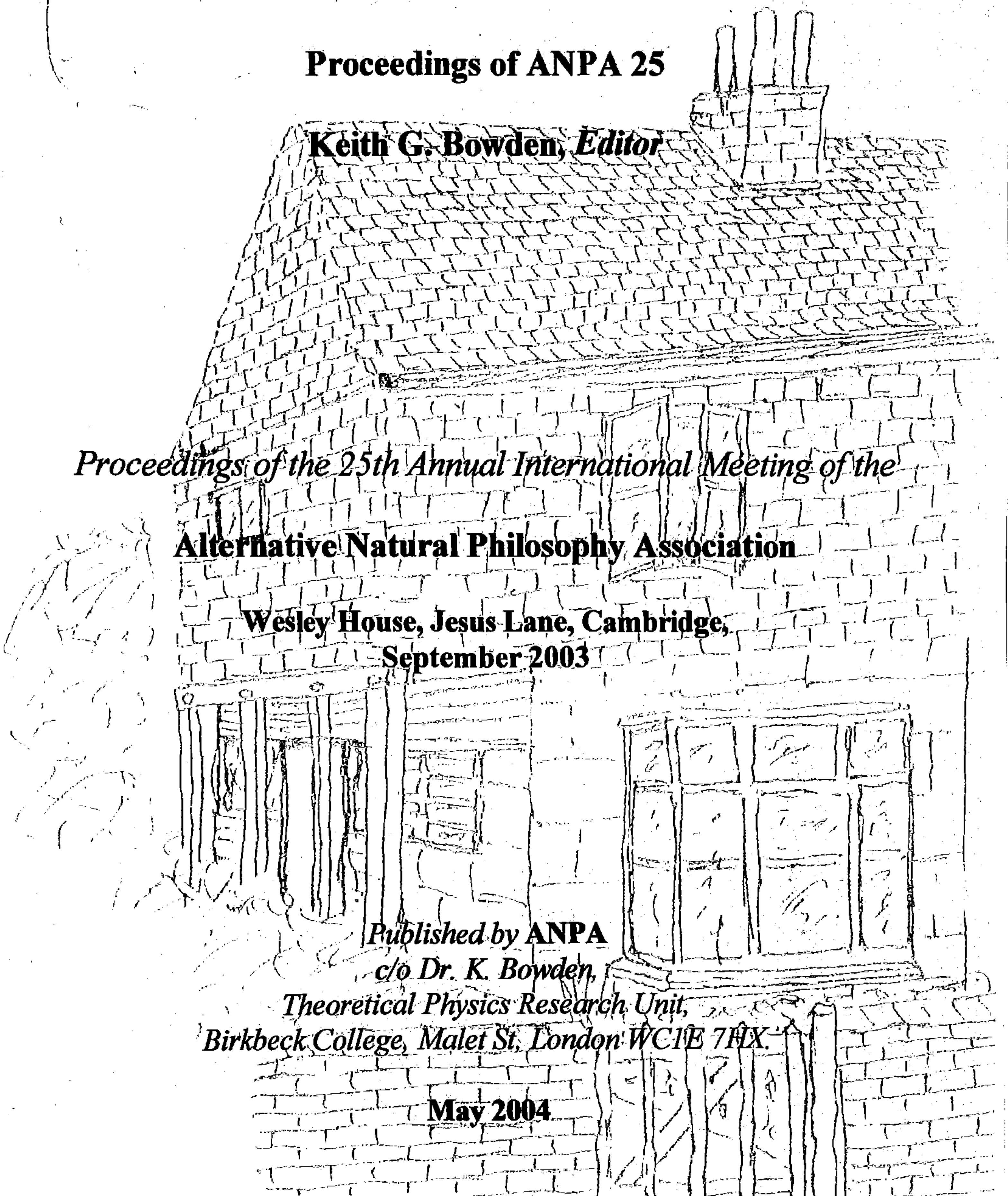
Spin

Proceedings of ANPA 25

Keith G. Bowden, ***Editor***



Proceedings of the 25th Annual International Meeting of the

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Editorial v2

I hear from Arleta that she has now booked - after tortuous negotiation - Saturday the 31st July until Thursday 5th August 2004 as the dates for the Silver Jubilee of ANPA. These dates are *exactly* the 25th Anniversary of the first annual meeting of the Alternative Natural Philosophy Association, which was held in Red Tiles Cottage, Lewes from Monday the 30th July evening to Friday the 3rd August 1979. Those present were Ted Bastin, Clive W. Kilmister, H. Pierre Noyes and A. Frederick Parker-Rhodes. John Amson was unable to attend. I hope and understand that he will be here for ANPA26.

I note from the minutes that ANPA was referred to as "*the* ANPA". Dues were set at five pounds subject to "increase when the market will bear it". I note how little they have risen in 25 years. Ted Bastin reported on negotiations with Cambridge University Press over the publication of a volume entitled “Combinatorial Physics”. To my knowledge these negotiations went on a further twenty years before finally the move was finally made to World Scientific.

Clive Kilmister reported that Berkeley, California were organising an Eddington Centenary and that ANPA would "obviously" cooperate in some fashion. It is a remarkable coincidence that a similar event is being organised in Cambridge this year at which Clive is presenting a paper (his notes are reproduced in this volume) entitled 'The Eddington Heritage - A Claim for ANPA'. It begins "It is 61 years since I heard Eddington give a semipopular talk just along the road from here but I had already succumbed to the Eddington magic much earlier. I had read the popular books... 'The Relativity Theory of Protons and Electrons' in - I think - 1941 was much more of a shock..." His last paragraph begins "There is much left to do...."

Other forerunners of ANPA, as many people know, were Cambridge Language Research Unit, funded by the US Air Force, ostensibly to develop Artificial Intelligence systems, but who actually funded much of the early research on the Combinatorial Hierarchy; and the Epiphany Philosophers group, again in Cambridge in the 1960's, who still subsidise ANPA today. (I have heard stories from Adam Parker-Rhodes about the mysterious ceremonies performed by this prestigious group, in a windmill in Norfolk I believe. He can remember that, as a child, his job was to wave the incense burner!) Also in Cambridge in 1968 was the Quantum Theory and Beyond conference (and CUP book) organised by Ted Bastin with a distinctly ANPA feel to it. Delegates included Roger Penrose (perhaps then at Birkbeck?), Bohm, Bub, Kilmister, Aharanov and Peterson, Chew, Ron Atkin, Mario Bunge, H R Post, O R Frisch and C F von Weizsacker.

I have something to say on the subject of Roger Penrose. One of the strongest continuing themes of ANPA (eg, see Bastin and Kilmister, Bowden and others, also Manthey, this volume) since its inception (and before) is the emergence of physical structure from information or thought like processes (eg, Topsy!) The International Journal of General Systems devoted a whole triple Special Issue to this subject.

One of the most studiously ignored theories of the twentieth century was the debunking by Roger Penrose of the opposite proposition, that consciousness as we know it can arise out of classical physical structure. This was published (somewhat interminably, due to the overwhelming opposition) in "The Emperor's New Mind" and then "Shadows of the Mind". Robots will never become intelligent. Machines will never think. Consciousness is NOT an emergent property of (for instance, complexity in) physical systems. It is proven. QED.

However this proof is now studiously ignored by the media, by popular science in general and most surprisingly by the Science Fiction community. They - perversely - *like* the idea that the world will be taken over by intelligent machines. Witness the popularity of "The Matrix" etc. It is part of our culture; it is very difficult to let go. Even Philip K. Dick was ambivalent on this topic, although he never read Penrose!

Nevertheless Penrose's proof will not go away. Mathematics abides. And if thought does (did) not arise from physical structure then either the two arose independently, albeit perhaps from a common cause (an idea more typical of the modern religions) or physical structure arose out of thought or process (as in the ancient religions). Occam indicates monism and perhaps forms of idealism. The world, as Eddington famously said, is made of *mind stuff*. The point I am making here is that Roger's proof is a very powerful tool for ANPA, in fact whether we do anything explicitly with it or not. Perhaps we should take another look.

Nominations for membership at this first 1979 ANPA meeting included Mike Arbib, Stafford Beer, Heinz von Forster, Fredkin, Brian Josephson, Mike Manthey, Gordon Pask, Jill Purce, Rupert Sheldrake, Martin Rees, David Bohm, Chris Clarke, Basil Hiley, Adam Parker-Rhodes, Nancy Cartright, Geoff Chew, Noam Chomsky, Freeman Dyson, Feyerabend, Thomas Kuhn, David Finkelstein, David McGoveran, Tom Phipps, Carl Pribram, Hank Stapp and Pat Suppes. It is remarkable how many of these first nominations have had a continuing influence on ANPA ever since.

We have had many famous (and infamous) members since. I was reminded when thinking about the past history of ANPA of the year in which Alex Comfort visited us. I remember spending a very enjoyable lunch with Alex during which he entertained us immensely. He told me afterwards (I don't believe that he would mind me revealing this now) that he had an ailment which resulted in him having a doctor's prescription for Benzedrine (or "speed") and that since starting this regime there was nothing that he enjoyed more than a pub lunch with a good audience! More recently I told this story to Clive Kilmister who responded by telling me something that I didn't know, that Alex had enjoyed this meeting so much that he had decided to make a charitable donation to ANPA, and that indeed for the ensuing few years ANPA was part funded by proceeds from the "Joy of Sex".

Later I remember in that same conversation with Clive Kilmister, which I believe took place in Arleta's room at Wesley House last year, after a few glasses of wine, we discussed possible titles for this year's Proceedings. At some point in time I think I rather pompously suggested

Revolutions. Clive, with an ear to recent political events, retorted that Spin had much the same meaning and rather more punch. So it was decided. I guess possible future titles must be Charm and Strangeness. Top and Bottom are perhaps a little more odd.

ANPA West was formed by Pierre Noyes and Tom Etter in Feb 1984 and ran until Feb 1997 (ANPAW13). The annual meeting was held at Cordura Hall, Stanford University. Regulars included Fred Young, Richard Shoup, Pat Suppes, David McGoveran, Mike Manthey, Vaughn Pratt, Irv Stein, Herb Doughty and Eddie Oshins. Occasionals included Geoff Chew and Jack Sarfatti. At this last meeting Lou Kauffman presented *four* papers (of his own).

I am very sad to have to report the death of Eddie Oshins. Eddie died Friday night, the 10th October 2003. He was at a party and had a heart attack. I spent time with Eddie at Stanford, but I knew him best for his extremely... lengthy and often very entertaining emails. If he got a bee in his bonnet about a project he would chase everybody involved incessantly until either he got his own way or he forgot about it and went on to the next obsession which would be equally wild and equally entertaining. I would like to say that when I go that is how I would like to do it - at a party on a Friday night! - but it must have been a shock and very upsetting for everyone there. My condolences to all those who were close to him. Eddie will be sorely missed.

In recent correspondence responding to a request by me for more information about early ANPA West, Pierre Noyes comments that "Receiving the first 'Alternative Natural Philosophy Award' was probably the most important and rewarding achievement of my professional career. Because it was given to me without warning of its existence I have no knowledge of its 'prehistory'. I suspect David is the one who was most instrumental in creating the idea and making it happen." This correspondence resulted in the document included in this volume.

A word about the Proceedings. I have bound this (last) year's Proceedings as one volume instead of the two that have been traditional recently. This disguises the fact that for the first year that I remember the

Philosophical Aspects would have vastly exceeded the Scientific Aspects in size. This is partly due to the fact that AP-R has finally been persuaded to write up all his talks into one massive paper and partly due to the fact that I haven't been able to complete my own paper due to the pressure of unexpectedly installing a new kitchen. Nevertheless even taking these into account it is most pleasing that we are finally getting a better balance of Scientific and Philosophical papers.

Finally, from the minutes of ANPA1 I note that the projected first three year's budget of ANPA (at 1979 prices) was 238,000 dollars. There were talks by Kilmister, Bastin, Parker-Rhodes and Noyes. The '"banquet"' (note carefully the quotes) was held on Wednesday evening the 1st August. I understand from Ted Bastin that the ANPA26 banquet is now booked for Saturday 31st July at King's. I thoroughly expect it to live up to its predecessor 25 years earlier. Have a great Conference.

Keith Bowden,
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Birkbeck College, London
April 2004

ANPA Electronic Mailing Lists

It is **YOUR** responsibility to subscribe to these groups. If you do not do so you may miss important announcements about the annual conference or Proceedings contributions.

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(this last not yet implemented I believe)

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Keith Bowden

ANPA Proceedings Editorial Policy

ANPA has been criticised in the past - in particular by members of its own Advisory Board - for having no formal editorial policy for its Proceedings. This has been balanced by a feeling within ANPA that we should keep ourselves open to all viewpoints. In the last few years as editor I have tried to tighten things up in such a way as I felt would satisfy our critics whilst not compromising our own position. This has been partially successful although for some time I have felt that it is time that there was a formally stated policy. The following has been approved by the Executive Council, although it is open to feedback from all. By "the editor" is meant the Editor or (an) appropriate nominated Referee(s) (note the capital R!)

1. The paper should make a new and original contribution to the fields of ANPA's interest. Survey papers are acceptable.

2. The default use of language for submitted papers in Physics {and Philosophy of Physics}* should be the common language of Physics as usually understood by Physicists {and, in particular, by Philosophers of Physics}*. Any other use of language should be carefully explained at the start of the paper and all appropriate definitions included there.
{* added by KGB}

3. The editor should be satisfied that the paper is ***presented*** in such a way that the majority of the readership will understand the author's intentions. In particular ***it should be clear*** that the author has a correct understanding of the subject matter. Please avoid forward references.

4. "Verbatim" reports will be accepted subject to the above three conditions only, regardless of whether the final draft is an accurate rendition of what was originally said. Other such reports are better submitted to the Newsletter.

5. Theories of any nature are acceptable material, provided they are compatible with the known facts, and provided they are deemed to be of interest to the readership. Theories of alternative, imaginary worlds are also acceptable, provided their nature is made clear.

ANPA Proceedings Notes for Authors

I would like to try to continue conformity of *style* for future issues of the Proceedings. Ideally I would like contributions to be submitted in International Journal of General Systems format (I have some copies of their Notes for Authors) or similar.

Times Roman, 14 point, is preferred. **10 point is TOO SMALL to be reduced to A5.** If you are sending hard copy please send two copies single sided. Main heading 20 point capitalised and centred, other headings 16 point capitalised to the left. Author's name(s) capitalised and centred. Address italicised and centred. No underlining. At least a one inch bottom margin for footers; page numbers NOT top centre. *Only copy in good English will be considered, and remember, this is a formal Proceedings.* **Remember also to include your name (surprising how many people omit this!), affiliation and full address, email address and the version number (even if it is 1.0) or date of the draft, centred below the main heading.** I often get sent more than one version of a paper and invariably mix them up! Send copy **to *KEITH BOWDEN, 139 SANDRINGHAM RD, BARKING, ESSEX IG11 9AH.***

The copy date for the ANPA2004 Proceedings is January 1st 2005. The issue will go to print on April 1st 2005. This will be adhered to rigidly this year.

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FOR EDITORIAL ADDRESS SEE THE PENULTIMATE PARAGRAPH OF THE TEXT

THE FINE STRUCTURE CONSTANT

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ABSTRACT. I show that a careful examination of Parker-Rhodes Combinatorial Hierarchy (the CH) and its elaborations yields a value of the reciprocal of the fine-structure constant of $1/\alpha = 137{\cdot}0360$, completely agreeing with the seven significant figures of the experimental determinations.

1. INTRODUCTION

Over forty years ago Frederick Parker-Rhodes devised a curious algebraic construction of a hierarchical system of four levels (graded group) of cumulative multiplicities 3, 10, 137 and 10^{38}. The process terminates at the fourth level. His construction is fully set out in (1) but may be summarised thus: Parker-Rhodes' basic entities were bit-strings - that is, vectors over the field $\mathbb{Z}_2$ of two elements. His initial bit-strings were two in number and of length 2. Addition in the field was called discrimination because two bit-strings added to zero if and only if they were the same. The zero string was therefore a special element which acted as a signal of equality. It was accordingly excluded from the original bit-strings. Thus Parker-Rhodes had recognised the importance of a discrimination system, that is, a structure in which there is a binary operation, here written as +, such that

$$a + b = 0 \Longleftrightarrow a = b.$$

The next important notion for Parker-Rhodes was that of the discriminately closed subset (dcs) defined as a set of bit-strings closed under the discrimination of any two different elements. His multiplicities were the number of dcs's. The other important notion for Parker-Rhodes was level-change defined in this way: given a collection of dcs's, define for each dcs of the collection a representative square matrix having exactly the dcs as its set of proper eigenvectors (i.e. with eigenvalues 1). These representatives are the elements (bit-strings) at the next higher level and are required to be chosen linearly independent. Each level was, if 0 is included, an abelian group with every element of order 2 and so, by a well-known theorem, was a direct product of cyclic groups of order 2. The termination at the fourth level is because there are not enough linearly independent matrices to continue the construction.

Because Parker-Rhodes devised this construction as part of a project to understand how there could be a physical theory with intrinsic "scale constants", it was natural for him to interpret 137 as the reciprocal $1/\alpha$ of the fine-structure constant. Similar considerations relate the large number to gravitation but I am not concerned with that here. Still less am I at present concerned with the two smaller numbers; one may conjecture a relation with the strong interactions. Two apparently different valid objections may be raised to Parker-Rhodes' identification. Firstly one may argue that although the construction came as part of a project to understand physics better, yet the connexion with physics is not at all clear. Secondly, one may argue that the value of $1/\alpha$ is known to differ from 137; the latest experimental values which agree with each other only to seven significant figures, centre on 137·0360. In this paper I show that these objections may be resolved by a more careful use of Parker-Rhodes' ideas, which corrects his value to agreement with observation to at least the seven significant figures available. The percentage error in Parker-Rhodes original conjecture was 0·026%, or, more conveniently, 260 parts per million (ppm).

The corrected version depends critically on a view of the logical status of scale constants such as α, that of a process theory. In this theory, which Ted Bastin and I have elaborated, information comes in from a statistical background and is fitted into a framework which itself generates an algebraic system. This system is not quite identical with Parker-Rhodes' but resembles it sufficiently to yield the same numbers. The fitting in to the framework is done in this way; explained more fully in (2), pp.57-90. (This reference will in future be quoted as CP.) A new entity is compared with one already in the system and it is determined whether they are the same or not. This is discrimination. There is also a procedure of level change in which sets of entities are treated as a single entity at the next level, much as in Parker-Rhodes' original system. Everything develops from these two procedures. This is possible because all is flux; the process keeps repeating itself and changing, randomly, within the stability provided by the framework. It is this stability which appears in the form of the numbers, called scale constants above.

The correction of the calculation of $1/\alpha$ falls naturally into three parts, the first two of which are to be found in CP. For the sake of completeness I begin with a very brief description of these two parts and then I deal with the third part in more detail.

2. CALCULATION: I

The number 137 in Parker-Rhodes' system is the number of dcs's there would be if the system were artificially constrained in some unexplained way not to rise to the highest level. In that situation, 1/137 is therefore the probability that, given any dcs, another one is identical to it. In saying this one is appealing to what is called in CP (p.58) a discrete ergodic principle: that probabilities of members of a set of outcomes are to be taken as equal in the absence of knowledge to the contrary. David McGoveran explained to me how to use the ergodic principle again to obviate the inexplicable constraint (CP, pp.106-7). At each level the process has to determine whether the element is at that level or not. At the first level it may be one of 3 dcs's or none of them, so the ergodic principle gives $\frac{1}{4}$ as the probability of not being at that level. Similarly it gives 1/8 as the probability of not being at the second level. Some subtlety enters at the next level but ignoring that for the present the probability of not being at the third level is 1/128. That of being at the top level is therefore $e = 1/(4.8.128)$ and so the probability of the constraint to three levels being satisfied without artificial interference is 1 - e. The original probability is therefore changed from 1/137 to (1 - e)/137, or, more conveniently, 137 has to replaced by 137·0334. The discrepancy between this and the measured value of $1/\alpha$ is therefore reduced by an order of magnitude to 19 ppm. The whole of the rest of this paper is simply a refinement of this constraint calculation.

3. CALCULATION:II.

The next part of the calculation is set out in CP, pp. 57-90. The original apparatus of bit-strings and matrices is discarded in favour of a process in which a growing set of elements comes into play. As new elements are added they have to be labelled and the process has to check whether the "new" element is really new or a repetition. This checking corresponds to Parker-Rhodes' discrimination and has to have a signal z to show if the element is not really new. The necessity of a signal is to avoid an infinite regress. Discrimination is a binary operation which CP follows Parker-Rhodes in taking as commutative and associative and so writing it as +. Labels are drawn systematically from an ordered set of strings in the ordinal numbers 1, 2, 3... As an abbreviation the string 1,2 is written as 12 (so long as 12 is not yet in play) and similarly for others. The ordering of the strings is then: 1,2,12,3,13,23,123,4,... The signal z is taken as less than any string in this ordering. In place of Parker-Rhodes' explicit construction, the labelling is done by what CP calls "Conway's construction", a modification of the generation of numbers in (3): a + b is to be given the least label not already used for a,b,a'+b or a+b' ,where a', b' are any strings less than a, b. The new system has a particular representation in terms of bit-strings which differs only slightly from Parker-Rhodes' system.But the process view does lead to three important changes, two of which I describe here and the remaining one in the next section.The first change, mentioned only in passing in CP (p. 67) is that, whereas for Parker-Rhodes the first level of the hierarchy was the quadratic group (so long as zero is included), here the process generates only the three non-zero members of the group, [1, 2, 12] and their relations (1 + 12 = 2 and so on). z + 1 and z + z are not generated by the process. It is a fairly obvious choice for the mathematician to adjoin the values z + 1 = 1, z + z = z but he does not have to. One could put it this way: discrimination systems can be open or closed, according as values for the operations including the signal are unspecified or specified. Parker-Rhodes' original system was closed; the process generates an open one. An open discrimination system can be closed by specifying further rules. Here z + z must be z if the closure is to be a discrimination system and 1 is the only obvious choice for z + 1.

The second change is more obviously significant. Reverting to Parker-Rhodes' construction for a moment, at the first level it is easily seen that the three dcs's can be labelled by 2 x 2 non-singular matrices in only one way. At the next level these three matrices generate $2^3 - 1 = 7$ dcs's. These are required to be chosen to be linearly independent to ensure that at the next level there will be the full set of 127 dcs's. Now the process view is that such a "choice" is interfering with the process from outside and so cannot be required. Instead any of the 74088 (CP p.110) sets of matrices is allowed. Only 61772 of these sets are linearly independent. The remaining sets give ensembles of 63, 31, 15 dcs's at the next level (CP p.111). This corrects the factor 1/128 in the calculation of probability. A lengthy calculation finishes up (CP p.109) with 1/(122·2167). Because it is at this point that the third new correction comes in, I will explain the calculation more fully in the next section. It gives a value of $1/\alpha$ of 137·03503, which reduces the error to less than 8 ppm.

4. CALCULATION: III

Take the three basis elements at the second level as 1,2,3 without loss of generality. Use the notation (p,q,r) for a matrix operator, where the p,q,r are strings in [1,2,3]. It is easy to see that (p,q,r)1 = p, (p,q,r)12 = p + q and so on. The three operators fixing the dcs's [1], [2],[3] have the form (1,a,b), (c,2,d), (e,f,3) where a,b,c,d,e,f are suitable strings in [1,2,3]. The effect of (1,a,b) on the whole set is given by

1	2	12	3	13	23	123
1	a	1a	b	1b	ab	1ab

Since the operator is to be non-singular, $a \neq 1$, $b \neq 1$, $a \neq b$, $ab \neq 1$. Since 1 is to be the only eigenvalue, $a \neq 2$, $b \neq 3$, $ab \neq 23$. These conditions limit the pair (a,b) to 14 possible values, and similarly for (c,d) and (e,f). In much the same way the operator leaving [2,3,23] invariant, (g,2,3), can have only three possible values for g, with similar results for (1,h,3) and (1,2,i). Lastly, the only operator leaving the whole set invariant is the identity, (1,2,3). The total number of sets of operators is, as said above, $14^3.3^3 = 74088$.

The problem is to find how many of these sets generate a space of dimension 7 (so containing $2^7 - 1 = 127$ elements), how many of dimension 6, and so on. If the dimension is less than 7, some of the elements generated by repeated discrimination will be zero. In the case of dimension 4 there will be a seven-fold zero, for dimension 5 a three-fold and for dimension 6 a single zero.It is quite hard to identify correctly the number of ways of being of dimension 6. To avoid error I count zeros, determining which are seven-fold, and which are three-fold. The remainder must then be single zeros corresponding to dimension 6. The results of this search for zeros are shown in CP (p. 113) in the table at the top of the next page. This table has been abbreviated by using some obvious symmetries. The counts in the first two columns must be multiplied by 3 to take account of this.

In CP an ingenious matrix method was used in some cases but this turns out not to be applicable in the third part of the correction so I will not describe it here.I give two examples to illustrate the construction of the table: firstly, in the notation used in CP, 12·1, since it is the largest contribution, 3 x 2352 = 7056 cases.

				1 a b		ce 2f 3d		1ce 2af 3bd	
0.								216	(3)
1.	1	2	3					216	(4)
2.	*g*	2	3					198	(4)
3.	1	*h*	3					198	(4)
4.	1	2	*i*					198	(4)
5.	1*g*	0	0					198	(5)
6.	0	2*h*	0					198	(5)
7.	0	0	3*i*					198	(5)
8.	1*g*	2*h*	0					132	(5)
9.	1*g*	0	3*i*					132	(5)
10.	0	2*h*	3*i*			378	(4)	132	(5)
11.	*g*	*h*	*i*			168	(5)	112	(6)
12.	1	*h*	*i*	2352	(4)	168	(5)	174	(6)
13.	*g*	2	*i*					174	(6)
14.	*g*	*h*	3					174	(6)
15.	1*g*	2*h*	3*i*			322	(6)	114	(7)

<u>12·1.</u> Here the only conditions for a zero are a = h, b = i. But the values of a which can be values of h as well will be found to be 12, 23, 123. Similarly those of b which can be i are 13, 23, 123. Then looking at the list of possible values of the pair (a,b) will show that the only possible pairs here are

a,b = 12,23, 12,123, 23,13, 123,13.

These can be taken with any values of (c,d), (e,f) and g so the number of cases is $4 \times 14^2 \times 3 = 2352$.

Secondly, I take 0·3 as a relatively straightforward case:

<u>0·3.</u> A zero comes if ce = 1, af = 2, bd = 3. Now it is clear that if 1,2 are interchanged, the new value of a becomes c, that of b, d and e,f are interchanged. Thus, given that a possible value of (a,b) is (12,23), one for (c,d) is (12,13). Similarly interchanging 1,3 means that the new a becomes f and the new b,e. So from (a,b) = (12,23) one gets (e,f) = (12,23). Since a ≠ 1 or 2, this symmetry gives f ≠ 3 or 2. If f ≠ 3, then from af = 2 one gets a ≠ 23. Next, b ≠ 1, 3 so d ≠ 2, 3 and if d ≠ 2, then from bd = 3 it follows that b ≠ 23. This leaves only eight values for (a,b) which are listed on the next page, with corresponding values of f, d. These then give rise to several values of e, c in the next two columns. The final column gives

the values of c allowed by ce = 1. There are eight cases, which may be accompanied by any values of g, h, i giving a total of 8 x 3^3 = 216.

TABLE FOR 0·3

a	b	f	d	e	c	ce = 1 gives
12	123	1	12	23, 123	3, 123	123
3	12	23	123	12, 123	12, 3, 23	23
3	123	23	12	12, 123	3, 123	-
13	2	123	23	2, 23, 13	123	123
13	123	123	12	2, 23, 13	3, 123	123, 3
123	2	13	23	2, 12	123	-
123	12	13	123	2, 123	12, 3, 23	12, 23
123	13	13	1	2,123	23, 123	23

There is one further complication to be noted here. It is trivial that a set of r linearly independent elements generates $2^r - 1$ elements and also generates the same number of dcss. But if the r elements are not linearly independent, this is no longer the case. For example, the set [1,2,12,3] obviously generates 7 elements but the dcss generated are:
[1], [2], [12], [3], [1,2,12], [1,3,13], [2,3,23] and the whole space of 7 elements, together with [12,3,123].

An investigation shows (CP, p.108) that the correct number of dcss depends on the number of basis elements giving rise to a zero and these numbers are given in brackets in the table on the previous page. The upshot is the formula (CP, p.109) which can be expressed more cogently in the form

$$\sum_{r=2}^{6} \frac{p_r}{s_r + 1} = \frac{74088}{E}$$

where p_r = the number of zeros produced by (r+1) elements making a zero,

s_r = the number of dcss corresponding.

The values of s_r are tabulated on p.109 of CP as

r = 2	3	4	5	6
s_r =79	95	107	115	120

Then the McGoveran correction factor 1 - e is 1 - 1/(4.8.E), and this gives the value quoted above, 137·0353 with error less than 8 ppm.

5. THE THIRD CORRECTION.

The third part of the corrected calculation is new, because it arises from the correction of an error in CP. Parker-Rhodes simply assumed that the discrimination operation was a commutative one. In the process analysis of CP a proof was given (p.84) that there was no loss of generality in assuming this to be true for the discrimination defined there. A careful inspection shows this proof to be fallacious. Moreover, the result is false, for if p + q = r and r denotes the operation of checking whether the "new" element q is the same as p or not, the result of finding r = 0 is to relabel all the q's as p's. But if q + p = s, then finding s = 0 requires relabelling all the p's as q's. A different notation is evidently desirable. I use pq where previously p + q was written. The signal, which was zero before, will now be written as z. Evidently the Conway construction will need to be modified. In doing this it will be necessary to take into account that, although there is a loss of generality in taking pq = qp, it is still an alternative , simpler process. One can ignore the distinction and this means that there must be a homomorphism back from the new algebraic system to the original one with the same number of generators. Call this homomorphism H. Then it defines an equivalence relation between elements in the usual way:

$$p \sim q \longleftrightarrow Hp = Hq \longleftrightarrow (Hp)(Hq) = z .$$

For example, if pq = r and qp = s, then $r \sim s$. This new equivalence relation has to be signalled in the process in the same way as before, so an additional signal, y say, is needed. The modified Conway construction is then:

pq = the least r different from p, q, p'q, pq', qp, subject to H.

I work out the algebraic consequences of this in the present section, and then deal with the results of this new algebraic system on the calculation of α in the next one.

One begins in the same way as before: 1.2 = 3; then 2.1 = 4. This will have the result that 3.4 = y = 4.3. In the mean time, though, 3.1 cannot be 3 or 4 but can be 2 and so is 2. Coming on to 1.3 this cannot be 1,2 or 3 but 4 is not immediately excluded. However, if 1.3 = 4, then 4 $\sim$2 and we already know that 4 $\sim$3, so 2$\sim$3, that is, 2.3 = y, whereas the table already has 2.3 = 1. Thus 4 is excluded, and so 1.3 = 5. It is in this way that the phrase "subject to H" makes it presence felt. Continuing in this way yields the table for the first level of the modified hierarchy:

	1	2	3	4	5	6
1	z	3	5	2	4	y
2	4	z	1	6	y	3
3	2	6	z	y	1	5
4	5	1	y	z	6	2
5	3	y	6	1	z	4
6	y	4	2	5	3	z

As in the commutative case, this is an open discrimination system. It is interesting to enquire whether it is associative and whether it has a closure (which would then be a group). It is easy to see there is a closure, which comes about by adding the extra rules

$z^2 = y^2 = y$, $zy = yz = z$, $yp = py = p$

The result will at once be recognised as the quaternion group, Q, which is well-known to be a semi-direct product, $C_2 \rtimes S \simeq Q$. This is just the mathematician's way of saying that the group can be written in terms of three basis elements and the identity taken with ± signs. Then the extra rules can be summarised by taking y = 1, z = -1.

But this group, though the closure of the open discrimination system, is not a closed discrimination system ; whereas p.p = z for p = 1,2,3..6 z.z = y. There is a closure that is a closed discrimination system and it is got by taking (in the abbreviated notation used for Q) z = 1, y = -1. This closed discrimination system is not associative. I call it Q*. It is not a group but a loop. Non-associativity is often seen as a warning signal of something that must be tamed, by mathematicians. The best known example of a non-associative but fairly tame system is octonions, or Cayley numbers. Here the loop is of order 16 having the form $C_2 \rtimes R$. Writing the group C_2 as [1, -1] in the same way as with Q, R has a table which can be totally constructed from the proviso that every sub-loop generated from two elements is isomorphic to Q . The non-associativity is made respectable by coming in only in the relations between these seven sub-groups.

Here the taming is achieved in a different way. The open part of Q^* is also the open part of a grouplet. This notion was introduced by Arleta Dylus. A set G^* of elements with a binary operation, $\cdot$ say, is called a grouplet by her if there is a bijection $G^* \leftrightarrow G$, where G is a group and the binary operation $\cdot$ is given by:

$$a^* \cdot b^* = (b^{-1}a)^*.$$

Every group gives a grouplet and vice versa although groups in which every element is of order 2 do not give proper grouplets, but just give themselves.

The next key idea in the hierarchy construction is that of the dcs and this also needs a little modification. The idea behind the definition ("any two different elements") is to avoid introducing zero, or, in the present case, any signal. So here the definition should say "any two non-equivalent elements". Now consider the way in which the original hierarchy represented a dcs by a square matrix. In the process re-telling of the story this becomes a linear operator. The linearity was required in order to preserve discrimination. In Parker-Rhodes' original formulation discrimination between the matrices was, again, addition over $\mathbb{Z}_2$. In the process reformulation this was recognised as the obvious induced binary operatiuon defined by

$$(A + B)u = Au + Bu \quad \text{for all } u.$$

Now that discrimination is expressed multiplicatively, a further reformulation is required. The operators representing the dcss must evidently be automorphisms of the multiplicative structure. The matrix notation is no longer available but something very similar is. Recall that, in section 4, a matrix (p,q,r) simply stated that its effect on 1,2,3 was, respectively,, p,q,r. So here, an automorphism on the lowest level just found can be specified by stating what 1,2,3 become under it. Thus, to leave the dcs [1,-1] invariant, possible operators are (1,3,-2) or (1,-3,2). Here the third member of each triad is determined by the condition of being an automorphism but it must be retained in the notation. This is because the combination of automorphisms in forming the next level will not usually give automorphisms. This simply reflects the position in Parker-Rhodes' original hierarchy, in which not all the elements at the next level were non-singular matrices. The automorphisms represent the dcss in the same way as before by the eigenvector condition:

$Au = u \longleftrightarrow u \in T$ means A represents the dcs T.

Turning to the definition of induced discrimination, it will be, much as before,

$AB(u) = (Au)(Bu)$.

Thus, if $A = (p,q,r)$ and $B=(s,t,u)$, then the definition gives $AB = (ps,qt,ru)$. This is not usually an automorphism because ru is not $(ps)(qt)$ but $(pq)(st)$.

I now need to construct the next level in some detail. Because I need to retain 1,2,3 as the elements at the lowest level, it is best to use, as a temporary expedient, other symbols at the next level, to avoid confusion. Working in the usual abbreviated notation for Q* gives the table:

	132'	321'	123	2'1'3'	z11	2z2	33z'
	A	B	C	D	E	F	G
A	I	D	k_1E	$-B$	C	G	$-k_1F$
B	$-D$	I	F	A	$-k_2G$	k_2C	E
C	E	k_2F	I	$-k_1G$	k_1A	B	$-k_2D$
D	B	$-A$	$-k_2G$	I	$-F$	k_3E	$-k_1C$
E	k_1C	G	A	k_3F	I	$-D$	$-k_2B$
F	$-k_1G$	C	k_2B	$-E$	k_3D	I	A
G	F	$-k_2E$	$-k_1D$	$-k_2C$	B	$-k_1A$	I

Here 2' is written for -2 and so on; k_1 denotes (z,y,y') and so on. The loop is of order 64, with a centre of order 8 (in fact, $C_2 \times C_2 \times C_2$). It is easy to see that this loop is a discrimination system and also that all the seven sub-loops generated by two elements have the same structure, Q*. This symmetry recalls the structure of octonions. Notice however, that the different copies of Q* have different elements of the centre as their "signs". For example, the table generated by A, B is

	A	B	C
A	z	C	−B
B	−C	z	A
C	B	−A	z

but that for A, E is

	A	E	C
A	z	C	k_1E
E	k_1C	z	A
C	E	k_1A	z

with k_1 playing the part for A, E that −1 played for A, B. On the one hand the non-associativity is more serious than with the smaller loop of octonions, as it extends down to the sub-loops, but on the other it is tamed in the other way, for it is obvious that the table represents a grouplet, because each element of each triad belongs to a grouplet.

6. THE FINAL CALCULATION.

I turn to the question of the way in which this generalisation of the CH modifies the calculation of α . At first sight it might be thought to make large changes. The lowest level now has eight rather than 4 elements, and the next one has 64 rather than eight. But it was explained in section 1 that it was the stability of the framework which was characterised by these numbers. Since now the construction is constrained by the proviso in section 5 that there should always be a homomorphism back to the original hierarchy, it is clear that the stability will be determined in the homomorphic image and this is carrying the same critical numbers.

None the less, the value of α is changed because the generalisation brings in the possibility (and therefore the necessity) of another application of the finite ergodic principle. In constructing the table in section 5 I remarked that there were two possibilities for the automorphism, A, leaving [1, −1] invariant. These were (1,3,−2) and (1,−3,2). The same applies to B; but for C there is only the one possibility, (1,2,3). In terms of the continual flux of elements described in section 1, both (1,3,−2) and (1,−3,2) have the same homomorph (1,3,2). From the ergodic principle, then, (1,3,2) will occur twice as often as (1,2,3). But when D is considered, it may turn up in the algebra multiplied by any of the elements of the centre and all these elements homomorph into (2,1,3), with similar results for E,F,G. Of course, A also may turn up in the algebra multiplied by any of the eight elements of the centre. But the important point is that only (1,3,-2) and (1,-3,2) are allowed by the eigenvector condition. There is no such restriction on D,E,F,G. The relative frequencies, or "weights" are therefore

A	B	C	D	E	F	G
2	2	1	8	8	8	8

In section 4 the counting of zeros was undertaken under the unstated assumption that all were equally likely. This was a tacit application of the ergodic principle. This counting now has to be altered to take account of the different weights. The way I have done this is to work out the weights for each of the individual types of zero in the table of section 4 and present the results in the form of "effective totals". Then these new totals can be used in the calculation in the same way as the original totals. To make the comparison with section 4 clearer it is desirable to change back from the temporary notation to 1,2,3 (for A,B,C). Then from the table in section 5 it follows that D = 12, E = 13, F = 23 and G = 123. The corresponding table of weights (with a more convenient normalisation) becomes:

1	2	12	3	13	23	123
¼	¼	1	1/8	1	1	1

The calculation can be most simply explained by looking first at the case 12·1 which was considered in section 4.Here c,d,e,f,g may have any values but the only possible values of a,b are, as stated there, 12,23; 12,123; 23,13; 123,13. As there are no isolated basis elements, each of these entries has weight 1.

Now the number of zeros are being counted here in order to assess the probabilities of various outcomes. The operator (1,a,b) can have only 14 possible values which are easily seen to be :

a,b = 12,23; 12,123; 3,12; 3,23; 3,123; 13,2; 13,23; 13,123; 23,2; 23,12; 23,13; 123,2; 123,12; 123,13. In section 4 these were treated as having equal probability 1/14. When 12·1 was discussed there the 4 possibilities meant that the probability of a zero of this type was 4/14 and then in the subsequent calculation this is, in effect, multiplied by 14 to give the 4 which (suitably multiplied by 588) gives the entry in the table.

Using the normalised table of weights it is easy to find the weights of any of the 14 potential values of a,b. An isolated 1 or 2 gives rise to weight $\frac{1}{4}$ and an isolated 3 to 1/8 but otherwise the weight is 1. Thus the three cases listed with a = 3 give 3/8, the three with b = 2 give 3/4 and the remaining eight give 8. The total weight is then 9·125. In the particular case of 12·1, in which each of the four entries has weight 1, the probability of a zero of this type is now not 4/14 but 4/9·125. In the subsequent entry to the calculation, this will again be multiplied by 14. The ratio q_1 = 14/(9·125) = 1·5342 will evidently be useful in this recalculation. Instead of $4.14^2.3$ we now have an effective total of $q_1.4.14^2.3$ = 6·1368.588. The effective total is in this case larger than the previous count because the zeros arise only with the elements of higher weight. Some of the other entries in the table will be increased, some decreased. I have not given the numerical value above because, as will be clear below, there is a little more to be said about 12·1. I have simply used the calculation as an explanatory guide.

To deal with other cases I must now analyse the 14 possibilities for c,d and those for e,f. The interchange of 1 and 2 turns a,b into c,d and e,f into f, e so, since 1, 2 have equal weights, the factor q_1 applies to c,d as well. The interchange of 1 and 3 turns a,b into f,e but this interchange also changes weights. Looking back at the list of a, b, the corresponding total weight for e,f will easily be seen to be 8 + 6/4 = 9·5. The corresponding factor 14/(9·5) is then q_2 = 1·4737. I shall now use these results to illustrate more fully the effect on the two cases of the table described in section 4.

<u>12·1.</u> The illustration above shows that the original total of 2352 gives rise to an effective total of q_1.2352 = 3608. But it will be recalled that the total in 12·1 was one of those which has to be multiplied by 3. This was because the total 2352 comes from asking how often a zero results from (1,h,i) + (1,a,b). Interchanging 1 and 2 converts this into asking for the number of zeros of the form (g,2,i) + (c,2,d), with, of course, the same

answer. This is not entered in the original table because it comes from this symmetry. Similarly for the interchange of 1 and 3. When the new weights are used, the first two cases both give effective totals of 3608. The third one, however, involves e,f and so in its case the effective total is q_2.2352 = 3466. To make the comparison with the original table more transparent, I will not enter these three numbers separately but instead will put their mean, 3561 under 12·1 with the same proviso as before that this corresponds to 3561.3 = 10683 zeros.

The other case considered in section 4 was:

0·3. I tabulate the eight values obtained before:

a	b	c	d	e	f	w
12	123	123	12	23	1	1/4
3	12	23	123	123	23	1/8
13	2	123	23	23	123	1/4
13	123	123	12	23	123	1
13	123	3	12	13	123	1/8
123	12	12	123	2	13	1/4
123	12	23	123	123	13	1
123	13	23	1	123	13	1/4

The last column gives the weights. As before the total weight, 3·25, must be multiplied by $q_1^2.q_2$ corresponding to all three pairs a,b, c,d, e,f being in play and this converts 216 into 304.

Since both of these cases lead to increases in the effective totals, I will give one more example, one which leads to a decrease. Consider:

11·2. The conditions for a zero are: g = ce, 2 = hf, 3 = id.
From 3 = id one gets (since i = 13, 23 or 123) that d = 1,2,12. But d = 2 is impossible, so d = 1 or 12. When d = 1, c = 23, 13 or 123 and when d = 12, c = 3, 13, 123. Next, from 2 = hf in the same way either f = 1 when e = 23, 12 123 or f = 13 when e = 2, 12, 123. These sets of values for c, e have to give ce as one of the values of g, i.e. 12, 13 or 123. It is easy to list the 12 cases that result and the table is as follows:

c	d	e	f	w
3	12	12	1	1/32
3	12	123	1	1/32
3	12	12	13	1/8
3	12	123	13	1/8
13	1	2	13	1/16
13	12	23	1	1/4
13	1	2	13	1/16
13	12	23	1	1/4
23	1	12	1	1/16
23	1	12	13	1/4
123	1	2	13	1/16
123	12	2	13	1/4

The fifth column gives the weights. The total weight is 1·5625. Since c,d,e,f, are all involved one forms $1{\cdot}5625.q_1.q_2$ giving 3·5327 in place of 12. Thus 168 is to be replaced by 49·46. But here again I am dealing with one of the cases in the original calculation which has to be multiplied by 3. Interchange of 1 and 2 will not affect the weights, so this gives another 49·46. But interchange of 1 and 3 turns e,f into a,b. The total weight comes out now to 1·2969 and this has to be multiplied by q_1^2 giving 3·0525 instead of 12 and so a new effective total of 42·74. Taking the mean of the three gives 47.

Each of the cases in the table has to be treated in this way. A considerable amount of (rather elementary) labour is involved. Some of the previous results were found by a matrix method, which is no longer applicable. These cases have to be set out at length. When this has been done, the following table of effective totals results:

	1 a b	ce 2f 3d	1ce 2af 3bd
0.			303 (3)
1. 1 2 3			749 (4)
2. g 2 3			414 (4)
3. 1 h 3			414 (4)
4. 1 2 i			429 (4)
5. 1g 0 0			134 (5)
6. 0 2h 0			134 (5)
7. 0 0 3i			143 (5)
8. 1g 2h 0			73 (5)
9. 1g 0 3i			77(5)
10. 0 2h 3i		866 (4)	77 (5)
11. g h i		47 (5)	104 (6)
12.1 h i	3561 (4)	53 (5)	238 (6)
13. g 2 i			238 (6)
14. g h 3			222 (6)
15.1g 2h 3i		412 (6)	52 (7)

The next step is to insert these totals into the formula of section 4. For this I need the effective totals for each of the bracketed figures (the p_r of the formula). Counting up gives:

r	2	3	4	5	6
p_r	304	15287	940	1968	52

corresponding to a total of 18555. This leaves 55536 of the original 74088 cases. Thus the formula is modified to

$$\frac{304}{80} + \frac{15287}{96} + \frac{941}{108} + \frac{1968}{116} + \frac{52}{121} + \frac{55536}{128} = \frac{74088}{E}$$

The McGoveran correction factor is then 1 - 1/32E and so the value of $1/\alpha$ becomes 137/[1 - 1/32E] = 137·036012. The values of $1/\alpha$ available in 1996 varied between 137·03601128 and 137·0359940, a range which is just outside the 1986 CODATA value of 137·0359895. I can claim that I have at least seven significant figures in agreement with any of these.

I want to make two concluding remarks. First, I am engaged in careful recalculation of the effective totals in the table but I must point out that the result is very robust against small errors. For example, if (as actually happened as I prepared this paper) the effective total for 11·2 is reduced by 1, the effect on 1/E is to reduce it by (1/108 - 1/128)/74088 and so to change $1/\alpha$ by about 8.10^{-8}. Second, I owe a great debt to many ANPA members over the years. If I have mentioned only David McGoveran, who has enlightened me, and Ted Bastin and Arleta Dylus, who have both pushed me and inspired me in their own ways, I must not be thought ungrateful to all the others for their help and encouragement.

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Observations on the Perception of Time

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Much has been written about both physical and psychological time. The modern view of physical time ranges from a continuum fourth dimension to a discrete, chronon-based quantum time. As is common throughout physics, both extremes have their experimental justification and their strong proponents, but little is done toward providing reconciliation. Curiously, time is also recognised as a component of psychological or perceptual description somehow differentiated from the physical concept. The well-known impact of observation on physical experiment as expressed via the uncertainty principle has long teased psychologists, suggesting that there must be a connection between these two kinds of time. Indeed, it led to synergy between such noted theorists as David Bohm in physics and Karl Pribram in psychology with a holographic theory of both the physical and the psychological worlds. Curiously, after years of denigration these theories have recently seen resurgent respect and interest.

The asymmetry or directionality of physical time was given a new important basis by Eddington when he noted that it could be correlated with increase in entropy. This principle of statistical thermodynamics serves well through most of physics, perhaps breaking down in the quantum region. That it does, in fact, break down is a widely held view, but one which I would like to call into question *en passant*. The problem is that it is difficult to define entropy in a consistent manner when the number of objects in a quantum system is quite small. In particular, what does it mean to speak of the statistical entropy of a single quantum?
If we relate entropy algebraically to other parameters such as some kind of "configuration" energy, we inherently assume a continuum background. In effect, we must change the definition of the statistical population in order to obtain something with enough granularity for the definition of entropy to work. We "rechunk" our description of a physical system as we move from macroscopic objects to molecules to

atoms to elementary particles and so on. As long as this "rechunking" is a classical reductionist or decomposition hierarchy, the definition of entropy and the corresponding arrow of time can be consistent. However, if it is not true that the objects counted at a lower (more granular) level of description can be composed to as to recover exactly the objects counted at a higher (less granular) level, then the meaning of entropy between the two levels cannot be made consistent. The key conclusion here is that the directionality of time as identified with increase in entropy cannot be made consistent at all levels of decomposition. In effect, we must observe this as a breakdown in the direction of time, with processes being able to move backward and forward in time once we cross into a descriptive realm in which the reductionist hierarchy has failed.

Curiously, chunking is a peculiarly psychological notion, having almost as much to do with description as anything inherent in the system observed. Chunking of psychological experience is an extremely important phenomenon and has many effects (beyond the scope of this note) on reasoning and memory. For example, almost everyone knows the "7 ± 2" rule of memory: human beings are much better at recall and reasoning if the number of items to be remembered (and differentiated) or reasoned about does not exceed the range of seven plus or minus two. We often forget that such chunking is volitional and so can be used to advantage or disadvantage. Observe that these considerations remain true even in describing physical systems, whether at macroscopic or quantum. Ultimately, chunking is about managing information, a fact that deserves much more attention than it has been given.

Shannon observed that entropy was equivalent to information. When combined with Eddington's observation, it follows that we can define the direction of time as increasing with increase in information. Although much was written starting in the early 1960s about Heisenberg uncertainty, entropy, and information as applied to psychological models (including computing the maximum amount of information the human brain can process without "boiling over"), this particular dictate has been applied to physical systems insofar as I am aware. A few moments consideration of the implications for psychological experience produce an interesting conclusion.

As we observe, presumably we acquire information. That is, our experience of the direction of time is inherent in our ability to take in information. Indeed, most of us have had the experience of time "dragging on" when we are bored and have little sensory input (i.e., are acquiring information at a lowered rate) and its converse of wondering "where the time went" when we are very busy (i.e., are acquiring information at an increased rate). Thus it seems that common experience would validate the combined Eddington-Shannon theory of time direction.

There is another way of looking at this phenomenon. Suppose, hypothetically, that we were able to observe a physical process going backward in time. By definition, we would know something at the end of this process that we did not know at the beginning of the process. Thus, information would be increasing and by Eddington-Shannon, time would be going forward. But this is contradicts our hypothesis, and it follows that – overall – *human beings cannot experience backward time.*[1]

This result applies to all physical systems, not just human beings, and says that *no physical system can detect backward time.* To detect backward time in some subject physical system, the detecting physical system must obtain information from the subject system and so its local time must be forward time. Thus any description of the subject system is, at best, one consistent with forward time. This gedanken experiment raises a curious paradox: How can a system moving backward in time communicate with a system moving forward in time? I leave the resolution of this paradox, or the discovery that one does not exist, as an exercise to the reader or listener as the case may be.

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[1] It is, of course, possible that the "observer" experiences a relative lose of total information due to other processes such as memory loss. Thus it might be possible in physical systems for there to be local processes that can be described as having backward time embedded in a matrix of processes, the global description of which would have forward time. I am uncomfortable with such descriptions and suspect that they will always involve some form of inconsistency between the local and global descriptions.

The Pope-Osborne Angular Momentum Synthesis

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ABSTRACT

It is postulated that all motion is naturally orbital and that angular momentum is holistically conserved. In particular, an isolated two-body system is considered and it is shown how conservation of angular momentum is sufficient in itself to explain orbital motion in such a system, without the need for assuming the existence of an *in vacuo* 'gravitational' force of attraction. This approach naturally incorporates any effects due to spin and leads us to predict certain measurable physical effects associated with spinning bodies. It is shown how this approach, in incorporating effects due to spin, is able to deduce the parameters of the hydrogen atom from considerations of angular momentum alone, without the need for introducing an *in vacuo* 'electrostatic' force. Accordingly, it is suggested that the postulated *in vacuo* 'forces' of static interaction held responsible for the 'orbital' motion of 'particles' on the micro- as well as the macro-physical levels may usefully be replaced by a common factor, angular momentum.

1. Holistic Conservation of Angular Momentum

A fundamental question to which the Pope-Osborne Angular synthesis (POAMS) seeks an answer is what mechanism causes 'freely moving' bodies in the 'universe', such as stars and the planets, to move in the way that they do, such that a holistic balance is maintained. In Newton's theory, all inertial, or force-free motion is naturally rectilinear, so that orbital motion, precisely the motion which is generally observed, is unnatural, caused by an invisible 'gravitational force' acting *in vacuo.* What are traditionally called Newton's three laws, together with his inverse square law for gravitational attraction, provides an equation of motion for orbiting bodies. In contrast, POAMS accepts, as it stands, the empirical and observational evidences of unmediated instantaneous correlation-at-a-distance as simply revealing the natural tendency of objects to move and distribute (balance) themselves in instant conformity with the various holistic conservation laws. In this way, POAMS concurs with Phipps, Graneau and Assis [1][2][3], that there is an urgent need to rethink our ideas of space, time and motion along more holistic lines proposed by Mach.

In particular, the POAMS approach to gravitation is to postulate that the natural paths of 'freely moving' bodies are closed, so that all 'force-free' motion (used here in the same sense as in General Relativity) is naturally orbital, with no need to postulate any *in vacuo* 'gravitational forces' being responsible for these curvilinear motions. In other words, for inertial or 'force-free' motion, all bodies seek to move, if allowed, in closed orbits with respect to one another, trajectories of the sort that we observe in astronomical space. It must be remembered that *in vacuo* forces are not observed in practice. Newtonian theory construed what we label '*in vacuo* forces' satisfactorily to explain observed trajectories in space and so implied that these actually exist. POAMS provides an explanation for what these apparent *in vacuo* forces actually are, by simply by switching from a rectilinear to a curvilinear force-free motion assumption. This approach also removes any necessity of having to contemplate the existence of any mediatory devices for the transport of these apparent 'forces', such as the 'ethers' and fields of classical imagining. It should be noted here that POAMS does not imply that vacuum forces do not exist, only that they are superfluous, in the same way as Special Relativity does not imply that the luminiferous ether does not exist, only that it becomes redundant. In contrast, 'real' forces, *i.e.*, forces which are actually experienced and measurable in some way, are important in POAMS.

The *natural orbit* of any particle may therefore be thought of as the path described by the particle when all restrictions on its motion are removed, that is, when the particle moves freely under the influence of nothing but its own angular momentum. In POAMS, it is a consequence of Mach's principle of universal connexivity of masses and their motions that all bodies in motion in the 'universe' at all times follow their natural orbit, except those in artificially constrained orbits such as a body in contact with the surface of a planet, or an artificial satellite equipped with a booster rocket.

According to POAMS, the holistic balance in the 'universe' is maintained by holistic conservation of angular momentum [4][5]. Of course, the fact that angular momentum is universally conserved cannot be expected to provide, in itself, the reason why bodies move in the way they do. It is quite clear that any conservation principle does not automatically provide an equation of motion. POAMS chooses conservation of angular momentum as the instantaneously correlating principle in action-at-a-distance since angular momentum is the natural measure to use in orbital motion and contains information about the fundamental empirical measures of spatial relations, mass, time and orientation. The other likely candidate for the instantaneously correlating principle in action-at-a-distance is conservation of energy. However, angular momentum provides more information than energy, being a vector rather than a scalar quantity. Also, conservation of angular momentum does not imply conservation of kinetic energy. For example, in an elliptical natural orbit of constant angular momentum, the kinetic energy of the orbiting particle varies.

In complete contrast to both Newtonian theory and General Relativity, POAMS makes no distinction between the 'universe' on the macro- and micro-levels. Whereas both Newtonian theory and General Relativity have an underlying continuum, in POAMS, the structure of the 'universe', at the microphysical level, is ultimately discrete, with angular momentum quantised in discrete units of $h/2\pi$. Other recent work, notably by Kanarev, also emphasises the central role of conservation of angular momentum, both in the effect of spin on macroscopic orbital trajectories, as is also indicated later in this chapter, and its quantisation in atomic structure [6].

2. The POAMS Approach to Orbital Motion

In order to illustrate the basic POAMS approach to orbital motion, we restrict ourselves to the simplest case of an isolated paired system consisting of two bodies, in which a particle *P* of mass *m* orbits a larger body *B* of mass *M*, where *M* □ *m*. In POAMS, all motion is naturally orbital and closed relative to some chosen origin, so we shall assume in this paired system that the orbit of *P* relative to *B* is closed. For initial simplicity, we shall also assume that *P* and *B* either do not spin or that they spin very slowly so that there is no or negligible contribution from the spin angular momenta of *P* and *B*. In this case, *P* can be considered as orbiting an origin *O*, which is the centre of mass of *B*. This is a special case of a more general situation, in which simply a particle *P* in orbit about a point *O* is considered. For the present, we shall not assume that *P* is freely moving, i.e. the orbit of *P* could be constrained in some way.

As in Newtonian dynamics, the *orbital angular momentum*, **L**, of the particle *P*, in orbit about a point *O*, is defined by

$$\mathbf{L} = \mathbf{r} \times m\mathbf{v} = \mathbf{r} \times \mathbf{p}, \qquad (1)$$

where $\mathbf{r}$ is the position vector of P, $\mathbf{v} = d\mathbf{r}/dt$ is the velocity of P and $\mathbf{p}$ is the linear momentum of P, all relative to O, at any instant of time t. Then the orbital angular momentum of P is a vector that lies perpendicular to the plane containing $\mathbf{r}$ and $\mathbf{v}$. Since, by hypothesis, the angular momentum of P is constant in time, it follows that $d\mathbf{L}/dt = \mathbf{0}$. Let $L = \|\mathbf{L}\|$ denote the magnitude of $\mathbf{L}$. Since $\mathbf{L}$ is a constant vector, it follows that the orbit of P lies in a plane and L is a constant. Also, since $d\mathbf{L}/dt = \mathbf{0}$, it follows by (1) that

$$d\mathbf{L}/dt = d\mathbf{r}/dt \times m\mathbf{v} + \mathbf{r} \times m d\mathbf{v}/dt = \mathbf{v} \times m\mathbf{v} + \mathbf{r} \times m\mathbf{a} = \mathbf{r} \times m\mathbf{a} = \mathbf{0},$$

where $\mathbf{a} = d\mathbf{v}/dt$ is the acceleration of P relative to O. Since $m \neq 0$, $\mathbf{r} \neq \mathbf{0}$ and $\mathbf{a} \neq \mathbf{0}$, ($\mathbf{a}$ cannot be $\mathbf{0}$ since $\mathbf{v}$ cannot be a constant in orbital motion), this implies that $\mathbf{a}$ must be parallel to $\mathbf{r}$ and so, clearly, the acceleration of P is directed towards O.

Since the orbit of the particle P lies in a plane, it is convenient to describe it using plane polar coordinates r and θ, where $r = 7\mathbf{r}7$ and θ is the angle between the radial vector $\mathbf{r}$ and some fixed radial axis. The *angular speed* of P is then denoted and defined by

$$\omega = d\theta/dt = \theta^{\bullet},$$

where a dot denotes differentiation with respect to t. Exactly as in Newtonian theory, it then follows that the *orbital speed*, $v = 7\mathbf{v}7$, of P is given by

$$v^2 = (r^{\bullet})^2 + r^2\omega^2. \qquad (2)$$

The magnitude of the acceleration, $a = 7a7$, of P is then given by

$$a = \left| r^{\bullet\bullet} - r\omega^2 \right| \qquad (3)$$

and the magnitude of the orbital angular momentum, L, of P is given by

$$L = mr^2\omega. \qquad (4)$$

The fact that L is constant implies that $r^2\omega$ is a constant, *i.e.*, $r^2\theta^{\bullet}$ is a constant, so that equal areas are swept out in equal times by the orbit of

P. In other words, this law due to Kepler does not depend on the existence of a Newtonian inverse square law.

So far, we have not considered the particular shape of the orbit of P about O. We consider, first of all, the case in which the orbit of P about O is perfectly symmetrical, so that P's orbit is a *circle* with centre O. Of course, circular orbits are readily observed. For example, if P is any fixed point on the surface of a rigid body that is rotated about a fixed point O, then P will have a circular orbit relative to O. In the case of a circular orbit, r is a constant, so that $r^{\bullet} = r^{\bullet\bullet} = 0$, and it then follows from (2) and (3) that

$$a = r\omega^2 = v\omega. \qquad (5)$$

Equation (4) then gives

$$L = mvr. \qquad (6)$$

Hence, in this case, v is a constant and from (1), $\mathbf{v}$ is perpendicular to $\mathbf{r}$ and hence to $\mathbf{a}$.

In general, in order to determine all possible closed orbits of P about O, we require, of course, an equation of motion. Since the acceleration of P is directed towards O, it must take the form

$$\mathbf{a} = -h(r)\mathbf{n}$$

for some function $h(r)$, where $\mathbf{n}$ is a vector of unit length in the direction of $\mathbf{r}$, *i.e.*, $\mathbf{n} = \mathbf{r}/r$. (It is possible that h is a function of both r and θ, but then in the special case of a circle, this would give $\|\mathbf{a}\| = h(r, \theta)$ where $\|\mathbf{a}\|$ and r are constant, giving a contradiction.) This provides the general equation of motion

$$\mathbf{a} = d^2\mathbf{r}/dt^2 = -(h(r)/r)\mathbf{r} \qquad (7)$$

for the orbit of P. Then, noting that $r^2\theta^{\bullet} = L/m$, a constant, (3), (4) and (7) give

$$a = |r^{\bullet\bullet} - r(\theta^{\bullet})^2| = |r^{\bullet\bullet} - L^2/(m^2r^3)| = h(r). \qquad (8)$$

Knowing $h(r)$, this equation can be solved for $r(t)$, at least in theory, in the usual way [7], and together with $\theta^{\bullet} = L/(mr^2)$ gives $r = r(\theta)$ as the equation of the orbit of P.

Since we require the orbit of P about O to be closed, it follows by Bertrand's theorem [8], that either $h(r) = \alpha/r^2$ or $h(r) = \alpha r$, for some positive constant α. It is interesting to note that in the case $h(r) = \alpha r$, (8) can be integrated using the standard procedure to give

$$1/r^2 = E/L^2 - \sqrt{(E^2 + \alpha L^2)}\sin(2\theta - 2\theta_0),$$

as the equation of the orbit of P, where θ_0 and E are constants, with E related to the total energy of the orbit. With appropriate scaling we can consider the equation

$$1/r^2 = K - \sin 2\theta, K > 1.$$

It is easily seen that this equation represents a closed curve that is ellipse-like in character in the sense that it is symmetrical about a major and a minor axis, but it does *not* represent an ellipse. Note that in this case, since $h(r) = \alpha r$, it follows that $\|d^2\mathbf{r}/dt^2\| = \alpha r$, so that $\|d^2\mathbf{r}/dt^2\|$ takes its maximum value at the point on the orbit of P *furthest* from O, which makes no physical sense and does not agree with empirical evidence.

Hence, the only choice left for $h(r)$ that agrees with observational and empirical evidence is $h(r) = \alpha/r^2$, and so it follows from (7) that in POAMS, the equation of motion for P in a closed orbit about O

$$d^2\mathbf{r}/dt^2 = -(\alpha/r^3)\mathbf{r}. \qquad (9)$$

This equation gives (8), with $f(r) = \alpha/r^2$, which can be solved in exactly the same way as in Newtonian theory [9] to show that since the orbit of P about O is closed, it must be an ellipse. In this case, since $\mathbf{a}$ is directed towards O, it follows by (4) and (8) that

$$L = \alpha m/r\omega + mr\ddot{r}/\omega. \qquad (10)$$

In the case of a circular orbit, this equation simplifies, using (5), to

$$L = \alpha m/r\omega = \alpha m/v. \qquad (11)$$

Let us now return to the more specific case of a freely moving particle P, of mass m, orbiting a body B, of mass M. Since we wish to compare (9) with the corresponding Newtonian equation of motion in this case, we shall re-label α as GM, where G is simply a new constant, introduced for the sake of convenience, so that the equation of motion of P is

$$d^2\mathbf{r}/dt^2 = -(GM/r^3)\mathbf{r}. \tag{12}$$

This, of course, is now the same as the Newtonian equation of motion, but it is important to note that this has been derived with no reference to arguments involving forces. In contrast, Newtonian dynamics implies that orbital motion is unnatural, caused by an *in vacuo* force of 'gravitational attraction' so that (12) is derived by equating the centrifugal force, $m\mathbf{a}$, on P with the artificially introduced 'gravitational force' $-(GmM/r^3)\mathbf{r}$, induced on P by B.

According to the POAMS thesis, the true (*i.e.* measurable) force on the particle P is not concealed in its natural orbit, but rather is *revealed* in the *difference* between that natural orbit and an 'unnatural' one, that is, some state of motion in which the particle is held by a *measurable* force, of magnitude F. Specifically, let P orbit B in an elliptical orbit which is *constrained* in some way, so that although P is not in a natural orbit around B, its equation of motion takes the form (9). (Of course, in general, a particle P may be artificially made to follow any 'unnatural' orbit about P and so may not even have an equation of motion of the form (9).) In general in (9), we let $\alpha = G'M$, where G' is a constant with the same dimensions as the Newtonian gravitational constant G, but only equal to G in any natural (unconstrained) orbit of P. Then the measurable 'gravitational' force on P in any constrained orbit (in the sense just described) has magnitude, F, given by

$$F = |G - G'|\, mM/r^2. \tag{13}$$

Otherwise, in a natural, 'force-free' orbit, $G' = G$ and so $F = 0$.

Let P orbit B in a constrained closed orbit (in the above sense). For a particular orbit, the parameters $r(t)$ and $\omega(t)$ are known. Then, since from (8),

$$\ddot{r} - r\omega^2 = -G'M/r^2 \tag{14}$$

in this case, and the mass M of body B is known, the constant G' for this orbit can be calculated. Note that G' does not depend on the mass m of P. The magnitude F of the measurable force exerted on P in its constrained orbit is then given by (13). In the context of the type of constrained orbits we have been considering, the obvious and most useful case to consider is the case in which P is constrained to the surface of a body B, and so orbits the center-of-mass O, of B, where the mass M of B is considered to be concentrated at O. In this case, P's constrained orbit is a *circle* with center O. It follows from (14) and (5) that, in this special case

$$r\omega^2 = G'M/r^2 \Rightarrow G' = rv^2/M, \quad (15)$$

where v is the orbital speed of P in its constrained circular orbit of radius r. By calculating G', we can determine immediately whether or not P is in a natural orbit, since P is in such an orbit if and only if $G' = G$.

Consider once again P in a general elliptical constrained orbit. Then since $r(t)$ and $\omega(t)$ are known, the angular momentum, L, of P's orbit may be calculated from (4). It is then possible to calculate any natural orbit of P, with the same angular momentum L, if the orbit of P were not constrained. Let the particle P, in its natural orbit, with angular momentum L, have angular speed $\omega_0(t)$ and radial coordinate $r_0(t)$. Then by (4) and (10) with $\alpha = GM$,

$$L = mr_0^2\omega_0 = (GmM/r_0 + mr_0r_0^{\cdot\cdot})/\omega_0 \quad (16)$$

$$\Rightarrow L = (GmM/r_0 + mr_0r_0^{\cdot\cdot})(mr_0^2/L)$$

$$\Rightarrow L^2 = m^2r_0(GM + r_0^2r_0^{\cdot\cdot}). \quad (17)$$

Knowing L, G, M and m, (17) is a second order differential equation for $r_0(t)$, and with a given set of initial conditions can be solved, at least in theory, to give the parameters of any natural orbit of the particle, *i.e.*, $r_0(t)$ may be calculated and then $\omega_0(t) = L/mr_0^2(t)$. Notice that any natural orbit of P is not unique, since it depends on a given set of initial conditions. However, given a constrained orbit of P, there is a unique *circular* natural orbit for P, with parameters r_0 and v_0, where v_0 is the orbital speed of P. In this case, it follows by (11), with $\alpha = GM$, that (16) reduces to

$$v_0 = GmM/L \quad (18)$$

and then, by (6),

$$r_0 = L/mv_0. \quad (19)$$

Note that the parameters v_0 and r_0 do not depend on the mass m, since (18) implies that $v_0 = GM/r^2\omega$ and then (19) gives $r_0 = r^2\omega/v_0$.

Example

Consider a mass of 1 kg, say, placed on a weighing-scale somewhere on the earth's equator. The speed of the mass in this constrained, very nearly circular orbit about the earth's centre, is the revolutionary speed of the earth, namely $v \approx 464.74$ m s^{-1}. The radius of this orbit is the earth's mean equatorial radius, given by $r \approx 6.372828 \times 10^6$ m. For this constrained circular orbit, G' is given by (15) where M is the mass of the earth. Notice that this is not exactly the usual figure quoted for M in Newtonian dynamics since in POAMS, according to (13)

$$M(G - G')/r^2 = g,$$

where g is the usual acceleration due to the earth's 'gravitational pull'. Hence, by (15),

$$gr^2 = M(G - G') = MG - rv^2 \Rightarrow M = r(rg + v^2)/G.$$

This gives $M \approx 5.989 \times 10^{24}$ kg and it then follows by (15) that

$$G' \approx 2.298254 \times 10^{-13} \text{ N m}^2 \text{ kg}^{-2}.$$

Now $G \approx 6.67259 \times 10^{-11}$ N m^2 kg^{-2}, so that 1 kg mass is not in any natural orbit about the earth's centre. It follows by (13), that the magnitude of the real, measurable force on this mass at the earth's equator is,

$$F = M(G - G')/r^2 \approx 9.81 \text{ N}.$$

This is the magnitude of the force with which the 1 kg mass seeks to oppose the intervention of the earth's surface and to take up its natural circular orbit. It is this force that produces 1 kg weight on the weighing-scale.

Since the motion of the mass in its constrained orbit is circular, the magnitude of its orbital angular momentum, L, is given by (6), so that $L \approx 2.961708 \times 10^9$ kg m^2 s^{-1}. With this relatively small amount of angular momentum, if we imagine the earth's radius shrunk to the size of a super-dense billiard ball, the natural orbital speed, v_0, of that kilogram mass, assuming its natural orbit to be circular around the earth's mass, is given by (18), so that $v_0 \approx 1.3493 \times 10^5$ m s^{-1}. Then the radius, r_0, of this natural orbit is given by (19), so that $r_0 \approx 2.195 \times 10^4$ m. This is approximately 1/290th of the earth's true radius. The mass seeks to orbit at this radius but is prevented from doing so by the Earth's surface.

3. Spin in POAMS

Recall that in POAMS, the total angular momentum of any system is holistically balanced, as opposed to that balance being mediated by any time-retarded causal interaction between bodies. In general, the total angular momentum does not just consist of contributions from orbital motion. It will also depend on the effect of other forms of angular momentum, and in particular, that due to spin. Of course, any effects due to spin can be taken into account in Newtonian dynamics. In this approach, Newton's laws may be applied to any system containing spinning bodies by treating the spinning bodies as themselves, being a system of many orbiting particles. However, the POAMS approach is entirely different. In POAMS, any effects of spin are instantaneously correlated with the total angular momentum. POAMS takes account of this by postulating that the orbital kinetic energy of an orbiting spinning particle depends on both its spin kinetic energy and the orbital kinetic energy it would have if it were not spinning. This new approach implies that the 'gravitational constant' G, has to be replaced by a more general function $\mathcal{G}$ of the orbital parameters.

Once again, only the case of a particle P, of mass m, orbiting a body B, of mass M, will be considered. According to POAMS, any orbital angular momentum is balanced against the total momentum of the system, so that introducing a spin at any one place has a direct and immediate effect on the system throughout. In other words, P's spin will cause it to take up a different natural orbit from that if it were not spinning. A convenient way of investigating the effect of P's spin on its orbit is by considering the kinetic energy of P.

Suppose for the moment that P does not spin and follows a natural elliptical orbit about B, with radial coordinate $r_0(t)$, and with angular speed $\omega_0(t)$. As in Newtonian dynamics, P's orbital kinetic energy, K_o, is given by

$$K_o = mv_0^2/2, \qquad (20)$$

where $v_0(t)$ is the orbital speed of P in its natural orbit. It follows by (2) and (4) that

$$K_o = m(r_0^{\bullet})^2/2 + L\omega_0/2$$

$$\Rightarrow \; L = 2K_o/\omega_0 - m(r_0^{\bullet})^2/\omega_0,$$

(21)

where L is the magnitude of P's orbital angular momentum. Hence, P's orbital angular momentum depends on its orbital kinetic energy.

Now let P spin on its axis while orbiting the body B. POAMS postulates that the orbital parameters of P's orbit will be directly affected by P's spin. Hence, by (21), when spinning, P follows a new natural orbit given by

$$L = 2\mathcal{K}/\omega_s - m(r_s^{\bullet})^2/\omega_s, \tag{22}$$

where now $\mathcal{K}$ is P's orbital kinetic energy, and $r_s(t)$ and $\omega_s(t)$ are the parameters of P's natural orbit, under the influence of its spin. POAMS postulates that $\mathcal{K}$ depends on both the orbital kinetic energy, K_o, of P of the natural orbit it would follow if it were not spinning and its spin kinetic energy, K_s. In other words, P's spin kinetic energy is transferred directly to its 'new' orbital kinetic energy.

Clearly, the maximum possible effect due to spin occurs when P either spins in the same direction as its orbital motion, so that K_s acts with K_o, or in the opposite direction to its orbital motion, in which case K_s acts against K_o. In the first case we postulate that

$$\mathcal{K} = K_o + K_s \tag{23}$$

and in the second case, we postulate that

$$\mathcal{K} = |K_o - K_s|. \tag{24}$$

We maintain that an intermediate effect takes place when P's axis of rotation is inclined at some angle to its orbital plane.

Once $\mathcal{K}$ has been determined, it follows by (22) and (4) that

$$L = 2\mathcal{K}mr_s^2/L - m^2(r_s^{\bullet})^2\, r_s^2/L$$

$$\Rightarrow L^2 = 2\mathcal{K}mr_s^2 - m^2(r_s^{\bullet})^2 r_s^2.$$

(25)

Since L can also be determined, (25) is a first order differential equation for the orbital parameter $r_s(t)$. Hence, at least in theory, $r_s(t)$ may be determined and then $\omega_s(t)$ can be found using (4). The fact that P's orbital parameters are affected by its spin gives rise to the fact that the equation of motion, (14), must be adapted to read

$$r_s^{\bullet\bullet} - r_s\omega_s^2 = -\mathcal{G}M/r_s^2, \qquad (26)$$

that is, G is replaced by a function $\mathcal{G}$ which depends on the orbital parameters r_s and ω_s. The function $\mathcal{G}$ reduces to G when and only when (26) describes any natural, spin-less orbit of P, so that $\mathcal{G} \to G$ as $\mathcal{K} \to K_o$. If r_s and ω_s are known, then the corresponding 'gravitational factor', $\mathcal{G}$, may be calculated from (26).

If the orbit of P is circular, this process is considerably simplified. In this case, it follows by (5) and (6) that (22) reduces to

$$L = mv_sr_s = 2\mathcal{K}r_s/v_s \qquad (27)$$

$$\Rightarrow v_s^2 = 2\mathcal{K}/m,$$

(28)

where v_s is the orbital speed of P. Note that (28) is, of course, just the definition of $\mathcal{K}$ as the orbital momentum of P. Since $\mathcal{K}$ is known, v_s may be determined by (28) and then r_s may be calculated from (27). Finally in this case, it follows by (5) that (26) reduces to

$$\mathcal{G} = r_sv_s^2/M. \qquad (29)$$

In summary then, we acknowledge that Newtonian mechanics provides all the means to deal with the more general situation in which self-rotation (spin) is included in the overall angular momentum account. However, the key difference between Newtonian Mechanics and POAMS is that in POAMS, the spin angular momentum and the corresponding spin energy are given as initial conditions and the change in the total angular momentum is ascribed to a modified 'gravitational factor', which replaces Newton's 'gravitational constant'. POAMS predicts that a change in spin causes a change in $\mathcal{G}$ in order that the holistic balance of the angular momentum of the system is maintained. A major difference then, between the POAMS account of orbital motion and Newton's 'gravitational' account, is that in POAMS, the spins of bodies affect their orbital parameters in a way that cannot be accounted for by purely 'gravitational' mass attraction.

Example

Consider a spherical steel ball of mass $m = 2.52$ kg and of radius 4.25 cm. (We are indebted to IEC in Poole, Dorset, who have told us that they can supply such a ball with these physical dimensions.) We shall assume that the ball is situated somewhere on the earth's equator, so that its orbit around the earth's centre of mass is approximately circular. As in the previous example, the orbital speed of this ball in its constrained orbit is the revolutionary speed of the earth, namely $v \approx 464.74$ m s^{-1}. The radius of this orbit is the earth's mean equatorial radius, given by $r \approx 6.372828 \times 10^6$ m. It follows by (6) that the magnitude of its orbital angular momentum is

$$L \approx 7.463504 \times 10^9 \text{ kg m}^2 \text{ s}^{-1}.$$

This ball, as an inert mass, would, if allowed to do so, follow its natural circular orbit with a radius r_0, at orbital speed v_0. It follows by (18), where M is the earth's mass as in the example in section 3, that

$$v_0 \approx 1.349294 \times 10^5 \text{ m s}^{-1}.$$

Then by (19),

$$r_0 \approx 2.195006 \times 10^4 \text{ m}.$$

Recall that that these parameters are independent of the mass m. It follows by (15), exactly as in the previous example, that for the mass's constrained orbit,

$$G' \approx 2.298254 \times 10^{-13} \text{ N m}^2 \text{ kg}^{-2}.$$

The orbital kinetic energy, K_o, of this mass, if allowed to follow its natural circular orbit is given by (20), so that

$$K_o \approx 2.293948 \times 10^{10} \text{ kg m}^2 \text{ s}^{-2}.$$

Now let the spherical steel ball spin at 2000 revolutions per second, in the same direction and in the same plane as its constrained orbit around the earth's center of mass. (We are indebted to Colin Evans of the Department of Physics, University of Wales, Swansea, who has told us that it is physically possible to spin such a ball at such a speed without distortion.) The spin kinetic energy, K_s, of the ball is given by

$$K_s = m(r\omega)^2/5,$$

where r is the radius of the ball and ω is its angular speed. Hence, in this case,

$$K_s \approx 2.52 \times (0.0425 \times 2\pi \times 2000)^2/5 \approx 1.437567 \times 10^5 \text{ kg m}^2 \text{ s}^{-2}.$$

It then follows by (23) and (28) that the ball's orbital speed, v_s, in its natural circular orbit, if allowed to follow that orbit, would be given by

$$v_s = (2\mathcal{K}/m)^{1/2} = (2(K_o + K_s)/m)^{1/2} \approx 1.349298 \times 10^5 \text{ m s}^{-1},$$

and the radius, r_s, of that orbit would be given by (27), so that

$$r_s = L/mv_s \approx 2.194999 \times 10^4 \text{ m}.$$

Since $r_s < r_0$, it follows that the spinning ball would naturally orbit at a smaller distance from the earth's centre of mass than if it were not spinning, so that the spinning ball is more 'attracted' to the earth's surface than if it were not spinning. In other words, the spinning ball should weigh more. This can be immediately seen, since by (29), (6) and (18), the 'gravitational factor', $\mathcal{G}$, for the ball's natural circular orbit when spinning is given by

$$\mathcal{G} = r_s v_s^2/M = Lv_s/(mM) = (GmMv_s)/(v_0 mM) = v_s G/v_0. \qquad (30)$$

Hence, in this case,

$$\mathcal{G} \approx 1.000003G.$$

Just how much more the spherical steel ball weighs when it is spinning, in the sense described above, can be calculated by using the technique of the previous example. The weight of the ball, when not spinning, is given by the weight equivalent of the force with magnitude F, where

$$F = mM(G - G')/r^2 \approx 2.52 \times 9.805865 \text{ N} = 2.52 \text{ kg}.$$

On the other hand, the weight of the ball, when spinning in the sense described above, is given by the weight equivalent of the force with magnitude F', where now

$$F' = mM(\mathcal{G} - G')/r^2 \approx mM(1.000003G - G')/r^2 \approx 2.52 \times 9.805896 \text{ N}$$

$$\Rightarrow F' \approx 1.000003 \times 2.52 \approx 2.520008 \text{ kg}.$$

In other words, when spinning, the weight of the ball is increased by approximately 8/1000$^{\text{ths}}$ of a gram.

Now suppose that the steel ball spins at 2000 revolutions per second but now in the opposite direction to its constrained orbit about the earth's centre of mass, and in the same plane. The spinning ball will have the same spin kinetic energy, K_s, as above, but now, according to (24) and (28), its orbital speed in its natural orbit would be

$$v_s = (2(K_o - K_s)/m)^{1/2} \approx 1.3492895 \times 10^5 \text{ m s}^{-1},$$

if the ball was allowed to follow that orbit. The radius of that natural orbit would be, by (27),

$$r_s \approx 2.195013 \times 10^4 \text{ m},$$

so that $r_s > r_0$, which means that the ball is less 'attracted' to the earth than if it were not spinning. From (30), the 'gravitational factor' for the ball's natural orbit is now

$$\mathcal{G} \approx 0.999997G$$

and the spinning ball has a weight equivalent of the force with magnitude

$$F' = mM(0.999997G - G')/r^2 \approx 2.52 \times 9.805834 \text{ N}$$

$$\Rightarrow F' \approx 0.999997 \times 2.52 \approx 2.519992 \text{ kg}$$

so that, when spinning, the ball's weight is decreased by approximately 8/1000$^{\text{ths}}$ of a gram. The same changes occur, of course, to the 'acceleration due to gravity' of a falling spinning ball. It follows from the calculations above that if the spherical steel ball is not spun and is dropped towards the surface of the earth, then its acceleration is about 9.805865 m s^{-2}, whereas if it is spun in the same direction and in the same plane as the earth's orbit and is then dropped, its acceleration is increased by a factor of about 1.000003. Similarly, if the steel ball is spun in the opposite direction and in the same plane as the earth's orbit and is then dropped, its acceleration is decreased by a factor of approximately 0.999997. It is to be noted that these are tiny effects and, according to our thesis, they are the maximum possible effects for the specified steel ball.

Although, to the best of our knowledge, experiments involving a spinning ball, as described above have not yet been performed, Hayasaka and his co-workers have performed dropping experiments involving a spinning disc [10][11]. In these experiments, a disc of weight 175 gm was used and was spun at 18000 revolutions per minute. Going through the above calculations shows that if the disc is spun in the same direction and in the same plane as its orbital motion, then its acceleration should increase by a factor of approximately 1.00000005 [12]. Similarly, if the disc is spun in the opposite direction and in the same plane as its orbital motion, then its acceleration should decrease by a factor of approximately 0.99999996. In the Hayasaka experiments, a difference in acceleration was detected when the disc was spun in a particular direction, but not in the opposite direction. Also, the difference in acceleration was very much larger than we predict. The experiments were not performed at the equator and, according to our thesis, the disc was not spun as to produce maximum and minimum effects, so that it is difficult to compare the results of these experiments with our predictions. It is encouraging to note however, that the experiments indicate that there is a change in weight of an object if it is spun. The fact that our theory predicts that the effects detected in these experiments should have been very much smaller than those actually reported accords with the findings of Quinn and Picard of the International Bureau of Weights and Measures [13], who have claimed that the Hayasaka results are spurious, although Hayasaka has taken issue with this view [11].

4. POAMS at the quantum level

The fundamental departure of POAMS from Newtonian dynamics as far as the treatment of the effects of spin is concerned may, at first sight, seem unnecessarily radical. Indeed, for normal situations involving macroscopic bodies, spin effects are very small since the total spin angular momentum is very much smaller than the total orbital angular momentum. For example, when studying planetary motion in the solar system, it is not necessary to take the spin of the planets into account. However, the conceptual change dictated by our approach to spin is fundamental to our attempt at unification of the classically conceived 'forces' on the macro- and micro-physical levels. For it must be remembered that according to the holistic approach of POAMS, conservation of angular momentum applies not only on the macro-scale but also on the micro-scale, where the structure of the 'universe', at the micro-physical level, is ultimately discrete, with angular momentum quantised in discrete units of $h/2\pi$. We shall demonstrate that in contrast to the macro-level, on the micro-level the intrinsic angular momentum (commonly called 'spin') predominates over the orbital angular momentum. This accords well with the fact that those elementary particles conventionally called 'electrically charged' are much more volatile in terms of the strengths of the classically conceived forces they exert on one another compared to those of 'gravity'. Indeed, so spectacularly different are the strengths of these forces that they have been imputed to 'powers' that differ not only quantitatively but also qualitatively from that of 'gravity'. This, we maintain, is why these powers have been allocated special units such as, for instance, 'coulombs' for 'electric charge'. All these other forces, however, are conceived by analogy with 'gravity' by the introduction of an inverse square law, such as Coulomb's law of electrostatics. We shall demonstrate that angular momentum at the micro-level is essentially a representation of charge.

The following example demonstrates that the parameters for the hydrogen atom can be derived purely from considerations of angular momentum alone, without the classical assumption of the existence of an electrostatic force (as in Bohr's original derivation). The hydrogen atom constitutes the simplest atomic system and, as such, it has played a crucial role in the history of modern physics as a testing ground for atomic theories. As is well known, Bohr's model of the hydrogen atom supplied an explanation of the Balmer formula for the spectral lines of hydrogen on the basis of the electrodynamical theories of Faraday and Maxwell, with the *ad-hoc* addition of the Planck relation to provide the essential discreteness of the possible associated energies [14]. In this electrodynamical approach, the behaviour of an electron in relation to a proton is explained by ascribing to each particle a purely static property called 'charge', of equal magnitude but of opposite sign. The resulting 'attractive force' between the two particles, by Coulomb's law, countered by 'centrifugal force', is thus thought to determine the orbit of the one around the other by analogy with Newton's account of planetary orbits. In contrast, in POAMS the need to imagine such a counterbalance between *in vacuo* forces becomes redundant. POAMS conceives the hydrogen atom as an angular momentum system of automatically paired and balanced masses, equivalent to the conventional 'electron' and 'proton'. However, it must be stressed here that we are not attempting to resurrect the Bohr model of the hydrogen atom. The essential point is that our model is consistent with the underlying philosophy of POAMS. It is not simply a mathematically convenient description of 'the atom' as in quantum mechanics. The fact that our model is not intended to provide a true *physical* description of the hydrogen atom does not prevent us from associating vectors with the quantities involved, in the same way that vectors can be associated with the operators of quantum theory [15].

Example

Consider once again, a particle P of mass m orbiting a body B of mass M, where now the masses are of micro dimensions. Suppose that P has a circular orbit about B and that $m = 9.1093897 \times 10^{-31}$ kg and $M = 1.6726231 \times 10^{-27}$ kg. These masses just happen to be the mass of the so-called 'electron' and 'proton' respectively, in the Bohr hydrogen atom. It must not be supposed that we are implying that the electron physically orbits the proton in a circular orbit, or indeed in any continuous classical orbit. If it is assumed that there are no spin effects present in this system, taking account of orbital angular momentum alone provides natural orbital parameters and hence the parameters of the hydrogen atom which are clearly nonsensical. For assuming that our two-body system has a total angular momentum whose magnitude is $h/2\pi$ and that there are no spin effects present, it follows by (18) that

$$v_0 = GmM/L = 2\pi GmM/h, \qquad (31)$$

where v_0 is the orbital speed of P. Taking the known values, $h/2\pi \approx 1.054572749 \times 10^{-34}$ Kg m^2 s^{-1} and $G \approx 6.67259 \times 10^{-11}$ N m^2 kg^{-2}, (31) gives

$$v_0 \approx 9.640627 \times 10^{-34} \text{ m s}^{-1}.$$

It then follows by (19) that the radius of the natural circular orbit of P about B is given by

$$r_0 = L/mv_0 = h/2\pi mv_0 \approx 1.2 \times 10^{29} \text{ m}.$$

Hence, in this scenario, the 'electron' orbits the 'proton' at a truly enormous distance, at almost zero speed! This clearly demonstrates that orbital angular momentum alone is not sufficient to explain the parameters of the hydrogen atom. The implication is that, in POAMS, the effects of spin become significant at the quantum level. Notice that the orbital kinetic energy of the mass P in its spin-free natural orbit is, by above,

$$K_o = mv_0^2/2 \approx 4.23 \times 10^{-97} \text{ J.}$$

The hypothesis that the 'electron', considered as an elementary particle, has an intrinsic angular momentum, as though it were spinning, was first introduced by Uhlenbeck and Goudsmit in 1926 [16]. This, of course, was not known to Bohr when he proposed his model for the atom in 1913. According to our approach, it is the presence of this spin angular momentum that provides the correct parameters for the hydrogen atom, without assuming the existence of an electrostatic force. Following the technique introduced in section 3, it is necessary to incorporate the intrinsic (spin) kinetic energy of the 'electron' P in the calculation of its orbital kinetic energy for its proper orbit about the 'proton' B. We proceed by finding the mechanical energy equivalent, E, in joules, of the magnitude of the conventional 'electron charge' e. This is given by

$$E = I_0 e, \tag{32}$$

where I_0 is the ionization potential for hydrogen, *i.e* the kinetic energy required to ionize the hydrogen atom [14]. Here, I_0 = 13.6 volts and $e \approx$ 1.60217733 × 10^{-19} coulomb. In POAMS, it is this mechanical energy-equivalent of the static charge which is interpreted as the intrinsic (spin) kinetic energy, K_s, of the 'electron' so that by (32), it follows in this case that

$$K_s = E \approx 2.179 \times 10^{-18} \text{ J.}$$

Notice that the purely orbital kinetic energy, K_o, of the natural orbit of P, without taking spin effects into consideration, is almost negligible compared to K_s. This means that in following the technique of section 3, we can take $\mathcal{K} \approx K_s$, where $\mathcal{K}$ is the orbital kinetic energy of the 'electron' P in its true 'orbit', *independent* of the direction of its theoretical spin in relation to the plane of its theoretical orbit about B. (According to quantum mechanics, there are only two possible spin states for the 'electron', which are conventionally labelled 'up' or 'down'. Analogously, in POAMS it is possible conventionally to assign an intrinsic angular momentum vector to the 'electron', which points either parallel to its orbital angular momentum vector or in the opposite direction. Since, from section 3, $\mathcal{K} = K_o + K_s$ in the first case and $\mathcal{K} = K_s - K_o$ in the second, $\mathcal{K} \approx K_s$ in either case, so that the direction of spin has no effect on the parameters of the orbit of P.)

It then follows from (28) that the orbital speed, v_s, of P in its natural orbit, taking spin effects into account, is

$$v_s = (2\mathcal{K}/m)^{1/2} \approx (2K_s/m)^{1/2} \approx 2.1877 \times 10^6 \text{ m s}^{-1}.$$

Then from (27), the radius, r_s, of the natural orbit of P is given by

$$r_s = (h/2\pi)/mv_s \approx 5.292 \times 10^{-11} \text{ m}.$$

The parameters v_s and r_s are the same as those predicted by Bohr's model [14] and by quantum mechanics for the hydrogen atom [17]. In POAMS, these parameters can be explained in terms of an 'equation of motion' of the form (29) for some particular value of the 'gravitational factor' $\mathcal{G}$. For this 'natural orbit' of the 'electron', (29) gives the value of $\mathcal{G}$ as

$$\mathcal{G} = r_s v_s^2/M \approx 1.5142 \times 10^{29} \text{ N m}^2 \text{ kg}^{-2}.$$

In this way, Coulomb's law of electrostatics is replaced with what is virtually the Newtonian gravitational inverse square law, but with a different value of G. It must be remembered however, that in POAMS, this is not due to some unseen force acting *in vacuo*. The reason, of course, for this huge increase in the value of G is the presence of the relatively enormous amount of the intrinsic kinetic energy of the 'electron'.

It needs to be stressed that our aim in this example is to demonstrate philosophically how those parameters, calculated by Bohr in terms of a Newtonian dynamics laced with the electrodynamics of Faraday, Maxwell, Coulomb and others, could *logically* have been derived from Newtonian-type dynamics alone, simply by altering the value of the 'gravitational constant' G. And again it must be emphasized that what we are proposing here is not some new concept of the atom to compete with the current physical models that superseded Bohr's. That is to say, it is by no means an exercise in up-to-date physics. It is essentially a logical demonstration of how, ahistorically, our concepts of motion and distant interaction might have developed in direct observational terms, without postulating invisible intermediaries such as 'gravitational' and 'electrostatic' forces, linking one atom or part of an atom to another.

5. Conclusion

If these arguments are correct, then there is no necessity for thinking of bodies being attracted and repelled by any sorts of invisible in vacuo field forces, either by "gravitational forces" on the macro scale or "electrostatic forces" on the micro scale. Our ultimate thesis is that with appropriate changes in the value of our generalised "gravitational factor" $\mathcal{G}$, the parameters of "orbits" may be explained in terms of angular momenta. Insofar as the angular momentum nexus may be regarded as a "field", it is of the instantaneous "action-at-a-distance" kind envisaged, in electromagnetic contexts, by Weber and Helmholtz, rather than the time-delayed "propagated" kind postulated by Faraday and Maxwell. [18]

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TIME

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ABSTRACT

This essay is the preamble to a much larger document, entitled Concepts Of Time In Quantum Mechanics (CTQM), which was, originally, a private communication. I have added some paragraphs from CTQM; and mathematical asides, drawn from CTQM, are relegated to appendices. I suggest that the asides should be skipped on the first reading.

'Nothing is constant in the whole world. Everything is in a state of flux, and comes into being as a transient appearance. Time itself flows on with constant motion, just like a river: for no more than a river can the fleeting hour stand still. As wave is driven on wave, and, itself pursued, pursues the one before, so the moments of time at once flee and follow, and are ever new. What was before is left behind, that which was not comes to be, and every minute gives place to another.'

Ovid (Publius Ovidius Naso 43BC-17AD)

Metamorphoses, Bk. XV- 'The Teachings of Pythagoras'

Trans. Mary M. Innes, Penguin Books, 1955

1. APOLOGY

Because my aim is to comment on the treatment of time, in Quantum Mechanics (QM), I feel the need to clarify my general view of time. Apologies then, in advance, for an essay that may well be naive!

2. LIVING CLOCKS

2.1 Perception

When we perceive either a change in our bodily sensations or a change in the external world we say that 'time has passed'; memory is essential to this judgement. We are particularly observant of cycles (i.e., sequences of change that repeat), their relative rates and whether they are speeding up or slowing down; and we are able roughly to quantify time elapsed. There is ample evidence that these senses of rhythm and elapsed time are essential to survival.

2.2 The Nervous System

But how do we recognise rhythm/ elapse? Evidently the nervous system incorporates clocks. These clocks seem to mediate between messages from the senses and the activities of higher levels of the nervous system; for example, they synchronise the body/ world picture created by the brain and the consequent instructions issued, as a result of those messages.

As part of the nervous system there are scanning clocks (α, θ and δ rhythms) analogous to the clocks that synchronise the operations of a computer; these have a resolution of the order 0.1s. There are clocks that work on a time-scale of seconds to minutes associated with movement, sensation and memory. There is the circadian clock that is synchronised to the day and its phases (sleep, hormones, hunger etc.). There are the monthly clocks associated with reproduction. There may also be clocks that synchronise with the seasons; (but it is difficult to tell, a propos of these, whether the dog is wagging his tail or his tail is wagging him!). Not all these body clocks reside in the brain alone; and we are certainly not conscious of all of them.

There is abundant evidence that other animals sense rhythm and the passage of time; and that, consequently, they possess clocks. Even plants have been shown to have a sense of time.

3. SORTS OF CLOCK

3.1 What is a Clock?

There are natural clocks and man-made clocks. But what is a clock? *It is a mechanism for measuring (i.e., quantifying) or parcelling-out time so as to label change.* There seem to be two distinct types of clock: *linear* clocks and *harmonic* clocks. The linear type is, as far as I know, a largely theoretical notion useful for understanding concepts. Man-made clocks and most of the natural clocks are harmonic.

3.2 The Linear Clock

The archetype *linear clock* is a ponderous particle moving in outer space, free from all disturbance by matter and radiation, past a lightweight, rigid, straight ruler. Newton tells us that the particle will move 'uniformly in a straight line'. We can verify the 'straight line' clause by suitable positioning of the straight ruler. But what does 'uniformly' mean? We can say that each time the particle moves past one of the equispaced fiducials, on the ruler, a unit of time has passed. This statement defines a unit of time. Units of time that are not equal would not be of much use; and our intuition tells us that, to make them equal, the fiducials must be equispaced. If the particle then travels the fixed distance between fiducials in unit time, surely, its motion can be said to be 'uniform'. But there is obvious tautology in this definition. Such tautology can, perhaps, be avoided given more than one linear clock, each in a different place and with a different orientation, all viewed by the same observer. [For simplicity suppose that the speeds are so low that Special Relativity (SR) effects are negligible and, further, that the 'observer' has no internal clock]. By counting the numbers of fiducials traversed over a long period the observer can determine whether or not the units of time defined by each clock are in fixed ratios; if all the ratios are fixed then the observer has little option but to assert that all the particle motions are uniform and that therefore, according to Newton, there are no forces. Conversely, if the ratios alter, there are forces. It may be possible, nevertheless, to dupe the observer by judicious placing of force fields; but this, presumably, becomes more difficult as the clocks increase in number.

3.3 Harmonic Clocks

An *harmonic clock* is a device which repeats a precise sequence of actions over and over again. The completion of each cycle can be taken to define a unit of time. A mechanism designed to detect the end of each cycle and to count the number of cycles can measure elapsed time in terms of this unit. Digital watches, with numerical displays, count oscillations and, in effect, show us the total. Traditional escapement clocks also create and count oscillations but, by means of gearing, they convert the accumulation into the displacement of a pointer over a numbered dial. Except in the case of a big clock this motion appears uniform and we might suppose, if we did not know better, that the clock was of a linear type. We encounter problems of definition with harmonic clocks similar to those we encounter with the linear clocks. Only by comparison of many clocks can we determine whether the time units that they define are in fixed ratios. In practice the long term uniformity of an harmonic clock is compromised by a legion of effects that disturb the cyclic mechanism.

