Conservation
and
Invariance

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CONTENTS

A Vector Semantics for Actions

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This paper is concerned with the - in contemporary computer science - radical notion of applying the concepts and methods of vector algebra to the problem of describing the semantics of concurrent computation. Its focus is to display how to capture in vector algebraic terms the act of observation and the inference of causal connections in a concurrent computational context. The central analogy is that of an observer equipped with nothing but binary sensors, a single causality inference rule called *the co-exclusion principle*, and a memory in which to accumulate its inferences. Our fundamental epistomological position is that an 'object' is something displaying invariance in the time domain, which leads to the ontological position that everything is made out of time.

This work should be viewed as well in the larger context of elucidating a mechanics of information, what we call the *neo*-mechanistic position. It is therefore particularly interesting that both the (computational) mechanism and the interpretation this induces on the vector algebra cast new and profound light on such fundamental physical matters as non-determinism, non-locality, the nature of light, the concept of half-integral spin, and the incremental appearance of spatial structure; there are also novel quaternion-like group structures at the elementary level. The structures exhibiting these interpretations exist (in the observer's memory, at least) as a self-similar hierarchy whose sensory boundary with the external world can be drawn arbitrarily, rather in the manner of Huygen's principle. The cardinal numbers of this hierarchy are essentially the same as those of the combinatorial hierarchy.

1 From Sensors to Actions

A sensor is viewed as a discrete vector-valued process. Hence, each such sensor process consists of the alternation of two [1] states, which states denote the *orientation* (in time) *in the time frame implicitly created by that process.*

[1] Which means that we will always be dealing with unit vectors.

Definition. A *sensor* is a discrete variable s which can take on ordered values $\sigma_1, \sigma_2, \ldots \sigma_k$. We restrict ourselves here, without any punishing loss of generality, to $k = 2$, and consider only sensors having finite values.

To maintain consistency with later analysis, we interpret $s = 0$ to mean a sensor value which cannot occur. The *orientation* of a sensor is positive when $\sigma_i > 0$ and negative when $\sigma_i < 0$. It is helpful to think of binary sensors as taking on the values ± 1. For our present purposes, we define a *sensor space* S^m as an m-dimensional vector space over the rational numbers with basis $(s_1, s_2, \ldots, s_m)$; hence $S^m = \{x_1 s_1, x_2 s_2, \ldots\}$.

We assume an inner product "$\cdot$" between these vectors, such that $s_i \cdot s_j = 1$ when $i = j$ and zero when $i \neq j$. The inner product expresses the projection of one sensor on another, and the fact that the inner product of two sensors is zero means that a given sensor in principle says nothing about another sensor.

Relative to its sensors, the observer can capture the (locally) simultaneously obtaining values of a set of sensors. Such a set of sensor values is called a *co-occurrence*. We assume that the observer's sensors are local to the observer, but one could also introduce Einstein simultaneity. In any event, we believe that the concept of a locally registered co-occurrence is well-defined.

[Jumping ahead temporarily, we have decided that the proper interpretation of "+" in the geometric (vector) calculus presented later is "co-occurrence", in that

- For both, the actuality of their components is indicated by their very presence, or rather, their absence can be ignored;
- Order is unimportant, or rather, the absence of order is necessary;
- Those sensor values which can co-occur correspond to those which can change or be changed concurrently;
- The additive identity "0" is interpreted to mean "does not occur", so reasoning involving equality, for example $X = Y$, which means $X + (-Y) = 0$, comes to mean that X and its inverse cannot co-occur.

Viewing therefore "+" as co-occurrence, a multi-vector consisting of sums and (geometric) products of sensors expresses all the necessary aspects of the simultaneous activity both present and possible.]

Less abstractly, a co-occurrence is denoted by a *tuple* $(\ldots)$ consisting of ordered pairs $(sensor, state)$. The *arity* of a tuple is the number of such pairs that it contains, and we assume that a given sensor-state pair occurs at most once in a given tuple. The *actions*

("XnX") which are inferred and which we seek to describe formally are referred to in terms of their arity, denoted by the n in "XnX". Hence, an X2X action is one which causes a particular change, from the 2-ary pre-condition indicated by one of its constituent co-occurrences to the 2-ary post-condition indicated by another such co-occurrence. We use an archetypical "blocks world" to exemplify the workings of our actions, but it should be obvious that our model applies very broadly.

We use the notation (sensor, state) to denote explicitly the association of some entity with its state. If there were two such, we would write the tuple ((sensor1, state1), (sensor2, state2)), but we could just as well write ((sensor2, state2), (sensor1, state1)). For example, s_1 = *(sensorA, valueA)*, where *valueA*$\in \{A, \tilde{A}\}$; and $(A, \tilde{B}) = s_1 + \tilde{s}_2$. Explicitly naming the sensors serves to keep us aware that even the *order* of items in a tuple is an assumption, since the usual 'short form' would be (state, state), wherein nothing but the order serves to differentiate the two entities.

What we call an *action* XnX characterizes - we believe - all causally valid changes, and is formed from two complementary co-occurrences via the *co-exclusion principle*, defined formally later. For n=2, these have two 2-ary co-occurrences as components, and the elements that can occur include both permutation of places and inversion; hence in general an XnX will have $2^n n!$ elements, and an X2X eight. These are

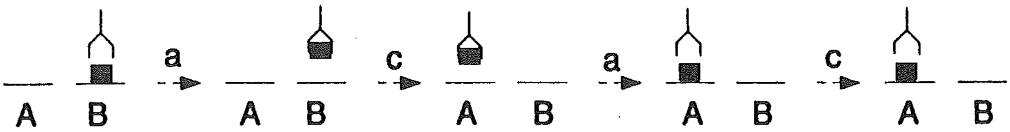
1	$\tilde{1}$	a	$\tilde{a}$	b	$\tilde{b}$	c	$\tilde{c}$
(A,B)	$(\tilde{A},\tilde{B})$	$(A,\tilde{B})$	$(\tilde{A},B)$	$(\tilde{B},A)$	$(B,\tilde{A})$	(B,A)	$(\tilde{B},\tilde{A})$

The table for their composition is

1	$\tilde{1}$	a	$\tilde{a}$	b	$\tilde{b}$	c	$\tilde{c}$
$\tilde{1}$	1	$\tilde{a}$	a	$\tilde{b}$	b	$\tilde{c}$	c
a	$\tilde{a}$	1	$\tilde{1}$	c	$\tilde{c}$	b	$\tilde{b}$
$\tilde{a}$	a	$\tilde{1}$	1	$\tilde{c}$	c	$\tilde{b}$	b
b	$\tilde{b}$	$\tilde{c}$	c	$\tilde{1}$	1	a	$\tilde{a}$
$\tilde{b}$	b	c	$\tilde{c}$	1	$\tilde{1}$	$\tilde{a}$	a
c	$\tilde{c}$	$\tilde{b}$	b	$\tilde{a}$	a	1	$\tilde{1}$
$\tilde{c}$	c	b	$\tilde{b}$	a	$\tilde{a}$	$\tilde{1}$	1

Suffice it to say that $\{1, \tilde{1}, a, \tilde{a}, b, \tilde{b}, c, \tilde{c}\}$ forms a group we will call $\mathcal{X}^2$, an *exclusion* group. Looking at this table, we can see that $aa = cc = 1$ but $bb = \tilde{1}$, and similarly for the inverses. In addition, $ab =\sim ba, bc =\sim cb$, and $ca =\sim ac$, and similarly for their inverses. An algebra of all possible sums and products of elements from a group with this anti-commutativity property forms a *Clifford* algebra. We will return to the algebraic analysis after an example (see Figure 1).

(a) View from the table: the hand moves.



(b) View from the hand: the table moves.

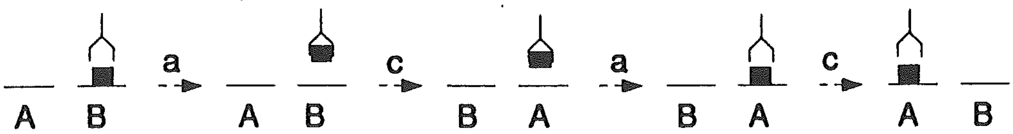


Figure 1: Block Movement from Two Points of View.

Let us take A to mean 'place A is full' and Ã to mean 'place A is empty', and similarly for B. Then the moving of a block from B to A, presuming A is empty, would be

$$(\tilde{A}, B) \xrightarrow{a} (\tilde{A}, \tilde{B}) \xrightarrow{c} (\tilde{B}, \tilde{A}) \xrightarrow{a} (\tilde{B}, A) \xrightarrow{c} (A, \tilde{B}) = (A, \tilde{B})acac.$$

If we rewrite the block movement in terms of Full and Empty, we have

$$(E, F) \xrightarrow{a} (E, E) \xrightarrow{c} (E, E) \xrightarrow{a} (E, F) \xrightarrow{c} (F, E).$$

The longer one looks at this, the stranger it seems, but what is going on is that, aside from the 'time' element, the two places - given precisely the indistinguishability of the sensory inputs - are permutations, i.e., rotations. But the time element, introduced by ac, is keeping track in a different dimension, namely the complex $(\sqrt{-1})$, which is also undergoing rotation.

Definition. Two co-occurrences *co-exclude* each other when there exists a symmetric mapping e between the co-occurrences, $C \xleftrightarrow{e} C'$, such that (1) there exist at least two sensors s_i, s_j which are members of both co-occurrences, and (2) $s_i \neq s'_i, s_j \neq s'_j$.

Hence in the figure, ((A,full),(B,empty)) mutually excludes with ((A,empty),(B,full)) because (A,full) mutually excludes (A,empty), and (B,full) mutually excludes with (B,empty). The same applies to the pair of co-occurrences ((A,full),(B,full)) and ((A,empty),(B,empty)), but we will focus for the time being on the first form.

Drawing now the crucial analogy with the two mutually exclusive states of a *single* sensor, the combined co-exclusive state of two sensors is itself a state, but at a higher level of abstraction. If we observe two co-exclusive states - say the first and the last in the figure - and wish to effect the transition between them, then accomplishing [in the first] (B,full)⟶(B,empty) and (A,empty)⟶(A,full) will necessarily lead to the last state, and similarly [in the last state], accomplishing (B,empty)⟶(B,full) and (A,full)⟶(A,empty) will lead to the first state. *Furthermore, because the two states co-exclude, it is sufficient to cause one of the changes - the other* must *then follow.* We say 'must' in the logical sense, since the co-exclusion is a valid inference from observation. However, *physically* changing just one of the two initial states will only lead *necessarily* on the other's changing if the environment does not interfere with interactions outside the purview of the given action. For example, pushing the gas pedal generally co-excludes with gas moving from the car's gas tank to the engine, but only if there is gas in the tank.

This then is our concept of causality, and notice that its inference can be carried out *no matter when or in what order* the co-excluding co-occurrences are observed. The pre-condition that such an action can be carried out is that the state of the environment matches one of the action's two defining co-occurrences; the other co-occurrence then constitutes the action's post-condition. *I claim that co-exclusion is the basis for every inference of causal relationships.*

An X2X action operates on a sub-space S^2 of the sensor space S^m, converting a given state (s_i, s_j) to its inverse, $(\tilde{s}_i, \tilde{s}_j)$. As we saw in the example this happens via third orthogonal dimension (the Hand) which is however *implicit* from the point of view of the action. This new dimension was created by the composition of (e.g.) ac, and will similarly be created by the other products - $ab, \tilde{a}b, a\tilde{b}$, etc. The entity denoting this third dimension is simultaneously both a vector expressing the *orientation* of the space spanned by its two multiplicands and an operator which (taken twice) can transform (s_1, s_2) into $(\tilde{s}_1, \tilde{s}_2)$.

The case of X3X's is more complicated, due to the odd number of elements. They too form an exclusion group, $\mathcal{X}^3$, and we can quickly calculate that $\mathcal{X}^3$ contains $2^3 3! = 48$ elements. Looking a little more closely, it can be demonstrated that $\mathcal{X}^3 = C_2 \times (S_3 \times Q)$, where C_2 is the inversion group $\{1, -1\}$, S_3 is the symmetric group on three elements, and Q is the so-called quadratic group $\{1, a, b, ab\}$. The C_2 factor captures the two fully complementary tuples (A, B, C) and $(\tilde{A}, \tilde{B}, \tilde{C})$, S_3 all the permutations of the three elements, and Q the partial inversions. Thus we might suppose that $C_2 \times (S_3 \times A)$ is the way to capture the structure of $\mathcal{X}^3$.

To this end, we can define the quadratic group Q as $1 = (123), a = (1\tilde{2}\tilde{3}), b = (\tilde{1}2\tilde{3})$, and $ab = c = (\tilde{1}\tilde{2}3)$. Notice that a, b, c resemble X3X's wherein one of the components remains constant. For the purposes of comparison, we show the product $C_2 \times Q$, which is:

1	$\tilde{1}$	a	$\tilde{a}$	b	$\tilde{b}$	c	$\tilde{c}$
$\tilde{1}$	1	$\tilde{a}$	a	$\tilde{b}$	b	$\tilde{c}$	c
a	$\tilde{a}$	1	$\tilde{1}$	c	$\tilde{c}$	b	$\tilde{b}$
$\tilde{a}$	a	$\tilde{1}$	1	$\tilde{c}$	c	$\tilde{b}$	b
b	$\tilde{b}$	c	$\tilde{c}$	1	$\tilde{1}$	a	$\tilde{a}$
$\tilde{b}$	b	$\tilde{c}$	c	$\tilde{1}$	1	$\tilde{a}$	a
c	$\tilde{c}$	b	$\tilde{b}$	a	$\tilde{a}$	1	$\tilde{1}$
$\tilde{c}$	c	$\tilde{b}$	b	$\tilde{a}$	a	$\tilde{1}$	1

This example, and in general the factorization $\mathcal{X}^3 = C_2 \times S_3 \times Q$, are all well and good, but notice that the crucial property of anti-commutativity is missing, i.e., it is nowhere the case in this table that $xy = -yx$. At the same time, this property should be in there *somewhere*, since we know that X3X's that hold one component constant have as their other component an X2X, which *is* an exclusion group.

It is important to notice that the concept of a 3-ary action where one of the components is held constant while the other two change plays a crucial conceptual role. To see this, consider a 2-ary action to `grasp` a block at place A while the Hand is in fact over place B. The action will think all is well and attempt the `grasp`, which will of course fail because there is no block where the Hand is. The same phenomenon occurs with `release` as well. Thus we see that 2-ary actions lack a crucial piece of information, namely that some third thing must be, and remain, in a *particular* state *while* the action is occurring. This constancy constitutes a condition which *connects*, as it were, the two places (A and Hand. As far as we can see, it is not possible to express this with 2-ary actions: the special 3-ary co-exclusion where one of the components is constant throughout is of fundamental character. We call such actions X2+1X actions.

Let us therefore write down the three X3X actions where, respectively, the A, B or C component is held constant:

	1	$\tilde{1}$	a	$\tilde{a}$	b	$\tilde{b}$	c	$\tilde{c}$
C_A^2 :	(123)	$(1\tilde{2}\tilde{3})$	$(1\tilde{3}2)$	$(1\tilde{2}3)$	$(13\tilde{2})$	$(12\tilde{3})$	(132)	$(1\tilde{3}\tilde{2})$
C_B^2 :	(123)	$(\tilde{1}2\tilde{3})$	$(32\tilde{1})$	$(\tilde{1}23)$	$(12\tilde{3})$	$(\tilde{3}21)$	(321)	$(\tilde{3}2\tilde{1})$
C_C^2 :	(123)	$(\tilde{1}\tilde{2}3)$	$(1\tilde{2}3)$	$(\tilde{1}23)$	$(\tilde{2}13)$	$(2\tilde{1}3)$	(213)	$(\tilde{2}\tilde{1}3)$

We see that, comparing with the table for $C^2 = \mathcal{X}^2$ earlier, each of the above individually is in fact an exclusion group. Taking C_C^2 as an example, we find that $ab = (\tilde{1}23)(\tilde{2}13) = (\tilde{2}\tilde{1}3) = c$ and $ba = (\tilde{2}13)(\tilde{1}23) = (213) = \tilde{c}$, and hence $ab = -ba$. Similarly, $ac = -ca$ and $bc = -cb$, and finally, $a^2 = c^2 = +1$ and $b^2 = \tilde{1}$. Hence C_C^2 is indeed an exclusion group, as are C_A^2 and C_B^2. However, each has its own concept of "-1". In this connection, we note that $\tilde{1}_A\tilde{1}_B = \tilde{1}_B\tilde{1}_A = \tilde{1}_C, \tilde{1}_B\tilde{1}_C = \tilde{1}_C\tilde{1}_B = \tilde{1}_A$, and $\tilde{1}_C\tilde{1}_A = \tilde{1}_A\tilde{1}_C = \tilde{1}_B$; and (e.g.) $a_A b_B a_C = (1\tilde{2}3)(12\tilde{3})(\tilde{1}23) = (\tilde{1}\tilde{2}\tilde{3}) = -1$.

We can furthermore observe that $C_A^2 \cap C_B^2 \cap C_C^2 = (123)$, and that $C_A^2 \cap C_B^2 = (12\tilde{3}), C_A^2 \cap C_C^2 = (1\tilde{2}3)$, and $C_B^2 \cap C_C^2 = (\tilde{1}23)$, which is to say that the three groups overlap - sharing the same identity element and pairwise an additional element. Notice also that (e.g.) $b_A c_B \tilde{c}_A = (13\tilde{2})(321)(132) = (\tilde{2}13) = b_C$, which is to say that we can get C_C^2 from products of elements of C_A^2 and C_B^2, or in general, given two of the groups, we can generate the third. Finally, given two such groups, one has eight elements, and the other - taking the two shared elements into account - brings six more, yielding a total of 48 elements.

We have therefore demonstrated that the complete set of actions on three (given) sensors, that is, X3X's, can be viewed as the product of all the possible X2X's on these same three sensors, taking any two distinct X2X's at a time.

Theorem. The exclusion group $\mathcal{X}^3 = \mathcal{X}_p^2 \amalg \mathcal{X}_q^2$, where $\amalg$ denotes the overlapping product of the two groups uncovered above, and p, q are different and chosen from $\{AB, BC, AC\}$.

We now move from X3X's to X4X's, and (following Eddington [Edd]) define the following operations:

$a = (2143)$ = interchange the 1st and 2nd, and also the 3rd and 4th fields.
$b = (3412)$ = interchange the 1st and 3rd, and also the 2nd and 4th fields.
$c = (4321)$ = interchange the 1st and 4th, and also the 2nd and 3rd fields.
$d = (1234)$ = do nothing.

and

$p = (12\tilde{3}\tilde{4})$ = invert the 3rd and 4th fields.
$q = (1\tilde{2}3\tilde{4})$ = invert the 2nd and 4th fields.
$r = (1\tilde{2}\tilde{3}4)$ = invert the 2nd and 3rd fields.
$s = (1234)$ = do nothing.

We choose now the five operations $I_1 = as, I_2 = dq, I_3 = cr, I_4 = ar, I_5 = I_1 I_2 I_3 I_4 = cq$.

There are six different ways of choosing this pentad, but there are always five such elements. Each I_k anti-commutes with each of the others, i.e., $I_i I_j = -I_j I_i$ for $1 \leq i, j \leq 5, i \neq j$. Finally, it turns out that $I_1 I_1 = I_2 I_2 = I_3 I_3 = +1$, but $I_4 I_4 = I_5 I_5 = -1$. Returning for a moment to X2X's, we saw that we could capture the movement of a single block as a purely spatial rotation with three dimensions, one for each of A, B, and the Hand. However, the fact that with this X4X family only the squares of I_1, I_2 and I_3 are +1 means that we cannot continue this trick indefinitely - after three spatial dimensions, we enter new territory, where true time-like changes cannot be avoided. In fact, I_4 corresponds to what we normally consider to be time-like changes, and I_5 can be taken to correspond to 3-dimensional spatial inversions, such as turning a glove that has no "hand hole" inside-out. This action family's group thus captures everything that (apparently) can happen in 3+1 space-time, and is therefore a strong reason to stop at X4X's - they appear to be enough to allow us to describe the physical world, though we hasten to add that we do not think that this is how $3+1$ space-time will in fact make its entry.

Howsoever, the space spanned by this C^4 contains only 192 elements - it does not capture all $2^4 4! = 384$ 4-tuples which can occur; in particular it lacks those in which an *odd* number of components are held constant. Hence we will analyze the situation as we did with X3X's.

The case where three of the four components remain unpermuted can in fact not occur (think about it), so we need only look at the case where one of the four is held fixed. Looking first at the permutations, we can arrange them according to whether the first, second, third, or fourth components are unchanged, or all are changed:

$1^{st}\,fixed$	$2^{nd}\,fixed$	$3^{rd}\,fixed$	$4^{th}\,fixed$	$0\,fixed$
(1234)	[1234]	[1234]	[1234]	(2341)
(1243)	[1243]	(2134)	[2134]	(2143)
(1324)	(3214)	(2431)	(2314)	(2413)
(1342)	(3241)	[1432]	[1324]	(3142)
(1423)	(4213)	(4132)	(3124)	(3412)
(1432)	(4231)	[4231]	[3214]	(3421)
				(4123)
				(4312)
6	4	3	2	(4321)

where the entries in brackets are duplicates (scanning the columns from left to right) and the numbers at the bottom give the number of unique elements in each column after this elimination.

The first four columns share the identity element (1234) and one other with each of the others, and contain in all 15 unique elements. Incorporating now inversion, each of these 15 can have 2^4 different values, so the first four columns represent $15 \times 16 = 240$ distinct tuples. The last column, which incidentally includes the operations a, b, c above, represents $9 * 2^4 = 144$ tuples, for a grand total of $240 + 144 = 384$ elements.

Looking at the first column and ignoring the 1's, we can see that it has $\mathcal{X}^3$ on (234) as a factor, which can be 'multiplied' by the two possible values of the 1-column. Similar reasoning can be applied to the next three columns. Furthermore, the third and fourth columns can be derived from the first two:

$$\begin{array}{rcl} (1324)(3214) & = & (2314) \\ (2314)(1324) & = & (2134) \\ (2314)(1423) & = & (2431) \\ (3214)(1324) & = & (3124) \\ (1432)(2134) & = & (4132) \end{array}$$

It is of course not possible to derive the second column from the first because the latter has no way to change the first element in a tuple. Finally, composing odd numbers of changes can lead to even numbers of changes, i.e., the fifth column is automatically included. For example, (1243) (3214) (1324) = (4123).

