Aspects I

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K. G. Bowden, *Editor*

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ANPA19 Editorial

Keith Bowden

This issue of the Proceedings is a year late, largely due to the timing of many of the contributions although also, in one case I know of ("mollified, but not yet lazy"), due to my mislaying copy during my house removal last year. My original intention after the nonappearance of ANPA19 was to publish an ANPA19/20 double issue this year. However, when all the copy for both issues was in, it became clear that there were well over 500 pages and that the new binding procedures at the University of London would not allow for this in a single volume. **Aspects** (with respect to Clive's remarkable paper in this issue) thus became split into **Aspects I** and **Aspects II** retaining, I hoped, some of the flavour of a double issue. An Appendix of the Proceedings of the two day extended session to discuss the paranormal is also available as **Paranormal Aspects**. I hope everyone likes the new wrap around, perfect bound format, (and I hope the glue holds better than before!) My thanks are due to Patrick and David at the Print Unit in SOAS for producing these volumes (and to Christine at Birkbeck Print Unit for the Appendix).

I have included, as the first paper in this issue, at the request of John Amson, "Mathematics of a Hierarchy of Brouwerian Operations" by Ted Bastin (September 1964, Cambridge Language Research Unit). John sent it to me with the following note. "This IS an important paper (I have just read it again). Please publish it your next Proc ANPA if at all possible!" I am trying to build an archive of older ANPA material with a view to (re)publishing some of it. In particular I am hoping to produce a "Best of ANPA West" and a "Best of ANPA Proceedings". If anyone can throw light on what happened at the earlier meetings and if any material remains I would much appreciate seeing it. I have also mentioned to some members of ANPA that I would like to publish a Bibliography of Combinatorial Hierarchy/ANPA related papers. I would much appreciate it if those who could make a personal bibliography available to me would do so (preferably electronically).

Please ensure all contributions to ANPA21 are submitted by the copy date, and please read the Notes for Authors below, taking particular care that fonts are large enough to be read after a 50% reduction in size during production. It is not necessary however to compensate quite as much as one contributor attempted in a submission to Arleta Griffor for the Appendix to this issue, using a font that measured approximately three feet across. It was only after discovering a large black area some fifteen feet from the top of the page that Arleta discovered that the document contained text at all...

ANPA Proceedings Notes for Authors

I would like to try to continue conformity of *style* for future issues of the Proceedings. Ideally I would like contributions to be submitted in International Journal of General Systems format (I have some copies of their Notes for Authors) or similar - look at my papers in any recent issue of the Proceedings. At least, Times Roman, 12 point, ***single sided, two copies,*** is preferred, although I will still accept typescripts in Courier. **10 point is TOO SMALL to be reduced to A5; 14 point is better for short papers.** Main heading capitalised, centred, other headings capitalised to the left. No underlining. At least a one inch bottom margin for footers; page numbers NOT bottom right. *Only copy in good English will be considered, and remember, this is a formal Proceedings.* **Remember also to include your name (surprising how many people omit this!), affiliation and full address, email address and the version number (even if it is 1.0) or date of the draft, centred below the main heading.** I often get sent more than one version of a paper and invariably mix them up! Send copy to *Keith Bowden, 139 Sandringham Rd, Barking, Essex IG11 9AH.* **Please help me by conforming to all this as closely as you can. THIS PLEA HAS LARGELY BEEN IGNORED IN THE PAST. PLEASE HELP BY READING IT CAREFULLY. As January 1st has proved impractical in the past the copy date for the Proceedings of ANPA21 is 1st April 2000 (often used as an academic deadline as any fool should know!) This will be adhered to rigidly this year.**

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MATHEMATICS OF A HIERARCHY OF BROUWERIAN OPERATIONS

TED BASTIN*

Original paper written in September 1964

Cambridge Language Research Unit

Re-typed in TEX, with some editing, 1998-FEB-27, by John Amson.**

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Editorial comment by John Amson

A typescript draft of this 1964 paper was found amongst old files in my attic. The original had been lost in the fire at Ted Bastin's home that destroyed so much of his archive material.

I believe the paper to be significant on both historic and practical grounds. Especially because it helps to recall what I remember to be the importance attached by Ted Bastin, when introducing me to this area of thinking, of the vital rôle of ideas of Brouwerian Intuitionism on the early development of what later became known as the Combinatorial Hierarchy — a rôle which, in my considered opinion, should once again be accorded primacy of place in current studies in this area.

Only the minimum of purely editorial changes have been made, occasional supplementary remarks and comments have been added, and the bibliography updated.

....

Section 1. Introduction.

The purpose of this paper (which is partly based upon work by A.F.Parker-Rhodes [see Refs.(1) & (2)] is to develop an algebraic system in which a set of operations upon a second set of quantities can be treated as operands for another set. The required algebraic system thus essentially has a hierarchical structure in respect of the operator/operand relationship.

The general reason for being interested in such an algebraic system is best explained by referring to the accounts given by Brouwer of his understanding of the nature of mathematics and of mathematical *creation.* This understanding was the original motivation — as far as Brouwer himself was concerned — for the technical developments of the Intuitionist School of mathematicians and logicians. I shall not be concerned much with these developments in the present paper, but my starting point is the same[1].

Brouwer describes two "acts of intuitionism" and writes [see Ref.(3), p.2] :-

> The first act of intuitionism *completely separates mathematics from mathematical language, in particular from the phenomena of language which are described by theoretical logic. It recognises that mathematics is a languageless activity of the mind having its origin in the basic phenomenon of perception of a* move of time, *which is the falling apart of a life moment into two distinct things, one of which gives way to the other but is retained in memory.*[2] *If the two-ity thus born is divested of all quality, there remains the common substratum of all two-ities, the mental creation of the* empty two-ity. *This empty two-ity and the two unities of which it is composed, constitute the* basic mathematical systems. *And the basic operation of mathematical construction is the* mental creation of the two-ity of two mathematical systems previously acquired, *and the consideration of this two-ity as a new mathematical system.*

The basic thought I wish to adopt and develop from this statement is that the juxtaposition of two mathematical entities generates a new mathematical entity, as a direct consequence of the fact that they are entertained in the mind in temporal succession. The mathematical system developed in this paper is intended to describe the generation of mathematical entities of this sort where complete freedom is allowed to the generating process. The mathematical entities considered are simple — being restricted to arrays of binary quantities. This approach has to be contrasted with intuitionist mathematics as that term is usually understood. In conventional intuitionist mathematics infinite choice sequences (in which some freedom is allowed at each stage) are shown to necessitate certain logical distinctions which are then taken as a sufficient representation of the creation process in mathematics. The creation process itself is then forgotten. In this way, indeed, intuitionist mathematics as developed by Heyting [see Ref.(4)] has ignored its own origins

[1] *Ed.* For a comprehensive account of Brouwer's ideas, and a bibliography, see Ref.(9).

[2] *Ed.* The following two sentences in this quotation were omitted in Ted Bastin's original paper, but are here included, for completeness.

and concentrated on the systematization of these logical distinctions in abstraction from the study of the Urphenomenon that they were introduced to illustrate. I take this view that this development has restricted the possible field of application of Brouwerian philosophy, and I shall attempt a more positive application of it in the present paper by systematizing the time concept directly.

The difference between the present approach and current intuitionist mathematics becomes important as soon as any operation is defined in which members of a set are selected and combined with other elements in any way. any such operation would be equivalent to a method of retrieving the elements in question, and any retrieval process must consist (by the original definition of the mathematical creation process) of a series of temporal juxtapositions each of which is a mathematical operation which must be represented within the system.

In the present approach the act of remembering the elements has to be part of the mathematics, and from this position our development of the formal properties of a mathematical system will begin. Originally [see Ref.(1)] such a mathematical system was proposed as a description of a hierarchically organized automaton, and Parker-Rhodes has applied the techniques of matrix algebra over the Boolean ring of two elements ($\mathbf{J}_2$) to describe such an automaton. In § 2 matrix multiplication which will be formally introduced later in this paper is used to illustrate the principles laid down in this present introductory section, by establishing a model of a Brouwer *spread* of infinitely proceeding sequences. These sequences are chosen subject to restrictions which make them appropriate to illustrate the principles in question, but in § 2 I do not aim at a deductive presentation, and therefore I shall not attempt to justify these restrictions in that section. In the subsequent sections I shall give a systematic development of the binary matrix algebra from a set of simple principles. The finiteness theorem of Parker-Rhodes [Ref.(2)] will be shown to be deducible from these principles without the arbitrary features of the form in which the theorem was originally proved. Finally, the finiteness theorem of Parker-Rhodes will be compared with the 'spread theorem' of Brouwer [Ref.(3)], and by that comparison further light will be thrown onto the relationship between the mathematics of this present paper and technical intuitionism.

Section 2. A Mathematical Model to Illustrate the Hierarchical Principle.

Suppose that data are presented to a system capable of manipulating them (in ways I shall presently describe) in the form of a time sequence of ordered pairs of quantities :-

$$\text{- - - - - - - - - -} \longrightarrow \textit{time}$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix}, \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}, \dots, \begin{bmatrix} x_j \\ y_j \end{bmatrix}, \dots \qquad (2.1)$$

where x_j, y_j stand for either 0 or 1. suppose further that the passage of time is from left to right as is shown by the arrow in diagram (2.1) so that at any given point in the sequence the generation of the next term on the right may be discussed before it is given. Moreover, in accordance with the principles of § 1, the sequence is considered two terms at a time in the first instance.

The act of considering two successive terms will be represented in the following way :-

> if the pairs are written V_i, V_j, then we seek a 2×2 matrix A_{ij} such that V_j is the transform of V_i under A_{ij}.

Following Parker-Rhodes [Refs. (1) & (2)] the defining operations used in these matrix transforms are

MULTIPLICATION

×	0	1
0	0	0
1	0	1

SYMMETRIC DIFFERENCE

+	0	1
0	0	1
1	1	0

The explanation of the reason for adopting the symmetric difference operation is deferred to the next section. In general there will be more than one matrix A_{ij} such that $A_{ij}(V_i) = V_j$. We choose any one of these <u>at will</u>. Having chosen it we see if it will fit the next pair of terms (*i.e.* V_j and V_k where V_k follows V_j). If it does (*i.e.* if V_k is the transform of V_j under A_{ij}) then we proceed to the next pair, and so on. If it does not fit, then A_{ij} has at this stage to be rejected as the specification of the <u>development</u> of the sequence. An example will make this clear.

Level $\mathbf{J}_2$

$$\begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix}, \dashrightarrow \qquad (2.2)$$

Level $(\mathbf{J}_2)^2$

$$\begin{pmatrix}0\,1\\1\,1\end{pmatrix}, \qquad \begin{pmatrix}1\,1\\0\,0\end{pmatrix}, \qquad \begin{pmatrix}0\,1\\1\,1\end{pmatrix}, \qquad \dashrightarrow$$

The notation " $(\mathbf{J}_2)^2$ " with an exponent '2' to represent the changed level, is a convenient one, but it should not be taken to mean that in any ordinary sense that $(\mathbf{J}_2)^2$ is the square of $\mathbf{J}_2$. However, the <u>number</u> of <u>elements</u> in the matrices in $(\mathbf{J}_2)^2$ is in fact the square of the number of elements in the vectors in $\mathbf{J}_2$, and this relationship persists at higher levels. The sequence of vectors labelled " Level $\mathbf{J}_2$ " is the original data sequence, and the sequence of matrices labelled " Level $(\mathbf{J}_2)^2$ " specifies limited portions of the development of the original data sequence $\mathbf{J}_2$.

The level $(\mathbf{J}_2)^2$ in diagram (2.2) is now regarded as a second order sequence of data for repetition of the process of analysis at a new level $(\mathbf{J}_2)^4$ in which 2×2 matrices have been replaced by 4×4 matrices. For this purpose each 2×2 matrix is to be treated as an array and replaced by a 4×1 array which can then be regarded as a vector for the new matrix transform. Provided that the rewriting of the arrays is always performed in the same manner, it is obvious that there is an order preserving 1 : 1 correspondence between the elements of the set of 2×2 arrays and elements of the set of 4×1 column arrays. A simple way to rewrite the 2×2 arrays is to put the second column beneath the first column and in general to 'string out' the columns one beneath another. This method is obviously applicable to all other levels. Thus the second level in Diagram (2.2) is rewritten as

$$\begin{pmatrix}0\\1\\1\\1\end{pmatrix}, \quad \begin{pmatrix}1\\0\\1\\0\end{pmatrix}, \quad \begin{pmatrix}0\\1\\1\\1\end{pmatrix}, \quad ---\rightarrow$$

I shall now compare the properties of this model with the requirements of the general principles of § 1. In the first place I recall that matrices were described in the model as being chosen "at will". This phrase needs further explanation. We may imagine that the choice is made by a random process[3] so far as any knowledge is concerned that can be described at level $(J_2)^2$. However, if we are to consider a more complex level, then a specification may exist of the choice which can only be regarded as random in the absence of the more complex level. In terms of this model, we imagine a system containing a series of levels of increasing complexity which correspond to the possibility of surveying — in stages — larger segments of the original sequence. In § 6, below, it will be shown that this sequence cannot be continued indefinitely, and that an infinite regress is avoided.

It is now necessary to make an important distinction between the cases where the original data sequence is considered as having an objective existence apart from the process of analysis provided by the model that I am describing, and cases where no such independent existence is granted to the data sequence. The present paper will be entirely concerned with the second of these cases (an essentially simpler situation). Thus in the case I shall consider, the length of the data sequence that can be considered at one time is determined by the number of levels of the model that have been developed, and elements of the data sequence that lie outside the reach of the system in this way are inaccessible and cannot be recalled, since to recall them would require us to specify the individual characteristics of each. If the power of recalling or retrieving an item when presented with another — related — item (as in memory association experiments) can be taken as a definition of memory, then the systems described in the present paper could be described by saying that within them no meaning can be attached to entities lying outside the memory defined in the system.

The model given in this section was constructed by Bastin and Kilmister [see Ref.(5)].

[3] By "random process" I mean a process (a) whose causation we are not prepared to look into further for the purpose in question, and (b) whose causation we have no reason to expect to be connected in any way with the purpose we have in mind for its application.

Section 3. The Matrix Model and Intuitionist Logic.

Brouwer [see Ref.(6), p.113] has defined a number, S_f, which enables him to demonstrate that the general philosophical principles of intuitionism make necessary an intuitionistic logic[4]. He has shown this, for example, by considering the assertion :

$$S_f = 0.$$

This demonstration consists in observing that though this assertion will be proved to be true by any proof that proves it to be non-contradictory, yet it does not satisfy the principle of the excluded third. It is easy to show that an analogous proof can be carried out for the model of § 2 — in which case all the conclusions regarding intuitionist logic adduced by Brouwer would follow also for that model — provided that we can assume that integers in natural order are automatically assigned to the elements of the sequences in the model, and that this assigning of integers does not have itself to be represented in the model.

Before I can consider this vitally important *proviso* I have just mentioned however, I shall construct S_f.

The definition of S_f requires first the definition of a *fleeing property*[5], which Brouwer defines to satisfy these conditions :-

(1) for each natural number n, it can be decided whether or not n possesses the property f;
(the adaptation of the model just described was necessary to satisfy this condition);

(2) no way is known to calculate a natural number possessing f;

(3) the assumption that at least one natural number possesses f is not known to be contradictory.

To relate these 'fleeing properties' to the rules of § 2 — expressed by matrix transformations — that were chosen at each stage of a sequence to specify the development of the sequence, we take f to be the property that a given matrix A ceases to specify the development of a sequence $V_0, V_1, V_2, \ldots$ at the elements of the sequence under consideration. Clearly this 'fleeing property' then has a sense for each natural number defining a term in the sequence, (condition (1) above); clearly also conditions (2) and (3) are satisfied.

Let us now return to consider the adaptation of the model of § 2 that made it possible to construct a number that had the properties of Brouwer's number $S{-}F$, within the model.

[4] *Ed.* In the later Ref.(9), Brouwer uses the symbol K_f here, and reserves S_f for a real number which is a limit of a sequence of rational numbers that depend on K_f.

[5] *Ed.* Brouwer [see Ref.(9), p.6] gives this example of what would be a fleeing property:- "There exists a natural number n such that in the decimal expansion of π the nth, $(n+1)$th, $\ldots, (n+8)$th and $(n+8)$th digits form a sequence 0123456789." The question as to whether this is true "relating as it does to a so far not judgeable assertion, can be answered neither affirmatively nor negatively. But then, from the intuitionist point of view, because outside human thought there are *no* mathematical truths, the assertion that in the decimal expansion of π a sequence 0123456789 either does or does not occur is devoid of sense."

We find at once that no provision has been made within the model for the adaptation in question — namely the assumption that a natural number can be assigned to each element of a sequence, without including the assigning process within the hierarchy of levels. So far as the model has been developed at present, the process of deciding that a sequence has fitted a given law of development has meaning only in so far as it is possible to compare more than two terms, any such comparisons have to be conducted in stages — built up from comparisons of pairs. Hence when we "decide, for a given natural number n, that n does possess or does not possess f," this decision can be taken at any one of a sequence of levels depending on how much of the sequence we are prepared to consider, and all the decisions of this sort that can be contemplated within the model have already been incorporated within the levels. As moreover the mathematical model of § 2 is intended to be applied to the understanding of the process of observation, though I do not develop this application in the present paper, the following point needs making. The mathematical model of § 2 is not yet a description of the process of observation; it is at a more primitive stage — being a formulation of the reference frame within which all observations have to be made. We have not a scheme rich enough to distinguish observing system from the system that is observed so long as further mathematical properties cannot be discussed in detail in future work and cannot be short-circuited by having an independent assignment of numbers to the elements of a sequence.

let us now turn to consider Brouwer's own position. It seems on the face of it that if the model of § 2 correctly represents Brouwer's time philosophy, and if as well we have to adapt the model in a way quite foreign to the spirit of this philosophy to define the number S_f within it, then Brouwer must have been wrong in thinking that an intuitionist logic would adequately exemplify the philosophical principles from which he started. I think this conclusion is correct. I can see no justification for Brouwer's implicit assumption that we have — as it were — an automatic sense of the order of the natural numbers (which would provide names for the terms in the data sequence, thus distinguishing it from the sequence of names for the terms provided by the hierarchy of levels). If such an explicit sense exists, then it needs to be introduced explicitly as a modification or extension of the account given by Brouwer of the faculties of mathematical creation — an account which we have seen to exclude the possibility of it.

If I am correct in insisting on a separate first stage in the development of a mathematical system from Brouwer's time philosophy,[6] then certain conclusions follow regarding the

[6] It was first pointed out to me by Margaret Masterman that in Brouwer's calculus of infinitely proceeding sequences as presented by Heyting [Ref.(4)], a logical distinction ought to be drawn between the numbers in the sequences used a "grid" or "reference system", on the one hand, and the use of the numbers to represent stages in the development of mathematical entities, on the other hand. In the algebraic hierarchy of the present paper there is a distinction which is analogous to this though not identical with it, for in the hierarchy (later called the ground hierarchy, § 7) in which all the mathematical possibilities are included, it is possible to represent only as it were a skeleton of mathematical meaning. More subtle concepts would have to be represented by imposing restrictions upon the possibilities allowed by the ground hierarchy. In this way the ground hierarchy plays the part of Masterman's "grid".

conventional link between Brouwerian mathematics and logic. My own summary of the situation is that the importance customarily attached by intuitionists to denying the law of the excluded third resulted in an unfortunate side-track in the development of brouwerian mathematics; successful conceptual innovations issue in special techniques, and it was felt that the Brouwerian innovation had already found its proper issue in intuitionist logic, and development of the Brouwerian mathematics into a general discipline applicable in empirical fields of an unfamiliar sort was thereby inhibited.

Before I leave the subject of the connexion of the model of s 2 with intuitionism as usually understood, I shall describe the counterpart of my model to the 'spread theorem' of Brouwer. The task of the present paper is to consider the ramifications of the total set of levels of entities constructed by specifying relations between entities existing at previously specified levels. In carrying out this task, great importance will naturally attach to circumstances in which there is a mathematically determined or automatic end to the process of the developing levels, and I shall devote the later sections of this paper to a discussion on a combinatorial basis that will show that when the greatest possible generality is given to the generation of new elements, the process does naturally come to an end — (Parker-Rhodes Theorem in Ref.(2)). This work has clearly a connexion with Brouwer's *Spread Theorem* [Ref.(3)][7]. Brouwer proves that if it is known that a number can be assigned to every sequence, within a spread of infinitely proceeding sequences, then that assignment will have to be made before any sequence can be stated. The proof of this theorem depends essentially upon the possibility of taking the *arguments* [Ref.(3), p.14] which lead to the conclusions that given sequences have numbers assigned under given conditions, themselves to have the structure of sequences in the spread.

Brouwer's proof applies to a hierarchical structure in the sense of the present paper, since Brouwer allows the possibility that the decision to consider a given sequence may generate a new sequence: indeed Brouwer deduces consequences from the fact that a connexion between statements that is established by any argument must itself be subject to the laws governing the spread calculus. Parker-Rhode's proof, also, describes a hierarchical situation, but by contrast with Brouwer's, its scope is restricted to the logically more primitive situation in which data sequences are not independent of the hierarchy that forms the subject of the present paper. Within this simpler context Parker-Rhodes' proof is stronger than that of Brouwer. It shows that a 'stop-rule' for the hierarchical generation process exists even when <u>all</u> <u>possible</u> arguments are included in it as elements, with no *protasis* concerning the assigning of numbers to such arguments.

Section 4. The Algebra of the Hierarchy : Binary Representation of Operators.

In preceding sections use has been made of an algebraic system to represent the structure provided by the total set of decisions that can be made about the progress of any Brouwer time-sequence. The concepts of level, element of level, and the hierarchy of

[7] *Ed.* Ref.3 has a 'Fan' Theorem, but does it have a 'Spread' Theorem ?

levels have been introduced. In addition it has been shown than an element of a given level determines a decision about the progress of a sequence by specifying two contiguous terms at the adjacent simpler level of the hierarchy. Now it is possible to consider each element of a given level as an operator upon the elements at the adjacent simpler level, whilst being in the relation of operand to the elements at the adjacent more complex level. With the aid of this idea I shall deduce some properties of a hierarchy from the principles that have been established earlier in this paper, and this deductive development will take the rest of the paper.

Let us consider operators p_μ in a finite family[8] $p_1, \ldots, p_n$ which define a level in a hierarchy. Without any knowledge of other levels of the hierarchy there is only one thing we can know about any element p_μ — namely whether at a given time the operation it describes is taking place or not. We represent a change by associating two operators, p_i, p_j, by a new operator P_{ij}. We call P_{ij} the *discriminator*[9] because we wish it to discriminate between the cases

(a) the operator has changed, and
(b) the operator has not changed.

We can write the two possibilities for the discriminator :-

$$P_{ij} \quad \begin{cases} \textit{exists} & p_i = p_j \\ \textit{does not exist} & p_j \neq p_j \end{cases} \tag{4.1}$$

The discriminator relation is really the basic Brouwer "two-ity" relation [Ref.(3)] in the primitive form in which we can distinguish the present thing we are considering from the past thing, and nothing else. It cannot tell us whether p_j is equal to any other element of the family at the same time. The choice of " 0 " to represent the null effect of the discriminator, is a suitable notation because it conveys the property of an operator that when it has no effect it might equally be considered not to exist. We shall also write the symbol " 1 " in Diagram (4.1) instead of "*exists*", but at this stage 'multiplication' is not defined. hence we write :-

$$P_{ij} = \begin{cases} 0\,, & p_i = p_j \\ 1\,, & p_j \neq p_j \end{cases} \tag{4.1a}$$

let us now define a composite structure to represent the situation of more than one discrimination process taking place at the same time. We form a family[10] of discriminators,

[8] *Ed.* Here and elsewhere, where need be, the original term 'set' has been replaced by 'family' since the members of a family need not all be distinct as they must were the collection a 'set'.

[9] *Ed.* This may historically be the first occurrence of the term 'discriminator' in this context.

[10] *Ed.* See the earlier footnote about 'set' and 'family'.

and I shall write such a family as a column with single square brackets on the left[11] :-

$$\left[\begin{array}{l}(P_{ij})_1\\(P_{ij})_2\\ \vdots\\(P_{ij})_n\end{array}\right. \quad = \quad \left[\begin{array}{l}a\\b\\ \vdots\\n\end{array}\right. \tag{4.2}$$

where $a, b, \ldots, n$ on the right take values 0 or 1. Such a *column* will be referred to henceforward as a *vector*.

It is clearly possible to write such a structure formally, provided we do not allow the notation to lead us into doing things that have not been defined for constituent discriminators. In particular, in writing such a composite structure as a column we assume that the elements of the structure are ordered in the sense that we know which element has been put into which place (so that — for example — it is possible to compare one structure with another, element by element)[12] Now to assume ordering (in this sense and not in the sense in which the integers are 'ordered' or 'simply ordered') is to beg the whole question that is to be discussed — namely the question of what mathematical operations are involved in recalling the elements of a segment of a time-sequence[13] nevertheless we choose deliberately to use a notation that does beg this question, because it will be convenient for our later demonstration that the order in fact exists, which will shall provide by defining a sequence of recursive steps between the levels of the hierarchy.

The question of order, in the sense above defined, will accordingly play a large part in the rest of this paper.

We now wish to extend the concept of a discriminator to apply to a vector, and we define the operation $\{P_{ij}\}$ on $[P_{ij}$, by :-

$$\{P_{ij}\} \quad = \quad \begin{cases} 0 & \textit{if } a = b = c = \ldots = 0, \\ 1 & \textit{otherwise}^*. \end{cases} \tag{4.3}$$

(* *i.e.* if any $a, b, c, \ldots,$ is $\neq 0$.)

This definition is appropriate to the idea of an operator, since if and only if each of the constituent operators produces no change can the combinations be said to produce no change.

We shall call the discriminator operations applied to pairs of vectors the *generalized discriminators*, and say that the discriminator has been *generalized*[14].

To determine the form of the discriminator in terms of ordinary algebraic operations we first prove

THEOREM 1. *The discriminator operation must be the symmetric difference operation.*

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[11] *Ed.* This notation was introduced by Parker-Rhodes in his 1962 paper (see Ref.(8))

[12] *Ed.* It is this requirement that prompted the change from 'set' to 'family' as noted in an earlier footnote.

[13] *Ed.* The original paper here has *"the elements of a set in a time-sequence"*.

[14] *Ed.* These notions do not appear in the sequel.

PROOF: Let A and B be distinct vectors, and write

$$(P_{AB}) \equiv A \star B$$

where " $\star$ " is the operation we wish to specify. We require

$$\begin{aligned} A \star B &= 1 \quad \text{if } A \text{ and } B \text{ are different, so that} \\ A \star B &= B \star A, \quad \text{and also} \\ A \star A &= B \star B = 0 \end{aligned} \qquad (4.4)$$

We divide the problem into two parts :-
(i) We first define a binary operation $\wedge$, which makes a vector correspond to any pair of vectors A and B. This vector (V) will be said to have a certain form *designated* if and only if A and B are identical.
(ii) Then we define a function, f, in such a way that $f(V) = 0$ if and only if V is designated.

For part (i) we have to consider a particularly simple binary operation (a 'separable' one[15]) such that if $A = [a_n$, and $B = [b_n$, then $A \wedge B = [c_n$ where $c_n = a_n \wedge b_n$.

The possible tables for $\wedge$ are of the form

$\wedge$	0	1
0	x	y
1	w	z

From our assumptions, $x = z$, $y = w$, and $x \neq y$. Hence possible tables are

$\wedge_{(a)}$	0	1
0	0	1
1	1	0

$\wedge_{(b)}$	0	1
0	1	0
1	0	1

For part (ii) we consider first (a) (the symmetric difference, or simply " + " in the field $\mathbf{J}_2$. the designated vector is then the null vector $(0, 0, \ldots, 0)$, and if $V = [V_n$ we can define

$$f(V) = \begin{cases} 0, & \text{if every } V_n = 0, \\ 1, & \text{otherwise.} \end{cases}$$

If we choose (b), which is related to " + " in $\mathbf{J}_2$ by

$$x \wedge_{(b)} y = x + y + 1,$$

[15] *Ed. I.e.* one that acts 'component-wise' on the components of its vector operands.

then the designated vector is $(1,1,\ldots,1)$, and so

$$f(V) = \begin{cases} 0, & \text{if every } V_n = 1, \\ 1, & \text{otherwise.} \end{cases}$$

The operation required is therefore either the symmetric difference or its complement. Between these two possibilities we are in a strict sense free to choose[16] without loss of generality (though we must stick to the choice we make). But if we choose the complement we do so at the expense of intuitive meanings of 0 and 1, since we should expect the effect of discriminating between two vectors to be zero if the vectors differ. Accordingly we choose symmetric difference. *This establishes the theorem.*

Section 5. Mappings.

I shall now establish the recursive relation between levels that will enable us to order the elements at a given level in the hierarchy (in the sense of "order" defined in § 4).

By means of the discriminator relation we can derive new vectors from ones which have already been found. But any pair of vectors corresponds to a third vector, and in a family of n vectors repeated use of the discriminator operation will produce in general

$$n + \tfrac{1}{2}n(n-1) + \cdots = 2^n - 1 \tag{5.1}$$

vectors, unless it happens that the process throws up vectors which have already occurred before. If the full number of vectors is produced by this process, we can call the original family *linearly independent* (this definition agrees with the usual definition for the field of two elements when multiplication has been defined, but is appropriate for the present stage of development of the theory before a second operation has been introduced). If the two vectors have m elements, a necessary condition for linear independence is clearly

$$n \leq m\,.$$

To establish recursion we prove

Theorem 2. *When a family of vectors is ordered in the sense used in § 4, the larger family derived from it by repeated use of the discriminator operation can also be regarded as ordered.* We establish some preliminary results before giving the proof. The discriminator relation corresponds roughly to the idea of "next" : if a family of vectors is ordered, then there must also be a mathematical representation of a "memory" which is capable of deciding whether the association of vectors defined by the discriminator operation is, or is not, in accordance with the sequence as it exists in the memory. This idea requires a store of pairwise associations of vectors. Such an association can be defined by an expression

$$V = f(U)\,,$$

[16] *Ed.* This concept of choice led eventually to other notions of what Pierre Noyes in 1987 called 'Amson Invariance'.

where a functional operator f defines a unique vector V for each vector U. The elements of U and V belong to $\{0, 1\}$.

LEMMA *The functional operation f is a matrix transformation.*
PROOF: We know from a result of Boole that if we define a new binary operation between[17] the 0, 1–elements u, v of the vectors U, V, which it is convenient to denote by multiplication as uv, and which is defined by the table

uv	0	1
0	0	0
1	0	1

(5.2)

then, with this operation and the original one, any function can be expressed. For example, if U has two elements u_1, u_2, then a general function of U will have elements of the form

$$a + bu_1 + cu_2 + du_1u_2 \tag{5.3}$$

Thus Boole's argument shows that (5.3) is the most general form fro the mapping. For the kind of algebra we are constructing, however, it is still too general, for we introduced the two kinds of terms — $a, b, \ldots$ and $u, v, \ldots$ — so that the two basic type of operation could be represented (discrimination and mapping), and it is therefore unnecessary to construct the quantity du_1u_2 : the mapping relation is represented between terms of the different type.

At this stage almost any other binary operation would do for the mapping; the only exception being the one which is complementary to addition (+), since this gives us essentially nothing more than we have already assumed when we take addition as the first operation. I shall discuss the choice of operation.

The next problem is to ensure that the operation that defined the mapping is consistent with the discriminator operation already considered in the sense that the "non-linear" term in (5.3) will not reappear even if the vectors to be mapped are considered as composite arrays. (An obvious necessary condition since vectors are meant to be considered as composite arrays.) For this purpose it is convenient to change the notation for a moment, and to write

$$D(U, V) \tag{5.4}$$

for the discriminator of two vectors U, V, and to write $\bar{U}$ for the vector corresponding to U in the mapping association. The condition of consistency can then be written as

$$\overline{D(U, V)} = D(\overline{U}, \overline{V}),$$

and when we translate this back into the notation with which we are familiar we get

$$f(U + V) = f(U) + f(V). \tag{5.5}$$

[17] *Ed.* This sentence is a clearer re-wording of the original.

Equation (5.5) is simply the familiar condition that f should be a linear[18] function, and we may notice that our development gets us into conventional algebra by developing the concept of linearity from the more primitive notion of an operation that is capable of representing a mapping (of levels).

It is now necessary to consider the form of the second binary operation. When we have rejected the choices that would give different results depending on which vector in the mapping of a pair of vectors we chose first, we are left with only one operation as a serious contender competing with "multiplication", (5.2). This is the set union

$u \cup v$	0	1
0	0	1
1	1	1

We reject the possibility because it gives us an algebra of mapping operations in which (as can easily be shown) there exists a zero element in the set of mappings but no unit element. Such an algebra could not be reduced to the simple form in which all the vectors represent binary situations.

Given the choice of the second operation, the condition of linearity confines us in fact to a matrix formalism, as we can see at once in the following way. If we choose a basis for the vectors so that any vector can be written in term of its components by

$$U = \sum_i u_i e_i \,, \tag{5.6}$$

where the basis elements are

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \qquad e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \qquad etc.$$

then we have at once that

$$V = f(U) = f\left(\sum_i u_i e_i\right) = \sum_i u_i f(e_i). \tag{5.7}$$

[18] *Ed.* Strictly speaking, an 'additive' function, but over the field of two elements, 'additive' and 'linear' are identical notions.

However, in this equation the functional operations on the basis elements give us vectors which must be expressible in terms of the basis (since any vector is so expressible). Accordingly we write

$$f(e_i) = \sum_j f_{ji} e_j , \tag{5.8}$$

and this at once gives us

$$v_i = (f(u_i)) = \sum_j f_{ij} u_j ,$$

which is simply an expression of the matrix algebra. *This proves our lemma.*

Theorem 2 follows directly from the lemma : each matrix transform of vectors X_1 and X_2 orders the pair (X_1, X_2), and if there are enough matrices to order all the pairs (X_i, X_j), then a 1 : 1 relation is established between the elements of the larger family and the matrices. Since the smaller family is ordered, the order of the larger family follows provided that there not more pairs than there are matrices — *i.e.* provided that

$$2^n \leq m^2 ,$$

where m is the dimension of the space. *This proves Theorem 2.*

In the following table these results are applied over and over again in successive stages of a hierarchy which begins with two linearly independent vectors each with two elements.

m	n	dim(f)	$2^n - 1$
2	2	4	3
4	3	16	7
16	7	256	127
256	127	$(256)^2$	$2^{127} - 1$

(5.10)

Section 6. Level Recursion.

It is now possible to specify some of the mathematical properties of a hierarchy of levels from the results established in Theorem 2. This we do by a recursive method, since to order the operations at a given level it is necessary to construct the contiguous simpler level, and to assume ordering of its elements. This in turn requires us to consider the level simpler than that, and so on.

We can see on general grounds and in an intuitive way that at a given level in a hierarchy the effective number of operators is not the number in the level in question; rather it is the number at that level together with the numbers at all simpler levels that is to be taken as the effective number. This we shall call the *multiplicity* of the hierarchy at a given level. The intuitive reason is that the hierarchy is a structure which occupies time, and therefore operators at different levels (which would be kept strictly incomparable by type distinctions in the class logic) can be taken together. One cannot say that if there are a horse and a cow in a field then there are three things in the field : the horse, the cow, and the (horse and cow). In a hierarchy one could (by analogy) say, however, that there was the perceiving cow event, the perceiving horse event, and the perceiving (horse and cow) event. More formally, the summation of numbers of operations can be understood if we consider that every individual process represented in the hierarchy is a binary one in which a mapping of two operations is represented by an operation at a different level. If many operations at one level are to be considered then they have to be brought under consideration by these binary processes which will in general involve a tree of operations at all the new simpler levels.

In applications of the theory of the present paper the multiplicities of hierarchies will be of considerable importance.

A recursive definition of the multiplicity, $\Lambda\lambda$, of level λ of a hierarchy (*i.e.* the total number of operations up to level λ) can be expressed by the relation :-

$$\Lambda\lambda = (2^n - 1) + \Lambda(\lambda - 1),$$

where n is the increase in the total provided at level $\lambda - 1$, in accordance with Theorem 2 and on the assumption that the largest family of operators that can be ordered is also the largest family of operators that can be enumerated by any means whatever at that level.

A hierarchy has now been defined in terms of a recursive rule. We now wish to apply the rule successively to find the form that emerges for hierarchies. There is clearly a hierarchy of specially simple form[19] in which the lowest level is based on a vector of two components; successive levels then have basis vectors of

$$2^2, \quad (2^2)^2, \quad \ldots$$

components. We shall call this the *ground hierarchy.* This name derives from the fact that more special hierarchies with distinguishing features can all be obtained by imposing restrictions on the ground hierarchy. to put it another way, the ground hierarchy is the hierarchy from which all distinguishing features have been removed. In the present paper we restrict our attention to the ground hierarchy. These properties of the ground hierarchy follow from a theorem to be proved below, but first we illustrate the first stages in the construction of the ground hierarchy.

We take a bottom level consisting of two operators. According to § 1 these are to be written 0, 1. And

$$\lambda_1 = \lambda_1(0, 1).$$

The level λ_2 of operators upon λ_1 is now constructed, according to our recursive procedure. From these we construct a further level λ_3 of $2^3 - 1$ operations and obtain the total number of operations Λ_2 at this level by adding the increments at the levels hitherto generated. This gives

$$\Lambda_2 = 3 + 7,$$

[19] If the lowest level has one element, no non-trivial hierarchy results. The level of operations upon this also contains one operation, and is formally indistinguishable from the first. And so on.

By continuing in this way we generate the following table :-

LEVEL	Number of elements of array	Increment in number of operators at level	Total number of operators
1	$2^1 = 2$	3	3
2	$(2)^2 = 4$	7	10
3	$((2)^2)^2 = 16$	127	137
4	$(((2)^2)^2)^2 = 256$	$\sim 10^{38.2}$	$\sim 10^{38.2} + 137$

The table immediately provides the following important

THEOREM 3. Parker-Rhodes' Theorem
The ground hierarchy has only 4 levels.

REMARK: This result follows at once from the fact that at level **4** there are more operations than the dimension number of the vector space at the new level. Therefore the construction terminates.

Parker-Rhodes' Theorem shows that the *protasis* of Brouwer's Spread Theorem is satisfied by the hierarchical algebra that has been developed in this paper. That is to say, in Brouwer's proof that we can "assign a number" to each sequence of operations defined in the hierarchical algebra. Brouwer's theorem, with its implications for ordering the continuum, follows, and we have shown that the program suggested by Bastin and Kilmister [see Ref.(7)] of developing a continuum suitable for physical theory from a Brouwer space is logically feasible. The original program lacked a suitable form fro the Brouwer spread, and this has been provided by the hierarchy algebra of the present paper. As, however, the hierarchy algebra is (in ways fully discussed in the opening sections of this paper) more general than the spreads contemplated by Brouwer, it has been necessary to show that sequences in the hierarchy can be treated as forming a spread for the purposes of the spread theorem.

In an earlier form of discussion of the hierarchy, Parker-Rhodes [Refs. (1) & (2)] used the device of mapping vectors at one level onto points in a vector space at a higher level, as the first method of hierarchy construction. This device led to a somewhat involved calculation of the hierarchy multiplicities which we have calculated in the present paper without any reference to a special choice of mapping since we have made no assumptions about the relation of the m-vectors to the vector space at the next most complex level.

Naturally, further applications of the hierarchy mathematics will make use of special forms of mapping, and in particular it will become important to consider mappings defined by eigenvalues as vectors at one level that are mapped onto the vector space at the higher level. this was the form originally used by Parker-Rhodes to define the hierarchy, but in fact the multiplicities that have been calculated do not depend on any such special forms.

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Some Thoughts on COMMUTATION RELATIONS and MEASUREMENT ACCURACY *

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Abstract

We show that measuring the trajectories of charged particles to finite accuracy leads to the commutation relations needed for the derivation of the free space Maxwell equations using the *discrete ordered calculus* (DOC). We note that the finite step length derivation of the discrete difference version of the single particle Dirac equation implies the discrete version of the p, q commutation relations for a free particle. We speculate that a careful operational analysis of the change in momenta occurring in a step-wise continuous solution of the discrete Dirac equation could supply the missing source-sink terms in the DOC derivation of the Maxwell equations, and lead to a finite and discrete ("renormalized") quantum electrodynamics (QED).

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1 INTRODUCTION

Our derivation of the free space Maxwell equations using the *discrete ordered calculus* (DOC)[2] mentioned that the postulated commutation relations between position and velocity could be interpreted as a consequence of a fixed discrepancy between first measuring position and then velocity or visa versa. However, these commutation relations were not given a careful physical justification in terms of our finite measurement accuracy philosophy [5]. A second deficiency, which in fact caused us to warn the reader that we had only derived one part of the *formalism* of classical electrodynamics rather than the theory itself, was that no attempt was made to identify the sources and sinks of the "fields" and derive the inhomogeneous Maxwell equations from them. We took a step in that direction by our derivation[3] of a finite and discrete version of the 1+1 free space Dirac equation from a fixed step-length *Zitterbewegung* postulate using finite difference equations. Although it was noted that an attempt had been made by me[4] to attribute the *Zitterbewegung* to the conservation of spin or particle number in the presence of random electromagnetic fluctuations, no attempt was made to relate these interactions to the source terms needed to complete the argument in the Maxwell equations paper. Neither Kauffman nor I have attempted to relate the non-commutativity known to arise from the Dirac equation to the commutation relations needed to derive the Maxwell equations in our finite and discrete context. In this paper I take a few steps to remedy both defects, but more work is needed.

2 ELECTROMAGNETIC MEASUREMENT OF A CHARGED PARTICLE TRAJECTORY

In earlier work I have made use of what I called "the counter paradigm" to cut the Gordian knot of specifying what a physicist means when he says that a particle was or was not present in a finite spacial volume for a finite time duration. As a first approximation, I assume that this volume is the "sensitive volume" of a counter, and the time duration is the time during which the recording device attached to the counter could have recorded an event, often called a "firing". This I call a NO-YES event, depending on whether the counter did not or did "fire". A more careful treatment specifies the probability of "spurious events", i.e. cases when the counter "should have fired" but did not (counter inefficiency), and the probability of cases when the counter "should not have fired", but according to the record did in fact fire (background events). Ted Bastin has often objected that this abrupt transition from the laboratory to Boolean logic sweeps too much under the rug, and I have often replied that to justify this way of talking about laboratory practice would require a book. Fortunately, Peter Galison has taken ten years to write the book I needed. He separates the history of the material culture of particle physics into a "logic" tradition contrasted with an "image" tradition. My "counter paradigm" finds its appropriate niche as part of the logic tradition. Galison shows that by now the two alternatives have fused in the mammoth "detectors" which are integrated into the accelerators in all high energy particle physics laboratories [1]. It took over a century for this language and practice to mature, and a decade to make a convincing argument as to why it should be accepted by philosophers. I now have a simple tactic open. I can ask any critic of my conceptual leap from counter firings to NO-YES events to first convince me that Galison's defense of the mainstream tradition is inadequate. Only then will I feel any need to take his or her criticism seriously.

This ploy allows me to use conventional language in my descriptions of laboratory measurement. In particular I can now construct a simple paradigm for what I mean by the measurement of the electromagnetic trajectory of a particle. First recall that by a "particle" I mean[5] "a conceptual carrier of conserved quantum numbers between events". I can take the simplest interpretation of two sequential counter firings a fixed distance L apart with a time interval T between them to be that a particle conserving mass, momentum and energy passed between them with velocity L/T. I assume available a "source" of particles which allows a large number of repetitions of these paired sequential events to occur. This data set is assumed to provide both statistical and systematic accuracy adequate for calibrating the *changes* in the magnitude and/or direction of this velocity caused by inserting electromagnetic devices into the path defined by sequential counter firings

The electromagnetic device we consider first, inserted between two counters previously used to measure velocity, is simply two parallel conducting plates with a hole through them across which a constant voltage can be applied. This voltage is measured by standard techniques. When the voltage is negligible, our original source and sink counters still give a velocity $v = L/T$ for each particle "passing through the two holes", showing that we can maintain the same particulate interpretation of the two sequential events with the plates in place, even though we do *not* "measure" the presence of the particles between the plates. We now apply a voltage V across the plates, which are large enough compared to the holes so that, according to standard electrostatic theory, the electric field between the plates and along the direction of motion of the particle is $\mathcal{E} = V/\Delta d$ where Δd is the separation between the plates. We now study the *change* in the velocity of a particle of the type being studied (i.e. produced in the same way or available from the same source) during a time when the voltage across the plates is held at V. Counter firings before the presumed arrival and after the presumed departure of the particle at the device allow us to say that the particle arrived at the position of the plates with velocity v_1 and left with velocity v_2. We then say that the particles have a charge e, a (rest) mass m, an energy E_1 before they enter the first hole, and an energy E_2 after leaving the second hole when, for various experiments, the velocity change produced by the device is equivalent to an energy change

$$\Delta E = E_2 - E_1 = \pm e\mathcal{E}\Delta q; \quad \mathcal{E} = V/\Delta d \tag{1}$$

with

$$E_1 = \frac{m}{\sqrt{1-(v_1^2/c^2)}}; \quad E_2 = \frac{m}{\sqrt{1-(v_2^2/c^2)}} \tag{2}$$

We then take this as our paradigm for the *measurement* of an electric field in a region of length Δd of strength $\mathcal{E}$.

We emphasize that this measurement requires a *change* in the velocity of the particle. The minimum change to which we can reliably assign a number *quantizes* our *measurement accuracy* at the level of technology we are using. Note that our paradigm assumes *constant* velocity between measurements in field-free regions. [Recall that we *derived* a discrete version of the constant velocity law from bit-string physics in our foundational paper[6], Sec. 6.5, pp 94-95.] Alternatively, if we know the field (or voltage) and the (constant velocity) trajectories before and after the device, we can use the *same* device as a paradigm for *position* measurement to an accuracy Δd. By fleshing out this paradigm, we can recursively use electromagnetic language to justify the construction of laboratory counters which have a conceptual connection to those used in our counter paradigm.

Our paradigm for magnetic field (or momentum) measurement assumes that we have two double plates across each of which independently adjustable voltages can be applied. We call the entrance hole of the first pair 1 and the exit hole 2, and for the second plate the entrance hole 3 and the exit hole 4; thus the gaps are d_{12} and d_{34}, and the trajectory is 1,2,3,4. The plates are located geometrically in the laboratory in such a way that a path connecting the exit hole 2 from the first pair to the entrance hole 3 into the other can be an arc of a circle of radius R whose center lies in a plane with the two gaps; the gaps between the plates are two (short) arcs of that circle. The arc between the two devices is of length $R\Delta\theta$. The magnetic field we wish to measure is perpendicular to the plane of the circle and is of constant strength $\mathcal{B}$, along this arc. This is "guaranteed" by the geometry and the standard theory of magnetostatic fields. According to electromagnetic theory, this field does not change the energy of the particle, or the magnitude of its velocity, but does cause the *direction* of the velocity to change. This change is simply described in terms of the momentum **P** of vector magnitude

$$\mathbf{P} = \frac{m\mathbf{v}}{\sqrt{1-(v^2/c^2)}}; \quad |\mathbf{v}| = \frac{R\Delta\theta}{t_3 - t_2} \tag{3}$$

where the time t_2 when the particle exits hole 2 and the time t_3 when it enters hole 3 are usually inferred rather than directly measured; **v** is the vector velocity of constant magnitude with a (varying) direction assumed tangent to the arc. The radius of the circle is related to the magnitude of the momentum by

$$R = \frac{eP}{c\mathcal{B}} \tag{4}$$

and the change in momentum (due to change in direction since the magnitude is constant) by

$$\Delta P = 2P\ sin^2\Delta\theta/2 = P(1 - cos\ \Delta\theta) \tag{5}$$

As as in the case of electric field measurement, we can consider this arrangement as either a measurement of the field $\mathcal{B}$ at (perpendicular to) the arc $R\Delta\theta$ geometrically defined or as a measurement of the velocity of the particle along that arc. But as a velocity measurement, it is important to realize that there is an ambiguity as to whether this is the measurement of velocity *after* the particle has traversed the first double plate 12, which could be a counter measuring position, or a measurement of velocity *before* it traverses the second double plate 34.

If all we have available are not individual particle detectors, but only devices that measure the charged current flowing along the trajectory, the arrangement discussed above can only be used to measure e/m and not charge and mass separately. Such experiments were, historically, sufficient to convince the proponents of various models of the charge distribution "within the electron" (Abraham, Lorentz, Poincaré) that their models were wrong, and that the Einstein equation connecting mass to velocity used above was correct even though it violated their way of thinking about space and time ([1], Sec. 9.6, pp 810-816). Galison shows by this historically examined case that experimental tradition and the material culture of physics allow theoretical physicists on opposite sides of what Kuhn would call a "paradigm shift" to agree on the significance of experimental results..

The fact that electric and magnetic fields acting on a moving charge effect changes in velocity along or at right angles to the direction of motion respectively allows one to build a "velocity selector" by setting up a region of electrostatic and magnetostatic fields in which the fields are at right angles to each other and both are at right angles to the direction of motion of the charge. The force on the charge due to the electric field is $e\mathcal{E}$ while the force due to the magnetic field is $ev\mathcal{B}/c$ and the geometry we have specified requires these forces to be in the same direction. Consequently there is a unique direction for which they cancel, provided the velocity has magnitude $v = c\mathcal{E}/\mathcal{B}$. A particle of that charge with any other velocity will be deflected away from this direction.

At first glance, such a device would seem to allow us to measure position and velocity "simultaneously". But this is not correct. So long as the charged particle has this velocity and the magnitude and direction of the fields does not change along this straight line trajectory, no force acts and the particle maintains constant velocity. However, we have no way of knowing *where* it is within this region, and hence when it enters and leaves it, without a measurement. But this measurement will change the velocity. So we must measure when the particle enters the region *and* when it leaves the region in order to know how long and when it is in the region with that velocity.

As before, we can first measure position and then velocity or first measure velocity, and then position but not both simultaneously. An extended discussion of this case should allow us to see that three points on the trajectory are needed to establish the field at the intermediate point, and four if we are to measure both **E** and **B**. On another occasion we hope to be able to go on to derive the free field commutation relations by such considerations (or directly from our DOC equations), and not just the uncertainty principle restrictions obtained by Bohr and Rosenfeld.

In closing we note that, even though we started out to devise a paradigm for electromagnetic field measurement, we have ended up deriving from this paradigm the DOC postulate that we can first measure position and then velocity or first measure velocity and then position, but not both simultaneously. We hope that this discussion makes it less of a mystery why the DOC postulate leads so directly to the formalism of free-field electromagnetism.

3 FROM FREE DIRAC PARTICLES TO FIELD SOURCES AND SINKS

The derivation of the finite difference version of the free particle Dirac equation[3] for fixed step length $\hbar/mc$ with step velocity $\pm c$ tells us immediately that we can cut the trajectory of a free particle into segments of constant velocity between "points" at which the velocity changes discontinuously. On the other hand our DOC equations for the free space electromagnetic field [2] support solutions corresponding to the propagation of crossed electromagnetic fields with velocity c and constant frequency which, for finite segments, can be interpreted as "photons" if they have the right amplitude. All we seem to need to produce a quantum electrodynamics which is finite and discrete, and hence "born renormalized", would seem to be to assign a charge to the massive particle which satisfies the Dirac equation in such a way that its discrete changes in velocity correspond to the emission or absorption of such photons. I hope to do this on another occasion. The details will obviously take some time to work out, but will provide a lot of fun along the way.

Since this amounts to solving a finite and discrete "three particle problem", an approach to the same theory which starts more directly from bit-string physics would be to treat the photon as a bound state of a particle-antiparticle pair in the relativistic three body theory now under active development [7].

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Path Integrals and Discrete Physics

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1 Introduction

This paper is a short introduction to a discrete approach to quantum mechanics. I point out that the Schrodinger and Dirac equations can be discretized and solved exactly in the discrete domain by summations over histories that are the analogs of the integrals over paths in the continuum model. This fact is well known to quantum mechanicians, but perhaps it is not so well appreciated that the solutions are indeed exact in the discrete framework. The point of this short paper is promote the point of view that these beautiful discrete equations and the corresponding discrete processes should be studied in their own right. This paper is a first step in that direction.

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2 Schrodinger's Equation and Sums over Histories

Lets begin by recalling Schrodinger's equation in one space variable and one time variable:

$$\imath\hbar\partial\psi/\partial t = H\psi$$

where H denotes the Hamiltonian of the physical system. In standard one-dimensional quantum mechanics, the Hamiltonian is given by the equation

$$H(x,t) = (-\hbar^2/2m)\partial^2\psi(x,t)/\partial t^2 + V(x)\psi(x,t).$$

Lets discretize the Schrodinger equation by replacing the time derivative of ψ by

$$(\psi(x,t+\tau) - \psi(x,t))/\tau$$

and the second space derivative of ψ by

$$(\psi(x-\epsilon,t) - 2\psi(x,t) + \psi(x+\epsilon,t))/\epsilon^2.$$

The result is

$$\imath\hbar(\psi(x,t+\tau) - \psi(x,t))/\tau =$$

$$(-\hbar^2/2m)(\psi(x-\epsilon,t) - 2\psi(x,t) + \psi(x+\epsilon,t))/\epsilon^2) + V(x)\psi(x,t).$$

Rewriting, we find that

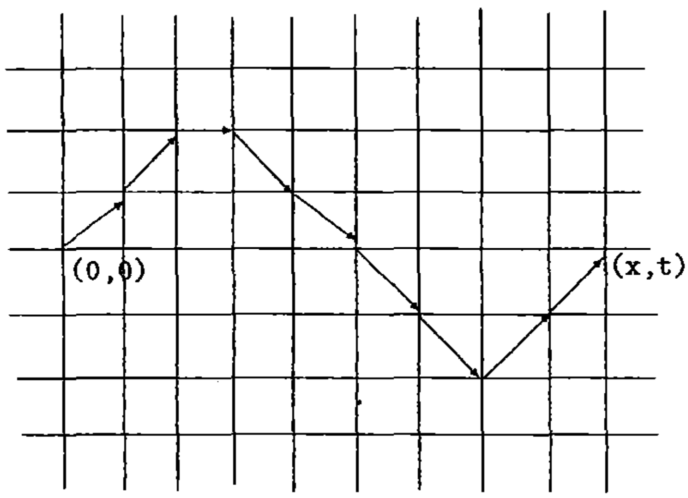
Theorem. The discrete Schrodinger eqation is equivalent to the formula

$$\psi(x,t+\tau) = A\psi(x-\epsilon,t) + A\psi(x+\epsilon,t) + B(x)\psi(x,t)$$

where $A = \imath\hbar\tau/2m\epsilon^2$ and $B(x) = (1 - (\imath/\hbar)(\hbar^2\tau/m\epsilon^2 + \tau V(x))$.

This equation gives a discrete form of the Schrodinger equation that has an *exact* discrete description as a summation over paths. for we can let $\psi(x,t)$ (when x is a multiple of ϵ and t is a multiple of τ) be the sum of contributions from all discrete paths p with time step τ and space step ϵ starting at $(x,t) = (0,0)$. The contribution of a path p, denoted $W(p)$, is the *product* of the *weights* A and $B(x)$ (as defined above) at the vertices of the path. Note that we really are dealing with discrete paths in the $\epsilon--\tau$ lattice since the nearest neighbors one time step back from a point $(x,t+\tau)$ are the points $(x - \epsilon, t)$, (x, t) and $(x + \epsilon, t)$. A typical discrete path is indicated in the diagram below.

$$\psi(x,t) = \sum_{p \in Paths} W(p).$$



It is important to understand that the discretized version of the Schrodinger equation is solved exactly by this path sum. This means that if we take the discrete physics seriously, then the Schrodinger equation and its corresponding quantum mechanics come equipped with an exact solution that can be studied via computer simulation and combinatorial mathematics.

The reader familiar with the Feynman path integral will be interested in the relationship of this path sum with the corresponding path integral with its weighting of the paths by the factor

$$exp((i/\hbar) \int_p S)$$

where $S(x,t) = mv^2/2 - V(x)$ denotes the classical action of the "particle" moving on the path with velocity v. That is, the Feynman path integral shows that ψ can be expressed as

$$\psi = \int_{p \in Paths} exp((i/\hbar) \int_p S).$$

The usual derivation of this path integral in relation to the Schrodinger equation uses a combination of local and global contributions from the continuous model. On the one hand the oscillatory nature of the action integral makes it clear that the contributions to $\psi(x, t+\tau)$ come mainly from values (x', t) where x' is near x. On the other hand, it is convenient in the mathematics to rewrite in such a way that one simplifies the local expression of the path integral by computing Gaussian integrals over all positive real values for $x - x'$. This local/global mixture produces the approximation that implies the Schrodinger equation from the full path integral. Such an approach is probably necessary since the velocity of the "particle" in the path integral is the classical velocity, and as such is not defined in the discrete model except as an average.

More remains to be done with this discrete model. It should be studied on its own grounds. Note that the sort of discrete path-sum model that we have derived here for the Schrodinger equation works perfectly well in three dimensions of space and one dimension of time.

In the next section we take a similar approach to the Dirac equation.

3 Discrete Spacetime and the Dirac Equation

We should remark on the basic formalism for amplitudes in quantum mechanics. The Dirac notation $\langle A|B\rangle$ [3] denotes the probability amplitude for a transition from A to B. Here A and B could be points in space (for the path of a particle), fields (for quantum field theory), or geometries on spacetime (for quantum gravity). The probability amplitude is a complex number. The actual probability of an event is the absolute square of the amplitude. If a complete set of intermediate states $C_1, C_2, \ldots C_n$ is known, then the amplitude can be expanded to a summation

$$\langle A|B\rangle = \Sigma_{i=1}^{n} \langle A|C_i\rangle\langle C_i|B\rangle.$$

This formula follows the formalism of the usual rules for probability, and it allows for the constructive and destructive interference of the amplitudes. It is the simplest case of a quantum network of the form

$$A - - - * - - - C - - - * - - - B$$

where the colors at A and B are fixed and we run through all choices of colors for for the middle edge. The vertex weights at the vertices labelled $*$ are $\langle A|C\rangle$ and $\langle C|B\rangle$ respectively. A measurement at the C edge reduces the big summation to a single value.