3.4 Do Harmonic and Linear Clocks Measure the Same Thing?

The question arises: is any harmonic clock measuring the same quantity as a true linear clock? Evidently by using a rapid escapement and much gearing we are trying to achieve uniform motion of a hand over the clock face (i.e., to convert the harmonic clock to a linear clock). We could use the many clocks test to validate this uniformity; but, nevertheless, unless at least one of the comparison clocks is a *genuine* and *validated* linear clock we cannot be sure that the uniformity achieved is of the same sort as that spoken of by Newton. This matter is thrown into sharp relief if we compare a single cycle of a big escapement (say that of Big Ben) with the many contemporary cycles of a small escapement (say that of my self-winding watch). The big fellow will exhibit all sorts of irregularities (all kinds of squeaks, clunks and bumps) before it gets to the end of its cycle. So, it seems, this cycle, although it is called a unit of time, may not be the same sort of thing as the unit defined by the linear clock which, presumably, can be subdivided indefinitely by intermediate fiducials.

3.5 Clock Types and Their Performance

The nervous system clocks are electrochemical. The chief man-made clock types are: (a) mechanical using a balance wheel and an escapement or one or more pendulii and escapements (1 in 10^6); (b) electromechanical maintaining the oscillation of a structure (e.g., pendulum (1 in 10^7), tuning fork, quartz crystal (1 in 10^{10})) by means of magnetic or piezoelecric forces generated by an electric circuit coupled with them (counting is by a synchronous motor or digital counter preceded, in the quartz crystal clock (100 kHz), by multiple frequency-halving circuits); atomic using, for example, microwave (10^{10} Hz) stimulated transitions in the molecules of caesium vapour to control the frequency of a microwave generator the oscillations of which are electronically counted (1 in 2×10^{12} to 1 in 5×10^{14}).

It is rumoured in the bazaar that a clock is being developed that uses the same basic ideas as the caesium clock but which works at optical frequencies! This may, supposedly, be even more stable.

3.6 Astronomical Phenomena as Clocks

Until the 20th century relatively simple astronomical phenomena provided the most reliable clocks. Observations of cycles (e.g., earth's rotation, moon's orbit, earth's orbit) resulted in various ways of defining the year/ day/ second as mean values. They also provided calibration for the best available clocks. Harrison, for example, used the earth's rotation to calibrate his marine chronometers. Gradually all sorts of irregularities have been discovered in motions, such as the rotation of the earth, previously supposed uniform. The quartz crystal and atomic clocks have been at the centre of this revolution.

It is still necessary to define the second in the most stable way possible so that it can be used as a standard unit for the best man-made clocks and for astronomical and terrestrial events alike. This has been done by defining the second in terms of Dynamical Time. This is the time parameter in a set of differential equations that are assumed to describe the motion of a collection of bodies in the solar system (principally the moon). Long term numerical solutions of these equations have been compared with measurements by optics, radio telescopes, radar and atomic clocks in order to correct (i.e., refine) the standard second.

4. WHAT DO CLOCKS MEASURE?

4.1 An Act of Faith

What are we to make of all this? Harrison's use of the earth's rotation, to calibrate his clocks, was an act of faith; a faith that the rotation of the earth is uniform in the Newtonian sense. [The earth is better thought of as a linear clock rather than an harmonic clock; but that is a question of how we count the degrees!]. We make the same act of faith when we calibrate any man-made clock against a process we believe to be uniform. It is a sobering thought that, if Harrison's clocks had been so sophisticated and so accurate that they were able to mimic the irregularities in the rotation of the earth, they would have been fitter for purpose than a perfect clock!

In the mechanical and electromechanical clocks we seek, somehow, to reproduce Newtonian uniformity, from gross matter, subject to the laws of classical mechanics. But a definition of that uniformity eludes us in exactly the same way as the rest of Newton's laws elude us; there is an element of tautology. These clocks can define reasonably stable units of time; but the units are arbitrary because they depend on arbitrarily chosen dimensions etc. of the mechanism. The atomic clocks, by contrast, define units of time that are *absolute* in the sense that they depend on supposed fundamental constants of physics and the configuration of the molecules of a particular substance. The recorded frequency, admittedly an average over an assemblage of many molecules, is the frequency of *one particular transition*; it is, therefore, a quantity derived on QM principles from a relatively simple structure (the individual molecule) itself determined by QM.

4.2 At Least Two Sorts of Time

The quantum theorists would claim that, strictly, the different ways of measuring time, involving as they do different procedures and apparatii, are, in principle, measuring different quantities. They would point to the stark contrast between mechanical and atomic clocks. It is, they might say, only good fortune that the numerical values agree (within error bounds)! Worse still, the modern theorists would say (firmly) that the time variable t that appears in the equations of QM is not an observable; it is a scalar that commutes with *all* observables. To be sure there are procedures, involving special devices called clocks, that purport to measure something called 'time'; but they are not directly measuring t. In QM t is the parameter of evolution *between* observations; and, since the measurement of 'time' is an observation it can have nothing exact to do with t; see Appendix A. Asher Peres (*Quantum Theory Concepts and Methods*), for example, is at pains to show this by analysing various archetypal quantum clock models. He includes in his models terms which represent the interaction of the clocks with their environment. He establishes relationships between the energy available and the theoretical resolution obtainable. Whatever these mechanisms measure it is not the quantum time t! Further, the accuracy of quantum clocks is never perfect; see Appendix B.

The postulates of physics are often of the kind that state that two or more different procedures measure the same thing. For example, a certain platinum-iridium bar, so many wavelengths of a well defined spectral line and light transit time (in vacuo) might all be said to measure length. Again, gravitational and inertial mass might be assumed equal, offering different ways of measuring mass. So, we should not be unduly suspicious about different time measurements. When methods that are assumed equivalent produce results that disagree we learn something. For example, if G (Newton's gravitational constant) drifts with respect to the other fundamentals then, eventually, Dynamic Time and Atomic Time will disagree; the drift in G may then be detectable.

The quantum theorists assertions about t confront us with a mystery. I am led to the conclusion that there are at least two sorts of time: (a) t the <u>quantum time,</u> '...that like an ever flowing stream bears all its sons away...', represents the fact that the world wags whether or not we choose to watch; (b) Q_0 the <u>time measured by clocks,</u> to label events, which is, necessarily, the result of observation. I have also come to the melancholy conclusion that the study and analysis of the different ways by which we measure time sheds no light on what time really is!

4.3 Which Time Do We Choose?

I suspect that failure to distinguish between t and Q_0 is one of the causes of the difficulties with quantum general relativity (GR). Einstein tells us, in the theories of relativity, that both coordinate time and proper time are *measured by clocks*. So, t is an 'horse of a different colour' from either coordinate time or proper time! Another difficulty with quantum GR may be that the space-time manifold is not, as GR assumes, primary. As you know I am a 'process man' as opposed to a 'manifold man'. I believe that the dynamics of matter and radiation are primary; that is, the manifold is just a symptom of the dynamics. Without matter and radiation there are no events; so time cannot then exist. Also without matter and radiation there is no means to partition space; so metrical space cannot then exist.

It seems, then, that Q_0 is peculiar to the observer and/ or his clock. Perhaps the quantum time t is not peculiar to the observer; perhaps it is best thought of as something like Newton's universal time (i.e., the same everywhere); if so it will always evade us being a strictly theoretical, but seemingly necessary, notion!

A question that vexes: Is the time in Maxwell's theory quantum time t or measured time Q_0? In his theory of the electron Dirac (*The Principles of Quantum Mechanics', 4th Ed.*) seems always to use t. Yet, at the start of the chapter, he clearly identifies the time with Einstein's fourth dimension. Much more modern accounts of quantum field theory, that invoke Maxwell's equations, also seem to use the quantum time t throughout. Another problem is that, in the Schrödinger representation, Dirac's quantized Hamiltonians do not give the correct (quantum) equations of motion. The problem is that the vector potentials a_j are, in the classical theory, functions of time. There is nothing in the Schrödinger representation that generates operators representing the rates $\partial a_j / \partial t$. Dirac gets away with it because, for the most part, he uses classical equations. Something has been glossed over!

I find that, at the very least, it is necessary to add to the QM Hamiltonian a term $-P_0$ where P_0 is conjugate to Q_0, has a continuous spectrum and commutes with the $\underline{P}$, the $\underline{Q}$ and (like all operators) t. This also allows me to bring into my ANPA20/23 theory functions $\theta(\underline{q})$ that depend *explicitly* on time; but there is still the ambiguity about which sort of time one should be using!

4.4 Is Time Multidimensional?

In higher flights of philosophical fancy it has sometimes been suggested that time may be multidimensional. What does this mean? I am not sure; but I suppose that the concept could be defined as follows!

Consider a classical particle contained in a metrical space for which two of the coordinates are timelike. There will be a *surface* (i.e., a 2-D subspace) spanned only by these coordinates. If the points occupied (from time to time!) by the particle lie only in this surface then the particle is stationary; but, if the particle points lie outside the surface the spacelike coordinates of the particle may be subject to change; the particle may be in motion.

Suppose that the trajectory is a continuous curve that has projections in both the timelike and spacelike subspaces. These projections might exhibit strange features; there could be, for example, loops in the timelike subspace. The trajectory itself could be strange; it might be, for example, a 'space-filling' curve (i.e., a fractal).

Under these conditions a continuous transformation of coordinates should always be possible that renders one timelike coordinate invariant along the trajectory and equates the other to the geodesic length. It would then be possible to describe the motion of the particle in terms of *a single time coordinate only*. This argument obviously extends to timelike subspaces of higher dimension. The reduction to a single coordinate does not preclude bizarre behaviour; but it may disguise it! For example the displacement caused by a loop, in multi-dimensional time, may be mistaken for an simple oscillation.

The Schrödinger evolution is deterministic. The array $\underline{q}$ (see (A.1) in Appendix A) can always be defined as the set of spacelike coordinates. All that is required of the unitary evolution of ψ is that it is a deterministic function of the timelike coordinates. But I am not sure that, in the quantum case, a transformation of coordinates can *always* bring about dependence on only a single coordinate.

4.5 Classical Analysis Versus Quantum Analysis

Of the types of man-made clock mentioned above only the atomic clocks can be said to be quantum mechanical; all the rest depend on macrophysical events that can be safely analysed by classical methods. The caesium clock measures a frequency associated with an undoubted quantum mechanical event: namely, the production of microwave photons by the stimulation of a transition between molecular energy levels. But the measurement of that frequency is made by a macrophysical instrument after amplification. The analysis of error sources in that instrument will be, therefore, classical. On the other hand, the explanation of the atomic/ molecular structure, the calculation of energy levels and the estimation of spectral width (pertaining to the stability of the clock) must employ QM.

In the light of these remarks it seems that *purely* quantum mechanical clocks are theoretical constructs; see Appendix B.

4.6 Resolution and Discreteness

All clocks must 'tick'; the tick is the moment at which the clock is read; and the period between ticks is the resolution. Harmonic clocks tick naturally; but even a linear clock must also, in practice, tick. In order to record the position of the ponderous particle or pointer, against the ruler, some sort of regular interaction is needed. For example the scene could be strobe illuminated by a light carried with the particle. At a certain frequency the pattern of fiducials would seem to stand still; the period of the strobe could then be taken as the unit of time and the number of flashes counted.

All measurements the results of which are encoded into finite length numbers are, necessarily, discrete. Strictly, quantum theory should always take account of this by representing the measured quantity by an operator with a discrete spectrum; see Appendix B. Theorists do not always bother; sometimes they prefer to assume continuous spectra (e.g., for coordinates) in circumstances where the method of measurement is neither specified nor analysed. These remarks apply, a fortiori, to the measurement of time.

Measured distance and time may be *fundamentally discrete*. So, it is certainly alright to assume or to deduce, in a theory, that measured space-time coordinates are discrete. If the theory is any good, eventually, measurement should confirm that they are indeed so. [Discreteness at the level of the Planck time/ distance may be always beyond our reach]. But I am not so sure about the validity of assuming that the quantum time t is discrete. No experiment should be able to demonstrate that it is so; because such a demonstration would amount to a measurement of t.

5. THE ARROW OF TIME

5.1 Classical Systems

The equations of mechanics, both classical and quantum, are time-reversible. Yet, although some simple systems are accurately cyclic, most systems of any complexity do not repeat themselves or go into reverse. A cup of tepid water does not become a cup of hot water upon which floats an ice cube; but the reverse process seems always to happen. Likewise, the old do not become young; but the young always, if they survive, become old.

The standard explanations of these phenomena are in terms of statistical mechanics and the increase in entropy. Even in the development of living things, e.g., the growth of an embryo, where entropy seems temporarily to decrease, the net effect is an increase. Mother and baby live by destroying other life-forms.

The study of deterministic chaos has shed further light on this matter. For trajectories within a *strange attractor* (SA) neither prediction nor retrodiction, based on *measured* initial conditions, is feasible. The consciousness of an observer, interacting with the system, is marooned. He cannot tell what *will be*; and although he remembers what *was* it does not relate well to what *is* according to deterministic rules!

5.2 Quantum Systems

The equations of the simple systems treated by the quantum theorists are completely time-reversible. This is worrying because, although the classical equations are now recognised as a grossly simplified description of nature, the quantum equations are supposed, at least for the simple systems treated, to be the real McCoy! So, if QM is at the bottom of all things, where does the arrow of time come from?

As Roland Omnes (*Quantum Philosophy*) has outlined, the quantum theorists seem now to have a coherent explanation of the arrow. My understanding of this is as follows. Time has direction in all but the very simplest isolated systems. Suppose that a complex system has subsystems each characterised by a few commuting variables. We can set up apparatus repeatedly to separate out those subsystems that are in a joint eigenstate of the few variables (so as to measure them); but we cannot control the myriad other variables that define the state of the rest of the system. Hence, even when the variables that interest us are constants of the motion, so that successive eigenvectors correspond to the same eigenvalues, the eigenvectors do not represent the same whole system states. Thus, the state of the whole system has evolved and, previous states cannot be recovered. See Appendix C for remarks on quantum chaos.

5.3 Our Sense of the Arrow

Our conscious minds seem to have an immediate short-term sense of the arrow of time. Why? Is 'now' instantaneous or does 'now' have depth? When is 'now'?

I can make sense of these questions only by supposing that 'now' is a label that our brains put on incoming sense data. The immediate fate of these labelled data (and I am not here speaking of conscious processes) is to be stored, i.e., recorded, in some form. This 'memory' allows our brains to predict. Without prediction we cannot survive; we cannot even move in a co-ordinated way. The ability to predict also confers the ability to retrodict; but the latter is less important to survival. Fortunately for us much of the mechanics of the world at, our bodily scale, is not short-term chaotic; thus our brains can learn the necessary tricks.

A simple example of what is involved, in deterministic prediction, is provided by a rudimentary 2D radar tracker. We assume that the radar is stationary and that it receives plots, at roughly equal intervals, from a single target. The task of the tracker is to provide best estimates of the target's position and velocity at an instant in the near future.

The tracker consists of five main elements: a *coordinate transformation algorithm*, a *push-down data store*, a *target motion model*, a *fitting process* and a *prediction algorithm*. We assume, for simplicity, that the push-down store has a fixed depth; it records a fixed number n of Cartesian X,Y,τ triplets derived from n successive Polar R,θ,τ plots; (τ is time on target). The transform algorithm continuously converts new polar coordinate pairs R,θ to Cartesians X,Y. We assume that for a short interval, that encompasses both the n triplets and the time of prediction, the motion of the target is uniform; this is the motion model. The fitting process fits the current n triplets (by, say, least squares) to three separate straight line (uniform motion) models, one for each Cartesian coordinate. These straight line models are then extrapolated to give a prediction of position; the gradient coefficients are taken as smoothed estimates of the velocity components. With the arrival of each new plot the push-down store re-assigns the time labels τ and discards the oldest plot; the whole procedure is then repeated. Notice that the coefficients derived from the fitting process can be regarded as just a smoothed and condensed form of the current data, i.e., the memory. There are, in effect, two clocks: one which measures the transmit/ return interval $2Rc$ and another which measures the time $\tau + Rc$ of a return. The head synchro sends a continuous stream of pulses to the processor; the synchro pulse that is closest in time to τ gives a measure of $\theta \bmod(360^0)$.

Why did we bother to convert to Cartesians? In R,θ space straight tracks would appear curved; so our motion model, that assumes that each coordinate is linear in time, would be in error. Only if the target is slow and/ or distant would these curvature errors be swamped either by measurement errors or manoeuvre. [All targets manoeuvre even when they intend not to!]. The model would be grossly in error if we failed to store the data in the order of receipt. Notice, however, that the model does not require plots to be received at strictly uniform intervals; it merely requires that the time (on target) τ is correctly recorded. [The effects of measurement errors are, however, more pernicious when the return intervals are not uniform].

In this simple example 'now' is the time of the latest plot. Direct knowledge of the past is contained in the triplets. We have no direct knowledge of the future. But we have an *estimate* (prediction) of the immediate future behaviour of the target; and we also have an *estimate* (retrodiction) of its immediate past behaviour. The latter should have a *lower* position variance than the raw data providing that the instant of retrodiction lies within the epoch of the memory; the former is likely to have an *higher* position variance.

To construct the tracker we appear to have no option but to choose the direction of the arrow of time. We also have no option but to process the data in the order of receipt; any other choice introduces delays. This, I believe, is what animal brains do to accomplish their enourmously more complex tracking tasks; and it is this which gives our brains the short term sense of the arrow. Does it also ensure that, for us, 'now' has depth; (i.e., sensor sampling period$\leq$ depth$<$duration of memory)?

6. Epilogue

We can examine our sense of time and study the ways in which it depends on the activity and physical condition of our brains. We can study the natural clocks including, of course, astronomical phenomena. We can devise and analyse many sorts of clock; and we can study to determine what they measure; we can relate their measurements. We can analyse the different concepts and uses of time in physics so to decide whether the sorts of time are one, a few or many. Yet the central mystery remains: why do things happen?

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APPENDIX A

A.1 The Schrödinger Evolution

What is referred to here is the Schrödinger Evolution

(A.1a) $\psi(\underline{q},t) = \exp(-itH/\eta)\psi(\underline{q},0)$; the $\underline{q}$ are Hilbert space variables

(A.1b) η is (Planck's constant)$/(2\pi)$

Starting in a quantum mechanical 'state' $\psi(\underline{q},0)$ it is supposed that an <u>isolated system</u> evolves to the state $\psi(\underline{q},t)$ in elapsed quantum time t. The operator H, called the Hamiltonian, is Hermitian or self-adjoint and represents the system. It acts on an Hilbert space of functions ψ that represent possible state vectors. An observation or any other kind of disturbance from outside, breaks the isolation, interrupts the Schrödinger evolution and forces the system into another state $\xi(\underline{q})$; if the system is then left alone ξ becomes the starting state for another Schrödinger evolution under H. It can be shown that an H can always be found such that (A.1) approximates *any* given unitary evolution $\psi(\underline{q},0) \to \psi(\underline{q},t)$.

A.2 The Born Interpretation Of ψ

The Born interpretation of $\psi(\underline{q},t)$ is that

(A.2) $\left|\left(\psi(\underline{q},t),\xi_a\right)\right|^2$; ψ and ξ_a normed

is the probability that a measurement, at time t, of a real variable represented by Hermitian operator A will result in eigenstate ξ_a with eigenvalue a; notice that the probabilities (A.2) sum to unity. From this interpretation it follows that the inner product $\left(\psi(\underline{q},t), A\,\psi(\underline{q},t)\right)$ is the statistical expectation of A at time t. For measurements of the coordinates (that are assumed mutually to commute and to have continuous spectra) the Hilbert variables $\underline{q}$ are assumed to be the (eigenvalues of the) coordinates; (A.2) is then replaced by the probability density function (PDF) $\left|\psi(\underline{q},t)\right|^2$. The derivative of (A.1) with respect to t is the Schrödinger equation.

A.3 ψ is a Figment

The function/ vector ψ is a purely theoretical construct; via (4.1) and (4.2) it is an engine for predicting the statistics of the outcomes of measurements of variables represented by given operators; it is not the 'state' of anything! The term 'state', as correctly used by engineers, signifies the set of instantaneous values of the dynamical parameters of a classical system. The nearest QM equivalent, to this set, is the collection of eigenvalues obtained immediately following the simultaneous measurement of a maximal set of commuting variables. This results in a common eigenvector (say) $\psi(\underline{q},0)$ at (say) quantum time $t=0$. Formulae (4.1) and (4.2) show that the probabilities associated with subsequent measurements quickly evolve away from certainty. Thus, although $\psi(\underline{q},0)$ is associated with the state (correctly so termed) at quantum time $t=0$, the variances increase from zero with $t>0$ and $\psi(\underline{q},t)$ is associated only with a probability distribution. It is a disaster that early authors called $\psi(\underline{q},t)$ the 'state' of the system.

Notice that if these ideas are to be consistent then, for an observation that starts at time t, the transition from state $\psi(\underline{q},t)$ to state ξ must be representable as the Schrödinger evolution of a larger isolated system. The larger system must include the original system, the apparatus, the environment (itself isolated) and, possibly, an observer.

APPENDIX B

B.1 Quantum Clocks

The measurement of time is not the measurement of t, which cannot be measured, but rather the measurement of some dynamical variable Q_0 characterised by uniform or regular cyclic behaviour related to t. Such variables can be defined in various ways; and, as we have seen, the concept of uniformity is not free from tautology. In what follows I discuss examples of simplified quantum clocks. Much of the material is gathered from Peres.

B.1.1 A Linear Clock

The ponderous particle that is the moving part of the linear clock defined in Section 3.2 has, in the classical analysis, mass m, momentum p and velocity p/m. It moves parallel to the ruler a distance q in time

(B.1) $q_0 \equiv mq/p$

where q is to be expressed in units l (the distance between the fiducials). We may be content to regard ml/p as the unit of measured time; in which case to measure q_0 we have only to measure q. But if we wish to express q_0 in terms of arbitrary units, such as MKS, then we must calibrate the clock by also measuring either the velocity $\dot{q} = p/m$ or the momentum p the mass m being given. Let us suppose that we wish to use arbitrary units. Then the rate of the measured time is

(B.2) $$\dot{q}_0 \equiv \frac{d(mq/p)}{d\tau} = m\dot{q}/p = 1; \quad \dot{p} = 0$$

which implies that

(B.3) $$q_0 = \tau + \textit{a constant}$$

showing, as we would expect, that q_0 is a true measure of the dynamical time τ.

A quantum analysis of the linear clock is less encouraging! We have

(B.4) $$H \equiv \frac{P^2}{2m}; \quad QP - PQ = i\eta I;$$
$$\dot{P} = \frac{i}{\eta}(HP - PH) = O; \quad \dot{Q} = \frac{i}{\eta}(HQ - QH) = P/m$$

and the quantisation of (B.1) is

(B.5) $$Q_0 \equiv \frac{1}{2}[Q(P/m)^{-1} + (P/m)^{-1}Q]$$

so that the rate of the measured time is represented by

(B.6) $$\dot{Q}_0 \equiv \frac{i}{\eta}(HQ_0 - Q_0H) = \frac{i}{4\eta}[-i\eta I - 3P^{-1}(-3i\eta P^2)P^{-1}] = I; \text{ see (B.2)}$$

Also, from (B.5), the eigenequation for Q_0 is

(B.7) $$Q_0\varphi = q_0\varphi \Rightarrow \frac{i\eta m}{2}\left(\frac{1}{p}\frac{\partial}{\partial p} + \frac{\partial}{\partial p}\frac{1}{p}\right)\varphi = q_0\varphi; \quad P \equiv pI; \quad Q \equiv i\eta\frac{\partial}{\partial p}$$
$$\Rightarrow \varphi(p) \propto p^{1/2}\exp[-ip^2 q_0/(2m\eta)]$$

in the momentum representation. It follows from (B.6) that (Heisenberg's Generalised Uncertainty Principle)

(B.8) $$\text{var}(H)\text{var}(Q_0) \geq \frac{1}{4} < i(HQ_0 - Q_0 H) >^2 = \frac{\eta^2}{4}$$

We conclude from (B.6) that the Heisenberg representation of Q_0 is the quantum time t. We conclude from (B.8) that measurements of time are accurate only in states for which the energy is very uncertain; i.e., (B.8) is the traditional Heisenberg uncertainty relation for energy and time. The eigenfunction at (B.7) is disappointing: because $|\varphi|^2 \propto p$ the eigenfunction φ is not normalisable. The very same problem arises when we use Schrödinger's equation (i.e., the time derivative of (A.1)) to calculate the plane-wave eigensolution for H given (B.4). The problem arises because H is not completely specified; i.e., the particle is not bound and/ or the space is not closed!

Notice that, although the motion of the particle described by the Hamiltonian is uniform, the set-up is deficient as a *clock*. The spectrum of Q_0 is continuous. The fiducials on the ruler are either infinitesimally close or they have been erased! Put another way the Hamiltonian includes no terms which tell us how the particle interacts with the ruler; there is no mechanism (photons etc.) for reading and recording the time; the clock does not 'tick'.

B.1.2 Requirements for an Harmonic Dial-Clock Having N States

If Q_0 is to be the time measured by a *uniform clock* then its spectrum must be *discrete* and *equispaced*; and because Q_0 depends, in general, on H the Hamiltonian must be chosen appropriately. Further, the Hamiltonian must be such that, starting in an eigenstate of Q_0, the Schrödinger evolution moves through or close to the successive eigenstates of Q_0. In this way the clock is almost sure to be uniform! If, in addition, the clock is a dial-clock then the number N of the eigenstates of the indicated time $Q_0 \equiv Q_c$ is finite.

This means that Q_c and the Hamiltonian of the clock $H \equiv H_c$ should be represented in an Hilbert space of finite dimension N. Taken together these requirements mean that the N eigenvectors $\underline{v}_\alpha$ of Q_c must satisfy (see (A.1))

(B.9) $\underline{v}_{\alpha+1} = \exp(-iH_c\tau/\eta)\underline{v}_\alpha;\quad \alpha = 1,2,....;\quad \alpha, \alpha+1$ are modulo N

where τ is the interval between successive 'ticks' and the $\underline{v}_\alpha$ form a complete orthonormal basis.

Given the eigenvectors $\underline{v}_\alpha$ we can express Q_c as an expansion in terms of the corresponding projection operators and the desired eigenvalues

(B10)
$$Q_c = \tau\sum_{\alpha=1}^{N}\alpha\underline{v}_\alpha\underline{v}_\alpha^+ \qquad \alpha,\beta \text{ are modulo } N$$
$$\Rightarrow Q_c\underline{v}_\beta = \beta\tau\underline{v}_\beta;\quad \underline{v}_\alpha^+\underline{v}_\beta = \delta_{\alpha\beta}$$

Because the eigenvalues of Q_c are both real and non-zero Q_c is Hermitian and has an inverse. Peres calls Q_c a *dial operator* and the eigenstates $\underline{v}_\alpha$ of Q_c *dial states.* It is also possible to prove that H_c satisfying (B.9) always exists. It is clear that (B.9) defines the 'tick' of an harmonic clock; and that the N eigenvalues of Q_c are the uniformly spaced positions of the pointer.

B.2 The Larmor Clock- An Example of An Harmonic Dial-Clock

B.2.1 The Pointer Angle as Coordinate

Return now, for a moment, to the concept of the linear clock discussed in Section (B.1.1). Define

(B.11) $$H \equiv \omega P;\quad QP - PQ = i\eta I;\quad \omega \equiv \frac{2\pi}{\tau N} \text{ is a scalar constant}$$

Results (B.6) and (B.8) still hold but the rate of the (now dimensionless) coordinate Q is a scalar constant $\dot{Q} = \omega I$; so, to measure the time $Q_0 \equiv (Q\dot{Q}^{-1} + \dot{Q}^{-1}Q)/2 = Q\omega^{-1}$, we have only to measure Q.

Recall that self-adjoint operators P and Q have real, continuous spectra. Acting on an infinite dimensional Hilbert space **H**, of bounded functions $\theta(q)$, they may be represented as

(B.12) $$P \equiv -i\eta\partial()/\partial q; \quad Q \equiv qI; \quad -\infty < q < \infty$$

Acting on the dual Hilbert space **H′**, of the continuous Fourier transforms $\vartheta(p)$ of the $\theta(q)$, the operators have representations

(B.13) $$P \equiv pI; \quad Q \equiv i\eta\partial()/\partial p; \quad -\infty < p < \infty$$

The connection between these two representations is, of course, the Fourier transform considered as a unitary operator.

If, however, we think of q as a continuous rotation, such that the end points of the range $-\pi \le q \le \pi$ are identified then, the spectrum of P (now angular momentum) is discrete. The eigenvalues of P are $p = m\eta$, where m is a $\pm$ integer or zero, and the eigenfunctions are

(B.14) $$\upsilon_m(q) \equiv (2\pi)^{-1/2}\exp(imq); \quad -\pi \le q \le \pi$$

To get these results we apply, to the eigenequation

(B.15) $$P\upsilon_m = -i\eta\frac{\partial\upsilon_m}{\partial q} = p\upsilon_m,$$

the conditions that $\upsilon_m(q)$ shall be continuous and single-valued at the points $q = \pm\pi$. This calculation is exactly that by which the eigenvalues and eigenfunctions of the azimuthal angular momentum are computed for the hydrogen atom. Notice that the $\upsilon_m(q)$ are orthonormal for q in the range $[-\pi,\pi]$.

B.2.2 A Compact Hilbert Space

Now consider the expansion of any function $\psi(q)$ in **H** with respect to the υ_m as basis. By restricting the expansion to a finite number of terms

(B.16) $$\psi(q) \equiv \sum_{-j}^{j} c_m \upsilon_m; \quad -\pi \le q \le \pi$$

we reduce **H** to an Hilbert space of finite dimension

(B.17) $$N = 2j+1; \quad -j \le m \le j$$

Call this space **CH** (i.e., compact **H**). Following Peres we can think of the angle q as the position of a pointer on a clock face. But the operator $Q \equiv qI$ is not a dial operator because its spectrum is continuous; and it is not defined on **CH** because if $\psi(q)$ can be expressed in the form (B.15) then the function $q\psi(q)$ cannot.

B.2.3 Definition of the Dial Operator and the Hamiltonian

We are now, however, in a position to define a dial operator Q_c together with the corresponding Hamiltonian H_c. Peres gives two methods; I describe only his final (and simpler) version. Consider the set of N functions of q

(B.18) $$\nu_\alpha(q) \equiv \sum_{m=-j}^{j} c_{\alpha m} \upsilon_m(q); \quad -\pi \le q \le \pi; \quad \alpha = 1,2,...N; \text{ see (B.15)}$$

We see that $\nu_\alpha(q)$ is represented in **CH** by the $N \times 1$ vector

(B.19) $$\underline{\nu}_\alpha \equiv \{c_{\alpha 1}, c_{\alpha 2}, ... c_{\alpha m}\}^T$$

So, if the matrix $c_{\alpha m}$ is unitary, the N vectors $\underline{\nu}_\alpha$ form an orthonormal basis in **CH**. Peres chooses

(B.20) $$c_{\alpha m} \equiv N^{-1/2} \exp(-2\pi i \alpha m / N)$$

which is the unitary matrix of the N^{th} order *discrete Fourier transform*. This discrete transform is the analogue of the continuous transform that relates the diagonal representation of P at (B.12) to that of Q at (B.13). Thus Q_c is defined by (B.19) and (B.20); see (B.10). Because

(B.21) $P\upsilon_m = m\eta\upsilon_m$; see (B.14)

it follows from (B.14),(B.15),(B.18) and (B.20) that in **H**

(B.22) $\exp(-2\pi iP/N\eta)\nu_\alpha = \nu_{\alpha+1(\mathrm{mod}\,N)} \Rightarrow H_c \equiv H$; ; see (B.9)

and in **CH** (B.9) holds where H_c is represented by an $N \times N$ matrix.

B.2.4 Statistics

With respect to the υ_m as basis the elements of this matrix are (see (B.14) and (B.22))

(B.23) $\langle \upsilon_m | H_c | \upsilon_n \rangle = \omega m\eta\delta_{mn}$; $m,n = -j,-j+1,\ldots.j-1,j$

so that with respect to the ν_α as basis the matrix elements are (form the similarity transform of (B.23) with the matrix (B.20))

(B.24) $\langle \nu_\alpha | H_c | \nu_\beta \rangle = \frac{\omega\eta}{N}\sum_{m=-j}^{j} m\exp[2\pi im(\alpha-\beta)/N]$; $\alpha,\beta = 1,2,\ldots N$

From which we conclude that, in any of the dial states,

(B.25) $\langle H_c \rangle = 0$; set $\alpha = \beta$ in (B.24) and complete the sum over m

Similarly

(B.26) $\langle \upsilon_m | H_c^2 | \upsilon_n \rangle = (\omega m\eta)^2\delta_{mn}$; $m,n = -j,-j+1,\ldots.j-1,j$

which implies that, in any of the dial states, (B.27) $$\mathrm{var}(H_c) = \langle H_c^2 \rangle = \frac{\omega^2 \eta^2}{N} \sum_{m=-j}^{m=j} m^2 = \omega^2 \eta^2 j(j+1)/3; \quad N = 2j+1$$

It follows that, in a dial state,

(B.28) $$SD(H_c) \equiv \left(\langle H_c^2 \rangle - \langle H_c \rangle^2\right)^{1/2} = \omega\eta[j(j+1)/3]^{1/2}$$
$$\approx \omega\eta j/\sqrt{3}; \quad j >> 1$$

which is approximately 58% of the maximum available energy $\omega m\eta$ (i.e., the greatest eigenvalue of H_c); see (B .23). NB A uniform PDF in the range [−1,1] has an SD of $1/\sqrt{3}$. This result contrasts with that for the linear clock considered in Section B.1.1 for which $\mathrm{var}(H)$ is infinite in an eigenstate of the measured time Q_0; see (B.8). The uncertainties in the clock reading are perhaps best understood by considering the probabilities of successive transitions between dial states.

The probability

(B.29) $$P_t \equiv \left|(\underline{v}_{\alpha+\gamma}, \exp(-iH_c x/\eta)\underline{v}_\alpha)\right|^2; \quad x \equiv t - \alpha\tau; \quad \alpha+\gamma \text{ is modulo } N,$$

where γ is integer or zero and x is now unrestricted, is the chance that a reading $q_c = \alpha\tau$ (i.e., a dial state $\underline{v}_\alpha$) will be followed by a reading $q_c = (\alpha+\gamma)\tau$ at quantum time $t > \alpha t$. I have calculated that

(B.30) $$P_t = \left(\frac{\sin[\pi y/\tau]}{N\sin[\pi y/(\tau N)]}\right)^2; \quad y \equiv t - (\alpha+\gamma)\tau$$

If t/τ is an integer then $y = 0$ for some γ and, given ideal measurement, the correct time $(\alpha+\gamma)\tau$ will be recorded for certain (i.e., $P_t = 1$). If, however, t/τ is not an integer then, in general, $0 < P_t < 1$ and *any* result is possible but not, necessarily, probable.

APPENDIX C

C.1 Quantisation Tends to Suppress Chaos

Modern studies of quantum perturbation focus on problems of stability, predictability and reversibility. These are also among the concerns of *classical chaotic dynamics*. So, the question arises: to what extent can QM be chaotic? It has been found (mainly by computer simulations) that quantisation tends to suppress chaos. That is, the quantisation of a classical chaotic system may not always exhibit chaos. This does not mean that quantum chaos does not exist; it is simply harder to demonstrate. Measurements of QM phenomena, when they indicate chaos, tend to do so by indirect means.

In some ways this is not surprising. Consider two possible initial states of a system at quantum time $t = 0$; these can be represented by two normalised vectors ψ_0 and ψ'_0. Because both vectors are subject to the same deterministic unitary transform, during the Schrödinger evolution $t > 0$, the transition probabilities $|(\psi(t), \psi'(t))|^2 = |(\psi_0, \psi'_0)|^2$ remain unchanged. Thus, using the transition probability as a distance measure in a phase space of the real and imaginary parts of ψ, orbits, starting at ψ_0 and ψ'_0, do not diverge; thus a primary symptom of chaos is not present.

The expectation of a constant of the motion is also constant during the Schrödinger evolution. Suppose, however, that an observable represented by (bounded) A does not commute with the Hamiltonian (bounded) H. The expectation $< A(t) >$ is then variable but deterministic. Under what conditions, if any, is it possible for $< A(t) >$ to participate in chaos? It turns out that if H has a point spectrum of *finite order* then $< A(t) >$ is harmonic with a finite period; it cannot be chaotic. If H has an infinite point spectrum there is a possibility; but the spectrum has also to be dense (but countable?).

Provided that H does not depend on the state ψ, Schrödinger's equation is linear and, therefore, the real and imaginary parts of ψ cannot be truly chaotic. In the Bohm-Hiley (*The Undivided Universe*) interpretation, however, Schrödinger's single particle equation is analysed into two non-linear differential equations, namely, those governing R and S defined by

(C.1) $\psi(\underline{q}) \equiv R \times e^{(iS/h)}$; R and S real

With the choice of the phase space variables

(C.2) $\underline{p} \equiv \nabla S$ and $\underline{q}$,

and with a suitable choice of potential function it is possible for the trajectories to exhibit chaos. A criticism, however, can be levelled. The variables $\underline{p}$ and $\underline{q}$ are theoretical fictions; they are not QM observables. So, perhaps, the results do not bear on the existence or otherwise of true quantum chaos.

C.2 Detection of Chaos by Measurement

In favourable circumstances we can also *observe a classical system* to detect and quantify chaos from a sufficiency of measurements. Baker and Gollub (*Chaotic Dynamics*) have done this with an instrumented forced pendulum. The theoretical parameters (i.e., Lyapunov exponents, fractal dimension, geometry of the attractor etc.) of the chaotic and regular motions agreed well with those determined from measurement.

I have analysed the question: could we detect chaos, by repeated measurements, on a system of particles each small enough to be subject to appreciable quantum effects? My conclusion is that, although we might conduct computer simulations to decide theoretical issues of quantum chaos in such a system, direct quantum experiments, for the same purpose, are probably impractical. It is likely that quantum effects will always be masked by experimental error.

C.3 Intimations of True Quantum Chaos

Baker and Gollub report on investigations involving hydrogen-like atoms with outer electrons that are close to ionisation (i.e., escape). For such electrons the energy levels (Hamiltonian eigenvalues) are both numerous and close together; (these, recall, are prerequisite conditions for the expectation of a QM variable to able to exhibit chaos). The effects of external fields (fluctuating electric and constant magnetic) were modelled and the rates of ionisation measured experimentally. It was found that the measured rates agreed well with the predictions made by *quantum models*; indeed, for weak fields, standard perturbation theory sufficed. But, because the principle quantum numbers were high (e.g., 60+), it was also found possible successfully to use *classical models* (Bohr Correspondence Principle). These always exhibited chaos when the external field was strong enough.

In the case of the electric field (Jensen 1985, 1987) the measured ionisation rate depends mainly on amplitude and only slightly on frequency. In the classical model the motion is regular only when the field is weak. With a strong field the chaotic trajectories are found to wander over large regions of the action-angle phase space allowing the electrons to acquire higher energies so, eventually, to escape. Remarkably, in one experimental regime, the threshold value of field strength required to produce ionisation agrees with the theoretical value required to produce classical chaos. In another regime, however, there appear to be quantum effects that reduce the measured ionisation rate below that predicted by the classical model.

In the case of the constant magnetic field there are again two regimes. When the field is weak the quantum problem can be analysed by perturbation theory. This predicts the well known Zeeman splitting of the energy levels; further, these levels are close together. At the same time the classical model exhibits regular motion. When the field is strong enough, however, the energy spectrum is complex because the atomic energy spacings are overwhelmed by the Zeeman splitting. The quantum calculation is much more difficult; but it shows that, when the field is strong enough to produce classical chaos, the small energy level spacings (in the quantum version) are suppressed. This phenomenon is called *level repulsion* (Gutzwiller 1992).

C.4 Peres' Simulations

Peres' investigations suggest that perturbation theory will always work for a quantum system having a classical analogue with a regular trajectory. But, if the classical trajectory is chaotic then, perturbation theory will fail correctly to describe the effects of a small, *specific* disturbance of the quantum system. Such a quantum system might exhibit chaos under the influence of a small but *unknown* disturbance.

Peres reports on a detailed simulation that compares a classical deterministic (discrete) map ($k = 0,1,2,.....$)

$$\text{(C.3)}\quad \underline{x}_{k+1} = \begin{pmatrix} 0 & 0 & 1 \\ \sin[\alpha(x_3)_k] & \cos[\alpha(x_3)_k] & 0 \\ -\cos[\alpha(x_3)_k] & \sin[\alpha(x_3)_k] & 0 \end{pmatrix} \underline{x}_k; \quad \underline{x} \equiv \{x_1 x_2 x_3\}; \quad \underline{x}_k^T \underline{x}_k = 1$$

with its quantum analogue

$$\text{(C.4)}\quad \psi_{k+1} = U\psi_k; \quad U \equiv \exp\left(-\frac{i\pi J_2}{2}\right)\exp\left(-\frac{i\alpha J_3^2}{2j}\right)$$

by numerical simulation. Here $\underline{x}$ is a vector of rotations about the Cartesian coordinate axes, the ψ_k represent successive states, U is a unitary operator, j is given (being either an integer or half an odd integer) and

$$\text{(C.5)}\quad \underline{J} \equiv \{J_1, J_2, J_3\}$$

is a vector of angular momentum operators (for a system of particles) expressed in units of η. The system, represented by (C.4), has been described as a 'kicked top'. Each 'kick', delivered by U, can be thought of as a 'tick' of discrete quantum time.

With a setting $\alpha = 3.0$ the classical orbit is chaotic and the quantum system is close to the breakdown of perturbation theory. With a setting $\alpha = 3.0003$ the classical orbit is still chaotic but the quantum map has radically different properties. When the controlling quantum number j is high, in particular, the Hamiltonian eigenvectors of the perturbed ($\alpha = 3.0003$) system are no longer in one-one relation with those of the unperturbed ($\alpha = 3.0$) system; and hence perturbation theory fails. It is possible to set up a correspondence between any point in the phase space of the classical system and a coherent state of the quantum system; (these coherent states optimise the variances of the non-commuting observables $\underline{J}$ that characterise the quantum system). For points in the regular regions of the classical attractor (small, compact and symmetrically placed) the coherent states are shown to be insensitive to the value of α; likewise the evolution of the quantum system starting with these states. But, for points in the classical SA, both the coherent states and the subsequent evolutions are highly sensitive. In consequence, because the classical evolution must remain in the SA, the quantum evolutions are irreversible.

A. M. Deakin 12/08/2003

Non Commutative Worlds

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1 Introduction

We present a view of aspects of mathematical physics, showing how the forms of gauge theory, Hamiltonian mechanics and quantum mechanics arise from a non-commutative framework for calculus and differential geometry.

In this paper we assume that all constructions are performed in a Lie algebra $\mathcal{A}$. One may take $\mathcal{A}$ to be a specific matrix Lie algebra, or abstract Lie algebra. If $\mathcal{A}$ is taken to be an abstract Lie algebra, then it is convenient to use the universal enveloping algebra so that the Lie product can be expressed as a commutator. In making general constructions of operators satisfying certain relations, it is understood that we can always begin with a free algebra and make a quotient algebra where the relations are satisfied.

We build a variant of calculus on $\mathcal{A}$ by defining derivations as commutators (or more generally as Lie products). That is, if for a fixed N in $\mathcal{A}$ we define $\nabla : \mathcal{A} \longrightarrow \mathcal{A}$ by the formula

$$\nabla F = [F, N] = FN - NF$$

then ∇ is a derivation. Note that ∇ satisfies the formulas

1. $\nabla(F + G) = \nabla(F) + \nabla(G)$
2. $\nabla(FG) = \nabla(F)G + F\nabla(G)$.

In $\mathcal{A}$ there are as many derivations as there are elements of the algebra, and these derivations behave quite wildly with respect to one another. If we have the abstract concept of *curvature* as the non-commutation of derivations, then $\mathcal{A}$ is a highly curved world indeed. Within $\mathcal{A}$ we shall build in a natural way a tame world of derivations that mimics the behaviour of flat coordinates in Euclidean space. We will then find that the description of the structure of $\mathcal{A}$ with respect to these flat coordinates contains many of the equations and patterns of mathematical physics.

Note that for any A, B, C in $\mathcal{A}$ we have the Jacobi Identity

$$[[A,B],C]+[[C,A],B]+[[B,C],A]=0.$$

Suppose that $\{\nabla_i\}$ is a collection of derivations on $\mathcal{A}$, represented respectively by $\{N_i\}$ so that $\nabla_i(F) = [F, N_i]$ for each F in $\mathcal{A}$. We define the *curvature* of the collection $\{\nabla_i\}$ to be the collection of commutators $\{R_{ij} = [N_i, N_j]\}$.

Proposition. Let the family $\{\nabla_i\}$ be given as above with $\nabla_i(F) = [F, N_i]$. then

$$[\nabla_i, \nabla_j]F = [[N_i, N_j], F]$$

for all F in $\mathcal{A}$. Hence the curvature of $\{\nabla_i\}$ measures the deviation of the cocatenations of these derivations from commutativity.

Proof. First,

$$\nabla_i(\nabla_j(F)) = [[F, N_j], N_i],$$

which becomes via Jacobi identity

$$= -[[N_j, N_i], F] - [[N_i, F], N_j]$$

$$= [[N_i, N_j], F] + [[F, N_i], N_j].$$

Hence

$$\nabla_i(\nabla_j(F)) = [[N_i, N_j], F] + \nabla_j(\nabla_i(F)).$$

Whence

$$[\nabla_i, \nabla_j]F = [[N_i, N_j], F].$$

This proves the proposition.

In the next sections we will see how these patterns interact with concepts of calculus and differential geometry, and with physical models. Section 2 shows how multivariable discrete calculus can be reformulated as a calculus of commutators. Section 3 sets up a general format for non-commutative calculus and associated physics. Section 4 discusses curvature and connection and includes a discussion of the Feynman-Dyson derivation of electromagnetism from commutator calculus and its generalizations. It this through these generalizations that we encounter differential geometry and the Levi-Civita connection in relation to abstract physical trajectories. In Section 5 we discuss the Jacobi identity and give a combinatorial proof that Poisson brackets satisfy the Jacobi identity. This proof is based on a Lemma that determines when commutators in a non-associative algebra satisfy the Jacobi identity. Section 6 gives a diagrammatic formulation of the Jacobi identity and gives a specific example of how this identity arises in the context of colorings of intersection graphs. The purpose of this example in the present context is to show how the structures that we are discussing can live in a multiplicity of contexts. These varied models suggest new intepretations for the physics, a topic that will be the subject of subsequent papers. The paper ends with an epilogue, and suggestions for new directions.

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2 Derivatives are Commutators

Consider a discrete deriviative $Df = (f(x+\Delta) - f(x))/\Delta$. It is easy to see that D does not satisfy the Leibniz rule. In fact, if

$$\tilde{f}(x) = f(x+\Delta),$$

then

$$D(fg) = D(f)g + \tilde{f}D(g).$$

In the limit as Δ goes to zero, $\tilde{f}$ approaches f and the Leibniz rule is satisfied in the limit. Now define a shift operator J that satisfies the equation

$$Jf(x+\Delta) = f(x)J$$

or equivalently

$$J\tilde{f} = fJ.$$

Note that the existence of J is accomplished by taking the commutative algebra C that we started with, and extending it to the free product of C with an algebra generated by the symbol J, modulo the ideal generated by $fJ - J\tilde{f}$ for all f in C.

Then, setting

$$\nabla = JD,$$

we have

$$\nabla(fg) = JD(f)g + J\tilde{f}D(g) = JD(f)g + fJD(g)$$

Hence

$$\nabla(fg) = \nabla(f)g + f\nabla(g).$$

The adjusted derivative ∇ satisfies the Leibniz rule.

In fact, this adjusted derivative is a commutator in the algebra C of functions, extnded by the operator J:

$$\nabla(f) = J(\tilde{f} - f)/\Delta = (fJ - Jf)/\Delta.$$

Hence

$$\nabla(f) = [f, J/\Delta].$$

Note however that

$$[x, J/\Delta] = (xJ - Jx)/\Delta = J(x + \Delta - x)/\Delta = J.$$

Thus $\nabla(x) = J$. This underlines the fact that these derivatives now take values in a non-commutative algebra. Note however, that if

$$x^{(n)} = x(x - \Delta)...(x - (n-1)\Delta),$$

then

$$\nabla(x^{(n)}) = JD((x^{(n)}) = Jnx^{(n-1)}.$$

Hence we can proceed in calculations with power series just as in ordinary discrete calculus, keeping in mind powers of J that are shifted to the left. That is, a typical power series should be expressed in terms of the falling powers $x^{(n)}$. We would define

$$exp_\Delta(x) = \Sigma_{n=0}^{\infty} x^{(n)}/n!$$

and find that

$$\nabla(exp_\Delta(x)) = Jexp_\Delta(x).$$

The price paid for having the Leibniz rule restored and the derivatives expressed in terms of commutators is the appearance factors of J on the left in final expressions of functions and derivatives.

Note that we have

$$\nabla(x) = [x, J/\Delta] = J,$$

and that this writes discrete calculus in terms that satisfy the Leibniz rule with a step of size Δ. It would be convenient to have an operator P such that $[x, P] = 1$. Then $[f, P]$ would formally mimic the usual derivative with respect to x, and we would have

$$[x^n, P] = nx^{n-1}.$$

Of course, we can simply posit such a P, but in fact, we can redefine J so that

$$fJ = J\tilde{f}$$

where

$$\tilde{f}(x) = f(x + J^{-1}\Delta).$$

Then

$$\nabla(x) = [x, J/\Delta] = J(x + J^{-1}\Delta - x)/\Delta = 1$$

and we can take

$$P = J/\Delta.$$

In this interpretation, $[f, P] = JDf = \nabla f$ where

$$Df(x) = (f(x + J^{-1}\Delta) - f(x))/\Delta.$$

This double readjustment of the discrete derivative allows us to transfer standard calculus to an algebra of commutators.

The cost for this double readjustment is that we must have a collection of functions in the original algebra C such that one can sensibly define $\tilde{f}(x) = f(x + J^{-1}\Delta)$. Polynomial and power series functions have such natural extensions. For other function algebras it will be an interesting problem in analysis, and algebra, to understand the structure of such extensions of commutative rings of functions to non-commutative rings of functions.

More Variables. In order to have multivariable calculus, it is best to introduce new shift operators (commuting with one another), one for each variable. For example, suppose we are working with functions of x and y. Let

$$S_x f(x, y) = f(x + \Delta, y)$$

and

$$S_y f(x, y) = f(x, y + \Delta).$$

Then we have discrete partial derivatives

$$D_x f = (S_x f - f)/\Delta$$

and

$$D_y f = (S_y f - f)/\Delta.$$

We define operators J_x and J_y such that

$$f J_x = J_x S_x f,$$

$$f J_y = J_y S_y f,$$

and

$$0 = [J_x, J_y] = J_x J_y - J_y J_x.$$

Then we can define

$$\nabla_x f = J_x D_x f = [f, J_x/\Delta]$$

and

$$\nabla_y f = J_y D_y = [f, J_y/\Delta].$$

In this version of two-variable calculus, ∇_x and ∇_y commute with one another, and we have

$$[x, J_x/\Delta] = J_x,$$

$$[y, J_y/\Delta] = J_y.$$

Just as in the one-variable case, we can accomplish the more desirable commutation relations by making the discrete derivatives carry an operator in their step. That is, we redefine

$$S_x f(x, y) = f(x + J_x^{-1}\Delta, y)$$

and

$$S_y f(x, y) = f(x, y + J_y^{-1}\Delta).$$

Retaining the above definitions and taking $P_x = J_x/\Delta$, $P_y = J_y/\Delta$, we have

$$[x, y] = 0,$$

$$[P_x, P_y] = 0,$$

$$[x, P_y] = 0.$$

$$[y, P_x] = 0,$$

$$[x, P_x] = 1,$$

$$[y, P_y] = 1.$$

In this way we can transfer multivariable discrete calculus to non-commutative algebra. One way to read the rest of this paper is to use the model, presented in this section, as the underlying non-commutative algebra. In that case the physical systems that are discussed will be discrete dynamical systems defined partially on the original underlying commutative algebra $\mathcal{C}$, and partially in the non-commutative algebra $\mathcal{A}$, that has been constructed around it. Detailed examination of these dynamical systems will be the subject of a sequel to the present paper.

Example. Before going on to more abstract calculus, it is worth looking at the simplest case of a commutator equation using this form of the discrete calculus. Consider a time series $\{X, X', X'', ...\}$ with commuting scalar values. Let $DX = J(X' - X)/\Delta t$. Consider the commutator equation

$$[X, DX] = k$$

where k is a constant and it is understood that the equality sign refers to the scalar evaluation of the expression after the J operator has been shifted to the left and removed. We have

$$[X, DX] = X(DX) - (DX)X = XJ(X' - X)/\Delta t - J((X' - X)/\Delta t)X$$
$$= J[X'(X' - X) - (X' - X)X]/\Delta t = J[(X' - X)^2]/\Delta t.$$

Thus we interpret the equation $[X, DX] = k$ as

$$[(X' - X)^2]/\Delta t = k.$$

This means that the process is a Brownian walk with spatial step

$$\Delta x = \pm\sqrt{k\Delta t}$$

where k is a constant. In other words, we have

$$k = (\Delta x)^2/\Delta t.$$

This identifies the constant k as the diffusion constant for the Brownian process. See [28] for a more detailed discussion of this example.

3 Non Commutative Calculus and Its Associated Physics

We now set up a framework for non-commutative calculus in an arbitrary number of dimensions. We shall assume that each derivative is represented by a commutator, and that the basic space and time derivatives commute with one another as is customary for the flat space of standard multi-variable calculus. This production of a flat space for calculus is the formation of a clearing in the complexity of the containing Lie algebra. Curvature remains present, ready to assert itself at any moment through other choices of algebra elements.

Since all derivatives are represented by commutators, this includes the time derivative as well. We shall assume that there is an element H in $\mathcal{A}$ representing the time derivative. This means that

$$dA/dt = [A, H]$$

for any A in $\mathcal{A}$. Note that it follows at once from this choice that H itself is time independent, since $dH/dt = [H, H] = 0$. We shall see that H behaves formally like the Hamiltonian operator in classical mechanics.

We will assume that there is a set of coordinates $\{X_1, ..., X_d\}$ that are as flat as possible. It is assumed that the X_i all commute with one another, and that the derivatives with respect to them commute with one another. The partial derivatives with respect to X_i will be represented by a set of elements $\{P_1, \cdots, P_d\}$ with

$$\partial_i F = \partial F/\partial X_i = [F, P_i]$$

for any F in $\mathcal{A}$.

The commutation of the derivatives is entailed in the commutation of the P_i and the fact that

$$\partial X_i/\partial X_j = \delta_{ij}$$

is the commutator equation

$$[X_i, P_j] = \delta_{ij}.$$

Thus the flat coordinates satisfy:

$$[X_i, X_j] = 0$$
$$[P_i, P_j] = 0$$
$$[X_i, P_j] = \delta_{ij}.$$

Note that we also have

$$\hat{\partial}_i F = \partial F/\partial P_i = [X_i, F]$$

so that

$$\hat{\partial}_j X_i = \partial X_i/\partial P_j = [X_j, X_i] = 0$$

and

$$\partial_j P_i = \partial P_i/\partial P_j = [X_j, P_i] = \delta_{ij}.$$

This formalism looks like bare quantum mechanics and can be so interpreted. (if we take $i\hbar dA/dt = [A, H]$ and H the Hamiltonian operator). But these coordinates can also be viewed as the simplest flat set of coordinates for referring the description of temporal phenomena in a non-commutative world. There are various things to note. For example

$$dP_i/dt = [P_i, H] = -[H, P_i] = -\partial H/\partial X_i$$
$$dX_i/dt = [X_i, H] = \partial H/\partial P_i.$$

Thus

$$dP_i/dt = -\partial H/\partial X_i$$
$$dX_i/dt = \partial H/\partial P_i.$$

These are exactly Hamilton's equations of motion. The pattern of Hamilton's equations is built into the system!

3.1 General Equations of Motion

A general description of dX_i/dt takes the form of an equation

$$dX_i/dt = \mathcal{G}_i$$

where $\{\mathcal{G}_1, \cdots, \mathcal{G}_d\}$ is a collection of elements of $\mathcal{A}$. If we choose to write $\mathcal{G}_i$ relative to the flat coordinates via $\mathcal{G}_i = P_i - A_i$ (this is a definition of A_i) then the formalism of gauge theory appears naturally. For example, if

$$\nabla_i(F) = [F, \mathcal{G}_i],$$

then we have the curvature

$$[\nabla_i, \nabla_j]F = [R_{ij}, F]$$

where

$$R_{ij} = [\mathcal{G}_i, \mathcal{G}_j]$$
$$= [P_i - A_i, P_j - A_j]$$
$$= -[P_i, A_j] - [A_i, P_j] + [A_i, A_j]$$

$$= \partial_i A_j - \partial_j A_i + [A_i, A_j].$$

This is the well-known formula that expresses the gauge field as the curvature of the gauge connection. From this point of view everything comes naturally from the assumption that all derivatives are represented by commutators, and that one is trying to refer all equations to the flat background coordinates (that look like bare quantum mechanics).

4 Curvature and Connection at the Next Level

The dynamical law is

$$dX_i/dt = \dot{X}_i = P_i - A_i = \mathcal{G}_i.$$

This gives rise to new commutation relations

$$[X_i, \dot{X}_j] = [X_i, P_j] - [X_i, A_j] = \delta_{ij} - \partial A_j / \partial P_i = g_{ij}$$

where this equation defines g_{ij}, and

$$[\dot{X}_i, \dot{X}_j] = R_{ij} = \partial_i A_j - \partial_j A_i + [A_i, A_j].$$

We define the "covariant derivative"

$$\nabla_i F = [F, P_i - A_i] = \partial_i(F) - [F, A_i] = [F, \dot{X}_i],$$

while we can still write

$$\hat{\partial}_i F = [X_i, F].$$

It is natural to think that g_{ij} is analogous to a metric. This analogy is strongest if we *assume* that

$$[X_k, g_{ij}] = 0.$$

By assuming that the spatial coordinates commmute with the metric coefficients we have that

$$[\dot{X}_k, g_{ij}] + [X_k, \dot{g_{ij}}] = 0.$$

Hence

$$\nabla_k g_{ij} = \hat{\partial}_k \dot{g_{ij}}.$$

A stream of consequences then follows by differentiating both sides of the equation

$$g_{ij} = [X_i, \dot{X}_j].$$

In the following we shall use D as an abbreviation for d/dt.

Note that by the Leibniz rule

$$D[A,B] = [DA,B] + [A,DB],$$

whence

$$\dot{g_{ij}} = [\dot{X}_i, \dot{X}_j] + [X_i, D^2 X_j].$$

Note also that we can freely use the Jacobi identity

$$[A,[B,C]] + [C,[A,B]] + [B,[C,A]] = 0.$$

In particular, the Levi-Civita connection

$$\Gamma_{ijk} = (1/2)(\nabla_i g_{jk} + \nabla_j g_{ik} - \nabla_k g_{ij})$$

associated with the g_{ij} comes up almost at once from the differentiation process described above. To see how this happens, view the following calculation where

$$\hat{\partial}_i \hat{\partial}_j F = [X_i, [X_j, F]].$$

We apply the operator $\hat{\partial}_i \hat{\partial}_j$ to the second time derivative of X_k.

Lemma $\Gamma_{ijk} = (1/2)\hat{\partial}_i \hat{\partial}_j D^2 X_k$

Proof.

$$\hat{\partial}_i \hat{\partial}_j D^2 X_k = [X_i, [X_j, D^2 X_k]]$$

$$= [X_i, \dot{g_{jk}} - [\dot{X}_j, \dot{X}_k]]$$

$$= [X_i, \dot{g_{jk}}] - [X_i, [\dot{X}_j, \dot{X}_k]]$$

$$= [X_i, \dot{g_{jk}}] + [\dot{X}_k, [X_i, \dot{X}_j]] + [\dot{X}_j, [\dot{X}_k, X_i]$$

$$= [g_{jk}, \dot{X}_i] + [\dot{X}_k, g_{ij}] + [\dot{X}_j, -g_{ik}]$$

$$= \nabla_i g_{jk} - \nabla_k g_{ij} + \nabla_j g_{ik}$$

$$= 2\Gamma_{kij}.$$

It is remarkable that the form of the Levi-Civita connection comes up directly from this non-commutative calculus without any apriori geometric interpretation.

The upshot of this derivation is that it confirms our interpretation of

$$g_{ij} = [X_i, \dot{X}_j] = [X_i, P_j] - [X_i, A_j] = \delta_{ij} - \partial A_j / \partial P_i$$

as an abstract form of metric (in the absence of any actual notion of distance in the non-commutative world). This calls for a reevaluation and reconstruction of differential geometry based on non-commutativity and the Jacobi identity. This is differential geometry where the fundamental concept is no longer parallel translation, but rather a non-commutative version of a physical trajectory. This approach will be the subject of a separate paper.

4.1 Electromagnetism and the Feynman - Dyson Derivation

It is useful to restrict to the case where $[X_i, A_j] = 0$ so that $g_{ij} = \delta_{ij}$. This is the domain to which the original Feynman-Dyson derivation [5, 21, 15, 38] applies. In order to enter this domain, we set

$$\dot{X} = DX = \hat{P}_i = P_i - A_i.$$

We then have

$$[X_i, X_j] = 0$$

$$[X_i, \dot{X}_j] = \delta_{ij}$$

and

$$R_{ij} = [\dot{X}_i, \dot{X}_j] = \partial_i A_j - \partial_j A_i + [A_i, A_j].$$

Note that even under these restrictions we are still looking at the possibility of a non-abelian gauge field. The pure electromagnetic case is when the commutator of A_i and A_j vanishes. With this interpretation, $\dot{X}$ satisfies the Lorentz force law $\ddot{X} = E + \dot{X} \times B$ where B represents the magnetic field and E the electric field (in the case of three space variables X_i with $i = 1, 2, 3$.) To see how this works, suppose that $\ddot{X}_i = E_i + F_{ij}\dot{X}_j$ and suppose that E_i and F_{ij} commute with X_k. Then we can compute

$$[X_i, \ddot{X}_j] = [X_i, E_j + F_{jk}\dot{X}_k]$$
$$= F_{jk}[X_i, \dot{X}_k] = F_{jk}\delta_{ik} = F_{ji}.$$

This implies that

$$F_{ij} = [\dot{X}_i, \dot{X}_j] = R_{ij} = \partial_i A_j - \partial_j A_i + [A_i, A_j]$$

since $[X_i, \ddot{X}_j] + [\dot{X}_i, \dot{X}_j] = D[X_i, \dot{X}_j] = 0$. It is then easy to verify that the Lorentz force equation is satisfied with $B_k = \epsilon_{ijk}R_{ij}$ and that in the case of $[A_i, A_j] = 0$ this leads directly to standard electromagnetic theory when the bracket is a Poisson bracket (see the next section for a discussion of Poisson brackets). When this bracket is not zero but the potentials A_i are functions only of the X_j we can look at a generalization of gauge theory where the non-commutativity comes from internal Lie algebra parameters. This shows that the Feynman-Dyson derivation supports certain generalizations of classical electromagnetism.

In regard to this last remark, the reader should note that in our [25, 24] algebraic and discrete version of the Feynman-Dyson derivation it was actually an additional assumption that $B \times B = 0$ where $B \times B$ denotes the (non-commutative) vector cross product of B with itself. (Note that $B = (1/2)\dot{X} \times \dot{X}$.) In the original Dyson paper this cross product vanished because of assumptions about the operators and their Hilbert space representations. With $B \times B$ as an extra term, the Feynman-Dyson derivation is indeed a non-commutative generalization of electromagnetism and includes forms of gauge theories among its models.

5 The Jacobi Identity and Poisson Brackets

Dirac [9] introduced a fundamental relationship between quantum mechanics and classical mechanics that is summarized by the maxim *replace Poisson brackets by commutator brackets.* Recall that the Poisson bracket $\{A, B\}$ is defined by the formula

$$\{A, B\} = (\partial A/\partial q)(\partial B/\partial p) - (\partial A/\partial p)(\partial B/\partial q),$$

where q and p denote classical position and momentum variables respectively.

Recall the genesis of Poisson brackets in classical mechanics: Position and momentum are given by q and p respectively. The energy of the system is given by

$$H = p^2/2m + V(q)$$

where the momentum p satisfies the equation

$$p = m\dot{q}$$

and Newton's law has the form

$$m\ddot{q} = -\partial V/\partial q.$$

Thus we have

$$\partial H/\partial p = p/m = \dot{q},$$

and

$$\partial H/\partial q = \partial V/\partial q = -\dot{p}.$$

This is the classical derivation of Hamilton's equations of motion. We then have, for any function F of p and q

$$dF/dt = (\partial F/\partial q)(\dot{q}) + (\partial F/\partial p)(\dot{p}).$$

Hence

$$dF/dt = (\partial F/\partial q)(\partial H/\partial p) - (\partial F/\partial p)(\partial H/\partial q).$$

Thus it follows directly from Hamilton's equations of motion that

$$dF/dt = \{F, H\}.$$

This is the classical physical background to the patterns that we have seen as tautologies in the non-commutative world. It is worth thinking through the message of the non-commutative world in respect to the existence of the Poisson brackets and their connection with continous differentiation and the commutative world of topology and differential geometry from which the classical and the quantum models of physics are derived. In that world there are specific point locations, and the notion of a trajectory is given in terms of a continous sequence of such locations. But there is no inherent operational structure intrinsic to the space. There is great freedom in the world of commutative and continous calculus, a freedom that allows the construction of many models of temporal evolution. Yet we have seen that non-commutative worlds have built in laws and built in patterns of evolution, yet these patterns of evolution do not lead directly to trajectories but rather to patterns of concatenations of operators. At first sight it would seem that there could be no real connection between these worlds. The Poisson bracket and the reformulation of mechanics in Hamilton's form shows that this is not so. There is a special non-commutativity inherent in the continuous calculus, via the Poisson Bracket.

It is easy to see the truth of the Jacobi identity for commutators. It is just a little harder to see the Jacobi identity of Poisson brackets. It is the purpose of this section to recall these verifications and to discuss the nature of the identity.

First let $[A,B] = AB - BA$. Then

$$[[A,B],C] = (AB - BA)C - C(AB - BA) = ABC - BAC - CAB + CBA$$

$$[[A,B],C] = ABC - BAC - CAB + CBA,$$
$$[[C,A],B] = CAB - ACB - BCA + BAC,$$
$$[[B,C],A] = BCA - CBA - ABC + ACB.$$

So

$$[[A,B],C] + [[C,A],B] + [[B,C],A] = 0.$$

This is the *Jacobi identity.*

More generally, a Lie algebra is an algebra $\mathcal{A}$ with a (non-associative) product ab, not necessarily a commutator, that satisfies

1. Jacobi identity $(ab)c + (bc)a + (ca)b = 0$ and
2. ba = -ab.

It follows that if we define $\rho_a : \mathcal{A} \longrightarrow \mathcal{A}$ by the equation $\rho_a(x) = ax$ for each a in $\mathcal{A}$, then

$$\rho_{ab} = [\rho_a, \rho_b],$$

so that products go to commutators naturally in the left-regular representation of the algebra upon itself.

Here is another point of view. We have the following equivalent form of the Jacobi identity (when $ab = -ba$ for all a and b):

$$a(xy) = (ax)y + x(ay)$$

for all a, x and y in the algebra. This identity says that each element a in the algebra acts, by left multiplication, as a derivation on the algebra. In this way, we see that Lie algebras are the natural candidates as contexts for non-commutative worlds that contain an image of the calculus.

5.1 Poisson Brackets and the Jacobi Identity

There are examples of Lie algebras where the non-associative product is not a commutator, the most prominent being the *Poisson bracket.* Here we start with a *commutative* algebra $\mathcal{CA}$ with two (or more) derivations on $\mathcal{CA}$. Let there be operators $\overline{\lceil \mathrm{a}}$ and $\overline{\mathrm{b}\rceil}$ acting on $\mathcal{CA}$ (ab is the commutative multiplication) such that these operators satisfy the Leibniz rule and commute with one another:

$$\overline{\lceil \mathrm{ab}} = \overline{\lceil \mathrm{a}}\, b + a \overline{\lceil \mathrm{b}}$$

and

$$\overline{\mathrm{ab}\rceil} = \overline{\mathrm{a}\rceil}\, b + a\, \overline{\mathrm{b}\rceil},$$

and

$$\overline{\lceil \overline{\mathrm{a}\rceil}} = \overline{\overline{\lceil \mathrm{a}}\rceil}$$

for all elements of $\mathcal{CA}$. Then we define the Poisson Bracket on $\mathcal{CA}$ by the formula

$$\{a,b\}_{\mathcal{CA}} = \lceil \overline{a\ \ b} \rceil - \lceil \overline{b\ \ a} \rceil.$$

We wish to see that this product satisfies the Jacobi identity. In order to do this we first prove a lemma about the Jacobi identity for commutators in a non-associative algebra. We then apply that lemma to the specific non-associative product

$$a * b = \lceil \overline{a\ \ b} \rceil.$$

Suppose that $*$ denotes a non-commutative and non-associative binary operation. We want to determine when the commutator $[A,B] = A*B - B*A$ satisfies the Jacobi identity. We first prove a lemma about the Jacobi identity for commutators in a non-associative algebra. Let $\mathcal{NA}$ be a non-associative linear algebra with multiplication denoted by $*$ as above. Let

$$J(a,b,c) = [[a,b],c] + [[c,a],b] + [[b,c],a],$$

and call this the *Jacobi sum* of a, b and c. We say that the Jacobi identity is satisfied for all elements $a,b,c \in \mathcal{NA}$ if $J(a,b,c) = 0$ for all $a,b,c \in \mathcal{NA}$. We define the *associator* of elements a, b, c by the formula

$$< a,b,c >= (a*b)*c - a*(b*c).$$

Let σ be an element of the permutaion group S_3 on three letters, acting on the set $\{a,b,c\}$. Let $a^\sigma, b^\sigma, c^\sigma$ be the images of a,b,c under this permutation. Let $sgn(\sigma)$ denote the sign of the permutation.

Lemma. Let $\mathcal{NA}$ be a non-associative algebra as above, then the the Jacobi sum $J(a,b,c) = [[a,b],c] + [[c,a],b] + [[b,c],a]$, for any elements $a,b,c \in A$ is given by the formula

$$J(a,b,c) = \Sigma_{\sigma \in S_3} sgn(\sigma) < a^\sigma, b^\sigma, c^\sigma > .$$

Thus the Jacobi identity is satisfied in $\mathcal{NA}$ iff the following identity is true for all $a,b,c \in \mathcal{NA}$.

$$\Sigma_{\sigma \in S_3} sgn(\sigma) < a^\sigma, b^\sigma, c^\sigma >= 0.$$

Proof. For the duration of this proof we shall write ab for $a * b$. Then

$$[[a, b], c] = (ab - ba)c - c(ab - ba) = (ab)c - (ba)c - c(ab) + c(ba),$$

$$[[c, a], b] = (ca - ac)b - b(ca - ac) = (ca)b - (ac)b - b(ca) + b(ac),$$

$$[[b, c], a] = (bc - cb)a - a(bc - cb) = (bc)a - (cb)a - a(bc) + a(cb).$$

Hence

$$\begin{aligned} &[[a, b], c] + [[c, a], b] + [[b, c], a] \\ &= (ab)c - (ba)c + (ca)b - (ac)b + (bc)a - (cb)a \\ &\quad - c(ab) + c(ba) - b(ca) + b(ac) - a(bc) + a(cb) \end{aligned}$$

$$\begin{aligned} &= ((ab)c - a(bc)) - ((ba)c - b(ac)) + ((ca)b - c(ab)) \\ &\quad - ((ac)b - a(cb)) + ((bc)a - b(ca)) - ((cb)a - c(ba)) \end{aligned}$$

$$\begin{aligned} &= \langle a, b, c\rangle - \langle b, a, c\rangle + \langle c, a, b\rangle \\ &\quad - \langle a, c, b\rangle + \langle b, c, a\rangle - \langle c, b, a\rangle . \end{aligned}$$

This completes the proof.

Remark. We discovered this lemma in the course of the research for this paper. Gregory Wene points out to us that a version of the lemma can be found in [36]. We now apply this result to prove that Poisson Brackets satisfy the Jacobi identity.

Theorem. Let there be operators $\lceil\overline{\mathrm{a}}$ and $\overline{\mathrm{b}}\rceil$ acting on a commutative algebra $\mathcal{CA}$ (ab is the commutative multiplication) such that these operators satisfy the Leibniz rule and commute with one another:

$$\lceil\overline{\mathrm{ab}} = \lceil\overline{\mathrm{a}}\, b + a \lceil\overline{\mathrm{b}}$$

and

$$\overline{\mathrm{ab}}\rceil = \overline{\mathrm{a}}\rceil b + a\, \overline{\mathrm{b}}\rceil,$$

and

$$\lceil\overline{\overline{\mathrm{a}}\rceil} = \overline{\lceil\overline{\mathrm{a}}}\rceil$$

for all elements of $\mathcal{CA}$. Define a non-associative algebra $\mathcal{NA}$ with product

$$a * b = \lceil a \;\; b \rceil.$$

Then the commutator in this algebra $[a, b]_A = a{*}b - b{*}a$ will satisfy the Jacobi identity. Note that this commutator is the Poisson bracket with respect to the above derivations in the original commutative algebra:

$$\{a, b\}_{\mathcal{CA}} = \lceil a \;\; b \rceil - \lceil b \;\; a \rceil = a * b - b * a = [a, b]_{\mathcal{NA}}.$$

This result implies that Poisson brackets satisfy the Jacobi identity.

Proof. Consider the associator in the non-associative algebra defined in the statement of the Theorem:

$$< a, b, c >= (a * b) * c - a * (b * c) = \Big| \lceil a \;\; b \rceil \;\; c \rceil - \lceil a \;\; \lceil b \;\; c \rceil \Big|$$

$$= \Big| \lceil a \;\; b \rceil \;\; c \rceil + \lceil a \, \lceil \;\; b \rceil \;\; c \rceil - \lceil a \;\; \lceil b \rceil \;\; c \rceil + \lceil a \, \lceil b \;\; c \rceil \Big|$$

$$= \Big| \lceil a \;\; b \rceil \;\; c \rceil + \lceil a \, \lceil b \;\; c \rceil \Big|$$

Note that an expression of the form

$$\big| \lceil a \;\; b \rceil \;\; c \rceil$$

will return zero when averaged in the summation

$$\Sigma_{\sigma \in S_3} sgn(\sigma) < a^{\sigma}, b^{\sigma}, c^{\sigma} >$$

since $\big| \lceil a \;\; b \rceil \;\; c \rceil = \big| \lceil a \;\; c \rceil \;\; b \rceil$ (the underlying algebra is commutative) and these terms will appear with opposite signs in the summation. Therefore we find that $Jac(a, b, c) = 0$ for all a, b, c in R. This completes the proof.

6 Diagrammatics and the Jacobi Identity

We have seen that a commutative world equipped with distinct derivations that commute with one another is sufficient to produce a non-commutative world (via the Poisson brackets) that is strong enough to support our story of physical patterns. Many combinatorial patterns mimic the Jacobi identity, and hence provide fuel for further study. In order to illustrate these connections, we give in this section a diagrammatic version of the Jacobi identity and an interpretation in terms of graph coloring. We will initially work in an Lie algebra $\mathcal{G}$ whose product ab satisfies $ba = -ab$ and the Jacobi identity $a(bc) = (ab)c + a(bc)$. In Figure 1 we show a diagrammatic interpretation of multiplication, consisting in a trivalent vertex labeled with a, b, and ab. As one moves around the vertex in the plane, clockwise, one encounters first a, then b, and then ab.

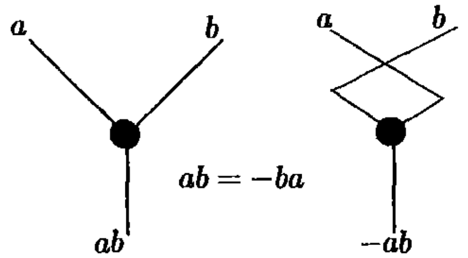

Figure 1 – Diagrammatic Multiplication

In Figure 2 we illustrate the Jacobi identity in the form

$$(ac)b = (ab)c - a(bc).$$

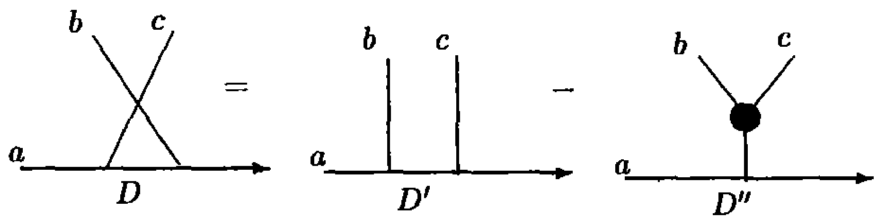

Figure 2 – Diagrammatic Jacobi Identity

To illustrate how this pattern can occur in a different context, consider diagrams D of intersecting chords on a circle as shown in Figure 3. By a circle we mean a curve in the plane without self-intersections that is a topological circle. By a chord, we mean an arc without self-intersections that is embedded in the interior of the circle, touching the circle in two distinct points. Let us suppose that we wish to color the chords from a set of q colors such that *if two chords intersect in an odd number of points, then they receive different colors.* Let $C(D,q)$ denote the number of distinct colorings of the chords of the diagram D, as a function of q. Call such a diagram of intersecting chords an *intersection graph.* We extend such diagrams by allowing internal trivalent vertices as illustrated in the abstract by diagram D'' in Figure 2 and by the diagram with the same label, D'', in Figure 3. Interpret the trivalent vertex as an instruction that *all chord lines meeting at a trivalent vertex receive the same color.* The diagrammatic Jacobi identity of Figure 2 corresponds directly to the logical coloring identity that says that if we have three diagrams D, D', D'' with two chords touching in an odd number of points in D, one point removed in D', and the two chords fused by a trivalent vertex in D'' so that they must receive the same color, then *the number of colorings of D is the number of colorings of D′ minus the number of colorings of D″*. This is just the coloring version of the logical identity

$$Different = Anything - Same.$$

For graph coloring problems, this identity was first articulated by Hassler Whitney [49]. In formulas, we have

$$C(D,q) = C(D',q) - C(D'',q).$$

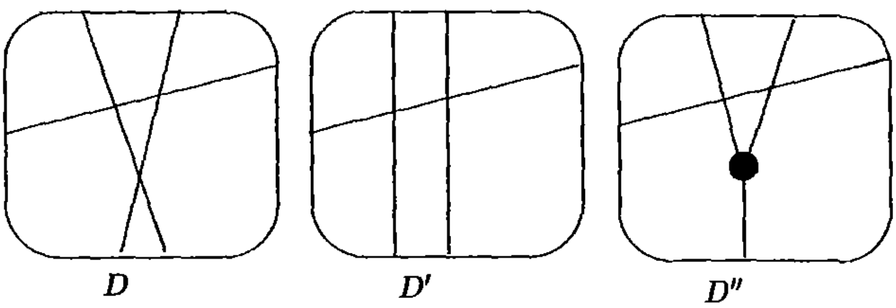

Figure 3 – Intersection Graphs

The convention that we have adopted here – that two chords are colored differently if and only if they intersect in an odd number of points, makes a demand on the interpretation of the trivalent nodes. All arcs entering a given node must receive the same color. After more nodes are added we will have connected components of the resulting graph that contain nodes (the outer circle is not regarded as part of the graph). Call such a connected component a *web* in a given diagram. Each web is colored by a single color. We regard a chord without nodes as a (degenerate) web. We take the convention that if the total number of intersections between two distinct webs is odd, then they must receive different colors. Of course, a web may have self-intersections; we define the sign of the coloring of a given web to be -1 if it has an odd number of self-intersections and $+1$ if it has an even number of self-intersections. The sign of the coloring of a diagram is the product of the signs of its component webs. Note the the sign of a chord is positive. With these conventions, the formulas in Figures 2 and 3 match perfectly and can be understood as indicating parts of larger diagrams that differ only as indicated. We see, as in Figure 4, that an extra self-intersection added to a trivalent vertex changes the sign of its web. This corresponds to the algebraic interpretation of such as vertex as $ab = -ba$. See Figure 1.

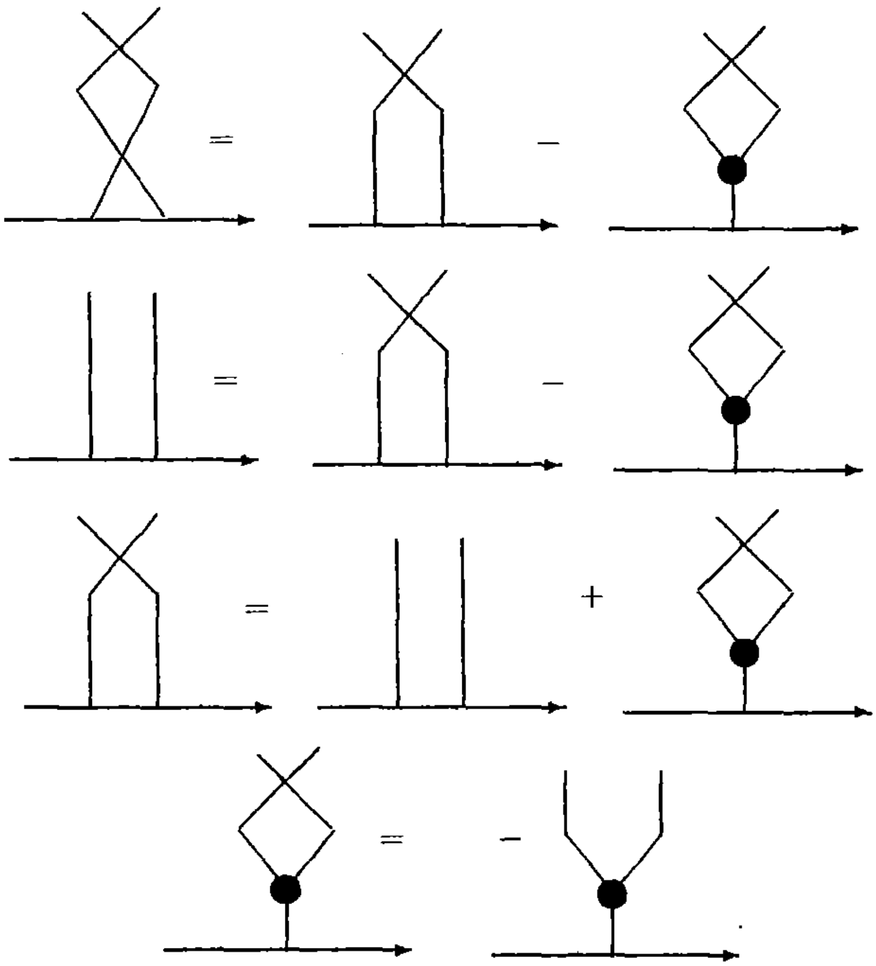

Figure 4 – Verifying the Twist Identity for Color Diagrams

In Figure 4 we illustrate how these sign conventions are consistent with the coloring formula/Jacobi identity. In this figure, we begin with the Jacobi identity with a twist (crossing) added to each diagram. The original diagram with one crossing now has two, and hence is equivalent to a diagram with none (no local requirement of difference). The original diagram with no crossing now has one, and is interpreted as a requirement of difference. Rearranging, we find the Jacobi identity again, but with an extra crossing and change of sign for the noded diagram. The conclusion is that adding a crossing to a node changes the sign of its diagram.

We see that the patterns of counting colorings of chord diagrams correspond formally to the axioms for a Lie algebra. This example indicates how a combinatorial context can lead to the very formalism on which this paper is based, but though different structures than one could have initially visualized. Diagrammatic Lie algebras similar to this example feature prominently in the theory of Vassiliev invariants [3, 35] of knots and links, and may form the basis for new models for the structures that we have discussed in this paper.

7 Epilogue

We have sought in this paper, to begin in an algebraic framework that naturally contains the formalism of the calculus, but not its notions of limits or spaces with specific locations of points and trajectories. It is remarkable that so many patterns of physical law fit so well in such a framework, and we believe that this is indicative of the secondary nature of point sets, topologies and classical differential geometries in physics (Compare [4]). In this paper we have dispensed with spacetime and replaced it by algebraic structure. But behind that structure the space stands ready to be constructed, by discrete derivatives and patterns of steps, or by starting with a discrete pattern in the form of a diagram, a network, a lattice, a knot, or a simplicial complex, and elaborating that structure until the specificity of spatiotemporal locations appear.

There are many ideas for producing location. Poisson brackets allow us to connect classical notions of location with the non-commutative algebra used herein. I believe that other aspects of algebra will be important in making this connection. A hint is given in the most general logical construction of fixed points for operators, for such fixed points are indeed the precursors of the geometric points of our experience. The logical construction of fixed points is usually called the Church-Curry Fixed Point Theorem [34, 16, 30, 31, 33] and it goes as follows. Let

$$Gx = F(xx).$$

Then

$$GG = F(GG),$$

and hence GG is a fixed point for F. We did not speak of the nature of F. We did assume that whatever the entities x were, they could act upon themselves and that one could define an entity G by writing an algebraic description of the action of $G : Gx = F(xx)$. Most important, we assumed that once G had been defined, it was a member of the collection of entities that were available for interaction. This is a form of bootstrapping that occurs in language all the time (a word is a word is a word) but not in formal mathematics where it can take the appearance of a structure being defined in terms of itself. This is exactly the point about the fixed point theorem: GG, defined in terms of itself, is the fixed point for F, and only secondarily must we consider the infinite concatenation of F upon itself, or the infinite concatenation of F upon a "seed value" to obtain the fixed point. The self-defined GG is the "Eigenform" [33] generated from F.

In order for locations to appear from process, one wants an appropriate degree of recursiveness, self-reference and re-entry. Lie algebras begin the process with their fully self-operant structure of derivations. Other searches for this cybernetic turn will lead outward into categories, functors and the comfortable, but large assurance of higher categorical structure in the category of all categories. Our guess is that it is just such bootstrapping that will fit into the basis of this program and produce the ways to make the spaces emerge, through process, from the abstract algebra. All this will be the subject of the next paper.

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A THEORY OF SAPIENCE

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"A time will however come when physiology will invade and destroy mathematical physics, as the latter has destroyed geometry. The basic metaphysical working hypothesis of science and practical life will then, I think, be something like Bergsonian activism. I do not for one moment suggest that this or any other metaphysical system has any claims whatever to finality." JBS Haldane, *Daedalus* 1923.

INTRODUCTION

Western philosophy seems increasingly unable to reconcile new scientific knowledge with our everyday ways of thinking about the ourselves. Physics has long been problematic, but biology is much closer to home and is causing far greater difficulties. Advances in micro-biology, neurophysiology and paleoarchaeology are bringing us right up against the most immediate questions about ourselves. What is consciousness and how does it relate to the brain? How did we evolve our cognitive powers and what does that evolution tell us about human nature? These are vital questions and traditional scientific attitudes and Western philosophy seems poorly equipped to answer them. Many people think that though the issues are real and pressing, we are asking the wrong kinds of questions about them because our whole underlying framework is wrong. As Mary Midgley says, there is something wrong with our philosophical plumbing. This is not just a matter being wrong about some facts and definitions. Philosophy is this level involves our whole world-view and sense of self. It is not just academic theory but part of what we are.

There are today two fundamental and contradictory attitudes one can adopt in confronting these problems. One stemming from the Enlightenment tradition of rationalism and natural science and the other from Idealism, today reduced to a disillusioned romantic relativism. It is easy to have sympathy for both. They are, I believe, the contemporary expression of two equally valid and complementary positions whose origins lie in the deep structure of cognition. To make sense of this one must go back to their roots and trace our philosophies from their origins, a genealogy or rational reconstruction of how, through history and evolution we reached our present dilemma.

The strength of Western philosophy stems from the prime position it gives to the idea of representation. It is also the main source of its difficulties. Its concern with epistemology and its analytical and logical attitude made it the ideal handmaiden to natural science, but its dualism and reductionism are a poor way of understanding human life, consciousness and culture. Cognitive science has major problems with its Cartesian conceptual framework. There is a growing sense that the way beyond traditional philosophy and metaphysics lies with a dynamic, systems theoretic attitude and a connectionist account of cognition. From that alternative perspective the mind is an embodied activity. Phenomenology and neuroscience are two aspects of one bio-social process. The movement in this direction has been going on in various ways for most of the last century, and it has now become clear to many people (*e.g.* Toulmin - *Renewing Philosophy*) that a prime need is for a *naturalistic* understanding of representation. In that way we can keep what is of value in Representationalism while placing it a wider, systems context. This is a major undertaking, but not so difficulty because much of the work has already been done. The ideas are mostly already to hand, but the problem is to frame them in an acceptable way. In what follows I try to do this by setting the present dilemma in its cultural historical context, and then offering a natural historical way out of it.

It is central to Western philosophy that the human mind, language and science are forms of representation. If these ideas are giving us trouble then we need a coherent account of the origin and nature of representation, and this is something that the representationalist tradition itself is has been quite unable to provide. In what follows, I argue that representation occurs in nature independently of human beings, but that for us, the fundamental form of representation, the one which all others are based, is the innate human ability to use one's own body as a representation of someone else's. The obvious kinds of representational system, and those in philosophy, are founded on, and developed out of, this deeper bodily representation. We fail to see this innate somatic representation because it is the underlying medium, the foundation on which all reflective thought is based. At the end of this essay I conclude that our cognitive capacities and conscious experience are the expression of meta-representational systems that emerge from the interaction between innate biological representation and the cultural representational systems.

Modern philosophy started when the Greeks formed the concept of *mimesis* and came to use the idea of representation as a model or metaphor for the relation between thought and the world. The origin of this powerful conception clearly lies with the world's first fully phonetic, and transparently representational, script. Western Philosophy has struggled with the implications of the idea ever since. Cognitive science is still trying to make sense of it today.

The so-called 'Hard Problem' of the nature of consciousness, is an expression of it. How does the mind represent the world? Is it images or language or some innate mentalese that do the representing? What is the relation between representations and the brain in which they are embodied? How can language represent the world? Or, on the contrary, is the fact that our culture uses representation any reason for supposing that conscious thought itself does? – couldn't we be direct participants in the world, our experience unmediated by representation at all? And what on Earth is consciousness?

The central role of representation in philosophy was made first made completely explicit by Kant, but after 200 years we still don't have a workable naturalistic account of what it is. In this essay I show that Neumann's theory of self-replicating systems provides us with one. DNA represents proteins in a way that is clearly analogous to the way phonetic script represents speech. Using a non-human example like DNA as our paradigm case makes it easier to disentangle the structure of, and relationships between, our own more intimate representational systems. With this model even the problem of representations that represent themselves becomes more tractable. From Plato's *Thaetatus* on, writing has been the dominant metaphor of Derrida calls Western 'logocentrism'. Though usually used in a reductive way, DNA is already becoming the dominant metaphor a non-dualistic, 'somacentric' philosophy.

In the essay that follows I first briefly outline the history that lead philosophy to its present impasse, emphasising the difference between representationalist and anti-representationalist orientations. On one hand there is the Epistemological Subject of mainstream philosophy in the tradition of Descartes and Enlightenment, and on the other the holistic or Romantic alternative with its Hermeneutic Self. I believe that these can serve as two ideal types to epitomise two equally valid modes of reflection, each with its own understanding of knowledge, meaning and truth. What we need is a theory that can give a naturalistic account of the real relationship between them.

Developing von Neumann's ideas shows that representations are the parts of self-reproducing systems that replicate themselves and regenerate the parts that cannot do this. Representation occurs in nature as a halfway stage in the process of self-reproduction. It allows structural forms to be replicated separately from their material embodiment. Self-reproduction is obviously a spontaneous natural process. The 'intentionality' of representation derives from the role in plays in it.

The von Neumann definition of representation can be shown to cover such uncontentious cases as our scripts, maps and measurement systems, while at the same time excluding various kinds of imprints, covariances, reflections and replications that can easily be mistaken for representation but which are not. This account supplies the concepts we need to give a plausible account of how humans could have evolved to have representational and meta-representational cognition. It offers new ways of understanding the relations between thought, culture and consciousness. Contrary to most cognitive science, perception in animals has to be regarded not as a kind of representation but as just one aspect sensory-motor activity in general. Dreams and imagination are not 'pictures in the head', but rather that covert or 'off-line' performances of sensory-motor activity, and as such not essentially different from overt action. The earliest real representation probably occurred among early *homo* species and was at first collective without being cognitive. Unlike even the hard-wired language of bees, it was not based on communication.

Homo sapiens emerged as a species specialised in co-operative and culture-dependent ways of life. A key adaptation for this was the evolution of brains with extensive cross-modal connections. Hard-wired connections between the visual to somato-sensory systems gave us the innate ability to imitate others. It allows us to use our own bodies as a medium for understanding the acts, feelings and intentions of others. This emulative ability leads to the replication of sensory-motor patterns in the cognition of individuals as well as in culture. *H. erectus* had a simulative imagination, but in *H. sapiens* this became the first real cognitive representation, with replicatable thoughts representing the sensory-motor actions of the self and others. This primary representation made it possible to develop language. All human representation is founded on our innate ability to represent one another, and from that, ourselves.

Emulative cognition a direct insight into intentions and acts of pointing. It opens up the way to more complex indicative gestures. On this basis we developed sophisticated signing systems that eventually lead to co-operative question-answering routines, with true linguistic representation emerging when questions and answers were combined to give propositional statements. Separately they are indexical and context-dependent, combined, they form statements that can circulate independently of particular individuals. Complex syntax evolved to maximise their question-answering capacity, the 'information' they could carried.

Human sapience results from the use of two distinct but inter-acting representational systems, one analogical and based on the innate understanding of others, the other symbolic and cultural. The two systems form a self-correcting couple in which each medium is able to translate and transform the content of the other. The over-all system can operate in relative independence of external conditions and allows an unprecedented control over attention and action. It gives us a capacity for unlimited internal reflection that we experience as consciousness and freewill.

The Epistemological and Hermeneutic modes of philosophy are the expressions of two different ways of relating to language and two different aspects of sapience. In one language is primary and the two cognitive media are used as a working model of cultural representational systems that exist independently of particular individuals. In the other emulation comes first, and they are used in parallel as a way of controlling attention and tuning the ways of perceiving from which action follows.

Both kinds of philosophy, one starting from enactive emulation and the other from symbolic depiction, provide us with important insights. The Western logocentric tradition gives us powerful analytical tools, and rightly asserts the possibility of impersonal knowledge and rational action. The somacentric orientation, prevalent in the East, provides a much better foundation for understanding people, culture, social relations and morality. These have an innate foundation and are ontologically prior to propositional representation.

A century ago, Bergson made the idea of evolution a source of progressive inspiration, but today scientism and radical relativism associate Darwinism with individualism and mechanical materialism: reductionism on one side and deconstruction on the other. I believe that these contradictory forms of nihilism are the result of one-sided thinking, and that a systems theoretic understanding of representation allows us to see that the two poles of philosophy are in fact complementary. Describing the evolution of cognition in the way appropriate to biology and neurophysiology leads to a phenomenological style of philosophy which understands the idea mind in terms of bodily performance, and offers a dynamic and holistic explanation of our dualistic and reductive attitudes. It opens up new cognitive and cultural possibilities and provide a firmer basis for the positive achievements of traditional philosophy.

TWO POLES OF PHILOSOPHY

Although these ideas are to be presented as a hypothetical history, it should be admitted from the start that there is an underlying presupposition that philosophy, like consciousness itself, is an embodied activity whose roots and deep grammar, lie in the functional structure of the human brain.

There is one very basic correlation between the structure of the brain and the structure of human thought, so obvious that its significance has not had the attention it deserves. It is the clear separation of sensory and motor functions. The generation and control of movement, organising actions and planning sequences of acts, are associated with the front half of the cortex, while perception and the grasp of simultaneous and spatial relations are performed at the back. In language every statement contains its noun phrase and verb phrase. Logic and maths depend on a distinction between operations and objects. As Kant saw, it is impossible to conceive of reflective thought and experience without a framework distinguishing the sensory from the motor, space from time. This basic polar structure is reflected not only in the subject matter of philosophy, but also in the kinds of philosophical orientation that it is possible for us to adopt. It is as if sapient human beings had two fundamental and innate modes of self-reflection, one starting from each pole of the brain. Though this distinction needs to be seen in various wider contexts which will be dealt with later, I will briefly sketch its form here.

From one vantage, action is understood in a perceptual context and in the other perception is taken as an activity. Cognition as such is a sensory-motor process involving the whole brain and its situation, but self-reflection is a specialised activity that has to start from one of two stances. In normal thought we use both orientations, but self-consistent theoretical reflection cannot always do this.

One basic orientation is the kind of reflection required by practical action: confronting a given situation, one decides on the action that will transform it into some other preferred, state of affairs (P→A→P). The other is a reflection on one's involvement in an on-gong activity in which one seeks the perceptual cues or perspectives that allow one to continue (A→P→A).

In more general terms: from one pole we perceive ourselves as actors in a perceptual world, and from the other, we re-enact ourselves as perceivers in a world of activity. If we start from perception we see ourselves as goal-directed agents in a world of objects. Starting from the performative, frontal lobes, we enact ourselves as points-of-view in a process.

Anyone studying the history of philosophy will become aware of this dialectical distinction in some form, and one can point to many manifestations of it in areas outside philosophy proper. Differently expressed in different contexts, the two orientations constitutes the most general possible division between philosophical attitudes. We see it the radical difference in attitude between Descartes and Spinoza, between Enlightenment and Counter-Enlightenment and between the earlier and later Wittgenstein. It may seem reductive to characterise the difference between philosophies in terms of the brain or grammar or cognitive function, but I think it should not. We are embodied beings and if these are complementary modes of physiological function, then they are also complementary as spiritual orientations and dialectical stances. We need to see them, not as competing theories but different modes of reflection or ways of being, each with its own distinct access to vital insights and ways of understanding. In the West, one of these, the view of intentional agents and objects located in space, is associated with the birth of literacy. It has the form of what Husserl called the 'Natural Attitude'. The other mode of reflection, the 'Wisdom of the Sages' or Huxley's 'Perennial Philosophy', emerges as the antithesis to this. I refer to these two as the Intentional and Participatory Attitudes. They are reflected in the different paths taken by Western and Oriental philosophy and in the modern opposition[7] between rational-empiricist and hermeneutic philosophies.