We are therefore led to conclude:

Theorem. The exclusion group $\mathcal{X}^4 = \mathcal{X}^3 \amalg \mathcal{X}^3$.

We are further led to conjecture that $\mathcal{X}^n = \mathcal{X}^{n-1} \amalg \mathcal{X}^{n-1}, n \geq 3$, and note that the relationship expressed by $\amalg$ is probably better expressed as the so-called *semi-direct* product of two groups, g_1, g_2 with operations τ_1, τ_2: $(g_1, \tau_1) \circ (g_2, \tau_2) = (g_1 g_2^{\tau_1}, \tau_1 \tau_2)$, where the two groups are the symmetric group and the 'trivial' group $\{1, -1\}$.

2 Vector Algebra.

Definition. Let $V^n_{(l)}, l \in Z^+, l \leq n$ be an n-dimensional vector space over the rationals with inner product "$\cdot$" and basis (e_i) such that

$$\begin{array}{ll} e_i \cdot e_j = 0 & i \neq j \\ e_i \cdot e_j = -1 & i = j = 1, \ldots, l \\ e_i \cdot e_j = +1 & i = j = l+1, \ldots, n \end{array}$$

The characterization of V^n according to l follows from the fact that some of the basis vectors may have square -1. Using for example (a, c) as the basis, $a^2 = c^2 = 1$ but $b^2 = \tilde{1}$, and the group $\mathcal{X}^2$ is covered exactly by $V^2_{(2)}$.

We emphasize that this new space is distinct from, and must not be confused with, the vector space S spanned by the sensors. V is a space of *operations* on S, and it is in terms of the structure of these operations that the algebra expresses correlations (ie. co-occurrences) observed among sensor values.

If we define a product uv on the vectors in $V^n_{(l)}$ which is associative and distributive with respect to addition and which satifies the condition $uv + vu = 2(u \cdot v)$, i.e.,

$uv = -vu$ when u, v are orthogonal, then the resulting algebra of all possible sums and products is called the *Clifford algebra* $C(V^n_{(l)})$, which we will abbreviate as $C^n_{(l)}$, and as C^n whenever the value of l is clear. It follows from these definitions that (1) $\mathcal{X}^2$ possesses exactly the algebraic structure found in the algebra C^2, and (2) the X2X action $s_1 + s_2 \longrightarrow \tilde{s}_1 + \tilde{s}_2$ is captured by the product $s_1 s_2$ taken twice, that is, $(s_1 s_2)^2 = -1$ and hence $(s_1 s_2)^2 (s_1 + s_2) = \tilde{s}_1 + \tilde{s}_2$.

Clifford algebras have a number of other properties[ChDe]. The linear subspace of C^n spanned by the $\binom{n}{p}$ products $(e_{J_1} e_{J_2} \cdots e_{J_n})$ is denoted C^p and can be decomposed into 'even' and 'odd' linear subspaces $C^+ = \overset{p\,even}{\oplus} C^p$ and $C^- = \overset{p\,odd}{\oplus} C^p$, both of dimension 2^{n-1}. C^+ is also a subalgebra of C. The algebra C^+ is isomorphic to the Clifford algebra $C^{n-1}_{(l)}$ only for certain values of l. Some examples of the even sub-algebra structures:

1. An even sub-algebra of the *Dirac algebra* (found in $\mathcal{X}^4$ and characterized by $C^4_{(3)}$), the algebra of 3+1 space-time, is the *Pauli* algebra $C^3_{(0)}$.

2. The even sub-algebra $C^2_{(0)}$ of the Pauli algebra is the algebra of *quaternions*;

3. The even sub-algebra $C^1_{(1)}$ of both $C^2_{(0)}$ and $C^2_{(2)} = \mathcal{X}^2$ is $\mathbb{C}$, the algebra of the complex numbers;

4. The even sub-algebra of $\mathbb{C}$ is the algebra of the real numbers;

5. Any Lorentz transformation can be expressed in terms of the center of C^1.

We next impose a vector space structure S over the ring $\mathbb{R}$ of reals[2], on sensors so $(s_1, s_2, \ldots, s_n)$ is a basis of S and any element of S is $s = \sum_{r=1}^{n} x_r s_r$. Here + is purely formal.

We now extend valuation to S by $v(s) = \sum x_r v(s_r)$. Since we interpret + as co-occurrence, transformations leaving + invariant (linear transformations) are important.

Now consider changes in the values of v for the various sensors: a "flux". The purpose of constructing S was to give a new *computational* way of talking about fluxes. If $v(s_i)$ changes from 1 to -1, let us write $s_i \dashrightarrow \tilde{s}_i$ where $v(\tilde{s}_i) = -v(s_i) = v(-s_i)$, so that when $v(s_i) = -1$, $v(\tilde{s}_i) = 1$. Call $\tilde{s}_i$ the complementary sensor. Make the convention that all sensors written down have $v = 1$ so that $s_1 + s_2 \dashrightarrow s_1 + \tilde{s}_2$ means that initially $v(s_i) = v(s_j) = 1$, but then $v(s_2)$ changes to -1. In this way a flux is "taken into the algebra". Note that $-s_i$ is interpreted as $\tilde{s}_i$ but no interpretation is given yet to ks_i, $(k \neq \pm 1)$.

[2]We may need only the rationals, but for now assume the worst.

So far we have an algebra of co-occurrences. The various group properties of XnX's that we saw in the preceding section prompt the choice of a *Clifford algebra* as the next step. As we will soon see, *products* in this algebra, not to mention other aspects, fit these properties extremely well.

Clifford algebras[3] can express a large number of fundamental and crucial aspects of physical reality, which we will discuss later. For the present, we will simply remark (and later exemplify) that the phrase "all possible sums and products" of the vectors in $V^n_{(s)}$ corresponds to all possible state and action configurations our action-concept can express. The intention of the following is to sketch how Clifford algebras can provide a good (and hopefully in the fullness of time, complete) formal characterization of the semantics of distributed processes.

A Clifford algebra is itself a linear space of dimension

$$\sum_{p=0}^{n} \binom{n}{p}$$

with basis $(1, e_{J_1}, e_{J_1}e_{J_2}, \ldots, e_1e_2\cdots e_n)$, where the J_j label ordered natural numbers, $J_j < J_{j+1}$. The semantics which we present below is that of a Clifford algebra over all possible sums and products of the combinations of the the states that S^m provides, i.e., all possible sums and products of

$$s_1,\ s_2,\ \ldots, s_m,\ s_1s_2,\ s_1s_3,\ \ldots,\ s_1s_m,\ s_2s_3,\ \ldots,\ \ldots,\ s_1s_2\cdots s_m$$

Hestenes [HeSo] has generalized Clifford algebras into a general calculus of space-time, called the *geometric calculus*. The basic element of this calculus is a *multi-vector*, which is a sum of *r-vectors*, which are themselves products of the (anti-commuting) basis vectors of the algebra. This product is defined on 1-vectors x, y as

$$xy = x \cdot y + x \wedge y$$

where $\wedge$ denotes the 'outer' product.[4] This product has the properties that $xy = -yx$, $xx = 1$ (a scalar) and $x\tilde{x} = \tilde{x}x = \tilde{1}$, and (as we have seen) the 2-vector xy can also be viewed as a 1-vector which is orthogonal to both x and y. The geometric sum and product of multi-vectors X, Y, Z have the properties

[3]Grassman or 'exterior' algebras are similar, except that $|ee| = 0$ instead of 1, due to the fact that the product ee is solely 'outer', rather than 'inner + outer' as in a Clifford algebra. Clifford algebras are a generalization of Grassman algebras, Hamilton's quaternion algebra, tensor algebra, and vector manifolds.

[4]The outer product is akin to, but distinct from, the vector cross product $x \times y = -ix \wedge y$. The magnitude of $x \wedge y$ is the area of the parallelogram x and y span, and hence $x \wedge x = x \wedge \tilde{x} = \tilde{x} \wedge x = 0$.

Addition is commutative	$X + Y = Y + X$
Addition and multiplication are associative	$(X + Y) + Z = X + (Y + Z)$ $(XY)Z = X(YZ)$
Multiplication distributes over addition	$X(Y + Z) = XY + XZ$ $(Y + Z)X = YX + ZX$
There are unique additive and multiplicative identities	$A + 0 = A$ $1A = A$
Every multi-vector X has a unique additive inverse	$\exists -X$ such that $X + (-X) = 0$
Every non-zero vector x has a multiplicative inverse	$\exists x^{-1} = x/\|x\|^2$

The purpose of constructing a Clifford algebra is two-fold. Firstly, the linear functions on S (i.e. transformations preserving co-occurrence) are of the form

$$s \to f(s) = \sum_{\alpha=1}^{2^n} a_\alpha s b_\alpha$$

where a_α, b_α are elements of the algebra i.e. expressions of the form

$$a_\alpha = a_{\alpha,0} + \sum_i a_{\alpha,i} s_i + \sum_{i<j} a_{\alpha,ij} s_i s_j + \sum_{i<j<k} a_{\alpha,ijk} s_i s_j s_k + \dots$$

Secondly, as a special case, either the a_α or the b_α can be taken to be the 2^n units

$$1, s_i, s_i s_j, \dots, s_1 s_2 \dots s_n$$

themselves[5]. The latter forms allow us to entertain the idea that products can represent *actions*. See also Figure 2.

The vertical dimension in Figure 2 corresponds to the product $s_1 s_2 = \frac{1}{2}(s_1 + s_2)(s_1 + \tilde{s}_2) =$ etc. The 2-dimensional vector space S^2 is uniquely determined by the non-zero 2-blade $s_1 s_2$ [HeSo, p.17]. There is an implicit assumption throughout this presentation that the underlying space is Euclidean, but this can probably be avoided.

[5]This theorem is due to Kilmister [1950], but the result for quaternions was known in the 1930's by A.W. Conway (no relation to J. Conway).

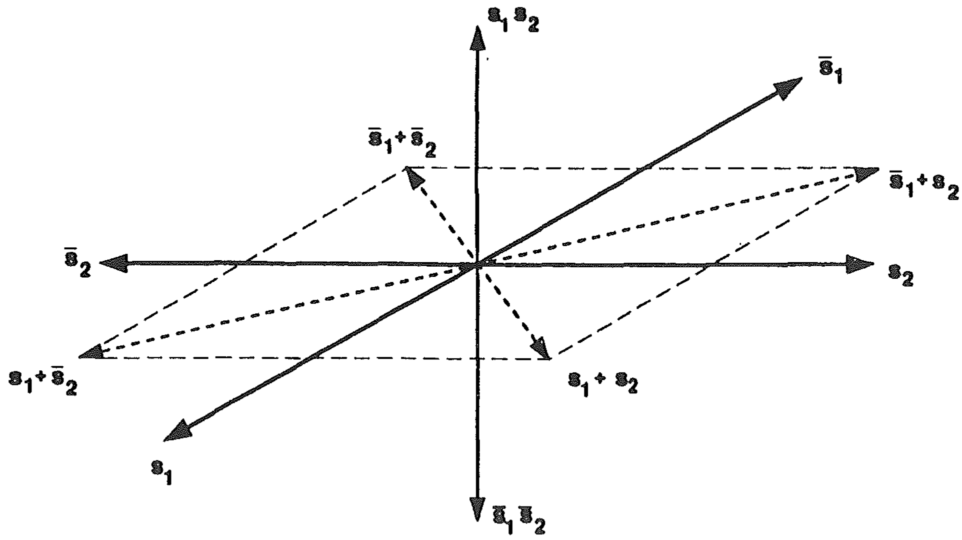


Figure 2: Two Possible Actions: $(s_1 + s_2) \leftrightarrow (\tilde{s}_1 + \tilde{s}_2)$ and $(\tilde{s}_1 + s_2) \leftrightarrow (s_1 + \tilde{s}_2)$.

3 The Boundary Operator and Co-Exclusion

Summing up the development to this point, we have an algebra which can express

- The existence of sensors and their changing values as a vector space;
- The co-occurrence of sensors as one of its fundamental operations: + ;
- The concept of 'cannot occur' as its zero;
- The our concept of 'action' as its fundamental product;
- The effect of these actions on co-occurrences preserves +, for example, it makes no difference whether we think of an action as $(s_1 + s_2)s_3s_4$ or as $s_1s_3s_4 + s_2s_3s_4$.

So far so good. We lack however a means of expressing exactly how the algebra's product expresses our concept of action. What we are looking for is a linear transformation which preserves products, i.e. an automorphism of $C(S)$. If S contains an even number of elements, then the required automorphism is *inner*, that is, of the form

$$s \rightarrow s' = asa^{-1}$$

In general, Clifford algebras exhibit various quirks when the basis contains an odd number of elements. Regarding the above automorphism, for *odd* algebras an automorphism is either inner, or it is the composition of an inner automorphism with $i \rightarrow -i$ (complex conjugation). It remains to be seen whether these quirks are a plague or a blessing, and a consideration of this is beyond us at the current state of development.

Consider as an example the case $n = 2$ and look at $s_1 + s_2$. The possible changes are:

$$s_1 + s_2 \rightarrow s_1 + s_2$$

$$s_1 + s_2 \rightarrow s_1 + \tilde{s}_2$$

$$s_1 + s_2 \rightarrow \tilde{s}_1 + s_2$$

$$s_1 + s_2 \rightarrow \tilde{s}_1 + \tilde{s}_2$$

Since $s_1^{-1} = s_1$, and we find

$$s_1(s_1 + s_2)s_1 = s_1 + \tilde{s}_2$$

and also $s_2^{-1} = s_2$, so

$$s_2(s_1 + s_2)s_2 = \tilde{s}_1 + s_2$$

and finally, since $(s_1 s_2)^{-1} = s_2 s_1$, a genuine *action* has the form

$$s_1 s_2(s_1 + s_2)s_2 s_1 = \tilde{s}_1 + \tilde{s}_2$$

Similarly, for 3- and 4-ary actions, we get

$$-s_1 s_2 s_3(s_1 + s_2 + s_3)s_3 s_2 s_1 = \tilde{s}_1 + \tilde{s}_2 + \tilde{s}_3$$

where the minus sign reflects the effect of the complex conjugation mentioned above, and

$$s_1 s_2 s_3 s_4(s_1 + s_2 + s_3 + s_4)s_4 s_3 s_2 s_1 = \tilde{s}_1 + \tilde{s}_2 + \tilde{s}_3 + \tilde{s}_4$$

Notice that the structure of the inner morphism contains the 'action' twice, reflecting what we noticed in the example of moving the block (cf. Figure 1, where the operation ac must be done twice). Furthermore, we have seen that an X2X is exactly captured by the product $s_i s_j$ (taken twice), and also (earlier) that X3X's can be expressed as products of X2X's, etc. So now we know how to express actions in our algebra.

Finally, this apparatus leads to a natural composition rule for actions, where by 'composition' we mean that one action acting on the result of another can be viewed as a single combined action. Suppose we have the actions $s_1 s_2 \longrightarrow \tilde{s}_1 + \tilde{s}_2$ and $\tilde{s}_2 s_3 \longrightarrow s_2 + \tilde{s}_3$, where it is the $\tilde{s}_2$ in the second action that expresses that it builds on the result of the first, and which therefore allows it to be composed with it. From above we know

$$s_1 s_2(s_1 + s_2)s_2 s_1 = \tilde{s}_1 + \tilde{s}_2$$

$$\tilde{s}_2 s_3(\tilde{s}_2 + s_3)s_3\tilde{s}_2 = s_2 + \tilde{s}_3$$

$$s_1 s_3(s_1 + s_3)s_3 s_1 = \tilde{s}_1 + \tilde{s}_3$$

where the third relationship is the desired result. Adding the first two equations and replacing their right hand side with the third then yields the composition rule

$$s_1 s_2(s_1 + s_2)s_2 s_1 + s_2 s_3(\tilde{s}_2 + s_3)s_3 s_2 = s_1 s_3(s_1 + s_3)s_3 s_1$$

wherein we have removed the superfluous inversions of s_2 in the second equation above. In a similar fashion, it is easy to derive

$$-s_1 s_2 s_3(s_1+s_2+s_3)s_3 s_2 s_1 - s_3 s_4 s_5(\tilde{s}_3+s_4+s_5)s_5 s_4 s_3 = s_1 s_2 s_4 s_5(s_1+s_2+s_4+s_5)s_5 s_4 s_2 s_1$$

where the two actions $s_1 s_2 s_3$ and $s_3 s_4 s_5$ overlap on s_3, that is, a single sensor; and

$$-s_1 s_2 s_3(s_1 + s_2 + s_3)s_3 s_2 s_1 - s_2 s_3 s_4(\tilde{s}_2 + \tilde{s}_3 + s_4)s_4 s_3 s_2 = s_1 s_4(s_1 + s_4)s_4 s_1$$

where they overlap on two sensors. The pattern should be clear by now - the overlapping, complementary sensors simply drop out, leaving the rest. The general composition rule is thus, for $2 \leq j \leq k < n$,

$$(-1)^k s_1 s_2 \ldots s_k(s_1 + s_2 + \ldots + s_k)s_k \ldots s_2 s_1 \ +$$

$$(-1)^{n-j+1} s_j s_{j+1} \ldots s_k \ldots s_n(\tilde{s}_j+\tilde{s}_{j+1}+\ldots+\tilde{s}_k+s_{k+1}+\ldots+s_n)s_n \ldots s_{k+1}\tilde{s}_k \ldots \tilde{s}_{j+1}\tilde{s}_j \ =$$

$$(-1)^{(j-1)+(n-k)} s_1 s_2 \ldots s_{j-1}s_{k+1} \ldots s_n(s_1+s_2+\ldots+s_{j-1}+s_{k+1}+\ldots+s_n)s_n \ldots s_{k+1}s_{j-1} \ldots s_2 s_1$$

Still remaining is the basic issue of capturing our basic co-occurrence $\Rightarrow$ co-exclusion inference. To begin, we can express the co-exclusion principle formally for X2X's as:

The ***co-exclusion principle*** says that if we observe $(s_1 + s_2)$ *and* $(\tilde{s}_1 + \tilde{s}_2)$, then it is correct to conclude that $(s_1 + s_2)xyxy = (\tilde{s}_1 + \tilde{s}_2)$, where $x : (s_i, s_j) \rightarrow (s_j, s_i)$ and $y : (s_i, s_j) \rightarrow (s_i, \tilde{s}_j)$.

Example. In terms of our earlier analysis, $xyxy$ corresponds to $caca = bb$. Suppose therefore we observe $(s_1 + s_2)$ and its inverse. Then

$$xyxy = (\tilde{s}_1 + \tilde{s}_2)(\tilde{s}_1 + \tilde{s}_2)(s_1 + \tilde{s}_2)(\tilde{s}_1 + s_2) = -1$$

where we indicate explicitly how the values being manipulated by x and y are mutating as the product is formed from right to left. We find therefore that $(s_1 + s_2)xyxy = (\tilde{s}_1 + \tilde{s}_2)$ as expected.

We can conclude two things from this formulation of co-exclusion:

(1) If we take the primitive sensor values as real numbers, the operator $xyxy$ will have the value -1.

(2) ***Axiom***: The concept of mutually exclusive states in the computational world corresponds to the concept of orthogonality in the mathematics of vector spaces.

The significance of the first item is that we can *reconstruct the world* on the basis of real information. The axiom rests on the interpretation that a given sensor represents the current state of some computational process, and the sequence of such states represents the time-wise evolution of that process. The co-exclusion principle then says that should it be the case that co-occurrences of the values of *two* sensors, representing the evolution of the *two* sensor processes, have the property that these state *pairs* exclude each other, then the changes these two processes make to these states cannot occur simultaneously, that is, the two processes exclude each other when making said state changes, which is to say that the changes take place "under mutual exclusion".

A persistent intuition has been that, given that the paradigm is so intimately connected with *change*, the concept of differentiation must have a role to play. One possibility, inspired by the geometric flavor of various things we have presented, is to look at algebraic topology, more specifically, homology theory.

It turns out that we can get the equivalent of (one concept of) differentiation via homology theory's boundary operator, ∂. Define

$$\partial(s_{i_1}s_{i_2}\ldots s_{i_k}) = \sum_{r=1}^{k}(-1)^{r+1}s_{i_1}\ldots s_{i_{r-1}}.s_{i_{r+1}}\ldots s_{i_k}, \quad k>1$$

and for $k=1, \partial(s_i)=1$ (rather than 0 - arbitrarily chosen - as in the usual theory). Then

$$\partial(a+b) = \partial a + \partial b$$

$$\partial(\partial a) = 0$$

and

$$\partial(ab) = (\partial a)b + (-1)^o a(\partial b)$$

where o is the order of a (i.e. if a is a p-simplex, $o=p+1$), just like the differentiation of exterior products.

As a simple geometric example of the boundary operation, consider an ordinary triangle ABC, where we specify it in terms of its vertices A, B, C and its edges are thus AB, BC, CA. Then

$$\partial(ABC) = BC - AC + AB$$

Since specifying the triangle's edges in terms of the vertices means that edge AC is oriented oppositely to edge CA, we can rewrite the above as $AB + BC + CA$, which is indeed the boundary of the triangle.

Notice, by the way, that the fact that a triangle's components are *discrete* entities plays no role. Indeed, despite the ingrained association of 'continuity' with 'differentiation', there is a surprising *isomorphism* theorem between the operations of the exterior differential calculus and the homology theory's boundary operator. This also puts [Petri], which proves continuity over partial orderings (read, discrete concurrent events), into context.