Consider the generalization of the previous example to the graph

$$A - - - * - - - C^1 - - - * - - - C^2 - - - * - - - ... - - - * - - - C^m - - - B$$

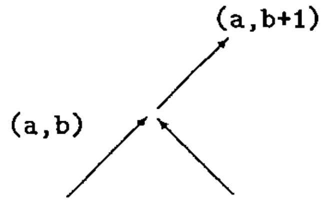
With A and B fixed the amplitude for the net is

$$< A|B >= \Sigma_{1 \leq i_1 \leq ... \leq i_m \leq n} < A|C^1_{i_1} >< C^2_{i_2}|C^3_{i_3} > ... < C^m_{i_m}|B >$$

One can think of this as the sum over all the possible paths from A to B. In fact in the case of a "particle" travelling between two points in space, this is exactly what must be done to compute an amplitude - integrate over all the paths between the two points with appropriate weightings. In the discrete case this sort of summation makes perfect sense. In the case of a continuum there is no known way to make rigorous mathematical sense out of all cases of such integrals. Nevertheless, the principles of quantum mechanics must be held foremost for physical purposes and so such "path integrals" and their generalizations to quantum fields are in constant use by theoretical physicists [4] who take the point of view that the proof of a technique is in the consistency of the results with the experiments. When the observations themselves are mathematical (such as finding invariants of knots and links), the issue acquires a new texture.

Now consider the summation discussed above in the case where $n = 2$. That is, we shall assume that each C^kcan take two values, call these values L and R. Furthermore let us suppose that $< L|R >=< R|L >= \sqrt{-1}$ while $< L|L >=< R|R >= 1$. The amplitudes that one computes in this case correspond to solutions to the Dirac equation [3] in one space variable and one time variable. This example is related to an observation of Richard Feynman [4]. In [7] we give a very elementary derivation of this result and we show how these amplitudes give solutions to the discretized Dirac equation, so everything is really quite exact and one can understand just what happens in taking the limit to the continuum. In this example a state of the network consists in a sequence of choices of L or R. These can be interpreted as choices to move left or right along the light-cone in a Minkowski plane. It is in summing over such paths in spacetime that the solution to the Dirac equation appears. In this case, time has been introduced into the net by interpreting the sequence of nodes in the network as a temporal direction.

More specifically, let (a, b) denote a point in discrete Minkowski spacetime in lightcone coordinates. This means that a denotes the number of steps taken to the left and b denotes the number of steps taken to the right. We let $\psi_L(a, b)$ denote the sum over the paths that enter the point (a, b) from the left and $\psi_R(a, b)$ the sum over the paths that enter (a, b) from the right. Each path P contributes $i^{c(P}$ where $c(P)$ denotes the number of corners in the path. View the diagram below.



It is clear from the diagram that

$$\psi_L(a, b+1) = \psi_L(a, b) + i\psi_R(a, b).$$

Thus we have that

$$\partial\psi_L/\partial R = i\psi_R$$

and similarly

$$\partial\psi_R/\partial L = i\psi_L.$$

This pair of equations is the Dirac equation in light cone coordinates.

This discrete derivation of the Dirac equation is simpler than the method used in [7]. I am indebted to Charles Bloom [1] for pointing this out to me. In fact, this form of the discretization is essentially Feynman's original method as is evident from the reproduction of Feynman's handwritten notes in Figure 8 of the review paper [9] by Schweber. It is still an open problem to generalize this exercise of Feynman to four dimensional discrete spacetime.

In this case of the Dirac equation in one dimension of space and one dimension of time, the discretization and the continuum limit agree perfectly. For this point see our paper [7] where we interpret the limit of the discrete paths with k corners as correspondent to terms of the form $r^k l^{k+1}/k!(k+1)!$ and $r^{k+1} l^k/(k+1)!k!$ More can be said about this agreement, as the paths with k corners and continuously varying segment length can be parametrized by points in the product of two simplices. This leads to a rigorous treatment of the measure theory associated with this special path integral. See [2] for more information on this point of view.

A similar measure theory for the full Dirac equation does not exist at the present time. This should not stop investigation of the discrete version of the equation and its corresponding path sums. These sums can be formulated just as we have done for the Schrodinger equation in the first part of this paper. A detailed version of these matters will appear in a subsequent publication.

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A CYCLICALLY–SCALED COSMOLOGY

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Abstract

The fundmanetal premise from which this paper starts is that in the physical world there are regions — *'terra icognito'* — paramaterized by values of time and/or space that are very small or very large, whose properties are essentially unobservable and hence unknowable. The implications flowing from this and the further idea that there is some cosmological limitation in the large analogous to the Heisenberg quantal limitation in the small and that time and spatial distance may not be single-valued are investigated. A novel 'Quanto-Cosmological Scale Indeterminacy Principle' is introduced, and illustrated by the new concept of an 'Ouroborosian Universe' in which 'scale' is no longer unbounded in both the small and the large but is instead cyclical. Such a universe consists entirely of singularities and can be thought of as being totally self-similar in that every space-time point in the Universe again contains the entire Universe. This situation is a manifestation of ultimate self-similarity — a much stronger property than being universally holographic because every part not merely contains structural information about the whole but contains the entire whole itself.

A 'Cyclical Scale Mapping' from a Lorentzian Space-Time Universe to a compactified complex Riemannian Sphere is developed in order to give formal meaning to the Quanto-Cosmological Scale Indeterminacy Principle. Two examples of Ouroborosian Universes are given, one a continuous, equiradial-toroidal model of Lorentzian Space-Time, the other a discrete, bit-string model, based on the 'Bi-Ouroborosian Recursive Hierarchy'.

The development of these ideas is at a rudimentary stage and suggests the emergence of such important but wholly novel concepts as *ouroborosian* topology, thermo-dynamics, gravity, entropic information theory and non-locality, to name but a few.

An Ouroborosian Universe

That which is the largest
Shall be the smallest,
And that which is the least
Shall be the greatest.
Growth and diminution
Shall be cyclical without end.
The Observer is the Observed,
And each Part shall contain the Whole
In its Entirety.
John Amson (1994)

To see a World in a grain of sand,
And a Heaven in a wild flower,
Hold Infinity in the palm of your hand,
Eternity in an hour.
William Blake, 'Auguries of Innocence', (1757-1827)

Cyclical scale
may be an uncomfortable concept,
Yet negative numbers
were once thought 'impossible',
And square roots of negative numbers
dismissed as 'imaginary'.
John Amson (1997)

In my beginning is my end. ....
Time present and time past
Are both perhaps present in time future,
And time future contained in time past.
T.S.Eliot, 'Four Quartets', (1943)

There is no reason to suppose
that the Universe exists to suit our convenience.
Fred Hoyle, 'Home is Where The Wind Blows', (1997)

Do you remember our Space-Time
before they thought of Singularities ?
Did you not foresee
That our Space-Time was
Nothing But Singularities ?
John Amson (1997)

Chapter 1 : CYCLICAL SCALE

Section 1. Introduction.

The inadequacies of extrapolating local world properties to regions remote in scale are familiar and have led to the idea that the quantum limitations "in the small" were somehow mirrored by "cosmological limitations in the large"[1]. On the one hand, the quantal limitations are due to the quantum of action being non-zero ("bounded below") and are made evident through the Heisenberg uncertainty principle. Whilst on the other hand, the cosmological limitations are due to the velocity of light being finite ("bounded above") and made evident in our inability to observe any distant part of the universe at anything other than a 'remote' time or to see any other remote part in the state at which it actually influenced the observed state of the first part. Even at first brush these two limitations appear to possess a feeling of something akin to "complementarity".

In this regard I want to suggest that a simple extrapolation of the notion of scales of size from regions of local experience to regions remote in scale — cosmological and quantal regions alike — involves a premise which is either (at least) unnoticed or (at worst) deliberately ignored, but which nonetheless most emphatically has no *a priori* validity. This premise in question is that scales of size are generally presumed to possess Asymmetry of Order[2] :—

greater than X is never *less than X*

I would also like to suggest that, if in dealing with scales of size the asymmetrical notion of *between-ness* were to be generalised to the symmetrical notion of *separation*[3], then upper and lower limits on observations would not simply be analogous but, in some sense, identical. The two "ends" of the macro–micro scale would, again in some sense, become indistinguishable :—

a system *larger than X* would no longer necessarily be *not smaller than X*

and some kind of generalisation of the Heisenberg uncertainty principle might arise which could apply symmetrically rather than being restricted just to systems much smaller than the locally-sized system.

It also seems important here to reflect that at each epoch in the history of Natural Philosophy actual observers find that the largest and the smallest measurements that they

[1] See *e.g.* E.W.Bastin [1961], W.H.McCrea [1960]

[2] See, *e.g.* Fraenkel [1966], Ch.3, §8.2. It is perhaps helpful to notice that 'asymmetrical' is the contrary, rather than the contradictory, of 'symmetrical': thus suppose "$|$" is a relation; if $x|y$ and the relation is symmetrical then we <u>always</u> have $y|x$; if asymmetrical then we <u>never</u> have $y|x$; if non-asymmetrical then we <u>might</u> have $y|x$. By definition, an 'order' has to be asymmetrical.

[3] See, *e.g.* Russell [1937], Ch.25.

can make in their universe are of orders of magnitude that are roughly reciprocals of each other, be they of Length, Time or Mass. Whilst this could be due to an accidental anthropocentrism (and hence of more value, perhaps, to physiologists and psychologists, say, than to physicists), it might simply be an instance of the Weak Anthropic Principle[4].

Or, crucially for this study, it might reinforce the idea of the existence of an invariant of the observable Universe —

it is as large as it is small

brought about by all observable magnitudes not obeying an asymmetrical ordering.

The simplest illustration of a such a 'non-asymmetrical' relationship is the familiar one illustrated by an orientated circle on which we would say that a point P is *before* a point Q (write P|Q) if the circle must be traversed clock-wise in order to pass from P to Q. This is plainly not asymmetrical, since by continuing around the circle we see that both P|Q and Q|P — the ordering is cyclical. If magnitudes in our observable Universe obeys such a non-asymmetrical, cyclical relationship — as well they might since a critical test seems unlikely with our present understanding — then quantities which we have traditionally regarded as 'larger than us' and therefore 'not smaller than us', or *vice versa*, would be capable of being both being 'larger and smaller than us'.

We already have some experience, direct or inferred, of such cyclical ordering. In a Schwarzchild-Einsteinian spherical 'Space-Time' universe (a 'bounded but infinite' universe possessed of an intrinsic curvature) the 3-dimensional sub-space of the 'spatial part' of the universe can be regarded as the surface of a 4-dimensional sphere. Here, geodesics leaving from the eye of an observer reach out through space further and further away from the observer until they finally arrive back at the observer : "given a sufficiently powerful telescope an observer can view the back of his or her own head" — "the point furthest from the observer is also the nearest". Or to put it another way : if E is 'Eye-of-observer' and B is 'Back-of-head-of-observer' and "|" denotes 'in-front-of', then we have both E|B and B|E. Spatial *distance* became cyclical : objects became 'further – nearer – further – nearer – ...' This is an instance of a closed spatial loop.

Then again, Gödel [1949] had discovered a solution of Einstein's gravitation field equations in which closed time loops can exist (even though they need not be geodesics). The investigation of closed time loops has since become a substantial part of recent cosmological studies[5].

In another direction, the conceptual dilemmas over the concepts of 'Action at a Distance' and 'Non-locality' may, at their deepest levels, be fundamentally as uncomfortable as the concept of cyclical scale. It should, perhaps, not go unnoticed that some of the unfamiliarness of the notion of *quantum potential* is on a par with that of cyclical scale. By this I mean that difficulties in interpreting what happens "in the small" or "locally" appear to be a little more explainable if what happens there does depend on what happens

[4] See, *e.g.* Barrow & Tipler [1986], Ch.4

[5] See *e.g.* Hawking & Ellis [1973], p.168 for a technical account; or Davies [1995], Ch.11 (Fig.5) for a popular account.

in the Universe as a whole. Support for this idea comes from David Bohm's arguments as the following extract from Hiley & Peat's discussion[6] shows, (the italics are their's, the underlining mine) :

> 'Unlike a classical potential, the quantum potential appears to have no point-like source. Moreover, since the field from which one derives the potential satisfies a homogeneous equation, the field is not radiated, as is, for example, the electro-magnetic field. But there are two further very important differences.'
>
> '1. The quantum potential does not produce, in general, a vanishing interaction between two particles as the distance between those particles becomes very large. Thus two distant systems may still be strongly and directly connected. This is, of course, contrary to the implicit requirements of classical physics, where it is always assumed that where two systems are sufficiently far apart, they will behave independently. This is a necessary condition if the notion of analysis of a system into separately and independent existent constituent parts is to be carried out. Thus the quantum potential seriously calls into the question the notion that all explanations of complexity must be understood by considering independent systems in interaction with each other.'
>
> '2. What is even more striking is that the quantum potential cannot be expressed as a universally determined function of all the coordinates of the particles. Rather it depends on the "quantum state" [....] of the *system as a whole*. This means that even if at some time the positions and momenta of two sets of particles are the same, but they are in different quantum states, then their subsequent evolution can be very different.'
>
> 'All of this implies that the relationship between the two particles depends on something that goes beyond what can be described in terms of these two particles alone. In fact more generally, this relationship may depend on the quantum states of even larger systems, ultimately going on to the universe as a whole. Within this view separation becomes a contingent rather than a necessary feature of nature.'

The large and the small challenge us by their sharing of the notion of being at the 'opposite ends' of our way of thinking about 'scale' in all of its manifestations. As Fred Hoyle has recently remarked [Hoyle, 1997, p.217] :

> 'I have a conviction that the large and the small are interlinked, whereas the usual view is that the small (that is, the laws) are primary and that the large (that is the Universe) is secondary. I suspect the two form a closed logical loop in which both exist together, without either being primary. There is a hint of this already in the Dirac equation.'

However, in elaborating upon this hint, Hoyle shows that within the conventional approach (avoiding any question of scale being cyclical as proposed in this study) one runs

6 In the introductory chapter to Hiley & Peat [1994;2] in Hiley & Peat [1994;1]

into awkward (even desperate) manouevres, that are at best unsatisfactory and at worst wilfully arbitrary[7] :

> 'At the level of the quantum mechanics, the mass m is the experimental mass of the electron, but, at the highest level of quantum electrodynamics, m is the so-called theoretical mass from which a quantity must be subtracted to obtain the mass measured in the laboratory, a procedure known as re-normalisation. The quantity to be subtracted is arbitrary, in that it depends on cutting off all electromagnetic fields at some arbitrarily chosen, very high frequency (that is, removing all field components with frequencies above some assigned value). The recommended scheme for avoiding this arbitrariness is to push the cutoff frequency ever higher and higher, until it goes to infinity. But then the m in Dirac's equation goes down and down until, eventually it goes to minus infinity — not a result to recommend the study of physics, you might think. Another way is to appeal to the universe to provide a definite cutoff, with m having a definite unique value determined in part by the electron and in part by cosmology. This is the solution I favor, and I think it applies to a very wide range of physical problems. Its only demerit is that one has to see deeper to apply it, which makes things harder to work through. But there is no reason to suppose that the Universe exists to suit our convenience.'

Might another alternative, of cutoff frequencies and related m masses progressing (somehow) cyclically, prove more rewarding ?

It is in the light of such tentatively questing ideas that I want to give some firmer form to my proposal of a cyclically-scaled universes. In doing so we will encounter many apparently bizarre situations in which concepts which are strictly speaking antimonies or contradictions in any familiar logical and physical sense (such as 'large' and 'small') become essentially equivalent. I shall call these our

Quanto-Cosmological Antimonies.

[7] Hoyle [*ibid*, p.215] briefly describes Dirac's equation as follows :
[...] write down two equations [...] The first defines a spinor field $X^{\dot\beta}$ from a given spinor field ξ_α, namely, $m\,X^{\dot\beta} = \left(i\,\partial^{\alpha\dot\beta} + e\,A^{\alpha\dot\beta} \right) \xi_\alpha$,
Here, e and m are the charge and the mass of the electron, $\partial^{\alpha\dot\beta}$ is the spinor form of the space-time derivatives, and $A^{\alpha\dot\beta}$ is the spinor form of the electromagnetic field, while i is the square root of -1. No work has yet been done. But now comes the big statement, a statement that controls the structure of the world, even down to the behaviour of the electronic gadgets we use routinely in our everyday lives.
The statement is, $m\,\xi_\alpha = \left(i\,\partial^{\alpha\dot\beta} + e\,A^{\alpha\dot\beta} \right) X^{\dot\beta}$, which is sophisticated but simple. You do the same thing again and get back to where you started. [...]

Section 2. The concept of Cyclical Scale.

The idea of closed spatial loops in a spherical universe has exhibited the first of my quanto-cosmological antimonies :

between 'far' and 'near'

Of course, it is not new. The existence of two contending 'distances' between two points in a space was recognised early in the development of Spherical Trigonometry, many centuries ago, when it became desirable for practical reasons to impose a conventional limit on the angular measure of the 'sides' of a spherical triangle to no more than the circumference of a semi-circle (the spherical angle π) deliberately discarding the possibility of assigning to it the measure of the larger angle 'round the back' [8].

The second of my quanto-cosmological antimonies will be seen to be between 'small' and 'large' A fruitful approach to this is to recall the well known representation of the Complex Plane **C** in the Riemann Sphere **S** *via* stereographic projection of the plane onto a unit sphere in 3-dimensional Euclidean space[9]. The projection point can be taken to be the 'north pole' of the sphere and the complex plane is represented as the 2-dimensional Euclidean plane touching the sphere tangentially at its 'south pole'. Each point on the plane is mapped in a unique way to a point on the sphere where the ray from the north pole to the point cuts the sphere. The north pole itself cannot be the image of any point of the plane, but it is declared to be the image of the plane's 'ideal point at infinity' in order to use up every point on the sphere. Euclidean distances in the plane (*planar distances*) can be matched to distances on the sphere (*spherical distances*) by at least two methods : the first uses the angular separation between two points on the sphere and the second uses the Euclidean length of the chord joining them[10]. But in each method two points near the south pole have a short spherical distance and a short planar distance, whilst two points near the north pole still have a short spherical distance but a vast planar distance.

To apply this illustration to the concept of 'scales of magnitude' rather than just to 'distances' we take advantage of the *logarithm* as an indicator of scale[11]. As trivial examples, offered simply to remind ourselves at what is at issue, the natural logarithms of the pair of large and small real numbers 3.5×10^{22} and 3.5×10^{-22} are $+22.5440\ldots$ and $-21.4559\ldots$. On the other hand, the complex logarithms of the large and small imaginary numbers $i \times 3.5 \times 10^{22}$ and $i \times 3.5 \times 10^{-22}$ will be of the complex number form $2N\pi i + 22.5440\ldots$ and $2M\pi i - 21.4559\ldots$ where N and M are any chosen positive,

[8] Todhunter [1901] (Ch.XIX) says that this conventional restriction was first lifted by Möbius [1846].

[9] This model is 'conformal but not projective' in contrast to the alternative 'projective but not conformal' model in which the projection centre is the centre of the sphere and each pair of antipodal points on the sphere is regarded as a 'single' point. For a readable account of the differences between these models, see Penrose [1978].

[10] A reminder of these standard details can be found in section 3.2

[11] The use of this idea in the present context of cyclical scale was first used by me in an article [Amson,1957] in a student magazine forty years ago.

zero or negative *integers*. We shall see later that the logarithms of imaginary numbers will be related, *via* such 'index' numbers N and M, to the (doubly infinitely many choices of) scale for time-like objects in Space-Time.

Suppose we now modify the stereographic projection of the plane onto the sphere by first of all mapping the plane to a second plane *via* the logarithm of the norms of distances on the first plane, and then mapping this second plane onto the sphere. I will fill in the details below (in Section 3). But it will be apparent that when we now talk of two points on the sphere near the north pole as 'being near' what is being implied is that the corresponding two logarithm values are 'near' in some sense – even though one corresponded to a large positive logarithm and the other to a large negative logarithm. This in turn implies that the original points to which they corresponded in the first plane were 'near in scale' even though the value of one was huge and the value of the other tiny.

Our next step is to regard the 'arctic zone' around the north pole of the sphere as special, because it is the place where we find the images of those points in the first plane whose values were either huge or tiny. If the first plane is thought of as an analogue of Space-Time, then huge and tiny can be thought of as 'cosmologically large' and 'quantally small', respectively. But these are just the sort of physical objects about which we 'know' very little. We may give formal expression to our ignorance by asserting a 'principle of indeterminacy', namely, that because we can but 'know' little about those points whose images are in this arctic zone, we are not at liberty to assume that the values of their magnitudes obey any naturally asymmetrical ordering — and hence that at the very least we should merely assume that their relative scales of magnitude are indeterminate. In other words, we are no longer at liberty to assume that those space-time objects which we customarily think of as 'quantally small' and those which we customarily think of as 'cosmologically large' are *a priori* different in scale. If that be the case, then we have exhibited my second quanto-cosmological antimony :

between 'small' and 'large'

We may encapsulate this line of thinking in what I shall call

A Principle of Quanto-Cosmological Scale Indeterminacy :
Space-Time objects whose magnitudes are to be found at the extreme limits of our observable scales of magnitudes are not necessarily structurally distinguishable.

Of course, in orthodox physics (which logically exists within the context of orthodox first-order predicate calculus), to say that two things are not distinguishable is equivalent to saying that they are in-distinguishable, meaning that they are "the same". But what I have in mind here is that if two things are not distinguishable then they may perhaps be "indistinguishable" in the sense of Parker-Rhodes [1981].

Such "indistinguishable" objects may, for example, behave as if they simultaneously possessed the structure both of a minute atomic nature and a vast cosmological nature.

Whilst this is clearly unlikely — indeed the mere thought of it may be abhorrent — in an "orthodox universe" (whatever that may be), it could certainly be the case if a universe were to be *ouroborosian*. What this means will be explained in Section (5).

Section 3. The Construction of a Cyclical Scale Mapping.

We begin with a Lorentzian space $\mathbf{L}^{1,3}$ of $(-1,+3)$ Sylvester type (or as some would prefer a Minkowskian space of $(-3,+1)$ type), so that we can think of it as 4-dimensional Space-Time with coordinates written either as x_0, x_1, x_2, x_3 or t, x, y, z, as convenient. It is a 4-dimensional real space $\mathbf{R}^4$ equipped with a Lorentzian inner-product and norm. (A useful account of Lorentzian Space, Elliptical Space and Hyperbolic Space can be found in Ratcliffe [1994]). The Lorentzian Norm $\|.\|_L$ on $\mathbf{L}^{1,3}$ is given in terms of the Lorentzian inner-product $\bowtie$ as follows :

$$\begin{aligned} x \bowtie y &= -x_0 \cdot y_0 + x_1 \cdot y_1 + x_2 \cdot y_2 + x_3 \cdot y_3 \\ \|x\|_L &= (x \bowtie x)^{1/2} . \end{aligned}$$

Note that $x \bowtie y$ is a real number which can be a positive, zero, or negative. Hence, because $\|x\|_L^2$ can be negative, the Lorentzian norm can be imaginary (it cannot, however, be a general complex number of the form $\xi + i\eta$ with both ξ and η nonzero together.). Nevertheless, the Lorentzian norm is a mapping from Lorentzian space into the complex numbers rather than into the real numbers (as is usual in Euclidean space and most Hilbert spaces) :

$$\|.\|_L \; : \; \mathbf{L}^{1,3} \longrightarrow \mathbf{C} .$$

The *Light Cone* $\mathbf{K}^3$ in $\mathbf{L}^{1,3}$ is the set of all points with zero Lorentzian norm; points in $\mathbf{K}^3$ are *light-like* and satisfy

$$x_0^2 = x_1^2 + x_2^2 + x_3^2 ,$$

and they are *positive* [*negative*] if and only if $x_0 > 0$ [$x_0 < 0$].

Points in $\mathbf{L}^{1,3}$ with positive Lorentzian norm are *space-like* and satisfy

$$x_0^2 < x_1^2 + x_2^2 + x_3^2 ;$$

the *exterior* of the Light Cone is the open subset in $\mathbf{R}^4$ consisting of all space-like points.

Points in $\mathbf{L}^{1,3}$ with imaginary Lorentzian norm are *time-like* and satisfy

$$x_0^2 > x_1^2 + x_2^2 + x_3^2 ;$$

the *interior* of the Light Cone is the open subset in $\mathbf{R}^4$ consisting of all time-like points; such points are *positive* [*negative*] if and only if $x_0 > 0$ [$x_0 < 0$].

3.1 The Scaling Map

We now define what we may call a *scaling map* from Lorentzian space to the extended complex plane $\mathbf{C}_\infty$ (*i.e.* the complex plane together with the 'point at infinity') :

$$\Phi \quad : \quad \mathbf{L}^{1,3} \quad \xrightarrow{\|\cdot\|} \quad \mathbf{C} \quad \xrightarrow{\text{Log}} \quad \mathbf{C}_\infty$$

by setting $\Phi(x) = \text{Log}\,(\,\|x\|_L\,)$ for all points x in the Lorentzian space $\mathbf{L}^{1,3}$ except those on the light cone and $\Phi(x) = \infty$ for all points x on the Lorentzian light cone (where $\|x\|_L = 0$).

The function 'Log' is the complex logarithm because its variables $\|x\|_L$ can be imaginary. This means that we must take into account the many-valued-ness of 'Log'.

Technically, 'Log' is a *set-valued* mapping

$$\text{Log(z)} \;=\; \{\,\text{w} \in \mathbf{C} \;:\; \exp(\text{w}) = \text{z}\,\}\;.$$

(A useful account of this particular aspect of 'Log' can be found *e.g.* in Beardon [1979].) There are infinitely many members $w \in \text{Log(z)}$, each one is called a *choice* (or *branch*) of Log(z). This will have important consequences for us when we come to talk about 'Parallel Universes'.

Writing a complex number z in its 'polar form' $z = |z|\cdot\exp(i\,\theta)$ where $\theta \in \text{Arg(z)}$ is a 'choice' of one from the infinitely many possible values all differing from one another by multiples of 2π, we see that

$$\text{Log(z)} \;=\; \{\,\log_e|\text{z}| + \text{i}\,\theta \;:\; \theta \in \text{Arg(z)}\,\}\;.$$

The particular choice of $\theta = \theta_0$ with $0 \le \theta_0 < 2\pi$ is usually called the 'Principal Value' of θ, and then the 'Principal Value' of 'Log' is

$$\text{Log}_0(\text{z}) \;=\; \log_e|\text{z}| + \text{i}\,\theta_0\,.$$

Since the values taken by $\|.\|_L$ are either purely real and nonnegative or purely imaginary, the values of θ in the expressions for $\Phi(x) = \text{Log}\,(\,\|\text{x}\|_\text{L}\,)$ for all points x in the Lorentzian space $\mathbf{L}^{1,3}$ will all be of the simpler forms

$$\theta \;=\; \begin{cases} 0 & \text{if x is space-like,} \\ \frac{1}{2}\,\pi + n\,\pi & \text{if x is time-like,} \\ 0 & \text{if x is light-like (by convention).} \end{cases}$$

The Logarithm function is defined on the whole of the complex plane except for the origin $z = 0$ (since there is no number w such that $\exp(w) = 0$). We extend it to the Extended Complex Plane $\mathbf{C}_\infty$ — including both the origin and the point at infinity, by agreeing, as usual, to set $\text{Log}(0) = \text{Log}(\infty) = \infty$. (This is similar to the convention that extends the real logarithm function to the extended nonnegative real line $0 \le x \le +\infty$ by setting $\log_e(0) = -\infty$ and $\log_e(+\infty) = +\infty$; but in the complex case there is but a single complex 'infinity' ∞.)

It should also be noted that although we are making use of Log on the extended complex plane we are actually using only part of that domain, namely the Real axis and the Imaginary axis. This is because the Lorentzian space is partitioned into three mutually exclusive categories of points (light-like, time-like, space-like) and the function $\|,\|$ on $\mathbf{L}^{1,3}$ takes only real values (for light- and space-like points) and imaginary values (for time-like points) and not any general complex values of the form $x+iy$ with both x and y nonzero.

3.2 The Sphericalisation Map

For our next step towards a Cyclical Scale we must utilise the classical stereographic projection

$$\Sigma : \mathbf{C}_\infty \longrightarrow \mathbf{S}^2$$

of the extended complex plane $\mathbf{C}_\infty$ to the 2-dimensional spherical surface $\mathbf{S}^2$ of unit radius in the 3-dimensional Euclidean ambient space $\mathbf{E}^3$. Coordinates in the latter space are denoted by (ξ, η, ζ). We identify the extended complex plane with the 2-dimensional real Euclidean plane $\mathbf{E}^2$ having $\zeta = 0$, extended to include the 'point at infinity'. A point $z = x+iy$ in $\mathbf{C}$ is therefore identified with a point $w = (\xi, \eta, 0)$ in $\mathbf{E}^2$ by setting $\xi = x$, $\eta = y$. The centre of the sphere is at the origin $\mathtt{0} = (0,0,0)$ of the ambient space, the centre of projection is the 'north pole' $\mathtt{N} = (0,0,1)$, and the plane $\mathbf{E}^2$ cuts the sphere along its equator which coincides with the unit circle in the plane. Therefore, the stereographic projection of a point $z = x+iy$ in $\mathbf{C}$ is the point $s = (\xi, \eta, \zeta)$ in $\mathbf{S}^2$ where

$$\xi = \frac{2x}{x^2+y^2+1}, \qquad \eta = \frac{2y}{x^2+y^2+1}, \qquad \zeta = \frac{x^2+y^2-1}{x^2+y^2+1},$$

in Euclidean coordinates, or, in complex coordinates

$$\xi = \frac{z+\bar{z}}{|z|^2+1}, \qquad \eta = \frac{z-\bar{z}}{|z|^2+1}, \qquad \zeta = \frac{|z|^2-1}{|z|^2+1};$$

and the stereographic projection of the point at infinity in $\mathbf{C}_\infty$ is the centre of projection $\mathtt{N} = (0,0,1)$.

If $\xi(z,w)$ denotes the *chordal distance* in the sphere $\mathbf{S}^2$ of the stereographic images of two points z, w in $\mathbf{C}_\infty$ (one or both of which may be ∞), then

$$\begin{aligned}
\chi(z,w) &= \frac{2\,|z-w|}{\sqrt{(1+|z|^2)}\cdot\sqrt{(1+|w|^2)}}, \\
\chi(z,\infty) &= \frac{2}{\sqrt{(1+|z|^2)}}, \\
\chi(\infty,\infty) &= 0.
\end{aligned}$$

This provides us with a metric for the sphere $\mathbf{S}^2$. Another distance function on the sphere is the natural spherical angle between two points $s = \Sigma(z)$ and $t = \Sigma(w)$ on $\mathbf{S}^2$ given *via* the arc-cosine function and the Euclidean inner product

$$\begin{aligned}
\angle(s,t) &= \cos^{-1}(s \bullet t) \\
s \bullet t &= \xi_s.\xi_t + \eta_s.\eta_t + \zeta_s.\zeta_t
\end{aligned}$$

using an obvious notation, and where the coordinates ξ_s *etc.*, can be given in terms of $z, \bar{z}, |z|^2$, *etc.*, as before.

The metrics determined by these two distance functions χ and $\angle$ on $\mathbf{S}^2$ can be shown to be equivalent.

3.3 The Cyclical Scale Map

We are now able to construct our Cycle Scale map as the composition of these two maps, resulting in a map Ξ from Lorentzian space $\mathbf{L}^{1,3}$ to the stereographic sphere $\mathbf{S}^2$:

$$\Xi \quad : \quad \mathbf{L}^{1,3} \xrightarrow{\Phi} \mathbf{C}_\infty \xrightarrow{\Sigma} \mathbf{S}^2$$

or, writing it out fully,

$$\Xi \quad : \quad \mathbf{L}^{1,3} \xrightarrow{\|\cdot\|} \mathbf{C} \xrightarrow{\infty} \mathbf{C}_\infty \xrightarrow{\mathrm{Log}} \mathbf{C}_\infty \xrightarrow{\Sigma} \mathbf{S}^2$$

We shall call the final image space $\mathbf{S}^2$ the SCALE SPHERE for the Lorentzian space $\mathbf{L}^{1,3}$.

There are some immediate consequences of our definition of the Cyclical Scale map.

The Light Cone $\mathbf{K}^3$ in $\mathbf{L}^{1,3}$, being the set of all points with zero Lorentzian norm, is mapped by the extended complex Logarithm to the point at infinity ∞ in the extended complex plane $\mathbf{C}_\infty$ (recall, the ordinary logarithm of zero, though undefined, 'is' $-\infty$), and hence onto the north pole N of the stereographic sphere :

- *The north pole of the Scale Sphere is the Cyclical Scale Image of the Light Cone.*

By an *Arctic Zone* of the Scale Sphere $\mathbf{S}^2$ we mean a neighbourhood A of the north pole N, small in the metric sense, *i.e.* an open set containing a disc (spherical cap) of some small radius $\epsilon > 0$, say, and centred on N.

Now, the quantally small spatial or temporal objects in Lorentzian Space-Time $\mathbf{L}^{1,3}$ have very small real or imaginary Lorentz norms, and the Logarithms of small quantities have real parts which are highly negative (*i.e.* near to $-\infty$). Hence :

- *An arctic zone of the Scale Sphere contains the Cyclical Scale Images of all quantally small objects in Space-Time.*

Again, the cosmologically large spatial or temporal objects in Lorentzian Space-Time $\mathbf{L}^{1,3}$ have very large real or imaginary Lorentz norms, and the Logarithms of large quantities have real parts which are highly positive (*i.e.* near to $+\infty$). Hence :

- *An arctic zone of the Scale Sphere contains the Cyclical Scale Images of all cosmologically large objects in Space-Time.*

The last two items reveal my sought for conflux of the 'large' and the 'small' :

- *A common arctic zone of the Scale Sphere contains the Cyclical Scale Images of all both quantally small and cosmologically large objects in Space-Time and of all objects in the Light Cone.*

Additional categorisations of zones on $\mathbf{S}^2$ include :

- *The southern hemisphere of the Scale Sphere contains the Cyclical Scale Images of all those objects in Space-Time* near *us in scale.*
- *The northern hemisphere of the Scale Sphere contains the Cyclical Scale Images of all those objects in Space-Time* increasingly remote from *us in scale.*

To give some idea of 'how small' the Arctic Zone is, we can estimate the 'latitude' of the image of a Space-Time object of Planck Length 10^{-31} metres : we have $\log_e(10^{-31}) = -73.68$ which is the ξ coordinate, say, of a point in $\mathbf{E}^2$, which maps stereographically into a point on $\mathbf{S}^2$ with 'latitude' 88.34°. Similarly, for an object the size of the 'radius of curvature of the universe', 10^{+26} metres, its 'latitude' on $\mathbf{S}^2$ is 88.16°. Again, for a duration equal to the Planck Time, 10^{-43} seconds, its 'latitude' on $\mathbf{S}^2$ is 88.82°. The age of some stars in our Galaxy may be 18 billion years, *i.e.* 10^{17} seconds; this duration has a 'latitude' on $\mathbf{S}^2$ of 87.24°.

Section 4. The Quanto-Cosmological Scale Indeterminacy Principle.

This section continues the development of the idea of Scale Indeterminacy which I introduced in Section 1 above as a 'Principle of Quanto-Cosmological Scale Indeterminacy'. For brevity I will use the abbreviation ' QCSI ' for this principle.

The above construction of a Cyclical Scale Mapping Ξ from Lorentzian Space-Time $\mathbf{L}^{1,3}$ to the Scale Sphere $\mathbf{S}^2$ shows that a small Arctic Zone AZ in the latter contains the images of all the quantally small and cosmologically large objects in $\mathbf{L}^{1,3}$ and the single image (the north pole) of all the objects in the Light Cone $\mathbf{K}^3$ in $\mathbf{L}^{1,3}$.

The Cyclical Scale Mapping Ξ is not one-to-one, however; many objects in its source $\mathbf{L}^{1,3}$ have the same image (scale) in its range $\mathbf{S}^2$. In particular, objects at the extreme limits (quantally and cosmologically) of our observability may have the same image (scale) in the arctic zone AZ. The QCSI Principle then asserts that such objects are not necessarily distinct. To give a definite meaning to the idea of being 'at the extreme limits of our observability' we may say that such is the case if such an object (X, say) had its scale image $\Xi(X)$ in an Arctic Zone AZ of some specific spherical latitude Ω and we may call this specific Arctic Zone the ' Ω-zone '. It would perhaps be a matter of professional opinion to fix on an actual working value for this Ω latitude, but presumably it would have to be a little over " 88° " to encompass objects of both Planck length and curvature radius, and of Planck time and Age of Universe, as noted above. Since the Ξ image of every point in the Light Cone is the north pole (the centre of the Ω-zone) the Ω-zone contains the image of the Light Cone. It could be important to consider Ω-zone without this north pole image of the Light Cone; we may call this the 'punctured Ω-zone', and denote it by Ω_0-zone.

In the light of these concepts of an 'omega zone' (Ω-zone) and a 'punctured omega zone' (Ω_0-zone) I can now suggest two stronger forms of the QCSI Principle, which together with the original version from Section 1 now read as follows :

(I) : The Weak Principle of Quanto-Cosmological Scale Indeterminacy :

Space-Time objects whose magnitudes are to be found at the extreme limits of our observable scales of magnitudes are not necessarily structurally distinguishable.

(II) : The Stronger Principle of Quanto-Cosmological Scale Indeterminacy :

Space-Time objects whose Ξ images lie within the Ω_0-zone are structurally indistinguishable.

(III) : The Strongest Principle of Quanto-Cosmological Scale Indeterminacy :

Space-Time objects whose Ξ images lie within the Ω-zone are structurally indistinguishable.

Of course, it may be important to replace the idea of a specific Ω-zone determined by a specific Ω latitude, by one in which the latter is determined intrinsically by the structure of the Space-Time in question (perhaps in relation to the total mass, for example), rather than empirically, as above.

And the significance of the multiple θ values of Log for cyclical time when it comes to identifying multiple copies of $L^{1,3}$ under QCSI-(I), QCSI-(II) or QCSI-(III) needs much further thought !

Chapter 2 : OUROBOROSIAN UNIVERSES

Section 5. The concept of an Ouroborosian Universe.

Recall that the *Ouroboros* is the mythical beast, usually a serpent, which devours itself by eating its tail[12].

By an *Ouroborosian Universe*, I shall mean a universe in which each point is, in some sense, (a copy of) the entire Universe.

Each point of an Ouroborosian Universe is evidently a singularity, of a peculiar and unfamiliar (not to say terrifying) kind. It is a much stronger kind of singularity than, say, the kind that would arise at each so-called 'ultra-sub-microscopic worm-hole' of Planck length and which would 'merely' link far distant regions of the Universe and allow the leakage of information.

An Ouroborosian universe also exhibits a novel, extreme form of *fractal* behaviour, one that is characterised by the inherent *cyclicality* of its 'fractal-ness' that is absent from the essentially *linearly directed* self-similarity of conventional fractal situations. As an instance of this difference we may use the familiar example of the fractal 'fern' in which each 'leaf' is self-similar to the whole fern, in a linearly descending scale of identification. However, in an ouroborosian fractal fern, at some stage in its unfolding self-similarity, each tiny, most recently expanded, leaf is no longer merely self-similar to but 'is' the original fern : the self-similarity has become cyclical.

Section 6. Examples of Ouroborosian Universes.

(1) A CONTINUUM OUROBOROSIAN UNIVERSE.

Earlier, in introducing the idea of a cyclically-scaled cosmology I started by noting the existence of closed spatial loops in the <u>spatial</u> subspace consisting of a 3-dimensional spherical hyper-surface in 4-dimensional Space-Time. But we may usefully start with any 4-dimensional Space-Time having a Lorentzian[13] structure of a $(-1, +3)$ Sylvester type (or, as some would prefer, of a (Minkowskian) $(+3, -1)$ type).

Suppose we now embed this 4-dimensional Lorentzian space as a hyper-sphere in a 5-dimensional space. In analogy to the construction of the Riemann sphere as a compact representation of the extended complex plane *via* stereographic projection, we may envisage the hyper-sphere as a representation of the extended 4-dimensional Space-Time.

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[12] From the Greek, *ουρας* – arse or tail, *βορος* – devour.

As a harmless diversion, not quite totally unrelated to the matter in hand, one may attempt to evaluate the 'time' needed for the Ouroboros to complete its auto-digestive evanescence, using ideal models and various hypotheses concerning such fantastical physiognomies and gastronomic processes.

[13] A useful account of Lorentzian Space, Elliptical Space and Hyperbolic Space, sufficient for our purposes, can be found in Ratcliffe [1994].

The stereographic projection involved has its projection centre at the north pole N of the hyper-sphere; it is the image of the point-at-infinity in Space-Time. It is therefore the place where the branches of the Light Cone meet-at-infinity; the place where the Past and the Future coincide; the place where any World-line becomes a closed loop.

It is now possible to take one more step to transform this closed hyper-sphere representation of Space-Time into an Ouroborosian Universe. Perhaps the simplest way we can do this is to identify the north pole N with its antipode at the south pole — the origin O — as a single 'Observer pode/antipode' which we denote by Ø and call the 'conflux' point.

Under this identification, the hyper-spherical surface becomes an *Equiradial Torus* (a mint-with-a-hole in which the radius of the hole has vanished)[14], and the Observer pode/antipode Ø is a singularity of infinite total curvature. For convenience we may refer to our Equiradial-Toroidal construction as our **ET** Universe.

The image in **ET** of the Light Cone re-enters itself at the conflux point Ø; small scale Time is simultaneously large scale Time; all World-lines and indeed Past and Future become cyclical. By repeated excursions around **ET** *via* Ø we repeatedly re-enter what are in effect identical copies of our universe : **ET** consists of what at first appears to be infinitely many sequential universes — in effect, an infinitely-sheeted Riemannian surface. But since the remotest Time 'is' the nearest Time (by virtue of our quanto-cosmological indeterminacy principle), entering each fresh cycle in remotest cosmological Time is the same as entering it in least quantal Time — that is, virtually instantaneously — so that any one of these infinitely many sequential universes are 'instantaneously' accessible from any other : what first emerge as *sequential* universes now take on the rôle of *parallel* universes. In **ET** each observer's conflux point has become a proto-wormhole through which access can be gained from any one to any other of these parallel universes.

The above construction has also delivered an example of my third quanto-cosmological antimony :

between 'sequential' and 'parallel'

(2) A DISCRETE OUROBOROSIAN UNIVERSE.

For this example we generate a discrete, recursive, binary structure which, so far as this present study is concerned, has no direct relationship with an "observable universe" but which neatly exhibits the extreme cyclically-fractal property, introduced earlier in this section, which is absent from examples of conventional fractal behaviour.

[14] Recall: a torus is the surface of revolution created by sweeping a circle (of radius a) around an axis of revolution lying in the plane of the circle; the centre of the circle sweeps out another circle (of radius b) whose plane is perpendicular to the axis. In **ET** the two parametric radii a and b are equal, so that the first circle is tangential to the axis at the centre of **ET**. A 3-dimensional version of **ET**can be got by rotating the boundary skin of *Schatz's Apple* (see *e.g.* [Wilansky, 1964], p.23, Example 2) around its 'stalk' and deleting the upper & lower segments of its stalk.

The binary structure developed here is fairly trivial, unlike the more elaborate one I introduced in a more special context[15], but it retains the "Bi-Ouroborosian" feature essential for its construction.

We start with two *primitive* symbols 'o' and '1' which are undefined except that they shall be distinguishable (initially in an elementary sense, but perhaps later in a the sense of being 'not Parker-Rhodes indistinguishable' [1981]). Although typographically similar to the ordinals '0, 1' the symbols o,1 should not automatically be identified with such ordinals.

We first define $\mathtt{H}_0$ to be the set $\{\mathtt{o}, \mathtt{1}\}$ together with 'discrimination' (denoted by '+', and logically equivalent to XOR) defined by the usual rules :-

$$\mathtt{o} + \mathtt{o} = \mathtt{o} = \mathtt{1} + \mathtt{1}, \quad \mathtt{o} + \mathtt{1} = \mathtt{1} + \mathtt{o} = \mathtt{u}.$$

At the same time we define $\mathtt{K}_0$ to be the same set $\{\mathtt{o}, \mathtt{1}\}$ together with 'anti-discrimination' (denoted by '$\mp$' – read as 'bar-plus' not 'plus-or-minus'), and logically equivalent to NXOR) defined by these "dual" rules :-

$$\mathtt{o} \mp \mathtt{o} = \mathtt{1} = \mathtt{1} \mp \mathtt{1}, \quad \mathtt{o} \mp \mathtt{1} = \mathtt{1} \mp \mathtt{o} = \mathtt{o}.$$

Note that anti-discrimination $\mp$ is the same as replacing discrimination + by the composite operation +1+. Plainly $\mathtt{H}_0$ and $\mathtt{K}_0$ are each the "dual" of the other, and o is the 'neutral' element for discrimination + on $\mathtt{H}_0$ and we can think of 1 as the 'unit' element; whereas their rôles are exchanged in $\mathtt{K}_0$:- 1 is the 'neutral' element for anti-discrimination $\mp$ on $\mathtt{K}_0$ and o is the 'unit' element. We can also think of 1 as the anti-neutral in $\mathtt{H}_0$. Of course, $\mathtt{H}_0$,+ and $\mathtt{K}_0$,$\mp$ are isomorphic algebraic structures, and to that extent indistinguishable, but we need to keep their constructional distinction in mind here.

Next we define $\mathtt{H}_1$ to be the system of 2-ples $x = [x_0, x_1]$ with $x_i \in \{\mathtt{o}, \mathtt{1}\}$ and extend discrimination + on $\mathtt{H}_0$ to the larger system $\mathtt{H}_1$ *via* components :- $x + y = z$ means $z_i = x_i + y_i$ for each index $i = 0, 1$. And in a similar way we define $\mathtt{K}_1$ to be the system of 2-ples $x = [x_0, x_1]$ with $x_i \in \{\mathtt{1}, \mathtt{o}\}$ and extend anti-discrimination $\mp$ on $\mathtt{K}_0$ to the larger system $\mathtt{K}_1$ *via* components :- $x + y = z$ means $z_i = x_i \mp y_i$ for each index $i = 0, 1$. Thus

$$\mathtt{H}_1 = \{[\mathtt{o},\mathtt{o}],\ [\mathtt{o},\mathtt{1}]\ [\mathtt{1},\mathtt{o}]\ [\mathtt{1},\mathtt{1}]\} \equiv \begin{bmatrix} \mathtt{o} & \mathtt{o} & \mathtt{1} & \mathtt{1} \\ \mathtt{o} & \mathtt{1} & \mathtt{o} & \mathtt{1} \end{bmatrix} \quad \text{equipped with} \quad +$$

$$\mathtt{K}_1 = [[\mathtt{1},\mathtt{1}],\ [\mathtt{1},\mathtt{o}]\ [\mathtt{o},\mathtt{1}]\ [\mathtt{o},\mathtt{o}]\} \equiv \begin{bmatrix} \mathtt{1} & \mathtt{1} & \mathtt{o} & \mathtt{o} \\ \mathtt{1} & \mathtt{o} & \mathtt{1} & \mathtt{o} \end{bmatrix} \quad \text{equipped with} \quad \mp$$

so that $[\mathtt{o},\mathtt{o}]$ is the neutral element for + in $\mathtt{H}_1$ and $[\mathtt{1},\mathtt{1}]$ is the neutral element for $\mp$ in $\mathtt{K}_1$.

We note that both $\mathtt{H}_1$ and $\mathtt{K}_1$ are commutative groups and (trivially) become vector spaces over the field $\mathbf{Z}_2$ of two elements $(0, 1)$, if we include scalar multiplication by $1.x = x$ for all x and $0.x = [\mathtt{o},\mathtt{o}]$ in $\mathtt{H}_1$, and by $1.x = x$ for all x and $0.x = [\mathtt{1},\mathtt{1}]$ in $\mathtt{K}_1$.

[15] Amson, [1985]

Each of the combined systems

$$\mathbf{H} : \mathrm{H}_0 \longrightarrow\!\!\!\!\rightarrow \mathrm{H}_1$$
$$\mathbf{K} : \mathrm{K}_0 \longrightarrow\!\!\!\!\rightarrow \mathrm{K}_1$$

(where the 'double-arrow' symbol $\longrightarrow\!\!\!\!\rightarrow$ denotes this construction of the group of 2-ples from the pair of elements 'o' and '1') can be thought of as an extremely primitive 'Hierarchy System' of just two Levels in the sense of the 'Discrimination Hierarchy' of Bastin *et. al.*[1979] — albeit without any of the more complicated embeddings of one level into the next and certainly with no consequential, ultimate 'Parker-Rhodes termination' at a critical level. We can think of the first one **H** as the 'direct hierarchy' and the second one **K** as the 'dual hierarchy'.

Furthermore we can combine them both into one 'combined hierarchy', written '$\mathbb{HK}$' and formed as the ordered pair $(\mathbf{H}, \mathbf{K})$ constituted of the direct hierarchy **H** and the dual hierarchy **K** :-

$$\mathbb{HK} = \begin{cases} \mathbf{H} : \mathrm{H}_0 \longrightarrow\!\!\!\!\rightarrow \mathrm{H}_1 \\ \mathbf{K} : \mathrm{K}_0 \longrightarrow\!\!\!\!\rightarrow \mathrm{K}_1 \end{cases}$$

Finally, we introduce a 'recursive' or 'ouroborosian' aspect into the combined hierarchy by now *identifying the Primitive Symbols* o, 1 *with the Highest Levels in the derived hierarchies* :-

$$\mathrm{o} \equiv \mathrm{H}_1, \quad 1 \equiv \mathrm{K}_1 \,.$$

The system that then results is our desired 'mini' BI-OUROBOROSIAN HIERARCHY of just two levels

$$\mathbb{HK}(\text{with } \mathrm{o} \equiv \mathrm{H}_1,\ 1 \equiv \mathrm{K}_1)$$

— albeit a very much smaller system than the full system introduced in Amson [1985].

The recursivity, and hence the discrete fractality, of $\mathbb{HK}(\text{with } \mathrm{o} \equiv \mathrm{H}_1,\ 1 \equiv \mathrm{K}_1)$ can be very imperfectly illustrated by considering the (infinite regression) of identifications in **H** illustrated in Figures 1 & 2 at the end of this section. The next stage corresponding *e.g.* to the one shown in Figure 1, for the second item in Figure 1, is illustrated in Figure 2.

At each stage of the 'forward' identification of each of the elements with the eight elements of H_1 the 'granularity' of the perceived structure increases 8-fold :

$$1 \rightarrow 8 \rightarrow 64 \rightarrow 512 \rightarrow 4096 \cdots$$

and in the 'reverse' identifications decreases 8-fold — but both ways 'without end'. Each time you identify 'forwardly' you enter an increasingly elaborate structure, and each you identify 'reversedly' the component you identify with is itself just one of a system of many components, and so on ... Each one of the systems you entered in reverse is itself one of the systems 'round the back' into which you could also eventually expand forwardly.

Of course, we have implicitly assumed that such a recursively ouroborosian construction is indeed doubly-infinite (infinitely regressive in both directions). It would be instructive to consider the feasibility of some 'Parker-Rhodes'-type termination which would force a finitely-ouroborosian construction of the kind that played such an essential rôle in the Bi-Ouroborosian Universe in Amson [1985].

Figure 1

$$\cdots\leftrightarrow \begin{bmatrix} o \\ o \end{bmatrix} \leftrightarrow \begin{bmatrix} H_1 \\ H_1 \end{bmatrix} = \begin{bmatrix} o & o & 1 & 1 \\ o & 1 & o & 1 \\ o & o & 1 & 1 \\ o & 1 & o & 1 \end{bmatrix} \leftrightarrow \cdots \leftrightarrow \cdots$$

$$\cdots\leftrightarrow \begin{bmatrix} o \\ 1 \end{bmatrix} \leftrightarrow \begin{bmatrix} H_1 \\ K_1 \end{bmatrix} = \begin{bmatrix} o & o & 1 & 1 \\ o & 1 & o & 1 \\ 1 & 1 & o & o \\ 1 & o & 1 & o \end{bmatrix} \leftrightarrow \cdots \leftrightarrow \cdots$$

$$\cdots\leftrightarrow \begin{bmatrix} 1 \\ o \end{bmatrix} \leftrightarrow \begin{bmatrix} K_1 \\ H_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & o & o \\ 1 & o & 1 & o \\ o & o & 1 & 1 \\ o & 1 & o & 1 \end{bmatrix} \leftrightarrow \cdots \leftrightarrow \cdots$$

$$\cdots\leftrightarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix} \leftrightarrow \begin{bmatrix} K_1 \\ K_1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & o & o \\ 1 & o & 1 & o \\ 1 & 1 & o & o \\ 1 & o & 1 & o \end{bmatrix} \leftrightarrow \cdots \leftrightarrow \cdots$$

and similarly for similar infinitely regressive identifications in **K**.

Figure 2

The next stage corresponding to the <u>second item</u> in Fig.1

$$\left[\begin{array}{cccccccccccccccc} o & o & 1 & 1 & o & o & 1 & 1 & 1 & 1 & o & o & 1 & 1 & o & o \\ o & 1 & o & 1 & o & 1 & o & 1 & 1 & o & 1 & o & 1 & o & 1 & o \\ \\ o & o & 1 & 1 & 1 & 1 & o & o & o & o & 1 & 1 & 1 & 1 & o & o \\ o & 1 & o & 1 & 1 & o & 1 & o & o & 1 & o & 1 & 1 & o & 1 & o \\ \\ 1 & 1 & o & o & 1 & 1 & o & o & o & o & 1 & 1 & o & o & 1 & 1 \\ 1 & o & 1 & o & 1 & o & 1 & o & o & 1 & o & 1 & o & 1 & o & 1 \\ \\ 1 & 1 & o & o & o & o & 1 & 1 & 1 & 1 & o & o & o & o & 1 & 1 \\ 1 & o & 1 & o & o & 1 & o & 1 & 1 & o & 1 & o & o & 1 & o & 1 \end{array}\right]$$

with similar expressions for the next stage of the other three items in Fig.1.

Chapter 3 : IMPLICATIONS & SPECULATIONS

We may begin to explore some of the implications of Cyclical Scale and Ouroborosian Universes. Many of the observations noted in this section are, for the present, unavoidably inconclusive questions and speculations which inevitably challenge almost all the assumptions we, in the Western scientific culture, have consistently developed and absorbed throughout the past in the context of a Linearly Scaled Universe.

For convenience, I will use the abbreviation CSC ('Cyclically Scaled Cosmology'), or CSC Universe, to denote any Space-Time Universe equipped with a cyclical scale mapping as described in Section 3, and, in particular, for any Ouroborosian Universe as defined in Section 5.

Section 7. Self-similarity & Holograhic Phenomena.

The coalescing in our CSC space of the Observer's 'Local Location' with the Observer's 'Remotest Location' when interpreted in terms of 'scale' means the coalescing of the quantally small with the cosmologically large. This in turn implies that each quantally individual 'point' is also the entire cosmological 'system' — and this holds for each and every Observer, each and every location in the CSC Universe. This in its turn implies, *via* the cyclical nature of scale, that as each point 'expands' into the whole Universe, each point in this expanded Universe is again the entire cosmological system, and so on ...

This is readily exhibited, by construction, in the discrete model of an Ouroborosian Universe described in Section 8 above.

Thus a CSC Universe is *cyclically self-similar* at every point, and in Space-Time not just in Space or just in Time. Every point in the Universe 'is' the entire Universe again — "the whole is contained in each part" — a quintessentially *holographic* concept. An immediate consequence is that *non-locality* is an intrinsic property of the CSC Universe, a situation which has immediate resonances with the Bohm's notions of Implicate and Explicate Order[16] ...

Section 8. Speculative Phenomena at Remote Times.

? A Cyclically Entropic Universe rather than Gold's Symmetrically Entropic Universe ? Instead of entropy growing from a 'big bang', then decreasing again to a 'big crunch', the largest values of entropy in CSC are also the smallest — in some sense (what sense ?) — so that 'big bang' = 'big crunch'. CSC is intrinsically (hologrammatically) a 'Steady State Universe' in which every component is a 'Big Bang Universe'.

? Why bother with *e.g.* "Wheeler-Feynman Absorber Theory of Radiation" attempts at showing that radiation is not intrinsically asymmetric, when, in CSC, radiation is both expanding from a source and contracting upon it "round-the-back" ?

[16] Bohm [1980], *etc.*.

Section 9. Speculative Phenomena at Large Distances.

? Quasar Radiation = Çerenkov / Synchroton Radiation 'leaking through' from the 'Small' into the 'Large' ?

? Olber's Paradox[17] revisited ... What is the status (and rôle) of Olber's Paradox in a cyclically ouroborosian cosmology ?

? Gravity force = strong force "as seen from the back", *i.e.* gravitons are none other than gluons "as seen from the back". (Would this numerical constraint impose a metrical bound on the "size" of the ouroborosian universe ?)

Section 10. Speculative Phenomena at Remote Times and Large Distances.

? The 'quantum of mass' = the mass of the Universe "as seen from the back", and *vice versa.* This speculation could be most significant in view of the opinion that "the basic quantization is the quantization of mass"[18]

? The 'quantum of time' (10^{-43}) = 'eternity', *i.e.* the time from the 'Big Bang' to the 'Big Crunch' "as seen from the back", and *vice versa.*

? The 'quantum of length', *i.e.* Planck length (10^{-38}) = the 'radius' R of the Universe "as seen from the back", and *vice versa.* (The 'radius' of the universe when the 'time' was the quantum of time.)

Section 11. Cyclical Potentials & Cyclical Gravitation.

? *Cyclical Gravitational Attraction between particles.*

For a very simple-minded illustration of Cyclical Gravitational Attraction we may imagine gravity acting — unconventionally — over our ordinary 2-dimensional 'spherical' universe, the sphere embedded in ordinary Euclidean 3-space, in which the metric is ordinary great-circle distance $|\mathtt{P}-\mathtt{Q}| = R.\alpha(\mathtt{P},\mathtt{Q})$ where R is the radius of the spherical universe and $\alpha(\mathtt{P},\mathtt{Q})$ is the angle subtended at the centre of the spherical universe by the two points P and Q.

