 4-component spinors, with the operator applied to a single phase term. And we can easily identify the meaning of the sign variations:

fermion / antifermion	$\pm E$
spin up / down	$\pm$ **p**

These options apply to E and **p** as either operators or amplitude eigenvalues.

Relativistic quantum mechanics is now hugely streamlined, because it now depends only on a single term:

$$(\pm\, ikE \pm \mathbf{i}\mathbf{p} + \mathbf{j}m)$$

taken either as operator or as amplitude. E and **p** are generic terms, identified by their quaternion labels, which can be covariant derivatives or include field terms or potentials of any kind. The expression

$$(\pm\, ikE \pm \mathbf{i}\mathbf{p} + \mathbf{j}m)$$

which is really a row or column vector, containing the four components

$$(\boldsymbol{k}E + \boldsymbol{i}\mathbf{i}\mathbf{p} + \boldsymbol{i}\boldsymbol{j}m)$$
$$(\boldsymbol{k}E - \boldsymbol{i}\mathbf{i}\mathbf{p} + \boldsymbol{i}\boldsymbol{j}m)$$
$$(-\boldsymbol{k}E + \boldsymbol{i}\mathbf{i}\mathbf{p} + \boldsymbol{i}\boldsymbol{j}m)$$
$$(-\boldsymbol{k}E - \boldsymbol{i}\mathbf{i}\mathbf{p} + \boldsymbol{i}\boldsymbol{j}m)$$

now contains all that can be known about any fermion state.

If we take E and $\mathbf{p}$ as generic operators, then the only way they can operate is by finding a phase term, such that the resulting amplitude is nilpotent, or squares to zero, i.e.:

$$(\pm\, \boldsymbol{ik}E \pm \mathbf{i}\mathbf{p} + \boldsymbol{j}m)\,(\pm\, \boldsymbol{ik}E \pm \mathbf{i}\mathbf{p} + \boldsymbol{j}m) = 0.$$

So, specifying the operator means that we also specify the phase and the amplitude, and the 'wavefunction' becomes redundant. But the spinor structure is also redundant. We don't need to specify

$$(\pm\, \boldsymbol{ik}E \pm \mathbf{i}\mathbf{p} + \boldsymbol{j}m)$$

as a row or column vector. Once we specify the first of the four terms, the others follow *by automatic sign variation.* So, we only need

$$(\boldsymbol{ik}E + \mathbf{i}\mathbf{p} + \boldsymbol{j}m)$$

for complete specification of the state. To take a simple example, specifying a state as

$$(\pm\, \boldsymbol{k}\partial / \partial t \pm \boldsymbol{i}\mathbf{i}\nabla + \boldsymbol{j}m)$$

(which is a free particle) means that we have automatically created the four linked equations:

$$(\boldsymbol{k}\partial / \partial t + \boldsymbol{i}\mathbf{i}\nabla + \boldsymbol{j}m)\,(\boldsymbol{ik}E + \mathbf{i}\mathbf{p} + \boldsymbol{j}m)\, e^{-i(Et - \mathbf{p}.\mathbf{r})} = 0$$
$$(\boldsymbol{k}\partial / \partial t - \boldsymbol{i}\mathbf{i}\nabla + \boldsymbol{j}m)\,(\boldsymbol{ik}E - \mathbf{i}\mathbf{p} + \boldsymbol{j}m)\, e^{-i(Et - \mathbf{p}.\mathbf{r})} = 0$$
$$(-\boldsymbol{k}\partial / \partial t + \boldsymbol{i}\mathbf{i}\nabla + \boldsymbol{j}m)\,(-\boldsymbol{ik}E + \mathbf{i}\mathbf{p} + \boldsymbol{j}m)\, e^{-i(Et - \mathbf{p}.\mathbf{r})} = 0$$
$$(-\boldsymbol{k}\partial / \partial t - \boldsymbol{i}\mathbf{i}\nabla + \boldsymbol{j}m)\,(-\boldsymbol{ik}E - \mathbf{i}\mathbf{p} + \boldsymbol{j}m)\, e^{-i(Et - \mathbf{p}.\mathbf{r})} = 0.$$

By comparison with the conventional Dirac equation, we have reduced the number of separate terms required by 98 %, because we have reduced a 4×4 matrix operator multiplied by a 4-component spinor wavefunction to an operator with only a single independent term. We have effectively shown that the matrix, the wavefunction, the spinor and the equation itself, are redundant. We need only a single operator.

But, what is the physical meaning of defining the fermion as an operator? What is it operating on? The indications are that it is *vacuum*, meaning the rest of the universe. The nilpotency codified in

$$(\pm ikE \pm \mathbf{ip} + jm)\,(\pm ikE \pm \mathbf{ip} + jm) = 0$$

has built-in Pauli exclusion. No two fermions can have the same state vector. But it also signifies that nilpotency is an expression of the totality zero that is fundamental to the universal rewrite system. Fermion and the rest of the universe (0 – fermion) together make a zero totality, with a zero totality state vector:

$$(\pm ikE \pm \mathbf{ip} + jm)\,(-(\pm ikE \pm \mathbf{ip} + jm)) = 0.$$

The fact that Pauli exclusion is not unique to free fermions brings us to the most revolutionary step within the nilpotent theory. We assume that *all fermionic amplitudes in all states* are nilpotent. We postulate that the most general form for a state vector is nilpotent, and that we should seek specifically nilpotent solutions for all problems.

For a 'free' fermion, the phase ($exp\ (-i(Et - \mathbf{p.r}))$ provides the complete range of space and time translations and rotations, but if the E and $\mathbf{p}$ terms represent covariant derivatives or incorporate field terms, then the phase term is determined by whatever expression is needed to make the amplitude nilpotent.

The operator which defines each fermion is thus a creation operator acting on vacuum (= the rest of the universe). In fact there are four creation operators:

$(ikE + \mathbf{ip} + jm)$	fermion spin up
$(ikE - \mathbf{ip} + jm)$	fermion spin down
$(-ikE + \mathbf{ip} + jm)$	antifermion spin down
$(-ikE - \mathbf{ip} + jm)$	antifermion spin up

The nature of the state is determined by which of these is the lead term. The others can be regarded as vacuum states representing ones into which it could transform. So, for example, a real antifermion spin down would be symbolized by a row vector with the following components:

$(-ikE + \mathbf{ip} + jm)$	antifermion spin down
$(-ikE - \mathbf{ip} + jm)$	antifermion spin up
$(ikE + \mathbf{ip} + jm)$	fermion spin up
$(ikE - \mathbf{ip} + jm)$	fermion spin down

Because of the way they are defined, nilpotent operators are specified with respect to the entire quantum field. Formal second quantization is unnecessary. We can consider the nilpotency as defining the interaction between the localized fermionic state and the unlocalized vacuum, with which it is uniquely self-dual. The phase is the mechanism through which this is accomplished. So, defining a fermion implies simultaneous definition of vacuum as 'the rest of the universe' with which it interacts. The nilpotent structure then provides energy-momentum conservation without requiring the system to be closed. The nilpotent structure is thus naturally *thermodynamic*, and provides a mathematical route to defining nonequilibrium thermodynamics. We can now see that the expression

$$(ikE + \boldsymbol{i}\mathbf{p} + jm)\,(ikE + \boldsymbol{i}\mathbf{p} + jm) \rightarrow 0$$

has at least *five* independent meanings.

classical	special relativity
operator × operator	Klein-Gordon equation
operator × wavefunction	Dirac equation
wavefunction × wavefunction	Pauli exclusion
fermion × vacuum	thermodynamics

We thus have an operator $(ikE + \boldsymbol{i}\mathbf{p} + jm)$ that potentially incorporates all the physical information available to the fundamental physical state. It is easy to show that we can use our operator to do conventional quantum mechanics, e.g. by defining a probability density by multiplying by its complex quaternion conjugate $(ikE - \boldsymbol{i}\mathbf{p} - jm)$. Fermionic spin is a straightforward derivation from the $\mathbf{p}$ component of the nilpotent structure. If we mathematically define a quantity $\boldsymbol{\sigma} = -\mathbf{1}$, then

$$\begin{aligned}[\boldsymbol{\sigma}, H] &= [-\mathbf{1}, \boldsymbol{i}\,(\mathbf{i}p_1 + \mathbf{j}p_2 + \mathbf{k}p_3) + ijm] = [-\mathbf{1}, \boldsymbol{i}\,(\mathbf{i}p_1 + \mathbf{j}p_2 + \mathbf{k}p_3)] \\ &= -2\boldsymbol{i}\,(\mathbf{ij}p_2 + \mathbf{ik}p_3 + \mathbf{ji}p_1 + \mathbf{jk}p_3 + \mathbf{ki}p_1 + \mathbf{kj}p_2) \\ &= -2i\boldsymbol{i}\,(\mathbf{k}(p_2 - p_1) + \mathbf{j}(p_1 - p_3) + \mathbf{i}(p_3 - p_2)) \\ &= -2i\boldsymbol{i}\mathbf{1} \times \mathbf{p}\end{aligned}$$

If $\mathbf{L}$ is the orbital angular momentum $\mathbf{r} \times \mathbf{p}$, then

$$\begin{aligned}[\mathbf{L}, \mathcal{H}] &= [\mathbf{r} \times \mathbf{p}, \boldsymbol{i}\,(\mathbf{i}p_1 + \mathbf{j}p_2 + \mathbf{k}p_3) + ikm] \\ &= [\mathbf{r} \times \mathbf{p}, \boldsymbol{i}\,(\mathbf{i}p_1 + \mathbf{j}p_2 + \mathbf{k}p_3)] \\ &= \boldsymbol{i}\,[\mathbf{r}, (\mathbf{i}p_1 + \mathbf{j}p_2 + \mathbf{k}p_3)] \times \mathbf{p}\end{aligned}$$

But $$[\mathbf{r}, (\mathbf{i}p_1 + \mathbf{j}p_2 + \mathbf{k}p_3)]\,\psi = i\mathbf{1}\,\psi.$$

Hence $$[\mathbf{L}, \mathcal{H}] = \boldsymbol{i}i\, \mathbf{1} \times \mathbf{p},$$

and $\mathbf{L} + \boldsymbol{\sigma} / 2$ is a constant of the motion, because

$$[\mathbf{L} + \boldsymbol{\sigma} / 2, \mathcal{H}] = 0.$$

Helicity ($\boldsymbol{\sigma}.\mathbf{p}$) is also a constant of the motion because

$$[\boldsymbol{\sigma}.\mathbf{p}, \mathcal{H}] = [-p, \boldsymbol{i}\,(\mathbf{i}p_1 + \mathbf{j}p_2 + \mathbf{k}p_3) + ijm] = 0$$

For a hypothetical fermion / antifermion with zero mass,

$$(kE + ii\, \boldsymbol{\sigma}.\mathbf{p} + ijm) \rightarrow (kE - iip)$$
$$(-kE + ii\, \boldsymbol{\sigma}.\mathbf{p} + ijm) \rightarrow (-kE - iip)$$

Each of these is associated with a single sign of helicity, $(kE + iip)$ and $(-kE + iip)$ being excluded, if we choose the same sign conventions for $\mathbf{p}$. Numerically, $\pm E = p$, so we can express the allowed states as

$$\pm E(k - ii)$$

Multiplication from the left by the projection operator

$$(1 - ij) / 2 \equiv (1 - \gamma^5) / 2$$

leaves the allowed states unchanged while zeroing the excluded ones.

We may note that nilpotent wavefunctions or amplitudes are automatically antisymmetric:

$$(\pm ikE_1 \pm \boldsymbol{i}\mathbf{p}_1 + jm_1)\,(\pm ikE_2 \pm \boldsymbol{i}\mathbf{p}_2 + jm_2)$$
$$-\,(\pm ikE_2 \pm \boldsymbol{i}\mathbf{p}_2 + jm_2)\,(\pm ikE_1 \pm \boldsymbol{i}\mathbf{p}_1 + jm_1)$$
$$= 4\mathbf{p}_1\mathbf{p}_2 - 4\mathbf{p}_2\mathbf{p}_1 = 8\, i\, \mathbf{p}_1 \times \mathbf{p}_2.$$

This is a particularly striking result, as it implies that all fermionic states have a *spin phase* which is unique.

Vacuum

The three quaternion operators ***i***, ***j***, ***k*** are not just passive mathematical objects in the nilpotent formalism. They have multiple meanings, acting almost as a kind of hypertext: (1) the primary meaning is as charge generators; (2) premultiplying the nilpotent gives vacuum; (3) pre- and postmultiplying the nilpotent transforms it via P, C or T.

If we take $(ikE + i\mathbf{p} + jm)$ and postmultiply it by $k(ikE + i\mathbf{p} + jm)$, the result is $(ikE + i\mathbf{p} + jm)$, multiplied by a scalar, which can be normalized away. This can be done an indefinite number of times. That is, $k(ikE + i\mathbf{p} + jm)$ behaves as a vacuum operator. So do $i(ikE + i\mathbf{p} + jm)$ and $j(ikE + i\mathbf{p} + jm)$. Previously we said that the vacuum state vector had the same structure, apart from a scalar factor, as that of the fermion: $(ikE + i\mathbf{p} + jm)$. How do these three vacua relate?

Here, we can see the three vacuum coefficients k, i, j as originating in (or being responsible for) the concept of discrete (point-like) charge. The operators act as a discrete partitioning of the continuous vacuum responsible for zero-point energy, i.e. $(ikE + i\mathbf{p} + jm)$. In this sense, they are related to weak, strong and electric localized charges, though they are delocalized.

The 3 vacua also help to explain the meaning of the 4 terms in the Dirac 4-spinor. There is 1 real state (the lead term) and 3 potential (vacuum) states into which the lead term can be transformed by one of the 3 interactions. All possible states are always present, either as real states or vacuum ones, e.g.:

	$(ikE + i\mathbf{p} + jm)$	$\rightarrow$	$(ikE + i\mathbf{p} + jm)$
strong	$i\,(ikE + i\mathbf{p} + jm)$	$\rightarrow$	$(ikE - i\mathbf{p} + jm)$
weak	$k\,(ikE + i\mathbf{p} + jm)$	$\rightarrow$	$(-ikE + i\mathbf{p} + jm)$
electric	$j\,(ikE + i\mathbf{p} + jm)$	$\rightarrow$	$(-ikE - i\mathbf{p} + jm)$

We can suggest specific identifications of the interactions on the basis of the pseudoscalar, vector and scalar characteristics of the associated terms.

$k\,(ikE + i\mathbf{p} + jm)$	weak vacuum	fermion creation
$i\,(ikE + i\mathbf{p} + jm)$	strong vacuum	gluon plasma
$j\,(ikE + i\mathbf{p} + jm)$	electric vacuum	$SU(2)$

The 3 additional terms in the Dirac spinor then become strong, weak and electric vacuum 'reflections' of the state defined by the lead term.

CPT symmetry uses the same operators, and this is, of course, not a coincidence.