We now ask the question, "What is the boundary of $s_1 s_2$", or rather, for reasons which will become quickly apparent, "What is $\partial(s_1 s_2 = s_2 s_1)$?" We know that $s_1 s_2 + s_2 s_1 = 0$, and working out ∂ of this we find

$$s_2 - s_1 + s_1 - s_2 = 0$$

Re-writing this in terms of our valuation conventions, this becomes

$$s_2 + \tilde{s}_1 + s_1 + \tilde{s}_2 = 0$$

and, given that + is commutative, we can rearrange this in two different but equivalent ways:

$$(s_1 + \tilde{s}_2) + (\tilde{s}_1 + s_2) = 0$$

and

$$(s_1 + s_2) + (\tilde{s}_1 + \tilde{s}_2) = 0$$

which is to say that $(s_1 + \tilde{s}_2)$ may not co-occur with $(\tilde{s}_1 + s_2)$, nor may $(s_1 + s_2)$ co-occur with $(\tilde{s}_1 + \tilde{s}_2)$. Notice that these are the initial premises (both forms!) of the co-exclusion principle. Now it is obvious that if $\partial b = 0$ then there is an element a such that $b = \partial a$. For, if not, this would give b as a 'cycle', coresponding to a 'hole' in the space (non-trivial homology) whereas the Clifford algebra has trivial homology. So, running the derivation in the opposite direction, we can conclude that ***the fact that a co-occurrence and its inverse cannot co-occur means that the product of the component vectors is anti-commutative. In other words, the co-exclusion principle is really a statement about integrating change to derive a whole, and the 'action' performed by such a whole is its boundary!***. Clearly, this generalizes to co-exclusion inferences of arbitrary arity, and thus gives the original intuition a firm formal basis.

We can interpret this result further. Think back to the fact that the computational foundation of co-exclusion is mutual exclusion, and that when two processes mutually exclude each other there exists a resource invariant. The resource invariant expresses the fact that the synchronization 'stick' must move on a closed orbit, which orbit corresponds to the boundary expressed by the operator ∂. The fact that the resource invariant is indeed *invariant* is captured by the fact that $\partial(\partial(a)) = 0$, or as often put, "The boundary of the boundary is zero".

4 The Combinatorial Hierarchy

There remains to offer a speculation on the connection between the model described here and the combinatorial hierarchy. As noted earlier, a sensor space S^n induces an algebra of dimension $\sum_{p=0}^{n} \binom{n}{p} = 2^n$. The $p = 0^{th}$ element (dimension) is the scalars, which we will ignore; hence, the number of dimensions of an n-algebra is $2^n - 1$.

For example, given sensors s_1, s_2, they will produce a third dimension characterized by the orientation of $s_1 s_2$, ie. $2^2 - 1 = 3$.

In the present context, however, we are not given n a priori - rather, we wish to see how to 'grow' n from the materials at hand. In other words, what we wish to show is how one can get the combinatorial hierarchy's cardinal numbers from the dimensionality of the spaces spanned by (successive recursive constructions of) a 'base' Clifford algebra. The "cut-off" of this progression produces different numbers from the traditional 2^{2n} sequence, although its origin, effect, and interpretation is cut from the same cloth; the discussion will return to this in a moment. First, let us simply sketch the combinatorial hierarchy's hyper-exponential growth in terms of the aforementioned dimensions spanned by a Clifford algebra:

$$\sum_{p=1}^{2} \binom{2}{p} = 2^2 - 1 = 3$$

$$\sum_{p=1}^{3} \binom{3}{p} = 2^3 - 1 = 7$$

$$\sum_{p=1}^{7} \binom{7}{p} = 2^7 - 1 = 127$$

$$\sum_{p=1}^{127} \binom{127}{p} = 2^{127} - 1 \approx 1.7 \times 10^{38}$$

The first sum expresses the number of dimensions spanned (or, better in the present context, *generated*) by the basis s_1, s_2, that is, *the dimensionality of the space generated by the co-occurrences* $s_1 + s_2$ *and* $\tilde{s}_1 + \tilde{s}_2$.

The second sum then expresses how many dimensions are spanned / generated by the three dimensions obtained from the first sum. Etc. Hence, this method of generating the combinatorial hierarchy's basic cardinals follows directly from our interpretation of the Clifford algebra's '+' as co-occurrence and the accompanying application of the co-exclusion principle (while, though, ignoring the necessity of observing complementary co-exclusions, which is assumed and implicit).

We now turn to the matter of how / why this hyper-exponential progression should end after four steps. This requires a brief return to some basic issues.

The combinatorial hierarchy is fundamentally concerned with the issue of *distinguishable dichatomic symbols*, where by 'dichatomic' is meant symbols having exactly *two* complementary forms:

Definition. Presuming an as yet undefined operator "|", two symbols σ_1, σ_2 are *distinguishable* if and only if $\sigma_1|\sigma_2 = \sigma_2|\sigma_1 = \sigma_2$ and $\sigma_1|\sigma_1 = \sigma_2|\sigma_2 = \sigma_1$.

Taking $\sigma_1 = 0, \sigma_2 = 1$ and "|" to mean either "exclusive or" or "addition modulo 2" fits the definition, as does $\sigma_1 = +1, \sigma_2 = -1$ and "|" = "multiplication". The latter fits the algebraic context above directly, whereas the former fits the usual presentation of the combinatorial hierarchy. We note that the second interpretation is a multiplicative algebra, whereas the first is additive, which means that anti-commutativity is more deeply buried in the traditional elaboration of the combinatorial hierarchy's physical interpretation.

We now recapitulate the combinatorial hierarchy's progression of values, given above, but focusing on the fact that combination can both yield 'synonyms' of existing symbols and *new* ones.

Step -1. We are given $\not s$, where the / indicates that s has (so far) shown only a single value. So far, this is the only value in the Universe. However, since it represents but a *single* value, $\not s$ is not dichatomic. The number of dichatomic symbols is $2^0 - 1 = 0$, where the exponent is the number dichatomic symbols we begin with, and the result (here, also zero) the total number of dichatomic symbols that can be generated from this 'basis'.

If there exist other $\not s$'s, we can generate co-occurrences with them. Let us put this off for the moment.

Step 0. $\not s$ shows a different value. Of course, this cannot co-occur with its previous value. But it may be that it's a different $\not s$ - due to indistinguishability, we can never know for sure, but we assume that we have (observationally) matched the two complementary pieces properly[6] and therefore now have a single dichatomic symbol s_1. The number of dichatomic symbols is thus $2^1 - 1 = 1$.

The ultimate source of information is the co-occurrence of otherwise indistinguishable entities, in that co-occurrence produces the information that the entities in question do not exclude each other (and hence are not aspects of what is in fact the *same* entity). Hence, at every step in the growth of structure, we rely on the Universe to supply us with - otherwise indistinguishable - s's, of which the first was s_1. (It would therefore seem to follow that the question of how many observationally distinct symbols we can ultimately generate depends in the last analysis on how many indistinguishable entities the Universe started off with.)

[6] If indeed it actually makes any difference at all...

Step 1. Hence, we now suppose that the Universe produces *two* otherwise indistinguishable dichatomic symbols, say s_1 and s_2. These in turn can produce the new symbol $s_1 s_2$, whose dichatomicity resides in the distinction between $s_1 s_2$ and $s_2 s_1$, and where the similarity of this notation to the Clifford algebra's product is wholly intentional. No other symbols can be generated at this step; for example $s_2 s_1 s_2 = \tilde{s}_1$, ie. an aspect of the symbol s_1.

Furthermore, we cannot in fact distinguish between, say, s_1 and $s_1 s_2$, since the act of trying to observe them (in their presumed simultaneous existence) requires the co-occurrence $s_1 + s_1 s_2$, which via co-exclusion induces the existence of the action $s_1 s_1 s_2$, which simplifies to s_2. The point is that although we *generated* the symbol $s_1 s_2$ from s_1 and s_2, *observing* s_1 and $s_1 s_2$ simultaneously requires that $s_1 s_2$ be distinct from its components. This requirement can only be fulfilled by assuming that the Universe supplies us with two new s's, say s_3 and s_4.

Hence we see that while the absolute number of dichatomic symbols is $2^2 - 1 = 3$, requiring the ability to observe their simultaneous existence requires *four* symbols. These four *independent* symbols, viewed as a state vector, require a 4×4 'mapping matrix' to express a transition to a given new state.[7]

Aside. We also see here (I claim) that a *term* or product[8] such as $s_1 s_2$ in the Clifford algebra corresponds to the combinatorial hierarchy's concept of a "discriminately closed subset (Dcs)", and as well to a 'dimension' of the given Clifford algebra. Hence, the denotations 'symbol', 'term' or 'product', 'dimension', and 'discriminately closed subset' all denote the same concept.

Step 2. We invoke once again the Universe's store of information in the form of indistinguishable additional s's. This produces, then, three new symbols s_5, s_6, s_7 and forming all of their products: $s_5, s_6, s_7, s_5 s_6, s_5 s_7, s_6 s_7, s_5 s_6 s_7$, yields an absolute total of $2^3 - 1 = 7$ symbols. By the same logic as earlier, however, requiring simultaneous observation requires *both* that these be distinct from the symbols of Step 1 *and* that they

[7]Which is quite a different, and smaller, affair than the question of *how many* such matrices there are: four independent dichatomic symbols can together take on sixteen different values, and given such a set of sixteen values, it will require a 16×16 mapping matrice to express all the transitions that might result. So the number of possible mapping matrices grows roughly as fast as - and in fact faster than - the combinatorial hierarchy itself!

[8]More precisely, what in the geometric calculus is called a *blade*.

be distinct from each other. This generates the re-labeling

$$\begin{aligned} s_5 &\rightarrow s_5 \\ s_6 &\rightarrow s_6 \\ s_7 &\rightarrow s_7 \\ s_5 s_6 &\rightarrow s_8 s_9 \\ s_5 s_7 &\rightarrow s_{10} s_{11} \\ s_6 s_7 &\rightarrow s_{12} s_{13} \\ s_5 s_6 s_7 &\rightarrow s_{14} s_{15} s_{16} \end{aligned}$$

so we see that in all we require $4 + 12 = 16$ symbols to satisfy the simultaneous observation criterion, and hence a mapping matrix of size $16 \times 16 = 256$.

Step 3. With the seven symbols from the preceding step, the Universe can generate an additional $2^7 - 1 = 127$ symbols, denoted (and including the re-labeling) $s_{17}, s_{18}, s_{19}, \ldots, s_{448}$. [The general formula for how many symbols will be needed is $\sum \binom{n}{p} p$.] The state vector will have $4 + 12 + 448 = 464$ elements, and the mapping matrix will correspondingly have size $464 \times 464 = 192096$. Notice the departure from the usual $256 \times 256 = 65536$.

Step 4. With 127 symbols, we can generate $2^{127} - 1 \approx 1.7 \times 10^{38}$ symbols. We leave it as an exercise for the reader to calculate the size of the mapping matrix! The important point is that, unlike the previous matrix-size cardinals, $192096 < 2^{127} - 1$, that is, the state space spanned by the transformation matrix from the preceding level can no longer span the space generated by the following level. This is the "cut-off".

That the hyper-exponential progression cannot be continued indefinitely is of course a natural consequence of such a growth's eventually overtaking a quadratic growth. The significance of this factoid lies in the fact that, as has been pointed out frequently in discussions of the combinatorial hierarchy, the two sums $3 + 7 + 127 = 137$ and $3 + 7 + 127 + 2^{127} - 1 \approx 1.7 \times 10^{38}$ are remarkably good, and presumably not accidental, approximations to (respectively) the fine structure constant (expressing electromagnetic 'connectivity') and the gravitational constant (expressing gravitational 'connectivity') [9]. Since we observe gravitational interaction, we can at least conclude that the Universe started off with information corresponding to at least 127 indistinguishable dichatomic entities.

[9] Or as Noyes has pointed out, the maximum number of electrons (resp. protons) that can be confined within a Compton wavelength before the number actually present becomes unknowable.

The traditional table summarizing all these numbers is

(a) level	(b) # dims per level	(c) cumulative $\sum(b)$	ANPA trad.	(d) # symbols per level	(e) cumulative $\sum(d)$	(f) mapping matrix	comment
0	(1)	(1)	(1)	(1)	(1)	(1×1)	
1	3	3	4	4	4	$4 \times 4 = 16$	$16 > 7$
2	7	10	16	12	16	$16 \times 16 = 256$	$256 > 127$
3	127	137	256	448	464	$464^2 = 192096$	$192096 < 2^{127} - 1$!!
4	$2^{127} - 1$	$2^{127} + 136$	$(256)^2$	$\sum_{p=1}^{(b)} \binom{(b)}{p}$	$(d) + 464$	$(d) \times (d)$	cut-off reached

As to the departure of the cut-off sequence $16 \rightarrow 256 \rightarrow 192096$ from the conventional ANPA version $16 \rightarrow 256 \rightarrow 65536$, I can at present only offer the fact that $448 - 256 = 192$, which number popped up in the analysis of $\mathcal{X}^4$ as the number of group elements falling outside the pale of 'pure' interactions. Presuming this has anything at all to do with the 'departure', why the corresponding '24' doesn't seem to apply in the $\mathcal{X}^3$ case at the preceding level is currently a mystery.

5 Conclusion

This paper has outlined

1. How a purely computational, process-oriented, *information mechanical* view of Nature can be pared down to the registration of the co-occurrence or exclusion of physical events;
2. How these can be combined using the co-exclusion principle to infer causal relationships;
3. How these also can be interpreted in terms of the geometric vector calculus; and
4. How within this interpretation (a) the inferences drawn via the co-exclusion principle correspond to integration, and (b) the 'action' that occurs corresponds to differentiation.

In the course of the presentation, we saw - implicitly or explicitly - the following stages in the *incremental growth* of the concepts of action and space:

i. The primitive concept of co-occurrence of indistinguishable entities - that is, entities whose only property is their presence or absence - carries with it information which *in principle* cannot be obtained within the confines of sequentiality;

ii. This information induces the primitive concept of 'separation' or 'difference', and therewith the seed of the properties of space per se, and of 'metric' as the measure of separation or difference;

iii. The observation of a pair of 2-ary co-occurrences obeying the co-exclusion principle - of which the movement of an electron between two orbitals is an example - yields an object with spin $\frac{1}{2}$, the primitive concept of 'action';

iv. The preceding step thus links intimately the concept of action to the concepts of light, co-occurrence, and metric;

v. Step three also introduces, simultaneously, chirality and the seed of parity's relationship with time;

vi. The case of X2+1X's, in which one of the components is held fixed, teaches us that in order for certain kinds of change to occur, there must be a shared dimension. Thus the primitive concept of space in [ii] is augmented with the concept of 'locality' or 'positional commonality';

vii. The complete set of X3X's in which one or another component is held fixed yields 24+24=48 actions, corresponding to the structure of the weak/strong, EM/weak, and EM/strong particle interactions;

viii. The complete X4X group similarly induces a 3+1 structure which can be identified simultaneously with space-time and the Dirac algebra.

ix. Although Pierre has warned me about the following speculations, in the sense of 'fools rush in where angels fear to tread!', I cannot resist:

  a. 2-ary co-occurrences correspond to photons, in that (being a permutation group), they will have spin 1; and given the inherent lack of ordering, fit nicely with the Wheeler-Feynman view of photons, which denies a difference between time of 'emission' and time of 'absorption', which in turn fits nicely with the views of V. Pope in this / preceding conferences: (in my words) that photons are better viewed as the Universe's way of expressing that two frames are instantaneously *connected*, and rather than as traversing classical 'empty' space;

  b. 3-ary co-occurrences correspond to gluons, again because their spin is 1, and because of the ubiquitous three-ness of quark-gluon structures;

  c. I am not yet quite willing though to extend this to associating 4-ary co-occurrences to 'gravitons', although I haven't yet discarded the possibility either;

d. X2X actions correspond to chiral neutrinos, because they are the smallest spin-half structure, and their tiny collision cross-section with other matter could be interpreted as an expression of their poorly defined locality when compared to X2+1X actions;

e. The various electron / neutrino families correspond to similar structures, but based on different 'sensors';

f. The particle-anti-particle division corresponds to the 'dual' forms of actions, for example $s_1 + s_2 \longleftrightarrow \tilde{s}_1 + \tilde{s}_2$ versus $s_1 + \tilde{s}_2 \longleftrightarrow \tilde{s}_1 + s_2$;

g. The fact that (computational) synchronization Signals are *not* information-bearing leads to a natural resolution of the EPR 'paradox'. See [Man].

That all this should fall out of a computational - that is, *information-mechanical* - analysis is, I feel, an elequent testimony to the value of the process view in general, and a mechanistic model in particular. But, note well, this is no ordinary mechanism - it is *time itself.* Or, as (I recall how) the Bard put it, "All is but airey nothinge".

Acknowledgements.

The serendipitous discovery of Eddington's 1933(?) popular article [Edd] on the structure of the Dirac group provided the key insight [the connection of 'double state-flips' to anti-commutativity] that enabled me to connect the computational metaphor I have long been working on to the mathematical structures of physics. This led me to Hestenes' geometrical calculus, which has similarly been crucial for its ability express these and related matters cleanly and generally enough to allow a direct computational interpretation.

The analysis of $\mathcal{X}^3$ and the use of the boundary operator ∂ were done in collaboration with Clive Kilmister, whose help and patience(!) were crucial. I would also like to thank H. Pierre Noyes for sharing, over many years, his knowledge of physics; and as well the members of ANPA in general for sharing their expertise in a most supportive and stimulating atmosphere. Eddie Oshins supplied [ChDe], whose discussion of Clifford algebras I have found very useful. Finally, I would like to acknowledge a special debt to C.A. Petri for assuring me, psychologically speaking (as I am unfortunately unable to follow his mathematics), that it was far from crazy to apply the computational concepts of concurrency and mutual exclusion at a fundamental level to the description of Nature.

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Spin Networks, Topology and Discrete Physics

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Abstract. This paper discusses combinatorial recoupling theory amd gemeraizations of spin recoupling theory, first in relation to the vector cross product algebra and a reformulation of the Four Color Theorem, and secondly in relation to the Temperley Lieb algebra, Spin Networks, the Jones polynomial and the SU(2) 3-Manifold invariants of Witten, Reshetikhin and Turaev. We emphasize the roots of these ideas in the Penrose theory of spin networks.

I. Introduction

This paper discusses combinatorial recoupling theory, first in relation to the vector cross product algebra and a reformulation of the Four Color Theorem, and secondly in relation to the Temperley Lieb algebra, Spin Networks, the Jones polynomial and the SU(2) 3-Manifold invariants of Witten, Reshetikhin and Turaev.

Section 2 discusses a simple recoupling theory related to the vector cross product algebra that has implications for the coloring problem for plane maps. Section 3 discusses the combinatorial structure of the Temperley Lieb algebra. We investigate an algebra of capforms and boundaries (the boundary logic) that underlies the structure of the Temperley Lieb algebra. This capform algebra gives insight into the nature of the Jones-Wenzl projectors that are the basic construction for the recoupling theory for Temperley Lieb algebra discussed in Section 4. Section 5 discusses the relationship of this work with the spin networks of Roger Penrose. Section 6 discusses the definition of the Witten-Reshetikhin-Turaev invariant of 3-manifolds. Section 7 explains how to translate the definition in section 5 into a partition function on a 2-cell complex by using a reformulation of the Kirillov-Reshetikhin shadow world appropriate to the recoupling theory of Section 4. The work described in Sections 4 and 7 is joint work of the author and S. Lins and will appear in [KL92].

The foundations of the recoupling theory presented here go back to the work of Roger Penrose on spin networks in the 1960's and 1970's. On the mathematical physics side this has led to a number of interrelations with the the 3-manifold invariants discussed here and theories of quantum gravity in two dimensions of space and one dimension of time. There has not been room in this paper to go into these relationships. For the record, the reference list includes papers by Penrose and also more recent authors on this topic (Hasslacher and Perry, Crane, Williams and Archer, Ooguri). It is the author's belief that the approach to the recoupling theory discussed herein will illuminate questions about these matters of mathematical physics.

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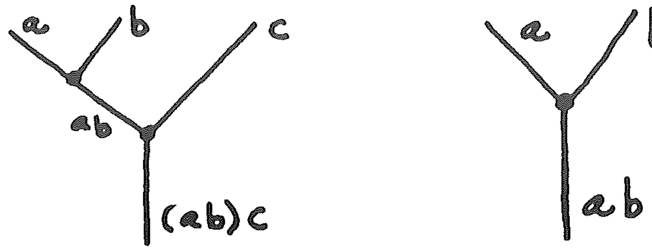
II. Trees and Four Colors

It is amusing and instructive to begin with the subject of trees and the Four Color Theorem. This provides a miniature arena that illustrates many subtle issues in relation to recoupling theory.

Recall the main theorem of [Kauffm90]. There the Four Color Theorem is reformulated as a problem about the non-associativity of the vector cross product algebra in three dimensional space. Specifically, let V = { i,-i,j,-j, k, -k, 1,-1, 0}, closed under the vector cross product (Here the cross product of a and b is denoted in the usual fashion by a x b.) :

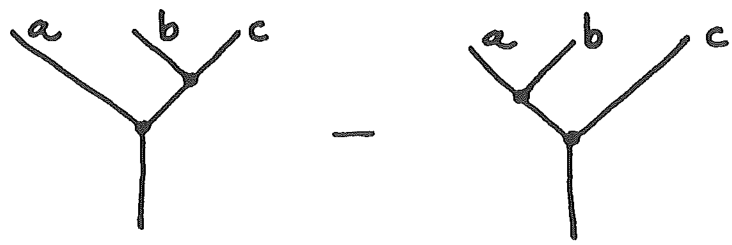
ixj=k, jxi=-k
jxk=i, kxi=-j
kxi=j, ixk=-j
ixi=jxj=kxk=0
0xa = 0 for any a
-0=0
-1xa = -a for any a
-(-a) = a for any a.

V is a multiplicatively closed subset of the usual vector cross product algebra. V is not associative, since (e.g.) ix(ixj) = ixk = -j while (ixi)xj = 0xj = 0.
Given variables a_1, a_2, a_3, ..., a_n, let L and R denote two parenthesizations of the product a_1 x a_2 ...x a_n. Then we can ask for solutions to the equation L=R with values for the a_i taken from the set V* = {i,j,k} and such that the resultant values of L and of R are non-zero. In [Kauffm90] such a solution is called a *sharp solution* to the equation L=R, and it is proved that *the existence of sharp solutions for all n and all L and R is equivalent to the Four Color Theorem.* The graph theory behind this algebraic reformulation of the Four Color Theorem comes from the fact that an associated product of a collection of variables corresponds to a rooted planar tree. This correspondence is illustrated below. Each binary product corresponds to a binary branching of the corresponding tree.