A logical and physiological account of these poles can explain how our fundamental philosophical disagreements are possible but I shall not attempt to unpack all that here. It explains the structure of philosophical disagreements but does not in itself help us to understand ourselves. Taken on its own it may seem to discredit philosophy rather than empower it, but my aim is not to deconstruct philosophy but help renew it. We need to naturalise our understanding of sapience in a way that keeps the best of both outlooks, and this is best done in a narrative form, a brief history of philosophy to explain our present predicament and an account of the evolution of sapient thought to put it in a naturalistic context. We need a plausible myth of origins – a story of the pre-history of sapient culture and cognition.

REPRESENTATION

The earliest explicit philosophies were a reaction against the Intentional Attitude. They probably arose as a response to the individualism and legalism of early urban life. These philosophies of the Participatory Attitude include the teachings of Buddha, Loa Tzu and Confucius, and the pre-Socratics in the West. They are more concerned with the moral and spiritual than with the practical and objective. They saw the world as flux and process, and humanity as the measure of all things. They tried to show the ways of seeing and being best suited to living a good life.

The birth of the characteristically Western style that lead to modern philosophy came when their Greeks first formulated the idea of *mimesis* or representation. The Greeks adapted the Phoenician script to make the world's first fully phonetic form of writing. With this they created a new kind of literature and a new way of relating to language, a fundamental shift in intellectual attitude. Unlike earlier more complicated forms of writing such as mixed pictographic scripts and syllabaries, the Greek phonetic alphabet was a form of representation that clearly displayed a precise, rule-governed correspondence between two separate and independent media. By representing the surface sounds of a language independently of its ideas, it introduced a awareness of a separation between language and thought. Using this script led naturally to the idea of representation in general. Writing, reading, and language could be seen as being like painting, sculpture, epic poetry and theatre, and the use of measurement and mathematics. The presence of these different depictions and reflections lead naturally to the idea that the abstract forms of things can exist independently of their material embodiment.

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Any writing changes our relation to language by externalising and objectifying it, making it visible and separate from spontaneous activity. It is well known (from the work Vygotsky and others) that becoming literate radically alters peoples' ways of thinking and categorising things. This must over time come to effect the whole character of a culture. Brains incorporate ideographic scripts very differently from our linear and analytic ones, and new magnetic resonance imaging (MRI) evidence shows that even spoken language is much more bilateral in Chinese people. There is a large body of experimental research showing that Oriental perception and judgement differs from ours in a way that is precisely in line with the traditional philosophical differences. It is more holistic, dynamic and concrete, showing a rationality that is more relational than analytic (R.E.Nisbett – *The Geography of Thought.*)

For Plato the idea of representation became a powerful metaphor for thought itself and the foundation of his metaphysical ideas. In his famous parable of the cave, bound prisoners know the outside world only through shadows cast on the wall in front of them. In the same way, he thought, the world of experience is just a flickering representation of reality itself – an eternal world of pure forms. As the written word is enduring and fixed in its relation to ideas compared to the fleeting and devious expressions of spoken language, so there is an absolute reality behind the superficial and unreliable experience of everyday life. This come with the idea that we can only begin to understand the world when we grasp the distinction between appearance and reality, between what we see with our senses and what reason tells us is really there. It was, and still is, a style of philosophy in which it was natural to suppose that consciousness was separate from the material world in the same way that representations are distinct and independent of what they represent.

Language and concepts became the object of a new kind of literate attention, no longer just as a means of making points with words chosen on pragmatic and aesthetic grounds, but as a means of permanent representation – with the generality and seriousness that that implies. It heralded an era of theory, analysis and philosophy, and also a new sense of what it was to be an individual in the world. The fundamental dualism of Western philosophy is founded on the metaphor of representation. Knowledge, meaning and truth are seen in terms of depiction, and the conscious minds that possess them are a medium for the representation of reality. The world is imprinted on perception and language *de-scribes* it.

Aristotle did not follow Plato into such extremely representationalist metaphysics. For him abstract terms stood for real properties, but they were always inherent in particular things. In a similar way, the mind existed in the *medium* of the body, its inseparable unifying form. He had the eye of a biologist, seeing nature and living things as dynamic self-organising systems. Aristotle had a dynamic idea of mimesis that went beyond the depictive idea of representation to include the more bodily and less obvious correspondences of music and dance. He did however believe that thought and perceptions were inner representations of an outer world beyond the mind. He understood that if language is to represent things in an accurate and reproducible way, one must have precise and agreed definitions of words and explicit criteria for using them. True descriptions need accurate and repeatable observations, thought that is analytical and logical, and the discovery of new ways to record, classify and depict facts. The Aristotelian concept of a concept is fundamental to Western philosophy in a way that was not the case in China.

The new representationalist concepts were grounded in the Intentional Attitude, and this new and more rigorous and practical style of thought eclipsed the relativistic, process philosophy of the pre-Socratics. The new conceptual and cognitive framework made possible the systematic, de-scription of the world, and so eventually, modern natural science and technology, but it did so at the expense of attempts to express and develop the social and spiritual implications of the Participatory Attitude.

REPRESENTATIONALISM

Greek philosophy founded representationalism, but it was far from being a pure expression of it. Aristotle's systems theoretic attitude and Plato's concern with 'Ways of Being' supported the Participatory Attitude within the new tradition for the next 2000 years.

Natural science developed slowly but the survival of representationalism was assured by the Judeo-Christian and Islamic traditions. It was natural for the Religions of the Book to see God as the *author* of the universe, with the world and the bible as his two great texts. Plato's representational dualism of heaven and earth, soul and body found new theological expressions, and the way was open to *de-scribe* the book of nature. The Renaissance brought advances in technology, arts and scholarship and, importantly, a cultural climate that encouraged their combination and cross-fertilisation of their different media.

This was the beginning of modern science, and with it Descartes laid out the terms of Western philosophy in its modern form. Among the great technical innovations were many important new instruments of representation: notations, telescopes, microscopes, printing, modern techniques of map-making and perspective drawing. The Greeks had the metaphor of phonetic script, but Descartes had printing, and his own algebraic geometry, and his experiments demonstrating that the eye functioned as a *camera obscura.* It was inevitable that his philosophy would be one that reached a new level of representationalism.

Although he is never explicit about it, Descartes dualism assumes that thought has a propositional form, and that the mind represents the material world. As representations are necessarily separate and independent from the things they represent – so is the mind in relation to the material world, conceived as a different non-material substance. With this Representationalism and the ontological separation of mind and matter Descartes established the basic conceptual scheme that has dominated the philosophy of mind ever since. Once a difficult, new idea, it is now so familiar and embedded in common-sense that primary school children can pick it up without difficulty, and as the so-called 'Hard Problem' in cognitive science shows, intelligent adults find it impossible to conceive of any alternative. Descartes conceptual framework provides a schema with room for endless philosophical dispute about the nature of the relation between mind and matter: idealists, materialists, dualists, identity theorists – and questions about just how the mind is associated with the brain that cannot in fact be settled within this framework. Descartes had a major problem with how the immaterial mind could have a causal effect on the material body. Kant, with his 'Copernican Revolution' turned it around, and made a problem of how anything outside the mind could come to be represented in it. It was the 'scandal of philosophy' that it could not prove the existence of the external world.

The disputes over mind and matter are easily extended to Knowledge, Meaning and Truth understood within the same representationalist framework. The Rationalist *vs.* Empiricist argument is still active as Nature *vs.* Nurture, and in cognitive science the relative importance of innate brain structure and sensory input.

ANTI-REPRESENTATIONALISM

Representationalism was the perfect partner to science and technology. As a sophisticated form of the Intentional Attitude it was well suited to the individualism of a growing mercantile and capitalist economy, but there have always been those who rejected its dualistic and abstract account as alienated and untrue to the human spirit. Spinoza, Vico, the Romantics, Nietzsche, Heidegger and Wittgenstein, up to the postmodernists of today, all in their different ways express this kind of holistic and dynamic way of understanding human nature. From this stance we are not contemplative intellects but primarily biological and social beings. Objective representation and its rational use are not taken as fundamental and primary but understood in their natural and cultural context. Where Descartes tried to break with all tradition, Spinoza, read medieval philosophy and studied the bible as human history. He did not use lenses, he made them. Vico studied Roman law. The Romantic reaction to the

Enlightenment was fascinated with old and alien ways of life. Nietzsche was a classical philologist. This is *hermeneutic* philosophy. Its primary concern is not with speaking the truth but with understanding human expression and culture. Representationalism is full of internal contradictions, the Romantics were quick to point these out, but when it comes to more serious philosophy, anti-representationalism is very difficult to express as a positive doctrine. Its stance is fundamentally opposed to the attitude of expository theory. Many thinkers must have cared more for consistency than communication and remained silent about what cannot, strictly speaking, be said. If they did write then, depending on the vocabulary they used, they were all too often mis-interpreted in representationalist terms as Idealists, Materialists, Subjectivists, or self-contradictory relativists. The Participatory Attitude calls for a new vocabulary in which knowledge, meaning and truth are related in a quite new way. Heidegger tried to express such a philosophy but the results became increasingly obscure. Wittgenstein was reduced to hints and awkward questions. Deconstruction refuses to take positive stances.

One is tempted to say that the Participatory Attitude cannot really be expressed in philosophy at all. But Representationalism has too many contradictions and absurdities to be left unaccompanied. Neither pole of philosophy is complete in itself. Recognising that there is a deep symmetry in the relation between the two primary attitudes gives us new ways of explicating just what constitutes the Participatory Attitude. In the introduction I suggested that these deep orientations have a biological basis in two alternative relations between Action and Perception. These can be expressed in a schematic way by the forms, P→A→P *vs.* A→P→A. These can be read as depictions of ordinary mapping *vs.* performatory mimesis, or in another way, as rational action and dynamic participation. But this is very abstract. One can get a better understanding of their philosophical meaning by interpreting them in terms of our everyday relations to language. The Representationalist pattern, P→A→P, is an expression of the reflective speaker's relation to language. Its alternative A→P→A, is that of the Listener. I will try and make this clearer in next two sections.

THE EPISTEMOLOGICAL SUBJECT

From the start, Representationalism was primarily concerned with knowledge and truth. Its stance is one in which the individual is presented as a mediator in the process of representation. It presupposes the idea of people whose primary role is to know and speak the truth. The conceptual framework it gives us has proved to be an immensely powerful tool for understanding and controlling the natural world. Descartes assumed that thought was essentially propositional in form and the model that follows is one of 'Man the Representor' or the 'Epistemological Subject' (ES).

The cognitive home-base for representational philosophy, the position from which it starts, is that of someone confronting a given subject matter and seeking the words to describe it, rather as one does when considering how to answer to a factual question. It is a stance which assumes without having to say it, that the world is prior to language and independent of anything we may say about it. Our cognitive content mediates in the activity of mapping the world into language.

This is the self of the Intentional Attitude, the individual as a goal-directed agent in a perceptual world, narrowed down here, to the performance of a particular kind of task: accurately describing a given state of affairs. The logic of this position is at the heart of a great deal of modern Western conception of self and mind. In essence it is quite banal, but it is still worthwhile making the paradigm case fully explicit here in order to contrast it point by point with its dialectical complement.

The ideal representor knows what has to be described. To know and judge the subject matter he must have cognitive representations, and hence a cognitive domain that is separate and independent of the material world. His subjective experience mirrors the objective world but never meets it. Like all representations it is necessarily separate and independent of what it represents. Seeing and speaking the truth does not need an audience. Hearsay and second-hand traditions are sources of error, as are rhetoric and sophistry. The ideal representor is an isolated individual who addresses a subject matter; informing other people is secondary and inessential. The Epistemological Subject (ES) is separated from the natural world by a single ontological gulf and from other minds by a double one. Communication between people is an uncertain process of coding thought into language, transmitting it, and having it decoded by a hearer. Talking and writing have the same logical form.

The ideal representor recognises an important distinction between the objective truth and subjective distortion. The perceptual, imaginative and conceptual representations that mediate in speaking are always subject to error. One must distinguish carefully between what belongs to the subject matter and what belongs to the Speaker's individual perspective. The things observed and logical thought must kept be separate from perceptual ambiguities, bodily conditioning and irrelevant desires. The activity of accurate representation leads to distinctions between reason and emotion, and between factual and evaluative statements. These distinctions reinforce the idea of the individual's dual nature as a mind and a body.

For the ES, the mental states of belief and desire are categorical. They are the input and output sides of cognitive representation. On one hand the Speaker represents real things in language and the mind, and on the other, does the reverse and, starting from imagined or planned situations, acts to make them a reality. In one direction he maps the world into the mind, in other he acts to make the world fit some mental representation of how he would like it to be. Rational action uses correct knowledge to carry out preconceived plans. Science maps the world, and technology uses its tools carry out plans. Representation and control, maps and plans, are in systems terms, two aspects of the same thing. Within the framework of ideas associated with the Speaker attitude and the ES, people are seen as an actors who live to achieve their personal desires and the increase of pleasure over pain. Human life is powered by its dissatisfactions. Means and ends are logically distinct.

The representor's task depends on there being discrete objects and state of affairs waiting to be described and a language with well defined terms. Where the another cognitive style sees or intuits processes, patterns of internal relations and open contexts, the ES concentrates on separable states of affairs made up of objects, their attribute and the relations between them. Describing begins with the separation of the subject matter into its nameable parts. The represented structure and its representation are must both be susceptible to rational analysis. Primary description and explanatory redescription follow the same analytical path. This results in reductionism, a natural and when appropriate, useful, result of the representationalist attitude.

Representation is a simultaneous or atemporal relation. Properly articulated statements are true (or false) independently, not only of particular individuals, but also of time. The ideals of accurate analysis and prediction are equally intrinsic to representationalism, and with them come the ideas of determinism and mechanism. From them it is natural to conclude that the conscious Subject and material object are not just independent: one is absolutely free and the other completely determined. The contradictions are obvious and insoluble.

The Epistemological Subject is of course an idealisation, but this simplified model does indicate a genuine cognitive archetype. Taken on its own it is a totally inadequate, even absurd account of what it is to be a human being. The logical requirements of the ideal representors form a useful tool-kit that has little to do with life as we live it. Representationalism's impersonal epistemology was in its time a momentous intellectual break-through, but its image of the Ideal Speaker, taken as the basic truth about the human condition, completely fail to express many things that we know are essential to our humanity. Starting with the neat model of phonetic writing we end with a monstrosity. One can dismiss it as an academic abstraction with little bearing on everyday life, but when we compare Western attitudes with those of other cultures it often looks as though we are, in many situations, being unconsciously ruled and misled by our image of Man the Rational Representer.

Representationalism cannot understand how mind can be intrinsically embodied. It separates the individual consciousness from the natural world, divides it against itself, and disconnects it from its roots in society, cultural history and in the natural world. Its patterns of reductive explanation remove things from the contexts that give them meaning and turn life into mechanism. Representationalism needs the idea of God the Author to supply life with meaning and values. Without this it cannot satisfy our intuitive understanding and spiritual needs, and it slides into nihilism.

As representation maps the world into language or some other medium, so the same attitude can map from representations back into the world. Every map has the potential to be a plan. Representationalism gives us an ideal for rational goal-directed action, mapping a given state of affairs to one envisaged and desired, and by a considered way of acting. Again P→A→P. It areas where it is appropriate, this an immensely useful cognitive schema. When it is applied to communication it gives us the idea of 'telementation' (the term is from Roy Harris, *The Language Myth*), that pictures encoded thoughts being transmitted from one mind another with listening as the simple inverse of speaking. It is true only in an abstract way that does not distinguish between live dialogue, literature and emails. Its assumption are those of Information Theory, black-boxes and the universal Turing machine. The origins and ontology of communication are left untouched. The idea of the Epistemological Subject is an abstract cipher created fill a gap in the theoretical framework. It has little relevance to the way people actually understand one another.

THE HERMENEUTIC SELF

The alternative to the Epistemological Speaker, in its own way equally schematic and partial, is the Hermeneutic Listener, the embodied mind participating in dialogue. Though it is often confused with it, the relation between these orientations is quite different from the representationalist's subjective-objective dualism. It is, in the literal sense, a dialectical relation. One is the systematic antithesis of other, not mirror images but as two different relations between action and perception, and so two alternative orientations of the embodied mind.

The Speaker starts from a particular perception and performs the rational action of mapping its structure into another medium or state. This is the Intentional Attitude from which the world is perceptual and the individual is an actor in it. The Listener starts from the position of someone situated in a dialogue, an on-going activity in which perceptions are reached only in order to open the way to new actions. It presupposes the Participatory Attitude of changing perspectives in an inter-active process.

Contrary to the representationalist 'telementation' model, real communication is not a matter of coding and decoding messages. The Listener follows what someone is saying in a shared social context. She re-enacts the expression of a perspective, going along with what is said in order to grasp its *point.* A contribution to dialogue shows us an act of attention within a way of seeing. For the Listener, language does not represent the world, but discloses some aspect of it. Looking, pointing and indicating with words, are all acts of orientation, and in real communication we cannot draw sharp distinctions between them. There is no dualism here between language and the world or subject and object because an act of indication is what it is by virtue of what it picks out or focuses on. Speech acts cannot be defined separately from their objects. For the Hermeneutic Self the shared world of a dialogue is a single bio-social activity with language as an integral part.

The Speaker starts from a position in which not only is the subject matter already given, but so too is a ready made and well defined language in which to describe it. For the listener it is not as simple as that. A statement discloses something, but it also shows the listener how language is, in this instance, being used. The relation between words and the world is never fully defined and always depends upon context. The Listener is continually re-assessing and re-learning. The use of language is always in the process of being re-negotiated. When one reads a text (as opposed to a message) there is no dialogue, and in this one-sided situation Listeners is in principle free to interpret in any way they like.

None of the Speaker's dualisms have any place here. The listener's attention is on the expressive activity of a whole person, an embodied mind. Direct perception is not separate from 'inner' imagination; nearby things can be seen directly, more distant ones must be shown by speech, but both lie in the shared horizon of discourse. There is no separation of reason and emotion, facts and values, as these are inseparable aspects of any act of meaning. Utterances are understood *from* their context and *to* the possibilities they open up; not as statements and commands, but as acts that begin as answers and end as questions. *Interpreting*, as its etymology suggests, is about finding the meaning-giving context to which something belongs.

Where the Speaker starts with an analytic and reductive attitude, the listener is a holist for whom any act or part of one, gets meaning from its role in a wider activity or larger whole. This is not the Speaker's rational goal-directed action but participation in an activity for its own sake and for that of the wider processes in which it is constituted. The significance of anything comes from its role in the system that constitutes it. All meaning is like that. In communication it is use, function, role. Don't ask what it stands for but what its part of – what needs it and what it is supposed to do.

The listener's epistemology, if one can call it that, is based not on metaphors of cultural representation like writing, but on human mimesis, our innate ability to recognise others as like ourselves and use our whole body and being as a dynamic perceptual system, a sensory-motor sounding board. (As I explain later, it is this physiological and largely unconscious somatic representation that makes the other kinds possible).

Knowledge from the Listener stance is, in the widest sense, a kind of know-how, a familiarity with things as they appear within cognitive practices. Skill in answering questions appropriately is only one among many such culturally-based abilities. Truth and meaning are hard to separate because understanding the meaning of a statement depends on recognising it what way it is true, what perspective it belongs to and in what form of language it makes sense. In a real dialogue, if you ask a speaker what they mean by a statement and they will give you an alternative formulation that makes the same point. Ask a listener if they understand what you mean and they will continue your thought, showing more of the orientation behind it and what pattern of ideas it is part of. One evokes correspondence and the other coherence.

One can describe the Ideal Listener but the Hermeneutic Self is harder to find than the ES. The Listener's position is not easily reached by direct self-questioning. It is perhaps, only sensed on the move and at the edge of or behind experience. It comes to light for the historian reading of an unfamiliar text or the experience of an anthropologist trying to understand an alien culture. The Romantics recognised it in the way literature, poetry and the arts can get behind the familiar surface of things and expand our ways of experiencing, and it is surely there in a young child's relation to the social world. The great moral teachers, especially in the East, call on us to put aside our ego-centric rationality and recover a participatory relation to people, situations and the world.

The attitude of the Ideal Listener is fundamental and important, but as the foundation for a philosophy in the conventional sense it only makes sense as response and antithesis to the Speaker's attitude. It is grounded in our interactions, and has no place for universal, context-free languages, or for the ideal of knowledge as something transcendent and independent of particular people and cultures. But no theoretical discourse or practical activity can accumulate experience over time without fixed meanings, and using them entails the idea of a disinterested truth and objective knowledge existing independently of our intentions. Taken as positive doctrine the Listener's holistic position is liable to deny the independent existence of the real world, and the reality of knowledge and truth, or even the self. Contrariwise, it can take any one of these as an aspect of everything or equal to the Totality itself. There is, in truth, often little point in trying to speak from the Listener position, and it is easy to become impatient who those try to express its insights.

Listener philosophy is anti-realistic. Its vantage tells us that the things we attend to exist for us only as parts of our social and cognitive practices. They are raised to consciousness by the our collective interests and activities, and disclosed to sapient reflection by language. There is nothing necessary about the way we happen to reify particular aspects of the total process of the world and treat them as having an existence independently of us.

Listener's concentration on active interpretation is not enough if it is only for the sake of a passive participation in society and nature. Hermeneutic Self is without fixed limits. Its cognitive activity unites it its environment, and it can be taken as an ungrounded, or even illusory embodiment of culture and history. This may seem better than the Speaker's solipsistic Subject, but it is surely not enough. When the demands of different social activities and attitudes conflict we must have somewhere from which to take a positive stand. The body is a dynamic process and a part of nature and history, but it is also a closed, self-organising system, whose essential nature is defined by the development and maintenance of the boundaries and separation between itself and the rest of the world. An embodied mind that loses touch with that fact is no longer true to itself or anything else.

Neither the Speaker nor the Listener position alone can give us an effective idea of consciousness and the substantive self. For that we need to understand just how it is natural and realistic to adopt a dialectical philosophy that embraces them both.

TWO KINDS OF PHILOSOPHY

There may have been times when Intentional and Participatory modes of reflection were taken as valid alternatives and any conflict between them was practical rather than theoretical. In the West, that ended when the Greeks generalised the idea of representation, and started the long ascent of Logocentrism and a metaphysical interpretation of the Speaker's relation to language. The Somacentric anti-representationalism that resisted it is relatively inarticulate, a negation without a synthesis. On the pre-philosophical level the two attitudes are naturally complementary, one as a view of everyday practicalities and the other a deeper but inarticulate understanding of their ontological context. One is interested in seeing and describing what the world is like and how things work, the other in exploring how things are what they are, and how to relate to them in a fitting way. They are valuable as alternative modes of reflection, but individually and subjected to metaphysical inflation they become disturbing and absurd. Either position taken on its own is one-sided and partial. One is epistemologically powerful but presents an impossible image of Man, the other opens our understanding of people and their world, but seems hopelessly unrealistic about representing the world as it is independently of us. Both accounts contain insights and methods that we cannot do without, and also incoherences and contradictions. If they were really incompatible world-views then we would have to choose one and do our best to subordinate the other to it. But that is not the case.

Compared with the unbridgeable ontological dualisms implied by Speaker Representationalism, the relation between the Speaker and Listener is a natural and creative dialectic. The need for consistency inclines philosophers to adopt one or other relationship to language, but all normal thought involves an inter-action of the two. Even in philosophy the richest systems of ideas involve some kind of interplay between them. No systematic thought or discourse is possible without some agreement about terms and ways of representing. Interesting theoretical problems arise when people take up positions within agreed contexts, fixing some terms in order to work on the relations between others. As Dummett shows, the frontiers between realism and anti-realism are always debatable.

Despite what some proponents of listener philosophy and even dynamic system theory have said, representation is a real cognitive process. Despite the confusions and contradictions of cognitive science, representational systems really are a powerful and pervasive aspect of human culture and sapient thought. Understanding and overcoming the contradictions of philosophy as it stands today requires a naturalistic, systems theoretic description of representation as it is independent of human cognition and culture, and an account of how it came to play its present role in human sapience.

THE THEORY OF REPRESENTATION

Modern cognitive psychology was founded as a deliberate break with Behaviourism on one hand, and dynamic systems theory on the other. It uses digital computers as its principle tool and metaphor, and describes cognition in terms of information processing by programs operating on internal representations, but its idea of cognition as representation is very much that of mainstream Western philosophy. Its formal models of cognitive process tell us very little about how we actually use our brains. Some of its investigations of perception have been illuminating, but computers are a very poor metaphor for human thought, and it give no coherent account of what representation actually is. Theories of dynamic systems, 'situated action' connectionism and even robotics, do not depend on undefined ideas of representation and are much better placed to explain how the mind works.

We cannot doubt that representation as such is real and natural. It is an integral part of our culture and cognition, and if it was not always an aspect of sapient thought, it certainly is now. And yet despite it being central to Western philosophy since Plato, and explicitly so since Kant, we do not have a satisfactory account of what it is. The traditional epistemological concern was with how and whether representations correspond rather than what kinds of correspondences count as representations.

Today the problem is posed in terms of 'intentionality', the mysterious property that makes something *about* something else. Marks found on an ancient edifice may or may not be symbols, it depends on the intention behind them. An accidental mark may look like a sign (*e.g.* '68' on the wing of a Caribbean butterfly) but we know it has no meaning in itself. Wittgenstein asked: "Every sign by itself seems dead. What gives it life? ... is the use its life?" The answer must be a guarded, Yes. But the point is still not clear. Is it the fact that a sign is used by consciousness minds that makes it an intentional representation? Or is representation is a natural phenomena and part of what makes us conscious beings in the first place? I shall argue for the latter view. Representation can occur independently of human beings, and our unique form of cognition and consciousness is a result of the way we use it. In the end it is not conscious intentionality that explains representation, but rather the reverse. Representation is not some magical property of signs, just a function of their place in some representational system. Our first task is to discover the defining properties of these systems.

Analytical approaches look for representation (as they look for consciousness) as a *property* to be explained in causal terms. But being a sign is a matter of playing a certain role, and understanding it requires a systems theoretic approach. One must look *from* the sign *to* the system it is part of. We usually think of representation as a process in which *we* map from a domain of objects to one of symbols, from the represented to the representing. A set of rules or consistent regularities defines a systematic correspondence between the structures in two separate domains. But there is something missing from this picture. The presence of a regular mapping is not enough, such an 'imprint' can only count as a representation if there is some way of *interpreting* it, if the original or represented entity can be in someway rediscovered or regenerated from the representation. What is important is not the action of representing something, but the re-presenting. A string of signs cannot be counted as a description if it cannot be de-scribed, *i.e.* read and understood, and the things it is about somehow reconstructed from it. Just what this recovery of the referent amounts will differ greatly according to the nature of the representational system, but making representations is not enough on its own. It is only one side of a loop that must be closed.

It is easy to see why this is so. The world is full of natural imprints and correspondences and very few of them are representations, unless human beings incorporate them into representational practices. A footprint does not in itself represent a foot, nor do a tree's growth rings represent its age, but these things can become representations for hunters or dendrochronologists. And even then a single imprint is not enough. They become representations for us when we can recognise the difference between prints of the same category, and interpret them in terms of difference between things in some other domain. Representations never occur as a one-off, but only in media that allow repetition and variation.

Von Neumann showed on logical grounds that self-reproduction, as opposed to replication (*i.e.* simple copying), is only possible where there is a two-part system in which one part is a blueprint or representation of the other, and where the blueprint, which can be copied directly, serves as a template for the other part which cannot. If the representation could not be read in two different ways (*i.e.* replication and translation) it would have to contain a representation of itself, which is impossible.

A representation is that part of a self-reproducing system which regenerates those parts unable to replicate themselves. Representation is an intermediate stage in self-reproduction, a means by which structural forms can be replicated or transformed independently of their final embodiment. We know what notations and maps represent but we cannot easily separate these structures from the intentionality we impose on them. Natural cases like the representation of protein by DNA (discovered after von Neumann formulated his theory), or the language of bees make much better paradigm cases. All of them, natural and cultural, including natural science itself, have the same basic form. In all cases a structure, (a protein, a bee's journey, a real magnitude) that cannot be directly replicated, is re-generated from one in another medium (a gene, a bee dance, a measurement) that can. The overall system that does this is reproduced over time because it has this property. The DNA–protein system provides us with a clear case of natural representation. Without it, it would be easy to doubt that our Western, representational approach to philosophy, with its logical and analytical methods, had any natural justification at all. With this case of molecular representation before us we can see how, on a higher level of organisation, cognitive representation is a 'natural kind', understandable as a perfectly natural phenomena.

Simple as it is, the systems account shows us representation in an unfamiliar light. DNA represents protein, and its interpretation is a simple mechanical process. The reverse process, that we tend to think of as representation itself, is not necessarily important at all. Von Neumann's blueprint is written before the reproduction starts. The 'first dogma of molecular biology' is that there is no mapping from protein to DNA. If maps were created not by cartographers but by some obscure natural process it would not alter the way we use them. It doesn't matter how science manages to map the world. Its results count as correct representations because they can be replicated and verified, i.e. because some form of the original phenomena can be recovered from them. The processes of discovery are distinct from those of use, and it's the use that counts. There are very many imprints and covariances in nature and it is science's job to find their inverse mappings and connect them with existing representational systems, preferably, but not necessarily, in a way that brings our individual cognitive systems into the loop (just because we have an effective theory doesn't mean we understand what we are doing).

A representational system must have at least two separate and independent material media such that objects in one can be isomorphic with those in the other. An object in one medium is a representation of one in the other if it can be replicated (and possibly transformed), and then used to regenerate a new example in the primary medium. Transformations and relations between representing entities correspond to those between the things they represent. Mutations in genes result in new proteins. Alterations in sentences change their meaning. Systems in which there is no independent replication or transformation of the representation is possible do not count as representational systems. They are simple self-replicating systems, like isolated RNA, or a manufacturing process that make moulds from objects and *vice versa*. In true representation there is replication within and for a self-reproducing system. Representational media and languages are always generative in two senses, of represented the entities and of new representations.

Much human representation is by symmetrical systems. Phonetic script is both inscribed and de-scribed (*i.e.* read aloud). In such cases either medium can be replicated and used independently of the other. Now that we have genetic engineering even DNA has become part of such a system. Such systems are very valuable but as we have seen, representation does not have to be reversible in this way.

Our representations are generally structures that we use or operate on, as surrogates for those in some other less tractable or accessible medium. This is of such importance that one may want to think of the primary case, with simple replication, as just the simplest case of making new structures from old.

Though the word is notoriously ill-defined, it seems best to say that the 'intentionality' of our representations comes only from our use of them. Natural representation is only purposeful in the sense that it is 'for', or a functional part of, a self-reproducing system that itself needs no final purpose. Our representational systems (as opposed to our representations) are often reproduced, or reproduce themselves, in culture, independently of the intentions of particular individuals.

This account, and the more detailed and precise definition one can derive from it (quite a long job to do properly) covers all our unambiguous cases of representation, *e.g.* maps, measurements, notations, phonetic script and those of the examples in Wittgenstein's *Tractatus* (4.014) whose nature is clearly uncontentious. Just as importantly it gives clear grounds for excluding things that are not representations but which can be and are, confused with it. *e.g.* natural imprints and covariances, projected images (*per se*), simple replications (copy machines, RNA), self-regulatory systems (homeostasis). These may be parts or aspects of what it takes to make a true representational system, but do not amount to one in themselves.

We can use this systems theoretic conception of representation to trace the important logical steps in the evolutionary process that lead from the first self-organising systems to the unlimited meta-representations that characterise sapient cognition and culture. The aim is to be able to deal in a solidly naturalistic way with Wittgenstein's more contentious examples: thoughts, statements, musical ideas.

It is not hard to think of evolutionary steps that could have lead to the emergence of bee language, but that is a very simple and limited system. Can we do the same for our own? In what follows, I try to give a plausible and logical account of the evolutionary stages that had to be traversed for molecular representational systems to give rise to the first cognitive representation and in due course the kind of sapient meta-representation we know today. Such a rational reconstruction can't be the definitive account of how mind emerged from matter, but it can hope to show how, even within the limits of present understanding, it is perfectly possible for it to have done so. Many of our present ways of looking at these issues offer no such hope at all.

Mainstream philosophy and cognitive science cannot understand how human thought *as we know it* fits into the natural world. Adopting a biological and systems theoretic approach to representation allows us to be scientific without loosing touch with the Participatory Attitude of Spinoza's *Natura Naturans.*

The following section outlines the story of how self-replicating molecules could give rise to cognitive representation and self-replicating cultures. We find that most of what cognitive psychology considers to be representation is not so according on the von Neumann model. The first representation in the human story is not cognitive but collective. Individual or cognitive representation emerged as a biological adaptation to a culture-dependent way of life. It is only when this had evolved that biological and cultural representation could work together to produce language and sapient meta-representation.

THE EVOLUTION OF REPRESENTATION

If the chemical and thermodynamic conditions are right, self-organising systems with self-replicating representation emerge spontaneously. In the pre-biotic environment, as anywhere else, the types of molecules that predominate are necessarily those with the greatest difference between their rates of creation and destruction. The appearance of self-replication greatly increases the former, and homeostasis minimises the latter. There is a symmetry and logical necessity to this: self-replication is the reproduction of structures in space with positive feedback, homeostasis is the reproduction of functions over time by negative feedback. Systems that have them necessarily prevail. The differentiation and elaboration of these two basic functions leads from simple replication to increasingly complex self-reproduction by the kind of two part systems predicted by von Neumann. The theory that an RNA world evolved into the one in which RNA mediated between replicating and representing DNA and a self-regulating system of proteins, fits this sequence well.

It is important to stress the presence, and the difference between, replication and homeostasis. They are both a material necessity, the yin and yang of self-organising systems (in the strong sense of Varela's 'autopoiesis'). They also mark a point at which thought about representation can start to become confused. We tend to describe self-stabilising systems in misleadingly intentional and representational language (error correction, messages, information, *etc.*,). This has lead some to suggest that even a household thermostat may have the first glimmerings of consciousness. Feedback controlled mechanisms can seem eerily intentional, but they are no more purposeful than a ball in a bowl. One can reasonably think of the parts of a self-organising system as having purposes, as being *for* the whole that could not exist without something to perform their functions – but that is another matter.

In this section I will describe the background to the emergence of cognitive representation by distinguishing between three levels of cognitive function. The behaviour of simple organisms can be understood entirely in terms of the homeostatic mechanisms that control their basic physiological functions. I call this *pre-sentience.* Everything we do and are is embodied in such systems. The body can be regarded as nothing but an immensely complex system of closed-loop feedback mechanisms – the particular matter from which it is made is secondary to its self-organising pattern. The next stage comes with organisms that have nervous systems acting as a self-organising cognitive buffer between its internal states and external variables. The brain stabilises the metabolism by way of stabilising the animal's relation to its environment. They are said to be *sentient.* Where animals can draw on past experience to look beyond the immediate situation and compare hypothetical ways forward, using a kind of disengaged or simulated activity to 'look before they leap' I refer to it as *percipience.*

Pre-sentient organisms have simple nervous systems that serve as a direct feedback link between inner and outer physical variables. Cognitive psychology supposes that simple animals like ants and snails can have internal 'maps'. Even without the demand for true von Neumann representation, it is easy to show that homeostatic mechanisms account for all their behaviour much more economically than any scheme involving 'cognitive maps'. Much of our own automatic and reflex behaviour is controlled in this way. Such simple organisms can be described as simple Stimulus-Response systems, black-boxes whose output is determined by input.

Animals with brains are *sentient.* They operate at a higher level of organisation and cannot be described in simple S-R terms. They are not simple input-output devices, but self-active autopoietic processes whose boundaries cannot be precisely defined. They actively control their own perceptual sensory input, rather than being wholly controlled by it. Acting as though they had curiosity, they gather experience pre-emptively, 'training' their own neural networks to form responsive systems that complement the properties of the environment. When we learn to negotiate our familiar world without thinking, avoiding obstacles, reaching for what we need, we display the sentience of our sensory-motor systems. Perception and emotion, in all animals including ourselves, are sensory-motor activities of this kind. Bergson, and others who took their lead from him, were right to hold that perception is not a dualistic process of inner representation but a part of the activity that fits the animal into its familiar environment without needing to mirror or picture it.

The traditional account of perception takes vision as its paradigm case, and describes it as a process in which information enters the eye and is processed to form internal, depictive 'representations'. It is becoming increasingly clear that even the visual system does not work in this way. The brain contains many separate systems that *analyse* in-put in different ways (line, shape, colour, *etc*..), but none that bring the parts together, except in so far as they are integrated into real sensory-motor inter-action with the environment. There are many general and specific systems for attending to and focussing on things that are significant for action, but no representations (even in the naïve sense). We have massively parallel systems in which top-down activity balances and controls the pattern of input. Real perception and emotion are integrated into on-going activity. Attention to things is an inter-active process in which the unity of the thing perceived is not reflected in any pattern or image in the organism, but only in the object itself and in the stability on an inter-active loop linking the organism and object together.

Theories that presuppose rather than explain the idea of inner representation lead to accounts that end in mystery and loose-ends. A realistic physiology has to use the language of 'dynamic system theory' (in Varela's bio-cybernetic sense). As Merleau-Ponty recognised, the most useful illustrative case to help us understand what it goes on, is not the almost instantaneous presentations of the visual system but the slower process of tactile exploration that turns an object in the hand until the pattern of inter-active experience becomes completely predictable. Haptic perception is the acquisition of a specific sensory-motor skill that *involves* the object. In essence all perceptual activity is like this. Experience of the perceptual world is not passively given but actively acquired and learned. Uncommitted neural-nets are put to the service of the body's pre-sentient needs. Given new kinds of sensory in-put, or deprived of old ones, organisms and especially human ones can develop completely new perceptual systems (*e.g.* perceptual prostheses; the acoustic or 'facial' vision of the blind). Cognitive science with its encapsulated modules and block diagram perceptual mechanisms, has no place for this kind of adaptability. Cross-cultural studies and easily demonstrated phenomenological experiences show that its approach is untenable.

Sensory-motor knowledge of the environment forms a stable background for action. Where pre-sentient organisms may be treated as a stable centre in a dynamic world, sentient animals, including ourselves, have to be understood as dynamic centres in a world held still. As sentient creatures our world is a stable, organised whole pervaded with forces and feelings. Knowledge of it is not an inner representation but the capacity for inter-action that fits us into it. We do not, at this level, represent, model, or mirror the environment, we complement its dynamic properties. In the systems theoretic terms appropriate to physiology there is no separation of subject and object. An act of perception is a skilled and specific sensory-motor orientation that unites an organism and its object of attention. We are so used to the representationalist metaphor of inner depiction that it takes some effort to recognise that perception is a process that must be understood as an active engagement, its relation to its familiar environment is like that of a tool to the person using it, or of partners united in co-operative activity.

Neurophysiologists have found correspondences between the activity of neural structures and outer states of the world that they sometimes class as representations. The first of these must have been Descartes' retinal images. Comparable patterns can be observed on the primary visual cortex. There are also the well-known 'body maps', and spatial maps in the hippocampus. With a coherent theory of representation these have to be understood as natural 'imprints'. They can linked into scientific representational systems but they do not exist in that way for the organism itself.

Simple sentience forms the basis for a more sophisticated level of cognition. When we scan the room, peripheral vision and past experience tell us how to move in order to bring familiar or unfamiliar things into direct attention. A few steps would take one to the window and to what is even more peripheral. Our world stretches out around us with no sharp distinction between what is 'imagined' and what is directly perceived. When we look at things we are tacitly aware of their physical properties and behind that, what one can do with them, the opportunities they afford. Perceptual space is structured by its sensory-motor potential. Sentient animals bear the imprint of a dynamic complementarity between themselves and the environment, but they are nowhere near having cognitive representation in the von Neumann sense.

Without implying that they have self-awareness, we can say that every sentient species has its own kind of tacit world, some far more extensive than others. It is a sensory-motor space of dynamic trajectories and their intersections in which the creature is guided by the immanent attractions and repulsions of how things feel. Dealing with such a world is not a matter of 'information processing', a misleading term implying, on the digital computer analogy, data structures and calculations performed on 'representations'. It is rather a process of self-organising, parallel processing networks working homeodynamically in real time, seeking balance, minimising stresses and optimising function. Unlike computers, biological systems are not nearly fast enough to do this with symbols, computations and mechanical inference.

There is however, a way in which some organisms manage to take their performance to a new and more advanced level. They use the kind of cognitive activity that we become aware of as imagination. All mammals have REM sleep and so probably dream. Their bodies go into a state of flaccid paralysis and their brains become highly active, like an engine running with its clutch disengaged. The function of dreaming is not understood. It may play a part in the consolidation of memory, and it is surely related to a potential. My own speculation is that dreaming and/or REM sleep, provide a way in which the brain can regularly exercise and keep separate, this cognitive ability to disengage itself and work in an 'off-line' mode. It simulates possible sensory-motor actions, trying them covertly before committing itself to their overt expression.

This is the percipient ability to 'disengage', to look before you leap, to scan situation ahead, survey its potential, and tacitly trying out actions to see how they fit. Our imaginations can range without limit, but animals for whom ego-centric percipience (far seeing) is the highest level of cognition are restricted to their immediate situation. As we shall see later, human beings can use this detached and abbreviated sensory-motor simulation as a kind of cognitive representation, but others animals cannot. MRI brain-scan studies show that imagines activity uses the same neural circuits as the corresponding engaged and overt activity. It has therefore none of the separation and independence that is essential to representation in the true sense. Imagination is not, in itself a representation of sensory-motor action, but the thing itself. The idea of 'pictures in the head' makes more sense here than in the case of direct perception, but it is still misleading.

Cognitive simulation is a kind of trial-and-error without the risk, an immensely powerful cognitive ability. Animals that have it are no longer restricted to the fully engaged involvement of merely sentient creatures. They have something equivalent to the contemplative gaze. We may suppose that they distinguish between outer perception and the inner sensations of physiological needs and emotional feelings, and thus have a tacit sense of self. They may not be conscious as we are, but they could be what Coleridge called 'scious' (pronounced *see-us*).

Many if not all, social species have some form of 'culture'. Traditional territories, roosting sites, hunting strategies, *etc.* are replicated from one generation to the next. A Sentient individual's way of life leaves an imprint on it, and on its environment, which includes other members of its species. A culture can be replicated over generations by a mutual imprinting without any need for direct copying or teaching. The traditional skills that Chimps pass on are quite complex, but given their intelligence, the way they are passed on is surprisingly unreliable and slow. They seem largely incapable of direct imitation and do not explicitly teach their offspring. Each individual must reinvent the arts of nut-cracking, *etc.*, for itself and not all do. Though apes are far more percipient than rats, the way in which their culture is reproduced is logically similar. The Australopithecines of 3 million years ago where much like apes, but unlike them they were probably dependent on their culture for survival. Traditional hunting techniques and the ability to make crude stone tools were not optional extras. Because of this, they started a process of evolutionary specialisation in acquiring and using culture. By 1.5 million years ago, various early *Homo* had given way to *Homo erectus*, a species who made elegant and very useful hand-axes, and probably other tools that have left no trace. There is evidence that they used fire. They spread from Africa, throughout Eurasia and into areas where they could only have survived through their material and social culture. Over the next million years their brain size steadily increased towards that of modern humans, but their material culture shows no sign of having changed. Having perfected the hand-axe it was copied and maintained virtually unchanged for a million years, ten times the age of our species. They did not go on to invent new kinds of stone tools for different purposes, nor any bone ones. They have left no trace of art, decoration or ritual practice. This cultural stasis is very odd, and needs to be explained. (Especially as Evolutionary Psychologists claim that these are the Middle Pleistocene ancestors whose adaptations constitute modern Human Nature). Our culture has advanced exponentially for 50 thousand years. Theirs stayed the same for twenty times as long. Their cognition must have been distinctly different from ours. The theory of representation suggests why this might have been so.

An animal with percipient cognition may look far into the possibilities of its familiar environment, but its cognitive processing is always tied to its present context and its percipience does not enable systematic copying. Parrots and *Corvidae* perform remarkable cognitive feats and many imitate sounds and songs. Special features of avian neuroanatomy suggests why this may be so, but no non-human mammals can do anything like that. Copying requires some function that forms an internal imprint, separates it from its immediate context, and uses it as a goal state or template to which the copied product must conform. If *H. erectus* could go beyond mere percipience, and formed images that were not just sensory-motor projection, but distinct and separate from its immediate context, then it could have used these as templates. It would have allow them to work on a stone until its shape matched either a given handaxe, or the ideal form of one held in its *simulative* imagination. It is just because perception is *not* a kind of representation, that copying is such a difficult thing to achieve. Unless they were totally unlike us and made their axes by instinct, like nest-building in birds, we must suppose that they used imaginative simulations of cultural objects and practices as cognitive templates for making new copies.

Beings with this kind of *simulant* cognition could hold the image of one thing along side the direct perception of another. It would allow them to become specialists in replicating culture. They would have a new level of 'rational' cognition, a problem solving, simulative imagination that not only imagined the way ahead in a percipient way, like the apes and Australopithecines, but could re-imagine absent but desired goals and work in a trial-and-error way to achieve them. What they could not yet do was directly *imitate* others. Direct imitation is so natural to us that it is easy to overlook what a remarkable activity it is. It involves a direct mapping between one sensory modality, vision, and another, motor activity. Simulent cognition only copies things within one modality.

Most people think that apes can imitate, but in fact the do it very poorly, extensive research has had great difficulty demonstrating that real imitation occurs at all. Apes cannot perform even simple cross-modal mappings like the visual-haptic matching of unfamiliar objects presented to hand and eye separately. For the same reason they cannot dance to music, draw shapes or directly imitate others. They can easily integrate the information from different senses, perhaps using sub-cortical connections, but they have no special need to perform the 'abstract' tasks that depend on major trans-cortical nerve tracts. The arts and learning that we call Culture in the everyday sense, are just those skills that involve the specialised use of such connections.

If *H. erectus* had simulant cognition in the sense I suggest they would be able to perform intra-modal copying but not inter-modal translation. Simulent cognition is a more focussed and stable form of percipience. Percipient imagery sees ahead but is still tied to present contexts. Simulant images are more autonomous and can be used as templates for goal-directed activity, but they cannot translate between separate cognitive media and is thus limited by its inability to replicate and transform its images. The problem solving ability *H. erectus* was far ahead of modern apes but still far behind modern humans. Simulative cognition gave them a capacity for long range goal-directed activity, for making things fit a pre-conception of how they ought to be, would be far more advanced than that of apes. But they were crucially lacking in cross-modal abilities.

They could copy hand-axes, and would be able match their own verbal calls to those of others. Though one cannot see one's own body, one can see another's hands in the same way as one's own. Even monkey's have cortical 'mirror neurons' that respond specifically to the sight of another individual's hand. In *erectus* they would have helped in acquiring manual skills. Though they could not imitate directly but they would have been able to apply their simulative copying ability to the behaviour of *other* individuals. They could reproduce the culture by shaping each other's behaviour to match the expected cultural norms. Unlike apes, they could train their off-spring to use technical skills in the same way as adults. Intra-modal, simulative cognition would allow this even though it did not allow them to directly match their *own* behaviour to someone else's (at least, not in the general way we do). Copying without imitating gave their culture great, and eventually excessive, stability. As with the hand-axe with their culture generally, all variation appeared as error needing to be corrected. We take direct imitation and the instant recognition of people's intentions for granted, but quite complex material cultures could be reproduced without these abilities. Intra-modal copying and mutual training do the job in a way that is logically and neurologically, simpler. It seems likely that they form the evolutionary link that bridges the huge cognitive gap between the apes and ourselves.

We should not forget that even apes show 'sympathy', in the sense of 'emotional contagion'. This ability to reflect another's emotions back to them provides organisms with an advantageous 'games theoretic' strategy in their social relations. It is also an ancient and powerful means for the transmission of cultural information. It generally pays to like and fear what others like and fear. Though apes give vocal expression to their feelings in some thirty different ways, but they do not seem to use this for deliberate communication. Among Australopithecines, early *Homo,* and *erectus,* the recognition, communication and use of emotional attitudes must have became increasingly sensitive and subtle, coming to play a key role in co-operation and culture.

The *erectus* (and *ergaster*) spread throughout Africa and Eurasia. They were dependent on culture for survival and dependent on each other for the training needed to acquire it. Cultural objects and practices were accurately replicated, their forms were imprinted in each individual's simulent cognition. The culture was precisely reproduced from one generation to the next without having to pass directly from one mind to another. Such a system meets all the requirements for genuine representation in the von Neumann sense, but this is collective, not individual representation. Individuals replicate the elements of culture, and these represent and regenerate the corresponding cognitive structures in individuals, but not *vice versa.* A large part of our culture is unconscious and of this kind. Where the *erectus* band specialised in accurately reproducing a technical culture and everyone acted alike, early modern man developed a more flexible and efficient way of life based on sharing work, food and knowledge. They could do this because they became specialists in the direct communication of intentions and information. The culture evolved rapidly through the interaction of collective and individual cognitive representation.

After one million years of cultural stasis, around one hundred thousand years ago, possibly in association with a severe population bottle-neck, early *sapiens,* appeared and started to produce new kinds of tools, innovation stated slowly and then increased until, after fifty thousand years, it reached the explosive point of the Palaeolithic revolution – the sudden appearance of a far more advanced and varied cultures, which made representations. *H. sapiens sapiens,* had definitely arrived. About 10 thousand years ago there was a final break through, the Neolithic revolution into the beginnings of pastoral, settled agriculture. Understanding the logic of how these stages is crucial to our understanding of human nature and our modern cognition and consciousness.

As I have said, *Erectus* culture would exert a steady selective advantage on those individuals and groups who were best able to learn from each other and pass on skills to their off-spring. Brain size gradually increased and with it the richness and strength of neural tracts that connect areas of cortex serving different sensory-motor modalities. Among these and of primary importance, were the visual-kinesthetic links. It was these connections that allowed the perception of others to become a fully developed imitative ability. Since evolution takes the simplest route this connection would almost certainly have appeared as just one among others, one element in a general increase in the nerve tracts between widely separate cortical areas. The hard-wired capacity for imitation brought with it other connections that were pre-adaptations for the development of the singing, dancing, painting and symbolising cultures that followed.

EMULATIVE COGNITION

Some Cognitive Psychologists describe cognition in terms of 'mentalese', an innate mental language whose grammar and logic have yet to be discovered. Without a theory of representation, the nature of its semantics is wholly mysterious. The approach accounts for cognition in terms of formal 'modules' and 'representations' largely unrelated to the physiology of the brain. This idea of representation as passive and open-ended is not coherent. The von Neumann, systems theoretic account gives representation a logical and biological interpretation. Defined properly, cognitive representation is seen to have arisen first in our own species or our immediate ancestors. Its occurrence depends on what may be called our *emulant* cognition, the expression of our innate ability to imitate one another.

Imitation depends on the translation of visual inputs into a kinesthetic form that can then be expressed as action. Apes cannot do it, but human beings start practising it almost from birth. It the foundation of human communication. The primate behaviourist, Michael Tomasello has described young children as 'imitation machines' (*The Cultural Origins of Human Cognition* – Harvard UP '99). Done overtly and automatically without any internal reflection, imitation is just 'echopraxia', the meaningless repetition of acts and utterances to which some autistics and touretters are prone. In normal circumstances this automatic translation is kept covert. It is there, but only as part of a perceptual process. We can imagine what it feels like to perform an action, and seeing what others do sets up a corresponding echo it us. We use our own bodies and past experience as the medium in which to understand the actions of other's. It is because of this that we automatically see what people do in terms of their intentions. This *emulative* perception is direct, and understanding generally comes without conscious reflection. The *intentionality* of an action is the way it is directed *to* a perception. The simplest intentional act is that of orienting or attending, the act of bringing something into conscious focus. An intentional act is one that leads to and brings about a perceptual end-point. More complex action involve several intermediate steps, perceptual stepping stones on the way to the final goal. Human beings have an innate ability to recognise the intentionality of orientations and actions. From infancy we have no difficulty following someone's gaze, their acts of indication, and the *point* of the things they do. Only human beings have this generalised, emulative ability to recognise intentions. Our distinctive eye-whites are an adaptation that helps others see what we are doing and thinking. Young chimps watch their mothers cracking nuts with hammer and anvil stones but it takes them several years to master such skills. A young child with its emulative perception understands them at once. Where the chimp sees only movements the child sees actions guided by perception. It emulates the crucial acts of attention and so learns to recognise the cues and end-points that structure the task into a meaningful whole. Chimps cannot draw useful deductions from the fact that another's eyes are covered up.

Simulant creatures like *erectus* could only replicate actions by learning them for themselves or by being trained, not by direct imitation. Emulant beings have innate insight into the correspondence between their own bodies and actions and those of others. They experience (and cognitively process) the acts of other's, not only visually, but also in terms of the kinesthetic and somesthetic sensations that go with them, i.e. how it feels and what it does. A direct connection between the vision and motor sensations allows the whole body, with its existing repertoire of skills to become a medium for perceiving and interpreting what others do. To talk of this in the representationalist way as a form of 'inference', 'analogy' or a 'Theory of Mind' makes an immediate and hard-wired process sound misleadingly intellectual. It is not the product of thought, it is the medium in which thought occurs.

Emulant mimesis is accompanied by other abilities such as specific face perception and our old and deep empathic abilities. Together they give a system of sensory-motor functions that can be treated as one faculty, 'homoception', or the capacity to understand people as thinking and feeling individuals. It is primarily a way of perceiving others, and by reflection a route to understanding oneself as a person. If there is one attribute that separates us from our forebears and characterises a distinctively human nature, it is this homoception, the foundation of our Humanity.

Individuals with good homoception were selected because it was essential for acquiring culture and surviving in a social environment. More complex cultures selected more homoceptive individuals, and more perceptive people created more complex cultures, a co-evolutionary, positive feedback between cognitive practices and brains needed to sustain them. This was the era of the domestication of humanity, the time when the *erectus* pack evolved into the *sapiens* community. Specialisation in so-called Machiavellian Intelligence may have been important for the evolution of our 'ego-centric' but ego-less Pleistocene ancestors, but as we shall see in connection with the evolution of language, it would have been an inefficient way of developing the potential of emulative cognition.

Imitation is made possible by strong neural connections between exteroceptive visual perception and interoceptive somesthesia and kinesthesia. They allow individuals to copy and understand others and to organise actions in imagination. *Erectus's* simulations were, on the individual level, still only imprints, while we have a form of personal, percipient and simulative imagination that has become genuinely representational.

To count as true cognitive representations emulative simulations must satisfy three main criteria. The must be properly separate and distinct from the things the represent; the things they represent must be recovered or regenerated from them; and they must be replicated in cognition as distinct and discrete entities. (A fourth condition, of different kind, is that the whole system should be reproduced because it can do things). Merely percipient sensory-motor simulation do not meet these criteria because they were, only an inhibited form or shadow of the corresponding overt activity. There is, on the physiological level, no proper separation between cognitive 'image' and its overt sensory-motor performance, between of represented and representing. *Erectus*'s simulant images meet the first and second criteria because they serve as templates for overt acts, but they are not replicated or transformed in cognition. The emulant images of human beings are genuine representations because they meet both these criteria. The representational systems that result are reproduced both at the level of culture, with replication of sensory-motor acts between individuals, and at the level of the individual, with the internal replication and transformation of ideas in individual cognition. The overall system has the creative power of what Giddens calls 'duality of structure'. The individual and collective representational systems reproduce one another.

One person's sensory-motor emulations correspond to another's action. I can imagine what it feels like to do what you are doing, and using the inverse connection, I can visualise what my own actions look like from outside. In the social context, One person's emulative image is clearly quite separate from the other person's action, and on the cognitive level, visual images are clearly separable from kinesthetic ones. We can transform cognitive 'images' (percipient, simulant and emulative), changing, combining and comparing them, testing them off-line. We can translating them from one cognitive modality and one perspective, to another. When we are satisfied, *i.e.* when we have decided what to do, we can 'pump up' the image and release them from inhibition, and expressing it as overtly, *i.e.* we can generate or regenerate the corresponding real sensory-motor action.

Our emulative simulations are thus genuine representations of things and events external to ourselves and they form distinct 'thoughts' that can be manipulated in cognition. We can translate our sensory-motor simulations back and forth between (for example) exteroceptive and interoceptive forms, visual images producing motor ones and *vice versa*, in a way that is analogous to the replication by the two strands of DNA. We can use our cognitive images as templates or models from which to generate real states of affairs in the world, in a way that is analogous to generation of protein from DNA. It is a necessary property of 'von Neumann' representations that they can be read in two different ways, by replication and transcription. Our emulant but pre-sapient ancestors had for the first time, *thoughts* that represented things in the environment, could be translated and transformed in cognition, and then 'interpreted' in order to regenerate environmental things and events. They could operate on simulated sensory-motor acts as distinct and relatively context-free, cognitive 'objects' or *ideas*. Their thought was the representational use of a compressed and inhibited form of action.

Without representation, cognition would be only by means of continuous processes, transductions and functional mappings. As cognitivists have pointed out, neural-nets alone cannot give rise to discrete cognitive objects or symbols. It requires a higher level of functioning, involving something equivalent to closed inter-modal loops and the internal replication and transformation not at the level of nerve impulses but of cognitive entities such as covert intentional action sensory-motor acts of and the representation of sensory-motor domains. Emulant thought is not symbolic but it is not entirely analogical either. The thoughts, like the actions they represents, are entities with internal structure, articulated by distinct points, perceptual goals, sub-goal and patterns of attention.

Thus it seem that our 'pictures in the head' really can represent, but our emulant ancestors were without symbolic language, and this made their thought in a very different way from ours. They had ideas and representational thought but had no way of knowing that they did. They would have been clearly aware of themselves as social individuals and members of their group, but they could not recognise thought as thought in the way we do. We ourselves don't draw as clear a distinction between imagination and direct perception as we think we do. Emulant people could draw no distinction at all. Deaf-people brought up without a sign language report later that they did not know they had thoughts.

Much of emulant culture would have been replicated in the same way as *H. erectus,* as simulant collective representation, and by direct imitation taking place without internal cognitive reflection, as even today we pass on styles of action, gestures, grammar, etc., without being consciously aware of doing so. What representational cognition did give them was a genuine sense of the self as one individual amongst others. The reflective part of their thought was that part they could *recognise* as shared with others. Emulative cognition thinks using images of enacted and visualised people, each with a different perspectives on in a shared world. Many animals have an innate understanding of movement in three dimensional space. *Erectus* had an egocentric perception and an in-depth, percipient understanding of its whole territory, but *sapiens* had the neural basis for a generalised allocentric grasp of a space in which each person has their own position and point-of-view. Perhaps even more than we who have language, early humans could directly apprehend a shared social space structured by the forces and trajectories of people's feelings, perspectives and intentions. Emulant hunter-gatherers co-operated and communicated in a world that was for them social as well as spatial. It was only because they lived in this kind of 'moral' universe that they could go on to develop language.

Understanding and representing human intentionality doesn't stop with sensory-motor acts but extends out into the non-human but culturalised environment. Anything that serves as an intentional object or goal becomes significant in itself and so potentially, an object of cognitive representation. Intentional understanding starts with human acts but can go on from there to grasp the causal relations between other things and events. Understanding that A causes B, is understanding that if one made A happen, B would follow as a result. In cognitive terms natural causation is an extension of human intention. Even today languages and cultures differ greatly in the way they understand different kinds of cause. As I shall mention later, even modern science can become confused in the way it ascribes causality to relations between different kinds of things.

LANGUAGE

Language cannot have been a representational system from the start. It must have developed out of the kind of emulative cognition I have described. Evolution tends to do things the easy way, and Occam's razor argues against the idea that propositional language depended on the sudden occurrence of one or more special mutations. It is more likely to have developed culturally from the simple signing that would have been a natural expression of emulant understanding. If one accepts that people could have a simple, hard-wired ability to understand one another then no special 'language instinct' or 'acquisition device' would have been required. Innately representational cognition could give rise, in a series of steps, to fully-fledged propositional languages that represented independently of individuals. One can draw plausible parallels between the logical stages in the development of language and the major advances in human pre-history, from the earliest *H. sapiens*, *via* the paleolithic and neolithic revolutions and finally literacy.

The need to co-operate and understand communication was the driving force behind the evolutionary development of homoception and emulant cognition. The difference between emulative imitation and mere echopraxia is that the former involves intentional understanding. We follow another's gaze, recognises orientations, and understand when things are pointed out, and see the point of actions. Through emulation acts are seen as having meaning, and some are recognised as especially meant to have meaning for others. Language developed naturally from emulative perception and inter-action.

Infants follow people's gaze in their first year. They instinctively look to see what others are looking at and how they react. They understand pointing and love to point things out. (Apes never do). Gestures and vocalisations add precision and scope to simple pointing. Words are used to separate things out, distinguishing what is meant from what is not. They can serve as ways of pointing out things only seen in percipience, things and events that are in the shared environment but are not immediately present or visible. Over generations early human culture developed sophisticated ways of communicating the kinds of information needed to find food, avoid danger, arrange meetings and generally organise a co-operative life. Such indicative practices would have been a vital addition to the culture of a hunter-gatherer community. Sharing information is the best way of increasing the efficiency of a way of life based on sharing food and labour.

After simple warnings and hunting cries, the earliest structured indicative systems must have dealt with the locations of people and things. These people knew their local territory and knew that their fellows did too. They had allocentric percipient vision of their shared environment. By pointing and naming, they could indicate the (invisible) route to where something was, the journey to be taken. Such sign systems could become quite elaborate. They could give instructions and draw the distinctions they needed without the kind of independently representational language systems we have today. Representation requires, at the very least, a clear separation of two media, but the speech acts of early man were culturally elaborated acts of indication, and like perceptual and other kinds of sensory-motor orientation, such as pointing with a finger, indicative acts are not separate from the things that they point to. They are individuated by what they pick out. Any gesture will do so long as it makes the intended point. People with a good mutual understanding, in a shared environment and with shared expectations, do not need a propositional language. Emulative culture became increasingly sophisticated and complex, but context-dependent verbal indication probably served well-enough for tens of thousands of years.

Pragmatic concerns would be paramount, but this was a creative culture in which cognitive and collective representations could inter-act. Indicating routes is like sharing knowledge of a journey, and is thus a form of narrative. Pre-sapient society would surely have produced and reproduce mythic and poetic narrative and broadly 'symbolic' ritual practices, dance and music that reinforce community. The development of a more complex culture would give a selective advantage to those who displayed the quickest understanding and most fluent performance.

An indicative language depends on speakers who have a spontaneous wish to impart information, and on their knowing what kind of information other people need. But a system that depends on this kind of mutual understanding and benevolence will obviously benefit from a complementary *questioning* system, a way for listeners to actively solicit the kind of information that they need. Against a background of mutual understanding, it is not hard to see how they would develop various kinds of standard question-answering routines (QARs) in which the business of communicating is more equally shared between both partners.

'*Where is* X...?' '...*by the* Y'.

A question and its answer are two alternative ways of indicating the same thing. In this case *the location of X*. The questioner raises a subject X, distinguishing it by signing or naming, and gestures an ignorance of it position, the other person answers with signs that indicate how one can get to it. Both are strictly indexical and tied to the social situation that gives rise to them.

The system, up to time when QARs were established, was driven by people's innate predisposition to give and share information without being asked. We are chattering species. When they had QARs, the people who had made spontaneous indications without waiting to be asked, could now volunteer information even more easily by asking and answering their own questions (sermocination). In due course these would be distinguished from ordinary questioned by combing combining the two parts and dropping the redundant interrogative, and thereby getting a *statement*: 'the X *is* by the Y'.

Question and answers taken alone are indexical and incomplete. Their sense depends on the context and the speaker's intentions, and they cannot be true or false. Combined they make a statement which can be. Its truth depends on whether or not the two parts indicate the same thing. Statements that were useful or interesting would circulate freely around the community and be reproduced possibly over generations. As long as it is grounded in a generally shared and coherent QAR, *e.g.* for locations, a linguistic representational system (LRS) will be reproduced in a community because its representational statements can be independently replicated and used to regenerate real states of affairs in the territory. Their truth is independent of particular individuals and situations.

Freely-circulating statements were now a special kind of cultural object and that could came under of selection pressure to make them more effective carriers of information. Early LRSs probably generated statements that answered just one kind (or 'category') of question - *where, when, who, what, etc.,* (Is it coincidence that they start with a sound like an empty breath?). With this established there would be a natural preference for statement forms with more complex syntax that enabled them to carry more information. It eventually became possible for a single sentence to say, who did what to whom, and when and where it happened. At the same time is always remains possible 'parse' complex sentences into the basic ones that belong to particular categorical QARs.

LRSs, are autonomous and objective representational systems that are reproduced in culture as such, independently of particular individuals. They have two genuinely separate and independent media, one of propositions and one of the things they are about, and a coherent practice for recovering one from the other. Their propositional statements satisfy all the conditions necessary to be genuine representations in von Neumann sense. They are replicated in and between individuals. They can be transformed according to the logical-semantic rules, that result from the fact that one can indicate the same thing in more than one. (Syntactic rules are to do with combining different basic statements). The states of affairs to which they correspond can consistently 'regenerated' from them (*e.g.* reconstructed, picked out, verified, *etc.*). And the whole representational system to which the representations belong is reproduced over time in a culture because it has these properties. Propositional Language really can represent the world. (But see also the final section on Epistemology that begins to discuss this in more detail).

SAPIENCE

The establishment of a second representational system took human cognition to a new, higher level of organisation, and the sapient consciousness we know today. The process must have occurred in a series of stages. Indicative skills must have developed with humanity from the beginning. The dramatic changes in material culture that start around 50,000 BP clearly mark a break-through in cognition and the most likely candidate for the appearance of systems of indicative communication sufficiently advanced to support the appearance of *questions*, the specific but open-ended indications that solicit information from others. The ability to ask questions lead to the development of increasingly efficient question answering routine. Used covertly for self-questioning they would enhance the effectiveness of on individual as well as social cognitive practices. They allowed a mode of dialectical cognition having much of the power of modern sapience. Questioning routines gave people a new level cognitive control, an articulate means of focussing attention, interrogating memory and organising action, first other people's and then the individual's own.

Fully propositional language may have appeared along with pictures, tallies *etc.*, some 20,000 years later. The final stage in which separate linguistic representational systems were finally integrated into a modern syntax may not have occurred until the time of the neolithic revolution. The evolution of ways of talking is continuous to this day. Such a history is obviously highly speculative, because we don't know the extent to which there was a continuing co-evolution of the brain along with culture. We know little of its details and time-scale. The theory of sapient cognition offers only a rational reconstruction of the logical stages. Human brains may have continued to evolve stronger and better tuned cortical connectivity and differentiation throughout this time, but it is also quite possible that the development of representational language was an entirely cultural process.

Fully sapient cognition is a dynamic system based on two distinct representational systems. One is the innate and biologically hard-wired emulative system based on the simulation of one person by another, the other, an autonomous symbolic system culturally developed as a tool for sharing information. Two kinds of representation, separate but inter-acting, give a completely new kind of cognition: a self-correcting, self-organising couple. The earlier levels of cognition, from sentience to emulance maintain their direct sensory-motor links to environment and other people, but sapience, built out of these, has a new order of autonomy. The inter-action of emulant cognition and language form a kind of inner dialogue, carried on in relative detachment from the body's physiological state and its environmental situation. Our reflective self-awareness and freedom to deliberate is the result of having two cognitive media with quite different origins, each representing the world in a quite different way. Each is able to control, represent and transform the content of the other. The routines for doing these things are implicit in the structure of dialogue, and the speaker and listener roles we master in learning to participate in it.

The QARs of language call up and control various kinds of emulant simulations: reflections on past and present events, and the deliberate planning of future acts. Sapient mental tools originate in the use of signs for communication and the organisation of co-operative activity, but also give each person their own creative imagination, our individual ways of combining emulance and cultural media. We use these cognitive tools in responses to the immediate situation, and also in a detached, reflective way, pragmatically and also playfully and for their own sake it. Two representational systems in combination give the control of attention and action necessary for us to develop as self-aware, social individuals, answerable to ourselves and others for what we do, and in a sense, for what we are.

Through the semantics of their QARs, the propositions of an LRS can be checked against emulative experience of the world, and against one another to see if they can be consistently combined in emulative representation. From the other side, imagined states of affairs can be expressed in words and checked for consistency by the rules and conventions of logic and language. The Representational systems inter-act to give a uniquely powerful, dynamic and self-correcting epistemic system.

When we meet with new events and situations, past simulative and emulative experience determines what we attend to and call up past linguistic experience, which in turn influences what and how we see. The two systems operate in parallel each serving to tune and modulate the other. Language points things out to us and the perception of them conditions our future use of words, and through them, the kinds of things we can tell ourselves to do.

The structure of representational systems implies also that the two inner or cognitive media can be taken as a closed couple and used as an inner model of the external representational systems that exist in culture independently of the individual. The representational correspondence between the cognitive use of language and simulant imagery can model that between public language and real things. The relationship between inner media *represents* that between the two outer media and is thus a meta-representation. It is this that allows us to judge the truth of statements and also, in principle, the coherence and value of the LRSs in which they arise. When we talk of knowledge, meaning and truth we are already using our two cognitive systems in this meta-representational way. Higher order representation allows us to compare different ways of representing, talking and perceiving. Such activity forms the basis for theoretical science and critical hermeneutics.

Sapient cognition is bio-social meta-representation. It exists at the dynamic interface of physiology and culture. Two (or more) representational systems inter-act to give higher-order cognitive functions that cannot be reduced to those of its components taken separately. This constitutes an embodied Mind that can operate in an unlimited number of different ways. Basic sapient cogitation replicates and transforms ideas which are signs with a dual aspect, part linguistic and part emulative. On the linguistic side it can reach out into a network of verbal and non-verbal cultural representational systems, and participate in objective representational practices that involve concrete objects, natural and cultural. On the emulant side sapience reaches down into the sentient and pre-sentient relations between itself, the body, other people and material things. Closed feedback loops using emulance and language can participate in or control any suitably responsive system to which it attaches itself, from single muscle cells to complex practical projects. They can stabilise attention on almost any distinct part of our activity, an infinite focus ranging over a world that stretches from particular sensations to the whole cosmos. At any point in its repertoire we can recognise the possibility of unlimited internal reflection on the relation between objects of attention and their context. Sapient cognition is a dual representational systems that gives us unlimited levels of reflection and degrees of indeterminacy. On the perceptual side we experience this as consciousness and on the performative side as free will. The representation of representation is not just a source of odd logical paradoxes, it is a lived reality.

Naïve Representationalism, (even the kind that recognises its own contradictions), tries to speak of consciousness as though in was just another subject matter, as some kind of a substance, a property of the brain or the function of some structure. Such accounts leads to the insuperable difficulties of the 'Hard Problem'. They may avoid talking about Mind as an unacceptably immaterial *thing*, but consciousness continues to be thought of as the essentially dualistic 'subjectivity'. The only way around this is to abandon the way of talking and line of questioning that gives rise to it. Representationalism, with its foundation in an unreflective and one-sided 'Intentional Attitude' cannot help reifying its subject matter. We need instead to take sapience as first of all a bio-social *activity*, made possible by a certain kind of biological self-organisation. Reflective self-awareness arises naturally as a relatively autonomous system of activity existing through the inter-action of biological emulance and social language. Thought of the activity of a meta-representational system there is nothing wrong with the idea of the Mind as a thing. It is the product of relatively autonomous, self-organising and self-reproducing representational systems, that exists embodied in the medium of the brain and whatever it is dealing with, organising its in-puts and controlling the body. We have no difficulty accepting that representations like print or pictures are real things in a material medium, so there should be no problem with the reality of dynamic systems existing as real things in the kinds of material media that can support them, brains today and perhaps one day, and strictly as parts of human culture, machines too. Bodies are self-organising and self-maintaining systems, and so are communities. Minds exist at the fluctuating boundary between the two.

Representationalism asks 'Where is consciousness and what is it made of?' and 'What is it *for*?', 'How does the brain *cause* conscious experience?', 'How does consciousness effect the brain?', 'Why are red qualia *red*?'. None of these questions make sense or are of interest when asked of a mind that is a self-organising meta-representation system, and with being self-consciousness as one the things that it can do. (Though with a bit of effort these questions can be reconstrued and answered after a fashion). It is not surprising that those who conceive the problem in the representationalist way either see no way forward, or postulate some way-out quantum mechanical or supernatural solution. Unlike these, the systems theoretic approach offers an understanding of mind that fits well with present day physiological knowledge. Its idea of the embodied minds finds a natural complementarity between phenomenology and neurophysiology, each providing a way of interpreting the other. Conscious experience is not a given datum, but something achieved by the social self (or mind) using its brain. The self-referential paradoxes are still there, not in our descriptions and questions, but as a necessary aspect of the process we are describing. One would like to say that consciousness is natural and finite, but in the complex and indefinable way of an infinite series converging on itself.

If one can be satisfied with that, there still remains, quite untouched, the unanswerable questions of why anything exists at all, and why any one of us should have turned out to be part of it. These questions can be hard to separate from our attempts to understand consciousness, but are actually separate issues. Sciences must have well defined objects of study. We can have a science, even a technology of cognitive function, but individual consciousness is not an object. It is an activity and an art.

RECAP

What we call consciousness is self-conscious sapience, a form of self-reflective meta-representational cognition. Whether or not it is, in itself, something very mysterious, its evolution was a natural process that can be understood quite easily. The logic of self-reproduction is the same for culture and cognition as for microbiology and entomology. It depends on replicating representations that regenerate their structure from one generation to the next. When a species comes to depend on culture to survive, its individuals will be under selective pressure to become better at acquiring it. With fluency at copying things, the simple cultural replication of apes goes on to become the full scale self-reproduction of *H. erectus.* The resulting collective representations give an advantage to individuals with a talent for true imitation and if this arises, then so too does individual cognitive representation. Individuals with the necessary inter-modal connections can represent and so understand, first each other and through them the things and relationships in their shared environment. Beings that simulate one another can simulate themselves in the same way, and the body acquires an idea of itself. (This is how Spinoza defines the mind.) This 'idea' is not just a shadowy notion but a working model that can be construed from a variety of perspectives. After this great cognitive break-through it was probably inevitable that people would go on to develop autonomous and cultural representational systems, primarily language. The dialectical inter-action between individual and collective representation can give rise to individual sapience and the self-reflective meta-representation we know as consciousness. Each step on the way to sapience involves a new level of self-organisation. Representation, and then meta-representation allow the establishment of new kinds of self-replicating cultural and cognitive entities, and a new levels of autonomy and control. But these necessarily occur against the background of what went before. A great part of human culture and cognition is still pre-sapient, collective and unconscious. Recognising what happens on these levels and bringing it into overt communication and cognitive representation is an important cultural activity, a major part of individual growth and social progress.

Both as representers and as interpreters of representation we necessarily use a tacit, meta-representational model of the representational systems we are using. The speaker uses it to judge the accuracy of their correspondences, the listener to identify and enact the particular kinds of representational system they use. Neither sees its limits because each takes on, and in a sense becomes, the system used. It is only as we develop *autonomous* meta-representational systems, *i.e. theories* of the different kinds of representation and their mechanisms, that we can come to do that. It may sound complicated, but to people immersed in Aristotelian and medieval ways of thinking Descartes was difficult and obscure. Many of us now find it hard to think in any other way. Literacy and printing lead us to a certain way of understanding the nature of language. But that seems to be changing. New media are putting writing into a wider postmodern context, and globalisation is bringing Western and Eastern ways of thought together. The theory of sapient cognition provides a way of understanding certain aspects of this process. The abstract theory may be turn out to be useful. If is coherent then, what is really important is an enacted or intuitive sense of what sapience is, a concrete recognition of how the traditional representational dualisms of subject-object, mind-body, *etc*., are only one side of an innate family of dialectical distinctions like Intentional-Participatory, Speaker/Listener, *etc*.. I have tried to define the main structure and logic of this here. It still remains to find simpler and more fluent and ways of expressing it.

I started by introducing the idea of a natural complementarity between Intentional and Participatory Attitudes. One starting from perception and the posterior cortex and the other from action and the frontal regions. New NMR studies allow us to extend this and relate it to the two hemispheres of the brain as well. Language has long been known to be usually associated with left hemisphere, while we tend to use the right we have music and spatial understanding and other non-verbal skills. Faces are recognised by the right visual area, and social perception and homoception are particularly associated with right frontal regions. The left hemisphere is said to be analytic because is it more concerned with perception and the identification of particular objects and attributes, the right is holistic, grasping spatial relations and is more important for emotion and organising actions. The separation of our two representational systems could have been merely abstract, but it now seems that they are, at least in part, actually separated in the brain. It is consistent with this account of sapience that the human brain has a distinct 'torque' or asymmetric twist such that the right frontal (seat of APA) and left posterior (seat of PAP) poles are disproportionately large. It is also perhaps not surprising that women have more wide-spread and stronger connectivity, and less marked left-right functional specialisation than do men, and that Chinese people show a considerable amount of language function in the right hemisphere as well as the left. Psychological tests of judgement and perception show corresponding differences of cognitive style which are consistent with the present account of sapience.

The Intentional and Participatory Attitudes, which I introduced at the beginning, have roots in pre-sapient cognition. The former first appeared in the simulant but 'rational' action of *H. erectus, i.e.* in the PAP copying ability that depends on imagining a goal state and working to bring a present state of things into alignment with it. The Participatory Attitude starts with the first *H. sapiens,* who were still unable to reflect on their own thought, but understood themselves and the world through reflection on their overt inter-actions with others. With propositional language comes full sapience and the Speaker and Listener relations to language. These came to be expressed in distinct kinds of philosophy, which should now be seen as complementary reflections of sapience. One could call them logosapience and somasapience. Literacy made possible a reifying reflection on language and so the philosophical expression of the two sapient attitudes. The Speaker-epistemological and the Listener-hermeneutic schools, each with its own concepts of Knowledge, Meaning and Truth and its own set of metaphysical presuppositions. One is logocentric and based on a model of the ideal representational relation between propositional language and the world. The other is somacentric and takes language as indexical and an integral part of consciousness, an expression of the human body and its social inter-actions.

Either taken on its own gives a partial and distorted view of how things are, and driven to its logical conclusion becomes increasingly abstract and absurd. We often see this today as a confused mixing together of scientific reductionism and nihilistic relativism. John Grey's recent book *Straw Dogs* is a good example of this. It is what you get when you try to force the two poles of philosophy together without understanding their origins. They cancel rather than complement one another. Interestingly, Grey still manages to conclude that we need to reach a Taoist vision of life. In the present state of the world we do indeed need a more profound sense of participation in the process of the world, not as an alternative to Western rationality and individual agency, but as the wider context that gives it meaning, responsibility and a coherent direction.

But how are they to be put together? How should we regard ourselves? The somacentric and logocentric, speaker and listener attitudes are after all, mutually exclusive perspectives, dialectically opposed and without a middle way. Faced with an account like the present one, adherents to the two sides may react according to their perspectives. Representationalists say that you can't have both ways, so that one side has to be wrong, and if we don't reject relativism, we are on a slippery slope to unreason and chaos. Relativists say that there are not two ways, but an infinity, and that the Theory of Sapience is blatant Structuralism and just another 'Grand Narrative'. One sees their points, but both sides have to accept the world that natural science - physiology, cosmology and Darwinist natural history, are showing us. Science has, in the end, to come to terms with the fact that representational systems, including its own, are not the product of a transcendental 'view from nowhere', but just as natural as the things they deal with. Relativists are right to point out that we are free to choose our own ways of viewing and participating in the process of the world, and that we are inevitably selective about what entities we recognise, but in the end they must accept the existence of people and a world that contains creatures, artefacts and natural things. Among these things are consistent LRSs that, like topographical maps, fit their subject matter and cannot be altered at will. If accounts like mine turn out to be consistent, then we can learn to live with a dialectical view of human thought and consciousness, at least until it is made irrelevant by another major shift in our cognition and culture.

The naturalistic approach that I adopt here leads to an account of sapient cognition as an emergent function of two very different forms of representation. It provides a model that is rich enough to cover our ordinary ideas of Mind, truth, knowledge, morality and freewill and shows why conventional philosophy leads to such contradictory metaphysical implications. Both kinds of traditional philosophies have glaring failures. The Representationalist attitude cannot find a secure place for morality and human values and is liable to treat them as subjective or even delusory, a civilised pretence to hide the necessary egoism of Darwinian animals. The Anti-representationalists are similarly nihilistic about objective truth, taking objects of study as constituted in and by their epistemological practices, and socially constructed knowledge that cannot be disinterested. A Naturalist approach can overcome both these kinds of problem.

In the final two sections I present very brief discussions of morality and truth as they appear in the light of the theory of sapience. Both deserve to be treated far more fully, but I put them in here to indicate the kind of direction I think one would follow, and for the light they throw on what has already been said.

MORALITY

Nowhere is the inadequacy of one-sided philosophies more obvious than in their accounts of morality. Without its religious faith, Western philosophy is unable to give us an account of morality that goes beyond the subjectivity of Emotivism and the unconvincing rationality of Utilitarianism. Postmodern, anti-romantic Romanticism, with its stress on the Freudian unconscious and social-historical determination is no better. Darwin and Nietzsche are used to reduce morality to psychologism and subjectivism. In their present incarnations, the antithetical poles of philosophy offer us a choice between a theoretical detachment and an ironic one. Neither gives us a sense of what it is to be naturally moral creatures.

Representationalist theorising about morality is impotent because its subject matter is precisely that area of cognition that symbolic thought and autonomous representation cannot reach, *i.e.* the pre-sapient foundation that makes its own propositional language possible. Like a torch unable to illuminate itself, its questions necessarily miss the point. The apparently nihilistic scorn of anti-representationalists is directed not against real morality, but to the absurdity of moral discourse that pretends to be dispassionate and disengaged.

Morality and conscience, like consciousness are a natural phenomena. Sapience could not have evolved other than as the practice of an intensely co-operative and communicative species. We are complex and contradictory beings, but if we have one defining and originitive attribute it is our homoception, our direct, emulative and empathic understanding of one another. Our moral perception is what makes us human, not in some poetic sense, but literally, as a fact of evolutionary natural history.

As a species specialised as carriers of culture, human beings have to be innately sociable. Infants are instinctive imitators, born with an intense interest in people and naturally developing the ability to understand others and their attitudes. Our sense of self and our reflective consciousness itself, derive from seeing others as like ourselves, and ourselves as we appear to others. The philosophy of cognitive science has emphasised the dualistic, subject-object, distinction between the first and third person perspectives, but it misses the primary role of the first and second person inter-action. It is the reflective and homoceptive you-me (or *I-Thou*) relation that is the foundation of human self-awareness. The internal reflection of this difference is at the heart of emulant cognition and human self-awareness. It is only because of it, that the body can have a genuine representation of itself.

We are hard-wired to understand others in the same way that we understand ourselves, and because of this we live in a moral world. Other animals have an innate understanding of their three dimensional spatial environment. We have that, but also, uniquely, we have the visual-kinesthetic connections that differentiates us from *erectus.* With it, we live in an allocentric space structured by people's intentions and feelings, and perspectives. We see and understand acts in terms of their meanings and feelings. As MRI scans show, we understand actions and recognise pleasure and pain in others, using the same cortical areas in which we perform and experience them in ourselves. This is not a matter of representation by dead symbols, but the direct representation of one person by another. Mental and physical actions and their feelings involve the same neural circuits. We can recognise the difference between kindness and cruelty, or joy and fear as directly and naturally as the difference between up and down, or tension and compression. Human action is moral because it is understood by, and follows from, emulative and empathic perception. Putting cynicism aside, we can say that in a real sense, loving one's neighbour as one self is something we are hard-wired to do. For a mimetic species such as ourselves, the greatest cause of evil is believing in the evil of others.

It has been proved mathematically that altruism is consistent with Darwinian principles. While bearing in mind that this conclusion was necessarily built into its premises, it shows that even the reductive approach that analyses evolutionary processes in terms of selfish genes and selfish individuals, has to accept that altruism and morality are be perfectly natural. It is only by retaining a Kantian dualism that one concludes they are thereby in some way unreal or invalid.

We are a species that has evolved to be expert in communication and the co-ordination of group activities. As we breed other species to have characteristics that are convenient to us, so also, have we bred ourselves - for each other's convenience. We are a self-selected, domesticated species. As we have seen, development of language would have been impossible without the expressive eyes, faces and a manner that openly displayed our intentions. Complex syntax evolved for the sake of a spontaneous desire to speak and share information with others. Given that the ability to recognise intentions is basic, and that questioning was an early development, a form of moral questioning probably emerged even before propositional speech. The 'ought' is older than the 'is'. Palaeolithic skeletons show that long before modern language, people were already looking after their sick and elderly, individuals who can only have been, on a 'rational' analysis, a hindrance to the group. This is Human Nature, the Humanity that made sapient consciousness possible.

The difference between good and bad, right and wrong, cannot be reached by some conceptual distinction between kinds of action, nor is morality reducible to rules or Commandments, nor generated by maxims or worked out by a eudaemonic calculus. It is something intrinsic to the way human beings perceive actions, the dimension within which we draw moral distinctions. Our understanding of intentionality and the meaning of actions in moral space is as innate and probably more fundamental than our understanding of mechanical causation. We recognise the difference between compassion and cruelty, selflessness and egocentricity, moral courage and cowardice, and we have a sense of fairness and 'natural justice' because we were bred to participate, co-operate and communicate. It is what defines us as a species.

When people were still only emulant, before the 'fall' into propositional language there could be no question of thinking in specifically moral terms. Their behaviour was less deliberately controlled than ours, perhaps more haphazard, but not necessarily worse. Simple emulance endows us with the tacit sense of a self to whom actions belong, but it does not have any way of recognising thought as such. Indeed it was not separate in the sapient sense. After the fall we had thought that was conscious of itself, not just in the epistemological sense of representationalism, but as the ontological somasapience that can, in principle, enact and compare ways of seeing and being, and consider different ways of representing events.

Our homoceptive and inter-emulative relations to other people are quite different from our pragmatic relations to inanimate things. The two are based on distinct neurophysiological functions. The latter involves actions that transform one state of affairs into another (P→A→P), the Intentional Attitude of rational goal-directed action and means-end analysis. It is the sapient orientation from which we represent the relations between different *ways of acting* as rules, methods, behaviour observed and described – action represented in a non-embodied, symbolic way. Morality can only be understood from the alternate, somacentric orientation, the A→P→A of mimesis and participation. Such action is not goal-directed but participatory, done for its own sake or for that of its meaning-giving context. Somasapience distinguishes between different *ways of perceiving*, dealing with attitudes, perspectives and construals.

This gives us the distinction between wrongs committed within a context of moral perception, and those that depend on rejecting it. The latter involves a disregard of other people's interests in the pursuance of some chosen end. Acts that are deliberately selfish or malign, even amounting to self-conscious wickedness, are immoral. Those, ranging from lack of consideration to gross exploitation, that treat people as instruments and as things are 'amoral', the expression on an immoral suspension of moral perception. This is evil as banal.

Homoception and emulant cognition allowed hunter-gathers to be good participants in the social life of the tribe, contributing to the success and well-being of the group, promoting its emotional harmony and organisational efficiency. This contribution had to include a responsibility to look after one's own well-being, and as Evolutionary Psychology rightly emphasises, contributing to the gene pool. There are obvious possibilities for conflict between different kinds and levels of participation. In more complex societies these become numerous. The nature of an action depends on its context, and it is necessarily ambiguous if it exists in several frameworks at once. Sapient morality has to deal with the fact that complete equilibrium and perfect participation is unattainable. Systems may achieve temporary homeostasis, but even at the most basic, sentient level, cognition is a homeodynamic buffer that never rests. It is still more so for emulant consciousness and the sapient mind.

The natural ethics of Buddha, Lao Tzu, Confucius and the Stoics, calls on us to restrain ourselves from seeking individual advantage at the expanse of the group, to act with due moderation and balance, optimising the equilibrium of the group and attending to the psycho-somatic good of all, using our innate abilities to promote homeodynamic stability, seeking inner and outer harmony.

Where every action and activity is part of some wider system we need a guiding sense of the highest level of participation, the final context in which all others are contained. The moral teachers give many names to this Totality, each showing it under different attributes – God, Nature, History, Tao, Dharma and so on. They all encourage us to attend to our somasapient side, to see that we are points-of-view in a process before we are goal-directed agents. Moral progress is not achieved by rule-following but by altering attitudes and awareness, and cultivating better ways of being in the world. Logocentric attitudes take people as ego-centric beings controlled by social pressure or religious belief. Somasapience sees them as beings whose departures from sociable behaviour are explained by external conditions. One emphasis rules, rewards and punishments, the other setting good examples and cultivating mutual concern.

We would not need moral teachers if we did not often depart from our innate ideals. Even those of us with the luck to avoid early catastrophes can fall into blind imitation, or allow logocentric ideologies and indoctrination to over-rule our emulant perception, or through ignorance and lack of awareness concentrate on the wrong level of stability. We will always need moral discourse, not as something transcendent but as an integral part of living together. However good our native ability at judging sensibilities and intentional trajectories, balancing relationships, harmonising the development of situations, we are always required to be participate in social systems that can call on us in conflicting ways. Propositional language aspires to be a representational system operating independently of individuals and their particular situations. But one cannot understand the moral quality of events at the same time as operating such a system. To perceive morally one has, oneself, to *be* an emulative representational system, in which language is at the service of emulance and not the other way round. For a moral assertion to be of value it must have an effect *in* the morally ambiguous situation with which it deals. It must *impinge* on, and be a causal part of, that situation – but to do that it has to give up being a disinterested representation of it. This does not mean that moral discourse cannot be rational, only that it must be based on a casuistry of perspectives, raising and comparing the relevance of different contexts and levels, and seeking the ways of seeing and understanding that are most fitting and best able to open up an appropriate way forward. In a world crying out for an ecological, Gaian view of our participation, we need this kind of biological perspective.

EPISTEMOLOGY

EPISTEMOLOGY The philosophy of knowledge forms the core of mainstream Western philosophy. But its understanding is severely compromised by its naïve representationalism and the confusion of knowledge with its dualistic conception of Mind and consciousness. Its confusions and contradictions can only be removed by reconstruing it as a special case in a wider, naturalistic and systems theoretic framework.

Knowledge understood as representation has its earliest roots in *collective* representations replicated socially and imprinted in the cognitive systems of individuals, and reproduced over generations by objective copying and mutual training. So it remained until biological changes allowed the *cognitive* representational systems of emulative people who shared perceptions by understanding one another's acts of orientation. Eventually there were independent *symbolic* representational systems, that combine distinct but convergent acts of indication into structured verbal representations of real states of affairs. These emergent representational systems, though cultural, and necessarily well adapted to the cognitive needs of human beings, were independent of particular individuals and situations. The forms of such representational statements evolved in a community over generations to give complex syntax and bodies of folk knowledge.

Autonomous representational systems are as natural to *H. sapiens* as handaxes and say, hand-shakes, were to our pre-sapient ancestors. All are, in their different contexts, systems that are reproduced because they contain replicators that regenerate entities that cannot be replicated directly. Cultural representational systems, and to a lesser extent the particular representations in them, operate by rules that do not need to be recognised consciously unless they are explicitly studied, (e.g. by philosophers, anthropologists and linguists). Symbolic representations are cognitive as well as cultural, existing in people's minds and memory, but also on paper and elsewhere, embedded in practices, and independent of particular individuals. They are the crucial currency of sapient discourse and thought. Symbolic representational systems, with their co-ordinated homomorphism of relationships between signifiers and relationships between signifieds are, in their embodying medium, as real and material as the mechanism that reproduces living cells. Many of them, e.g. measurement practices, that operate with a machine-like precision, the classification of 'natural kinds', the prediction of experimental results, have their consistently replicatable interpretations are reproducible in any culture that has a use for them. There is no question of relativism within a well defined representational systems.

Human representational language systems come in many different forms. The paradigm case is the location system, based on the *Where* routine, and used as the verbal equivalent of a map. It is probably the ancestor of all systematic QARs. (The theory that language was once primarily a kind of social grooming underestimates both human sociability and the pragmatic power of language.) There are cortical areas and connections in the brain that specialise in object identification (the *What* system) and spatial location (The *Where* system). The MRI evidence supports the idea that a QAR is a practice whose function is to set up new functional connections between them. One person's indication raises a subject and refers to a directed connection, and an interlocutor points out its terminus. In the associated LRS, truth is the unambiguous and impersonal matter of the correspondence between the relations of terms in a statement and relations of things in the world. The spatial pattern can be recovered from the verbal one, in imagination or by verifying it overtly. Systems of measurement, timetables and scientific classifications are representational systems that can be precisely defined, but not all systems are as straight forward as this. Sapience is flexible and adaptive, and cultures naturally develop some representational systems that have only metaphorical or formal systems of reference – mythology, maths *etc.*. systems whose syntax is derived from other media, and do not have objective referents of their own. But mythology is easily imagined even if maths its. There are also many linguistic practices that may seem to pose as LRSs, but cannot in fact support a coherent QAR. Some may be fraudulent and survive only through social prestige and constraints on full communication (we all have own opinions about which these are). Others are useful, such as those that use abstract or metaphorical language to work on the ill-defined meta-level of the relations *between* more object-like LRSs. All representational language is indexical, but not all indexical language forms part of even a formal representational medium. Emulative communication can use any sign to mean anything – the only criteria is whether it contributes to sharing orientations. We use such indexical language in an open and dialectical way in trying to forge new LRSs. It hardly needs saying that statements do not have to be in the service of an independent LRSs. They can indicate attitudes, express feeling, organise experience, and evoke and coach ways of perceiving, *etc.* in ways that make sense only to our emulative understanding of other people in particular inter-actions.

Despite the opinions of postmodernists, representational knowledge and truth are not just a matter of consensus or power. They are the coherent and independent LRSs that we have been using in an objective and disinterested way for tens of thousands of years. Our lives depend on them. Recognising this does not commit one to some Platonic or metaphysical 'Truth' with a capital T. Our knowledge is powerful not because it is transcendent, but because of the way it fits precisely into the real world and engages with it. True statements arise when two *different* indications (e.g. a question and its answer) pick out the same real state of affairs or relation between things. Truth is a correspondence property of replicatable representations in a consistently reproducible system with two independent domains. We can, as individuals, make reliable judgements about the truth of statements because sapient cognition gives us two kinds of 'inner' or cognitive media that can be used to form meta-representational models of the relations between the objective or 'outer', media of our cultural representational systems.

Critical hermeneutics has an essential role in the search for truth. Its task is to explore and make explicit our intuitive performatory knowledge of representational systems. There is a particular need for it to elucidate systems on the edge of formal science and the ill-defined relations between different representational systems within it. Being a representational medium is an associative property, i.e. if a new medium is mapped on to an existing representational medium, it too can carry representations (e.g. messenger RNA between DNA and protein). Natural science maps its domains of objects to elaborate structures of notational, instrumental and cognitive media. Covariances in nature become inter-linked via representational media to form structured and reliable systems. Take for example, the astonishing reconstruction of the pre-historic past built from connections between dendrochronology, sedimentation layers, geology, geography, biology, and the rest. Such systems are coherent and not, in any basic way, contentious. There are areas however where the relations between different representational systems is not well defined, in particular with respect to the way they are used by us, i.e. the practical connections between cultural representational systems and the sapient cognition of the people using them.

In biology and cognitive science, quite apart from the absurdities that arise from not having a theory of representation, there are also major confusions about the relation between different representational systems. When *Why* questions ask for explanations, they are demands for a re-description, and as such they deal in the relations between LRSs, and these are very often without any precise definition. *Where, When, etc.* questions are uncontentious, but there isn't and cannot be a single kind of QAR for Why questions. These relations lie at the root of our cognitive systems and are often at the very edge of what can, at present, be thought explicitly. (Which is doubtless why kids go through a stage of incessantly asking *Why*?). Where specialists are working within the constraints of an agreed set of representational systems (a paradigm), discourse is generally coherent. It is when people make unjustified steps from *How* to *Why*, that things go wrong. Where logic is absent, ideology fills the gap. Cognitive psychology and philosophy is replete with explanatory pronouncements in which causal relations are imputed between incompatible categories, and arguments in which ontology and epistemology become hopelessly confused. In cognitive science much of the trouble comes from not distinguishing between covariances ('imprints') that already belong to representational systems and those which only become representations when they are incorporated into our systems. One must separate 'real' causal relations from those that are only the expression of conventional relations (e.g. involving the prestige of different disciplines) between distinct representational systems. All representations are necessarily mediated by representational systems, and statements only have meaning if they are possible answers to questions. There is no relativism within a representational system, but between different ways of representing things there has to be relative relativism. If human cognition and knowledge are natural phenomena then this is a fact of life that we have to live with. It does not detract from the reality of knowledge, meaning and truth, in the correspondence sense, but recognising it may help us see where they belong in order of things.

In quantum mechanics, for example there are major problems that seem (to me) to be due to a failure to understand how different kinds of mathematics are thought and used, some being taken depictively, as cognitive objects, while others are enacted as analogical activity. I am no quantum mechanic, but I cannot believe that Bohr's complementarity is not a rational reflection of the basic cognitive distinction between A→P vs. P→A. The apparent contradictions are not in the world but between our different ways of relating to it via our notational practices.

Cognitive science my in the future make major contributions to what we might call practical epistemology, the devising of new representational systems that are better suited to our innate cognitive equipment. Natural languages are generally well adapted to the people who use them, but the same cannot be said of all our scientific and technical representational media. We may hope that electronic media will make it possible for people to develop new kinds of representational system better able that bridge the gap between written symbols and dynamic spatial forms. Devising media that are better tools of thought and communication would involve and artistic as well as scientific and technical creativity. As things are today only people with a special aptitude for spatial thought can build the right-hemisphere cognitive models needed to understand mathematical systems. Each individual has to do this for themselves (like chimps re-discovering the are of nut-cracking). It is hard work and few succeed. Sapient thought involves having systems of bio-social cognitive representation. In maths and mathematical physics only the notation is shared, and to this extent it is still at a pre-sapient level. It is that encourages so many to have a Platonic idea of maths and naïve realist view of physics.

Overcoming this view may have been part of what Haldane had in mind when he predicted that physiology would replace mathematical physics as the dominant paradigm of science. Physics, like all knowledge, is natural phenomena, the activity of bio-social beings working in a evolutionary-historical context. It does not stand outside the world but is rather, necessarily in the world and part of it. To properly understand this involves adopting the somasapient, systems theoretic style of thought that Haldane knew in the form of Bergsonian activism.