Recall that the ordinary 3-dimensional Newtonian gravitational attraction, between two masses $\mathtt{m}_1$ and $\mathtt{m}_2$ located at the two points P and Q on the sphere, would be

$$G\frac{\mathtt{m}_1\,\mathtt{m}_2}{R^2} \times \mathbf{S}$$

where G is a gravitational constant and the 'form function'

$$\mathbf{S} = \frac{1}{\alpha(\mathtt{P},\mathtt{Q})^2} = \frac{1}{4\sin^2(\alpha/2)}$$

---

[17] see *e.g.* Jaki [1969].

[18] A remark attributed in 1973 by H.P.Noyes to E.W.Bastin.

has a pole when P and Q coincide and $\alpha = 0$.

Now assume Gravitational Attraction to be **cyclical** — acting only *within* the spherical universe itself, *i.e. around* the surface of the sphere and certainly not *through* it. To start with there is the mutual attraction between the two masses due to them being at their 'first' distance from each other; then there is what appears to be a mutual repulsion due to them attracting each other 'round the back of the great-circle'; then there is another, weaker attraction like the first but in which the intervening distance has picked up its first cyclical addition of one full circumference; then another weaker 'repulsion' due to another full circumference 'the other way round', and so on ... We see that their total mutual attraction is a force

$$G\frac{m_1 m_2}{R^2} \times \left(\mathbf{S}_I + \mathbf{S}_A - \mathbf{S}_R\right)$$

the three terms making up the form function on the Right-Hand-Side being due to

(I) : the Immediate 'short distance' attraction,
(A) : the sum of the sequence of diminishing cyclical Attractions, and
(R) : the sum of diminishing cyclical 'Repulsions'.

Indeed,

$$\begin{aligned}
\mathbf{S}_I &= \frac{1}{\alpha^2} \\
\mathbf{S}_A &= \sum_{n=1}^{\infty} \frac{1}{(2n\pi + \alpha)^2} = \frac{1}{4\pi^2}.\Psi\left(1, 1 + \frac{1}{2}\frac{\alpha}{\pi}\right) \\
\mathbf{S}_R &= \sum_{n=1}^{\infty} \frac{1}{(2n\pi - \alpha)^2} = \frac{1}{4\pi^2}.\Psi\left(1, 1 - \frac{1}{2}\frac{\alpha}{\pi}\right)
\end{aligned}$$

the values of the sums being given by the first PSI function (one of the 'Polygamma functions' or derivatives of the logarithm of the Gamma function, see *e.g.* Char *et. al.*[1991], p.170 and Abramovitch & Segun [1968], p.260). In fact, $\Psi(1, z) = \frac{d}{dz} \log \Gamma(z)$.

The form function $(\mathbf{S}_I + \mathbf{S}_A - \mathbf{S}_R)$ is seen to be almost symmetrical for values of $\alpha \in [0, .., 2\pi]$ with poles at $\alpha = 0, 2\pi, 4\pi, \ldots$ (*i.e.* where P and Q coincide or differ by a cyclical angular difference of multiples of 2π, and which have the values (with α written in degrees) :-

000°	060°	120°	180°	240°	300°	360°
∞	0.933	0.277	0.202	0.292	0.963	∞
360°	420°	480°	540°	600°	660°	720°
∞	0.970	0.306	0.225	0.310	0.978	∞

and so on ...

'Ordinary' attraction in the Euclidean plane (*i.e.* inverse-distance attraction) or in Euclidean 3-space (*i.e.* inverse-distance-squared Newtonian attraction) has the characteristic property that the attraction on a particle within a ring or shell, due to a uniform distribution of mass over the ring or shell, is zero. This cannot be the case in the corresponding situation for attraction "over" the surface of a sphere, for the following reason. Any ring on the spherical surface divides the spherical surface into two caps, arbitrarily labelled 'interior' and 'exterior' respectively. Suppose a point mass lay within the interior of a ring of uniformly distributed matter and experienced a zero attraction from the ring. Then, exchanging the arbitrary rôle of "interior' and 'exterior', the point lies in the exterior of the ring but still experiences no attraction from the ring. Thus a point located anywhere on the spherical surface other than on the ring itself is unattracted by the ring. If the point should lie on the ring itself, the attraction of the ring has an infinite singularity there. As a corollary, a point exterior to a uniform distribution of matter over a spherical cap will be unattracted by that matter, and in the limit, a point exterior to any distribution of matter over an irregularly shaped cap will be unattracted by that matter. We are forced to conclude :

> *If the spherical attraction on a particle within a ring, due to a uniform distribution of mass over the ring, is zero then the spherical attraction is null everywhere outside any distribution of matter over any region (connected or not) of the spherical surface, and has an infinite singularity elsewhere.*

To avoid such an unsatisfactory conclusion, it will be necessary to develop a coherent account of the analogue of Poisson's Law for spherical and cyclical attraction over the 'surface' of a spherical universe ...

Section 12. Cyclical Geometry & Topology.

? What would constitute Axiomatic Systems of Cyclical Geometries and Topologies ...

Section 13. Cyclical Physics & Thermodynamic.

? Cyclical & Ouroborosian Orbit Theory ... Any theory of orbits 'in the small' would have to take account of the idea that each orbiting 'particle' ('planet', 'star') was itself comprised 'in the large' of (many) orbiting systems (multiply 'nested'). And that 'gravitational attraction in the large' "was equivalent to" 'strong attraction in the small".

? Cyclical Mass, Energy, Temperature ...

? In the absence of "Action at a Distance", two objects may affect one another through the exchange of, say, electromagnetic signals propagating at the speed of light. If A and B are two objects spatially remote from each other (and hence almost contiguous in CSC) a signal from A will reach B at (at least) two distinct times : one almost instantaneously and the other aeons in the future (and hence again almost instantaneously, but perceptibly later, in CSC) . If B on observing A's signal is obliged to respond in some way, then it will do so almost at once, and then be in a changed state when the first of the cyclical signals arrives 'later' from A. And then the other cyclical signals will arrive each equally remote ('instantaneously later') in time, so that the state of B is the result of the application of an infinitely drawn-out sequence of copies of the original signal from A. But, in the meanwhile the state of A may have been affected by a similarly drawn-out sequence of response-signals from B, each of which, in turn, may cause A to emit another signal towards B, so re-affecting B, and so on, *ad infinitum ...*

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Emergence, Complexity and Integrative Levels

Diagrams and Figures by Jens WALDECK

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1 Emergent Evolution of Integrative Levels

The Philosophy of EMERGENT EVOLUTION[1] found its first thorough elaboration in the works of Conwy LLOYD MORGAN and Samuel ALEXANDER, the founders and central figures of the BRITISH EMERGENTISM. It originated from the then newly established concepts of evolution in Charles DARWIN's theory of 'Natural Selection' and Herbert SPENCER's 'Synthetic Philosophy'[2]. But from its beginnings there always was and – still unresolved – certainly is a core question connected with the concept of evolution, the question of the continuity versus the discontinuity of the evolutionary process:

> „Was it possible to combine full recognition of qualitative novelty, particularly as concerned life and mind – as WALLACE did – without rejecting the continuity of the evolutionary process and its naturalistic framework – as required by Darwin? This was the basic problem that Lloyd Morgan faced in his development of EMERGENT EVOLUTION."[3]

Soon the notion of 'emergence' superseded that of 'evolution' as the central concept, when LLOYD MORGAN influenced by BERGSON and SPAULDING tackled the problem of qualitative novelty. In his accurate study on the history of EMERGENT EVOLUTION David BLITZ dated this turn to the year 1912[4]. From then on EMERGENT EVOLUTION was no more mainly attached to the paradigms of biological theory, but the matrix for a revival of an ontological question which dates back at least to PLATO and ARISTOTLE. It is the question if and how radical novelty – i.e. a novel form, substance or natural kind - may become real.

1.1 The Qualitative Access to Emergrgce

Other than PLATO and ARISTOTLE which – although with different rigor – both rejected the possibility of qualitative novelty (cf. e.g. PLATO, Timaios 49a -50d; ARISTOTLE, De generatione et corruptione 337b34 - 338a19) the Emergent Evolutionists not only took this possibility for granted but also took it as the focus of their ontological considerations. No longer change and becoming were accidental properties of transcendent ideas or eternal substances but on the contrary all such substances or complex entities were mere stages in an ongoing process of the emergence of qualitative novelty.

Neither LLOYD MORGAN nor ALEXANDER invented the concept of emergence[5] or that of different levels of reality, but it undoubtedly had been them which not at least by a careful but systematic combination of these concepts laid the foundations all later theories of emergence are based on. This combination was effectively expressed by LLOYD MORGAN:

> „ ... there are levels or orders of reality in respect both of intrinsic and of extrinsic relatedness [of pertinent components, *the authors*]. This does not, of course, imply a scale of more or less reality, as such, for relatedness as a mark of reality obtains at all levels. It does, however, imply (1) that there is increasing *complexity* [italics by us, *the authors*] in integral systems as new kinds of relatedness are successively *supervenient* [italics by us, *the authors*]; (2) that reality is ... in process of development; (3) that there is an ascending scale of what we may speak of as richness in reality; and (4) that the richest reality that we know lies at the apex of the pyramid of emergent evolution up to date."[6]

Most of that what mattered in the early stages of EMERGENT EVOLUTION - and a lot of what mattered later as well - is pointed out in these lines. The concept of levels which LLOYD MORGAN stresses so frequently in his *Emergent Evolution*[7] is mentioned as well as that of complexity[8] which had still not been tackled successfully until today. And complexity is carefully amalgamated with the emergence of new levels when he speaks about electrons, atoms, molecules, quartz-crystals and organisms as a series of entities with respectively higher yet „different kinds of complexity"[9].

But also the concept of supervenience may have contributed to LLOYD MORGAN's rather 'continuist' access to the problem of continuity in the emergent process. Though several other concepts which proved to be significant for the theory of emergence had also been addressed in his wide-ranging research on EMERGENT EVOLUTION which led from physiology and biopsychology to epistemology and even touched upon the depths of theology, we for now just want to take a look at his somewhat untypical view of continuity versus discontinuity.

> „There may often be resultants [i.e. nonemergent results of change, *the authors*] without emergence; but there are no emergents that do not involve resultant effects also. Resultants give quantitative continuity which underlies new constitutive steps in emergence. ... In that sense there is not the discontinuous break of a gap or hiatus. It may be said, then, that through resultants there is continuity in progress; through emergence there is progress in continuity."[10]

Though reasonable, we regard this position as slightly over-continuist. For we consider the possibility of emergent events as deeply rooted in an intrinsically discontinuous character of even the seemingly static stages of nature. But it should be viewed by the historical background of its antagonism against strongly maintained saltationist convictions. And it also had been somewhat qualified by LLOYD MORGAN:

> „There is increasing richness in stuff and in substance throughout the stages of evolutionary advance; there is redirection of the course of events at each level; this redirection is so marked at certain critical turning-points as to present "the apparent paradox" that the emergently new is incompatible in "substance" with the previous course of events before the turning-point was reached."[11]

In marked contrast to LLOYD MORGAN's aporetic approach to EMERGENT EVOLUTION ALEXANDER's in his two-volume *Space, Time, and Deity*[12] is a systematical. This treatise bears the spirit of 19th century idealism of a hegelian or at least bradleyan kind when it builds up a graded system of categories[13] as an abutment for its ontotheological construction. Yet its intentions are rather realistic. Even though we easily admit that the word „must not be too closely pressed" when he claims his method being „empirical"[14] we also concede it in a sense being constructive, i.e. not a priori but trying to tally the scientific findings of his time.

In a passage where ALEXANDER directly refers to „Mr. LLOYD MORGAN, with whom on this matter I believe myself to be in general agreement"[15] we find his approach to EMERGENT EVOLUTION – including some of that about complexity and levels what we've already learned – conclusively comprised, – and we also find some differences to LLOYD MORGAN's view:

> „Empirical things or existents are ... groupings within Space-Time, that is, they are complexes of pure events or motions in various degrees of complexity. ... New orders of finites [i.e. the empirical things, *the authors*] come into existence in Time; the world actually or historically develops from its first or elementary condition of Space-Time, which possesses no quality except what we agreed to call the spatio-temporal quality of motion. But as in the course of Time *new complexity of motions* [italics by us, *the authors*] comes into existence, a new quality emerges, that is, a new complex possesses as a matter of observed empirical fact a new or emergent quality. ... The emergence of a new quality from any level of existence means that at that level there comes into being a certain constellation or collocation of the motions belonging to that level, and possessing the quality appropriate to it, and this collocation possesses a new quality distinctive of the higher complex."[16]

Here we find again the already familiar increasing complexity along the ascending grades of levels. But there is also something effectively new about complexity. ALEXANDER's is a complexity of a very special sort – a *complexity of motions*. And the revelation of this intrinsic nature of complexity – which then of course is of a dynamic kind – links it directly to that particular problem somehow mistaken by LLOYD MORGAN – the problem of continuity versus discontinuity in the emergent process. Since ALEXANDER struggles from the beginning of his study with the reciprocity of the continuity and the dynamics of his spatio-temporal substratum[17] he finally must come to the conclusion:

> „Space as a whole we have seen is neither immoveable nor in motion. But neither can a place be at rest if Space is only one element of Space-Time. Rest, in fact, appears to be purely relative and to have no real existence. Every place has its time-coefficient and is the seat of motion. ... It seems, in fact, clear that if anything could be absolutely at rest everything must be at rest. For if any point in space retained its time, this would dislocate the whole system of lines of advance within Space-Time, a point being only a point on such a line.
>
> Thus if absolute rest means the negation of motion, there is no such thing in reality. Rest is one kind of motion, or better, it is a motion with some of its motional features omitted."[18]

So the prevalence of continuity as tentatively favored by LLOYD MORGAN can no longer be maintained. Motion is unveiled as the intrinsic discontinuity – yet *not* discreteness – of space-time.[19] Therefore emergence then is possible just because the new complexities of higher level integration stem from such older ones which had been intrinsically dynamic and variable and thus the link between the old and the new or *the emergent transition itself is discontinuous*. We regard this as one of the most profound insights in the analysis of emergence which hardly had been matched in the subsequent discussions of this subject.

But by all their merits for describing (the ontology of) nature by concepts and categories of a hierarchy of integrative levels the British emergentists as well as such counterparts in related research like Roy Wood SELLARS[20] or as their immediate successors like Joseph NEEDHAM[21] and Alex B. NOVIKOFF[22] did not give

a) an explanation of how the presumed emergence of these integrative levels would actually work or how it formally or physically could be explained, and

b) an answer to the question "Why are there levels of nature?".

Obviously the possibility to give an answer to this question is intimately connected with the capability first to give such an required explanation. At least the latter shall be tried in this paper.

1.2 Why are there Levels of Nature?

Nature is organized in a hierarchical way. There are levels of nature's organization getting more complex which show new entities and properties being supervenient on the lower level, this very one being less complex. But here the fundamental question arises Why is nature structured hierarchically ?, the very same nature which on the basis of that layered composition makes novelty possible. At this point we find ourselves confronted by the central problem of emergence that we discussed already in connection with the emergentists. On the one hand every new event has to be identified as a new one separated from the other events, so it is discontinuous; on the other hand the concept of the New as such requires a sort of quasi-continuity because a (discontinuous) chain of events ignores the necessary conservation properties proving the new event not to be a miracle. But strata demand a special sort of discontinuities often ignored by philosophers of nature.

For example, Nicolai Hartmann, the level-ontologist, takes new groups of categories for granted that refer to new and higher strata respectively, namely the spatially extended sphere of the physical and the biological (res extensa), and the non-spatial one of the psychological and the spiritual (res cogitans). Without these discontinuities there would be a „big single continuum of forms“ making up nature, „and any difference of ontic rank would be a difference of complexity only.“[23] Then he states apparently clear: „There would be no scope for radical diversity.“ But this statement confounds „emerging“ and „stratifying“: no strata without emergence, but there is emergence without strata.

One can imagine a universe consisting of one stuff and one force exclusively that brings about very big agglomerations of that stuff taking various, even bizarre, structures but holding no ones of a different sort arranged out of closed structures of the first kind- think of one kind of particles and one kind of attracting interaction only, the latter decreasing in strength with distance. In such a universe no strata will emerge because all agglomerations are of the same nature. If now a second, attracting force with a much smaller strength will be added to our universe nothing radically new will happen, because just as much as in the first case there will be no real parts constituting wholes of another quality that could agglomerate either and form new strata. Of course emergent events may happen in that simple world, antimorphic action can occur unlimited: Every increase of the topological genus of the universe's stuff is described as emergent. These actions can be increased beyond all measure keeping at level 0, they never lead to any stratification.

A minimal stratified universe, we think, must be made of one sort of particles interacting by one force to combine into wholes of order1 - these components radiating as parts a second, different force that binds them together to other wholes, now of order2. A little bit more „particles" and „forces" so behaving result in the hierarchical richness of the universe we are living in up to galaxies, black holes, sandstorms, presidents and stuffed pig's stomachs. Most of these complex things are products of a special kind of morphological complexity, for the difference of topological forms that represent a non-hierarchical system on the one hand, and a hierarchical one on the other hand, cannot be grasped by topology alone but here morphology[24] must give some help. In Fig.7 of this paper (subtitled as 'Alternative Morphological Complexities Generated by the Same Kind of Strong Antimorphic Action') the „column" at the right end shows three topologically equivalent figures, namely triple tori, the lower has is „holes" linearly arranged, the middle one is formed like trefoil, whereas the triple torus at the top looks rather strange, because two of the „simple" tori are intertwined, so that the sausage shaped and bent surfaces are pulled through their respective „holes" like a piece of thread through the eye of a needle, a piece of thread being so thick that it touches the eye's inside. The surfaces of the two simple tori (being parts of the triple torus) are contiguous in the technical sense of the term. One cannot tell these figures apart except by means of morphology, and it will be shown soon that strata must be handled in the same way. But let us now come back to our initial question.

Herbert SIMON has shown that hierarchical systems are „fitter" as „one-level" systems[25]. He used a very plausible metaphor going as follows. A clever watchmaker who divides his composition of springs, screws and wheels in stable sub-assemblies of say 100 such parts screwed together (an ordinary clock usually has about 1000), is luckier than his colleague gifted differently who tinkers with his watchhands and -glasses linearly building up the watch cheerfully until the critical threshold of 100 parts is passed, where the partly done watch will fall apart when the work is interrupted (the interruption comes for both craftsmen just having 150 components put together).

> „We conclude that hierarchies will evolve much more rapidly from elementary constituents than will non-hierarchic systems containing the same numbers of elements. Hence, almost all the very large systems will have hierarchical organization. And this is what we do, in fact, observe in nature."[26]

SIMON speaks of systems in nature, but what about nature itself as a whole? When nature as such is stratified, where are the other non-layered systems which evolved slower? Had there ever been such other - universes? Let us play with a toy model in the following - but you are reminded that from this thesis no serious cosmological conclusions will be drawn in this paper.

Accept an ensemble of all sorts of universes evolving differently so that the hierarchically organized ones are fitter in the struggle for the existence, just as the respective watches are in

SIMON's example, the least complex and stratified universes then dying without offspring. This strange phraseology refers to a very speculative hypothesis given birth to by Lee SMOLIN proposing that universes reproduce themselves by producing black holes, which grow up into other *kosmoi*. This way complex universes have more offspring because only universes occupied by big stars can develop these singularities and spread - provided that these baby-*kosmoi* have the same physical laws. Clearly the formation of black holes is a necessary condition for the generation of complex strata, for instance biological systems.

To state our ansatz more precisely let us push ahead the analogy of SMOLIN who equates organisms and genes from the sphere of biology with universes and elementary particles from the sphere of cosmology. Remember that the number of black holes in a universe depends on parameters referring to properties of elementary particles, for example on the strength of the fundamental interactions: If the gravitational force is too weak no black holes can be formed. In the toy model universes generate baby-universes by means of black hole dynamics the biological counterpart consists in self-reproduction by means of autocatalytic functions. The mutability of amino acids has its structural analogue in the mutability of properties of elementary particles guaranteeing many baby-universes if the interactions are calibrated in a way. Universes do not compete directly with one another because they do not share common (external) environments (but instead are merely rooted in the same ground state). So they have no external in- and output executing their metabolism but as a substitute for this their internal history ('big bang'≈high order≈anabolism; 'big crunch≈low order≈katabolism) by internal degrees of freedom.

Only in their old age universes give birth having reached a level of complexity for black hole dynamics. This level defines in a sense a non-tautological selective value: In a biological sense a selective value means an optimal combination of structural stability and efficiency of faithful replication.[27] But what does that mean in cosmology?

At this point we must come back to SIMON's theory of hierarchical systems. Exactly the hierarchical structure , the fact that nature forces itself to emerge in levels, means nothing else than a variational principle, a principle of optimization referring to an ensemble of universes so that this ensemble will survive and have offspring which transgresses a critical threshold of complexity and achieves a certain degree of stability. *Here we have the precise selective value for universes*: optimal differentiation of their internal structure, i.e. complexity by means of hierarchy. Universes which immediately fall back into the ground state or do not develop into a hierarchical level structure will die out, for example the sort of universes producing nebulae only.

So we tried to answer the question: 'Why are there levels of nature?' in a first attempt. The answer goes as follows: 'Because level-structured universes are the offspring of complex universes optimal differentiated and formed in a hierarchical way sharing gene-equivalence'.

Tentatively having answered the question why nature is stratified we proceed in working out our general theory of emergence by formulating several theses preparing for the general theory of emergence we are delivering. Emergence means always emergence of complexity, and especially it means the emergence of levels in nature, namely the so called inter-level emergence. Furthermore the mathematical framework of the theory still has to be stated. Thus here a preview in five theses:

- Emergence has to be mathematically represented as a not structure preserving mapping, that is, it has to be represented as a non-topological mapping
- Topology as the theory of transformations and equivalence classes of forms and structures is a natural framework for a theory of emergence
- Emergence means emergence of complexity (of forms and structures); which forms and structures then should appropriately be characterized by means of topology
- A non-topological mapping turns out to be an antimorphism
- Therefore category theory, the general theory of morphisms and equivalence classes of any mathematical structures is intrinsically involved in such non-topological mappings.

2 Emergence, Complexity and Integrative Levels

In a previous paper[28] we already tried to give a rather formal description of how emergence could be tackled mathematically. In that previous paper as well as in this present one we prepared ourselves with the means of category theory which we regard as a natural candidate for the exploration and subsequent formalization of the mathematics of emergence and complexity. And so we stressed a strong similarity of underlying intuitions between an older concept of *antimorphic mapping*[29] used by us to clarify the essence of emergent actions and the concept of *antimorphic action* introduced by W.H.CORNISH to describe anti-endomorphisms of algebraic structures with the means of category theory. For reasons of even more conceptual and terminological clarity we now will show some aspects of how emergentic antimorphic mapping - or as we will now call it: ***strong antimorphic action*** - differs from *antimorphic action* as described by CORNISH. And then we will try to show how this difference matters in formally describing emergence as the formation of new integrative levels.

2.1 Categorial Antimorphic Action

In 1986 W.H.CORNISH introduced a specific form of antimorphisms - the antimorphic action[30]. An *anti*-morphism of a structure S into another structure S' in the precise mathematical sense means a morphism of S into the structure S", the *conjugate* of S':

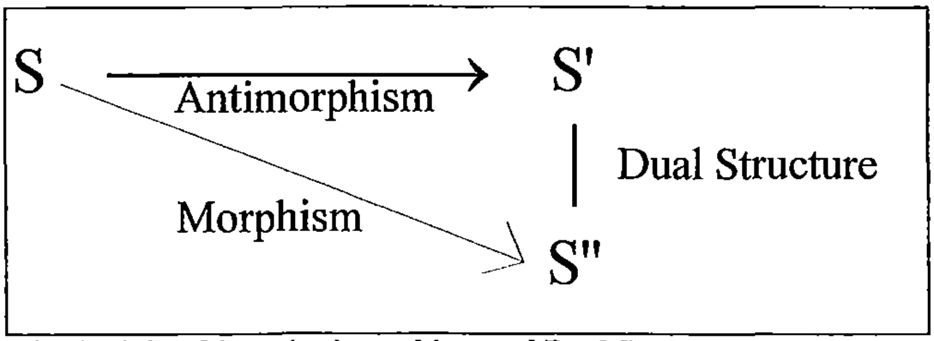

Fig. 1a: Morphism, Antimorphism and Dual Structure

So an anti-homomorphism of a ring R into a ring R' is a homomorphism of R into the ring R" ('conjugated' relative to R') which differs from R by multiplication: a × b = b · a (with a × b in R" and b · a in R'):

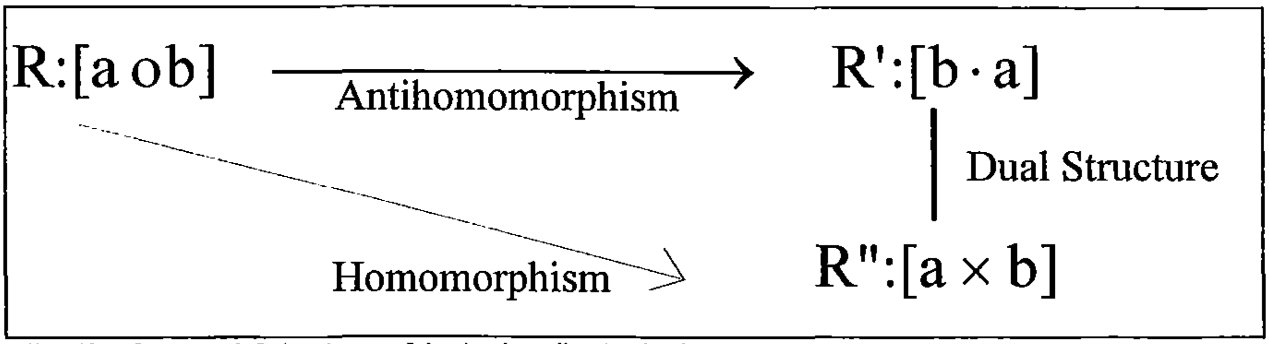

Fig. 1b: Categorial Antimorphic Action (in Action)

Only in category theory one can generalize anti-homomorphisms, anti-endomorphisms, anti-homeomorphisms, etc., so that one gets the general concept of an anti-morphism.

Thus CORNISH paved the way for a general concept of an antimorphism: he extends the notion of an antimorphism to an arbitrary category. His essentials run as follows:

The category $\underline{X}$ is a subcategory of $\underline{X}^{\pm}$ in the way that they consist of the same objects but differ in their respective morphisms: „$Ob(\underline{X}) = Ob(\underline{X}^{\pm})$, and $Mor(\underline{X}^{\pm}) = \underline{X}^{+} \cup \underline{X}^{-}$, where $\underline{X}^{+} = Mor(\underline{X})$“. Afterwards a functor T is defined which transforms the objects and the morphisms of a category into their conjugate; applying T twice gives the initial state ($T^2(A) = A$). Furthermore a „metafunctor“ δ is introduced acting on T as a natural transformation and yielding $\delta : T^2 \rightarrow T$, where $\delta^{-1} = \delta T$, more precisely it is a natural isomorphism. CORNISH calls the morphisms in $\underline{X}^{-}$ the antimorphisms of $\underline{X}$ and the quadruplet ($\underline{X}$, $\underline{X}^{\pm}$, T, δ) an expansion (by antimorphisms) of $\underline{X}$; when $\underline{X}^{+}$ and $\underline{X}^{-}$ are disjoint, the expansion has *separate* antimorphisms.

2.2 Beyond CORNISH's Antimorphism: Strong Antimorphic Action

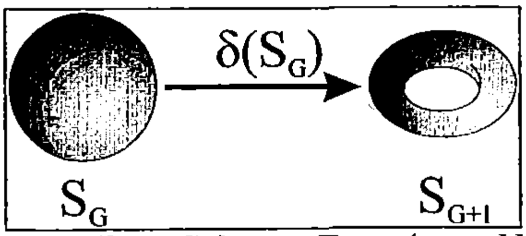

Fig. 2: From Sphere to Torus (or „cable“-torus): A kind of antimorphism.

For our purpose of explaining emergence as strong antimorphic action, we use a modification of the antimorphic structure, described by CORNISH.

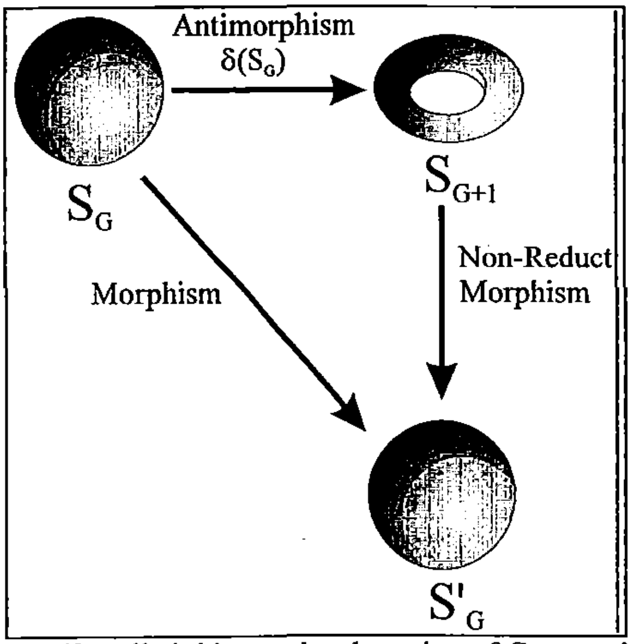

Fig. 3: Non-Reduct Morphism

Differing from CORNISH's definitions, there is a further antimorphism leading to T^2 (f), and then a next antimorphism T^3 (f) and so on, in our approach, such that we have: $\forall\ n \neq m \in \mathbb{N}$: $T^n \neq T^m$. This means, applying T twice, never gives the initial state.

We called this restricted version of CORNISH's concept of antimorphism a *strong antimorphism.* The restricting condition T^n (f) $\neq T^{n+1}$ (f) is a generalized version of CORNISH's concept of *separate antimorphism.* Hence, for the antimorphic action δ, the condition $\delta^{-1} \neq \delta$ T holds, which means, there will be no inverse of the antimorphic action. Given n and f, the *antimorphic action* δ leads from T^n (f) to T^{n+1} (f).

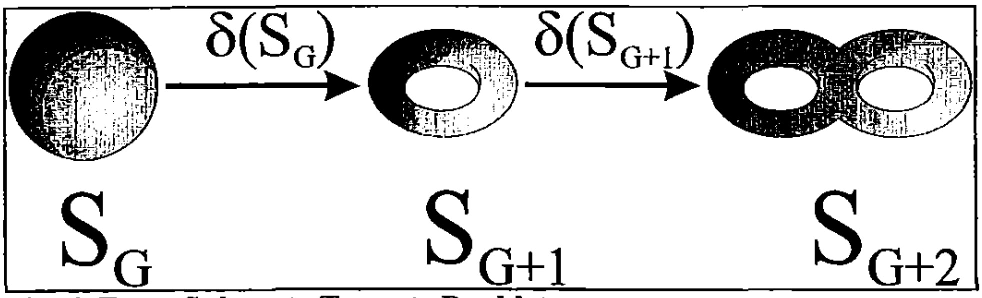

Fig. 4: From Sphere to Torus to Doubletorus

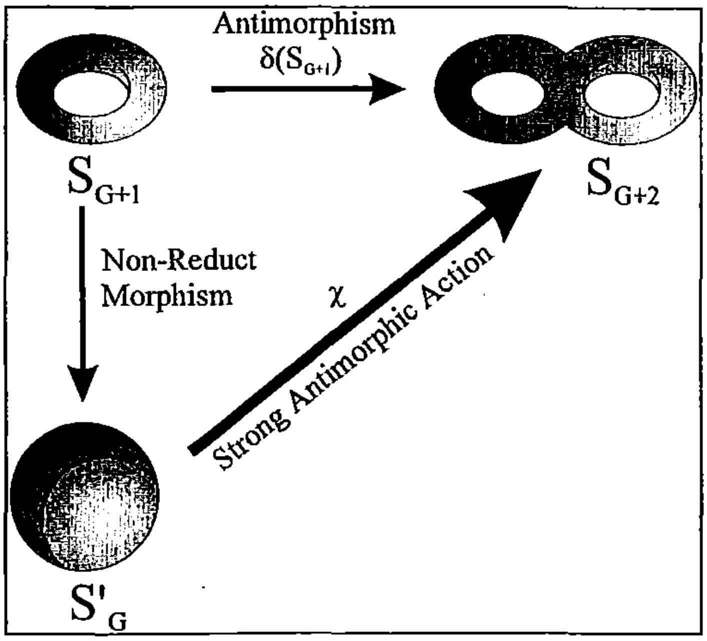

Fig. 5: From Torus to Doubletorus

The iterated form of the antimorphic action, which we named as χ with $\chi = \delta^2$ leads from T^{n} (f) to T^{n+2} (f). For an emergent transition, there at least one such iterated antimorphic action, the generalized version of CORNISH's separate antimorphism is required. According to our terminological conventions we called this generalized version of CORNISH's concept of antimorphic action a *strong antimorphic action.*

2.3 Topological Enrichment: Emergence as Increase of the Topological Genus

Thus the *strong antimorphic action* χ must then be described as an 'irreversible' iteration of the partially separating antimorphic action δ.

Now in Fig. 6 the real significance of χ will become clear, i.e. χ strictly indicates the iterated operation of δ and thus the irreversibility of the transition from a state T^n to a state T^{n+2}.

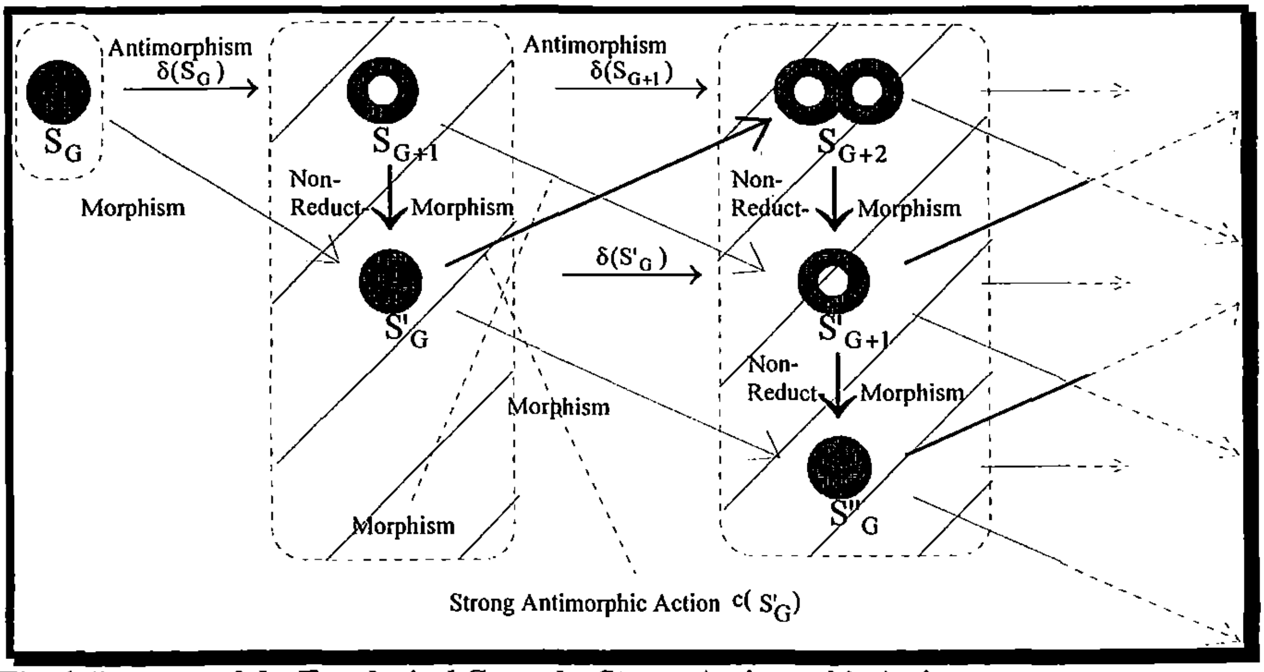

Fig. 6: Increase of the Topological Genus by Strong Antimorphic Action

If S_G is a space with a certain genus G, the conjugate T (S_G) = S_{G+1} will be a space 'emerged' from S_G. Thus the strong antimorphic action of the (iterated) strong anti-anti-identity χ in this 'special case' is to increase the genus of the topological space at least by one and therefore to describe the process of emergence.

Here the strong antimorphic meta-functor δ again causes a violation of the bijectivity and continuity of the mapping, now representing a change of the local discrete structure of S_{G+1} compared to that of S_G. Vulgar: the number of holes in the respective representation space increases.

This increase represents the emergence of complexity and leads us to:

2.4 A General Definition of Emergent Complexity (EC):

The complexity of a natural object is the amount of the emergent steps (strong antimorphic actions) accumulated in the natural history of its formation. Natural or Emergent Complexity consists of Topological and Morphological Complexity. Emergent Complexity is measured by the hierarchical rank and the topological genus of the object. The hierarchical rank of an natural object is indicated by the integrative level of the origin of the natural kind to which it belongs. A natural object of a higher hierarchical rank, i.e. which belongs to a higher integrative level, is thereby always more complex than a natural object of a lower hierarchical rank, even if this lower rank object has a higher topological genus.

The formation of new integrative levels, and thus also higher hierarchical ranks is due to a special kind of strong antimorphic action, which generates tied or chain-like discontinuities.

The presumed hierarchical structure of nature is an outcome of the intrinsic heterogeneity or complicatedness of natural objects and represented in a particularly interlocked structure of their internal organization. This organization is characterized by the embedding of substructures in superstructures, which themselves might again be integrated as substructures in superimposed super-superstructures and so on.

2.5 Topological and Morphological Complexity

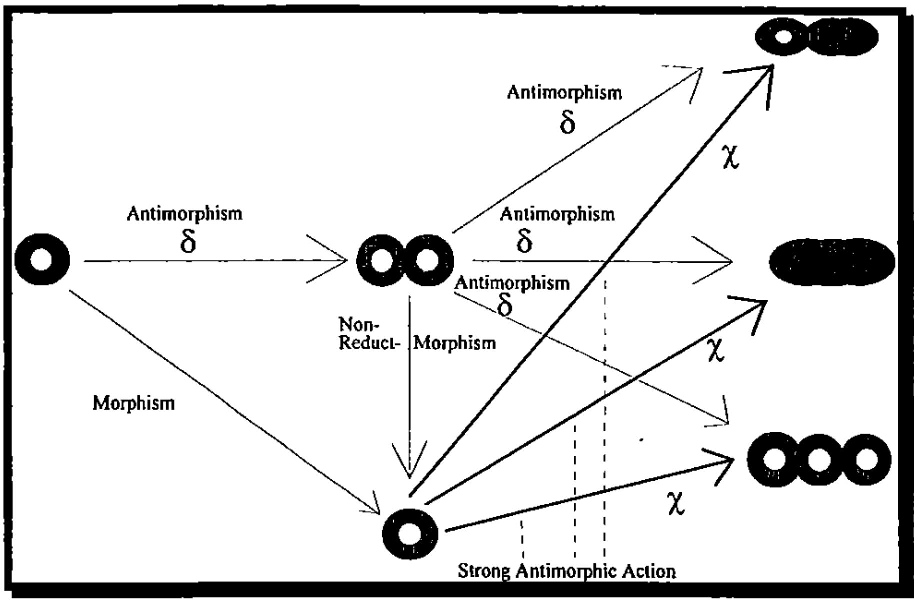

Fig. 7: Alternative Morphological Complexities Generated by the Same Kind of Strong Antimorphic Action

Considering this embedding of structures one has to take notice of two different kinds of transformations which are of crucial importance for our strong antimorphic action: On the one hand we have the usual *topological* mappings like homeomorphisms and isotopic transformations, on the other hand there are *morphological* transformations taking into account several forms topology cannot distinguish. So by means of homeomorphisms one is not able to tell the difference between the three right dedles in Fig.7 (cf. section 1.2). In the next section we will sketch the morphological issues concerning 'heterogeneity'; 'inseparability' and 'transformation by surgery'.

3 Measuring EC

Morphological Complexity is specifically a measure of a certain kind of *inseparability* which we are convinced is the underlying topological feature of the *heterogeneity* or *complicatedness* which then again by themselves are the essential characteristics of what is assumed to be the *structural or functional organization* of natural objects or systems. We propose that the whole of any of such *structural or functional organizations* can be produced or generated just by strong antimorphic action and thus is *emergent* by origin and destination.

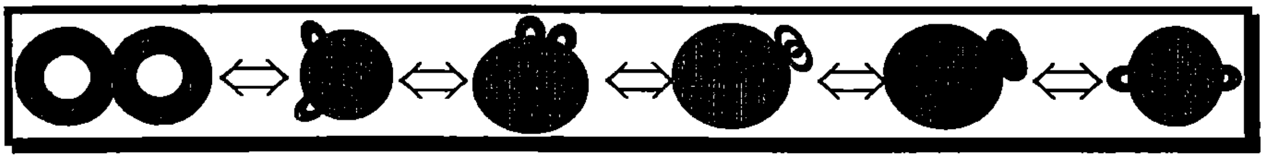

Fig. 8: Morphological Variations of the Two-Torus with Separability

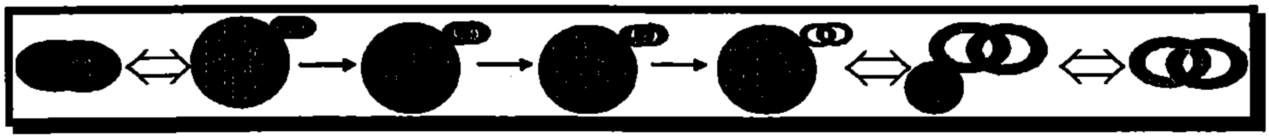

Fig. 9: Morphological Variation of the 'Dedle' with Inseparability developing to a Chain

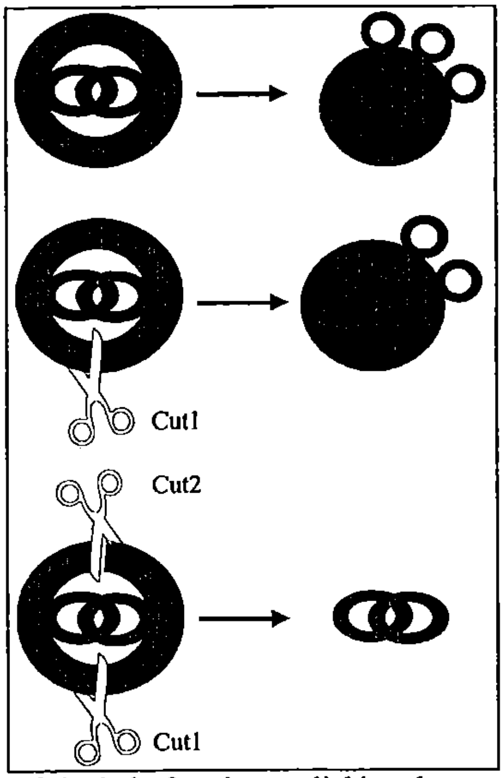

Fig. 10: Generating an Inseparability (Chain) by two Cuts

Furthermore the mentioned topological inseparability and therefore also the heterogeneity or complicatedness is specifically generated by a special kind of strong antimorphic action, namely that which leads at first from a fully connected compact topological entity like a n-torus to an entity which is rather similar to a „cable"-torus or a chain. Such chain-like entities usually are called links and therefore characterized by their linking numbers.

The transformation from a torus to a chain by surgery goes as follows:

Pull out a circular piece of „skin" that covers the interior of the hole forming a pliable tube, go round a meridian and glue the tube's end to the place (of the „under-side") where you started to draw. Circumcise the so formed „dedle" to get two linked tori, „cable"-tori, or a chain.

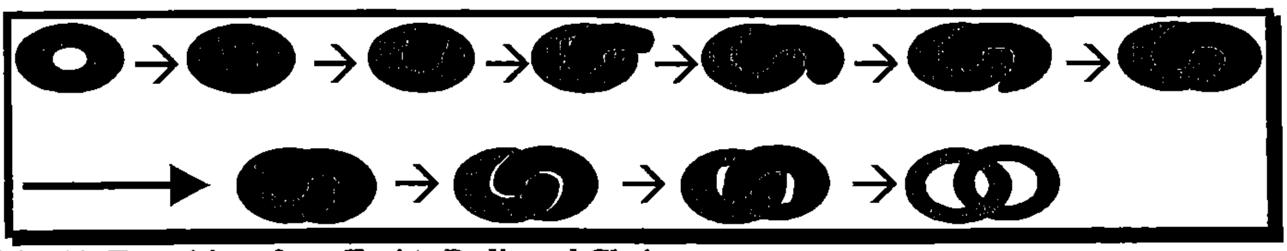

Fig. 11: Transitions from Tori to Dedle and Chain

In a next step the emergent enlargement of complexity produced by strong antimorphic action then leads to entities which we will call *interlockings*. Such interlockings could roughly be described as a kind of chains which elements are chains themselves. A famous and rather amazing example of such an interlocking is the well known Antoine's necklace.

It is essential to be fully aware of the fact that such chains as well as such interlockings can all be generated simply by strong antimorphic action (and nearly nothing else)! This is essential just for the reason that it means that *all hierarchical structures* or *all integrative levels* of nature can be generated simply by strong antimorphic action (and nearly nothing else), as shown in Fig. 11.

Being *structural or functional organized* or being *hierarchical structured* or being an *integrative level* of nature thus always means nothing but being characterized by an relatively high amount of *iterated internal embeddings of such discontinuities* as the mentioned chains and interlockings. And just to be precisely: an interlocking is by definition nothing else than just such an iterated internal embedding of such discontinuities. The following figures show a variety of some of such chain-like linkings and that higher order links or rather interlockings:

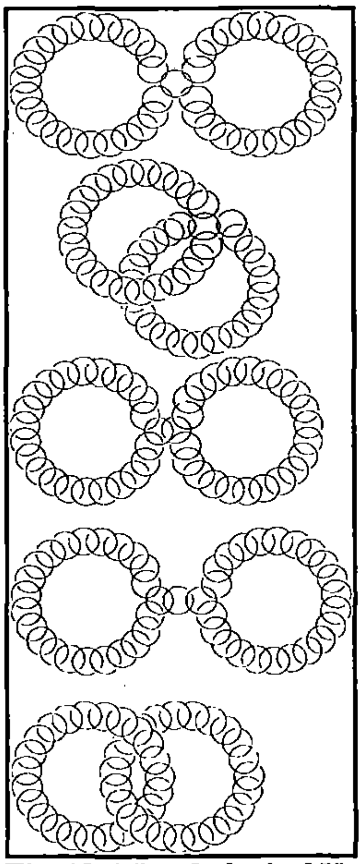

Fig. 12: Morphological Variations in Linking of Closed Chains

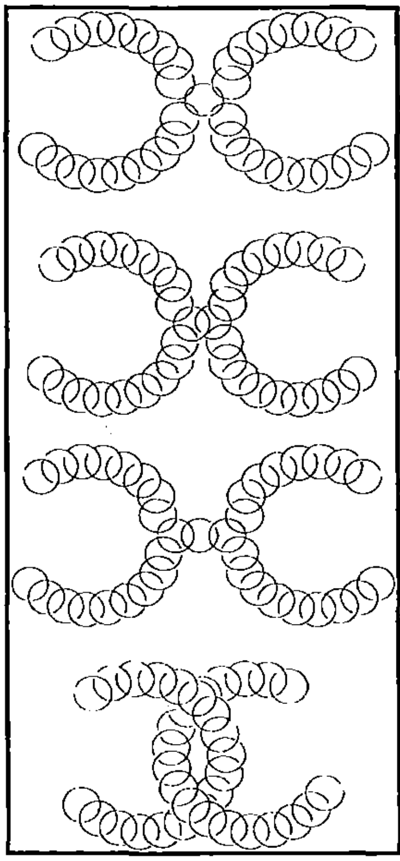

Fig. 13: Morphological Variations in Linking of Open Chains

Now after having the basic concepts of Emergent Complexity (EC) sorted, we can proceed from our previous rather qualitative 'General Definition of Emergent Complexity', where we said that „Emergent complexity is measured by the hierarchical rank and the topological genus of the object" to a even more solid quantitative measure of Emergent Complexity, which shall give a respectively solid meaning to all such seemingly vague concepts like 'hierarchical rank', 'structural organization' or 'integrative level', which despite Algorithmic Complexity's inability to deal with them are the thorough essence of complexity:

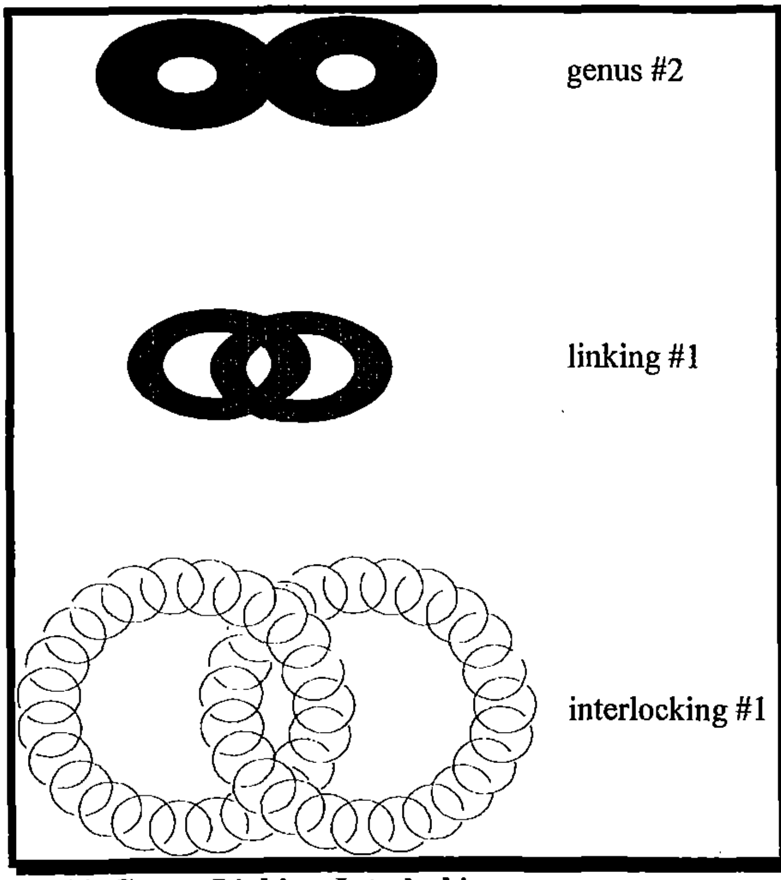

Fig. 14: Genus, Linking, Interlocking

Emergent Complexity then can be measured by a numerical expression composed of three separated numbers (arranged in the following order), namely

1) the interlocking number indicating the *integrative level* or *hierarchical rank* (or - as this 'complexitiv' feature alternatively may be called - the *'emergentic height'*) of the respective object,

2) the linking number, and

3) the topological genus indicating the complexity of the most elementary structures regarded, i.e. of level-0 structures.[31]

Thus the measure of EC is of the following form:

$$\mathbf{n_x;\ n_y;\ n_z}$$

(for n_x being the interlocking number, n_y being the linking number and n_z representing the topological genus of the respective object.)

The meaning of the topological genus and the linking number should be easy to understand. The topological genus, the change of which had been the original and immediate result of strong antimorphic action represents a most elementary or basic stratum of complexity. We call it level-0 complexity, indicating that by having a topological complexity represented by the topological genus alone, how great a number ever, the respective object is of a very low complexity. In the physical world such a complexity has to be lower than the complexity of the elementary particles of the standard model, and might be even confined to a sub-quark level.

The linking number again represents solely a first stratum of such a discontinuous interlocking or - looked at it top down - such an iterated internal embedding as mentioned above. Therefore we call the complexity expressed by the linking number, how high this number ever might be, a level-1 complexity. In the actual world such a level-1 complexity is obviously confined to very elementary physical reality.

The interlocking number then is of far greater relevance for the particular problems of a theory of complexity. The main reason for this is that the interlocking number contains and expresses all hierarchical ranks of all integrative levels higher than level-1. Therefore one should expect some peculiarities concerning the interlocking number.

Such expectations come true in the sense that the interlocking number, other than the linking number or the topological genus, is not just a simple number but itself a rather complicated expression:

$$\mathbf{n_x\,(k)\,/\,n_x\text{-}i_1\,(k)\,/\,n_x\text{-}i_2\,(k)\,/\,\ldots\,/\,n_x\text{-}i_j\,(k)}$$

(for $n_x \geq 1$ and $n_x > i_j$ and $n_x - i_j > 0$)

This complicatedness is due to the fact that the interlocking must not just - one might say vertically - represent the highest level or rank (in our measure represented by the $\mathbf{n_x}$) which can be assigned to the object in question, but as well - in a stratified horizontal perspective - the amount of all interlockings (represented by the various $\mathbf{k}$s (of the respective ranks $\mathbf{n_x\text{-}i_j}$)) of the same kind, i.e. of the same level, and this for all levels higher than level-1, which are contained or incorporated in the internal composition or organization of the respective object. And by definition any object of a higher level Emergent Complexity contains in one way or the other all lower level strata of Morphological Complexity down to the lowest stratum of Topological Complexity.

Whilst any such object or system may be composed of many more than one interlocked substructure, which substructures then again might be linked horizontally, its proper hierarchical rank is principally indicated by the ('emergentic') height of the highest vertically interlocked substructure, expressed by the respectively highest vertical interlocking number, representing its proper hierarchical rank.

Therefore the interlocking number is the genuine expression of the structural heterogeneity or complicatedness which is the characteristic of the Emergent Complexity of all natural objects.

Thus of course most significant in the three numbered measure of Emergent Complexity is the interlocking number which in itself represents most of the essential level-aspect of complexity.

To better understand these peculiarities of the interlocking number (or rather the interlocking itself) one must see that an interlocking essentially is to be a special kind of linking, i.e. a linking of links in such a manner that the elementary links of arbitrary scale are actually integrated within each other and within the comprehensive interlocking itself. Such interlocking must not be confused with a mere additional linking of links e.g. in a sequence of such links as typically in most chains.

Then this finally is the explicit version of the measure of EC (the peculiarities of the interlocking number now included):

$$\mathbf{n_x\,(k)\,/\,n_x\text{-}i_1\,(k)\,/\,n_x\text{-}i_2\,(k)\,/\,\ldots\,/\,n_x\text{-}i_j\,(k);\ n_y;\ n_z}$$

Now at the end of this paper we will give an illustration of this finally derived measure of EC. For this reason we will choose a topological example which in a perfectly natural manner contains more than any other topological figure exactly these characteristics which we highlighted as the essential ones for describing and measuring complexity and thereby explaining the intrinsically level-organized structure of the emergent nature. This topological figure is known as Antoine's necklace. Antoine's necklace is homeomorphic to the even more popular Cantor discontinuum. The very fragment of an Antoine's necklace on an arbitrary scale which is shown in Figure 15 then has the EC measure of:

2 (1) / 1 (2x24); 2x24x24; 2x24x24

(at least this seems to be something like a rational average because repeated counting by the authors led to different results).