We tie the tree for L, T(L), to the mirror image tree , T(R)*, of the tree for R to form a planar graph T(L)#T(R)*. It is the region coloring of this graph that corresponds to a sharp solution to the equation L=R. (A region coloring of the graph corresponds to an edge coloring it with three colors (i,j,k) so that each vertex is incident to three distinct colors. See [Kauffm90] and [Kau92].)

There is in this formulation of the coloring problem an analogue to the sort of recoupling theory for networks that is common in theories of angular momentum. In particular, we are interested in the difference between the following two branching situations:



In terms of multiplication, this asks the question about the difference generated by one local shift of associated variables:

$$a \times (b \times c) - (a \times b) \times c = ?$$

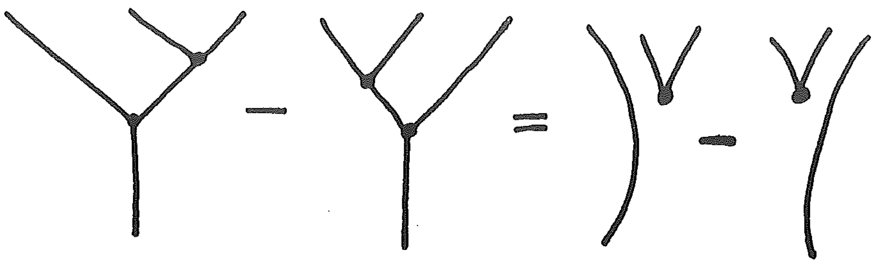
Since our algebra is the vector cross product algebra, we are well acquainted with the answer to this question. The answer is

$$a \times (b \times c) - (a \times b) \times c = a\,(b.c) - (a.b)c$$

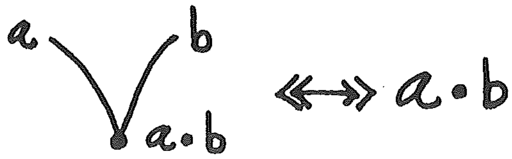
where a.b denotes the dot product of vectors in three space. (Thus i.i = j.j = k.k = 1, i.j = i.k = j.k = 0 and a.b =b.a).

(This formula is easily proved by using the fact that quaternion multiplication is associative, and that the formula for the product of two "pure" (i.e. of the form ai+bj+ck for a,b,c real) quaternions u and v is given by the equation uv= -u.v + u x v.)

We can diagram this basic recoupling formula as shown below



where it is understood that a concurrence of two lines represents a dot product of the corresponding labels:



We can use this formulation to investigate the behaviour of tree evaluations under changes of parenthesization. It also provides a way to investigate purely algebraically the existence of sharp solutions, and hence the existence of map colorations.

Example: Find sharp solutions to the equation ax(bxd) =(axb)xd.

Since $ax(bxc) - (axb)xc = a(b.c) - (a.b)c$, we see that the difference can be zero if $b.c = 0$ and $a.b=0$. Thus we can try $a=i$, $b=j$ and $c=i$ or k. In this case both of these work.

Example: Find sharp solutions to the equation

$$ax(bx(cxd))) = ((axb)xc)xd.$$

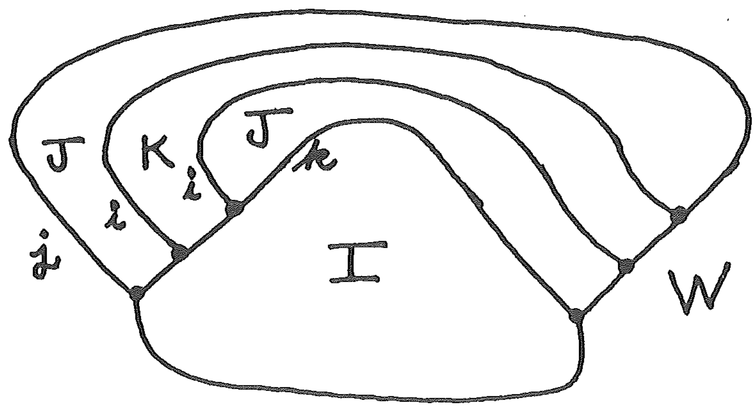
Three applications of these recoupling formulas yield the equation

$$ax(bx(cxd))) - ((axb)xc)xd =$$

$$-(a.b)\ cxd + (c.d)axb - (a.(bxc))d + ((bxc).d)a.$$

From this we are led to try $a=j$, $b=i$, $c=i$, $d=k$, and this indeed works.

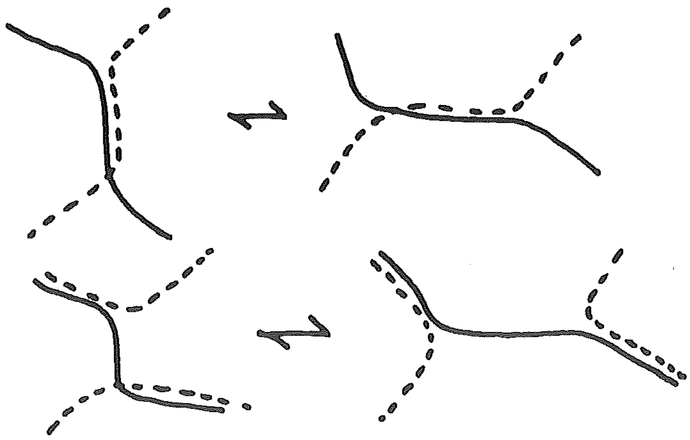
Remark. We hope to find a way to use the recoupling algebra associated with vector cross product to illuminate the original coloring problem. These are basically simple structures, but there arise gaps in language from one context to another. Thus the same problem as in the last example is very quickly and confidently solved by coloring the map shown below. This map represents the tied trees T(L)#T(R)*, and we color the regions of the map with colors W, I, J, K. The region coloring gives rise to an edge coloring by coloring an edge by the product of the colors for its adjacent regions with IJ=k, JK=i, KI=j, WA=AW=a for all A, AA=w for all A. (We use lower case letters to designate the colors of the edges.) Note that if the map is colored so that adjacent regions receive different colors, then the edges receive only the colors i,j,k.



The connection between the coloring approach and the recoupling theory of the vector cross product depends crucially upon our addition of signs to the products of colors (via the vector cross product algebra). That sharp solutions do derive from coloring *including the sign* also involves the quaternions. For an equation L=R to have a sharp solution up to sign (as it does from bare map coloring) implies the agreement of the signs. This is because both sides of the equation (being non-zero) can be viewed as products in the quaternions. Since the quaternions are associative, this implies that the two sides are equal, hence the signs are the same [Kau92]. Both sides must be non-zero in order for this argument to work.

The Kryuchkov Conjecture: It is interesting to examine the conjecture of Kryuchkov [Kry92] in the light of these diagrammatics. He is concerned with the possibility that one could find a coloring of L and of R (i.e. a choice of values i,j,k for the variables) forming an *amicable solution.* By an amicable solution I mean a choice of values so that that it is possible to go from L to R by a series of elementary recombinations (such as (xy)z -----> x(yz)) maintaining sharp solutions throughout the procedure. Kryuchkov conjectures that there is an amicable solution for every choice of L and R.

The notation of formations (see [Kauffm90]) allows a diagrammatic view of amicability. In this form we represent one color (red) by a solid line, one color (blue) by a dotted line, and the third color (purple) by a superposition of a dotted line and a solid line. The elementary forms of amicability are then as shown below:



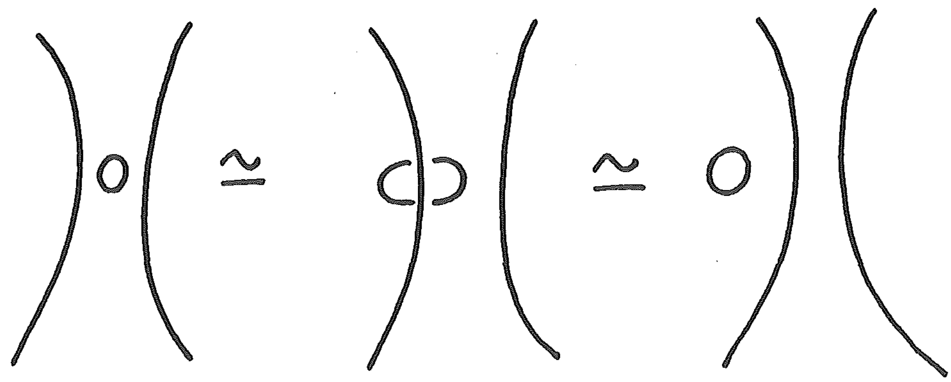
Clearly, more work remains to be done in this version of the Four Color Theorem and its generalizations. We have begun this paper with a short review of this arena both for its intrinsic interest, and for the sake of the possible analogies with other aspects of mathematics and mathematical physics.

III. The Temperley Lieb Algebra

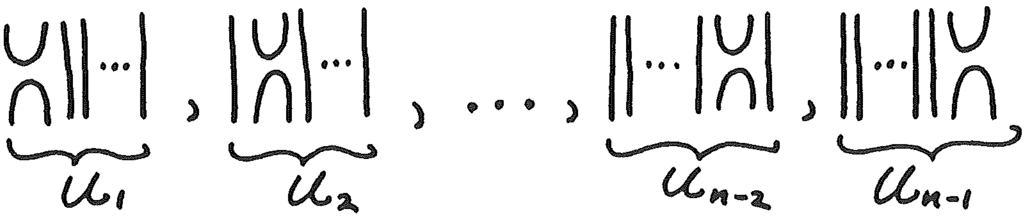
We now turn to the combinatorial underpinnings of the Temperley Lieb algebra.

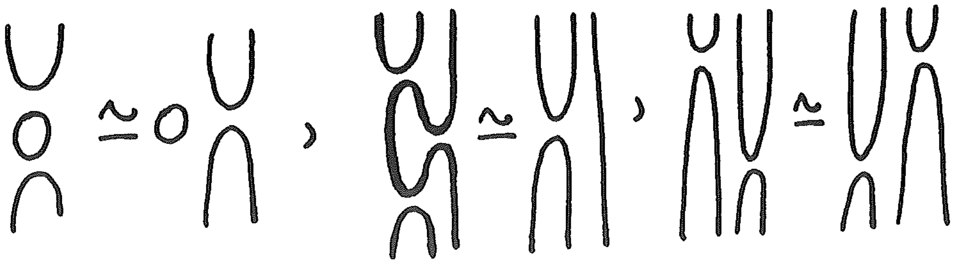
First recall the tangle-theoretic interpretation of the Temperley-Lieb Algebra [KA87]. In this interpretation, the additive generators of the algebra are *flat tangles* with equal numbers of inputs and outputs. We denote by T_n the Temperley Lieb algebra generated by tangles with n inputs and n outputs. A flat n-tangle is a an *embedding* of disjoint curves and line segments into the plane so that the free ends of the segments are in one-to-one correspondence with the input and output lines of a rectangle in the plane that is denoted the *tangle box*. Except for these inputs and outputs, the disjoint curves and line segments are embedded to the interior of the rectangle.

Two such tangles are *equivalent* if there is a regular isotopy carrying one to the other occurring within the rectangle and keeping the endpoints fixed. Regular isotopy is generated by the Reidemeister Moves of type II and type III for link diagrams. (See [KA87].) The reason we adhere to regular isotopy at this point is that it is necessary to be able to freely move closed curves in such a tangle. Thus the two tangles illustrated below are equivalent via a regular isotopy in the tangle box that has intermediate stages that are out of the category of flat tangles.



The Temperley Lieb algebra T_n is , for our purposes freely additively generated by the flat n-tangles, over the ring $C[A,A^{-1}]$ where C denotes the complex numbers. A closed loop in a tangle is identified with an element d in this algebra to be specified later. The familiar multiplicative generators of the Temperley Lieb algebra then appear as the following special flat tangles $U_1,U_2,\ldots,U_{n-1}$ in T_n.



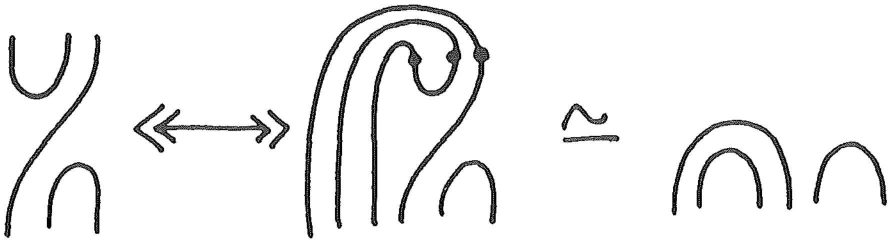


These generators enjoy the relations

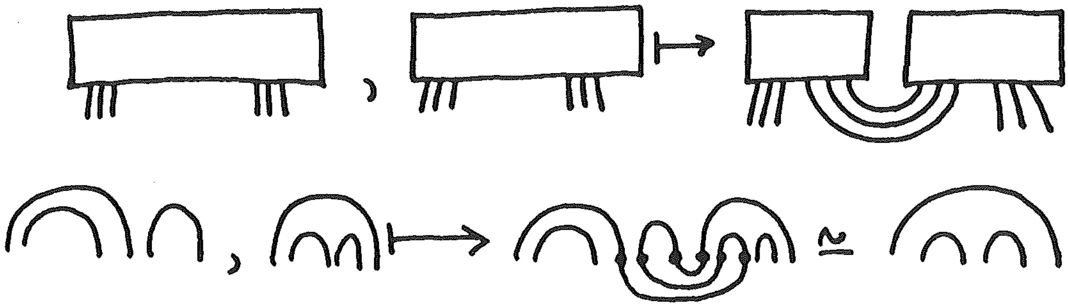
$$U_iU_{i+1}U_i = U_i$$
$$U_iU_{i-1}U_i = U_i$$
$$U_iU_i=dU_i$$
$$U_iU_j=U_jU_i \text{ for } |i-j|>1$$

and these relations generate equivalence of flat tangles [KA90].

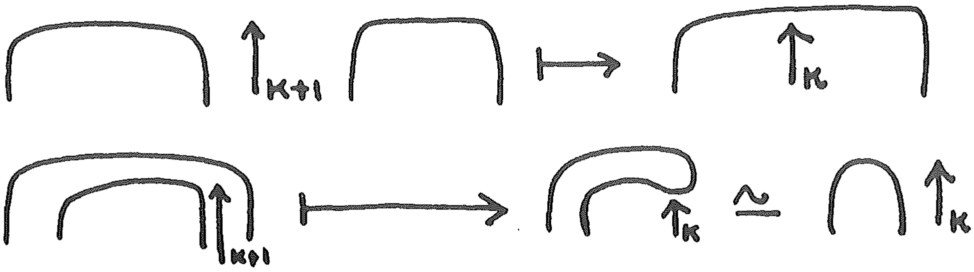
The purpose of this section is to point out a combinatorial algebra on parenthesis structures - *the boundary logic* - that forms a foundation for the Temperley Lieb algebra. First note that we can convert flat n-tangles to forms of parenthesization by bending the upper ends downward and to the left as illustrated below.



Call such a structure a *capform*.
In this way we convert the tangles to capforms with 2n strands restricted to the bottom of the form. These are capforms with n caps. The tangle multiplication then takes the form of tying the rightmost n strands of the left capform to the leftmost n strands of the right capform.

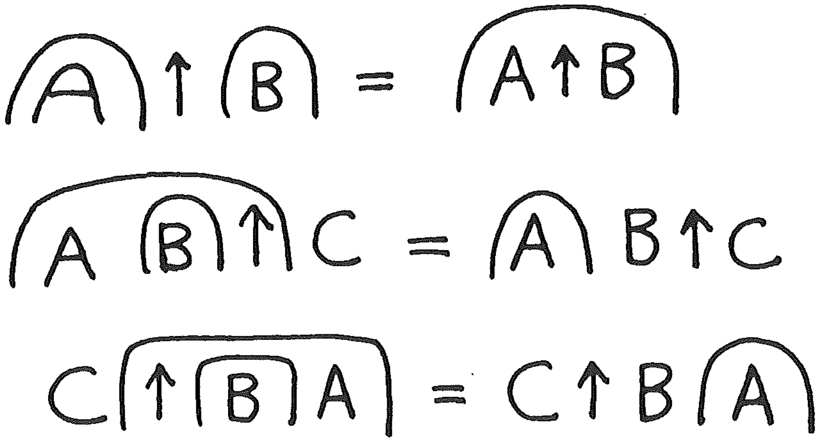


It is this operation that we shall convert into a series of more elementary operations: Regard the capform multiplication as done one pair of strands at a time. Then for a single pair of strands the pattern is to join them as shown below:



We denote this joining operation by a vertical arrow between adjacent strands. The resultant of the operation inherits an arrow in the place where the next joining can occur. An arrow with a subscript k+1 has an arrow with subscript k as its immediate descendant. An arrow with the subscript 0 (zero) is equal to the empty arrow. In this language the multiplication in Tn becomes, for the corresponding capforms: X , Y -----> X $\uparrow_n$ Y.

It is sometimes convenient to omit the subscript on the arrow, in writing identities and also in specific calculations where the count of operations is being performed separately. We shall accordingly omit the subscripts in the text that follows.
The boundary logic of the Temperley Lieb algebra is based on the joining and breaking of adjacent boundaries in capforms. Note the following basic equations in this boundary logic:



If we wish , we can re-express this formal structure in terms of ordinary brackets rather than the arches that have been given us by the capforms. In this form the basic equations look like

<A> | <B> = <A | B>

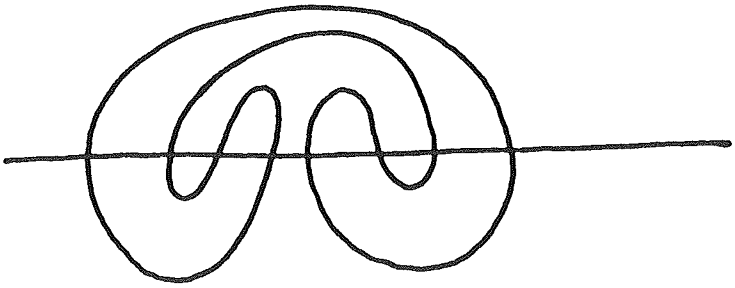
<A<B> | > = <A>B |

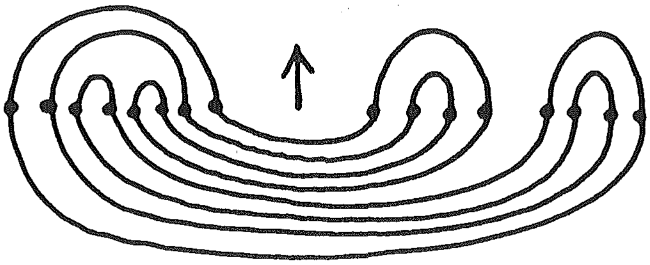
< | <A> B > = | A <B>

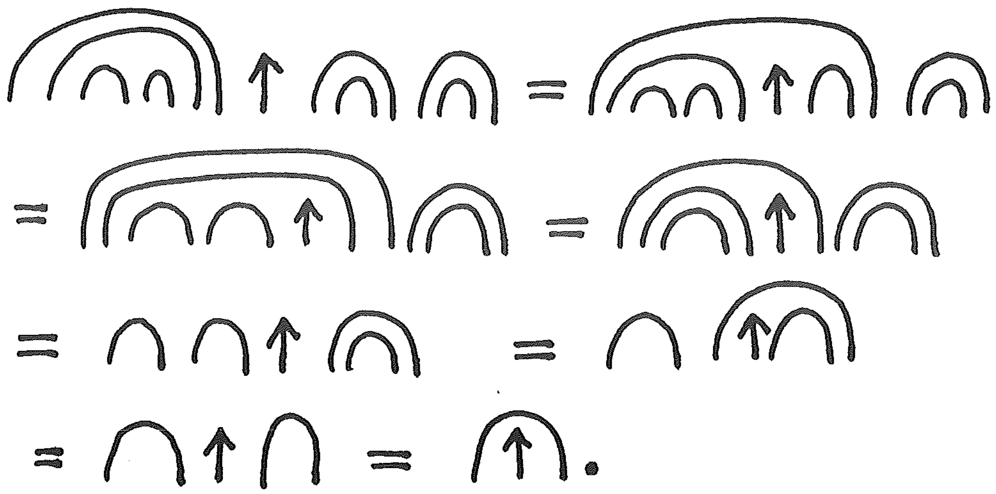
Here the vertical arrow indicatiing the join operation has been replaced by a bold face vertical line segment.

Boundary logic encapsulates the Temperley Lieb Albegra in a specific symbolic formalism that is suitable for machine computation. This formalism "knows" about the topology of Jordan curves in the plane! For example, take a Jordan curve, slice it by a line segment and regard the two halves as capforms. Then successive joining of these two halves will compute a single component.

Compare this symbolic computation with the complexity of the drawing that results from using joining arcs in the usual topological mode.







Finally, here is the same verification of connectedness performed in the boundary logic.

```
<<<><>>>|<<>><<>> =
<<<><>>|<>><<>> =
<<<><> |>><<>> =
<<<>> |><<>> =
<><> |<<>> =
<>< |<>> =
<>|<> =
<|>
```

We have left the vertical bar in the last entry, denoting a single loop. In this form it is interesting to note that with the extra rule

$$|| = |$$

the formalism contains an image of Dirac brackets:

$$P = |><|$$

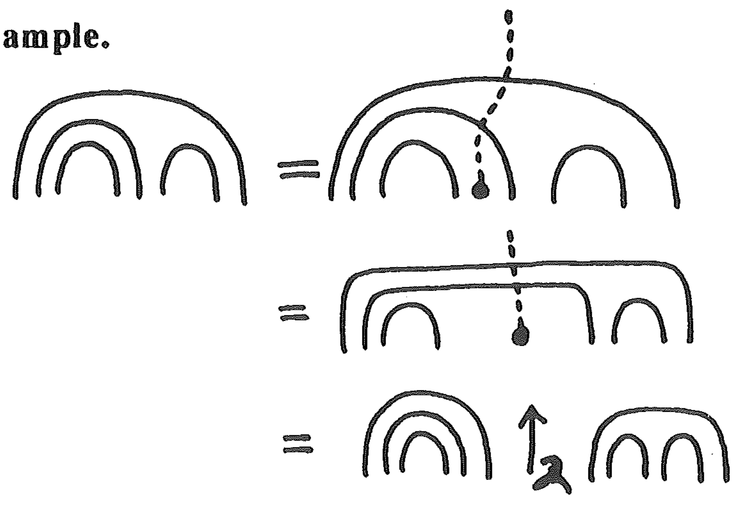
$$PP = |><||><| = |> <|> <| = <|> |><| = d\ P.$$

Aside from the advantages of formalization, certain structural features of the Temperley Lieb algebra are easy to see from the point of view of the boundary logic. For example, consider the folllowing natural map $T_n \times T_n$ -----> T_{n+1}
given by the formula A,B -----> $A*B = A \uparrow_{n-1} B$

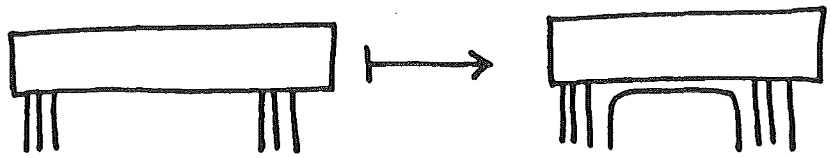
Theorem. Every element in T_{n+1} other than the identity element is of the form $A*B = A \uparrow_{n-1} B$ for some elements A and B in T_n. This decomposition is not unique.