P $\quad i\,(ikE + i\mathbf{p} + jm)\,i = (ikE - i\mathbf{p} + jm)$

T $\quad k\,(ikE + i\mathbf{p} + jm)\,k = (-ikE + i\mathbf{p} + jm)$

C $\quad -j\,(ikE + i\mathbf{p} + jm)\,j = (-ikE - i\mathbf{p} + jm)$

CPT $\quad -j\,(i\,(k\,(ikE + i\mathbf{p} + jm)\,k)\,i)\,j = (ikE + i\mathbf{p} + jm)$

The nilpotent structure is particularly significant, in that the *CPT* theorem is designed to connect relativity with causality, and it is only in the nilpotent form that the causality term *jm* (or, equivalently, $j\tau$ for the space-time invariant) becomes an integral component in the structure.

Particle states and interactions: the origin of the Coulomb term

An ongoing research programme has shown that nilpotents can be used for QED, QFD (weak interaction calculations), QCD and QID (quantum inertial dynamics) (Rowlands 2005). A very signficant result is that renormalization is not needed for free particle and that there no hierarchy problem in the nilpotent theory. Again, for propagators, there is no infrared divergence, but there is a distinction between different bosonic propagators which is a distinct theoretical advantage. Extensions have also been made to apply the theory to basic condensed matter physics and chemistry. For our purposes, however, it is more significant to ask whether the fermionic nilpotent, if is the most fundamental structure in physics – in effect, its fundamental unit, can reproduce the fundamental particle states and their interactions.

These two questions are not independent of each other, and the first stage in answering them is to see if the *structure* of the nilpotent operator can give us any insight into the nature of fermionic interactions. In fact, this is precisely what it can do. But, first, assuming that the constraint of spherical symmetry exists for a point particle, we need to express the momentum term of the operator in polar coordinates, using the Dirac prescription, with an explicit spin term:

$$\boldsymbol{\sigma}.\nabla = \left(\frac{\partial}{\partial r} + \frac{1}{r}\right) \pm i\frac{j + ½}{r}$$

We need the spin term because the multivariate nature of the **p** term cannot be expressed in polar coordinates. The nilpotent Dirac operator now becomes:

$$\left(kE + \boldsymbol{i}\left(\frac{\partial}{\partial r} + \frac{1}{r} \pm i\frac{j + ½}{r}\right) + ijm\right)$$

Now, whatever phase we apply this to, we will find that we will not get a nilpotent solution unless the $1 / r$ term with coefficient $\boldsymbol{i}$ is matched by a similar $1 / r$ term with coefficient k. So, simply requiring *spherical symmetry* for a point particle, requires a term of the form A / r to be added to E. If all point particles are spherically symmetric sources, then the minimum nilpotent operator is of the form

$$\left(\mathbf{k}\left(E-\frac{A}{r}\right)+\mathbf{i}\left(\frac{\partial}{\partial \mathbf{r}}+\frac{1}{r}\pm i\frac{j+½}{r}\right)+\mathbf{i}\mathbf{j}m\right) \qquad (1)$$

To establish that this is a nilpotent, we must now find the phase to which this must apply to create a nilpotent amplitude. This procedure is quite straightforward, for we know that the phase factor is

$$F=e^{-ar}r^{\gamma}\sum_{\nu=0}a_{\nu}r^{\nu} . \qquad (2)$$

If we now apply the operator (1) to phase factor (2) and equate the resulting squared amplitude to zero, we obtain:

$$4\left(E-\frac{A}{r}\right)^2=-2\left(-a+\frac{\gamma}{r}+\frac{\nu}{r}+...\frac{1}{r}+i\frac{j+½}{\mathrm{r}}\right)^2-2\left(-a+\frac{\gamma}{r}+\frac{\nu}{r}+...\frac{1}{r}-i\frac{j+½}{\mathrm{r}}\right)^2+4m^2$$

We then constant terms, so that

$$E^2=-a^2+m^2$$
$$a=\sqrt{m^2-E^2} ,$$

then terms in $1/r^2$, with $\nu=0$, so:

$$\left(\frac{A}{r}\right)^2=-\left(\frac{\gamma+1}{r}\right)^2+\left(\frac{j+½}{r}\right)^2 .$$

From these, excluding the negative root (as usual),

$$\gamma=-1+\sqrt{(j+½)^2-A^2} .$$

and, assuming the power series terminates at n', and equating coefficients of $1/r$ for $\nu=n'$, we obtain:

$$2EA=-2\sqrt{m^2-E^2}\,(\gamma+1+n'),$$

the terms in $(j + ½)$ cancelling over the summation of the four multiplications, with two positive and two negative. From this we may derive

$$\frac{E}{m} = \frac{1}{\sqrt{1 + \frac{A^2}{(\gamma + 1 + n')^2}}} = \frac{1}{\sqrt{1 + \frac{A^2}{\left(\sqrt{(j+½)^2 - A^2} + n'\right)^2}}} .$$

We have, of course, without mentioning anything about potentials or interactions, or anything physical at all, and only using the structure of the nilpotent operator, needed to maintain the spherical symmetry of a point-particle source, created the classic solution for the Coulomb or inverse linear potential. And we have shown that it is absolutely necessary to any fermionic state described as a point source, regardless of what other potentials may be present. We can now proceed to show that another fundamental potential can be derived from the structure of the nilpotent operator alone.

The strong interaction: the vector term

The vector part of the nilpotent has three components. So a significant question might be: can we have a 3-component state vector? Clearly, nilpotency makes a term such as

$$(ikE + \mathbf{ip} + jm)\,(ikE + \mathbf{ip} + jm)\,(ikE + \mathbf{ip} + jm)$$

zero immediately. But the following would be possible:

$$(ikE + \mathbf{ip} + jm)\,(ikE + jm)\,(ikE + jm) \rightarrow (ikE + \mathbf{ip} + jm)$$

$$(ikE + jm)\,(ikE + \mathbf{ip} + jm)\,(ikE + jm) \rightarrow (ikE - \mathbf{ip} + jm)$$

$$(ikE + jm)\,(ikE + jm)\,(ikE + \mathbf{ip} + jm) \rightarrow (ikE + \mathbf{ip} + jm)$$

So we could have a nonzero state vector if we use the vector properties of **p** and the arbitrary nature of its sign (+ or –). There is an obvious way in which this could be accomplished using the vector properties of **p**. A state vector of the form, privileging the components of the **p** vector:

$$(ikE \pm i\mathbf{i}p_x + jm)\,(ikE \pm i\mathbf{j}p_y + jm)\,(ikE \pm i\mathbf{k}p_z + jm)$$

has six independent allowed phases, i.e. when

$$\mathbf{p} = \pm\ \mathbf{i}p_x\ ,\ \mathbf{p} = \pm\ \mathbf{j}p_y\ ,\ \mathbf{p} = \pm\ \mathbf{k}p_z$$

but these must also be *gauge invariant*, i.e. indistinguishable, or all present at once. In addition, we must interpret the E, $\mathbf{p}$, m symbols as belonging to a totally entangled state, rather than the subcomponents. In principle, we would be using the concept of spatial (rather than temporal) separation to represent the arbitrary nature of the direction of fermionic spin.

One method of picturing the arbitrary nature of the phases (gauge invariance) is to imagine an automatic mechanism of transfer between them.

$$\begin{array}{ll}
(ikE + \boldsymbol{i}\ \mathbf{i}p_x + \boldsymbol{j}\ m)\ (ikE + \boldsymbol{i}\ \mathbf{j}p_y + \boldsymbol{j}\ m)\ (ikE + \boldsymbol{i}\ \mathbf{k}p_z + \boldsymbol{j}\ m) & +RGB \\
(ikE - \boldsymbol{i}\ \mathbf{i}p_x + \boldsymbol{j}\ m)\ (ikE - \boldsymbol{i}\ \mathbf{j}p_y + \boldsymbol{j}\ m)\ (ikE - \boldsymbol{i}\ \mathbf{k}p_z + \boldsymbol{j}\ m) & -RBG \\
(ikE + \boldsymbol{i}\ \mathbf{i}p_x + \boldsymbol{j}\ m)\ (ikE + \boldsymbol{i}\ \mathbf{j}p_y + \boldsymbol{j}\ m)\ (ikE + \boldsymbol{i}\ \mathbf{k}p_z + \boldsymbol{j}\ m) & +BRG \\
(ikE - \boldsymbol{i}\ \mathbf{i}p_x + \boldsymbol{j}\ m)\ (ikE - \boldsymbol{i}\ \mathbf{j}p_y + \boldsymbol{j}\ m)\ (ikE - \boldsymbol{i}\ \mathbf{k}p_z + \boldsymbol{j}\ m) & -GRB \\
(ikE + \boldsymbol{i}\ \mathbf{i}p_x + \boldsymbol{j}\ m)\ (ikE + \boldsymbol{i}\ \mathbf{j}p_y + \boldsymbol{j}\ m)\ (ikE + \boldsymbol{i}\ \mathbf{k}p_z + \boldsymbol{j}\ m) & +GBR \\
(ikE - \boldsymbol{i}\ \mathbf{i}p_x + \boldsymbol{j}\ m)\ (ikE - \boldsymbol{i}\ \mathbf{j}p_y + \boldsymbol{j}\ m)\ (ikE - \boldsymbol{i}\ \mathbf{k}p_z + \boldsymbol{j}\ m) & -BGR
\end{array}$$

This has exactly the same group structure as the standard 'coloured' baryon wavefunction made of R, G and B 'quarks',

$$\psi \sim (RGB - RBG + BRG - GRB + GBR - BGR).$$

That is, it has an $SU(3)$ structure, with 8 generators, and, since the E and $\mathbf{p}$ terms in the state vector really represent time and space derivatives, we can replace these with the covariant derivatives needed for invariance under a local $SU(3)$ gauge transformation. This $SU(3)$ symmetry or strong interaction is entirely nonlocal. That is, the exchange of momentum $\mathbf{p}$ involved is entirely independent of any spatial position of the 3 components of the baryon. We can suppose that the rate of change of momentum (or 'force') is constant with respect to spatial positioning or separation. A constant force is equivalent to a potential which is linear with distance, exactly as is required for the conventional strong interaction.

We can now identify our structures as those that would be required of a baryon in the nilpotent formalism. If we now construct a nilpotent operator, in which spherical symmetry still applies, we will find that the requirement for a term of the form A / r, added to E, remains unchanged – physically, it is the one gluon exchange term, and occurs in standard treatments of lattice gauge QCD (Takahashi *et al*, 2001) – but that we now require another term of the form Br. If we now try to solve for phase and amplitude, we will find that our solution has the characteristics of infrared slavery and asymptotic freedom that we apply to quarks and the strong interaction.

We note that the full symmetry between the 3 momentum components can only apply if the momentum operators can be equally + or –. With all phases of the interaction present at the same time (perfect gauge invariance), this is equivalent to saying that left-handedness and right-handedness must be present simultaneously in the baryon state. In other words, the baryonic state *must have non-zero mass via the Higgs mechanism.*

The weak interaction: the pseudoscalar term

The other significant component of the nilpotent is the pseudoscalar term (ikE). The particular significances of this term are:
(1) its necessity to nilpotency
(2) the necessity of removing it by a 'squaring' operation, or multiplication by a complex conjugate
(3) the dipolarity it creates between fermion and vacuum, etc.

Ultimately, this leads to the necessity for a term of the form Cr^n, where n may be, say –3, to be added to the E term, and we can show that *any* potential with terms of this form added to the Coulomb term will produce a harmonic oscillator solution for a nilpotent operator, as long as the added term is not linearly proportional to distance, like the strong interaction term. As with the Coulomb and strong interacting terms, a dipolar term, at least, is required by the *actual structure* of the nilpotent operator. This time, it is the 4-component spinor aspect, which requires a virtual switching between the components to provide the *zitterbewegung* which Schrödinger originally showed was a necessary consequence of the free particle Dirac state. This means that the actual structure of the nilpotent operator requires an in-built harmonic oscillator / dipolar / interaction vertex arrangement in which one state is created while another disappears.

Since a harmonic oscillator solution is the result of any polynomial potential other than Coulomb or Coulomb plus linear, there are therefore only three point-particle (spherically symmetric) fermionic ikE operators which give us the desired nilpotent solution, and these have the characteristics of the three interactions:

A / r	Coulomb	electric interaction
$A / r + Br$	confinement	strong interaction
$A / r + Cr^n$	harmonic oscillator	weak interaction

We can identify the A / r term with the scalar part of the three operators jm, $\mathbf{i}\mathbf{p}$, ikE (the coupling constant; Br with the vector part of $\mathbf{p}$; and Cr^n with the pseudoscalar part of iE.

The pseudoscalar part brings us to the consideration of interaction vertices. Because the state vector always represents four terms with the complete variation of signs in E and $\mathbf{p}$, an interaction vertex between any fermion / antifermion and any other

$$(ikE_1 + \mathbf{i}\mathbf{p}_1 + jm_1)\,(ikE_2 + \mathbf{i}\mathbf{p}_2 + jm_2)$$

will remove the quaternionic operators, leaving only scalars and vectors. When the E, $\mathbf{p}$ and m values become numerically equal, the vertex can be defined as a new *combined* bosonic state, with a single phase. Where there is an interaction vertex between two fermionic / antifermionic states, the signs of E and $\mathbf{p}$ of the second term, with respect to the first, will also determine the nature of the bosonic or combined state which may be created.

Because there are three operators involved – $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ – there are also three possible bosonic states, with three possible transformations (T, C or P) of the second component. Any transformation of a fermionic state can be represented as a bosonic state in which the old state is annihilated and the new one created. The spin 1 boson will have the form:

$$(ikE + \mathbf{i}\mathbf{p} + jm)\,(-ikE + \mathbf{i}\mathbf{p} + jm) \qquad T$$

The spin 0 boson becomes:

$$(ikE + \mathbf{i}\mathbf{p} + jm)\,(-ikE - \mathbf{i}\mathbf{p} + jm) \qquad C$$

while the third boson-like state will be the fermion-fermion combination, which is manifested in Bose-Einstein condensates, and instances of nonzero Berry phase, etc.:

$$(ikE + \mathbf{i}\mathbf{p} + jm)\,(ikE - \mathbf{i}\mathbf{p} + jm) \qquad P$$

The fermion-fermion boson-like state has many physical manifestations: the Aharonov-Bohm effect, the Jahn-Teller effect, the quantum Hall effect, Cooper pairs, even-even nuclei. Even spin 1 He^3 can be accommodated (via $\boldsymbol{\sigma}.\mathbf{p}$) if its physically separated components are considered to have opposite momenta, in a harmonic oscillator arrangement. This ensures that they can have the same spin but opposite signs of $\mathbf{p}$.

Significantly, the spin 0 bosonic state cannot be massless, because, if it is nilpotent it automatically becomes zero.

$$(ikE + i\mathbf{p})\,(-\,ikE - i\mathbf{p}) = 0$$

This becomes a significant factor in the Higgs mechanism. It also implies that massless fermions cannot have the same handedness as massless antifermions. The conventional derivation of spin assigns left-handedness to fermions.