It has been said that we now live in a 'neurocentric' age (*Soul Made Flesh* – Carl Zimmer). Neurophysiology is giving us better models of the brain and new imaging techniques are beginning to give us a understanding of our what Russian scientists used to call our 'neurophysiological functional organs', the acquired connectionist structures that mediate biological and cultural skills, our cognitive practices. Pre-sapient human consciousness came when inter-modal connections gave the body a visual image of itself and somesthetic image of others, 'the bodies idea of itself'. A day may come when this idea has extended to embrace the structure and function of the brain itself. Buddhist and other traditions of meditation and conscious control of the brain have been explored such things for centuries, but advances in neuro-cognitive techniques, with bio-feedback and computer mediated communication could allow people to make connections between phenomenology and neurophysiology. It may take centuries more but the movement is towards a situation in which people have and share simulative and functional models not just of their bodies but of their brains as well. It would be what David Bohm (in, *On Dialogue*) called mental proprioception. What kinds of post-sapient consciousness could emerge from this is impossible to say, but one imagines that it would involve an increased control of cognitive processes and neurophysiological process, and a deepened empathic understand. Consciousness and freedom could, in theory, exist at higher level to anything we know now. One should suppose that such developments would have to be for the good. The problem is not that our biological nature cannot be trusted with such power, but because human culture can so easily develop in ways that are entirely unfitting to that nature.

Sapience can aspire to higher levels of emulance, but it could just well be lead to kinds of internal division even more aliened than those we see today. Western dualism and modern technology has lead some people to think of a cognitive future in which we become semi-mechanised individuals, cyborgs with an electronic super-cortex. We are already have a situation in which children substitute private computer games for real-life activities and social inter-action. Email communication approximates to the logocentric model of pure telementation. One can imagine the development of ever greater social fragmentation and an autistic culture in which technology and the economy take people further and further from their pre-sapient, emulative roots.

I believe that our best hope lies in cultivating the way of talking and thinking that 'Bergsonian activism' and I have described here in terms of a dialectical philosophical framework that recognises the ontological primacy of a somacentric thought. The Sages and spiritual teachers were undoubtedly right to tell us that individual consciousness and collective well-being are best served by a participatory attitude. Individuals are most in tune with human nature and culture when the recognise their essentially moral nature as active points-of-view in the activity of an on-going bio-social world. The intentional and epistemological attitude that observes individuals as representors and goal directed agents is of immense practical value. We cannot do without its analytic and problem solving abilities, but if it rules alone the results can be disastrous. The important goals are holistic ones: individual and collective psychosomatic health, social harmony and ecological stability. These are dynamic states of homeostatic systems. Understanding them and participating in them are dynamic and enactive processes.

We cannot predict how Neurocentrism and new electronic media will change cognitive practice, but we will surely understand that process better and increase the chances of a good outcome if we cultivate the somacentric and dialectical philosophy and ways of talking. One powerful way of strengthening our emulative side would be to encourage the use of signing systems as a universal second language. When groups of deaf people who use quiet different systems (e.g. Japanese and American), come together on joint projects they can establish full inter-communication within a matter of days. Although these systems have syntactical structures that are just as complex as ordinarily speech, their practical properties and neurophysiological bases are complete different from spoken language. There is a 'Babel theory' of the evolution of language that emphasises their ability to unite individuals in groups while keeping separate groups apart. Places like the New Guinea highlands and Nepal have a stable system in every local group is distinguished by its own language. A USL would be of obvious value in the EU and the world generally. Schools should be linked by a world-wide web devoted to it. On a cognitive level it could in time have an effect even more profound than literacy. The power of sapience comes from the inter-action of two representational systems. If modern culture has shifted too far in logocentric direction, then the addition of this more immediately somatic language could help restore cognitive balance and promote emulance. Minds with a new symbolic systems with properties lying between those of somatic emulation and verbal symbolism could be expected to become more integrated, both within and between individuals. With only writing as a second kind of language we came to identify ourselves with our own cultural media rather than with one another. The mind was seen as like a text rather than like a person. If my account of sapience is correct, having two distinct kinds of directly inter-personal language, each able to reflect on the other on and represent the other, would be a great benefit to understanding and what Gregory Bateson called 'The Ecology of Mind'.

The Impossibility of a Universal Relativistic Temperature Transformation

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1. Historical Introduction

This paper has its roots in the work by Einstein which, while done about 96 years ago [1], is still controversial. In this reference he reviewed the special theory of relativity 'and the consequences which can be drawn from it'. He had already derived the now standard transformations of pressure and volume for an inertial frame I in uniform motion with respect to the rest frame I_0, assuming a relative velocity w along the common x-axes of the frames. He then noted that the entropy satisfies $S=S_0$ by citing verbatim half a page from a paper by Planck.

$$TdS = dQ = dQ_0 / \gamma = (T_o / \gamma)dS$$

From this he inferred $T = T_o / \gamma$, i.e. a moving body appears relatively cool [2].

In the next 56 years the topic remained pretty well unchanged: no radically new points of view or experiments were proposed, but a new famous summary was published by Pauli [3] when he was only 20 years old. The next development occurred in 1963 when H. Ott [4] proposed

$$T = \gamma T_o$$

as the appropriate transformation, suggesting that a moving body appears relatively hot. The idea that the temperature might be invariant was also suggested, for example in [5].

Regarded superficially, this problem seems to be mathematically simple and there have therefore been many publications about it. For the period 1963 - 1968 alone, more than 60 papers have been noted [6]. That it was not a triviality is also shown by the fact that it represents one of Einstein's worries towards the end of his life, as captured in an Einstein – von Laue correspondence of 1952 – 53 [7].

2. Our Proposal

In an earlier paper by us [8] we proposed a simple solution, namely that a universal and continuous Lorentz transformation of temperature does not exist. This solution is here simplified by omitting some of the mathematical details of the previous paper, as these are not essential.

The only result from special relativity needed to explain our solution of this problem is the following. An observer at rest in some frame of reference S' looks at black body radiation in a frequency interval $d\omega'$. He finds a photon number

$$n'_{\theta'}(\omega',T'_{\theta'})d\omega' d\Omega' = \frac{\omega'^2/c^3}{2\pi^2(e^{\hbar\omega'/kT'_{\theta'}}-1)}d\omega' d\Omega' \qquad (1)$$

which come from a solid angle interval $d\Omega'$. Here θ' is the angle between the direction of motion in S and the direction of observation. This result involves a 'directional' temperature

$$T'_{\theta'}(T,v,\theta') = \frac{T\sqrt{1-v^2/c^2}}{1-(v/c)\cos\theta'} \xrightarrow{\theta'\to 0} T\sqrt{\frac{1+v/c}{1-v/c}}$$

These formulae are effectively already in Pauli's famously relativity article [3]. Of course, a proper "thermal bath" has to be isotropic in its rest system. In the other reference systems it is not isotropic as seen in (1). A moving observer in a heat reservoir can therefore not detect a black-body spectrum, and hence cannot find a parameter which can be identified as 'temperature'. It is this circumstance which makes it impossible to devise a Lorentz transformation of temperature.

Summarizing, we see that the fact that a thermal bath or heat reservoir in S becomes a non-thermal bath in a relatively moving inertial frame is the circumstance which impedes one from devising a Lorentz transformation of temperature.

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Braiding, Quantum Groups and Quantum Mechanics

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Abstract.

Fourier transformations take a function space into another, however, it is not possible to introduce them on arbitrary function spaces since they need some kind of homogeneity on the underlying space. In simple cases, they can only act on functions defined on Lie groups and their homogeneous spaces. Furthermore, the target spaces of Fourier transformations are not of the same kind in the general case, spaces of functions on Lie groups or homogeneous spaces. We see that they are spaces of functions defined on larger structures called *quantum groups*, or more precisely, *Hopf algebras*. Inverse Fourier transformations take functions on these Hopf algebras into functions on other Hopf algebras, which include the original groups. Two spaces whose functions are related by a *Fourier transformation* and its *inverse* are said to be *Fourier dual* to each other. We see that, in the final analysis, this duality is a relationship between Hopf algebras. The simplest non-trivial cases of quantum groups are given by matrices of matrices, that is, matrices whose entries are other matrices. Non-commutative entries mean non-commutative coordinates, a signal of the presence of non-commutative geometry. As a result, spaces formed by these *megamatrices* have non-commutative geometries. The connection of matrices to Fourier transformations is *hidden* by seeing them as sets of rows and columns. We see that this is a convenient convention, and comes from the preferred use of a special basis in matrix space. Other bases, in particular those formed by Weyl's operators, give prominent role to the diagonals. As a result, the matrix product is seen in a new light and many Fourier related properties become prominent. We arrive now to Quantum Mechanics, whose non-commutativity reduces to a simple phase factor. Finally, we shall see that the Yang-Baxter braid equation reflects the preservation of associativity in a space of *megamatrices* and that it has non-trivial solutions in quantum phase space.

0. Introduction

We assume that Quantum Mechanics is the principal mathematical structure of nature. Classical structures will emerge in the process of taking the semi-classical limit and can be viewed as being the quantum structures which are left over in taking this process. Namely, we are pointing out in particular symplectic structures that are basic to the Hamiltonian formalism of Classical Mechanics. Symplectic structures are also basic in Quantum Mechanics. We will see how spaces of matrices can be endowed with a differential geometry and how this differential structure leads to a quantum symplectic space, from which the classical phase space inherits its makeup.

Consider the classical phase space E^{2n} of some simple mechanical system with generalized coordinates $q=(q^1,q^2,...,q^n)$ and conjugate momenta $p=(p^1,p^2,...,p^n)$. A mechanical system is called *simple* when its phase space is topologically trivial, as is the Euclidean $2n-$dimensional space, E^{2n}. This is the simplest example of phase space, and can be used in cases in which the configuration space is a vector space. The dynamic quantities are functions $F(q,p),G(q,p),...,$ defined on E^{2n}, and constitute an associative algebra with the usual pointwise product, $(F\bullet G)(x)= F(x)G(x)$. Given any associative algebra, leads to a Lie algebra with the commutator as operation. Note that, because of its commutativity, the classical Lie algebra of dynamic functions, coming from the pointwise product, is trivial. In Classical Mechanics, we use the non-commutative Lie algebra, defined by the Poisson bracket, to obtain physical correspondence. We note that this setup is not standard, from the mathematical point of view, since natural brackets are precisely commutators in associative algebras and the Poisson bracket does not emerge in such a way. However, we know that the basic geometric background of the Hamiltonian, that is, the symplectic structure, lies behind the Poisson bracket, giving to its algebra a deep content.

In Quantum Mechanics, the product in the algebra of dynamic functions, that is, the operators, is non-commutative and the resulting commutator Lie algebra plays a fundamental role. Moreover, despite the insight of Dirac, who calls commutators *quantum derivations*, the non-commutativity of Quantum Mechanics is more algebraic than geometric. The difference lies in the lack of geometric structures in the algebra of operators, such as differential forms, connections, metrics, that is, in the absence of a *differential* geometry.

The recent advances in non-commutative geometry have resulted in research to make explicit the quantum symplectic structure. For this research, see for instance, Hiley's *shadow spaces* and Manin's *quantum space*, which are attempts towards this explication.

We can briefly sketch the principle route into non-commutative geometry. The complex-valued functions defined on a differentiable manifold M constitute an associative algebra $C(M)$ with the pointwise product. $C(M)$ contains all the information about M, most importantly, everything in regards to its topology and differentiable structure. We see that, for instance, that the differentiable structure, is expressed in terms of vector fields, which are derivatives acting on $C(M)$.

Note that on usual manifolds, like point manifolds, as in the phase space above, this algebra is commutative. The route into non-commutative geometry is done by working out everything on $C(M)$ *as if* the product were non-commutative, while keeping associativity. In the case of E^{2n}, this would mean that $F \bullet G$ is transformed into some non-abelian product $F \text{o} G$, with a new operation "o". A manifold is basically a space on which coordinates, an ordered set of real, commutative point functions into the real line, can be defined. When we take $C(M)$ to be non-commutative, then also the coordinates, like the other functions, become non-commutative. We see that the consequent geometry of M will as a result *emerge* to be non-commutative.

We know that differentials are got at in a simple fashion through the commutator. The associativity of the product $F \text{o} G$ implies the Jacobi identity, that is, the basic character of Lie algebra, and this results in the commutator $[F,G] = F \text{o} G - G \text{o} F$, with fixed F, to be a derivative with respect to F. We can see that the Jacobi identity

$$[F,[G,H]] = [[F,G],H] + [G,[F,H]]$$

can be viewed precisely as the Leibnitz rule for the *product* defined by the operation $[,]$. The product "o" that is related to quantisation is the *star-product* and therefore the quantum-classical relationship has more clarity in the Weyl-Wigner picture of Quantum Mechanics. We see that in this picture, quantum operators are obtained from classical dynamic functions through the Weyl prescription. The quantum formalism is formulated in terms of Wigner functions which are *c-number* functions, that we note, multiply each other through the star product and not the pointwise product. This method is in one-to-one correspondence with the operator point of view in which, the matrix and not the function algebra operates.

We need to primarily establish a general notion for the Weyl prescription, by casting the expressions of quantum operators in a certain convenient basis of unitary operators. As a result of this, we look at the differential geometry of the space of quantum operators, and in particular, concentrate on its symplectic structure. This convenient unitary basis, was formulated by Schwinger to present Weyl's interpretation of the Heisenberg group. When we use these unitary basis, operators appear as the quantum counterparts of the classical dynamic quantities, whenever the latter exist. In other words, this basis provides a direct realisation of the correspondence principle. The Weyl-Wigner transformations will more precisely clarify our understanding of the limit between classical differential geometric concepts, of which the symplectic form is an example, and their quantum counterparts. The operators belonging to Schwinger's basis are labeled by a double-integer index whose values span a lattice torus, which we call *Quantum Phase Space*. The coefficients in the operator expansions are functions defined on Quantum Phase Space and, moreover, the main characteristic of this space are understood for the simplest cases.

We note that, two mathematical procedures will be crucial in setting up a differential geometry on the space of quantum operators, that is, on the space of Wigner functions. The first of these procedures will be the Weyl-Wigner transformations. After introducing the necessary notation, we find several main properties of Schwinger's basis. Operator expansions in that basis are related to Weyl-Wigner transformations. The notions of twisted convolutions and products are normally introduced with the aim of making clear the interpretation of the Weyl prescription as a Fourier operator expansion. After this short sketch, we will briefly discuss quantum groups, as well as their relationship to braiding and to the Yang-Baxter equations. They appear as algebras of matrices whose entries are themselves matrices. The second of these procedures will be the differential geometry of matrix space.

The mathematical structure that corresponds to the quantum commutator, in the context of a c-numbers, is the *Moyal* and *not* the Poisson bracket. We find that this bracket is a signature of matrix differential geometry. We can also show, the isomorphism that exists between this operator approach and the formalism employing the Wigner functions and the corresponding star product.

We will show below, braiding behind the above formalism and work out non-trivial solutions for the braid and Yang-Baxter equations. The existence of these solutions is related to a property that pertains to the quantum situation, since, in contrast to its classical counterpart, that gives rise to the Poisson structure, the inverse quantum symplectic matrix is not an antisymmetric matrix.

1.Weyl's Method

$N \times N$ matrix is a circulant if it has the *cyclic* form

$$A = \begin{matrix} a_N & a_{N-1} & a_{N-2} & . & . & . & a_1 \\ a_1 & a_N & a_{N-1} & . & . & . & a_2 \\ a_2 & a_1 & a_N & . & . & . & a_3 \\ . & . & . & . & . & . & . \\ . & . & . & . & . & . & . \\ a_{N-1} & a_{N-2} & a_{N-3} & . & . & . & a_N \end{matrix}$$

where the a_k's are complex numbers.

Consider the circulant corresponding to $a_j = \delta_{1j}$. It will be a unitary matrix U with entries

$$U_{rs} = \delta_{r,s+1},$$

$$U = \begin{matrix} 0 & 0 & 0 & \dots & \dots & \dots & \dots & \dots & 1 \\ 1 & 0 & 0 & \dots & \dots & \dots & \dots & \dots & 0 \\ 0 & 1 & 0 & \dots & \dots & \dots & \dots & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & \dots & \dots & \dots & 1 & 0 \end{matrix}$$

U is a displacement operator, or a shift operator on any matrix A, $(UA)_{ij} = A_{i-1,j}$.

A is a polynomial in U:

$$A = a_1U^1 + a_2U^2 + a_3U^3 + ... + a_NU^N = \sum_{n=1}^{N} a_nU^n,$$

$$V = \begin{matrix} \omega & 0 & 0 & 0 & ... & ... & ... & ... & 0 \\ 0 & \omega^2 & 0 & ... & ... & ... & ... & ... & 0 \\ 0 & 0 & \omega^3 & 0 & ... & ... & ... & ... & 0 \\ 0 & 0 & 0 & \omega^4 & 0 & ... & ... & ... & 0 \\ ... & ... & ... & ... & ... & ... & ... & ... & ... \\ 0 & 0 & 0 & 0 & ... & ... & ... & 0 & \omega^N \end{matrix}$$

Weyl's realisation of the Heisenberg group are built up in terms of two conjugate unitary operators U and V satisfying the basic relation

$$V\ U\ = \omega\ U\ V,$$

where ω is a complex number.

It follows immediately that

$$V^m\ U^n\ =\ \omega^{nm}\ U^n\ V^m$$

We see that circulants are matrices that underly elementary Fourier analysis. We see also that there is a basic matrix U, which is the cornerstone of all the theory and functions as a shift operator. From the above equation, we note that every circulant can be expressed as a sum of weighted shifts. Performing a Fourier transform is *equivalent* to diagonalising a circulant matrix.

We ask ourselves, do such circulant matrices have some deeper role in a more *general Fourier analysis*, that is to say, in discrete abelian harmonic analysis? The answer to this question we find is in the affirmative and is given in Weyl's approach to Quantum Mechanics on phase space. It emerges that the above pair of matrices U,V play a crucial role in our understanding, since we see them as the building blocks of general operations.

The equation $V^m\ U^n\ =\ \omega^{nm}\ U^n\ V^m$ is fundamental for *discrete* Quantum Mechanics, as it gives a representation of the discrete Heisenberg group. The passage into the continuum case, at least in the simple q,p case, is well known.

Furthermore, we note the interesting fact that V, acts as a shift operator on the U eigenkets.

The operator V is defined by

$$V|u_k\rangle = |u_{k-1}\rangle.$$

so that

$$V^n = \sum_{k=1}^{N} |u_{k-1}\rangle\langle u_k|$$

Also, $V^N = 1$. V has eigenvalues $v_k = \omega^k$ and its eigenkets $|v_k\rangle$, are such that

$$V|v_k\rangle = v_k|v_k\rangle$$

Also U can be defined by

$$U|v_k\rangle = |v_{k+1}\rangle$$

therefore,

$$U^m = \sum_{k=1}^{N} |v_{k+m}\rangle\langle v_k|$$

The cyclic condition imposed on the kets, implies $U^N = 1$. U's eigenvalues $u_k = \exp[i(2\pi/N)k] = \omega^k$ correspond to eigenkets $|u_k\rangle$, with $|u_{k+N}\rangle = |u_k\rangle$ and fixed by

$$U|u_k\rangle = u_k|u_k\rangle.$$

The duality that exists between U and V, gives rise to an insight, into a deep and complex background structure, standing behind Quantum Mechanics, at least for simple systems. We have seen that this structure is better seen in the light of discrete, finite matrix version Discrete Quantum Mechanics as initiated by Weyl.

The dynamical quantities of Classical Mechanics are commutative functions on phase space and are considered as classical number or *c-number* , whilst the dynamical quantities of Quantum Mechanics, on the other hand, are *operators*. The Weyl method is a Fourier-based procedure to avoid the ambiguities arising in the translation of classical into quantum quantities. It provides a quantum operator in correspondence with a certain classical quantity, called the *Wigner density* and it is possible to represent the operators as functions on phase space, with the *Wigner functions*. These are Fourier transforms of the Wigner densities.

The Weyl-Wigner transformations, gives rise to a two way relationship, between the Wigner functions and the quantum operators. The Wigner functions multiply each other, through a special operation, called the *star product*, giving rise to their algebra differing from that of usual functions. We can say that the dynamical quantities, are the same in Classical and in Quantun Mechanics, with the exception, that the product defining their algebra are different. We see that in working with Wigner functions, we are in actual fact, doing matrix algebra with c-number functions.

Quantisation can be seen as either a translation from classical dynamic quantities into operators, or, as a deformation in the algebra of the coefficients, or finally, as a deformation in the algebra of the basic functions, from pointwise product to twisted product.

We will see that this duality transposes to the Hamiltonian structure.

2. Braiding and Quantum Groups

General spaces that allow for a two way movement of Fourier transformations, are in actual fact, *quantum groups*, or more precisely, *Hopf algebras*. The remarkable fact is that these algebras show a natural tendency to have a *braided* underlying structure.

2.1. Emergence of Hopf algebras

We note that, one of the most incredible and fascinating developments, taking place in mathematics and physics, in recent times all the way to the present day, has been the discovery of the relation of Hopf algebras, that is, bialgebras and quantum groups, to the Yang-Baxter equation and, as a consequence, to braid groups.

We see that the most elementary example of bialgebras in Physics, appears in the sum of two angular momenta, written as $J = J_1 + J_2$. In actual fact, this operator acts on the direct sum kets $|Y\rangle = |Y_1\rangle \oplus |Y_2\rangle$, which implies that it should actually be formulated as $J = J_1 \otimes I + I \otimes J_2$. As a consequence, an object of this form, belongs to a Hopf algebra.

We note that there are several paths in which we can approach Hopf algebras apart from the purely algebraic path. In physics, the most common method involves group deformations. Another, useful and intuitive method of approach is through free calculus, as in knot theory. What we want to emphasise here, is that the entries of the usual matrices appearing in *classical* subjects, become non-commutative.

We will now sketch briefly the process that take place here. The entries t^i_j of usual matrices consist of real or complex numbers and consequently commute with each other, for instance, $t^i_j t^m_n = t^m_n t^i_j$. Matrices can constitute Lie groups, that is, smooth manifolds. To each group element corresponds a matrix. Each point on a manifold, each matrix in the case, is entitled to have coordinates, a set of real numbers describing it, hence, each matrix will have its coordinates, which are just the entries t^m_n . If now we take the entries of the matrices as non-commutative, the new *coordinates* will be of a new kind. Non-commutative geometry comes to the fore and is represented in the form

$$R^{rs}_{im} \; t^i_j \; t^m_n \; = \; R^{pq}_{jn} \; t^r_p \; t^s_q$$

where the R^{ij}_{kl}'s are complex coefficients.

Imposing restrictions, in the form of constraints on these coefficients, we will ensure a minimum level of compatibility for the new algebraic structure emerging. Significantly, the imposition of *associativity* leads to the *Yang-Baxter equation*, whose form and relationship to *braid groups* will be shown. The new structure that emerges is a *Hopf algebra.* Hence, we see that the term *quantum groups* is a labeling for sets of matrices whose entries are themselves non-commutative. We see that they are *not* groups at all, but structures genaralising them. We will find *megamatrices*, that is. matrices whose entries are themselves matrices.

We will now mention briefly another approach that is part of the overall picture. It stresses the role of *Fourier transformations* and it deals with *harmonic analysis* on groups. This approach takes as starting algebras the spaces of functions on groups. We know that the classical Fourier transformations, for instance, on a line or on Euclidean 3-space, establishes a duality between the space of functions on the original space and the space on which the Fourier transforms are defined. The latter is the Fourier dual of the original space. The original space is actually a translation group T and its dual $\hat{T}$ is the space of, that is, equivalence classes of, unitary irreducible representations of T. In the classical, abelian case, the dual $\hat{T}$ is another group. It happens that, when the original group G is locally compact commutative and compact, or discrete, the dual set $\hat{G}$ is a locally compact commutative group, which is furthermore respectively discrete or compact, in this order. We know this as the *Pontryagin duality*. The quantisation in a box, that implies q-space compactness, leads to discrete momenta. We note some classic examples of Fourier-dual spaces like the group R of the real numbers, dual to itself, continuum q, idem p; the circle S^1 and the group Z of integers, dual to each other, angle ϕ, angular momentum component J_z any integer; and the cyclic group Z_N, that is also self-dual. We remain in the same Z_N while going to and fro by Fourier transformations.

We see a completely new scenario when the original group is non-commutative, in that the corresponding dual space is *no* more a group. The case in which G is compact is well understood. The dual $\hat{G}$ is then a category, that of the finite dimensional representations of G, that is, a category of vector spaces, more precisely, modules. We note that this new duality, between a *group* and a *category*, is known as the *Tanaka-Krein duality*. We find that our understanding and the special simplicity of abelian groups that arises is because their unitary irreducible representations have dimension one and the tensor product of two such representations, gives another one-dimensional representation. Each such representation may be considered simply as a complex function f,

$$f: G \to C, \quad g \to f(g), \text{ with } f(g_1 g_2) = f(g_1) f(g_2).$$

Note that, characters are representations of this kind, $f: G \to S^1$. *Tensor product* gets reduced to the simple *pointwise* product of functions whilst the set of inequivalent unitary irreducible representations is itself a group. Here we have a property which does *not* generalize to the non-commutative case.

We wonder if it is somehow possible to construct and enlarge the notion of a group to create a new more general object, so to ensure that its dual is an object of the same kind? We find that the answer has been found in the case of *finite groups*, where the more general objects required are precisely *Hopf algebras*. We see that *quantum groups* are those *generalizations* of groups that admit for the classic notion of Fourier duality.

Through a mathematical device of absorbing the non-commutativity in the coefficients, we are ready to perform the Fourier transformation starting from the commutative group $Z_N \otimes Z_N$. The Heisenberg group is *non-abelian*, but with the twisted-convolution prescription we can avoid this problem. The algebra of functions on $Z_N \otimes Z_N$ with convolution given by "$*_c$" is Fourier transformed into the algebra of functions on $Z_N \otimes Z_N$ with multiplication "o" consequently it becomes possible to work, as if the domain space were, in both cases, commutative groups. As a result, the method we have used is, actually a device that allows use to use the simpler formalism of Pontryagrin duality and avoid the Tanaka-Krein duality. In other words, it allows us to avoid the explicit use of Hopf algebras.

3. Background of Quantum Phase Space

The non-trivial solutions of the Yang-Baxter equations are *equivalent* to the relations that define braid groups. We will commence with some general observations on braid groups. Afterwards, we will show how the Yang-Baxter equations are related to braid groups and to associativity. At the end, these solutions are shown. We will see that they are quantum in nature, that is, they are related to the non-antisymmetric character of the inverse quantum symplectic form, which as a result, implies the existence of some kind of weaving pattern in the background of quantum phase space.

3.1. The Braid Groups

We note that braid groups, which *generalise* the permutation groups, describe real braids. Let us look at an exchange of two particles. First, let us take a plane projection of the exchange. Imagine that a string links the initial to the final position of each particle. If we exchange two of them, there are two possible ways of going around each other. Either, the first string passes over or below the second. Demonstrations, with two strands, will show that the two ways are *inverse* to each other, in other words, performing the exchange one way, then the other, brings back the initial set up. A member of the group represents one of them, whilst, its inverse represents the other. The two string case is described by the smallest non-trivial braid group, which we denote by B_2, and has only one generator σ_1. There are only two ways of braiding once, that correspond to σ_1 and σ_1^{-1}. However, even in this simplest of all cases, the group is infinite. We can twine one of the strands around the other n times in the same way, which corresponds to σ_1^n or σ_1^{-n}, depending on the way we have chosen, for any value of n. This implies that the group has infinite members. The identity, describes no braiding and is the only element of the one string group B_1. The *generator* means that any other group element can be obtained as a power of σ_1. If we add a 3rd particle and consider the case of 3 strands, then the corresponding group B_3 will have two generators σ_1 and σ_2. We see that the second generator σ_2 describes the exchange of the second and the third particles, in a similar manner as σ_1 describes the exchange of the first and the second. Note that an exchange of the first and the third particles is not independent since it can be obtained by a composition of the previous generators. Any other group element is some product of powers of the two generators. Case becomes more interesting when 4 particles are present. We have a third generator σ_3, that is responsible for exchanging once the third and the fourth particles. If we perform with 4 strands, we see a few remarkable facts, that can be best described by algebra as

$$\sigma_1\sigma_2\sigma_1 = \sigma_2\sigma_1\sigma_2 \quad \text{and} \quad \sigma_2\sigma_3\sigma_2 = \sigma_3\sigma_2\sigma_3, \quad \text{but } \sigma_1\sigma_3 = \sigma_3\sigma_1.$$

Only nearest-neighbours have non-trivial relationships.

These results can be extended to any number of strands, and have been formalised by Artin. For the N-strand braid group B_N, there exists a basis $\{\sigma_j\}$ of $(N-1)$ generators obeying the relations

$$\sigma_i\sigma_{i+1}\sigma_i = \sigma_{i+1}\sigma_i\sigma_{i+1} \text{ for } i = 1,2,\dots,N-1;$$

and, $\sigma_i\sigma_j = \sigma_j\sigma_i$ for $|i-j| \geq 2$.

The *symmetric group* S_N is the special case, when all the σ_i's satisfy the additional conditions

$$(\sigma_i)^2 = I.$$

This would mean that each generator is equal to its own inverse, in other words, the difference between the two ways of curling two strands is not taken into account.

Braid groups are effectively present whenever some braiding process is at work. Braid groups classify weaving patterns, conversely, finding solutions of the above equations, signals the presence of some kind of weaving. We shall now show that the Yang-Baxter equation is *equivalent* to the above braid equations and that they imply associativity. Finally, we shall exhibit a non-trivial solution involving the structure of quantum phase space.

3.2. The Yang-Baxter equation

We have seen matrices whose entries are themselves $N \times N$ matrices. The home of such *megamatrices* are direct product spaces.

In direct product index notation, the product $A \otimes B$ of two matrices has entries

$$\langle ij|A \otimes B|mn\rangle = \langle i|A|m\rangle\langle j|B|n\rangle,$$

The direct product of three matrices will have elements

$$\langle ijk|A \otimes B \otimes C|mnr\rangle = \langle i|A|m\rangle\langle j|B|n\rangle\langle k|C|r\rangle,$$

and so on.

If $R = A \otimes B$, and E is the $N \times N$ *identity matrix*, then

$$R_{12} = A \otimes B \otimes E,\ R_{13} = A \otimes E \otimes B,\ R_{23} = E \otimes A \otimes B, \text{ etc.}$$

Write matrix elements as

$$\langle ij|R|mn\rangle = R^{ij}_{mn}$$

and,

$$\langle ijr|R_{12}|mns\rangle = \langle ijr|R\otimes E|mns\rangle = \delta^r_s R^{ij}_{mn}.$$

This notation applies to the case in which the R are not next to each other, as,

$$\langle ijr|R_{13}|mns\rangle = \delta^j_n R^{ir}_{ms}.$$

Let us find a direct product representation of the braid equations. We need to find braid group generators σ_1 and σ_2 in the forms $\sigma_1 = B_{12}$ and $\sigma_2 = B_{23}$, where B is some direct product. In this case,

$$\langle kji|\sigma_1\sigma_2\sigma_1|mnr\rangle = \langle kji|B_{12}|abs\rangle\langle abs|B_{23}|ucx\rangle\langle ucx|B_{12}|mnr\rangle$$

$$= B^{kj}_{ab}B^{bi}_{cr}B^{ac}_{mn}\ ;$$

whilst,

$$\langle kji|\sigma_2\sigma_1\sigma_2|mnr\rangle = \langle kji|B_{23}|abs\rangle\langle abs|B_{12}|ucd\rangle\langle ucd|B_{23}|mnr\rangle$$

$$= B^{ji}_{bd}B^{kb}_{mc}B^{cd}_{nr}.$$

The braid equation is given by

$$\sigma_1\sigma_2\sigma_1 = \sigma_2\sigma_1\sigma_2,$$

or,

$$B_{12}B_{23}B_{12} = B_{23}B_{12}B_{23},$$

which becomes

$$B^{kj}_{ab}B^{bi}_{cr}B^{ac}_{mn} = B^{ji}_{ca}B^{kc}_{mb}B^{ba}_{nr}.$$

If B satisfies this equation, the matrix $R = PB$, defined by the index permutation P and which is fixed by $R^{ij}_{mn} = B^{ji}_{mn}$, will satisfy

$$R^{jk}_{ab}R^{ib}_{cr}R^{ca}_{mn} = R^{ij}_{ca}R^{ck}_{mb}R^{ab}_{nr}.$$

Here we have the Yang-Baxter equation, which in short hand, is given by

$$R_{23}\ R_{13}\ R_{12}\ =\ R_{12}\ R_{13}\ R_{23}.$$

3.3. Quantum emboidery

We note that there exists a direct realisation of the above relations in terms of a metric. We find that, $R^{ij}_{mn} = \left(\delta^i_n\delta^j_m + ak^{ij}k_{mn}\right)$ is a solution, for constant a in some range. Here we have a trivial case. A quantum group can be constructed in a similar way to a classical group, as a Hopf algebra of megamatrix transformations preserving a given bilinear form, in other words, the R matrix is related to the preservation of the metric k_{mn}.

We shall, however, show the presence of another kind of solution.

We know that, $V^nU^m = \omega^{mn}U^mV^n$. and that it is invariant under the simultaneous changes $U \to V, V \to U^{-1}$, $m \to n$, $n \to -m$. This symmetry leaves also invariant the *Schwinger operators*

$$S_{(m,n)} = e^{i\frac{\pi}{N}mn}U^mV^n = \omega^{\frac{mn}{2}}U^mV^n,$$

which are unitary. The set $\left\{S_{(m,n)}\right\}$ constitutes a complete orthonormal matrix basis. Using a short hand notation in terms of 2-vectors, with $m = (m_1, m_2)$ we have the basis members as

$$S_m = e^{i(\pi/N)m_1m_2}U^{m_1}V^{m_2}.$$

The product satisfies

$$S_rS_m = e^{2i\alpha_2(m,r)}S_mS_r = e^{i\alpha_2(m,r)}S_{m+r}$$

with

$$\alpha_2(m,r) = \frac{\pi}{N}(m_1r_2 - m_2r_1)$$

The solution is non-trivial because of the multiplicity hidden in the above equation. When we write $S_rS_m = e^{2i\alpha_2(m,r)}S_mS_r$, we do not actually see the whole picture since this equation gives only the commutation condition for two fixed matrices S_r and S_m. We find that S_rS_m is actually related by phases to any other product of pairs of basic matrices for which the sum of indices is $r+m$.

Let us define the set $B = \left\{B^{ij}_{kl}\right\}$ of commutation coefficients between the members of the algebra basis by the following,

$$S^i S^j = \sum_{mn} B^{ij}_{mn} S^m S^n$$

Then the general solution is

$$B^{ij}_{mn} = \delta^{i+j}_{m+n} e^{i[\alpha_2(m,n)-\alpha_2(i,j)]}$$

We see that this satisfies the above braid equation.

We may use another procedure which shows clearly how representations of the braid groups come up very simply from any *associative* algebra.

We shall here follow a simple method leading directly to the braid equations for the Schwinger basis, but it will be clear that the procedure is valid for more general cases. The only condition will be that all indices are double.

Let us impose coherence of the commutation coefficients with associativity and compare two different ways of bracketing the S^i's.

1st set of bracketing, gives

$$S^k S^j S^i = \left(S^k S^j\right) S^i = B^{kj}_{ab} S^a S^b S^i = B^{kj}_{ab} S^a \left(S^b S^i\right)$$
$$= B^{kj}_{ab} B^{bi}_{cr} S^a S^c S^r = B^{kj}_{ab} B^{bi}_{cr} \left(S^a S^c\right) S^r = B^{kj}_{ab} B^{bi}_{cr} B^{ac}_{mn} S^m S^n S^r$$

2nd set of bracketing, gives

$$S^k S^j S^i = S^k \left(S^j S^i\right) = B^{ji}_{ca} S^k S^c S^a = B^{ji}_{ca} \left(S^k S^c\right) S^a$$
$$= B^{ji}_{ca} B^{kc}_{mb} S^m S^b S^a = B^{ji}_{ca} B^{kc}_{mb} S^m \left(S^b S^a\right) = B^{ji}_{ca} B^{kc}_{mb} B^{ba}_{nr} S^m S^n S^r$$

Comparing,

$$B^{kj}_{ab} B^{bi}_{cr} B^{ac}_{mn} = B^{ji}_{ca} B^{kc}_{mb} B^{ba}_{nr}$$

which is the braid equation.

We can see the role of B as a sort of grand matrix, representing the exchange of matrices, with each S related to one string of the braid.

The subindices in the braid equation obtained, $B_{12}B_{23}B_{12} = B_{23}B_{12}B_{23}$, reflect the order used in the bracketing. We see, to obtain the left-hand side, take first the 1st and the 2nd of the S's, giving B_{12}, then the 2nd and the 3rd giving B_{23} and finally, again 1st and the 2nd of the S's, giving finally B_{12}.

The same applies to the right-hand side. We can see that the braid equation is a presentation of the 3rd braid group B_3, corresponding to only three strands. For higher number of strands, we may consider higher order direct products and obtain elements of higher order braid groups. Adjacent bracketings give expressions of the type $\sigma_i\sigma_{i+1}\sigma_i = \sigma_{i+1}\sigma_i\sigma_{i+1}$ above. Noncontiguous bracketings will give exchanges in which the S^j's ignore each other, and hence expressions of the type $\sigma_k\sigma_i = \sigma_i\sigma_k$ for $|i-k| \geq 2$. S^j's that are not affected correspond to non-exchanging strands.

Examining the matrix elements of $B^{ij}_{mn} = \delta^{i+j}_{m+n} e^{i[\alpha_2(m,n)-\alpha_2(i,j)]}$ shows that in the diagonal, $m = i$ and $n = j$, we have the identity, and that in the crossed case $m = j$, $n = i$, with

$$B^{ij}_{ji} = \delta^j_j \delta^i_i e^{2i\alpha_2(j,i)} = \delta^j_j \delta^i_i e^{2i\alpha_2(i,j)},$$

we have correspondence to $S_i S_m = e^{2i\alpha_2(m,i)} S_m S_i$. This case is *trivial*, since $B_2 = I$ and it is not the braid but the symmetric group that is involved. The other cases are *non-trivial* and result from the above fact that the product $S^i S^j$ is, up the compensating phases of $B^{ij}_{mn} = \delta^{i+j}_{m+n} e^{i[\alpha_2(m,n)-\alpha_2(i,j)]}$, equal to any other product $S^m S^n$, with the condition that $i + j = m + n$.

The existence of non-trivial matrices B is directly related to the fact that r^{ij} is not simply antisymmetric. We have from before,

$$S^i S^j = \sum_{m,n} B^{ij}_{mn} S^m S^n$$

equivalent to,

$$r^{ji} = \sum_{m,n} B^{ij}_{mn} r^{nm}.$$

If the matrix B were a negative diagonal megamatrix, then $B^2 = I$ and only the symmetric group would be present.

Braiding as shown above is consequently a specifically quantum effect.

We have seen that the Yang-Baxter equation itself comes from associativity.

Quantum Mechanics differs from Classical Mechanics by the *non-commutativity* of its dynamical quantities giving rise to two algebras that are different. However, the difference is not as complete as it could be. It is not important to stress the associativity of an abelian algebra, nevertheless, it remains a fundamental algebraic property that is present in classical algebra.

Quantum algebra *breaks* commutativity while *preserving* associativity.

The braiding presented is a result of this preservation of associativity.

Quantisation appears directly as a deformation of the algebra of classical observables, that is, as a manifestation of the non-commutative character of the geometry of physical observables. The most striking aspect of the formalism above is precisely this directness, an obvious consequence of working at the same time with Weyl-Wigner transformations and with matrices. The expression of each operator is automatically a discrete and finite version of the Weyl prescription relating classical dynamic quantities to their quantum representatives through their Fourier components.

We see that Quantum Mechanics is a non-commutative symplectic geometry. Structures of Classical Mechanics emerge, in principle, from quantum mechanical structures, however, something may be lost or distorted in the limiting process. The classical symplectic structure results in the long run from the cocycle α_2 and is, as a consequence, a relic signal of the basic non-commutative character of quantum mechanical geometry. The mathematically weird Poisson bracket appears so because it did not come from an associative product. However, it comes out now as a limiting case of the quantum bracket, hence as part of the quantum history of Classical Mechanics.

The existence of non-trivial solutions of the Yang-Baxter equation signals the possible presence of braiding in Quantum Phase Space and behind the scene which classical phase space, as its limit, represents.

Conclusion

The paper has discussed the relationship between braiding and the Yang-Baxter equation, on one hand, and its connection to the symplectic structure, that is present in both Quantum Mechanics and Classical Mechanics, as shown through the use of the Weyl-Wigner method. What emerges in the overall picture is the importance of discreteness, non-commutativity and associativity, within the present theoretical framework of theoretical physics. We have left out of the discussion, the connection of thermodynamics, as used through the thermo quantum field theory, with the above strands of research. Further work needs to be done to establish Quantum Mechanics in the framework of braided categories.

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Gravitation in the fractal $D = 2$ inertial universe: New phenomenology in spiral discs and a theoretical basis for MOND

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Abstract

A particular interpretation of Mach's Principle led us to consider if it was possible to have a globally inertial universe that was irreducibly associated with a non-trivial global matter distribution, Roscoe [2002a]. This question received a positive answer, subject to the condition that the global matter distribution is necessarily fractal, $D = 2$. The purpose of the present paper is to show how general gravitational processes arise in this universe. We begin by showing how classical Newtonian gravitational processes arise from point-source perturbations of this $D = 2$ inertial background. We are then able to use the insights gained from this initial analysis to arrive at a general theory for arbitrary material distributions. We illustrate the process by using it to model an idealized spiral galaxy. One particular subclass of solutions, corresponding to *logarithmic* spirals, has already been extensively tested (Roscoe [1999a], [2002b]), and shown to resolve dynamical data over large samples of optical rotation curves (ORCs) with a very high degree of statistical precision.

Whilst the primary purpose of the data analysis of Roscoe [1999a] was to test the predictions of the logarithmic spiral theory, it led directly to the discovery of a major new phenomenology in spiral discs - that of *discrete dynamical classes* - initially reported in Roscoe [1999b] and comprehensively confirmed in Roscoe [2002b] over four large independent samples of ORCs. In this paper, we analyse the theory more comprehensively, and show how the discrete dynamical classes phenomenology has a ready explanation in terms of an algebraic consistency condition which must necessarily be satisfied.

Of equal significance, we apply the theory with complete success to the detailed modelling of a sample of eight Low Surface Brightness spirals (LSBs) which, hitherto, have been successfully modelled only by the MOND algorithm (Modified Newtonian Dynamics, Milgrom [1983a], [1983b], [1983c]). The CDM models have failed comprehensively when applied to LSBs. We are able to conclude that the essence of the MOND algorithm must be contained within the presented theory.

Mach – Spiral Galaxy – Rotation Curve – MOND – Gravitation – Vacuum physics

1 Introduction

1.1 Review of Preliminary Work:

General Relativity can be considered as a theory of what happens to predetermined clocks and rods in the presence of material systems (material here is understood in its widest sense). By contrast, the theory being discussed here can be considered as a primitive, but fundamental, theory of how clocks and rods arise in the first place within material systems.

The preliminary work (Roscoe [2002a]) was driven by the idea that it is impossible to conceive of physical (metric) space in the *absence* of material systems, and that it is similarly impossible to conceive of physical time in the absence of *process* within these material systems. Following upon this, we took the point of view that any fundamental theory of space & time must then necessarily have the property that, within it, it is impossible to talk about metrical space & physical time in the absence of any material system. In effect, this is the view that notions of material systems are logically prior to notions of clocks & rods, and that these latter notions are somehow *projected* out of prior relations which exists between the individual elements which make up the former.

We began by noting that the most simple form of space & time we can conceive is that of inertial space & time - that is, a space & time within which the relations between clocks and rods are fixed. And then, keeping to the spirit of the basic idea, we posed the questions:

- *Can a globally inertial space & time be associated with a non-trivial global matter distribution ?*
- *And, if so, what are the general properties of this distribution ?*

These questions were addressed within the context of an extremely simple model universe populated by particles which possessed only the property of *enumerability*, and within which there were no predetermined ideas of clocks & rods. It was required that these concepts should emerge from the general analysis. To simplify the initial development, it was originally assumed that the model universe was stationary. It transpired that this assumption was equivalent to choosing a one-clock quasi-classical model, whilst relaxing the assumption was equivalent to choosing a two-clock relativistic model. The work of this paper is based upon the one-clock quasi-classical development.

The original questions were then answered as follows: a globally inertial space & time can be associated with a non-trivial matter distribution, and this distribution is necessarily fractal $D = 2$. However, it transpires that the particles in this matter distribution cannot reasonably be identified with ordinary matter since (in crude terms) the particles all appear to be in states of randomly directed uniform motion, but with identical speeds of magnitude v_0. In other words, the distribution has some of the attributes of a quasi-photon gas and, for this reason, we interpreted it as a rudimentary model of a material vacuum. A closer investigation then shows that, in fact, whilst v_0 has the dimensions of speed, it is more properly interpreted as a conversion factor between length scales and temporal scales - in this sense, v_0 is more like Bondi's interpretation of c. It follows from this that the material vacuum itself appears to have the role of arbitrating between length scales and temporal scales.

We then noted that, if ordinary matter could, somehow, condense out of this $D = 2$ material vacuum then we would have a universe of ordinary matter which is in close accord with what is actually observed in galaxy counts out to medium distances - that is, on these medium scales, the material distribution in the form of galaxies has vanishingly small accelerations and is distributed in a quasi-fractal form with $D \approx 2$.

1.2 The Emergence of Gravitation

Once one has a particular conception of 'inertial space & time' (for example, that of Newtonian theory, or that of Einstein's special relativity), a theory of gravitation effectively follows as a perturbation of that particular inertial space & time.

For example, in the Newtonian context, such perturbations are interpreted as the introduction of *gravitational forces* into a previously force-free environment whilst, in the Einsteinian case, they are interpreted as the introduction of *curvature* into a flat spacetime manifold. In the present case, they are literally perturbations of the $D = 2$ distribution of material in the rudimentary model vacuum. Since the $D = 2$ distribution is irreducibly associated with clocks & rods in a fixed relationship with each other, it follows that any perturbation from $D = 2$ will necessarily entail distortions of the clocks & rods relationship, and will therefore give rise to what are conventionally called gravitational processes.

1.3 Qualitative Gravitational Mechanisms

Newton introduced the idea of ***force*** into discourse about the world through his mechanics, and his Law of Gravitation can be viewed as a recipe quantifying the amount of ***force*** acting between two massive bodies. However, Newton famously said that he had no idea what constituted the fundamental essence of this ***gravitational force*** and today, the whole theory is simply accepted as an enormously successful and useful means of describing the phenomenology.

Exactly the same can be said of General Relativity: that is, gravitational processes are said to arise in this theory when the flat spacetime manifold becomes curved by the presence of mass. The field equations can be viewed simply as the recipe which quantifies the amount of curvature created by a given distribution of mass. But, as with Newtonian theory, the ***mechanism*** by which this is achieved is absent. So, at one level, General Relativity can be considered as simply an alternative means of describing the phenomenology - albeit one which applies in far more extreme circumstances.

The case of the present theory is different: the search for a ***gravitational mechanism*** was emphatically not part of our original thinking and, in the main body of this paper, we pay no attention at all to the likely nature of any such mechanism. We simply perform a formal perturbation analysis of the $D = 2$ inertial universe, and the analysis is played out geometrically on a curved manifold using largely familiar techniques. However, regardless of this, it is impossible not to realize that a genuine mechanism has automatically presented itself. Specifically, we have mentioned that the material in the idealized $D = 2$ inertial universe behaves like a quasi-photon gas, and that gravitational processes arise when this $D = 2$ distribution is perturbed - in particular, point-wise spherically symmetric perturbations are interpreted as being due to the presence of conventional point masses. One is then immediately forced to conclude that the point-mass perturbs by acting as either a ***source*** or a ***sink*** of quasi-photon gas. Suppose it is a sink, and that its mass simply quantifies the amount of absorption going on; then, when two such absorbing masses are placed in the $D = 2$ universe, they will partly shade each other from the global quasi-photon flux, and will therefore experience a net external pressure acting to push them together. That is, a ***gravitational force*** arises. We merely note that such explicit mechanisms have been proposed before: the difference here is that the ***absorber mechanism*** has arisen from deeper considerations which were not themselves concerned with mechanisms at all.

1.4 Overview of Results

The theory, as we have so far developed it, is quasi-classical insofar as it is a one-clock model. This restriction is not structural - as we point out in Roscoe [2002a] - but was imposed in order not to obscure the central arguments of the original development. However, it does mean that any gravitation theory which is based upon this particular formal development of inertial space & time can only be applicable to weak-field regimes. Hence, here we restrict ourselves to showing how classical Newtonian theory is reproduced, and to the application of the theory to model an idealized spiral disc - one without a central bulge and with perfect cylindrical symmetry in the disc. A particular subclass of solutions, corresponding to logarithmic spirals, predicts that the circular velocities should behave according to $V = AR^{\alpha}$ where (A, α) are parameters which vary between discs, and where α must necessarily satisfy a certain algebraic consistency condition. In the first instance, we ignored this consistency condition and focused on the basic power-law model. This simplified model has been extensively tested on several very large samples (900, 1182, 497 and 305 objects respectively) in Roscoe [1999a] and Roscoe [2002b], and shown to resolve the data with a remarkable statistical precision.

However, in the course of the initial study, a major new phenomenology was discovered - that of *discrete dynamical classes* in spiral discs. This has been the subject of an initial study, Roscoe [1999b], and a comprehensive later study involving the four large samples, Roscoe [2002b], which confirmed the phenomenology at the level of statistical certainty.

This latter discovery caused us to reconsider the initially ignored α-consistency condition, and we found that its algebraic form provides all the structure required to understand the *discrete dynamical classes* phenomenology. Thus, we can potentially understand the new phenomenology as the physical manifestation of an algebraic consistency condition that must necessarily be satisfied in order for solutions of the complete system to exist.

Finally, we apply the theory with complete success to the detailed modelling of a sample of eight Low Surface Brightness spirals (LSBs). The point of choosing such objects is primarily because they are so diffuse that, according to the canonical viewpoint, they must consist of > 99% dark matter to be gravitationally bound. This is to be compared with, typically, > 95% dark matter for ordinary spirals.

1.5 The MOND Connection

Finally, we infer from the success of the theory that it must intrinsically provide a theoretical basis for the MOND (Modified Newtonian Dynamics) programme of Milgrom [1983a], [1983b], [1983c]. In considering the flat rotation curve problem of spiral galaxies, Milgrom had the idea that, perhaps, in extremely weak gravitational fields ($g << 10^{-10}ms^{-2}$), the nature of Newtonian gravitational mechanics changed in such a way that flat rotation curves were the natural result. The notable thing about MOND is that, whilst it was designed to address one particular phenomenology - that of the flat rotation curve in galaxy discs - it has enjoyed impressive success in a variety of quite distinct circumstances, making several predictions that have subsequently been verified - and any one of which could have falsified the theory. On any objective measure, the performance of the one-parameter MOND model is superior to that of the multi-parameter Cold Dark Matter model. See for example, de Blok & McGaugh [1998] and McGaugh & de Blok [1998a], [1998b]. The primary difficulty for the MOND programme has been that there is no underlying theory to support it. By virtue of the present theory's complete success in modelling the LSB sample, we infer that it must contain the quantitative essence of the MOND algorithm.

2 Review of Fundamental Arguments

Although the mathematical machinery used in Roscoe [2002a] is essentially straightforward, the ideas and forms of argument used in that development will be unfamiliar to most readers. Therefore, a short review of the process employed will probably be useful here.

The basic aim was to give expression to a particular form of Mach's Principle - essentially the idea that within the framework of a properly fundamental theory it should be impossible to conceive *empty* physical space & time. This was achieved, briefly, via the following process:

- Note that, on a large enough scale (> 10^8 lightyears, say) it is possible, in principle, to write down an approximate functional relation giving the amount of mass (determined by *counting* particles) contained within a given spherical volume, $M \approx F(R)$;
- Since M will be a monotonic function of R, we can invert this latter expression to get $R \approx G(M)$;
- Whilst it is conventional to suppose that $M \approx F(R)$ is logically prior to $R \approx G(M)$, there is no natural imperative dictating that we *cannot* reverse the logical priorities;
- That is, we are at liberty to suppose that $R \approx G(M)$ is the prior relationship so that, in effect, the radius of a sphere is defined in terms of the amount of mass contained within it. In other words, we have made it impossible to define a radial measure in the absence of material. This is the first critical step towards the Machian theory required;
- Once we have a radial measure, defined in terms of M, we can define a coordinate system allowing us to label points in the space and to specify displacements in the space. But we have no means of associating an ***invariant length*** to any such displacement - there is no metric yet;
- To obtain a qualitative idea of metric, we made the following thought experiment: An observer floats without effort through a featureless landscape. But this observer will have no sense of distance travelled - there is no metric. By contrast, suppose now that the landscape possesses many distinctive landmarks. Now there will be a powerful sense of distance travelled imposed by the continually changing relationships between the landmarks and the observer. In other words, the observer's ***changing perspective*** of the landscape provides him with the means to make qualitative judgements of the magnitudes of his displacements in that landscape. We are able to use this idea to obtain a quantitative definition of a metric within the model universe described below;
- Define a model universe consisting of particles (*not* assumed to be in a static relationship to each other) possessing *only* the property of enumerability (we have in mind that *mass* is fundamentally a measure of the *amount* of material, and therefore determinable by a counting process), and suppose that there is at least one point about which the distribution is spherical;
- There is no notion of *time* yet but, even so, we have to distinguish between the possibilities of *non-evolving* and *evolving* model universes. This distinction turns out to be the distinction between a non-relativistic universe and a relativistic one and, to simplify matters in the first instance, we supposed that the model universe was non-evolving;
- The latter two points give $R = G(M) \rightarrow M = F(R)$. The particles within any given level surface of M are then taken to define the landmarks within our landscape, and a straightforward modelling

process then allows us to use the ***changing perspectives*** idea above to define a metric within the model universe. We found:

$$g_{ab} \equiv \nabla_a \nabla_b \mathcal{M} \equiv \frac{\partial^2 \mathcal{M}}{\partial x^a \partial x^b} - \Gamma^k_{ab} \frac{\partial \mathcal{M}}{\partial x^k},$$

where $\mathcal{M}$ is a simple linear function of $M(R)$, and Γ^k_{ab} is chosen to be the metric affinity. This choice is made because it guarantees that appropriate generalizations of the divergence theorems exist - which is necessary if we are to have conservation laws in the model universe.

3 Gravitation: General Comments

3.1 Newtonian Theory and Point-Mass Perturbations

We have stated, in §1.1, that $\mathcal{M}$ in the $D = 2$ equilibrium universe can be properly considered as a classical representation of a material vacuum within which the particles (or quasi-photons) arbitrate between length and time scales in a way which is reminiscent of Bondi's interpretation of c.

But what about gravitational processes in such a universe? If they exist at all, they can only arise via perturbation processes in the material vacuum generated by conventionally understood point-mass sources. Given this, and thinking crudely in terms of the quasi-photons each moving with a speed v_0 (remember, v_0 is actually a conversion factor having dimensions of speed), it quickly becomes apparent that *gravitational effects* between two such point-masses are most readily understood in terms of each point-mass partially shading the other from vacuum particle collisions, so that each picks up a net momentum towards the other, as if attracted by a gravitational force. Of course, the emergence of Newtonian Gravitation for the case of the single point-mass perturbation is the first necessary condition that must be met, if the foregoing picture is to be given credence. This condition is shown to be met in appendices §A, §B and §C.

The N-body theory is developed by considering the perturbations generated in the material vacuum by finite ensembles of point-mass particles in appendices §D, §E, §F and §G. However, in the course of this development, a subtlety arises concerning the interpretation of $\mathcal{M}$: specifically, in the equilibrium universe, $\mathcal{M}(r)$ simply describes the distribution of vacuum mass about any centre (it is fractal, $D = 2$). But, when the equilibrium universe becomes perturbed by a finite ensemble of point-mass particles, then:

- In order to describe the changing states of motion of a specified point-mass particle, the vacuum mass 'seen' by that particle (that is, that component of vacuum mass which acts to cause a change in motion) must be defined;

- The unique association of a specified point-mass particle with a particular vacuum mass distribution is accomplished in the analysis of §D, which shows how the requirement for linear momentum conservation within the finite point-mass ensemble implies that the vacuum mass 'seen' by a particle of mass m at position $\mathbf{r}$ (defined with respect to the ensemble mass centre) must have the functional form $\mathcal{M} \equiv \mathcal{M}(m\mathbf{r})$. Equivalently, this implies that the vacuum mass 'seen' by a point-mass particle of *unit* mass at position $\mathbf{r}$ is given by $\mathcal{M}(\mathbf{r})$.

3.2 Continuum Perturbations: A Necessary Conjecture

We show, in appendix §B, how a perfectly workable quasi-Newtonian point-source theory arises from a point-mass perturbation of the equilibrium vacuum. However, our primary interest is in gravitational processes within extended material distributions. But, for these, we have no means of giving a quantitative definition to the corresponding vacuum mass distribution, $\mathcal{M}(\mathbf{r})$ - we have only been able to write down a probable general structure in §F. Therefore, we must necessarily rely upon the following qualitative arguments:

Bearing in mind that the vacuum mass 'seen' by a particular point-mass particle is actually some measure of the *discrepancy* between the $D = 2$ equilibrium distribution and the perturbed distribution, and that this discrepancy is generated by the finite point-mass ensemble, then the vacuum mass 'seen' by a particular point-mass particle must also be a measure of the total of other point-masses within the ensemble. This leads us to the following conjecture:

> In the limit of the finite point-mass ensemble becoming a continuum distribution, then this distribution traces the vacuum mass distribution 'seen' by a test particle to the extent that one can act as a proxy for the other.

As we shall see in the later sections, this conjecture (or something very similar) is strongly supported on the data for Low Surface Brightness (LSB) galaxies analyzed in later sections.

4 A Model For An Idealized Spiral Galaxy

In the following, we consider the application of this theory to model an idealized spiral galaxy - defined to be one without a central bulge, and without the irregularities that are routinely present in the discs of real galaxies, and without the bars that are present in many spirals. We model this idealized spiral galaxy as a mass distribution possessing perfect cylindrical symmetry.

In order to motivate the detailed analysis of §9 and §10 and the appendices more strongly, we begin by considering the solutions which arise and the high quality of their fit to the data. Furthermore, because, in our view, the work presents a very interesting example of the idealized cycle *theory* $\rightarrow$ *test* $\rightarrow$ *discovery* $\rightarrow$ *more theory* $\rightarrow$ *more test*, we present our solutions in chronological order of their calculation to illustrate the way in which the work as a whole has progressed.

4.1 The Power-Law Model

In the first instance, we were only able to progress by making use of the empirical knowledge that the spiral structure of spiral galaxies is essentially logarithmic. This led directly to the idealized disc solutions:

$$V_{rot} = AR^{\alpha}, \quad V_{rad} = BR^{\alpha}, \quad \rho = \frac{C}{R^2} \tag{1}$$

where R is the radial position within any given galaxy disc, V_{rot} is the rotational velocity, V_{rad} is the radial velocity, ρ is the mass density, A, B and C are arbitrary constants, and α is constrained to satisfy

$$\alpha = 1 - \frac{p}{Sq} - \frac{p^2 + q^2}{qK_1} \text{ where } p K_1^2 \pm (1 + p^2 + q^2) K_1 + p = 0, \tag{2}$$

where (p, q) are undetermined constants and $S = \pm 1$. This solution (ignoring the constraint equations for α) led directly to the data analysis described in detail in Roscoe [1999a] and described briefly in §5.

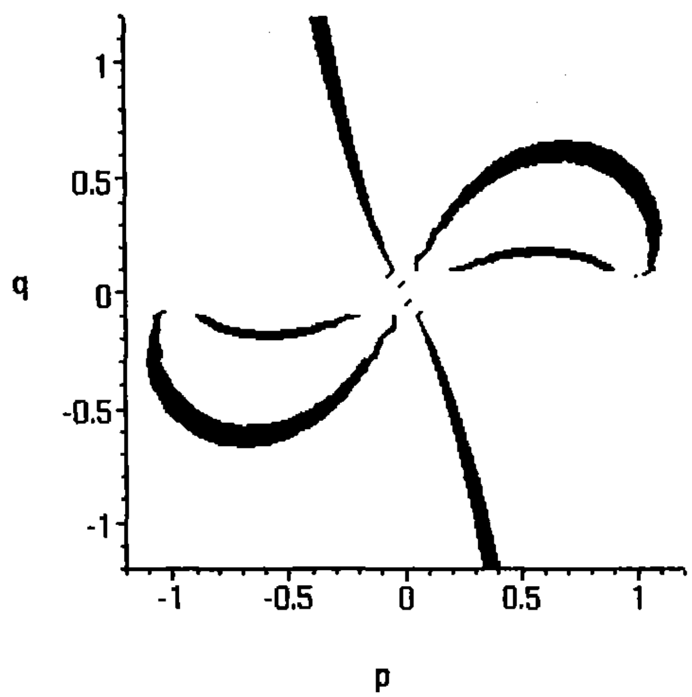

Figure 1: Intersection of three surfaces in $\alpha = 0.5$

4.2 The α-Constraint

However, the data analysis of Roscoe [1999a] led, in its turn, directly to the discovery of the *discrete dynamical states* phenomenology in galaxy discs, briefly reviewed in §6. This phenomenology was reported initially in Roscoe [1999b] for the case of one large data set, and subsequently confirmed in Roscoe [2002b] for three further large data sets. This discovery led us to reconsider the role of the α-constraint (2) which we had previously ignored.

Bearing in mind that there are four possible solutions for K_1 in terms of (p, q) and that the expression for α contains $S = \pm 1$ then, at face value, there are eight possibilities fo α. However, a closer investigation shows that there are only six *distinct* possibilities, so that we have the general structure

$$\alpha = F_i(p, q), \quad i = 1..6 \tag{3}$$

Thus, in principle, any given galaxy disc is potentially associated with one of six distinct surfaces in (α, p, q) space. Figure 1 shows the intersection of three of these surfaces with the plane $\alpha = 0.5$. The three omitted surfaces are mirror images (about either axis) of those shown so that the total figure is perfectly symmetric about the two axes. (The poor figure quality is a function of the limitations of the Maple graphics package). We show in §6 that the surfaces (3) provide all the structure required to arrive at a qualitative understanding on the discrete dynamical states phenomenology.

4.3 General Solutions

The big open question concerning spiral galaxies is whether dark matter exists or not and, amongst the class of spiral galaxies, it is the Low Surface Brightness (LSB) galaxies which present the most extreme problems. Typically, these objects are estimated to consist of more than 99% dark matter, according to the canonical viewpoint. Therefore, it is these objects which must be successfully modelled by any new theory if that theory is to have maximum credibility.

Numerical techniques appear to offer the only realistic possibility for modelling specific galaxies. But, there are serious problems even here: specifically, the first galaxy in the LSB class was only discovered in the late 1980s and, even today, there are only a few tens of them with detailed and accurate rotation velocity measurements. Of these, only a minor fraction have detailed estimates of the ordinary matter in their discs and none have any radial flow measurements. In general, we would therefore expect that the detailed modelling of any specific LSB would necessarily require sweeping assumptions to be made about radial flows - thereby considerably reducing the value of any such modelling process (especially so when the equations concerned are numerically stiff).

However, in the case of the present theory, it transpires that the radial velocity component can be algebraically *eliminated* from the complete system, leaving a reduced system in rotation velocity and mass density only.

4.3.1 Mass Distribution: The Beautiful Equation

Not only is it possible to eliminate the radial velocity explicitly from the complete system, but the mass equation can be expressed in a form which is independent of velocity at all! In this form, it is given by:

$$(\psi^2 - 1)\epsilon^2 + \frac{2}{q}\left[p(1+\psi^2) + (1+p^2+q^2)S\,\psi\right]\epsilon - (\psi^2 - 1) = 0\,, \tag{4}$$

$$\text{where}\quad \epsilon^2 \equiv -1 - \frac{R}{\rho}\frac{d\rho}{dR} \quad\text{and}\quad \psi^2 \equiv 1 - \frac{2\pi\rho}{m_0}R^2. \tag{5}$$

Here, $S = \pm 1$ whilst the parameters (p, q, m_0) are integration constants. Of these (p, q) are as in §4.1, whilst m_0 can be fixed to have a magnitude of $4M_{gal}$ where M_{gal} is the mass of the object being modelled estimated from measurements of stars, gas and dust. Since this is fixed independently of any calculations then there are, in effect, only two disposable parameters, (p, q), for the mass and circular velocity equations. It is the algebraic structure of the mass equation above which, ultimately, allows the theory to model the various complexities manifested by rotation curves as a class, and which leads us to refer to it as *the beautiful equation*.

At face value - and when the signature $S = \pm 1$ is taken into account - equation (4) is satisfied by any one of four distinct distributions for ρ. However, the situation is more complex than this: reference to (4) shows that if $\epsilon = \pm 1$, then ψ assumes one of two possible constant values. It then follows, by the second of (5), that $\rho \sim 1/R^2$. But any ρ of this form also satisfies the first of (5). The net result is that the mass equation, (4), is satisfied when ρ satisfies any one of *six* possible distributions.

However, closer analysis then reveals that of the two possibilities associated with $\epsilon = \pm 1$, one is not physical (it corresponds to negative densities) and of the four possibilities associated with $\epsilon \neq \pm 1$, a further one is also not physical. The net effect is that (4) is satisfied by any one of four physically realizable distributions for ρ - so that, in principle, the distribution of material in any given galaxy disc can satisfy different differential equations over different anular sections. For example, the single admissable ρ distribution corresponding to $\epsilon = \pm 1$ actually corresponds to the solution of §4.1, and we find that six of our sample of eight LSBs have such segments embedded within their discs.

4.3.2 The Rotation Velocity Distribution

The rotation velocity equation is given by given by

$$\frac{1}{V}\frac{dV}{dR} = \frac{1}{R}\left[\frac{1}{2}(1+\epsilon^2) - \frac{p}{q}\epsilon - \left(\frac{p^2+q^2}{q}\right)\frac{\epsilon}{S\psi}\right] \tag{6}$$

where we note that ρ appears through ϵ and ψ.

In practice, we have found that one (or two) switches between different ρ distribution laws - and hence between different velocity distribution laws - occur somewhere in every LSB disc, and it is this mechanism which allows the theory to model the complex behaviour of rotation curves. We show the results of a detailed modelling exercise on a sample of eight LSBs in §7.

4.3.3 General Comments

It is interesting to note that, if $V = V^*$ is a solution of (6), then so is $V = kV^*$, for any constant k. This is a useful property in the present context since, because galaxy discs are generally not seen edge on, we can only estimate rotation velocities when we have estimated disc-inclination - and the effect of getting this wrong is simply to scale the true rotation velocities by an unknown constant factor. Thus, (6) is indifferent to knowledge about disc-inclination angles.

Finally, we note that the existence of a switching mechanism in the theory is reminiscent of the similar thing which is a necessary component of the MOND algorithm, discussed in later sections.

5 Power Law Dynamics: The Observations

5.1 General Comments

In this section, we give a brief review of the published evidence supporting the view that velocity distributions in *idealized* logarithmic discs behave according to the basic power law solutions, (1). There are four preliminary comments:

- Firstly, it is essential to understand that, so far as the phenomenology is concerned, we are only talking about rotation curves over their interior segments on which they are rising strongly - that is, we are explicitly excluding the exterior flat parts. As it happens, practical considerations ensure that *optical* rotation curves are generally confined to this region anyway, and the analyses which follow are all confined to optical rotation curves (ORCs).

- Secondly, whilst a large amount of V_θ (circular velocity) data exists in the form of several large samples of published ORCs, there is no corresponding body of data for radial velocity flows. The reason is simply that such flows are, typically, an order of magnitude smaller than the circular flows and the techniques to measure them have only recently become available.

- Thirdly, our analysis applies only to idealized discs defined to possess perfect cylindrical symmetry. Since spiral galaxies typically possess bulgy central regions then our model can, at best, only have validity in those parts of the disc which are *exterior* to the innermost central regions and (by the initial comment) *interior* to the very outermost regions where the rotations curves become flat.

- Fourthly, since galactic discs are generally complete with all manner of irregularities then the model can only have a *statistical* validity. It is for this reason that we confine our selves to the analysis of very large samples only.

5.2 The ORC Samples

Mathewson, Ford & Buchhorn [1992] published a sample of 900 optical rotation curves (ORCs) which provided the basis for our first large scale analysis of disc dynamics from a power-law point of view. The results of this analysis, (Roscoe [1999a]) showed that the power-law resolves disc dynamics in the outer part of optical discs to a very high degree of statistical precision. We have subsequently analysed three further large samples, these being those of the 1182 ORCs published by Mathewson & Ford [1996], the 497 ORCs published by Dale et al. ([1997] et seq) and the 305 ORCs published by Courteau [1997]. This last sample differs from the previous three in being the only one using R-band photometry rather than I-band photometry. For associated technical reasons, the modelling process for the Courteau sample differs in its details from the others and so is excluded from the first part of the present discussion.

5.3 The Basic $(\alpha, \ln A)$ Plot

The basic question is: does the power-law $V_{rot} = AR^{\alpha}$ provide a good resolution of ORC data on the exterior part of optical discs? (Remember, practical considerations ensure that ORCs generally to not extend to the far exterior regions where rotation curves become flat). The first problem here is to give an objective definition of what is meant by the *exterior part of the optical disc*. This is provided originally in Roscoe [1999a], and more clearly in Roscoe [2002b]. Once this is done, the analysis uses linear regression to estimate the parameter pair $(\ln A, \alpha)$ for each of the ORCs in the sample. Figure 2 gives the scatter plot for all of the 2405 usable ORCs in the three I-band samples. (About 7% of the total sample of 2579 ORCs were lost to the analysis because of objective reasons associated with data quality.) There are three significant points to be made:

- The first point, which came as a shock, is the existence of a very clear and powerful correlation between $\ln A$ and α, since there is no obvious a priori reason why any correlation should exist at all;

- Secondly, although there is no a priori reason to expect an $(\alpha, \ln A)$ correlation, equation (2) states that α is correlated with the dynamical parameters (p, q). But, by definition, $\ln A$ is strongly determined by the dynamics and so it follows that, in qualitative terms, the $(\alpha, \ln A)$ correlation of figure 2 is consistent with the existence of a relation like the α-constraint of (2);

- Thirdly, except for what is probably statistical scatter, α appears to be strongly confined in the region $\alpha \in (0, 1)$. In section §7.5, we see how this α confinement has an elegant association with certain topological transitions in the (α, p, q) parameter space.

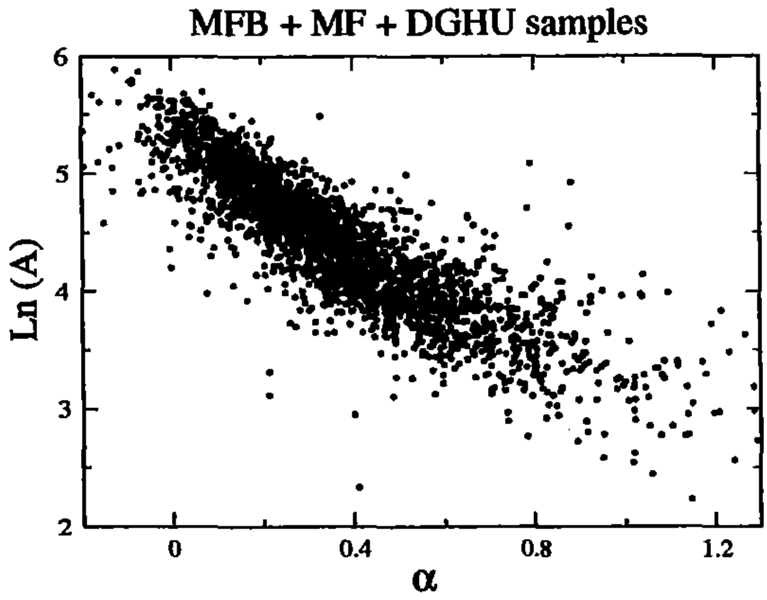

Figure 2: Plot of ($\ln A$, α) for 2405 galaxies

5.4 A Detailed Model of the $(\alpha, \ln A)$ Plot

A more detailed analysis of this diagram showed that the luminosity properties of galaxies vary very strongly through the plot. Specifically, consider the power-law model, $V = A\, R^{\alpha}$, in dimensionless form:

$$\frac{V}{V_0} = \left(\frac{R}{R_0}\right)^{\alpha} \rightarrow A = \frac{V_0}{R_0^{\alpha}}. \tag{7}$$

Then, defining M as the absolute I-band magnitude and S as the absolute I-band surface brightness of an object, a detailed modelling of the MFB (Mathewson et al. [1992]) and MF (Mathewson & Ford [1996]) data in figure 2 shows that the particular model

$$\begin{aligned} \ln A &= \ln V_0 - \alpha \ln R_0, \\ \ln V_0 &= -1.596 - 0.316\, M \\ \ln R_0 &= -7.614 - 0.474\, M - 0.0050\, S \end{aligned} \tag{8}$$

accounts for about 93% of the total variation in the figure. It is to be noted that, in the model, the t-statistic for each of the model parameters satisfies $|t| > 11$, so that all the included variations are powerfully present. This is, in itself, a strong demonstration of how effectively the power-law model resolves ORC data. The model fit can be improved by including the DGHU (Dale et al. [1997] et seq) data in the modelling process - but we choose to exclude it to provide an independent test, discussed below.

5.5 An Alternative Visualization of the Model-Fit

A very effective alternative way of visualizing the fit of the model (8) to the data can be obtained as follows: Suppose we use the definitions of (8) in the dimensionless form give at (7), so that the measured $(R,\ V)$ data for each ORC is scaled by the luminosity models for $(R_0,\ V_0)$ for that ORC, and then regress

$\ln(V/V_0)$ on $\ln(R/R_0)$ for each ORC. Then, if the power-law model (7) is good, we should find a null zero point for each ORC - except for statistical scatter.

Figure 3 (left) gives the frequency diagram for the actual zero points computed for the combined Mathewson et al. ([1992], [1996]) samples from which the model (8) was derived, whilst a wholly independent test of the model is given by figure 3 (right) which gives the frequency diagram for the zero points derived from the DGHU sample using the model (8) - which, of course, was derived *without* DGHU data. It is clear that there is absolutely no evidence to support the idea that these zero points are different from the null position. Thus, the power-law model is strongly supported, and (7) with (8) can be considered to give a high precision statistical resolution of ORC data in the exterior part of the disc.

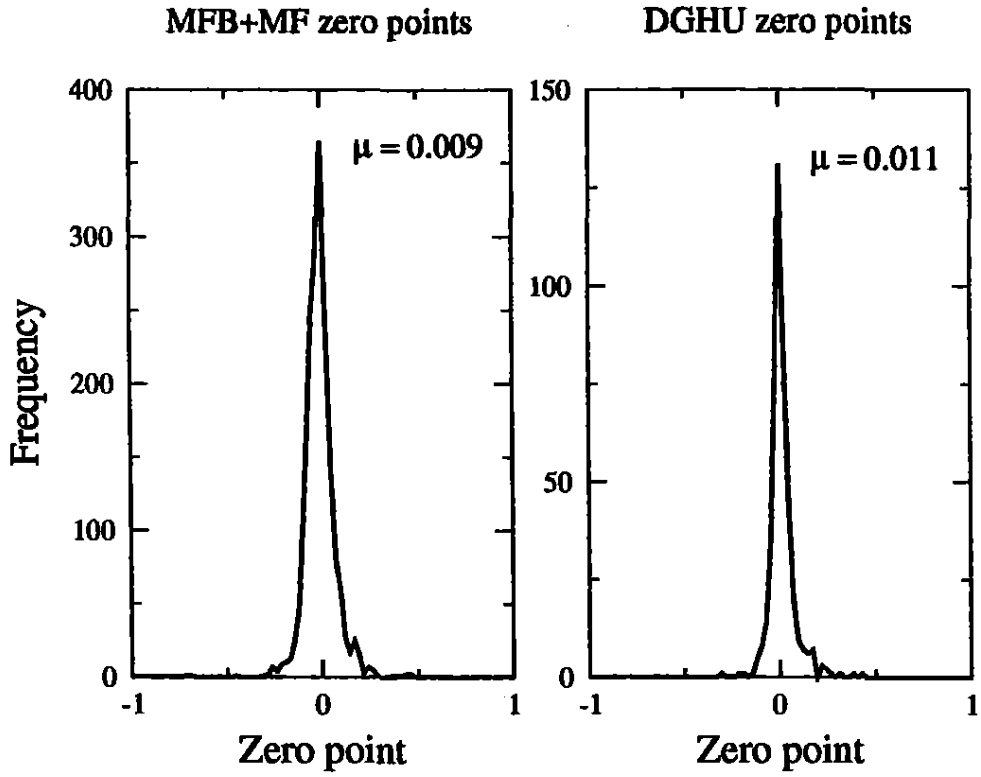

Figure 3: Plot of zero point for 1951 MFB+MF galaxies and 454 DGHU galaxies

To summarize, we have so far shown that the rotational part of the power-law model (1) is very strongly supported on the data and have indicated that the data does not yet exist to consider the validity - or otherwise - of the radial component of the model.

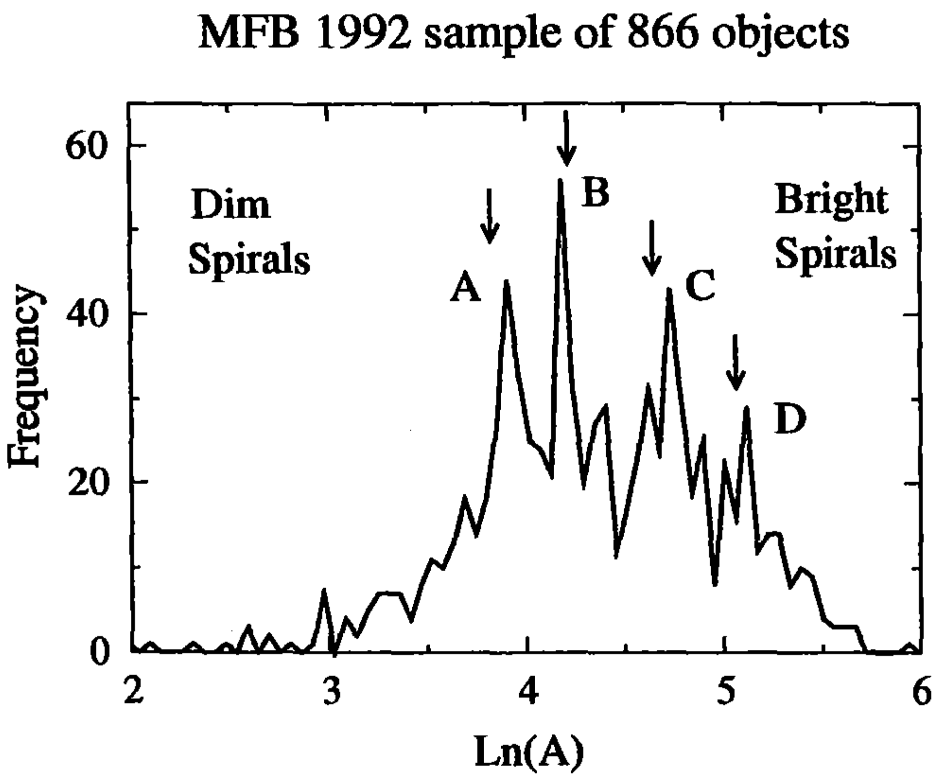

Figure 4: Vertical arrows represent predicted positions of peaks from an analysis of 12 Rubin et al. ORCs

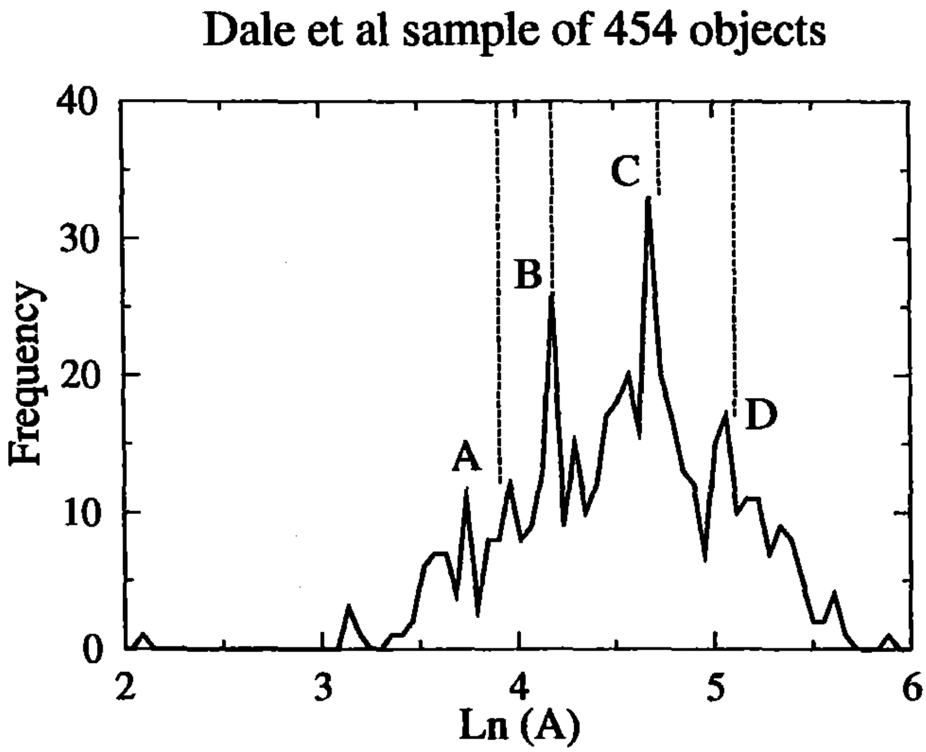

Figure 5: Vertical dotted lines indicate peak centres of figure 4.

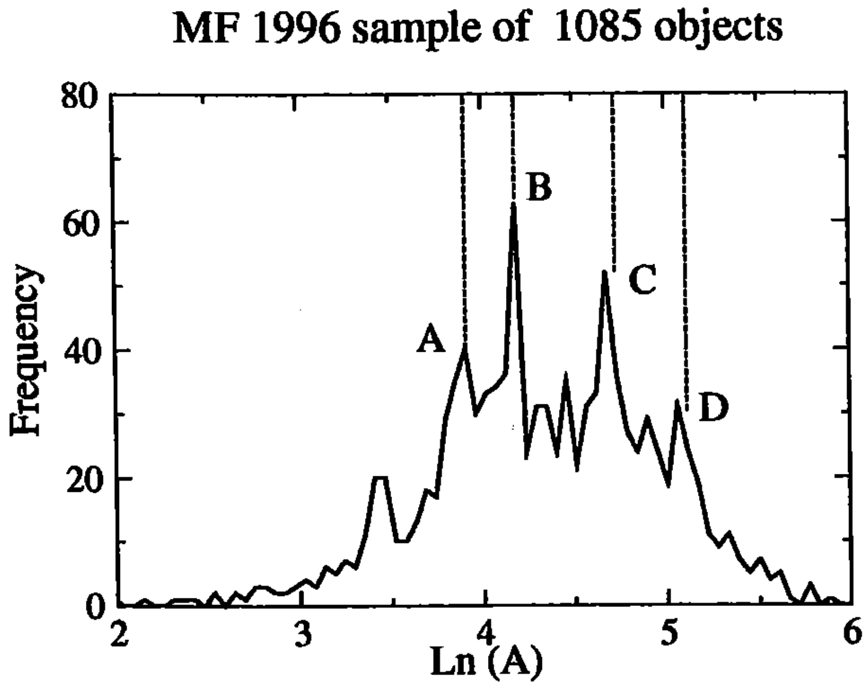

Figure 6: Vertical dotted lines indicate peak centres of figure 4.

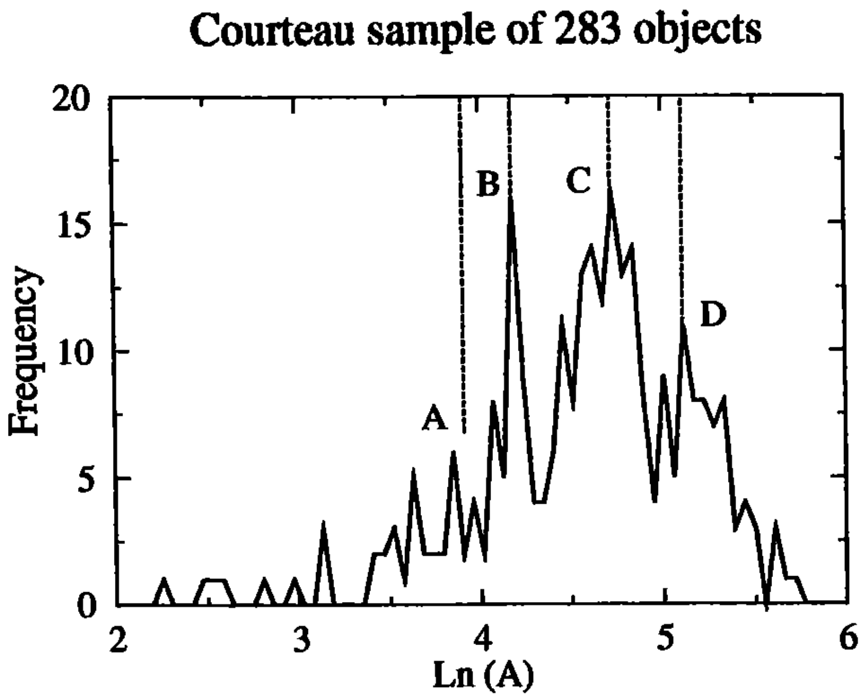

Figure 7: Vertical dotted lines indicate peak centres of figure 4.

6 Discrete Dynamical Classes: The Observations

6.1 General Comments

In this section we consider the exact solutions of §4.1 - specifically, we consider the extremely strong statement, made at (3), to the effect that α, calculated for any given spiral, is constrained to occupy one of a set of distinct surfaces in (α, p, q) space, where (p, q) are dynamical parameters in the model.

This statement has interest on many levels - not least because nothing like it features in any other extant theory of disc dynamics. The effect was first noticed - tentatively and prior to us recognizing the potential significance of the α-constraints - during a pilot study of a small sample of Rubin et al. [1980] ORCs, and this initial identification was used to define a hypothesis which was subsequently tested on the Mathewson et al. [1992] sample, and reported in Roscoe [1999b]. The effect was subsequently confirmed in Roscoe [2002b] on three further large samples (Dale et al. [1997] et seq, Mathewson & Ford [1996] and Courteau [1997]). For completeness, we give a brief review of this evidence here.

In practice, the evidence takes the form of the $\ln A$ frequency diagrams for each of the four samples, and then interpreting the meaning of these diagrams. The computation of the $\ln A$ parameter is as it was for figure 2 (but see Roscoe [2002b] for a complete discussion).

6.2 The $\ln A$ Frequency Diagrams

Figure 4 shows the $\ln A$ distribution arising from the analysis of the Mathewson et al. [1992] sample, and the short vertical arrows in that figure indicate the predicted positions of the peaks, based on a pilot study of a sample of twelve ORCs from Rubin et al. [1980]. Given the very small size of this initial sample of Rubin objects, we can see that the match is remarkable.

Figures 5, 6 and 7 show the corresponding distributions for the Dale et al.([1997] et seq) sample, the Mathewson & Ford [1996] sample and the Courteau [1997] sample. In each of these cases, the vertical dotted lines indicate the peak centres of figure 4.

The A peak in figures 5 and 7 are more-or-less absent because this peak corresponds to very dim objects, and these are very much under-represented in the two samples concerned.

The joint probability of the observed peaks in the four samples arising by chance alone, given the original hypothesis raised on the small Rubin et al.sample, has been computed in Roscoe [2002b], using extensive Monte-Carlo simulations, to be vanishingly small at $\approx 10^{-20}$.

6.3 Interpretation of the $\ln A$ Frequency Diagrams

It is clear from the four diagrams that $\ln A$ has a marked preference for one of four distinct values, say $\ln A = k_1, k_2, k_3, k_4$. However, we also know, from §5.4, that $\ln A$ is a strongly defined function of the galaxy parameters (α, M, S), so that $\ln A = F(\alpha, M, S)$. Putting these two results together gives

$$F(\alpha, M, S) = k_i, \quad i = 1, 2, 3, 4. \tag{9}$$

Consequently, the $\ln A$ frequency diagrams imply that spiral discs are confined to one of four distinct surfaces in (α, M, S) space.

But M is a measure of absolute galaxy luminosity, and therefore a measure of the corresponding total galaxy mass, m say (assuming no dark matter). Similarly, the surface brightness parameter, S, which is a measure of the density of absolute galaxy luminosity, can be considered as a measure of mass density, ρ,

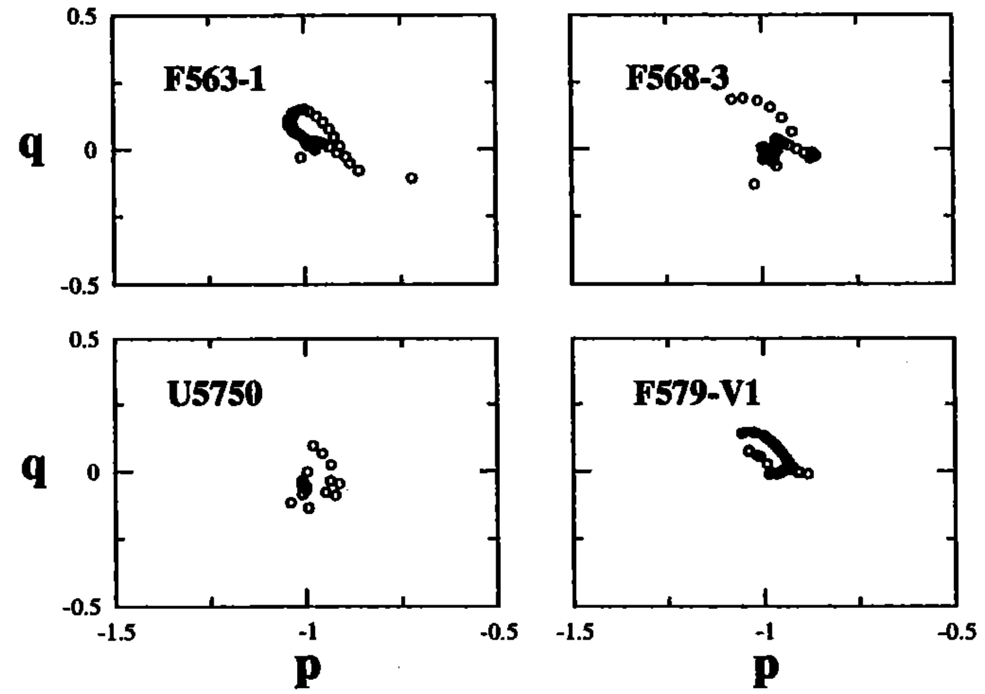

Figure 8: Numerical estimates of (p, q). Note: Each box is a unit box in the (p, q) plane.

in a galaxy. That is, $(M, S) \approx (m, \rho)$ and these latter two parameters are, a priori, important dynamical parameters for any given system. If we now presuppose the existence of a mapping $(m, \rho) \rightarrow (p, q)$ where (p, q) are the dynamical parameters in our model equations, then we see that (9) is consistent with (3) - the difference being that (3) allows up to six surfaces, whereas we have only identified four in the phenomenology.

We reasonably state, therefore, that the theory contains all the structure required to explain, *in principle*, the phenomenon of discrete dynamical classes for disc galaxies - subject to the existence of appropriate mappings $(m, \rho) \rightarrow (p, q)$. However, various significant details uncovered in the analysis of §7 give us further insight into the problem, and these are briefly discussed in §8.

7 Detailed Modelling of LSB Galaxies

In this section, we apply equations (4) and (6) of §4.3 to the detailed modelling of a sample of eight LSB galaxies [1] which have already been successfully modelled by MOND (deBlok & McGaugh [1998] and McGaugh & deBlok [1998b]). We shall not dwell on the computational problems involved in solving these equations, but we shall discuss, briefly, the means by which the parameters (m_0, p, q) were determined.

7.1 The Parameter m_0

Referring to (4), and noting that ρ in this equation is the density of material in a disc of unit thickness, we can deduce from the definition of ψ given there that m_0 has dimensions of mass. This suggests that it is

[1] Provided by Stacy McGaugh, of the University of Maryland, USA

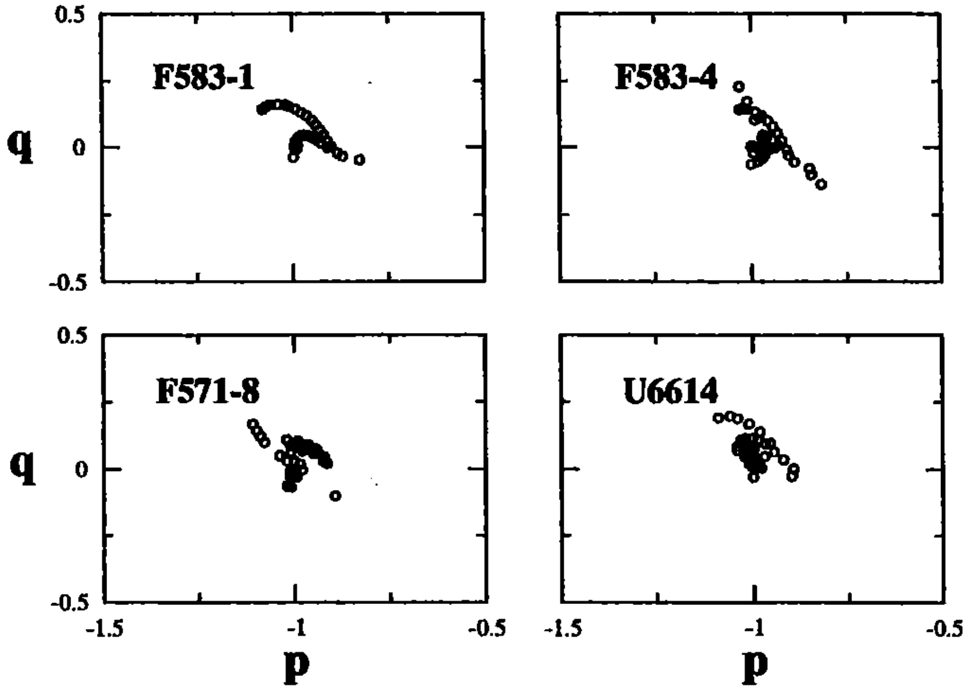

Figure 9: Numerical estimates of (p, q). Note: Each box is a unit box in the (p, q) plane.

likely to be a simply related to the total mass of the object being modelled. In fact, we found that, defining M_{gal} as the total or ordinary mass (stars + dust + gas) in the galaxy being modelled (as estimated by the original observing astronomer - Stacy McGaugh in the present case), then $m_0 = 4M_{gal}$ worked extremely well for all eight cases.

7.2 Estimating The Parameters (m_0, p, q)

We comment firstly that the theory is indifferent to the signatures of p and q but, for the sake of being explicit, we take them to be always negative. With this proviso, then as a means of (a) obtaining estimates of (m_0, p, q) for each LSB in the sample and (b) providing a preliminary consistency check on the model equations (4) and (6), we proceeded as follows:

- Assume that m_0 is some simple multiple (fixed for all the objects) of estimated total mass (obtained by integrating McGaugh's data for each object);
- For each LSB, use the McGaugh mass-distribution estimates and rotation velocity measurements to obtain smooth cubic spline models of mass and velocity distributions;
- Use these smooth cubic spline models to obtain estimates of density, density gradient, velocity and velocity gradient at a sequence of distinct points across the disc of each LSB;
- With these estimates, the differential equations , (4) and (6), at any given point in a disc become two *algebraic* equations for determining (p, q);
- For each LSB, solve this pair of algebraic equations at several points across the disc, and look for consistency in the resulting sequence of estimates of (p, q).

A necessary condition of the theory's consistency is that, for some reasonable choice of m_0, the foregoing process will lead to a consistent set of (p,q) estimates across each LSB disc.

In fact, we found that setting $m_0 = 4M_{gal}$, where M_{gal} is the estimated total of ordinary galaxy mass (stars+dust+gas) for each LSB, gave a very consistent picture for estimates of (p,q). The big surprise was that, not only was this the case for each individual object, but that the set of (p,q) pairs for the whole sample lies in the same very small neighbourhood of the (p,q) plane. Figures 8 and 9 show the results of this latter exercise for each of the eight LSBs in our sample, and we see that, for each object, the (p,q) estimates all lie in the neighbourhood of $(-1,0)$ in the (p,q) plane. Note: for each LSB, solutions were sought in a very large (20×20) region of the (p,q) plane. The only solutions found are those indicated.

This exercise allowed us to conclude that the model equations were highly consistent with the phenomenology, and also gave us a good starting estimate of $(p,q) \approx (-1,0)$ for the detailed modelling process of each object in the sample.

Table 1:

Galaxy	p	q
F563-1	-0.990	-0.038
F568-3	-0.970	-0.004
U5750	-0.955	-0.158
F579-V1	-0.997	-0.022
F583-1	-0.980	-0.047
F583-4	-0.950	-0.084
F571-8	-0.952	-0.041
U6614	-0.995	-0.032

7.3 The Detailed Models

The integrated solutions for the rotational velocities and the mass distributions (using the (p,q) estimates listed in Table 1), together with the corresponding observational measurements, are given in Figures 10 & 11. In every case, we see that the fit of the computed rotation velocities (solid lines) to the measured rotation velocities (filled circles) is, for all practical purposes, perfect.

Except for discrepancies near galactic centres, where our modelling assumptions become less good, our computed density distributions (dotted lines) provide reasonable fits to the estimated densities (crosses). However, it is important to realize that, by contrast with the accuracy of Doppler velocity measurements, mass estimation in galaxy discs is subject to very great uncertainties - which is why mass-modellers never quote error-bars for their estimates. We have used McGaugh's mass-models to estimate our parameter m_0, mentioned earlier. But, for the mass-density integration, we have chosen the initial conditions to ensure that the predicted mass distributions over the discs provide good qualitative fits to MacGaugh's models.