Proof. If C in T_{n+1} is not the identity element then it is possible to draw an curve from the midpoint of the base of C to the outer region crossing less than n arcs of C. This is easily modified in a non-unique way to cross exactly n-1 arcs of C (possibly crossing some arcs twice). The curve so drawn then divides into the desired product. //

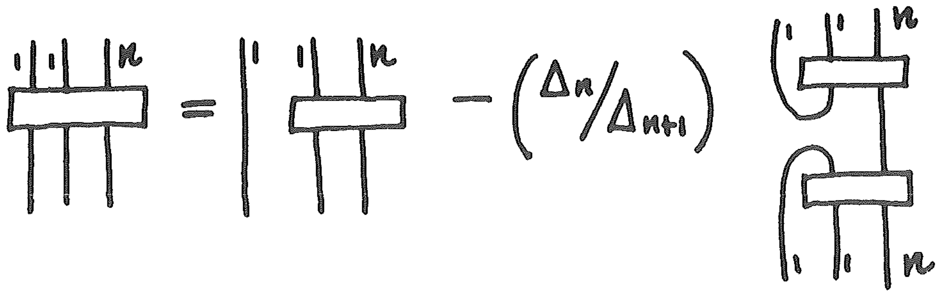
Example.



Another operation for going from T_n to T_{n+1} is x -----> x' where x' is obtained by adding an innermost cap as in

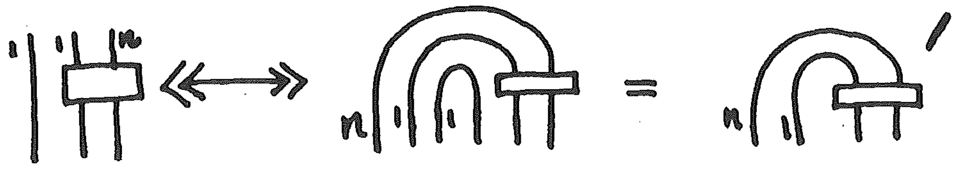


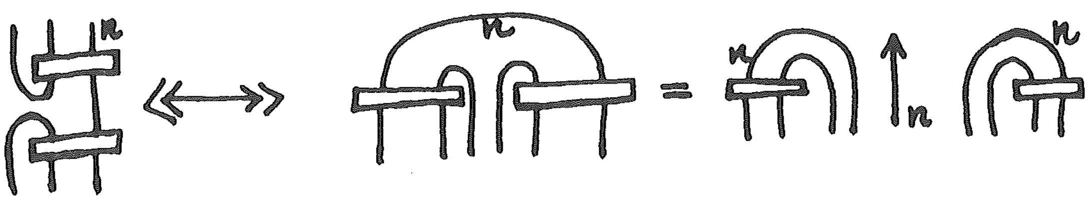
The operation x -----> x' takes the identity to the identity, and so together with x ⇑n-1 y encompasses all of T_{n+1} from T_n. This suggests combining these operations to produce inductive constructions in the Temperley Lieb algebra. An example that fits this idea is the well known ([JO83],[KA91], [LI91]) inductive construction of the Jones-Wenzl projectors. In tangle language these projectors are constructed by the recursion



where Δ_n is a Chebschev polynomial. These projectors are nontrivial idempotents in the Temperley Lieb algebra, and they give zero when multiplied by the generators U_i for i=1,...,n+1.

Now note the capform interpretation of the terms of this summation.





In the capform algebra the projectors are constructed via the recursion

$$g_{n+1} = g_n' - (\Delta_n/\Delta_{n+1})\, g_n * g_n.$$

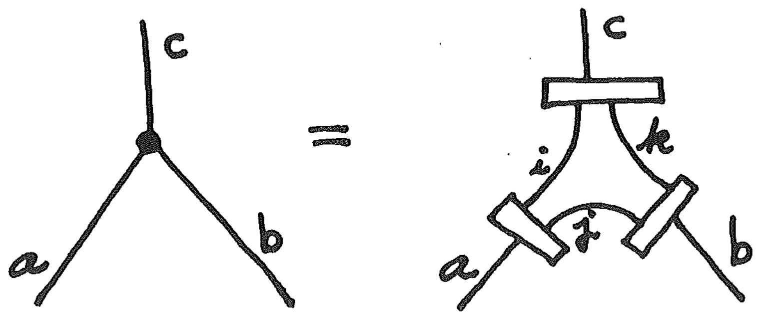
We shall return to these projectors in the next section.

A second use of the formalism is another reformulation of the Four Color Theorem. There is ([KS92] , [Kau92]) a completely algebraic form of the Four Color Theorm via the Temperley Lieb algebra. Boundary logic approach to the Temperley Lieb algebra provides a new way to look at the combinatorics of this version of the coloring problem. This relationship will be discussed elsewhere.

IV. Temperley Lieb Recoupling Theory

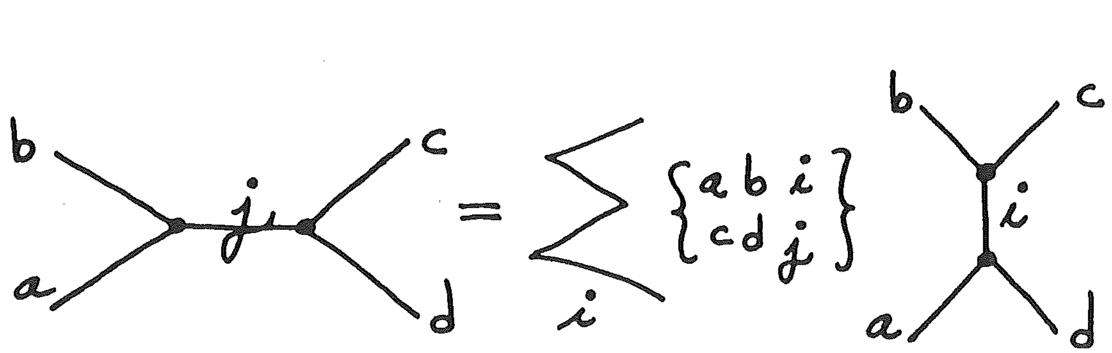
By using the Jones-Wenzl projectors, one builds a recoupling theory for the Temperley Lieb algebra that is essentially a version of the recoupling theory for the SL(2)q quantum group.
(See [KR88],[KAU92],[KL90], [KL92]). From the vantage of this theory it is easy to construct the Witten-Reshetikhin-Turaev invariants of 3-manifolds.

We begin by recalling the basics of the recoupling theory. The 3-vertex in this theory is built from three interconnected projectors in the pattern indicated below.



The internal lines must add up correctly and this forces the sum of the external lines to be even and it also forces the sum of any two external line numbers to be greater than or equal to the third.

With these 3-vertices, we have a recoupling formula

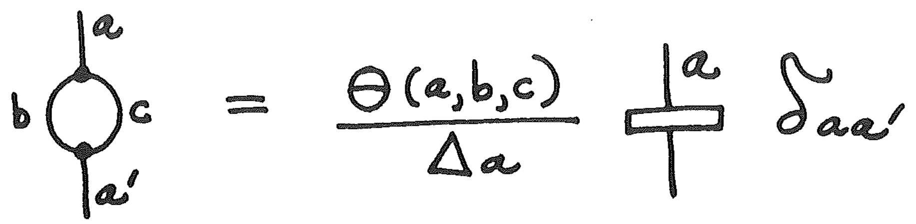


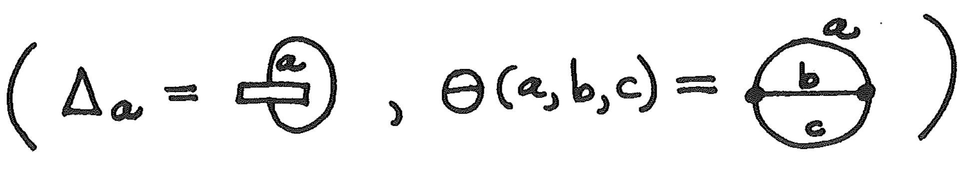
Here the symbol

$$\begin{Bmatrix} a & b & i \\ c & d & j \end{Bmatrix}$$

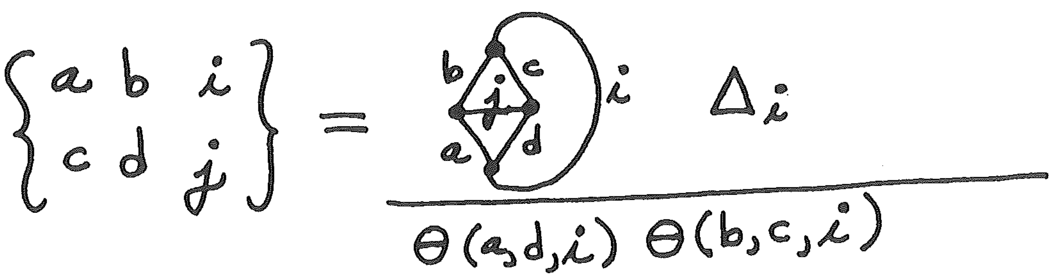
is a generalized 6j symbol.

A specific formula for the evaluation of this 6j symbol arises as the consequence of the following identity (See [KA91], [Kau92],[KL92]):





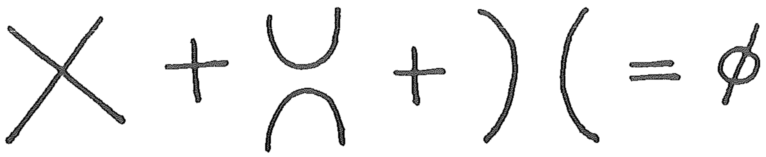
From this idetity it is easy to deduce that the 6j symbol is given by the network evaluation shown below:



The key ingredients are the tetrahedral and theta nets. They, in turn, can be evaluated quite specifically. (See [KL92],[LI91],[MV92]) There are a number of methods for obtaining these specific evaluations. For the general case one can induct using the recursion formula for the Jones-Wenzl projectors. In the special case where d=-2 there is a method to obtain the results via counting loops and colorings of loops in the networks. See [PEN79], [MOU79], [KA91], [KL92]. It should be mentioned that the case d= -2 corresponds to the classical theory of SU(2) recoupling.

V. Penrose Spin Networks

As mentioned at the end of the previous section, the Penrose spin networks are that special case of the recoupling theory where d=-2. In this case the projectors have a particularly simple epression as antisymmetrized sums of permutations, and there is an identity (the binor identity) that eliminates crossed lines as shown below:



The Binor Identity

The source of the binor identity in this special case is algebraic. It is a reformulation of the basic Fierze identity relating epsilons and deltas:

$$\varepsilon^{ab}\,\varepsilon_{cd} = \delta_c{}^a\,\delta_d{}^b - \delta_d{}^a\,\delta_c{}^b$$

Here the epsilon ε_{ab} denotes the alternating symbol where the indices a and b range over two values (say 0 and 1). Thus $\varepsilon_{ab} = \varepsilon^{ab} = 0$ if a=b, 1 if a<b, -1 if a>b. The delta is the Kronecker delta $\delta_a{}^b = 1$ if a=b, 0 if $a \neq b$.

Here we use the Einstein summation convention - *sum over repeated occurrences of upper and lower indices.*

The story of how the Fierze identity becomes the binor identity, and how this translation effects a relationship among SU(2), diagrams, spin and topology is the subject of this section. It is possible to make diagrammatic notations for tensor algebra. These diagrams can be allowed, in specific circumstances, to become spin networks, link diagrams or a combination of structures. In fact, in the case of spin networks alone, there are remarkable topological motivations for choosing certain diagrammatic conventions. These topological motivations provide a pathway into the beautiful combinatorics underlying the subject of spin angular momentum.

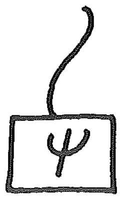
Let us begin by recalling that the groups SU(2) and SL(2) have the same Lie algebra structure. It is more convenient in this context to speak of SL(2) because it has such a simple definition as the set of complex 2x2 matrices of determinant equal to one. If ε denotes the 2x2 matrix $\varepsilon = [\ \varepsilon_{ab}\ ]$, then SL(2) is the set of 2x2 complex matrices P such that $P\varepsilon P^T = \varepsilon$. This follows from the fact that for any 2x2 matrix P, $P\varepsilon P^T = DET(P)\,\varepsilon$ where DET(P) denotes the determinant of the matrix P. Thus SL(2) is the set of matrices for which the epsilon is invariant under conjugation.

Recall also that a *spinor* is a complex vector of dimension two. That is a spinor is an element of the vector space V = C x C , C the complex numbers, and V is equipped with the standard left action of the group SL(2). If Ψ is a spinor and P belongs to SL(2), then $P\Psi$ is the vector with coordinates $(P\Psi)^A = P_B{}^A \psi^B$. Here we use the Einstein summation convention, summing (from 0 to 1) on repeated indices where one index is in the upper place, and one index is in the lower place.

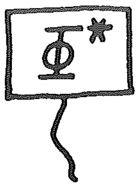
With this action, there is an SL(2) invariant inner product on spinors given by the formula $\Psi\Phi^* = \psi^A \varepsilon_{AB} \Phi^B$. In other words, we define $(\Phi^*)_A = \varepsilon_{AB}\Phi^B$ and take the standard inner product of the (upper and lower indexed) vectors Ψ and Φ^* to form this inner product. It is easy to see that this inner product is invariant under the action of SL(2).

This basic algebra related to SL(2) and to spinors is sufficient to use a case study in the art of diagramming tensor algebra. We let a spinor (which has one upper index or one lower index if it is a conjugate spinor) be denoted by a box with an arc emanating either from its top or its bottom. The arc will take the place of the index, and *common indices will be denoted by joined arcs.* Thus we represent Ψ^A and Φ^*_B as follows:

Ψ^A < ----->

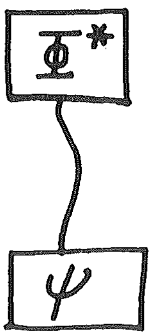


Φ^*_B < ----->

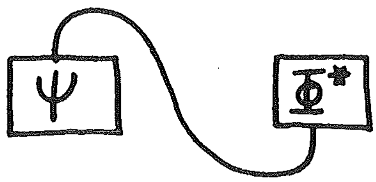


Consequently, we represent $\Psi\Phi^* = \Psi^A \Phi^*_A$ as the interconnection of the boxes for each term - tied by their common index lines.

$\Psi^A \Phi^*_A$ <----->

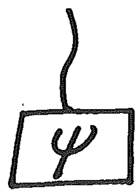


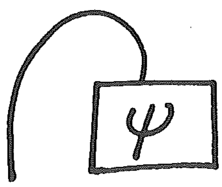
In translating into diagrammatic tensor algebra, it is convenient to have a convention for moving the boxes around so that, for example, we could also represent $\Psi^{A}\,\Phi^{*}{}_{A}$ by a picture with the two boxes next to one another. The connecting line must be topologically deformed as shown below:

$\Psi^{A}\,\Phi^{*}{}_{A}$ <-----> 

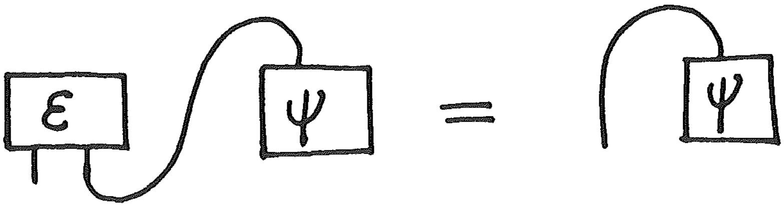
Since the lines in the tensor diagrams only serve to indicate the positions of common indices, *the expressions are invariant under topological deformations of the lines.*

On the other hand, it sometimes happens that a very compelling convention suggests itself and appears to contradict such topological invariance. In the case of these spinors that convention is as follows: *Indicate the lowering of the index in the passage from* Ψ^{A} *to* $\Psi^{*}{}_{A}$ *by bending the upper "antenna" downwards.*

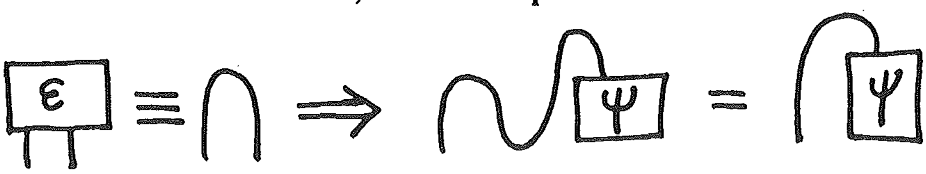
Ψ^{A} <-----> 

$\Psi^{*}{}_{A}$ <-----> 

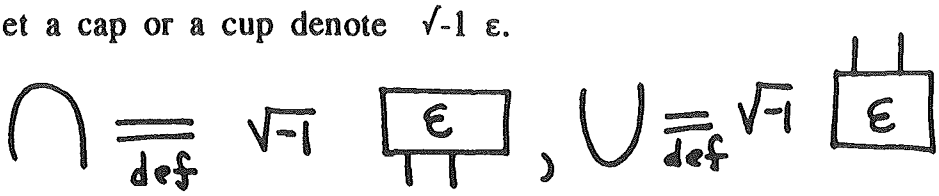
In our case we have $\Psi^{*}{}_{A} = \varepsilon_{AB}\Psi^{B}$. Therefore, if ε_{AB} is represented in dagrams as a box with two downward strokes, then we have the identity



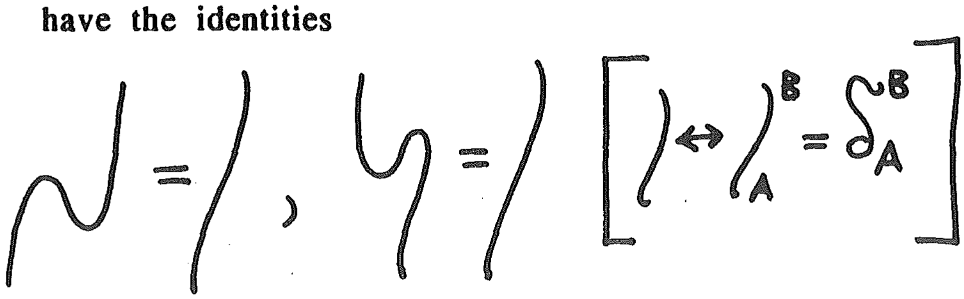
and consequently we must identify a "cap" (a segment with vertical tangents and one maximum) as an epsilon:



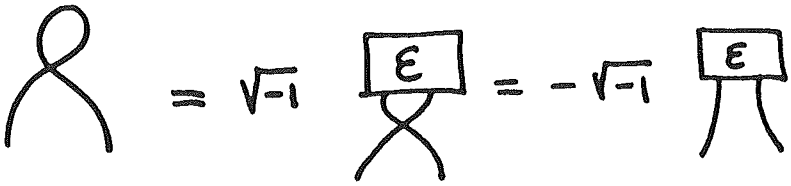
However, this convention is *not* topologically invariant! The square of the epsilon matrix is minus the identity. This paradox is remedied by multiplying the epsilon by the square root of minus one. We let a cap or a cup denote $\sqrt{-1}\ \varepsilon$.



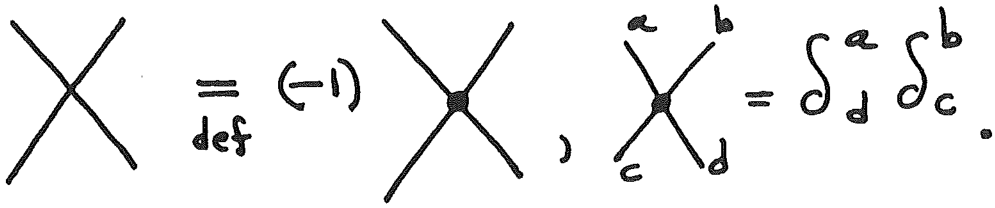
Then we have the identities



However it is still the case that a curl as shown below gives a minus sign.

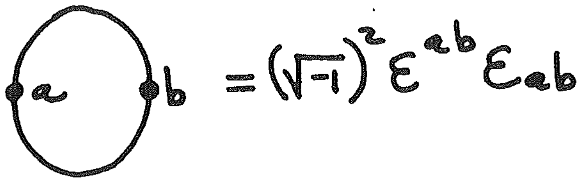


Therefore associate a minus sign with each crossing.