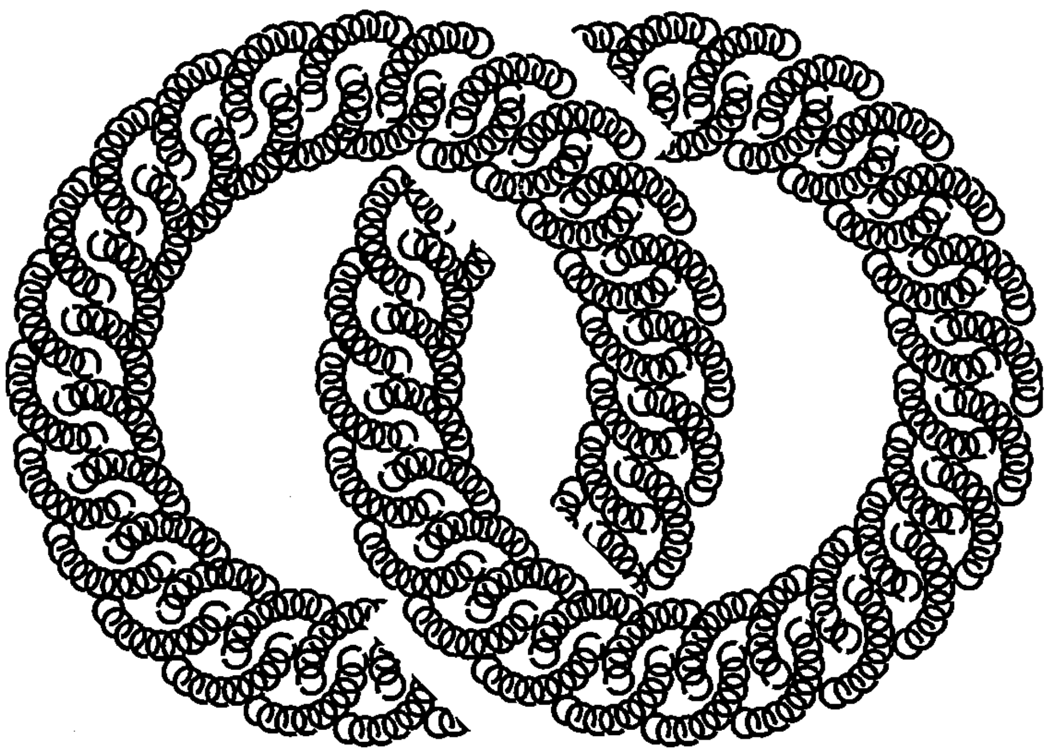

Fig. 15: Antoine's Necklace: A Topological Fractal and a Symbol for Emergent Complexity

[1] A comprehensive survey of this subject and its detailed history can be found in the careful study of David BLITZ: BLITZ, David [1992], *Emergent Evolution. Qualitative Novelty and the Levels of Reality.* Dordrecht, Boston, London 1992: Kluwer Academic Publishers.
[2] BLITZ illustrates the philosophical background of which the concepts of Emergent Evolution sprang from in his following remarks: „The 19th century debate in England over the philosophical framework for evolutionary theory posed the question which Lloyd Morgan answered with his theory of emergent evolution ... The debate among Darwin, Romanes, Wallace, Huxley, and Spencer concerned the continuous or discontinuous character of evolutionary change, the role of quantity and quality in that process, the relation of the organic and the inorganic, the relation between the natural and the supernatural, the mind-body problem, and the restricted or universal scope of the evolutionary process. This established the intellectual context for the work of Lloyd Morgan, who attempted to account for qualitative novelty within a continuous, natural, and universal evolutionary process. The result was the theory of emergent evolution." BLITZ [1992] p 175
[3] BLITZ [1992] p 56
[4] cf. ibid. p 92
[5] The concept of emergence was foreshadowed in J.S.MILLS 'Logic' as 'heteropathic laws' of causation. The term 'emergent' was used for the first time by G.H.LEWES in his 'Problems of Live and Mind'. Cf. LLOYD MORGAN, Conwy [1923], *Emergent Evolution*, London 1923 p 2,3.
[6] LLOYD MORGAN, Conwy [1923], *Emergent Evolution*, London 1923 p 203
[7] cf. ibid. e.g. p 4 - 6, 15 - 18, 20 - 22, 27 - 32, 37 f, 41, 45, 59, 62, 106 - 110, 199, 131, 135 f, 203 -207, 297 -299
[8] cf. ibid. e.g. p 11 f, 15, 31, 47 f
[9] ibid. p 12
[10] ibid. p 5
[11] ibid. p 207
[12] ALEXANDER, Samuel [1920], *Space, Time, and Deity — The Gifford Lectures at Glasgow 1916 - 1918* vol.I, II, repr. London 1966
[13] ibid. vol.I, p 322-323
[14] ibid. vol.I, p 4
[15] ibid. vol.II, p 46

[16] ibid. vol.II, p 45
[17] cf. ibid. vol.I, p 42-56, 66
[18] ibid. vol.I, p 84
[19] cf. also ibid. vol.I, p 320-323. Here ALEXANDER states that: „ ... in fact the category of motion is but another expression of the fact that every existent is a piece of Space-Time.“ ibid. p 320; or that: „ ... every *thing* besides substance is motion.“ ibid. p 323.
[20] Cf. SELLARS, Roy Wood [1922], *Evolutionary Naturalism*, repr. New York 1969
[21] Cf. NEEDHAM, Joseph [1937], *Integrative Levels: A Revaluation of the Idea of Progress*, Oxford 1937
[22] Cf. NOVIKOFF, Alex B. [1945], »The Concept of Integrative Levels and Biology«, in: SCIENCE, 101, 1945, 209-215
[23] HARTMANN, Nicolai [1942; [5]1968], *Neue Wege der Ontologie*, Stuttgart 1968, p. 64
[24] CASATI, Roberto and VARZI, Achille C. [1994], *Holes and Other Superficialities.* Cambridge, Mass.: MIT Press introduced this concept. We independently used it already.
[25] SIMON, Herbert A. [1969], *The Sciences of the Artificial.* Cambridge, Mass.: MIT Press, ch.4; SIMON, Herbert A. [1977], »The Organization of Complex Systems«, in: *Models of Discovery.* Dordrecht: D.Reidel
[26] SIMON[1977], p. 248f
[27] EIGEN, Manfred; SCHUSTER, Peter, »The Hypercycle. A Principle of Natural Self-Organization«, Die Naturwissenschaften 64 (1977) 541, 65 (1978) 7; 341; paraphrase referring to p. 357
[28] EISENHARDT, Peter, KURTH, Dan, WALDECK, Jens, »Emergence as Antimorphic Action«, in: Philosophies, Proceedings of ANPA 17 (ed. Keith Bowden), September 1996
[29] Cf. EISENHARDT, Peter, Dan KURTH [1990], »Aufriß einer Theorie der Emergenz«. In: SALTZER, Walter G., *Zur Einheit der Naturwissenschaft in Geschichte und Gegenwart*, Darmstadt 1990; and EISENHARDT, Peter, Dan KURTH [1993], *Emergenz und Dynamik. Naturphilosophische Grundlagen einer Nichtstandard Topologie*, Cuxhaven Junghans-Verlag
[30] Cf. CORNISH, William Hugh [1986], *Antimorphic Action, Categories of Algebraic Structures with Involutions or Anti-Endomorphisms*, Berlin 1986
[31] The topological genus of 'simple' links like the Hopf link or the Borromean rings is zero; therefore you must take trefoils instead of trivial knots as basic entities. We do not use trefoils for the sake of simplicity. Because of this devotion to a more descriptive use of these topological entities our terminology slightly differs from the conventional.

Topoi of Emergence: On the Metaphorization of Geometry[1]

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Abstract

With arguments based on topos theory (topoi) it has been shown recently that it is possible to express the degree to which the logic of an observer (in the physical universe) determines the observation, thus to formalize rational behaviour. In particular, a formal way of unifying logical, physical, and psychological templates of perception and cognition has been outlined. (Trifonov 1995) Hence, for the first time, the logical structure of physics (in a topos-theoretic sense having been investigated by Isham 1996 e.g.) can be generalized in order to explicitly cover the relationship between what thinks and what is being thought about (as representing two different aspects of the same underlying material substrate, dynamical status, and process of signification in the semiological sense). This empirically confirms an idea that Jean Piaget already mentioned in the seventies, pointing to this close relationship between perception and cognition on the one hand, and the mathematical concepts of morphism and categories, on the other. (ed. Brown 1992) Meanwhile, fractal aspects of cognition have been recently discussed by MacCormac, Stamenov (eds. 1996) et al. - thus establishing a link to the evolutionary structures on which the celebrated theories of self-organization and the formation of structure (chaos theory) in the sense of Prigogine and others are principally based. Looking at these results in terms of a philosophical "rationale", we can say the following: If reflexion (i.e. propositional thinking) is visualized as a human characteristic in the anthropological sense of a "definition" (of humans), and if humans are visualized as a product of nature at the same time, then reflexion is nothing but a form of (physical) matter (though of a very elaborate, complex kind). Humans thinking about nature can be thought of in turn as nature thinking about itself. Research is then nature's permanent effort of "reading" its own text within its own con-text (or equivalently: telling its

[1] Tucson III: Toward a Science of Consciousness, 1998, section 04.03 (Integrative Models), Abstract No. 942. Cf. http://www.zynet.co.uk/imprint/Tucson/1.htm.- Also delivered in a preliminary version at ANPA 19, Cambridge (UK), 1997.

own (hi-) story). Hence, this kind of self-narration is nothing but the wordly process itself when viewed in strictly onto-epistemic terms. So the unfolding of (worldly) being is practically equivalent to the cognitive process of collecting knowledge about this same unfolding. By means of an explicit mathematical concept ("negator algebra") the emergence of cognition as that kind of self-narration shall be illustrated. Within this context, the basic idea is to represent the production of metaphors (on the epistemic level) as a systematic (verbal) conceptualization of inhabited geometry (of space-time-matter) whose nucleus is the production of analogies. Hence, it is also the epistemic foundation of social poetics (or of the poetic geometrization of worldly references - which is basically the same - in turn based on narrative structures). In this sense, metaphors can be interpreted as "miniature poems" with characteristic topoi as their concepts. (Ricoeur 1975) Hence, the double meaning of "topoi". It is this metaphorization of geometry (the latter being at the roots of the worldly process itself) which expresses nature's permanent effort towards an adequate self-narration, the process structure of metaphors (MacCormac 1985) reflecting the very process structure of this worldly unfolding of meaning.

1 Philosophy Today

The chief objective of philosophy today can be visualized as a critical taking in sight of scientific results and their raising to a structural level, thus re-constructing the totality of references of meaning and establishing sense. By gaining distance to local problems and their detailed fine structure, philosophy is able to achieve insight into new perspectives from a global point of view. Philosophy can thus put forward heuristic offers to (interdisciplinary) science. Visualized this way, philosophy is not more one which establishes foundations for the sciences, but it is following up rather the results of the sciences, and its orientational function for humans is being met by means of a *post-hoc* unification of world views.
It is philosophy's intrinsic axiomatic structure however which establishes three important points serving as consistency criteria for world views: a) the *thesis of totality and unity of the world* (which means that there is one universe in one piece only governed by a set of laws which are universal), b) the *principle of intrinsic self-reference* (which means that the universe refers to itself during its evolution), c) the *principle of substrate unity* (which means that evolution is basically nothing but the unfolding of the

originally available matter to ever greater complexity such that thinking itself is a form of that same matter).
There is a decisive anthropomorphic tendency in this approach which is mainly due to the fact that everything which is being thought (about) at a time is depending on explicit anthropological and cognitively structured processes of signification linking the self-narrating nature and its symbolic self-representation to each other. But the mediators in this process of self-communication are the humans themselves (at least at present).

2 Geometry's Relevance for Cognition

The anthropomorphic aspects mentioned above are related to the relevance of space-time geometry proper for the very process of cognition. Some time ago, Leroi-Gourhan has shown in some detail that there has been a co-evolution of language and tools within the framework of human development which explicitly relates gesture with tool and maps this relationship in terms of signs which are mythograms rather than pictograms (meaning that both gesture and tool carry a narrative structure into this mapping procedure). The process of reflexion can be characterized then by abstracting symbols from concrete reality in order to constitute a parallel world of language which is able to act upon reality more efficiently. Obviously, there is an inbuilt feedback loop of reflexion and action. The association of temporal rhythms with spatial forms, preferrably within a unifying network structure of concepts, leads then to a domestication (i.e. a taking into possession) of space and time. It is this underlying reason (basically of a semiological nature) which introduces social space as a mapping of space-time which preserves geometry in a metaphorical form. Within the framework of self-unfolding cognitive processes, mappings of the mentioned kind co-develop either as morphisms or transformations, resepectively (the former conserving the structure of forms, the latter not). Piaget calls rough systems of morphisms *pre-categories*, pointing to a mathematical connotation and connecting it to a philosophical concept: Originally, categories refer to a continous process of signification with respect to a human mode of the grasping-comprehending of the world. The knowledge achieved would be worthless, if it were not communicable in one way or another. Hence, categories (in the philosophical sense) have to represent a bundle of structurally stable concepts in order to be able to communicate their basic contents. (One could say: the means of communication itself have to be adequate with respect to the geometric structure of the social space into which the real world is being mapped.) This basic idea is reflected in detail within the conventions of categories in the mathematical sense. Structural morphisms between categories are called functors. By developing the notion of functor categories (as categories of categories) the concept of an internal logic can be introduced which is of central importance for the foundations of mathematics itself. In the following we try to illustrate this interrelationship in more detail, referring basically to the recent works of Trifonov (1995), Isham (1996), Grinkevich (1996), and Vickers (1997).

3 Mathematical Topoi

With a view to quantum mechanics, Isham has recently addressed the problem that there are many d-consistent sets that are mutually incompatible. In this sense, a complete set of history propositions C: {α, β, ...} is said to be *d-consistent*, if the decoherence function d(α, β) vanishes for all possible pairwise entries. The probability that α be realized is then identified with the real number d(α, α). (Note that this historical context of propositions has been discussed similarly from another point of view in Abner Shimony's paper on potentiality and actuality in Penrose, 1997, referring here to a perspective which points to a more classical philosophy.) If the sieve on C gives the semantic value of some proposition within the context of C, then Isham can show that the set of all possible semantic values possesses the structure of a Heyting algebra: Be Set^P the category of varying sets over P. A *subobject* then, is a varying set {A(p), pεP} with A(p)⊆X(p) for all p, and with A_{pq} being the restriction of X_{pq} to A(p), whenever X_{pq}: X(p) → X(q) such that there is an identity and a composition. We call then Ω(p), pεP, the collection of all upper sets lying above p. Hence, a *sieve* on p in P is any subset S of P such that if rεS, then a) r≥p and b) r'εS for all r'≥ r. For each pεP, the set Ω(p) of all sieves on p can be shown to be a Heyting algebra. What is the use of knowing this set? If we know the appropriate Ω, if A is a subobject of X, then there is an associated characteristic morphism x^A: X → Ω which in each branch of the poset going up from p, picks out the first member q in that branch for which X_{pq} lies in A, and the commutative diagram on subobjects guarantees that X_{pr} will lie in A(r) for all r ≥ q. Hence, each morphism x defines a

subobject of X, and therefore Ω in Set^P is known as the *subobject classifier* in this categories. This indicates that the latter can be made a topos.
The generalizations of this view of topoi in terms of Trifonov's approach are shown in more detail in the appendix. For our purpose here it suffices to state that the basic idea is now to formalize the above mentioned self-narration in terms of an algebraical formulation which can be associated with these aspects of topoi, in order to produce a suitable relationship to the logic underlying categories. To this end, negator algebra is being introduced. This means that evolution is visualized as constituting a set of distributed self-compositions such that $f^n(x) := f \circ \ldots \circ f(x)$, n-times, defines a sequence of iterates $\{f^0(x), f(x), f^2(x), \ldots\}$. We call the first term *ground state*, and the second *initial state* of the unfolding system, and identify the latter with the set of attainable world states. The mathematical details can be found in the appendix already mentioned. The idea is now to give this sequence the meaning of a global dynamical system, of the general form $(dW/ds)^n = N^n (W)$, where W refers to the ground state of the development. Effectively, the dynamics is produced by means of the operator N called *negator* (negation operator), and it is proposed (but not yet clearly proven) that NEG (the category of negators) can be made a topos.
We have a parallel then between philosophy and physics (with respect to modelling the real world) in that there are fundamental concepts on the one hand, such as substance, attributes and so forth, telling something about the foundations of being as deriving it in some sense out of non-being which is usually identified with the field of possibility or potential of being. On the other hand, physics uses a formal terminology such as *dim* and *sign*, for describing the (metric) properties of space-time-matter and aims at pre-geometric models which enable us to eventually derive these properties out of the foundations these models offer. Twistor theory (of Penrose - including its approximation which is Newman's Heaven theory), M-Theory (or superstrings theory, introduced by Green, Schwarz, and Witten and a number of others) as well as loops (Ashtekar) and knots (Kauffman) lie on this line of thought. Both in philosophical and physical terms, the question is one for *the structure of non-being*, or to put it differently: for the condition for the possibility that N(W) exists consistently such that a universe might emerge. This can be thought of as establishing a generalized method according to Spinoza's idea of a philosophy "more geometrico".
I have discussed a biological system in adequate detail at another place (Selbstreferenz und poetische Praxis, 1991, 87sqq.), in order to illustrate the praxis of negation. See particularly numbers 7 through 9 of the appendix for more information, especially for some relationship to the "bit bang

evolution" discussed by Mike Manthey with respect to the notion of computing anticipatory systems (CASYS). A more detailed exposition of how to model evolutionary processes in the above mentioned sense is given shortly in another work. (The Klymene Principle, 1998)

4 Semiology and Metaphorization

The basic semiological idea of language centres around a dualism between language as system (langue) and spoken language (parole). Although the latter is in principle prescribed by the structure of the former, it is nevertheless able to influence the former by a straightforward innovation called *metaphor*. If a metaphor is associated with a "carrying over" of meaning from one word to another, we call it *metonymy*, if this translation remains within the same context. If this is not the case, we speak of a metaphor in the strict sense. In the process of signification then, the signifier (S) refers to a significate (s). But the signifier can never grasp the complete meaning of the significate, this being symbolically expressed by the quotient S/s. This means that there is a certain kind of reciprocity between the two, but that there is also a crucial gap between them. In principle, metaphorization (substituting one word by another in order to actually produce a new meaning) can then be expressed as a generalized quotient (S'/S) (S/s) indicating the space of free play for new meanings. It can be shown that the process of signification is closely related to psychoanalytic categories in that the metaphor can be visualized in terms of the condensational aspects of a symptom, while a metonymy can be visualized in terms of displacement (or shifting) aspects of desire (following here the terminology of Lacan and Kristeva). Hence, speaking as well as symbolizing/signifying at all is always - as is any kind of interpretation/translation - *mis-representation*/distortion or transposition (Ent-stellung). (Note that the same is true for using any mathematical formalism for expressing symbolically a process taking place in the real world.) If science can be visualized as the search for minimizing mis-representations, it is metaphorization the appropriate method to achieve this goal by means of testing the stability of

innovative metaphors. Wheelwright and MacCormac have discussed in more detail the relationship of metaphorization (being charged with all hermeneutic implications just mentioned) and the physical sciences.[2]
Within another line of argument, Deleuze has shown that metaphorization has an important meaning for the concept of differentiation. The ancient idea of "omnis determinatio est negatio" comes to a new life when thinking of the *differentiation of differences* which is a valid activity on both the ontological as well as the epistemological levels (namely in that it happens really within permanent evolution, and in that its modelling pursues in very much the same way). Abstraction in the traditional sense, namely as the task to separate substance from attribute, is based on signification visualised as *expressio* which is *explicatio and complicatio* at the same time. Hence the hermeneutic aspect: what is expressed is veiled and unveiled to equal parts. The traditional metaphors of *expressio* visualize therefore a mirror that reflects, but also a germ which expresses itself in unfolding. In this sense, Derrida has introduced the concept of "différance" (incorporating both différencier = to differentiate, and différer = to delay, to suspend) characterizing the nature of what is being produced in the process of signification by differentiating. Recently, MacCormac and Stamenov have discussed these aspects with a view to the mathematical connotations of fractals, relating these ideas to concepts of self-organization in the sense of Prigogine and others.
What we have as a (preliminary) result is a scheme for a recursive production of structures, both in ontological as well as epistemological terms. Modelling the world in physical or in philosophical terms means in anyway to deal with the foundations of language. The formal method (called here geometrization) is opposed by the heuristic method (metaphorization in the strict sense) according to whether one concentrates on the macroscopic or microscopic foundations of language, respectively, the former being dealt with in terms of formal logic and social philosophy, the latter in terms of hermeneutic logic and psycho-analysis:

<u>Recursive production of structures</u>

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[2] Especially, the epiphoric aspects of a metaphor are compared with its diaphoric aspects: The former has the primary function to express and to imply an obvious meaning, while the latter has the primary function to suggest and to imply possible new meanings. A famous example for this is the notion of "tachyon" which is a diaphor so long as empirical evidence is found for such particles when it changes to become an epiphor. Hence, as Ricoeur has noted, the metaphor is a process in itself with an equivalence between the semantic and the cognitive process structures. In this sense, metaphors generate historical contexts: Compare e.g. the proposition "quarks are coloured". We confirm that Ricoeur is right when calling a metaphor an "impertinent predication" in so far the innovative aspect of metaphors unfolds their heuristic intentions.

-----------------------------→ Physics → --------------------------

↑ ↓

heuristic formal

Semiology

Mathematics

Logic (hermeneutic) *Logic*

(formal)

metaphorization *geometri-*

zation

↑

↓

microscopic foundations macroscopic founda-

tions

of language (psycho-analysis) of language (social

phil./

anthropology)

↑

↓

--------------------------------← Philosophy ← --------------------------

The conclusion for a cognitive theory of consciousness, especially with a view to the latter's physical foundations is obvious: It is an *onto-epistemic approach towards a transcendental materialism* which offers itself as a solution to the classical forms of the body-mind problem on the one hand and the positioning of what thinking can mean within a physical theory of real world praxis on the other. As David Papineau has phrased it: "The true puzzle is not that some physical states produce states with the character of an experience (what would one expect, as they are identical?), but that it continuedly appears to be necessary to explain something ..."[3]

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Appendix

Topoi of Emergence

Part I (Introduction)

In the first part of the paper presented here, I have concentrated on discussing the basically philosophical aspects of emergence by mentioning relevant criteria for the appropriate philosophizing of today, the aspect of self-narration of nature, and geometry's significance for cognition, respectively. Theories have been visualized within this framework as sets of propositions with the latter being "charged" with metaphoric meanings such as to lay the grounds for (self-) creative (i.e. innovative) properties of language in general and of formal modelling languages in particular, with a view to both unveiling the concrete real processes underlying nature (on the ontological level) and the mode of their grasping and mapping in terms of human reflexion (on the epistemological level), this being a basically semiological problem, in fact. In this sense, the heuristic value of modern philosophizing has been stressed and explicitly discussed in some detail, preparing the starting points for mathematical formalization when shedding some light onto the difference between morphisms and transformations and the semiological relevance of this difference, according to a conception having been introduced first by Jean Piaget.[1]

In this present second part, I would like to concentrate on the formal aspects of the process of mathematical modelling. The problem of formalizing the basically philosophical discussion on the transition from potentiality to actuality has become a new centre of attention recently within various contexts, mainly with respect to quantum theory.[2] It has been Christopher Isham who utilized the concepts of topos-theory for this discussion when relating them to the decoherence function in quantum theory's many-world-view. If C is a complete set of history propositions which are d-consistent (i.e. consistent with respect to the decoherence function), then Isham can show that for any context C, the sieve on C constitutes the "truth" (or: semantic) value for propositions in C, and that the set of all

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[1] J.Piaget: Morphisms and Categories. Comparing and Transforming. (ed. T.Brown), Erlbaum, Hillsdale (N.J.), Hove, London, 1992.- Note in particular the contribution by G.Henriques: Morphisms and Transformations in the Construction of Invariants, ibid., ch.13, 183-206.

[2] See e.g. A.Shimony: On Mentality, Quantum Mechanics and the Actualization of Potentialities. In: R.Penrose, The Large, the Small and the Human Mind, Cambridge University Press, 1997, 144-160.

possible semantic values possesses the structure of a Heyting algebra. With a view to the basic decoherence problem (which says that there are many d-consistent sets that are mutually incompatible), the idea is to show how the mathematical structure of the collection of all complete sets of history propositions can be exploited to provide a novel logic with which to interpret the predictions in the many-world-view's context where all d-consistent sets are handled at once, this hinting straightforward to aspects of intuitionistic logic (which is the logic of topoi). In fact, Isham can construct a subobject in the category of varying sets and find an appropriate subobject classifier so as to identify the probability set of history propositions with a topos.
This approach can be generalized to other, non-quantum (and even non-physical) contexts when applying the idea Trifonov has introduced: He is interpreting topoi in terms of abstract worlds, universes of mathematical discourse, whose inhabitants may use non-Boolean logics in their reasoning. The idea is to show that a non-Boolean logic is more generic in a sense than a classical (Boolean) logic, this having profound implications towards physical (and other) applications.[3] As Vickers has shown[4], topoi can even be used for classifying geometric theories "in toto". Hence, they can be visualized as spaces whose points are the models of such theories, geometric morphisms being transformations of points then of one such space into points of another. This leads to a topology-free treatment of domains as topoi.
These results are important when looking for a general foundation of truly emergent processes in nature. In the following, the concept of emergence is introduced in terms of such an abstract process of self-founding subsequent negation operations. A possible relationship to Mike Manthey's conception of algebraic action is outlined.

Part II (Details)

1 *Self-Composition*: Starting from a basically philosophical point of view[5], how namely the transition from potentiality to actuality (emergence) could be described in a consistent way, which is presently a central topic of

[3] An economical example is discussed in a recent paper by Wolfram Völcker and myself: Topoi of Emergence. Theoretical Foundations and Practical Applications. In: Nonlinear Dynamics, Chaos, and the Formation of Structure, Proc. 7th Annual Meeting, Chaos Club (TU München), Z.Naturforsch. A (1998), in press.
[4] S.Vickers: Topical Categories of Domains, Res.Rep. Doc. 97/1, Imperial College London, 1996.- Also id.: Localic Completion of Quasimetric Spaces, Res.Rep.Doc. 97/2, Imperial College London, 1997.
[5] R.E.Zimmermann (1997): Topoi of Energence (I). The Logic of (the) Matter, op.cit.

theoretical physics[6], the concept of "self-composition" gains a decisive meaning: Be Y a for the time being unspecified, appropriate space, f: Y $\rightarrow$ Y a mapping on this space. Then a mapping of the form $f^n(x) := f \circ \ldots \circ f(x)$ (n times) with $x \in Y$ is called *self-composition* or self-iteration of f. The iteration sequence $\{f^0(x), f(x), f^2(x), \ldots, f^\omega(x)\}$ characterizes then the ground state ($f^0(x)$), the initial state ($f(x)$), and all further states up to a final state $f^\omega(x)$ of a given system, if we assume that f defines a dynamic on Y which formally renders the pair (Y, f) to be some such system.

2 *Chaotic Mappings*: Mappings f of the above mentioned type are called *chaotic* on Y, if 1) f is sensitively dependent on boundary conditions (unpredictability) and 2) f is topologically transitive (indecomposability), i.e. if there is for each pair of open sets U,V of Y a positive k such that $f^k(U) \cap V$ is nonzero, finally 3) the periodical points of f lie dense in Y (nucleus of regularity): i.e. if for $P := \{f^n(x) = x$ of period n$\}$ and $P \subset Y$, the closure of $P = Y$.

3 *Ground state*: Let us refer the iteration to the formal ground state of the sequence. Let us call it $W_0 \not\subset \{W\}$ and the mapping N, calling W_k with $k>0$ "world state" in general and N "negation". Then the state $W_k \in M$, any k, is element of a suitable "world space" of states M, usually a smooth manifold with a metric of dimension m and signature s. Thus we obtain a new sequence of the form $\{N(W_0) = W_1 = A, N(W_1) = W_2 = N(N(W_0)) = N^2(W_0), \ldots, N(W_{n-1}) = W_n = N^n(W_0) = \Omega\}$. This describes the evolution of states of a given world, starting with an initial state and ending with a final state.

4 *Global Boundary Conditions:* For such a gobal evolution it is useful to specify boundary conditions which characterize the boundary states A and Ω in more detail. In this way we can obtain results on the conditions for the possibility of such a world (transcendentality). For a realistic, physical world in the sense of modern cosmology, we can easily choose the "Penrose conditions" for this purpose: $A = \Phi(F)$; $\Phi(F) \rightarrow \infty$; $\Psi(P) = 0$ and $\Omega = \Psi(P)$; $\Phi(F) = 0$; $\Psi(P) \rightarrow \infty$, respectively. In this case, $\Phi(F)$ and $\Psi(P)$ are the Ricci and Weyl components of the Riemann curvature tensor from

[6] C.Isham (1996): Topos Theory and Consistent Histories: The Internal Logic of the Set of all Consistent Sets, gr-qc 9607069 (referred to in the introduction).- A.Shimony: On Mentality, Quantum Mechanics and the Actualization of Potentialities, op.cit.

Einstein's theory such that the relationships *Riemann* = *Ricci* + *Weyl* = $\Phi(F) + \Psi(P)$ and *Ricci* = *Energy-Momentum* are valid.

5 *Evolution Equation (Unfolding of the Ground State)*: Generally, we can describe the evolution of a world with the above mentioned boundary conditions as a formal dynamical system, of the characteristic form $(dW_0/ds)^n = N^n(W_0)$. Note that the exponent at the derivation has here a combinatorial function only: It signifies the initiation of a generic "frequency of cadence" (of a rhythmical beat) with which the successive negations "control" the evolution in acting upon the ground state. Hence, this action is one of a multi-contextual kind and surpasses the frame of a merely analytical description. The main reason for this is that the internal logic of the process is not one of the Boolean type. Also, it is important to note that what is unfolded (the ground state itself) is per definitionem *outside* of the world which actually emerges by this unfolding in first place.

6 *The Category of Negators:* Operators actually acting as negations are called "negators". Their category NEG, in which they are the morphisms in first place while the world states are the objects, can be visualized in terms of the category of varying sets over an index set P: Set^P - in analogy to the argument given by Isham (1996) on the complete set of d-consistent history propositions in quantum theory (if d means the decoherence function). In this sense it can be shown that for subobjects of NEG there is a subobject classifier such that NEG is a topos with an internal logic which secures that the semantics of negation operators has the structure of a Heyting algebra. Even more: After Trifonov has shown[7] that the paradigm A of an R-xenomorph is Grassmannian (or supersymmetric), if A(F), as category of linear algebras over a partially ordered field, has paradigms as its objects which are themselves non-trivial Grassmann algebras, it is straighforward to assume (but not yet explicitly proven) that NEG is an R-xenomorph with a Grassmannian paradigm itself (this actually securing the applicability of an intuitionistic logic). Note the relevant definitions for this in the appendix given below.

7 *Formalization of the Scheme:* The formalization of this scheme of analysis referring to local actions of NEG can be illustrated e.g. by the well-known Keller-Segel scenario from Biology.[8] Is E the evolution operator inherent to a given system, and is N(E) its negation, then the formal

[7] V.Trifonov (1995): A Linear Solution of the Four-Dimensionality Problem, Europhys. Lett. 32 (8), 621-626.

[8] R.E.Zimmermann (1991): Selbstreferenz und poetische Praxis, Junghans, Cuxhaven, 87sqq.

scheme can be established in a cyclic manner such that N(E) gives the transition from stability to instability and $N^2(E)$ the transition to a new stability (of local structures). Obviously, it is secured that $N^2(E) \neq E$. Such a local cycle in three steps corresponds to a complete transition from old actuality through potentiality toward new actuality. Such a cycle we call "sandwich layer". We can see clearly now that world states are not really static but inherently dynamical (hence evolutionary by themselves). But because we can interpret this ansatz also in global terms, the underlying dynamics constitutes two different perspectives which can be utilized in order to discuss the nature of sandwich layers: on the one hand the *local* perspective which concentrates on the layers one by one, on the other the *global* perspective which visualizes the totality of several layers at a time, according to whether or not this is adequate with a view to the reserach actually undertaken.

8 *Concatenation of Sandwich Structures:* Hence, the evolution of a given world can be visualized as concatenation of sandwich structures. According to the actual research undertaken a given structure has to be analyzed according to its "fine structure". The original form of a sandwich is conserved in the sense that independent of the "strength of magnification" as it is applied to a local fine structure, the basic dynamical scheme will always be reproduced. This is the way in which the *fractal* pattern of the processes is mirrored, reflecting the fact that their internal logic is basically of Grassmannian type. There is thus a universal structure of mediation of worldly evolution which relates the singularity to the totality (on microscopic as well as on macrsocopic levels). Note that the question of the complexity of a system can be answered easily with a view to this structure of mediation.

9 *Anticipation and Duality:* Mike Manthey has shown[9] that in terms of a pure process view anticipatory systems can be characterized by an explicit *project structure* (which can be interpreted in the sense of existential concepts of productivity and subjectivity) and a *hierarchy* which does not only regulate the levels of abstraction, but also those of the real concretion (of evolution). In particular, it can be shown that there is a "morphic" level of abstraction which corresponds to a *self-reflexion* of the system. Manthey models events (alterations of states of a system) and processes (as sequences of events) in terms of a self-composition of a suitable operator mapping as well. The representation of "perception" (i.e. composition of

[9] M.Manthey (1997): Distributed Computation, the Twisted Isomorphism, and Auto-Poiesis, R-97-5007, Aalborg Univ. (DK).

structures from sensorial gain of information) can be expressed in this sense by cohomology operations (of the co-boundary), the representation of "actions" (i.e. de-composition of structures to the purpose of reducing complexity) can be expressed by homology operations (of the boundary). Both of them are mediated dually by a "twisted" isomorphism. In case of self-reflecting systems both of these operation components can be visualized as the dual ground structure of the negator model introduced here.

10 *Temporality:* We can say now that anticipation (in terms of an action of systems) can be visualized as result of the mentioned underlying duality. Insofar anticipation in turn actually "anticipates" the generic time of the system (called s within the frame of the evolution equation), duality *implies* temporality. With a view to the evolution equation given in section **5**, we can call the upper index n *global age* of the system. However as *local age* of the system, we introduce the number j of actually emerged sandwich layers of a system (always referring to a suitable cycle). Hence, we follow the terminology of Prigogine. Obviously, it is straightforward to assume that n be a superposition of the j's, although its explicit form is not yet known.

11 *Algebraic Action:* Is a process expressed by a sequence $\{s_j\}$, if attributed to the group operations of a Manthey action, then this action can be written in the form $s_1 s_2 (s_1 + s_2) s_2 s_1 = \bar{s}_1 + \bar{s}_2$ where the barred quantities are duals and formal multiplication and addition are the group operations of the action. Generally, this can also be written in terms of a Clifford product such that this product takes on the form $s_i s_j := s_i \circ s_j + s_i \wedge s_j$. Because the Grassmann-Algebra is a sub-algebra of the Clifford algebra, as e.g. Freund has shown in the case of the important algebra of quadratic forms[10], we presently assume that the discussion of an R-xenomorph with a *Clifford Paradigm* is of central importance for the discussion of systems which describe worldly evolution as self-differentiation and self-reflexion of its own foundation (ground).

12 *Appendix* (Xenomorph): Be F a partially ordered field. An F-*xenomorph* is a category A(F) of linear algebras over F. *Paradigms* of an F-xenomorph are A(F)-objects, actions are A(F)-arrows. A paradigm is called *rational*, if the space of motions M(A) is a monoid. We apply here the terminology of Trifonov's: Topoi are introduced in this sense as abstract worlds which represent universes of mathematical discourse whose

[10] P.G.O.Freund (1986): Introduction to Supersymmetry, Cambridge University Press, 15, 34.

inhabitants can utilize non-Boolean logics for their argumentation (propositional structures). In contrary to the *sensory space* which mainly deals with the observations of researchers, the space of motions is the set of actions of the researcher. Hence, the paradigm is the set of states of knowledge. In particular, it can be shown that the set of all possible actions of a researcher is a topos whose arrows are those mappings which conserve realizations of the mentioned monoid (of the space of motions). It can also be shown: If A is a rational paradigm and the topos of all possible actions is Boolean (non-Boolean), then the paradigm A is classical (non-classical). For a xenomorph F = R of a generic type of "psychology" of an observer, Trifonov can show that an R-xenomorph implies a classical Einstein-paradigm, i.e. dimenionality 4 and signature 2. (The proof goes by quaternion algebra!) Also: If A is a non-trivial Grassmann algebra, then the paradigm is the Grassmannian of an R-xenomorph. Because A has a zero divisor, M(A) cannot be a group. Hence, the logic of a Grassmannian paradigm is always non-Boolean, and mathematics is non-classical.

MEASUREMENT and SPIN

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I want to talk about measurement. So far our discussions and writing have paid lip-service to the idea that the ultimate scale for measurement is given by our combinatorial considerations of one form or another, but it has been left vague just how the values of these constants can be seen as the same kind of thing as the numbers obtained by conventional measurements. Alternatively people have supposed that they could start from a separate point and postulate some operational argument with appeal to laboratory practice in the hope that the two would meet in the middle. So in fact you could say that we have all been doing numerology, even though it may be mystifyingly successful numerology.

Most of the discussions of combinatorial or discrete or computational physics give a unique place to the coupling or 'scale' constants in describing the relation of quantum physics to continuum physics. If one starts from the classical standpoint then the calculation of these constants may be said to explain the origin of the quantum world. However what holds the coupling constants and the experimental values of classical physical variables together as like things? We say both are the results of measurements, yet the two seem to have a different logic altogether. We have been mostly content with a rather vague sense that experimental procedure is the unifying feature in one way or another. I have mostly been content to say that the scale constants have the place they do because they are the limits or bounds to measurements of a given class. In these remarks I shall try to get this limiting notion clearer and in so doing I shall be led to some new vistas.

To get off the ground I give a unique place to angular momentum in the representation of measurement. If, as is usually held, measurement is based on quantum specified units, then what I am saying accords with the commonly held position that the quantum unit is a unit of angular momentum, and at first sight sounds so 'old hat' as to be suspect, though I would suggest that there is no automatic harm in being conventional. My innovation consists in placing the angular momentum concept within the scope of the Combinatorial Hierarchy. Spin, I shall contend, is uniquely the physical quantity - the quantum number - which is brought into existence by the simplest possible change of level, and therefore becomes the primaeval quantum number. You remember that the spin argument says that a level jump is needed to transform two two-strings without overriding the requirement that up-down-ness is preserved. (Or any other ordering of course). The point is that you have a conceptual unit which depends for its existence on two levels. This is briefly described in CP, p.133, and Mike Manthey takes a related view as central to his formulations in terms of co-occurence. I hope that the isomorphism which he and we have claimed for his development and the combinatorial hierarchy will be clarified by what I shall be saying.

This account of spin of mine has been about a long time. It appeared in "Quantum Theory and Beyond", and others have tolerated it, though there are apparent objections to it which Clive has explained in his paper. It is part of that program of giving combinatorial form to the quantum numbers independently of the classical concepts which usually have the same names (at least in the basic cases of spin and charge). Most people have been content to assume that everyone knows what spin is, even though the only available definitions are entirely classical and in spite of the fact that the same people will assure you that the quantum theoretical spin concept has nothing whatever to do with what goes under that name in classical physics.

My original account of spin with its half-integral character says that a level jump is needed to transform two two-strings without overriding the requirement that up-down-ness is preserved. (Or any other ordering of course). The point is that you have a conceptual unit which depends for its existence on two levels. This is briefly described in CP, p.133, and Mike Manthey uses a construction with his manipulation with coins which we think is isomorphic. The argument then used was that if we were to impose upon the 2-vectors an ordering represented by upwards and downwards in the case of column vectors, then a mathematical property identifiable with half-integral spin would emerge from the asymmetry. Thus we might investigate transformations of the 2-vectors which - for example - had no zeroes in the bottom place. We require two discriminations to perform one of these, whereas one 2×2 matrix will do the trick. The argument then ran that the frequency of the first process would be half that of the second and therefore that any measure of the physical quantity attached to the quantum number (angular momentum) would be halved, giving the familiar half-integral spin.

Clive said that though he had gone along with this in Combinatorial Physics he was not easy with it because it meant a departure from basing things entirely on the dcs. However he then thought about what was involved in making a class distinction between sets of two-strings at a fundamental level and to his surprise after a lengthy piece of work found that this amendment amounted to a level-change. I think that the most important piece of this work may be what it has to say about the level changes. There are many ways to look at level changes which prove them to be inevitable, and yet one may feel a bit uneasy because they look too much like a mathematical device. If it is really the case that their existence is needed to make measurement possible then one feels the whole structure much more solid.

We are after something more than a definition of spin however. We need to be able to take spin as a unit. In fact we have shown that a spin unit is always associated with a change of level. In past thinking we have been much influenced by the thinking of Brouwer, who considered a collection of *infinitely proceeding sequences* which he called a *spread.* It is convenient to use these terms in connection with measurement. Brouwer was bringing the temporal ordering of mathematical steps into the mathematical structure itself by insisting that mathematical truth was dependent on the epoch of its formulation. We can put this another way round and say that there is no automatic objectivity in temporal ordering, and derivatively of course, none in the less fundamental spatial orderings. My conjecture is that the establishment of a unit or limit point is the condition under which we can have any agreement on time ordering. This comjecture leads into my discussion in terms of spin units and necessarily, (as I discuss later) of some form of countiig of them.

I suggest that in the development of a spread one or more spin units are generated whenever a level change is forced upon us. There must be a very early limit to what can be encompassed by defining new quantum numbers, and there is something simpler we can do. This is to add them up to form multiples of the unit spin quantities. Here I appeal to an idea like Tom Etter's where the big problem is to go from *composition* to *extension.* It seems we should take the counting as equivalent to frequencies. (This gives us easy access discussion of the red shift.) We can probably assume that the only automatic time objectivity is provided by the assumption of universal 'spectral' counts. All else has to be constructed and is corrigible.

Next, we ask what is now needed to get to extension? How do we make the big jump? Well, I introduce the idea of c as a limit point in what I will call a measurement space and I take that as the essential first step in setting up physical space. In conventional physics it is natural to think of measurements as having some goal -call it the *result* of the measurement towards which things progress and so that the sequence of measurements has a natural and automatic unity. The reason why this all seems so natural is that we think in terms of a background of space or extension against which the ordering of the approximation has meaning. In our picture, however, there is no such background. Therefore there is no progression until we put in a limit point. The limit point is the condition under which alone we can proceed to extension.

I imagine your saying "Isn't it a bit corny dragging in the velocity of light here?" I answer that it is only the way physics happens to have developed that piles the logic of the limit point onto the velocity of light, though it does have to be somewhere. To put it another way we can think of the measurements as being 'nested' once we know there is a limit point. The rest of the main idea of this paper is that the spin unit is the switch which anchors the individual steps of the measurement sequence.

All this elaborate construction of a nested sequence of measurements resulting in a limit point has a very homely equivalence in conventional thinking. The limit point corresponds simply to the basic unit of measurement in terms of which we state our results.

We may ask what, in our context, corresponds to the generality which Brouwer imagined which needed the spread theorem. I think that the sense of Brouwer's spread theorem lies in his view of the fluidity of temporal relations until they have been tied down in some way. He, and we, imagine many distinguishable configurations where someone mapping them onto a classical background time sees them as all the same. If there is no change, then the steps which appear to denote a change can be collapsed onto one, and since the system is finite the labelling of the sequences will be achieved after a calculable number of steps. If we suppose we have a hierarchy as Parker-Rhodes imagined it then the finiteness is trivial -or at best reduces to Koenig's theorem. We are reminded of Brouwer's Fixed Point Theorem which proves the existence of a fixed point (like my limit point) using some sort of geometrical construction. Pierre helped me to get away from this geometrical crudity by calling the nesting property 'insidedness'. Of course the Fixed Point Theorem theorem put the cart before the horse, which is why Brouwer himself later repudiated the theorem.

It helps me to think to say that in the general situation, the time (or perhaps one should say the perceived time) can change so that we cannot automatically be sure whether we are contemplating a memory or the physical 'reality'. This idea helps me to say what we are really doing, and yet I can only express it in terms of the universal time which the whole theory rejects.

The need for a limit point - or perhaps rather a limiting process - is now much clearer. How do we recognise the separate points established this way? Do they have steps of frequency? Presumably they do, and then we get the Heisenberg relation or a primitive form of it, and as well we get a spatial ordering of some sort because of the sequential definition of unit, and the insidedness comes from the limiting process and is essential for the notion of a space: or perhaps to eliminate the possibility of jumping out of order by changing the time. Once we can assume the progression to a limit the creation of spin units is like a ratchet mechanism: it enables us to draw the net tighter whenever there is some slack, and it stays that much tighter, and so we get the idea of measurements which achieve ever greater accuracy.

I had hoped to develop these ideas to a point where they had evident relevance to some profound questions -particularly the basic foundations of relativity given that the passage of a ray of light is essentially a *quantum* event, as well as to an understanding of the red-shift. However I have had such trouble getting any clarity of vision even thus far, that I have only disconnected notes on these matters.

I finish with some remarks which go off at a tangent rather but which do have some bearing on the topics which I should like to have mastered. A friend of Clive's whom I will call X objects to the use of electro-magnetic propagation in special relativity on the grounds that wave propagation with velocity, *c* does not describe the production or the reception of signals. He puts his argument in a form in which it looks as though he is simply ignorant of the distinction between group and phase velocities. However he could say that special relativity has introduced something really new. It defines length in terms of the velocity of light and therefore of the uniform wave-train. Hence it cannot describe anything except the far field. It could not for example set a range for the near field.

In the near field, the electric and magnetic vectors are no longer at right-angles as they are in a long train of waves. This condition *can* be assimilated to the Fourier method of course, but bit really looks to a more primitive situation before the conceptual apparatus is there to set up the Fourier method. Perhaps one should look further at the implications in the more primitive situation. Consider the perfect hydrodynamic fluid. The force on a closed curve drawn in the fluid moving through the fluid is proportional to the velocity and to the *circulation* in the fluid enclosed by the closed curve. This leads of course to the definition of the curl vector. One can relate the force to the number of unit vortex filaments enclosed, and the treatment is formally the same for the force experienced by a conductor carrying a current as it is for the lift on an aerofoil or on a spinning (cylindrical) cricket ball.

In the case of the fluid, one asks what actually generates this force, and the answer has to be that it is the inertia of the particles of fluid which are accelerated. We may imagine Maxwell, for whom the mechanics was very real, looking for the analogy of this inertia in the magnetic case and coming to the conclusion that it was the electric displacement. One can be quite phenomenological about that and say that we have now introduced the real difference between the electric force and the magnetic. The electric force is indeed a kind of elastic effect which introduces a relaxation time. Maxwell said that the product of the elastic constants should define a propagation speed, and the rest of the conventional treatment follows, but we should notice that he hasn't produced the dynamics in detail. I have carried on this discussion without introducing length explicitly.

I return briefly to my hydrodynamics. My argument demands units of circulation whose only metrical property is that they can be counted. We may not ask whether the units are the same size as each other because that would lead to a circular argument. In this little detour I have just been giving a bit of practical background to the notion of counting of angular momentum units.

Excluded material

7. Something is needed in the K-calculus to get the degree of spatial objectivity that is always assumed but which your discussion shows to be logically unattainable. I suggest that a guide to this is that we have forgotten that a 'light signal' has to be a quantum event.

8. Then let us apply our new picture of the quantum event with complexes.

9. We are now talking about the field over Z_n, whatever n is -see previous discussion- whereas we thought we were talking of normal space.

10. This is not an error provided we follow out the consequences of the quantum event. We are here discussing the reconciliation of quantum physics and relativity. Even with the K-calculus we had to make some jumps into the conventional imagination to get recognisable physics, and now we suggest that the complete story is that the jumps are really quantal. My idea is that at the relativistic/classical end, the spin unit is needed to specify the unknown length which the 'other' observer is at, at a given time. At the relativity end of the picture this accords with the classical idea that an angular momentum is associated with a fixed length (the moment). We seem to have left the hierarchy and the level change out of the picture, but this is only because we are dealing with the relativity end. If we look at things quantum-wise then we are back there. So we can work either of two ways, and of course both are needed and they are not two independent worlds because of the spin theory.

If the answer is "yes", does this mean for the single observer or more generally? I think the K-calculus in our form gives the answer - namely that the mysterious constant c can be interpreted as a signal (perhaps not even yet as a velocity) if and only if there is a physically defined progression to a limit.

DISCRIMINATION WITH ASPECT.

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This paper describes general ideas. Technical developments are relegated to the notes at the end. I explain where there is a serious error in the book Combinatorial Physics[1] by Ted Bastin and me (hereafter referred to as CP): then I show why it doesn't matter quite so much as one might fear. Finally I sketch how it can be put right - only a sketch because a whole new theory is waiting to be born.

The 1996 Proceedings[2] contains Pierre Noyes' historical sketch from his point of view, as well as my summary, so there is no need to recapitulate details of the Parker-Rhodes construction of the "combinatorial hierarchy" (here, the CH). It is sufficient to say this: In 1961 Ted Bastin already saw the importance of what I have called Planck's Problem, the puzzle of the existence in physics of scale constants, of which I take $\alpha = e^2/\hbar c \simeq 1/137$ as a paradigm. Frederick Parker-Rhodes proposed to Ted a peculiar algebraic construction in terms of vectors and matrices over the field $\mathbb{Z}_2$ with two elements 0 & 1. At the time it seemed as if the peculiarity lay in the field $\mathbb{Z}_2$ but really the strange thing was that the construction gave rise to exactly four levels of structure, characterised by the numbers 3, 10, 137, 10^{38}, the matrices at one level being the vectors at the next. One can follow the multitude and dismiss this as coincidence or numerology - but not if one knew Frederick personally. The alternative is to surmise that it is physics in disguise and to try to understand it. Hitherto I have seen Frederick's construction as the Ark of the Covenant, to be preserved and reached somehow. This year I've come to see it more like the models of dinosaurs where one foot or one tooth has been found, the rest is re-construction. (The one foot is the CH, the rest what I am beginning to see as needed.)

A technical aside: Frederick's construction uses vectors and matrices of the field $\mathbb{Z}_2$, so introduces two operations. From early on + got the meaning "discrimination" because $x + y = 0$ if and only if $x = y$, but multiplication, which was essential in Frederick's matrices, never got a physical meaning.

My present preoccupations arose from a paper by Ted some 7 years later in Quantum Theory and Beyond[3] (published only later) which tried to fit the physicists' idea of "spin-$\frac{1}{2}$" into the CH. This paper failed (i) because it had to introduce concepts which did not fit into the CH, (ii) because, as I shall show below, it was an impossible task. My approach, guided by many at ANPA, has been to see the CH as arising naturally and inevitably from process. Ted and I, in CP, speak of a division of the world into 2, the unknown and the known and of entities becoming known. I have talked recently of the process of a scientific investigation. That is too specific: I don't want a theory whose truth depends on men in white coats. But I am inclined to think even CP is too specific: to whom are these entities known? It is better to grasp the bull by the horns as Whitehead does in Process and Reality[4]: "That the actual world is a process, and that the process is the becoming of actual entities...That in the becoming of an actual entity, the potential unity of many entities in disjuctive diversity - actual and non-actual - acquires the real unity of the one actual entity...". But for my purposes today we need not debate this. The theory is sufficiently abstract to apply to each.

The process is autonomous but we also give a theoretical analysis of it (what we used misleadingly to call "mathematics" because it involves symbols written on paper). Different analyses may be given but any, to be acceptable, must preserve the open, continually developing quality of the process. This restriction I call "openness" and it has a number of consequences. When an entity becomes known (or, for Whitehead, simply becomes) it enters the theoretical analysis as a symbol. I did not want to call this symbol its name because that would, I thought, import too much unwanted philosophical baggage, so I called it a label. I did not know enough to realise that this imported unwanted computing baggage. On balance I wish I had stuck with name.

We didn't really get the naming business quite right in CP. Each new entity has to be checked to see if it is really new or a copy of an earlier one (repetitions are the essential basis for the statistical inference about the still unknown). I think I had too much in mind a model of a baby arriving nameless at the font. The right procedure, which also takes up Whitehead's point about potential, is to give each entity, straight away,the first unused label. Then, when it is tested: if it is new, it continues to have this label, if not, it takes whatever label is appropriate. An example later on will make this clear. An entity becomes: call it a. Then another: call it b. At some stage in the process, not necessarily next, it determines whether a certain equivalence holds or not. Openness means there is only a procedure for determination, not a finite specification. If the determination results in the conclusion "a, b are different", this procedure + result is a new entity c, or,say, Dab. So when it is translated into the theoretical analysis, naming appears as a binary operation. This is important because way back in 1954 Ted[5] introduced this notion of a binary operation and Frederick did it again independently, but it was never clear to me why there should be such an operation.

Two useful interventions were made at this point. Rainer Zimmermann contrasted this approach with Schelling's, in which the realisation of the identity of A with B is an A of higher order.[6] But Schelling in 1810 was concerned with the arising of something new which has to be understood; we are concerned only with the conceptually simpler situation of a naming procedure, for which c = Dab is evidently a name, just like a and b. Then Arleta Griffor raised a related question about logical type. Of course one must avoid any serious errors of logical type but an explanation of the CH and its level-changes, in which whole sets of elements at one level become single elements at the next, cannot have the purity of logical type urged by Russell and Whitehead. Understanding naming leads to another bit of clarification. It is obvious that Dab must have a different name from a or b. We put this difference into CP as a "convenient assumption" but now we see it is not a separate assumption at all.

Returning to Dab: if the process determines a, b to be the same, no new entity results and so Dab cannot be a name; call it a <u>signal</u>. An equivalence relation must be reflexive, symmetric and transitive and this has consequences for D. Firstly two D's which agree in giving signals do the same job and so are interchangeable. Then we argued correctly in CP, leaning slightly on the reflexive condition, that any D is interchangeable with one in which all signals are the same, z say. We followed this in CP by a totally wrong argument leading to a false result:that symmetry:

Dab = z implies Dba = z,

could be used to show any D to be interchangeable with a symmetric D, for which Dab = Dba. With hindsight, this result must be false because Dab is the name of a process which may result in b being renamed a, not _vice versa_ as in Dba. If you have CP you might like to pick out the error in the proof. Another, easier question is: how did the error come about? Because I was so intent on reproducing the CH that I persuaded myself it was right.

To change D for a symmetric D is to ignore a certain difference; I call this difference that of _aspect_. Now this false theorem set alarm bells ringing! Because, to look ahead, the process theory, according to CP, was going to derive something near enough to Frederick's CH to get his 1/137 in a rational context and even to do better by showing it should be 1/137·035...But matters are not _so_ serious. To ignore aspect is not useless - it concentrates on some features of the process to the exclusion of others. Any theoretical analysis is selective. It's a bit like writing history. So it makes sense to look at symmetric D's, which is what CP does. Moreover, even if D is not symmetric, there will be an operation of "symmetrization" when the differences are ignored. [Technical aside to mathematicians: there will always be non-trivial homomorphisms into symmetric structures - non-trivial because of how the homomorphism arises.]

Let us go along with the symmetric assumption for the moment. Then openness means that the transitive condition, which has the form:

if Dab = z and Dac = z, then b = c,

can have a recursive procedure to fulfil it only if we strengthen it to

if Dab = Dac, then b = c,

and that is then fulfilled by "Conway's trick":

Dab = least c different from a,b,Da'b,Dab'

where a', b' are earlier names than a, b. Observe that this gives a closed set:

a,
b, Dab = c,
Dac ≠ a, c, = b
Dbc ≠ b, c, = a.

If z is included in a suitable way this is the quadratic group, or Klein's Viergruppe, S and so one concludes:

D is an associative operation.

Now that deserves comment. On the one hand, it is a very convenient outcome, for if D were a general operator, then when more than 2 elements were involved many different forms would result. There would be 2 for 3 elements, DaDbc and DDabc, and the number ~~rises~~ sharply:2,5,14,42,132,449,...[7]. Associativity means only one in each case. But it is a curious result: it says, if c is the name of Dab, then when one comes to discriminate Dbc, the act must be named a. I can't see any _a priori_ reason for this; it is just a consequence of openness and discrimination.

The set [a, b, c] is the first example of a _discriminately closed subset_ (a dcs): any two different elements discriminate to another element of the set.Going on, the next new element is named d. What of Dad? A priori it could potentially be z, a, b, c, d or a new one, e. But a, d are impossible and if Dad = b, then d = c, if Dad = c, then d = b and finally if d is indeed new, Dad = e and so on. This nicely illustrates at a very simple level Whitehead's "potential unity of many entities in disjunctive diversity". The next dcs has 7 members and is again associative. The notion of dcs is absolutely crucial in the CH; the dcss at one level are represented by single elements at the next. These elements serve to capture, in addition, the equivalent to discrimination when, instead of testing a new element against one old one, the process tests it against a whole set of old ones (in fact against a dcs). The argument on to this stage is still as set out in CP, introducing linear functions on the set of elements.

To go back to Ted's spin-$\frac{1}{2}$ argument, the first reason given against it could now be amplified: it rests on dividing elements at the first level into two sets, which cannot (from his argument) be dcss.

Returning to CP, the result of the argument is a system which differs from the CH only in the way that openness, which has been the midwife to its derivation from process, survives to say that the vectors over $\mathbb{Z}_2$ cannot be of fixed length. In short, the resultant system, which has the advantage that its origin is clear, is nearly isomorphic to Frederick's, and it is near enough to get 3, 10, 137, 10^{38} just as he did. But it is not quite isomorphic and so gets 137·035.. instead of 137. What we write as Dab is what Frederick writes as a + b and what Ted was writing[5] as ab back in 1954. Then the part played by matrices for Frederick is played here by linear functions, ie. those preserving +.

A partial answer to the "second binary operation" question noted earlier: x is an artefact of the expression of the linear functions in matrix form. Now the group of transformations is just that of (non-singular) linear functions. This is not a group with two-valued representations. That is why Ted's attempt to introduce spin-$\frac{1}{2}$ was impossible, as mentioned above.

So that is the story so far, suitably modified by hindsight: an actual discrimination process will have a non-trivial symmetrized skeleton yielding scale-constants. I spend the rest of the paper in going back to the symmetrization error and sketching the way ahead to non-symmetric systems. We shall find there that Ted's argument about spin-$\frac{1}{2}$ can be re-introduced. John Amson long ago asked the question whether there could be a non-symmetric discrimination but he started from the expression of the CH in terms of bit-strings, looked for an "element-wise" operation and found nothing interesting. We shall find a system for which the bit-string representation (except as mere coding) is ruled out. Indeed, the situation is more extreme than this. It will turn out that the lowest level of the hierarchy is more complex than in CH but still easy to work with. There is a surprise about the next level; but after that the theoretical analysis lands one into a morass of non-associative algebras. Nor is this to be lamented; it shows how quickly the complexities of the real world set in to obscure the symmetrized skeleton of the CH from view.

With aspect, Dab = c and Dba = d are different; but with this restriction - if Dab = z then Dba = z = Dab, so one cannot simply lay down that Dab, Dba are always different. In fact, it's a bit worse: because symmetrization must, as we have argued, be possible, there is an equivalence relation which holds between Dab and Dba. [Technical note: it is just the equivalence relation which determines the homomorphism.] Now I'm obviously very near here to Mike Manthey's co-occurrence and I think it is repeated discussions with him that have led to my idea. The new relation, I write it as $\sim$, has to be a strictly weaker one than = . If this were classical mathematics, I would simply write:

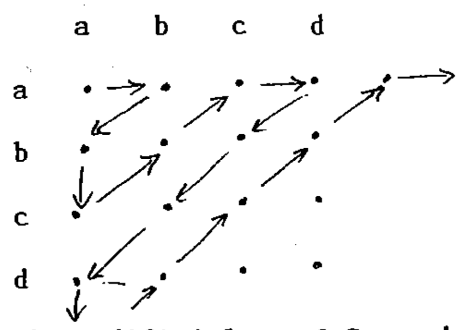
$u = v \rightarrow u \sim v$ but not vice versa.

But remembering openness, a procedure to produce $\sim$ like that must use the same D and be consistent with it:

$u \sim u'$ & $v \sim v' \rightarrow Du'v' \sim Duv$.

Thus Duv is a signal if $u \sim v$. If Duv = z, then u = v so there must be other signals, and by the same argument as in CP, it is sufficient to take one, y say, for which Duv = y $\rightarrow$ $u \sim v$ but $u \neq v$. Of course, Duv = y $\rightarrow$ Dvu = y.

The argument about the transitive law holds just as in CP but to apply (a modified form of) Conway's trick one has to do things in a definite order, and because of openness, move step-wise. One possible order which I have used[8] is shown in the figure; to use others will give somewhat different results, but not, I think, essentially different.



Then the modified form of Conway's trick is to define:

If $a \not\sim b$, then

Dab = least c such that $c \sim$ Dba, $c \neq$ a,b,Da'b,Dab',Dba.