The eight required mediators of the strong force will consist of six gluonic bosons of the form:

$$(ikE - i\mathbf{i}p_x)\,(-\,ikE - i\mathbf{j}p_y)$$

and two combinations of the three bosons of the form:

$$(ikE - i\mathbf{i}p_x)\,(-\,ikE - i\mathbf{i}p_x)$$

These structures are, of course, identical to an equivalent set in which both brackets undergo a complete sign reversal. The important thing here is that applying any of these mediators will produce a sign change in the **p** component that leads to mass.

The 3 types of bosonic state can be related to vacua produced by the 3 charge operators:

weak vacuum spin 1

$(ikE + i\mathbf{p} + jm)\, k\, (ikE + i\mathbf{p} + jm)\, k\, (ikE + i\mathbf{p} + jm)\, k\, (ikE + i\mathbf{p} + jm) \ldots$

$(ikE + i\mathbf{p} + jm)\, (-ikE + i\mathbf{p} + jm)\, (ikE + i\mathbf{p} + jm)\, (-ikE + i\mathbf{p} + jm) \ldots$

electric vacuum spin 0

$(ikE + i\mathbf{p} + jm)\, j\, (ikE + i\mathbf{p} + jm)\, j\, (ikE + i\mathbf{p} + jm)\, j\, (ikE + i\mathbf{p} + jm) \ldots$

$(ikE + i\mathbf{p} + jm)\, (-\,ikE - i\mathbf{p} + jm)\, (ikE + i\mathbf{p} + jm)\, (-\,ikE - i\mathbf{p} + jm) \ldots$

strong vacuum B-E condensate

$(ikE + i\mathbf{p} + jm)\, i\, (ikE + i\mathbf{p} + jm)\, i\, (ikE + i\mathbf{p} + jm)\, i\, (ikE + i\mathbf{p} + jm) \ldots$

$(ikE + i\mathbf{p} + jm)\, (ikE - i\mathbf{p} + jm)\, (ikE + i\mathbf{p} + jm)\, (ikE - i\mathbf{p} + jm) \ldots$

All these discrete vacuum states produce virtual boson states which have no effect on the fermion $(ikE + i\mathbf{p} + jm)$. So, each fermion becomes *its own* supersymmetric bosonic partner, and vice versa. This removes the need for renormalization in the case of free particles, while 'renormalization' of interacting particles becomes rescaling – charge values being determined by their interactions with all the others in the universe. In principle, with negative fermion loops cancelled by positive boson loops, it should remove the divergent terms altogether.

Particle states

We have seen that the nilpotent operator produces the characteristics of the three fundamental interactions purely from its own mathematical structure. This must also be the case with the spectrum of fundamental particles. We can clearly show the generic types of particle states – fermion, antifermion, quark, baryon, etc. – but can we generate the specific ones? Here we can reproduce the particle structures already derived for fundamental particles based on charges, directly using nilpotents. The tables previously produced for quarks (A, B, C) and leptons (L), reduced to one generation (Rowlands and Cullerne, 1999, 2001), give the following charge structures:

A

		B	G	R
u	$+e$	$1j$	$1j$	$0i$
	$+s$	$1i$	$0k$	$0j$
	$+w$	$1k$	$0i$	$0k$
d	$-e$	$0j$	$0k$	$1j$
	$+s$	$1i$	$0i$	$0k$
	$+w$	$1k$	$0j$	$0i$

B

		B	G	R
u	$+e$	$1j$	$1j$	$0k$
	$+s$	$0i$	$0k$	$1i$
	$+w$	$1k$	$0i$	$0j$
d	$-e$	$0i$	$0k$	$1j$
	$+s$	$0j$	$0i$	$1i$
	$+w$	$1k$	$0j$	$0k$

C

		B	G	R
u	$+e$	$1j$	$1j$	$0k$
	$+s$	$0i$	$1i$	$0j$
	$+w$	$1k$	$0k$	$0i$
d	$-e$	$0j$	$0k$	$1j$
	$+s$	$0i$	$1i$	$0k$
	$+w$	$1k$	$0j$	$0i$

L

		$\bar{e}$	$\bar{e}$	ν_e
	$+e$	$1j$	$1j$	$0j$
	$+s$	$0k$	$0i$	$0i$
	$+w$	$0i$	$0k$	$1k$
				e
	$-e$	$0i$	$0k$	$1j$
	$+s$	$0j$	$0i$	$0i$
	$+w$	$0k$	$0j$	$1k$

In A, B, C, the variation of the charges is

$0 \quad 0 \quad -e$ plus $e \quad e \quad e\,(u)$ $\quad 0 \quad 0 \quad 0\,(d)$

w

s on any

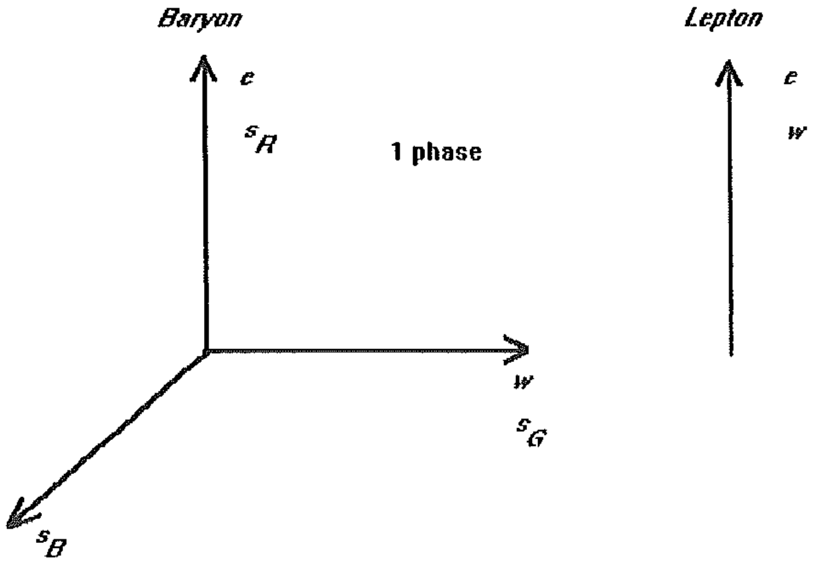
In L it is

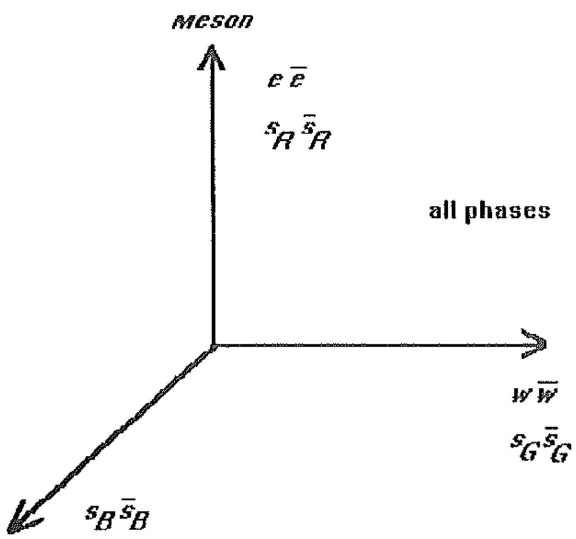
$0 \quad 0 \quad -e$ plus $e \quad e \quad e\,(n)$ $\quad 0 \quad 0 \quad 0\,(e)$

w

s on none

The first three columns can be regarded as possible *phases*, and we can also use phase diagrams to show the lepton, baryon and meson charge structures that result from these (Rowlands and Cullerne, 2002).





For the strong interaction, only one component of angular momentum is well-defined at any moment, 'privileging' one direction out of 3 independent phases; defined only through the directional variation of **p**. For weak / electric, we define a relative 'privileging' of phase. Here, we have two options. If the 'privileged' or 'active' phases of E and m (or w and e) coincide, then this determines the 'privileged' phase of **p**. There is no 'privileged' relative phase, so there is no no strong interaction. If they are different, then this information can only be carried through **p** (or s), and the strong interaction must be present.

The same phases can be used with the nilpotent structures. Here $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$ represent the three phases of the strong interaction. The three charges s, w, e are now represented by the respective $\boldsymbol{i}$, $\boldsymbol{k}$, $\boldsymbol{j}$ operators and the three vacuum states within the nilpotent that they represent.

s	is represented by	$\boldsymbol{i}\mathbf{p}$	$\rightarrow -\boldsymbol{i}\mathbf{p}$
w	is represented by	$\boldsymbol{ik}E$	$\rightarrow -\boldsymbol{ik}E$
e	is represented by	$\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p}$	$\rightarrow -\boldsymbol{ik}E - \boldsymbol{i}\mathbf{p}$

For baryons / mesons, we arrange it so that the weak and electric phases never coincide with each other. So, for example, the spin ½ baryon nilpotent representations become:

$$\begin{matrix} inertial \\ strong \\ weak \\ electric \end{matrix} \begin{pmatrix} ikE \pm i\sigma.\mathbf{p}_1 + jm \\ ikE \mp i\sigma.\mathbf{p}_1 + jm \\ -ikE \pm i\sigma.\mathbf{p}_1 + jm \\ -ikE \mp i\sigma.\mathbf{p}_3 + jm \end{pmatrix} \begin{pmatrix} ikE \mp i\sigma.\mathbf{p}_2 + jm \\ ikE \pm i\sigma.\mathbf{p}_2 + jm \\ -ikE \mp i\sigma.\mathbf{p}_3 + jm \\ -ikE \pm i\sigma.\mathbf{p}_2 + jm \end{pmatrix} \begin{pmatrix} ikE \pm i\sigma.\mathbf{p}_3 + jm \\ ikE \mp i\sigma.\mathbf{p}_3 + jm \\ -ikE \pm i\sigma.\mathbf{p}_2 + jm \\ -ikE \mp i\sigma.\mathbf{p}_1 + jm \end{pmatrix}$$

For leptons, we arrange it so that the weak and electric phases always coincide with each other; $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$ now represent a purely temporal cycle with no spatial element, and, hence, no strong interaction. So, a typical nilpotent representations for leptons would be:

$$\begin{matrix} inertial \\ strong \\ weak \\ electric \end{matrix} \begin{pmatrix} ikE \pm i\sigma.\mathbf{p}_1 + jm \\ ikE \mp i\sigma.\mathbf{p}_1 + jm \\ -ikE \pm i\sigma.\mathbf{p}_1 + jm \\ -ikE \mp i\sigma.\mathbf{p}_1 + jm \end{pmatrix}$$

SU(2) weak isospin and neutrinos

To represent, the alternative *SU*(2) weak isospin states, representing up and down quarks, or neutrinos and electrons, we need to postulate a transition from a 'filled' *e* background to an 'empty' one:

$$\begin{matrix} e & e & e \\ 0 & 0 & 0 \end{matrix}$$

In principle, a weak isospin transition can be seen as a change of the form:

$$\underset{isospin\ up}{(ikE + \mathbf{i}\mathbf{p} + jm)} \rightarrow \alpha_1\, \underset{isospin\ down}{(ikE + \mathbf{i}\mathbf{p} + jm)} + \alpha_2\, (ikE - \mathbf{i}\mathbf{p} + jm)$$

Here, the down state introduces a degree of right-handedness which is not present in the up state, and which is not weak in origin. Alternatively, we could have something like:

$$\alpha_1\, (ikE + \mathbf{i}\mathbf{p} + jm) - \underset{isospin\ up}{\alpha_2\, (ikE - \mathbf{i}\mathbf{p} + jm)} \rightarrow \underset{isospin\ down}{(ikE + \mathbf{i}\mathbf{p} + jm)}$$

or a combination of the two. It would be reasonable to suppose that one of these represents the *u* / *d* type variation, and the other the increasing amount of mass for states of the second and third generations. A term like $-\alpha_2\, (ikE - \mathbf{i}\mathbf{p} + jm)$ has the characteristics of a vacuum operator.

Fermion states with mass also carry a degree of right-handedness. A transition from left- to right-handedness, involving only fermionic states (not antifermionic), requires a vacuum which we can describe as 'electric'. Only the electric vacuum carries a transition to right-handedness where the vector character (strong interaction) is absent. And, to produce a pure transition from left- to right-handedness (and *vice versa*) without a change from fermion to antifermion requires an electroweak combination (***jk***, equivalent to ***i***):

$(ikE + \mathbf{i}\mathbf{p} + jm)$	left-handed fermion
$(-ikE + \mathbf{i}\mathbf{p} + jm)$	weak transition to right-handed antifermion
$(ikE - \mathbf{i}\mathbf{p} + jm)$	electric transition to right-handed fermion

Neutrinos are a little more difficult to explain than the rest of the lepton sector, but a Majorana neutrino might be considered as a superposition of state and antistate:

$$\alpha_1\,(ikE + \mathbf{ip} + jm) + \alpha_2\,(-ikE + \mathbf{ip} + jm)$$

It would be a result of the violation of weak charge-conjugation symmetry, and would be naturally *CP* violating. Majorana neutrinos relate to the spin 0 'bosonic' state involved in Berry phase:

$$(ikE + \mathbf{ip} + jm)\,(ikE - \mathbf{ip} + jm)$$

which would be required in a pure weak transition from $-ikE$ to $+\,ikE$, or its inverse. Because the spin 0 state is necessarily massive, time reversal symmetry (the one applicable to the transition) must be broken in the weak formation or decay of states involving the Berry phase. A possible experiment to observe this might involve the quantum Hall effect in graphene.

The Higgs mechanism

We can imagine a virtual fermionic state with no mass in vacuum and with structure

$$(ikE + \mathbf{ip}).$$

An ideal vacuum would maintain exact and absolute *C*, *P* and *T* symmetries. Under *C* transformation, $(ikE + \mathbf{ip})$ would become

$$(-\,ikE - \mathbf{ip})$$

with which it would be indistinguishable under normalization. No bosonic state would be required for the transformation.

If, however, the vacuum state is degenerate in some way under charge conjugation (as supposed in the weak interaction), then $(ikE + \mathbf{ip})$ will be transformable into a state which can be distinguished from it, and the bosonic state $(ikE + \mathbf{ip})\,(-ikE - \mathbf{ip})$ will necessarily exist. However, this can only be true if the state has nonzero mass and becomes the spin 0 'Higgs boson':

$$(ikE + \mathbf{ip} + jm)\,(-ikE - \mathbf{ip} + jm)$$

The coupling of a massless fermion, say $(ikE_1 + \mathbf{ip}_1)$, to a Higgs boson, say $(ikE + \mathbf{ip} + jm)\,(-ikE - \mathbf{ip} + jm)$, to produce a massive fermion, say $(ikE_2 + \mathbf{ip}_2 + jm_2)$, can be imagined as occurring at a vertex between the created fermion $(ikE_2 + \mathbf{ip}_2 + jm_2)$ and the antistate $(-ikE_1 - \mathbf{ip}_1)$, to the annihilated massless fermion, with subsequent equalization of energy and momentum states.