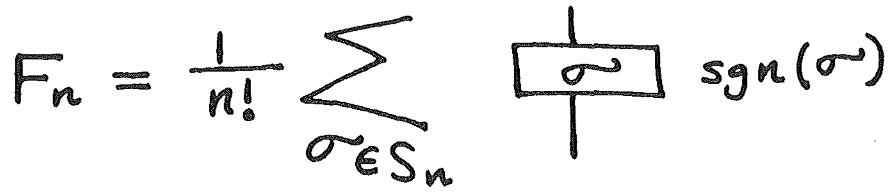
With this change of conventions, we obtain a topologically invariant calculus of modified epsilons in which the Fierze identiy becomes the binor identity mentioned previously, and the value of a loop is equal to minus two (-2) rather than the customary 2 that would result from $\varepsilon^{ab}\varepsilon_{ab}$. Here the loop corresponds to

$$(\sqrt{-1})^2\,\varepsilon^{ab}\,\varepsilon_{ab} = -2.$$

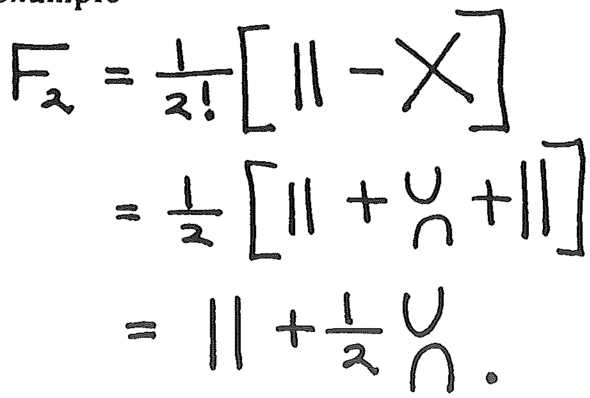


In any case, The result of this reformulation is a topologically invariant diagrammatic calculus of tensors associated with the group SL(2). We can now explain an important special case of the Temperley-Lieb projectors in terms of this calculus.

Consider the Temperley Lieb algebra where the loop value is -2. Form the *antisymmetrizer* F_n obtained by summing over all permutation diagrams for the permutaions of {1,2,...,n} multiplied by their signs, and divide the whole sum by n!.

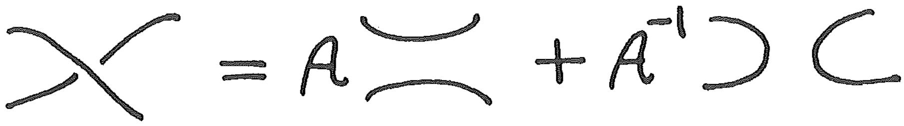


Transform this sum into an element in the Temperley-Lieb algebra by using the binor identity at each crossing.
For example



It is then not hard to see that $F_nF_n = F_n$ and that F_n annihilates each generator U_i of the Temperley Lieb algebra on n strands, for i= 1, ..., n-1. Thus F_n is the unique projector in this case of loop value equal to -2. These projectors serve as the basis for a recoupling theory as outlined in the last section. In this case, it is equivalent to the recoupling theory for standard angular momentum in quantum mechanics. Penrose reformulated angular momentum recoupling in this way, although he did not use the Temperley Lieb algebra nor did he extend the recoupling theory to arbitrary values of the loop variable as we have done.

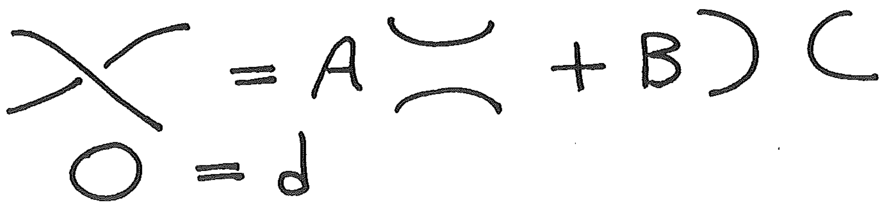
In the next section we shall use the bracket polynomial [KA87] to explain how the general recoupling theory is related to the topology of 3-manifolds. In that setting the binor identity becomes replaced by the bracket identity



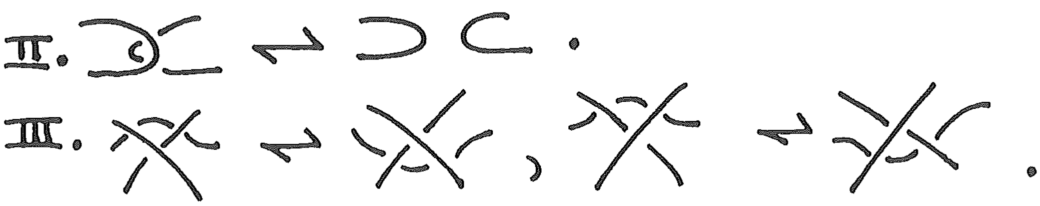
and the loop value is $-A^2-A^{-2}$. Note that when A=-1, the bracket identity reduces to the binor identity and the loop value equals -2. At -2 there is no longer any distinction between an overcrossing and an undercrossing and the bracket calculus of knots and links degenerates to the planar binor calculus.

For the remainder of this section I wish to discuss some of the issues behind the binor calculus and spin network construction from the point of view of discrete physics. Penrose originally invented the spin networks in order to begin a program for constructing a substratum of spacetime. The substratum was to be a timeless domain of networks representing spin exchange processes. From this substrate one hoped to coax the three dimensional geometry of space and the Lorentzian geometry of spacetime (perhaps to emerge in the limit of large networks). The program was partially successful in that through the Spin Geometry Theorem [PEN79] the directions and angles of 3-space emerge in such a limit. But distances remain mysterious and it is not yet clear what is the nature of spacetime with respect to these nets. Furthermore, from the point of view of the classical spin nets, the move to the topologically invariant binor calculus is a delicate and precarious descent into combinatorics. One wants to understand why it works at all, and what the specific meaning of this combinatorics is in relation to three dimensional space.

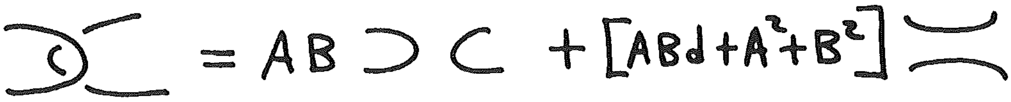
There is a radically different path. The radical path is the elucidation of the bracket polynomial invariant of knots and links. Start with a generalization of the binor identiy in the form



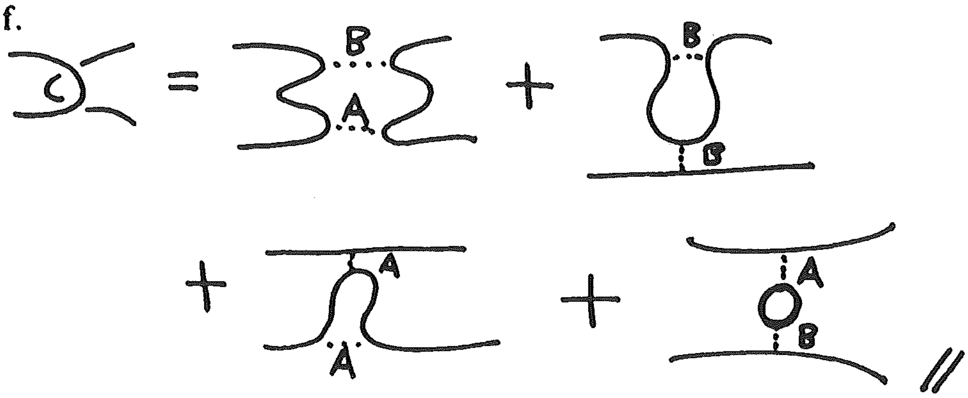
and ask that it be topologically invariant in the sense of regular isotopy generated by the knot diagrammatic moves II and III.



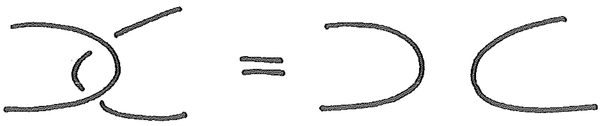
We see at once that



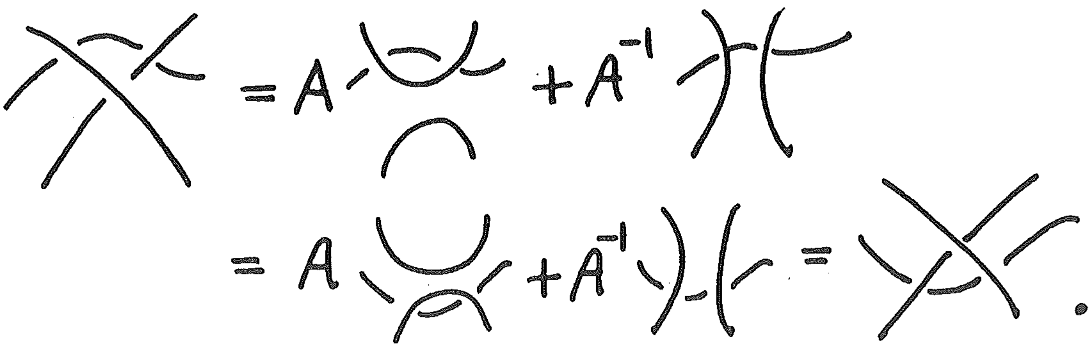
Proof.



Hence we can achieve the invariance



by taking $B=A^{-1}$ and $d = -A^2 - A^{-2}$. Then, a miracle happens, and we are granted invariance under the triangle move with no extra restrictions:

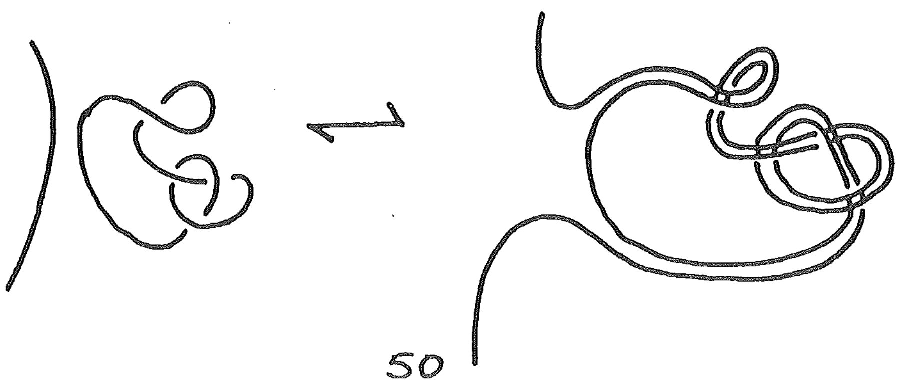


The rest of the mathematical story is told in the sections of this paper that precede and follow the present section. Let us stay with the discussion of discrete physics. All of the basic combinatorics of spin nets is met and generalized by a strong demand for network topological invariance (in three dimensions). The *topology* of three dimensions can be built into the system from the beginning.

The groups SL(2) and SU(2) (and the corresponding quantum groups [KA91]) emerge not as symmetries of metric euclidean space, but as internal symmetries of the network structure of the topology. Furthermore, it is only through the well-known *interpretations* of the knot and link diagrams that the combinatorics becomes interpreted in terms of the topology of three dimensional space. The knot-topological spin networks (with weave built in) become webs of pattern in an abstract or formal plane where the only criterion of distinction is the fact that a simple closed curve divides the space in twain. The knot theoretic networks speak directly to the logic of this formal plane. The significant invariances of type II and type III (above) are moves that occur in the plane plus the very slightest of vertical dimensions. The vertical dimension for the formal plane is infinitesimal/imaginary. Angular momentum and the topology of knots and links are a fantasy and fugue on the theme of pattern in a formal plane. The plane sings its song of distinction, unfolding into complex topological and quantum mechanical structures.

A link diagram is a code for a specific three dimensional manifold. This is accomplished by regarding the diagram as instructions for doing surgery to build the manifold. Each component of the diagram is seen as an embedding in three space of a solid torus, and the curling of the diagram in the plane gives instructions for cutting out this torus and repasting a twisted version to produce the 3-manifold. (See [KA91] for a more detailed description of the process.) Extra moves on the diagrams give a set of equivalence classes that are in one-to-one correspondence with the topological types of three-manifolds. These moves consist in the addition or deletion of components with one curl as shown below

plus "handle sliding" as shown below and in the next section



We also need the "ribbon" equivalence of curls shown below.

With these extra moves the links codify the topology of three dimensional manifolds. As we shall see in the next two sections, the recoupling theory and spin networks appear to be especially designed for handling this extra equivalence relation.

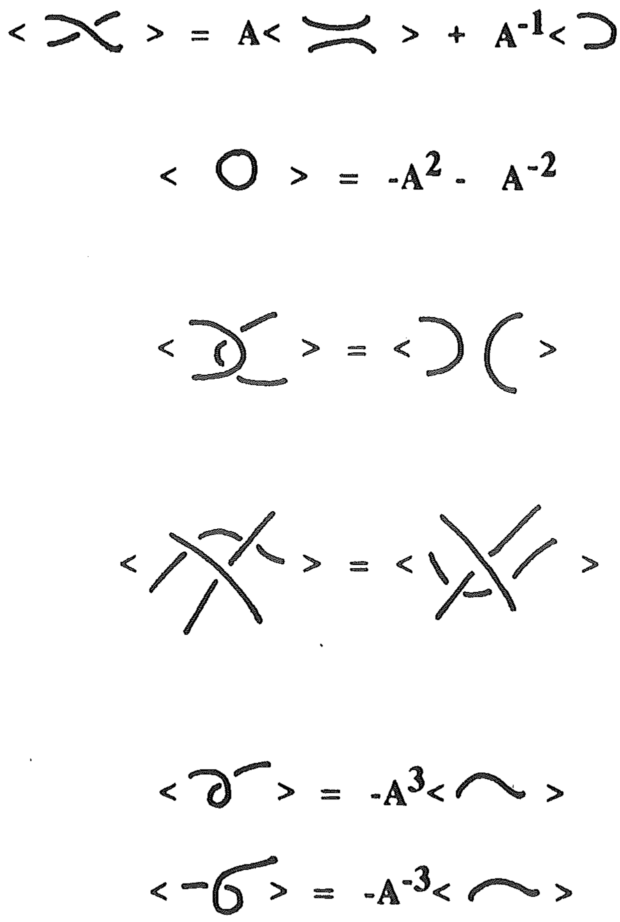
This is the real surprise. The topological generalization of the spin network theory actually handles the topology of three dimensional spaces. Each such space corresponds to a finite spin network, and there are no limiting approximations as in the original Spin Geometry Theorem. From the point of view of topology, each distinct three dimensional manifold is the direct correspondent of a finite spin network.

Where then is the geometry? Topology is neatly encoded at the level of the spin networks. Geometric structures on three manifolds, particularly metrics are the life blood of relativity and other physics. It is interesting to speculate on how to allow the geometry live in spin nets. One possibility is to use an appropriately fine spin network to encode the geometry. There are many spin networks corresponding to a given three manifold. We would like to have a mesh in the spin network that corresponds to the points of the three manifold. This could be accomplished by "engraving" the three manifold with many topologically extraneous handles, and then reexpressing the net as a coupling of these handles. This would make a big weaving pattern that could become the receptacle of the geometry. But really, this matter of putting in the geometry is an open problem and it is best stated as such.

The moral of our story is that the spin networks most naturally articulate topology. How they implicate geometry is a quest of great worth.

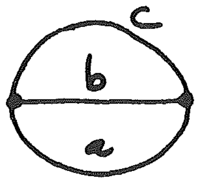
Knots and 3-Manifolds

This recoupling theory extends to trivalent graphs that are knotted and linked in three dimensional space. One way to delineate the connection is via the bracket model for the Jones polynomial [KA87]. In this model one obtains an invariant of regular isotopy of knots and links with the properties shown below.



Braids expand under the bracket into sums of elements in the Temperley Lieb algebra. In this context the Jones-Wenzl projectors can be realized via sums of braids corresponding to all permutations on n-strands. (See [KA91], [KL92].) In this context, the coefficient Δ_n is given by the formula $\Delta_{n-1} = (-1)^{n-1}(A^{2n} - A^{-2n})/(A^2 - A^{-2})$.

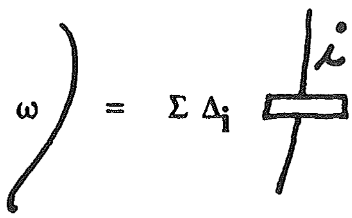
When $A = \exp(i\pi/2r)$ is a 4r-th root of unity, then the recoupling theory goes over to this context with an extra admissibility criterion for each 3-vertex with legs a, b, c. We require that a+b+c <= 2r-4. With indices ranging over the set {0,1,2,..., r-2}, everything we have said about recoupling goes over. In particular, the theta net evaluations

$\Theta(a,b,c) =$ 

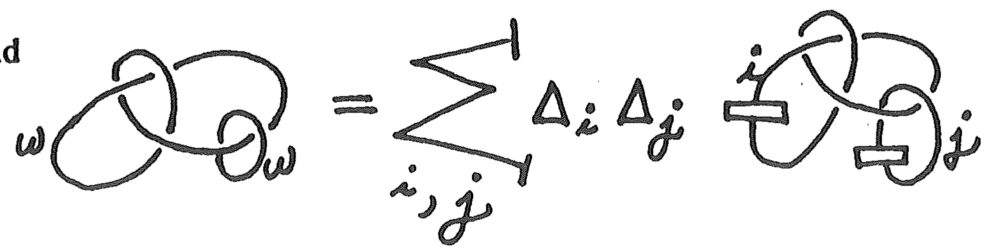
are non-zero in this range so that the network formulas for the recoupling coefficients still hold.

It is at the roots of unity that one can define invariants of 3-mainifolds. There are many ways to make this definition. A particularly neat version using the Jones-Wenzl projectors is given by Lickorish [LIC92]. We reproduce his definition here:

Let a link component ,K, labelled with ω (as in ω*K) denote the sum of i-cablings of this component over i belonging to the set {0,1,2,...,r-2}. A projector is applied to each cabling. Thus

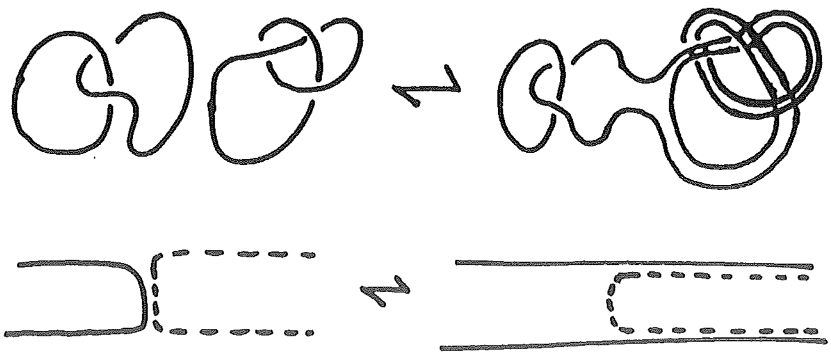


and

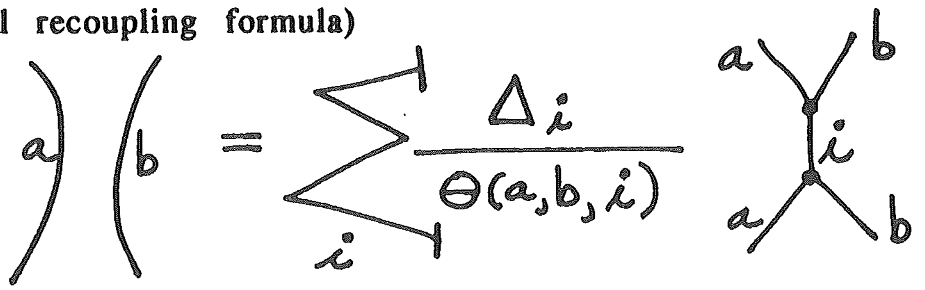


Then <ω*K> denotes the sum of evaluations of the corresponding bracket polynomials. If K has more than one component, then ω*K denotes the result of labelling each component, and taking the corresponding formal sum of products for the different cabelings of individual components.

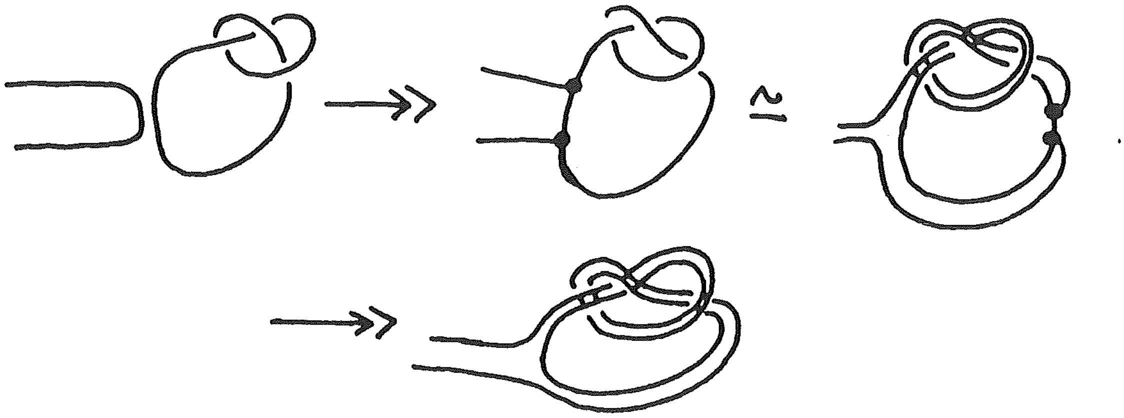
It is then quite an easy matter to prove that <ω*K> is invariant under handle sliding in the sense shown below.



This is the basic ingredient in producing an invariant of 3-manifolds as represented by links in the blackboard framing. (A normalization is needed for handling the fact the 3-sphere is returned after surgery on an unknot with framing plus or minus one.) A quick proof of handle sliding invariance using this definition and recoupling theory has been discovered by Justin Roberts [LIC92]. An even quicker proof using less machinery is given by Lickorish in [LIC92]. Roberts proof is based on the formula (a special case of the general recoupling formula)

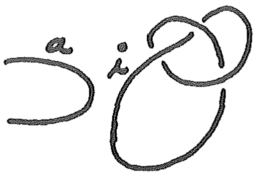


and the sequence of "events" shown below.

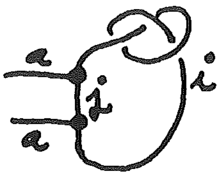


We turn these events into a proof of invariance under handle sliding by adding the algebra.