(As usual with Conway's trick, if Dab is being named before Dba, some of the requirements are nugatory.) You can work through the arguments and you will find a closed table of 6 rather than 3 elements:

	a	b	c	d	e	f
a	z	c	e	b	d	y
b	d	z	a	f	y	c
c	b	f	z	y	a	e
d	e	a	y	z	f	b
e	c	y	f	a	z	d
f	y	d	b	e	c	z

and this is easily seen to be a disguised form of the quaternion group, Q, by taking

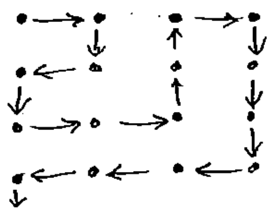
a,b,c = i,j,k

f,e,d = −i,−j,−k

z = −1, y = 1.

This affords a nice example of the symmetrization result above. For Q ⟶ S is provided by the well-known homomorphism "ignore signs". So, once again, the scale constants are safely locked away in the symmetrized skeleton. But there are other striking things too: The new discrimination is associative at this level, with similar comments to before. The fact that Q, rather than some more complicated algebra, arises is an instance of why non-commutative operations in physics are almost always anti-commutative instead of something more complicated. After all, there was no a priori reason why, if ab ≠ ba, the structure should require that ab = −ba. It will become clear below that the transformation group of Q has two-valued representations, so Ted's argument has more chance here.

There are qualifications. Firstly, the algebra produced depends on the ordering principle. If one uses another well-known primitive recursive function to provide:



one gets, not Q but Q*, the algebra generated by elements 1,2 such that 12 = 3, 31 = 2 but 23 = −1. If one continued to insist that 11 = 22 = 33 = z then one would lose associativity. More seriously, when a new element is introduced, insistence on non-commutativity also loses associativity (for,if a, b, c are 3

anti-commuting elements, then abc belongs to the centre of the algebra - commutes with every element.)[9] Initial hopes that I had of producing the higher Clifford algebras and so making contact with the work of Mike Manthey in earlier ANPA meetings and with that of the Bohm-Hiley (Birkbeck) school have had to be shelved for the present. Still, I have hopes for next year.
Two concluding remarks:
(i) evidently there is no bit-string representation, only one of the symmetrized skeleton. This seems to be a major difference between this and Program Universe.
(ii) Since for the original CH D was written + and there was a x, whereas here D is written x and there is a + "further back", I must be in Herb Doughty country, though I have not quite got double fields.

For the rest of the paper I confine myself to the lowest level, Q, only. The first problem is to get at the transformation group of Q. Now the mathematician knows that all the automorphisms of quaternion algebra are inner, that is, of the form

$$q \rightarrow q' = aqa^{-1}.$$

But here a is a general element of the algebra, so that if we follow him we resurrect the earlier problem of a second binary operation (here +) and its meaning. I leave this on one side for the moment, because it turns out that the other problem in Ted's spin-$\frac{1}{2}$ argument leads to something that solves this too.

So let me turn to Ted's use of sets of elements in the CH which are not dcss. Why should one need these? One reason is that, as I mentioned earlier, the process by which new elements are immediately discriminated with old ones (as suggested in CP) was a gross over-simplification, which we used there to derive the CH as quickly as possible. The new element has a label when it arrives; in due course this may be changed but in the meantime it "waits for discrimination". I call a set of elements awaiting discrimination a complex and use square brackets, [a,b,c] for it. I use an obvious formal trick to extend D to complexes:

D[a,b][c,d] = [Dac,Dad,Dbc,Dbd]

and so on. NB: Health warning; though this makes sense in trying out each possible pair, it is not a discrimination operation between complexes.

Now if a complex is a set of elements awaiting discrimination, some of the elements must be repeated, since discrimination operates on two elements. So a complex is not a set but a multiset. However, it cannot be a general multiset, or it would be possible to have [a,a,a,...] and the finite character of the process would evaporate. Some finite restriction is needed. One repetition is essential and a larger constraint than two would be arbitrary, so I define a complex so as to allow at most 2 appearances of any element; any triad can be dropped. So [a,a,a] = [] = ϕ, the empty complex. Two remarks:

(i) it is worthwhile noting here that again Mike Manthey's notion of co-occurrence is very close to this. Moreover we shall find shortly that his formal expression of it is equivalent.

(ii) I am aware that such complexes by themselves won't serve as complete waiting-rooms for discrimination. Something else is needed, which will need more thought.

Now I can rewrite the complex [a,b] as a + b and because 3a = 0, the set of complexes of any group G is the group algebra A(G) over $\mathbb{Z}_3$. The usual notation for the field $\mathbb{Z}_3$ is 0, 1, 2 but since 2 + 1 = 0, a better notation for 2 for our purposes is -1 and so A(Q) is just what Mike Manthey has often used in ANPA.As we remarked above, in A(Q), even over $\mathbb{Z}_3$, all automorphisms of the algebra, and therefore of Q, are inner and so we have solved the problem above about the interpretation of the second binary operation needed to express the automorphisms. For example, the transformation:

$$i, j, k \rightarrow i, k, -j$$

is provided by an inner automorphism with $a = i + 1$, $a^{-1} = i - 1$. These a's play the same rule here that the square matrices did in the CH; they are elements at the next level. So here, rising a level is only going from the group Q to the group algebra A(Q) over $\mathbb{Z}_3$.This is somewhat surprising; but anyway, further level changes will not come in that way.[10]

I come back to Dirac's treatment of spin in 1928. He locates it in the so-called half-angle transformations in the rotation group. When

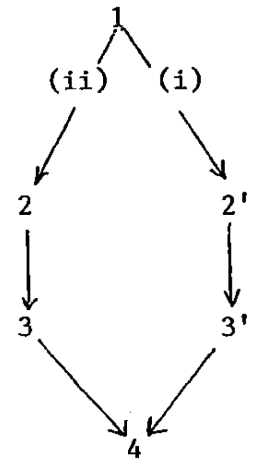
$$q \rightarrow q' = aqa^{-1},$$

what are we to say about an element s for which

$$s \rightarrow s' = as?$$

This is indeed a two-valued representation, since replacing a by $-a$ does not affect q' but changes s' in sign. We have, as Ted guessed, located the spin-$\frac{1}{2}$ all right, and free of all spatial considerations. But the symbolism has more to tell us; even if s is a group element, i, j or k, s' is not. For the a in the example above, i, j, k become $i - 1$, $j + k$, $k - j$ if i, j, k are spinors. Evidently spinors are elements at the next level, which may seem odd as one often thinks of spinors as more basic than vectors, as building bricks. (In this notation, if $s_1' = as_1$, $s_2' = as_2$, then $(s_1 s_2^{-1})' = a(s_1 s_2)a^{-1}$ and the corresponding thing in orthodox spinor notation is well-known.)

My argument has had rather a complicated structure, so a diagram may help. The shape of the argument is like this:



where the numbers refer to the stages in the argument listed below:

1. Attempt to describe spin-$\frac{1}{2}$ in CH fails because: (i) use of non dcss,

(ii) impossible because spin-$\frac{1}{2}$ involves 2-valued representations of the transformation group.

2.CP gives a process reformulation of CH which justifies it and so gets $\alpha = 1/137$, and shows that process in fact gives 1/137·035... But careful re-examination of CP shows an error in asserting that CH is the unique result of process analysis. Rather, the process analysis gives a structure with a non-trivial homomorphism to the CH.

3. At level 1 the structure turns out to be uniquely the quaternion group, which now has 2-valued representations of its automorphism group.

2'. How can non-dcss enter and be dealt with? They are complexes of entities waiting to be discriminated (Co-occurrence in Manthey's phrase). They turn out to be the group algebra over the field $\mathbb{Z}_3$.

3'. Instead of linear functions to generate the next level, as in CH, the next level is the set of automorphisms of the group.In general the formalism is obscure.

4. For the lowest level, where the group is Q, all automorphisms are inner (even for $\mathbb{Z}_3$) and so have two-valued representations. And so spin-$\frac{1}{2}$ is possible without using the rotation group

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1. Combinatorial Physics. T.Bastin & C.W.Kilmister. Singapore: World Scientific. 1995, especially Chapter 6.

2. Mereologies. Edit. T.Etter. Lewes: ANPA. 1997. Especially pp.11-17 and 21-23.

3. Quantum Theory & Beyond. Edit. T.Bastin. Cambridge: University Press. 1971. Ted's argument on spin-$\frac{1}{2}$ is on pp.217-8. He divides vectors at level 1 into two classes and exploits this division to produce a "1 or 2" character which is identified with the "1 or 2-ness" of spinors. The sets of vectors cannot be dcss. But it also seems desirable to take account of the physics context: Dirac (Proc.Roy.Soc.(A) 117,610,1928), starting from the context of spectral lines, says:"The discrepancies consist of 'duplexity' phenomena, the observed number of stationary states... being twice the number given by the theory." Then, expressing dissatisfaction with the "little spinning particle" view, he suggests looking for "some incompleteness in the previous methods" and identifies this with a failure to agree with the transformation theory of the Lorentz group in space-time (for he still espouses a space-time receptacle although eschewing the little particle).. He notices, in fact, that the group has 2-valued as well as 1-valued representations. This is easily seen in terms of the quaternion algebra of the present paper, only over the reals instead of $\mathbb{Z}_3$. It is sufficient to consider the rotation group. For a position quaternion $r = ix + jy + kz$, the transformation $r \rightarrow r' = qrq^{-1}$, with $q = \cos\theta + i \sin\theta$ is easily seen to give rotation through the angle 2θ. But then entities s going under $s \rightarrow s' = qs$ will give

$$s' = s_0\cos\theta - s_1\sin\theta + i(s_1\cos\theta + s_0\sin\theta) + \text{etc}$$

the so-called "half-angle transformations".

4. Process and Reality. A.N.Whitehead. New York: Macmillan, The Free Press. (Corrected edition, edit.D.R.Griffin & D.W.Sherburne) 1978. The extract is from his "27 categories of explanation".

5. E.W.Bastin & C.W.Kilmister. "The Concept of Order I: the Space-Time Structure" in Proc.Camb.Phil.Soc.50,278,1954.

6. Thus Schelling, in his Stuttgarter Privatvorlesung (1810) uses the symbolism:

$$\frac{.\quad A^3}{A^2 = (A = B)}$$

not in a mathematical sense, according to Zimmermann, but "Vielmehr handelt es sich um die (metapherisierende) Symbolisierung eines zureichend komplexen Kontextes,der die Formel selbst als blosse Abkürzung von Sachverhalten dient, die "Klarsprachlich" nur mit umständlichen Aufwand erfasst werden können." He notes also that in 1989 Horgrebe gave a useful reading of the formula as "Das, was dasselbe ist, wenn A = B, ist A^2 and dass das so ist, ist A^3..." These details are expanded in R.E.Zimmermann's Habilitätsionsschrift: Die Rekonstruction von Raum, Zeit u. Materie. 1997.

7.The number of well-formed forms with r elements increases quickly with r. Let u_r be the number. Consider any r-element form, which must be of the form DXY where X is a well-formed s-element form and Y a well-formed (r-s)-element form. Evidently

$$u_r = \sum_{s=1}^{r-1} u_s u_{r-s} \quad \text{with } u_1 = 1,\ u_2 = 1,\ u_3 = 2$$

$$u_4 = 1.2 + 1.1 + 2.1 = 5,$$

$$u_5 = 1.5 + 1.2 + 2.1 + 5.1 = 14$$

and $u_6 = 42$, $u_7 = 132$, $u_8 = 449$, ...

8. The enumeration for the ordering in the text is provided by a function f: f(0) = (1,1), f(1) = (1,2), f(2) = (2,1), f(3) = (3,1)... This is obviously recursive; in fact it is primitive recursive, as can be seen from the following definitions:

1.(a) Parity function: $\mathrm{pr}\ 0 = 0$, $\mathrm{pr}\ a' = 1 \dot{-} \mathrm{pr}\ a$

where $\dot{-}$ is the well-known primitive recursive function defined by $a \dot{-} 0 = a$, $a \dot{-} b' = \mathrm{pd}(a \dot{-} b)$, where pd is the predecessor.

1.(b) $\overline{\mathrm{pr}}\ a = 1 \dot{-} \mathrm{pr}\ a$.

1.(c) For convenience, if $f(n) = (p,q)$, write $p = Lf(n)$, $q = Rf(n)$ and define $pr[f(n)] = pr[Lf(n) + Rf(n)]$.

2.(a) $s(p,q) = (pd\ p,q')$, $t(p,q) = (p',pd\ q)$.

2.(b) For convenience, write $sf(n)$ for $s[Lf(n),Rf(n)]$, and so for t.

3.(a) A modified successor function: $\phi(1,1) = (1,2)$,

$\phi(p,q) = [1 \dot{-} (q \dot{-} 1)](p',q) + [1 \dot{-} (p \dot{-} 1)](p,q')$.

Then f can be defined by: $f(0) = (1,1)$,

$$f(n') = [pr\ f(n)](t[f(n)]sg[Rf(n) \dot{-} 1] + \overline{sg}[Rf(n) \dot{-} 1]\phi[f(n)]) + [\overline{pr}\ f(n)](s[f(n)]\,sg[Lf(n) \dot{-} 1] + \overline{sg}[Lf(n) \dot{-} 1]\phi[f(n)]).$$

9. It is easily verified that, with the original ordering, the introduction of a new element gives rise to a table of 14 elements (+z and y) which can again be condensed by introducing + and − signs to one with 8, including the unit. One might ask whether, since this is non-associative, it is octonions? (I am indebted to Keith Bowden for pressing this question.)It is not, for the following reason: with octonions one can choose any two elements and their product and so get a sub-group. This sub-group is always Q. The algebra generated here has less symmetry; for 4 of the sub-groups are Q and the remainder Q*. The second type of ordering considered gives rise to a system as symmetric as octonions, but all 7 of the sub-groups are Q*. The extent of non-associativity for octonions is 38·4% For these related algebras, it is somewhat larger.

10. It is important in the CH that amongst the dcss at one level are single element ones. The representatives of these at the next level serve to retain a "picture" of the level below. It is the same here with the complexes, because [a][b] = [ab] and so on.

Poincaré-Covariant Electromagnetism, Electrodynamic Reaction and Massive Photons

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Abstract

The basic question posed is whether or not Maxwell's equations can be reformulated in an explicitly covariant form which retains the Poincaré invariance (that is, three-dimensional rotations, translations and relativistic boosts) of the classical formalism. Whilst this question is answered in the affirmative, and the analysis succeeds in representing the electromagnetic field tensor, F^{ab}, as a Poincaré-invariant operator space operating directly on the classical magnetic vector potential, it also leads to the conclusion that F^{ab} is *irreducibly* associated with a symmetric field, G^{ab} say, which appears to play the role of the reaction field in electrodynamic interactions between charged particles - a role which hitherto has been unfulfilled in classical electrodynamics. The G^{ab} field is shown to decompose into a massive vector boson which, because of its apparent role in electrodynamic reaction processes, is interpreted as a non-zero mass photon.

1 Introduction

Although history tells us that the Lorentz transformations were originally derived as those transformations which leave Maxwell's equations unchanged in structure, the classical formulation of Maxwell's equations, given purely in terms of **E**, **B** and a current **J**, is actually covariant with respect to the much more general Poincaré group which, of course, consists of three-dimensional rotations, translations and boosts. Now, as Wigner [1] first showed, systems which are symmetric with respect to the Poincaré group are characterized by mass and spin, and so it follows that the classical formulation of Maxwell's equations does not exclude the possibility of the non-zero mass photon. It then becomes natural to ask wherein lies the concept of the *massless* photon in modern electromagnetic theory. The answer, of course, is simply that the process of making Maxwell's equations manifestly covariant in the conventional formulation required the introduction of an arbitrary scalar field; this had the effect of transforming the theory into a $U(1)$ gauge theory and, as a standard result, such gauge fields are massless. Of course, a non-zero photon mass can be introduced here via the process of spontaneous gauge symmetry breaking - but then photon mass becomes yet another arbitrary parameter to be admitted, or disposed of, as conditions require. So the massless/disposable-mass photon concept became embedded in modern electromagnetic theory more-or-less accidently.

The immediate question addressed here is if the process of covariantly formulating Maxwell's equations can be managed *without* changing the basic character of the system from one which is characterized by mass and spin (Poincaré invariance) to one which is characterized by a massless field ($U(1)$ gauge invariance). The general argument is structured as follows: it is well known that the homogeneous Maxwell equation is an identity under the four-vector potential; however, it is less well recognized that the inhomogeneous Maxwell equation also supports an interpretation as an identity - as is shown in §2. Using symmetry arguments, we are able to construct that general class of Poincaré-invariant variational principles which contains all those theories up to the order of Maxwell's equations *which reduce to identities in the sense of Maxwell's equations*; by definition, one of the theories in this class must then correspond to the required Poincaré-covariant formulation of Maxwell's equations. It transpires that this (six parameter) general class of variational principles contains a great deal of redundancy, and leads finally to a one-parameter class of field-equations; distinct theories of the required general type emerge for certain discrete values of the parameter. One particular value of this parameter defines a system for which general solutions are irreducibly composed of two parts, $\Psi^{ab} \equiv F^{ab}+G^{ab}$ say, where the F^{ab} is a skew-symmetric tensor and G^{ab} is a symmetric tensor. The F^{ab} part is easily shown to be the required explicitly Poincaré-covariant formulation of the electromagnetic field tensor, whilst the irreducibly connected G^{ab} part is initially uninterpreted. This interpretation is made by considering a particle, with charge e, moving with four-velocity V_a in the field Ψ^{ab}: for this particle we find $eV_i\Psi^{ai}/c \equiv eV_iF^{ai}/c + eV_iG^{ai}/c$; since the eV_iF^{ai}/c term gives the Lorentz force acting on the particle, the obvious question is what force does eV_iG^{ai}/c represent? Since it is well known that any mechanism describing electrodynamic reaction is absent in classical theory, then the *irreducible* association of the force eV_iG^{ai}/c with the force eV_iF^{ai}/c strongly suggests that the former should be interpreted as the reaction felt by the particle with charge e arising from its own action on the generator of the field F^{ab}; in other words, the particular variational principle underlying the $F^{ab} + G^{ab}$ structure can be understood as giving an electromagnetic theory which includes the fields of both electrodynamic action and electrodynamic reaction - and is therefore dynamically complete. It is subsequently shown that G^{ab} decomposes into a massive vector boson field; given the interpretation of V_iG^{ai} as the electrodynamic reaction force and the empirical knowledge that accelerating electrons radiate in the electromagnetic spectrum, we are led to the natural interpretation of this massive vector boson as a non-zero mass photon which takes up electrodynamic reaction.

2 The Identities Of Electromagnetic Theory

2.1 The $U(1)$-Gauge Formalism

The nature of the homogeneous Maxwell equation,

$$\frac{\partial F^{rs}}{\partial x^t} + \frac{\partial F^{tr}}{\partial x^s} + \frac{\partial F^{st}}{\partial x^r} = 0, \tag{1}$$

as an identity under the four-vector potential is well understood; however, the possibility of interpreting the non-homogeneous Maxwell equation,

$$\frac{\partial F^{ai}}{\partial x^i} = \frac{4\pi}{c} J^a, \tag{2}$$

for a conserved current **J**, as an identity under the four-vector potential is less well recognized. By reminding ourselves of this fact, we are able to identify a form of analysis by which the primary objective might be achieved.

Because **J** in (2) is conserved then we have

$$\frac{\partial F^{ai}}{\partial x^i} = \frac{4\pi}{c} J^a \iff \frac{\partial^2 F^{ij}}{\partial x^i \partial x^j} = 0 \tag{3}$$

so that these last two equations are mutually equivalent. The second of these equations is an identity for *any* skew-symmetric tensor, F^{ab}, and so is necessarily an identity for the electromagnetic field tensor. Since the second of these equations is an identity then the first one must also be an identity - the physics, of course, comes in when the conserved current, J^a, is identified with the physical flow of charge.

It is useful to consider the source of the identity structures in Maxwell's equations in a little more detail: Equation (1) is a differential relationship defined over a second-order tensor field which involves only *first* orders of differentiation whilst the second of (3) is a differential relationship defined over a second-order tensor field which involves only *second* orders of differentiation. The identities occurs because, when F^{ab} is defined as the four-curl of a four-vector potential, the two relationships define self-cancelling sums of permutations of fixed-order differential operations on the arbitrarily defined four-vector potential.

2.2 Electromagnetic Theory in the Poincaré-Covariant Form

Although we do not yet possess an explicitly Poincaré-covariant formalism for electrodynamics, we know that the electromagnetic field tensor will remain as skew-symmetric so that the second of (3) remains as an identity; furthermore, the homogeneous Maxwell equation, in the form of (1), is really the Jacobi identity which, for group-theoretic reasons, will necessarily be satisfied by the field tensor in any properly formulated expression of electrodynamics. It follows that, one way or another, the equations of an explicitly Poincaré-covariant electromagnetism will be identities in the same sense that the equations of $U(1)$-gauge electromagnetism have been shown to be. This perspective provides the means by which the required formalism can be generated.

3 Identity Relationships Out Of Variational Principles

The identities which occur in the four-vector formulation of electromagnetism arise because, when F^{ab} is defined as the four-curl of a four-vector potential, the field equation and the Jacobi identity define self-cancelling sums of permutations of fixed-order differential operations on the arbitrarily defined four-vector potential. It is straightforward to see that, in the general case, a necessary condition for a differential expression to become an identity in this way is that the expression concerned must be differentially homogeneous and, when such expressions arise from variational principles, then the principles concerned must also be differentially homogeneous.

In the case of electromagnetism, the field equation arises from a variational principle, whilst the Jacobi identity (the homogeneous equation) is guaranteed on the grounds of group-theoretic principles. Since the necessary skew-symmetry of the electromagnetic field tensor in *any* covariant formulation ensures that the identity

$$\frac{\partial F^{ij}}{\partial x^i \partial x^j} = 0$$

can be considered as a form of the field equation, then we are led to consider only those variational principles which give rise to equations which are second order in the tensor field over which they are defined. Additionally, we must add in the general constraint that any variational principle must be invariant with respect to the interchange of the tensor-field indices - this is necessary to ensure that changing the labels of axes has no effect.

Putting these considerations together, the most general variational principle which is (a) differentially homogeneous, (b) gives rise to a second order differential equation in the tensor field over which is defined, (c) is invariant with respect to the interchange of the tensor-field indices, is given by the density

$$\begin{aligned}
\mathcal{L} &= \alpha_0 \frac{\partial \Psi^{ij}}{\partial x^k}\frac{\partial \Psi^{ji}}{\partial x^k} + \alpha_1 \frac{\partial \Psi^{ij}}{\partial x^k}\frac{\partial \Psi^{ij}}{\partial x^k} \\
&+ \beta_0 \frac{\partial \Psi^{ik}}{\partial x^i}\frac{\partial \Psi^{kj}}{\partial x^j} + \beta_1 \left(\frac{\partial \Psi^{ik}}{\partial x^i}\frac{\partial \Psi^{jk}}{\partial x^j} + \frac{\partial \Psi^{ki}}{\partial x^i}\frac{\partial \Psi^{kj}}{\partial x^j}\right) \\
&+ \gamma_0 \frac{\partial \Psi^{ik}}{\partial x^j}\frac{\partial \Psi^{kj}}{\partial x^i} + \gamma_1 \left(\frac{\partial \Psi^{ik}}{\partial x^j}\frac{\partial \Psi^{jk}}{\partial x^i} + \frac{\partial \Psi^{ki}}{\partial x^j}\frac{\partial \Psi^{kj}}{\partial x^i}\right),
\end{aligned}$$

where $(\alpha_0, \alpha_1, \beta_0, \beta_1, \gamma_0, \gamma_1)$ are arbitrary constants. The Euler-Lagrange equations arising from the corresponding variational principle are given by

$$(\alpha_0 + \alpha_1)\Box^2 \Psi^{ab} + (\beta_0 + \beta_1 + \gamma_0 + \gamma_1)\frac{\partial}{\partial x^i}\left[\frac{\partial \Psi^{ai}}{\partial x^b} + \frac{\partial \Psi^{ib}}{\partial x^a}\right] = 0, \quad \Box^2 \equiv \frac{\partial^2}{\partial x^i \partial x^i}$$

so that the original Lagrangian density contains a large amount of redundancy. In view of this redundancy, these equations are more conveniently written as

$$-\lambda \Box^2 \Psi^{ab} + \frac{\partial}{\partial x^i}\left[\frac{\partial \Psi^{ai}}{\partial x^b} + \frac{\partial \Psi^{ib}}{\partial x^a}\right] = 0, \quad \Box^2 \equiv \frac{\partial^2}{\partial x^i \partial x^i}, \tag{4}$$

for free parameter $-\lambda$; the minus has been used for later convenience.

3.1 Equation (4) As An Identity

Suppose we define $\Psi^{ab} = U^{ab}\alpha(\mathbf{x}, ct)$ where U^{ab} is some (as yet) undetermined differential operator and $\alpha(\mathbf{x}, ct)$ is some sufficiently differentiable function; then (4) becomes

$$\left(-\lambda \Box^2 U^{ab} + \frac{\partial}{\partial x^i}\left[\frac{\partial U^{ai}}{\partial x^b} + \frac{\partial U^{ib}}{\partial x^a}\right]\right)\alpha(\mathbf{x}, ct) = 0,$$

where the whole structure $(\cdot)$ is considered to operate on $\alpha(\mathbf{x}, ct)$. This equation can be seen as *identically* satisfied for arbitrary $\alpha(\mathbf{x}, ct)$ if the differential operator U^{ab} is such that

$$\left(-\lambda\Box^2 U^{ab} + \frac{\partial}{\partial x^i}\left[\frac{\partial U^{ai}}{\partial x^b} + \frac{\partial U^{ib}}{\partial x^a}\right]\right) \equiv 0. \tag{5}$$

The nature of this problem is more easily seen after defining the notation $X_a \equiv \partial/\partial x^a$, $a = 1..4$ where $x^4 = ict$, $\mathbf{U} \equiv (U^{11}, U^{12}, U^{13}, ..., U^{44})^T$ and sixteen symmetric matrices σ^{mn}, $m, n = 1..4$, each of dimension 16×16 whose elements in row (i, j) and column (r, s) are given by

$$\sigma^{mn}_{ij:rs} = \delta_{im}\delta_{rn}\delta_{sj} + \delta_{jm}\delta_{sn}\delta_{ri}.$$

In this notation, (5) can be written as

$$\sigma^{ij} X_i X_j \mathbf{U} = \lambda\Box^2 \mathbf{U}, \tag{6}$$

where summation is assumed over i and j. Treating $\sigma_{ij}X_iX_j$ as an algebraic matrix, it is easily seen (on the grounds of general invariance requirements) that its eigenvalues must be simple multiples of the d'Alembertian, $\Box^2 \equiv X_iX_i$; it then follows that (6) has the formal structure of an algebraic eigenvalue problem. Consequently, non-trivial solutions for $\mathbf{U}$ can only exist when $\lambda\Box^2$ is an eigenvalue of $\sigma_{ij}X_iX_j$; in this case, $\mathbf{U}$ is the corresponding eigenvector, and must have the structure of a column of differential operators.

3.2 The $\lambda = 1$ Identity

There are three distinct eigenvalues of (6), given by $\lambda = (0, 1, 2)$. The relevant choice for the present discussion is $\lambda = 1$ since, as we shall show, the corresponding system contains the electromagnetic sector. The eigenspace of (6) which corresponds to $\lambda = 1$ consists of two distinct subspaces of operators described as follows:

The Skew-Symmetric Eigenspace $(S)_{sk,3}$

$$U^{ab}_k = (X_a\delta_{rb} - X_b\delta_{ra}), \quad k = 1, 2, 3$$

where, for $k = 1, 2, 3$ then r takes any three distinct values from the set $(1, 2, 3, 4)$ - for example, $r = (1, 2, 3)$. However, note that choosing any particular basis in this way has the effect of distinguishing the index '4'; this is equivalent to making a correspondence between the label '4' and the temporal axis.

The Symmetric Eigenspace $(S)_{sy,3}$

$$V_k^{ab} = X_a(X_r\delta_{sb} - X_s\delta_{rb}) + X_b(X_r\delta_{sa} - X_s\delta_{ra}), \quad k = 1, 2, 3$$

where, for $k = 1, 2, 3$, then (r, s) is three distinct pairs chosen from $(1, 2, 3, 4)$, where the choice is made by picking any one of the four digits, and pairing it with the remaining three - for example, $(r, s) = (1, 4), (2, 4), (3, 4)$. Note that choosing the basis in this way has the effect of distinguishing the index '4'; this is equivalent to making a correspondence between the label '4' and the temporal axis.

3.3 The General Solution

These two eigen-subspaces generate a general solution of (4), for the case $\lambda = 1$, given by

$$\begin{aligned} \Psi^{ab} &= F^{ab} + G^{ab} \\ F^{ab} &= \sum_{k=1}^{3} U_k^{ab} A_k(\mathbf{x}, ct) \\ G^{ab} &= \sum_{k=1}^{3} V_k^{ab} A'_k(\mathbf{x}, ct) \end{aligned} \tag{7}$$

where the functions (A_k, A'_k), $k = 1, 2, 3$ are arbitrary.

4 The Electromagnetic Field In The $\lambda = 1$ Identity

Defining $X_a \equiv \partial/\partial x^a$, then the skew-symmetric part of the $\lambda = 1$ system is defined by

$$F^{ab} = \sum_{k=1}^{3} U_k^{ab} A_k(\mathbf{x}, ct), \tag{8}$$

$$U_k^{ab} \equiv (X_a\delta_{rb} - X_b\delta_{ra}); \quad k = 1,2,3$$

where $A_k(\mathbf{x}, ct)$, $k = 1,2,3$ are arbitrary (to within differentiability), r takes any three values from $(1,2,3,4)$ and where, because of the skew-symmetry of U_k^{ab}, then F^{ab} is also skew-symmetric. Choosing the basis according to $r = (1,2,3)$ in (8), and expanding, we find

$$F^{ab} = \begin{pmatrix} 0 & X_1A_2 - X_2A_1 & X_1A_3 - X_3A_1 & -X_4A_1 \\ X_2A_1 - X_1A_2 & 0 & X_2A_3 - X_3A_2 & -X_4A_2 \\ X_3A_1 - X_1A_3 & X_3A_2 - X_2A_3 & 0 & -X_4A_3 \\ X_4A_1 & X_4A_2 & X_4A_3 & 0 \end{pmatrix} \quad (9)$$

which we recognize as the classical electric field tensor for the case of no scalar potential and magnetic vector potential (A_1, A_2, A_3). Given the very general nature of the foregoing analysis, it is clear that the Poincaré-invariant electromagnetism is *characterized* by the absence of the scalar potential.

5 The Electrodynamic Reaction Field

The complete solution of the $\lambda = 1$ system is given, by (7), as

$$\Psi^{ab} = F^{ab} + G^{ab}$$
$$F^{ab} \equiv \sum_{k=1}^{3} U_k^{ab} A_k(\mathbf{x}, ct), \quad G^{ab} = \sum_{k=1}^{3} V_k^{ab} A'_k(\mathbf{x}, ct)$$

where, as shown in the previous section, F^{ab} is the required form of the electromagnetic field tensor. However, G^{ab} is, as yet, completely uninterpreted.

It is well known that, according to the Lorentz force law of classical electrodynamics, the net electromagnetic forces generated by two charged particles on each other are not equal and opposite - that is, even in the case of non-relativistic motions, the classical electrodynamic description of a mutually interacting charged particle-pair does not satisfy Newtonian conservation principles. Consequently, *dynamical reactions*, and the freedom to include them, are missing from classical electrodynamics. In the following, we show that the irreducible association of the G^{ab} field with the electromagnetic field, F^{ab}, provides the mechanism by which the electrodynamic reaction forces can manifest themselves.

Since, according to the Lorentz force-law, the four-force generated by an electromagnetic field on a charged particle, e, with four-velocity $\mathbf{V}$ is given by $F^a = eV_iF^{ai}/c$ then, in the irreducible structure of $\Psi^{ab} \equiv F^{ab} + G^{ab}$, the total force on such a charged particle must be given by

$$F^a = \frac{e}{c}V_iF^{ai} + \frac{e}{c}V_iG^{ai}.$$

But what force can eV_iG^{ai}/c represent? The only possible answer, given the irreducible association of G^{ab} with F^{ab}, is that it describes the *reaction* on the particle of charge e of its own action on the source of the field F^{ab}. Thus, suppose that a non-relativistic system consists of just two charged particles, e_1 and e_2 with respective four-velocities $V_a^{(1)}$ and $V_a^{(2)}$, and that each particle generates electromagnetic fields F_1^{ab} and F_2^{ab} respectively, and generates a reaction to the actions on itself through the reaction fields G_1^{ab} and G_2^{ab} respectively. Then, the respective forces acting in the vicinity of each particle are:

$$\begin{aligned} F_1^a &= \frac{e_1}{c}V_i^{(1)}F_2^{ai} + \frac{e_1}{c}V_i^{(1)}G_2^{ai}, \\ F_2^a &= \frac{e_2}{c}V_i^{(2)}F_1^{ai} + \frac{e_2}{c}V_i^{(2)}G_1^{ai}. \end{aligned}$$

If action and reaction are to be equal and opposite in this non-relativistic system, then we must have

$$\begin{aligned} V_i^{(1)}G_2^{ai} &= -V_i^{(2)}F_1^{ai}, \\ V_i^{(2)}G_1^{ai} &= -V_i^{(1)}F_2^{ai} \end{aligned}$$

which, given the fields F_1^{ab} and F_2^{ab}, represent constraints on the reaction fields G_1^{ab} and G_2^{ab}.

6 Electrodynamic Reaction and Non-Zero Mass Photons

We have seen that the G^{ab} field has a natural interpretation as the field of electrodynamic reaction; in the following, it is shown how this field can be decomposed in a massive vector boson field which implies that the reaction occurs in the form of radiated massive vector bosons. Since electrons are known to radiate photons when accelerated, one is led to conclude that the massive vector bosons produced in the electrodynamic reaction should be identified as non-zero mass photons.

6.1 The Massive Vector Boson

The symmetric part of the $\lambda = 1$ identity is defined by

$$G^{ab} = \sum_{k=1}^{3} V_k^{ab} A'_k(\mathbf{x}, \mathbf{ct}), \tag{10}$$

$$V_k^{ab} \equiv X_a \left(X_r \delta_{sb} - X_s \delta_{rb}\right) + X_b \left(X_r \delta_{sa} - X_s \delta_{ra}\right); \quad k = 1, 2, 3 \tag{11}$$

where for $k = 1, 2, 3$, then (r, s) is any three distinct pairs chosen from $(1, 2, 3, 4)$ by picking any one of the four digits, and pairing it with the remaining three: for example, $(r, s) = (1, 4), (2, 4), (3, 4)$.

If we define

$$P_t^{rs} = X_r \delta_{st} - X_s \delta_{rt} \tag{12}$$

then the V_k^{ab}, given above, can be written as

$$V_k^{ab} = (X_a P_b^{rs} + X_b P_a^{rs}); \quad k = 1, 2, 3.$$

With this notation, and defining

$$\Phi^a = \sum_{k=1}^{3} P_a^{rs} A'_k(\mathbf{x}, ct), \qquad \Phi^b = \sum_{k=1}^{3} P_b^{rs} A'_k(\mathbf{x}, ct), \tag{13}$$

where we remember that, for $k = 1, 2, 3$, then $(r, s) = (1, 4), (2, 4), (3, 4)$, then (10) can be expressed as

$$G^{ab} = \left(X_a \Phi^b + X_b \Phi^a\right). \tag{14}$$

Using the easily proven operator identity $X_i X_j V_k^{ij} \equiv 0$, then we have immediately

$$X_i X_j G^{ij} \equiv \frac{\partial^2 G^{ij}}{\partial x^i \partial x^j} = 0 \iff \frac{\partial G^{aj}}{\partial x^j} = J^a \text{ where } \frac{\partial J^i}{\partial x^i} = 0,$$

for some unspecified current $\mathbf{J}$. Using (14) this latter equation can be expressed as

$$\frac{\partial}{\partial x^j}\left(\frac{\partial \Phi^j}{\partial x^a} + \frac{\partial \Phi^a}{\partial x^j}\right) = J^a. \tag{15}$$

However, since, from (12), $X_i P_i^{rs} \equiv 0$ then, from the definition of Φ^a at (13) and the definition of P_t^{rs} at (12), we can easily see that

$$X_j \Phi^j \equiv \frac{\partial \Phi^j}{\partial x^j} = 0, \tag{16}$$

so that (15) becomes

$$\Box^2 \Phi^a = J^a. \tag{17}$$

However, since $\partial J^i/\partial x^i = 0$ and $\partial \Phi^i/\partial x^i = 0$, we can write $J^a = m\Phi^a + J_0^a$ for some constant m and conserved current J_0^a; finally, therefore, (17) can be written as

$$\Box^2 \Phi^a = m\Phi^a + J_0^a, \tag{18}$$

so that the symmetric part of the $\lambda = 1$ identity has been decomposed into a massive vector boson field.

From (13) it is easily shown that

$$\Phi \equiv (\Phi^1, \Phi^2, \Phi^3, \Phi^4) = (-X_4 \mathbf{A}', \nabla \cdot \mathbf{A}').$$

Since $\partial \Phi^i \partial x^i = 0$, then we can conclude that the conserved quantity associated with the massive vector boson field is $\nabla \cdot \mathbf{A}'$.

7 Discussion: Implications of a Massive Photon

It has been shown in §6 that, according to $(R)_{sy,3}$, a massive vector boson can be constructed from the electromagnetic field, so that it can only be interpreted as a non-zero mass photon. Now, of course, a non-zero mass photon will have a speed which is less than the 'speed of light' and which will therefore differ between different inertial frames. This conflicts with a commonly held misconception that the Michelson-Morley showed the speed of light to be the same in all frames; this experiment was performed to test a particular hypothesis concerning 'aether drift' and, although the results - which actually measured fringe shifts - discounted the specific hypothesis, they were not null-results in the sense of recording no effect at all. The record actually shows that Michelson, Morley & Miller (variously) performed a long series of experiments in the years 1881-1925 ([4], [5], [6], [7]) which, by any reasonable standard, can only be interpreted as indicating a very strong sidereal effect of $12hr$ period in the measured fringe shifts. Miller's account [5] is probably the most easily accessible of the cited references, and provides a very detailed description of the several thousand measurements made in these experiments. Vigier [8] gives a good summary of these results, and points out that the observed effects are consistent with a photon mass of approximately $10^{-68}Kg$.

8 Conclusions

We began by noting:

- that Maxwell's equation in the classical $\mathbf{E}, \mathbf{B}$ formulation have the symmetries of the full Poincaré group, implying characteristic properties of spin and mass in electromagnetism;
- that the modern formulation of electromagnetic theory, as a $U(1)$ gauge theory, excludes the possibility of a characteristic photon mass.

This led to the question of whether it is possible to provide a formulation of Maxwell's equations which explicitly exhibited the Poincaré group symmetries, thereby preserving the original characteristic structures. This paper has shown this to be possible, and has provided a Poincaré-invariant formulation of electrodynamics.

A Linear Dependencies In The Operator Spaces

Eigenspace $(S)_{sk,3}$
With the definition

$$P_c^{ab} \equiv X_a \delta_{bc} - X_b \delta_{ac}$$

there follows easily the relation

$$X_i P_i^{ab} \equiv 0.$$

Eigenspace $(S)_{sy,3}$
With the definition

$$Q_{rs}^{ab} \equiv X_a \left(X_r \delta_{sb} - X_s \delta_{rb} \right) + X_b \left(X_r \delta_{sa} - X_s \delta_{ra} \right); \quad k = 14, 15, 16$$

then there exists the relation

$$X_r Q_{st}^{ab} + X_s Q_{tr}^{ab} + X_t Q_{rs}^{ab} \equiv 0,$$

where (r, s, t) is any distinct triple chosen from $(1,2,3,4)$; for example, $(1,2,3)$. It follows that three operators Q_{rs}^{ab}, defined by three distinct choices of the pair (r, s), can only be linearly dependent if the union of these three pairs contains each of the four digits $(1,2,3,4)$ at least once. Thus $(r,s) = (1,4),(2,4),(3,4)$ gives three independent operators which span the space, whereas $(r,s) = (1,4),(2,4),(1,2)$ does not.

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Infinity and unattainability: A case study

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ABSTRACT: This is a study of how to *remove* infinities in thermodynamics when they cause conceptual problems, and how to *introduce* them for some boundary points in physics generally which are in some sense unattainable. An analogy between temperature in thermodynamics and time in cosmology is pointed out for the case when boundary points are involved.

Draft of a paper for Rolf Landauer's 70th birthday.

1 INTRODUCTION

The ordinal number 0, 1, 2, ... go on, and the number of them is called alef-zero. It denotes a denumerable infinity. Its odd properties were illustrated by David Hilbert's hotel. Suppose a hotel has that many rooms, and that many guests, each guest occupying one room. An additional guest can, however, always be accommodated. He is placed into room 0, while the occupant of room zero is moved to room 1, the occupant of room 1 is moved to room 2, etc. Hilbert's hotel can even accommodate 2 groups of aleph-zero guests. The first group is assigned all the even-numbered rooms, of which there are aleph-zero, and the second group is assigned the odd-numbered rooms. In this way one can begin to learn about the marvels of infinity. Non-standard analysis was introduced in 1960s to avoid some of the problems of the infinite in mathematics.

These problems are not confined to mathematics. When an infinity turns up in physics it conveys a warning that something is wrong in the physical modelling, or, if not actually wrong, that something is excessively idealised. We shall meet several examples of this effect. Certainly, infinities cannot be *measured*; I therefore wish to advocate the view that whenever they occur the underlying modelling should be questioned. There are many books on infinity, e.g. [1]-[5]. So to make a contribution to the general subject here, it seemed desirable to start by limiting the discussion quite severely. It is for this reason that I shall deal first with a classical and well-known branch of physics to see how infinities occur and how they are exorcised. I then turn to the lessons to be learnt form this. The branch of physics I shall choose is thermodynamics, which is one of many topics of continuing interest to Rolf Landauer; see, for example, [6]-[8].

2 THE ZERO AND INFINITY OF THE ABSOLUTE TEMPERATURE

A thermodynamic system involves variables of various types:

(i) mechanical (pressure p, volume v...)

(ii) non-mechanical and non-thermal (magnetic field H, electric field E, ...)

(iii) thermal (entropy S, temperature T, ...).

Do infinities arise among these variables? To see that they do, consider a two-particle, four-level system (Fig. 1) and let us pump energy into it. In its lowest state T is zero, and the system energy U is also zero. The chance of finding a particle in an upper level at single-particle energy E_1 increases with U and eventually there is an equal chance of finding a particle in the upper level and one in the lower level. Such a situation of equal chances is always one of maximum entropy, with $\Delta S = 0$. But we can still add or subtract an increment of heat ΔQ. Since

$$\Delta Q = T\Delta S \tag{2.1}$$

and $\Delta S = 0$ at a maximum, this must mean $T = \infty$. As the system energy U is increased from E_1 to its maximum value $2E_1$ the entropy drops so that in (2.1) $dQ > 0 > dS$, and this means that T is negative (Fig. 2). Thus such a simple system has already furnished us with an infinity in the temperature and indeed with an infinite discontinuity in it at $U = E_1$. The evil phenomenon has occurred. How are we to remove it?

Here is a simple device: change the variable! Let us define *hotness* $\tau \equiv -1/T$. Instead of having *negative* absolute temperatures T lying *above* the infinite temperatures, one now has a smooth variation in τ from $-\infty$ to $+\infty$ as the energy U in the system is increased. That is much more satisfactory. There seems to be a snag, however, since the two extreme energies 0 and $2E_1$, of U correspond to infinite values of τ:

$$(U, \tau) = (0, -\infty) \quad \text{and} \quad (2E_1, +\infty)$$

This is very interesting: the infinities arise for the two values of U $\left(U = 0 \text{ and } U = 2E_1\right)$, which are such that one cannot get to "the other side" of them. The range of definition of the system simply does not allow one to have $U < 0$ or $U > 2E_1$. The question arises:

> Are the values of a variable which lie at the end of the range of variation of that variable actually attainable?

It is here that one sees that the variable τ is just right: its values are $\pm\infty$ at these end points.

The change of variable from T to τ has also reduced the infinite discontinuity in T at $U = E_1$ to a very prosaic and uninteresting value $\tau = 0$, and furthermore τ is continuous throughout.

3 THE PHASE SPACE OF A PARAMAGNET AND THE THIRD LAW OF THERMODYNAMICS

Let us next analyse the phase space of a paramagnet (which is capable of exhibiting negative temperatures), by utilising a formulation which goes back to 1956 [9, 10]. One uses a set β to describe a set of points in thermodynamic phase space such that between any two points an adiabatic change is physically realisable, and no point that can be in the set is excluded (Fig. 3a) [9, 10 and 11]. The interior set of a set β is called γ [Fig. 3(b)]. Thus any point in the set γ has a neighbourhood lying entirely in the set γ, i.e. it has a "γ-neighbourhood". The frontier or boundary points of γ, $F(\gamma)$, do not have this property of having a neighbourhood lying entirely in the set γ, as shown by point Q of Fig. 3(b). Consider now the thermodynamic phase space of an ideal spin system. Every equilibrium state of the system is represented by a point in this phase space. If one describes states by values of magnetic field B and the temperature T, one can show curves representing the entropy $S(B,T)$ for $B = 0$ and for $|B| > 0$ (Fig. 4).

The point $A = S(B,\infty)$ can be reached only as $T \to \infty$ and so lies in $F(\gamma)$. Similarly a point R requires $B \to \infty$ and so it also lies in $F(\gamma)$. Thus the set γ consists of two non-overlapping portions γ_1 and γ_2. Entropies for finite parameters B, T can be associated with each point of γ_1 and of γ_2. Quasistatic adiabatic changes are continuous sequences of equilibrium points, and as such are represented by curves in γ_1 and γ_2. However, no such curves can link γ_1 and γ_2 as the $(\tau = 0)$-axis is neither in γ_1 nor in γ_2. The set $F(\gamma)$ consists of this axis and of the points of the curve $B = 0$. The reason is that one cannot get to the other side of this curve: all values of $B \neq 0$, whether B is positive or negative, always lower the entropy and so lie below the $B = 0$ curve. Now the second law is about entropies and these can be defined in the first place only for points in γ, since they require a γ-neighbourhood [2, 10, 11]. For points of $F(\gamma)$ they can be defined only as *limits*, for example for point A we need a limit as T goes to infinity; for point R we need a limit as B goes to infinity. The question arises if the points of $F(\gamma)$ can be adiabatically linked with the rest of γ. This question cannot be decided by the first and second laws. As emphasized [9-12], a third law is needed to tell us which (if any) points of $F(\beta)$ and $F(\gamma)$ are in β, and thus represent states to which thermodynamics can be applied. In a generalised form the third law of thermodynamics states that these points *are* in β unless they belong to the absolute zero of temperature. Thus the points of $F(\gamma)$ such as R and A are in β. This is an application of the third law in the language of simple set point theory.

Since γ_1 and γ_2 are part of a single set β they can be linked adiabatically. *Quasistatic* adiabatic linkage is not possible as the $(\tau = 0)$-axis requires $B = \infty$ and the point A requires even $T = \infty$. There remain only the *non-static* adiabatic processes. It is these processes (described often as *adiabatic fast passages*) which stitch together γ_1 and γ_2 into a single set β.

One sees questions of infinities in thermodynamics lead us to the precise language of sets of points in thermodynamic phase space. They also show that a third law is a *logical necessity* once the first and second laws have been formulated. It is interesting that questions of *infinity* have led us to the unattainability of the *absolute zero*.

These examples suggest the following rules:

R_1. If a variable v reaches a limit v_0 which it cannot exceed, a new variable such as

$\phi_1 \equiv (v_o - v)^{-1}$ or $\phi_2 \equiv v_o - v$ is appropriate.

The physical fact that $v \equiv v_0$ is a limit, is then reflected by $\phi_1 \to \infty$, or $\phi_2 \to 0$. Similarly

R_2. If a variable w cannot drop below a value w_0, a new variable

$\psi_1 \equiv (w - w_0)^{-1}$ or $\psi_2 \equiv w - w_0$ is appropriate.

Now the physical fact about w is represented by $\psi_1 \to \infty$ or $\psi_2 \to 0$.

4 ARE THERE OTHER UNATTAINABILITIES?

Perhaps there is more to be recognised here: Are limiting points, representing average macroscopic states such that one cannot go beyond this limit *always* unattainable?

Consider for example the mechanical variables p and V. Both quantities are normally positive. If therefore one excludes negative values, one is tempted to guess that the system can exist at zero pressure only if negative pressures are also accessible to it. Similarly it clearly cannot exist at zero volume. This corresponds to the case of $\psi_2 \to 0$ in rule R_2.

Consider next the non-mechanical, non-thermal variables H and E. They are vectors and can be reversed in direction: E can go over to $-E$ and H can go over to $-H$. In these cases one can, in this sense, go "beyond" the values $E = 0$ and $H = 0$, which are therefore possible and attainable values.

5 CAN TEMPERATURES BE INFINITE?

Consider next a system which has no upper bound to its energy levels. One can pump energy into such a system indefinitely and it has never negative temperatures in equilibrium states. One is now restricted to positive absolute temperatures extending from zero to infinity. In such a case a change of variable $T \to \tau \equiv -1/T$ is still possible, but it is not needed for the purpose of this paper. So let us use the original absolute temperature scale, and ask:

Q: Can temperature T be infinite in this case?

One suggestion which has been made is the following. As one raises the temperature of a gas of elementary particles such as hadrons, the increased energy goes initially into raising the kinetic energy of existing particles as well as in creating new hadrons. However, in due course, the energy goes entirely into creating new particles. At that stage addition of energy can no longer raise the temperature, and a maximum temperature has been attained. This has been estimated at about $kT_{max} \sim 160 MeV$.

The phenomenon of creating more and more particles has been enshrined in a rest mass spectrum of an exponential type

$$\rho(m) = K m^{a} \exp\left(\frac{mc^2}{kT_0}\right)$$

where K, T_0 and a are constants. For a system of particles of rest masses m (extending from $m = m_0 > 0$ to $m = \infty$) each with spin degeneracy g in a d-dimensional volume V and in equilibrium at temperature T, the grand partition function Ξ satisfies [13]

$$\ln \Xi = \frac{2gV(kT)^{(d+1)/2}c}{(2\pi)^{\frac{d+1}{2}}\hbar^d} \int_{m_0}^{\infty} \rho(m) m^{\frac{d+1}{2}} \sum_{j=1}^{\infty} \frac{(\pm 1)^j \exp j\gamma}{j^{\frac{d+1}{2}}} K_{\frac{d+1}{2}}\left(\frac{jmc^2}{kT}\right) dm$$

where γ is the chemical potential divided by kT, and K_ν is the Bessel function of the second kind. The upper limit of the integral can cause a divergence. In order to examine it, use the approximation

$$K_\nu\left(\frac{jmc^2}{kT}\right) \sim \left(\frac{\pi kT}{2jmc^2}\right)^{\frac{1}{2}} \exp\left(-\frac{jmc^2}{kT}\right)$$

which holds for large m/T. This leads to $\ln\,\Xi$ depending on integrals of the type

$$\int_{m_0}^{\infty} m^{a+d/2} \exp\left[\frac{mc^2}{k}\left(\frac{1}{T_0} - \frac{j}{T}\right)\right],$$

which converge only if $T < jT_0$ $(j = 1, 2, ...)$. Most stringently, this means that for all d one requires that

$$T < T_0 \equiv T_{\max}.$$

In this way a maximum temperature is found, though the above numerical estimate comes from experiments. These can be done by studying the distribution of transverse moments of the particles as a result of high energy particle collisions. For a review see [14], where additional reference can be found. In this way the possibility of infinite temperatures seems to be removed.

One can approach the notion of a different maximum temperatures, $T'_{\max}$, in the following way [15]. Let λ be the linear dimension of a wave packet referring to an object of mass M_S. Then the uncertainty relation for momentum and distance gives (c = speed of light)

$$\lambda\, M_S c \geq \hbar \quad \text{i.e. } \lambda > \lambda_{\min} \equiv \hbar/M_S c\,.$$

Thus λ cannot drop below a minimum value. A Schwarzschild black hole has a radius $R_S = 2M_S G/c^2$, where G is the gravitational constant. The condition for our object to be able to form a black hole is $\lambda < R_S$ and therefore certainly

$$\lambda_{\min} < R_S .$$

This gives a condition on M_S

$$M_S > M_{\min} \equiv (\hbar c/2G)^{\frac{1}{2}} = 2^{-\frac{1}{2}} m_{Pl}$$

where m_{Pl} is the Planck mass ~ $5 \times 10^{-5}\ gm$. A minimum mass for collapse to a Schwarzschild black hole implies also a minimum radius by the above argument:

$$R_S > R_{S\min} = 2M_{\min}\, G/c^2 = \left(2\hbar G/c^3\right)^{\frac{1}{2}} = 2^{\frac{1}{2}}\, l_{Pl}$$

where the Planck length is $l_{Pl} \equiv \left(G\hbar/c^3\right)^{\frac{1}{2}} \sim 10^{-33}\ cm$.

Now the temperature normally associated with a Schwarzschild black hole is

$$T = \frac{\hbar c^3}{8\pi GMk} .$$

Hence minimum mass implies a new maximum temperature

$$T < T'_{\max} \equiv \frac{\hbar c^3}{8\pi G M_{\min} k} \doteq \frac{1}{4\sqrt{2\pi}}\, \frac{1}{k} \left(\frac{c^5\, \hbar}{G}\right)^{\frac{1}{2}} \sim T_{Pl} \sim 1.4 \times 10^{32}\ K \sim 3.2 \times 10^{19}\ GeV$$

This is the Planck temperature [16] which here emerges as a maximum temperature. The quantity T'_{max} was also suggested as a maximum temperature by Sakharov [17], who did not however refer to the Planck temperature. Summarising, we see that for our object to be able to form a black hole it must have a minimum mass, and the black hole which results must have a temperature below a maximum value.

Cosmologists discuss, of course, scenarios for the very early universe which involve such high temperatures.

6 INFINITIES IN PHASE TRANSITIONS

Returning to observable thermodynamic systems, Fig. 5 shows a typical specific heat curve in the neighbourhood of a critical temperature at which a phase transition occurs. There are many examples of such curves, also in the general area of magnetic susceptibilities. The theories speak of divergencies (for example in a heat capacity) as the critical point is approached. The experiments can of course not trace such curves beyond some large but finite number. The question of whether there are infinities must therefore be raised [19]. The most reasonable, but rarely expressed, view is that these infinities can occur only as theoretical possibilities resulting from a limiting procedure carried out in theory, but that they cannot be realised in actual systems.

Consider, as an illustration, the Bose-condensation phenomenon which is responsible for the superfluid properties of liquid helium at low temperatures. In equilibrium at temperature T the mean occupation number of energy level j (energy E_j), assumed g_j-fold degenerate, is for an ideal Bose gas

$$n_j(T, \mu) = \frac{g_i}{\exp\left[(E_j - \mu)/kT\right] - 1}$$

where μ is the chemical potential. Suppose the energy levels are numbered in increasing order: $E_1 < E_2 < E_3 \dots$. Since we need $n_j > 0$ it follows that

$$\mu \le E_1 < E_2 < E_3 \dots .$$

The equality sign is the crux of the matter and is related to our problem of infinities, as will now be explained.

If the average total number of particles per unit volume is fixed in the system, then

$$\frac{N}{\upsilon} = \frac{1}{V} \sum_j n_j (T, \mu).$$

As the temperature is lowered at constant volume, the lower levels gain and the higher levels lose particles. Hence μ rises. There exists an energy, $E_c(T)$ say, which divides the spectrum into these two parts at *any* temperature. It drops as the temperature is lowered. Now Bose condensation is said to occur when μ has risen high enough to coincide with E_1. This is not possible in all ideal Bose systems, for example it cannot occur in one-and two-dimensional systems of bosons having non-zero rest masses. But it is always possible for three dimensions and then $n_1(T_1, E_1) = \infty$, and the gas is said to have suffered condensation.

The condensation temperature is T_1, and the condensate is present for the range $(0, T_1)$ of absolute temperatures. But how can this be? Only as a theoretical limit when the total number of particles is allowed to go to infinity. In order to avoid infinite densities, the volume must then also be allowed to go to infinity so as to keep the ration N/v constant. This actual limit can at best be *approached* in experiments - it can of course not be actually reached. The thermodynamic limit is again seen to be obviously a calculational device which tends to yield results which, though not directly applicable to real systems, are often good approximations.

7 EXTENSIVE AND INTENSIVE VARIABLES

The concept of infinity is deeply imbedded in other aspects of thermodynamics. Where it occurs, it is a device which helps one to make simpler statements; they do not apply to the real world! This is obvious even in the basic distinction which one makes between intensive and extensive variables. Suppose, for example, a gas is doubled in that two identical boxes of gas, each of column V, are collected in a single box of volume $2V$. The energy of each original box is now doubled, so is its entropy, and these quantities are therefore called extensive. But what of surface effects on the energy and the entropy? If they were important, neither energy nor entropy would be doubled. So, basic to these notions is that we consider the original boxes in the limit when the surface effects can be neglected compared with the volume effects. This limit is in fact the limit of an infinite volume. If one wants to exorcise infinities therefore one must regard the distinction extensive-intensive as an approximation which holds in a limit which is convenient and plausible, but which, strictly speaking, does not occur in nature.

8 CONCLUSION

What have we learnt from this study of infinities in thermodynamics?

H^*. If one cannot get above or below a certain value of a variable υ, then (a) that value can be regarded as unattainable, (b) a change of scale is desirable which banishes this value to 0 or to $\pm\infty$.