If we imagine a vertex involving a fermion superposing $(ikE + i\mathbf{p} + jm)$ and $(ikE - i\mathbf{p} + jm)$ with an antifermion superposing $(-ikE + i\mathbf{p} + jm)$ and $(-ikE - i\mathbf{p} + jm)$, then there will be a minimum of two spin 1 combinations and two spin 0 combinations, meaning that the vertex will be massive (with Higgs coupling) and carry a non-weak (i.e. electric) charge. So, a process such as

$$\underset{\textit{isospin up}}{(ikE + i\mathbf{p} + jm)} \rightarrow \alpha_1 (ikE + i\mathbf{p} + jm) + \alpha_2 \underset{\textit{isospin down}}{(ikE - i\mathbf{p} + jm)}$$

requires an additional Higgs boson vertex (spin 0) to accommodate the right-handed part of the isospin down state, when the left-handed part interacts weakly. This is, of course, what we mean when we say that the W and Z bosons have mass. The mass balance is done through separate vertices involving the Higgs boson.

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Editors' Note: I have asked Peter for permission to publish the following two short notes as I believe that they are helpful in better understanding Peter's work. Keith Bowden

Idempotents and nilpotents

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The idempotent aspect of the nilpotent quantum mechanics is obvious from the vacuum operators. If I take a nilpotent amplitude like

$$(ikE + \mathbf{i}\mathbf{p} + jm) \qquad (1)$$

it is effectively unchanged by postmultiplication by, say, $k\,(ikE + \mathbf{i}\mathbf{p} + jm)$ (which is a vacuum operator). In other words, we can write $(ikE + \mathbf{i}\mathbf{p} + jm)$ as

$$(ikE + \mathbf{i}\mathbf{p} + jm)\, k\, (ikE + \mathbf{i}\mathbf{p} + jm)\, k\, (ikE + \mathbf{i}\mathbf{p} + jm)\, k\, (ikE + \mathbf{i}\mathbf{p} + jm) \ldots \qquad (2)$$

The only change is a scalar multiple which can be normalised away.

Exactly the same is true for

$$(ikE + \mathbf{i}\mathbf{p} + jm)\, i\, (ikE + \mathbf{i}\mathbf{p} + jm)\, i\, (ikE + \mathbf{i}\mathbf{p} + jm)\, i\, (ikE + \mathbf{i}\mathbf{p} + jm) \ldots \qquad (3)$$

and

$$(ikE + \mathbf{i}\mathbf{p} + jm)\, j\, (ikE + \mathbf{i}\mathbf{p} + jm)\, j\, (ikE + \mathbf{i}\mathbf{p} + jm)\, j\, (ikE + \mathbf{i}\mathbf{p} + jm) \ldots \qquad (4)$$

Multiplying (2) from the left by j, thus, for example, creates an idempotent:

$$j\, (ikE + \mathbf{i}\mathbf{p} + jm)\, j\, (ikE + \mathbf{i}\mathbf{p} + jm)\, j\, (ikE + \mathbf{i}\mathbf{p} + jm)\, j\, (ikE + \mathbf{i}\mathbf{p} + jm) \ldots \qquad (5)$$

and similar procedures could be applied to (2) and (3).

Now the differential operator for (1) is something like

$$(ik\partial / \partial t + \mathbf{i}\nabla + jm) \qquad (6)$$

So, we can write the Dirac equation as, say,

$$(\boldsymbol{i}k\partial / \partial t + \boldsymbol{i}\nabla + \boldsymbol{j}m)\,(\boldsymbol{i}kE + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\, e^{-i(Et - \mathbf{p}.\mathbf{r})} = 0. \qquad (7)$$

Here, we have written the equation for a free particle, but for the purposes of the argument that follows, it doesn't matter whether it's free or bound. All that would change is the phase. But, since $\boldsymbol{ij}\ \boldsymbol{ij} = 1$, we could equally well write (7) in a form such as

$$(\boldsymbol{i}k\partial / \partial t + \boldsymbol{i}\nabla + \boldsymbol{j}m)\ \boldsymbol{ij}\ \boldsymbol{ij}\,(\boldsymbol{i}kE + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\, e^{-i(Et - \mathbf{p}.\mathbf{r})} = 0. \qquad (8)$$

which becomes equivalent to:

$$-\,(\boldsymbol{i}\partial / \partial t + \boldsymbol{i}k\nabla - im)\,(\boldsymbol{i}iE + \boldsymbol{i}k\mathbf{p} + im)\, e^{-i(Et - \mathbf{p}.\mathbf{r})} = 0. \qquad (9)$$

Here, the amplitude, $(\boldsymbol{i}iE + \boldsymbol{i}k\mathbf{p} + im)$, is simply i times the idempotent $\boldsymbol{j}$ $(\boldsymbol{i}kE + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ (5), and we can, of course, remove the i if we want to. So the nilpotent equation (7) actually incorporates an idempotent equation (9). The equations are precisely the same – only the interpretation is different. There isn't even a transformation required, just a redistribution of algebraic operators between differential operator and amplitude. So the alternative interpretations are:

IDEMPOTENT

$$[(\boldsymbol{i}k\partial / \partial t + \boldsymbol{i}\nabla + \boldsymbol{j}m)\ \boldsymbol{ij}]\ [\boldsymbol{ij}\,(\boldsymbol{i}kE + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\, e^{-i(Et - \mathbf{p}.\mathbf{r})}] = 0. \qquad (10)$$

operator *wavefunction*

NILPOTENT

$$[(\boldsymbol{i}k\partial / \partial t + \boldsymbol{i}\nabla + \boldsymbol{j}m)\ \boldsymbol{ij}\ \boldsymbol{ij}]\ [(\boldsymbol{i}kE + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\, e^{-i(Et - \mathbf{p}.\mathbf{r})}] = 0. \qquad (11)$$

operator *wavefunction*

A discrete version of the nilpotent Dirac equation

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Let $\hbar = 1$, $c = 1$. Use the symbol E for H, and define

$$F = \psi = \boldsymbol{ik}E + \boldsymbol{i}\mathbf{i}P_1 + \boldsymbol{i}\mathbf{j}P_2 + \boldsymbol{i}\mathbf{k}P_3 + \boldsymbol{j}m, \tag{1}$$

where bold symbols represent multivariate vectors and bold italic symbols represent quaternions. Algebraically,

$$\begin{aligned}
-\boldsymbol{ik}\,\psi\,\boldsymbol{ik} &= \psi - 2\,\boldsymbol{ik}E \\
\boldsymbol{i}\mathbf{i}\,\psi\,\boldsymbol{i}\mathbf{i} &= \psi - 2\,\boldsymbol{i}\mathbf{i}P_1 \\
\boldsymbol{i}\mathbf{j}\,\psi\,\boldsymbol{i}\mathbf{j} &= \psi - 2\,\boldsymbol{i}\mathbf{j}P_2 \\
\boldsymbol{i}\mathbf{k}\,\psi\,\boldsymbol{i}\mathbf{k} &= \psi - 2\,\boldsymbol{i}\mathbf{k}P_3 \\
\boldsymbol{j}\,\psi\,\boldsymbol{j} &= \psi - 2\,\boldsymbol{j}m.
\end{aligned}$$

Now define the operator

$$D = -\boldsymbol{k}\frac{\partial}{\partial t} - \boldsymbol{i}i\mathbf{i}\frac{\partial}{\partial X_1} - \boldsymbol{i}i\mathbf{j}\frac{\partial}{\partial X_2} - \boldsymbol{i}i\mathbf{k}\frac{\partial}{\partial X_3},$$

with

$$i\frac{\partial F}{\partial t} = [F,H] = [F,E] \quad \text{and} \quad -i\frac{\partial F}{\partial X_i} = [F,P_i],$$

as in Kauffman's discrete version of differential calculus.[1-2] The negative operators are those which produce amplitude (1) in the continuum version. (The negative signs can be removed if we use a complexified version of (1).) This means that

$$\begin{aligned}
-\boldsymbol{k}\frac{\partial\psi}{\partial t} &= \boldsymbol{ik}[\psi,E] = \boldsymbol{ik}\psi E - \boldsymbol{ik}E\psi \\
&= \boldsymbol{ik}\psi\boldsymbol{ik}\boldsymbol{ik}E - \boldsymbol{ik}E\psi \\
&= -\psi\boldsymbol{ik}E - \boldsymbol{ik}E\psi + 2\boldsymbol{ik}E\boldsymbol{ik}E \\
&= -\psi\boldsymbol{ik}E - \boldsymbol{ik}E\psi + 2E^2.
\end{aligned} \tag{2}$$

$$-i\mathbf{i}\frac{\partial\psi}{\partial X_1} = i\mathbf{i}[\psi, P_1] = i\mathbf{i}\,\psi P_1 - i\mathbf{i}P_1\psi = -i\mathbf{i}\,\psi i\mathbf{i}i\mathbf{i}P_1 - i\mathbf{i}P_1\psi$$
$$= -\psi i\mathbf{i}P_1 - i\mathbf{i}P_1\psi + 2i\mathbf{i}P_1 i\mathbf{i}P_1 = -\psi i\mathbf{i}P_1 - i\mathbf{i}P_1\psi - 2P_1^2 \qquad (3)$$

$$-i\mathbf{j}\frac{\partial\psi}{\partial X_2} = i\mathbf{j}[\psi, P_2] = i\mathbf{j}\,\psi P_2 - i\mathbf{j}P_2\psi = -i\mathbf{j}\,\psi i\mathbf{j}i\mathbf{j}P_2 - i\mathbf{j}P_2\psi$$
$$= -\psi i\mathbf{j}P_2 - i\mathbf{j}P_2\psi + 2i\mathbf{j}P_2 i\mathbf{j}P_2 = -\psi i\mathbf{j}P_2 - i\mathbf{j}P_2\psi - 2P_2^2 \qquad (4)$$

$$-i\mathbf{k}\frac{\partial\psi}{\partial X_3} = i\mathbf{k}[\psi, P_3] = i\mathbf{k}\psi P_3 - i\mathbf{k}P_3\psi = -i\mathbf{k}\psi i\mathbf{k}i\mathbf{k}P_3 - i\mathbf{k}P_3\psi$$
$$= -\psi i\mathbf{k}P_3 - i\mathbf{k}P_3\psi + 2i\mathbf{k}P_1 i\mathbf{k}P_3 = -\psi i\mathbf{k}P_3 - i\mathbf{k}P_3\psi - 2P_3^2 \qquad (5)$$

Now, if m is a scalar, we may use the identity

$$0 \equiv j\psi m - jm\psi = -j\psi jjm - jm\psi$$
$$= -\psi jm - jm\psi - 2jmjm = -\psi jm - jm\psi - 2m^2. \qquad (6)$$

Combining equations (2)-(6), term by term, we obtain

$$\not{\partial}\psi = -\psi(ikE + i\mathbf{i}P_1 + i\mathbf{j}P_2 + i\mathbf{k}P_3 + jm)$$
$$-\quad (ikE + i\mathbf{i}P_1 + i\mathbf{j}P_2 + i\mathbf{k}P_3 + jm)\psi + 2(E^2 - P_1{}^2 - P_2{}^2 - P_3{}^2 - m^2).$$

When is ψ nilpotent, then

$$\not{\partial}\psi = 0.$$

This can also be written

$$\left(k\frac{\partial}{\partial t} + i\mathbf{i}\nabla\right)\psi = 0 \qquad (7)$$

This is the discrete form of the nilpotent Dirac equation. Significantly, it does not require a mass term (which is redundant when we have exact knowledge of E and P), although, for reasons of symmetry, we could define a mass operator $j[\psi, m] \equiv 0$. With only one well-defined direction for spin, the minimalist representation of $\not{\partial}$ in equation (7) would seem to be in accordance with the holographic principle, where a bounding 'area' contains all the information relevant to a canonically defined 'system'. It is significant also that phase plays no part in this discrete form of relativistic quantum mechanics, the 'wavefunction' ψ being specified by the amplitude alone.

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1 Louis H. Kauffman, Non-commutative worlds, *New Journal of Physics*, 6, 2-46, 2004.
2 Louis H. Kauffman, Differential geometry in non-commutative worlds, in K. G. Bowden (ed.), *Conceptions*, ANPA, London, 2006, 193-213.

The Evolutionary Anthropic Semantic Computational Principle II

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Abstract. The principal argument for an Evolutionary Anthropic Semantic Computational Principle and a confirmatory prediction methodology in its favour are summarized.

The Argument

Einstein said 'the most incomprehensible thing about the universe, is the fact that it is comprehensible'. However, this key (fact) to which Einstein directs us (of the accelerating widening of scientific understanding through human intelligence) can be explained, if intelligent life exemplified by the human brain, is not some fluky optional extra to the cosmos, but the result of a **predictable** evolutionary staircase of natural hierarchical structural complexity (testable at each stage against already established and future experimentally validated scientific fact) that arises from an universal computational organizational principle (UCOP):
i) which specifies the origin of the universe as an empty state (i.e. is the boundary condition essential to the solution of a physical problem) (Rowlands and Diaz, 2002; Diaz and Rowlands, 2005)
ii) which gives rise to the desired staircase of evolutionary complexity (Rowlands and Diaz, 2002; Diaz and Rowlands, 2005), explainable as a well defined quantum thermodynamic process in terms of the Quantum Carnot Engine (QCE) (Scully *et al*, 2003) and
iii) which has a computational order code, that of the nilpotent generalization of Dirac's famous quantum mechanical equation, D(N) see diagram I, **predicted** from the principle itself (Rowlands and Cullerne, 1999; Rowlands and Diaz, 2002; Diaz and Rowlands, 2005), where this order code specifies the fundamental level of quantum mechanical structure 'nature's building blocks', see diagram II in terms of which all further hierarchical levels of the universe's architectural complexity are described and are decomposable. 'Decomposable' because in the entangled quantum universe described, each quantum subsystem's

interaction with the rest of the universe must and is taken into account in describing its behaviour by the **calculable** use of the mathematical criterion nilpotent, as is the case in diagram II, and

iv) where D(N) **predicts** the initial simultaneous emergence by spontaneous symmetry breaking from its empty state (Rowlands, 2004), of 3 + 1 relativistic space-time and the strong, weak, and electromagnetic (inertial) quantizations including spin of Standard Model elementary particle physics (Rowlands, 2006) – 'the building blocks' diagram II as currently experimentally validated.

Diagram I

The nilpotent generalization of Dirac's famous equation D(N)

$$(\mp k\partial/\partial t \pm \boldsymbol{i}\nabla + jm)\ (\pm ikE \pm \mathbf{ip} + jm)\ exp\ i(-Et + \mathbf{p.r}) = 0$$

where E, $\mathbf{p}$, m, t and $\mathbf{r}$ are respectively energy, momentum, mass, time, space and the symbols $\pm$ 1, $\pm$ i, $\pm$ $\boldsymbol{i}$, $\pm$ j, $\pm$ k, $\pm$ $\mathbf{i}$, $\pm$ $\mathbf{j}$, $\pm$ $\mathbf{k}$, are used to represent the respective units required by the scalar, pseudo-scalar, quaternion and multivariate vector groups.