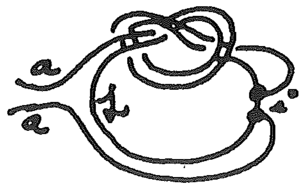
$$= \sum_i \Delta_i$$



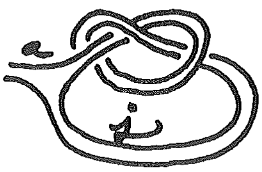
$$= \sum_i \Delta_i \sum_j \left(\frac{\Theta(a,i,j)}{\Delta_j}\right)^{-1}$$



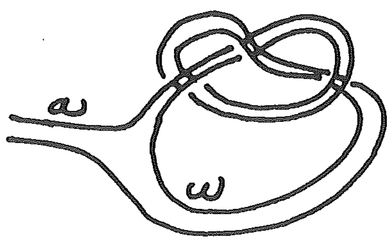
$$= \sum_j \Delta_j \sum_i \frac{\Delta_i}{\Theta(a,i,j)}$$



$$= \sum_j \Delta_j$$



$$=$$



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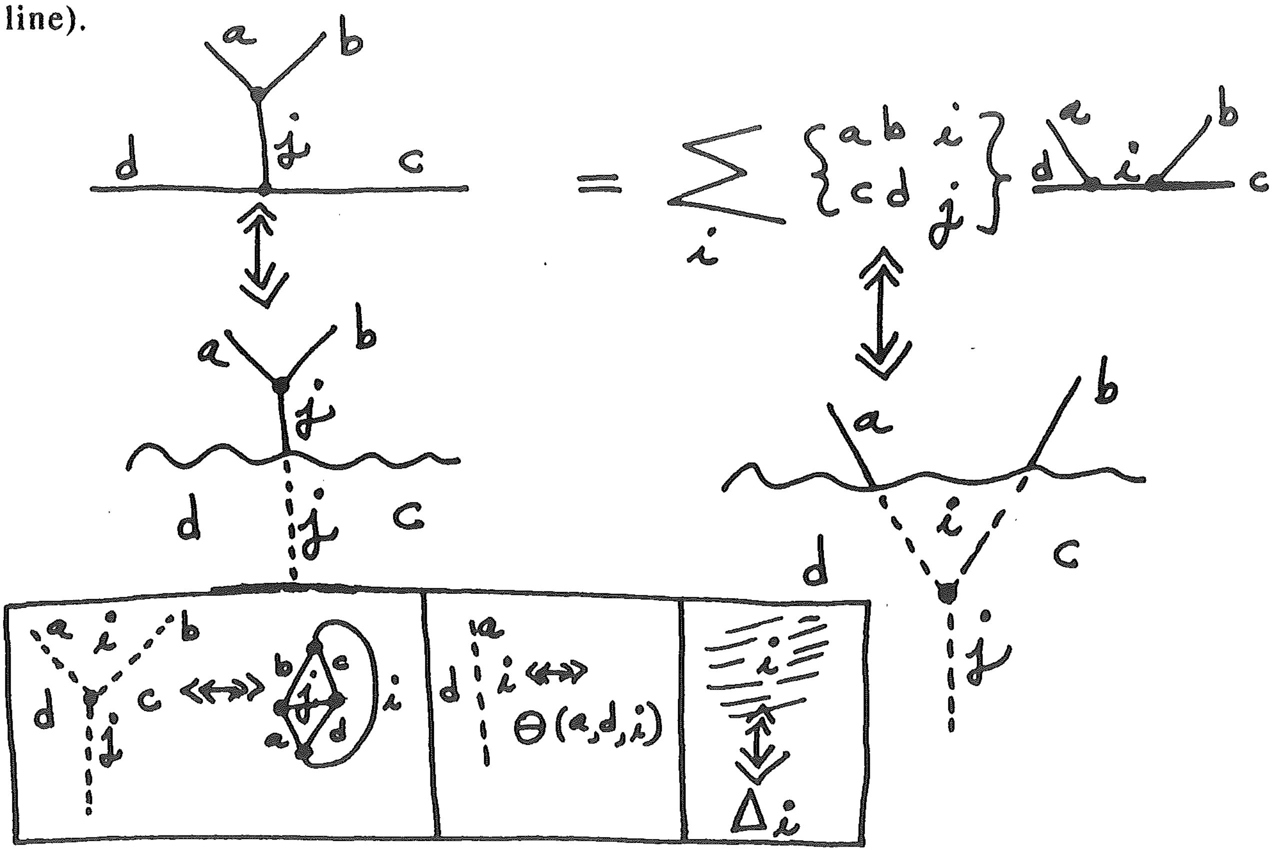
VII. The Shadow World

In this section we give a quick sketch of a reformulation of Kirillov-Reshetikhin Shadow World [KR88] from the point of view of the Temperley Lieb Algebra recoupling theory. The payoff is an elegant expression for the Witten-Reshetikhin-Turaev Invariant as a partition function on a two-cell complex.

Shadow world formalism rewrites formulas in recoupling theory in terms of colorings of a two-cell complex. In the case of diagrams drawn in the plane this means that we allow ourselves to color the regions of the plane as well as the lines of the diagram with indices from the set $\{0,1,2,\ldots, r-2\}$ (working at $A = \exp(i\pi/2r)$, as in the last section). In this way a recoupling formula can be rewritten in terms of weights assigned to parts of the two-cell complex. The diagram below illustrates the process of translating between the daylight world (above the wavy line) and the shadow world (below the wavy line).



In this diagram we have indicated how if the tetrahedral evaluation is assigned to the six colors around a shadow world vertex (either 4-valent or 3-valent) , the theta symbol is assigned to the three colors corresponding to an edge (the edge itself is colored as are the regions to either side of the edge) then we can take the shadow world picture as holding the information about 6-j symbols that are in the recoupling formula.

To complete this picture we assign Δ_i to a face labelled i, and there are phase factors corresponding to the crossings. We omit discussion of these phase factors here. The shadow world diagram is then interpreted as a sum of products of these weights over all colorings of its regions edges and faces.

See [KL92] for a complete treatment of this topic.
The result is an expression for the handle sliding invariant $<\omega^*K>$ (of section 5) as a partition function on a two-cell complex. The extra Δ_i's coming from the assignments of ω to the componments of the link can be indexed by attaching a 2-cell to each component. The result is a 2-cell complex that is a "shadow" of the 4-dimensional handlebody whose boundary is the 3-manifold constructed by surgery on the link. This marks the beginning of the newer shadow world techniques of Turaev [TU92], and is a good place for us to conclude this discussion.

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Quantum Theory and Consciousness

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Abstract It is argued that quantum theoretical ideas can solve many of the problems of consciousness; in particular, those concerning the nature of qualia and their externality.

1. Quantum theory and consciousness?

I want to begin by denying my title. To begin with 'Quantum theory'; like most of those here, I doubt that there is such a theory, in the sense of a generally recognised, fully articulated and interpreted, coherent formulation. What there is, however, is a fairly clear idea of 'doing quantum mechanics', as a teachable procedure, and a fairly clear range of 'quantum phenomena', for which doing quantum mechanics produces good results. Consequently, the ideas I shall be surveying fall into two classes: conservative theories, claiming that the sort of things that theoretical physicists do in predicting quantum phenomena can also be used to shed light on consciousness; and radical theories (or rather, prototheories) which aim at embracing both quantum phenomena and consciousness in a way that makes sense of both of them.

I will return to such theories later. But first, I need to query the second term in my title. The word 'consciousness' is misleading, being a noun that suggests some single object or faculty responsible for the great variety of phenomena that are usually associated with the word. Consequently I shall try to use instead the adjective 'conscious' or the verb 'aware'.

With these qualifications, I can start to examine my topics, beginning with the second.

2. The conscious/non-conscious division

Given the great range of available approaches to consciousness, I want first to try to pose the problem in a way that does not beg too many questions at the outset. We know that, at any given moment, we are aware of a whole range of things: external objects, abstract thoughts, bodily sensations, emotions, memories and so on. On the other hand, there are lots of activities in which our brain is involved which do not seem to correspond to anything we are aware of: from the 'background' running of our bodies to the quite sophisticated ways in which things picked up by our senses modify our behaviour without our being aware of it. (Marcel, 1983). There is thus a division between brain processes that do correspond to things we are aware of, and those that do not. Any theory of 'consciousness' must, at the very least, give some understanding of this division, and it is on this that I shall focus. It is worth noting that a great many theories fail even at this first hurdle.

A word of caution is needed before proceeding. We are not aware of brain-states (with the possible exception of migraines, etc.) - we are aware of trees, chairs and the rest. Of course, it could well be that my awareness-of-a-tree is directly caused by, or simply is, a brain state; or even that there is in reality no tree there, but only an ingenious arrangement of projectors and mirrors (so that I am aware-of-a-tree without there being a real tree that I am aware of; we have to include figmentary trees as objects of awareness). 'Aware-of-x' is here purely a psychological description, not necessarily implying any actual relationship with x. I shall, for brevity, call those (components of) brain states that correspond to things I am aware of conscious states; but it is not thereby implied that I am conscious (aware) of those states.

There is a certain fuzziness in this division into conscious and non-conscious states. While some, such as the control of blood pressure, are almost invariably non-conscious, there are other things going on that could be conscious if we attended to them. And there is a grey area: when asked about an experience, we might well reply 'Yes, now I think back I did see that, even though I wasn't really aware of it at the time'. But the fuzziness of the division does not detract from its existence.

If our awareness is entirely a function of our brain-activity - and this is a big 'if' to which I will return - then there is presumably something, or some dynamical process, in the brain that is responsible for the conscious/non-conscious division. Having eschewed the word 'consciousness', I call this the 'C-structure'. By this I do not imply that it is a fixed physical object. Nor do I imply that it is what Dennett (1991) calls 'the Cartesian Theater': a supposed internal screen on which all our percepts are projected so as to bring them all together. I would agree with Dennett that the reasons for requiring such a thing are misguided. But neither is there any reason to rule out such possibilities. At one extreme, the C-structure could be (a) just an ad-hoc collection of particular information handling processes; or, at the other extreme, (b) a particular physical entity or anatomical structure.

3. Arguments for a concrete C-structure

Consider first (b). There is strong intuitive support for this via the idea that consciousness is somehow special, and that there is, according to some, an essential difference between conscious and non-conscious brain processes. This difference, it is held, involves the presence of qualia in our awareness, and the fact that the things we are aware of are somehow unified into a single act of awareness. Let us look at these, the arguments from qualia and unity, in turn.

(i) Qualia.

What is a quale? Perhaps the only comparatively pure example is that of colour. Suppose I pick up a specimen of wallpaper and pronounce it to be red. Those who deny qualia would say that the psychological processes leading to this judgement involve forming the intellectual concept of 'redness', which is a compound of 'what blood, British letter boxes and sunsets have in common' together with emotional overtones that are 'hard-wired' in our brains through evolution. On this view, when I view the wallpaper a number of comparison-mechanisms in my brain bring up 'blood', 'letter boxes' and so forth, which are then stirred-together by the speech-producing mechanisms which eventually produce the utterance 'red', as an appropriate summary of all these comparisons.

Now, while it may turn out that this is an appropriate description of some neurological processes, it is certainly not what I am aware of. When I say 'red', I am not reporting an awareness of letter-box-likeness; I am reporting an awareness of redness and nothing else. At the level of awareness, if not at the neurological level, redness is irreducible. Such irreducible components of our awareness are called qualia.

The aim of Science should be to explain our world, the world we are aware of, and this world contains qualia. If the best that science can do is to produce a scientific model which is declared to be the 'real' world, followed by a dismissal of qualia as things that do not 'really' exist, then science has failed its proper task.

There still appears, however, to be a problem with the notion of qualia. If 'red' means something irreducible, subjectively, then one is prompted to ask, does it mean the same thing to Dr Smith as it does to Dr Jones? If 'red' were just 'blood-letter-box-sunset' then the question would pose no great difficulty: they could discuss their use of the word in these terms and agree that they concurred. But if 'red' is an irreducible internal sensation, as I believe it is, then the possibility arises that the sensation experienced by Dr Smith when he looks at a letter box may be the same as that experienced by Dr Jones when he looks at the cloudless sky, and vice versa. And, alas, there would appear to be absolutely no way of determining whether or not this was the case. Thus the notion of qualia seems to saddle us with a collection of words - 'red', 'blue', etc - which could be used totally differently by all speakers, and thus in fact mean nothing whatever. This is often taken as a reductio ad absurdum disproof of the existence of qualia.

This argument is actually oversimplified: the fact that 'red' may mean different things to different people does not imply that it is meaningless, but that it is token-reflexive: a word (like 'now' or 'I') whose meaning depends on the particular instance of its utterance. Unlike 'I', however, it seems that even when I know the instance of utterance of 'red', I still will not know the primary meaning of the word to the speaker. The word nonetheless continues in use, both because its secondary meanings (namely, the blood-letter-box etc complex) are communicable, and also because the logical relations between different colour-words enable them to mean something communicable when used together.

Thus we are not reduced to an absurd situation by this argument, but we are left in a decidedly unsatisfactory position. I will be arguing at the end that if the C-structure is quantum mechanical, then there is in fact a basis for interpersonal comparisons, and the situation is made much better.

So, if qualia exist, does that mean that the C-structure has to have some very special qualia-producing physical properties? Unfortunately, or perhaps fortunately, this is not the case. For there is one obvious and enormous difference between conscious and non-conscious brain states that is quite enough to explain a difference in quality, without having to invoke special properties of the C-structure: we <u>are</u> our conscious states, but we <u>infer</u> our non-conscious states. In other words, the distinction between having qualia and not having qualia is the distinction between experiencing a state from the inside and examining it from the outside.

This principle will be the core of my account of awareness. But it raises difficulties at this stage that can only be resolved later by introducing quantum mechanics. For the qualia we have been talking about, such as redness, are not 'inside' but 'outside' on the wallpaper. It would seem that, having experienced 'redness' by participating in certain brain-states from the inside, we then have to somehow project it outside, to marry it with the spatial perception of an external piece of wallpaper. While this account is logically acceptable, it starts to be an account of hypothetical neural processes rather than of direct experience. We need, if possible, to do better.

(ii) Unity

Our awareness is not of a collection of disconnected entities, but of sensations that are interlinked, it seems, into a whole. The experiences are all 'mine', 'now'; it seems as if there is, moment by moment, a single act of apprehension that grasps all the environment, external and internal, together. This idea will be decisive for subsequent theories.

Lockwood (1989) has argued that this view is in fact spurious. There is a certain inter-linking of our percepts - the external objects are connected by adjacency between neighbours; emotions may be 'latched on' to particular objects - but, he argues, the perceived situation is not one of everything being linked to some central point (an 'I'), but of a patch within a continuum of percepts linked in a complex fashion to one another. I shall call these the centralised and extended view, respectively.

The problem in deciding between these is that both correspond to experience at different times. Consciousness can be shifted (particularly, but not only, through meditation techniques) from being highly centred with both percepts and discursive thoughts peripherally linked, to being entirely absorbed in external percepts in an extended way. The argument from unity thus remains an open question.

4. Arguments for an ad-hoc C-structure

We have noted that, while qualia exist, they do not demand a physically distinctive C-structure; while the argument from unity leaves the question open. We can thus entertain the option (a) at the end of §2 that the C-structure is simply a collection of neural processes that are linked together by some sort of functional consensus. This suggestion, which is favoured by Lockwood and would be supported by many of Dennett's arguments, would mean that the experience of being aware of things, with their qualia, would be had by any suitable collection of neural processes, and hence that there may be many different such patches of awareness associated with different groupings of processes in our brain. The one that is named as 'I' is simply the one that has most immediate access to language centres of the brain. Thus on this view the C-structure is just a set of neural connections between certain information-processes and the language centres.

There is considerable economy in this approach, and I believe it deserves to be taken very seriously. In the present context, I must stress that it does not rule out the possibility that all the higher nervous processes are in fact quantum mechanical. It is to this that I turn next.

5. Quantum Mechanical Process

I shall now survey a few recent approaches to quantum mechanical accounts of consciousness, which derive support from the arguments either in section 3 or in section 4. Discussion of this issue is confused by the way different authors take different default positions. Some assume that everything is classical and Newtonian unless proved otherwise, others assume that the universe is fundamentally quantum and regard the emergence of Newtonian physics as an unsolved mystery. The burden of proof is interpreted very differently by each. Both would agree that classical physics is the relevant choice when we are dealing with an array of quantum elements linked together with random quantum-phases, whereas quantum mechanics must still be used if the phases of the elements are correlated (a coherent assemblage). The classical-default authors take the atomistic (reductionist) position that all organisms are built up by assembling atoms at random, and so are classic unless a special mechanism intervenes to correlate the phases. The quantum-default authors regard the organism as having a quantum state in its own right, so that the phases of the atoms are correlated by virtue of the fact that they are also constituents of the state of an organism.

(i) Marshall

Iain Marshall (1989) has proposed as the source of consciousness, a specific mechanism for establishing quantum coherence in the brain, which would meet the demands of the classical-default authors. It is a 'conservative' theory, based on the accepted procedures of 'doing quantum theory'. This mechanism is the pumped phonon model, first introduced by Fröhlich. For its operation we need an array of electrically polarised molecules (with positive charge at one end and negative at the other), each of which is able to vibrate mechanically. This could be realised either by the molecules making up the cell-membranes in the nervous system, or the proteins embedded in these membranes. The array need not be ordered. Because of the electric charges, the oscillations are linked together, and the motion of the whole system can be described in terms of electro-mechanical waves (normal modes) permeating the whole array. The frequencies of these modes, $(\omega_1,\omega_2, ..., \omega_N)$ - though not the energies with which they are excited - are determined purely by the geometry of the array and the nature of the molecules.

The Fröhlich model also requires a source feeding energy to the array (the 'pump'), and a bath maintained at a constant temperature into which this energy drains. In the cell the pump might be the processes driving the electrical activity of the neurones, and the bath will be the cellular and inter-cellular fluids. If the system is treated quantum mechanically then it is found (either by solving simple examples or simulating more complex ones) that at large pumping rates a large fraction of the energy is concentrated in the normal mode of lowest frequency, in which all the molecules move in phase, with the rest of the energy distributed over all the other modes. This result depends on the Bose statistics of the 'phonons' making up the normal modes - it is called a Bose condensation.

One technical drawback is that it appears to require that the spacing of the frequencies $\omega_1,\omega_2, ...$ be non-regular: if $n(\omega)\delta\omega$ is the number of $\omega's$ between ω and $\omega + \delta\omega$ in the continuum limit then it is required that $\int_{\omega_1}^{\infty} \frac{n(\omega)}{\omega-\omega_1} d\omega$ be convergent in order for this result to hold exactly. In practice this condition may not be met, so that the Bose condensations may be only approximate.

If there is in the brain a highly energised motion in which all the membrane-molecules are oscillating phase, then this would be an excellent candidate for the C-structure if one takes the centralised view of §3ii and requires a single system that can be linked to a wide range of different neural processes. As is involves coherence there is no way of avoiding treating it quantum mechanically.

Regrettably, there have so far been no direct experiments on any aspect of the Fröhlich mechanism, though there are several indirect observations.

(ii) Penrose

In his book 'The Emperor's New Mind', Penrose (1989) introduced ideas that could lead to the sort of theory that I earlier classified as radical, rather than conservative. It involves bringing together ingredients from many disciplines: computer science, relativity, quantum mechanics, neuropsychology.

His starting point is a combination of scientific realism (the view that the 'objects' talked about by science correspond to entities in some absolute reality) with a view that quantum mechanics (more precisely, a major modification of it which incorporates gravity) is the basic language for describing the universe. Thus the wave function is a real physical entity and is applicable to systems of all sizes, including the human brain. Since we do not experience superpositions of widely different perceptions, this means that the wave function of our brain is subject to collapse as well as to Hamiltonian (Schrödinger equation) evolution.

The central hypothesis is that this collapse is a non-algorithmic process mediated by quantum gravity. The book gives arguments for both parts of this hypothesis, which are too detailed to summarise here. It is then likely that this is used by the brain for non-algorithmic problem solving, and in particular for the insightful thinking characteristic of consciousness. Hence quantum collapse plays a central role in conscious thinking.

Moreover, since quantum gravity is involved, the collapse is determined by gravitational properties. The simplest criterion for collapse along these lines is that when a state is in a superposition

$$\psi = \psi_1 + \psi_2$$

then a collapse takes place whenever the number of longitudinal gravitons in the gravitational field $g_1 - g_2$ exceeds unity (where g_1 and g_2 are the gravitational fields of ψ_1 and ψ_2 respectively).

The interesting thing about this theory, which some would regard as too speculative to be entertained seriously, is that it is capable of being tested experimentally.

Calculations suggest that this is only very marginally satisfied by brain states. Thus if the gravitational fields of brain states could be enhanced, then collapse would become very much more likely and so conscious thinking should be affected in some way. The gravitational effects of brain states can be enhanced by many orders of magnitude simply by connecting to the scalp a conventional EEG with a pen recorder. Thus having an EEG connected should in itself affect thinking, without there being any normal electrical (or other) feedback from the EEG to the subject. Remarkably, an experiment now being carried out at Southampton General Hospital under the direction of Dr Chris Nunn suggests that this is indeed the case.

(iii) Lockwood

The Theory presented in Lockwood (1989) is the most interesting, and radical, of all. He starts from the extended view of consciousness reviewed in §4, in which a kind of consciousness is associated with many clusters of brain-processes. He applies this to a universe described by a quantum mechanical wave function in which no collapse takes place: the evolution is purely quantum mechanical. Thus many different versions of 'me' are present, superposed, in this wave function. Each of these has a consciousness, just as each logical region of my mental processes has a 'consciousness' (though only the one linked with speech has the properties of an 'I'). Thus consciousness becomes a particular perspective on a global uncollapsed quantum universe. It appears to any one perspective that a collapse has taken place.

Reviewers of the book have complained that Lockwood has not produced sufficient evidence for the mind being a quantum entity. But this is to misunderstand his argument completely. He is not using quantum theory to explain mind, but the reverse - using a theory of consciousness to make sense of quantum theory. The theory would be vindicated to the extent that, in the end, it did make sense.

6. Quantum mechanics and qualia

I have shown that a quantum mechanical origin for the C-structure is a live scientific issue that is capable of experimental test. I want to go on from this to argue that, if mental processes do have a quantum mechanical basis, then we can resolve many of the philosophical problems associated with qualia that I reviewed in §3(i). The key to this is the very simple distinction existing in quantum mechanics between systems that, in a given state, are separable - where the state is a tensor product of system-states - and systems that, in the given state, are not separable. If the two systems are engaged in an interaction then they are not separable until the interaction is over and a collapse has taken place (or whatever replaces 'collapse' in a different formalism).