If one applies the hypothesis H^* to the universe, one discovers a remarkable analogy between temperature in thermodynamics and time in cosmology. If one regards the big bang as a point -event in a developing space-time, then the time axis is like a set β and its origin is a boundary point whose status has to be regulated by a new, temporal, analogue of the third law of thermodynamics (see §3). One can regard the origin of the time axis (t) as a limiting value of times existing in a four-dimensional space-times, but as possibly not itself a time. It was presumably never really a part of the developing universe nor will it occur in the future: it is merely a convenient fiction. Any realistic early surfaces of space-time to emerge from the big bang then occur at positive values of t. As suggested by (b) of H^*, it may therefore be convenient to pass to a scale $\tau_1 = \ln t$ which banishes $t = 0$ to $\tau_1 = -\infty$ [20]. Alternatively one can use $\tau_2 = \ln\left(V^{-\frac{1}{3}}\right)$ where V is a dimensionless volume of the universe at a given epoch [21], and this would have the same effect.

The big bang is supposed to exhibit other infinities: an infinity of space-time curvature and an infinity of energy density. There are complicated singularity theorems in "classical" relativity (i.e. in relativity to which quantum theory has not yet been added) which give the conditions for their existence. They are easily appreciated (in an intuitive way) by Newtonian mechanics [22]. One merely needs to note that a Newtonian system of particles, expanding subject to Hubble's law under gravitational attraction, can be projected back in time to an exploding point of infinite density which imparts different velocities to different fragments.

Various investigations have suggested that if quantum effects are considered then one can justify models of the universe without the initial singularity (for example by including quantum free matter fields) [23, 24]. Oscillating models in which expansion follows contraction without the intervening singularity of classical relativity are also possible [25]. Most surprisingly of all, quantum cosmology enables one to contemplate a situation in which bubbles of space-time appear and disappear spontaneously by quantum tunnelling from the vacuum state [25, 26], which has to be pictured as in a seething state of quantum fluctuations which give rise to creation and annihilation of particle - antiparticle pairs.

These possibilities are thus superimposed on the two basic alternatives offered to us by classical cosmology: The ever-expanding universe from which life as we know it will ultimately disappear, and the oscillating universe [27] in which life can conceivably regenerate during some period in each cycle. The removal of the initial singularity in quantum cosmology by having a special "no-boundary condition" has also been advocated [28].

In conclusion one may perhaps apply H^* to the limited time extension of the world line of a human being whose birth and death occur at "normal" time t_B and t_D respectively. A new time scale, τ_3 say, assigns 0 to the birth and $+\infty$ to the death of the individual. The times t_B, t_D themselves can (but need not) be regarded as just unattainable *by this individual as boundary points on his time axis*. In analogy with special relativity, one might define a (psychological?) proper time

$$\tau_3 \equiv \ln \frac{t_D - t_B}{t_D - t} \qquad \left(\text{or } t = t_D - \left(t_D - t_B\right) e^{-\tau_3}\right).$$

Any given time interval is lengthened in τ_3-time and more so the greater the age (Fig. 6).

One could also use $\tau_4 \equiv \dfrac{t_D - t_B}{t_D - t}\left(t - t_B\right)$ and there are other possibilities. This is a *jeu d'esprit* that can just possibly give solace to mathematically recondite persons contemplating their own lives.

Actually this paper was not just about removing infinities, but more about (i) removing them where they cause conceptual problems, and (ii) also about introducing them for boundary points which are in some sense unattainable.

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CAPTIONS

Fig.1 A simple 2-particle 4-level system. It is approximated by nuclear spins (in *Cu* for example), regarded as thermally isolated from the lattice. When all the nuclear spin magnetic moments are anti-parallel to the field the nuclear spin energy has a maximum, when they are parallel to the field it has a minimum. This corresponds to the top levels being fully occupied and empty respectively.

Fig.2 Temperature T and "hotness" $\tau \equiv -1/T$ as a function of the energy U in the system.

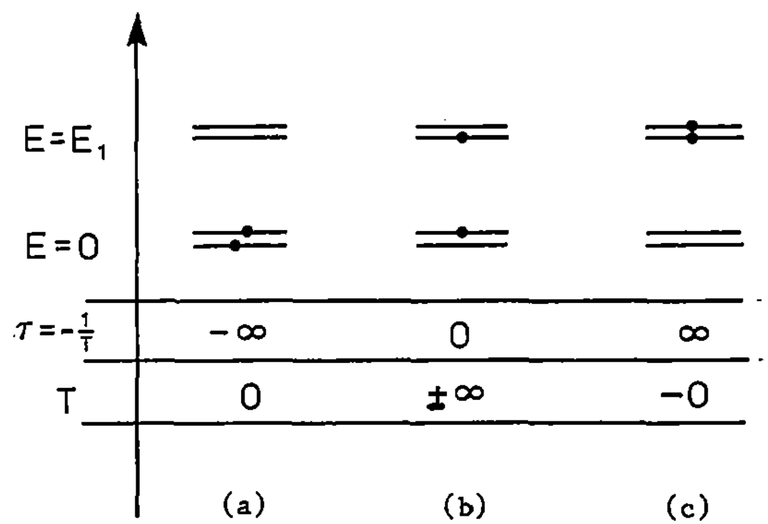
Fig.3 On removing the boundary points (solid line) from a set β one is left with a set γ. Any point P of γ has a neighbourhood (however small) consisting of points of γ. Every neighbourhood of a point Q on the boundary has, however, some points indicated by shading) which lie outside γ.

Fig.4 A thermodynamic phase space of a spin system.

Fig.5 The specific heat of fluid argon at its critical density.

Fig.6 A time-scale on which a human life is infinitely extended.

Figure 1



P T Landsberg Figure 2

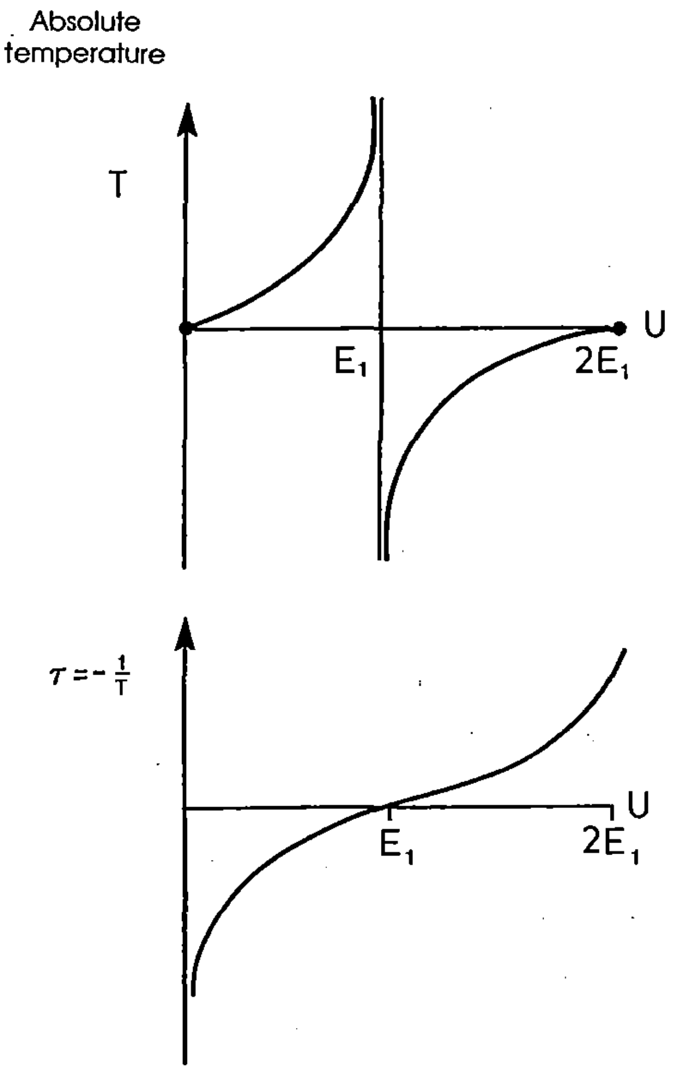
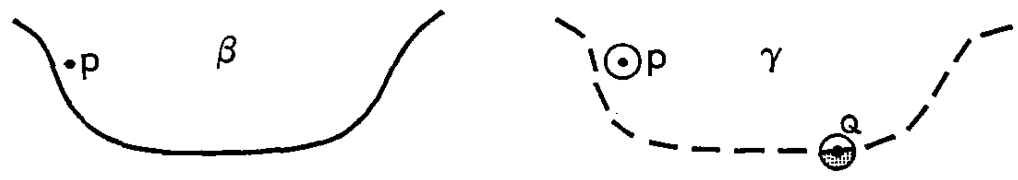


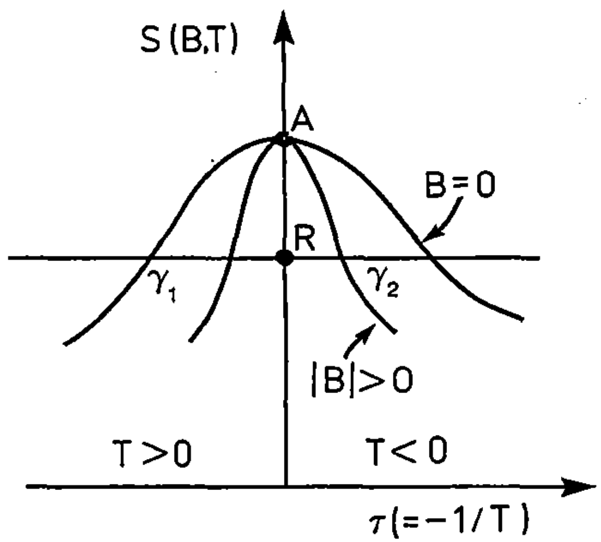
Figure 3

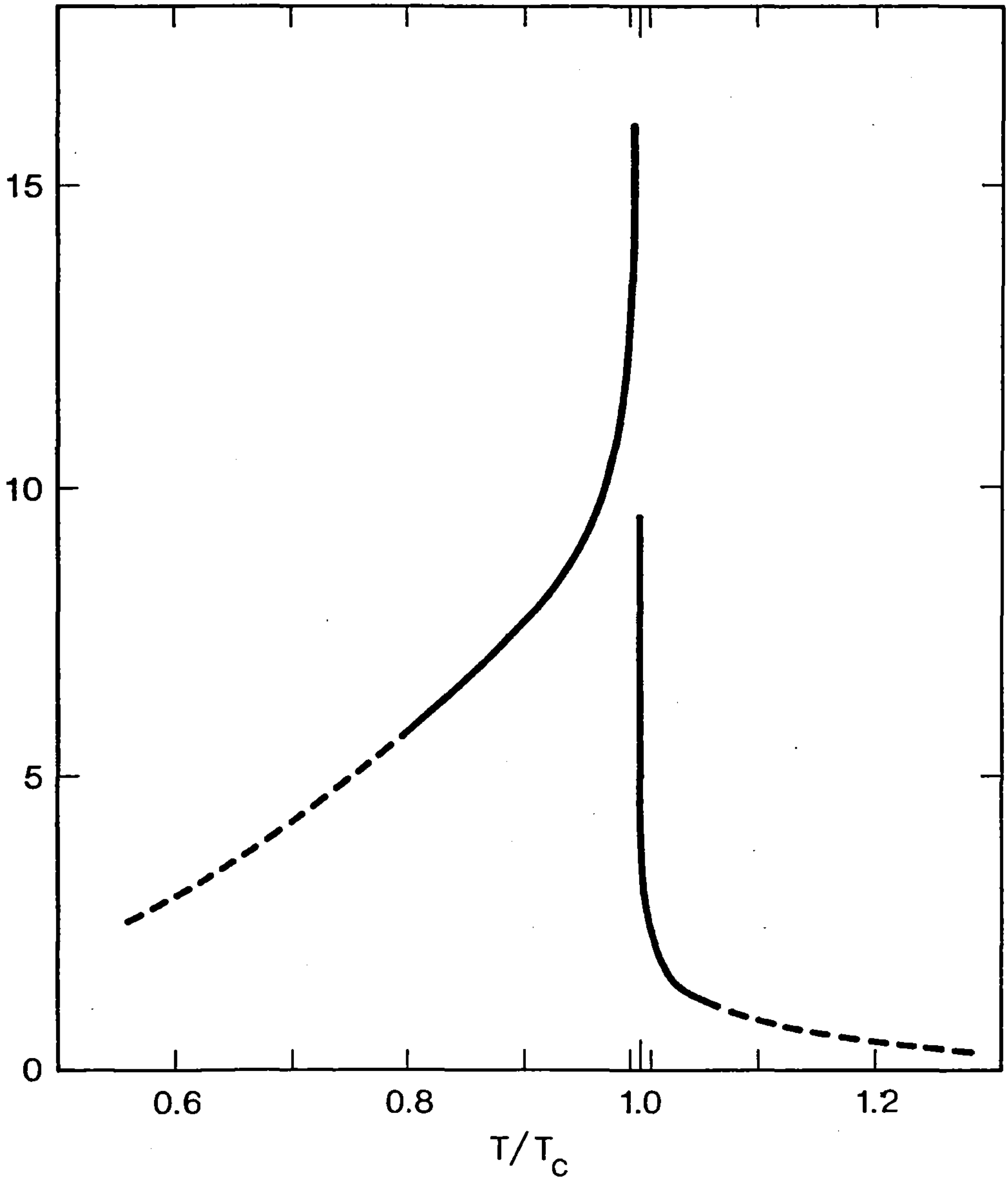


(a) (b)

P T Landsberg

Figure 4

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10
5
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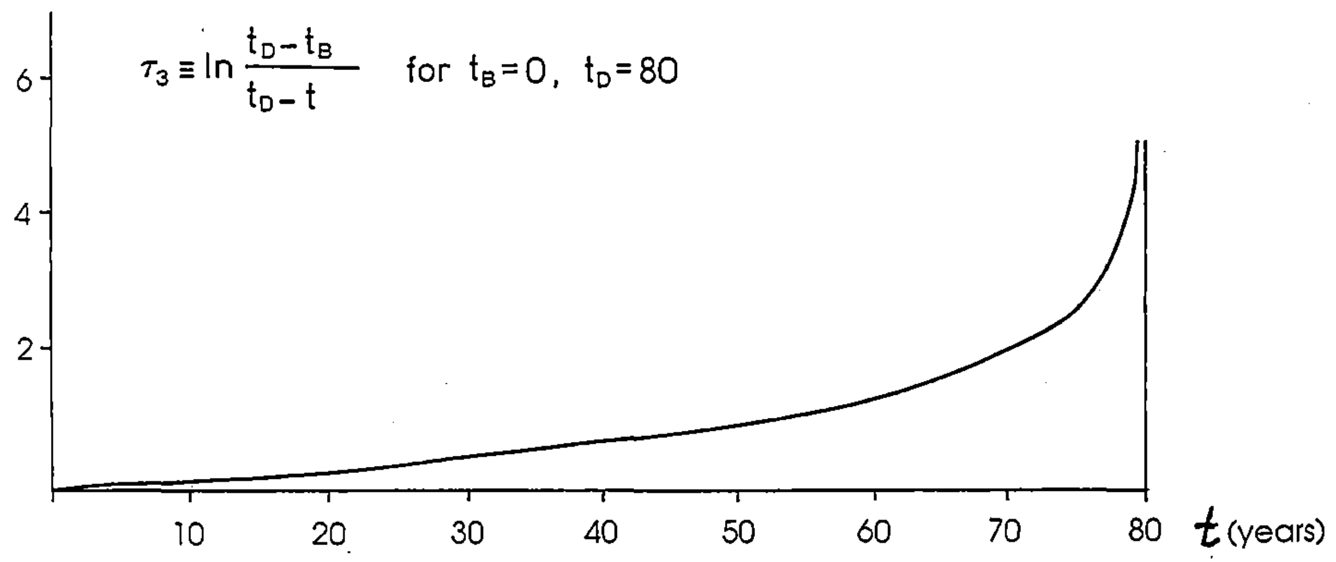
0.6
0.8
1.0
1.2
T/T_C

Figure 6



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Presidential Address

Classical Computation can be Counterfactual 12/8/99 DRAFT 3.1

(*or* Can Schrodinger's Cat Collapse the Wavefunction?)

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We show that at least some classes of classical computation can be carried out counterfactually in a particular sense. By counterfactually we mean that, given a quantum superposition which includes the possibility of a (classical) computation being carried out, then an observation can be made such that, even if the computation is not carried out, the ***result*** of the computation can be obtained. (In David Deutsch's [1997] view the computation is carried out in a parallel universe.) That is, on some observations, the output of a computer run can be obtained without the computer even being switched on and depends only on the existence of the computer and the possibility of the computation being carried out. As with all counterfactual measurements the proportion of "successful" trials (ie, those in which the computation does not occur, although the result of the computation is obtained) can be made arbitrarily large [Kwiat, 1996], but the time taken to get the output is the same as that which would be needed in order to carry out the computation. The interest is in circumstances where there is a reason not to carry out the computation (such as the likelihood that it would permanently change the system) but we still wish to know the result.

Although the computation is classical, the overall setup including the measuring device constitutes a quantum computer, and our result is essentially a special case of Jozsa's algorithm [Jozsa, 1995] which shows that all quantum computation [Deutsch, 1985] can be carried out counterfactually. However today's technology is some years away from building a universal quantum computer in Deutsch's sense. Our paradigm demonstrates that by considering a quantum computer to consist of a combination of classical and nonclassical parts, and by restricting the quantum part to observation and the classical part to computation, we can build interesting devices now. We consider how we can widen the class of counterfactual classical computations and come across some unexpected results and interesting speculations.

INTRODUCTION

Consider a terrorist who has obtained a collection of bombs from some dubious source, so that he is not certain which, and indeed how many, of the bombs are dud. As a "card carrying" terrorist he needs to know which are good (usable) bombs. [Abner Shimony and David Deutsch have each independently suggested to me that this story should be told with a bomb disposal expert as the protagonist.] Clearly one could accumulate dud bombs by throwing the bombs, one by one, at a wall, and collecting the ones that do not explode. Unfortunately this process does not work in reverse; the only way of finding out which bombs are usable destroys them. Just to make the problem a little harder we will specify that the trigger of each bomb is sensitive to a single photon. Specifically the bombs either absorb the photon and explode, or transmit the photon and are duds. It is, in principle, impossible to identify usable bombs by any nondestructive classical process.

In 1993 Elitzur and Vaidman published a paper that showed that, unexpectedly, a readily available device (actually a Mach-Zender interferometer, but here and elsewhere dubbed a Bomb Tester) could be used that would identify usable bombs, nondestructively and conclusively, at least half of the time. A Mach-Zender interferometer consists of two silvered and two half silvered plane mirrors arranged (as in Fig (1)) such that one of the half silvered mirrors splits a light beam into two; the two silvered mirrors are arranged such that they deflect the two half beams which then rejoin at the second half silvered mirror. The path lengths of the two half beams are set equal (or an integer multiple of wavelengths of the light apart). It can easily be seen that interference will occur such that the input beam seems to pass directly through the device, albeit with a lateral shift. Consider the case when the input beam enters horizontally, then the output beam will exit horizontally, and photons from the two half beams arriving at the second half silvered mirror in any other direction will either be corrected by reflection or lost by destructive interference.

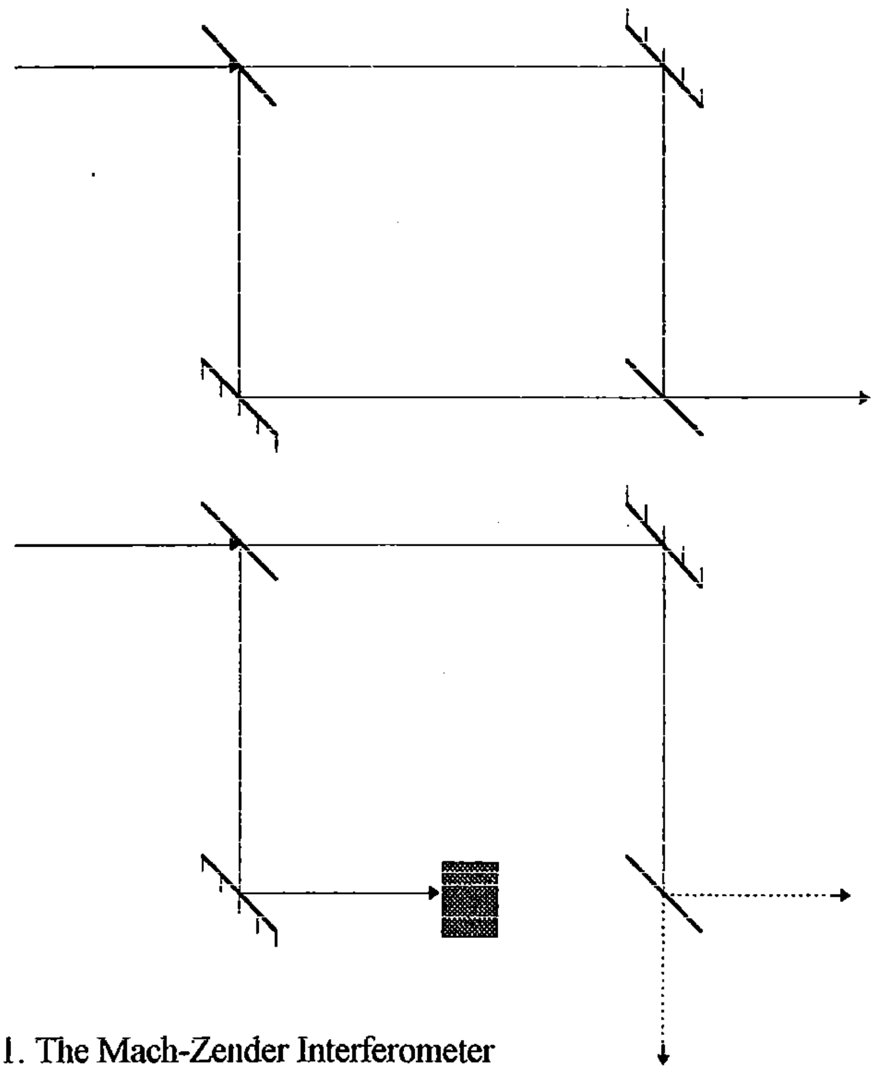


FIGURE 1. The Mach-Zender Interferometer

Consider the case when one of the half beams is blocked (say the lower one in the diagram). Interference can no longer occur at the output. The remaining half beam will be split by the second half silvered mirror such that a quarter of the input photons leave horizontally, and a quarter vertically. Now consider what happens if we turn the intensity of the light beam down so that only single photons are passing through the device in a measurable time interval. Quantum mechanics says that if neither of the beams are blocked interference will still occur and if the input beam is horizontal, then photons will only leave the device horizontally. If the lower beam is blocked as before, then a quarter of the photons will leave the device horizontally and a quarter vertically (that is, any single photon stands a one in four chance of leaving the system in each of these two directions). Now the crux of the argument is that whenever a photon leaves the device vertically we know that the lower half beam is blocked, ***even though the photon did not traverse this route in order to find out!***

This mode of observation is known as ***counterfactual*** (or sometimes *nondemolition*) measurement. To resolve the tale with which we began this section, the bomb disposal expert can use the device described by blocking the upper beam (either half beam will do) with the trigger of a bomb, then there are four cases. Either

1. the input photon travels the upper route and the bomb is good in which case the photon is absorbed and the bomb explodes and is lost, or

2. the input photon travels the upper route and the bomb is dud, in which case it does not explode and the photon is transmitted, destructive interference occurs at the output and the photon leaves the device horizontally, or

3. the input photon travels the lower route and leaves the device horizontally in which case we cannot distinguish between this case and the previous one and the result in each case is inconclusive with respect to the status of the bomb, or

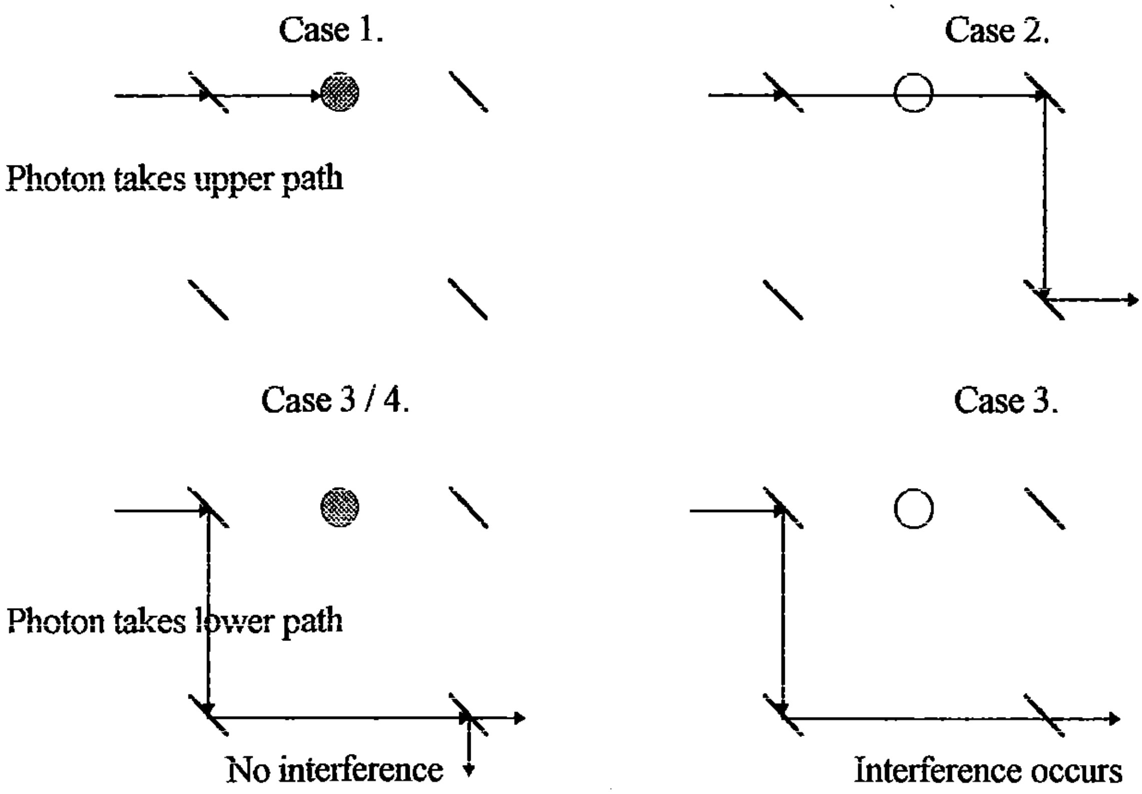
4. the input photon travels the lower route and leaves the device vertically in which case we know that interference has not occurred and that the bomb would have absorbed the photon and exploded if it had instead followed the upper route.

So, by sacrificing half the usable bombs, we can identify the other half from the duds. Kwiat et al [1996] have devised a method, using a sequence of polarising devices, that efficiently increases the yield rate to a level arbitrarily close to one. It is only required that there is a possibility of interference for the system to work. Finally note that in the indistinguishable cases 2. and 3. in which interference occurred that we cannot identify the route that the photon took.

Note that in describing the Bomb Tester we have used what is essentially the Bohm-Hiley causal interpretation of Quantum Mechanics. In a more conventional interpretation we might have said something like "if the particle is not absorbed then its amplitude takes both paths around the device and interference occurs".

Conversely Deutsch [1985, 1997], in his seminal paper on Quantum Computation and in his book The Fabric of Reality, uses the Many Worlds interpretation in which the photon is considered to go a different way in each of two parallel universes.

FIGURE 2. The Elitzur-Vaidman Bomb Tester

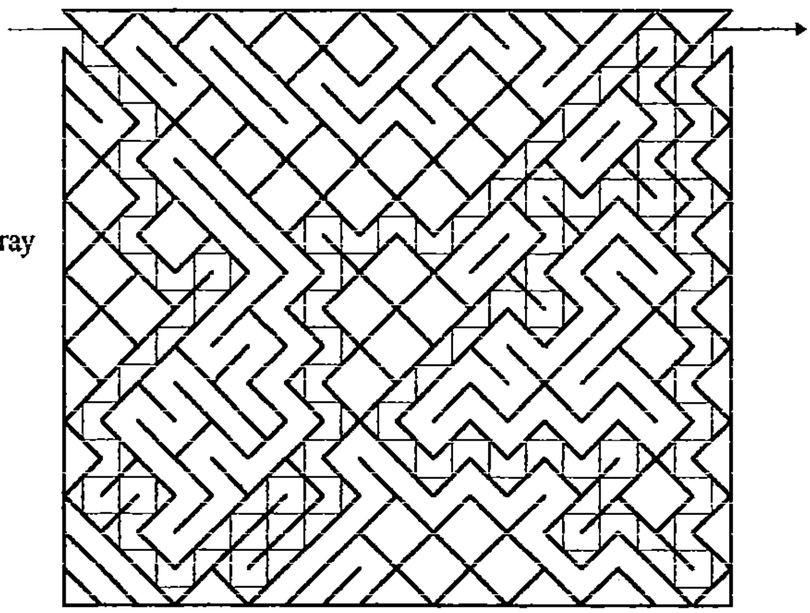


THE MIRROR ARRAY

At a workshop, at the Isaac Newton Institute, in November 1995, Richard Jozsa presented a paper [Jozsa, 1995] showing that any quantum computation can be carried out counterfactually. The quantum computer was invented by David Deutsch and described in a celebrated Royal Society paper [Deutsch, 1985]. It consists of a quantum device which is initially set in a superposition (in a similar way to the Bomb Tester) arranged so that when a measurement is taken, information is collated from computation which is counterfactually occurring in each of the superposed states. Deutsch thinks of it as parallel computation in which part of the computation is occurring in each of a number of parallel universes. Jozsa's presentation led us to consider if it would be possible to replace the bomb in the E-V Bomb Tester with a classical computer that could be "bomb tested". It was not initially obvious if this could be achieved with a Turing Machine but certain restricted classes of computation are obvious candidates. The mirror array described below demonstrates clearly that there are classes of ***classical*** computation that can be carried out counterfactually in this sense. The device is a bit string classifier, which actually classifies certain kinds of random walk.

The computation that we will consider involves the classification of a set of bit strings of length *nm*, by associating with each an integer, the output, with value 0 or 1. The device with which we will carry out the classification consists of an *n* by *m* Cartesian array, in the x-y plane, of plane mirrors mounted such that they can swivel on axes in the z-direction and can take angles of 45 degrees and -45 degrees only to the grid. The spacing of the grid is an integer multiple of the wave-length of the photons to be used. The bits in the bit strings are mapped to the settings of the mirrors in the obvious way, 45 degrees for a 1 and -45 degrees for a 0. The mirrors are silvered on both sides and the array is enclosed in a BLACK square box with sides parallel to the axes of the grid. An opening at the top left of the box allows the entry of photons in a beam parallel to the x-axis and a similar opening at the top right allows exit.

FIGURE 3.
The Mirror Array



Then a horizontally incoming photon will eventually either travel right through the box on a kind of random walk until it leaves through the door marked EXIT or be absorbed by the black surface of the box. It is easy to see that the photon cannot reverse its path and leave the device. Then for every allowable bit string and associated set of mirror settings the computation has a one bit output 1 if the photon leaves the device and 0 if it gets absorbed.

This is clearly a bit string classifier. It can also clearly be bomb tested and the output obtained counterfactually. To remove the possibility of classical usage we can connect a bomb to the absorber in the usual way. Note that the output can only be obtained counterfactually if it is 0 and not if it is 1. This restriction can be overcome by carrying out the measurement twice with the output of the mirror array "inverted" in the second experiment. Note the further restriction on the device such that the path lengths of the two arms must be equal, or differ by an integer number of wavelengths of the light frequency used. This situation is easily created by ensuring that the spacing of the mirror array is such a multiple and that the interferometer is otherwise symmetrical. By relaxing this restriction it would be possible to set up the device such that it classified by path length.

The overall device of interferometer plus mirror array can itself be simulated by a larger mirror array with more possible "states" for each "mirror". For instance putting

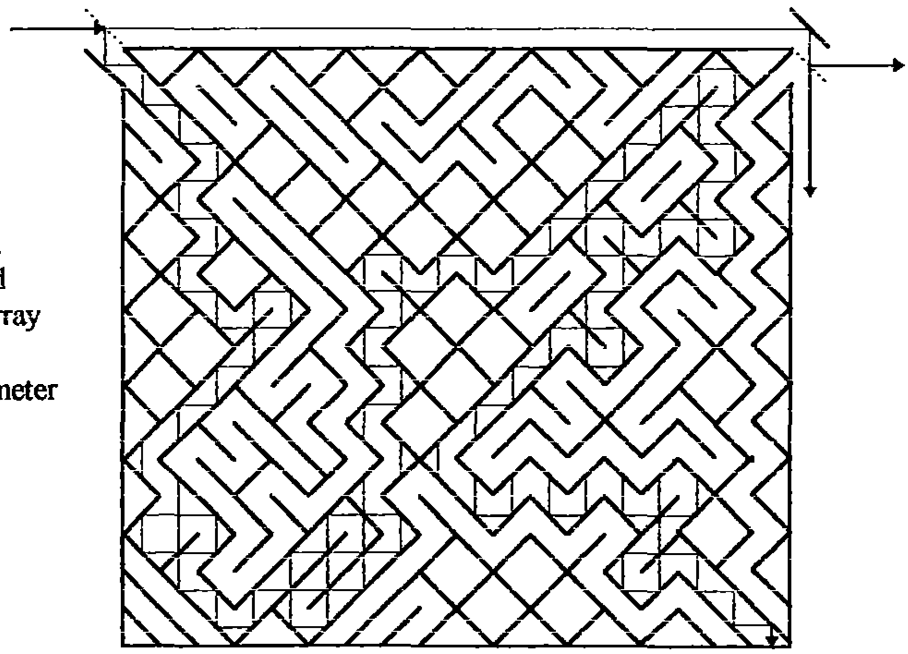
00 = transparent
01 = mirror at -45 degrees
10 = mirror at 45 degrees
11 = half silvered mirror at -45 degrees

is enough to simulate the overall device above. Adding further states

100 = half silvered mirror at +45 degrees
101 = mirror with black back at -45 degrees
110 = mirror with black back at + 45 degrees
111 = black blob

and so forth allows the creation of other interesting devices.

FIGURE 4. Combined Mirror Array and M-Z Interferometer



A COUNTERFACTUAL TURING MACHINE

In the last section we showed that it is in principle possible to perform at least some classes of classical computation counterfactually and with today's technology. In this section we consider, in principle, how wide the class of counterfactual classical computations might be made. Consider an *electron* interferometer set up as described above. It does not matter for now what a half silvered electron mirror looks like, the technology is possible in principle. The advantage of electrons is that they are slow and could, in principle, be delayed long enough to allow a conventional computation to take place. Consider the situation when an electron travelling the upper path is caused to initiate the running of a classical computer. The electron is slowed down to allow the computation to continue, and then absorbed or transmitted and accelerated according to whether the (one bit) output of the computer program is 0 or 1 respectively. The same changes in travelling arrangements are suffered to an electron in the other symmetrical path of the device. Then intuitively we have made a device that will bomb-test a PC. What next? A human being? The stock market? The UK economy?

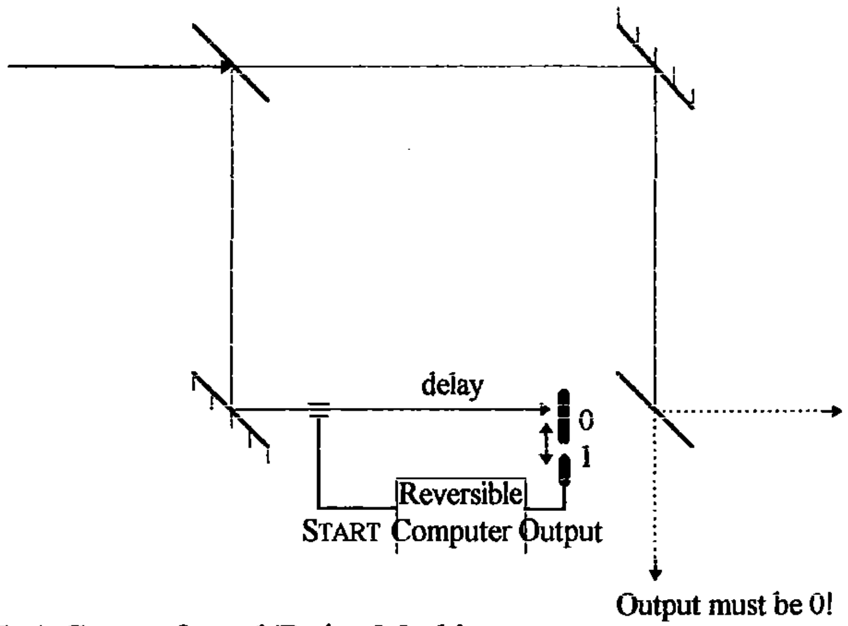


FIGURE 5. A Counterfactual Turing Machine

But hold on... this speculation contradicts one of the basic laws of quantum mechanics which is that if you can tell which path the particle travelled then interference cannot occur. If a PC runs it gets hot; this is one of a variety of ways in which we can identify whether or not the electron has triggered a real computation. If interference cannot occur in case 2. above, in our description of the bomb tester, then we cannot distinguish between case 2. and case 4. and our strategy is confounded. So what class of computations can we perform such that we cannot tell whether the computation has been physically performed or not? One criterion is that **the computations must be reversible**, that is that, after the computation has been performed, it must be possible to return the system to its initial state, or at least one that is (in principle?) indistinguishable from its initial state. Clearly this is true of the mirror array which undergoes no change during the computational process. It is also true of the class of quantum computers, because quantum computation is defined by a unitary transformation. We can, if we wish, define this reversibility as part of the computational process and write computation : input × registers=R → output × registers=R

where during the individual steps in the process the values in the registers (computer memory) can vary, but at the end of the computation they have to be set back to their initial values, and only the output bit can vary, and that is held nonclassically in our scheme. There are a number of ways to implement such a system with classical technology, eg helical logic [ref].

As noted above the interferometer and the classical computer together constitute a trivial (not universal!) quantum computer. By combining classical devices with interferometers in networks it is possible to build other interesting quantum machines.

CONCLUSIONS

In a small article in the Guardian, the week I wrote the original draft of this paper, BT announced their development of a technology known as Quantum Encryption. This scheme allows the, in principle, secure transmission of an encryption key between two remote people with old fashioned names. Similar schemes allow quantum teleportation, the ability to transmit unknown quantum superpositions down a conventional communications channel, and dense encoding, effectively giving two bits for the price of one through a noisy environment. Quantum computing is now riding on the back of the ability of quantum factoring algorithms to destroy conventional encryption security (as well as its theoretical interest). Quantum Key Encryption restores the balance by giving a new in principle secure encryption technique. (The idea of quantum key encryption was first devised by Gilles Brassard and others at PHYSCOMP92, Peter Shor's factoring algorithm was first announced at PHYSCOMP94, both in Dallas, Texas.) Kwiat at al (1996) have considered counterfactual measurement as a means of photographing a hazardous environment (such as a plasma) without disturbing it, and shown that the rate of counterfactual measurement cold be made arbitrarily high. Jozsa extended this idea to show that any quantum computation could be carried out counterfactually. In this paper we looked at restricting Jozsa's result to the classical case and show that at least some interesting classical computations can be carried out with today's technology. A major advance on Kwiat's results is the ability not only to observe a system counterfactually, but to observe its evolution counterfactually.

It is clear that, as electromagnetism drove the technology of the 20th century, quantum theory may well drive the technology of the 21st. It is strange, however, that a theory that has been around since the first decade of this century did not find application until the last decade. It is interesting to speculate on the form that some of these inventions may take. Increasing financial restriction in the UK academic sector has led me to wonder whether the bomb tester could be used to guarantee that one day a week could be reserved for academics for research time. In fact a Counterfactual Clocking-In Machine could be used to give an even greater dividend, at least for work restricted to computer usage. We have considered in this paper the idea of a human being "bomb testing" a classical computer. If the input to a day's work is a bit string, and the output is a bit string, there is no reason why this should not be carried out in reverse, the computer bomb-testing the human being. The bomb tester is set up as a "clocking-in" machine; on arrival at work the lucky employee looks through a pair of lenses into the machine. If he sees a photon he does his day's work, if not he goes home and takes the day off, or catches up on his research. The bomb tester then counterfactually obtains the bit string the employee would have produced had he stayed at work for the day. In a similar way a bomb tester connected to a terminal in the stock market could look at the effect on the stock market of, say, a large investment in QM Devices Incorporated, or even a change in the interest rate. The bomb tester in this case is not just bomb testing a human being, but the whole of the international financial system. The fact that these systems are not reversible does not prove that they cannot be measured counterfactually. Indeed the field is open theoretically. It would be interesting to have a result proving the limitation of counterfactual measurement to finite or to reversible systems or otherwise. More serious applications may arise from counterfactual holography, counterfactual EPR, or a counterfactual Bohm-Aharanov effect.

The idea that it might be possible to bomb test a human being, that is to find out the answer he would give to a particular question without ever asking him that question, led me to the conclusion that it might be possible to counterfactually observe the classical part of such a complex system as a human being, to find out what he would compute mechanically and methodically, but not the part associated with intuition, creativity and willpower, often thought to be the "quantum part" of thinking. For instance consider a PC running an algorithm that uses random numbers to achieve a result. It is well known that any particular set of pseudorandom numbers as generated by most computer languages, can quickly have limitations when it comes to many complex applications such as graphics. By modifying a PC with a simple plug-in board that generates true quantum random numbers from a device such as a noisy back biased diode it is possible to create a device that overcomes these limitations. My first thought was that this modified PC could not be bomb tested. However the following argument easily shows that quantum noise can be measured counterfactually as readily as any other information stream. Consider the bomb tester as described in the first section operating on a bomb modified such that instead of it being predetermined that the bomb is good or a dud, the decision is made by the decay of a radioactive material enclosed within the bomb. If decay occurs within a certain period (in which there is a fifty-fifty chance of it occurring) then the bomb will be made safe, otherwise it is usable. This decay can clearly be measured whilst the photon is in flight, before it reaches the bomb. QED. I find this a strange idea, that we can generate quantum noise from a series of events that never happened. All that is required is the existence of the source of the noise and the ***possibility*** (however small) that it could generate it.

Finally we mention that we are looking at the work of Etter [1996] and Lewis [1996] on parts and wholes in quantum mechanics to understand how to build quantum computational devices out of components (see also [Barenco, 1995] and

[Deutsch, 1997]).

ACKNOWLEDGEMENTS

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A Mathematical Definition of Intelligence

Back to the Future: The Machines of Bletchley Park

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Abstract

From the archival evidence and reconstructions produced by the BCS Computer Conservation Society, there is little doubt as will be shown, the decoding of Axis encoded messages which took place at Bletchley Park before and during World War Two, was performed by an adaptive/leaming process corresponding to topological computation/quantum holography utilising phase dynamics. The sheer scale and complexity of Axis encryption performed on the Enigma and Lorentz machines during the War precludes the possibility of decryption by purely digital methods especially in relation to the state of technology as it existed then.

It is therefore possible to make the true remarkable assertion that Alan Turing and the teams at Bletchley Park not only fathered the now ubiquitous digital computers of today, but also fathered and built, prototypes of the (quantum) holographic computers of tomorrow. These astonishing facts were lost apparently because Bletchley Park technology was destroyed on the instructions of Churchill at the end of World War Two; because Alan Turing, who may have been the only scientist there who truly appreciated the significance of what had been achieved (the evidence for this are his papers on the theory of types, the word problem and others) committed suicide in the early fifties; because Bletchley Park Staff were subject to the Official Secrets Act, and archival material was subject to the 30 year rule.

The evidence below, and the reconstruction of the Bletchley Park technology of Colossus, and now the so called Turing Bombes, therefore stand not only as evidence that quantum holographic/topological computers are possible and capable of tackling problems of exponential complexity (or it can be shown towers of exponential complexity since topological computation concerns the Lie diffeomorphism or differentiable mapping with a differentiable inverse) but that such computers were responsible for shortening by several years (or even of winning) World War Two, and therefore of saving perhaps millions of lives.

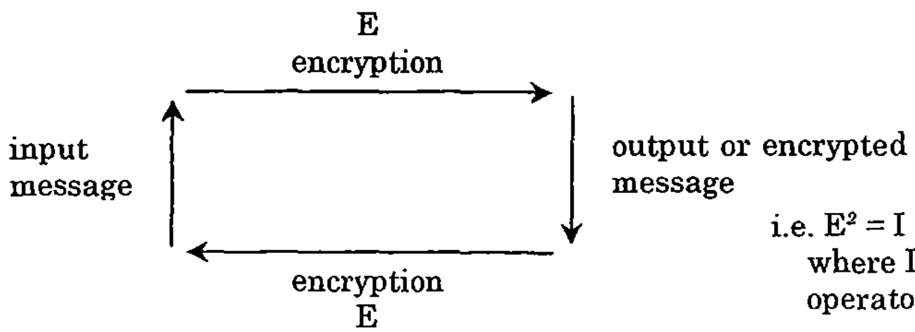
Evidence that the Processes of Encryption/Decryption Learned and Applied at Bletchley Park in World War Two Using Various Machines including the Turing Bombes and Colossus Utilised Phase Dynamics and Quantum Holography

The evidence to be outlined is based on category theory where the arrows of categorical diagrams constitute mappings/morphisms to and from objects in the categorical mathematical sense. Such a mapping/morphism therefore represents a generalised logical relation between objects, even where no specific representation of the mapping/morphism is known.

The facts are these: -

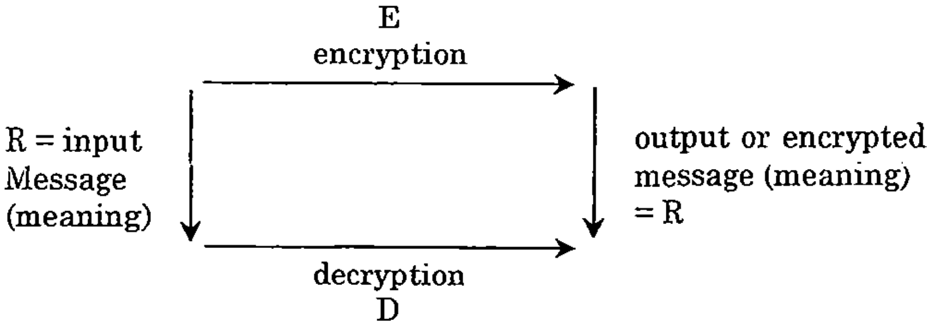
The Enigma and Lorentz coding machines use 'geared wheels'; that is, employ a phase dynamics with a discrete component, i.e. an alphabet. This phase dynamics is realised geometrically by means of the wheels which are phased locked together to form the machine. The processes of encryption and decryption are therefore geometrically encoded and decoded. Such a process of encryption/decryption therefore exhibits all the features of a quantum holographic process where information is geometrically encoded and decoding by means of phase.

The key to such encoding/decoding or reversible information processing can be expressed in the form of a categorical diagram



i.e. $E^2 = I$
where I is the identity operator

since the key allows the encrypted message to be 'decrypted' by the machine to reveal the input message as well as vice versa. This message would be in some natural language be it English, German, Italian or Japanese, and would be recognisable by its meaning. Hence the input and the output messages while quite different since one is a highly randomised version of the other, carry the same meaning and so are equivalent to one another. Thus since decryption is the inverse of encryption, the diagram can now be rewritten



The diagram therefore says that

$$E = RDR^{-1}$$

and hence that the key to the decryption of the messages, is a diagonalization procedure analogous to those performed in quantum mechanics. E is also an operation, whereby the object image or encrypted/output message is reflected back so that it coincides with the object or input message, and so E concerns phase conjugation. E is essentially a unitary operator, so that the phase dynamics it represents is such that the increments of phase summed over the cycle represented by the first categorical diagram is a closed loop invariance, and hence there is a Berry [19881 or geometric phase associated with the dynamics. For example, since the field propagators or alphabet in the natural language carry meaning and thus the model say have a natural spectral frequency of occurrence appropriate to that natural language, this must also be the case for the field propagators of the encrypted language. This is because the messages in both languages natural and encrypted are equivalent (i.e. carry the same meaning) and so have the same properties. This shows that not only can the process of encryption be reversed, but that since the diagram commutes, it constitutes a Lie group germ or Lie product in a Lie transformational system. This is indeed the case for quantum holography [Schempp 19921 where the group is the Heisenberg nilpotent Lie group which defines the requisite geometrical encoding/decoding, i.e. how the machine in this case should be differentially geared and phase locked to carry out the processes of encryption and decryption.

Thus the Lie methodology which concerns the natural Lie diffeomorphism or differentiable mapping with the differentiable inverse (and is therefore analytic) represents a means to reverse the process of encryption. In particular the natural Lie diffeomorphism is an exponential mapping and so has the capability to deal with the exponentially complex encryption that the Enigma and Lorentz machines produce, by means of the corresponding Lie algebra of derivations L which allows the inverse to be defined. The above diagrams therefore show that as long as the encrypted messages have meaning, real time decryption is possible and that once the key to the encryption/decryption process is found, encrypted messages can be inserted into their originating type of machine, and the decoded messages will emerge until those encrypting the messages move to a new key. The above Lie methodology therefore represents a generalised means by which to break the Enigma and Lorentz encoding systems. A possibility discovered by the Poles who devised the first 'bombe' for this purpose as early as 1932.

A Definition of Intelligence and of Creativity

The Lie methodology described therefore represents a mathematically specified means by which meaning or a description of order can be realised in a situation previously considered disordered or without meaning, which is the definition of intelligence proposed by Fatmi and Young [1970]. The methodology has the ability to replace the natural Lie diffeomorphism or exponential mapping by its differential inverse or logarithmic mapping thus allowing a problem expressible in terms of the Lie algebra L to be linearized and thus in principle facilitating problem solution in real time, which is a prerequisite for the survival of intelligent machines in a complex world. For if L_i are the associated infinitesimal transformations of the Lie algebra L, then the Lie commutator/product or Lie group germ determines a Lie group

GL, which is a topological group, such that by Lie's second fundamental theorem [Cohn 1957].

$(L_i,L_j) = L_iL_j - L_jL_i = c^k_{ij}L_k$ over all k belonging to G_L where c^k_{ij} are constants or invariants of the dynamics.

Further, quantum holography defined by means of the Heisenberg nilpotent Lie group, as stated above, specifies in principle how such encryption/decryption machines are to be constructed/designed, i.e. differentially geared and phase locked so as to be able to carry the processes of encryption/decryption. It can therefore be postulated that since the process of encryption/decryption is an example of a model of computation where an output replaces an input, that quantum holography specifies a model of topological computation/machines employing phase dynamics and geometrical encoding/decoding or generalised holography, such that by analogy with the definition of intelligence, there is a creation of order/structure, where none previously existed. This is borne out by the fact that quantum holography concerns the Bargmann-Fock model of quantics [Schempp 1992], which is based on the quantum field theory annihilation and creation operators for bosons, and on harmonic analysis of the three-dimensional isotropic Heisenberg nilpotent Lie group. That is to say quantum holography so defined is computer constructor universal. Such computer constructors are able to construct replicas of themselves, where these can be quantum computer universal, i.e. perfect simulators in the sense of Deutsch [1985].

The design of machines [Jessel 1988] to carry out the Lie methodology which is the key to the decryption procedure or learning process, concerns Jessel's formalization of Huygens' Principle of secondary sources which he and Resconi [1986] were able to put into the form of the commutative diagram like that already given. Huygens' Principle which describes the generation of wavelets from the mother wavelet or source, says that,

"... the perturbation of the field F that goes out through a surface S containing a wave source S_{or} is identical to the perturbation that can be obtained by cutting off the source and substituting it by appropriate secondary sources S^{s}_{or} distributed on the surface S"

formalized as follows

$$\begin{array}{ccc} F = F(o) & \xrightarrow{\quad OP \quad} & S_{or} = OPF \\ T \downarrow & & \downarrow \\ F(t) = TF(o) & \xrightarrow{\quad OP \quad} & S^{s}_{or} = (OPT\text{-}TOP)F(o) = \\ & & (OP,T)F(o) \end{array}$$

where $F(t) = TF(o)$ describes the nature of the perturbation, and $OPF = S_{or}$ describes the nature of sources which give rise to the field.

Thus the Lie methodology specifies the basis of a phase dynamics of a cybernetic/intelligent machine [Resconi, Marcer 19871[Fatmi et al 1990] by which using the secondary sources of a wavefield on a surface one is able to "perfectly simulate" the source of the field which in this case is the Enigma or Lorentz machine.

And this is exactly what the bombes devised by Turing do, they use multi copies or secondary sources of 'the machine' to simulate 'the machine itself' or source wired up so as to investigate the relevance of particular keys to a particular message set. The technique therefore is to wire in the coded messages, get the machine to stop revealing a key, take the key to the appropriate Enigma or Lorentz machine and test if the key works, i.e. produces meaningful messages. Human intervention is necessary at this point to decide if the messages are meaningful and are what might be expected from their origin.

It is worth noting here that the bombes have a self-similar structure which in phase dynamics indicates their holographic nature and that in an isolated wavefield system, the propagation of the field corresponds to the conditions

$$F(t) = TF(o)\ ;\ dT/dt = 0\ ;\ T = \text{constant}$$

which are precisely the conditions appropriate to recursion in a Turing machine, and explain why such a perfect simulation can be expected to halt on a key. These are therefore in effect the eigenvalues of the trace transform of the phase dynamics. Such phase dynamical systems therefore fall within the orbit of David Deutsch [1985] Church-Turing Principle - that 'every finitely realizable physical system can be perfectly simulated by a universal model computing machine operating by finite means' but where in general phase dynamics extends the meaning of a universal model computing machine from that of the Turing machine to that of universal quantum computer [Deutsch 1985, Feynman 1985]; where it must be noted from the above analysis that such machines contain a class capable of solving exponentially/arbitrarily complex problems by means of the methodology above and where encryption/decryption is in this class. Thus universal quantum computation as defined by Deutsch [1985] is capable of solving the class of problems known as NP complete in classical computation in polynomial time.

These conclusions are confirmed from the archival information at Bletchley Park. Steps in the decryption process involved

a) what is called the compilation of Zygalski sheets first produced by the Poles and then independently found by Welchman. In quantum holography this concerns the holographic transform $H(\psi, \phi; x, y)$ which in the case of encryption/decryption defines the matrix/hologram/discrete interference pattern in the hologram plane $R \oplus R$ where x and y function as indices ranging over the field propagators/alphabets of the message field/language ψ and the corresponding encrypted field of meaning ϕ. The matrix is therefore a correlation matrix of probabilities that can be expected to be observed in relation to a particular message set under a particular key.

   The next step quantum holography says is to find the Holographic trace transform or diagalization of the Zygalski sheet which gives eigenvalues or keys to the decryption. And this is indeed what happened at Bletchley Park, and

b) it was noticed that the processes of encryption/decryption were ternary rather than binary and this is as already pointed out in Lie transformation approach to encryption/decryption concerns the Lie group germ, and topological computation, where the nature of the topological spaces may be determined by what is known as triangulation. It therefore concerns simplicial complexes and categories in relation to the Lie or commutative categorical diagram of replacement computation, and homology and cohomology theory. A view supported by the work of Bowden [1990] who has related the formalization of Huygens' Principle to Kron's [1963] problem solving methodology of tearing, the basis of which is, as shown by Amari [1962] and Roth [1955], grounded in algebraic topology and homology and cohomology theory and which led to much of the methodology of the solution of general engineering problems in use today. Bowden too, expresses Kron's methodology as a commutative diagram, illustrating its relationship to both Huygens' principle and holography.

c) Colossus was built to carry out what in a recent paper is called bulk spin-resonance quantum computation where in effect two streams of data are compared producing an adaptive resonance which can then be detected by eye if necessary, and provides the key to the encryption/decryption process. In the latest American research rather than the electromechanical system of Colossus, nuclear magnetic resonance is applied to chemical soup. However Schempp [1977] has shown that nuclear magnetic resonance is the basis of functional magnetic resonance tomography where the resonant coupling of classical electromagnetic fields with the spin populations of soft tissues is used to extract brain and body scan images as diffraction patterns. These are then translated using fast Fourier transforms and current digital technology to display the images for medical diagnosis. A technique which is now widely used throughout the world in medical centres. Such tomographic machines thus contain linked digital and quantum coprocessors, as is probably the case for Colossus, but where the 'quantum coprocessor' may be essentially mechanical unless Colossus itself employs ternary rather than binary logic, so itself uses in effect topological computation.

Quantum holography introduces signal theory into quantum physics by means of the transformational theory of the Heisenberg nilpotent Lie group, specifying a paradigm of information processing where there is three dimensional geometric encoding and decoding utilising phase dynamics. This paradigm of quantum computation is essentially analogue, working by phase conjugate adaptive resonance or learning and hence is basically non-algorithmic. It however, because of its essentially ternary nature, includes binary computation as a subset.

Appendix

To the scientist unfamiliar with category theory in which the mathematics of this paper is expressed, it may be still unclear that what is set down relates directly to quantum mechanics since it is not expressed in the standard quantum mechanical formalism. The fundamental spectral theorem of Hilbert and Von Neumann at the heart of quantum physics is therefore re-stated below in the form of a commutator diagram or Lie Group germ. The theorem defines how in quantum mechanics, via a rotation R in the Hilbert space, the self-adjoint operators F can be expressed in the simplest possible way as the action of a multiplicative weighting operator W for any vector f in the Hilbert space, i.e.

$$\begin{array}{ccc} f & \xrightarrow{\quad OP = R \quad} & R(f) \\ {\scriptstyle F}\downarrow & & \downarrow{\scriptstyle W} \\ F(f) & \xrightarrow{\quad OP = R \quad} & R(F(f)) = W(R(f)) \end{array}$$

That is, the fundamental theorem concerns a Lie transformational system where the formalization of Huygens' principle applies so describing the phase dynamics that operate throughout the Hilbert space for every vector f.

Furthermore as pointed out encryption/decryption is a process of what Zurek [1989 a & b] calls replacement computation where an output replaces an input which he relates to the thermodynamic cost of computation, algorithmic complexity, physical entropy as an information metric and algorithmic randomness. This is important because it can be shown [Coveney, Jessel, Marcer 1990] that the commutative diagram of replacement computation can be expressed in terms of the entropy productions which can create or destroy states in a thermodynamic machine so that if S is the entropy and t is again the time

$$\begin{array}{ccc} S = S(o) & \xrightarrow{\quad OP \quad} & \sigma^s = OPS \\ T_1 \downarrow & & \downarrow \\ S(t) = T_1 S(o) & \xrightarrow{\quad OP \quad} & (OP, T_1)S(o) = S(o)\partial T_1/\partial t + J^s.\nabla T_1 \end{array}$$

since in an isolated entropic system, the entropy is governed by the continuity equation

$$\partial S/\partial t + \nabla .J^s = \sigma s$$

where J^s is the flux of entropy across a surface.