Diagram II

The table of the nilpotents D(N, X_i), where the nilpotent operators $X_i^{\,2} = 0$, but $X_i \neq 0$ specify the calculable quantizations of the experimentally validated particles of the Standard Model of elementary particle physics:-

Baryons (spin 3/2):

$$\begin{matrix} inertial \\ strong \\ weak \\ electric \end{matrix} \begin{pmatrix} ikE \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + jm \\ ikE \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + jm \\ -ikE \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + jm \\ -ikE \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + jm \end{pmatrix} \begin{pmatrix} ikE \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + jm \\ ikE \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + jm \\ -ikE \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + jm \\ -ikE \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + jm \end{pmatrix} \begin{pmatrix} ikE \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + jm \\ ikE \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + jm \\ -ikE \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + jm \\ -ikE \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + jm \end{pmatrix}$$

Baryons (spin ½):

$$\begin{matrix} inertial \\ strong \\ weak \\ electric \end{matrix} \begin{pmatrix} ikE \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + jm \\ ikE \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + jm \\ -ikE \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + jm \\ -ikE \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + jm \end{pmatrix} \begin{pmatrix} ikE \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + jm \\ ikE \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + jm \\ -ikE \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + jm \\ -ikE \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + jm \end{pmatrix} \begin{pmatrix} ikE \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + jm \\ ikE \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + jm \\ -ikE \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + jm \\ -ikE \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + jm \end{pmatrix}$$

Leptons:

$$\begin{matrix} inertial \\ strong \\ weak \\ electric \end{matrix} \begin{pmatrix} ikE \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + jm \\ ikE \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + jm \\ -ikE \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + jm \\ -ikE \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + jm \end{pmatrix}$$

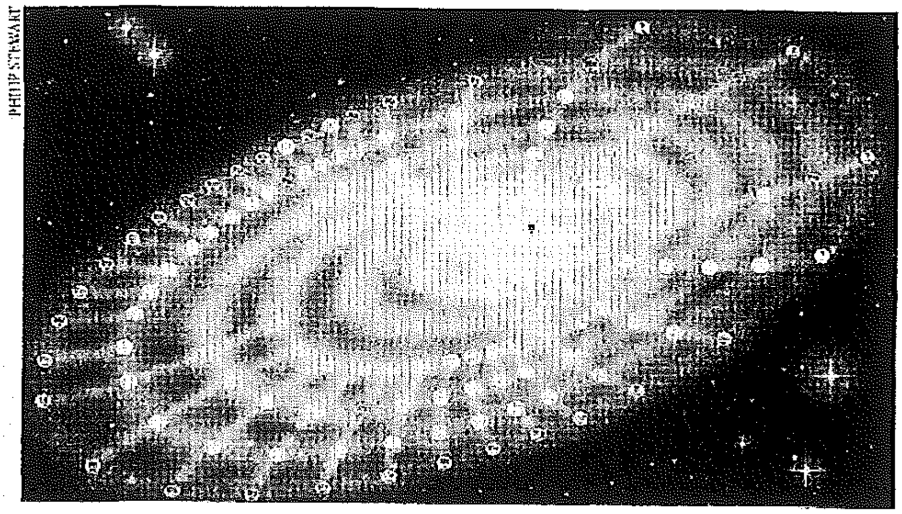
However the critical feature needed to complete the proposed solution, is that the DNA/RNA genetic code constitutes a replication of the UCOP implemented at further hierarchical level of the evolutionary staircase, able to **predict** the code's double / single helical nucleotide architecture and function as this is currently known and understood (Marcer and Schempp, 1986; Gariaev *et al*, 2001, 2002; Rowlands and Hill, 2006; Clement *et al*, 1993). For then, in agreement with the known fact of the genetic code's ability to generically describe both the architecture and functioning of all living systems, including that of the intelligent human brain, the solution proceeds from a simple repetition of this argument. That is, it can be inferred that the human brain (encoded within the genetic code) constitutes a further implementation of the UCOP at a yet further hierarchical level of complexity on the evolutionary staircase. For such a **prediction** – which must describe the human brain's neural / gial structure and its functioning as it is known from experimental observation to be valid (Deutsch, 1985; Eccles, 1986; Noboli, 1985, 1987; Marcer, 1986; Schempp, 1986, 1992, 1993; Hoffman, 1989; Penrose, 1990, 1998; Pribram, 1991; Clement *et al*, 1992; Marcer *et al*, 1997, 1998 a, 1998 b, 2001; Tuszynski *et al*, 1998; Sutherland, 1999; Perus *et al*, 2003) – would explain both the human brain's innate natural semantic language abilities and its capability to understand the universe that conceived it (i.e. its human scientific intelligence) in terms of the UCOP.

The UCOP's Initial Confirmatory Predictions

From diagram I it is seen that the UCOP proposed gives rise to the basic laws of physics in a recognizable quantum mechanical form – that of the nilpotent generalization of Dirac's famous quantum mechanical equation D(N) – as its universal computational order code. This equation **predicts** the simultaneous emergence by spontaneous symmetry breaking from its empty state, of 3 + 1 relativistic space-time and the strong, weak, and electromagnetic (inertial) quantizations including spin of Standard Model elementary particle physics diagram II as experimentally validated. That is to say these constitute the fundamental level of quantum mechanical structure in terms of which all further higher levels are to be described, so that the quantizations specified are the sole elementary particle sources and sinks of the now 3 + 1 relativistic space-time quantum field.

Moreover the periodic 'table' (the determinant for the whole of chemistry) of the (atomic) elements for which the universal organization principle predicts a spiral evolutionary atomic staircase, as the consequence of 'fusion' and 'fission' (the universal organizational principle's two hypothesized fundamental computational productions at this hierarchical level of structure where these act on the simplest UCOP composite neutral atomic state / unit, neutronium) is shown in diagram III.

Diagram III



This new spiral presentation for the periodic table begins with neutronium, not usually considered an element (but which cosmically is as abundant as oxygen). It situates hydrogen next to carbon which chemically it most resembles. Such a spiral emphasizing the fact that the elements form a continuum, rather than a series of blocks, is in excellent accord with single heat bath thermodynamic quantum Carnot engine[2] where the quantum phase θ follows such a time reversal asymmetric spiral behaviour. (*Illustration, courtesy of Philip Stewart.*)

The Proposed UCOP

The UCOP proposed is that of nilpotent universal computational rewrite system (NUCRS) which has a universal grammar as discovered by Rowlands and Diaz. where each new symbol of its alphabet can stand for itself, a sub-alphabet or its infinite alphabet (Rowlands and Diaz, 2002; Diaz and Rowlands, 2005). That is, the NUCRS has required ability by means of the introduction of a new symbol which can stand for its infinite alphabet to replicate itself at further hierarchical levels of the evolution staircase of complexity, as is required above.

All the evidence, we have so far, including that cited below in diagrams I, II and III thus supports the hypothesis that the nilpotent quantum mechanical language description, NUCRS, constitutes 'Nature's rules' so as to formalize the Premise and Mission Statement of the British

Computer Society's Cybernetics Machine Group, that 'In science, Nature sets the rules, but it must never be forgotten, that it is only because life has exploited these rules successfully for billions of years to our evolutionary advantage, that human brains are able to understand them. The mission, at the physical foundations of computing / information processing if one accepts the premise, is therefore to identify how these rules were exploited to achieve this end.' That is to say, the NUCRS turns this premise into an **Evolutionary Anthropic Semantic Computational Principle,** which can indeed be identified with the principal stages needed to accomplish the mission, as set out in the premise. It also strongly advances the claim that the NUCRS generalization of the computational rewrite concept can be taken as a new fundamental computational foundation for both quantum mechanical and mathematical language description, so as to constitute, we would propose, from these stages, a likely basis for a 'Theory of Everything'. We may thus hypothesize that the universal grammar for semantic quantum mechanical mathematical language description that constitutes the nilpotent quantum mechanical formalism is a candidate for 'alternative (a)' in Leggett's incisive analysis of 'The Quantum Measurement (QM) Problem' (Leggett, 2005), which says that 'QM is the complete truth about the physical world (in the sense that it will always give reliable predictions concerning the nature of experiments) at all levels and describes an external reality'. It also says that the nilpotent methodology must generate all the physical constants so that they can known without empirical determination, in accordance with the statement of Einstein that 'In a sensible theory, there can be no numbers whose values are determinable only empirically. I can, of course, not prove that .... dimensionless constants in the laws of nature, which from a purely logical point of view can just as well have other values, should not exist. To me in my 'Gottvertrauen' (faith in God) this seems evident, but there may well be few who have the same opinion.' (Einstein., 1982)

That is to say, that this methodology for prediction would be, we can infer the totally exhaustive means of testing NUCRS's correctness, at each stage of the evolutionary staircase, where it the staircase will repeatedly exhibit nilpotent closure.

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Synchronization - the Font of Physical Structure [1]

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Abstract. **The computational operation called *sychronization* is vital for realizing multi-process systems. I show that expressing synchronization in Clifford algebra yields entire families of causal entities, as well as profound insight into basic process structure: identification of *the* primitive causal atom, and the *emergent* mechanism of process non-determinism. Synchronization's fundamental nature, which is independent of its physical realization, implies that these results should be mirrored in physical systems, such as quantum mechanics, and I present evidence for this novel *synchronizational* interpretation of quantum mechanics.**

Keywords: synchronization, causality, non-determinism, emergent, quantum

1. Introduction

Synchronization is unique among the instructions routinely executed by contemporary computers, in that unlike all the others, it is by definition *transparent* to the computation executing it. This is so because synchronization addresses the interaction *between* sequential programs, which interaction must not affect the correct operation of the individual interacting programs themselves. A typical use of synchronization is to assure that two or more program processes exclude each other in their access to some shared entity, eg. a printer, a disk or memory block, an I/O port, etc. Operating systems, real-time systems, and the internet would be literally impossible to construct without synchronization instructions.

A primitive synchronizer T consists of a notional internal binary flag - Open or Closed - that can be changed by two operations: *Wait* and *Signal*, denoted hereafter by W and S. [The restriction to binary behavior implies no loss of generality.] A synchronizer must supply the following behavior:

1 To appear in the Proceedings of CASYS 07, Liege, Belgium, August 2007.

S_out
↑
W_in → T → **W_out**
↑
S_in

A Signal sets T to Open, and passes the Signalling process thru;
Successive Signals are the same as a single Signal;
A Wait on Closed T fails & the waiting process is not passed thru;
A Wait on Open T sets T to Closed; pass waiting process thru;
Simultaneous Waits on the same T => max 1 Waiter passes thru;
Simultaneous Signals on the same T = a single Signal.

In the above diagram, Waits enter from the left and exit to the right; similarly, Signals enter from the bottom and exit at the top. The exclusion of processes over (say) a printer is realized by placing the use of the printer on the W_out leg, and thereafter directing the process to perform a corresponding S_in before exiting entirely. This arrangement guarantees that processes will use the printer *serially* - otherwise, output from different processes would be meaninglessly interleaved on the paper record, which is why synchronization is necessary in the first place. More complex examples can be found in any good operating system textbook.

Implicit in such arrangements is the requirement that synchronization be *transparent* to the participating processes: it would be unacceptable for the correct operation of a program to be dependent on whether it "really" waited to acquire some resource because some other process(es) *happened* to be present. Hence, no information in the Shannon sense is conveyed between two processes via the act of synchronization. Rather, synchronization induces/enforces a non-deterministic ordering of the processes as they pass through the synchronizer.

Mathematically, a synchronizer establishes a partial order on the events W and S, such that a Wait never succeeds unless it has been preceded by a Signal. Physically, this ordering is tantamount to imputing a causal relationship between the S and the subsequent W. Thus one would expect that a mathematical treatment of the synchronization mechanism will cast new light on such matters as causality and it's quantum cousin, non-determinism. This expectation is grandly satisfied, as will become clear.[2]

The analysis of synchronization presented here approaches the issues via a Clifford algebra G [3,4,5] whose generators, the 1-vectors $\{a,b,c,\ldots,x,y,z\}$, represent the boundary[3] of the system or entity in question *vis a vis* its surround. These vectors take their values from $Z_3 = \{0,1,2\} = \{0,1,-1\}$. In practice, a 1-vector will only have magnitude ± 1; *zero*, on the other hand, denoting the mutual

2 But no discussion here of the uncertainty principle; the Event Window mechanism of [1] is however my basis for understanding it.

3 'Boundary' in the homological sense; this will not be elaborated further here.

exclusion of these values, $x + (-x) = 0$, implies the interpretation "can/does not occur". One can think of the 1-vector x as a one bit "sensor", with $x = +1$ denoting the current existence, in the surround, of whatever x senses; and $x = -1$ denoting oppositely that whatever x senses does *not* currently exist in the surround.

These definitions imply the following:

$|x| = 1$
$1+1 = -1$, whence
$X+X+X = 0$ for any expression X in the algebra

The algebra's + operation denotes *concurrent* existence, understood as the opposite of "enforced mutual exclusion". This makes sense because co-existence is implicitly mutual and commutative: $x+y$ and $y+x$ both mean that x and y co-exist. The use of + to represent the formal parallel composition of processes is common in the CS theoretical literature (though the use of vector algebras, as here, is not). Cf. also $x + (-x) = 0$ just above.

[*Some asides:*

0. Notation: lower case letters $\{a,b,c, ..., x,y,z\}$ denote ordinary 1-vectors; upper case letters {A,B,C, ..., X,Y,Z} denote arbitrary expressions ("multi-vectors") in the algebra; the product xy is a 2-vector, xyz is a 3-vector, etc. Expressions written with $\{x,y,z\}$ are *generic* forms, with x,y,z chosen from $\{a,b,c,...\}$ without duplication and with arbitrary sign (modulo local context in the case of ambiguity). Thus $x+yz+xyz$ can represent $a-bc-abc$, $-b+ac+abc$, etc; in that the algebra is exceedingly symmetric, it is common that expressions having the same form also have the same algebraic properties. Nested parenthesized expressions specify more complex computations.

1. The 1-vector generators of the algebra are the "logical bottom", so x cannot take on the superposed value ± 1; superposition enters the picture with the algebra's anti-commutative product.

2. The choice of $Z_3 = \{0,1,-1\}$ removes the ambiguity present in $Z_2 = \{0,1\}$, where zero wears two hats: the opposite of one, and Void. In Z_3, the opposite of +1 is -1 and vice versa; zero (Void) is a meaningless 'value' for a vector, and occurs only as the result of sums or products (ie. multi-party computations).

3. The restriction to Z_3, disallowing such expressions as $x+2y+3z$, is not viewed as such, since this expression can be re-written as $(x+y+z)+(y+z)+z$, which contains the same information regarding bottom-line existence, "ground states" so to speak; the algebra's distributivity and associativity guarantee that the only effects that will be missed are those that depend on cancellation mod 3, ie. coefficients larger than unity function as amplitudes. Clearly, expansion to Z_n, n prime, is desirable at some point, but as will be seen, fundamental outlines appear when the picture is unconfused by matters of multiplicity. I see no pressing need for expansion into the real continuum.
End asides.]