Finally, it should be remarked that all dark matter models fail comprehensively when applied to LSBs - MOND (described in detail in §11) has been successfully applied to all of the objects considered here (deBlok & McGaugh [1998] and McGaugh & deBlok [1998b]) and to a great many more.

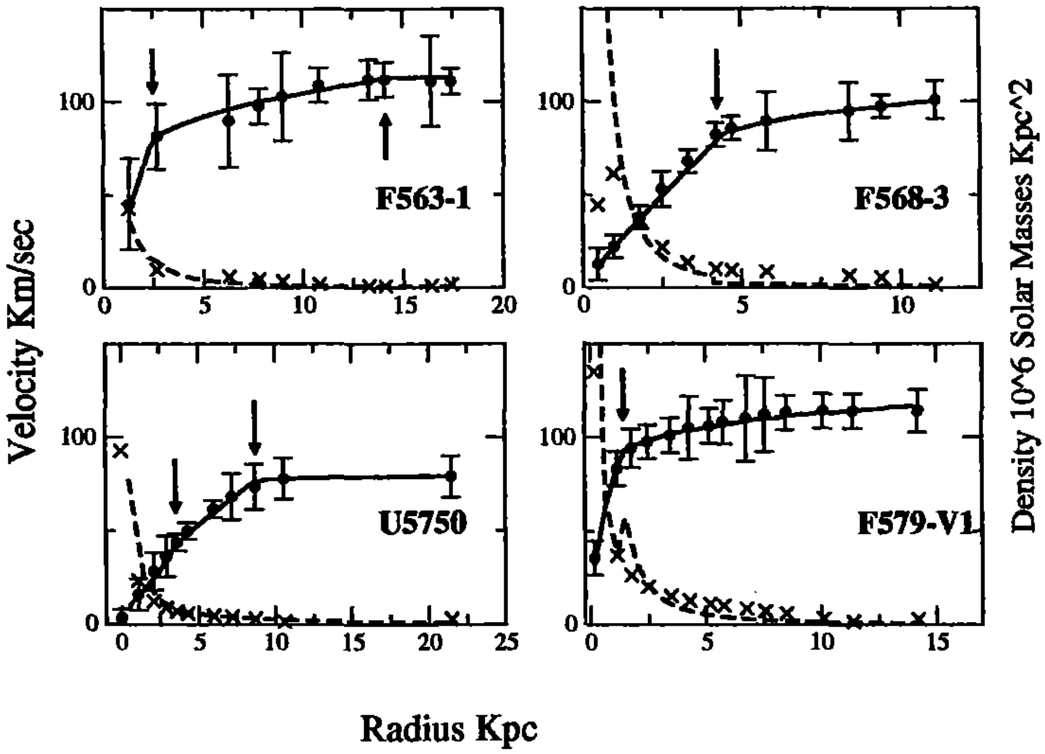

Figure 10: Solid line = calculated velocity; dotted line = calculated mass density;

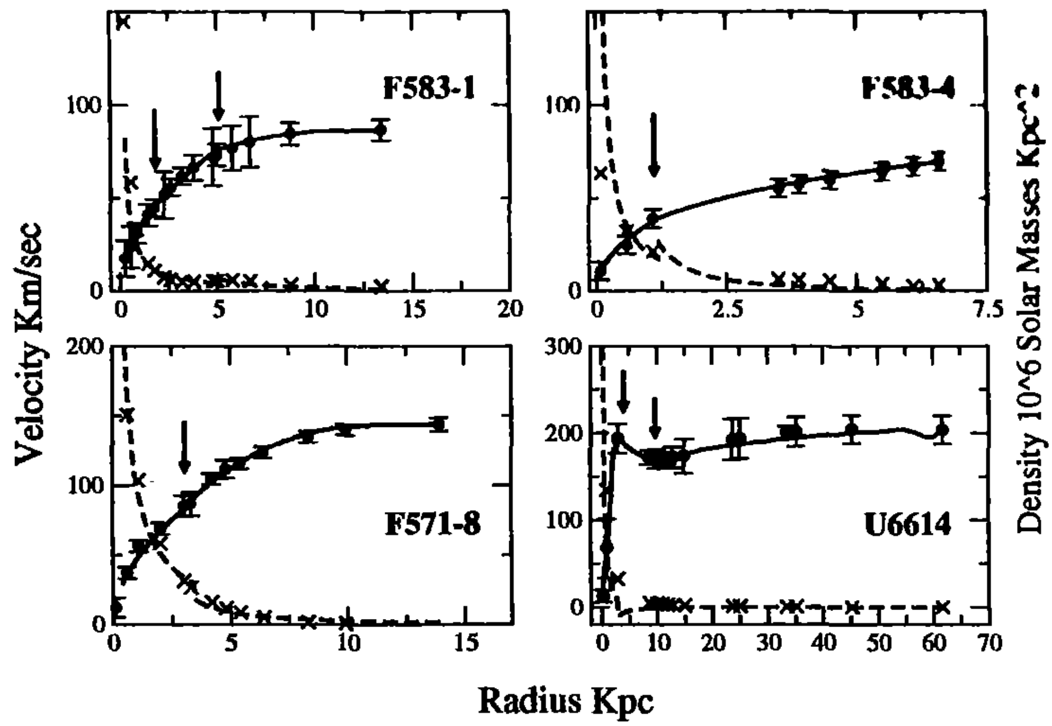

Figure 11: Solid line = calculated velocity; dotted line = calculated mass density; circles = measured velocities with error bars; crosses = estimated mass density. Arrows = switch-points.

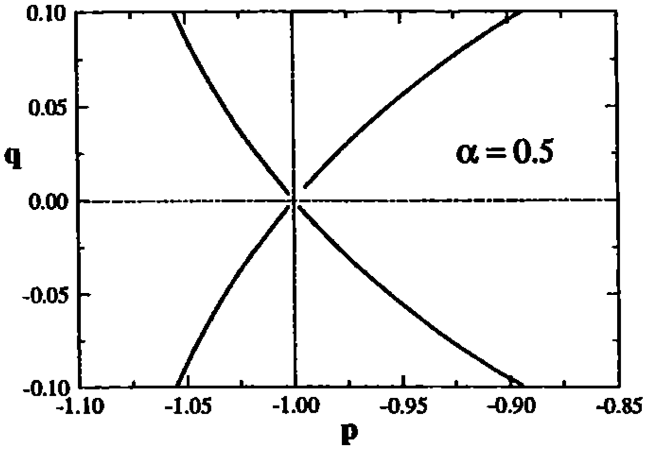

Figure 12: Intersection of $\alpha = F_i(p,q),\ i = 1..6$ in the plane $\alpha = 0.5$

7.4 Remarkable Circumstantial Evidence

We have already noted how, for each LSB in the sample, $(p,q) \approx (-1,0)$. This circumstance allows us to discover a remarkable connection between the phenomenology, as represented by these particular (p,q) values and the theory as represented by the surfaces $\alpha = F_i(p,q),\ i = 1..6$ defined at (2).

Specifically, we know that, in practice, it is virtually always the case that $0 < \alpha < 1$ (see Figure 2, for example) so that we might expect the intersection of these surfaces with the plane-surface $\alpha = 0.5$ to give a fairly typical cross-section. Figure 12 shows this cross-section in the neighbourhood of $(p,q) = (-1,0)$, which contains our LSB sample. We see immediately that the point $(p,q) = (-1,0)$ enjoys a very special status in this $\alpha = 0.5$ plane - it is, in fact, the point of intersection of *four* distinct surfaces from the set $\alpha = F_i(p,q),\ i = 1..6$ with the plane $\alpha = 0.5$. Closer investigation reveals that $(p,q) = (-1,0)$ retains its status as a special nodal point for all $(0 < \alpha < 1)$, and is therefore a *distinguished axis* for the theory.

So, we have the circumstance that our LSB sample lies in the neighbourhood of a distinguished axis of the theory defined by the intersection of four particular surfaces. Whilst it is probably not possible to say what the meaning of this is at present (we need larger samples of LSBs and corresponding samples of ordinary spirals with complete mass models), there is a very good chance that it represents a circumstance of considerable significance in the overall context of galactic evolution and dynamics.

7.5 Global Complex Structure and α Phenomenology

Figure 13 shows the evolution of three of the six surfaces as α varies in the range $(1.5, -0.5)$ (the three omitted surfaces are mirror images of the three shown). At $\alpha = 1$ (not explicitly shown) there is a degeneracy as the three curves merge identically along the $q = 0$ axis whilst, at $\alpha = 0$ (explicitly shown), there is a degeneracy in which two of the curves merge identically, although not along any axis. Thus, the two values $\alpha = 0$ and $\alpha = 1$ which, according to figure 2, appear to bound the phenomenology, are also each associated with degenerate transitions of the surface topology.

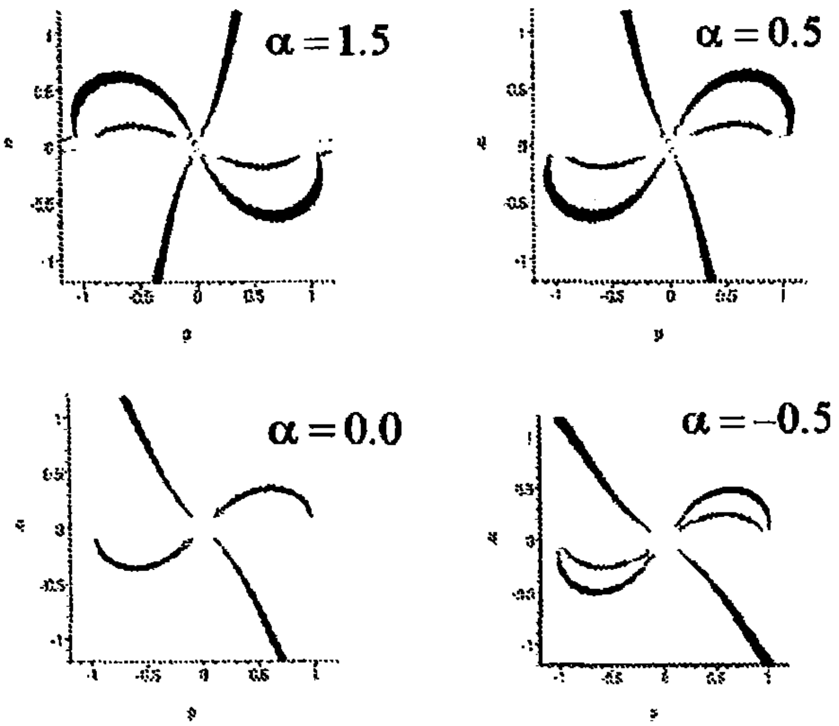

Figure 13: Intersection of $\alpha = F_i(p,q),\ i = 1..3$ in the planes $\alpha = 1.5, 0.5, 0.0, -0.5$

8 More On The Discrete Dynamical Classes Phenomenology

The LSB modelling exercise of §7 was significant at two levels:

- firstly, it showed how the theory successfully models the details of LSB dynamics and mass distributions;
- secondly, it led us to the significant discovery that LSBs are strongly associated with the distinguished axis $(p,q) = (-1,0)$ in the (α,p,q) parameter space.

The second point poses questions relevant to gaining a detailed understanding of the discrete dynamical states phenomenology. Specifically, according to the discrete dynamical states phenomenology, *low-luminosity* spirals are associated with the A-state typified in figure 4. The LSBs of our sample are, by definition, low-luminosity spirals and are therefore, presumably, A-state spirals. But, we have also seen that these LSBs are strongly associated with the distinguished axis, $(p,q) = (-1,0)$, and so the obvious question now is: what is the distribution of higher-luminosity spirals (that is, B, C and D-state spirals) in the (p,q) plane?

This question must be answered before we can begin to understand the discrete dynamical states phenomenology in any detailed way. But, at the moment, we cannot address this question rigorously, simply because we do not have available a sample of higher-luminosity spirals with detailed estimated mass distributions (stars+dust+gas). The reason for this lack of data is simple: since, according to the canonical view, $> 95\%$ of galaxy mass is CDM then there is no point in constructing detailed maps of the distribution of ordinary matter in galaxy discs. This exercise has only been done systematically (and for LSBs only) by astronomers interested in the MOND vs CDM debate.

However, there is a possible way forward: for every one of the $\approx$ 2500 ordinary spirals used in the four discrete dynamical states analyses we have I-band (or R-band) photometry. In principle, such photometry can be used to derive broad-brush estimates of ordinary mass distributions which, combined with large statistics, might be sufficient for the task at hand. This is for future work.

9 The Detailed Dynamical Theory

In the following, we finally detail the application of the basic theory to the modelling of an idealized spiral galaxies - defined to be one without a central bulge, and without the irregularities that are routinely present in the discs of real galaxies, and without the bars that are present in many spirals. We model this idealized spiral galaxy as a mass distribution possessing perfect cylindrical symmetry.

We know, from the considerations of §F.1, that the general form of the metric tensor, for any given material distribution, is

$$g_{ab} \equiv \nabla_a \nabla_b U \equiv \frac{\partial^2 \mathcal{M}}{\partial x^a \partial x^b} - \Gamma^k_{ab} \frac{\partial \mathcal{M}}{\partial x^k},$$

where Γ^k_{ab} represents the metrical affinity and where, by the conjecture of §3.2, the value of $\mathcal{M}(\mathbf{r})$ is to be generally understood as a measure of the amount of (ordinary) mass contained within the level surface passing through the point with position vector $\mathbf{r}$ defined with respect to the mass-centre of the ordinary perturbing matter distribution. In the present case, of course, the level surfaces are infinite cylinders, and so we have to modify the definition so that $\mathcal{M}(\mathbf{r})$ refers to the mass contained within cylinders of unit thickness. With this understanding, and remembering that $\mathcal{M}' \equiv d\mathcal{M}/d\Phi$ and that $\Phi = R^2/2$, it is easily shown that $\mathcal{M}' \equiv 2\pi\rho$, where ρ is the mass density in a disc of unit thickness.

Noting that the geometry on any level-surface of a cylindrical distribution is Euclidean, it follows that the Γ^k_{ab} are identically zero in the present case. Consequently,

$$g_{ab} = \frac{\partial^2 \mathcal{M}}{\partial x^a \partial x^b} \equiv \mathcal{M}'\delta_{ab} + \mathcal{M}'' x^a x^b, \tag{10}$$

where we remember $\mathcal{M}' \equiv d\mathcal{M}/d\Phi$ and $\Phi \equiv R^2/2$ (this latter notation simplies the algebra). Defining the Lagrangian density

$$\mathcal{L} \equiv \sqrt{g_{ij}\dot{x}^i \dot{x}^j} = \left(\mathcal{M}' < \dot{\mathbf{R}}|\dot{\mathbf{R}} > + \mathcal{M}''\dot{\Phi}^2\right)^{1/2},$$

where $\dot{x}^i \equiv dx^i/dt$ etc., the equations of motion are found from the Euler-Lagrange equations as

$$2\mathcal{M}'\ddot{\mathbf{R}} + \left(2\mathcal{M}''\dot{\Phi} - 2\frac{\dot{\mathcal{L}}}{\mathcal{L}}\mathcal{M}'\right)\dot{\mathbf{R}}$$
$$+ \left(\mathcal{M}'''\dot{\Phi}^2 + 2\mathcal{M}''\ddot{\Phi} - \mathcal{M}'' < \dot{\mathbf{R}}|\dot{\mathbf{R}} > -2\frac{\dot{\mathcal{L}}}{\mathcal{L}}\mathcal{M}''\dot{\Phi}\right)\mathbf{R} = 0. \tag{11}$$

However, it is obvious that the Lagrangian density, defined above, will lead to a variational principle which is degree zero in the 'time' parameter. It follows that the equations of Euler-Lagrange pair, above, cannot be linearly independent. Whilst either equation can therefore be chosen, it transpires that the $\hat{\theta}$ component equation is algebraically less complicated.

9.1 The $\hat{\theta}$ component

We have

$$2\mathcal{M}'(R\ddot{\theta} + 2\dot{R}\dot{\theta}) + R\dot{\theta}\left(2\mathcal{M}''\dot{\Phi} - 2\frac{\dot{\mathcal{L}}}{\mathcal{L}}\mathcal{M}'\right) = 0.$$

Multiply through by $R/2\mathcal{L}$ and use $\mathcal{M}''\dot{\Phi} \equiv \dot{\mathcal{M}}'$ to get, after some rearrangement:

$$\left(\frac{\mathcal{M}'}{\mathcal{L}}\right)\frac{d}{dt}(R^2\dot{\theta}) + R^2\dot{\theta}\left(\frac{\mathcal{L}\dot{\mathcal{M}}' - \dot{\mathcal{L}}\mathcal{M}'}{\mathcal{L}^2}\right) = 0\,.$$

This integrates directly to give:

$$\frac{d}{dt}\left\{R^2\dot{\theta}\left(\frac{\mathcal{M}'}{\mathcal{L}}\right)\right\} = 0,$$

from which we see that angular momentum is *not* generally conserved. Consequently, the net disc forces are not, in general, central forces so that, correspondingly, there exists a mechanism for transferring angular momentum through the disc. This latter equation integrates to give:

$$(\mathcal{M}')^2 R^2 V_\theta^2 = m_0\left[(\mathcal{M}' + \mathcal{M}''R^2)V_R^2 + \mathcal{M}'V_\theta^2\right] \qquad (12)$$

where $V_R \equiv \dot{R}$ and $V_\theta \equiv R\dot{\theta}$.

9.2 Completion of the Dynamical System

The cylindrical symmetry of the idealized spiral galaxy implies that there is net zero force out of the plane of the galaxy. It follows that the 'self-similar' dynamics condition (cf §F.2), which must be used in conjunction with (12), to close the system can be written as:

$$\frac{Transverse\ Accn}{Radial\ Accn} = k_0. \qquad (13)$$

where, because of the radial symmetry of the system, k_0 has the same value along all radial directions. To obtain the quantitative form of this, we need expressions for the radial and transverse accelerations in the disc geometry. These are derived in appendix H, and we find that (13) becomes

$$\begin{aligned}
&- \quad k_0 S\sqrt{\frac{\mathcal{M}' + \mathcal{M}''R^2}{\mathcal{M}'}}V_R\frac{dV_R}{dR} + V_R\frac{dV_\theta}{dR} = \\
&- \quad \left(S\sqrt{\frac{\mathcal{M}' + \mathcal{M}''R^2}{\mathcal{M}'}} + \frac{\mathcal{M}''}{2\mathcal{M}'}R^2\right)\frac{V_R V_\theta}{R} \\
&+ \quad k_0\left(S\frac{3\mathcal{M}'' + \mathcal{M}'''R^2}{2\sqrt{\mathcal{M}'(\mathcal{M}' + \mathcal{M}''R^2)}}R V_R{}^2 - \frac{V_\theta^2}{R}\right) \qquad (14)
\end{aligned}$$

where $S = \pm 1$.

Remembering that $\mathcal{M}' = 2\pi\rho$ (cf start of §9), we now note that the expression $\mathcal{M}' + \mathcal{M}''R^2$ is *negative* if ρ drops off more quickly than $1/R$ - which, in practice, always seems to be the case. The implication of this practical reality is that k_0 is actually complex so that the two equations (12) and (14) represent three equations in the three unknowns $(V_\theta, V_R, \mathcal{M}')$, and not just two. It is when V_R is eliminated between these three equations that we obtain equations (4) and (6) of §4.3.

However, we initially overlooked the possibility that $\mathcal{M}'+\mathcal{M}''R^2$ might be negative and, consequently, believed we needed a further equation to close the system completely. As it happens, our approach to obtaining this extra equation gave $\mathcal{M}'+\mathcal{M}''R^2 < 0$ anyway, and effectively picked out a special solution of the above system, and is described in the following section.

10 A Special Case Solution: The Logarithmic Disc

Our oversight led us to believe that a further equation was required to close the system. We argued as follows: It has been recognized for a very long time that, if the obvious irregularities which exist in spiral discs are ignored, then the spiral structure of spiral galaxies is essentially logarithmic. In the context of a classical disc, the most direct, way to interpret this phenomenology is to write

$$\frac{V_\theta}{V_R} = K_1 \tag{15}$$

since this implies directly that disc streamlines are logarithmic spirals. Substitution of this into (12), and using $\mathcal{M}' \equiv 2\pi\rho$, gives immediately

$$\mathcal{M}' \equiv 2\pi\rho = \frac{k_2}{R^2} \tag{16}$$

for some constant k_2. That is, the density of matter in the logarithmic disc behaves as an inverse square law. We quickly find that this implies $\mathcal{M}'+\mathcal{M}''R^2 < 0$, so that we are back to the point we overlooked initially - that (14) represents, in fact, two distinct equations which must both be satisfied. More particularly, putting (15) and (16) into (14) we find

$$(K_1 - k_0Sj)\frac{1}{V_R}\frac{dV_R}{dR} = -\frac{1}{R}((Sj-1)K_1 + k_0(Sj+K_1^2)),$$

where $S = \pm 1$ and $j = \sqrt{-1}$. Since solutions must be real, we must have k_0 complex. Putting $k_0 = q+pj$, where (p,q) are real parameters, then gives the two equations

$$\frac{1}{V_R}\frac{dV_R}{dR} = \frac{1}{R}\left(\frac{K_1 - qK_1^2 + pS}{pS+K_1}\right), \quad \frac{1}{V_R}\frac{dV_R}{dR} = \frac{1}{R}\left(\frac{SK_1 + pK_1^2 + qS}{qS}\right) \equiv \frac{\alpha}{R} \tag{17}$$

which must be identical. This latter requirement quickly gives that the parameters (K_1, p, q) must satisfy

$$pK_1^2 + S(1+p^2+q^2)K_1 + p = 0. \tag{18}$$

If we use this to eliminate the explicit appearance of K_1^2 in the second of (17), we get

$$\alpha = 1 - \frac{p}{Sq} - \frac{p^2+q^2}{q}K_1 = 1 - \frac{p}{Sq} - \frac{p^2+q^2}{qK_1}.$$

The second expression for α, which is the form given at (2), occurs since, if K_1 is a solution of (18), then so is $1/K_1$.

11 The MOND Programme

11.1 Overview of MOND

Modified Newtonian Dynamics (MOND) is an empirically motivated modification of Newtonian gravitational mechanics which can be interpreted as either a modification of Newton's gravitational law, or as a modification of the second law (the law of inertia). See Milgrom [1994] for a comprehensive review. The basic idea was conceived by Milgrom [1983a], [1983b], [1983c], as a way of understanding the flat rotation curve phenomenology of spiral galaxies without recourse to the dark matter idea. The basic hypothesis is that, in extremely weak gravitational fields ($g << 10^{-10}ms^{-2}$), the nature of Newtonian gravitation changes in such a way that flat rotation curves are the natural result.

The quantitative idea can be briefly described as follows: if we use $g_N(\mathbf{R})$ to denote the gravitational acceleration at any position $\mathbf{R}$ in a material distribution *according to Newtonian theory*, then the MOND prescription says that the actual gravitational acceleration is given by $g = \sqrt{a_0\, g_N(\mathbf{R})}$ when the field is *extremely weak*. Here a_0 is a parameter which has been fixed (Begeman et al.[1991]) for all applications of MOND to the value $a_0 = 1.2 \times 10^{-10}\, ms^{-2}$. Thus, for a particle in a circular orbit about a point source of mass M, the acceleration-balance equation in the MOND limit of the very weak field would be given by

$$\frac{V_{rot}^2}{R} = \sqrt{\frac{\gamma M}{R^2} a_0} \;\rightarrow\; V_{rot} = (\gamma\, M\, a_0)^{1/4},$$

where V_{rot} is the circular velocity.

This simple idea, which in practice, leads to a model with only one free parameter (the so-called mass-to-light ratio, $\gamma^* \equiv M/L$, which gives the conversion factor from the observed light to the inferred mass), has been remarkably successful in explaining the phenomenology associated with a very wide variety of circumstances (de Blok & McGaugh [1998]). Furthermore, MOND makes several strong predictions (Milgrom [1983b]) and is therefore eminently falsifiable. It is notable, therefore, that it never has been although several attempts have been made (de Blok & Mcgaugh [1998]). This circumstance is to be compared with that of the conventionally favoured CDM (Cold Dark Matter) model. This latter model is a multi-parameter model, which makes no predictions - other than that of the existence of CDM - and can therefore never be falsified in the classical sense.

11.2 General Comments on MOND and the Present Theory

If the MOND prescription is an accurate reflection of the reality, then there necessarily exists an underlying theory of gravitation which provides flat rotation curves in the weak field regime independently of dark matter distributions. Conversely, any such theory - should it exist - must necessarily give rise to the basic MOND prescription, $g = \sqrt{a_0\, g_N(\mathbf{R})}$, in the weak field when interpreted from a Newtonian perspective.

In the present case, flat rotation curves correspond to the degenerate $\alpha = 0$ case discussed in §7.5. Thus, flat rotation curves solutions do exist in the presented theory, and are associated with a degeneracy, and are therefore special in some - yet to be understood - way. The inclusion of the qualitative aspects of MOND in the theory is already guaranteed by the simple fact that flat rotation curve solutions are admitted.

11.3 LSBs - Extreme Objects in MOND and the Present Theory

According to any given theory, when the mass/dynamical relations in a given spiral object cross a certain threshold specific to that theory, the object ceases to be gravitationally bound. In the early 1980s Milgrom realized that, according to MOND, there should exist spiral objects which were so diffuse that they could not possibly exist according to the canonical theory. These objects, now known as Low Surface Brightness galaxies (LSBs), were subsequently observed in the late 1980s. Of all astrophysical objects, these present the most critical conditions for the CDM models since, according to these models, LSBs must typically consist of more than 99% dark matter. Even so, it is now well recognized that the CDM models have suffered comprehensive failure when applied to model LSBs.

Beyond the initial prediction, the importance of LSBs to MOND can be summarized as follows: a common criticism of MOND is that, since it was designed to yield flat rotation curves in the very weak-field regimes of spiral exteriors, it is hardly surprising that it does so. However, LSBs are so low-mass and diffuse that virtually the whole of the typical LSB disc is in the MOND regime - including the rising segments. Thus, the very successful application of MOND to LSBs has completely undermined such criticisms.

However, we have shown in this paper that a sample of LSBs of widely differing dynamical properties are modelled virtually perfectly by the presented theory. We can therefore conclude that the quantitative aspects of MOND must also be included within it. In practice, of course, the theory is very much more difficult to apply than MOND because, unlike MOND which uses the observed mass distributions to make its dynamical predictions, it requires the mass distributions to be calculated using (4) - and this is a very difficult equation to integrate because the switching points must (currently) be found by trial and error.

12 Conclusions

The fractal $D = 2$ inertial universe (Roscoe [2002a]) provides an entirely new way of understanding the idea of 'inertial space & time', and can be considered as the strongest possible realization of Mach's Principle. We have considered how gravitational processes might arise in such a universe, and have indicated how to derive the dynamical equations for extended high-symmetry mass systems. The process has been explicitly illustrated by applying it to derive the model equations for an idealized spiral galaxy, defined as one possessing perfect cylindrical symmetry.

The parameter space, (p, q, α), of these dynamical equations has a complicated topology, and we have shown how various aspects of the phenomenology have a ready qualitative explanation in terms of it - in particular, the *discrete dynamical states* phenomenology falls into this category.

The theory has been very successfully applied to model the dynamics and mass distribution of eight Low Surface Brightness spiral galaxies which, hitherto, have been successfully modelled only by the MOND algorithm introduced by Milgrom [1983a], [1983b], [1983c]. The CDM models inevitably fail badly in this context. Of equal significance to the theory's success in this context is the fact that the values necessarily assigned to the parameters (p, q) for each of the eight LSBs are all in the neighbourhood of $(p, q) = (-1, 0)$ - which, it transpires, is a *very special* distinguished axis in the (p, q, α) parameter space. As well as providing further circumstantial support for the theory, this latter fact suggests the possibility of intimate connections between the topology of the parameter space and galactic evolution.

To summarize, we have presented a theory with a very richly structured parameter space and have shown how several aspects of spiral galaxy phenomenology fit beautifully into this structure. There would appear to be every prospect that the theory can form the basis of understanding spiral galaxies and their evolution in a way that has hitherto not been possible. Only very much more work involving the analysis of many more spirals of all types will show if this bold claim can become a reality.

A Preliminaries

Mass in the $D = 2$ equilibrium universe of Roscoe [2002a] is distributed according to

$$M = \frac{m_0}{r_0^2} r^2,$$

so that m_0 is the amount of mass contained in side a sphere of arbitrary radius r_0, and $g_0 \equiv m_0/r_0^2$ is a global constant of this equilibrium universe.

However, this model universe is, in fact, a particular case of a class of non-equilibrium model universes possessing a general spherical symmetry. But, since we are primarily interested in non-spherical systems, we use the $D = 2$ equilibrium solution as our starting point, and consider how to perturb that in progressively complex ways.

The general spherically symmetric model has an associated potential function defined in Roscoe [2002a] as

$$V \equiv C_0 - \frac{v_0^2}{4\, d_0^2\, g_0} A + \frac{B}{2A} r^2 \dot{r}^2, \tag{19}$$

where C_0 is the arbitrary constant usually associated with potential functions, v_0 is a global constant having the dimensions of velocity, $g_0 \equiv m_0/r_0^2$ is the global constant defined above, d_0 is a dimensionless constant evaluated below, and

$$A \equiv \frac{2\mathcal{M}}{r^2}, \quad B \equiv -\left(\frac{2\mathcal{M}}{r^4}\right) + \frac{\mathcal{M}'\mathcal{M}'}{2d_0\mathcal{M}}, \quad \mathcal{M}' \equiv \frac{d\mathcal{M}}{d\Phi}, \quad \Phi \equiv \frac{r^2}{2} \tag{20}$$

where the notation $\Phi \equiv r^2/2$ is introduced to simplify the algebra later on, and

$$\mathcal{M} = d_0 M + m_1, \quad M = m_0 \left(\frac{r}{r_0}\right)^2 + 2\sqrt{\frac{m_0 m_1}{d_0}} \left(\frac{r}{r_0}\right). \tag{21}$$

As before, M quantifies the mount of mass inside a sphere of radius r whilst m_1 an arbitrary constant having units of mass which quantifies the perturbation from the $D = 2$ equilibrium universe. The dimensionless constant d_0 can be determined by noting that the special case $m_1 = 0$ must recover the equilibrium (inertial) case, which requires $A = const$ and $B = 0$. Reference to the above shows that this can only happen if $d_0 = 1$, and so this value is assumed from hereon.

A.1 Interpretation Issues

The equilibrium case ($V = const$) of globally inertial conditions arises when $d_0 = 1$, $m_1 = 0$ and $\mathcal{M}(r) = m_0 r^2/r_0^2$ about any centre. Since $\mathcal{M}(r)$ (in the form of M) is interpreted as the amount of mass in a sphere of radius r, then it follows immediately that globally inertial conditions are irreducibly associated with a fractal $D = 2$ mass distribution. However, it was noted in Roscoe [2002a] that this mass exists in the form of a 'quasi-photon' gas - that is, it consists of primitive particles moving in arbitrary directions but in otherwise identical states of motion, in direct analogy with photons in a vacuum. For this reason, we interpreted it as a 'quasi-classical' vacuum gas and noted that, assuming conventional material 'condenses' out of this vacuum gas in some way - perhaps by collision processes - then the theory provided a direct way of understanding the observed $D \approx 2$ distribution of galaxies on medium scales.

In the present case, we consider perturbations of $\mathcal{M}$ that are not generally spherically symmetric. This has the direct result that our interpretation of $\mathcal{M}$ must necessarily evolve such that our original understanding is included as a special case.

B Newtonian Gravitation For Test Particles

In this section, we establish the principle that classical Newtonian gravitation can be recovered as a point-mass perturbation of the $D = 2$ equilibrium universe, $\mathcal{M} = m_0 r^2/r_0^2$ where we remember $g_0 \equiv m_0/r_0^2$ is a global constant. But, as a by-product of this analysis, we also establish that the point-mass perturbation necessarily picks up its conventional mass properties via a global interaction - as required by conventional interpretations of Mach's Principle.

We consider the most simple possible perturbation of the $D = 2$ distribution (in fact, the original one given in Roscoe [2002a])which (21) (with $d_0 = 1$) shows is given by

$$\mathcal{M} = \left(\frac{\sqrt{m_0}}{r_0} r + \sqrt{m_1}\right)^2 \qquad (22)$$

where the coordinate origin, $\mathbf{r} = 0$, is the position of the perturbing mass.

The general form of the potential function, V, generated by an arbitrary spherically symmetric mass distribution, $\mathcal{M}$, is given by (19) with (20); consequently, to consider the circumstances under which (19) gives rise to Newtonian gravitation - if at all - it is only necessary to consider the structure of this potential when $\mathcal{M}$ is defined by (22). However, this form of the potential function is an explicit function of r and $\dot{r}$, which makes analysis more difficult. A more convenient form, expressed purely in terms of r, is given in [2002a]. For the particular case $d_0 = 1$, this is given by:

$$V(r) \;=\; -\frac{2v_0^2}{r}\sqrt{\frac{m_1}{g_0}} - \frac{1}{r^3}\sqrt{\frac{m_1}{g_0}}\left(\frac{2m_1 v_0^2}{g_0} - h^2\right) - \frac{1}{2r^4}\frac{m_1}{g_0}\left(\frac{m_1 v_0^2}{g_0} - h^2\right) \qquad (23)$$

where h is the classical angular momentum. This can only become a *first order* approximation for potential of classical Newtonian theory if

$$2v_0^2\sqrt{\frac{m_1}{g_0}} = \gamma M_S \qquad (24)$$

where M_S is the conventional mass of the central disturbing distribution, γ is the usual gravitational constant and we remember that m_1, which has units of mass, is a quantitative measure of the perturbing disturbance in the equilibrium universe. This relation is extremely interesting since, if γ is a global constant as we believe, then it effectively states that

$$M_S = \sqrt{M_0 m_1}$$

where M_0 is a global scaling constant with dimensions of mass. In other words, the perturbing disturbance, quantified by the parameter m_1 originally, picks up its conventional mass properties via a global interaction - which is a common interpretation of Mach's Principle.

In terms of (24), then (23) becomes

$$V(r) = -\frac{\gamma M_S}{r} - \frac{\gamma M_S}{r^3}\left[\left(\frac{\gamma M_S}{2v_0^2}\right)^2 - \frac{h^2}{2v_0^2}\right] - \frac{\gamma M_S}{r^4}\left[\left(\frac{\gamma M_S}{2v_0^2}\right)^2 - \frac{h^2}{v_0^2}\right]\frac{\gamma M_S}{8v_0^2}.$$

It is clear from this expression that standard Newtonian results will be reproduced, provided v_0^2 is sufficiently large.

C The Two-Body Problem

We know from the Newtonian analysis that the two-body problem is essentially spherically symmetric. Consequently, the two-body mass function $\mathcal{M}$ necessarily has a structure similar to the one-body case, given at (22), except that an extra degree of freedom must be accounted for. This can be most easily accomplished by a mass-function similar to:

$$\mathcal{M} = m_0\left(\frac{r}{r_0}\right)^2 + m_1\left(\frac{r}{r_0}\right) + m_2 \qquad (25)$$

where (m_1, m_2) have the dimensions of mass, and will be functions of the masses of the perturbing sources.

A similar analysis to that of the one-particle case produces a relation similar to (24) - with the major difference that M_S is replaced by the Newtonian expression for the effective gravitational mass at the mass-centre of a two-body system.

D Global Momentum Conservation And Consequences For $\mathcal{M}$

Having established how classical Newtonian gravitation can, in principle, arise as a point-mass perturbation of the inertial fractal $D = 2$ universe, we can consider the question of point-mass perturbations by an ensemble of N conventional particle masses. But this automatically raises the question of momentum conservation in the ensemble, which we consider here.

We find the remarkable result that momentum conservation in the finite ensemble of particles requires that the distribution of vacuum mass, $\mathcal{M}$, must be expressible as an *even* function of $m\mathbf{r}$, where m is the inertial mass of an arbitrarily chosen ensemble particle, and $\mathbf{r}$ is its position defined with respect to the ensemble mass centre. This then leads to a *tentative* reinterpretation of $\mathcal{M}$ as a measure of the total vacuum mass detected by a particle of inertial mass m at position $\mathbf{r}$ - or, to be more specific, it suggests that $\mathcal{M}(m\mathbf{r})$ is a measure of the total vacuum mass contained with the level surface which passes through the point $\mathbf{R} \equiv |m|\mathbf{r}$.

However, since we are also accustomed to thinking that, within a gravitating particle ensemble, the orbit of a particular mass m is a function of the distribution of the other masses within the ensemble, then it would appear that the measure of total vacuum mass, $\mathcal{M}(m\mathbf{r})$, within the level surface *must also* be a measure of the total inertial mass of the finite particle ensemble. The implication is that there is a deep association between the inertial masses of the particles in the ensemble and the vacuum mass.

We adopt the tenative working interpretation that, for the case $m = 1$, then $\mathcal{M}(\mathbf{r})$ is a measure of the inertial mass contained within the level surface which passes through the point which has position vector $\mathbf{r}$ with respect to the ensemble mass centre.

D.1 The Details

Firstly, since we have a finite ensemble embedded in an equilibrium background, we can suppose that all discussion of momentum conservation can be referred to the mass centre of the ensemble, and that this mass centre is in dynamic equilibrium with the background.

For any system of particles of masses $M_1, ..., M_N$, described from a centre-of-mass frame, the integrated momentum-conservation equation becomes

$$M_1\mathbf{R}^1 + M_2\mathbf{R}^2 + ... + M_N\mathbf{R}^N = 0.$$

The masses appearing in this equation are now arbitrarily partitioned into the pair of ensembles $M_1, ..., M_{k-1}$ and $M_k, ..., M_N$. Defining the mass of the whole system as M, and the mass of the ensemble $M_1, ..., M_{k-1}$ as m, then the foregoing equation can be written as

$$m\mathbf{r} + (M - m)\mathbf{R} = 0,$$

where $\mathbf{r}$ and $\mathbf{R}$ are the respective mass-centres of the two, arbitrarily defined, particle ensembles defined with respect to the mass centre of the whole ensemble. Any interaction can then be considered as being between the particle ensemble of mass m (which can represent a planet or star or galaxy, etc) and the rest of the ensemble, having mass $M - m$. Whatever the details of this interaction, these two particle ensembles must, together, evolve from their initial state in such a way that linear momentum is conserved for all $t > 0$ so that, always,

$$m\mathbf{r} = -(M - m)\mathbf{R}. \tag{26}$$

Now, we know from classical Newtonian theory that the two-body problem can be reduced to spherically symmetric form, and so we can expect the same here. Consequently, from the point of view of either of the two bodies, perturbations of the mass function, $\mathcal{M}$, are also spherically symmetric about the ensemble mass centre. But, it is easily shown that the equations of motion for the spherically symmetric system (Roscoe [2002a]) are scale-invariant under $\mathbf{r} = \lambda\mathbf{R}$ for non-zero constant λ, up to unspecified $\mathcal{M}$. Consequently, under (26), the structure of the equations of motion remains unchanged up to $\mathcal{M}$ not specified. It follows that if $\mathcal{M}$ is an *even* function of $m\mathbf{r}$ then the equations of motion for $m\mathbf{r}$ will transform into *identical* equations of motion for $(M - m)\mathbf{R}$ under (26) so that, with the initial condition $m\mathbf{r}(0) = -(M - m)\mathbf{R}(0)$, the calculated trajectories will satisfy $m\mathbf{r} = -(M - m)\mathbf{R}$ for all time. It is easily seen that no other form of $\mathcal{M}$ has this property. It follows that, for global momentum conservation, $\mathcal{M}$ must be an even function of $m\mathbf{r}$ - as stated.

E The Three-Body Problem

We now consider the structure of $\mathcal{M}$ appropriate to the non-spherical case. A 'degrees of freedom' argument clarifies the situation - note that we make the fundamental assumption that individual particle masses in the perturbing ensemble have no intrinsic angular momentum.

Firstly, consider a two-body system: notionally, in the absence of intrinsic angular momentum, each particle mass in the system has six degrees of freedom consisting of three positional and three kinematic. However, in this simple system, once these are set for one mass, momentum conservation fixes everything for the second mass and so there are only six degrees of freedom in total for this case. But the equations of motion for the 'free' body in this system are second order in three components, and therefore require all six of these degrees of freedom for their closure. It follows that $\mathcal{M}$ need contain no freedom to describe any positional or kinematic qualities of the detected mass distribution it describes, and this is reflected in the simple structure of (25).

Now consider what happens when a third perturbing mass is introduced: If m_1 is any one of the three masses in the system, then $\mathcal{M}(m_1\mathbf{r})$ quantifies the total effective gravitational mass detected by m_1 at position $\mathbf{r}$, and provides the basis for the equations of motion of m_1, and these equations of motion still require only six degrees of freedom for their closure. However, the introduction of the third perturbing mass has introduced six extra degrees of freedom into the system which, since these degrees of freedom are not required by the equations of motion for m_1, must therefore be incorporated into the mass function $\mathcal{M}$ itself.

Introducing the notation $r \equiv <\mathbf{r}|\mathbf{r}>^{1/2} \equiv <\mathbf{r}|\mathbf{I}|\mathbf{r}>^{1/2}$, where $\mathbf{I}$ is the unit matrix, and bearing in mind the idea that, at large distances from the mass centre, we might expect $\mathcal{M}(m\mathbf{r})$ for any finite ensemble to behave like a point-perturbation, then the most obvious perturbation of (25) having the structure $\mathcal{M}(m\mathbf{r})$, and which incorporates the required six degrees of freedom, is given by

$$\mathcal{M} = \kappa_0 m^2 r^2 + \kappa_1 m <\mathbf{r}|\mathbf{I}|\mathbf{r}>^{1/2} + \kappa_2 + \kappa_3 m^{-1} <\mathbf{r}|\mathbf{A}|\mathbf{r}>^{-1/2}, \tag{27}$$

where m is the mass of the chosen particle and $\mathbf{A}$ is a positive (semi) definite 3×3 matrix which provides the required extra degrees of freedom. The restriction of $\mathbf{A}$ to positive (semi) definiteness is imposed by the square-root operation and this, in turn, is what limits $\mathbf{A}$ to contain only six free parameters.

F The N-Body Problem

The generalization to N bodies (each one requiring six degrees of freedom) is now fairly obvious, and is given by

$$\mathcal{M}(m\mathbf{r}) = \kappa_0 m^2 r^2 + \sum_{j=1}^{N-2} \kappa_j m^{2-j} <\mathbf{r}|\mathbf{A}_k|\mathbf{r}>^{1-j/2}, \quad N > 2, \tag{28}$$

where κ_j, $(j = 1, ..., N-2)$ are constants which are independent of the chosen mass, and $\mathbf{A}_j$, $(j = 1, ..., N-2,\ N > 2)$ is a class of positive (semi) definite matrices and $\mathbf{A}_1 \equiv \mathbf{I}$.

The equations of motion for m are defined in terms of this $\mathcal{M}$, and there will be similar definitions for each of the other masses in the system. Since the mass detected by any chosen mass is the whole system sans the chosen mass itself, it follows that, in the most general situation, each individual particle mass in the perturbing ensemble will have its own unique set of $\mathbf{A}_k$ matrices. However, situations of high symmetry can be imagined in which every mass in the ensemble will detect identical things; it can also be expected that circumstances will exist for which $\mathbf{A}_1 = \mathbf{A}_2 = ... = \mathbf{A}_{N-2}$ for any chosen particle mass; in such a case, $\mathcal{M}$ will attain a maximal simplicity for the chosen particle mass.

F.1 The Time-Invariant Subset

It remains to define the equations of motion which correspond to the mass function defined at (28). To this end, we note that the arguments of Roscoe [2002a] which lead to the definition of the metric tensor, g_{ab}, from the mass function, $\mathcal{M}$, are strictly independent of any assumptions of spherical symmetry. Consequently, we still have

$$g_{ab} \equiv \nabla_a \nabla_b \mathcal{M} \equiv \frac{\partial^2 \mathcal{M}}{\partial x^a \partial x^b} - \Gamma^k_{ab} \frac{\partial \mathcal{M}}{\partial x^k}, \tag{29}$$

where Γ^k_{ab} is determined by the metric affinity. This choice was made because it guarantees that appropriate generalizations of the divergence theorems exist - which is necessary if we are to have conservation laws.

Assuming this system implies a unique determination of g_{ab} in terms of $\mathcal{M}$ (as it does for the case of arbitrary spherical symmetry), then the Lagrangian density $\mathcal{L} = \sqrt{g_{ij}\dot{x}^i\dot{x}^j}$ can defined, and the equations of motion defined as the Euler-Lagrange equations. However, as pointed out in Roscoe [2002a], since the variational principle arising from this $\mathcal{L}$ is invariant with respect to arbitrary transformations of the 'time' parameter, then physical time has yet to be defined.

F.2 Physical Time Defined By The Self-Similarity Of Forces.

This problem of 'physical time' was resolved in Roscoe [2002a] by the application of the condition that *all accelerations are directed through the global mass-centre* - but this condition arises partly from the circumstance of spherical symmetry, and so is not appropriate to the general case being considered here. However, as indicated in Roscoe [2002a], this latter condition actually represents an integrated form of the fundamental Newtonian condition:

C0 *The action between any two material particles is along the shortest path (a straight line, classically) joining them.*

Consequently, it is this part of the physics which has to be given appropriate expression in the present circumstance, and which will complete the equations of motion.

The first relevant point in this connection is the recognition that the incomplete equations of motion are scale invariant up to the specification of the mass function $\mathcal{M}$. The second relevant point is the recognition that the statement **C0** above is also a scale-invariant statement. It follows that when the equations of motion are completed by an appropriate application of **C0**, they will remain scale-invariant up to the specification of the mass function.

Let us therefore consider the hypothetically completed equations of motion for the chosen mass m, and suppose that they are represented as

$$\mathbf{F}(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}}, \mathcal{M}(m\mathbf{r})) = 0, \tag{30}$$

so the trajectory of m is represented by $\mathbf{r}(t)$. Since $\mathbf{F}$ must be scale-invariant up to the specification of $\mathcal{M}$ then, under the change of scale $\mathbf{r} = \lambda\mathbf{R}$, (30) becomes

$$\mathbf{F}(\mathbf{R}, \dot{\mathbf{R}}, \ddot{\mathbf{R}}, \mathcal{M}(\lambda m\mathbf{R})) = 0, \tag{31}$$

which can be interpreted to mean that the trajectory of a chosen mass λm is given by $\mathbf{R}(t)$.

Now consider a very special case: since $m\mathbf{r}(t)$ in (30) satisfies the same equation as $\lambda m\mathbf{R}(t)$ in (31) then, for properly matched initial conditions, $m\mathbf{r}(t) = \lambda m\mathbf{R}(t)$, $t \geq 0$. That is, under these specially chosen initial conditions, the trajectory $\mathbf{R}(t)$ of mass λm is geometrically similar to the trajectory $\mathbf{r}(t)$ of mass m.

But we now note that a sufficient *dynamical* condition for the trajectories to be geometrically similar in this way (given the special initial conditions) is that the force system acting on mass λm is geometrically similar to the force system acting on mass m. That is, if the resolved force components acting on the chosen mass are denoted by (F_1, F_2, F_3), then along any radial drawn from the system mass-centre we have

$$\frac{F_2}{F_1} = k_0, \ \frac{F_3}{F_1} = k_1 \qquad (32)$$

for parameters k_0, k_1 which are constant along any given radial direction. Of course, in cases of perfect radial symmetry (k_0, k_1) will be global constants. Since, generally speaking, we do not expect the dynamical constraints acting in a system to depend on the initial conditions, then we can take (32) to be a general dynamical law in the system.

Finally, we note that, since (32) can only be deduced from the completed dynamical system (30), then the *incomplete* dynamical system can be completed by augmenting it with (32). In other words, the application of (32) to the system is equivalent to applying the Newtonian law **C0**.

G The Level-Surface Geometry For Very Large N-Body Systems

The mass function for a chosen particle, mass m, in a system of N massive particles is defined at (28). For $N = 0$ (the inertial fractal D = universe), the level surfaces of $\mathcal{M}$ are spherical surfaces centred anywhere. For $N = 1$ they are spherical surfaces centred on the single perturbing mass. For $N = 2$ they are spherical surfaces centred on the mass-centre of the two perturbing masses.

The addition of perturbing masses beyond $N = 2$ made it necessary to introduce successive ellipsoidal perturbations of $\mathcal{M}$ so that its level-surfaces will be successively perturbed to form a sequence of differing level-surface structures. It is obvious that a *closed* description of the geometry of $\mathcal{M}$ (and hence of an analytic determination of these level surfaces) will be difficult (if not impossible) to calculate even for $N = 3$, and almost certainly impossible for any N which is moderately greater than this value.

However, there exists an alternative means of determining - at least qualitatively - the level-surface geometry of $\mathcal{M}$ for high-symmetry very large N systems. Whatever these level-surfaces represent (for example, surfaces of constant $\dot{g}$-magnitude, as in the spherically symmetric case), they must be determined by the bulk spatial and kinematic properties of the distributions concerned, and must therefore reflect any symmetries possessed by these properties. Nature tells us that many large N distributions (in the form of galaxies) naturally form such structures as spiral discs, ellipsoids, rings etc which possess various forms of high-symmetry. For example, in spiral galaxies, the spatial and kinematic information says 'rotating disc' and so we can reasonably deduce that the level-surfaces of any corresponding $\mathcal{M}$ must likewise possess plane rotational symmetry. Similarly, the rotational symmetry of an elliptical galaxy about its long axis and its reflective symmetry about the plane through the mass centre which contains the two minor axes, provides corresponding information about the internal symmetries of its $\mathcal{M}$ function.

Once a geometry for the level-surfaces in a given distribution has been identified (at least qualitatively), then these level-surfaces can be used to parametrize $\mathcal{M}$ - exactly as was done for the spherical analysis of Roscoe [2002a]. For simple geometries such as spheres, discs and ellipsoids, this parametrization allows (29) to be solved for the explicit form of the metric tensor, g_{ab}, in terms of $\mathcal{M}$ and its derivatives with respect to the parametrization. The equations of motion can then be determined in the same form, and the system completed by the 'similarity of forces' condition developed in §F.2.

H Completion of the Dynamical System: Details

We require a quantitative expression for (13) in the particular geometry of our disc. We begin by noting that, from (10), we have

$$\begin{aligned} ds^2 &= \mathcal{M}' dx^i dx^j \delta_{ij} + \mathcal{M}'' x^i x^j dx^i dx^j \\ &= \mathcal{M}' \left(dR^2 + R^2 d\theta^2 \right) + \mathcal{M}'' R^2 dR^2 \\ &= \left(\mathcal{M}' + R^2 \mathcal{M}'' \right) dR^2 + \mathcal{M}' R^2 d\theta^2 \\ &= \left\{ S_0 \sqrt{\mathcal{M}' + R^2 \mathcal{M}''}\, dR \,\hat{\mathbf{R}} + S_1 \sqrt{\mathcal{M}'}\, R\, d\theta\, \hat{\theta} \right\}^2 . \end{aligned}$$

where we have introduced the orthogonal unit vectors, $\hat{\mathbf{R}}$ and $\hat{\theta}$, and where $S_0 = \pm 1$ and $S_1 = \pm 1$ and are independent. Consequently,

$$d\mathbf{s} = \left\{ S_0 \sqrt{\mathcal{M}' + R^2 \mathcal{M}''}\, dR \,\hat{\mathbf{R}} + S_1 \sqrt{\mathcal{M}'}\, R\, d\theta\, \hat{\theta} \right\} ,$$

so that the vector velocity is given as:

$$\mathbf{V} = \left\{ S_0 \sqrt{\mathcal{M}' + R^2 \mathcal{M}''}\, \dot{R} \,\hat{\mathbf{R}} + S_1 \sqrt{\mathcal{M}'}\, R \dot{\theta}\, \hat{\theta} \right\} . \tag{33}$$

From this,we can now calculate the vector acceleration as:

$$\begin{aligned} \dot{\mathbf{V}} &= \left\{ S_0 \frac{3\mathcal{M}'' + 2\Phi \mathcal{M}'''}{2\sqrt{\mathcal{M}' + 2\Phi\mathcal{M}''}} R \dot{R}^2 + S_0 \sqrt{\mathcal{M}' + 2\Phi \mathcal{M}''}\, \ddot{R} - S_1 \sqrt{\mathcal{M}'}\, R \dot{\theta}^2 \right\} \hat{\mathbf{R}} \\ &+ \left\{ S_0 \sqrt{\mathcal{M}' + 2\Phi\mathcal{M}''}\, \dot{R}\dot{\theta} + S_1 \frac{\mathcal{M}''}{2\sqrt{\mathcal{M}'}} R^2 \dot{R}\dot{\theta} + S_1 \sqrt{\mathcal{M}'} \left(\dot{R}\dot{\theta} + R\ddot{\theta} \right) \right\} \hat{\theta}. \end{aligned}$$

Hence, using $2\Phi \equiv R^2$, the condition (13) becomes:

$$\begin{aligned} & \left\{ \sqrt{\mathcal{M}' + \mathcal{M}'' R^2}\, \dot{R}\dot{\theta} + S \frac{\mathcal{M}''}{2\sqrt{\mathcal{M}'}} R^2 \dot{R}\dot{\theta} + S\sqrt{\mathcal{M}'} \left(\dot{R}\dot{\theta} + R\ddot{\theta} \right) \right\} \\ = \;& k_0 \left\{ \frac{3\mathcal{M}'' + \mathcal{M}''' R^2}{2\sqrt{\mathcal{M}' + \mathcal{M}'' R^2}} R\dot{R}^2 + \sqrt{\mathcal{M}' + \mathcal{M}'' R^2}\, \ddot{R} - S\sqrt{\mathcal{M}'}\, R\dot{\theta}^2 \right\} \end{aligned}$$

where $S = \pm 1$. This can be rearranged as:

$$\begin{aligned} & S\sqrt{\mathcal{M}'}\, R\ddot{\theta} - k_0 \sqrt{\mathcal{M}' + \mathcal{M}'' R^2}\, \ddot{R} \\ = & - \left[\sqrt{\mathcal{M}' + \mathcal{M}'' R^2} + S \frac{\mathcal{M}''}{2\sqrt{\mathcal{M}'}} R^2 + S\sqrt{\mathcal{M}'} \right] \dot{R}\dot{\theta} \\ & + k_0 \left\{ \frac{3\mathcal{M}'' + \mathcal{M}''' R^2}{2\sqrt{\mathcal{M}' + \mathcal{M}'' R^2}} R\dot{R}^2 - S\sqrt{\mathcal{M}'}\, R\dot{\theta}^2 \right\}. \end{aligned}$$

Using the identities

$$\ddot{R} \equiv \dot{R}\frac{d\dot{R}}{dR}, \quad \ddot{\theta} \equiv \frac{\dot{R}}{R}\frac{d(R\dot{\theta})}{dR} - \frac{\dot{R}}{R^2}(R\dot{\theta})$$

together with $V_R \equiv \dot{R}$ and $V_\theta \equiv R\,\dot{\theta}$, then this last equation becomes:

$$- \; k_0\, S \sqrt{\frac{\mathcal{M}' + \mathcal{M}''\, R^2}{\mathcal{M}'}}\, V_R\, \frac{dV_R}{dR} + V_R\, \frac{dV_\theta}{dR} \;=\; - \left(S \sqrt{\frac{\mathcal{M}' + \mathcal{M}''\, R^2}{\mathcal{M}'}} + \frac{\mathcal{M}''}{2\mathcal{M}'}\, R^2 \right) \frac{V_R\, V_\theta}{R}$$
$$+ \; k_0 \left(S\, \frac{3\mathcal{M}'' + \mathcal{M}'''\, R^2}{2\sqrt{(\mathcal{M}')^2 + \mathcal{M}'\,\mathcal{M}''\, R^2}}\, R\, V_R{}^2 - \frac{V_\theta^2}{R} \right). \qquad (34)$$

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An Interpretation of QM based on the Holographic Principle and M-Cosmology

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Abstract. A quantum information interpretation of QM based on the Holographic Principle and related to the Horava Witten model of M-cosmology and the ensuing AdS/CFT duality will be proposed. This interpretation resumes the central idea of the relative state interpretation of H. Everett namely, that the wave function (of the universe) designates a fundamental reality. Yet it sharply differs from B. DeWitt's (Many Worlds) version of the original relative state interpretation and it has a different understanding of what makes up that fundamental reality. Bringing together features of the Holographic Principle and such of M-cosmology the basic assumption of the proposed quantum information interpretation is, that the observed universe should be seen as the 3+1 dim surface 'part' of a not observed 4+1 dim bulk 'part', where the wave function of the universe eventually relates to the *entire combined system*. The bulk space then can provide the (hitherto not located) 'storage dump' for the (informational equivalent of the) not observed quantum states. A further result of these considerations is, that the quantum theoretical feature of duality, which once had been introduced by Einstein in his work on the nature of 'Lichtquanten', is seen – yet closely together with the feature of yet again 'concealing' these dualities – as the very essence of QM.

The implicit connection of interpretative and cosmological aspects made by the quantum information interpretation then reinforces again what from their respectively very different angles already had been claimed by the relative state interpretation as well as D. Bohm's holistic interpretation of QM namely, that an appropriate interpretation of QM has a) in the perspective of the relative state interpretation directly to relate to the wave function of the universe or b) in the perspective of the holistic interpretation to the universe as a whole. These different perspectives are normally not seen as being directed to the same picture. Changing this perception is one purpose of this paper.

Motivations

The original motivation for this paper came from the belief, that the intricacies associated with the interpretation of QM have not to do with a lack of more or less exotic philosophical ideas[1], but with a severe lack of knowledge with respect to what quantum mechanics ultimately relates to, which is probably just another phrasing of the insight, that these intricacies have essential to do with the fact, that QM still seems to be 'eine unabgeschlossene Theorie'. The meaning of this insight probably has changed since it had been expressed for the first time. There is no more much doubt, that all the formalism and all technicalities of QM are pretty well understood, but it is rather the (making of the) fundamental physical reality it relates to, which simply seems to be still not fully unveiled.

As a consequence of that on the one hand the assumption of such a postulated fundamental reality has been questioned and on the other some substitutes for the still missing insight into that reality have been proposed. The former consequence led to all the different more or less subjectivist interpretations, which have their origin in the Copenhagen interpretation.[2] The latter then led to these non-subjectivist interpretations, which brought respective physical actions into play, which however unfortunately share the same drawback with the originally missing fundamental reality, namely to be undetectable.

Yet there is one objectivist interpretation, which took QM simply serious and this means first of all which took QM literal. As a consequence of that it – at least in its original formulation – hold back with any of such suitably postulated

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[1] Quite on the contrary I tend to think, that some of the problems – though not the really puzzling ones – of the interpretation of QM had been first of all created by philosophical ideas. This relates especially to such 'problems', which – in this paper – I will refer to as 'quantum mystifications'. The perhaps most important reason to be against such interpretations that are based on philosophical assumptions is, that in my view a proper interpretation of QM will have to be falsifiable. Since it depends on specific cosmological assumptions the interpretation proposed in this paper matches this standard – perhaps too good.

I also must admit, that it is not without some irony, that this is a paper by someone with a philosophical background and – even worse – a paper containing almost no professional physical argumentation. But then the ideas put forward here are at least not derived from philosophical presumptions, but meant to be measured by their consistency with physical models.

[2] The – by its proponents always highlighted – fact, that the original Copenhagen interpretation stood all experimental tests is simply due to the fact, that the existence or presence of an observer, who him/herself is a decoherent object, *indicates* (i.e. is an index for) the observable universe the observer is a part of. Thus the seeming experimental corroboration of that interpretation is with respect to its significance equivalent to the significance of the weak anthropic principle, which is 'true' just in the sense that it is purely trivial. In the same sense the connection made by the proponents of the Copenhagen interpretation between the measurement process (and attached observer activities) and the behavior of quantum phenomena is trivial because measurement induces decoherence and quantum phenomena obey the quantum algebra. What the Copenhagen interpretation as well as any other subjectivist interpretation doesn't provide is even a hint of a proper *physical* explanation, why and how this observed behavior of quantum phenomena works as it works.

or fabricated hypostasized physical 'substitute realities'. That interpretation of QM is the relative state interpretation of QM, which had been originally introduced by Hugh Everett [1].
As the essential merit of this interpretation I regard the fact, that it directly and as focused as possible targeted the essential question of QM, namely to what fundamental reality, i.e. to what reality thoroughly transgressing the observable universe, it actually relates. And it did so under the premises, that this reality then was undoubtedly unknown in any of the details concerning its intrinsic build-up. Furthermore it had in admitting this even a central argument. By this the relative state interpretation left the question partly open, for which the time of complete answering had not yet been come, but was just for this reason apt to formulate the only precise and correct version of the very question itself. The answer it then gave was rather simple: 'The fundamental reality, to which QM relates to, is the implied ontology of the wave function (of the universe)'. I.e. the 'implied ontology' of the Schrödinger equation (of the universe) in the Born interpretation as a probability density function.[3]
That had several important consequences, one of which was, that the relative state interpretation overcame the dualism of the then still prevailing Copenhagen interpretation. And consequently it came to the result, that there are no classical objects, but only proper quantum objects. This result is of course equivalent to the solution, which the relative state interpretation provides for the so-called cut problem namely, that there is no cut, let alone a problem. The statement of the relative state interpretation, that there are no classical objects, will be a leitmotiv in the following. Yet concerning the more specific ingredients and makings of the mentioned implied ontology of the wave function (of the universe) the original relative state interpretation then still had to remain quite silent or 'unabgeschlossen'.
This doesn't hold for the Many Worlds interpretation [2]. This interpretation has for almost all the time since shortly after the introduction of the relative state interpretation been seen as its proper final version. And indeed the Many Worlds interpretation is a perfectly consistent model of the relative state interpretation and it is beyond that also as explicit as possible concerning a presumably implied ontology of it. And that presumed ontology proposed by the Many

[3] The central statement of the Everett interpretation is, that the Born interpretation of the Schrödinger wave equation has to be taken *literally*, i.e. that it has to be taken as designating a fundamental reality excelling if not superseding the reality of the observable part of the universe. By this Everett tacitly indicated a difference in the meanings of 'real' on the one hand and 'empirical' (or 'observable') on the other. Yet this not less radical than plain assertion also evoked the question of how the physical composition of this fundamental reality would stand. The hitherto prevailing answer to this question is known as 'Many Worlds interpretation', which is rather a further interpretation of Everett's relative state interpretation this time due to Bryce DeWitt.

Worlds interpretation is for many minds (including that of the author of this paper) regarded as being – quite literally – unbearable.
J. A. Wheeler once referred to that ontology as "too much metaphysical baggage", by that refusing the consequence, that the Many Worlds interpretation requires and reserves for every decoherent or otherwise observed state a well as every seemingly nonlocal action an entire (copy of the) universe of its own. Wheeler's cut was obviously to the point, but however it was not an argument against the theoretical consistency of the Many Worlds interpretation and it was of course also not meant to be that.
To overcome this excessive generosity but sticking to the very merits of the relative state interpretation, which is taking QM literally and that is taking the wave function as designating the – yet still unknown – fundamental reality (QM relates to), then motivates to look for a *parsimonious relative state interpretation.* Such a parsimonious relative state interpretation can't by any means be without severe intricacies, since after all there was a very good reason for taking the Many Worlds interpretation as the proper version of Everett's original proposal, namely avoiding nonlocal actions (with respect to the problems of interpretation brought forth by the EPR thought experiment).

Intricacies of a parsimonious relative state interpretation

The relative state interpretation says, that all quantum states of a respective quantum object are equally real, but that an observer normally, i.e. if looking at decoherent objects, only will see one particular of these equivalent (i.e. equally entitled to be taken as *real*) 'complementary' states of the object. The fact, that at one time only one of these states can be actually observed then has nothing to do with any presence or activity of any observer, but with the fact, that at this time the unobserved state is *not here*, i.e. is somewhere else, i.e. somewhere, where it is not observable.
Such semantics of 'not here' or 'somewhere else' are in my view not idle subtleties, but they go directly to the core of the answer Everett once gave to the questions concerning the interpretation of QM. This basic answer of the relative state interpretation, i.e. the answer, which all further interpretations of this particular interpretation have to be consistent with, is – in the light of the semantics of 'not here' – the following: the fact, that the unobserved quantum state 'isn't here' does not at all mean that 'it isn't there' (in the sense, that it does not exist).[4]

[4] Since the semantics of 'not here' and 'not there' will play a certain role in the following, now then a short terminological annotation: I will in that particular context use 'not here' as somewhat synonymous to 'not being located in the observable universe' and 'not there' as somewhat synonymous to 'not being existent (at all)'.

In one respect this marks the already mentioned difference to the so-called collapse picture of the Copenhagen interpretation and the various rephrasings of that by its numerous offspring. In this respect the fact, that 'isn't here' doesn't mean 'isn't there' just says, that the non observed quantum state has the same reality as the respective observed state, i.e. that there is not a problem of existence, but of location. The problem then however returns by the fact, that that location in question might be laid not just beyond almost imagination, but also even beyond almost everything.

The inequality of 'isn't here' and 'isn't there' also marks a difference to some realist interpretations, which could be understood as saying, that the unobserved quantum state 'isn't there' by this implying 'is strictly hidden' or 'can by no means be found' but still 'is here', i.e. – so to speak – it 'is somewhere around here, but unfortunately strictly undetectable'.

Such an ambivalence concerning the maybe a bit confusing questions of 'existence' and/or 'location' of the non observable quantum states could even still be found, when – introducing their decoherent histories proposal – M. Gell-Mann and J. Hartle said, that "the off-diagonal terms in the decoherence functional vanish, in other words, decoherence results" [3, p 443][5]. What then is exactly the meaning of 'vanish'? Since it hardly can mean just a technical canceling out, it can either mean ceasing or a rather drastic change of 'location'. And since in the decoherent histories approach of Gell-Mann and Hartle one no longer is provided with an excess of universes to get rid of the off-diagonal terms, it seems the 'off-diagonal terms' have to vanish to a non-observable location somewhere in the observable universe. And – in the case, that that it is, what they've meant – that would be a graphic example for an interpretation of QM, where 'isn't there' has to be read as 'is somewhere around here, but can unfortunately not be found'.

But if one would read 'vanish' as 'a drastic change of location' and then place this location just across the edge of the observable part of the universe, so to speak 'in its aura or halo', then the decoherent histories approach could be seen as a decisive step in the direction to overcome the reason, why the relative state interpretation intrinsically tended to become the Many Worlds interpretation instead of becoming parsimonious.

Thus let me point out, that I think, that an essential difference in the approach to an interpretation of QM between other so-called 'realist' or – as I would prefer to say – objectivist interpretations of QM and the quantum information interpretation that I will introduce in the second part of this paper shows already in the analysis of the situation in question. I.e. these realist interpretations tend to

[5] That paper of M. Gell-Mann and J. Hartle has been a most important step in the direction of a parsimonious relative state interpretation, primarily for introducing the concept of decoherence in the context of this type of interpretation.

say, that the not observable quantum states – though in a sense they are 'not there' or are 'vanished' – yet still are somehow 'here', whereas the quantum information interpretation, which will be the version of a parsimonious relative state interpretation that I will propose, says, that the not observable quantum states are quite and thoroughly 'there', notwithstanding the fact, that they are – thoroughly again – 'not here'.

Decoherence

One important reason, why the relative state interpretation intrinsically tended to become the Many Worlds interpretation instead of becoming parsimonious, was Everett's assumption of the inseparability of the observer and the observed quantum state as cocooned elements of one branch of the wave function, i.e. the neglect of the fact, that actual observers are decoherent systems. This can hardly be a reproach to Everett, since doing otherwise would almost inevitably have been seen as a relapse to the Copenhagen dualism or at least the classical-quantum cut problem in general, what he was just out to overcome.

Yet – on the other hand – the inseparability of the observer and the observed quantum state was also a sort of quantum mystification, which Everett – to my understanding - unfortunately had been enticed to adopt as well by the then still predominant obsession with the observer to (quantum) object relation, which however is a rather self created problem in the Copenhagen & Successors camp, as by the proper universalistic scope of his own claim.

The first of these two reasons is now only of historical interest, the second however has to be taken seriously. If the wave function of a quantum object in question (or under possible observation) is primarily seen as just an element of the wave function of the universe then certainly it should also be in whatever way 'correlated' (to avoid to say: 'entangled') with that part of the wave function of the universe, which relates to its observer. Obviously this was one of the two features of Everett's original proposal, which implied its natural propensity for becoming the Many Worlds interpretation. The other was the task to consistently deal with – or rather dispose of – nonlocality.

Yet to come to a parsimonious relative state interpretation one has to overcome this propensity. One of the means to do so – at least in the cases, where nonlocality is not primarily involved – is to take decoherence seriously.

Overcoming the observer inclusion

The decoherence functional, which had been discussed by Gell-Mann/Hartle and others [4] has serious implications for dealing with sufficiently large complex objects – formerly known as 'classical objects'– in general and such complex 'classical' objects as for example observers in particular. Since it follows from the decoherence approach, that such complex 'classical' objects can very well be treated as quantum objects without having to accept some absurd consequences, which earlier heavily contributed to what I call quantum mystification. A particularly confusing one had been the assumption, that treating a complex 'classical' object appropriately as a quantum object would strictly relate to all its typically coarse grained, i.e. supervenient features too. In the original relative state interpretation that then led to the excess of copying – beginning with the allegedly entangled 'observer plus quantum object' branch of the wave function and ending with entire universes.

But now we fortunately know, that there are anyway no classical objects in that former mystified sense (of the original Copenhagen dualism), but only proper decoherent objects, having disposed their off-diagonals. With respect to the problem of the observer inclusion in the original relative state interpretation that leads to the solution, that not the observer as such has to be copied for the reason to be appropriately taken as a quantum object, but that rather the vast amount of off-diagonal terms, which are associated with the observer (as a quantum object) has to be disposed – somewhere else. And it also leads to the insight, that, wherever that 'somewhere else' may be located, the disposed off-diagonals at least won't decohere again 'over there' to make a copy of that original observer, since off-diagonals just don't decohere.

This then also shows, that it doesn't contradict the unrestricted validity or universality of QM, if the emergent complexity features of sufficiently large and complex decoherent objects are *as such* not accounted for as proper quantum states (yet this does not mean, that they are seen as being 'classical'), since these supervenient complexity features are – so to speak – utterly superficies from the quantum state perspective.[6] In a sense all this follows immediately just from the very nature of such sufficiently large and complex objects as being decoherent objects. I.e. decoherent objects can – as entire objects – never be found in a state of superposition. But this does *not* say, that not *any* of their respective sufficiently small and less complex parts could regularly be found in superposition. Just the off-diagonals then have kindly to vanish, when these parts join to build a sufficiently large and complex decoherent whole.

Taking decoherence strictly and properly into consideration then effectively solves the seeming problem of the observer inclusion. It does so, since it leads to the insight that observers (as well as any other sufficiently large and complex emergent objects) already and – so to speak – *a priori* are decoherent objects. I.e. they must not and cannot branch off by observing quantum states being actually in the process of decoherence. That's for the reason that the observers

[6] In somewhat traditionally sounding philosophical terms this could be expressed the following way: applying quantum features to the *supervenient characteristics* of sufficiently large complex objects is mistaking *essence*, i.e. the respective supervenient complexity features, for *existence*, i.e. – in a most radical reductionist perspective or description – the physical composition of these objects.

already are 'splitted', i.e. decohered, objects. The various 'copies' or rather *'ensembles of off-diagonal elements'* of these observers then must not be conceived as either being stored in universes of their own or as being here (somehow hidden in this particular observable universe). But they should instead be seen as being stored somewhere else and not necessarily stored in the same physical setting as the actual observer (in this particular observable universe) is characterized by or from which he or she is built up.

A snapshot of that cat

With that we are prepared to see how another notorious cause of quantum mystification[7] would look in the perspective of a parsimonious relative state interpretation. That is the thought-experiment once brought forward by Erwin Schrödinger known as Schrödinger's cat[8] [5]. But by taking the effects of decoherence into proper account nearly every quantum mystification, which once had been associated with this experiment will vanish – and this time to nowhere. Observers, 'steel chambers' or isolation boxes, cats, 'small flasks of hydrocyanic acid' and Geiger counters, hammers or other mechanical devices are all proper decoherent objects and will never be found in superposition. Yet that of course doesn't hold for "radioactive substances".

But no matter if this thought-experiment will be performed by the decay of radioactive substances or rather the measurement of a polarization of elementary particles (as a trigger for possibly killing that cat) in any of such cases the only question that matters for the task of interpreting QM would be *where* the 'vanished' states of the respective elements or particles have gone to.

And by taking decoherence properly into account, i.e. by admitting, that the observers, steel chambers, cats etc. already *are* decoherent objects, which means they are proper quantum objects, which yet cannot be observed in a state of superposition, one inevitably comes to the result, that the psi-function of the entire system would not relate to a mix of dead and living cats "smeared out in equal parts" and that the psi-function would also not relate to perhaps compara-

[7] One has however to point out, that Schrödinger's cat became such a cause of quantum mystification directly against the intention, which drove Schrödinger to present this thought-experiment. His intention was to ridicule the overdrawn observer inclusion. Yet the later reception of his contribution tended predominantly in the opposite direction.

[8] Here the relevant passage of Schrödinger's paper: "A cat is penned up in a steel chamber, along with the following device (which must be secured against direct interference by the cat): in a Geiger counter there is a tiny bit of radioactive substance, so small, that perhaps in the course of the hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer which shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives if meanwhile no atom has decayed. The psi-function of the entire system would express this by having in it the living and dead cat (pardon the expression) mixed or smeared out in equal parts."[5, p 157]

bly confused mental states of respective observers – at least it would not relate to a confusion caused by the unpredictability of the radioactive decay.
The mental state of an observer of the originally proposed thought-experiment of Schrödinger's cat is anyway not differently affected than if there would no such explicit quantum feature be involved in the attempted killing of that cat, but instead the possible smashing of the poison gas ampoule would be triggered by a so-called 'classical' random generator, provided, that the respective algorithms could not be (separately) calculated in the time the experiment would take. After all there is a lot of so-called 'classical' lack of knowledge in the world.
This isn't meant to say either, that there is no 'psi-function of the entire system' (in the sense Schrödinger referred to) or that there are any classical objects (in the sense of the original Copenhagen interpretation). Quite on the contrary I hold, that there is not only a psi-function of the entire system Schrödinger refers to, but also even a psi-function of the *really entire* system as it has pointed to by Everett. And I also hold – again in *some* accordance with Everett – that there are no proper classical objects at all, but – now branching off from Everett's route – that there are decoherent objects, which then simply *are no classical objects*, but rather quantum objects their vast amount of off-diagonal quantum states vanished out of the scope of observableness and –since that is not due to a lack of observational power – out of the observable part of the universe to *somewhere else*.

Decoherence and interpretation

Decoherence provides the essential 'mechanism' for the emergence and further evolution of sufficiently large structurally integrated complex objects in the observable universe and thus it properly accounts for all of what once the unclear (if not utterly mystified) concept of the so-called collapse ever could have reasonably stood for. And it fortunately does not stand for any of the unreasonably claimed powers of that collapse as for example to create so-called classical objects. Decoherence just explains, how and why certain features of quantum objects vanish – due to interaction with the environment – out of the scope of observableness in the observable part of the universe or in a more strict relative state interpretation view: out of the particular reality of the *observable* universe.
Yet then the decoherence 'mechanism' also seems in the eyes of some – and perhaps sometimes in the eyes of Gell-Mann and Hartle too – instrumental for the 'origin of the classical domain' [3, p 430]. Fortunately they then make clear: "There are no classical domains, only quasiclassical ones." [3, p 445]
I would like to stress, that a major significance of the concept of decoherence is actually due to the fact, that it allows us to finally get rid of that old Copenhagen dualism, i.e. to get finally rid of any notions of classical or quasiclassical

domains or objects. Therefore decoherence must clearly not be misunderstood as something like a technically advanced version of Heisenberg's 'objective collapse'. Decoherence leads simply to no collapse whatsoever.

To overcome the Copenhagen dualism had also been an essential motive of Everett for presenting his original relative state interpretation, namely to show, that *there are no classical objects*, but only quantum objects and that the only solution of the cut-problem is, that there is no cut at all. Yet the fact, that there are such objects – formerly known as 'classical' or 'quasiclassical' – which for example can never be observed in superposition then contributed to the ontological overkill in the final version of his proposal. This was for two reasons: a) he didn't take decoherence into consideration (though the basic idea was already introduced by Nevill Mott in 1929 [6]) and b) even if he would have taken decoherence into consideration, he would still have had the problem where to *store* the vanished off-diagonals (if not in universes of their own). But, even if b) might in the end stand for a far greater problem, the consequence of a) already helps a lot.

Since by *taking decoherence seriously*, we now can say, that the decoherence 'mechanism' is absolutely instrumental for the origin or rather the emergence and evolution of a domain or rather a world of sufficiently large structurally integrated complex objects. I.e. it turns out, that the decoherence mechanism is absolutely instrumental for a world of *decoherent objects*. The seeming triviality of this statement is misleading. Since we can gain important information from it, namely, that we actually got rid of the stubborn illusion of 'classical' or quasiclassical domains or objects. There is neither a conceptual nor an ontological reason left to adhere to this illusion, since decoherent objects stand perfectly for all of that, what once had been reserved to the domain of 'classical' objects, yet decoherent objects are proper quantum objects, just quantum objects, for which the question "Where to store the vanished off-diagonals (if not in universes of their own)?" is still unanswered.

This brings us of course back to the reason 'b)' just mentioned above and to the conclusion, that decoherence despite its major significance for an encompassing (just to avoid to say 'holistic') interpretation of QM (i.e. an interpretation without a cut) does not provide a major contribution to crack the hard nut any possibly successful interpretation of QM will be confronted with. The reason for that is, that decoherence just doesn't tell us anything about a more fundamental reality encompassing the reality of the observable universe.[9] It doesn't even give us the slightest hint, *if there is such a fundamental reality or not.* And – upholding

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[9] And it is for the reason, that decoherence doesn't answer the questions a) where the vanished quantum states are left (notwithstanding the fact, that decoherence tells us, *how* they vanish) and b) to what fundamental reality QM relates, why decoherence, despite its enormous significance, does *not* provide an essential contribution to the very question of the interpretation of QM.

the suspicious question and the following postulate[10] of Einstein [7] as well as Everett's basic idea of addressing that postulate – the author of this paper holds, that just *this* is the very question that matters for the interpretation of QM.
Thus let us come to a first summary:
Decoherent objects are proper quantum objects. They differ from quantum objects, which can be observed in superposition, just by the fact that decoherent objects – by definition – cannot be observed in a coherent state. The *process of decoherence* however can be observed in appropriate cases.
Since decoherent objects are proper quantum objects, the question, which matters for the interpretation, then is: "Where are the not (or no more) observed quantum states?" In the frequent cases, where the decoherent objects are sufficiently large structurally integrated complex objects like rigid bodies, sufficiently large organisms or observers, there is a very large amount of not observable quantum states (or off-diagonal terms) associated with the respective object, and also in these cases the question is: "Where are these not observed quantum states left?"
In the framework of the intended parsimonious relative state interpretation then the question is *not*, if there is anywhere a copy (or even more than one) of such a sufficiently large structurally integrated complex object (with or without an universe attached).
Thus *provisionally* it seems, that the desired parsimonious version of the relative state interpretation then could perhaps go as follows: the non observable quantum states become stored outside of the observable universe, just across its edge, i.e. – so to speak – in a kind of halo or aura of the observable universe, instead of being stored in these exorbitant Many Worlds.

Yet still a splitting force: nonlocality

But even if the problem of the observer inclusion, which in the original relative state interpretation came up for the reason of not proper accounting for the fact, that observers are decoherent quantum objects, had been an obstacle on the way to a parsimonious relative state interpretation that had eventually been overcome, then there still looms another obstacle much harder to overcome. That is the problem of nonlocality that once had been brought into the – since then enduring – debate by Einstein, Podolsky and Rosen [7], best expressed in the original EPR thought experiment, which later had been realized in a veritable physical experiment [8] as well.
Frankly, one has to admit, that in this case the hitherto intended version of a parsimonious relative state interpretation does not work and cannot work. Since it

[10] That postulate was, that QM could only be considered complete, if it could be formulated as a realist and local theory.

doesn't help, that the respectively vanishing states of the objects entangled at arbitrary distance might be stored in whatever aura or halo. For it is simply not about these states of the entangled objects, but about the instantaneous 'action' seemingly taking place in the experiment, i.e. an action, which appears to be nonlocal. Such a nonlocal action, seemingly taking place in the observable universe, then is neither subject to decoherence nor to storage procedures.

Therefore only two equally unpleasant 'solutions' seem to be left, namely either to attach a nonlocal action (taking place in the observable universe) to the hitherto sketched version of a parsimonious relative state interpretation, which then anyway would be no more that parsimonious, or simply falling back to the Many Worlds interpretation, and it doesn't make things better, that the implied lavishness of the Many Worlds interpretation might this time even be motivated by another ideal of frugality, namely to avoid any such spooky interactions and additions to proper QM, which had been frequently associated with nonlocality.

The only thing that really could help out would be, if this nonlocal action would not be *here*, i.e. if whatever effects these nonlocal appearances would not be in the observable part of the universe.

And since for example pilot waves in whatever disguise are by definition 'hidden' or undetectable, it would perhaps seem rather prudent to come to the conclusion, that whatever effects that nonlocal appearance might just exactly be characterized by that what it so manifestly shows just by hiding, namely that it is undetectable simply because it isn't there, what however in this special case rather means to say: isn't here. And 'isn't here' then doesn't mean 'isn't there', but – again – it rather means 'is somewhere else, where it is not observable'.

Before we will start to have an even closer look at the semantics of 'not here' and their relevance for the announced quantum information interpretation of QM, which shall be chosen to redeem the expectations, which hitherto have been laid upon a parsimonious relative state interpretation, I shortly will address a conceptual aspect concerning the interpretation of QM.

The opposite of the subjectivist interpretations of QM stemming from Bohr's original Copenhagen and carried to such extremes as Wigner's friend is in my view not (as it almost always had been thought by most of the contributors to this debate) a 'realist' interpretation – at least not in any epistemological sense of 'realism', i.e. in a sense closely associated with either empiricism or naturalism – but a non subjectivist, i.e. an *objectivist* interpretation.

An objectivist interpretation then would in contrast to a realist interpretation – if in fact 'realism' here refers to a sort of (how hidden or concealed ever) intended physical entity or actuality – not necessarily ask for any intended ordinary physical entities or actions – as e.g. pilot waves or quantum potentials – at all to do the job of the dismissed nonlocal action. And of course an objectivist interpretation doesn't bother where that, whatever does this job, should be considered being located. Especially the proponents of an objectivist interpretation should not be ontological protectionists concerning the universe they happen to stay in. Quantum states may be traded free across such – at least for the power of theoretical imagination – contingent borders as the edge of an observable universe is one of.

The quantum information interpretation of QM

The semantics of 'not here': From a parsimonious relative state interpretation to the quantum information interpretation

Since "somewhere else" just means nothing else and nothing more than: "not here" (where "here" again means: "in this observable universe") it might be the case, that the 'vanished' i.e. the not observable quantum states are just 'outside' i.e. just outside of this observable universe or just across its edge. But "somewhere else" does *not* mean or imply, that these vanished quantum states 'at the outside' or 'just across the edge' of the observable universe would or should have to be associated with another world or universe.

Yet there is one more seeming tacit implication, which might be not much less misleading than the one, which claims to need entire universes to store minuscule quantum states. This one has to do with the understanding of 'what is outside' or 'what is the edge (of what)'.

For most of the time, when the relative state (or the Many Worlds) interpretation had been discussed, the undisputed self-evident understanding was, that the observed universe would be the obvious *inside state* and the "somewhere else" laid storage of the vanished quantum states – wherever or whatever this may be – would have to be considered to be *the outside* of this universe. With respect to this 'outside' of the universe the difference between a parsimonious relative state interpretation and the prodigal Many Worlds interpretation would only be, that in the relative state interpretation this 'outside' could be located just across the edge of this universe – so to speak – in a sort of quantum halo, whereas the Many Worlds interpretation won't do the job for (or with) less than other universes as storage bin. Yet both these versions of the Everett interpretation of QM would in a rather unquestioned way hold the *observable universe as being the inside, the kernel or the bulk in this relation.*

And that's the hitherto unquestioned premise, which will be rejected by the quantum information interpretation of QM based on the Holographic Principle and M-cosmology, which is proposed in this paper. In accordance with the Holographic Principle and a certain model of M-cosmology the observable universe will not be seen as the inside or bulk or the kernel in relation to the storage bin for vanished quantum states, but rather as the outside, the surface or just as being at the edge of *a bulk (or kernel) with one additional dimension.* And that five (or 4+1) dimensional inside or bulk is proposed to be the "somewhere else", which had been the synonym for the "not here", which had been the answer to the question: "Where are the vanished (i.e. not observed) quantum states?" That 4+1 dimensional bulk or kernel then is proposed to be the looked for storage bin of the vanished, not observed quantum states.

The quantum information interpretation of QM based on the Holographic Principle and M-cosmology

Edward Witten once suggested, that M-theory[11] "may entail an explanation of quantum mechanics" [9]. The quantum information interpretation, which now will – at least as best the capabilities of the author allow – be sketched then perhaps could be seen as an attempt to contribute a bit to the efforts to make this anticipation come true.

One of the – together with decoherence and the Holographic Principle – three essential cornerstones this attempt recklessly utilizes had been presented by Petr Horava and Edward Witten in the context of the compactification of higher dimensional superstring domains to respectively lower dimensional ones [10]. Based on their results then a cosmological model called the 'Horava Witten model' gained some prominence in *string cosmology* (and might just by that also have indicated the turning point, from which on one should perhaps better speak of *M-cosmology*) [11]. In M-cosmology the Horava Witten model then gave a major impulse to the development of the so-called AdS/CFT duality, where already aspects of the Horava Witten model became somewhat amalgamated with the features of the Holographic Principle [11a].

A major result of the Horava Witten approach was that the origin of our universe could be envisioned as a five dimensional bulk space-time coming as the result of a compactification of an eleven-dimensional supergravity and that the observable part of the universe, i.e. the part we are actually living in, then has to be conceived as the (faster than the original five dimensional bulk part) expanding four – i.e. 3+1 – dimensional surface part of that original five dimensional universe.

Such a M-cosmological model would imply a drastic change to all our previous understandings of cosmology. There would neither be anything remotely similar to a singularity (not even to the – in the Pre Big Bang scenario of Gabriele Veneziano and Maurizio Gasperini – again by means of superstring theory altered configuration of a 'triviality'[12]) nor to the idea, that the observable

[11] Witten actually referred to "superstring theory after the second superstring revolution". [9, p 137]

universe started – so to speak – by a kind of concentric expansion from a tiny spheroidal beginning.[12]
But in the M-cosmological model based on the Horava Witten theory the observable universe rather started at the edge or *as the surface* of a respectively small five dimensional structure, not with a Big Bang and not with inflation, since the compactification from eleven dimensional supergravity is just meant to care for all of that instead.[13]
Yet in this paper I'm for obvious reasons neither concerned with the cosmological impact and consequences of the Horava Witten model nor with its M-theoretical underpinnings. Here the M-cosmological aspects of that model are just taken as presuppositions and it is made use of them for the proposed quantum information interpretation of QM. That has the – perhaps rather dubious – advantage, that such a proposal based on a secondary utilization is definitely wide open to falsification.
With respect to the task of presenting an interpretation of QM that
a) shall be based on proper physical assumptions and
b) follow the central ideas of Everett's relative state interpretation, namely that the wave function of the universe relates to a fundamental reality and that there is no dualism or quantum – classical cut and then
c) shall still operate parsimoniously,
the Horava Witten model of M-cosmology then eventually provides the long awaited 4+1 dim storage space for the not observable quantum states.

In FIG.1 a lower dimensional analogy of the situation I refer to is shown.

[12] It must hardly be stressed, that such a M-cosmological model would have a drastic change of our understanding of the observable universe (formerly known as 'the universe') as a consequence. The observable universe would then undergo a change of status from the be-all and end-all of a post-eleatic onto-centrism to something like a sort of peripheral 'ectoplasm' of the unobservable bulk (part of the entire combined universe).
[13] For a discussion of these implications of the Horava Witten model in the context of a critical assessment of the ekpyrotic model of P. Steinhardt, N. Turok et. al., which also got essential inspiration from the Horava Witten theory, cf. [13].

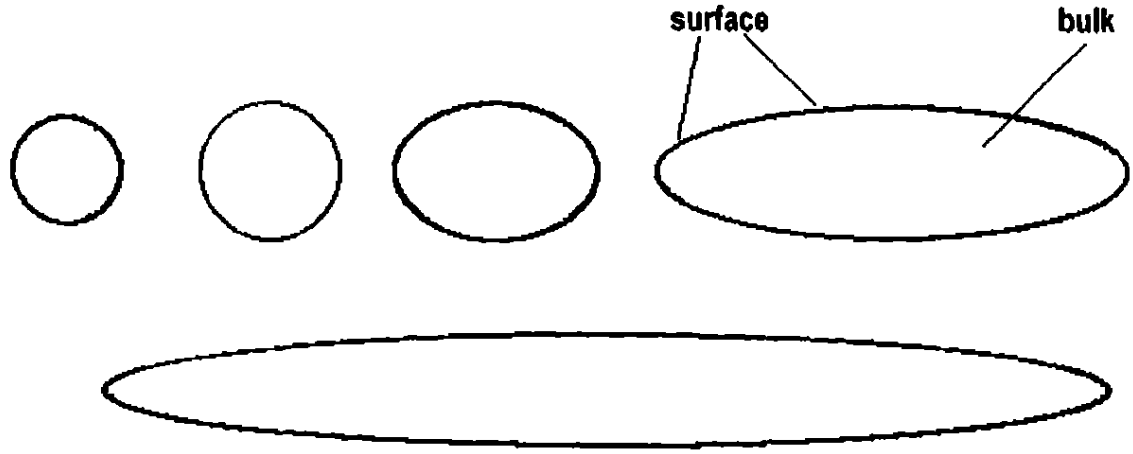


FIG.1

FIG.1 shows in the most simple diagrammatic manner the 'expansion of a two dimensional 'universe' with a one dimensional surface' (or equivalently some cuts through the respective stages of the expansion of a three dimensional universe with a two dimensional surface). What differs from other illustrations with a comparable 'object' in focus is, that this two dimensional 'universe' somehow seems to loose its proper shape or just seems to become a bit rambling at the third stage of its evolution. By that the original circular shape and concentric expansion will change to a rather – yet not necessarily regular – elliptic shape and in FIG.1 that is additionally correlated with a faster increase of the perimeter in relation to the volume. In the two later stages of that evolution shown in FIG.1 that faster increase of the perimeter compared to the volume goes on and that leads to an increasing *flattening* of the ellipsoid in question.
Now, in the two-dimensional case and also in the three dimensional case (with two dimensional surface) that might possibly be not of much interest for many people. But that could perhaps change in the case, where a comparable expansion is considered for a 4+1 dimensional bulk space with a 3+1 dimensional surface, especially when the latter becomes somehow associated with the observable universe. The different expansion rates of the 3+1 surface and the 4+1 bulk part of the universe as shown in FIG.1 could also be the result of other conditions as those supposed in FIG.1. They would also follow from a scenario, in which the bulk part is assumed to come to an end of its expansion and/or in which it maintains a constant volume[14], or perhaps even another one, in which it

[14] The five dim bulk part could in such alternative scenarios either also keep its original size or volume from its beginning as a result of compactification or keep a particular volume reached at any later

would – slowly – decrease from an original size, i.e. volume. Yet it would in all cases be required, that the surficial part (i.e. the equivalent to the observable universe in such a model) would have to increase and, if both parts increase, then to *increase significantly faster* than the bulk part.[15]

If such a cosmological model, somewhat 'associated' with M-theory, could possibly stand the test of cosmological evidence, it then would also immediately suggest itself to become explored with respect to the applicability of the Holographic Principle.
The Holographic Principle[16] had been introduced by Gerard t'Hooft [15] and Leonard Susskind [16] in the context of black hole thermodynamics. L. Susskind then also discussed the Holographic Principle with respect to its significance in superstring theory, where it soon turned out to be a generic feature of that theory [16]. It soon also became intensively discussed in the particular context of superstring- and M-cosmology [17]. And it is in this context, where it possibly connects to the quantum information interpretation.

In its original formulation by t'Hooft and Susskind the Holographic Principle said, that the information content of an appropriate spatial region, e.g. a black hole, can be fully described by the information content of the area of its surface (with one bit of information per one unit of the Planck size of the surface). That original Holographic Principle then could possibly be generalized to state, that the information content of any appropriate *n* dimensional stable spatial region can be found represented in its *n-1* dimensional boundary.
If the Holographic Principle could possibly be applied to a cosmological model in line with the toy model shown in FIG.1, then this would imply, that the observable 3+1 dimensional part of the universe is something like a display of the information content of the 4+1 dimensional part of the universe, of which it is the boundary. And this relation would of course also hold vice versa. The basic assumption of the quantum information interpretation then is, that the puzzling features of QM – so to speak: the 'quantum phenomenology' – are due to the significantly different ways this *equivalent information* is stored at the surface part on the one hand and in the bulk on the other and by that, that these features

stage of a possible (and highly probable) expansion. The quantum information interpretation of QM proposed in this paper would be consistent with any of such M-cosmological scenarios.
[15] Even though in this paper quantum information states are just the stuff we are interested in, that must not preclude, that such a five dim bulk part of the universe might also be useful to store other things, which share the feature of stubbornly denying to be observed with the ‚vanished' quantum states. Since after all the capacity of that five dim bulk might well be apt to store enough dark energy and dark matter to also serve as the *heart of darkness* of the entire, i.e. combined universe.
[16] For a comprehensive review of the Holographic Principle cf. [14].

are not less due to the *filtering*, which takes place in the mapping of this information from the surface to the bulk and vice versa.
One important difference may very well be, that the bulk way of storing information is more 'effective' in a sense. I.e. that in the assumed 4+1 dimensional bulk part all 'complementary' or rather *dual* quantum information states are actually copresent, whereas in the 3+1 dimensional surface part in many cases only one of such complementary states can be observed at one particular time. That is not to say, that the information content on the surface would be less than that in the bulk, since the observation of one of two 'complementary' states entails the information about the not observed one as well. There would then be no difference in the information relating to the *reality* of the not observed quantum state, but just a difference with respect to its *actuality*, i.e. the question of how that reality is actually expressed or – in a less favoured way of describing this – if this state is actually observed or not. And 'information' telling us, that an observed state is observed and a not observed state is not observed is not particularly telling us much at all.
The aspect of different efficiency in storing the equivalent information content with respect to either the surface or the bulk part of the universe seems in my view to further support the assumed link between the proposed M-cosmological model hinted at in FIG.1 and the Holographic Principle. The increasing variable in the surface part of the universe as well as in the bulk part appears to be their respective entropy, i.e. the thermodynamical history of the surface (i.e. the observable universe) on the one side and its informational equivalent in the bulk on the other. That the five dimensional bulk part of the encompassing entire universe is a far more effective storage for the quantum informational equivalent of the thermodynamical entropy of its surficial part is then quite in line with the assumed (and now also required) faster increase of the expansion of the surface part of the universe compared to its bulk part.
If the applicability of the Holographic Principle to a cosmological model consisting of a five dimensional bulk part and a four dimensional surface part of the – both parts equally encompassing – entire universe, where the surface part would correspond to the observable universe, could be possibly corroborated, then the observable universe itself should be seen as something similar to a 'hologram', quite in line with the paraphrase of "The World as a Hologram" once used by L. Susskind.
Concerning this metaphor of a 'hologram' I however think, that the situation of an observer in the 3+1 dim universe observing quantum phenomena is even more different from the situation of an observer looking at an ordinary hologram than it obviously appears. The main difference comes of course from the fact, that the observer in the 3+1 dim universe is him/herself just a part of that 'hologram' he or she is assumed to look at as well. Taking this unavoidable participancy into account it seems, that the Holographic Principle rather invites us to

see the 3+1 dim observable universe as a kind of complicated 'interface' of the 4+1 dim bulk part of the universe on the one side and the decoherent observer as a proper part of the 3+1 dim surface part of the universe on the other.
Therefore it then requires in the *quantum information universe* case much more abstraction than in the case of an ordinary hologram to account for that unavoidable involvement of the observer or maybe better: to cancel it out.[17] Since for the participant observer the observable universe appears less to be a hologram than rather to be a kind of semitransparent mirror, at which it seems hard to distinguish between, what it shows, from, what's behind, and, what it reflects, from, what is and what *happens* in front of it.

Quantum through the looking glass[18]: concealed duality

Thus now we will try to take a look at what's behind that mirror the eyes or rather the mind aided with trust in the proposed M-cosmological model as well as with hope, that that mirror behaves according to the Holographic Principle. In particular we want to see, if the problem of nonlocality as exposed in the EPR experiment and still unresolved even by taking decoherence into account may find a better explanation.
The quantum information interpretation based on the proposed M-cosmological model and the Holographic Principle just claims, that the entire quantum information state relating to the two arbitrarily distant yet entangled particles like e.g. electrons with respectively two dual (also known as 'complementary') quantum states like spin up and spin down is *actually copresent* in the 4+1 dimensional bulk part of the universe and – in accordance with the Holographic Principle – mapped (or projected) at (or to) the 3+1 dimensional surface part of the universe. Yet obviously here it doesn't show its proper actual copresence, when being manipulated with or sufficiently sharp looked at. In such an experimental situation the looked at one shows only one of its two dual states and an alleged nonlocal action is assumed to take place seemingly instantaneously transmitting the necessary information to the other sufficiently distant one to make this one showing the respectively dual state.
And that already implies the quantum information interpretation of the EPR situation. The quantum information interpretation reduces the seeming nonlocality in the surface part of the universe to the concealing of an underlying dual-

[17] The relative state interpretation of H. Everett was in my view the first one, which dared to face the challenge of the 'required abstraction', whereas the many subjectivist interpretations of QM took that unavoidable involvement or participancy of the observer to shield their anthropocentric resentment, which historically often was closely associated with a resentment at mustering the respectively 'required abstraction'.
[18] That phrase might perhaps induce some to see the quantum information interpretation as a sort of 'quantum in wonderland'.

ity of dual quantum information states. This unconcealed duality is yet still unreduced copresent in the corresponding five dimensional bulk part of the universe.