And thus in regions where entropy production is reversible, entropy acts as an information metric and produces a natural process of encryption and decryption, in relation to Maxwell demons or holes in a membrane. It can therefore be postulated that quantum holography defines the class of biological machines.

Equally the new computing principle of Fatmi and Resconi [1988] can be put into the form of such a diagram. In the late fifties, Dennis Gabor [1960,1948] specified the design for an advanced computer - the universal non-linear filter, predictor and simulator that optimises itself by a learning process, a prototype of which was built at Imperial College, and then forgotten. The mathematical specification of the machine was reformulated in the eighties, by Fatmi and Resconi (the former a colleague of Gabor) using Lie algebras and geometric probabilities to give the new computing principle as follows

$$\begin{array}{ccc} \prod_{i=1}^{p} f_{mi}(o) & \xrightarrow{\quad G_j \quad} & G_j \prod_{i=1}^{p} f_{mi}(o) = \prod_{i=1}^{p} f_{mi}(t) \\ G_k \downarrow & & \downarrow G_s \\ G_k \prod_{i=1}^{p} f_{mi}(o) & \xrightarrow[\quad G_j \quad]{} & G_s G_j \prod_{i=1}^{p} f_{mi}(o) \end{array}$$

That is the principle can be represented by a topological structure where G_j is a topological Lie group and can therefore specify translation, rotation, Euclidean movement, affine, homographic and other topological transformations, such that there is a set of operators $O_j(f(t))$ constituting a Lie algebra of derivations of a vectorial field U whose elements $f = (f_1 ...... f_{m1}, ..., f_{m1} f_{m2}$ ... n terms are the input and output signals.

The above principle is therefore once again a model of replacement computation where outputs replace inputs and vice versa and where $G_j = \exp(t0_j(f(t)))$ is an automorphic group in the input output vectorial field U. From the diagram it is seen also that there must be an equivalence relation between groups that is to say $G_k \cong G_s$.

Finally Charles Babbage 1791-1871 designed mechanical computers called the difference and analytic engines using the phase dynamics of geared wheels, such that the phase dynamics is realised geometrically, i.e. is geometrically encoded and decoded. It is clear that intuitively at least he understood the complete nature of computation, and not simply that of digital system. He is therefore indeed the father of computation in all its forms. He even used the correct words to describe his engines which under the modem mathematical formalism presented in the paper concerns the natural Lie diffeomorphism or differentiable mapping with a differentiable inverse, i.e. difference and analytic.

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The Theory of Quantised Variables: an Investigation into the Magnetic Moments of Elementary Particles.

Geoffrey Constable

Abstract

Use is made of the Theory of Quantised Variables and the Quantised Structure Model to provide a theoretical explanation of electromagnetic force. Predictions are made of the magnetic moment of the electron and, as a result, a calculation is made of the age of the universe.

The outcome of such a calculation (13.66 billion years) is compared with a parallel prediction (13.86 billion years) that arises from the Combinatorial Hierarchy.

A further outcome is a proposal that the currently accepted measurements of nucleon magnetic moments may need to be revised by a factor of $4/\pi$, such an adjustment yielding an indication that pions may be present in nucleonic structures.

1) Introduction

The object of this paper, the seventh in the current series being presented to ANPA, is to test the findings of previous papers in the light of observed electromagnetic phenomena. In particular, it is hoped to provide a theoretical understanding of Ampere's Law (known also as Biot and Savart's Law) by predicting the magnetic moments of certain elementary particles.

A brief summary of previous papers was given in the Introduction to Paper 6 'An Investigation, arising from the Theory of Quantised Variables, into Inter-particle Gravitational Forces' and is not, therefore, repeated. However, the arguments now presented arise from the content of Paper 6, the salient findings of which are summarised as follows.

Use was made of the Theory of Quantised Variables and Schrödinger's Equation to deduce the Quantised Structure Model of the electron (and other fermions) - see fig 1.

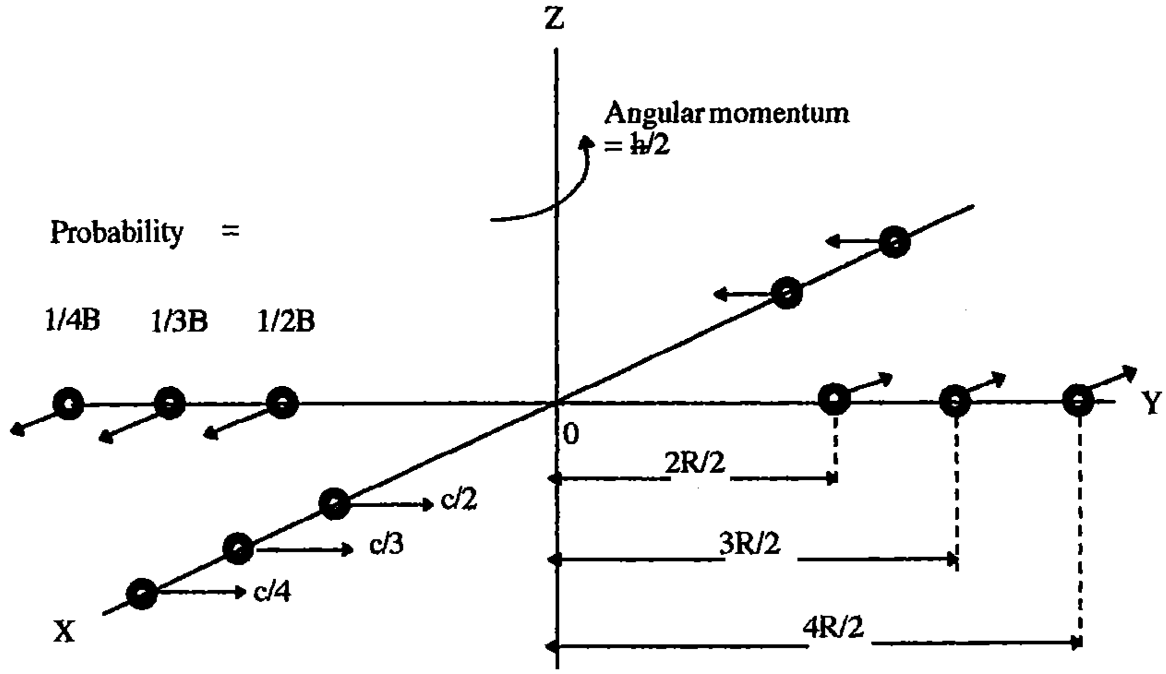

Fig 1 The Quantised Structure Model

In this model the probability that a particle in free space is momentarily at a distance r from its mean location is inversely proportional to r. Furthermore, r cannot be less than the Compton radius, R, of the particle and is aligned to OX or OY (OZ being the axis of spin). Each quantum of r is equal to R/2. As the angular momentum, $\hbar/2$, of the particle has to be conserved irrespective of location, it can be shown that the tangential velocity of a particle 'element' is given by c/n, where

$$r = nR/2.$$

It was then shown in Paper 6 (using the principle of indistinguishability to predict the interaction of the elements of one particle upon those of another) that two electrons as described by this model, but separated by a distance d, would repel each other with a force that varies inversely as d^2 and is identical to that given by the conventional law of electrostatic attraction/repulsion (Coulomb's Law).

It was also shown that the value of the Fine Structure Constant is given to good accuracy by the equation, summed from n = 2 to n = cT_o/R. - T_o being the age of the universe,

$$1/\alpha = \frac{\sum \frac{1}{n}}{\sum \frac{1}{n^2}}$$

.

It was then argued that the Quantised Structure Model gives rise to a weak, inverse-square law, attractive force that corresponds to the force of gravity, given the existence of pions in the atomic nucleus.

2 Classical Electromagnetic Theory

The basic law that relates magnetic field strength to a flow of electric current is known as Ampère's Law or the law of Biot and Savart. If H denotes 'magnetic flux density',

$$dH = \frac{idl \sin\theta}{r^2}$$

where i is the current that flows in a conductor of length dl, located at a distance r from the point of measurement and inclined at an angle θ to the radius vector. This equation, as all others in this paper, is expressed in cgs units.

As is well known, it follows from this law that H inside a circular current-carrying loop of radius r is given by the equation

$$H = {}^{2\pi i}/_{r}$$

If a particle of magnetic moment μ is acted upon by a magnetic field of magnitude H perpendicular to the axis of the moment, it will (by definition) experience a torque of magnitude μH.

The magnetic moment μ of a particle is conventionally assumed to arise from an orbiting electronic charge. This is assumed to behave as an electric current which flows in a loop of area A. (μ can be positive or negative to take account of the direction of current flow).

If an electronic charge, e, describes at speed c a circular path of radius r, the magnetic moment, which is given by given by $\mu = IA$

$$= {}^{ec}/_{2\pi rc} \times \pi r^2$$

$$= {}^{er}/_{2}$$

If r is the Compton radius R of the electron, where $R = \hbar / m_e c$,

$$\mu = e\hbar / 2m_e c \text{ ,}$$

a value that is known as the Bohr Magneton, μ_B.

In fact, the measured magnetic moment of the electron is greater than μ_B by a factor of 1.001159652193.

Since a spinning electron possesses angular momentum, the presence of a torque (such as would be caused by the presence of a magnetic field) can lead to precession, as with a top or gyroscope. As an electron has spin either 'up' or 'down', the increment of energy attributable to an electron changing from being parallel to antiparallel to a magnetic field is - 2μH. This increment is conventionally equated with hv (by Planck's Law) to calculate the precession frequency of a particle. Thus

$$\nu = 2\mu H / h \qquad (1)$$

It is current practice for the frequency ν to be measured in conjunction with a known magnetic field H in order to determine the magnetic moment of such a particle.

3 Ampère's Law in the light of gyroscope theory and the Quantised Structure Model

Ampère's Law is, essentially, an empirical law. It has been accepted by generations of physicists and engineers as fundamental, and is now used as a definition of magnetic field strength and as a means of calibration.

We now attempt to explain this law in terms of interaction between the extended structures of two electrons. In order to replicate the existence of an electric current and a resulting magnetic field, one electron has to be in motion with respect to the other and has to be shown to generate an appropriate torque . This torque has to equate with the torque that would be created by a magnetic moment (equivalent to that of an electron) when acted on by a magnetic field (as calculated by Ampere's Law) of appropriate strength. If one torque is identical to the other, Ampère's Law will acquire a theoretical basis.

Imagine two electrons A and B as depicted in Fig 2. The mean position of Electron B describes a circular orbit about the mean position of electron A and has a tangential velocity V parallel to 0Z. It is separated from A by a distance d.

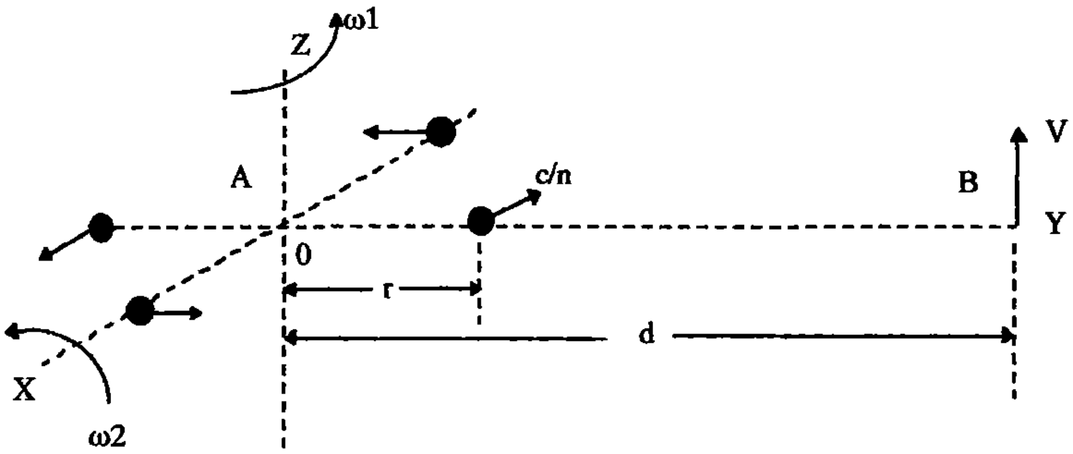

Fig 2 Interaction between electrons A and B using gyroscope theory

It is assumed that Electrons A and B have structures that accord with the Quantised Structure Model - as shown for electron A in fig 2 but not for electron B. Electrons A and B will, therefore, be located remotely from their mean positions. Fig 2 shows elements of A located momentarily at a distance r from the mean location. However, irrespective of location, electron A will at all times possess an instantaneous angular momentum $\hbar/2$. When investigating whether a torque is generated by electron B on electron A, we shall regard electron A as a small gyroscope or top, which is acted upon in some manner by electron B, as described by the well-known vector equation:

$$\mathbf{T} = I\omega_1 \times \omega_2 = \hbar/_2 \times \omega_2 \quad (\omega_1 >> \omega_2) \qquad (2)$$

where $\mathbf{T}$ is the torque that is generated, and $I\omega_1$ is the angular momentum (about 0Z) of electron A ($\hbar/_2$). The proposed angular velocity ω_2 results from the orbiting of electron B about electron A. This proposal can be considered in more detail as follows.

Suppose electron A is momentarily at a distance r from its mean position, where $r = nR/2$. In accordance with the Quantised Structure Model, the probability that the four elements of A shown in fig 2 will be thus located is $^1/_{nB}$, where B is the normalisation constant that is needed if probabilities are to sum to unity.

The mean position of electron B has an angular velocity of V/d about the mean position of A (shown in fig 2 as ω_2) - aligned along 0X.

We now use the principle of indistinguishability to argue that, when electron B is located at the mean position of A, the elements of A are indistinguishable from those of B and *vice versa*. (If B were located elsewhere, the principle of indistinguishability would not apply; B would not be at the centre of rotation of the elements of A and would thus possess motion along OZ, thereby differing from all elements of A, which do not).

Thus, when B is located at the centre of rotation of A, the elements of A interact with those of B as though they were all part of a single, integrated body. However, with respect to A, the angular momentum of B (aligned along 0Z) rotates with angular velocity V/d about 0X, and *vice versa*. According to the law of gyroscopes - see equation (1) - a torque is generated about 0Y that is the vector product of this angular momentum and the associated orthogonal angular velocity ω_2.

As explained above, this torque will only be present when B is located at the mean position of A. Thus the angular velocity of B has to be factored by the probability that B is so located. This probability is 1/pB, where $d = pR/2$.

As stated above, the probability that electron A is positioned at a distance r from its mean location is 1/Bn, where r = nR/2. Consequently, the element of torque dT about OY caused by electrons A and B being located as proposed is given by

$$dT = \hbar/_2 \times {}^1/_{nB} \times {}^V/_d \times {}^1/_{pB}$$

This value of dT has to be summed for all possible values of n when, of course,

$$\Sigma\, {}^1/_{nB} = 1$$

Thus $$T = {}^{\hbar V}/_{2dpB} \qquad (3)$$

However, the angular velocity of B is not necessarily orthogonal to the angular momentum of A - ie the spin axis of A. We have, therefore, to consider the case when the spin axis of electron A is inclined at an angle α to the OZ axis and, thus, is similarly inclined to the velocity v of B. (Note: this spin axis has to remain orthogonal to OY in order that B, similarly inclined as necessary, can at all times be a 'quantum observer' of A. An electron at all times possesses half-integral spin; thus the act of observing an electron requires the spin axis to be orthogonal to the line of observation). There is no means of determining the value of the angle α and we choose to assume, since all inclinations seem possible and equally probable, that we should include a mean value in our calculations. Averaging all possible values of torque from $\alpha = 0$ to $\alpha = \pi/2$ yields the following expression for mean torque

$$T = 2/\pi \times \hbar V/2dpB \qquad (4)$$

Substituting $p = 2d/R$ yields

$$T = \hbar VR/2\pi Bd^2$$

Substituting $R = \hbar/m_e c$ yields

$$T = \hbar^2 V/2\pi m_e cBd^2$$

Substituting $e^2 = \hbar c / 137.036$ yields

$$T = 137.036 \times e^2 V\hbar/2\pi m_e c^2 Bd^2$$

Substituting $\mu_b = e\hbar/2m_e c$ yields

$$T = 137.036 \times eV\mu_b/\pi cBd^2$$

Note: such a torque is exerted on the element of B when it is located at the mean position of A, and an equal but opposite torque is generated at this point upon the electron A. We must also take account of the interaction of an element of A when it is located at the mean position of B. A, being in motion relaltive to B, gives rise to an equivalent pair of torques. Thus the net torque upon either A or B is twice the value given in equation (3), and the total torque is given by

$$T = 2 \times 137.036 \, {}^{eV\mu_B}\!\big/_{\pi c B d^2}\,, \qquad (5)$$

ie (in the light of equation 4) ,a factor of $4/\pi$ greater than might initially have been supposed.

Equation (5) is consistent with Ampère's Law in that it describes an inverse square law. The constant of proportionality of this law will remain undetermined until we have a value for the constant B.

4) Ampère's Law in the light of Coriolis Acceleration.

A parallel argument to that given in (3) can be constructed by considering the Coriolis acceleration caused by interaction between electrons A and B, as depicted in fig 2. Coriolis acceleration 'a' is given by the exact vector equation

$$\mathbf{a} = 2\omega \times \mathbf{v}$$

As before, we observe that the angular velocity (ω_2) of electron B about the mean location of A is V/d.

In accordance with the Quantised Structure Model, the tangential velocity of an element of A (as shown in fig 3) is c/n.

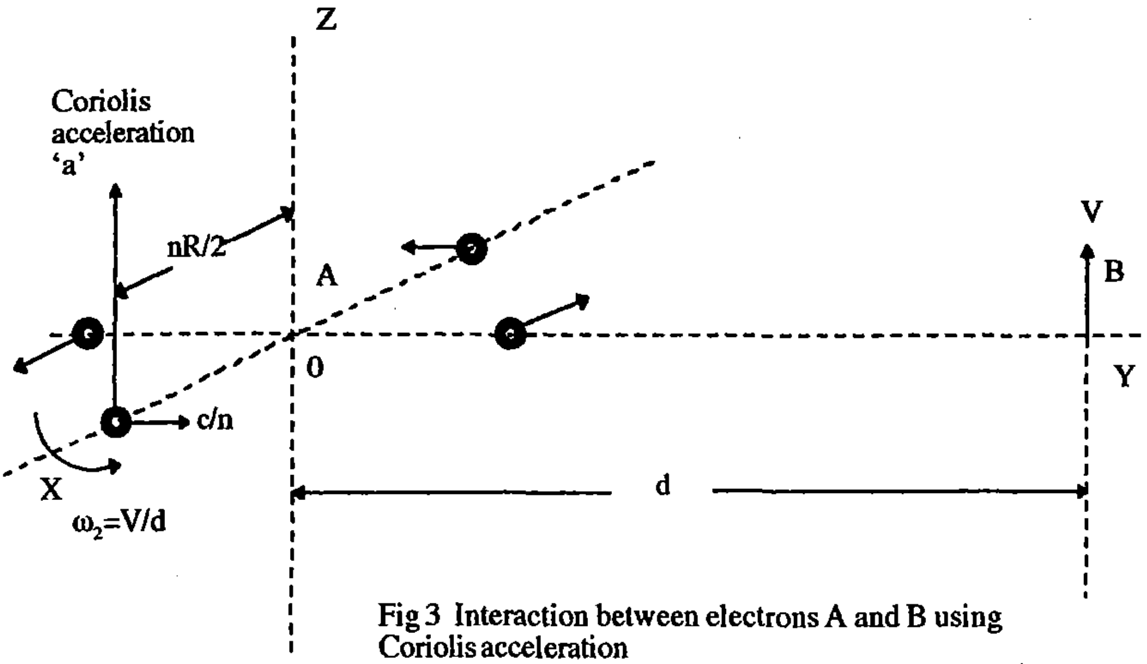

Fig 3 Interaction between electrons A and B using Coriolis acceleration

The tangential velocity (c/n) of the elements of electron A that are located on the 0X axis is orthogonal to the angular velocity V/d. Thus the Coriolis acceleration 'a' of an element of A located on 0X caused by the indistinguishability effect (taking full account of the probabilities involved) is given by

$$a = 2 \times V/d \times 1/Bp \times c/n \times 1/Bn = 2Vc/B^2 n^2 dp,$$

and is parallel to 0Z.

However, the elements of electron A that are located on the 0Y axis experience no such acceleration. The tangential velocity c/n of such elements is parallel to the angular velocity V/d, and the resulting vector product is, therefore, zero. Thus, only two of the four elements of electron that are specified in fig 3 experience Coriolis acceleration a.

The corresponding net Coriolis force on one element is thus

$$m_e a/4$$

and the element of torque that arises from the Coriolis acceleration of both elements located on the 0X axis is

$$dT = m_e/2 \times 2Vc/B^2 n^2 dp \times nR/2$$

$$= m_e cRV/2B^2 ndp$$

As in section (3) above, summing for all values of n, and observing that $\hbar = m_e Rc$, yields

$$T = \hbar V/2Bdp$$

This expression is identical to equation (4) ,and can be subjected to arguments identical to those given in section (3) to yield equation (5).

The significance of this section is that the Coriolis relationship is exact for all values of ω and v. The gyroscope law is an approximation and, for true gyroscopes, is only valid when ω_1 is much larger than ω_2. We regard the Coriolis calculation as the more rigorous, though the gyroscope approach is simple and, thus, helpful.

5) Calculation of the Normalisation Constant B

Mathematicians and physicists are accustomed to working with variables that are assumed to be both continuous and of unlimited value. Length, for example, is normally considered to be continuously variable and, if necessary, infinite.

The theory of quantised variables conflicts with such a view. It teaches that length cannot have any value and, in particular, must be less than the radius of the universe, cT_o.

The probabilities associated with the Quantised Structure Model are of the form: 1/2, 1/3, 1/4...... . Specifically, the probability that an electron (say) will be located one Compton radius from its mean position is 1/2; that for 3/2 Compton radii is 1/3; that for two Compton radii is 1/4; and so on.

Thus the normalising constant B (which is needed to reduce the sum of all probabilities to unity) is given by the series:

$$B = \tfrac{1}{2} + \tfrac{1}{3} + \tfrac{1}{4} + \ldots\tfrac{1}{n} + \ldots\tfrac{1}{N}$$

At first sight, it might seem that $N = {}^{2cT_0}/_{R_c}$, ie the number of half Compton electron radii contained in the radius of the universe.

In this particular case, however, such thinking has to be modified. A magnetic field is generated by electrons that travel in a loop, usually taken as circular. In contrast to the calculation of electrostatic force given in paper 6, we are here concerned with the effects of rotation rather than displacement. If electron B travels in a circle, the maximum circumference of such a circle would equate with the maximum length cT_0. The maximum length of the circle radius would, therefore, be ${}^{cT_0}/_{2\pi}$.

This leads to a modified value of N given by $N = {}^{cT_0}/_{\pi R_c}$

(The introduction of 2π into this fraction is analogous to the use of the 'reduced' version of Planck's Constant for situations that involve rotation).

All factors needed to calculate N are now known precisely, with the exception of To, the age of the universe, which is believed to have a value between 10 and 20 billion years.

A value of 15 billion years (or 4.73 x 10^{17} s) is used for calculating the value of N, yielding a value of N = 1.1688 x 10^{38}.

In order to calculate the normalisation constant B, we make use of the formula that defines Euler's Number, 0.5772157

$$Lim_{(n\to\infty)}(1 + 1/2 + 1/3 + ... 1/N - \ln.N) = 0.5772157$$

or $$B = 1/2 + 1/3 + ... 1/N = \ln.N - 0.4228 = 87.2314$$

We now use B = 87.2314 to determine the magnetic moment of the electron.

6 Calculation of the Magnetic Moment of the Electron

Fig 2 illustrated two electrons, one with a tangential velocity V with respect to the other. One electron will experience a torque and will precess with an angular velocity given by (5)

$$T = 2 \times 137,036 \times eV\mu_B / 87.2314 \times \pi c d^2 \qquad (6)$$

Such a precession is conventionally attributed to the magnetic field generated by an electric current, as caused by the motion of electron B, acting on the magnetic moment of electron A. Thus, conventionally,

$$T = \mu_e H = \mu_e \times 2\pi i/d = \mu_e \times 2\pi/d \times eV/2\pi c d = \mu_e \times eV/cd^2$$

Substituting $T = \mu_e eV/cd^2$ in (6)

yields $$\mu_e = \mu_B 2 \times 137.036/87.2314\pi$$

$$= \mu_B 1.0001$$

This prediction is accurate to about one part in a thousand, but is not absolutely correct. Importantly, this calculation indicates that $\mu_e/\mu_B \neq 1$.

7 The age of the universe

The calculation given above is no more than an approximation, for there is an obvious source of error. We assumed that the age of the universe is 15 billion years. This assumption may be sufficiently precise to indicate whether our calculation is on the right lines, but is most unlikely to provide predictions that are entirely accurate. We choose, therefore, to calculate the age of the universe using equation (6), and the belief that measurements of the magnetic moment of the electron are correct. The resulting calculation yields the prediction that the age of the universe is 13.66 billion years.

8 Corroborative Indicators

The arguments outlined in this paper differ substantially from conventional thinking. It is prudent, therefore, to enquire whether any indicators come to hand that support the conclusions reached. Four such indicators can be proposed, none of which provide firm confirmation, although all (in different ways) suggest that the arguments presented may have some credibility.

8.1) First, the precise age of the universe To has proved difficult to measure. Most astronomers were agreed upon an estimate of 15 billion years until recent revisions to the Hubble Constant caused estimates of this age to be reduced. Unfortunately, such estimates were reduced to the point where the universe was younger than the oldest stars - an impossibility. Fortunately, though, recent work at the Universities of Glasgow and Sussex in analysing data from the Hubble telescope and from the Hipparchos satellite now indicates that the universe has sufficient age to resolve this paradox. Apparently, the age of the oldest stars - some 12 billion years - is indeed within the age of the universe, which is now calculated as lying between 13 and 14 billion years. (The Sunday Times of 4 May 1997).

This estimate of T_o is in good agreement with the prediction given in (7) above.

8.2) Second, the teaching of ANPA and the findings of the Combinatorial Hierarchy may be of considerable relevance in this context. A key part of this teaching is that the large number

$$2^{127} = 1.70141 \times 10^{38}$$

is of special significance with respect to the fundamentals of the universe.

When calculating a value for the normalising constant B, we notice that

$$cT_o/2R = 1.672 \times 10^{38},$$

(using a value for T_o of 13.66 billion years).

These two numbers are very similar and differ by approximately 2%. We proceed by assuming that the ANPA value is correct, and calculate on this basis that the age of the universe is 13.86 billion years. If the magnetic moment of the electron is calculated using this assumption, we find that

$$\mu_e = 1.00098\mu_B$$

This result is in error by more than the accuracy of experimental measurement, but is remarkably close in that it differs from the measured value by only one part in ten thousand.

Possible explanations for a small discrepancy such as this will be explored in detail in a future paper. For example, relativistic effects associated with the Quantised Structure Model may have to be taken into account. Alternatively, if the elements of an electron have finite size, the centre of such an element will not coincide with the radius of gyration about the mean location of the particle. Small errors to probabilities would result, which could cause small changes to the normalisation constant.

8.3 Thus far we have attempted to justify Ampère's Law by examining the quantised situation that arises from two electrons, one in motion with respect to the other. We now examine whether the same outcomes can be derived using a 'macro' model involving the interaction between two current loops. Such a situation is illustrated in fig 4.

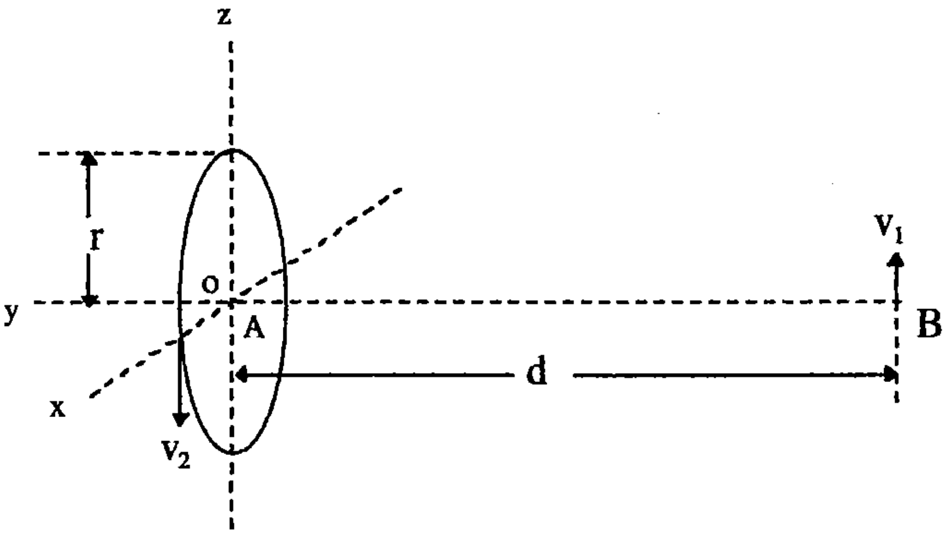

Fig 4 Interaction between electrons A and B, both describing current loops.

Electron B has a velocity V_1 and describes a circular path of radius d about OX. Electron B has a velocity V_2 and describes a circular path of radius r about OY. We use the gyroscope model to calculate the interaction of electron B upon electron A.

The angular velocity of B about the origin is V_1/d. Using the Quantised Structure Model, the probability that electron B will be located at the origin of the axes is 1/pB, where B is the normalising constant and d = pR/2 (R being the Compton Radius of the electron).

The angular momentum of electron A about the origin of the axes is mrV_2.

The torque yielded by the interaction of B upon A is given (see equation 2) by

$$T = {V_1}/{d} \times {1}/{pB} \times m_e V_2 r$$

(B denoting the normalisation constant).

Note: this relationship is only valid provided that the radius of the loop of electron B is small, and is much less than the radius of the loop of electron A.

Substituting $p = 2d/R$ yields

$$T = \frac{V_1 V_2 m_e R r}{2Bd^2}$$

Substituting $R = \hbar/m_e c$ yields

$$T = \frac{V_1 V_2 r \hbar}{2Bd^2 c}$$

Substituting $e^2 = \hbar c/137.036$ yields

$$T = \frac{137.036}{B} \times \frac{e^2 V_1 V_2 r}{2d^2 c^2}$$

There is, however, a complication to this calculation. Although constrained to travel in a circular orbit, the precise orientation of electron A - and the associated observed orientation of electron B - is not determined. If the spin axis of electron A is parallel to OZ, there will be full coupling between electrons A and B when B lies on the X axis. When B lies on the Z axis, there will be zero coupling. At intermediate points, the coupling will follow a cosine law. Thus, the torque as described above has to be modified by a factor of $\pi/2$ to give the mean torque, averaged over all possible locations within the orbit of electron B.

Thus:

$$T = \frac{2 \times 137.036}{\pi B} \times \frac{e^2 V_1 V_2 r}{2d^2 c^2} \qquad (6)$$

According to conventional thinking, the torque exerted on electron B in fig 4 would be calculated as follows.

The torque on a body of magnetic moment μ in a magnetic field of strength H is given by $T = \mu H$

In the case of an electron (such as electron B) travelling with speed V_2 in a circular orbit of radius r and area A, thus creating a current, the magnetic moment is given by

$$\mu = iA = \frac{eV_2}{2\pi rc} \times \pi r^2 = \frac{eV_2 r}{2c}$$

The field yielded by the orbit of an electron such as electron A is given by

$$H = \frac{2\pi i}{d} = \frac{2\pi e V_1}{2\pi c d^2} = \frac{e V_1}{c d^2}$$

Thus, the conventionally calculated torque is given by

$$T = \frac{e V_2 r}{2c} \times \frac{e V_1}{c d^2} \tag{7}$$

Equation (7) is identical with equation (6), provided that the constant

$2 \times 137.036/\pi B$ is equal to unity. As was shown in (7), this fraction equals 1.00116, provided that the age of the universe is 13.66 billion years. Thus, this 'macro' model appears to explain Ampère's Law in that the inverse square law property is confirmed. A difference of one part in one thousand with respect to the constant of proportionality may provide an interesting opportunity to test the arguments presented in this paper by experimental means. Is it possible, for example, that the constant of proportionality of this law should be modified, and that such a change has been masked by the use of this law for calibration purposes? Such a change would imply that the magnetic moment of the electron is exactly one Bohr Magneton, but would not alter the prediction of the age of the universe given above.

8.4 The arguments outlined in this paper may be relevant to the magnetic moments of particles other than the electron. Several particles, in addition to electrons, have been discovered to possess magnetic moments. These moments are measured in a unit known as the nuclear magneton μ_n. This unit is similar to the Bohr Magneton, but has the mass of the electron replaced by the mass of the proton. Thus

$$\mu_n = \frac{e\hbar}{2 m_p c}$$

The moments that are derived by experiment are puzzling. They are either positive or negative and are far from being an integral number of nuclear magnetons.

For example:

the moment of the proton	=	2.79284739 μ_N
the moment of the neutron	=	-1.9130427 μ_N
Net change	=	4.70589009 μ_N

When a neutron decays into a proton (plus an electron and neutrino) the net change of moment is as shown above.

However, we saw in section (3) that an unexpected factor of $4/\pi$ had to be introduced when calculating torque. Such a correction resulted in the torque exerted on a particle being somewhat greater than might be expected.

The magnetic moment of a nucleon is measured by detecting the precession frequency that is caused by the torque arising from the presence of a particle with magnetic moment in a magnetic field. We speculate that the torque generated in such circumstances is also enlarged by $4/\pi$, with the consequence that magnetic moments as currently measured may have to be increased by the same factor.

Introducing such a factor into the neutron decay yields a net change of moment of

$$4.705899999009 \times 4/\pi = 5.991730416$$

However, it would be logical to measure this net change in units that are appropriate to the neutron, not to the proton. This value should, therefore, be adjusted by 939.565663/938.27231, which yields a value of

5.999989664

It is reasonable to assume that the correct value, thus calculated, is exactly six.

What is the meaning of such a finding? Let us suppose that the neutron is not a monolithic particle but has some form of structure, being composed of smaller parts. Let us suppose, too, that the decay of the neutron hinges around the decay of a constituent primary particle. A particle that is less massive than the neutron can be expected to have a larger magnetic moment. For other particles (eg the electron and the muon), magnetic moment is inversely proportional to particle mass. This conclusion accords with fig 2 and the associated theory, according to which the probability that electron A will be coincident with electron B is inversely proportional to the mass of A.

For the proton,

$$\hbar/2 = m_p R_p c/2$$

If the proton were a simple primary particle, we would expect it, by simple scaling, to possess a magnetic moment of approximately one nuclear magneton.

For another particle we can write $\hbar/2 = m/6 \times 6Rc/2$

Such a particle would have a mass six times less than that of the proton, ie 156.38 MeV but would possess a magnetic moment of six nuclear magnetons. Such a mass is not recognised as an existing particle.

However, we might expect such a particle to be in motion and, in accordance with the theory of quantised variables, to possess a velocity of the form c/n, where 1/n is a simple fraction. Such a form of velocity quantisation is fundamental to the quantised structure model.

If this particle were to have a velocity of c/2, its rest mass would be

$$156.38 \times \sqrt{1 - 1/2^2}$$

$$= 135.42 \text{ MeV},$$

which is instantly recognisable as the mass of a pion.

Such an approach is relevant to other particles. For example, the Σ^+ particle has a mass of 1189.37 MeV and a magnetic moment of 2.42 ± 0.004 nuclear magnetons. One of its decay modes is to a neutron and pion; the associated change of magnetic moment, calculated as above and measured in units appropriate to this particle is

$$6.9932\ \mu_\Sigma,$$

or seven, to all intents and purposes.

One seventh of the particle mass is 169.9 MeV which, once more, seems unrelated to any known particle. However, this is almost exactly the mass of a pion that is travelling at three fifths of the speed of light. (169.9MeV x 0.8 = 135.9MeV).

The Λ particle has a mass of 1115.6 MeV and a magnetic moment of -0.613 ± 0.004 nuclear magnetons. It decays to a proton and a pion. The change of magnetic moment, as calculated above and measured in units appropriate to this particle approximates to five.

One fifth of the particle mass is 223.12 MeV - almost exactly the mass of a pion that is travelling at four fifths of the speed of light. (223.12MeV x 0.6 = 133.8MeV).

Thus multiplying the measured magnetic moments of nucleons and other particles by $4/\pi$ appears to produce values that have meaning.

8 Summary

This paper should be regarded as speculative and exploratory. Further study will be required if the approaches discussed are to be adequately investigated and evaluated. Nevertheless, some tentative conclusions may at this point prove helpful.

1) It seems likely that interaction between magnet and electric current can be explained by interaction between the extended structures of electrons. Taking this finding in conjunction with the outcomes of paper six concerning electrostatic force and the force of gravity, it seems reasonable to assert that conventional field theory is merely a model which, though useful for calculating effects that are common to our experience, fails to explain causes.

2) The presence and value of the electron magnetic moment can be explained by interaction between the structures of electrons as described by the Quantised Structure Model.

3) It seems probable that the age of the universe lies between 13.5 and 14 billion years.

4) Second thoughts may be needed concerning the conventional view of baryon magnetic moments. It is possible that currently aaccepted values should be revised by a factor of $4/\pi$.

5) The 'standard theory' of particle structure involves 'quarks'. In the light of this and preceding papers, it seems possible that pions feature, at least to some extent, in nuclei

and nucleons. Such an outcome would seem reasonable, being anticipated by Yukawa some sixty years ago.

6) Another issue concerns the implications of the equation

$$2^{127} = {cT_o}/{2R}$$

The left-hand side does not vary with time, which implies that either the speed of light or the Compton Radius of the electron must vary with time in order to balance the ever-increasing age of the universe T_o. (The alternative explanation that the age of the universe remains constant seems implausible but not impossible).
Further, if we rewrite this equation as:

$$2^{127} = \frac{m_e}{2\hbar/c^2T_o},$$

this number appears as the ratio between the mass of the electron and the smallest mass that can exist $2\hbar/c^2T_o$. Is it possible that the mathematical process that yields the combinatorial hierarchy can also describe a physical process that relates these two masses? If so, new light will be shed on some of the intractable problems of modern physics.

Iterative Methods

E. Grey.

Let $f(y+t) = a_0 + a_1 t + a_2 \frac{t^2}{2!} + a_3 \frac{t^3}{3!} + \ldots$

Then $a_0 = f(y)$, $a_1 = f'(y)$, $a_2 = f''(y)$, $a_3 = f'''(y)$, ...

i.e. $f(y+t) = f(y) + t f'(y) + \frac{t^2}{2!} f''(y) + \frac{t^3}{3!} f'''(y) + \ldots$

which is Taylor's Series.

Let D_y be the operator which differentiates with respect to y

then $f(y+t) = f(y) + t D_y[f(y)] + \frac{t^2 D_y^2}{2!}[f(y)] + \frac{t^3 D_y^3}{3!}[f(y)] + \ldots = e^{tD_y}[f(y)]$

Let $x = f(y)$ then $\frac{dx}{dy} = f'(y) = f' \circ f^{-1}(x)$

If D_x takes derivatives with respect to x then $e^{tD_y}[f(y)] = f(y+t)$ becomes:

$e^{t \frac{dx}{dy} D_x}[x] = e^{t f' \circ f^{-1}(x) D_x}[x] = f(f^{-1}(x) + t)$

Similar to the above $e^{tD_y}[H(f(y))] = H(f(y+t))$

$\therefore e^{t f' \circ f^{-1}(x) D_x}[H(x)] = H(f(f^{-1}(x) + t))$

2 Let $\underline{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, $\underline{t} = \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$ and $\underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ 2

Also given $\underline{x} = f(\underline{y})$ equivalent to:

$$x_1 = f_1(y_1, y_2)$$
$$x_2 = f_2(y_1, y_2)$$

and $\underline{y} = f^{-1}(\underline{x}) = g(\underline{x})$

equivalent to:

$$y_1 = g_1(x_1, x_2)$$
$$y_2 = g_2(x_1, x_2)$$

Now we have:

$$e^{(t_1 D_{y_1} + t_2 D_{y_2})}[f(\underline{y})] = f(\underline{y} + \underline{t})$$

$$= f(g(\underline{x}) + \underline{t}) = f(f^{-1}(\underline{x}) + \underline{t})$$

$$t_1 D_{y_1} + t_2 D_{y_2} = \begin{bmatrix} t_1 & t_2 \end{bmatrix} \begin{bmatrix} D_{y_1} \\ D_{y_2} \end{bmatrix}$$

$$= \begin{bmatrix} t_1 & t_2 \end{bmatrix} \begin{bmatrix} \frac{\partial x_1}{\partial y_1} & \frac{\partial x_2}{\partial y_1} \\ \frac{\partial x_1}{\partial y_2} & \frac{\partial x_2}{\partial y_2} \end{bmatrix} \begin{bmatrix} D_{x_1} \\ D_{x_2} \end{bmatrix}$$

$$= \begin{bmatrix} t_1 & t_2 \end{bmatrix} \begin{bmatrix} \frac{\partial x_1}{\partial y_1} D_{x_1} + \frac{\partial x_2}{\partial y_1} D_{x_2} \\ \frac{\partial x_1}{\partial y_2} D_{x_1} + \frac{\partial x_2}{\partial y_2} D_{x_2} \end{bmatrix} = t_1\left(\frac{\partial x_1}{\partial y_1} D_{x_1} + \frac{\partial x_2}{\partial y_1} D_{x_2}\right) + t_2\left(\frac{\partial x_1}{\partial y_2} D_{x_1} + \frac{\partial x_2}{\partial y_2} D_{x_2}\right)$$

3

Given a moving body (in a straight line) initial position, velocity, acceleration etc. respectively $a_0, a_1, a_2, a_3 \ldots$

Let τ be the smallest unit of time we can measure then in time τ the new set of parameters for the body will be given by:

$$\begin{bmatrix} 1 & \tau & \tau^2/2! & \tau^3/3! & \tau^4/4! & \cdots \\ \cdot & 1 & \tau & \tau^2/2! & \tau^3/3! & \cdots \\ \cdot & \cdot & 1 & \tau & \tau^2/2! & \cdots \\ \cdot & \cdot & \cdot & 1 & \tau & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ \vdots \end{bmatrix} = \begin{bmatrix} a_0 + \tau a_1 + \frac{\tau^2}{2!} a_2 + \ldots \\ a_1 + \tau a_2 + \frac{\tau^2}{2!} a_3 + \ldots \\ a_2 + \tau a_3 + \frac{\tau^2}{2!} a_4 + \ldots \\ a_3 + \tau a_4 + \frac{\tau^2}{2!} a_5 + \ldots \\ \vdots \end{bmatrix}$$

or more conveniently $T \underline{a}$

After an interval of $n\tau$ the new parameters will be $T^n \underline{a}$.

4

1 In trying to get $f_0 f^{-1}(x)$ from $f[f^{-1}(x)+1]$

Finite difference methods will sometimes work.

Let: $F(x) = f[f^{-1}(x)+1] = f \circ S \circ f^{-1}(x)$

then $(t) F(x) = f \circ (t) S \circ f^{-1}(x) = f[f^{-1}(x)+t]$

Take sequence of different iterates of F in increasing order for same 'x'.

(This is the 'orbit' of 'x')

$x \quad F(x) \quad F \circ F(x) \quad F \circ F \circ F(x) \quad F \circ F \circ F \circ F(x) \ldots$

$f(y) \quad f(y+1) \quad f(y+2) \quad f(y+3) \quad F(y+4) \ldots$

Now take finite differences.

$f(y+1)-f(y) \quad f(y+2)-f(y+1) \quad f(y+3)-f(y+2) \quad f(y+4)-f(y+3)$

$f(y+2) - 2f(y+1) + f(y) \qquad f(y+3) - 2f(y+2) + f(y+1) \qquad f(y+4) - 2f(y+3) + f(y+2)$

$f(y+3) - 3f(y+2) + 3f(y+1) - f(y) \qquad f(y+4) - 3f(y+3) + 3f(y+2) - f(y+1)$

...

Now $e^{D_y} f(y) = f(y+1) = Ef(y)$

$(e^{D_y}-1) f(y) = f(y+1) - f(y) = \Delta f(y)$

~~So~~ $e^{D_y} - 1 = \Delta \quad$ ~~So~~ $1+\Delta = e^{D_y}$

$D_y = \log_e(1+\Delta) = \Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} + \frac{\Delta^5}{5} - \frac{\Delta^6}{6} + \frac{\Delta^7}{7} \ldots$

$D_y f(y) = f'(y) = \quad f' \circ \frac{2}{3} f^{-1}(x)$

Since the operator $e^{t\, f' \circ f^{-1}(x) D_x}$
$= e^{t\phi(x) D_x}$ takes $H(x)$ to $H \circ F(x)$

where $F(x) = f(f^{-1}(x)+1) = f \circ S \circ f^{-1}(x)$

another approach is to use the eigenvalues and eigenvectors of this operator:

Let $\omega = e^{t\phi(x) D_x}$

Then we have $\omega H(x) = H \circ F(x)$

If $L(x)$ is an eigenvector of this operator and λ is the corresponding eigenvalue we get:

$\omega L(x) = \lambda L(x)$

But $\omega L(x) = L \circ F(x)$

So we have: $L \circ F(x) = \lambda L(x)$

$\therefore F(x) = L^{-1}(\lambda L(x))$

A similar approach can be taken with the multivalued functions.

1 Given $x(t) = A \sin \omega t$ 6

Represent it as a probability distribution if the time is taken at random.

The chance of finding a particle moving according to the above rule is inversely proportional to its speed.

Now speed is $\dot{x}(t) = A\omega \cos \omega t$

with reciprocal $\frac{1}{\dot{x}(t)} = \frac{1}{A\omega \cos \omega t}$

This needs to be represented as a function of 'x' and then divided by the integral over the maximum range to get a probability density.

We get $\frac{1}{A\omega \cos \omega t} \div \int_{-A}^{A} \frac{1}{A\omega \cos \omega t} dx$

$$= \frac{1}{A\omega\sqrt{1 - \sin^2 \omega t}} \div \int_{-A}^{A} \frac{1}{\dot{x}(t)} dx$$

$$= \frac{1}{A\omega\sqrt{1 - \frac{x^2}{A^2}}} \div \int_{-A}^{A} \frac{dt}{dx} dx$$

$$= \frac{1}{\omega\sqrt{A^2 - x^2}} \div \Big[t\Big]_{-A}^{A} = \frac{\frac{1}{\omega\sqrt{A^2-x^2}}}{\left[\frac{1}{\omega}\sin^{-1}\frac{x}{A}\right]_{-A}^{A}} = \frac{1}{\pi\sqrt{A^2 - x^2}}$$

Given $x(t) = A \sin \omega t$

Represent it as a probability distribution if the time is taken at random.

The chance of finding a particle moving according to the above rule is inversely proportional to its speed.

Now speed is $\dot{x}(t) = A\omega \cos \omega t$

with reciprocal $\frac{1}{\dot{x}(t)} = \frac{1}{A\omega \cos \omega t}$.

This needs to be integrated and then divided by the integral over the maximum range to get a probability (from the density).

We get $\int_{x_1}^{x_2} \frac{1}{A\omega \cos \omega t} dx \Big/ \int_{-A}^{A} \frac{1}{A\omega \cos \omega t} dx$

$$= \int_{x_1}^{x_2} \frac{1}{\dot{x}(t)} dx \Big/ \int_{-A}^{A} \frac{1}{\dot{x}(t)} dx = \int_{x_1}^{x_2} \frac{dt}{dx} dx \Big/ \int_{-A}^{A} \frac{dt}{dx} dx$$

$$= \frac{\left[t \right]_{x_1}^{x_2}}{\left[t \right]_{-A}^{A}} = \frac{\left[\frac{1}{\omega} \sin^{-1} \frac{x}{A} \right]_{x_1}^{x_2}}{\left[\frac{1}{\omega} \sin^{-1} \frac{x}{A} \right]_{-A}^{A}} = \frac{\left[\frac{1}{\omega} \sin^{-1} \frac{x}{A} \right]_{x_1}^{x_2}}{\frac{1}{\omega}\left(\frac{\pi}{2} - \left(-\frac{\pi}{2} \right) \right)} = \frac{1}{\pi}\left(\sin^{-1} \frac{x_2}{A} - \sin^{-1} \frac{x_1}{A} \right)$$

3 When the co-ordinates of F change so that we have $h \circ F \circ h^{-1}$ 8

Since $F = f \circ S \circ f^{-1}(x) = g^{-1} \circ S \circ g(x)$

$g(x)$ becomes $g \circ h^{-1}(x)$

So $\psi(x) = g'(x)$ becomes: $g' \circ h^{-1}(x) \frac{d(h^{-1}(x))}{dx}$

$$= \frac{g' \circ h^{-1}(x)}{h' \circ h^{-1}(x)} = \frac{\psi \circ h^{-1}(x)}{h' \circ h^{-1}(x)}$$

The value of the expression

$$\frac{\psi \circ h^{-1}(x)}{|h' \circ h^{-1}(x)|} = \frac{g' \circ h^{-1}(x)}{|h' \circ h^{-1}(x)|}$$ represents a probability density

for x given ψ is the probability density of $h^{-1}(x)$.

Thus ψ transforms under a change of coordinates like a covariant tensor of rank 1.

$x(t) = A \sin \omega t$

$\dot{x}(t) = A\omega \cos \omega t = \frac{dx}{dt} = f' \circ f^{-1}(x) = \frac{1}{g'(x)}$

$\frac{dt}{dx} = g'(x)$ $\quad t = \int g'(x)\,dx = g(x)$

Now $g(x) = \frac{1}{\omega}\left(\sin^{-1} \frac{x}{A}\right)$

$x(t) = f\left[f^{-1}(x) + t\right] = g^{-1}\left[g(x) + t\right]$

The probability density is:

$$\frac{\Delta x\, g'(x)}{(\pi/\omega)}$$

and the probability of being in the range (x_1, x_2)

is $\int_{x_1}^{x_2} \frac{g'(x)}{(\pi/\omega)}\,dx = \left[\frac{g(x)}{(\pi/\omega)}\right]_{x_1}^{x_2} = \frac{\omega}{\pi}\left(g(x_2) - g(x_1)\right)$

Let $F(x) = f(f^{-1}(x) + 1)$

and using the successor function

$S(z) = z+1$

we can write $F(x) = f(S(f^{-1}(x)))$

Let $\{t\}F$ denote the function F iterated 't' times then we have:

$\{t\}F(x) = f(S(f^{-1}(f(S(f^{-1} \ldots (x)$ 't' times

$= f(\{t\}S(f^{-1}(x))) = f(f^{-1}(x) + t)$

Also $\{t_1\}F \circ \{t_2\}F(x) = \{t_2\}F\,\{t_1\}F(x)$

$= \{t_1 + t_2\}F(x) = f(\{t_1+t_2\}S(f^{-1}(x)))$

$= f(f^{-1}(x) + (t_1 + t_2))$

Any function which commutes with F is of the form $\{t\}F$ for some t.

Let $\phi(x) = f' \circ f^{-1}(x)$

Let us look at $\frac{d}{dx}F(x) = F'(x)$

This is $f'(f^{-1}(x)+1) \cdot \frac{d f^{-1}(x)}{dx} = f'(f^{-1}(x)+1)$

$= \frac{f'(f^{-1}(x)+1)}{dx/dy} = \frac{f'(f^{-1}(x)+1)}{f' \circ f^{-1}(x)} = \frac{\phi(u)}{\phi(x)}$ where $u = F(x)$ $= \frac{du}{dx}$.

Under a change of coordinates

$F(x)$ becomes $h \circ F \circ h^{-1}(x)$

i.e. $h \circ f \circ s \circ f^{-1} \circ h^{-1}(x)$

So $f^{-1}(x)$ has become $f^{-1} \circ h^{-1}(x)$.

Now $\phi(x) = \left[\frac{df^{-1}(x)}{dx}\right]^{-1} = f' \circ f^{-1}(x)$

So the new value is $\left[\frac{d f^{-1} \circ h^{-1}(x)}{dx}\right]^{-1}$

$= \left(f' \circ f^{-1} \circ h^{-1}(x) \right) . h' \circ h^{-1}(x) = \left[h' \circ h^{-1}(x)\right] \phi \circ h^{-1}(x)$

Let $\phi(x)$ be called $\mathcal{F}(x)$

A change of coordinates then gives:

$\left[h' \circ h^{-1}(x)\right] \left[\mathcal{F} \circ h^{-1}(x)\right]$

So that if we take $\mathcal{K}(x) = \mathcal{F}(x) + \mathcal{G}(x)$

A change of coordinates gives

$\left[h' \circ h^{-1}(x)\right]\left[\mathcal{F} \circ h^{-1}(x)\right] + \left[h' \circ h^{-1}(x)\right]\left[\mathcal{G} \circ h^{-1}(x)\right]$

$= \left[h' \circ h^{-1}(x)\right] \left[\mathcal{K} \circ h^{-1}(x)\right]$

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Given a function of both time and position how does it transform under a change of coordinates.

Let $U = F_t(x)$ represent a function of time 't' and position 'x'.

Composition of such functions is always with respect to 'x'.

Two sorts of derivative are defined:

1) with respect to the time variable and denoted by a dot above the function name.

2) with respect to the composition variable and denoted by a dash.

Two operators are defined:

1) $\Omega F_t(x) = \dot{F}_t \circ F_t^{-1}(x)$

Where $F_t^{-1}(x)$ denotes the inverse with respect to the composition variable.

2) $\omega F_t(x) = F_t' \circ F_t^{-1}(x)$.

$\Omega F_t(x)$ is of particular importance representing as it does a velocity of the time varying field represented by $F_t(x)$.

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A change of coordinates is represente by a pre-composition by another function.

i.e. $F_t(x)$ becomes $H_t \circ F_t(x)$.

What happens to $\Omega\, H_t \circ F_t(x)$?

Start with $\frac{\partial}{\partial t} H_t \circ F_t(x) = [H_t' \circ F_t(x)] \cdot \dot{F}_t(x)$

$+ \dot{H}_t \circ F_t(x)$

Since the inverse of $H_t \circ F_t(x)$ is $F_t^{-1} \circ H_t^{-1}(x)$ we have:

$$\Omega\, H_t \circ F_t(x) = [H_t' \circ F_t \circ F_t^{-1} \circ H_t^{-1}(x)][\dot{F}_t \circ F_t^{-1} \circ H_t^{-1}(x)]$$

$$+ \dot{H}_t \circ F_t \circ F_t^{-1} \circ H_t^{-1}(x)$$

$$= [H_t' \circ H_t^{-1}(x)] \cdot [\dot{F}_t \circ F_t^{-1} \circ H_t^{-1}(x)] + [\dot{H}_t \circ H_t^{-1}(x)]$$

$$= [\omega H_t(x)] \cdot [\Omega F_t \circ H_t^{-1}(x)] + [\Omega H_t(x)]$$

Let $K_t(x) = H_t^{-1}(x)$ then from above:

$\Omega H_t \circ K_t(x) = \Omega I(x) = 0 = [\omega H_t(x)] \cdot [\Omega K_t \circ H_t^{-1}(x)] + \Omega H_t(x)$

where $I_t(x) = x$ is the identity function and not involving t!

$$\therefore\ \Omega\, H_t \circ F_t(x) = [\omega H_t(x)] \cdot [\Omega F_t \circ H_t^{-1}(x)]$$

$$- [\omega H_t(x)] \cdot [\Omega K_t \circ H_t^{-1}(x)]$$

PANPSYCHISM, THE CONSCIOUS BRAIN, AND EXO-BIOLOGICAL AWARENESS

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ABSTRACT

Brain awareness doesn't come out of thin air. There is evidence aplenty that it critically depends on a set of neural activities. What is more, recent findings compellingly suggest that consciousness is a function of an identifiable neural architecture. This, to be sure, affords an invaluable insight into the inner workings of our awareness-begetting brain. However, this *descriptive* account falls woefully short, *explanation*-wise, of providing any telling clue. This being so, can we go any further towards unlocking the neural underpinnings of consciousness, and get an inkling as to *what* physical processes yield some awareness - and above all, *why* they do so? This raises the so-called hard problem of consciousness.

In a bid to unfold it, I put forward a panpsychic approach based on the psychomatter hypothesis. According to it, what we are wont to take for plain matter would actually shroud an oft-latent - and hence hidden, unseen - "psychic" or 'psi' content. The 'psi' parts of elementary particles would be the raw building blocks of higher-level, macropsychic conscious experience. This tentative outlook leads to the *cognitive iceberg* model of perceptual awareness.

This *iceberg* is made of an underwater - or rather, "underaware" - part, where incoming sensory stimuli are coded in the shape of what I call the *suprels.* Visual *suprels*, for instance, are typically made in the visual cortical areas, where they remain unconscious. (This is meant to account for *preconscious* brain processing, on the one hand.) Then they dash into the "tip", where they target specific microstructures (dubbed the *paralgens)* that readily turn them into the variegated subjective contents of experience, known as *qualia.* (This is to deal with *conscious* brain processing - on the other hand.)

(It turns out that we are far from clueless as to the putative nature of both the *suprels* and the *paralgens.* Indeed, some of these paralgens are likely to be tucked inside the postsynaptic NMDA receptors found on the dendritic synapses of large pyramidal cells in the neocortical fifth layer...)

It will be argued that the above model sheds new, if provisional, light on such conundrums as: the binding problem (which deals with the uncanny, seamless unity of conscious experience); the nature of our conscious recalls; the parallel (and unconscious) versus serial (and conscious) processing problem; and the 'upshot problem', whereby what we are conscious of appears to be the result, or upshot, of neural computations rather than the computations themselves.

Finally, I shall touch on the tantalizing possibility, in keeping with the foregoing, to think up and carry out full-blown exo-biological brains endowed with genuine awareness. (This perspective is implicit to the panpsychic idea.)