Besides co-existence, the other thing that can happen with two processes is that they interact, ie. they "operate" on each other. For this we use the algebra's anti-commutative multiplication, which denotes *action*:

$xy = -yx$ for distinct 1-vectors x,y [The canonical ordering is alphabetical]
$xx = +1$

The anti-commutative property applies *only* to 1-vectors; in general, $XY \neq -YX$, though simple non-commutativity is common. Application of the above rules for addition and multiplication yields the fact that $(xy)(xy) = -1$, that is, xy is a representation of $i = \sqrt{-1}$. Thus the algebra implicitly incorporates all the felicities of complex numbers, and indeed, exhibits a plethora of i's, often called pseudo-scalars, ... and spinors. The n-vectors $x...y$ expresses logical *xor* compactly, and other expressions in G_2 express the logical *and* and *or* operators [2].

Finally, the algebra is associative and distributive as usual:

$$X+Y+Z = (X+Y)+Z = X+(Y+Z)$$
$$XYZ = (XY)Z = X(YZ)$$
$$X(Y+Z) = XY+XZ \quad \text{and} \quad (Y+Z)X = YX+ZX$$

This very simple Clifford algebra over $Z_3 = \{0,1,-1\}$ is remarkably expressive, containing

Idempotents, XX=X, eg. $(-1+x)$, $(-1+x+y+xy) = -(1-y)(1-x)$
Nilpotents, XX=0, eg. $x+xy$, $x+y+z$, $xy+xz+yz$
Bell and magic operators, cf. entanglement [2].

Given this algebraic apparatus, *computational processes are represented directly and literally by the expressions of the algebra.* Sums express concurrent activity; this is a formal addition: subtraction X-Y is understood as addition of the negative: X+(-Y).]. Products express *action.*

The following general properties of Clifford algebras should be noted:

- The full specification of a Clifford algebra is G(p,q), where p is the number of generators that square to +1, and q the number that square to -1. Our algebra is thus G(n,0) = G_n. The Pauli algebra, which spans quantum mechanics, is isomorphic to G_3.

- The set {*1, x, y, z, xy, xz, yz, xyz*} forms an ortho-normal basis for a $2^3 = 8$ dimensional space; similarly, n generators produce a space of 2^n dimensions. These spaces express *distinctions*[1], and *must* not be confused with relativity's 3+1 space, which latter I insist must be constructed from the former.

- ***Theorem.*** For any expression X in the algebra, XX ≠ 0, X has no inverse iff X has an idempotent factor.

Having mentioned quantum mechanics, the reader may have noticed that nothing has been said of probability distributions and the like: our Z_3 algebra is finite and discrete, quite unlike the continuous [-1:+1] space of correlations. On the other hand, concurrent computational systems are *inherently* non-deterministic, and it is argued that the present computational view, via its discrete and finite combinatorics, obviates the current source-less probabilistic skin over the actual goings-on. It will become clear that in the end, although mechanism endures, *one must give up determinism.* Period.[4]

It is of course the author's hope that the approach will ultimately translate into a novel *computational* physical theory. Being computational, such a theory is necessarily *constructive*, and hence can supply the (non-material, information-based) synchronization *mechanism,* which lack of mechanism has for so long hampered our understanding of the quantum world. The story begins with ordinary sequential processes.

4 Also, the "many-worlds" hypothesis is obviated, though I do not argue the case here.

2. Sequential processes

A *sequential* program, unrolled into its future, forms a system consisting of a *single* process, namely itself - there is no talk of other processes: even if they're present, any synchronization is transparent, and any interference oblique and unrecognized. The single most important property of a process is that it is a *sequence*: the *order* in which its events take place is crucial, *defining* in effect what the process does. As will be seen, it is crucial not to confuse the three concepts of ordering/sequence, determinism, and causality, as occurred in the early years of quantum mechanics.

Let X,Y,Z be arbitrary expressions in the algebra, and consider the product XYZ, which states the process "do Z, then do Y, then do X", that is, we *always* operate on the left. If any of X,Y, or Z has an inverse, we could algebraically manipulate XYZ to produce some other order. This will not do! Rather, to enforce sequence, we will require that none of X,Y,Z has an inverse. In physical terms, this means that they are irreversible and *time-like,* and we will intend these three terms interchangeably, as well as their opposites, possessing an inverse = reversible (which expresses wave-like activity) = *space-like.* [Again, this is *not* physical 3-D space, just space-like rotations.]

Taking this reasoning further, if X,Y,Z have any reversible factors, they can all be moved to (say) the end of the sequence, leaving the sequence to consist of only irreversible factors with a final reversible postlude, which can therefore be excluded entirely from consideration without loss of generality. Therefore, *a sequential process is represented by the product of irreversible factors, namely idempotents* (ie. SS=S), whose order therefore cannot be changed.

For example, suppressing *much* detail, the vanilla sequential program *Add A; Add B; Add C* would translate to the sequence *(-1 + AddC)(-1+AddB)(-1+AddA),* where it is assumed that *(-1+AddX)* is idempotent. However distant this may seem from an actual implementation, it captures the fact that ordinary computation *is* irreversible at each step due to its use of logical AND in some form.[5] So, invoking the above-mentioned theorem and the factoring-out of reversible factors, the example is then not all that far from computational reality after all.

So far, so good. To get a feel for how to use this algebraic representation of computation, analyzing the ***if-then-else*** construction is a good warming-up exercise. We will write ***if*** V ***then*** X ***else*** Y, where V,X,Y are arbitrary expressions representing arbitrary computations. For simplicity and with

5 'Infinitely slow' computation can be reversible, but a discussion would take us too far afield.

no loss of generality, take V = a, a 1-vector ("sensor").

"***if*** *a*" implies a probing of the current state of a: is it +1, so do X; or is it -1, so do Y.

Given that the only relevant states of a are ± 1, the next question is how to ascertain which of these obtains. Clearly, said 'ascertaining' requires measuring a, where again idempotent operators play the central role. Consider the following identities:

$(1+a) = (1+a)(a)$	$(-1+a) = (-1+a)(-a)$	$(-1+a) = (1-a)(1-a)$
$(1-a) = (1-a)(-a)$	$(-1-a) = (-1-a)(a)$	$(-1-a) = (1+a)(1+a)$

Taking P: $(1+a) = (1+a)(a)$ as an example, multiply P's rhs out to get $a+aa$, whence we see that the +1 in the lhs can be seen as the product of a with itself. It follows, and this is the key point, that if the a we have in hand - in the rhs's "$(1+a)$" factor - has the same sign as the a we probe - the rhs's "(a)" factor - then the *sign* of the scalar will be +1, whereas if the a we probe is actually $-a$, then the scalar will be -1. This also applies if P, oppositely, specifies "$(-a)$" and we find $-a$ ($\Longrightarrow$ +1), or we find $+a$ ($\Longrightarrow$ -1). Finally, take *just* the scalar value from $(1\pm a)(\pm a)$ to complete the measurement (one can only actually measure scalars... like a meter reading).

This is the basic act of measurement. Because $(1+a)$ has no inverse, the act of measurement is irreversible, in accordance with contemporary understanding of the equivalence of energy and (Shannon) information. Furthermore, successive measurements using the idempotent form yield no new information, in that PP=P. [6]

So now we know how to do "***if*** *a*": we will write $(1+a)(a)$ or suchlike, depending. The next issue is to choose the correct continuation depending on what the measurement on a produces.

The basic idea is to arrange for the conjugate forms (1+a) and (1-a), whose product is zero, to collide on the unwanted branches of the ***if***, thus eliminating those continuations. A zero means the computation's future is empty - zero*product = 0, ie. it does not occur; generating a zero to express/denote an empty continuation is a key tool in the following.

Therefore, write the test in the ***if*** as a probe: $1+a$ or $1-a$, acting on the actual a, which can be plus or

6 Actually, $(1+a)$ is the square root ("sqert") of an idempotent, cf. the third column above, but this is unimportant here.

minus. The ***then*** and ***else*** branches apply respectively $1+a$ or $1-a$ to the result of the test, whence one of them should yield 0 (because conjugate) and the other the correct continuation based on the observed value of a. There are four possibilities (the '|' marks off visually (only) the common ***if***-header/probe, copied from the 'probe' column, rightmost because it occurs first):

	probe	left branch	right branch	
	if ***then***	X ***else***	Y	
1	$(1+a)(+a)$	$X(1+a)\|(1+a)(a)$ $= -X(1+a)$	$Y(1-a)\|(1+a)(a)$ $= 0$	Correct-left branch only.
2	$(1+a)(-a)$	$X(1+a)\|(1+a)(-a)$ $= X(1+a)$	$Y(1-a)\|(1+a)(-a)$ $= 0$	Not correct -need both 0
3	$(1-a)(+a)$	$X(1+a)\|(1-a)(a)$ $= 0$	$Y(1-a)\|(1-a)(a)$ $= Y(1-a)$	Not correct -need both 0
4	$(1-a)(-a)$	$X(1+a)\|(1-a)(-a)$ $= 0$	$Y(1-a)\|(1-a)(-a)$ $= -Y(1-a)$	Correct-rightbranch only

In situation 1 above, we probe for $+a$ with $(1+a)$, and a is in fact $+a$; situation 2 has the same probe, but discovers $-a$; situation 3 probes for $-a$ but discovers $+a$; and situation 4 probes for $-a$ and discovers $-a$. Notice that if we consider all four possibilities concurrently (ie. Left + Right), we get zero: this situation (namely, a having both values simultaneously) cannot occur. So instead, combine 1&2, 3&4 by subtraction to get the desired terms to double instead of cancel: 1-2 = $+X(1+a)$; 4-3 = $+Y(1-a)$, and move the | -cue to the right, eliminating the common probe-preface of the previous version:

1 minus 2: $-X(1+a) \mid (\pm a)$

4 minus 3: $-Y(1-a) \mid (\pm a)$

Finally, run 1-2 and 4-3 concurrently (ie. add): $-X(1+a) \mid (+-a) - Y(1-a) \mid (\pm a)$

$$= [\, -X(1+a) - Y(1-a) \,] \mid (\pm a)$$

If $a = +1$ then the Y term drops out leaving +X; and if $a = -1$ then the X term drops out, leaving +Y. Just as we wanted! Push the minus-signs into the parentheses:

$$= [\, X(-1-a)+Y(-1+a)\,] \mid (\pm a)$$

and we see that doing ***if-then-else*** necessarily invokes observation, ie. idempotents, not sqerts, consistent with thermodynamic and quantum measurement theory. The form also makes good computational sense when multiplied out:

$$= X(-1-a)(\pm a) \quad + \quad Y(-1+a)(\pm a)$$

which transparently describes two independent processes X and Y, each independently and concurrently testing for its own condition, only one of which will succeed.

NB: if one measures simultaneously with $1+a$ and $1-a$, one gets (summing) an inversion ($1+1=-1$), but no knowledge of a's actual value, in accordance with quantum measurement theory: if one is to get information, one must specify *exactly* what it is one is looking for ... $+a$ or $-a$, and this *cannot* be finessed.

3. Synchronization in the algebra

Having warmed up with ***if-then-else***, we now tackle synchronization's Wait and Signal. From the introduction, the required behavior is

a. A Signal sets T to Open, and passes the Signalling process thru;
b. Successive Signals are the same as a single Signal;
c. A Wait on Closed T fails, ie. the Waiting process is not passed thru;
d. A Wait on Open T sets T to Closed, and passes the Waiting process thru;
e. Simultaneous Waits on the same T result in at most one Waiter passing thru;
f. Simultaneous Signals on the same T are the same as a single Signal.

Items *e* and *f* refer to situations where there is competition between multiple Waiters and/or Signallers; this complication will be deferred for the moment.

The first step comes from item *b*, which in effect says SS = S, ie. S must be *idempotent.*

Item *d* says that WT must succeed if T is Open. Therefore initialize T to Open, which we can do via item *a* by setting T=S. Item *d* then reads WT = WS, which must be non-zero to succeed.[7]

Item *c* in effect says (together with item *a*) that successive Waits without an intervening Signal must fail. That is, WW=0, so W must be *nilpotent.* So now we know the shapes of both W and S, and very special ones at that.[8]

These considerations imply that a sequence like WSSSWSWSST = WSWSWST = WSWSWS, and any sequence with consecutive W's yields zero: WWSWST = 0.

Process-wise (see figure just below), there is process P1, which after a sequence of arbitrary irreversible operations *X* issues the signal S, creating a so-called 'synchronization token'; and then there is process P2 which after a sequence of Y's consumes this token by Waiting on it, whereafter P2 continues, executing *Y*'s (read right-to-left: things begin on the right!]:

P1: ...XXXS*XXX*... <–
\ X, Y, *X*, *Y* are arbitrary distinct irreversible actions
P2: ...*YYY*WYYY... <–

Despite the visually implicit timeline in the above two sequences, the Wait can occur any time 'before', 'simultaneously with', or 'after' the Signal, but unless the Wait occurs 'after' the Signal, process P2 is logically halted at the W. Whichever of these circumstances obtains, the ultimate result is a logically and physically seamless transition from P1's S*XXX* to P2's *YYY*W. *This sequence too is a process*, process P3:

P3: ...*YYY*WS*XXX*...

The fact that W *must* be nilpotent means that 'whenever' the WS mating actually occurs, it is just as

7 Initializing T to W (ie. T is initially Closed) doesn't work: WT = WW = 0, whence SWT also yields zero, which it shouldn't. Initializing T to 1 (which is idempotent) is indiscriminant - *any* W will succeed.

8 I am embarassed at how easily this (finally!) goes, considering the time spent considering the problem. My big mistake was thinking that WW=W, ie. that successive unsuccessful Waits are a no-op, just like successive Signals; the error is that the point-of-view must be from *inside* T, whereas the WW=W view, typical of the computational world, is from *outside* T.

though P3 occurred seamlessly. An example: when one absorbs a photon in the retina, at that very instant one is exactly connected with the state that generated the S - even if the star that generated the photon has 'long since' disappeared.[9]

P1 and P2 are *classical,* in that we imagine them to be deterministic - good old-fashioned Newtonian / Einsteinian processes. (We might think of the state preparations preceding an actual quantum experiment, which are classical.) P3, on the other hand, is *non-deterministic,* because it was precisely P2's Wait that succeeded, leading to the *Y*'s. If instead it had happened that some P4's Wait occurred ahead of P2's, then P3's continuation would be in P4 - ie. entirely different. Notice that even though P3 is non-deterministic, every one of its constituent events has a distinct causal basis.

This *emergent* non-determinism is old news in computer science, though it is most often noted in the form of unwanted *values* (cf. the interleaved printer output example earlier), rather than the entirely proper non-deterministic *ordering* induced by serialization as just described.[10] In both cases - *order* or *value* non-determinism - the root is the *asynchrony* of the interaction of two independent processes. Said a bit differently, *if* one is to use 'process' as a conceptual primitive, *then* one necessarily must accept into the bargain the consequent, unavoidable *emergent* non-determinism born of the asynchronous interaction of these same processes.[11] Both non-deterministic *values* and non-deterministic *order* are produced by asynchrony. I therefore advance the claim that *asynchrony is the very source of QM's non-determinism.*

Order-non-determinism forms the *coarse-grained* skeleton of physical non-determinism. Suppose now that one has guaranteed that only a *particular* Wait-continuation will match a given Signal, so *order* is out of the picture. One *still* doesn't know what one will get from the measurement, cf. ***if-then-else***'s measurement earlier. So within the *order*-skeleton is a second, *fine-grained* source of non-determinism, *value*-non-determinism, induced by the measurements encapsulated in the Signals. For example, the idempotent $-1+xy+xz$ expresses a *value*-changing intrusion into the entity $xy+xz$, which in principle "lives its own (reversible) life" both prior to and subsequent to the measurement.