That concealment of one of the dual states of the 'entangled at distance' quantum system in the EPR situation is caused by the – in the surface part of the universe *local* – action reducing the dual quantum state of the particular particle (by manipulating e.g. polarizing) to one of the original two pictures. By this – so to speak – 'surficially' local action the dual state of the whole entangled system becomes instantaneously accordingly reduced.

This follows a dictum of Erwin Schrödinger, who already has said a long time ago, that quantum objects once entangled remain entangled not regarding how far remote these objects become later. That statement was directed as well against any miraculous observer power as against the assumption of any nonlocal action carried by hidden forces in the observable universe.

The 'mechanism' the quantum information interpretation proposes to be responsible for that nonlocal appearance or effect (in the surface part of the universe) of the seeming transmission of the reduced quantum state of the particle 'at hand' to the whole entangled at distance quantum system is no mechanism at all, but rather a direct consequence of the Holographic Principle. Since the Holographic Principle states, that the entire information of the quantum states of a respective bulk space is mapped on (or projected at or represented in) a respective surface, it also directly follows, that any change of the informational status of the quantum states at the surface is itself again mapped to the bulk space. That happens, if in the EPR situation the quantum state of the particle 'at hand' becomes reduced by a surficially *local action*.

Yet since the Holographic Principle unweariedly states, that the entire information of the quantum states of the bulk part of the universe is mapped to (etc.) the respective surface part, the then respectively altered bulk information – now including the additional information about the change of the informational status of the quantum states at the surface caused by that surficial local action – becomes also instantaneously mapped again 'back' to the surface. And since the entangled system is an entangled system this 'mapped back again' information relates to that entangled system as a whole. And that's the quantum information interpretation of the nonlocal appearance in the EPR situation caused by one local action in the surface part of the universe and actualized in that surface part by two instantaneous mappings – forth and back.

The result of that playing the information with rebound is, that – caused by the surficially local action – one of the dual states of the whole entangled quantum system has vanished or rather is concealed in the observable 3+1 dimensional part of the universe.

Then the question comes up what 'happens' in the bulk space, when the information about the reduction of the dual quantum state of the particle manipulated

by that surficially local action is processed in the bulk. That is in my understanding the question, if there takes another action place in the bulk part of the universe or not. In the case that this question would have to be answered positively then a next question would come up, namely the question, if such an action in the bulk part of the universe would be a nonlocal action or not.
In the attempt to find answers to these questions we will try to support our imagination by having a look at the following FIG.2.

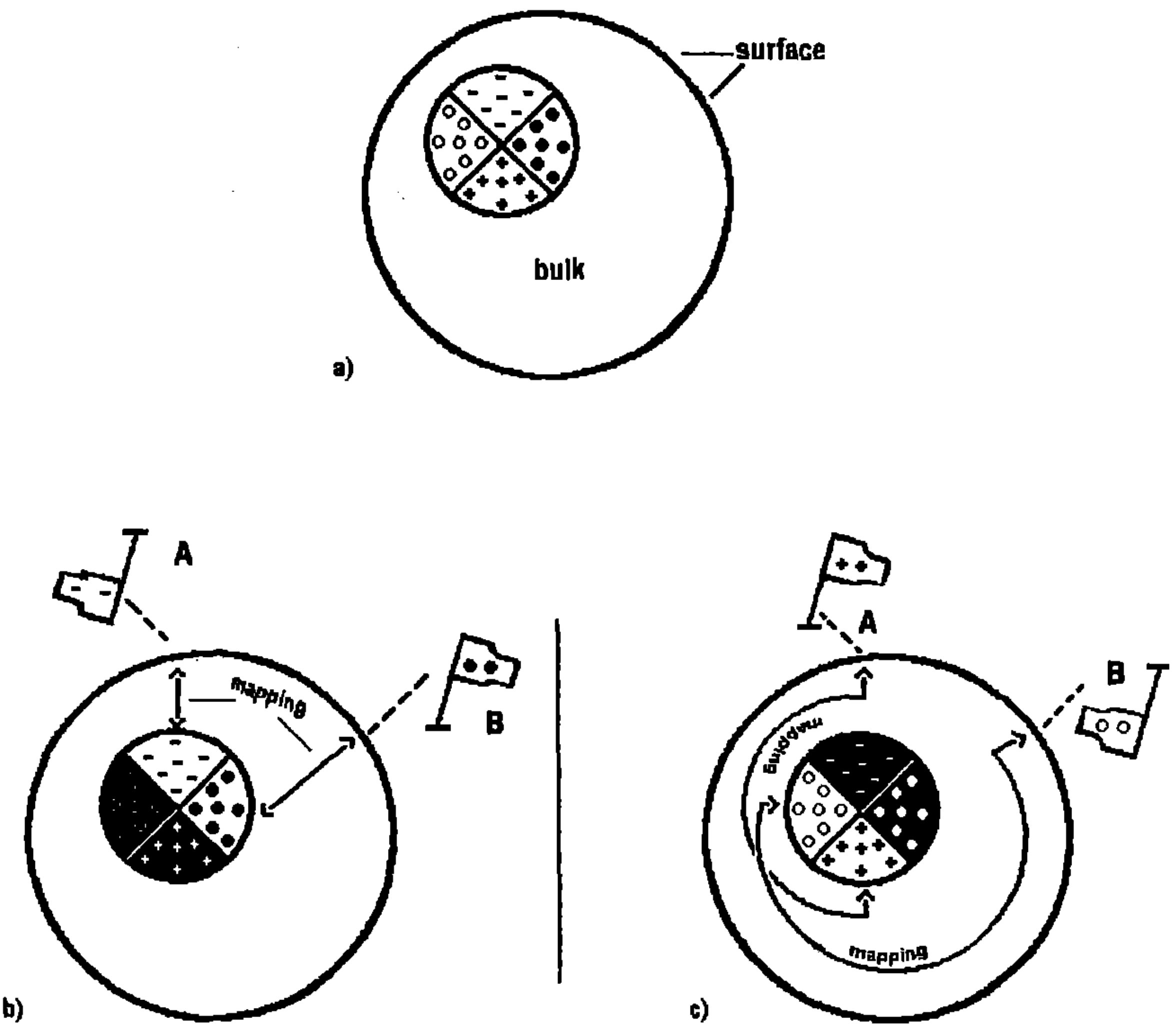


FIG.2

FIG.2a shows the nonreduced or nonobserved coherent quantum information state(s) of a pair of entangled particles, which have to be supposed or imagined at the surface, but are of course not shown or visible, since after all they are supposed not to be observed. The respective dual quantum information state(s) of one of the two particles is laid in (the angular domains of) one of the two corresponding angles shown in the inner circle in the bulk, and accordingly so for the other. I.e. '+' and '–' symbolize the dual quantum state of one of the two assumed particles and accordingly '●' and 'O' symbolize the dual quantum state of the other assumed particle. What FIG.2a implies is, that the entire quantum information state in the bulk, which is correlated to the whole entangled system (assumed to be) on the surface, is simultaneously copresent and – so to speak – activated at least as long as the correlated entangled system on the surface is not reduced.

What according to the quantum information interpretation should happen, if the entangled particles in the surface undergo an EPR treatment, then is shown in FIGS.2b and 2c. In FIG.2b one can see an object (or particle) A, which shall be the double of the particle 'at hand', i.e. manipulated by a local action, in an EPR situation, showing a (quantum) state 'flagged down', which is characterized by '–'. From this follows as well, that the respective dual quantum state of particle A (characterized by '+') cannot be observed, i.e. is vanished or concealed, as also, that the entangled particle B shows the 'complementary' (to the one observable at particle A) of *its* respective dual quantum states, namely the state 'flagged up' (characterized by '●'), whereas the respective dual quantum state of particle B, namely 'flagged down' (characterized by 'O') then of course can also not be observed, i.e. is vanished or concealed.

Accordingly in FIG.2c the particle A then presents – probably after a further manipulation – the previously concealed state 'flagged up' (characterized by '+') and has its state 'flagged down' (characterized by '–') concealed. Then the entangled particle B now shows – again accordingly – its quantum state 'flagged down' (characterized by 'O') and has *its* respective dual quantum state, namely 'flagged up' (characterized by '●'), concealed.

In the bulk the entire information about what happens in that EPR situation could possibly be stored or represented – and then *instantaneously mapped back to the surface* – in the way shown in FIGS.2b and 2c[19], namely by marking the quantum information states equivalent to the not observable, vanished or concealed states at the surface (alternatively also the quantum information states equivalent to the observable or not concealed states at the surface could be marked and the dual ones then left unmarked). In FIGS.2b and 2c that marking comes in the form of inverting the symbols (and the background) in the respective angle domains. In FIG.2b that are the *inverted graphic representations* of '+' and of 'O' and in FIG.2c the *inverted graphs* of '–' and '●'.

What happened then in the inner circles of quantum information representation in the bulk? Was there a local or nonlocal action involved? It *seems* at least, as if there happened three things.

[19] In fact that information can be stored much more effectively as will soon be shown below.

In FIG.2b the 'process' started with the quantum information state characterized by '–' getting the information *mapped* from the surface, that its equivalent state at the surface namely 'flagged down' of particle A became actualized or observed. Therefore at first its dual bulk state '+' had to be marked or inverted and secondly its neighbour in clockwise direction '●' had to be informed to be activated or in charge for mapping its quantum information state back to particle B, which then would show the surface state 'flagged up', and then thirdly the dual bulk state of '●', namely 'O', had also to be marked or inverted.[20]

Accordingly in FIG.2c the surface state 'flagged up' of particle A became mapped to the quantum information state characterized by '+', then the dual state '–' of this state had to be inverted and then its neighbour in clockwise direction 'O' had to be informed to be activated for mapping its quantum information state back to particle B, which then would show the surface state 'flagged down', and then the dual bulk state of 'O', namely '●', had again to be marked or inverted.

Obviously the mapping of the information from the surface forth to the bulk and back again are no actions at all. But what's about the marking or inverting and what's about the transmission of the information to the neighbour in clockwise direction (in the bulk), that it is now in charge a) to mark (invert) its dual twin and b) to map its quantum information state to particle B at the surface. It is not so obvious, that these three happenings in the bulk are actions at all and it is even less obvious, that, if there would be an action involved, such action would have to be a nonlocal action.

Yet one thing at least is obvious, namely that the FIGS.2a, 2b and 2c contain much too much redundant information due to a too much surficial – or observer infected – view of the bulk state, which after all is as such first of all a space of – from an observer's point of view – utterly concealed states. And due to this there also may even to many seeming actions – nonlocal or other – be contained in these figures.

Concealing the quantum information duality by highlighting the equivalent to the observable state

When we look again in FIGS.3a, 3b and 3c at the same EPR situation in the quantum information interpretation environment as above, but now not from the surficial observer point of view (or with this view overly in mind), then we will see a certain slimming down of the number of symbols symbolizing the four particular states of the two dual quantum information states in the bulk. I.e. we will only see two symbols, namely '●' and its marked (or inverted) image. And

[20] That kind of enumeration must not and shall not imply a time sequence of whatever actions or else.

even these two symbols are again actually redundant and only kept for reasons of (a more contrasting) illustration and easier description.

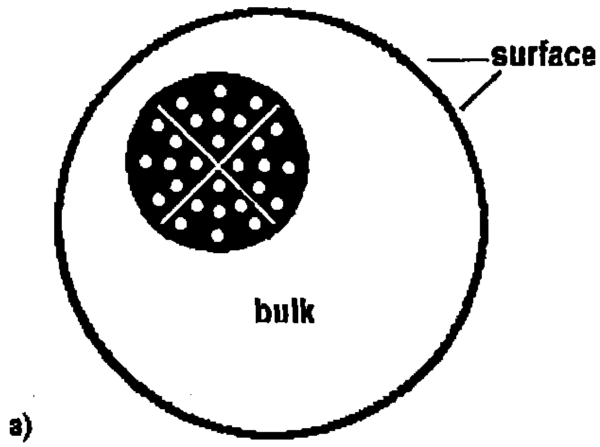


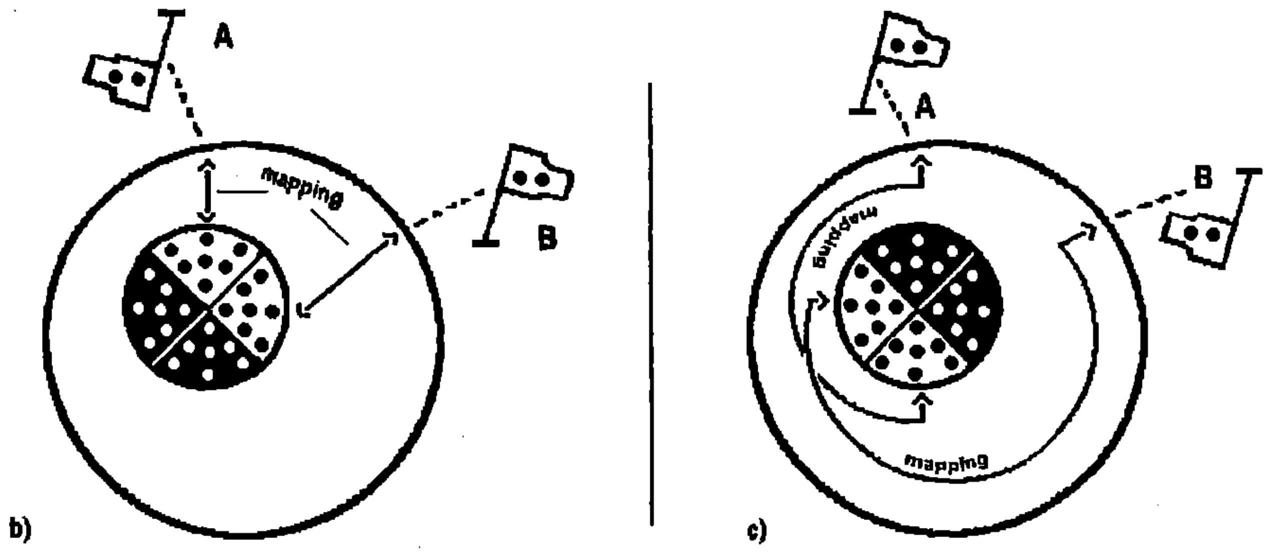
FIG.3a

In FIG.3a all four angle domains of the two corresponding angles, which stand for the two dual quantum information states in the bulk, which are equivalent to the two dual quantum states of the not observed whole entangled system on the surface, i.e. the two dual states that system is actually in, when it is not observed, show even only one symbol, namely that marked (or inverted) image of '●'.

That is – in difference to the representation chosen in FIG.2a – the appropriate representation of the quantum information states in the bulk, since FIG.3a is meant to represent these quantum information states as equivalent to the *non-observed* quantum states on the surface. I.e. quantum information states in the bulk equivalent to non observed quantum states on the surface have to be shown as – in the terminology introduced in the preceding paragraph – *concealed* quantum information states, since the concealed (or marked or inverted) quantum information states are already in FIGS.2b and 2c consistently treated as being the bulk equivalent of the not observed quantum states at the surface. The mistake in FIG.2a just was to start with two dual *not* concealed etc. quantum information states in the bulk, which would rather be the equivalent of the whole entangled quantum system in the EPR situation *manipulated and still observed in superposition.*

Thus in FIG.3a we will do better and start with the proper equivalent to the quantum state(s) of the whole entangled system on the surface, when it is neither manipulated nor observed, i.e. with the respective quantum information state(s) concealed or – graphically – inverted.

And by starting properly then the number of actions involved in the EPR situation decreases to its possible minimum.



FIGS.3b and 3c

In FIG.3b we can see, that – caused by the surficial local action, which is the only relevant action the entire EPR experiment consists of – the surficial quantum state 'flagged down' of particle A becomes *actualized* and that information becomes mapped to the bulk, where by that the respective quantum information state becomes *highlighted* or activated. Then the neighbour in clockwise direction in the bulk becomes highlighted or activated and the information about that becomes mapped to particle B at the surface, which then shows quantum state 'flagged up'. In FIG.3c, where it is supposed, that choosing quantum state 'flagged up' of A starts the experiment, everything happens accordingly.

In this scenario only one proper action occurs in the bulk. That is the transmission of the information, that it now has to become activated, to the so-called 'neighbour in clockwise direction', i.e. the bulk quantum information state equivalent to – in FIG.3b – the surficial quantum state 'flagged up' of particle B.

Some of the work the hypostasized 'nonlocal action' (on the surface) had been designated for then is done by the mappings forth (to the bulk) and back (to the surface) in line with the Holographic Principle. *And these mappings are no actions at all.*

But the really hard job is done by that action of activating the quantum information state in the bulk equivalent to the observable quantum state of particle B. It is that action, which effects the appearance of nonlocality at the surface, i.e. to the surficial observer perspective. And there is – at least for me – no obvious reason, why that action in the bulk should not be a perfectly local action (in the bulk).

The notions of 'locality' and perhaps also 'nonlocality' will anyway undergo a certain change in meaning, if applied to a five dimensional quantum information storage. Yet, since the bulk is – at least in the perspective of its utilization in the quantum information interpretation of QM – first of all a five dimensional quantum information storage, that could even further strengthen the anti nonlocality cause. At first a 4 (+1) dim space(time) provides much more opportunity for close contact or immediate neighbourhood than a 3 (+1) dimensional one. And, secondly, storing of information about arbitrarily distant states does not require in any sense *distant storing*. What it probably would require, would rather be information about that particular distance. And in this case that information can again be stored in the immediate neighbourhood of the other relevant quantum information states in question.

Thus it seems, that there is – quite literally – no space left for nonlocality, since immediate neighbourhood is not just an allowed but almost a required feature of effective quantum information storing in the bulk part of the universe.

The fixative for quantum information: dodging dualities

Since it becomes more and more obvious, that the feature of duality as well as the ways of in one instance concealing the dual states or in another dimming them seems to be central for the quantum information interpretation, we will now turn even more directly than we already did to the original or paradigmatic duality, namely the wave – particle duality of quantum entities. At first we will shortly consider the question, if there would quantum information states in the bulk related to the double slit experiment, in which the wave – particle duality indirectly, yet nevertheless clearly, is shown, have to be expected. I think the answer has to be 'yes'. But the quantum information states equivalent to the surficial situation of the double slit experiment performed would then be not so directly related to the nature or features of the particles, i.e. photons, involved as for example in the EPR situation. That is so for the already mentioned reason, that the double slit experiment only indirectly relates to the dual nature of the photons.

In the case, when only one slit is opened, one can find a pattern of traces or imprints on the detector screen, which are perfectly in line with the assumption that photons are proper particles. But that accordance comes entirely from that *pattern*, which definitely shows no indication of the wave nature of photons. On the other hand any *individual trace* on that detector could of course equally be caused by a hit of a tiny wave as by a hit of a tiny particle just because there would be no difference then in the appearance of the individual trace at all.

In the case, when both slits are opened, the experiment becomes interesting. Now one can find a pattern of traces or imprints on the detector screen, which indirectly prove the assumption, that photons have a wave nature as well. Again that accordance comes entirely from that pattern, which now needs a bit more explanation than in the 'one slit open' case is required. That explanation says, that the photons of both – by passing through the slits aligned – lightbeams interfere. And that interference then causes the pattern (on the detector), which indirectly shows the wave nature of the photons involved. Yet again any individual trace caused by an individual hit could equally be caused by a particle or a wave for the same reason as above.
As a consequence of that the quantum information state equivalent in the bulk would hardly look related to any surficial experiment. In the 'one slit open' situation it would – related to the wave-particle duality – show nothing particularly different from that, what it would normally show, namely that both natures exist with equal rights. In the 'two slit open' situation it would show the same, but there would probably also information be stored about occurring interferences, since these would influence surficial quantum states. Yet that these interferences occur in the surficial context of an experiment related to the duality of the wave and the particle nature of photons, would rather be unnoticed in the bulk.

That the wave particle duality is only indirectly shown by the double slit experiment, does however not diminish at all the central role and the eminent significance that the feature of duality has in and for the quantum information interpretation.
In the light of the quantum information interpretation the feature of duality is rather the essential feature of QM. It dates back to its original discovery by Einstein as the duality feature of Lichtquanten as discrete entities as well as waves in (one of) his famous paper of 1905 [18]. This duality then had been further explored by Einstein in two papers of 1909 [19a, 19b]. Einstein didn't then fully see this duality as one of proper particles and waves, but he preferred to speak of Lichtquanten as "voneinander unabhängig beweglichen, punktförmigen Quanten" [19a], a view which comes very near to the later emerging concept of a photon. By introducing the *momentum* of Lichtquanten [20] he then – albeit somewhat reluctantly – clearly exposed the *particle nature* (as one of the two dual natures) of the Lichtquant.
In direct continuation of that work of Einstein L. de Broglie then extended that original duality of Lichtquanten to (other) particles and stated it explicitly as the duality of corpuscles and waves [21]. Since then the wave particle duality has been the paradigmatic duality in QM.

In the quantum information interpretation the feature of duality showed its importance in the interpretation of the EPR situation, where it occurred as the concealing of either one or the other of two 'sides' of a dual state or – when looking at the entire entangled system – of two combined dual states.
Yet the feature of duality or rather the feature of its concealing or suppressing also plays an important role in the apparently quite different case of decoherence. That is for the reason, that decoherence doesn't lead to a world of classical or semi-classical (whatever that shall be) objects, as I already stressed above in the respective paragraphs. The decoherent objects are rather proper quantum objects, but just such quantum objects, of which a vast amount of quantum states is not observable. And many if not all of these not observable quantum states are just belonging to these sides of dual quantum states, of which the other sides are actualized in the deeply entangled and intensively interacting ensemble of quantum entities, of which the decoherent object eventually consists of and is made up. I.e. the decoherent object must be seen as the result of a vast amount of intensively interacting quantum entities, where however this interaction itself at first induces the 'vanishing' or concealing of the superposed or dual states of the quantum entities involved.

Yet, in my view, the centrality of the feature of duality in the quantum information interpretation also indicates something more far reaching. It indicates the fundamental incompleteness of the observable reality. And by that it already unobtrusively transcends the reality of the observable universe, quite in the spirit of Everett's relative state interpretation. But for Everett the quantum cosmological domain his interpretation related to was then utterly uncharted territory, whereas for the quantum information interpretation it is just an – assumed – presupposition.
The Horava Witten model of M-cosmology not only provides the five dim bulk part of the universe for storing the quantum information states as equivalents of the vanished quantum states (of the observable universe), but it also links the traces of dualities, which almost in obscurity reach down to the particularity (also known as 'reality') of the observable universe, to the domain of their veritable presence. At least the six outposts of this domain are meanwhile charted and occupied by various superstring theories and 11-dim supergravitation. Yet since it had been said, that unknown monsters might hide in the middle of that domain, I will not take further risks and let it rest there.
Yet such as the breaking of symmetries is an accepted way of understanding the precipitation of the low energy domain of the so-called semi-classical world also the concealing (or suppressing or dimming) or – in general – the filtering of the 'complementary' states of dualities should not only be seen as an even more fundamental principle than that of symmetry(breaking), such as it is– to my understanding – done in M-theory, but also as an not less important element of

the very nature of the fundamental reality of the particular universe, to which any objectivist or ontological interpretation of QM tends to relate to.

And by that transcending of the physics of the observable universe the feature of duality also reinforces the underlying philosophical motivation of the objectivist and ontological interpretation of QM, as which the quantum information interpretation has to be seen. Namely overcoming the – more or less tacit – anthropocentrism of the Copenhagen and other subjectivist interpretations, which have been a last bastion of a somehow pre-Copernican observer-centric resentment. That observer-centrism then always preferred to take the projections of its epistemological presumptions or the scope of its operationalist reach as the ultimate limit of conceivability than instead consider the reality transcending the narrow limitations of these projections and that scope.
But by pursuing the aim of overcoming that observer-centrism we have been – again quite in the spirit of the relative state interpretation, yet in the end even going much further – carried away and got the presentiment, that not only the observable (part of the) universe, but even the *entire universe* (as proposed in that paper) is neither the totality nor the paragon of fundamental physical existence, but just a particular instance in the – in this respect somewhat Meta-cosmological – M-cosmology.

'Interpreting' the quantum information interpretation

And such ideas lead us to the somewhat self-referential twist of trying to 'interpret' the quantum information interpretation. The original motivation, by which we became instigated to all of that, what followed, was to look for a parsimonious relative state interpretation of QM. And therefore we will at first have a look, if the quantum information interpretation qualifies for that.
The problem of Everett's relative state interpretation was, that it had a natural inclination to become the Many World interpretation, if and when it became scrutinized with respect to its implied ontology. And the task of a parsimonious version of the relative state interpretation then had to be to provide an alternative to the unbearable prodigality of the Many Worlds interpretation.
For the quantum information interpretation being fit for that task three things had been required: a) a storage for the seemingly 'vanished' or not observable quantum states, b) a new understanding of the nature of quantum states regardless, if they are observed or not observed and c) an appropriate link between the world, in which – some – quantum states are sometimes observed, and the domain, in which the not observed ones then are properly stored.
The storage then was found by (ab)using the five dim bulk space of the Horava Witten model of M-cosmology for that purpose. And the link was provided by the Holographic Principle. And that had consequences for the understanding of

the nature of the quantum states involved. At least the not observable bulk stored quantum states formerly known as 'vanished' had to be reinterpreted as *quantum information states*[21], and that is again warranted as an implied consequence of the Holographic Principle. In a more philosophical or rather 'ontological' perspective this reinterpretation could, in my view, have the most far-reaching consequences. And it will soon turn out to mark the decisive difference of the quantum information interpretation to other objectivist interpretations of QM.
But first let's again come back to the question in what sense the quantum information interpretation then is parsimonious compared with the Many Worlds interpretation. The answer is not only related to the 'substance', i.e. quantum information states instead of quantum states, which is used in either model, or what has to be assumed as domain(s) for storing the not observed quantum states, i.e. infinitely many copies of the observable universe versus a five dim bulk space, but this question is not less related to the way of 'operating' the intercourse of the observed and the not observed quantum states.
In the Many Worlds interpretation there is not much of such intercourse and there is also not much need for it, since the many different relative states are each one for itself realized as a (copy of the) proper universe. In the quantum information interpretation that intercourse is however managed as it should be in the case of information states, i.e. it is managed by projecting or mapping the respective quantum states from the surface forth to the bulk and then back to the surface or vice versa just as respectively required. And since the quantum information interpretation requires really very many of these mappings it could also justifiedly be called the many mappings interpretation of QM. And since mappings are undeniably less substantial than universes or ontologically much cheaper, the quantum information interpretation can justifiedly be seen as the looked for parsimonious relative state interpretation of QM.

It is then also this ontological 'lightness' or the prevalence of the informational aspect, which gives a slight indication in what sense the quantum information interpretation differs from other objectivist or ontological interpretations of QM, including the holistic interpretation of QM of David Bohm [22] resp. the later versions of that known as the quantum potential approach [23], which on the other hand shows also some features somewhat familiar to the quantum information interpretation.
The ontological difference of the quantum information interpretation to these and other objectivist or ontological interpretations has at least two different yet connected aspects. And these aspects are apart from the differences in the physi-

[21] And to be honest, if the quantum information interpretation will stand firm, that will also fully apply to the observable quantum states as well.

cal consequences, which are attached to them, of a genuine philosophical kind. I.e. they are related to the question, if the respective interpretations have a dualistic or monistic ontological setting and – far more interestingly than that – to the question, which of the various possible kinds of a monistic setting they imply.
Related to the first question the quantum information interpretation differs sharply from such ontologically utterly overloaded interpretations as the 'old' quantum potential (combined with the concept of a so-called 'objective collapse') once proposed by Heisenberg, which had been explicitly Platonistic and took that particular kind of quantum potential as the 'place' of preexistence of the totality of all quantum states, which ever had occurred, are occurring and will occur in the (history of the) universe. Since such Platonism doesn't play any role anymore in the interpretation of QM, it will do for that to make clear, that the quantum information interpretation has not the slightest tinge of dualism, but instead it is an utterly monistic approach to the interpretation of QM.
Yet that then doesn't mean, that 'monism' is such an unequivocal notion, as one should expect for a term referring to something so seemingly uniform. Furthermore the quantum information interpretation is also an actualistic interpretation of QM, it doesn't require or allow any pre-existence or pre-storing of quantum information states, which are not actualized or respectively concealed and by that instantaneously connected with the quantum states at the surface being observed or decoherent or respectively not observed or in superposition.[22] That instantaneous actualism of the quantum information interpretation is again directly due to the Holographic Principle.
And though not with respect to the question of dualism and monism then at least with respect to the question: 'What kind of monism?' the quantum information interpretation also differs strictly from the holistic interpretation and the later version of that known also as 'quantum potential'[23].
These partly identical interpretations are – at least intended to be – soberly monistic. However they widen or extend the ontological scope of physics by adding either new kinds of actions or – in the quantum potential approach – a domain or reservoir for hidden quantum states to be projected as needed. And that's fairly similar to the way the quantum information interpretation proceeds. But the result differs severely.

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[22] I.e. there is no quantum potential required in the quantum information interpretation neither in the appalling Platonistic version once proposed by Heisenberg nor in the rather sober monistic way of the later versions of the holistic interpretation of David Bohm and Basil Hiley, since in the quantum information interpretation there must be nothing, i.e. no quantum state neither real nor virtual nor potential nor whatsoever, added or be presumed as preexistent. And the reason, why in the quantum information interpretation – quite as in the original relative state interpretation – nothing must be added in the end is, that from the beginning nothing is lost.
[23] Which should not be mistaken for that former dualist interpretation with the same name.

It differs with respect to the particular ways of widening or extending the ontological scope of the observable universe. The holistic as well as the quantum potential interpretation are definitely not observer-centric in any traditional way, but maybe being universe-centric is the very last bastion of being even involuntarily observer-centric.
The extension of the scope of the physical ontology of the observable universe goes in the holistic as well as the quantum potential interpretation in a way of as well embedding as permeating or pervading the substance of the observable universe with a kind of ethereal quantum-state-substrate, i.e. a nonlocally omnipresent aura in and of the observable universe and being in undivided communion with it. And also being as detectable as it fits for ethereal quantum substances.
The quantum information interpretation instead presupposes an orderly divided universe and – what makes even more of a difference to the otherwise philosophically not so unfamiliar sounding quantum potential interpretation – it is not universe-centric. Universes are in the view of the M-cosmologically prepossessed quantum information interpretation rather particular instances or occurrences in a more fundamental pre-universe(s) or hyperspace physics.
The particular way of dividing the universe is directly related to its beginning as a 5 dim space, which came into existence as a compactification in a 11 dim hyperspace, and of which then 'later' the 4 dim surface overproportionally expanded. The storage dump for the quantum information states correlated to the quantum states of the 4 dim surface part then is the 5 dim bulk part of the universe and that might perhaps justifiedly be called an 'inner aureola of the entire universe', but it could hardly be called an 'ethereal quantum substance', since it is just – meant to be – an ordinary chunk of (more or less) ordinary physical space. That is to say, that hyperspace ontology is more ordinary and sober than Hilbert space ontology.
The multidimensionality and higher dimensions D. Bohm and B. Hiley referred to, thereby explicitly pointing out, that it could go up to infinity, and which plays the role of the space of the ethereal quantum-state-substrate in the holistic interpretation are of course the dimensions of the Hilbert or quantum mechanical representation space, whereas the higher dimension required by the bulk space in the quantum information interpretation is not that of a representation space but of a 'proper' physical space. An *ontological* interpretation of QM however should then probably be better built in and based on proper physical space – though of an a bit advanced dimensionality – than being implicated in the infinite dimensionality of an ethereal quantum substance derived from ontified secondary attributes.
The monism of the holistic or the quantum potential interpretation of QM then is – as it should be expected – quite in accordance with their respective ontology, i.e. a widened or extended physicalistic monism, the proper substance of which

consisting of the ordinary physical substrate of the observable universe as provided by the respective theories of high energy physics on the one hand and general relativity on the other, but also with this ordinary physical substrate somehow 'completed' by that pervading ethereal quantum substance of ontified Hilbert space states.

The monism of the quantum information interpretation however differs essentially from all of that. It is simply not a physicalistic monism and it is also not a sensualist, mentalist or idealist monism in any way. In philosophical terms the monism of the quantum information interpretation has, in my view, to be seen as a quite new and pretty strange kind of 'neutral monism'. That is the name of a concept, which once had been introduced by Bertrand Russell [24], yet in a context completely different from that of the ontological implications of an interpretation of QM and also with a distinctively different meaning of 'monism' compared with that; what one has to expect in the case of the ontology of the quantum information interpretation.

The monism implied by the quantum information interpretation will have to be as objectivist as this interpretation claims to be and, since it also claims to be no physicalistic monism, it seems to have a somewhat hybrid nature, i.e. it seems just to be ontologically – somehow neutral. Yet a hybrid nature means sterility and neutrality sooner or later turns out not to be a stable position. Therefore being neither physicalistic nor subjectivist in any sense can only go together with a comfortable ontological position, when it can be – so to speak – substantiated, i.e. being founded in a respectively *neutral substance.*

Asking for a substance independent from any physical or – in the broadest sense – mental or subjectivist substance must appear quite weird or being a case of philosophical agitation, but interestingly enough the answer to this question is possibly a suggested implication of the Holographic Principle.

What I mean by that I can perhaps make at best clear by 'demonstrating' that, what I think the Holographic Principle eventually will do to the content of a well known statement of Rolf Landauer, namely "Information is physical" [25]. The Holographic Principle however is, in my view, the *first indication*, that Landauer's statement eventually will have to be inverted and then go as *"Physics is informational"*, and that is of course not meant 'informally', but utterly ontologically.

And since the Holographic Principle, in particular in connection with that informational aspect, which sees quantum states – at least if vanished or not being observable – becoming equivalently quantum information states (in the bulk), plays such an important role in the quantum information interpretation, I therefore hold that the proper monism of the quantum information interpretation is an *informationmonism*.

Yet I will at once admit, that the assumption of an informationmonism being the ultimate ontological setting can not be sufficiently corroborated solely by referring to the Holographic Principle. The Holographic Principle just is, as I said above, a first indication that the ontology of an ultimate theory should be of such a kind. The domain of the possible corroboration of that hypothesis yet lies even beyond the domain of pre-universe physics but rather in the domain of the emergence of primordial partons up from 'something else' – i.e. something that could perhaps be called 'pre-physics'. [24]
Yet since an interpretation of QM should be concerned with the question of, where 'somewhere else' may be laid, and not with the question of, what 'something else' may be, I now should better leave this ontological speculation.
Talking of speculation, a quantum information interpretation should perhaps say something about quantum information processing. D. Deutsch recently [27] argued for the Many Worlds interpretation of QM by suggesting, that the expected effectiveness of quantum computation would be due to exploiting the computational capacities of the many worlds – taken literally. I think he pointed out something really important by that argument and I also strongly agree, that the expected effectiveness of quantum computation is further evidence against any subjectivist interpretation. This effectiveness then is also an indication, that quantum computation must have a reach to a resource somewhere beyond the reach of observation, or better: beyond the limits of the observable universe.
But then I would also hold, that quantum computation would work not less effective on – so to speak – a 4+1 dim 'bulk hard disk' than it would do on a sequence of copies of the (3+1 dim) observable universe in parallel connection.

And since we are run out of speculations for now, we will come to the end of this paper – with a bit of 'revisionism' attached.
The quantum information interpretation of QM as proposed in this paper has some rather unexpected implications. So it is intriguingly double faced with respect to the universal validity or significance of QM. On the one hand it absolutely uncompromisingly follows Everett's proposal, that the wave function (of the universe) designates a fundamental encompassing reality (of the entire universe, now consisting of a 3+1 dim surface part called the observable universe and a 4+1 dim bulk part, which could be called the 'dark (side of the) universe'[25]). And it also strictly follows Everett's proposal, that there are no objects, which are no quantum objects (in the entire universe).
But on the other hand it also seems to confine QM – at least with respect to the quantum phenomenology, which plays such an important role in the questions

[24] More about the possible relations between 'pre-physics', informationmonism and the mêontology of an ultimate theory can be found in [26].
[25] Which again might by some be seen as a more or less fitting paraphrase of 'inner aureola'.

concerning the interpretation of QM – to such rather incidental and particular objects as universes (including delicately combined ones) are or at least to such objects, which would satisfy the Holographic Principle, in a way comparable to the one proposed in this paper, i.e. for surfaces of up to 3 spatial dimensions. That obviously leaves some (hyper)space open for pre-universe physics, which doesn't relate to universes at all, i.e. – so to speak – for pre-compactification physics, which then would not necessarily show effects of quantum phenomenology comparable to such, which play a role in the EPR situation, at all.

These effects then could possibly turn out just to be something like an epiperipheral after-image of the intrinsic dualities at the core of M-theory. An after-image appearing after at first these dualities have been broken down to the 11 dim supergravity and then secondly this domain again becomes compactified to the 5 dim emergent integrated universe and then eventually actually emerging, when growing out of this 5 dim universe a 3+1 dim surface part comes up, called the observable part of the universe (and covering the 4+1 dim bulk part). And then the Holographic Principle would come into play as a kind of 'interface' – or projection area for what's eventually left of the original dualities – for the observer in the end.

This somehow leads me to a final remark. If the quantum information interpretation of QM proposed in this paper would turn out to be right, then this would consequently imply a rehabilitation of the consistently mocked at position of Einstein in the debate (with Bohr and others) about the interpretation of QM. The quantum information interpretation is thoroughly non-subjectivist, i.e. objectivist and therefore – in any reasonable sense [26] – 'realist' and it also doesn't require any nonlocal action taking place *in the observed part* of the universe. Yet it nevertheless strictly accounts for the nonlocal appearances or effects, which are observed. I.e. the answer to that question, which was the title of the famous article of Einstein, Podolsky and Rosen: "Can quantum-mechanical description of physical reality be considered complete?" [7] then would – at that historical time – have had to be a sounding "No!". Which incidentally is exactly the answer Einstein and his co-authors then suggested.

Yet in the light of the proposed quantum information interpretation this would have been the right answer not for the same reason, which had been assumed to be in question by all the participants in that original debate, namely that there either was something missing or not *on the side of the theory*, which then again would either way have had consequences for its ability to more or less perfectly or completely describe physical reality.

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[26] I.e. *not* in an epistemological sense! For a more detailed discussion of epistemological, ontological and other aspects of anti-realism, objectivism and realism cf. [28].

But the answer would rather have had to be "No!" for the actually *not* deliberated reason, namely that there was something missing *on the side of the physical reality*, and not just a major chunk of knowledge about its pertinent (in)completeness, but – even worse – the biggest chunk of that physical reality itself.

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THE DISEMBODIED OBSERVER

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In the ANPA meeting of '03, 'time' was a recurrent theme. In a process theory such as the combinatorial hierarchy, things happen one after another and their sequentiality is what the theory consists of in the first place. If one asks 'do they happen in time?' then the obvious answer is 'yes'. However it is wrong to speak as though there has to be a 'time' formulated for them all to be *in.* Moreover questions about the directionality of the time become otiose. If things happen in some particular order it is a mere necessity of grammar to say that the order is in the forward direction. These questions have often provoked the idea that one should import the unidirectional increase in entropy to explain the apparent arbitrariness of the direction of time, but from our point of view this is clearly wrong-headed.

There is a natural place for the process view in high energy physics, where the initial information is always of the sequentiality of events and where everything has to be built up from that sequentiality. Thus in studies of gamma ray spectra there may be a profusion of evidence of 'particles' out of which alone one can get any assurance of their individuality (their 'reality' really). One makes inferences from their appearance in an observed sequence.

In a process physics there is no obvious meaning for a universal time: there are only processes that are disconnected until we prescribe otherwise, and we cannot force a connectivity on them by just saying that they all go on 'in time'. People might say that the relative places of things in this 'time' are discovered by an observer, or perhaps by an observing system. There may be no harm in that as long as we realize that there has to be an observer postulated for each process that is discovered. Thus though we may speak of an observer for each sequence, we still have to give meaning to the universal observer and to be able to attach objectivity to the manifold of observations.

It may seem that it would be better not to introduce the term 'observer' at all in view of the limitations on it. This is not necessarily the case since there is a central step in the development of the hierarchy at which self-reference becomes possible, and at that point the logical foundations for the observer are laid. I shall not discuss this here, except to notice that the change goes with the first change in level construction.

I mean to explore how far our primitive observer satisfies what is needed in what we know of 'observation' of paranormal things. For this limited purpose I shall speak of *the disembodied observer*.

The first and most obvious thing about the disembodied observer; and one which accords with the massive evidence of the paranormal; is the fragmented nature of the spatial intuition. We expect that we can *imagine* near or distant things and scenes, and switch without trouble form one to another. When however we claim to have actual knowledge of those scenes, or even to influence them psychically, we run headlong into our deepest physical assumptions. Action and interaction are assumed to be in the first place between *contiguous* things, and even if physics assumes interactions between spatially separated things, that assumption is always accompanied by assumptions about fields or other ways of transmitting action. Fields are thus a tangible substitute for contiguity. That interactions, or effects of 'mind over matter' should depart from this assumption of contiguity is what causes most people to find them astonishing and causes many to reject any possibility of their being true because they would be 'illogical'. The position can be put very clearly by saying that in the massive experience of the paranormal, spatial relationships are simply irrelevant. Spatial separation seems to make no difference. This could not be in stronger contrast with conventional physics where the whole corpus of knowledge, including all such things as laws of force, is built up from distance dependence between objects. I give an example. Perhaps the most easily accessible paranormal experience is that of dowsing. Indeed it is so common that people often imagine that it is caused by forces that resemble other forces such as the electric attraction but that have not been studied in a systematic way. However the lie is given to this impression by the existence of *map-dowsing*. It is found that the dowser who is trying to find objects can work in much the same way using a map of the environment of the object, even if that is thousands of miles away. To my knowledge there are no paranormal phenomena for which any consistent dependence on the spatial separation of observer and target has been, shown to exist.

Time is very different from space. True, one can do such things as card guessing experiments directed at guessing cards which have yet to be turned up, in which one may be successful, and it is remarkable that the feeling of what is going on is not markedly different from what happens when the guessing does not involve any step into the future. One might conclude from this that time differences are also irrelevant to the success of the phenomenon. In fact this would be inaccurate, and no such sweeping statement can be made. The most one can say with any certainty is that forward connections between events which we should normally describe as separated in time cannot be denied as having happened. We have to make the best sense that we can of the very great amount of experimental evidence that is available to us –mostly anecdotal.

Again, one might argue that if sensitive people can sometimes transcend the time barrier then the barrier is not absolutely there at all and might not be psychically observable. Here again one has to pay careful attention to the evidence, and I believe there is no clear answer. To say that there is no barrier is probably wrong: sensitives who are looking for a missing person usually have a pretty clear idea of whether the person is alive or dead *at that moment,* so that they do seem to be working in real time to that extent. Would we expect this complicated relationship between space and time from our hierarchy? Yes, I think so.

Our psychic world evidently starts from an intuition of space. However that intuition is very different from what the physicist uses. You could say that he imagines himself, for the purpose of physics, as an animated object like the other objects that physics deals with, moving about among spatially distributed objects. It is usual to suppose that every significant statement about the physical world is built using this intuition of our place in a space which is there to start with. I say, by contrast, that that picture has *to be built up*; we have to construct it from the beginnings. To say that one rejects the continuum helps in a way, but really the discrete/continuum discussion presented as a dichotomy, is something else again.

My experience makes me think that unless the cards are put on the table in this brutal way readers will find all sorts of ways to wriggle round the vital points and temporize with more conventional images to mitigate my radicalism. They will then get distracted from the logical progression of the argument. I of course realize that some people who are concerned with the hierarchy work will strongly disagree and will think that fiddling about with the conventional philosophy –particularly where perception is involved- is irreducibly wrong. A larger number will hope that more than one set of principles can be pursued at once in a relaxed manner.

What has become of the *objectivity* of the world that I describe? Where are the massive permanences of history? Clearly our theory allows a great deal of adjustment which you might call subjectivity. However we do not go all the way to subjectivity since our combinatorial framework is not adjustable. For example it is very inflexible on the three-dimensionality of space, and the basic numbers would remain in any application. Thus some basic things are invariant under the statistical input and it is these that give us fixed points in our theory constuction. They do not need observer universality.

Contingency arises from the special configurations that dictate the choices that are presented to our construction process, and, while that is inscrutable as far as physics is concerned we must realize that it may be subject to psychic influences. The existence of psychokinesis shows that this idea is not absurd. Whether it is spoon bending or the movement of physical objects or whatever, the importance of a *thought form* is usually manifest. The delineation of the objects manipulated seems to follow human ideas of what constitutes an object, and the psychic phenomenon keeps the objects intact –observing what we think their limits to be. Physical forces couldn't care less about how we see fit to divide up matter. Thus with psychokinesis there seems to be a linking of perception or volition with 'reality', so that thought forms are directly manifested.

The philosophical implications of the views that I have put forward are too all-pervading to go into much, but I mentions two very general ways in which experience of the disembodied observer seems to suggest two traditional philosophical positions that fit well with our physics. Firstly the place of space and time accords in general with the Kantian synthetic a priori. However different things seen paranormally may be, we still SEE them in space and time. Secondly both the paranormal and the physics need the relational space of Leibniz. It is hard to fit the *milieu divin* into absolute space.

PROOF

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What is proof? And what is reality? I am reminded of the unfortunate foreigner with an imperfect grasp of English use of the definite article who inquired of a stranger "Vat is time?" Unfortunately for him he happened upon a philosopher rather that a policeman, and the philosopher, stroking his beard, said "Sir, you have asked a very profound question." That is rather the way I have come to feel about proof. If you ask a difficult question you get a difficult answer.

It was in connection with psychical research that I first asked this question, because paranormal phenomena distinguish themselves from most other forms of experience by being sporadic, unreliable unpredictable, unrepeatable, demonstrable only by one or two people in every generation, if that, and by normal standards acausal. And, of course, impossible. So how do we prove that paranormal incidents have reality? In the light of this problem I asked the question.

I started by taking down a book called Philosophy made Simple, which is how it has to be if I am going to understand it. To my great dismay PROOF was not indexed. But if you regard something as proved, or proven, if you prefer, you are (I think) making a statement about your state of knowledge, and "Philosophy made Simple" reminded me that Hume - David, not Daniel - brought philosophy to a lurching halt by proving that you cannot know anything. This is called the problem of knowledge. After that scientists took over from the philosophers, and scientists do not have that problem. (Nor, come to that, do lawyers). But scientists have their own problem, a variety of the Jowett syndrome:

"I am the Master of this College;
What I don't know isn't knowledge."

What scientists tend to assert is that nothing can be regarded as known unless it is susceptible to scientific modes of proof, and has been shown to be established by those methods. 'Known' is equated with scientifically proven. That is surely a great mistake, because most things that actually happen in the world, or are supposed to happen, are not open to scientific investigation, and yet they are real events. To-morrow no one will be able to prove scientifically that today's events took place, but would you consider yourself to be lacking in scientific rigour if you were here and now convinced that yesterday you woke up, had breakfast, etc. etc. There must be ways of knowing, and being sure, other than by scientific proof; otherwise we shouldn't know much.

At this point please imagine that up there on the wall is a visual aid, a diamond shaped diagram. Sitting there aloft, nearest to heaven and furthest from the ground, you see the words **Mathematical proof**, that is to say, proof by incontestable logic. Once you have learned the lesson of Pythagoras and his theorem you can sit back and be one hundred percent sure that the square on the hypotenuse really does equal the sum of the squares on the other two sides. You have an absolute proof, and therefore absolute certainty. And note that we are not talking about any particular triangle that existed in the past, or that will exist in the future. We are really talking about a platonic idea of a triangle, something detached from the specifics of time and place.

I gather that mathematicians, happy at last with a proof of Fermat's last theorem still have to suffer uncertainty when it comes to goldbach's conjecture - a proposition that even I can understand, that all even numbers can be broken down into two prime numbers. It seems that computers can furnish umpteen examples to illustrate this great truth, but trial and error is not the same thing as proof in the first degree. Because that tells you exactly why, even if it takes a few hundred pages of argument.

Sometimes the proof seems to arrive before anyone knows that there is anything to prove. Among anecdotes I pick up on Open University TV is the feat performed by the 19th century Irish mathematician William Hamilton. He proved that if you had a prism with bi-axial symmetry, which he did not have, then light focused on the surface would pass through as a single ray but emerge the other side in conical form, whereas light striking at another angle would break up to form a hollow cone and emerge the other side as a hollow cylinder. He was later proved to be right when the requisite prism came to hand.

But which was the proof? If two groups working with different prisms came up with conflicting results then mathematics, the supreme manifestation of truth, would prove that one lot had got it right and the other lot had not. But supposing everyone working with a bi-axially symmetrical prism found no signs of a cone or hollow cylinder, then who would you believe? That the prism experts were wrong, that the experimenters were wrong or that the mathematical proof was wrong? In the real world, removed from on high, proof would not always win the day.

Outside mathematics - and most of life does seem to carry on without any mathematical counterpart - there are explanations and partial explanations that look rather like equations. My chemistry book tells me that if you put an iron nail into copper sulphate solution the result will be a nail coated with copper and a green liquid where before you had a blue liquid. I suppose this must have been a fairly reliable sequence of events long before anyone had the slightest idea why this might be so.

Comes the day when someone can write on the board Fe + CuSO4 = FeSO4 + Cu, and that looks very similar to some simple equation like $(a+b)^2 = a^2+2ab+b^2$, but despite the similarity these two explanations are not quite *in pari materia* . The chemical equation is a mere description of what is happening, that the iron is pushing the copper out of the solution, and in terms of chemistry it is a very partial explanation, one that would be amplified in the course of time by further explanations in terms of atomic structure and core charge and I know not what (and had better not try to say).

The point is that before any of the explanations people who put nails into copper sulphate would have been in no doubt as to what was going to happen and what did happen. They were convinced because it happened and they would have considered it a demonstrable fact of science though there was no theory to back it up. And on the basis of past experience they could predict future events.

An explanation is a lovely thing, and one can see why scientists find it so satisfying. My brief acquaintance with chemistry was very short on explanation. I knew that salt dissolved in water and even understood that the components separated into ions. I was thrilled years later to see a new school textbook with marvellous illustrations showing the break up of sodium chloride, two water molecules heading towards the positive sodium, negative oxygen leading the way like two red-nosed policemen, green negative chlorine surrounded by pale but positive hydrogen prongs protruding from an asymmetrical water molecule; here I felt was something entirely convincing, something that didn't just happen, but something that **had** to happen. Without actually seeing any of this take place, you feel that when an explanation hangs together so perfectly it has to be right.

John Stuart Mill thought that the then emergent atomic theories might transform chemistry from an inductive science reliant on observation and experiment into a deductive science where the scientist just thinks and reasons.

I think he was saying or implying that this would raise it to a status close to the mathematically exact. But it can ever get there. In school geometry there would seem to be no way Pythagoras could be unseated, but in applied science theories can indeed prove to be wrong or faulty, however well they seem to fit the facts.

As an example of faulty theory Lavoisier classified heat as an element, so that a body was hot because it contained heat. But regardless of why, things got hot when they were heated. The demonstrable proof that salt dissolves in water is really to show that it does. The supporting explanation adds weight. There is always room for weight, because outside maths there is no such thing as 100% certainty. Strictly speaking 'proof' should be eliminated from popular speech, but that means losing a short and useful word; it should however be understood to have quotation marks around it. What is beginning to show a nose above the water is that a scientifically proved principle and a real effect are not identical.

The weight added to a demonstration is rather like the weight added by motive to a sequence of objective clues. The detective finds that the supposed murderer has been identified as leaving the scene, his fingerprints are there, the victim's blood is on his clothes, but there is no shred of motive. This leaves a big hole in the detective's case and in his sense of certainty. Now he finds the motive and it all hangs together. Motive is a very probative factor, but standing alone it proves nothing.

You can put salt into water any day of the week, but there are many established facts of nature that you cannot demonstrate except by waiting for them to happen. Some of them happen with frequency and regularity. In pre-scientific times as now the sun (presumably) rose on one side of the horizon and sank on the other. The tides came and went. People may have theorised that the regularity of the heavens were arranged by the gods to assist astrologers and sundials, and they may have thought that Thor was gathering up the waters to take a bath, but however wrong they were about the explanations could they have doubted that sunrise and high tide were proven facts of nature?

Certainty that high tide would happen about six hours after low tide rested entirely on past performance and the ability to predict on the basis of regular observation. Proof, in the sense of being able to show that your prediction was correct, would have to wait upon the event, and is really better described as validation of a prediction. But as you stood on a dry beach looking out on a barely visible strip of sea at midday you would surely be entitled to tell a sceptical visitor from a Mediterranean country that within six hours the Atlantic waters would be raging over your heads if you stayed where you were.

Would or should your certainty about this cycle of events be any more securely based because you know, or more probably have a very vague idea, that tides are something to do with the gravitational pull of sun and moon? If regular natural events are reliable to the point of invariability, an explanation adds very little weight to the certainty you are entitled to feel about a frequently verifiable event. Whether or not you understand what is meant by gravitation - and it seems that those who do are in acrimonious disagreement with one another - you do not doubt the reality of alternating tides or falling apples.

So far all the events considered here have been predictable, either because they happen with ordered regularity or can be made to do so, and repeated observations enable you to enunciate a principle to the effect that they can, do and will take place. **Demonstration** and **Prediction** can now be placed to left and right of the diamond, so that they form a triangular pediment with mathematical proof sitting aloft on its own. As I said, mathematical propositions are timeless and placeless, but Demonstration and Prediction are concerned with events in the real world that are to take place at a definite time in the future; in a demonstration someone has arranged things so that the event will take place as a result of these arrangements, and in a prediction the predictor has identified the conditions that will lead to the predicted event.

But we have not finished with reality yet - in fact, we have hardly started. Life as we know it consists almost entirely of apparently random happenings, just one thing after another, but some of nature's surprises may in time move into the predictable class when more is known about them. Eclipses have moved up a class, so to speak, but tidal waves and earthquakes still take people by surprise. A tidal wave, or tsunami was reported in Hawai a few years ago, but speaking for myself I have never seen a tidal wave, and have never seen Hawai, come to that. Tidal waves do not seem to happen round the coast of Britain, so how do I know that they happen at all? How would you set about proving to me that they do happen? This is where tidal waves differ from dissolving salt and falling apples. The only way we can know that they **can** happen is to become convinced by evidence that tidal waves **have** happened, and to be sure of that you must be sure that a particular earthquake here and there **did** happen. So the answer to the original question depends on information gained from identifiable past events.

You could explain various theories onwhat is thought to cause tidal waves, and that might sound plausible, but I am not well informed enough to know whether your explanation is good science. You certainly can't arrange for one to take place. I am not much better off than if you told me that they happened because God was angry with the wicked, and nobody knew what would set him off. So why should I accept that tidal waves happen? And if I do, on what basis? On the basis, I have to say, of the rankest hearsay; but then hearsay can be entirely veracious. Tidal waves, I tell myself, are talked about in serious books as if they were a recognised class of geophysical event. Several have been reported recently in quality newspapers. I can think of no plausible reason for these stories being invented, and I should think there would be an enormous international row if news media reporters faked shocking tragedies and got people to subscribe to imaginary funds. I have actually seen on television what have purported to be people fleeing from tidal waves, though they might have been pure fiction, and heard people tell their dramatic stories.

This is just the sort of evidence we have to rely on to be sure that tidal waves actually happen, and when all things are considered the evidence for the existence of tidal waves is somewhat less copious than evidence for the existence of poltergeists. All in all, is the case for tidal waves, and for Poltergeists proved? Can we speak here of proof? It is more a case of **Verification,** being satisfied or convinced by the testimony. I do not actually find myself significantly less sure about the reality of tidal waves, which I have not witnessed, than I am about high and low tide, which I have. So I reckon that when you're sure, you're sure - anything above 99% sure will do, and the fact that you might add a few .999s onto your sureness about the sunrise, tides or earthquakes does not make all that difference.

But beyond the sunrise, tides and earthquakes are those singular events that fall into no familiar framework or context, because they are unexpected, unlikely, and sometimes one-off events. Selecting one at random, what about thousands of jellyfish that arrived a few years ago in the North Sea and blocked the water intake of a power station, a startlingly improbable event that with any luck may never happen again. How would you satisfy yourself that it ever happened at all?

It is clear enough that you cannot prove it by laying on a demonstration, and unless you have the sort of gifts we are always looking for you cannot make a prediction to say when it will happen again, though you might say in a general way that if the climate gets warmer we may expect to find some very obnoxious marine life in northern waters. You also have to say against its veridical status that jellyfish blocking cooling inlets is not something that you commonly read about in serious books. It was just a specific thing that happened, or is claimed to have happened, and you want to be satisfied that it was a real event, as real as the salt in the sea, and not just a sensational story made up by the media.

Psychical researchers will recognise this problem and know how to go about it. First I refer to my source of information, which is again *The Times* itself. But if you did not read about it you are hearing about it from me, so you have more questions to ask about it than I have; you have to consider how likely it is that I have invented the whole story. An explanation was forthcoming at the time, though I do not remember the details, but it sounded convincing.

I argue that *The Times* is usually reliable as a source of information, and though you shouldn't believe everything you read in the newspapers they are not likely to have published a report about a named power station if the story had been substantially untrue; perhaps the size of the jellyfish might have been exaggerated by a small margin, but jellyfish there must surely have been. If it had been a complete invention there would have been an indignant rebuttal from the director of the power station, leading to retraction and even apology from *The Times*.

That is good enough for my purposes, but if my whole view of life depended on it I should make further inquiries - look up the back numbers of *The Times*, find the name of the power station, write to the director, question employees, examine the site, seek out expert opinion, ask around in the neighbourhood, speak to the reporter, and (crucially) assess all the witnesses as purveyors of truth. I might finally be satisfied.

And when I am satisfied, how satisfied am I entitled to be? Am I sure that this unusual event happened? If my expressing or holding a false view of its ontological status would result in my being sent to serve a term of life imprisonment in Saudi Arabia, or spend ten years in the salt mines of Siberia, or even in our own salubrious open prison, I should certainly say 'Yes, it happened.' The *Siberia test*, as I term it, is a wonderful way of focussing the mind on how reasonable it is, and how likely to be *right*, for one to adopt a Humean scepticism in the assessment of evidence.

The Siberia test forces a decision on a balance of probabilities, which might be a mere 51:49, not a very high standard. I am in fact more certain than that. I am sure beyond reasonable doubt, a standard closer to 99:1. However, it is clear enough that my certainty has nothing to do with scientific proof. There may once have been jellied remains to be seen at the site, but I should have to take someone else's word for it that they were there.

We approach the awesome truth, which is that most facts in life are 'proven' by the methods I have outlined, which are the methods of the historian, the lawyer, the policeman and any other rational person who wants to know what has actually happened - not what *can* happen under certain circumstances, not what *will* happen on defined occasions, but what *did* happen. This is how you 'prove' that the Thames used to freeze over so that horses and carriages could cross it. This is how you prove that the Queen's Hall was hit by a bomb in the 1940s.

This is, of course, how you prove that filing cabinets, chandeliers and that pictures moved about of their own volition in a lawyer's office in Rosenheim and the misbehaviour of the telephone system baffled scientists from the Max Planck Institute. But it is not only in psychical research that the reality of events has to be proved by these methods. This is also the way you prove that Eddington and his team took some rather blurry photographs that proved the bending of light in the gravitational field of the sun.

It is true that science, which tells you what can happen, and seeks to tell you why, aims to move on beyond historical fact. Its business is to re-enact the event using improved technology, verify the the cause and enunciate the principle. Rutherford's demonstration of particles passing through atoms does not have to be verified by reference to his personal experiments, because the effect can be demonstrated by other scientists.

But let us note that the quest for a principle has to be grounded on data from reports of past events, events that have to be evaluated in just the same way as those displays of paranormality at Rosenheim. Scientists can't usually generate their own data, because no one can be an expert in every field; so scientific inquiry starts with reliance what other people say. Once a principle has been propounded it is again only experts in the field who will be able to verify it - others have to depend on them and assess their reliability. Each verification in turn becomes a past event, and other people have to rely on it as a report.

So to navigate reality we are all dependent on evaluation of things that are said to have happened. The verification of past events is not by considering whether they ought to have happened according to rules, regularities of principles, but by **inquisition**, which is directed to discovering whether they did in fact happen. Inquisition is a tedious process, and you never reach the end of your inquires, though you can reach a vanishing point, the point of satisfaction.

Can you ever be as satisfied as you are about Pythagoras? In theory, no, but then remember that there are very convincing ways of showing that 1=2, and if you cannot spot the fallacy in the argument you might do better to feel sure that there really was a hurricane here in England in 1988, or that you did indeed listen to the radio news this morning, though that event has now passed into history, beyond the reach of science.

Back to earthquakes and other things that never happen to most of us. Some readers may be bursting to point out that earthquakes do in fact leave objective traces behind so that archaeologists can bring scientific certainty to the hearsay of history. That is true, and it leads me to point out that the sort of past events I have mentioned so far belong to the world of things. But non-geographical events also belong in the web of history and have reality. When you bring in behaviour, problems of proof proliferate, and the paranormal does appear to be a behavioural phenomenon. Even the most physical paranormalities are not ordinary anomalies such as belong in the realms of unsolved science; psychical anomalies appear to be related to will, mood and personality.

So I am on the way to bringing in the paranormal, but first I think a short recapitulation is in order, and I shall return to the invisible visual aid, which has mathematical proof, which we can now forget, sitting at the top with Demonstration on the left and Validation on the right. **Inquisition** may now be placed at the lower extremity of the diamond, and the visual aid is now complete. However, these terse descriptions can do with some amplification. Ignoring mathematical proof, the three classes stand as follows:

1. **Demonstration.** The **ascertainment** of **a replicable and reliable effect**; and demonstrating the effect necessarily entails sufficient understanding of its cause.

2. **Validation.** The **ascertainment** of a **recurrent and predictable effect;** and predicting the effect may or may not entail understanding of its cause.

3. **Inquisition.** The **verification** of **an occasional or singular**

event, regardless of any causal context that may be attributed to it. This is the historical mode of fact finding. It includes personal observation, the testimony of personal observers, the testimony of more remote informants and any objective exhibits from which deductions can be made.

It will be seen that Class 3 covers a range of incidents, from occasional to singular. An **occasional** event is an activity of a recognised type known to occur from time to time, while a **singular** event is *sui generis*, a one-off surprise. But just as an unpredictable event can move up a class and become a predictable effect with a recognisable cause, so may a singular event turn out to be occasional rather than singular.

Scientists in general don't have to deal with events that seem to have no cause, because the paranormal lies outside the usual scope of science. Those scientists who do study it have rather limited scope for strictly scientific methodology, and are frequently assailed by the ultimate insult that what they are doing is not science. But as we have seen, scientists dealing with normal events also have to depend on historical modes, in other words they have to make the best of observation, evidence and testimony.

Personal observation necessarily plays a very limited role, since the observer cannot be everywhere observing everything, nor is an observer equipped to make reliable observations in specialities outside his own. and unpalatable though it may be to the scientific spirit, the status of all events not personally and efficiently observed rely ultimately on testimony. The very laws of science, such as they are believed to be at any time, rest on the testimony of other people, and so long as they report events that fit in with the current paradigms those witnesses are believed, and believed in just the same way and for the same reasons as providers of historical source material.

Most people know next to nothing about the laws of science, but feel that there is great strength in the ability of scientists to check up on one another by replicating any effect as to which they feel some doubt. Well, a team of scientists are reported to have descended into the bowels of the earth, somewhere in Europe, to try to register a blip that just might happen this year or next year, their purpose being to prove that most of the matter in the universe is invisible. Try replicating that.

I should now like to move on from the domain of physical events that just happen and deal incidents that are brought about by volition or at least by behaviour. We are now well out of the mainly mineral-vegetable world and into the mainly animal world, so much more complicated. What are the routes to certainty here.

Demonstration. Can anything be demonstrated about people? Biology is not all that different from chemistry, so that clearly you can seize anyone in the street and show that his heart is beating. But can you say with scientific certainty that because he is wearing a cloth cap he will definitely vote left? Obviously not. You may have picked on a titled country gentleman on a day out in town. Once mind and will enters the picture you can forget about geography, chemistry and biology. We are into psychology.

The DMT test can, it is claimed, be applied with a high degree of confidence that the result will be a reliable indicator of the behavioural tendency known as defence mechanism. This is apparently something about people that can be demonstrated, but still no one claims that it will yield a correct result every time without fail. Perhaps it will work out with 70%, 80% even 90% reliability. That is not certainty, but psychology is recognised as a science, so it seems that statistical degrees of certainty are acceptable as proof of an objective effect. But your demonstration is reliable only on a probabilistic basis.

Validation. This concerns effects that can be predicted because they have a degree of recurrence in their character. Predictions are certainly made about behaviour, but again only in terms of statistical certainty. Insurance companies remain in business because they know roughly how many people will set their houses on fire, crash their cars or die just when they are about to draw their pension. Students of the Poisson distribution were able to calculate, presumably in pre-war days, how many years were due to go by before it was time for a horse to kick a Prussian army officer to death - and the horse seemed to know, too. It is all scientific methodology, just like the theory that tells you how long it will take for half the radiation in a radioactive substance to break free, though no one can say which particle will go and which will stay.

The moment has come to say a few words about the paranormal. It does seem that demonstration based on the supposed talents of the average person, or the average student, could lead to statistical certainty. It may well be that in the normal way, if one can put it like that, powers of ESP and PK are so thinly distributed that we have to be treated as amounting to a swarm of particles, insubstantial as individuals but sturdy taken en masse.

Great efforts have made by meta-analysis to bring these numerous experiments into a framework demonstrating an acceptable degree of reliability. This is an exciting development. If the availability of computers makes it possible to establish by means of millions of responses that a scintilla of psi lurks somewhere in human consciousness that is certainly the beginning of quasi-scientific proof. Whether the scientific establishment will ever accept probabilistic proof of psi remains to be seen, and in view of the establishment's resistance to the implications of the paranormal the prospects are not very hopeful. Logic has little to do with the matter when belief systems are at stake, and scientific fundamentalism dies hard.

Clearly the aim of parapsychologists is indeed to have their experiments accepted as items in a chain of probative results. But pending that acceptance, and for what comfort it may bring, it should be pointed out that experiments yielding significant results also take an honourable place as Class 3 occasional events, where a recognised type of activity takes place at irregular intervals. Alongside earthquakes and statistically significant experiments would be a myriad historically verifiable events of the sort that happen from time to time - births, deaths, marriages, divorces, murders, trials, lectures, concerts, levitating tables, poltergeists *and* highly significant experimental results - the whole pageant of life, things we prove, if we need to, by looking at the evidence.

It is not only the parapsychologists with their statistically based methodologies who suffer disappointment in having their experiments dismissed or ignored as proof by demonstration and end up in what they feel to be the inferior class that has to be assessed by verification. When Crookes found that Daniel Home could make the end of a wooden board dip down and pull on a spring balance while Home placed his finger tips on the other end he thought he would be able to deliver scientific proof of psychic force. He invited the Secretaries of the Royal Society to witness the demonstration. One refused, and the other seized on a small detail in the set-up that certainly could not have accounted for the magnitude of the effect, and refused to take any further interest. Crookes went on to try various methods in which Home altered the weight of the board without touching it at all.

As a class 1 demonstration it was possibly one of the clearest ever devised, and Home was fairly reliable in his ability to perform, but as class 3 reporting it leaves something to be desired in that Crookes asks the reader to credit him with some common sense. He expected his competence and his word as a scientist to be accepted; but no psychical researcher can afford to make these assumptions.

For class 3 reporting nothing must be assumed, and every precaution taken must be noted, not by asserting that all necessary steps were taken, but by painfully and tediously enumerating those steps, so that they can be scrutinised and assessed by the historical methodologies of verification. If he had succeeded in getting the Royal Society representatives to observe and then to replicate his experiments he would have had, for the time being, a class 1 scientific demonstration. But with the passing of Home, the whole package would pass with him into class 3 - things that once happened.

We come now to the **singular**, the paranormal events that equate with the invading jellyfish, unusual events belonging to no established class of recurrent incidents, things that just happen now and then, here and there.

These are in in the same class as the less routine facts of history, taking history to be the unfolding of events from moment to moment. Here above all it is ludicrous to think in terms of scientific proof. As in all other departments of verification, technology plays a part in dating documents, authenticating signatures and so on. But of course you do not look to science to prove that James VI of Scotland was the same person as James I of England, or that Charles I was beheaded, a highly improbable event generally accepted without question as real on purely historical evidence. Nor can science establish the abdication of Edward VIII, another unlikely happening.

Whatever means we use to establish the reality of these events is the same means that we apply to the Kathleen Goligher's table, Kluski's hands, the Cheltenham Ghost, the first encounter between Hodgson and Mrs. Piper, the Rosenheim Poltergeist and the Talking Mongoose , i.e. inquisition, examination of records, all the complex procedures of verification. If we can be sure about normal events that are improbable and prone to the evils of falsehood, error, exaggeration, embellishment and other distortions, then it ought to be possible to arrive at the certainty of satisfaction or conviction when it comes to paranormal events.

It may be useful at this stage to look at some examples to see how one might go about establishing the reality of some of the past events that constitute the case for the paranormal. And it is in the past that our case lies. If we wipe it out as we go, we eliminate the subject matter of psychical research. So let us take a look at some of the jewels in the crown.

One can draw up some sort of ideal specification for a Class 1 demonstration of the paranormal - I should certainly say quasi-Class 1, because in the course of time it will inevitably become a class 3 event. I suppose I have to call the person demonstrating the medium, though this covers a multitude of sinners. The medium would be able to perform a replicated task with a high degree of reliability. He should also be able to vary his repertoire to minimise the possibility that he has a limited routine worked out to deceive. He would be able to demonstrate his powers to several experienced and reputable researchers, and also to other responsible citizens brought in to test him. He should preferably earn a living by some normal means take no payment for his demonstrations of the paranormal.

On the few occasions when he failed he would be no worse than an actor who very occasionally forgets his lines or gives a wrong cue; no doubt there would be a reason, though no one would be able to say for sure what it was. One must say that even demonstrations of chemistry are far from being exact copies of one another, but in the aggregate they confirm whatever principle they are supposed to be demonstrating. Has there ever been such a paragon?

In the field of mental mediumship the supreme psychic was Stefan Ossowiecki, who meets all these desiderata. As there has apparently never been any other to match up to him for variety and reliability it is of considerable importance to establish that what is reported of him really happened. It would of course be even nicer if there were another Ossowiecki alive today, but then he too would be dead and gone 50 years from now. We are all historical characters.

So we have to do here very briefly what needs doing at enormous length, look at the man himself, at what he did, and at the people who say that he did the amazing things he is said to have done. Ossowiecki was a Polish engineer of high social standing, who performed acts of mediumship to oblige researchers, friends and other people he liked. Most of his work was done in Poland, much of it with well known and respected researchers, such as Szmurlo, the President of the Polish SPR.

Born in 1877, he was already approaching or into middle age in the 1920s and 1930s, when he worked in Paris with researchers whose names are, or ought to be, familiar to us. In his younger days he is reputed to have been an astounding physical medium, but when he practised one aspect of his mediumship the other went into remission.

As a mental medium his repertoire was extensive, expanding to meet the imaginative demands of the inquirer. His most typical performance was to take a sealed envelope from the researcher and proceed to give a very good description of the writing or drawing on folded paper inside the envelope. Under the strictest conditions of control the researcher would not know the contents of the target paper nor would he know the identity of the donor.

When I say that he would give a good description of the target writing or drawing I mean he would in nearly every case be about 80% to 90% right, so that you did not need to consult statistical tables to know that he had hit the target. Here is an example of a test organised by Charles Richet, prof. of physiology and leading member of the Institut Metapsychique International (IMI), where most of this research took place.

Richet sometimes used target papers supplied by other members of the IMI, or from his family or from outsiders, some of them well known, among them the French writer Anna de Noailles. She supplied three target papers, each sealed by her into identical envelopes. Richet put each one into outer identical envelopes and handed one chosen at random to Ossowiecki, who said immediately that the writing was taken from a great French poet, and he correctly named Rostand. Then he said - and though I do not reproduce every word here he did not say anything that was irrelevant - he said: " ...something of Chantecler...the cockerel... there is an idea about light during the night." He also said that below the name of Rostand there were two further lines written. In English translation the lines were:

> It is at night that it is good to believe in light.
> Edmond Rostand
> Verse to be found in Chantecler and spoken by the cock.

So he had light, night, Rostand, Chantecler, the cockerel and two lines below the name of Rostand. (Actually there were 3 lines, the words "by the cock" being on the third line). He seldom fell below that standard and sometimes surpassed it. It was almost as if he were reading from a faded photocopy and getting it nearly word for word.

In case anyone thinks he was (heaven knows how) peeking into the envelope or looking through it I must point out that on some occasions he deciphered scrawl more clearly than capital letters, and he could never give a reading for typewritten words. He could also give a reading if the target was screwed up into a ball, he could describe objects enclosed in a box, or a lead pipe, he was able, in a most fascinating experiment to describe a sentence written in invisible ink, and most telling of all he would usually launch into a preliminary description of the person who provided the target, not only their appearance but their life history and the actions they made when preparing the target material. So how could we account for this performance on a non-paranormal basis, this being the question you have to ask before concluding that it had to be paranormal.

Could he have been in league with target donors? But the identity of absent persons donating material was never disclosed to Ossowiecki. Further, he had no way of knowing which of the three letters would be picked at random by Richet. If Ossowiecki operated by obtaining secret information we should surely hear that one day he gave a full description of a letter or drawing that happened to be in a non-target envelope, but no such incident was ever reported.

So if we eliminate simple, or even complicated, trickery and also eliminate confederacy that leaves Richet as the only source or error or fraud or delusion. In the simple protocol of this test I can see no room for error, so that leaves fraud or delusion. Now if Richet had been the only one to report such marvellous results with Ossowiecki it might be reasonable for some people - especially those who have not read his publications to think that he might be fraudulent or deluded.

This is where the inquisitorial method is so convoluted. You must first of all appraise Richet, and though he was inclined to make the sort of error that comes from relying on memory rather than checking the records (bearing in mind the vast scientific and literary output of this industrious Nobel prize winner it is an understandable failing) I appraise him as a researcher of the greatest integrity.

That is not the end of the matter, for now you must go on to consider and assess the other researchers who tell the same story. One will be kept quite busy. There were the Russian and Polish academics, there was Gustave Geley, author of one of the most significant works in psychical research, "From the Unconscious to the Conscious", there were sundry respectable citizens, doctor, civil servant, League of Nations Delegate, the President of Poland and, my favourite, of course, Prof. Barrington-Emerson.

There was a test carried out at the Warsaw Congress (1923) where the target was prepared by our own Dr. Eric Dingwall and held by the German Professor Schrenck-Notzing. There were many others. Unless all these people were fraudulent or deluded then why should Richet be fraudulent or deluded?

That is the network of logic applied by people who try to determine the truth about a past event.

Let me say again that a past event includes one that happened very recently and may be part of an ongoing story. There was, or is alleged to have been, not so long ago in the 1990s, a boy called Stephen Wiltshire, who at aged 11 was mentally limited to the point that he could not solve the sort of problem that a normal child of 3 or 4 might find simple; but he had an extraordinary talent for drawing buildings, and could reproduce a mass of architectural detail with strict accuracy from memory after seeing the building for a relatively short time, and all in beautiful style. There have been quite a lot of *idiots savants* reported from time to time with various talents - extracting cube roots or making other monstrous calculations - but this is probably a unique case of a fairly retarded boy being a master draughtsman.

So we have here a singular event, perhaps the only one of its sort that most of us will ever encounter. Nevertheless I expect most people who have seen this case described in books and on television quite reasonably accept its authenticity; one would seriously doubt the judgment (and the plain common sense) of those who did not. But the published evidence supporting the reality of Ossowiecki's clairvoyance is much more copious and robust in every way.

The creators of these past events often used scientific methodology in framing their inquiries, just as scientists investigating atomic and subatomic phenomena have to use the evidential methodology of lawyers to establish a case. Let me quote from a popular science book by Heinz Haber:

"And so, more than 100 years ago, scientists were already hot on the trail of the atom. Hardly a single reputable scientist remained who was not convinced that the atom exists; all believed in the atom, though nobody had ever seen one. Yet the belief in its existence was based entirely on what a lawyer would call circumstantial evidence. It was as though the atom was the defendant in a trial. The judge, the jury, and the witnesses were all scientists. The witnesses brought to court a tremendous number of observations from the scene of the crime, and these facts could only be explained if there was such a thing as an atom. The jury weighed the facts. The circumstances were such that it could only conclude: the atom exists."

If the scientific jury weighed the facts of the paranormal in an equally rational way they would conclude that Ossowiecki had actually demonstrated paranormal cognition. They would draw similar conclusions if they gave due weight to documents signed by dozens of scientists together with dozens of other reputable observers, stating their certainty that telekinesis had been demonstrated to them by the medium Willi Schneider in decisive tests carried out by Schrenck-Notzing. Similar attestations to physical phenomena were obtained at the IMI.

One must ask how many attesting signatures would carry conviction with someone who was not disposed to accept the reality of the paranormal? If 100 signatures carry no weight would 1000 persuade anyone to accept this powerful testimony against his inclinations? The answer is plainly, sadly, 'No.'

Dr. John Beloff, a past President of the SPR, has pointed out what a fine thing it would be if we had a permanent paranormal object that could prove itself in that it could not be faked by any known process, so that testimony as to its provenance would not have to be involved in the verification process. There may be objects approaching the status of PPO already in the world, no further away than Paris. I refer to the 'hands' of Kluski. Kluski is not a household name, though he should be. He is probably the most amazing and the best tested physical medium of all time, perhaps outstripping even D D Home.

Like Ossowiecki, he was a successful professional man who demonstrated mediumship to oblige his friends, though it impaired his health. When he came to Paris in the 1920s he was already well known for materialisations, and one of ways he delivered tangible evidence of this was to get the 'spirit' hands, or feet, or bits of face, to plunge into molten wax, then into water, then to dematerialise leaving an unbroken wax mould from which a plaster cast could be taken showing lines, wrinkles and other skin surface markings indicating that the hand, foot, chin or whatever had been inside the wax mould.

He is said to have produced scores of wax moulds, mostly of hands, and we have some of the casts made from the moulds in the SPR, but the specimens in Paris are of greater interest having been produced there under known conditions. The casts are distinguished by a highly surprising feature, viz. they consist largely of adult hands scaled down to various child sizes. This extraordinary property, so inexplicable in normal (i.e. fraudulent) terms, was attributed during the Polish sittings to undersized phantoms that could sometimes be increased to full size by sitters breathing in unison with the medium. Fortunately they mostly remained undersized, and the casts of scaled-down hands with adult shape and markings remain a challenge to the psi-denialist.

Suppose it proved to be that there was no such thing known in nature as a child size hand with adult markings? The hands of dwarves are certainly not like that, and though I have never seen midget hands at close range they also appear to be rather chubby, whereas the Kluski hands are delicate and shapely. This is a matter for expert evidence, and I have not succeeded in obtaining any clear statement either way. If in the end the expert opinion were to be that midget hands are never as shapely as the Kluski hands, then you would have to go on to the next question, and ask how possible it would be to fashion some plaster and impose on it hand and finger markings that would convince an expert that it could be a cast taken from a wax mould of a genuine hand.

When all questions had been asked and answered, let us suppose that as a result these casts could constitute samples of PPO, what then? Could we sit back and congratulate ourselves on having produced a scientific demonstration of the paranormal, a sure route to proof? We could not. The weight of expert opinion may have favoured our proposition, viz. that midgets do not have Kluski-like hands. But there was, and always is, expert opinion on the other side. That is good enough for those who want to reject the proposition. So the PPO would fare no better, and be no more conclusive, than a well attested event. Its status ultimately rests on testimony, and on our judgment of the testimony.

Therefore we must return to the position where the truth about the hands of Kluski has to be established by reference to the conditions under which they were produced, which means we have to assess the testimony of Geley, Richet and the Polish researchers. We have to consider how likely it is that Kluski, or a collaborator, chose to make up wax moulds using a group of midgets (rather than more easily available adults or children), that Kluski was able to introduce them into the sitting and manipulate them while his hands were held by the IMI researchers, and so on and on. When all circumstances are taken into account then the reasonable conclusion may be drawn that the wax moulds must have been of paranormal origin. Ultimately we have to rely on verification by inquisition, and though a person who accepts the testimony as compelling cannot claim to have an incontestable proof on hand he can echo the views of Sir William Crookes, who said that the occurrence of paranormal was as well established as any other fact in life.

It is very understandable that many researchers, especially those engaged in parapsychological experimentation, yearn for the apparent certainties of scientific modes of proof. Verification relies on judgment, a fuzzy concept compared with the clarity of the eye-witness experience that validates demonstration and prediction. The endpoint of scientific proof presents itself as knowledge and certainty, whereas the endpoint of verification is varying degrees of satisfaction or conviction. The unease engendered by assessment-based conclusions reminds me of my early car driving experience, and the shock of realising that when one vehicle passes another the driver cannot actually *see* the edges of the two vehicles, but must make an assessment of their position and judge the distance between them.

But as we have shown, the seemingly solid rock of certainty based on scientific proof is extremely limited in scope. Apart from common and daily effects that are manifest to all of us, scientific certainty is available only to those investigators who personally observe the outcome of their experiments. That has to be a mere sliver of experience. All other quasi-certainties rely on the testimony of other people, so that scientific knowledge for the overwhelmingly greater part also depends on those soft-centred concepts of satisfaction and conviction based on the arts of assessment and judgment.

So if in our appraisal of the paranormal we cannot offer 'proof' on demand, let us make it clear to those who do the demanding that the methods of verification applied to paranormal effects and events precisely mirror the methods applied to effects and events in the normal world. If we can feel sure that, as reported recently, passengers in a transatlantic flight were informed, due to an inadvertently broadcast recorded message, that an emergency landing was to be made in the sea and they should put on their life-jackets (surely a most unusual and possibly singular error) then we are entitled on the same principles of testimony assessment to be equally sure that on repeated occasions Ossowiecki demonstrated clairvoyance to numerous witnesses. We can be sure that these things happened on the same reasonable grounds, namely, that the evidence compels acceptance. These were real events in the real world, with real implications. Anyone who ignores them on the grounds that they are not the subject of scientific proof is, to put it politely, making a serious category error, and equating the domain of science with the whole reality.

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A Combinatorial Perspective on the Standard Model

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ABSTRACT
Combinatorial physics provides a scheme of numbers identified with coupling constants. The general task is described and discussed of deriving quantum physics, with the standard model of the particles as the crucial step first, and of considering what is required to get to continuum physics. The following topics are discussed in a general way so as to show how they fit into the general task: 1. What are coupling constants? 2. The statistical background of the hierarchy and the vacuum and virtual particles. 3. How we get to quantum theory? The harmonic oscillator. 4. What is mass? Quarks. 6. The photon.

Introduction

The hierarchy algebra and the work associated with it claims a variety of successes that bring it directly into the ambit of the standard model for the particles. Since the work demands a quite new general dynamical background and philosophy, I propose here to look at the various critical successes of the standard model to see how far we have to go to incorporate them or satisfactorily replace them with our methods, and to see whether there are some which effectively rule out our approach altogether. Some results which are used or implied in the standard model are not specific to that model but are general results of quantum physics. It will be necessary to include such things in this survey.

This seems a tall order. Perhaps it is, but we have to remember that people thinking about the standard model usually imagine it embedded in a whole background of quantum mechanics which is, as it were, prescribed thinking. The philosophy that has grown up with the combinatorial studies sets the dimensionless constants as the first physically interpretable things to be encountered. This general principle requires, in particular, a change in the usual order of procedure, taking the line that quantum mechanics is derivative, to be added on after the principles governing the dimensionless constants have been established. In fact it will be important to see how much of quantum mechanics is needed for the standard model. We shall find that cutting ourselves free from a great deal of this dead weight leads to important clarification.

In pursuing this programme it will be necessary to show that there are no important results that have to be taken at face value because they follow unambiguously from the quantum mathematics and cannot be got otherwise. You may be inclined to say "Why can't we follow quantum mechanics (and its connections with relativity) in the ordinary way and just get on with it without having to pull everything up by the roots?" I have given the answer to this many times. It is because quantum physics does not work unless we agree to use different mathematics for two classes of things according as they have, or have not, to do with measurement or observation. The resulting principle amounts to an intrusion of a linguistic trick to provide understanding of the discrete/continuum dichotomy. It is not impossible that an understanding of observation could be found to explain the trick, in which case it would cease to be a trick, but that would take us into strange waters, and none has been offered by the quantum theorists that holds water. As things stand, a muddle has been introduced into a would-be articulated sphere of discourse, and once let in such a muddle and it proliferates to permeate the whole sphere.

The main thrust of this paper is to consider how it is that there are a variety of questions one would ordinarily ask about the particles and the standard model which the development of that model shows to be inappropriate. It seems one has to learn what are the proper questions to ask. One is reminded of a remark I have heard attributed to Dyson who said that a large part of learning quantum physics in general was to realize that there are some questions that you may ask and others that you may not, and to learn not to demand answers to the second sort.. Not very satisfactory as a general principle, one would have thought. I shall be maintaining that in an important range of such questions not merely does the hierarchy approach explain why these questions are inappropriate, it provides a theoretical background in which they never arise in the first place.

To follow this programme we find ourselves having to answer two kinds of question The first is defensive and so negative; the second goes forward positively.

FIRST. Are there any calculations which are essential to the field but which we could never see our way to obtaining combinatorially? If there are, then that would vitiate our claim to be able to advance toward as much as we need of the 'continuum' constructively. We could not break free of the quantum theoretical presentation of classical physics.

SECOND. What results or methods are there which arise naturally for us, but which the standard model has to strain to get or cannot get at all?

What are the coupling constants?

We have to get any dynamics we want for the purposes of comparison with the standard model from the bare dimensionless numbers of the hierarchy and from such pretty basic ideas as come from the construction of those numbers. This theoretical policy follows because we see these numbers as the paradigm case of measurement and from which the ordinary kinds of measureable numbers –being more complicated- ~~numbers~~ have to be constructed. In this way we get as near to the continuum as we find the need, and the continuum is a word meaning that that road is open to go further.

It is usually said that coupling constants specify the strengths of the fundamental fields or fundamental interactions. This could mean that they are *ratios* of some experimental measures of those fields. This interpretation works approximately for the comparison of the electromagnetic and the gravitational fields since the hierarchy factor 2^{137} or approximately 10^{39} is of the correct order of magnitude. However there is no obvious meaning for the pure numbers found from the hierarchy standing alone. One is tempted to use the two simplest levels (giving 3, 10) to try to form ratios with the strengths of the strong field, but the experimental interpretation for these coupling constants are at best vague.

Spin

The unique change made by quantum theory is the appearance of spin, with its characteristic factor 2. The combinatorial programme requires us to search for its combinatorial origins and leave such questions as whether it represents angular momentum to be settled later, when more structure has appeared. A long time ago we maintained that spin arose because there were certain changes at the level of the 2x2 vectors which require two operations to secure them at that level but which could be performed in one operation at the level of 4-vectors or 2x2 matrices. Though at the time we associated this fact with the half-angle rotations that were introduced for spin, we now argue that it is the appearance of the factor 2 , purely combinatorially, that is vital, and that the geometrical interpretation can be added on afterwards to taste. However the argument from the algebra of discrimination which we tried to use was in fact circular because we appealed to a process of doing something at one level and then comparing that with what happened when we did it at the adjacent level. Essentially we put a time step into the formal process. Though this change was at the heart of the interpretation of the hierarchical structure it had not been introduced formally. .

To do better, I use the idea of iterant due to Kauffman, though I do not presume his agreement with that use. I am using correspondence with him. Full 2 x 2 matrix operations correspond to pairs of iterants with [a, b] + [c, d]P corresponding to the 2x2 matrix with a b as the diagonal and c d as the anti-diagonal. This is a special case of his definition of a matrix as a sum over permutations. The vital case is when one is confined to 2x2 matrices. There seem to be the following elements in play. 1. Time steps are integral. 2. There comes a change in basic logic which for the hierarchy is the first level change and which Kauffman sees as a vital logical extension that is built into the 2x2 matrix. 3. At this stage self-referentiality becomes possible. 4. Lou sees this as the way to the observer concept. 5. The 2x2 matrix (in particular) is generated by permutations rather than by transformations.

For our present purpose we need to sidestep the usual track which Kauffman expects to follow of developing quantum mechanics and expecting particle theory to arise out of that mechanics. The alternative path is to treat the hierarchy numbers as the inescapable first steps toward measurement in a space. There is a deep correspondence with particle physics which this path illuminates. The baryons and the strong interactions are characterized by an incomplete logic not having advanced to my step 2. above. Full measurement comes at the next level with the fermions. Note that the neutron and proton are separated by isospin which has not appeared at the simplest logic. I know this sounds like the standard model spiel, but there is a profound truth hidden there I think. In any case I still pursue my 'unconquerable hope' that the duality of the iterants when you get to self-referentiality explains the inevitability of the mystic factor two which is interpreted as double helicity of spin and isospin.

A bit more on my point 5, above. Kauffman's change from matrix as transformations to matrix as permutations probably needs more analysis than I can give here. I have always stood firmly by the notion that a matrix element at a new level is given its meaning by the fact that it specifies the closure of a set of elements at the simpler level, so that random changes can go on without the closure being broken. In that case and only in that case does the new entity have a physical reason for existence. In the first place these ideas seem to fit very naturally with the matrix construction from permutations. Moreover they seem to avoid a difficulty that went with matrices qua transformations. We were compelled to think that our though our point of reference had to be instantaneously at one level, we had nevertheless to hold in our minds the potential for switching levels. There is a time step implied here, but it seems to be unhappily associated with the intervention of a mathematician.

Now the famous factor 2 can be seen in a more general light, or at least in a way that ties up with high energy physics. It is necessary to have a residual ½h of action which cannot be eliminated. This ½h is called the zero-point energy. Normally this is attributed to the Heisenberg uncertainty relation but from the point of view that I am proposing it is better to do things in reverse and to use it to introduce Heisenberg for the first time. Its physical reality is re-emphasized by the fact that its interaction with the electron gives rise to very small but extremely exact spectral displacements at low energies called the Lamb shift. These are very small because of their appearance in expansion series with increasing powers of the fine-structure constant. This understanding has impelled a more general belief in a background of particles collectively called the vacuum. Since the vacuum has no ordinary spatial concommittants the particles are said to be 'virtual'. Something of this sort, however, seems to be a logical necessity. I quote from Peter Rowlands who puts the cards on the table in a way that you do not find much in most treatments :-

"The idea of an alpha is that if you have an isolated charge it still interacts with the vacuum to produce virtual bosons (photons or whatever) by emission or absorption, and, where another charge is present, the bosons can be emitted by one and absorbed by the other in a two way process.... Virtual interaction with the vacuum is similar to real interaction with a real field (which, of course, is simply a distribution of real charge of some kind) -the vacuum acts as a virtual field." And:- "The measurement of the coupling between charge and field is the coupling constant alpha. Now, in the case of a coupling involving one charge emitting (or radiating) a photon and another charge absorbing it (in a mutual process that goes both ways simultaneously), the rate of the interaction, or probability per unit time that the process will happen, is proportional to alpha." He quotes Mike Houlden as saying that 'a particle with charge e randomly radiates photons at a rate that is a constant of nature'. Scattering and emission /absorption are essentially the same process on a Feynman diagram."

This is enough to present the half integral spin as a very real physical thing. It requires only a completely general intrusion of discreteness, and anticommutativity appears only in a bare form. This fits with Kauffman's seeing it as the first algebraic step in quantum physics. His and our immediate concerns differ. Whereas he wants to show how much of quantum theory is already prefigured, I want to show how little *must* be drawn in to start with. The vision that emerges from this about the two interlocking time steps which introduce self reference really gets to the heart of things. We can take the resulting 'basic twoity' (an expression of Brouwer's) as the origin of that doubling at the basis of measurement that gives the half-integral spin. My position on spin demands a digression.

People will say that we all know the history of the concept and that whereas it may be helpful to have these combinatorial ideas they can only add to what has been well-known since Sommerfeld, Goudsmid and Uhlenbeck, Pauli and the rest. Underlying such comments there is an outlook that I think is at best misleading. It seems to me essential to ask where a fundamental innovation REALLY comes from, and to be told that it has several different kinds of root cannot be right. I expect that this complacency about a multiplicity of origins reflects the common view that quantum theory unifies everything as a solid base. I think most people would say that there is an understanding of spin within mainstream quantum theory (with its continuum basis) and that therefore to see its arising purely combinatorially must be wrong. Obviously I have to explain why they are wrong.

All the discussions of the origin of alpha (e^2/hc) require a ratio of the strengths of the mechanical to the electromagnetic force, but a variety of mechanisms to which this ratio has to be applied exists, and they are all different. The first was due to Sommerfeld who introduced a correction for the relativistic change of mass of the rotating electron into the Hamiltonian. It was at this stage that the Bohr model was superceded. Sommerfeld's mechanism was replaced (not corrected for but scrapped) by Goudsmid and Uhlenbeck who assigned an angular momentum to the electron the better to fit the spectral observations At that stage the necessity for the factor 2 emerged which they did not explain and which appeared as a puzzle. It is usual to say that Dirac put the electron angular momentum on a sound basis as a deduction from quantum mechanics by taking a 4-dimensional linear form for the wave equation for the Hamiltonian.

How to get to quantum theory

Let me suggest a commonsense way we can get to what we need from the quantum theory. We find that systems which are periodic with discrete periods are at the heart of the matter. We therefore look for mathematical ways to impose the discreteness of this sort. We might as well use the device that is used in quantum theory by making the discrete values come as the special case when complex quantities become purely real. We have the levels of energy in spectra to start with and we know that we need three sets of real integers called quantum numbers – n, l, j or whatever you call them which represent the degrees of freedom that would characterize the physical application if it were in the continuum.

I now make extensive use of an account of the place of quantization given in Clive Kilmister's book "Eddington's search for a fundamental theory" (P. 86 et seq. and the note 4 on P.95). I strongly recommend reading this whole discussion which is very pertinent to my contention, though Kilmister stops short of the radical position that I am taking. He says that for the free particle one may take p, q with the usual meanings and then bring in the complex algebra as I mentioned to give the discreteness or quantization with the rule

$$pq - qp = h/i,$$

He says "Here i is the usual imaginary quantity of algebra, ... and the way in which this plays an essential role is still a mystery". Of course it is here that the hocus pocus about states and observables comes in, but we can forget about all that by sticking to our programme of representing known facts by algebraic devices that reproduce them. Of course if we cut things down to size like this we do not get an understanding of how we go from the discrete variables to continuous ones starting with the free particle. Well no: but whoever thought we would? Next we consider the simplest case of a periodic system about which there is no hocus pocus, and that is the harmonic oscillator, where the energy is

$$E = ½.(p^2/m + m\acute{\omega}^2q^2).$$

where m is the mass and $\acute{\omega}$ is the frequency. It is then possible to show that the condition Es = es is satisfied if the number e is of the form

$$e = (n + ½)h\acute{\omega}$$

for integral values of n = 1, 2, 3.... Now write the observable A = (x + iy)(x – iy) replacing E to conform with Dirac's notation, and require

$$As = as.$$

We then get $(x - iy)(x + iy) = x^2 + y^2 + 1 = A + 2$.

Thus if a is a value of A it is also a value of A + 2 and the values of E are

$$(n + ½)h\acute{\omega}.$$

So this is how we insert the notorious half into quantum theory, though we are still a way from being able to link it with electron angular momentum. Its appearance evidently depends on the quadratic form for energy, which I take as something that needs to be introduced explicitly. (I note that Tony Deakin would probably consider that he shows the quadratic form to be necessary anyway. He, however, starts with the intuitive continuum, which we do not.) I observe that it is at this point that we may begin to install the physical variables of quantum theory with their continuum language.

If it is actually possible to do what we have done at all, then I ask with incredulity why we have all had to spend so much time, and sustain our persistence through so much confusion and perplexity in coming to terms with the current philosophical stance of the quantum theory.

There are many trenchant consequences of the position that we have reached. Mostly they arise through seeing clearly what we have NOT had to introduce. I assume that our argument will take a similar form in the case of the hydrogen atom. It is commonly said that Dirac put the theory on a sound footing by introducing a linear wave-function along with Lorentz invariance. If my sketch is right then all this was missing the fundamental point anyway since it was not getting to the heart of the matter. The linear wave-function was needed to produce the matrices, and the Lorentz invariance hasn't much to do with objects travelling at high speeds, but is to satisfy the need that people feel for a formal symmetry between length and time. which again is needed for the matrix representation. This, taken with the requirement that the development of the wave-function has to have a first-order time-derivative, makes possible a matrix representation bringing in the Pauli matrices. I point out that if the hierarchy algebra is right then it is a mistake to use space and time to identify the 4-vectors at all.

Finally, inspection of Dirac's treatment reveals that the argument for the factor 2 (duplexity) is complex –depending on selection from several possibilities. Kilmister commented on the position, in his deceptively mild way, thus:- "Dirac's approach in this paper ("The quantum theory of the electron' P.A.M. Dirac, Proc. Roy, Soc. (A), **117**, 610.) is admirably summarized in the first paragraph; to find why nature had chosen, not a point charge, but a spinning electron, as a building brick. He then proceeds to relate this with the necessity of making the theory Lorentz invariant. It is clear now (what was perhaps not clear in 1928) that neither requirement involved the other uniquely; it is not necessary for Lorentz invariance to have half-integral spin –other spins are possible and the choice is experimental. Similarly Pauli and Darwin had already fitted a spinning electron, at least in a normal manner into a pre-relativistic theory. Dirac's achievement is to have shown how the two requirements can be combined."

The way now seems to be open to begin to connect a purely combinatorial account of spin with current views. Enough has been said to give us freedom of action in considering what part of the train of classical dynamical associations is appropriate for, for example, the interpretation of spin as angular momentum in high-energy physics.