Foreword

If we peer inside a living skull, straight into the shifty privacy of a feeling, sensing, thinking brain, we are bound to find matter, matter anew, and matter again. And nothing else, ever! What then about the mind - to say nothing of the soul? No hint, no inkling, no whiff of it. Not the flimsiest shred of evidence. To all outwards appearances, the mind doesn't exist.

However, seeing is believing. And this is preciely the point: when we see through the private, inner eye of consciousness, we realize that there may be more to reality than meets the... biological eye. This throws open the intriguing prospect of panpsychism, which posits that the seeds of mind are at once truly nonmaterial, tightly knit to matter, and all-pervasive in our 'mineral' world. (Or is the mind just an emergent property, pulled off by some eery trick of plain matter?)

Panpsychism, as I take it to be, rests on the tentative insight that there is something, in the raw substance we call matter, that could occasionally be kindled into yielding awareness (in much a similar fashion as chunks of matter can be ignited into giving off light). On this account, our universe would be richer than we tend to think. And matter would qualify as *psychomatter.*

If so, the living brain is to consciousness what a lamp is to light: a catalyzer of sorts. Brains and lightbulbs alike make a potential property of "matter" come forth - whether it be light or awareness. (This property, in both cases, is overwhelmingly hidden: here's the catch!)

Let me make one thing clear at the outset. In this paper I shall make no attempt at defining consciousness. I shall restrict my focus to the conscious brain, and shall only seek to address the following question: *Why and how it is that the brain is the organ of awareness?* To that end, I don't need to define consciousness. (For simplicity's sake, I take the two words of awareness and consciousness as synonymous, without further ado.) I only need to *characterize* it operationally in a way that disambiguates it from matter.

Needless to say, the conscious brain is a dreadfully tough nut to crack. Indeed, a number of people believe that "it will always be impossible to demonstrate unequivocally and empirically how brain cells *cause* consciousness" (Greenfield 1995). I hope that my attempt, for right or wrong, will convince a few that this impossibility... is only in the mind!

Is Panpsychism Worthwhile?

A whole wealth of data is available, today, as regards the neural correlates of arousal, wakefulness, alertness, consciousness, and the multifarious facets of subjective experience. It lends strong support to the thesis, recently upheld by Baars and Newman, that "consciousness is a function of an identifiable neural architecture" - which would be "the extended reticular-thalamic activating system (ERTAS)" (Newman, 1997).

That consciousness is somehow a "product" of the living brain is further evidenced e.g. by the so-called focal disturbances of consciousness (Sacks 1986, Flohr 1992) where, following partial lesions of the reticular formation in the brain, a person exhibits specific, and well documented, deficits in his/her conscious experience. (A larger lesion of the same brain structure, on the other hand, elicits no less than a global loss of consciousness: it leads to coma.) A striking example of brain-based deficit is hemi-neglect, where patients consistently ignore whatever relates to one side of their body.

Thanks in no mean part to our sophisticated non-invasive scanning techniques (e.g. PET, positron emission tomography, and MRI, magnetic resonance imaging), the brain is no longer the totally uncharted territory it once was. By and large, we can boast to know a hefty lot about its inner workings, even though much remains to be found. However, for all our hard-won knowledge, impressive as it is, we still fall woefully short of addressing the hard problem of consciousness (Chalmers, 1995, 1996). (It deals, roughly speaking, with explaining the occurrence - as opposed to merely nailing down the neural underpinnings - of brain awareness.)

As things stand, no truly winning, operative idea concerning the conditions under which physical processes give rise to consciousness is anywhere near in sight. There are words and talks galore - but very little to really go for, at the end of the day. (Were it not so, one can safely bet that artificial consciouness wouldn't be a long way off....)

Could it be that present-day science bites off rather more than it can chew, given its current framework? Perhaps. When it comes to consciousness, we are faced with a choice. Either we decide to hollow it out so that science, as we know and are used to it, is not called into question. We may then assume, somewhat squarely, that "consciousness is not only caused by neurobiological states but actually *is* these neurobiological states" (Damasio 1997).

(Incidentally, this reductionist stance is the loose consensus of the day, within the neuroscience community. However, and as emphasized by Velmans (e.g. Velmans 1990, 1996), reductionist arguments typically confound correlation, causation, and ontological identity; or they rely on false analogies; or both.)

Or we may contrariwise seek to broaden science - and thus change it, perhaps in a deep way - so that it can allow for consciousness (deemed non-material). This admittedly entails something of a sea-change in our outlook. But science is no absolute; it is not cast in a timeless, definitive, 'stiff-as-dead' mold. Like any human-made artefact, it is a living, evolving, ever-shifting entity. It is even prone to sweeping paradigm shifts (Kuhn 1970).

Should we then stick to a hard-nosed, fuss-free but perhaps inadequate materialism? Or should we take on board some additional axiom, some new hypothesis or ingredient, if we are to grasp and grab more of the world? Should we go for Ockham (a.k.a. Occam, of razor fame) or Gödel (forever undecided)?

It appears that a disconcerting number of people would invoke, when pressed to buttress the reductionist option, Ockham's razor. This 'razor' refers to the sound principle that, upon explaining something, you must dispense with any assumption that is not strictly necessary.

In the present context however, any claim based on this heuristic principle has no real bite. It only begs the question at issue; for it is very unclear, in the first place, whether materialism can be up to the job at all. It is interesting in this respect to note that physics, according to authors such as Nagel (1986, p.7), is incomplete and "bound to leave undescribed the irreducibly subjective character of conscious mental processes, whatever may be their intimate relation to the physical operation of the brain."

As Chalmers would have it, the hard problem of consciousness is hard indeed!

If we go for the second horn of the alternative - for Gödel instead of Ockham - we acknowledge or suspect that the elusive and baffling phenomenon of awareness confronts us with a genuine unknown, which flies all too glaringly in the face of materialism. This brings us to a situation akin to stumbling on one of Gödel's undecidable statements... that science is demonstrably unable, in its current state, to handle (Gödel 1981, Smullyan 1987).

The way out is to award consciousness some of the wiggle room it deserves, within and on a par with matter itself. It is therefore to broaden our framework. The right thing to do - as we learnt from Gödel, too - is to enrich science with some new (and hopefully relevant) axiom or hypothesis. *Psychomatter* or *panspychism*, we'll soon find out, is just such a tentative axiom.

My homespun brand of panspychism qualifies as proto-panpsychism - or, even better, as "pan-crypto-protopsychism"! It rests on the assumption that matter is actually *psycho*-matter; and that the conscious brain is an effect, viz. a causal outcome, of *psychomatter*. (The linchpin is that the "psycho" or 'psi' side evades scrutiny for it is overwhelmingly latent - and hence inert, utterly inobservable.)

This view is in line with speculations made by David Chalmers (Chalmers 1995, 1996), who pondered that an ontology that includes consciousness requires a new physics. (Incidentally, I touched on the broader issue of consciousness - as seen from the wilder shores of an ontological, meta-physical perspective - in (Ransford 1997).)

New Concepts are Best Tagged with New Words

(The scope of this short Section is to familiarize the reader with some of the nuts and bolts of my panpsychic approach. It just anticipates what will be developed in the next Section. It can therefore easily be skipped at first, and read after this next Section.)

As a rule, new entities request new labels, if a host of unnecessary confusions are to be eschewed. Thus, even though I am loath to do so, I shall coin a few neologisms to tag the new (but intuitively straightforward) features that go with the psychomatter hypothesis. I shall do my best to keep it as clear and as simple as possible.

We gather from its very name that psycho-matter is bi-dimensional, and contains a "psycho" (or 'psi', for short) part. The fact that this 'psi' shuns detection is a strong hint that it is overwhelmingly latent. Accordingly, psychomatter takes on two alternative guises, depending on whether its content is active or latent: this Janus-like substance - "one stuff, two faces" - breaks down into what I propose to christen matter (i.e. *matter proper*) and *paral*. (Simple enough: it is like water, that can also be ice.)

This being so, a *paralling* or *paral phase* labels this particular state psychomatter is in, when its 'psi' is off-latent - and thereby becomes fully active. When a speck of psychomatter undergoes a paralling, its 'psi' is set alight, as it were. It is made to glow, like a tiny spark.

Next, there is the concept of *supralness*. *Supralness* is about binding, blending or aggregating the 'psi' parts of several chunks of psychomatter. A *supral link* is a kind of 'psi' thread, running through sundry specks of psychomatter. It is noteworthy that it cannot be seen either, any more than the 'psi' itself.

Now, two additional concepts can be drawn from the foregoing. The first one is that of *suprel*. A *suprel* is an elementary bit, or unit, of a brand-new type of information; which is embedded in supralness. It is encoded in supral patterns, that (quite simply) are webs or tangles of 'psi' threads. What stands out is that these patterns bring structure, *and hence information*, at the 'psi' level. (This structure is combinatorial and topological in essence.)

The complexity and variety of this supral information is virtually boundless. (Think of all the patterns that can be wrought by linking a basketful of beads with threads!) We should again take heed that these (nonlocal) supral patterns do not, as a rule, show at the material level. They are not anchored in the physical side of psychomatter. It is little wonder that we failed to spot them in the brain....

Lastly, there is the concept of *paralgen*. Neural or brain *paralgens* are alleged microdevices in our brains that turn or kindle matter into paral. (Crudely put: they "wake up" the 'psi' contents of psychomatter).

One can summarize all of the above in a short-hand way:

Matter = psychomatter whose 'psi' content is latent

Paral = psychomatter whose 'psi' is "off-latent" (i.e. it is astir, or akindle)

Paralling, paral phase = 'psi' spark ; Supral link = 'psi' thread

Suprel = basic 'psi' pattern *(a* suprel *encodes data that we may become aware of)*

Paralgen = microdevice poised to turn matter into paral *(a* paralgen *arouses or kindles the 'psi' in psychomatter)*

All these words and phrases link either with the paral interaction (viz. the 'psi' spark) or with the supral interaction (viz. the 'psi' thread), which will soon take on an immediate visual meaning. (See the *M-P-S diagram* underneath.) The "paral" bit underscores the low-level, microphysical, roots of awareness; whilst the "supral" bit underpins its wider, distributed or holistic nature.

As with light, so with awareness: there is a threshold below which awareness is just too dim to be worth the name. Due to its supral side, consciousness is a truly emergent property that is not available at the raw level of the unsupralled (or poorly supralled) paral. It takes a whole slew of suprally bound 'psi' sparks to make a full-blown *macropsychic* entity, such as human consciousness. Likewise, it takes many dewdrops to make a puddle!

In line with this, the wakeful brain can be thought of as a 'psi'-triggering machine, bent on turning an ever sufficient amount of matter into paral. We can therefore write, in a nutshell:

Stream of consciousness = stream of supralled paral

Awakened brain = wide-scale 'psi' catalyzer (that brings about an ongoing, above-threshold flow of supralled paral - this flow comes down to a trickle during sleep)

And, last but not least:

- *Supralness* can seamlessly bind a host of 'psi' parts of psychomatter into forming macroscopic wholes (e.g. the human mental selves). It moreover brings information-laden patterns to bear at the 'psi' level. This may account for the diversity - and sheer existence - of the mental contents of subjective experience.

- The presence of (myriads of) *paralgens* in some brain areas holds the key to the amazing and puzzling fact that "mental events are a feature of neurophysiological systems with certain properties." (Zeki 1993, p. 345)

By way of partial and provisional conclusion, the understanding of the conscious brain which goes with my strain of panpsychism can be encapsulated in the following statement:

The brain is the organ of awareness because it is able to yield, on wide enough a scale, an ongoing stream of supralled paral. (It owes this stunning ability to its paralgens.)

This spells out my panspychic answer to the riddle of the conscious brain. Interestingly, it basically agrees with the by now widespread opinion that "consciousness is generated when vast groups of neurons work together collectively under specific conditions" (Greenfield 1995, p. 191). Were it on target, it could pave the way for an exciting new era. That of artificial consciousness....

The M-P-S Diagram

(This Section can be read independently of the previous one.)

For a start and as already said, matter (call it φ, or 'phi' - for physical) may turn out to harbour within itself an unseen spare content, or dimension (call it ψ, or 'psi' - for psychic). The two, 'phi' and 'psi', would be qualitatively different (i.e. their respective features would sharply differ).

The *psychomatter* hypothesis takes this insight at face value. It posits that what we call matter is really twofold, or bi-dimensional; that it shelters, along with the physical part, a psychic, or 'psi', one. As I already pointed out, this view is akin to that of panpsychism or panprotopsychism, as proposed e.g. by Seager and Rosenberg (see Chalmers, 1997).

To get this idea off the ground, three ingredients are needed. They are:

1 - As a rule, the 'psi' component of psychomatter does not show - for it is overwhelmingly latent, or dormant; whereby it remains stark unconscious. It requires peculiar conditions to be stirred out of its latency - thereby becoming conscious. If I call ***paralling conditions*** these "psi-stirring" conditions, one can readily surmise that the brain is conscious because it carries out the paralling conditions on a grand scale.

2 - When the 'psi' is stirred out of its latency, it forthwith becomes conscious and active. It is thus made to interfere with the 'phi' part of psychomatter, in a highly idiosyncratic (and hence easy to pinpoint) way. In this "off-latent" state, full consciousness or consciousness proper mainly boils down to a matter of scale, or threshold. I call ***paral interaction*** or ***paral phase*** - or else, ***paralling*** - this ψ-φ ('psi'-'phi') interaction. My bet is that it holds the key to the mind-body interaction (at its barest).

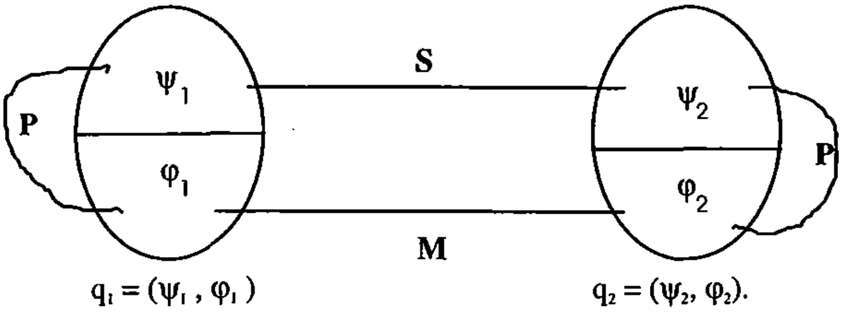
3 - Split brains notwithstanding, the conscious self has a knack to experience itself as a unified, globally coherent whole. However and as of today, there is no conclusive underlying neural mechanism that can thoroughly account for this. How then is the uncanny oneness of the self achieved? My assumption is that the 'psi' parts of, say, various chunks and specks - or 'elementary particles' - of psychomatter can be bundled together so as to form an overall 'psi' entity. I christen ***supralness*** this ability. I call ***supral link*** or ***supral interaction*** any existing

bond of *supralness* between elementary particles. Being a quintessentially ψ-ψ ('psi'-'psi') interaction, it allows nature to shape up macro-psychic clusters or entities.

Now, it should be stressed that the 'psi' needs not be a spooky stuff of sorts. Indeed, the main difference between ψ and φ is that the former is endo-causal whereas the latter is exo-causal. (These notions have been discussed at some length in (Ransford 1997); endo-causation comes down as an element of indeterminacy while exo-causation is deterministic.)

A good picture being worth over one thousand words, the *M-P-S diagram* below provides a concise (and uneschewably, exceedingly sketchy!) summary of my approach.

If $q_1 = (\psi_1, \varphi_1)$ and $q_2 = (\psi_2, \varphi_2)$ represent two elementary particles, or quantons - which both allegedly own a 'psi' (ψ) and a 'phi' (φ) parts - then one gets:



One has:
- M : *material* interaction (between φ_1 and φ_2)
- P : *paral* phase or interaction; or else, *paralling* (between ψ_i, φ_i) [i = 1, 2]
- S : *supral* link or interaction (between ψ_1 and ψ_2)

The M-P-S diagram raises an immediate question. It is: How can we tell M, P and S apart? This really is no big deal. Given that these interactions put either ψ or φ (or both) into play, telling them apart amounts to find out - or rather, to surmise - what diffentiates ψ and φ. So the question becomes: How can we tell the 'psycho' and the 'matter' (that is, the 'psi' and 'phi') contents of psychomatter apart? This is a key issue, that I take up now.

Hints and Clues from Science

The psychomatter hypothesis rubs off on two domains of human knowledge, which are physics and neuroscience. This is fairly obvious. Indeed there are - very briefly, and to start with physics (neuroscience will be broached in the next Section) - three *a priori* possibilities regarding their mutual relationship, namely:

- Both the *paral* and *supral* interactions do not exist. Chances are, in this case, that physics will never come up with any worthy candidate for them!
- These nonmaterial (*viz.* non-purely-material) interactions do exist, *but* physics hasn't yet stumbled on them. If so, it might be just a matter of time - and efforts - before we unearth them.
- These interactions exist, *and* physics has (unwittingly) spotted them. Then, given its current and too narrow materialistic framework, it cannot handle them properly. It can describe, but adamantly *not* interpret, them. Thus they are bound to wreak some havoc in our conventional understanding of matter, which no longer adds up!

I believe that, with quantum physics, we are in the last case. If so it be, expect supralness and paral phases to raise a sprinkling of unwieldy problems and paradoxes of their own, wherever they hit (and rock) materialism. *(Doesn't this ring a bell - or rather two?...)*

Now, in order to proceed meaningfully, I must be able to draw a sharp and watertight line between ψ ('psi') and φ ('phi'), as well as between matter and paral. This in turn requires that I lay down some clear-cut criteria. With an eye on contemporary physics, I propose that:

- φ is thoroughly deterministic, i.e. it is exo-determined. It complies with the principle of least action. (Relativity provides another conspicuous example of "φ-determinism".)

- ψ is just the opposite. It embodies an element of endo-determinacy. Furthermore, it is free from any of the "φ-determinisms".

The principle of least action governs φ alone, it holds its sway for matter only. Where it applies, a system evolves or behaves so as to minimize the 'action'. (The physical 'action' equals an energy times a duration.) The (wavelike) behavior of least action is fully deterministic, and is therefore inherently uncreative: it doesn't spawn any new element of reality. I express this by saying that it is 'deedless' (Ransford 1997).

Conversely, the 'psi' part, ψ, is endowed with a degree of self-determinacy, that shows as nondeterminism. The processes during which it becomes 'off-latent' or active are thereby 'deedful', i.e. they are genuinely creative. This (partial) self-determinacy can roughly be likened to an extremely low and raw measure of "free-will".

Free will? Free will is and has always been a highly controversial notion. Can we still believe in it, when some neuropsychologists (e.g. Christopher Frith in London) carry out experiments which show that the nervous system initiates an action *before* we consciously decide to do something? Granted, free will could be less than what it's cracked up to be.

Arguably, though, there is still room for a genuine, if limited, free will. (Were it not so, it is unclear why nature would have bothered to create something as mammothly complicated as perceptual awareness, with the attendant 'free will-bending' pain/pleasure polarities.) As William James once quipped: "My first act of free will shall be to believe in free will." The logic thereof is nearly unassailable, and I make it mine!

In light of the foregoing, let me recap this Section with three items:

- The 'phi' part (φ) of psychomatter behaves according to the principle of least action, whereas the 'psi' part (ψ) displays an unequivocal nondeterministic tinge. This 'psi' evolves regardless of the least action and, broadly speaking, of any "φ-determinism" (including relativity). On top of that, ψ is hidden when latent - which means well-nigh all the time! Its unmistakable properties come to the fore, on the contrary, whenever a paralling is triggered.

Therefore: A paralling is, *for one*, irreducibly nondeterministic; it, *for two*, breaches the least action principle and, for that matter, any "φ-determinism". It likewise breaks free, *for three*, of relativity. This consistently pins the paral phases down as what is known as quantum jumps and wavefunction collapses. (The idea is that all which is not strictly least action, or wavelike, micro-evolution involves one paralling or more.)

A paral phase, as I wrote in (Ransford 1995), "jolts a subatomic particle out of its smooth and steady wavelike motion (of 'least action')." For instance, the emission and absorption of a photon are run-of-the-mill paral phases. Similarly, a particle that decays or disintegrates - sometimes in the merest fraction of a second, without ever bothering to wait for someone to observe - does so through paral phases.

(Incidentally, the 'relativity-blindness' of these events is backed by faster-than-light tunnelling, observed by Chiao and coworkers in Berkeley. What is called tunnelling really spans two realities. It is a pure wavelike phenomenon as long as it remains reversible; as with the ammonia molecule. However, it is paralling-driven when it becomes irreversible.)

No paral phase, no decay. That simple! Were it enforced throughout, the principle of least action would stand in the way of all events of that ilk, and starkly preclude their happening. Indeed, any discrete, sudden, irreversible microphysical event bears the hallmark of a paralling: it is "paral-driven" (or paralling-driven). Any microphysical event that displays either discreteness, irreversibility or nondeterminism - and of course, expect to find *all* of them together - stems from a paralling. (Technically, it is a "nonunitary" event.)

These telltale features are in marked contrast to the smooth, reversible, fully deterministic least action motion. How might we fail to spot the difference? Besides, paral phases breach conservation laws (Burgos 1993).

- As for supralness, we recall that a supral link amounts to weld, or blend, hitherto unrelated 'psi' parts into a unified, coherent whole: it *correlates* them, regardless of the physical distance. Withal, these correlations are hidden as long as the 'psi' remains latent. Furthermore, supralness is not constrained by relativity. (It is ψ-related through and through, which frees it from any "φ-determinism". To be accurate, the relativistic notion of light-cone does not run its writ as far as it goes.)

Therefore: Supralness, *for one,* binds hence correlates various 'psi' parts within psychomatter - this *(for two)* being on display only when a paral phase comes off. *For three,* it is relativity-blind. All of the above pins supralness down as what is known as quantum entanglement, a.k.a. nonseparability.

- As for the paralling conditions (without which no paralling can occur), they are met in a variety of situations, which includes measurement processes as a sub-category (Ransford 1998). An interesting question is: Why in the world should there be paral phases, or parallings? I believe the root cause for them to be inherent in *quantumhood.*

Quantumhood, in short, is a all-or-nothing affair. It is epitomized by the fact that only integer numbers make sense at the microlevel (e.g. we may find 2 or 17 electrons, but never 2.3 or 16.362 of them). It is a 'straitjacket' forced upon micro-objects. As such, quantumhood is occasionally challenged and threatened - by the *paralling conditions* precisely. (Technically, they relate to the fact that some microsystem is not in a pure, or eigen, state of its energy operator.)

If, under some paralling condition, things were left to themselves (that is, to the least action motion) they would eventually run foul of quantumhood. (An example of it is when, through an appropriate setup, we insist on splitting an electron into two separate, not-mutually-interfering bits; e.g. 0.38 of it here, 0.62 there.) So a paralling is in order, and may swiftly arise. It saves the day because the 'psi', on becoming active and 'deedful', makes a fitting 'choice' as it were, that overcomes the hurdle: it is specifically designed to remove the quantumhood-threatening situation. (See (Ransford 1996, 1998) for a more detailed account.)

The drift here is that quantumhood would be all but untenable without parallings! (Were it not for them, physics - and worse still, nature herself - would be inconsistent. Here is why another evolution law was badly needed, further to that of the least action.) I believe that this state of affairs - which has no counterpart in the macroworld - unwraps much of the weirdness and mysteries surrounding microphysics.

As an aside, let me mention in passing that my approach bears some loose resemblance to that of Hameroff and Penrose (1996), which singles the *microtubules* out as something remotely akin to the paralgens. Apparently the microtubule idea was first formulated by Ezio Insinna (Insinna 1992).

Does (crypto-)panpsychism, as sketched out so far, bring any novel inkling as regards the underpinnings of the higher cognitive functions (which are our higher-level mental faculties by another name)? This is the question I now seek to answer. For the sake of specificity, I narrow my focus to our sensory abilities (e.g. vision).

The Cognitive Iceberg

(This Section dwells on some aspects of conscious perception, of which I shall give the skimpiest of outlines. Its meat is the cognitive iceberg *model of sensory awareness.)*

Sight is perhaps the most important of our senses. Much depends on it, as far as our ability to cope with the world 'out there' is concerned. Indeed, we know a great deal about how the 'wetware' of our brain deals with sight. We can accurately say where the visual information goes, and (up to a point) how it is processed (Crick 1994, Shepherd 1994, Zeki 1983).

Very roughly, the brain visual processing goes from the retina on to the optic nerve, through to the lateral geniculate nuclei and the visual cortex. This cortex includes a number (say, 20-odd) of functionally specialized cortical areas; each of which undergoes separate but parallel "submodalities" processing (e.g. area V4 for color, area V5 for motion). The abstract visual "image" is therefore distributed over many different zones in the brain.

This comes as a surprise; all the more so that there seems to be no conclusive mechanism - apart from the 40-Hertz oscillations highlighted by Crick and Koch (1990), of which more later: *but, if anything, are they a cause or a consequence?* - for combining all the operations of this separate and parallel processing into the coherent representation of what is felt as a mental image. This lack of acknowledged mechanism is known as the *binding problem.* As we read in Crick (Crick 1994, p. 159):

> Although there are many different visual regions, each of which analyzes visual input in different and complex ways, so far we can locate no single region in which the neural activity corresponds exactly to the vivid picture of the world we see in front of our eyes. ... In short, *we can see how the brain takes the picture apart, but we do not yet understand how it puts it together.*

It turns out that this problem does not stand in isolation, but has many siblings. Like, to name a few, the following ones: the bewildering existence of qualia (e.g. of the felt redness of red); the poorly understood origin and nature of declarative memory (i.e. of our conscious, reportable recalls); the upshot problem (it refers to the tantalizing fact that visual awareness only knows of the outcomes, or upshots, of neural computations); the parallel/serial problem (see hereafter).

What I dub the "parallel/serial problem" revolves around the intriguing fact that, whereas preconscious processing is massively parallel, the conscious processing appears to be sequential: "Seeing is both a constructive and a complicated process. Psychological tests suggest that it is highly parallel but with a serial, "attentional" mechanism on top of the parallel one." (Crick 1994, p. 203)

Can we, from the vantage point of psychomatter, hope to piece together some of these puzzles? I think we can, with a helping nudge from the *cognitive iceberg.* (Again, this model is no more than a schematic sketch, highly simplified for purposes of illustration.) Let me, before I present it, recall two ideas that come forth as paramount. They are:

- The threads of supralness weave unseen 'psi' webs within psychomatter. These webs (made of elemental patterns, the *suprels*) bring structure at the 'psi' level. Owing to this, the ψ side of psychomatter is information-laden (in a nonlocal, distributed or 'holographic' way).

- The brain is conscious because it yields "psi-stirring" paralling conditions on wide enough a scale. These conditions are brought about by a vast array of *paralgens.*

The guess, in short, is that awareness stems from *supralled paral.* On this ground, the conscious brain needs swaths and throngs of these specific paral-triggering micro-biological structures that are the *paralgens*. Herein would lie its utmost secret. By the same token, proto-experiential properties (Chalmers, 1997) are prompted by isolated (i.e. "unsupralled") paral phases; whereas full-blown experience, or consciousness proper, requires a huge amount of simultaneous parallings, seamlessly linked together by their overall supral binding.

The origin and tremendous diversity of qualia is explained by this presumed mapping between the *objective* structural properties of the suprels and their *subjective* information contents. Supralness would thus address Seager's combinatorial problem (Seager, 1995).

Now, these notions of *suprel* and *paralgen* are the cornerstones of the *cognitive iceberg* model of sensory awareness (Ransford 1995). (Alternative models - of which I'll say nothing here - will cater for other cognitive skills, such as spoken language use, 'pure' thinking and willed motor control.)

The cognitive iceberg, unsurprisingly enough, is made of two parts, namely: the "underwater" (or rather, as we shall see: the "underaware") part, and the "tip". (Again, what follows is a very sketchy account that cannot do justice to the skein of issues at stake.) Each corresponds to a particular stage of neural 'calculations', typified by their respective input and output:

A- In the "underaware" part: *input:* sensory stimuli ; *output:* suprels

B- In the "tip": *input:* suprels ; *output:* qualia, conscious recalls

In the underaware part, afferent sensory stimuli are encoded in the shape of *suprels*. (For instance, visual *suprel*s arise in the relevant cortical areas, like V4 for the 'color' sub-modality.) Here, for want of paralgens, they remain unconscious: this former stage belongs to the preconscious brain processing. Once made, the suprels are sent to the "tip", through specific neural pathways embedded in the brain's hard-wired architecture that project to it.

Here the inflows of successive suprels gush through the *paralgens*, which readily turn them into qualia. This latter stage is therefore that of the felt, or conscious, brain processing. The tip, in short, is teeming with paralgens. They collectively *enparal* the flows of suprels pouring in from the underaware part, whereupon these suprels enter the mind. (They typically encode present sensory stimuli or past memories.)

In summary, the proposal is that our vivid, "technicolor" mental images are arrived at by means of the following two-stage process:

- The *unconscious* or *pre-conscious* processing, to begin with. It takes place in the "underaware" part of the cortex. It is scattered over a number of areas, each simultaneously dealing with a submodal aspect (e.g. color, shape, depth, motion) of visual data. The whole 'suprel-yielding' process is therefore parallel. It falls short of reaching consciousness since the suprels are left unparalled throughout.

- The *conscious* processing, then. It is carried out in the "tip" of the cognitive iceberg, where paralgens turns the incoming flows of suprels into globally coherent streams of successive qualia. This end-stage processing is thus serially conscious.

This, by the way, settles the aforementioned parallel/serial problem. Now, what happens next? The next stage is the reverse of what occurs in the tip, where the 'psi' was kindled or awakened. No sooner do the suprels spurt out of the paralgens, their 'psi' becomes latent anew. They accordingly turn nonconscious all over again, and the matching qualia all but vanish. This is straightforward enough. However, inasmuch as these suprels have not been shattered along the way, they end up stashed-in-waiting somewhere in the brain (in a nonlocal, distributed way - we deal with supral entities). Their information load lives on, unheeded.

In short, the qualia are (unconsciously) *memorized.* Yet they are recalled whenever, for some reason, they are sent back on to paralgens, to be enparalled afresh. (The weight of empirical evidence points to an involvement of the hippocampus in the neurophysiological underpinnings of the recall.) We thus reap a potential explanation of the declarative, a.k.a.

explicit, memory (which is that of our conscious recalls). In a nutshell: Our mental memory is supral in essence. (This insight is a far cry from the commonly accepted Hebb or Hebb-Hopfield model of memory - which arguably concerns learning rather than declarative memory. I'll come back to this important question later on.)

This brief outline of how sensory awareness could be achieved in the brain is, at best, a mere scratching of the surface. (What goes on in the living brain is breathtakingly more intricate and complicated!) I nevertheless believe that, for all its yawning shortcomings, this analysis throws some fresh and fruitful light on the conundrums already hinted at, to wit:

- The binding problem. It is likely, at least in part, to be a straightforward outcome of supralness. (Recall: supralness is about binding together the 'psi', or 'psi' parts, of an arbitrary number of microparticles, or specks of psychomatter - regardless of the distance.)

- The origin and nature of qualia. They are accounted for by the two-step process of: (a) suprel-breeding (in the underaware part); and: (b) suprel-enparalling (in the tip). This latter stage links up (suprally) with the whole mind.

- Declarative memory. As suggested earlier, this memory appears to be ascribable to the sea of (usually unparalled) extant or abiding suprels, that once released their psychic information load into the wider supral network that goes with the conscious mind.

- The upshot problem. It boils down to the fact that the bulk of the brain processing takes place in the preconscious underaware part. Only its final or "upshot" stage is carried out in the tip, where suprels give rise to countless whits of fickle awareness of their own.

- The parallel/serial problem. As we saw, it raises no difficulty. Suprels are 'hatched' by means of *parallel* preconscious processes, distributed across several areas within the underaware part. In the tip however, they are successively enparalled as they flood in: aware, focal-attentive modes of functioning are elicited by this ongoing *serial* processing.

Furthermore, the 'causal paradox' surrounding the interactions between consciousness and brain seems all but wiped out. This 'paradox' sets in when considering the first-person versus the third-person accounts (Velmans, 1997). Briefly stated: subjective mental causation is necessary, in the former case, to explain behaviour (e.g. I go to the grocer's shop *because* I am hungry); whereas it can be blithely ignored in the latter case (e.g. Bob goes to the grocer's *because* of all the purely biochemical processes that crop up in his central nervous system).

The panpsychic solution to this paradox is obvious: the (third-person) biochemical processes enshroud within themselves the (first-person) psychic causation that goes with the unacknowledged production of supralled paral....

I conclude this Section on two points. The first one has to do with the paral threshold. We recall that there should be a threshold below which awareness is just too dim to be worth the name. So how does the brain manage to beget enough supralled paral? There is a strong empirical suggestion that it does so putting swarms of paralgens to work together through synchronous neuron firing. This phenomenon, whose detailed underpinnings are still far from clear, has been widely reported in the literature. For example (Flohr 1992):

> If focused arousal in a specific sensory or multisensory circuitry is induced in human or animal experiments it is accompanied by characterisitc event-related EEG changes ... Typically, the arousing signal gives rise to a so-called 40 Hz EEG ... [that is a shorthand for a] high frequency gamma range (35-85 Hz). These large-amplitude, highly synchronous bursts have been observed in different focused arousal paradigms, different species, and different sensory modalities, such as the olfactory, visual, and auditory systems.

The second issue I'd like to (quickly) bring up is that of declarative memory (I shall yet come back to it later). Are we beginning to uncover the roots of memory, as it is oft trumpeted? Really, the truth of the matter is that this phenomenon is still poorly (if at all) understood. However, we know (and learnt from the pioneering works of Karl Lashley, back in the 1940's) that memories are distributed throughout the brain. They somehow reside everywhere and nowhere in the brain (Rose 1994).

This really is just what one must expect if it is supral in essence! (A similar remark applies to the acknowledged associativity of our recollections.)

What and Where in the Brain?

My goal here is to figure out whether it is possible to say something definite and tangible about paralgens, with a view to come nearer to an experimental assessment of the panpsychic approach. The question is twofold. It breaks down into: Where in the brain are the paralgens? and: What is their nature, what are they like? The hunt for putative paralgens is doubtlessly a tall order. Nevertheless, neuroscience gives off many a pithy clue; and I have much to feed on and get started with. Besides, I know from the outset that:

- paralgens ought to be sought where consciousness arises in the brain;
- paralgens ought to be sought where suprels are enparalled in the brain.

Actually, I only need to pinpoint *some* paralgens, not *all* of them. The first order of my business is thus to narrow the search down to a few auspicious loci in the brain. If, within the panspychic framework, it clearly makes sense to look for the detailed neural correlates of consciousness, a few caveats are however in order, since (Greenfield 1995, p.152):

> ... consciousness must be generated somewhere in the brain, but at the same time there seems to be no obvious single area, no Cartesian theater. Undoubtedly we cannot treat all areas of the brain the same.

Accordingly we can press ahead and garner promising leads (Ransford 1995):

> As William James once observed, attention depends on consciousness. By the same token, consciousness involves very short term memory (this type of memory, which surrounds or 'frames' our ever-fleeting perceptual moments, is also called the *working memory*). And then ... what we are conscious of are the results or upshots of neural computations held in the cortex. So [we] have three pointers *(attention, working memory and the processing upshots)* to get on and elaborate from!

Thus,

> ... the weight of evidence consistently points to the cerebral cortex as the neural substrate of consciousness. ... Most interestingly, the frontal lobe is ... "involved in circuits which construct an internal representation of the visual information about the place of an object, and then read out that information to control a motor response at an appropriate later time" (Shepherd 1994).
>
> This 'reading-out-and-delayed-motor-control' bears the hallmarks of *conscious* decision-making. This unambiguously brands this region as a prime location for the higher mental functions. In fact, modern research indicates that the operations of working memory are carried out in the prefrontal part of the cortical frontal lobe. ... we can be fairly confident that the prefrontal (and parietal) associative areas loom large in the brain production of awareness: these are places where paralgens should eagerly be sought after.

(The so-called 'associative areas' in the cortex are not directly related to a specific sensory or motor function. They are polysensory and multimodal, and deal with the outcomes of the sensory information processing. On the whole, the association cortex is where the brain's most abstract and integrated analysis of the sensory environment takes place (Chuchland 1993). This keenly smacks of the tip of the cognitive iceberg!)

Finally,

... most brain neurons fit into two classes: principal neurons and interneurons. It turns out that the cortical information is first processed by the interneurons, which project to the principal neurons - which then "decide" what kind of message they will send out to other regions.

In short, it means that the processing upshots (and the possible responses thereof, by way of neural - *and mental?* - "decisions" or initiatives) are the business of principal neurons, not of interneurons. This is yet another telling and clearcut lead in my search for paralgens.

The principal neurons of the neocortex are the pyramidal cells, which are excitatory (I expect most paralgens to be on fast excitatory rather than of inhibitory pathways). It appears (Shepherd 1994) that they are involved in the highest levels of processing sensory and motor systems, in memory mechanisms and in higher cognitive (i.e. mental) functions - such as intentional and willed attention.

Moreover, and given that the cortex reveals six distinct layers (Crick 1994, p. 251):

Consciousness is associated with certain neural activities. A plausible model could start with the idea that this activity is largely in the lower cortical layer (layers 5 and 6). This activity expresses the local (transient) results of "computations" taking place mainly in other cortical layers.

Not all cortical neurons in the lower cortical layers can express consciousness. The most likely types are some of the large "bursty" pyramidal cells in layer 5, such as those that project right out of the cortical system.

Venturing still farther afield, down to subcellular level (Ransford 1995):

It is fitting ... that the organization of the pyramidal cells has been found to favor a great deal of computational complexity in their dendrites (*esp.* the distal ones), and that the dendritic spines are semi-independent metabolic subunits. (Shepherd 1994)

Chances are that many paralgens are tucked inside these distal dendrites; whose 'computational complexity' would then account for much of our higher mental capabilities. The very fact that dendritic spines play a prominent role in the after-birth brain development (and are affected in certain kinds of diseases that produce mental retardation) further substantiates this.

To cut a long story short, here is where I propose that we eventually land, at the molecular level:

[Post-synaptic] receptors (and what goes with them: effectors and channels) make a very compelling target for speculation about paralgenic microsites [i.e. paralgens]. ... Of particular interest is the so-called NMDA receptor found on the dendritic synapses of pyramidal cells. It is excitatory, and has several critical properties suggesting that it may be involved in a wide range of neurophysical and pathological processes ... In addition, the NMDA channel is highly nonlinear, is a prime candidate to explain the synchronous oscillatory behavior in the cortex ... The conclusion I draw is that at least **some NMDA channels in the dendritic spine synapses of the large bursty pyramidal cells of the cortical fifth layer do function as paralgens.**

The above chimes in with other converging clues that make me keep placing my bets, today, on the very same postsynaptic NMDA receptors found on the dendritic synapses of large glutamatergic pyramidal cells in the fifth layer of the neocortex. (NMDA stands for N-methyl-D-aspartate. It is customary, by the way, to name a receptor/channel compound after the substance that activates it.)

What are they, what makes these receptors likely candidates (according to me) to play host to some paralgens? Most of the many compelling reasons for this choice cannot be properly grasped unless we get at least a rough idea of what a paralgen should be like. To that end, as I again wrote - admittedly, a tad jejunely - in (Ransford 1995):

We can think of the paralgen as a sort of biological device - e.g. *an allosteric protein molecule?* - that would be akin to a channel endowed with a snare; into which, say, ions and molecules are sent by the relevant assemblies of neurons ... whence they undergo a paral phase before being released and 'unparalled' again...

The rationale here is that paralgens, in order to enparal the flows of incoming suprels, must somehow stand in the way of these flows. As for the suprels they can, for all practical purposes, be thought of as (invisible) threads that bind together clusters of ions, molecules and the like, as they roam about in the brain. So, a fitting and hence likely locus for paralgens is near, across or inside certain synaptic pores and channels. They make perfect spots to latch on to - and enparal - the successive ions and perhaps also molecules, as they flood through.

Talking about ions, of particular interest are the calcium ones (Ca^{++}). Calcium is one of the key substances driving nerve signalling. (The NMDA receptor-linked channel, it turns out, is a calcium channel; generally speaking, it is permeable to Na^+, K^+ and Ca^{2+} ions.) It is an ubiquitous intracellular second messenger, is released as a wave (Cooper *et al.* 1991) and plays a role in the 40 Hz oscillations. Moreover (Levitan & Kaczmarek 1997, p.124):

> Calcium channels are of particular interest because calcium is far more than simply a charge carrier across the plasma membrane. ... intracellular calcium ion regulates the gating of several types of ion channel, and can even feed back and participate in the inactivation of its own channels. In addition, an essential characteristic of neuronal signaling, the release of chemical neurotransmitters at synapses, is controlled directly by intracellular calcium. In this sense calcium can be thought of as the transducer of an electrical signal, depolarization, into chemical signals inside the cell. All of these features set calcium apart.

On the face of it, I deeply suspect that at least a fair share of these ions, in the relevant pathways and brain areas, are part and parcel of the suspected information-laden suprels. (I shall label them, for short, the "sub-suprel" ions.) If so, these bits of suprels, on simultaneously whisking through the relevant calcium channels, would be collectively *enparalled* - thereby becoming aware, or consciously felt. We already know this story.

Another point is that a "paralgenic" channel ought to exhibit a high degree of selectivity. It should emphatically let in the "sub-suprel" ions only, as they reach either from the underaware part or from the memory hoard. (Were it not so, the psychic information, sensory and otherwise, would be unredeemably blurred, swamped in a wasteland of meaningless noises.)

Indeed, the NMDA receptor channel, as both ligand-gated and voltage-dependent, seems expressly designed to display such a sharp selectivity (Levitan & Kaczmarek 1997, p. 258):

> ... an interesting property of the NMDA receptor channels [is] that they are blocked by extracellular magnesium ions [Mg^{++}] in a voltage-dependent manner. ... When a neuron is near its resting potential, magnesium ions bind to the outside of the [NMDA receptor channel] and effectively prevent the movement of other ions through the pore. ... when the cell is depolarized in the presence of glutamate, calcium (as well as sodium) flows into the cell through the NMDA receptor channels. ... A moderate stimulus produces only a small depolarization, and no calcium entry ... . With more intense stimulation the depolarization becomes sufficient to relieve the magnesium block of the NMDA receptor channels, resulting in further depolarization and calcium entry.

This "more intense stimulation" might be a cue that the sub-suprel ions are pouring in from the underaware part, and that the paralgenic channel must swing open to let them flood through....

Various data strengthen my 'NMDA' claim, but I shall limit myself to point to a handful of particularly engaging clues (adapted form Ransford 1995):

- Paralgens should be linked to fast excitatory synaptic transmission, for at least a fair proportion of them. (Glutamate is the primary fast transmitter in the brain - and the NMDA channel is glutamatergic.)

- Paralgens should be sought for somewhere along the neurobiological pathways of mood-altering substances in the cortex. (Alcohol and psychoactive drugs are known to act on receptor channels; and also on the 'second messenger system'.)

- Paralgens should be linked to some apparently random and acausal (and/or highly nonlinear) brain events, as a token of 'free will' or decision-making. (This points specifically to the NMDA channel.)

As concerns mood-altering substances, glutamate is - by far - not the only neurotransmitter involved. Dopamine, norepinephrine and serotonin, to name but a very few, play an outstanding role (Cooper *et al.* 1991, Ryall 1989). (Is could be a clue that some paralgens are to be found along certain of the matching, e.g. dopaminergic, pathways.)

Yet it is worth noting that "data are being accumulated that suggest a role for NMDA receptors in anxiety, depression, schizophrenia, psychomotor stimulation, psychomimetic behavioral and subjective effects" (Witkin 1995). In a similar vein, the NMDA system has a well-researched role in mental retardation and in degenerative conditions such as Alzheimer's disease, where scientists have observed a "decreased population of the receptor for glutamic acid of the ... NMDA type" (Ryall 1989).

Finally, anæsthesiology appears to bring an interesting insight of its own (Flohr 1992, 1995, 1996). For instance, we read in (Flohr 1996) that "General anæsthetics have a common operative mechanism: they directly or indirectly affect the function of the NMDA system."

As regards the non-linearity issue, the reasoning goes roughly as follows. Paralgens yield an ongoing production of supralled paral in some (distributed) areas of the brain. Paral being, as I surmise, endo-causal, it is bent on wielding a kind of 'decision-making' power. The hallmark of 'decisions' being made is a high non-linearity. It goes with a 'willful' and 'deedful' mode of functioning, which rules out sheer randomness. This criterion seems to pinpoint just these cells where the NMDA channels are (Crick 1994, p. 235):

> ... some of the pyramidal cells in layer 5 [of the cortex] ... can fire in a special way. A number of neuroscientists have found that such neurons tend to be "bursty". (These neurons do not produce axonal spikes in a completely regular way, nor at random time intervals; instead, they tend to produce short bursts of several spikes at a time, with longer intervals having only a few or no spikes between the bursts.)

The notions of paralgen and suprel are putative but factual. They hold out the promise that, one day, we might put the M-P-S model of psychomatter to the test. But can we realistically move forward on the broader issue of falsification?

Putting it to the Test?

The conscious brain is so intricate and elusive that no one should expect any theoretical explanation to lend itself to hard and fast experimental checks. Moreover, it should be borne in mind that our scientific knowledge encompasses what was comparatively easier to arrive at. What is still left out is the hardest part ever! There is a kind of incremental law at work. To give an example of the growing difficulties that are met on our way to scientific discoveries, let me recall in a few words the saga of supralness (a.k.a. of quantum nonseparability, or entanglement).

It got under way in 1935, due to a seminal paper co-signed by Einstein, Podolsky and Rosen. Then, after a slight reformulation by David Bohm, a breakthrough takes place in 1964, with Bell's theorem. (With it, supralness lends itself to experimental appraisal. It can no longer dismissed as "just philosophy".) The last episode takes us to 1986 with the (nearly) conclusive experiments carried out by Alain Aspect and coworkers.

The whole saga spans a full 51 years! This illustrates the emblematic difficulties that theoreticians and experimentalists alike encountered. (In 1997, a new experiment involving microparticles flying a few miles apart has been made on supralness, by N. Gisin in Geneva.)

Now, when it comes to the conscious brain, the difficulty is all too obvious and all too wrenching. The trouble is, what we are after (viz. supralled paral) is a most wayward aspect of reality. Suprels and supralness, for instance, are outright invisible! The upshot is that no direct evidence will ever be within reach. One has only indirect probing to fall back on - whatever that means.

Talking about experimental tests, a few ideas readily spring to mind. They target either suprels and paralgens, or paral and supralness as such. Here are several possible ideas.

First and foremeost, the reality of paral could be fathomed through one or another of its outlandish properties, such as its 'relativity-blindness'. This feature, which bears on both special and general relativities, entails that paral does not contribute to the gravity field. The presumed "gravitylessness" of paral should be amenable to experimental assessment.

Another idea is to check if the NMDA receptor channels yield, as I claim, some of the brain's supralled paral. One can imagine this being done by means of watching the selective impact, on global awareness, of some high affinity molecule (or ligand) specifically geared to them. (Such an impact would begin to bear witness to the 'paralgenic' nature of these channels.) This is much in the spirit of monitoring the detailed effects of psychotropic drugs and anæsthetics.

Another tack would be to undertake an in-depth survey of the role of the NMDA system in the 40 Hz oscillations observed in attentional tasks (linked to short-term memory), for which there is as yet no known neural mechanism (Black 1994, Crick 1994, Levitan & Kaczmarek 1997, Shepherd 1994).

Can we tease out the mysteries behind these oscillations? My hunch is that this 40 Hz correlated firing *is both the consequence and the cause* of the overall supral binding that relates the untold paral sparks that are spawned by the brain. It is rooted in supralness, and at the same time bolsters it. Supralness is what keeps the neurons firing in synchrony - across several cortical zones and even between the two halves of the cortex (Crick 1994). If so, one should *not* find any straight neural mechanism that would explain it all.

Going back to the suprels, a number of promising leads can be weighed. One could take advantage of their absolute disregard for distance and geometry to check it and play tricks with it. (Suprels are topological, not geometrical, patterns.) Incidentally, the mapping between qualia and suprels is worth investigating. I propose to name *suprology* the study of this alleged correspondence between suprels and qualia.

We could tamper with the topological structure of suprels, as they arise from the brain, to see whether we can induce a kind of artificially engineered synesthesia. (Synesthesia is a rare condition where different senses mingle. Synesthetes will typically experience 'colored hearing', where a perceived sound is also a perceived color, and vice-versa.) We can also think of transfering suprels, of grafting them onto someone else's brain. (A cross-species grafting would be most interesting and revealing.)

There is no shortage of suchlike ideas, but they don't by themselves carry us very far: designing and implementing actual experiments is an entirely different story....

To conclude this Section, I want, for the last time round, to delve somewhat into the question of our declarative memory; which is that of our conscious recalls. Understanding the essence of memory will speak volumes about the true nature of the mind. (There are many sides to it, that I cannot mention. For the record, its main stages are: coding, storage, and retrieval. There is a short-term memory and a long-term one; the passage from the former to the latter entails a procedure known as of consolidation.)

A great deal of empirical data are available on memory. For example the hippocampus - a brain structure that belongs to the limbic system - appears to play a well-documented role: "Damage to the hippocampus, it emerges, blocks the transfer of information from short-term into long-term memory." (Chuchland 1993)

As of today, the prevailing theory on memory is that of Hebb (Hebb 1949). This author assumed that synapses on a neuron that are active whilst the neuron discharge will be strengthened, whereas inactive synapses will be weakened. Synapses that are active at the same time on the same neuron will tend to be reinforced and selected over others. In short, they will keep a trace of their past track record of joint pre- and post-synaptic firings.

The Hebbian phenomenon of long-lasting increase in synaptic strength has been observed in the excitatory afferents to hippocampal neurons (Black 1994), and elsewhere in the cortex - but by no means everywhere (including in places where we'd like to find it).

So, should we deem the memory issue thoroughly settled in view of Hebb's thesis, once for all? This is unfortunately far from sure, and the evidence at hand is to say the least not exceedingly supportive (Levitan & Kaczmarek 1997, pp.499 & 505):

> Donald Hebb suggested that synaptic strength might be enhanced by concurrent activity in the pre- and postsynaptic neurons. He postulated further that this might provide a mechanism for associative learning. .... There is widespread agreement that the onset of LTP ... is *triggered* by events occurring in the postsynaptic neuron. What about strength and expression? Is there a long-lasting change in the release of the neurotransmitter glutamate from the presynaptic cell, or a change in the responsiveness of the postsynaptic target, or perhaps both? In spite of extensive work, this issue remains unresolved and contentious ....

A first hint that Hebb's rule does not add up to fact is that the hippocampal LTP is no memory trace indeed (Levitan & Kaczmarek 1997, p. 481):

> It is believed that the long-term memory traces themselves are not stored [in the hippocampus], but rather that the hippocampus participates in memory acquisition, and in establishing an enduring and retrievable memory elsewhere.

Besides, Hebb's learning rule, if anything, deals with... learning. It is not geared, to put it bluntly, to be relevant for memory proper. The long term strengthening or weakening of the synaptic activity that would result from past track record is assuredly not a one-off affair - as can be our recollections. (At times we remember a single event decades afterwards!)

What is more, the Hebbian theory goes with the hidden assumption that mental states are identical with purely neurological brain states. It ties-in with a computational approach to mental realities. And this materialist approach, further enriched by Hopfield and others, requires a means to back-propagate the errors (in order for the learning to be effective). This requisite is fine with artificial networks, but it doesn't sit well with living brains (Chuchland 1993, p.164-5):

> With artificial networks, we can build in appropriate systems for calculating output errors and for modifying the weights accordingly. ... But what pathways, in a *real* brain, is the output propagated back to the relevant set of synaptic connections, so their weights can be modified and learning can take place? ... we do not yet understand how [target cells] might be doing this. Nor are we really sure that they do anything remotely like this. ... neuroscientific data may show that an appealing theory of learning ... (back propagation of errors) cannot possibly be right.

So, where are we now? (Some hope for brain back-propagation have lately been pinned on NO molecules, but no proof thereof is forthcoming.) It looks like we are very nearly back to where we started. The arresting logic of Hebb's rule doesn't seem to hold water in the real world of real nervous systems. Hippocampal LTP, for one thing, "is not clearly associated with any known behavioral modification" (Levitan & Kaczmarek 1997). And, generally speaking (Greenfield 1995, p.47):

> ... LTP can occur rather promiscuously in a variety of totally different situations. For example, LTP can take place before a learning task, thereby facilitating it, rather than being the learning event itself. ... The physiologist Rodolfo Llinás has further demonstrated that the physical

> events of LTP do not match up exactly with the phenomenological process of memory, since they can be generated under conditions totally different from those associated with memory.

On the whole, we are forced to the conclusion that "LTP is not the essence of memory but a possible requirement for it" (Greenfield 1995). Isn't this more or less a flat refutation of the Hebbian theory - and of the connectionist approach that goes with it?

Reverting to my proposed panpsychic theory of memory, let me end up this Section by airing my lingering suspicion - which could perhaps be mathematically validated - that the nonphysical data storage capacity inherent in supralness could be hugely superior to the matter-rooted one that goes with the Hebb-Hopfield scheme. (That too could set the stage for some experimental tests.) As I mused in (Ransford 1995),

> ... there is every reason to suspect that a vast (and virtually unlimited) number of *suprels* can be stored in the brain. *Supralness would offer much more room and flexibility than plain neural storage.*
>
> Indeed, the staggering and immeasurable capacity of our visual memory bears this out. Moreover, as Crick [Crick 1994] points out, "There are not enough neurons in the brain to code the almost infinite number of *conceivable* objects. The same is true of language. Each language has a large but limited number of words, but the number of possible well-formed senstences is almost infinite." Still, our brain manages to cope with objects and sentences almost flawlessly!
>
> Supralness comes in handy, to free our mental selves from these materials limits. Thanks to supralled paral, our minds ... can go way beyond the restrictive shackles of sheer biochemistry.

The ultimate, nearly clinching corroboration of panpsychism (if it not hopelessly wide of the mark) would of course come from evolving the technical know-how of artificial consciousness along its lines. The proof, as the saying goes, is in the pudding....

Towards Exo-Biological Awareness

What is at stake, with panpsychism, is no less than the eventual feasilibity of *exo-biological awareness.* Its prospect lurks at the very heart of the psychomatter idea, since within its framework, the seeds of awareness are sown in all things (*psycho*)material. Therefore awareness might glow from virtually all things material; not just from the biological brains. Of course one should expect, before a man-made artificial mind can spring to existence at all, many hurdles and pitfalls to arise, and need to be patched up or smoothed out.

Such a technological feat would - assuming it is a real possibility - rest heavily on the twin abilities to carry out artificial paralgens and to cope with the pivotal issue of supral binding and patterning. Learning to handle and harness supralness is a critical aspect. Remember: supralness is the hidden gateway to the inner contents of subjective experience!

A sound starting point is to fathom what nature has so successfully achieved, to study it at close quarters with a view to mimick it. We can avail ourselves of the wealth of empirical data that can be plucked from the animal kingdom's central nervous systems. We should manage to wring out of them deep and fruitful insights about both natural paralgens and the suprel-qualia relationship.

As seen from the window of psychomatter, exo-biological awareness might be arrived at by plodding through the following broad-brush 7-step program:

1- In-depth study of biological paralgens (as templates for would-be artificial ones)
2- Artificial paralgens design and making
3- Sweeping survey of suprel breeding and patterning (suprology)
4- Modality-specific (e.g. sight, hearing, etc.) sensory units design and manufacture
5- Motor modules design and manufacture
6- Memory module making (allowing for consolidation, associative retrieval, etc.)
7- Full-fledged artificial brain, gotten by putting all these modules together

This "modular brain" idea fits in with current findings (Black 1994, pp. 101 & 134):

> Extensive work in neurology, psychology, and neuroscience suggests that brain structure and function are organized into discrete modules. ... Observations based on many different clinical disorders and experimental paradigms support the contention that behavior and underlying brain function are organized in a modular fashion.

As regards the (long-term) memory module, let me quickly point out that physics teaches that supralness is lessened by the onset of paral phases (to the point of being all but wiped out if a series of parallings corresponding to a complete set of communting observables is undertaken...). Against this untoward backdrop looms the necessity of memory-shielding processes - known as consolidation. Without it, newly hatched suprels would be very short-lived, killed off in less than a jiffy by the detrimental paral activity that goes with awareness.

As for the "motor module", we may find guidance and draw inspiration from the brain's "cortico-basal ganglia-thalamo-cortical loop". It is a massive reverberatory circuit, modulating all motor inputs. A growing body of evidence suggests that portions of this circuit may provide the essential substrate for conscious volition (Newman 1997).

Leaving the wry question of technological hubris aside, the intellectual and human challenge of exo-biological awareness is enormous. Exo-biological awareness is no plaything. It has the potential to spawn an exciting new world of boundless opportunities, a brave new world that would change our world view and lives beyond recognition. This outlook is truly staggering and enthralling!

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Alternative Natural Philosophy Association

Statement of Purpose

1. The primary purpose of the Association is to consider coherent models based on a minimal number of assumptions, so as to bring together major areas of thought and experience within a natural philosophy alternative to the prevailing scientific attitude. The Combinatorial Hierarchy, as such a model, will form an initial focus of our discussions.

2. This purpose will be pursued by research, publications and any other appropriate means including the foundation of subsidiary organisations and the support of individuals and groups with the same objective.

3. The Association will remain open to new ideas and modes of action, however suggested, which might serve the primary purpose.

4. The Association will seek ways to use its knowledge and facilities for the benefit of humanity and will try to prevent such knowledge and facilities being used to the detriment of humanity.

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1. The Executive Council is the governing body of the Association. It consists of:

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2. Members of the Association are (a) members of the Executive Council and (b) others nominated by the members and approved by the Executive Council.

3. Vice-presidential candidates are nominated by the membership and the Executive Council during the first year of the President's term of office. Any nomination must be accompanied by a statement from the nominee that he will serve a full term if elected. If there is more than one nominee, selection will be made by mail ballot to the Membership decided by plurality of votes. The Vice-President is elected to serve concurrently with the President during his last year of office. He will then serve as President for at most five years and cannot run for re-election until three years after his initial term has elapsed. If the President decides to stand down before his five year term has elapsed, he should give the Executive Council one year's notice of his intention, so that a Vice-President may be elected.

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