Popping up conceptually, imagine now P3's form as it evolves into its future. Its sequence of *Y*'s is

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9 It's pretty limited time travel tho - you only get the single bit of information that the photon carries ... not much of a view!

10 Both are the source of the most difficult bugs, because they are namely not repeatable; cf. Ullman's fine novel, "The Bug".

11 It is the *necessity* for exclusion, at *every* step, that dictates that processes be discrete, cf. Planck's constant.

just shorthand for an arbitrary sequence of idempotents, for example $(-1+a)(1-+b)...(-1+r)$. Being idempotents, each of them can act as a Signal to some matching Wait 'out there'. [It is important that they are idempotents, because this means that the event that the Wait is dependent on has actually physically occurred.] Ultimately, if every idempotent in P3 satisfies a Wait, and all those Waits' continuations do the same, the universe will be populated entirely by utterly non-deterministic processes that look like (WS)(WS)(WS)...(WS) - these W's and S's being notionally distinct. In fact, we see that our classical view of P1 and P2 as deterministic processes puts them in a miniscule minority - namely that minority inhabiting/forming classical 3+1 space-time (plus all ordinary sequential computer programs).

Finally, consider the issue of competing Signals and/or Waits for a given synchronizer. Taking the case of multiple *identical* Signals, we can consider combining them concurrently (addition) or as interacting (multiplication). This yields S+S = -S and SS=S, so both possibilities yield the same result, S, in that a sign difference is irrelevant in this context.

Reasoning similarly for *identical* Waits yields W+W=-W and WW=0. Thinking in physical terms, the choice between the two ways of combining can be made in terms of energy. If the event S is such that it is appropriate that it trigger multiple continuations (like a race-starting pistol shot), then additive combination of Waits is the choice. If on the other hand it is appropriate that only one continuation arise from a Wait, then construing the collision of Waits as an actual interaction yields WW=0 and no continuation for either Waiter, which is entirely acceptable, computationally speaking (since they can just try again). Lacking further knowledge or insight, it seems best to assume the worst and define competing identical Waits multiplicatively: WW=0.

There is one more variant, namely that the various competing Waits and/or Signals are *not* identical. It is a fact that for a given T (=S), *several* different W's will yield on-going continuations, and similarly for S's. However, the sum W_1+W_2 need not be nilpotent, and S_1+S_2 need not be idempotent; nor their products (though S_1S_2 is always irreversible[12]). Given that what happens is therefore dependent on more details than the nilpotent or idempotent properties themselves provide, this case requires careful algebraic investigation, leading presumably to the symmetries that define conservation laws. Howsoever, the discussion below ignores this complication, but should not be misleading for that; and I will assume that for a given T, all competing Signals and Waits are respectively *identical*, with their combination as described.

12 Interestingly, (x+xy)(y+xy) = -1+x+y+xy: the product of two nilpotents producing an idempotent.

4. Structures induced by synchronization

I argued above that the processes under examination are in fact all of the form $(WS)^*$, where the * indicates one or more repetitions, and the W's and S's are notionally distinct (ie. not identical, but not competing). Thus the processes $(WS)^*$ are 'sentences' whose 'words' are the various possible juxtapostions of the 'phoneme' W to the 'phoneme' S, each such word being a *primitive causal act.*

With this in mind, the algebra seems to imply that *any* nilpotent W will work with *any* idempotent S, but although many WS pairs are 'compatible', this is not always so. For example, in G_3:

$$S = -1+a+b+c+ab+ac, \quad T = S$$
$$W = a+b+c$$
$$WT = 0$$

That is, WT = WS = 0. Logically, the process ("P3") simply ends; this collision of phonemes might describe an annihilation, but we can at least say that this particular WS pair produces no future - the computation simply ends. The physical interpretation then is that this particular S will not enable a process requiring this particular W as a pre-condition; for example, given that $a+b+c$ is a photon, this means that the condition established by S is unaffected by electro-magnetism.

In G_4, $W = a+b+c$ and $S = -1+d = T$ turn out to produce SWT = SWS = 0. The interpretation would here seem that the interaction WS negates the further existence of T - it can no longer be Signalled. Whatever the interpretation, it is clear that this WS expresses a one-shot event.

Here is a 'compatible' solution in G_3:

$W = a+b+c$	WW = 0
$S = -1+b$	SS = S (the b is arbitrary - could also be a or c)
T = S	Synchronizer T is initially open.
$ST = SS = -1+b$	ST = SS = S, and T is still open.
$WT = WS = 1-a-b-c+ab-bc$	T is now closed...
$SWST = SWS = 1-b$	Ie. SWS = -S
$WSWST = WSWS = -1+a+b+c-ab+bc$	Ie. WSW = -W

Notice here that, unlike the two preceding examples, this WS pair cycles indefinitely between ±S and ±W, what I called 'compatible'. Note that the synchronizing occurs independently (as it were) of the signs of S and W: the synchronizing relationship is one of *orthogonality*, whereas sign differences are 180° apart, ie. in the *same* dimension. The cyclicity reflects the *external* view of T, that it cycles between being Open and Closed, and as well that the virtual synchronization token created by S and consumed by W is continually conserved.

5. Stepping back - Implications

The lesson of these examples is that the mathematics itself - representing actual computational cum physical processes - imposes restrictions - a grammar - on what can happen. It tells us that only certain WS combinations produce on-going processes. Given that WS pairs express causal events, and hence $(WS)^*$ is a causal (though non-deterministic) process, such processes represent the real world of irreversibility, energy expenditure, and entropy creation. These processes are what we see when we experience the world around us, even though we constantly try to fit them into a deterministic, classical framework.

Do the processes described by $(WS)^*$ exhaust the realm of causal events? By the preceding analysis, a sequence of irreversible actions represents what we traditionally mean by 'causality'. Consider the simplest such sequence: (-1+y)(-1+x). Recalling that we always operate on the left, one would say that the action (-1+x) *caused* (-1+y), in that (-1+x) establishes[13] the pre-condition for (-1+y) to occur. But observing that (-1+x) = -*x*(-1+x), we see that (-1+y)(-1+x) = (*x*-*yx*)(-1+x), where (*x*-*yx*) is namely nilpotent. Since this same trick can be used pairwise *ad libitum* on a longer such sequence, we see that *any* such even causal sequence can be expressed in $(WS)^*$ form; in the odd case, one S is left, so the final result is S(WS)*.[14] Furthermore, it can be shown that the G_3 idempotents -1+*xy*+*xz* and -1+*x* + *(y+z)* +- *x(y+z)* are time-like boundaries of the same simplest sequence; the overall analysis generalizes to higher-level sequences and idempotents. I therefore claim that the form WS is *the* causal atom, and that there are no others. Note however that WS, is time-like, which we associate with causality, which in turn is confused with change. .]

Given that the algebra reflects the quantum world - though not in the usual terms - it does not seem

13 *Somehow*... the story is tellingly vague; one could ask, "What prevents writing (1+y) *plus* (1+x) here?"

14For arbitrary X,Y, XX = YY = 1, and for Z = (X-YX): ZZ = 0 if XY = -YX; ZZ = -1+Y if XY = YX; and otherwise mixed (though of course always irreversible), often conjugate in appearance.

unreasonable to try to connect a little more explicitly to the physics. Of course, most of the following hypotheses are probably at least partially wrong, and yet probably also partially right.[15]

Since the algebra in any particular case is finite, we can *mechanically* generate *all* its idempotents S and nilpotents W and directly calculate which pairs produce which processes. This list should then be an exhaustive catalog of what can happen, and by implication, of the 'particles' that are possible. The appendix therefore exhibits a complete list of the nilpotents and idempotents of G_3. Regarding nilpotents, their forms are:

Nilpotents:

Column:	1	2	3	4	5
	$x+xy$	$x+y+z(x+y)$	$x+y+z$	$x+y+z+xyz(x+y+z)$	$xy+xz+yz$
		$=x+y+xz+yz$		$=x+y+z+xy-xz+yz$	
Count:	24	24	8	16	8
Out of:	24	128	8	64	8

Column 2 consists of particular pairs (namely those that form a nilpotent) from column 1:

$$(x+xz)+(y+yz) = (x+yz)+(y+xz)$$

If we instead take triples from column 1, we get column 4, which is itself formed from particular pairs from columns 3 and 5. Thus both sets emergently exhibit pairs with the form $x+yz$, either in two's or in three's. Also, the cube roots of -1 are identical to column 1 with a scalar component: $-1+x+xy$, which (in multiplicative combination) expresses a reversible transition from $x+y$ to $-x-y$; added together in pairs or triples with the scalars cancelling again yields pairs $(x+yz)+(z+xy)$ and triples $(x+yz)+(z+xy)+(y+xz)$. The three 'singlets' $(x+yz)$, and pairs and triples thereof, are all boundaries of xyz, the top element of G_3 (which is isomorphic to the Pauli algebra).

Shifting from nilpotents to idempotents, the criterion for $-1+X$ to be idempotent is that $XX=1$, that is, X is unitary, and thus a persisting entity. That is, X is a *particle*. So, extracting from the list in the Appendix, the particles (in ***bold***) specified by our analysis are:

15 I'm sure some friendly physicist will be pleased to point out any errors, which would be most welcome!

Idempotents:

Count:Of		
6:6	$-1 \pm x$	3 families of 2
24:24	$-1 + x + y + xy$	3 families of 8
12:12	$-1 + xy + xz$	3 families of 4
48:96	$-1 + x + (y+z) \pm x(y+z)$	3 families of 16 [16]
	$= -1 + x + (y \pm xz) + (z \pm xy)$	

[Notice how the algebra neatly glosses individual differences among particles via its variables, which permits this purely *symbolic* statement of familial commonalities.]

Noting that the form $x+yz$ does not exist *alone* in either {W} or {S} - in that it *emerges in pairs* from G_2 forms - causes me to see so-called "quark confinement", and thus to believe that the form $x+yz$ is the basic quarkish 'atom'. The number 48 is also characteristic of this family of particles.

I therefore advance the hypothesis that among these various forms with $x+yz$ and their precursors are to be found the quarks, gluons, and mesons of the standard model of QM. The spin of the 2-vectors *xy, xz, yz* would then express the magnetic component, and the spin of the 3-vector *xyz* the charge, of the entity generated by any particular such combination.

The appearance of photons with G_3 invokes the physics of electro-magnetism, so one can reasonably infer that the nilpotents and idempotents of the G_2 solutions will reflect the physics of this simpler level. Similarly, this reasoning opens the interesting possibility that higher-level nilpotents and idempotents (ie. G_n, n>3) will throw light on the mechanism of gravity and more. The hierarchy of algebras presents a natural and elegant path to unification, though neither attribute guarantees success.

Howsoever, what might we elucidate regarding another mystery, namely dark matter? We know that it is 'dark' because it does not interact with electro-magnetism, so (in particular) W = a+b+c must yield WT = WS = 0. On the other hand, it *does* interact with gravity, so the right combination of W and S must yield non-zero continuations. I hold the view [1] that 3+1 space (and the gravity that shapes it) cannot emerge before a fourth level of complexity, ie. G_4. A weighty argument for this view is that just as superposition and spin 1/2 emerge in G_2 and *exhaust* the information-carrying

16 *(y+z)* must be the same in both positions.

capacity of that level; and that further structure (especially charge) can therefore first emerge in G_3, which exhausts *its* information-carrying capacity; so similarly, gravity can first emerge in G_4.

Thus, we seek level 4 (or higher) nilpotents and idempotents that can mediate our putative gravitational interaction. Consider the following table of powers of n-vectors:

level n	n-vector	(n-vector)^2	
0	1	+1	scalars
1	x	+1	vectors
2	xy	-1	spinors, quaternions (in distinction space)
3	xyz	-1	
4	wxyz	+1	
5	vwxyz	+1	

Clearly, the pattern + + - - + + - - ... is that of powers of $i = \sqrt{-1}$, hence the 4-cycle. Many algebraic properties therefore repeat mod 4 - for example 1-vectors and 5-vectors (with no shared variables) both anti-commute. More to the point, the mod 4 cycling of the algebra means that G_4 is implicitly and inherently scalar-like (G_0), and mass, too, is a scalar quantity. Also noteworthy is that 4- and 5-vectors both square to +1, indicating a non-polar form of interaction, as opposed to the -1 of 2- and 3-vectors, indicating the polarity characteristic of electro-magnetism. So G_4 and G_5 are likely candidates on this score as well.

Unfortunately, G_4 contains $3^{16} \approx 40$ million different expressions, too many for the exhaustive search that produced the G_3 table in the appendix. So although we are stymied at this point, this approach is both promising and pointed.

6. Conclusions

The overall approach described above, of applying vector algebra to computation *qua* computation, seems to have been this author's path alone. This is perhaps not surprising, since computation as commonly understood, ie. ordinary sequential programs and systems thereof, is dominated by an automata-theoretic view that leaves little room for a physics of computation. Nevertheless, whatever theoretical view is taken, the constant fact is that computation is, at bottom, about *mechanism.* As such, any computation-based theory is fundamentally *constructive* - at every stage, it must be specified who does what to whom, and how. For this reason, any physics of computation is *non-redundant*: every statement in the theory must correspond 1-to-1 to the reality it describes. At the same time, computation's way of describing processes is independent of its way of realizing same: how an Add instruction is implemented has zero impact on its actual operation (aside from speed, which is logically irrelevant to this consideration).

In this context, quantum mechanics is famous for the inscrutability of its mechanism - after all, how can one have a finite mechanism that generates the unbounded information inherent in 'randomness'? Furthermore, careful analyses of the formalism of quantum mechanics have limited the scope of any 'hidden mechanism' (cf. "hidden variables") quite severely. It is therefore noteworthy that the present analysis produces non-determinism as a phenomenon that *emerges* when one, tellingly, moves from the consideration of isolated deterministic processes (which isolation is implicitly, but unobviously, classical) to *interacting collections* of same. As noted earlier, the inherent non-determinism of interacting computational processes is well-known in computer science, but connecting this solidly to physics, as here, is (to my knowledge) new.

That the present characterization of synchronization - the key mechanism of multi-process systems - as the product of nilpotent and idempotent forms then generates what appear to be entire realms of insight - a unique primitive causal form and whole emergent families of structure - is perhaps to be expected from such a foundational approach, but satisfying and encouraging nonetheless. Less rosily, the description above suffers greatly from the absence of both a group-theoretic anatomy and concrete input from physics; hopefully others will be encouraged by the results so far to contribute. In any case, the identification of WS as *the* primitive causal atom is new.