What is mass? Quarks

The standard model theorists admit they have no theory of the origin of mass (as distinct from fitting it into a classificatory scheme). We may suspect that they won't have even if they find the Higgs particle. It is a case where one has to stand outside the range of continuum concepts for insight. We ought to be able to do just that. I adopt a general picture of Peter Rowlands who has a step-by-step creation of a digital numerical value of mass and charge treated mutually. He assumes the build-up to stop with 137, in which case e and h and therefore m fit in. He has a complicated combinatorial theory to account for the step-by-step-ness, and we do it differently. Of course we have an advantage in knowing why we have to stop at that particular number. Fine so far, but how do we make it look like a continuum concept for the purpose of giving different masses to the menagerie of high energy particles?

Clive Kilmister points out that inertial mass is really an intruder into classical mechanics and relativity because we start with six *moments and products of inertia* from the classical picture, and these are the precursors of mass as a measure of field strength rather than of inertial mass. The distinction between inertial and source (gravitational) mass exists in the usual accounts, but is left rather mysterious.

We cannot think about inertial mass without imposing some sort of link between extension and composition –a distinction whose fundamental status was always stressed by Tom Etter. Thus we need a first step in going from composition that will lead into a 'spatial distribution'. Let us consider the quarks. They are parts of the total 'energy-momentum tensor' rather than the whole. Of course they have no spatial extension, and it is just that that gives us flexibility to extend the mass/charge construction to describe variety. As a first step this variety merely consists in our being able to use more than one. If we can add together constituent masses then that implies that mass already has an extensive content. What we seem to want is the have a quark as an *incomplete* level structure so that mass as composition can fill out into mass as a field or rather as an interaction strength. This provides us with the necessity for the quark splitting and has the right metaphysics. My reasons for pursuing the ideas about mass and quarks are, firstly, that at this degree of simplicity the choices are few and nothing else offers, though other offers are warmly invited. Secondly, there has to be a decisive innovatory step before which no kind of articulated spatial picture can exist and after which there is for the first time, spatial distribution which is separated from some numerically specifiable physical thing. Obviously that 'thing' will involve mass because mass is the primordial non-dimensional quantifiable property. It would seem reasonable simply to define it as mass, though if we do that we have to explain carefully that each new attribute that would normally be thought to be a part of the concept has to be added explicitly and separately.

We can probably placate the standard modellers by saying that they have already been forced to postulate a 'vacuum' which has no more of a tangible quality than our pre-space/mass manifold.

To get further we must pick up on Rowland's view that the alphas are diversified by values of e-squared. Now we could not do all this if the quarks were spatially defined. Of course the quarks have partial numbers attached and these make the jump into extension somehow. . Thus mass and charge should now become *separable*. It is still sensible to think that the measureable nature is conventionally allocated to mass, leaving the charge quantized.

What about lifetimes? One argument is that they are a different problem since we are not concerned in the first place with high energies. However it may be that the disappearance of extension happens automatically when the life-time is finite, though it would be difficult to accommodate near stability. The key seems to be the vacuum. Incoming interactions have to be such that anything mass-like is open to continual reassessment. The model for mass stability is broken when statistical fluctuation governs the choice of 'working alpha'. This can only happen when the level is incomplete.

I try to draw together these arguments in the following model of the origin of inertial mass.

We are working at the electromagnetic level and therefore with the alpha (1/137). However we consider the part alphas corresponding to the quarks which are seen as partially established structures.

The natural place for the three primitive quarks in the hierarchy algebra is the existence of three the primary discriminately closed subsets. They have flexibility to change, which the basic alpha does not. This change, happening statistically as dictated by the discrimination input or by the vacuum, destroys the mass/spatialization or in other words the composition/extension separation. This is our picture of the origin of mass through the relinquishment of space. We seriously propose this complete mutual disappearance of the two concepts. We also note that the principle explains the experimental unobservability of the quarks as particles.

To start with, we rely on Rowland's characterization of the alphas by their e-squareds. I do not know how far this helps us with calculations of mass, or where the e-squareds come in. Obviously we need something from the hierarchy: what we get from the variety of the dcss is sparse but cannot be non-existent. There must be some way of specifying those differences that persist under change of representation. One way in which I start to think is to consider sets generated by one dcs rather than the others combined with a subset at the next level chosen so as to have a ready interpretation as a biased or truncated system representing, say, absence of spin.

Very much depends on where we start. Rowlands excludes charge units progressively in tandem with mass so as to understand the appearance of the electromagnetic alpha as the limit. This process naturally appeals to me since we believe in the possibility of calculating the reciprocal of that alpha, and therefore have an automatic maximum. The charge exclusion depends on Rowlands' detailed theory, of course.

It is really just the first steps that I am concerned with. I find these first steps so important because they encapsulate the logical change from 'composition' to 'extension'. The discrete/continuum follows, but is less fundamental. We want to go from seeing masses as distinct numbers to seeing them as measures of something in a space.

The numbers giving the alphas arise from the maximum occupation of the states allowed by the algebra, when every possibility is given equal weight. We want to give physical significance to other possibilities since without that fundamental step we are not using the numbers so as to interpret them as in an extended space. We need to split up the level that is under discussion (what we might figuratively call the total energy-momentum tensor) and such parts are the quarks. There is a case anyway for identifying the three basic quarks with our three basic discriminately closed subsets, which exist within the envelope structure when all statistical probabilities are allowed. It is important to realize that spatial distinctness is not allowed for these structures because they are, as it were, free to take all values within the enveloping structure. This picture recalls the vacuum of the standard model. That, too, does not represent a real space, but assumes interactions just as our statistical background does. The difference arises because the standard model has normal physical space as well as the vacuum virtual space, whereas we start with only the virtual manifold and are free to construct the physical space as we need it

I hope that the basic physical picture emerges clearly. Where there is to start with no real space the numbers arising from the quarks also have no way of being interpreted as mass or anything else. It is only when the distinction comes into play that we have what is necessary for inertial mass, and by sticking at the stage before we have got to the limiting alpha value we bridge the logical gap between composition and extension. *En passant* this explains why the quarks appear without a spatial significance, and we are far better off than the current theorists who have to invent special mechanisms to explain why the quarks do not appear experimentally.

The Photon

We have imported particle and interacting photon as a packaged deal. Not even in the most classical thinking is it supposed that the photon has meaning apart from an interaction, so we are not breaking new ground yet. There is the further question whether we can ever speak of a given photon's taking part in more than one interaction- building up its individuality gradually, as it were. We shall consider that later and separately. If we can associate rest mass with the interacting particle then we can at the same time say that velocity entails momentum and get the Lorentz form for momentum. To associate the scaling constant c with the velocity of the photon is only a *facon de parler* which causes no trouble. Similarly the question whether the photon has mass or not is only a convention to be settled when we introduce rest-mass. We adhere to the convention that it is zero. Clive Kilmister conjectures that the photon is imported into the theory at an early stage in the hierarchy algebra when it is necessary to define a signal which is symbolized by zero or 0.

At the stage of which he is talking we have no space defined. However we claim that the mathematics contains from the outset this symbol which will inevitably dictate profound consequences for any subsequent construction of spatial ideas, and that it will have enough properties to make it identifiable with light, or the photon. Of course this characteristic of the constructive method is controversial, but it is part and parcel of such identifications as we have already made including in particularthe identification of the quarks. As with the quarks our procedure makes some things much easier. Thus the fundamental worry why such a thing as the photon should exist in view of its being completely unlike all other particles, comes to be seen in a quite different light even if it would be going too far to say that it had been resolved.

This way of thinkingwill probably leave us open to sudden incredulity fits along the lines that in fact there really are photons that are, at least in most respects, objective particles. One has to recall again and again that in spite of the language inculcated largely by special relativity, we have no grasp, however experimentally, of the photon separated from its interaction.

The standard model group symmetries

My conclusion so far is that all the really crucial and precise calculations from the physics that leads up to the standard model come exclusively from the pattern of coupling constants. For example the very exactness of the calculation of the Lamb shift depends on the expansion in powers of the fine-structure constant. This comes as a surprise since one might naively have expected to find all sorts of quite different continuum-based quantum dynamics coming into play: but none does.

This surprise can reasonably be taken as a significant argument in favour of our approach.

There is however another, though different, source of numbers in the standard model, namely the group symmetries. The group symmetries used in the standard model are all understood by analogy with the structure of space-time. Since the combinatorialists advance to space and time by a carefully constructed set of arguments from an algebra which has its roots in binary distinctions, the space-time does not have its compulsive intuitive immediacy. So what of the analogical groups?

These groups are representation groups, while any groups that arise in a combinatorial theory have to be abstract groups in which one would start by asking for the multiplication table.

There should surely be a potential programme of finding and elucidating the structures within the combinatoric picture that could replace the extended space-time groups and define the new competences of that study. For example in the standard model it is noticed that the classical requirement of time reversal has to be abandoned. So also for parity. Our policy obviously has to be first to observe that there never was any requirement to follow the classical assumptions and therefore it is putting unnecessary limitations on the search for replacement structures to require that they be plausible generalizations. We have a clean sheet.

Arguing as I have before in this paper, I say we should try to assess the success of the standard model in its use of the group symmetries when we cut the whole effort adrift from the space-time groups. On the one hand it may be that there really isn't anything to show for the effort. At the other extreme one may still have to contemplate real numerical successes from the use of the groups. In the latter case we stand in need of some unassuming but well-considered estimate of what it really all boils down to.

Do the symmetry groups SU(3), U(1) and SU(2) merely produce the number of symmetrical components that are available to identify with numbers of quarks, say, or is there more mathematical content? Since the very idea of symmetry is based on the dimensional character of space, does that constrain us only to contemplate the choice of certain types of group, and indeed of groups at all. I.e. could other mathematical structures compete if the spatial connection were missing?

More generally we need to be told what we demand of a group for it to be appropriate to provide numbers that may be given an interpretation in terms of particles and quarks. Then we want to know what numbers are considered in the nature of the case suitable for interpretation. Then we need a compendium of such groups. On a different tack we need to be told the list of the numbers that appear in particle studies that are proper candidates for interpretation

One is told that the most inclusive symmetry is that described as CPT invariance. Whereas each of the component symmetries fails in certain experimental cases we always find that in each such case symmetry can be restored if we are allowed to change all three of the charge conjugation, parity, and time invariance. This observation gives us a starting point, and I shall look at each separately. Charge conjugation speaks of descriptors –things that can have just two values. These will have to be interpreted in any combinatorial scheme in terms of occupancy of the vectors, and no matter of principle in involved. Parity is not so straightforward. The hierarchy vectors will certainly have to have some way of interpreting transformations but we shall not get any help in doing that from the notions of continuous space. It may be said though that embedding the whole in continuum ideas leaves huge problems of understanding for the orthodox. I used to look in wonder at the way people spoke of spin –wondering whether the whole world was supposed to go round and round and to change its motion when the spin changed.

Then we turn to time reversal, to find the situation even more extreme. Here nobody thinks that time means physical time. Or do they?

The Origin and Meaning of 3-Dimensionality

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Abstract: 3-dimensionality is identified as one of the most profound and fundamental concepts in physics. With its origin in ideas of anticommutativity, which are antecedent to the concept of number, it is responsible for all discreteness in physical systems, and in particular for quantization. It is responsible for symmetry breaking between the forces, for many significant aspects of particle structure, and for most of the manifestations of the number 3 that are considered fundamental in physics. It is responsible for the selection of the fundamental parameters that we use in the most basic physical explanations, and for their special properties, and the Dirac equation is specially structured to accommodate it. No other dimensionality, not even that of '4-dimensional' space-time, has any fundamental physical significance, a fact which has extremely profound consequences for a unified theory.

1 Discreteness and dimensionality

String theorists regularly talk of 10 and 11 dimensions. Special relativity uses a 4-dimensional space-time. Kaluza-Klein theory introduces a fifth dimension to general relativity to account for the electromagnetic field. All this is done on the basis that the origin and meaning of dimensionality in nature are matters still to be decided, that, *a priori*, no particular number of dimensions is more likely than any other, that no number associated with dimensionality can be privileged, and that the actual number of dimensions is still negotiable.

Yet 3-dimensionality is very special. It has a mathematical as well as physical validity, which should make us wary of cavalierly expanding the number of dimensions in our system to meet the immediate needs of defining a physically inclusive theory. And the number 3 appears everywhere in fundamental physical contexts – 3 dimensions of space, 3 nongravitational interactions, 3 fundamental symmetries (*C*, *P* and *T*), 3 conserved dynamical quantities (momentum, angular momentum and energy), 3 quarks in a baryon, 3 generations of fermions. Could these be in any way related, and could the explanation of their common 'threeness' somehow lead to a deeper explanation of physical 'reality' than is apparent from the more complex attempts at explanation represented by string theory? What could be special about the number 3 which could unite these apparently disparate manifestations of its application?

To answer this, I need to draw upon an idea which I have long held and often written about. (Rowlands, 1983, 1991, 1999, 2001) This is the idea that the most fundamental concepts in physics are the parameters space and time, and the sources of the four fundamental interactions, namely mass (or mass-energy) and three types of 'charge' (electromagnetic, strong and weak), which it is convenient to describe as orthogonal dimensions in a 'charge space'. Seemingly, these parameters are symmetric according to a Klein-4 scheme, with the following properties and exactly opposite 'antiproperties':

mass	conserved	real	continuous nondimensional
time	nonconserved	imaginary	continuous nondimensional
charge	conserved	imaginary	discrete dimensional
space	nonconserved	real	discrete dimensional

A remarkable aspect of this symmetry lies in the last column, where there are *two* properties and *two* antiproperties, which, if the symmetry is exact, must be linked. We are obliged, it would seem, to suppose that discreteness and dimensionality are intrinsically related properties. In addition, though it is apparent that discreteness and continuity can be considered as a genuinely opposite pairing of property and antiproperty, it is not quite so obvious that the same applies to (3)-dimensionality and nondimensionality, or, as it is sometimes called, 1-dimensionality.

However, '1-dimensional' quantities are not really dimensional at all, and it is relatively easy to see why an absolutely continuous quantity cannot be dimensional. Dimensionality requires an origin, a cross-over point or zero position, that is a distinct discontinuity of some kind, which is of course incompatible with the kind of absolute continuity which makes time irreversible and mass-energy unipolar and ubiquitous in the form of the vacuum or Higgs field.

If the linkage here seems relatively obvious, it is far from obvious that discrete quantities must be dimensional, and specifically 3-dimensional at that. However, each of the two known dimensional quantities seems to supply half of the required explanation. Thus, the discreteness of space is associated with its use as the unique channel of physical measurement because the *nonconserved* nature of space means that its discreteness can be endlessly restructured. Measurement would, of course, be impossible in one dimension. A continuous line would offer no possibility of measurement unless it could be drawn in a 2-dimensional space with the other dimension providing the marking off of the zero points or origins. At the same time, the imaginary nature of charge would appear to imply that any dimensionality associated with this quantity must be 3-dimensionality, as required by the algebra of quaternions.

The link between discreteness and 3-dimensionality would appear to be a result of the other properties associated with the parameters which we have defined as discrete, and there appears to be no direct route to be found leading from discreteness itself to dimensionality. However, this is not the case with the reverse process, and it is in fact possible to show that *dimensionality is really the primary property*, and that all discreteness in nature results from dimensionality, and that *only 3-dimensional quantities* are discrete.

2 The consequences of zero totality

To find the route from dimensionality to discreteness, we again refer to the table of properties for the fundamental parameters, and observe that, in the conceptual sense, it represents a *zero totality*, every property in a parameter being negated by the corresponding antiproperty in another. Remarkably, it is from this perceived need to preserve a zero totality in nature that 3-dimensionality ultimately springs. If we suppose that the only logical condition which incorporates no special assumptions is a zero totality, we may assume that any attempted deviation from this state will automatically generate its own zeroing mechanism, exactly as we observe in the physical world when we conserve momentum or angular momentum.

Let us suppose that we describe deviations from zero by a nonunique term $\mathfrak{R}$, which remains simply unspecified and undefined, and which can only be examined in relation to itself in such a way that it forces an attempt at recovering the original zero totality. The immediate outcome of the 'self-examination' $(\mathfrak{R}) \times (\mathfrak{R}) = (\mathfrak{R})$ will be a *conjugate*, or zero-producing, term, which we may represent by $-\mathfrak{R}$, without making any assumptions about a specific mathematical meaning for the symbol. Again, without making mathematical assumptions, we can represent the process in the form:

$$(\mathfrak{R}) \times (\mathfrak{R}) = (\mathfrak{R}) \rightarrow (\mathfrak{R}, -\mathfrak{R})$$

Let us describe objects of the form $(\mathfrak{R})$ and $(\mathfrak{R}, -\mathfrak{R})$ as 'alphabets', and any representations of components as 'subalphabets' (Rowlands and Diaz, 2002). Then we may suppose that any examination of an alphabet in relation to a subalphabet will produce the original alphabet in the same way as $(\mathfrak{R}) \times (\mathfrak{R}) = (\mathfrak{R})$. So we have, for example:

$$(\mathfrak{R}) \times (\mathfrak{R}, -\mathfrak{R}) = (\mathfrak{R}, -\mathfrak{R}) \qquad \text{and} \qquad (-\mathfrak{R}) \times (\mathfrak{R}, -\mathfrak{R}) = (\mathfrak{R}, -\mathfrak{R}) \; .$$

We may assume, however, that the process of producing a conjugate is no more unique than the symbol and that the same will apply to the new symbol. So examining this in relation to itself will produce a new conjugated alphabet, for example:

$$(\mathfrak{R}, -\mathfrak{R}) \times (\mathfrak{R}, -\mathfrak{R}) = (\mathfrak{R}, -\mathfrak{R}) \rightarrow (\mathfrak{R}, -\mathfrak{R}, ¢, -¢)$$

where

$$(\mathfrak{R}) \times (\mathfrak{R}, -\mathfrak{R}, ¢, -¢) = (\mathfrak{R}, -\mathfrak{R}, ¢, -¢)$$
$$(-\mathfrak{R}) \times (\mathfrak{R}, -\mathfrak{R}, ¢, -¢) = (\mathfrak{R}, -\mathfrak{R}, ¢, -¢)$$
$$(\mathfrak{R}, -\mathfrak{R}) \times (\mathfrak{R}, -\mathfrak{R}, ¢, -¢) = (\mathfrak{R}, -\mathfrak{R}, ¢, -¢)$$
$$(¢, -¢) \times (\mathfrak{R}, -\mathfrak{R}, ¢, -¢) = (\mathfrak{R}, -\mathfrak{R}, ¢, -¢) \text{ , etc.}$$

From such expressions, we derive also the results of the 'actions' of subalphabets upon each other. For example:

$$(-\mathfrak{R}) \times (-\mathfrak{R}) = (\mathfrak{R})$$
$$(\mathfrak{R}) \times (¢) = (¢)$$
$$(¢) \times (¢) = (-\mathfrak{R})$$
$$(¢) \times (-¢) = (\mathfrak{R})$$

Further conjugated alphabets will be constructed with appropriate subalphabets so that the correct rules will automatically apply.

$$(\mathfrak{R}, -\mathfrak{R}, ¢, -¢) \times (\mathfrak{R}, -\mathfrak{R}, ¢, -¢) = (\mathfrak{R}, -\mathfrak{R}, ¢, -¢, ¢¢', -¢', ¢¢', -¢¢') \, .$$

In effect, we will generate a series of ¢-type terms, ¢, ¢', ¢'', ¢''', etc., and their 'actions' upon each other, such as ¢¢'; and the 'self-action' of the ¢-type terms will always result in $(-\mathfrak{R})$. The series may be imagined as continuing to infinity, and generating itself in the supervenient, not temporal, sense, with all the terms existing at once. If, however, we take any pair of terms in the ¢ series, say X and Y, then *either*

$$(XY) \times (XY) = (-\mathfrak{R}) \qquad \textit{or} \qquad (XY) \times (XY) = (\mathfrak{R}) \; .$$

These may be described as the respective anticommutative and commutative options. The significant point here is that the anticommutative option can be used only once. For any given *X*, there is only a single *Y* and a single *XY*. *X*, *Y* and *XY* form a closed cycle. This is exactly what we mean by 3-dimensionality: it is the direct manifestation of closure through anticommutativity, and has no other fundamental significance. On the other hand, the commutative option remains open to infinity with an unlimited number of *Y* terms for any given *X*.

Clearly, there are an infinite set of available options within this alphabet-generating mechanism, but, going for maximum efficiency, i.e. the minimum generation of rules, we may set the default condition at the anticommutative option as the automatic choice whenever this is available. We then produce a regular sequence of closed finite-dimensional systems taking us to infinity, which has exactly the same form as the set of finite integers in conventional enumeration, and can be used exactly for this purpose, occurring even in a ready-made binary form. We create numbers at the same time as we create finite (3-)dimensionality, exactly as physics seems to suggest that we must.

3 The alphabet becomes algebra

With the concept of numbers established, we can now *choose* to interpret $\mathfrak{R}$, which is not itself defined in terms of finite enumeration, as the Cantorian or non-denumerable set of real numbers and the ₵-series as an infinite set of complex forms, whose real 'magnitudes', when closed, are represented by the constructible real numbers of Robinson's non-standard analysis or Skolem's non-standard arithmetic, and by the Cantorian reals when open. The − sign can be seen as referring to arithmetic or algebraic negation and × as arithmetic or algebraic multiplication. These definitions do not retrospectively limit the generality of the argument in the preceding section to a mathematical one, and only apply where we choose an option which introduces the counting of discrete numbers. We can now define counting units within $\mathfrak{R}$, ₵, ₵′, ₵″, ₵‴, ... as, say, 1, $\boldsymbol{i}_1$, $\boldsymbol{j}_1$, $\boldsymbol{i}_2$, $\boldsymbol{j}_2$,..., in which $\boldsymbol{i}_n$, $\boldsymbol{j}_n$, $\boldsymbol{i}_n\,\boldsymbol{j}_n = \boldsymbol{k}_n$, and so on, are independent sets of *quaternions*, following the usual rules determined by anticommutativity:

$$\begin{gathered}
\boldsymbol{i}_n \boldsymbol{j}_n = -\boldsymbol{j}_n \boldsymbol{i}_n = \boldsymbol{k}_n \\
\boldsymbol{j}_n \boldsymbol{k}_n = -\boldsymbol{k}_n \boldsymbol{j}_n = \boldsymbol{i}_n \\
\boldsymbol{k}_n \boldsymbol{i}_n = -\boldsymbol{i}_n \boldsymbol{k}_n = \boldsymbol{j}_n \\
\boldsymbol{i}_n^2 = \boldsymbol{j}_n^2 = \boldsymbol{k}_n^2 = \boldsymbol{i}_n \boldsymbol{j}_n \boldsymbol{k}_n = -1 \;.
\end{gathered}$$

All other products, however, follow the rules of commutativity. For example, when $m \neq n$,

$$\boldsymbol{i}_m \boldsymbol{i}_n = \boldsymbol{i}_n \boldsymbol{i}_m$$
$$\boldsymbol{i}_m \boldsymbol{j}_n = \boldsymbol{j}_n \boldsymbol{i}_m$$
$$\boldsymbol{j}_m \boldsymbol{j}_n = \boldsymbol{j}_n \boldsymbol{j}_m$$

and

$$(\boldsymbol{i}_m \boldsymbol{i}_n)(\boldsymbol{i}_m \boldsymbol{i}_n) = 1$$
$$(\boldsymbol{i}_m \boldsymbol{j}_n)(\boldsymbol{i}_m \boldsymbol{j}_n) = 1$$
$$(\boldsymbol{j}_m \boldsymbol{j}_n)(\boldsymbol{j}_m \boldsymbol{j}_n) = 1$$

even though

$$\boldsymbol{i}_m^{\,2} = \boldsymbol{i}_n^{\,2} = \boldsymbol{j}_m^{\,2} = \boldsymbol{j}_n^{\,2} = -1 \quad .$$

We can further interpret this algebraic series as a dualistic doubling of terms in each conjugation process, so that the series:

order 2 $(1, -1)$
order 4 $(1, -1) \times (1, \boldsymbol{i}_1)$
order 8 $(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1)$
order 16 $(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2)$
order 32 $(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2) \times (1, \boldsymbol{j}_2)$
order 64 $(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2) \times (1, \boldsymbol{j}_2) \times (1, \boldsymbol{i}_3)$,

generates the terms:

order 2 ± 1
order 4 $\pm 1, \pm \boldsymbol{i}_1$
order 8 $\pm 1, \pm \boldsymbol{i}_1, \pm \boldsymbol{j}_1, \pm \boldsymbol{i}_1\boldsymbol{j}_1$
order 16 $\pm 1, \pm \boldsymbol{i}_1, \pm \boldsymbol{j}_1, \pm \boldsymbol{i}_1\boldsymbol{j}_1, \pm \boldsymbol{i}_2, \pm \boldsymbol{i}_2\boldsymbol{i}_1, \pm \boldsymbol{i}_2\boldsymbol{j}_1, \pm \boldsymbol{i}_2\boldsymbol{i}_1\boldsymbol{j}_1$
order 32 $\pm 1, \pm \boldsymbol{i}_1, \pm \boldsymbol{j}_1, \pm \boldsymbol{i}_1\boldsymbol{j}_1, \pm \boldsymbol{i}_2, \pm \boldsymbol{i}_2\boldsymbol{i}_1, \pm \boldsymbol{i}_2\boldsymbol{j}_1, \pm \boldsymbol{i}_2\boldsymbol{i}_1\boldsymbol{j}_1,$
$\pm \boldsymbol{j}_2, \pm \boldsymbol{j}_2\boldsymbol{i}_1, \pm \boldsymbol{j}_2\boldsymbol{j}_1, \pm \boldsymbol{j}_2\boldsymbol{i}_1\boldsymbol{j}_1, \pm \boldsymbol{j}_2\boldsymbol{i}_2, \pm \boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{i}_1, \pm \boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{j}_1, \pm \boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{i}_1\boldsymbol{j}_1$
order 64 $\pm 1, \pm \boldsymbol{i}_1, \pm \boldsymbol{j}_1, \pm \boldsymbol{i}_1\boldsymbol{j}_1, \pm \boldsymbol{i}_2\boldsymbol{i}_1, \pm \boldsymbol{i}_2\boldsymbol{i}_1, \pm \boldsymbol{i}_2\boldsymbol{j}_1, \pm \boldsymbol{i}_2\boldsymbol{i}_1\boldsymbol{j}_1,$
$\pm \boldsymbol{j}_2, \pm \boldsymbol{j}_2\boldsymbol{i}_1, \pm \boldsymbol{j}_2\boldsymbol{j}_1, \pm \boldsymbol{j}_2\boldsymbol{i}_1\boldsymbol{j}_1, \pm \boldsymbol{j}_2\boldsymbol{i}_2, \pm \boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{i}_1, \pm \boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{j}_1, \pm \boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{i}_1\boldsymbol{j}_1$
$\pm \boldsymbol{i}_3, \pm \boldsymbol{i}_3\boldsymbol{i}_1, \pm \boldsymbol{i}_3\boldsymbol{j}_1, \pm \boldsymbol{i}_3\boldsymbol{i}_1\boldsymbol{j}_1, \pm \boldsymbol{i}_3\boldsymbol{i}_2, \pm \boldsymbol{i}_3\boldsymbol{i}_2\boldsymbol{i}_1, \pm \boldsymbol{i}_3\boldsymbol{i}_2\boldsymbol{j}_1, \pm \boldsymbol{i}_3\boldsymbol{i}_2\boldsymbol{i}_1\boldsymbol{j}_1,$
$\pm \boldsymbol{i}_3\boldsymbol{j}_2, \pm \boldsymbol{i}_3\boldsymbol{j}_2\boldsymbol{i}_1, \pm \boldsymbol{i}_3\boldsymbol{j}_2\boldsymbol{j}_1, \pm \boldsymbol{i}_3\boldsymbol{j}_2\boldsymbol{i}_1\boldsymbol{j}_1, \pm \boldsymbol{i}_3\boldsymbol{j}_2\boldsymbol{i}_2, \pm \boldsymbol{i}_3\boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{i}_1, \pm \boldsymbol{i}_3\boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{j}_1,$
$\pm \boldsymbol{i}_3\boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{i}_1\boldsymbol{j}_1$

We recognise the algebraic groups in this series as those of:

order 2	real scalars
order 4	complex scalars (real scalars plus pseudoscalars)
order 8	quaternions
order 16	complex quaternions or multivariate 4-vectors
order 32	double quaternions
order 64	complex double quaternions or multivariate vector quaternions

Another way to look at the series is in terms of an endless succession of just three processes: conjugation (introducing opposite algebraic signs), complexification (multiplying by a single imaginary term); dimensionalization (multiplying by the imaginary term which will complete the quaternion set):

order 2	conjugation	$\times (1, -1)$
order 4	complexification	$\times (1, \boldsymbol{i}_1)$
order 8	dimensionalization	$\times (1, \boldsymbol{j}_1)$
order 16	complexification	$\times (1, \boldsymbol{i}_2)$
order 32	dimensionalization	$\times (1, \boldsymbol{j}_2)$
order 64	complexification	$\times (1, \boldsymbol{i}_3)$

The conjugation process only occurs once, because further applications would not change the character set, but the complexification and dimensionalization processes alternate to infinity. It is notable here that complex numbers are merely incomplete quaternion sets. It is significant, also, that order 16 is the point at which repetition begins.

4 The algebra applied to physics

Each of the processes involved in the generation of this algebra appears to have a realisation in physics, for we can easily identify the process of conservation with conjugation (meaning that the acquisition of a + value in a conserved or conjugated quantity can only happen if accompanied by the equivalent – value, and vice versa), and so write the table of parameters in the following form:

mass	conjugated	real	nondimensionalized
time	nonconjugated	complexified	nondimensionalized
charge	conjugated	complexified	dimensionalized
space	nonconjugated	real	dimensionalized

We can also see that, in addition to thus encoding the three processes involved in the algebraic structure on an equal basis, the parameters also incorporate the first four stages in the emergent algebra itself, and so reach the point of repetition:

order 2	real scalar	1	mass
order 4	pseudoscalar	i	time
order 8	quaternions	$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$	charge
order 16	multivariate vectors	$\mathbf{i}, \mathbf{j}, \mathbf{k}$	space

Here, the multivariate vectors (or Pauli matrices) which appear to apply to space, with multiplication rules:

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = 1$$
$$\mathbf{ij} = -\mathbf{ji} = i\mathbf{k}$$
$$\mathbf{jk} = -\mathbf{kj} = i\mathbf{i}$$
$$\mathbf{ki} = -\mathbf{ik} = i\mathbf{j}$$
$$\mathbf{ijk} = i\ .$$

are completely isomorphic to the complex quaternions which appear at this stage in the algebra:

$$(i\boldsymbol{i})^2 = (i\boldsymbol{j})^2 = (i\boldsymbol{k})^2 = 1$$
$$(i\boldsymbol{i})(i\boldsymbol{j}) = -(i\boldsymbol{j})(i\boldsymbol{i}) = i(i\boldsymbol{k})$$
$$(i\boldsymbol{j})(i\boldsymbol{k}) = -(i\boldsymbol{k})(i\boldsymbol{j}) = i(i\boldsymbol{i})$$
$$(i\boldsymbol{k})(i\boldsymbol{i}) = -(i\boldsymbol{i})(i\boldsymbol{k}) = i(i\boldsymbol{j})$$
$$(i\boldsymbol{i})(i\boldsymbol{j})(i\boldsymbol{k}) = i\ .$$

To incorporate the four algebras of space, time, mass and charge as independent units of a single comprehensive algebra, however, requires us to take our series to order 64, in the complex double quaternions or multivariate vector quaternions which appear in the Dirac equation, the fundamental equation needed to structure the whole of physics; and the Dirac equation, we will find, allows us a means of making an immediate return to zero at the same time as extending the algebra to infinity as required. This equation produces one of the most remarkable manifestations of 3-dimensionality in the whole of physics.

5 The Dirac state and 3-dimensionality

The conventional Dirac equation is structured on the 32-part algebra of the gamma matrices. Taking both + and – values, this forms a group of order 64 entirely isomorphic to the complex double quaternions or multivariate vector quaternions which emerge from the algebra discussed in section 3. A mapping between the two algebras can be established by forming composite terms within the multivariate vector quaternions, for example:

$$\begin{array}{llll} \gamma^0 = -ii & \text{or} & \gamma^0 = ik & \\ \gamma^1 = \mathbf{i}k & & \gamma^1 = \mathbf{i}i & \\ \gamma^2 = \mathbf{j}k & & \gamma^2 = \mathbf{j}i & \\ \gamma^3 = \mathbf{k}k & & \gamma^3 = \mathbf{k}i & \\ \gamma^5 = ij & (1) & \gamma^5 = ij\ . & (2) \end{array}$$

The binomial products of either of these 5-part sets or pentads will generate the entirely algebra in exactly the same way as the gamma matrices. When applied to the Dirac equation, set (1) can be converted to set (2) by multiplying the Dirac equation from the left by the equivalent of γ^5, to produce a nilpotent equation.

From the point of view of the fundamental algebra discussed in the previous sections, the pentad structures exemplified by (1) and (2) are the most efficient way of compactifying space, time, mass and charge into a single package. They are also a characteristic expression of the fundamental nature of 3-dimensionality and its relationship to closure. They produce a new infinite series of closed systems, which incorporate all the different processes and algebraic structures on an equal basis in such a way as to produce an immediate return to zero totality in each. Mathematically, the creation of a pentad involves taking the units of one of the two 3-dimensional components (the quaternion charge or vector space) and imposing each on the units of the other three parameters. We begin, for example, with:

time	space	mass	charge
i	$\mathbf{i}\ \mathbf{j}\ \mathbf{k}$	1	$i\ j\ k$

and impose each of the three charge units onto one of the algebraic expressions representing time, mass or space:

i	$\mathbf{i}\ \mathbf{j}\ \mathbf{k}$	1	$i\ j\ k$
k	i	j	

to give the following combinations:

$$ik \qquad i\mathbf{i}\ \ i\mathbf{j}\ \ i\mathbf{k} \qquad j$$

though, for mathematical convenience and for compatibility with the conventional way of writing the Dirac algebra, we will often write them in the form:

$$k \qquad ii\mathbf{i} \quad ii\mathbf{j} \quad ii\mathbf{k} \qquad ij \;.$$

We might expect the new, composite units to represent entirely new physical parameters, incorporating both the properties of charge, namely conservation and discrete quantization, and those of the respective parent parameters, time, space and mass. In fact, the very act of structuring the new quantities on a 3-dimensional (charge) substrate requires the resulting 3-dimensional combination to be discrete or quantized, though the new composite parameters, which we call the Dirac energy (E), the Dirac momentum ($\mathbf{p}$) and the Dirac rest mass (m), will also retain the respective pseudoscalar, multivariate vector, and real scalar properties of time, space and mass:

$$\begin{array}{ccc} ik & i\mathbf{i} \; i\mathbf{j} \; i\mathbf{k} & j \\ E & \mathbf{p} & m \end{array}$$

The concept of 'rest mass' only emerges in this process of quantization, and only exists in classical physics because it also exists in quantum physics, while the quantization of the directional properties of the vector term is expressed by relation to another quantized quantity, the Dirac angular momentum.

Dirac himself, on the basis of the quantization of angular momentum incorporated in the Dirac equation, apparently predicted that a magnetic monopole could exist with charge automatically quantized in integral multiples of fundamental constants, and that the existence of one such monopole anywhere in the universe would explain charge quantization. However, it would seem that it is rather the fundamentally quantized nature of charge that *explains* the quantization of angular momentum, and other quantities, in the Dirac state; so, the position is actually reversed.

As already stated, the use of a conserved quantity as substrate leads to E, $\mathbf{p}$ and m being conserved quantities in the Dirac state, but it is also possible to express the same superposition in the context of nonconservation, in terms of the quantum (or differential) operators, relating to the parent quantities, time and space:

$$\begin{array}{ccc} ik & i\mathbf{i} \; i\mathbf{j} \; i\mathbf{k} & j \\ \partial / \partial t & \nabla & m \end{array}$$

although the object on which they act must be so structured as to produce the same conserved state as is represented by E, $\mathbf{p}$ and m. Exactly such a result is obtained by a differential operator acting on the exponential or 'wave' term, $e^{-i(Et\ -\ \mathbf{p.r})}$, which can be seen as a mathematical representation of the group of space and time translations and rotations, which provide the maximal variation or 'nonconservation' for space and time coordinates in the most idealised or 'free' state. Because they lead to the same result, we can describe $\partial/\partial t$ as the operator E and ∇ as the operator $\mathbf{p}$.

By necessity, quantization, in connecting all four parameters within a single dimensional structure necessarily establishes direct and inverse numerical relationships between their units. Through this process, E and t, and $\mathbf{p}$ and $\mathbf{r}$, become conjugate variables, that is, ones which exchange statements about conservation into equivalent statements about nonconservation, and vice versa. The relationships between the units of E and $\mathbf{p}$, and those of t and $\mathbf{r}$, then lead to the introduction of the constants $\hbar$ and c, while a third constant, G, is required when we involve m. These constants, as has long been known, have no intrinsic meaning; they are simply the inevitable consequence of creating a composite state. With the explicit introduction of $\hbar$, the operator E becomes $i\hbar\partial/\partial t$, while the operator $\mathbf{p}$ becomes $-i\hbar\nabla$, though the usual convention is to choose units such that $\hbar = 1$ and $c = 1$.

Since the three components of the Dirac state, E, $\mathbf{p}$, and m, are, from the fundamental properties of their parent-parameters time, space, and mass(-energy), specified by unrestricted real number values (though space's are countable in the Löwenheim-Skolem sense), it is possible, using the anticommuting properties of the quaternion and vector operators, and the presence of at least one complex term, to find values of the Dirac state, ($\pm$ ***k**E* $\pm$ ***ii*p** + ***ij**m*), which square to a *zero numerical solution.* We may then use this property to define those states in which the conservation of E, $\mathbf{p}$, and m, applies at the same time as the absolute nonconservation of space and time. The expression which results is the nilpotent Dirac equation, which in its purest form, for the free state, is given by:

$$\left(\pm i\mathbf{k}\frac{\partial}{\partial t} \pm \mathbf{i}\nabla + i\mathbf{j}m\right)\psi = \left(\pm i\mathbf{k}\frac{\partial}{\partial t} \pm \mathbf{i}\nabla + i\mathbf{j}m\right)(\pm \mathbf{k}E \pm i\mathbf{i}\mathbf{p} + i\mathbf{j}\,m)\, e^{-i(Et-\mathbf{p.r})} = 0$$

The conjugate nature of E, $\mathbf{p}$ and t, $\mathbf{r}$ means that, through the Dirac equation, we can also establish a nilpotent structure connecting t and $\mathbf{r}$, with another term τ (described as 'proper time') in the position occupied by m. The theory we know as 'special relativity' is merely the working out of the consequences of this structure, which we may write in the form ($\pm$ ***ik**t* $\pm$ ***i*r** + ***j**τ*), under classical conditions. In fact, all other laws of physics may be seen in some sense as consequences of or approximations to the nilpotent Dirac equation.

Significantly, introducing the proper time term in the nilpotent expression ($\pm\, \boldsymbol{ik}t \pm \boldsymbol{i}\mathbf{r} + \boldsymbol{j}\tau$) also introduces the principle of *causality*, and this, along with relativity, is conventionally held to be the reason for the validity of the *CPT* theorem. The derivation of this theorem in section 7 depended on the necessity of the three terms in the nilpotent structure, ($\pm\, \boldsymbol{k}E \pm i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m$) or ($\pm\, \boldsymbol{ik}t \pm \boldsymbol{i}\mathbf{r} + \boldsymbol{j}\tau$), having equal dimensional status, that is, on each having a quaternion operator. Causality, of course, is effectively a way of ensuring the irreversibility of time, and this, according to the symmetry group between the parameters presented in section 1, is equivalent to the unipolarity of mass, the term which occupies the 'proper time' slot in the energy-momentum nilpotent.

The nilpotency of the Dirac or fermion state, the fact that ($\pm\, \boldsymbol{k}E \pm i\boldsymbol{i}\, \mathbf{p} + i\boldsymbol{j}\, m$) squares to zero, gives us the opportunity of achieving the return to zero which was the original reason for the creation of the entire algebra, but we also need to extend the algebra to infinity. Here, we may consider the nilpotents ψ_1, ψ_2, ψ_3, …, with coefficients which are unrepeated but arbitrary units or strings of units of the form $\boldsymbol{i}_s$, as forming an infinite-dimensional Grassmann algebra, with successive outer products defined by the Slater determinant, and so requiring $\psi_1 \wedge \psi_1 = 0$ and $\psi_1 \wedge \psi_2 = -\, \psi_2 \wedge \psi_1$, etc. To create such an algebra, it would seem, the state vector units ψ_n must be both nilpotent and antisymmetric. The generating algebra which we have created from first principles can then be extended to infinity, through an algebraic and nonlocal superposition of fermionic states throughout the entire universe. Because we have an infinite range of real number values, we can consider each individual nilpotent to be unique, with superposition otherwise producing immediate zeroing – in a sense, an infinite number of individual or unique fermionic states, puts off the final return to zero, as each is examined against each other. It is this mathematical interconnectedness that allows us to group the nilpotents as closed 'units' of this even higher algebra, which is exactly equivalent to the conventional complex Hilbert space, and the nilpotency allows us to identify a part of the sequence without having to specify more than a finite number of the infinite number of terms which we know must exist. If the nilpotents were not themselves 3-dimensional, then this level of closure would not be possible.

6 Baryons

A classic example of the significance of 3-dimensionality occurs in the case of baryons. These structures only exist because the Dirac state vector incorporates the 3-dimensional term $\mathbf{p}$. While it is clear that a state vector of the form

$$(\boldsymbol{k}E \pm \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E \pm \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E \pm \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)$$

would immediately zero itself, and so could not exist, this would not be the case with one of the form

$$(\boldsymbol{k}E \pm \boldsymbol{ii}\,p_x + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E \pm \boldsymbol{ii}\,p_y + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E \pm \boldsymbol{ii}\,p_z + \boldsymbol{ij}\,m)\,,$$

where $\mathbf{p}$ may be imagined as having allowed phases in which only *one* of the three components of momentum, p_x, p_y, p_z, is nonzero and represents the total $\mathbf{p}$. The products

$$(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)$$
$$(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E - \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)$$
$$(\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)$$

would then become equivalent to the characteristic fermionic structure, $-p^2(\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)$, while

$$(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E - \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)$$
$$(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)$$
$$(\boldsymbol{k}E - \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)$$

would result in $p^2(\boldsymbol{k}E - \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)$.

Assuming perfect gauge invariance, it is clear that these phases have exactly the same structure and $SU(3)$ symmetry as the conventional representation of the baryon, composed of three 'coloured quarks':

$$\psi \sim (BGR - BRG + GRB - GBR + RBG - RGB)\,,$$

with the mappings:

$$\begin{array}{rcl}
BGR & \rightarrow & (\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m) \\
-BRG & \rightarrow & (\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E - \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m) \\
GRB & \rightarrow & (\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m) \\
-GBR & \rightarrow & (\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E - \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m) \\
RBG & \rightarrow & (\boldsymbol{k}E + \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m) \\
-RGB & \rightarrow & (\boldsymbol{k}E - \boldsymbol{ii}\,\mathbf{p} + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,(\boldsymbol{k}E + \boldsymbol{ij}\,m)\,.
\end{array}$$

The behaviour of the strong interaction can then be understood in the most simple terms by mapping it onto the identical behaviour of the momentum or angular momentum operator **p**. We can even understand the perfect gauge invariance as a nonlocal 'transfer' of momentum between the phases at a rate independent of the separation of the component parts, which becomes equivalent to the linear potential used in the theory of the strong interaction. The principle may be expected to operate in relation to any states based on the 'quark' principle, or explicit use of the 3-dimensional properties of the **p** operator, including quark-antiquark as well as 3-quark states.

7 *CPT* symmetry

CPT symmetry is an even more obvious result of 3-dimensionality. The *P*, *T* and *C* transformations are equivalent to reversals in the signs of space, time and mass-energy, and can be accomplished by using the ***i***, ***k***, and ***j*** operators which connect these to the three dimensions of charge in the Dirac state vector:

Parity (P): $$\boldsymbol{i}\,(\pm\, \boldsymbol{k}E \pm \boldsymbol{i}i\,\mathbf{p} + i\boldsymbol{j}\,m)\,\boldsymbol{i} \;=\; (\pm\, \boldsymbol{k}E \mp \boldsymbol{i}i\,\mathbf{p} + i\boldsymbol{j}\,m)$$

Time reversal (T): $$\boldsymbol{k}\,(\pm\, \boldsymbol{k}E \pm \boldsymbol{i}i\,\mathbf{p} + i\boldsymbol{j}\,m)\,\boldsymbol{k} \;=\; (\mp\, \boldsymbol{k}E \pm \boldsymbol{i}i\,\mathbf{p} + i\boldsymbol{j}\,m)$$

Charge conjugation (C): $$-\boldsymbol{j}\,(\pm\, \boldsymbol{k}E \pm \boldsymbol{i}i\,\mathbf{p} + i\boldsymbol{j}\,m)\,\boldsymbol{j} \;=\; (\mp\, \boldsymbol{k}E \mp \boldsymbol{i}i\,\mathbf{p} + i\boldsymbol{j}\,m)$$

Obvious consequences of these are the combined transformations:

$CP = T$:
$$-\boldsymbol{j}\,(\boldsymbol{i}\,(\pm\, \boldsymbol{k}E \pm \boldsymbol{i}i\,\mathbf{p} + i\boldsymbol{j}m)\,\boldsymbol{i})\,\boldsymbol{j} = \boldsymbol{k}\,(\pm\, \boldsymbol{k}E \pm \boldsymbol{i}i\,\mathbf{p} + i\boldsymbol{j}m)\,\boldsymbol{k} = (\mp\, \boldsymbol{k}E \pm \boldsymbol{i}i\,\mathbf{p} + i\boldsymbol{j}m)$$

$PT = C$:
$$\boldsymbol{i}\,(\boldsymbol{k}\,(\pm\, \boldsymbol{k}E \pm \boldsymbol{i}i\,\mathbf{p} + i\boldsymbol{j}m)\,\boldsymbol{k})\,\boldsymbol{i} = -\boldsymbol{j}\,(\pm\, \boldsymbol{k}E \pm \boldsymbol{i}i\,\mathbf{p} + i\boldsymbol{j}\,m)\,\boldsymbol{j} = (\mp\, \boldsymbol{k}E \mp \boldsymbol{i}i\,\mathbf{p} + i\boldsymbol{j}m)$$

$TC = P$:
$$\boldsymbol{k}\,(-\boldsymbol{j}\,(\pm\, \boldsymbol{k}E \pm \boldsymbol{i}i\,\mathbf{p} + i\boldsymbol{j}m)\,\boldsymbol{j})\,\boldsymbol{k} = \boldsymbol{i}\,(\pm\, \boldsymbol{k}E \pm \boldsymbol{i}i\,\mathbf{p} + i\boldsymbol{j}m)\,\boldsymbol{i} = (\pm\, \boldsymbol{k}E \mp \boldsymbol{i}i\,\mathbf{p} + i\boldsymbol{j}m)$$

and the fact that $TCP \equiv$ identity, because:

$$\boldsymbol{k}\,(-\boldsymbol{j}\,(\boldsymbol{i}\,(\pm\, \boldsymbol{k}E \pm \boldsymbol{i}i\,\mathbf{p} + i\boldsymbol{j}m)\,\boldsymbol{i})\,\boldsymbol{j})\,\boldsymbol{k} = -\boldsymbol{k}\boldsymbol{j}\boldsymbol{i}\,(\pm\, \boldsymbol{k}E \pm \boldsymbol{i}i\,\mathbf{p} + i\boldsymbol{j}\,m)\,\boldsymbol{i}\boldsymbol{j}\boldsymbol{k}$$
$$= (\pm\, \boldsymbol{k}E \pm \boldsymbol{i}i\,\mathbf{p} + i\boldsymbol{j}\,m).$$

8 Symmetry breaking and 3-dimensionality

The combination of space, time, mass and charge in creating the Dirac state has another important physical consequence, as the quaternion units, ***i***, ***j***, ***k***, are changed from being symmetrical and indistinguishable representations of independent charges into composite units whose symmetry is broken, by being associated with quantities with different mathematical properties (pseudoscalar, vector and real scalar). From the composition of ***ik***, the combined (***ii***, ***ij***, ***ik***), and ***j***, it is possible to derive the respective *SU*(2), *SU*(3) and *U*(1) symmetries associated with the weak, strong and electric charges.

ik	***ii ij ik***	***j***
w	*s*	*e*

But there is something even more fundamental at work here. *Any* 3-dimensional structure which has individually identifiable components is, in principle, a broken or chiral symmetry, and it is always broken in the same way. If we take, say, a quaternion system and identify ***j*** (the label is arbitrary, but this choice will be convenient), then we have, typically, a magnitude or a level of complexification. If, but only if, we bring in a second term, say ***i***, we will introduce dimensionalization, and it will necessarily be 3-dimensionalization, automatically generating ***k***. This will mean that the ***k*** term now has nothing left to do, except determine + or – values, or right- or left-handed axes. In a sense, ***k*** has been made redundant, except for 'book-keeping'. Of course, where we don't identify the axes, as for example in the usual description of space rotation, the perfect symmetry is preserved, and it appears that the symmetry-breaking has a close association with the use of a concept of conservation or conjugation in connection with the axes, the 'book-keeping' term being specifically concerned with this, and being of the opposite complexity to the rest to ensure the zeroing of the squared nilpotent quantity, while keeping open the two possible sign options.

The separate roles for the three axes in a 3-dimensional system with identifiable components has a remarkable similarity with the processes involved in creating the infinite algebra. The role of ***j*** is essentially that of complexification, the beginning of a new and as yet incomplete new quaternion system. The role of ***i*** is to introduce dimensionalization, while ***k*** is restricted to the 'book-keeping' role of conjugation or conservation. These also run parallel to the roles of scalar, vector and pseudoscalar quantities (which an extra *i* factor has transformed from the sequence pseudoscalar, quaternion, scalar). This is not, in fact, a coincidence, because the key properties of the fundamental parameters have been chosen, by a process of physical 'natural selection' of what can be made to 'work', to reflect the 3-dimensionality which makes it possible to define them at all. The same also applies to the parameter sequence mass, space, time, whose algebraic structures effectively reflect those attributable to the components of charge, the only fundamental 3-dimensional system with identifiable, i.e. independently conserved, components. It is this parallelism which makes it possible to create a closed parameter system with zero totality and in-built repetition.

A fundamental difference between charge and space, as 3-dimensional parameters, is that the first is a conserved quantity, whereas the second is not. One aspect of the nonconservation of space is its rotation symmetry, the principle that the laws of physics are invariant to the arbitrary rotation of spatial axes, a property which leads clearly to space's *affine* structure, the infinite number of possible resolutions of a vector into dimensional components. Clearly, this cannot apply to charge, whose conservation property must be exactly opposite. The axes of charge, that is, the electromagnetic, strong and weak types, must be rotation *a*symmetric: they cannot rotate into each other, and effectively constitute a non-affine 'space'. Charge type must be conserved. In fact, this principle is over and over again the message of particle physics. Despite the combined electroweak theory developed by Weinberg and Salam, and the proposed GUT unification of the electroweak with the strong force, the three nongravitational interactions each behave as if the others did not exist, and much of particle physics (lepton flavour conservation, baryon conservation, $SU(2)$ weak isospin, nondecay of the proton, etc.) is simply a statement of some aspect of this fact.

Now, the rotation symmetry of space, although an expression of nonconservation, is still responsible for a conservation law. This is a result of Noether's theorem, which states that, for every global transformation preserving the Lagrangian density, there exists a conserved quantity. This, however, is effectively a result of the exactness of the oppositeness of conservation and nonconservation in the parameter group. Noether's theorem has been taken, for instance, to imply that the translation symmetry of time is precisely identical to the conservation of energy, and that the translation symmetry of space is precisely identical to the conservation of linear momentum, while the additional rotation symmetry of 3-dimensional space becomes identical to the conservation of angular momentum. We can see how the conservation / nonconservation connection operates in the case of the first relation. Since energy is related to mass by the equation $E = mc^2$, then the translation symmetry of time will also be linked to the conservation of mass. So, the nonconservation of time is responsible for the conservation of mass, exactly as the parameter table would suggest.

We can, however, extend the interpretation of Noether's theorem even further by linking the conservation of the quantity of charge (of any type) with the nonconservation, or translation symmetry of space, and consequently with the conservation of linear momentum; and, by the same reasoning, we can make the conservation of *type* of charge linked to the rotation symmetry of space, and so to the conservation of angular momentum, as in the following scheme:

symmetry	**conserved quantity**	**linked conservation**
space translation	linear momentum	value of charge
time translation	energy	value of mass
space rotation	angular momentum	type of charge

Using the last connection, we can propose that, if conservation of angular momentum is also taken to represent conservation of type of charge, then any symmetry-breaking which differentiates types of charge will also be applicable to the quantized angular momentum relevant to particle physics. Remarkably, this appears to be the case, as the three charge types (electric, strong and weak) seem to be responsible for conveying different aspects of angular momentum conservation, as though these were representable by different identifiable dimensions of the quantity. In the electric case it is the magnitude; in the strong case the direction; and in the weak case the orientation. Again, we recognise the symmetry-breaking pattern which is characteristic of 3-dimensional systems with identifiable components: the magnitude, or complexifying, term; the dimensionalizing term; and the 'book-keeping' term providing the orientation. It has nothing to do with mysterious physical characteristics possessed by these interactions: it is a result of 3-dimensionality alone.

9 Fermionic structures

In section 6, we associated the $SU(3)$ symmetry for the strong charge with the dimensional behaviour of the **p** operator. From the structure of the Dirac state vector, it is clear that this will be affected by the combination of the other two charges. However, these charges are actually governed by quite separate symmetries, the weak charge being attached to *iE* and the electric charge to *m*, in the Dirac state, and we can expect that the pseudoscalar nature of *iE* and the scalar nature of *m* will determine the respective characters of the weak and electric forces.

Now, the complex algebra determines that we have two sign options for *iE*, with two mathematical solutions, and consequently two helicity states; and it is, of course, the weak interaction that is concerned with this aspect of angular momentum conservation. However, the weak interaction has the special property of being confined to a single, left-handed, helicity state for fermions, with the right-handed state reserved for antifermions. This is entirely a result of the fundamental parameter group structure requiring mass-energy to be a continuum or non-dimensional quantity, and the consequent generation of a filled vacuum state; and it parallels the single physical (as opposed to mathematical) direction available to time.

In principle, there is no *physical* state corresponding to –*E*, although the use of a complex operator ensures that –*iE* has the same *mathematical* status as *iE*. Charge conjugation, however, or reversal of the signs of quaternion labels, *is* permitted physically, because charge is dimensional. So the –***ik****E* states can be interpreted as antifermion or charge-conjugated states; and the mass-energy continuum becomes a filled vacuum for the ground state of the universe, in which such states would not exist. The filled ***k*** or weak vacuum for the –*iE* fermion states, however, leads to a charge conjugation violation for the weak interaction, which manifests itself in the indifference of the interaction to the sign of weak charge, though not to the fermion / antifermion status. To preserve *CPT* symmetry, either parity or time-reversal symmetry must also be violated. In addition, when both *w* and *e* are present to affect **p**, the helicity state is no longer that of the pure *w*, and a mass term is generated, representing the scalar or magnitude part of the broken symmetry.

The $SU(2)_L$ or 'isospin' symmetry for the weak interaction now follows from the very principle which ensures that the 3-dimensional symmetry between the charges is a broken one – the fact that its component axes are separately identifiable because the three charges are conserved independently of each other. The mutual independence of weak and electric charges creates the $SU(2)_L$ weak isospin: the weak component acts in the same way, whether or not charges are present. The two $SU(2)_L$ states define the weak interaction, with and without electric charge. If we take the mixing of *E* and **p** terms, or right-handed and left-handed components, as being also equivalent to the mixing of *e* and *w* charges, this mixing will not affect the weak interaction as such. So, the weak interaction will be simultaneously left-handed for fermion states and indifferent to the presence or absence of the electric charge, which introduces the right-handed element.

The weak interaction must behave in such a way that the two possible isospin states are indistinguishable. Conventionally, these two states are described by 'the third component of weak isospin', t_3, by analogy with the $SU(2)$ of spin, whose value is such that $(t_3)^2 = (½)^2$ in half the total number of possible states, that is, in the left-handed ones. The relevant quantum number for electric charge (Q) is determined by its absence or presence, and, for free fermions, takes the values 0 and –1, equivalent to the charges 0 and $-e$, the negative sign being purely historical in origin. So, once again, $Q^2 = 1$ in half the total number of possible states (though this time it is a different half, including the right-handed ones), and 0 in the others. Using the standard argument of Georgi and Glashow (1974), it can be shown that, if the weak and electric interactions are described by some grand unifying gauge group, irrespective of its particular structure, then orthogonality and normalisation conditions require the parameter describing the mixing ratio, $\sin^2\theta_W$, to be precisely determined by Tr (t_3^2) / Tr (Q^2), which in this case must be 0.25.

The ratio cannot apply only to free fermions, as the weak interaction must also be indifferent to the presence or absence of the strong charge, or the directional state of the angular momentum operator. This means that the same mixing proportion must exist also for quark states, and separately for each 'colour' phase, so that colour is not directly detectable through w. Assuming that the same weak isospin states can be created for one lepton-like colour or phase, that is with alternative Q values of –1 and 0, or charge values of $-e$ and 0, we now find that the only corresponding isospin states for the other colours that retain both the accepted value of $\sin^2\theta_W = e^2 / w^2$ in a system which allows the instantaneous existence of only one quark phase in three, are 1 and 0 (or e and 0). So, the charge variation 0 0 $-e$ is taken against either an empty background or 'electric vacuum' (0 0 0) or a full background (e e e), so that the two states of weak isospin in the three colours become:

$$\begin{matrix} e & e & 0 \\ 0 & 0 & -e \end{matrix} .$$

In this interpretation, the weak interaction has again performed its 'book-keeping' role, while the electromagnetic interaction takes on the required $U(1)$ structure for a pure scalar magnitude by introducing a required phase. In more conventional terms, if $SU(2)$ breaks parity, group structure and renormalizability require the incorporation of $U(1)$. This also becomes significant in defining a Higgs ground state which is nonsymmetric and parity violating through identification of the one such state that $SU(2)$ and $U(1)$ have in common. The next section will show, however, that the origin of the phase term is evident in the solutions of the Dirac equations that preserve angular momentum conservation for single charges.

The symmetries, as defined in this section, effectively specify all possible fermion (that is, quark and lepton) states. Such states can be defined as all those *which are indistinguishable from each other in terms of the weak interaction.* Because of the 3-dimensional character of the strong interaction, quarks are not independent fermions, but merely phases of them. The phases are made explicit in the presence of the strong charge, in baryons and mesons, but are absent when the strong charge is absent, in free fermions or leptons. The weak and electric charges, unlike the strong charge, have no dimensional character, and only one phase, and so, where the strong charge is present, their phases cannot be aligned, as this would also confine the strong charge to a single phase. If the strong charge is absent, however, this alignment becomes necessary. This is the main distinction between quarks and leptons.

In terms of the weak interaction, however, quarks ought to be indistinguishable from leptons. The emergence of fractional *e* charges in QED phenomenology can therefore be taken as an expression of the perfect gauge invariance of the strong interaction. In this case, the 3-dimensional axes are not specifically identifiable and the symmetry remains unbroken. We may therefore propose that the charge structures for fundamental fermions are represented in the following tables, the left-handed quarks being represented by *A*, *B*, *C* and the leptons by *L*:

A

		B	*G*	*R*
u	+ *e*	1*j*	1*j*	0*i*
	+ *s*	1*i*	0*k*	0*j*
	+ *w*	1*k*	0*i*	0*k*
d	− *e*	0*j*	0*k*	1*j*
	+ *s*	1*i*	0*i*	0*k*
	+ *w*	1*k*	0*j*	0*i*

B

		B	*G*	*R*
u	+ *e*	1*j*	1*j*	0*k*
	+ *s*	0*i*	0*k*	1*i*
	+ *w*	1*k*	0*i*	0*j*
d	− *e*	0*i*	0*k*	1*j*
	+ *s*	0*j*	0*i*	1*i*
	+ *w*	1*k*	0*j*	0*k*

C

		B	*G*	*R*
u	+ *e*	1*j*	1*j*	0*k*
	+ *s*	0*i*	1*i*	0*j*
	+ *w*	1*k*	0*k*	0*i*
d	− *e*	0*j*	0*k*	1*j*
	+ *s*	0*i*	1*i*	0*k*
	+ *w*	1*k*	0*j*	0*i*

L

		$\bar{a}$	$\bar{a}$	ν_e
	+ *e*	1*j*	1*j*	0*j*
	+ *s*	0*k*	0*i*	0*i*
	+ *w*	0*i*	0*k*	1*k*
				e
	− *e*	0*i*	0*k*	1*j*
	+ *s*	0*j*	0*i*	0*i*
	+ *w*	0*k*	0*j*	1*k*

The filled nature of the weak vacuum and the consequent violation of charge-conjugation symmetry for the weak interaction, however, requires yet another application of the principle of 3-dimensionality to the tables. The fact that the weak interaction is indifferent to the sign of the weak charge, and responds (via the vacuum) only to the status of fermion or antifermion means that we must, additionally, define two further generations, replacing w by $-w$, and introducing respective violations of parity and time-reversal symmetry. The three quark-lepton generations are a consequence of the 3-dimensionality of the C, P and T symmetries.

It is possible to generate all the information incorporated in these tables using the angular momentum connection to provide a single unified representation for the entire set of charge structures for quarks and leptons (and their antistates) (Rowlands, 2003a):

$$\boldsymbol{\sigma}_z \cdot (\boldsymbol{i}\, \hat{\mathbf{p}}_a (\delta_{bc} - 1) + \boldsymbol{j}\, (\hat{\mathbf{p}}_b - \mathbf{1}\delta_{0m}) + \boldsymbol{k}\, \hat{\mathbf{p}}_c (-1)^{\delta_{1g}} g) \ .$$

The quaternion operators $\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ are respectively strong, electric and weak charge units; σ_z is the spin pseudovector component defined in the z direction (here used as a reference); $\hat{\mathbf{p}}_a$, $\hat{\mathbf{p}}_b$, $\hat{\mathbf{p}}_c$ are each units of quantized angular momentum, selected *randomly* and *independently* from the three orthogonal components $\hat{\mathbf{p}}_x$, $\hat{\mathbf{p}}_y$, $\hat{\mathbf{p}}_z$. These represent the phases of the respective direction, magnitude and orientation components of the angular momentum, determining the respective presence / absence of the units of strong, electric and weak charge. The other terms in the expression are merely codified ways of representing the divisions between fermions and antifermions, quarks and leptons, $SU(2)_L$ weak isospin, and the charge-conjugation violation associated with the weak interaction. The significant aspect of the expression, for our purposes, is the way it links the conservation of charge type and angular momentum through a broken 3-dimensional symmetry.

The quark tables may be taken as an illustration of the fact that broken 3-dimensional symmetries always incorporate unbroken ones. Thus, the tables are derived by assuming that an unbroken 3-dimensionality for colour phases lies within a broken one for charge, and can be derived, alternatively, by assuming that an irrotational 3-dimensional symmetry (charge conservation) is specified by a rotational one (quaternion algebra). (Interestingly, this alternative derivation *forces* the weak charge into adopting an ambiguous ± state.) It would seem, from fundamental considerations, that the broken 3-dimensionality represented by the Dirac state or charge conservation will necessarily include an unbroken one, such as the rotational symmetry of the **p** operator or the quark system. It is a characteristic, of course, of *nonconserved* or unbroken 3-dimensional structures that the dimensions themselves show the same structure, and this is the property responsible for the affine structure of space. In a sense it applies also to the non-vector terms in the broken symmetry. For example, m-$\mathbf{p}$-E could be described as a 3-dimensional mass or energy and even as a 3-dimensional time (like τ-$\mathbf{r}$-t), showing that 3-dimensionality is, in some sense, inherent within the whole parameter system described by the fundamental algebra. Unlike that of the vector term, however, these symmetries are not unbroken.

10 Spherically symmetric solutions of the nilpotent Dirac equation

According to the conception of Noether's theorem outlined in section 8, it ought to be possible to identify solutions of the nilpotent Dirac equation which involve spherically-symmetric distance-dependent potentials $V(r)$ as also being those which conserve angular momentum and therefore charge type. The procedure is relatively simple (Rowlands, 2003a). Using the standard conversion of the ∇ term to polar coordinates, with explicit introduction of fermionic spin (which is necessary only when we write ∇ as an ordinary vector), we set up an equation of the form:

$$\left(\boldsymbol{k}(E + V(r)) + \boldsymbol{i}\left(\frac{\partial}{\partial r} + \frac{1}{r} \pm i\frac{j + \frac{1}{2}}{r}\right) + \boldsymbol{i}\boldsymbol{j}m\right)\psi = 0\,,$$

where $V(r)$ is the r-dependent potential, and

$$\psi = \left(\boldsymbol{k}(E + V(r)) + \boldsymbol{i}\left(\frac{\partial}{\partial r} + \frac{1}{r} \pm i\frac{j + \frac{1}{2}}{r}\right) + \boldsymbol{i}\boldsymbol{j}m\right)F(r, t)$$

and find the form of the phase term function $F(r, t)$, which will make ψ or its amplitude nilpotent. In a sense, we can avoid using the equation altogether and simply define the state vector, in differential form, as being a nilpotent. This will then uniquely determine both amplitude and phase in a way which is unique to the nilpotent method.

The simplest solution is found for the case where $V(r)$ is inverse linear ($\propto 1 / r$). This is characteristic of the electromagnetic or Coulomb interaction and emerges because of the inverse linear terms (due to spherical symmetry and spin) which are present in the $\boldsymbol{i}$ component of the state vector. A term of this kind in $V(r)$ is the minimum required for spherical symmetry and no such solution can be found without its presence. In effect, this potential gives the scalar part of the interaction, and it results in the characteristic scalar phase or $U(1)$ solution associated with the electromagnetic interaction. The phase term of the wavefunction becomes

$$F = e^{-ar} r^{\gamma} \sum_{\nu=0} a_\nu r^\nu ,$$

and the available energy levels can be calculated from

$$\frac{E}{m} = \left(1 + \frac{(Ze^2)^2}{(\gamma + 1 + n')^2}\right)^{-1/2} = \left(1 + \frac{(Ze^2)^2}{(\sqrt{(j+½)^2 - (Ze^2)^2} + n')^2}\right)^{-1/2} ,$$

which, for the case when $Z = 1$, becomes the standard 'hydrogen atom' solution.

If we now take $V(r)$ as a direct linear potential, combined with the inverse linear term which we know must be present, we obtain a solution with the characteristics of the strong interaction. The state vector has a functional component

$$F = \exp(\mp iEr \pm iq\sigma r^2/2)\, r^{\pm iqA - 1} .$$

in which the imaginary exponential terms can be seen as representing asymptotic freedom, the exp ($\mp iEr$) being typical for a free fermion. The complex r^{r-1} term can be written as a phase, $\phi(r) = \exp(\pm iqA \ln(r))$, which varies less rapidly with r than the rest of ψ. We can therefore write ψ as

$$\psi = \frac{\exp(kr + \phi(r))}{r} ,$$

where $k = (\mp iE \pm iq\sigma r/2)$. At high energies, where r is small, the first term dominates, approximating to a free fermion solution (asymptotic freedom). At low energies, when r is large, the second term dominates, with its confining potential σ (infrared slavery). The Coulomb or inverse linear term, which is required to maintain spherical symmetry, is, as we would expect, the component which here defines the strong interaction phase, $\phi(r)$, and this can be related to the directional status of **p** in the state vector. The direct linear term can be seen as equivalent to a force or rate of change of momentum which is constant with separation, and hence to a quantity which is determined by the vector nature of **p**.

The solutions for direct linear and inverse linear potentials, however, appear to be special cases, and they correspond exactly to the special cases found in classical physics, where they are held to be characteristic of 3-dimensional systems in steady state. In the case of the nilpotent Dirac equation, there appears to be only one other spherically symmetric solution, and it appears to be the same for any potential depending on r^n, where $2 \geq n \geq -2$. Any such potential, or one containing any combination of such terms, gives a harmonic oscillator solution, with energy levels

$$E = -\frac{m}{(j + \frac{1}{2})}(\tfrac{1}{2} + n') \ ,$$

but only when combined with an inverse linear or Coulomb term of the opposite complexity, and the form of the solution is indifferent to the particular value of n chosen.

The harmonic oscillator, of course, can be expressed in terms of the creation and annihilation operators which characterize the unique behaviour of the weak interaction in creating and annihilating fermion-antifermion states, and it is entirely within our expectations of the principle of 3-dimensionality that one interaction should have this conjugative or 'book-keeping' role, after the others have dealt with the scalar magnitude and vector aspects. It corresponds to the position of the energy term in the Dirac operator which only has meaning in connection with fermion / antifermion, right- or left-handed, + or –; and relates to the fundamental process of conservation or conjugation. (This is why the weak interaction responds only to the status of fermion / antifermion and not to the sign of weak charge.) And, of course, values of n different from 1 or –1 will be expected from an interaction which is *invariably dipolar*, as the weak interaction certainly is. The dipolarity is a characteristic expected of a state determined by a pseudoscalar or imaginary quantity, with a ± mathematical duality, and this pseudoscalar aspect appears to be reinforced by the relative complexity of the r^n and inverse linear potential terms required for this solution. It is the same dipolarity as is found in the energy terms in the Dirac equation and in the time term in time-reversal symmetry.

It would seem from our analysis that if we take the Coulomb terms relating to all three interactions to be an expression of the real scalar magnitudes of the charges with which they are associated, then we may suppose that the additional potentials required by the 'strong' and 'weak' solutions are expressions of the respective vector and pseudoscalar terms associated, in the Dirac equation, with their charges. It would also seem that the Dirac equation produces three spherically symmetric, i.e. 3-dimensional, solutions because of its fundamental structural basis in a 3-dimensional object with individually identifiable components.

11 '4-dimensional' space-time

Minkowski famously said about space and time, after his introduction of 4-vectors (1909): 'From now on, space by itself, and time by itself, are destined to sink into shadows, and only a kind of union of both to retain an independent existence'. Of course, space and time still *are* connected, but the connection is not privileged as Minkowski believed it to be. The connection between space and time is no more significant than that between space and mass and mass and time, or all these parameters and charge. And *there is no fundamental 4-dimensional link between space and time.* There is, however, a 3-dimensional one!

The space-time 4-vector has always run into the problem that one component is physically different from all the others, and it is essentially on account of these differences that the problem of wave-particle duality developed. Wave theories made space timelike (i.e. continuous) while particle theories made time spacelike (i.e. discrete) to fit the two parameters into a single physical model or dimensional structure. The dichotomy even manifested itself in nonrelativistic quantum mechanics, with Schrödinger's timelike theory opposed to Heisenberg's spacelike one. However, the problem, in fundamental terms, is that it cannot be done. The true picture is restored in the Dirac nilpotent theory which is neither wavelike nor timelike, but incorporates elements from both Schrödinger and Heisenberg.

What this theory tells us is that space and time are not a 4-vector. We do not add a pseudoscalar to a pure vector, because each term is premultiplied by a gamma factor or a quaternion before addition. Space and time are actually two dimensions of a 3-dimensional structure, whose third dimension is a mass-related term, the 'proper time', which is of course premultiplied by the remaining quaternion. The 'proper time' is not a time term; it is not a pseudoscalar. It gets its name simply from the fact that it becomes *numerically* equal to the time variable if we equate the space component to zero. We could just as easily describe the actual time variable as the 'proper space' in investigating systems, such as photons, in which the proper time (or rest mass) becomes zero.

Of course, when we take a scalar product, as we invariably do in classical special relativity, the quaternion terms disappear and time acts, to all intents and purposes, as an imaginary fourth dimension of space, fulfilling the role of pseudoscalar needed to complete a mathematical vector theory. However, it is important that it is not exactly that pseudoscalar, and no physical quantity exists which can fill this role. The algebraic structure which we have created as a representation of physical 'reality' has no place for 4-dimensional physical quantities. It forces us over and over again into a 3-dimensional pattern. Our quantized, i.e. 3-

dimensional, picture denies us the opportunity of representing time as a fourth dimension, denying it status as a physical observable. In a *3-dimensional* theory, time occupies the place of the 'book-keeper', as energy does in the Dirac state, the quantity which preserves conservation or conjugation, but adds only the information of + or –. We only know the direction of the sequence that preserves causality, not a *measure* of time in the same sense as we measure space, in the same way as energy only tells us whether the system is a fermion or antifermion. This fact is well known as a stumbling block to proponents of a quantum theory of gravity, which automatically incorporates time as a physical fourth dimension. It is likely to prove equally damaging to string theories in which a spatio-temporal dimensionality is automatically assumed to be possible.

The fact that the number system we use in mathematics has a 3-dimensional origin is of profound significance. It means that we can't arbitrarily choose the number of dimensions we apply to quantities like space and time without contradicting the principles on which these concepts, and related ones, such as quantization and conservation, were founded. The number of dimensions is not negotiable once we have decided to use the fundamental parameter group and the number system which emerges from 3-dimensionality. Only at the level of classical approximation can we even contemplate any interference in the number of dimensions which nature appears to have thrust upon us.

Appendix I: Table of 3-dimensional systems with identifiable components

pseudoscalar	quaternion	scalar	(1)
scalar	vector	pseudoscalar	(2)
mass	space	time	
m	$\mathbf{p}$	E	
τ	$\mathbf{r}$	t	
e	s	w	
C	P	T	
$\boldsymbol{j}$	$\boldsymbol{i}$	$\boldsymbol{k}$	
magnitude	direction	orientation	
complexification	dimensionalization	conjugation	
complexification	dimensionalization	conservation	

Here, the 'dimensional' term is in the second column and the 'book-keeping' term in the third. (1) = (2) × i and (2) = (1) × i. It may be that we can also include momentum-angular momentum-energy and space translation-space rotation-time translation. The last row refers to the properties of the parameter group, whose fundamental 3-dimensionality is displayed in the diagrams included in Rowlands (2003b).

Appendix II: Quantum gravitational inertia

The principle that 3-dimensionality is the sole source for discreteness in physics, and that no other dimensionality exists at the fundamental level has consequences for the development of a mathematical theory of quantum gravity, or, in more accurate terms, a mathematical theory of quantum gravitational inertia. According to the argument presented here, there is no fundamental 4-D, and, though there is a mathematical object called a 4-vector, there is no physical realisation of it, except in the classical approximation. The key structure then becomes the *3-dimensional* nilpotent structure, variously represented by $\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m$ and $\boldsymbol{ik}t + \boldsymbol{i}\mathbf{r} + \boldsymbol{j}\tau$, which is both fully quantum and fully relativistic, and the 3-dimensionality of the structure is essential to its complete quantization.

There is no true 5-dimensionality in the structure, as we might at first think, because the nonconserved 3-dimensionality of the **p** and **r** terms is of a different nature to the conserved 3-dimensionality of $\boldsymbol{k}$, $\boldsymbol{i}$ and $\boldsymbol{j}$, though we can, if we choose, relate the nilpotent information in $\boldsymbol{ik}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m$ and $\boldsymbol{ik}t + \boldsymbol{i}\mathbf{r} + \boldsymbol{j}\tau$, as defining the ten 'degrees of freedom' concerning any fermionic state which lie at the basis of the 10- and 11-dimensional superstring and supermembrane theories. The possibility of establishing such a connection, and the outline method of achieving it, are discussed in Rowlands (1998) and Rowlands *et al* (2001). The Grassmann algebra linking the nilpotent fermionic states (which is equivalent to the conventional Hilbert space) would provide the so-called 'eleventh dimension'. However, although it is worth showing that this is possible, it is not worth pursuing in detail, as there is no point in developing a more limited superstructure, whose ultimate purpose is to provide a route to a more fundamental basis, when that basis is already available. Thus, although various larger algebraic structures, for example octonions and even classical Minkowski space-time, have been shown to produce some of the results that are required in a fundamental theory, this is always at the price of producing others which are invalid, and it would seem that the 3-dimensional pattern is the one that nature prefers, and that in identifying this as the true fundamental context we are likely to discover more universally valid results.

We can now, for example, immediately relate $\boldsymbol{ik}t + \boldsymbol{i}\mathbf{r} + \boldsymbol{j}\tau$ to the discrete gravity theory presented in Koberlein (2001), which is based on the fact that a single object (particle or field) at two points in Minkowski space-time (represented by the 4-vector x) must satisfy the causality constraint $\Delta\tau^2 + \Delta x^2 = \Delta(\boldsymbol{ik}t + \boldsymbol{i}\mathbf{r} + \boldsymbol{j}\tau)^2 = 0$, which defines a hypercone for the object. 'Extended causality' then applies when we shift τ and x by infinitesimal steps $d\tau$ and dx. Using Koberlein's procedure, we can then apply a massless scalar field to obtain a discrete field equation, and a field source represented by a scalar charge to generate a 'graviton'-like object and a metric for a discrete gravitational field. It is clear that if we apply the quantum $ikt + i\mathbf{r} + j\tau$ for a Dirac particle in the appropriate places in place of (x, τ), then we can produce a fully quantum version of this discrete gravity, with the discreteness referring to an interaction between fermions as discrete particles defined by a 3-D Dirac nilpotent. Significantly, the discrete theory also dispenses with the transverse directions, to create a 1 + 1 space-time, paralleling the fact that a quantum Dirac particle, with conserved charge (the kind of object to which quantum gravity or quantum gravitational inertia will apply), requires only $\mathbf{r}$, and a single well-defined direction of spin, rather than the classical x, y, z.

It is already apparent, from previous quantum gravity theories, that any attempt at quantizing 4-D space-time is a lost cause, because time is not an observable in quantum mechanics as it is in classical relativity theory; it merely plays the 'book-keeper' role of specifying the direction which preserves causality. This means that, for a fully quantized theory, we need a metric other than the 4 × 4 representation using x, y, z, t, with added curvature, which is used in classical general relativity. The obvious one that suggests itself is a 3 × 3 representation, with diagonal terms ikt, $i\mathbf{r}$, $j\tau$, in the absence of the curvature resulting from a gravitational field. This formalism would have the distinct advantage of being a natural 2 + 1 theory of gravity (the 2 representing the 'real' terms $\mathbf{r}$ and τ, and the 1 the imaginary term it), and such theories are already known to be renormalizable, unlike those with a higher number of dimensions. A preliminary investigation of the method suggests that it works exactly as expected.

Now, Bell *et al* have presented a preliminary approach to a QED-like quantum gravity (2002) by using a quaternionic mapping of the four solutions of the Dirac equation onto a space which, without curvature, is equivalent to that provided by the usual 4 × 4 representation of the Lorentzian metric. The natural result of this mapping is the production of the Bohr-Sommerfeld orbitals for the electron in a scalar electrostatic potential in a purely classical way, thus providing a 'natural' generation of space-time curvature, which can be extended when gravitational curvature terms are directly applied to the metric of the four Dirac solutions (Bell *et al*, 2000).

In terms of the theory presented here, of course, any version of the 4 × 4 Lorentzian metric will be neither fully quantized nor fully relativistic, but the 3 × 3 'quantum metric', based on ***ik**t*, ***i*r**, ***j*** τ, will fulfil both these criteria, and, in the spirit of Bell *et al* (though using a different set of dimensional quantities), can be mapped onto a 'phase space' metric based on ***ik**E*, ***i*p**, ***j**m*, which gives the full information about the Dirac state, and produces the full Dirac 'atom' solution and $U(1)$ QED-type behaviour, with a corresponding photon-like mediating boson, merely on the assumption of spherical symmetry and the multivariate vector nature of the spin term **p** (or, equivalently, conserved charge) (Rowlands, 1992, 1994). A purely 'Lorentzian' metric would not, of course, automatically include spin, unless the vector term was assumed to be multivariate, but, more seriously, would exclude the fundamental nilpotent relations between the parameters space, time, mass and charge which are responsible for both quantization and relativity.

The phase space metric has the direct advantage that it can be obtained directly from the 'quantum metric' (and vice versa) via a Fourier transform, and we can thus imagine the quantum metric as being generated by and carried along with the state which defines it. Evidence for 'curvature' (i.e. the effect of a gravitational field on the inertial metric) can then be seen in the functional term through which this transformation occurs – which will be the usual complex exponential for a free particle, but distorted in the presence of a field or 'curvature'. Since the Dirac state directly determines the nature of the vacuum which responds to it, this process will be equivalent to the Davies-Unruh effect, where a nonaccelerating system sees a plane-wave version of the zero-point field but an accelerating system sees a distorted one.

In the case of phase space, the reduction of the metric to 3 × 3 reflects the fact that, in the nilpotent formulation, the specification of four separate solutions becomes redundant information in the Dirac spinor, because knowledge of the signs of ***ik**E* and ***i*p** in the first term automatically gives us the entire pattern which follows – this is equivalent to separate specification of *x*, *y* and *z* being redundant in the quantum context. In addition, if the basic metric is 3 × 3, rather than 4 × 4, the mediator responsible for any curvature terms becomes spin 1 (as Bell *et al* require for a renormalizable theory), rather than spin 2.

The need for a spin 1 mediator and QED-like theory in 'quantum gravity' has been discussed in many previous publications. There, it has been suggested that the continuity of mass-energy, the filled vacuum, the Higgs field, and the need for instantaneous correlation between Dirac states, together with the fact that energy does not actually move (as opposed to the form of its realisation in connection with a discrete state), require an instantaneous gravitational force, which is undetectable by direct observation, and only ever observed through the *c*-dependent inertial reaction on discrete fermionic or bosonic states. Being repulsive, this force requires a mediator of spin 1. In this context, we may note that the nilpotent representation significantly makes the Dirac state identical to its own gravitational vacuum, at $1(ikE + \mathbf{i}\mathbf{p} + jm)$, whereas the vacuum responses to the weak, strong and electric charges can be represented, respectively, by $\mathbf{k}(ikE + \mathbf{i}\mathbf{p} + jm)$, $\mathbf{i}(ikE + \mathbf{i}\mathbf{p} + jm)$, and $\mathbf{j}(ikE + \mathbf{i}\mathbf{p} + jm)$.

The standard mathematical representation of the gravitational force incorporates no information relating to speed, but the description of gravity as an undetectable property of the vacuum would *require* it to be instantaneous. The *c*-dependence of the inertial reaction, however, determines that, though linear and renormalizable, this force will itself be affected by gravity, giving rise to the 'curvature' terms in the metric tensor, as in general relativity. It is, however, 'curvature' of the metric for inertia, not for gravity, which has no metric. Previous work has shown that, if we equate the inertial reaction numerically with the undetectable gravitational attraction (so defining an equivalence principle), we justify a form of Mach's principle, and obtain gravomagnetic effects, redshift, acceleration of the redshift, and perhaps even the cosmic microwave background radiation (Rowlands, 1992, 1994, 2002). In the simplest case of 'curvature', provided by a point source, we will generate the Schwarzschild metric and a factor 4 in the gravomagnetic equations by comparison with those for QED. This factor (incorporating 2 for space 'contraction' and 2 for time 'dilation', if we adopt the usual convention of making *c* constant, is evident in the factors of 2 which appear in the mass and field terms in the Schwarzschild solution presented by Bell *et al* (2002).

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Fundamental Questions in Information Science- Part I

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Abstract

The Cybernetic Machine specialist Group concerns itself with fundamental questions at the frontiers of Information Processing Science (IPS), but more specifically with the nature of information and the information processing principles by means of which brains work.

Premise and Mission Statement

In science, Nature sets the rules, but it must never be forgotten, that it is only because life has exploited these rules successfully for billions of years to our evolutionary advantage, that we are able to understand them. The mission, at the frontier of IPS, if one accepts the premise, is therefore to identify how these rules were exploited to achieve this end.

Introduction

Some fundamental facts of the BCSCMsG's mission, so far:-

A. With the publication in 1985 of David Deutsch's more general quantum theory of universal computation [1] within which the theory of digital computation was shown to be contained, it was realized that computation is fundamentally a quantum physical process. The mathematical theory of digital computation based on recursive functions, described in terms of, for example, the idealized Turing Machine published in 1935/6 by Alan Turing [2], shown by Deutsch to correspond with classical physics, was thus superseded. It follows that information must have:- (a) an essential physical embodiment, and (b) an explanation as to how it, as a signal theory, is incorporated into quantum physics.

Information can thus be hypothesized to be a further fundamental physical phenomenon of nature, which, like energy, is of general/universal application to nature's understanding.

B. But it is a phenomenon of nature, that any physical wave field (quantum mechanical, electro-magnetic, acoustic, etc) incident on a physical object results in local changes to the amplitude and phase of the field, so as to spontaneously encode the whole 3 dimensional spatial image of the object, up to some degree of resolution. The three basic elements of any information processing procedure identified by Schmacher [3], are therefore in this case,

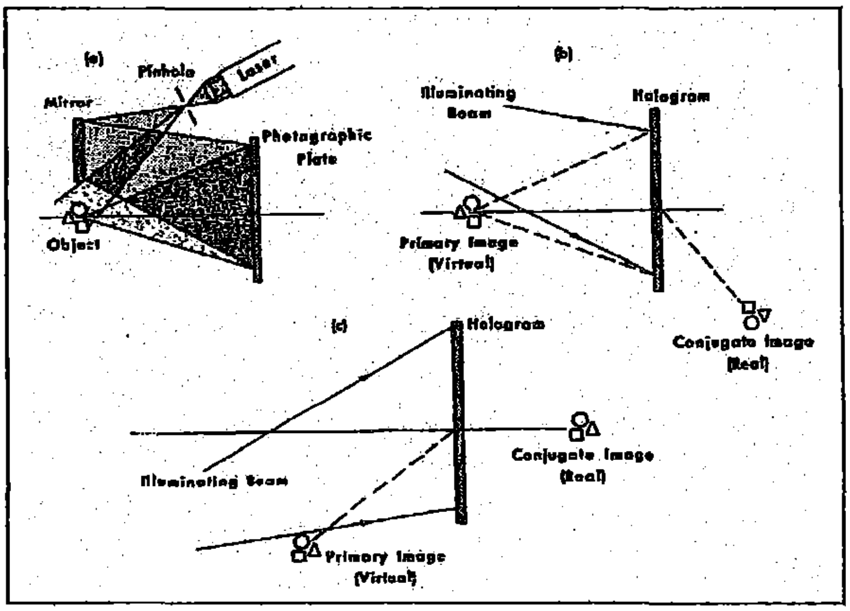
(i) a physical resource, ie any wave field

(ii) an information procedure, ie the spontaneous natural phenomenon of image capture just described, such that the wave field in question becomes "object image" bearing and

(iii) a required third element, the successful completion of an information processing task, which in this case, consists of the proof that (ii) actually happens. But this proof is a well known demonstrable fact [4] see Figure I, since any such object image can be recovered/decoded (so as, for example, in the case of visible illumination, to be viewable by the human eye 3 dimensionally) from its hologram. This is the physical wave interference pattern formed by suitably combining the object image bearing field (ie the illumination previously incident on the object) with a corresponding physical reference field usually of non object image bearing illumination (so that it records not just the object image, but object image bearing field itself [4]).

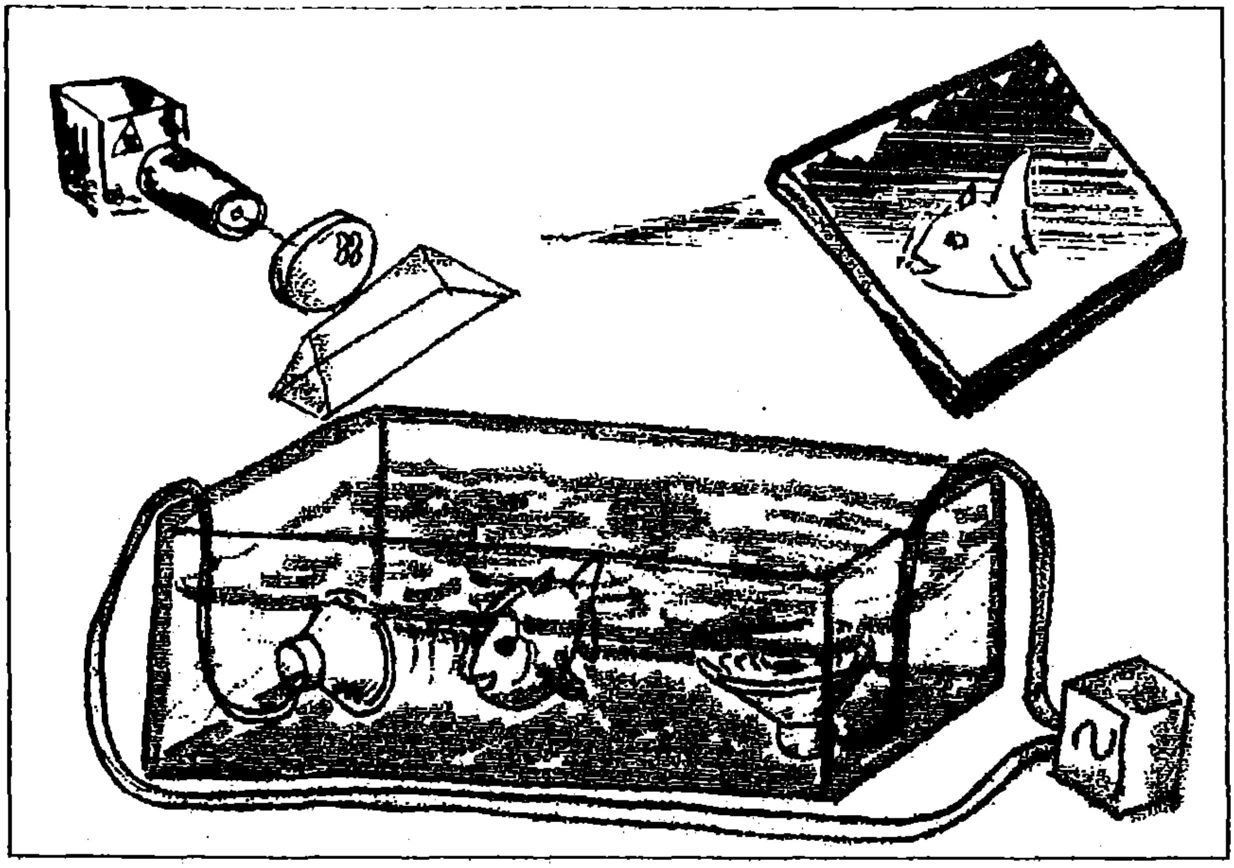
Three dimensional object image encoding and decoding are just two of the basic procedures within the information pattern recognition science, to be called here holochory, better known in the case of electro-magnetic and acoustic fields, as holography and holophony, respectively.

Figure I



The recording of a hologram and the subsequent wavefront reconstruction of an object image is shown in the figures above. In (a) a laser beam is first expanded by a pinhole and then divided by a mirror, which beams part of the illumination directly onto say a photographic plate, while the rest of it is reflected from the object. This plate, after processing then allows the hologram formed to be replaced in its original position, figure(b), after the object has been removed. Still retaining the illuminating beam, its light diffracted from the hologram then forms, in part, the same wavefront that was originally scattered by the object, which as the source of the object image is thus replaced in accordance with Huygens' principle, by this wavefront - its system of secondary sources. This allows the viewer looking through the hologram an undistorted view of the object, just as if it were present. In addition to this virtual or primary object image, a real or conjugate image is also formed on the viewer's side of the hologram. This image will appear unsharp and highly distorted and it will be reversed back to front as shown in (b). However a distortion-free real image can be formed by changing the position of the illuminating beam, known as the reference beam, so that all its rays are reversed in direction, and a real three dimensional image of the object appears in front of the hologram as in (c).

Figure II



C. Some of holochory's other "required third elements" are, for example, that holograms can function:- (i) as transducers of 3 dimensional spatial images across suitable physical interfaces between one physical medium and another, see Figure II above or (ii) as a record or a filter in an associative holochoric memory and filter bank, into and from which object imagery can be both written and read. Furthermore, since the object images encoded in the holograms represent actual behaviour taking place in the real 3 dimensional spatial world, these constitute actual knowledge of the world. And so, in the case of the human brain postulated here to work by holochory, such knowledge would constitute a record of its actual sensory experience, consisting of empty (non image bearing) internal sensory reference fields, on which the initial and all further sensory experience is imprinted so as to form holograms. It is to be emphasized that this record is not encoded via strings of bits or qubits [3], but as physical signals - the actual wave interference pattern or patterns, themselves.

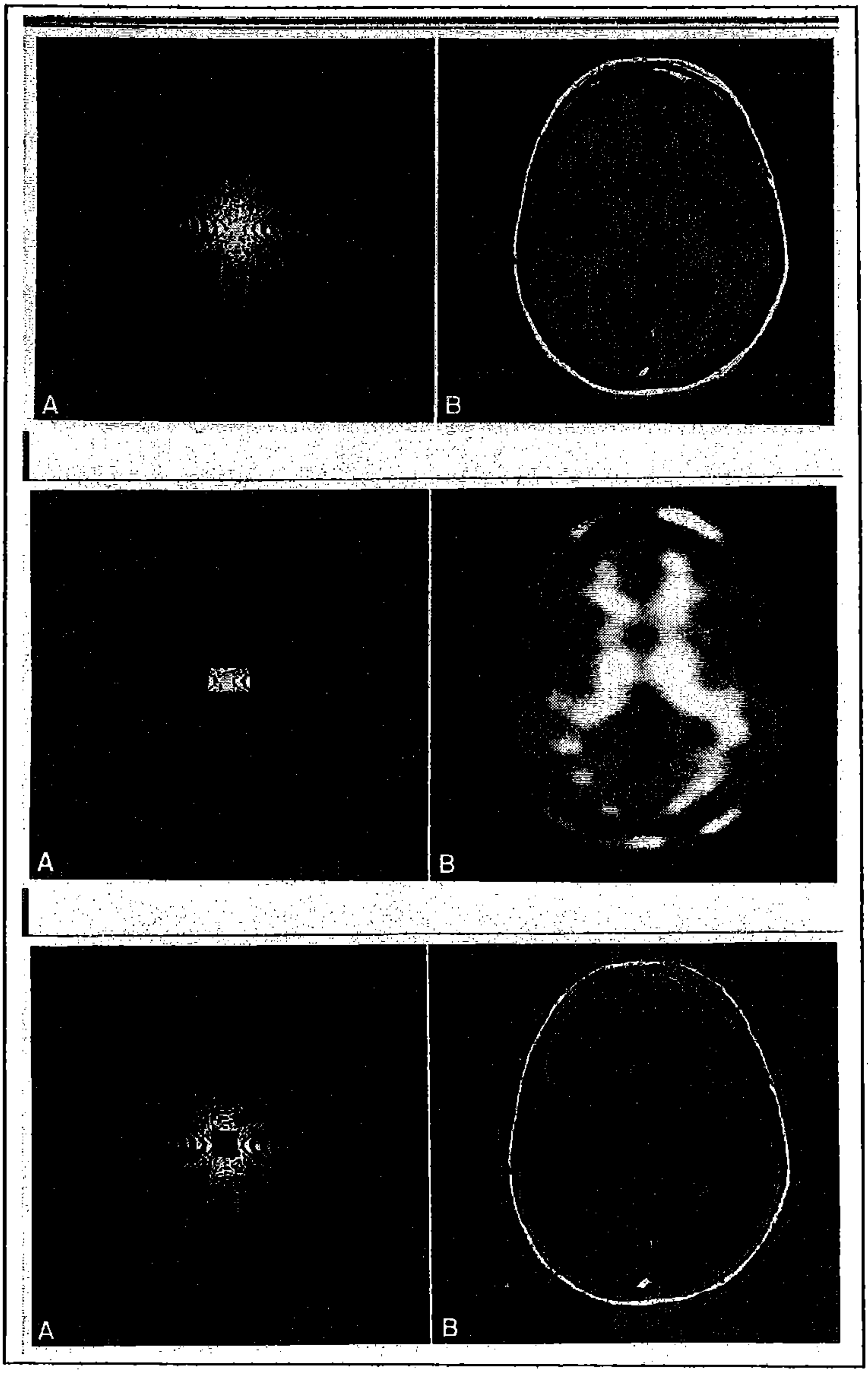
Contrary to the accepted view in neuroscience and biology, therefore, one is lead in IPS in the 3 dimensional spatial world to a quantum hypothesis, in view of (A) above, that the basis of biological vision or any of the biological senses must be fundamentally holochoric in nature. For to discard this entirely practical possibility, which makes optimum use of all the information of both amplitude and phase arriving in the sensory illumination at the sensory organs, would be to place any organism at an evolutionary disadvantage in its race for survival. For example, holochory allows the immediate full 3 dimensional spatial wave front reconstruction (up to some degree of resolution and from a state of no initial prior experiential knowledge) of the whole of the 3 dimensional object imagery arriving at a sensory apparatus, a real time requirement, often fundamental to survival of living systems in the 3 dimensional world, in which we live.

A conclusion, which in relation to the possible fact of holochory in quantum physics, gains further credence from the following :-

D) that while practical quantum computers using qubits [3] are only foreseen to exist, quantum information processing systems using quantum holography are already in worldwide medical diagnostic use in magnetic resonance imaging (MRI) systems [5,6,7]. In these systems, the diffraction (wave interference) patterns extracted by the magnetic resonant nuclear spin choreography to produce the desired medical images are easily shown to be holochoric in nature, ie to be quantum holograms, see Figure III, as predicted by the theory of quantum holography, as identified by Schempp [8], which is described in terms of the 3 dimensional nilpotent Heisenberg Lie group. See also http://wwwcivm.duke.edu

E) that quantum holographic information processing of three dimensional spatial imagery presents in principle no unsolved quantum measurement°° problems since the conditions for its success are well understood in relation to MRI cited above, and offers similar quantum computational advantages, as are predicted for qubits as against bits. That is to say, that quantum coherence in relation to quantum holochoric filter banks and associative memories, enables quantum parallelism rather than its very significantly less computationally efficient classical counterpart [3]. (°°Technicalities, (a) Such measurements are, in this case, phase measurements, that constitute those of the second class of quantum observables ie the gauge invariant phases of the quantum state vector (ie of the

Figure III



whole holochoric system in this case) discovered in the 1980s by Berry and called the geometric phase [9,10] and (b) quantum decoherence, becomes the actual means to carry out the desired object image computation, rather than being the often cited actual obstacle to it. The problem is rather to control the computation in such a way as is done in MRI for example by spin choreography to achieve the required object image resolution.)