It should be noted that Rowlands et al [6] have over the last decade developed what has been called a "nilpotent factorization" of quantum mechanics. I am tempted to believe that their approach and the present paper both describe the same line of thought, and to hope that this is so. At the present time, however, this must remain speculation, however unlikely it would seem that they, both featuring nilpotence, would be entirely different characterizations of reality.

Finally, the word "font" in the title of this paper has two meanings. One is the once obscure but now familiar type-setting term denoting the physical, re-usable form underlying actual printed letters. The various forms that W, S, and WS can take are indeed the font that Nature uses to write out physical processes.

The second meaning for "font" is a *fountain*, a source, here, of insight: for each level, and each higher 4-cycle of the algebra, ad infinitum, a constantly self-renewing well, from which we *try* to drink. But for all our theories, Nature has the final say; as Emily Dickinson, the supreme lyricist of the English language, wrote [7]:

Tell all the truth but tell it slant-
Success in Circuit lies
Too bright for our infirm Delight
The Truth's superb surprise

As Lightning to the Children eased
With explanation kind
The Truth must dazzle gradually
Or every man be blind -

7. Acknowledgments

The algebraic calculations were done using a purpose-built symbolic Z_3 Clifford algebra system programmed by Doug Matzke, whose thesis [2] connecting the standard QM formalism to this algebra I am also pleased to acknowledge. Thanks also to Peter Marcer for encouraging me to write this paper; and to the members of the Alternative Natural Philosophy Association (ANPA), especially Clive Kilmister, Ted Bastin, H. Pierre Noyes, and Keith Bowden.

8. Appendix

G_3 contains $3^8 = 6561$ expressions, including 0 and 1; in all there are 81 nilpotents (including 0), and 92 idempotents (including 0 and 1).

<u>G_3 nilpotents</u>

$x + xy$:

8	$a+ab$	$-a+ab$	$a-ab$	$-a-ab$	$a+ac$	$-a+ac$	$a-ac$	$-a-ac$
8	$b-ab$	$-b-ab$	$b+ab$	$-b+ab$	$b+bc$	$-b+bc$	$b-bc$	$-b-bc$
8	$c-bc$	$-c-bc$	$c+bc$	$-c+bc$	$c-ac$	$-c-ac$	$c+ac$	$-c+ac$
=24								

$x + y + z(x + y)$:

4	$a+b+ac+bc$	$-a-b+ac+bc$	$-a+b+ac-bc$	$a-b+ac-bc$	All have even # of -'s
4	$-a-b-ac-bc$	$a+b-ac-bc$	$a-b-ac+bc$	$-a+b-ac+bc$	"
4	$-a+c+ab+bc$	$a-c+ab+bc$	$a+c-ab+bc$	$-a-c-ab+bc$	All have odd # of -'s
4	$a-c-ab-bc$	$-a+c-ab-bc$	$-a-c+ab-bc$	$a+c+ab-bc$	"
4	$b+c+ab+ac$	$-b-c+ab+ac$	$-b+c-ab+ac$	$b-c-ab+ac$	All have even # of -'s
4	$-b-c-ab-ac$	$b+c-ab-ac$	$b-c+ab-ac$	$-b+c+ab-ac$	"
=24					Just 24 of 8 x16 = 128

$x + y + z$:

8	$a+b+c$	$-a+b+c$	$a-b+c$	$-a-b+c$	$a+b-c$	$-a+b-c$	$a-b-c$	$-a-b-c$
=8								

$xy + xz + yz$:

4	$ab+ac+bc$	$-ab+ac+bc$	$ab-ac+bc$	$-ab-ac+bc$
4	$ab+ac-bc$	$-ab+ac-bc$	$ab-ac-bc$	$-ab-ac-bc$
=8				

$x + y + z + xyz(x + y + z)$:

4	$a-b+c+ab+ac+bc$	$-a+b-c+ab+ac+bc$	$-a+b+c-ab+ac+bc$	$a-b-c-ab+ac+bc$
4	$a+b+c+ab-ac+bc$	$-a-b-c+ab-ac+bc$	$-a-b+c-ab-ac+bc$	$a+b-c-ab-ac+bc$
4	$-a-b+c+ab+ac-bc$	$a+b-c+ab+ac-bc$	$a+b+c-ab+ac-bc$	$-a-b-c-ab+ac-bc$
4	$-a+b+c+ab-ac-bc$	$a-b-c+ab-ac-bc$	$a-b+c-ab-ac-bc$	$-a+b-c-ab-ac-bc$
=16				Just 16 out of $2^6 = 64$

G_3 idempotents

Count:Of

92:140	In all.	Below: *x,y,z* taken (distinctly) from {*a,b,c*}
2:2	0, 1	
6:6	*-1+-x*	3 families of 2
24:24	*-1+x+y+xy*	3 families of 8
12:12	*-1+xy+xz*	3 families of 4
48:96	*-1 +x + (y+z) ± x(y+z)*	3 families of 16; *(y+z)* must be the same in both positions:

-1+a+b+c-ab-ac = -1+a + (b+c) - a(b+c)
-1-a+b+c-ab-ac = -1-a + (b+c) - a(b+c)
-1+a-b-c-ab-ac = -1+a + (-b-c) + a(-b-c)
-1-a-b-c-ab-ac = -1-a + (-b-c) + a(-b-c)

The other 48 of the 96 are reversible; in fact, they are 40th roots of unity, w/ -1@^20.

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A partial List of CLRU Working Papers

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Cambridge Language Research Unit was one of the antecedents of ANPA. Here is a partial list of their largely unpublished working papers. It is hoped to update the list soon. Any contributions or other information on the work of CLRU would be gratefully received. Yorick Wilks has recently published a book on the work of Margaret Masterman. Jeremy Kessler is working on an MPhil (for HPS,Cambs) on CLRU.

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Masterman ???. "The EEC test sentences: an exercise in pre-editing" CLRU, Final Report of TH-17, Annex 5.

Masterman and Smith, R.J. ???. "The Subsensu Experiment" CLRU, Final Report of TH-17, Annex 2.

Masterman and Smith ???. "The 'Skeletal# Nature of Systran" CLRU, Final Report of TH-17, Annex 3.

Masterman and Smith ???. "The Role of Humour" CLRU, Final Report of TH-17, Annex 4.

Masterman ???. "Note for Translators: How to Read Hex" CLRU, Final Report of TH-17, Annex 5.

Masterman ???. "Routine for finding the stress pattern of English" CLRU, Final Report of TH-17, Annex 7.

Masterman and Williams, B. 1983. "Patterns of emphasis and heterodynes of meaning" CLRU, CLT007, March.

Masterman, Parker-Rhodes, Blackmore, R. M., Sparck Jones **1958.** "Description of the tests carried out on methods of constructing sentence lattices" CLRU unnumbered work paper

Masterman and Needham 1966. "Natural Language Analysis as **non-numerical data processing**" CLRU unnumbered workpaper, January.

ANPA29

The next meeting of ANPA will be held in the New Common Room, Wesley House, Jesus Lane, Cambridge CB5 8BJ from Monday 30th July to Friday 3rd August 2007 (*five* days only). The Executive Council Meeting will be held Sunday 29th July, venue to be arranged.

The House Manager there is Iain Murton. His email is idm23@cam.ac.uk and the telephone number is 01223 741033, fax 01223 321177. Only contact Iain directly in an emergency, or if there is a need that *cannot* be satisfied by either Arleta Griffor or Keith Bowden. The Bursar is Jim Sturmey and the Principal is Philip Luscombe.

Could all members please adhere to the following rules

1. NO smoking anywhere indoors under pain of death.

2. The shower and kitchen facilities near the rooms in Wesley House are communal and you may use them. Please make sure that you leave the kitchen and all other facilities clean and tidy and that you do all your washing up promptly. Do NOT take other people's shampoo, conditioner, milk, bread etc without their permission!

3. Please read the Fire Regulations in the corridors and follow them accordingly.

4. Be aware of security issues. Please keep all downstairs windows and doors locked at night. Please keep the main doors to each corridor locked all the time.

5. *Nobody* is to be brought onto the premises without agreement with Keith Bowden specifically. The same goes for any expensive personal or professional equipment, with the exception of laptops. There are important insurance and security issues here.

6. The gates to Wesley House are locked between 10.30 and 11.30 pm and are not reopened until morning. You are provided with a key to the side entrance ten yards to the right of the main gates when facing Wesley House. You may use these out of hours but make sure that you return QUIETLY. ANPA has been criticised in the past for making a row in the evenings. There will be other people staying at Wesley House. Please respect their needs.

7. Breakfast is provided as usual ONLY for those who have paid for it.

8. Please pay any money owing, to either Arleta Dylus or Keith Bowden. Spare copies of the Proceedings are £25 each. I have some spare copies of the Special ANPA Issue of IJGS. These are £25 each payable to me (Keith).

9. There may be someone recording talks in the meetings. If you do not wish to be recorded please make this clear before your talk.

10. Copies of the timetable and maps of Wesley House and Cambridge will be made available. Coffee and tea will be provided in the breaks. For those that want to lunch together the Bun Shop on the corner of Malcolm St and King St (five minutes from Wesley) is a popular venue.

11. You may use the chapel with the obvious provisos.

12. You may use the facilities in the New Common Room at any time that there are no talks. There is table tennis, pool, TV, video and tea and coffee making equipment. Please give yourself adequate time to put everything way before the next talk. Please try to keep the Common Room clean and tidy at all times and do any washing up promptly.

13. Any extra requirements that Keith or Arleta cannot satisfy, such as irons, mirrors etc. may be addressed to Iain Murton, but please remember that he has other responsibilities as well. In emergencies contact any one of us.

14. The usual niceties at such meetings, such as sitting down or stopping talking when the Chairman of the Meeting requires it, WILL be observed.

15. At the end of the meeting please RETURN ALL KEYS either to me, Arleta or Iain Murton.

Have a great meeting.

Keith

Alternative Natural Philosophy Association [28]

Statement of Purpose

1. The primary purpose of the Association is to consider coherent models based on a minimal number of assumptions, so as to bring together major areas of thought and experience within a Natural Philosophy alternative to the prevailing scientific attitude. The Combinatorial Hierarchy, as such a model, will form an initial focus of our discussions.

2. This purpose will be pursued by research, publications and any other appropriate means including the foundation of subsidiary organisations and the support of individuals and groups with the same objective.

3. The Association will remain open to new ideas and modes of action, however suggested, which might serve the primary purpose.

4. The Association will seek ways to use its knowledge and facilities for the benefit of humanity and will try to prevent such knowledge and facilities being used to the detriment of humanity.

Organisation (altered to reflect the current situation by KGB)

1. The Founder of the Association was Pierre Noyes. The Founder Members were Pierre, John Amson, Ted Bastin, Clive Kilmister and Frederick Parker-Rhodes. They will be known herein as the Founders. The Executive Council is the governing body of the Association. It consists of:

   (a) The Founders and all past Presidents of the Association, the President, the Co-ordinator and the Treasurer,

   (b) Ordinary members nominated by classes (a) and (b), who serve for three years, with the possibility of re-nomination.

2. The Members of the Association are (a) the members of the Executive Council and (b) others nominated by the Members and approved by the Executive Council.

3. The President is the official representative of the Association in external affairs, and has the responsibility for calling meetings of the Executive Council, at least annually, for the determination of overall policy.

4. The Treasurer is the responsible financial officer of the Association for the receipt and disbursement of funds and shall maintain and make available appropriate records, including annual accounts.

5. The President and the Co-ordinator may be paid an appropriate salary for their services, funds permitting. These services will include the organisation of meetings and the editing of the Proceedings of such meetings for publication, co-ordination of, and participation in, the research activities of the Association, preparation when appropriate of research reports and publication of such reports, and other such duties as may be assigned.

6. Members of the Executive Council may as appropriate receive funds for travel, expenses, etc.

7. The Executive Council has selected an independent Advisory Board. It may adopt its own rules for the operation and replacement of members. The Executive Council may nominate candidates to the Board. Any member of the Board, or the Board collectively, may make recommendations to the Executive Council, or directly to the Membership. Action taken on such recommendations must be promptly reported by the Executive Council to the Board in writing.

8. No one of the general public who has booked and paid the appropriate fess should be excluded from attending ANPA meetings except as decided by a majority of the Executive Council in a formal meeting. Notification of ANPA meetings will be sent to all members.

Executive Council: Dr. John Amson, Dr. Ted Bastin, Dr. Keith Bowden, Arleta Dylus, Dr. Tom Etter, Dr. Mike Horner, Prof. Louis Kauffman, Prof. James Lindesay, Dr. Michael Manthey, Dr. David Roscoe, Dr. Fredric S. Young.

President: Dr. Keith Bowden, 139 Sandringham Road, Barking, Essex, IG11 9AH, UK.

[Tel: 0208 594 5064 Email: k.bowden@physics.bbk.ac.uk].

Co-ordinator: Arleta Dylus, 50A Grove Rd, North Finchley, London N12 9DY.

[Tel: 0208 369 5865 Email: a.griffor@physics.bbk.ac.uk]

Treasurer: David Roscoe, Department of Applied Mathematics, Sheffield University, Sheffield S3 7RH.

Advisory Board: Mike Horner (Chairman), Profs. G.F. Chew (Berkeley), C. Isham (Imperial College), M. Redhead (Cambridge and LSE), N. Cartwright (LSE), C. W. Kilmister (London), H. Pierre Noyes (Stanford), Rev. Dr. Philip Luscombe (Wesley House, Cambridge).

ANPA Electronic Mailing Lists

It is **YOUR** responsibility to subscribe to these groups. If you do not do so you may miss important announcements about the annual conference or Proceedings contributions.

anpa-list

This group is for general announcements ONLY and is moderated by Keith Bowden

To subscribe to this group send **ANY** email to:
anpa-list-subscribe@yahoogroups.com

To post messages to this group email:
anpa-list@yahoogroups.com

To read messages on the web, go to:
http://groups.yahoo.com/group/anpa-list/

To unsubscribe from this group send **ANY** email to:
anpa-list-unsubscribe@yahoogroups.com

anpa-discussions

This is a group for general discussion and is moderated by Keith Bowden

To subscribe to this group, send **ANY** email to:
anpa-discussions-subscribe@yahoogroups.com

To post messages to this group email:
anpa-list@yahoogroups.com

To read messages on the web, go to:
http://groups.yahoo.com/group/anpa-discussions/
(this last not yet implemented I believe)

To unsubscribe from this group, send **ANY** email to:
anpa-discussions-unsubscribe@yahoogroups.com

Keith Bowden


Roasted
"EVERY YEAR WE LOSE AN AREA OF RAIN FOREST THE SIZE OF WALES."
"AN ICEBERG THE SIZE OF WALES HAS BROKEN AWAY FROM THE ANTARCTIC ICE SHEET."
IT'S ALWAYS WALES WITH THESE THINGS, ISN'T IT?
YEP.
IT'S VERY THOUGHTFUL OF THE ENVIRONMENT TO COLLAPSE IN SUCH AN EASILY VISUALISABLE WAY.
Andy Riley