F) that this concept of the quantum holochoric brain, first proposed on the basis of his experimental findings by neurophysiologist Karl Pribram [11], has, for example, been further independently confirmed, by the theoretical work of Perus [12]. This work shows that if one restricts the complex quantities, which are able by the use of phase to encode 3 dimensional holographic information in the quantum mechanical formalism to their corresponding real quantities, so to exclude phase, that there is a one to one correspondence with the neural net formalism, where weighting functions currently provide the best established means IPS has for modelling learning in the brain. Moreover this ability to encode 3 dimensional holographic information by phase in accordance with the precepts of quantum mechanics is the theoretical basis of a highly efficient holographic classical pattern recognition technology used by the ANDCorporation [13] see also http://www.andcorporation.com

G. This is to say, that quantum neural nets, as quantum holographic theory shows, record not only object images and their weighting functions such that the objects images become weighted in line with sensory experience [14], but are such that the objects themselves can each be given a unique name [15]. Each such name (written or read say by alphabet, encoded by the more usual kind of information, a string of bits or qubits) would then be such that its corresponding holochoric pattern is its assigned meaning, so allowing the name to be capable of being properly understood. Such well labeled quantum holographic information processing, where the individual holographic patterns define its semantics, constitutes computation [1], since in all computation, the computational objects require a unique labeling or naming schema ("canonically labeling" 15,1]) so as have a valid syntax.

It is therefore virtually certain that quantum mechanics itself can and should be regarded as constituting a semantic theory of holochoric pattern recognition, involving not one but two specific kinds of physical information resource, the hologram or generalized holochoric pattern as a signal, and implicitly a symbol, its name, which consists of a representation in the form of a bit string which could label it. An example, that can therefore be hypothesized, is DNA where the alphabet of symbols identified by molecular biologists, known as the genetic code is well known. This then immediately raises the questions, as to the nature and the whereabouts in DNA of the corresponding generalized holochoric patterns for which the genetic code would supply the labeling. An answer [16] is that, these holochoric physical wave patterns internal to the DNA, have to be those essential to the 3 dimensional geometric construction of the embryo of it's *organism, in accordance with that embryo's programmed development as laid down in the DNA control structure ie its genetic code. And experimental evidence that such physical holochoric information is located within DNA has already been partially validated [17]. Further the presence of these holochoric patterns, which assign actual geometrical meaning to the associated genetic code, indicates that the genetic code is much closer to human natural than to a digital computer language, since it, like natural language, has its own inherent semantics (see [18] for actual experimental evidence of this), and thus DNA can function not just as a code but as an actual holochoric 'neuralnetwork/brain', enabling it to run the simplest living cells, for example[19]. Thus subject to further extensive experimental validation, a working hypothesis – quantum holochory - for the Group's mission is now essentially in place.*

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Viv Pope's Pythagorean Intepretation of Special Relativity

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Viv Pope [1, 2] likes to say that the ancient Greeks could have discovered relativity if they had had the concept of geometric spacetime and the idea that the speed of light c is nothing more than a constant for transforming distance into time. With that idea, a distance D is associated with the time D/c (the time it takes a photon to traverse this distance). Then you add to this the principle that *the time between two points in spacetime is given by applying the Pythagorean Theorem* (square on the hypotenuse is equal to sum of squares on the other two sides) *to the temporal differences of their coordinates.*

For example, suppose you are standing at the origin, and I zip past you at velocity V, holding my watch. Suppose that for me time t passes and that for you time T passes. Then for you there is a *space passage* of VT which is a temporal passage of VT/c (remember we use c to transform space to time). So for you there is the temporal passage of VT/c and the (perpendicular) temporal passage of t (the movement in time perpendicular to your space). The total temporal interval from your point of view is therefore equal to

$$\sqrt{(VT/c)^2 + t^2}$$

But this is equal to the amount of time that you reckon has passed. But the time you reckon has passed is in fact equal to T. So we have the equation

$$T = \sqrt{(VT/c)^2 + t^2}.$$

This is Viv's formula, and indeed it is a correct formula in special relativity, and usually derived in a somewhat different way.

The thing that struck me about Viv's formula is that it is self-referential in regard to T. You have to know T in order to find it, but of course you can solve the quadratic equation and get the usual formula

$$T = t/\sqrt{1 - (V/c)^2}.$$

On the other hand you can view Viv's formula as a instruction to make a recursion

$$T = \sqrt{aT^2 + b}$$

where

$$a = (V/c)^2$$

and

$$b = t^2.$$

Then

$$T = \sqrt{a\sqrt{a\sqrt{a\sqrt{\cdots} + b} + b} + b}$$

and you can see the time dilation as a recursive process emanating from a Pythagorean self-referential relationship!

Note that so long as $0 < V < c$ then we have that the T-process converges, and if $V \geq c$ then the process blows up. The strange appearance of the square root of minus one for $V > c$ does not happen in the recursion.

What about $V = c$?

Then

$$T = \sqrt{T^2 + b},$$

whence

$$T^2 = T^2 + b$$

whence $b = 0$.

So no time can pass in the time flow t. But T can be anything! We have passed into the imaginary world of the photon. From the point of view of the recursion

$$T = \sqrt{\sqrt{\sqrt{\sqrt{\cdots}}}}.$$

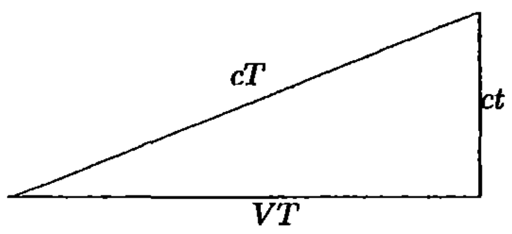
There is neither convergence nor divergence here, but just a formal fixed point.

We are in the cleft of matter/mind.

Remark. The usual derivation of the formula goes like this. Consider a photon that moves directly upward for the moving observer. This photon starts at ground level at time 0 (agreed upon by both observers). For the moving observer, the photon travels a distance ct. For the stationary observer the photon travels a distance cT Note that T is the time interval for the stationary observer, and t is the time interval for the moving observer. Note also that the speed of the photon is c for each observer (Einstein's principle of relativity). From the point of view of the stationary observer the photon moves along the hypotenuse of a right triangle with base VT. See the Figure below. If we divide each side of the right triangle by c we get that

$$T = \sqrt{(VT/c)^2 + t^2},$$

expressing the temporal length of the hypotenuse as the square root of the sum of the squares of the two sides.



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Gulliver encounters Conscious Machines

by Louis Gidney

Gulliver now came to the Land of Conscious Machines where he met a young man who understood how to make and fix Conscious Machines as easily as shelling peas.

This happened shortly after a disconcerting encounter with a similar young man in the Land of Electric Currents. There Gulliver watched him build a pile of metal discs and slices of lemon, and been somewhat startled when he unexpectedly drew a mighty bolt of lightning from it.

When he had recovered from the shock he saw the young man sitting as cool as you please, as if nothing had happened, busily winding some wire around a bottle cork. When he had finished he placed the cork on two pins whereupon it started to spin of its own accord and continued thereafter to propel itself as if pushed by invisible spirits.

He had asked what Fluences this young man was conjuring and had been given the answer: Electric Currents. Some substances are Conductors he was told, others are not, and the Key Idea to grasp was that of Flow around a Circuit or a completed loop. That was all he remembered. Having seen people burnt at the stake for less Gulliver felt reluctant to stay longer and shortly thereafter he took his leave.

En route he ruminated on the power of ideas in general, and marvelled what Key Ideas were these that could guide even mischievous boys to get such startling results from simple manipulations of everyday materials.

Now, finding himself in the Land of Conscious Machines, Gulliver determined to ask more persistently what currents of what stuff circulated to make machines conscious.

Unfortunately the young apprentice spoke only gobbledegook:

There are no objects in the world except in experience. The ground of the world is not objects but Perpetual Activity which is real but unable to exist. Without Action there can be no Existing, and all Existing is Relative ('Existence To'). If 'Existing To' is amplified enough you have Consciousness. Something 'exists to' the conscious machine. This has nothing to do with intelligence. Activity is composed of discrete acts such as the photon, which in its own proper time accomplishes everything instantly. Hence Existing is necessarily intermittent on the macro and the micro scale. Everything perishes. Whenever you check you may see the tree in the quad but it will fade if you stare at it for too long. When no one is looking at it there is no tree. But its knot of activity, which cannot exist, continues until that too perishes. Then it will rot and produce mushrooms. In the distant past no one understood that Existing was Relative. Relative Existing can only be amplified into Consciousness within machines built according to the principles of Empathetic Technology which employs Spontaneous Activities of substances. In the Dark Ages they made machine parts by forcing substances into forms that they dislike. They frequently broke down as soon as the substances got a chance to do their own thing. Iron rusts for example. The nearest they got to Empathetic Technology was the cathodic protection of steel bridges standing in water.

Later Gulliver fell in with a Sage who told him some History: Before the Relativity of Existing had been understood, which happened some centuries after the Relativity Of Simultaneity had been understood, that is to say before 'Existence' was seen to be a meaningless superstitious relic of Old Thinking, almost everything that the Schoolmen of the twentieth century studied was said to 'Exist' as it was called. It is difficult for us nowadays to put ourselves into the mindset of those Dark Ages. Perhaps 'Persist' is quite close to what they meant by 'Exist', or perhaps what it meant was close to 'Real'. It seems to have meant something mystical.

You can get some grasp of how they used to think by meditating on Permanence and Change. While most Things in Nature can be seen to change, some Things seem to be permanent. Now we understand that there are no Things that are Permanent other than the Laws of change and activity.

Well we think they used this word 'Exist' (without much precision) as a sort of predicate for all the things that seemed to them to be Permanent and unchanging. Indeed it is no exaggeration to say that they valued Permanence above all else and wasted centuries in a quest to find something that was truly Permanent that could be seen as the Ground of the whole world. Well we now know their search was fruitless, because Activity (or Process) is the basis of everything.

However you can be sure that if they had found this ultimate Philosopher's Stone and Ground of the World, they would have said that it 'Existed'. Meanwhile they would say of even ordinary practical substances that they 'exist', even though they perish.

If you had said in those days that only Perpetual Activity is tangible and Real, and that Permanence is an illusion, they would have told you that it is impossible for Activity to occur unless first there 'Exists' something that can change or act. Roughly speaking that is how their minds worked.

The theories of their erroneous science had penetrated everyday language, thought and feeling to such an extent that you could almost say they were unable to see what was in front of their noses; unable to to recognise their own everyday reality.

For example instead of experiencing wood, rock, metal, water and so on, as substances in their own right, people automatically regarded them as forms of something called 'matter' which was what theory said. Similarly, they also distrusted their own immediate experience (even though they believed their science to be based on observation) because theory told them that 'secondary qualities' were unreal and that only 'matter' (which only science understood properly) was real. In a way there was still some authoritarianism in their attitude.

It seemed quite irrational to them to think of Activity as real and Permanence as illusion even when the evidence was staring them in the face.

It was for this reason (this fixation on Permanent Stuff) that an understanding of genuinely Consciousness Machines which are now commonplace and simple to understand was so slow in coming about. This fixation controlled their researches and the character of their technology. It pointed them in a certain direction so that the very 'stuff' they needed to understand, which as not a 'stuff' at all but 'Existence' itself, which had been there under their noses all the time, but to which they paid little heed, and had no understanding of.

It was only after The Second Copernican Age, when the very idea of 'Absolute Existence' was seen to be a misconceived and meaningless idea that progress on Consciousness started. Before this had been clarified it was easy to slip into idealism and confusion.

Once it was seen that Pure Activity was the basis of Nature it was understood that a new direction of research was needed. If the very idea of 'Existent' things was meaningless; that there could be no objects in the world except what is present to Sentience, and that Sentience IS just the Relative Existing of things thrown up in the flux of Perpetual Activity, people became curious to know why it should happen in some circumstances and not others, and become more amplified in some cases than others. They learned afresh the art of how to observe their own immediate experience which they had lost. And so a fresh return to direct observation came about which anyone could do.

The excitement of that New Enlightenment is difficult to recapture. But the fact was that after centuries of chasing an illusion, a new 'Empathetic Science' and technology was born.

There arose the idea of 'Spontaneous Tendencies' of substances as a field of study. That is to say substances are not inert. Left to themselves they do something. They respond in some way to their environment. Iron may rust. A log on the forest floor will rot and grow fungi and so on. Even rocks and bricks may produce an efflorescence of lichens appropriate to their natures. Put them in different environments and they do something else. These are only simple examples.

This New Direction was based on looking at substances and other phenomena from their own point of view; that is to say 'empathetically' - instead of, as before, 'objectively' whatever that had meant. (My own theory is that 'objective' was connected with one of the mythological creatures in their pantheon called "The Impartial Objective Observer" who they revered as being superior to humankins in being able to see things as they "really are" which was different from what you see in front of you. They seem to have imagined him situated outside the universe. Of course that was a contradictory idea since by "universe" we means Everything including all possible viewpoints.)

But I digress. After the Second Faraday had noticed enough of these Spontaneous Tendencies, it became possible to concoct arrangements of substances in which some formed the environment of others, and so on, and by stringing substances together, tricking them into 'spontaneously' doing what they did in such a way that the overall result was useful. Conscious Machines did not come out of his work immediately. But at least he had demonstrated the basic principles of how to use and amplify Relative Existing, now understood to be an aspect of all physical processes - and the feature of the physical that makes consciousness possible at all in its forms.

It is only in technologies of this kind that the natural Relative Existing inherent in the Perpetual Activity of Nature can be moulded and amplified to the level of Consciousness. In the Old Days most artefacts were made by forcing substances 'reluctantly' into forms that were not natural to their Spontaneous Tendencies, so they could never have made use of Relative Existing, and consequently could never have built conscious machines - even though they built some very good imitations of conscious behaviour. That is why the machines of those days frequently malfunctioned. The substances of which they were made quite soon start acting spontaneously.

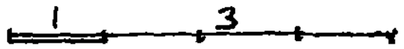
Louis Gidney, Strontian, 28/08/03

THE EDDINGTON HERITAGE - A CLAIM FOR ANPA.

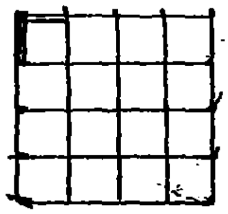
It is sixty-one years since I heard Eddington give a semi-popular talk just along the road from here . In retrospect I suppose I should have been disappointed at his dry manner. I wasn't because I had already succumbed to the Eddington magic much earlier. I had read the popular books, as many did, and I toiled through THE MATHEMATICAL THEORY OF RELATIVITY during the early days of the War. Getting hold of THE RELATIVITY THEORY OF PROTONS AND ELECTRONS in - I think - 1941 was much more of a shock. It was another eight years before George McVittie showed me that Eddington had already formed views on general relativity which amounted to turning the usual interpretation on its head, before he wrote the first of those two books in 1923, but they became stronger and more unconventional in that book. None the less, his achievement in introducing general relativity almost single-handed to a sceptical British scientific community, coupled with the part he played in the eclipse expedition of 1919, seen as confirming general relativity, earned him very great and widespread respect.

Eddington's views about the completeness of the analysis of general relativity were suddenly upset in 1928 by Dirac's publication of his equation for the electron, for this did not at all fit into the pattern that Eddington had constructed. He interpreted this as an indictment: that he had neglected certain algebraic structures (Clifford algebras) which Dirac had used. Eddington concentrated on these and thought he had found in them the source of an important number - 136 - near to a measured number which characterises the weakness of the electromagnetic field in quantum mechanics. Quantum mechanics had a long history of obsession with spectroscopy, and this number (or rather its reciprocal, written as α) first comes to prominence in an experimental context. The sharp lines observed in the spectrum of hydrogen were found, by a more refined spectrometer, to have a fine-structure - each line split into a number of lines. The parameter describing this splitting, α, was called "the fine-structure constant", but it occurs in many different parts of quantum physics. The name is only of historical significance. It is agreed by everyone that this number and a few others like it are of basic importance, but their values which are determined by experiment, are a mystery. This mystery was first noticed by Planck at the turn of the century though not so much as a mystery as an opportunity. He observed that once the constant that bears his name was taken with other well-known physical constants such as the speed of light, the possibility arose of setting up what he called an absolute system of units for physics; the Paris standard metre would be unnecessary. But this possibility depends on specifying the values of certain numbers, like α. Why they should have these values and not others is the mystery. Dirac himself went on record much later, saying that quantum electrodynamics was an incomplete theory because it had failed to calculate α. His insistence on this arises from the fact that the very successful computational techniques of that theory depend critically on the smallness of α.

I must say something about Eddington's quite indefensible calculation of 136, not only to show its inadequacy, which explains its reception by other physicists but also because it has a positive feature which I want to hold on to. But don't bother too much over the details since I shall be rejecting them later. The bare bones of Eddington's argument are as follows: he noted that in the Clifford algebra there were three elements of one kind and one of another and he associated this (reasonably enough) with the three dimensions of space and one of time. I can represent this in a diagram like this:



The algebraic tie-up then encouraged him to consider two such structures multiplied together, which I can put in the diagram like this:



Now there will be 16 elements of which $3^2 + 1^2 = 10$ will be of one kind:- they do not multiply spatial with temporal elements The other 6 will mix spatial and temporal elements in the products. Call these "mixers". So now, says Eddington, you have your 16 divided up into 10 non-mixers and 6 mixers, rather like the original situation where there were 3 spatial and 1 temporal elements. He associates this again with

algebra, which allows him to repeat the association of two such structures by multiplication. This time there are $10^2 + 6^2 =$ 136 non-mixers and 120 mixers. It is this notion of repeated association of structures with algebras, the more complex structures with larger algebras, which was crucial for Eddington's calculation. This is the positive feature which I want to take from Eddington's effort.

Now I'm pretty sure that Eddington saw a lot more beneath the surface than he was able to spell out in words. But others saw only the weakness of the argument and not surprisingly teh calcualtion was not well received by the scientific community. Matters were not helped by the way in which Eddington argued for an extra "degree of freedom" to give 137 rather than

136 when the experimental value was revised to about 137. Whatever the validity of this new argument, as a PR move it was fatal. But because of his position of eminence, contrary views were expressed in a less hostile public manner than they might have been. For example, the acerbic Pauli , in a letter to Oskar Klein in 1929, says "Note, by the way: I now regard the Eddington 136-work as complete nonsense; more precisely, as romantic poetry, not as physics." But his public statements were much more restrained. The last eighty years have not seen any great change in general opinion.This limited criticism suggests a reason why Eddington's response to criticism was not to seek to clarify his position but to go on to find other such numbers and so by 1936 to make an extreme claim: "there is nothing in the whole system of laws of physics that cannot be deduced unambiguously from epistemological considerations". Eddington had earlier expressed his great affinity with Kant. The three-dimensional nature of space is part of Kant's sythetic a priori characterisation of the concept. Eddington saw the 3 + 1 and so the 136 as also synthetic a priori. But "nothing in the whole system of laws..."? It is true that he qualifies the statement by excepting the nuclear realm but such a conclusion still surely comes from someone who, as a small boy, started to count the words in the Bible. Such a thoroughgoing Pythagoreanism is clearly untenable.

One could, however, imagine a less extreme position in which, rather than the whole of physics, some fundamental numbers were found in this way. This modification still has major troubles. It has no clear criterion to determine which numbers are to be designated as fundamental, but a more serious defect is simply that there is no proper basis for using numbers in the way that Eddington did. He tried to justify some of his calculations by using the algebraic structure to chase through the rigmarole of differential equations in the orthodox quantum theory so as to say: "You see, it comes in here". In doing this, Eddington had, of course, to work in quantum theory, for which he had no sympathy or expertise. I remember once suggesting to George Temple, who had been a student of Eddington, that Eddington's view of quantum theory was like that of Temple's little Methuen Monograph, "An introduction to quantum theory" of 1931.

Temple replied "Oh, he never knew so much!" But this was not why the attempt never worked. It was just not possible. I should mention in passing that at that time - the mid-thirties - questions of accuracy of these numbers were not so important as they are now.because the experiments were much less accurate.

Now Bastin and I were convinced that what we isolated as Eddington's programme of constructing a rationale for placing these numbers prior to measurement, and therefore casting their character on all subsequent measurements, was correct. We therefore set about understanding where the numbers came from. In some papers in 1954 and later we drew attention to different algebraic structures (groups). We had the idea, partly from Eddington, that there would be a sequence or hierarchy of such groups, each derived in some way from the elements of the last but we didn't know their precise relationship. Eddington's way of doing it did not work for us. This missing link was provided by A.F.Parker-Rhodes, who devised a peculiar hierarchical structure into which our groups fitted. This structure had successive multiplicities of 3, 10, 137 and a large number, about 10^{39}. The construction of the hierarchy could not continue beyond this fourth stage. Now I should mention that, just as 137 measures the weakness of the electromagnetic field in quantum mechanics, so the large number measures that of the weakest field, gravitation. Parker-Rhodes asked us if we thought the 137 was significant but he had not made the connection with gravitation via the large number. We saw that the termination of the sequence would correspond to gravitation being the weakest field.

Personally we found these two numbers a strong indication that we had here something relevant to physics but what confirmed this for us was the termination of the sequence. The constructions used by Eddington could have been repeated over and over again. He was forced to use a rather weak argument that "physics does not seem to go any further". I ought to recur to the numbers 3, 10 but I would prefer not to. They are surely connected in some way with the strong nuclear forces but we don't know how. That is a problem for the future. For the present, major problems still needed answering:

1) Exactly how was this peculiar algebraic structure related to physics?

2) Now that greater experimental accuracy had been achieved, why was 137 not a very good approximation to the inverse fine-structure constant?

The first one is a problem in physics or the philosophy of physics or both. The second one is a mathematical problem.

This work has occupied us for almost half a century. Parker-Rhodes' work was pre-eminent amongst several important contributions from John Amson, David McGoveran, Pierre Noyes, Gordon Pask and others. Twenty-five years ago Pierre Noyes proposed setting up an organisation centred on the work, which would be called The Alternative Natural Philosophy Association, the ANPA of the title of this talk. This has been a loose and open organisation. At any one time it has comprised 20 - 30 active full or mostly part-time members. I suppose over the 25 years some hundred have been involved at different times. Some of them have been from physics, some from mathematics, some from computing and some (to my mind too few) from philosophy. Some of the activity has been difficult to connect with the central problem at all but a continuing core has been directed at it. The name of the organisation was chosen to place it in the broad context of natural philosophy as Newton understood it. Perhaps "Alternative" was a mistake - it can conjure up unfortunate New Age suggestions. But discussions in the group about a different name have been inconclusive.

The first question is answered in this way: Our theory is a "process" theory. Contrary to Eddington, we do not begin with classical space and time. Here "process" is not to be understood as having any temporal significance; perhaps "sequential" would be a better word. The notion of a process theory is notoriously hard to pin down but one thing I do want to say at this point is that we are not using the word in the way that Whitehead did. Information comes in. This is just a statement about scientific investigations. If you insist on asking where it comes from, I would have to say, from a background. Then a statistical analysis of the information tells us something about the background. For this reason, we often refer to it as a statistical background. The information

is fitted into a framework which itself generates the algebraic scheme. This scheme turns out to be not very different from Parker-Rhodes' original one, so the numbers are preserved. The fitting in is done in this way: a new entity is compared with one already in the system and it is determined whether they are different or not. This one primitive act, which we call discrimination, is represented by an operation in the algebra. There is also, corresponding to Eddington's repeated operations, a procedure of level change in which certain sets ... are treated as a single entity at the next level. This definite mathematical form for the hierarchical step comes directly from the crucial contribution that Parker-Rhodes made to our thinking. It would be out of place here to linger over the technical details of this part of the process. What should be emphasised, however, is that these two operations are the only ones, from which the whole theory develops. One might well wonder how all the complexity of the physical world could develop from such simple operations. It comes about because there is a continual accession of elements and the process keeps on repeating itself and changing, randomly, within the stability provided by the algebraic scheme. This stability is characterised by the numbers pinpointed by Eddington. Since the system whose stability is in question is just the information input, which may be by experiment, this answers the damaging accusation that we are being "a priori"" instead of working from experiment, that was often levelled against him.

Now Eddington did not envisage a process theory and his "selective subjectivism" is different, although there are certain analogies. As I said before, our view runs counter to the objective space of classical and most quantum physics but can well be seen as an abstract description of what goes on in high energy particle theory. The "vacuum" of high energy physics is very like our statistical background and the step-by-step incorporation of empirical material from it by the elementary discrimination in the structure is what goes on there. Some might think of this as the becoming known of new information, but the analogy is a tenuous one because there is nothing which does the knowing.

To those prepared to accept our unconventional philosophy the internal coherence of our scheme should be compelling.

But I must emphasise that a major change of thinking is needed. As I said, we do not begin with space and time. They must come as constructs in the theory. It is necessary, we claim, simply to accept that there are certain numbers, we call them scale constants, whose values depend on the framework and its stability. Such a claim can be defended because these numbers are logically prior in the structure. This statement throws up its own question of what is the overall logical structure in which this makes sense? Oddly enough, this was the very thing with which Bastin and I were concerned back in 1954. We argued there that physical science needs to be seen as a network of theories of different degrees of complexity and that it will not be correct to use experiments in a more complex theory to criticise a less complex one. Logically prior will then mean in the least complex theory. But this is not the place to go into such details, the more so as it issues in a philosophy of physics unlike Eddington's.

I turn instead to the second of the two major questions, the lack of accuracy of the Parker-Rhodes calculation.

If you don't want to buy the whole philosophy you may still be persuaded by the empirical success of the theory. I have analysed the calculation of the inverse fine-structure constant in more detail. Without the addition of any further ad hoc assumptions, the corrected value is in complete agreement with the most accurate experimental value, correct to seven significant figures, 137·036012 .

There is much left to do, not only in tackling some other scale constants but also in getting some understanding, if these numbers are distinguished as different in some way, of how one can find a place for the "ordinary" numbers which can be found experimentally to have a range of values. But on the basis of our progress we think we have made a case for thinking that Eddington's argument for giving the central place in theory that he does to the scale constants, was correct, even though we have had to replace his actual calculation. This is the part of the Eddington heritage we are claiming.

Clive Kilmister.

A COMBINATORIC HIERARCHY WITH INDISTINGUISHABLES ? WHY NOT ?

A twenty-fifth anniversary essay for ANPA 25

Alternative Natural Philosophy Association
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The view backwards

To those of us who knew him, it was clear that two major strands dominated the thinking of Frederick Parker-Rhodes, a founder member of our Association, throughout the last two decades of his life: one, the so-called Combinatoric Hierarchy with all its half-understood implications for a finite discrete physics, and two, his novel theory of Indistinguishables with its emphasis on two things being not merely the same or different but instead possibly indistinguishable.

What never fails to surprise me is that no substantial attempt was made by him, nor subsequently by anyone else, to bring these two strands together.

There was a faint glimmer of a possibility as far back as 1965 but that was nothing if not premature, not least because Frederick had barely launched himself on his Indistinguishability Odyssey at the time. What happened was this:

Frederick and myself were working on the way his fledgling Combinatoric Hierarchy 'stopped' at the third stage — in an 'essential' way. It was the 'essentiality' of this stopping that led so many of us think that something very special had been uncovered by Frederick. Especially as it stopped at the 'third' stage (we all wondered whether there was any connection with the dominance of three dimensions in the familiar world of physical experience) and the combinatoric counts of the number of members in each Level matched various non-dimensional ratios that were known to have great significance to theoretical physicists.

The 'essentiality' was a consequence of the 'mapping of one Level into the next higher Level'. The 'mapping' in question was later seen to be the result of identifying members of one Level as (linear) operators acting on the immediately lower Level in such a way as to preserve the 'discriminately closed subsets' in that lower Level; of course we did not know that at the time. All that was really visible was that for the algebraic constructions to take place certain numerical quantities could grow from Level to Level but only up to the point where some crucial numerical inequality was flouted.

We were looking for similar situations that might arise if we put aside the fundamental premise that we <u>must</u> work only over the field of two elements ($\mathbf{J}_2$, $\{0, 1\}$) with its usual <u>associative</u> addition and <u>distributive</u> multiplication. We tried various modifications to the Level-to-Level procedures and different number fields. Significantly we tried working over the field of three elements ($\mathbf{J}_3$, $\{0, 1, 2\}$) — significant in hind-sight because of the rôle of '3' when we move into the jungle of Indistinguishability. A number of these experiments were reported in a paper we prepared in 1966[2] and which was published posthumously for Frederick in 1998[3]. Admittedly, our paper was technically far too obtuse. Apart from the last section 6 which illustrated how everything worked out fine numerically for the binary field and gave us the bare bones of what became the Combinatoric Hierarchy, it could all do with rewriting and a readily readable summary.

The "essential" point of our paper — at that time — was that you may not get a useful version of the Combinatoric Hierarchy when you leave the binary field and the world

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[2] Rejected out of hand by *Camb.Phil.Soc.*!

[3] J.C.Amson, A.F.Parker-Rhodes, 'Essentially Finite Chains', *Int.J.General Systems*, Vol. 27(1-3), pp.81-92, 1998.

of 'bit-strings'. At least not if you arbitrarily force yourself to retain the more familiar algebraic and logical processes.

The view forwards

Any attempt to produce a Combinatoric Hierarchy-with-Indistinguishables must start out from a basic entity consisting of 'three things'.

The collapse of our attempts (Frederick's and mine) to produce a comparable — and *interesting* — 'essentially finite chain' using the three-field $\mathbf{J}_3$ in place of the binary field $\mathbf{J}_2$ suggests that we must review our mathematical and logical habits.

We must not always take the usual route and assume that the algebraic and logical and structural systems we employ must be the familiar ones of Groups (with their pervading Associativity) and Rings (with their pervading Distributivity of addition over multiplication), and Lattices (with their pervading Linear or Partial Orderings) and Logics dominated by the principle of the Excluded Middle.

In recent years my experiments are leading me towards thinking in terms of systems that display an intrinsic 'cyclical ordering'. Frederick's theory of Indistinguishables[4] emphatically rejected cyclical-ness, and this may have inadvertently precluded any attempt on his part to bring Indistinguishables into contact with Combinatorial Hierarchies of a different flavour.

It is also curious that (as far as I am aware, and there is a huge literature[5] to examine) the logicians who introduced and have developed the notions of 'many-valued logics'[6] have never made use of a 'cyclical negation' — under which the more-than-two Truth-States are ordered cyclically by the successive application of a 'Negation Connective' — but have always tended to use a 'linear ordering'.

With only two Truth-States (F, T) to work on, negation is trivially cyclical (indeed, involutory): thus *not False is true, not True is False*, whence *not(not True) is True*; $\neg F = T$, $\neg T = F$, $\neg\neg T = T$. This is mirrored in Binary (Boolean) arithmetic with 'complement' playing the rôle of negation and defined by $x' = 1 - x$, thus: $0' = 1$, $1' = 0$.

But, for example, with three Truth-States (F, N, T, say, in that order), the usual negation is nothing but a re-ordering: $\neg F = T$, $\neg N = N$, $\neg T = F$. A reversal of (F, N, T) into (T, N, F).

[4] See (i) A. F. PARKER-RHODES, *The Theory of Indistinguishables: A Search for Explanatory Principles Below the Level of Physics*, Vol. 150, Synthese Library, D. Riedel Publishing Company (Dordrecht, Holland; Boston, USA; London, England), 1981; and (ii) JOHN AMSON, 'On Discrimination; the first A.Frederick Parker-Rhodes Memorial Lecture', in *Discrete and Combinatorial Physics, Proceedings of ANPA 9, Cambridge, 1987.* Ed. H.P.Noyes; pp.141-157.

[5] See the valuable text: SIEGFRIED GOTTWALD, *A Treatise on Many-Valued Logics*, Vol.9 in King's College London Series in Logic and Computation, Research Studies Press Ltd (Baldock, Hertfordshire, England), 2001.

[6] Beginning *e.g.* with J.Lukasiewicz (1920) and E.L.Post (1921).

This too is mirrored in a Trinary arithmetic[7] on the set $\{0, \frac{1}{2}, 1\}$ with 'complement' again playing the rôle of negation and defined by $x' = 1 - x$, thus: $0' = 1$, $\frac{1}{2}' = 1 - \frac{1}{2} = \frac{1}{2}$, $1' = 0$.

Moreover, logicians seem very unwilling to forego the Excluded Middle. The involutory nature of 'negation' is paramount: $\neg(\neg P) = P$ — *if I am not not dead, then I am dead, and there is no other alternative state in which I can exist.*

Gottwald in his Historical Outline[8] suggests that many-valued logics may be traced back to Aristotle "who discussed the problem of future contingents. [....] the problem to determine 'today' the truth value of a proposition which asserts some future event. [...] and is closely tied with the philosophical problems of determinism and the understanding of modalities". And "modalities" (Modal Logics) qualify assertions about the truth of statements, *e.g.* we may say that something is 'necessarily' / 'possibly' / 'ought to be' / 'has always been' true, and so on. The assertion that some future event is "possible" or "undetermined" may well be seen as our willingness to accept a third "truth value" besides F 'False' and T 'True'.

If we are to depart from our familiar binary state of thinking by the very least amount, into a trinary state, then we need a third Truth-Value and an algebra with three elements. But if, as Frederick and I found, that the usual algebra and arithmetic over the field $\mathbf{J}_3$ led us to a dramatically dull version of a Combinatoric Hierarchy with at most two Levels, then we have to be bold enough to change our tools.

To this end I am inclined to favour first a background logic of three states $\{F, T, N\}$ equipped with a Cyclical Negation connective. My definition of this unary operation of 'negation' (NOT_3, symbol $\neg$) is crucial, and I believe, novel. It specifies the 'flavour' of the resulting logic, which accordingly I am calling a *Tricycle logic.*

Any such negation is a logical function $f(P) = Q$ where for each of the three values of the argument P there are three possible functional truth-values Q choosable from $\{T, N, F\}$. Thus there are $3^3 = 27$ possible 'negations', one of which being trivially the 'identity' mapping has no 'negatory' rôle. For my purposes, I am choosing 'negation' to be the **cyclical** 'successor' mapping under the cyclical ordering :

N *is 'later' than* T, F *is 'later' than* N, T *is 'later' than* F

viz. $$T \xrightarrow{\neg} N \xrightarrow{\neg} F \xrightarrow{\neg} T$$

which is arbitrary to the extent that we could have chosen the cyclical 'direction' to be the opposite of that selected here.

Thus : $\neg T = N$, $\neg N = F$, $\neg F = T$, so that, applying 'negation' a second time gives : $\neg(\neg T) = \neg N = F$, $\neg(\neg N) = \neg F = T$, $\neg(\neg F) = \neg T = N$; showing that 'negation' is tri-potent: $\neg(\neg(\neg P)) = P$ for all propositions P. We naturally write $\neg(\neg F))$ as $\neg\neg F$, and $\neg(\neg(\neg F))$ as $\neg\neg\neg F$, *etc.*.

Then there is the need to design the other 'fundamental' Logical Connectives analogous

[7] Trinary arithmetic over the set $\{0, 1, 2\}$ is isomorphic to that over the set $\{0, \frac{1}{2}, 1\}$ under the obvious identifications.

[8] *loc.cit.*. Ch.4

to AND, OR, *etc.*. Obviously there is a surprisingly large choice. And different schemes and persuasions seem to drift in and out of fashion[9]. In the case of a Binary Logic in which each Logical Connective can be represented as 2×2 Truth-Table there are 4 cells, each of which can hold one of F, T. There are $2^4 = 16$ such tables, corresponding to AND, OR, XOR, NAND, NXOR, NAND,... and so on (terms very familiar to engineers working in electronic switching-gate technologies). In the case of Trinary Logic each Truth-Table is a 3×3 table, each of whose 9 cells can hold one of F, T, N. There are $3^9 = 19,683$ such tables, a "35-decibel increase" over Binary Logic, so-to-speak!

So far as the algebra and arithmetic over the set $\{0, 1, 2\}$ is concerned, there are again 19,693 possible functions. Ordinary addition and multiplication and given by the usual tables:

ADDITION

$+_3$	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

and

MULTIPLICATION

$\times_3$	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

and both these are associative, symmetric, and together create a Ring.

I computed a census of all 19,683 possible such operational tables and found that only 113 of them are associative (about half of 1percent), 63 of them being symmetric. There are 729 symmetric ones. Only three of the 19,683 tables corresponded to an operation that formed an Abelian Group but these differed only in having a different distinguished element (a neutral, or 'zero') namely 0 or 1 or 2 in turn. The one with 0 as 'zero' was ADDITION above.

Sticking with this + operation, there were just three more tables which gave rise to a multiplication that formed a Ring. One of them was trivial (its table had 0 in every cell) the other two differed in having either 1 or 2 as its multiplicative 'unit'. The one with 1 as 'unit' was MULTIPLICATION above. It corresponds in a natural way to the 3-Logic Connective AND_3 (symbol $\wedge_3$ or simply $\wedge$)

In Binary Arithmetic and the Binary Logic used in the Combinatoric Hierarchy we know that

ADDITION

+	0	1
0	0	1
1	1	0

corresponds to

XOR
'DISCRIMINATION'

⊕	0	1
F	F	T
T	T	F

[9] Gottwald, *loc.cit.*, Ch.5.

But this does not carry over into our Trinary Logic. Addition $+_3$ is associative, but every attempt to design an appropriate 'DISCRIMINATION' operation on our a Tricycle Logic turns out be non-associative[10]

What are the criteria for such a new discrimination operation, DISC_3 ?

Looking at the XOR table above, for 'discrimination' in the Binary (Boolean) Combinatoric Hierarchy it can be said that 'discrimination' takes place when the truth-value of a pair of logical states is T; the pair in question have their value in a cell off the diagonal axis in the table. Indeed, 'discrimination' and 'being-off-axis' go together.

At this stage of the study there is of course more than one candidate for a suitable 3×3 DISC_3 truth-table each taking into account the additional rôle played by the intermediate truth-value 'N'. Any arrangements of the truth-values in each of the nine cells of a tricycle truth-table in order that the truth-function it represents retains some resemblance to that of XOR in Binary Logic should be guided by these considerations :-

(i) the truth-function should be symmetric in its arguments, *i.e.* the table should be symmetric about its principal diagonal;

(ii) none of the truth-values on the diagonal should be 'T';

(iii) the truth-values of the 'extreme pairs' (T,F) and (F,T) should both be 'T';

(iv) the truth-values of two of the 'like pairs', namely (T,T) and (F,F), on the principal diagonal should both be 'F'.

Filling in a truth-table to meet these suggested requirements leads to this partially completed table :-

F	a	T
a	?	c
T	c	F

in which the three entries a, ?, c, are to be decided. The value ? in the central cell, for the pair (N,N) could either be emphatic and low, F, or neutral, N. The value a in the two cells for the 'lower pairs' (F,N), (N,F) could also either be emphatic and low, F, or neutral, N. The value c in the two cells for the 'higher pairs' (T,N), (N,T) could also either be emphatic but high, T, or neutral, N. This gives us eight candidates for our tricyle DISC_3 truth-table :-

...continued overleaf...

[10] By that I mean the corresponding algebraic operator (under $F \leftrightarrow 0,\ T \leftrightarrow 1,\ N \leftrightarrow 2$) is non-associative.

1

F	F	T
F	F	T
T	T	F

|N| =0
R 2, D 0

2

F	F	T
F	F	N
T	N	F

|N| =2
R 2, D 0

3

F	F	T
F	N	N
T	N	F

|N| =3
R 4, D -2

4

F	N	T
N	F	N
T	N	F

|N| =4
R 3, D 8

5

F	F	T
F	N	T
T	T	F

|N| =1
R 3, D -2

6

F	N	T
N	F	T
T	T	F

|N| =2
R 3, D 4

7

F	N	T
N	N	T
T	T	F

|N| =3
R 3, D 2

8

F	N	T
N	N	N
T	N	F

|N| =5
R 3, D 6

Each candidate has below it a label from 1 to 8; the number |N| of N's in the table, from 0 to 5, draws attention to the degree of 'neutrality' in the discrimination output; then the table's Rank and Determinant (as an integer matrix using the association : F=0, T=1, N=2) by way of extra information.

The table labelled **4** has four neutral states N (*i.e.* $^4/_9$ -th of all possible outcomes) but there is no neutrality about the 'absence' of discrimination since all three cells on the principal diagonal return F as the signal for "No Discrimination" between two input states.

On the other hand, it does allow input states to return <u>two</u> separate levels of discrimination : "Complete Discrimination" for the two extreme pairs of states (T,F) and (F,T), and what we might call "Neutral Discrimination" for the four 'mixed' pairs of states with precisely one member neutral, (T,N), (N,T), (F,N), (N,F). The cells for these pairs lie in the 'upper' and 'lower' diagonal bands adjacent the principal diagonal (top-left to bottom-right). It is arguably the most suitable choice for our Tricycle Discrimination DISC_3 table, and the one that we should prefer to adopt :-

DISCRIMINATION 'DISC_3'

$\oplus_3$	F	T	N
F	F	N	T
T	N	F	N
N	T	N	F

corresponding to

TRIDISCRIMINATION

Δ_3	0	1	2
0	0	2	1
1	2	0	2
2	1	2	0

It certainly does not correspond to addition $+_3$. Nor to a 'subtraction'. Nor to a 'symmetric difference' (as does binary XOR).

It is symmetric (abelian), but it is not associative, it has no neutral, no left-neutral, no right-neutral, it has no inverse; its 'Numerical Determinant' = 8. Its algebraic and arithmetic properties and its trinary logical properties are as yet largely unexplored.

Tentatively, we can use it like this. In place of Bit-strings we must use Trit-strings —

finite sequences ['lists'] of 'trits' (the trinary digits $\{0, 1, 2\}$). Given two trit-strings $X = (x_1, \ldots, x_n)$ and $Y = (y_1, \ldots, y_n)$ both of the same length $n > 0$, their 'tridiscrimination' (*alias* tricycle discrimination) $X \Delta Y$ is defined to be the result of a trit-wise application of the tridiscrimination operation $x_i \Delta y_i$ (for $i = 1...n$). A 'tridiscriminately closed subset' (`tdcs`) is a set of trits of common length n containing the tridiscrimination of every pair of its members. And so on....

This of course begs the question of whether 'binary set theory' is appropriate in this context. My own on-going studies are leading to a related theory of 'Trisets' in which an elemental notion of 'belonging' interacts with a tricycle negation so that we have three 'states of belonging' : $\in$, $\neg\in$, $\neg\neg\in$ which is tri-potent : $\neg\neg\neg\in\ = \in$. Consequently a subset may possess not only an 'interior' and an 'exterior' but also a "neutral-erior". Frederick might have been interested — is this where Indistinguishables really live ? It is becoming likely that the appropriate algebraic and arithmetic machinery for Trit-strings will have to be manufactured against the fundamental background of Trisets.

Envoi

'Discrimination' has played a principal, indeed paramount rôle in the Binary Combinatoric Hierarchy from its inception and has continued its primacy throughout the last four decades in the thinking that began amongst the original pre-ANPA activists : Clive Kilmister, Ted Bastin, Frederick Parker-Rhodes, Margaret Masterman, and myself. (Of which three of us survive today). It was the 'Ur' notion from which the entire edifice of the Combinatoric Hierarchy grew and has been worked upon by all of us who came under the aegis of ANPA following its foundation by Pierre Noyes in 1976.

If there is to be any conflux between the Combinatoric Hierarchy and any workable theory of Indistinguishables, a conflux that might in my view arise in what it is my conceit to call a 'Tricycle Combinatoric Hierarchy', then the notion of a trinary form of 'Discrimination' — 'tridiscrimination' — will again have to play a similar 'Ur' rôle.

The fact that any putative Tricycle Discrimination $\texttt{DISC}_3$ (logic) or Δ_3 (algebra) appears unavoidably to be a non-associative connective may have a far-reaching and important advantage here. There has always been some disquiet about what purpose or functionality there could be for 'associativity' in a system which was formulated to be intrinsically and fundamentally a 'process' — as the Binary Combinatoric Hierarchy has steadily been held to be by its first progenitors. The very idea of associativity suggests that some kind of 'meta-memory' was available to the system whereby the result of processing three states in one 'meta-temporal order' could be recalled for a comparison with the processing of the three states in a different 'meta-temporal order'. Which is far from being an acceptable situation. The current studies by Clive Kilmister into systems which deliberately eschew associativity give substantial support to this move away from associativity.

But the waters ahead are deep and clouded, and we sail into them with few charts to guide us. To attempt to create a powerful Tricycle Combinatoric Hierarchy will require much effort. It may well be worth it.

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Towards the Homology of the Phase Web Paradigm[1]

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The Phase Web paradigm is a genuinely radically new concept in computation. Nowadays some would call it *space-like* to distinguish it from the sequential approach (includes "parallelism") embodied in Turing machines and von Neumann architecture, which is *time-like.*

The basic insight giving birth to this new approach is *co-occurrence*, which is the term applied to two or more events that occur simultaneously (locally, and modulo a Δt in practice). Co-occurrence is an exemplar of Leibniz's concept of indistinguishability, but in the time domain: co-occurring entities are indistinguishable *in time* because they by definition occur neither before nor after each other, and constitute the germ of the concept of space. Oppositely, *exclusion* implies distinguishability in this domain, and constitutes the germ of the concept of time. The 'space' of our discourse is thus the space of the *distinctions* it is possible to draw from this conceptual base.

The events just mentioned are represented by (changes in sign of) the elements of the Clifford algebra. Simultaneity is represented by sums (+), and exclusion (via the precedence of $\times$ over +) by products, of these same elements.

In our computational view, the greater the grade ('arity') of a product, the more complex a transformation it can effect when viewed as an operator (always applied on the left). Therefore we imagine arranging products in chains of increasing grade, meeting in shared elements, to form the lattice-like structure we call the Ladder hierarchy[1].[2] It is the goal of this paper to show how this hierarchy is defined and constructed, and demonstrate some of its properties, using an inside-the-algebra version of

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[1] To appear in Proceedings of ANPA 25, 2003. K. Bowden, editor.
[2] A representation of the Combinatorial Hierarchy [1,2,3]; see also [4-9].

homology's boundary and co-boundary operators.

Sets and Algebras.

The first finite discrete set of interest is Z_3:

$$Z_3 = \{0,1,-1\} = \{0,1,2\}$$

$Z_3(\times,+)$ is the ring of coefficients of the elements of our algebra, and Z_3 is the range of these elements' scalar magnitudes. We have found $Z_3 = \{0,1,-1\}$ to be superior to $Z_2 = \{0,1\}$ because it separates the concept of 'nothing' or 'Void' (ie. *zero*) from that of 'the opposite of one' (ie. *-1*), which separation has a wide and subtle ripple effect;

The second set of interest is the symmetric set (and group), combinatorial in origin:

$$\mathcal{S}_n = \{a_1, a_2, ..., a_1a_2, ..., a_1a_2a_3, ..., a_1a_2...a_n\}$$

The third set, also finite and discrete, is $Z_3 \; \mathcal{S}_n$:

$$\mathcal{A}_n = \{-1, 0, 1, a_1, a_2, ..., a_1a_2, ..., a_1a_2a_3, ..., a_1a_2...a_n\}$$

which constitutes the actual *elements* of our algebra. The a_i, which are the generators of the algebra, will be replaced by a, b, c, ... when subscripts would clutter. Expressing multiplication as juxtaposition, $\mathcal{A}$ (juxta,+) forms a ring.

Our Clifford algebra $\mathcal{G}_n$ is the R-module (Z_3(juxta,+), $\mathcal{A}_n$(juxta,+)).

We take this opportunity to define some properties of multiplication in our Clifford algebra:

$$(1)a = a(1) = a; \quad (-1)a = -(1)a = -a; \quad (0)a = a(0) = 0;$$
$$a_ia_i = 1$$
$$a_ia_j = -a_ja_i \quad , i \neq j$$

and note that the algebra contains, non-trivially, both nilpotents (eg. $(a+b+c)^2 = 0$) and idempotents (eg. $(-1+a)^2 = -1+a$).

Notation and terminology.

A *term* is a *single* element of $\mathcal{A}$, eg. 0, ±1, a_3, $a_2a_7a_8$, etc.

Grade refers to the number of (multiplicative) operands in a term, eg. $a_2a_7a_8$ has grade 3. Scalars have grade 0. The term *grade* is used in preference to *arity* or *dimension.*

Lower case latin letters denote the grade 1 elements of $\mathcal{G}$.

An *expression* is a sum of terms, eg. $1 + a_2 + a_2a_7a_8$.

Capital letters denote expressions; generally, $XY \neq -YX$. Also denoted with a capital letters are single terms $X = X_r$ of arbitrary grade $r > 0$, called *singletons.* Note that $X_rX_r = (-1)^{r+1}$.

The notation $\langle X \rangle_r$ means the sum of all grade r terms of X.

We will occasionally write C | -C to denote the *co-exclusion* (defined below) of C and -C.

Co-exclusion.

We now introduce the concept of *co-exclusion* [1], wherein, when we conclude the existence of ab from the complementary co-occurrences -a+b and a-b, we say that the *co-exclusion of -a+b is ab* (and similarly, that of a-b is ba). That is, the *boundary* of ab (defined below) is -a+b, and the boundary of ba = -ab is a-b. Inversely, we would like to express co-exclusion using the concept of *co-boundary.*

Unfortunately, neither a+b nor -a-b are the boundary of ab from the viewpoint of singular homology, but they *are* possible co-occurrences that co-exclude to produce ab.

Nor do all complementary co-occurrences co-exclude to produce a single expression in $\mathcal{G}$, in that sometimes the calculation loops (ie. there is a *loop*, defined after theorem *Disjoint* below). We will later give some theorems and restrictions on which co-occurrences are in this sense well-behaved.

Our approach to these matters requires an expansion of the traditional definition of the boundary operator ∂. We begin with that definition, then expand it, and then use this expanded definition to define the co-boundary operator δ and examine therewith the Ladder hierarchy's construction and properties.

Define ∂

We begin by stating the definition of the boundary operator ∂ as it is usually given:

$$\partial(a_1a_2a_3...a_n) = (\partial a_1)a_2...a_n - a_1(\partial a_2)...a_n + a_1a_2(\partial a_3)...a_n - ... \pm a_1a_2...(\partial a_n)$$

That is, move the ∂-operator through its argument, alternating signs.[3] To get further in the calculation, one *defines* that $\partial a_i = 1$. Thus ∂ reduces the grade of each term by one:

$$\partial(a_1a_2a_3...a_n) = a_2a_3...a_n - a_1a_3...a_n + a_1a_2a_4...a_n - ... \pm a_1a_2a_3...a_{n-1} \quad (1)$$

For use a bit later, calculate $\partial(a_1 + a_2 + a_3 + ... + a_n)$: ([4])

$$\begin{aligned} \partial(a_1 + a_2 + a_3 + ... + a_n) &= \partial a_1 + \partial a_2 + ... + \partial a_n \\ &= 1 + 1 + ... + 1 \\ &= n \textit{ mod } 3 \qquad (2) \end{aligned}$$

wherein we see that ∂ distributes over addition.[5]

[3] More precisely, the sign flips, from + to - to + to - etc. for each element ∂ moves past. Ie. ∂ anti-commutes with the a_i, whence ∂ may be construed as an element of $\mathcal{G}$, an observation that will gain in relevance.

[4] The signs *don't* alternate here because ∂ never 'moves' past a multiplicand in any of the terms, they all having grade 1.

[5] Varying the signs of the a_i gets n=0 via ± pairs or Z_3's 1+1+1=0.

<u>*Re-express ∂ using ·*</u>

This way of expressing how ∂ works is, nevertheless, ill-suited to our needs. Instead, for arbitrary terms $A_r = a_1a_2...a_r$ and $B_s = b_1b_2...b_s$

define the operator · as

$s \cdot s = 0$	s = a *scalar*
$a_i \cdot s = 0$	
$a_i \cdot a_i = 1$	
$a_i \cdot a_j = 0$	$i \neq j$
$A_r \cdot B_s = \langle A_rB_s \rangle_{\lvert r-s \rvert}$	r,s>0; r,s=0 implies *scalar*

where A_rB_s is $\mathcal{G}$'s product, and the subscript on the pointy brackets instructs us to accept only a term generated by this product whose grade is exactly $|r-s|$; if there is no such, then $A_r \cdot B_s = 0$.

The · operator is *not* associative, but does satisfy:

Commutativity: $A_r \cdot B_s = (-1)^{r(s-1)} B_s \cdot A_r$ when $r \leq s$ ([6])

Distributivity: $A \cdot (B+C) = A \cdot B + A \cdot C$ ([7])

Now calculate

$$(a_1+a_2+a_3+...+a_n)\cdot(a_1a_2a_3...a_n) = a_1\cdot a_1a_2a_3...a_n + a_2\cdot a_1a_2a_3...a_n + a_3\cdot a_1a_2a_3...a_n +...+ a_n\cdot a_1a_2a_3...a_n$$
$$= a_2a_3...a_n - a_1a_3...a_n + a_1a_2a_4...a_n - ... \pm a_1a_2a_3...a_{n-1}$$

where the ± is resolved depending on whether n is odd (+) or even (-). Notice that this result is *exactly the same* as that for ∂ (cf. eqn. 1), but without all the fussy little rules about alternating signs while moving ∂.

Let us now calculate

$$(a_1 + a_2 + a_3 +...+ a_n)\cdot(a_1 + a_2 + a_3 +...+ a_n) = a_1\cdot a_1 + a_2\cdot a_2 +...+a_n\cdot a_n$$
$$= 1 + 1 +...+ 1$$
$$= n \; mod \; 3 \quad (3)$$

---

[6] $B_s \cdot A_r$ is of course allowed: move the sign-term to the other side.

[7] $(B+C) \cdot A$ can be derived, provided one adheres to the preceding footnote.

and compare the result to equation 2 above: they are also identical.

Let therefore $Y = a_1 + a_2 + a_3 + \ldots + a_n$. This shows that we can *replace* the notation ∂X with the notation $\partial_Y X = Y \cdot X$. In effect, we have "unloaded" the detailed operation of ∂ into the $\cdot$ operator, and can now think of what remains of ∂ as being merely one of $\cdot$'s operands. The preceding calculation shows us that these remains are $a_1 + a_2 + a_3 + \ldots + a_n$, namely Y.

We now introduce an important notational convention: $\partial X \equiv \partial(\{X\})$ will always denote the traditional homological definition of ∂. In contrast, $\partial(\{X,Y\})$ denotes the above dot-based equivalent, namely

Definition. $\partial(\{X,Y\}) \equiv \partial_Y X \equiv Y \cdot X$.

$\partial\partial X$ will denote either $\partial(\{\partial(\{X\})\})$ or $\partial(\{\partial(\{X,Y\}),Y\})$, according to context. Finally, this change in notation reflects a change in point of view that allows us to say "the boundary of X *with respect to* Y", where the traditional transliteration "the boundary of X" has nothing to say on this relative aspect of ∂'s operation.

The effect of introducing $\cdot$ is to replace the homological view of ∂ as an operat<u>or</u> existing *outside* $\mathcal{G}$ with that of an opera<u>tion</u> *within* $\mathcal{G}$. Both points of view are valid; in the following, we take the latter.

<u>Expand the meaning of ∂</u>

The expansion of ∂ is now at hand: simply allow Y to be *any expression at all*, and not restrict it to always being $a_1+a_2+a_3+\ldots+a_n$. That is, we will want to set Y to be one half (say C) of some co-exclusion C | -C . To do this, re-express $\partial(\{X\})$ as $\partial(\{X,Y\})$, and then allow *any* Y. Since Y (and X) will often be sums, we can exploit the distributivity of $\cdot$ over + to write

Theorem (*∂-linearity*). $\partial(\{X,P+Q\}) = (P+Q) \cdot X = P \cdot X + Q \cdot X$ and
$\partial(\{W+Z,Y\}) = Y \cdot (W+Z) = Y \cdot W + Y \cdot Z$

This concludes our definition of $\partial(\{X,Y\})$.

<u>Check if $\partial\partial=0$</u>

We now check if $\partial\partial=0$. In our new notation this means that X=Y, that is, $\partial(\{X,Y\}) = \partial(\{Y,Y\})$, so $\partial\partial$ is written

$$\partial\partial = \partial(\{\partial(\{Y,Y\}),Y\}) = Y\cdot(Y\cdot Y)$$

Recalling equations 2 and 3:

$$\begin{aligned}\partial(\{Y,Y\}) = Y\cdot Y &= (a_1 + a_2 + a_3 + ... + a_n)\cdot(a_1 + a_2 + a_3 + ... + a_n) \\ &= a_1\cdot a_1 + a_2\cdot a_2 + ... + a_n\cdot a_n \\ &= n \textit{ mod } 3\end{aligned}$$

The aforementioned special property of ∂ appears when we do it a *second* time:

$$\begin{aligned}\partial\partial &= \partial(\{\partial(\{Y,Y\}), Y\}) = Y\cdot(Y\cdot Y) \\ &= (a_1 + a_2 + ... + a_n)\cdot[(a_1 + a_2 + ... + a_n)\cdot(a_1 + a_2 + ... + a_n)] \\ &= (a_1 + a_2 + ... + a_n)\cdot[\textit{scalar}] \\ &= 0\end{aligned}$$

This example is important because this particular value for Y both duplicates and explicates what the traditional $\partial(\partial)$ means, in terms of $\cdot$. But what about other values of Y?

Theorem (*Disjoint*). $\partial\partial = 0$ for any $Y = (Y_1+Y_2+...+Y_m)$, each Y_i a singleton, and for all $i,j \le m$, $i \ne j$, $Y_i \cap Y_j = \varphi$, ie. the Y_i don't overlap.

Proof. Recall the definition of $\cdot$ between two singletons: $A_r \cdot B_s = \langle A_r B_s \rangle_{|r-s|}$ where $r,s > 0$. Since when $i \ne j$, $Y_i \cap Y_j = \varphi$, the grade of their product will be $r+s \ne |r-s|$, and so such 'cross products' will always yield zero. Products when i=j will yield ± 1, and $Y\cdot(\pm 1) = 0$.

Disjoint thus excludes co-occurrences like ab+abc, where $\partial\partial = 1+c$, and subsequent applications of ∂ produce, successively: ab, -1-c, 1+c. That is, not only is $\partial\partial \ne 0$, it also *loops*; we examine this later.

Theorem (*Un0*). $\partial\partial = 0$ for any singleton Y.

Proof. For any singleton, $Y\cdot Y = \pm 1$, so $Y\cdot(Y\cdot Y) = \pm Y\cdot 1 = 0$.

Theorem (*ZapOnes*). $X\cdot Y = (1+X)\cdot Y = X\cdot(1+Y) = (1+X)\cdot(1+Y)$.

Proof: Follows immediately from the fact that $X\cdot 1 = 1\cdot Y = 0$.

For any Y,
Corollary 1: If Y·(Y·Y) = 0, then (1+Y)·[(1+Y)·(1+Y)] = 0 as well.
Corollary 2: If Y·(Y·Y) loops, then so does (1+Y)·[(1+Y)·(1+Y)].

Non-theorem. For any Y, if Y·(Y·Y) = 0 then Y·Y is a scalar.
Counter-example: Y = ab+abc+abcd, so Y·Y = -1+c+cd, and yet Y·(Y·Y) = 0.

Non-Theorem. For any Y≠0, if Y·(Y·Y) loops, then so does (X+Y)·[(X+Y)·(X+Y)], where X≠0 is any expression not containing terms of Y. *Counter-example*: same.

Conjecture. For any Y, either ∂∂ = 0 or else ∂∂ loops.
We prove this as our penultimate theorem.

We can conclude for the moment that ∂∂ = Y·(Y·Y) = 0 holds for the cases of greatest importance, namely those covered by *Disjoint, UnO* and *ZapOnes'* first corollary. In particular note that the traditional formulation's Y = $a_1 + a_2 + ... + a_n$ exactly satisfies *Disjoint's* criterion.[8] *We therefore restrict co-exclusion to these cases*, and those (eg. ab+ac+bc, see below) that are otherwise demonstrably well-behaved, so we are assured that ∂∂ is *always* 0. We call these cases *legal*. This is good news, and allows us to proceed, under this restriction, with the examination of our algebra. *We will therefore, from this point, always take 'boundary' and ' ∂ ' to denote our expanded version of ∂ unless stated otherwise.*

<u>*Define δ via ∂*</u>

The next part of our program is to use our expanded ∂ to define co-exclusion as the (inside-the-algebra) co-boundary operator δ.

We are given the two complementary co-occurrences C and -C and wish to define their co-exclusion X. The key to defining X is to observe that the boundaries of X and -X must be the respective two co-occurrences of the co-exclusion, since they are what we want X (resp. -X) to be built out of. That is, C and -C are to be the boundaries of X and -X respectively *by construction*. We therefore conclude:

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[8] Which congruence can be taken as testimony for our expanded definition of ∂, and thus ultimately for the validity of the co-exclusion principle.

Definition (co-ex1). C co-excludes to X iff $\partial(\{X,C\}) = \partial_C X \cong C$.

[Important Note: for our present purposes, the boundary of X with respect to A, $\partial_A X$, equals B *whenever* A *and* B *contain the same terms, though perhaps differing in their signs; we denote this where relevant with* $\cong$.]

The next step is to see that, like the traditional ∂, the traditional δ *also* contains an implicit parameter, namely the (unique) top element of the (sub-)algebra specified by C. One can see this from the point of view that while ∂ *decreases* the grade of its operand, δ as the inverse of ∂ *increases* the grade beyond that of its operand, and the top element is that with the highest achievable grade.

We will denote this top element by Q, and define it as follows: let C be some expression in $\mathcal{G}$, and let $\{a_{J1}, a_{J2}, ..., a_{Jm}\}$, $a_{Ji} \neq a_{Jk}$, be all of the unique grade 1 elements of $\mathcal{A}$ that occur in C. Then $Q = a_{J1}a_{J2}...a_{Jm}$ is the *top element* of the smallest sub-algebra of $\mathcal{G}$ containing C. Given now Q as the top element, we can write

Definition. Co-Boundary (cob).

$$\delta(\{X\}) = \delta_Q X \equiv \delta(\{X,Q\}) = \begin{cases} \pm 1 & \text{if } X = Q \text{, ie. } Q \cdot X \\ Q & \text{if } \partial_Q X = Q \cdot X \cong X \\ Q \cdot X & \text{otherwise} \end{cases}$$

Note that while ∂_Y allows many possible relative boundaries, there is only *one* relative (ie. implicit) *co*-boundary, namely Q. [9]

It follows from *cob's* definition of δ in terms of $\cdot$ that $\delta(\{0\}) = \delta(\{1\}) = 0$. We now unpack *cob* into usable form.

Theorem. ***(co-ex2).*** X is the co-exclusion of C iff $\delta(\{X,Q\}) = \pm 1$.
Proof. Trivial from definition *cob*.

Theorem. X is the co-exclusion of C iff $\partial(\{\delta(\{C,X\}),C\}) \cong C$.
Proof.
a. $\partial(\{\delta(\{C,X\}),C\}) \cong C$ implies X is the co-exclusion of C: Let $W = \delta(\{C,X\})$ whence $\partial(\{W,C\}) \cong C$ and thus W=X by definition *co-ex1*.
b. X is the co-exclusion of C implies $\partial(\{\delta(\{C,X\}),C\}) \cong C$: By definition *co-ex1*, $\partial(\{X,C\}) \cong C$; and by theorem *co-ex2* and definition *cob*, $\delta(\{C,X\}) \cong X$. Substituting

[9] Unless one counts $\pm 1+Q$, cf. *ZapOnes*, in that δ is defined via ∂.

the latter in the former, $\partial(\{\delta(\{C,X\}),C\}) \cong C$.

Theorem. $\partial(\{\delta(\{C,X\}),C\}) \cong C$ iff $X = Q$.
Proof. Let $W = \delta(\{C,X\})$, then $\partial(\{W,C\}) \cong C$, hence, by definition *co-ex1*, C is the co-exclusion of W. Then theorem *co-ex2* implies that $\delta(\{C,W\}) = \pm 1$, which via definition *del* implies that C=W, that is, X=Q. The X=Q implication is trivial.

Theorem (*co-ex3*). X is the co-exclusion of C iff X = Q.
Proof. Directly from the preceding two theorems; also trivially from *cob*.

The critical property of δ, corresponding to $\partial\partial = 0$, is that $\delta\delta = 0$. We can check this easily:

$$\delta\delta = \delta(\delta) = \delta(\{\delta(\{Q,Q\}),Q\}) = Q\cdot(Q\cdot Q) = 0$$

due to *Un0*: Q·Q is a scalar because Q is by definition a singleton, and Q·(*scalar*)= 0. With this established, we can now write out all four of the combinations of the ∂ and δ operations:

$$\partial\partial = C\cdot(C\cdot C) = 0$$
$$\delta\delta = Q\cdot(Q\cdot Q) = 0$$

To calculate $\partial\delta$ and $\delta\partial$, we apply the *legality* criterion that $\partial(\{Q,C\}) \cong C$ whence $\delta(\{C,Q\}) = Q$, since these are the only values of C that we allow:[10]

$$\partial\delta = \partial(\{\delta(\{C,Q\}),C\}) = C\cdot(\delta(\{C,Q\}) = C\cdot(Q) \cong C$$
$$\delta\partial = \delta(\{\partial(\{Q,C\}),Q\}) = \delta(\{\partial(\{Q,C\}),Q\}) = Q\cdot(C) \cong C$$

which reveal $\partial\delta$ and $\delta\partial$ as identity operations on legal values of C. It is in this sense that one can say $\partial\delta = \delta\partial = 1$, ie. they are pseudo-inverses.

Properties of co-exclusion.

Theorem (δ-*linearity*). $\delta(\{C+C'\}) = \delta(\{C\}) + \delta(\{C'\})$.
Proof. Invoking the distributivity of · over +, Q·(C+C') = Q·C + Q·C' ; now apply definition *cob*.

[10] Includes, indirectly, C's like a+b+c and ab+ac+bc, cf. the *Chain Rule*, below.

Corollary. $\delta(\{1+C\}) = \delta(\{C\})$.

From the discussion so far, one might think that $\delta(\{C,Q\}) = Q$ always, but there is no guarantee of this being so: Q is in fact virtually always *given* in the theorems above. In this connection, we state

Theorem ***(Chain Rule)***. $\partial(\{Q,C\}) = C'$ and $\partial(\{Q,C'\}) = C$ iff Q is the co-exclusion of C+C'.

Proof. By *∂-linearity* and adding the two premises together,

$$\partial(\{Q,C\}) + \partial(\{Q,C'\}) = \partial(\{Q, C+C'\}) = C + C'$$

the right-most two expressions of which satisfy definition *co-exl* for Q to be the co-exclusion of C+C' ; the 'if-and-only-if' follows according to which direction one reads the above equations. If $C = C'$ this reduces to $\partial(\{Q,C\}) \cong C$.

Example. Let $C = a+b+c$ and $C' = ab+ac+bc$. Then abc is the co-exclusion of $a+b+c+ab+ac+bc$. Note incidently that C, C', and C+C' are nilpotent.

The next theorem establishes a very useful special case.

Theorem (*Qdot*). For any $X = \Sigma_r \langle X\rangle_{>0}$, $Q_r X = Q_r{\cdot}X$ and $XQ_r = X{\cdot}Q_r$

Proof. By the construction of Q_r (from X), every term of X has *all* of its factors in common with Q_r and no term with grade >r. Therefore we can write $X = \langle X\rangle_1 +...+ \langle X\rangle_r$, where $\langle X\rangle_i$ is the sum of all grade i terms of X. By distributivity, $Q_r X = \Sigma_{i=1}{}^r Q_r\langle X\rangle_i$; and by definition, and excluding $\langle X\rangle_0$ by hypothesis, $Q_r{\cdot}X = \Sigma_{i=1}{}^r \langle Q_r\langle X\rangle_i\rangle_{|r-i|}$. Since each $X_i \in \langle X\rangle_i$ shares all its factors with Q_r , $Q_r\langle X\rangle_i$ *must* have grade r-i, so the outermost ⟨ ⟩'s are redundant, thus $Q_r X=Q_r{\cdot}X$. Same argument for XQ_r .

This allows us to accompany the *Chain Rule* with

Theorem. For all Q_q , q>1, there exist X such that $X \cong Q_q{\cdot}X$.

Proof. Let $Q_q = Q_r Q_s$, where Q_r and Q_s connote only grade: they share *no* grade 1 elements. Let $X = Q_r + Q_s$ and observe that $Q_q Q_r \cong Q_s$ and $Q_q Q_s \cong Q_r$ by construction. Adding these, $Q_r + Q_s \cong Q_q(Q_r + Q_s)$. Now apply *Qdot*.

Since $Q_r{\cdot}X \cong X$ is the context in which we will most often be using *Qdot*, it is well to note that, for $X=-1\pm Z$, $Q_r{\cdot}(-1\pm Z) = \pm Q_r{\cdot}Z$, so the restriction $X = \Sigma \langle X\rangle_{>0}$ is violated in this case. With this in hand, we turn to an interesting side-view of matters.

Since $\mathcal{G}$ is over Z_3 , the powers X^n of *arbitrary expressions* X can only yield one of four possibilities: ±1, 0, or a loop. We examine loops first.

We encountered loops first in the context of $\partial\partial = X\cdot(X\cdot X)$, but our interest in ∂ here is in the context of meeting the co-exclusion criterion $X = Q\cdot X$. We will approach from the direction of *Qdot*, in that we delay the transition from X=QX to $X=Q\cdot X$.

Definition. Let $X \neq \langle X\rangle_0$ be any expression in $\mathcal{G}$. Then a *loop* is the sequence $[X] = [X = X_0, ..., X_i = YX_{i-1} ..., X_{n-1}]$, Y being some non-scalar, followed by $X_n = X$, where there is no j<n such that $X_j = \pm X$, ie. a loop is *minimal.*[11] We say that an X *generates* a loop.

Note that we can generalize $X_i = YX_{i-1}$ to get $X_{n-i} = YX_{n-i-1}$.

Ordinary idempotents are loops where n=1 and $Y=X_0$: $X_{n-1} = X_0$, so the loop is X_0, $X_1 = X_0 X_0 = X_0$, ie. trivial. [It is worth noting that it can be shown that if P (not nilpotent) has no inverse, then P contains an idempotent as a factor, whence irreversibility becomes associated with idempotence.]

Theorem. All loops in $\mathcal{G}_m$ are finite.

Proof. Excluding zero (see below), there are 2^m-1 elements in $\mathcal{G}_m$, and each can be ±, leading to a total $2(2^m -1)$ expressions in $\mathcal{G}_m$.. Hence no loop can be longer than $2(2^m -1)$.

Theorem (*QloopsCoEx*). All loops $[X \mid Y=Q_X]$ co-exclude to Q_X.

Proof. Since a loop is finite, there exists an X_k in the loop [X] such that $X_k = X_0$. Adding the equations $X_i = YX_{i-1}$ therefore yields for any Y

$$X_0 + X_1 +...+ X_{k-1} = YX_0 + YX_1 +...+ YX_{k-1} = Y(X_0 + X_1 +...+ X_{k-1})$$

Noting that either X_i satisfies *Qdot* or X_i contains a ±1,[12] let $X_0 = \pm 1+Z$, $Z = \Sigma \langle Z\rangle_{>0}$. Substituting, and separating out the 1's from the various Z-terms, we get (writing *s* for the Σ1's)

$$\Sigma_{i=0}^{k-1}\mathbf{1}_i + \langle Z\rangle_1 +...+ \langle Z\rangle_{k-1} = Y(s + \langle Z\rangle_1 +...+ \langle Z\rangle_{k-1}) = sY + Y(\langle Z\rangle_1 +...+ \langle Z\rangle_{k-1}) \quad (^{13})$$

whence *Qdot* implies $X_0 + X_1 +...+ X_{k-1} = sY + Y\cdot(\langle Z\rangle_1 +...+ \langle Z\rangle_{k-1})$. The $Y\cdot()$ expression rightmost can produce a scalar, so we cannot draw any conclusions about X_0. If $s = 0$ then

$$X_0 + X_1 +...+ X_{k-1} = \langle Z\rangle_1 +...+ \langle Z\rangle_{k-1} = Y\cdot(\langle Z\rangle_1 +...+ \langle Z\rangle_{k-1})$$

[11] NB: The subscripts here denote ordering, *not* grade.

[12] If zero, simply apply *Qdot* to the equation just above and the result follows.

[13] The ⟨⟩ -notation is inadequate, but Z's details are not important, as follows.

so when $s = 0$ and $Y=Q_X$, [X] co-excludes to Q_X. Suppose then that $s = \pm 1$, whence (rearranging from earlier) and setting $Y=Q_X$ we get

$$X_0 + X_1 + ... + X_{k-1} - sQ_X = Q_X \cdot (\langle Z \rangle_1 + ... + \langle Z \rangle_{k-1})$$

The right side, being the grade-reducing · operator, cannot yield a term Q_X, which has maximal grade. Therefore the left side cannot either, so it must be that $s \neq \pm 1$. Therefore, when $Y = Q_X$ all loops *eventually* co-exclude after k-1 steps.

Closer examination of this result reveals, at least for $Q_X = Q_2$, that Q_X, viewed as a spinor, is bookkeeping the 'orientation' of the 'rotation' through states, which state transitions are the classical[14] -a+b ↔ a-b ones *only*, the superpositions a+b ↔ -a-b *not* appearing. Howsoever, the case Y=Q contains many (all?) n>1 idempotents, and is clearly the one of interest here. In fact, since we can always write Z=QY, whence $Y=Q^{-1}Z$, the Y=Q case is the *only* case.

Lemma. X_i and X_j, $0 \leq i,j \leq n-1$, generate identical loops.
Proof. Match the elements of loops X_i and X_j one for one, in order.

This allows us to replace any loop with any other congruent to it.

Theorem. A loop forms a modular arithmetic group ({X}, juxta) of order n-1.
Proof. Let the identity element e be $X_0 = X_n$, whence $X_0X_0 = X_0$. Then the inverse of any X_i is X_{n-i-1}, as follows. Using the identity $X_i = YX_{i-1}$ rewrite the claim, for any i, that $X_iX_{n-i-1} = e$ as X_iX_{n-i-1} . By the lemma, we can replace the loop X_i beginning at i with the loop X_0 beginning at i=0, yielding $X_iX_{n-i-1} = X_0X_{n-1} = X_0$.

Since the X's combine according to the laws of (mod n-1) exponent arithmetic, the product X_iX_j is both commutative and associative and *we will henceforth feel free to write the subscripts as exponents.* Closure is via the fact that in modular arithmetic, any exponent k will be $0 \leq k \leq n-1$. Note that this characterization does not yield a method for predicting *which* X's will loop and which will not.

Theorem (*NoNilCycles*). No element of a loop is nilpotent.
Proof. A 0 simply collapses the loop, and so cannot appear.

We now examine nilpotents.

Theorem. No nilpotent contains ±1 as a term.
Proof. Let XX=0, X≠ 0; we must show that X is not of the form ±1+Z, where $Z = \Sigma \langle Z \rangle_{>0}$. So suppose that X = 1+Z, so XX = (1+Z)(1+Z) = 1- Z+ZZ = 0, whence ZZ-Z = -1, which requires that ZZ = -1+Z , whence Z must be idempotent (or its

[14] Classical because -a+b excludes a-b, which <u>is</u> sequence, ie. sequentiality.

square root = its negative), because all idempotents/roots are of the form $\pm 1 \pm Z$, which violates the premise for Z. Same for $X = -1+Z$.

Corollary. Nilpotents satisfy *Qdot*.

Corollary. For any $Y=QX$, X nilpotent, then $XY = YX = YY = 0$.
Proof. Right-multiply by X => YX=0; left-multiply for XY=0; right-multiply by Y for YY=0.

Corollary. For nilpotents $XX=YY=0$, $(X+Y)^2 = 0$.
Proof. $(X+Y)^2 = XX+XY+YX+YY=0$.

Theorem (*NilsCoEx*). All nilpotents co-exclude.
Proof. For any $XX=0$ we can write $Y=QX$ whence $X = Q^{-1}Y \cong QY$. The preceding corollaries guarantee that Y is nilpotent and hence does not contain ± 1; and that X+Y is nilpotent. Adding $Y=QX$ and $X \cong QY$ and noting that nilpotents satisfy *Qdot*, we get $X+Y \cong Q \cdot (X+Y)$.

Underlying this proof is the syllogism $Y = QX \Rightarrow X \cong QY \Rightarrow X+Y \cong Q(X+Y)$. Note that this cannot be done with $Y=Q \cdot X$ unless X satisfies *Qdot* because $\cdot$ is not associative. We expand on this later.

Having established the properties of loops and nilpotents, we turn to the roots of unity. Our overall aim is to connect co-exclusion to these roots.

For $X^n = \pm 1$, $n>0$, note first that $n=1$ implies that $X = Q_0 = 1$, and $n=2$ implies that X is self-inverse, eg. a, ab, abc, ab+bc, ...[15] (X is obviously not nilpotent.)

Theorem. If for $X \neq \pm 1$, $X^n = \pm 1$, then $X^n = \pm Q^{2/n}$.
Proof. $X^n = \pm 1 \Rightarrow \pm Q_r = Q_r X^n \Rightarrow \pm Q^{-1}Q = X^n \Rightarrow \pm(-1)^{r(r-1)/2}Q^2 = X^n \Rightarrow$

$$X = \pm((-1)^{r(r-1)/2}Q^2)^{1/n} = \pm(-1)^{r(r-1)/2n}Q^{2/n} = \pm Q^{2/n}$$

So the case of interest is the roots of Q.

Theorem (*Qroots*). All Q_q have at least one non-scalar square root, $q > 2$.
Proof. Let $Q_q = Q_r Q_s$ where r,s connote only grade: Q_r and Q_s share *no* grade 1 elements. Then $(Q_r \pm Q_s)^2 = Q_r^2 + Q_r Q_s + Q_s Q_r + Q_s^2$, which can take the values $\pm Q_q$, $\pm 1 \pm Q_q$, ± 1, or zero. Since we can always take r=0 and s=q, there will always exist the roots $\pm(1-Q_q)$, but this is only necessary when q=2 because for q>2, there exist combinations of r+s = q that produce more substantial combinations, as follows. For q=3, $(a-bc)^2 = (b-ac)^2 = (c-ab)^2 = abc$. If either (or both) of r and s is (are) even, then

[15] In fact, generally, $X^{-1} = X^{n-1}$.

$Q_rQ_s \cong Q_sQ_r \cong Q_q$. If both are odd then q is even, which means that for even q>3, we can set r=s, whence the signs of Q_rQ_s and Q_sQ_r are assuredly the same, whence there always exists a non-scalar-producing formulation of $Q_r + Q_s$. Note the echo of theorem *Disjoint*.

Corollary (Qn2). The cases $X^n = Q$ are all cases of n=2: $(Q_r + Q_s)^2$.

Now, all of the various roots Y, Z of Q's share some common set of grade 1 elements from $\mathcal{G}$, so the only thing that varies between different roots is the composition and grade r of their respective $Q = Q_r$'s. So for arbitrary $Y^i = Q_r$ and $Z^j = Q_s$ we can write

$$Y^iZ^j = Q_rQ_s = Q_q$$

Since every X^n has an inverse $\pm X^{2n-1}$ (see lemma below) and the preceding shows closure, we need only the associativity of exponent arithmetic to conclude that the roots of various Q's form a group:

Theorem. The set $\{X \mid X^n=Q_q ; q,n>0\}$ forms the group ({X}, juxta).
Proof. The inverse of any element X is $X^{-1} = \pm X^{2n-1}$ and always exists; the identity element is $XX^{-1} = 1$; closure follows from $Y^m Z^n = Q_q$ above, and associativity from exponent arithmetic.

Lemma. For any X such that $X^n = Q$, n>1:

*L*a. $X^{-1} = \pm X^{2n-1}$ *Proof.* $X^n=Q \Rightarrow X^{2n}=X(X^{2n-1})=\pm 1 \Rightarrow X^{-1}=\pm X^{2n-1}$
*L*b. $X = QX^{1-n}$ *Proof.* Right-multiply $X^n = Q$ by X^{1-n}
*L*c. $X^{-1} = \pm QX^{n-1}$ *Proof.* Rewrite (a): $X^{2n-1} = X^nX^{n-1} = QX^{n-1}$
*L*d. $QXQ^{-1} = X$ *Proof.* w/ $Q^{-1}=X^{-n}$, rewrite (b): $QX^{1-n} = QXX^{-n}$
*L*e. $QX = XQ$ *Proof.* Right-multiply (d) by Q
*L*f. $XQX^{-1} = Q$ *Proof.* Right-multiply (e) by X^{-1}

Note that because X is well-defined, so is X^{2n-1}, thus X^{-1} is always defined. Since any given root X is finite (ie. is some expression in $\mathcal{G}$) and the grade r of its co-exclusion $Q = Q_r$ therefore likewise, so is $X^{-1} = X^{2n-1}$ -just multiply X by itself 2n-1 times. This same computation determines if X is either nilpotent or idempotent.

Theorem (*CoExRoots1*). For all roots X of Q: $X^n = Q$, n>0, $X+X^{-1}$ co-excludes to Q, ie. $X+X^{-1} \cong Q\cdot(X+X^{-1})$.
Proof. For n=1, $X^1 = Q = Q_1$, a trivial case. Add *Lb* and *Lc* above, yielding for *any* root X

$$X+X^{-1} \cong QX^{1-n} + QX^{n-1} = Q(X^{1-n} + X^{n-1}) = Q\cdot(X^{1-n} + X^{n-1})$$

which satisfies *cob* (by equating exponents) for precisely n=2. And by corollary *Qn2* we need not consider other n.

The converse is not true because X can be nilpotent (eg. a+b+c+ab+ ac+bc co-excludes to abc). Note that when X is self-inverse, the sum $X+X^{-1}$ simplifies to $X \cong Q{\cdot}X$.

Theorem (*CoExRoots2*). For all roots X of Q: $X^n = Q$, n>0, if $X_r = X^i = Q_r$ and $X_s = X^j = Q_s$ co-exclude, then X_rX_s co-excludes to Q_q.[16]

Proof. The preceding theorem allows us to conclude that for n=1,2, X_r and X_s individually co-exclude. We now show that X_rX_s co-excludes by continuing the preceding proof:

$$X_rX_s + (X_rX_s)^{-1} \cong Q_q(\,(X_rX_s)^{1-n} + (X_rX_s)^{n-1}) = Q_q(X_r^{1-n}X_s^{1-n} + X_r^{n-1}X_s^{n-1})$$

For the trivial n=1 case, $X_rX_s = Q_q$. For n=2, substituting in the right-most:

$$X_rX_s + (X_rX_s)^{-1} \cong Q_q(X_rX_s + (X_rX_s)^{-1})$$

Theorem. All roots of unity are ultimately co-exclusions.

Proof. $X^n=1$ implies that there exists, underneath, a Q such that $X=Q^{2/n}$. Theorem *CoExRoots1* guarantees the existence of Y such that $Y = Q{\cdot}Y$.

Theorem. All co-exclusions are roots of unity.

Proof. Every co-exclusion produces a Q, and $Q^2 = \pm 1$ (square again if -1).

Note that nilpotents like a+b+c+ab+ac+bc (co-excludes to abc), under *cob*, appear as roots of unity; as do Q-loop elements.

Theorem. The roots of unity $\{Q \mid Q^2=1 \text{ or } Q^4=1\}$ form the group ({Q}, juxta).

Proof. Let *e* = 1. All Q^{2k} 's are self-inverse. Closure and associativity via $\mathcal{G}$.

All the above culminates in our penultimate theorem:

Theorem (*Coverage*). Either the *Chain Rule* yields a co-exclusion within two steps, or the calculation forms a non-trivial loop.

Proof. Since both Q-loops and *Chain Rule* calculations have finite length, we can write both together as the *finite* sequence $Q{\cdot}X_1 = X_2$, $Q{\cdot}X_2 = X_3$, ... where each $X_{i+1} = Q{\cdot}X_i$. Because the sequence is finite, there exists an X_k in the finite set $\{X_i\}$ such that $X_k = X_1$, and so adding the equations $X_{i+1} = Q{\cdot}X_i$ we get

[16] NB: Superscripts denote exponents, Q-subscripts *grade* (and *not* membership), the X-subscripts echoing their corresponding Q.

$$X_1 + X_2 + ... + X_{k-1} \cong Q \cdot X_1 + Q \cdot X_2 + ... + Q \cdot X_{k-1} = Q \cdot (X_1 + X_2 + ... + X_{k-1})$$

which satisfies the definition of co-exclusion. So, as theorem *QloopsCoEx* showed in the same way, even a loop [X] eventually co-excludes after k-1 steps. On the other hand, substituting $Q \cdot X_i$ successively for X_{i+1} in the *sequence* yields

$$X_2 \quad X_3 \quad X_4 \qquad X_2 \quad X_3 \cong X_1 \quad X_4 \cong X_2 \quad X_1 \, ...$$

$$Q \cdot X_1 \, , Q \cdot (Q \cdot X_1) \, , Q \cdot (Q \cdot (Q \cdot X_1)) \, , ... = Q \cdot X_1 \, , \ \pm X_1 \, , \ \pm Q \cdot X_1 \, , \ X_1 \, , ...$$

↑

whence it is seen, by invoking δ-linearity and thence theorem *Qdot* to do the simplification, that we get the desired congruence after the second step.

The difference between the two equations in the proof is the difference between Q·X and QX as embodied in theorem *Qdot*. This is most clearly seen from theorem *Qroots* and its corollary, which establishes that all Q's have non-trivial square roots, whence XX=Q, and given that X^{-1} exists, then $X = QX^{-1} \Rightarrow X + X^{-1} = Q(X + X^{-1})$. On the other hand, many (all?) loop elements (cf. idempotence) have the form $X = (1 \pm Z + ZZ)^k$ where $Z^2 = 1$, and exactly these scalars are the elements of $\mathcal{G}$ that are eliminated via *Qdot*, which in effect only treats the Z case (we recover ZZ by its being self-inverse, and thus ensure co-exclusion), which in turn is the 'n=2' criterion in another guise.[17]

Our final theorem wraps it all up in a neat package:

Theorem. X co-excludes ($\delta_Q X = Q$) iff $\partial_X \partial_X X = 0$.

Proof. a. $\delta_Q X = Q$ implies $\partial_X \partial_X X = 0$: Since X co-excludes, $\partial_X Q = Q \cdot X = \underline{X} \cong X$, where $\underline{X}$ is conjugate to X due to the action of Q, whence $X \cdot \underline{X} = s$, a scalar. Multiplying both sides of $Q \cdot X \cong X$ by $\partial_X \partial_X$ yields $\partial_X \partial_X (Q \cdot X) \cong \partial_X \partial_X X$; whereas expanding just $\partial_X \partial_X (Q \cdot X)$ gives $\partial_X \partial_X (Q \cdot X) = \partial_X \partial_X (\underline{X}) = \partial_X (s) = s \cdot X = 0$.

b. $\partial_X \partial_X X = 0$ implies $\delta_Q X = Q$: $\partial_X \partial_X X = 0$ means that there is no loop, and *Coverage* then implies that X co-excludes to Q.

We can conclude from theorem *Coverage* that there is a very simple algorithm (*is* $X_3 \cong X_1$ *?*) for determining *either* the co-exclusion of some X and therefore that $\partial\partial = 0$ *or* the fact that that co-exclusion doesn't exist.[18]

This is interesting, because given the idempotent nature of loops, which implies irreversibility (a.k.a sequentiality), one is reminded of the undecideability of the Halting Problem for Turing machines: there exists no sequential algorithm (includes "parallelism") that can decide if a given

[17] Also, QM's unitarity criterion and amplitude-squaring (cf. Pythagoras' thm).

[18] That is, under the restriction that $\partial\partial = 0$.

program, the latter's definition provided as the former's input, will halt when executed on given input. If we allow that (segments of *long*) loops model sequential computation - not a big assumption - then we have shown that a phase web can solve the Halting Problem (either it loops; or $X_3 \cong X_1$ and $\partial\partial=0$, so it can only halt), not surprising, given the close connection with quantum mechanics.

The poisonous consequence that one will first discover a loop when it occurs is therefore also avoided.[19] In the present context, the loops themselves appear to be singularities, and their further analysis might throw light on such interesting matters. Even though homology requires $\partial\partial=0$, it appears that Nature wastes nothing: even loops [read: classical processes] display symmetry.

Howsoever, since loops generalize idempotents, we have three cases: nilpotents, loops, and co-exclusions. Furthermore, theorem *NoNilCycles* establishes that loops cannot contain nilpotents and the *Coverage* and *NilsCoEx* theorems both establish that nilpotents also co-exclude. So the three cases become the two of theorem *Coverage*, those expressions X in $\mathcal{G}$ that co-exclude in two steps (whence $\partial\partial=0$) and those that loop. We have shown that both of these form (distinct) multiplicative groups, whose respective multiplicative operations connect them directly to the calculation of δ, and expressing the same distinction.

Because they can co-exclude, nilpotents can form structures by themselves (bosonic matter). They serve individually both to connect the reversible space-like unitary world (pun intended) to Void, and to connect Void to time (in that they have no inverse); whereas idempotents connect the space-like to the time-like, and longer loops represent irreversible sequential processes in a classical world.

Summary and Conclusions.

In this paper we have formally identified boundary and coboundary operators that square to zero in the phase web theory, so our system forms something like a "sufficient category for a homology theory". In fact strictly speaking there is a potentially unbounded number of homology theories, one for each legal expression X in arbitrary $\mathcal{G}$. This could

[19] This brings to mind David McGoveran's observation that if the length of the loop exceeds the memory capacity of the system in question, then the loop *cannot* be detected from *within* said system.

possibly be interpreted as some kind of relative homology theory, but that is for another paper.

What we need to do now is look at the homology of our system under these boundary operators, that is, understand how different instances or instantiations or implementations of Topsy[20] are classified as equivalent, or otherwise, by their homology groups. Further, this emergence of homology out of our abstract theory can be interpreted as space-like properties arising out of the purely mind-like process theory that is Topsy. This may be related to the homology groups of the Clifford Algebras themselves.

Howsoever, to the best of my knowledge, the preceding mathematics expresses as precisely as I am able what I mean by co-exclusion, the hierarchy it produces, and some of the latter's engaging properties. Given that the hierarchy describes the distinctions that can be constructed on the basis of a primitive "time-like" vs. "space-like" distinction (ie. exclusion vs. co-occurrence), the categorization that it yields into, precisely, light-like (nilpotents), reversible (unitary space-like), and irreversible (classically, and probably relativistically) time-like[21] processes, all within a unified framework, is very encouraging.

In closing, I would like to express my gratitude to my long-suffering guide on the path to homological enlightenment, Keith Bowden, for his persistence regarding rigor; any lapses are assuredly mine.

[20] Topsy is the self-organizing 'strong AI' program that inspired all of this. No refs just yet.

[21] In that since $\mathcal{G}$ has a representation of the symmetric group, the discrete Lorentz group, if such exists, must be a sub-group thereof [Cayley] (tho with potential loss of substructure). The same applies to $U(1)\times U(2)\times SU(3)$.

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Feb 2004.

Dear Keith, David, Tom, Suzanne, and Fred,

Here is the completed document. Thank you all for your cooperation in preparing this report. It also has reminded me, once again, of how much your collaboration in this joint enterprise enriched my life over those years, and continues to do so. I will always be grateful to you.

Love,
Pierre

A BRIEF HISTORY OF ANPA WEST

The 1st Annual Western Regional ANPA Meeting (start of ANPA West) was held on November 23-25, 1984 at Ventura Hall, Stanford University, which was made available to us thanks to the interest of Prof. Patrick Suppes (Philosophy, etc.) The only funding was a minimal conference fee. The speakers who provided texts of their talks were H.P.Noyes, H.Partovi, M.Manthey, G.Chew. D.McGoveran, I.Stein, S.Sirag, D.Adelson and E.Oshins. The discussion was so lively that David McGoveran (without whose enthusiastic support HPN would not have dared to have the meeting) volunteered to bring out a Proceedings (Entitled "Discrete Approaches to Natural Philosophy"). This ambitious endeavor produced a volume, about a quarter of which consisted of those texts of the talks which the speakers were willing to provide, and about three times as much ANPA and ANPA-related material from various sources. This was distributed at the production cost to those interested. In the early years the very informal organization could be described by saying that Pierre Noyes served as president and David McGoveran served as secretary/treasurer. Others, including Mike Manthey and Chris Gefwert also helped out.

David McGoveran made Herculean efforts to get the this fledgling organization off the ground in a timely and well-organized manner. In particular, he even created a not-for-profit corporation with the name "Alternative Natural Philosophy Corporation" in order to pursue grants and accept tax-free donations. Unfortunately none of the rest of us were willing to put in the time and effort needed to make it fly.

However, those interested continued to have annual meetings, thanks to the continuing hospitality of Prof. Suppes, and the good-natured help of the attendees. A major reason for our success was that David realized that that we could have proceedings each year if (a) the speakers who wanted their work to appear came to the meeting with camera ready copy, (b) all attendees who wanted a copy would pay the estimated Xeroxing cost in advance and (c) someone would trundle the papers over to an all-night Xerox after the last session on Saturday and pick them up to distribute at the Sunday morning session. David did this for the next four years and has copies of the ANPA West Proceedings 1 -5 in archival storage. Fred Young then took on this task, yielding Proceedings for ANPA West 6-13. We called these "Instant Proceedings". I (HPN) possess single copies of 1, 5-13. David McGoveran also comments that: "I produced a yearly newsletter for the first four or five years, this being a natural extension of my `duties' since I had taken over as editor of the newsletter (and later as secretary/treasurer) from John Amson for ANPA in 1982 or 1983".

In the heyday of this effort Suzanne Etter took over the onerous duty of collecting conference fees, orders and funds for the Proceedings and reservations for the banquet, as well as acting as gatekeeper of the conference room. Eddie Oshins often arranged special banquet sites. Tom Etter and Suzanne also succeeded in creating a quarterly ANPA West Journal which published some very interesting articles. Abstracts of all the articles published are available at www.stanford.edu/~pnoyes or www.geocities.com/tometter_2000/anpawest.htm.

In 1989 David McGoveran, in conspiracy with Eddie Oshins and Chris Gefwert as a way to honor the efforts of Pierre Noyes, created the annual Alternative Natural Philosopher Award. The Award consisted of a certificate inscribed to the recipient and a trophy. The trophy was created and donated by McGoveran, to be passed from recipient to recipient each year. The trophy consists of a "wizard" (aka natural philosopher) figurine mounted on a walnut pedestal. The front face gave the name of the award, while the other three sides of the pedestal were reserved for brass plates for inscribing recipients' names. The awardees, in chronological order and starting in 1989, were: '89 H. Pierre Noyes, '90 David 0. McGoveran, '91 Eddie Oshins, '92 Irving Stein, '93 Tom Etter, '94 Fredrick S. Young, '96 Louis H. Kauffman. The physical trophy is now possessed by Pierre Noyes.

ANPA West came to an end in 1997 when Tom and Suzanne could no longer donate their time to keep it running, and the rest of us could not find a way to raise enough money to pay in the future for what had been a most generous free gift from Tom and Suzanne for many years.

Alternative Natural Philosophy Association

Statement of Purpose

1. The primary purpose of the Association is to consider coherent models based on a minimal number of assumptions, so as to bring together major areas of thought and experience within a Natural Philosophy alternative to the prevailing scientific attitude. The Combinatorial Hierarchy, as such a model, will form an initial focus of our discussions.

2. This purpose will be pursued by research, publications and any other appropriate means including the foundation of subsidiary organisations and the support of individuals and groups with the same objective.

3. The Association will remain open to new ideas and modes of action, however suggested, which might serve the primary purpose.

4. The Association will seek ways to use its knowledge and facilities for the benefit of humanity and will try to prevent such knowledge and facilities being used to the detriment of humanity.

Organisation (altered to reflect the current situation by KGB)

1. The Executive Council is the governing body of the Association. It consists of:

    (a) The Founders and all past Presidents of the Association, the Co-ordinator and the Treasurer,

    (b) The Executive Officers (the President and the Chairmen of the Executive Council and the Advisory Board),

    (c) Ordinary members nominated by classes (a) and (b), who serve for three years, with the possibility of re-nomination.

2. The Members of the Association are (a) the members of the Executive Council and (b) others nominated by the members and approved by the Executive Council.

3. The President is the official representative of the Association in external affairs, and has the responsibility for calling meetings of the Executive Council, at least annually, for the determination of overall policy.

4. The Treasurer is the responsible financial officer of the Association for the receipt and disbursement of funds and shall maintain and make available appropriate records, including annual accounts.

5. The President and the Co-ordinator may be paid an appropriate salary for their services, funds permitting. These services will include the organisation of meetings and the editing of the Proceedings of such meetings for publication, co-ordination of and participation in the research activities of the Association, preparation when appropriate of research reports and publication of such reports, and other such duties as may be assigned.

6. Members of the Executive Council may as appropriate receive funds for travel, expenses, etc.

7. The Executive Council has selected an independent Advisory Board. It may adopt its own rules for the operation and replacement of members. The Executive Council may nominate candidates to the Board. Any member of the Board, or the Board collectively, may make recommendations to the Executive Council, or directly to the Membership. Action taken on such recommendations must be promptly reported by the Executive Council to the Board in writing.

8. Dues are currently £20.00 per annum.

Executive Council: Mr. Anthony M. Deakin (Chairman), Dr. John Amson, Dr. Ted Bastin, Dr. Tom Etter, Prof. Louis Kauffman, Dr. Michael Manthey, Dr. David Roscoe, Dr. Fredric S. Young.

President: Dr. Keith Bowden, 139 Sandringham Road, Barking, Essex, IG11 9AH, UK.

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Co-ordinator: Arleta Dylus, 37a Shepherds Way, Rickmansworth WD3 7NN.

[Tel: 01923 461679, Email: a.griffor@physics.bbk.ac.uk]

Treasurer: David Roscoe, Department of Applied Mathematics, Sheffield University, Sheffield S3 7RH.

Advisory Board: Mike Horner (Chairman), Profs. G.F. Chew (Berkeley), C. Isham (Imperial College), M. Redhead (Cambridge and LSE), N. Cartwright (LSE), C. W. Kilmister (London), H. Pierre Noyes (Stanford).