Perception is an interaction. So while perception is under way, while I am still deciding what it is that I am perceiving, I am not separable from the system that represents those aspects of the external object that I am perceiving. Quantum theory thus predicts that, for example, the physical colour of an external object is, in the momental of perception, an inseparable part of my own mental state.

I have explained in §3(i) how the qualia are brainstates 'from the inside'. We now have a natural way of including certain aspects of external objects along with these states. Thus brain states 'from the inside' will include external states 'from the inside'. These are clear candidates for the qualia of external perception. We see that they will automatically be external, with no need of the projection that we found so unsatisfactory at the end of §3. Moreover, since the redness of an object as perceived from the inside by me is the same external physical as state as that perceived by you, it is a corollary of the theory that perceived qualia are in fact the same for different people, even though this is not directly verifiable.

There is, however, a severe difficulty which implies that this picture, though it contains the essence of a solution to the problems of consciousness, is not the whole truth. Namely, perception can be radically mistaken. We may think we are seeing a rose, when it is only a picture. Or worse it may only be an after-image or a similar neurologically based delusion. And yet so long as we are convinced that it is the real thing, all the qualia are perceived, and are perceived as external, though there is no object there in whose quantum state we are participating.

While it complicates the situation, the difficulty is not fatal: there are at least two solutions between which further investigation could decide, but both take us beyond the 'conservative' area of quantum theory and force us radically to revise our physical concepts, as well as our psychological ones.

(i) We could regard qualia as in essence learnt by an initial contact with the actual object. Behind this idea lies the observation that, in the awareness of an object, the centre of gravity - so to speak - can lie anywhere between the observer (when he/she is absorbed in the abstract and conceptual significance of the object) and the object (when the being of the object fills attention to the exclusion of the observer's own ego). Only in this last extreme, a mode of consciousness that, after early childhood, can only be captured by special meditative techniques or in a sudden breakthrough, does full participation in the object itself take place. At other times we have a mixture of the actual object with warmed-up memories of earlier encounters.

This then requires us to explain what is happening when we experience a memory. If it is accompanied by qualia, and if qualia arise from participation in a quantum state, then we have to suppose that we are participating in (i.e. physically combining with) a past quantum state. Most conventional physicists would be unable to accept such a possibility, on the grounds that physical interaction is always local in space-time. Yet we have come to accept that the quantum state is not local in space, as demonstrated by the ERP-Aspect phenomenon, and relativity teaches us that what goes for space goes for time also. Though we do not at present have a formulation of quantum theory that involves states that are non-local in time - the 'Newtonian-ness' of time in quantum theory is notorious - there seems no fundamental reason against it.

Support from the idea of participation in past states would come from Sheldrake's (1989) theory, in which such participation is part of the dynamics of the universe. I would see it more as defining what a state is, leaving the dynamics (in the strict sense) to the Schrödinger Equation. Taking the evidence presented by Sheldrake into account, there seems to be positive advantages in looking for a reformulation in which participation in past states is possible. This would be what Zohar (1990) calls quantum memory, a concept that does not exist in the formalism but is clearly called for by the interpretation.

(ii) The first account has been firmly within a realist setting. I have been assuming that present and past external objects are simply given, to be participated in. There is an alternative account, however, which veers a little to the idealist end of the spectrum. We could see the universe as involved in a constant act of self-creation in which a key role is played by consciousness; or, more generally, by that aspect of things 'from within' of which consciousness is the human example. I am thinking of a hybrid between Lockwood's picture and Penrose's picture, in which the world is always a superposition of possibilities, with a plurality of perspectives, but a plurality that is constantly creatively resolving itself through an internal dialogue.

On such a view there is no hard distinction between an apparent rose that is really an hallucination and a real rose waiting to be smelled. Nor between perceptions in the light of day and the fantasies of dreams. All contribute, more or less fleetingly, to a shifting plurality of the possible.

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A Philosophical Focus ANPA 92

Ted Bastin

An essential characteristic of the ANPA endeavour (I argue) is to make certain traditional philosophical issues come alive again as relevant to physics. At the same time we may see some new vistas in information studies. The Newtonian conceptual framework is so entrenched that to ask for a change at the philosophic level may seem to be asking for the moon, but there is a hope that the combinatorial hierarchy can provide the necessary leverage. This, to my mind, is why it has a central place in our discussions, and should continue to have until someone can come up with something else which offers a better hope of cracking the carapace. There seems to me to be a distinction between new theories (or programmes for new theories) which have could force a change at this philosophic depth, and those which do not and are technical innovations. My opinion is that the first kind should be the focus of our endeavour.

2. It is very difficult to get the traditional philosophical questions which I have in mind treated as other than historical curiosities by the philosophers. This seems to be because the Newtonian outlook which still provides the conceptual framework for physicists in spite of modern changes, has to a remarkable extent been implicitly accepted by the philosophers. They accept that there is a world of material things and that there is mental activity which is concerned with information about those things, and that the distinction between them is commonsense, merely. All suggestions which would revise this line of demarcation are relegated to the archaic, for those areas of discussion have been relegated to the technical, and what remains is a small part of what philosophy once was. (Mainly to do with logic and language).

Pope has pointed out that a change in philosophy took place at the hands of Moore, Ryle and others, and of the later Wittgenstein, which put the kind of metaphysical questioning which I am hankering for out of date. Now I entirely accept that the sublety of the new philosophy can never be dispensed with, now that we have it, but I would turn the argument round. The exponents of the new philosophy <u>would</u> find themselves having to come to terms with the old metaphysical puzzles (and doubtless their conclusions would have suffered a sea change) if they opened themselves to the whole range of knowledge. In my experience they consistently refuse to do this by the device of refusing to handle what they call the technical; as I have already argued.

3. There is some interest at the moment in generalizing the combinatorial hierarchy (henceforward CH) beyond physics. Alvarez

de Lorenzano (ANPA 91) uses the term "evolutionary systems" which is convenient and I hope he will allow it to be used without strict reference to his study. He picks up my attempt to characterize evolutionary systems as depending on a boundary between the known and the unknown, and comments on it in a very penetrating way. I think he sees the characterization as being appropriate to any evolutionary or constructive theory. Now with that starting point we find we are committed to a view which won't fit into the straight-jacket of current philosophy. This known/unknown boundary is certainly needed in Kilmister's constructive form of CH, though I think it is implied already in the use of the discrimination operation. Anyway according to the CH view, the most fundamental properties of the physical world -those that provide the fixities of the elementary systems and hence what are conventionally thought of as the building blocks for everything material- depend for their existence on the construction at the boundary. Hence it follows that the world of the empirically given, or the material, is derivative from principles describing how we know things, and in general from a mentalist position.

4. I shall be happy if serious discussion sets in about which of the traditional philosophers best represents constructive physics. Some of the founders of the quantum theory thought that Kant got nearest. It is indeed interesting as an example of the discussion I would like to see, to think about the critical philosophy. Recall that in CH we make the important assumption that we will treat all the choices of constructed elements that are allowed by the mathematics indifferently. If we do not do that we do not get our numbers right. However we are making an assumption about the unknown background which is not forced on us. Indeed, what we could call truly empirically knowledge will always come through some sort of weighting of the input -the order in which the unknown elements present themselves. That is our only access to the unknown. These therefore are like the Kantian noumena, the <u>Dinge an sich</u>. However when we look at space and time which for Kant are the a priori necessary forms of intuition we find we have changed from his position. Space, at least, is a product of CH and therefore dependent on the indifference assumption. It is not a priori necessary, and our thinking encourages us to look for other ordering principles if we want (as some of us here do) to apply constructive thinking to a wider class of evolutionary situations. So our view is Kantian only up to a point, though it seems that Kant was righter than the more modern philosophers.

Among philosophers, the figure of Leibniz is overshadowing. It would be, if only because inception of combinatorics was his, but there is much else and I cannot go into it now. Alison Watson has written on Leibniz in the course of her head-on encounter with the academic philosophers.

5. Some of the founders of the quantum theory were well aware that a radically new philosophy (Kantian or other) was implied by it, and they worked to provide epistemologically appropriate foundations for that. Unfortunately physicists in general were so anxious to resume their dogmatic slumbers in a Newtonian conceptual frame work that they have adopted a trivialized version of the epistemology which is nonsense. I and others have

discussed this situation in previous meetings and elsewhere.

6. Many people at ANPA meetings are interested in concepts of information and in what we may call information structures. One might think that to recognize the place of information will of itself mitigate the domination of the Newtonian conceptual framework by introducing a mental aspect to counterbalance the material. Thus Stonier argues emphatically ("Information and the internal structure of the universe") that 'information' is a thing in its own right as much as matter or energy. In the language of the scholastic philosophers he sees it as a distinct substance. He develops this view in terms of the definition of information provided by Shannon and others who use the model of the telephone line and its capacity for handling messages consisting of on/off symbols. However the Shannon definition presupposes that we have the physical world there always to start with (if only to provide the telephone lines) and this is just what we have had to reject.

7. If we are to use the information concept it seems we need a definition which is relational -meaning that we speak of things as units of information when a certain relationship obtains with other things which are not to be seen as information. If I drive past another person in another car and see him/her make a hand movement past his/her nose, I may suspect that they were making a signal, though there may have been an irritating fly in the car. How do I decide? Well not by any kind of inspection to see if there were really information present or not. Skarratt broke new ground when he insisted that in order to use the language of information there needs to be a relationship of recursion. An information structure is one where recursion is defined in such a way that one can work to-and-fro between levels. Without this provision it is meaningless to ask whether a thing is informational or not. I need hardly point out that CH is an information structure in this sense.

8. I have argued that physics is a special case, even among evolutionary systems which start off as it does, because of our assumption of indifferent choice. If one wishes to get a more general vision of evolutionary systems one may be tempted to follow the physics because it gets hard answers, but one has to be careful: its forms of orderings may not be the only ones, and if we assume they are we may rule out the discovery of everything we are looking for. Lorenzano (loc,cit.) has drawn attention to some things in CH which he thinks apply more generally:-

(1) I have mentioned his discussion of the boundary between the known and the unknown at which novel construction takes place. At that boundary we become aware of novelty through the persistent occurrence of sequential relationships because they would be improbable on the indifference assumption. Such relationships are our model for empirical discovery.

(2) Lorenzano derives a form of dimensional structure (the 3D) from the existence of recursive levels which will occur in any evolutionary theory. This is in his discussion of 'neural networks' -a phrase which he seems to use innocently of any connexion with brains. If I read him rightly he relates the number of dimensions to the level structure (connecting the

argument with Rosenblatt's perceptron theorem). This agrees with my discussion of the origin of dimension structure of last year, when, you may remember I had to produce a rather complicated argument to explain the apparent over determination of the 3D since it occurs as the smallest complete number of discriminately closed subsets (you can't prove the same thing twice from principles which are unconnected). Lorenzano produces what may be an important twist in the argument, claiming (rather too briefly) that the discrimination operation "cannot be defined in a two-dimensional network". (I suppose this means a two-level network.) This could be the step I have missed.

(3) I quote a very suggestive passage from Lorenzano dealing with general evolutionary systems:-
"Keeping in mind the dichotomous nature of constructibility within evolution (undifferentiated vs differentiated), I have taken the view that our problem requires three basic principles upon which the construction can be defined and take place: two principles of systemic unfolding (one dealing with the undifferentiated, the other with the differentiated), plus a third dealing with the way in which their conjunction will take place." We have long been familiar with the two principles of unfolding in the special case of CH. I shall end by returning to physics to show the third principle in action in McGoveran's hydrogen atom theory.

9. We have had McGoveran's treatment of the hydrogen atom and his associated calculation of the correction term to the fine-structure constant with us for several years, but perhaps there are those of us who, like me, have not found it altogether plain sailing. I will give a bit of detail about how I have found it easiest to approach it. In the first place the theory is very important to understand not only because of its success but also because it forces us to be unambiguous about the mathematical implementation of the object-system/environment ambiguity. This ambiguity is what enables McGoveran to consider he is dealing with an indefinite number of hierarchy constructions whereas with Kilmister's construction (and certainly with any of the earlier less explicitly constructive accounts of CH -following Parker-Rhodes) one had the embarrassment that one could only construct the hierarchy once. One could not claim one had acheived a view of the physical world, even though one might argue that the construction had to take place somewhere to get the vital numbers. You will remember too that in some early forms of 'Program Universe' a virtue was made of this seeming necessity by identifying the once-for-all construction with cosmogenesis.

McGoveran's theory is an astonishing piece of virtuosity. Though he says that once started he really only followed Dirac's account, in fact almost the whole is new. Contrast this with any of the familiar pieces of mathematical innovation in physics (Dirac's relativistic extension of the wave equation, for example) where the changes from what has been done before amount only to one or two steps. It is very difficult partly because McGoveran uses the structure of the quantum numbers which are usually attributed to the proton-electron system forming the hydrogen atom (longitudinal, orbital and azimuthal) partly, but only partly, abstracted from their geometrical and dynamical interpretations, as a guide to enable him to select the bits of the hierarchy (numbers of elements or strings) to combine to represent the correction due to the ambiguity. It is very near to the heart of our whole approach that we are seeing the particles systems as things thrown up as the informational limits of our modes of description, and so we are right to be sensitive to the dangers in extrapolating our dynamics down to and beyond this limit -the more so as our language persistently tries to lead us down that path.

I became convinced that it couldn't be as complicated as that, and then I saw from McGoveran's paper "Deriving the fine structure of hydrogen" dated March 1989, a way of separating the parts of the argument:-

First, the arithmetic.

the experimental alpha : the first order alpha (which is 1/137) = 3810 : 3810-1. (1)

(colons are signs of proportionality)

the 3810 is made up as 2x15x127 (2)

now we get from (1); alpha(experimental)=3810/(137x3809) =1/137.0359674... (3)

Second, the combinatorics.

1/137 is the frequency of a coulomb-type interaction. (Noyes' theory of the coupling constants).

We consider the transition from the level at which there are 16 elements to that at which there are 256 in the linear (matrix) progression of the hierarchy. At the transition itself the elements have been ordered and the linear form established in the less complex level, but at the more complex level the ordering has not been done and we are in the other kind of progression. Hence the 127 in the arithmetic. Of course as in the hierarchy construction itself we do not count the null element and so the number is 15. The cross product 15x127 now represents the number of ways of associating labels (in the linear system) with events. The factor 2 appears because the strings which represent the elements can be turned end for end to give twice the number of possibilities. Of course this is a combinatorial not a spatial turning.

Now we bring in the possibility that the construction is really two because we start our counting from some different point in it, and (from the original point of view) confuse labels with entities. The smallest difference we can make to establish a difference is to reduce one of the cross products by one, which changes alpha in the ratio we have calculated in the arithmetic. If you ask why we don't consider changes which are not the smallest, the I answer that we could do that, but that we are really concerned to establish the 'grain size', or unit of measurement.

To my mind this bit of combinatorics stands on its own logic and should come <u>before</u> any association with dynamics. Having got it in its pure form it will then be comparatively easy to develop McGoveran's theory. Moreover it becomes more natural to apply similar logic to other coupling constants as McGoveran and Noyes have already done.

SPACE, TIME , DISCRETENESS

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Introduction.

This paper is a contribution towards removing an idea that seems to have got around : roughly, that there is a clear set of notions called CH or ANPA, which is clearly understood by (at least) the founder members but which, through negligence or inability (or even malevolence) they do not explain to others. It seems a pity if this idea is around because it is so much like the idea that I (and others) used to have about quantum mechanics. Today's effort will remove this idea by showing the depth of my own confusions.

Before getting on to the main point of the paper, I will take up the same theme at a more pedestrian level. I refer to my paper[1] at the last meeting, where I said that only 1/10 of the possibilities at level 2 of the CH were deficient in the sense that they gave rise to less than 127 dcss at level 3. The calculation was wrong. It is nearer to 1/9 and the calculation is more tedious than I thought.

Turning now to my main argument, the combinatorial physics programme has always had two prongs: to reformulate quantum theory in terms of process so as to have the discreteness in from the start, and to go on to construct a discrete approximation to classical space and time. Success has eluded us so far as the second prong is concerned. I argue that this is because there are at least two severe logical difficulties about the concept of discrete space and time. What I have to say is partly back-tracking on remarks I have made in previous years[2] and partly it is complementary to what Pierre is doing, because his path, as I understand it, confronts these difficulties by making exceedingly small demands on the construction. So he is going to finish with a space-time <u>so</u> discrete as not to seem worthy of the name to me. I, on the other hand, avoid the logical difficulties by "devices" which he, I suspect, would regard as selling out. My path to these so-called devices did not start out from the problems of discrete space and time, but from trying, after the last ANPA meeting, to answer Alison Watson's question - she put it in the form: were the entities I was labelling there before they came up in the process?[3] I took this to be hiding a different question - what were the labels labelling?

The logical difficulties.

A discrete approximation to space and time must finish with either a finite or an enumerably infinite set of points. The difficulties come up in either case. They have not always been recognised - e.g. Russell[4] remarked that "rational geometry" was not observationally different from "real geometry" (1903). This is true only so long as you confine yourself to rather simple geometrical actions. But outside the stockade lurk Zeno-like monsters. Grünbaum[5] draws attention to the most immediate of these: that a line of length L, from the origin to the point L, cannot be considered as a union of its points if, as in rational geometry, there is only an enumerable infinity of points and each point has length zero. That does not worry much but to my mind a more serious difficulty of the same kind is the one exhibited in measure theory. Since the rationals are enumerable, enumerate the points between 0 and L. Situate the first point of this enumeration at the middle of a segment of length L/4, the second point similarly in one of length L/8 and so on. In this way all the points are potentially covered by line-segments. Some segments overlap, so that a more subtle covering could be provided by shortening some, but this is not necessary for my argument. All the points of the line are covered by a set of line-segments of length $(L/4)(1 + \frac{1}{2} + \frac{1}{4} + \ldots) = L/2$ and this contradicts the original assumption that the line is of length L. This is a more serious difficulty because it does not depend on bizarre requirements of constructing the length of a line from its points, only from its line-segments. This is where I differ from Pierre. He would be content _not_ to have the line made up from its segments. If it is the join of two events, then he demands nothing about its intermediate points unless another event intervenes. It is clear how this would be fine in high-energy physics or even in old-fashioned quantum mechanics. It is not clear how it can get to even special relativity, not to mention general relativity.

The second acute difficulty arises over dimensionality. If space is described as a manifold over the reals, its three-dimensionality is a well-defined property. The existence of monsters like Cantor's space-filling curve does not upset this, for it represents a transformation which, though continuous, is not bi-continuous. But the whole notion of three-dimensionality slips away if the manifold is defined only over the rationals, or _a fortiori_ over a finite field. For the enumerable character of the rationals means that there is a one-to -one mapping between the set of triples of rationals and the set of rationals themselves. Continuity will no longer serve to prohibit such mappings for it plays no important part in a rational theory.

I hope to persuade you that the deeper understanding of the CH that I believe I have reached provides something with a recognisable similarity to R^3. The dimensional difficulty is solved and I shall make very tentative proposals about the Zeno-like monsters.

Process.

I propose to construct space and time from a more primitive beginning. What could this beginning be? When Kant characterises space, not as a conception derived from outward experience (since such experience needs space already as a form of representation) but as a representation _a priori_ which is necessary for external experience to be possible, and similarly, with internal replacing external, for time, he is drawing attention to an important aspect which is independent of his particular philosophical position. It is that a construction of personal space and time must be in terms of the process of experience and the operative word here is not experience but process. This encapsulates Kant's point.

My own starting point, as you will guess, is the Combinatorial Hierarchy, seen from the point of view of a discrete process. This has been explained so many time in ANPA meetings that I simply give a reference[6] and instead use the time to discuss what Frederick Parker-Rhodes's original construction and the amended form really do. They show that any such process is built on an operation called discrimination and that there is a limit to the amount of self-organisation that can go in the process. This limit corresponds to a hierarchy of four levels, having 3, 7, 127, and 10^{38} elements. The last of these levels is different from the earlier ones in that the process of rising to new levels cannot be further continued. This stoppage draws attention to the fact that similar barriers might have arisen unnoticed at an earlier stage. In fact, the three characteristic functions at the first level are uniquely determined and do give rise to 7 discriminately closed subsets. The 7 characteristic functions at this level are not uniquely determined but, as I said earlier on, in about 8/9 of the cases they give rise to 127 dcss. It is more difficult, but possible, to prove that some choices of the 127 characteristic functions at the next level do indeed give rise to 10^{38} dcss. The argument for this has been given in previous years .

Dimensionality.

I begin by summarising the argument up to here. I have assumed that information about the world increases by means of a discrete self-organising process. Such a process leads to algebraic structures on several levels. The mathematicians' language is to speak of a graded algebra whose elements are of the form

$$u = u_1 \oplus u_2 \oplus \dots$$

the u_i being elements at different levels. Operations between elements, especially discrimination, take place individually:

$$u + v = (u_1 + v_1) \oplus (u_2 + v_2) \oplus \ldots$$

The extent to which the higher levels come into play is a measure of the process's self-organisation but there is a limit to how much self-organisation can take place, specified in the combinatorial hierarchy construction. The result of this limit shows that the graded algebra is of finite type, as it is called; it has only four levels. The elements at the first level form an algebra of dimension 3, then those at the next level come in as well so that a graded algebra of dimension 3 + 7 = 10 results. Those at the third level adjoined to this give a graded algebra of dimension 137; and the fourth level is then exhibited as different in nature from the other three. My contention is that, because this abstract process (i) has this unique 3 + 1 bounding structure, and (ii) represents the process of increasing information about the world, this provides an explanation of why we describe all our experience as taking place in a particular framework, which we call space and time. The nature of the construction overcomes the dimensional objection because the hierarchy levels play the same role here that bi-continuity plays in the theory with the real field. Indeed it does much more, for it offers an explanation of why the dimensionality is three and not any other number.

Because of the ambitious nature of my claim, I must expect objections. One of these I wish to disarm at once because I earlier held it myself. It is that it appears bizarre to identify the very different three initial levels of the hierarchy with the three dimensions of an isotropic space. But to urge this is to ignore the process aspect of the theory. The first level is pursued until it is no longer possible or convenient to fit in the new information coming in. Then the next level is brought into play; the previous overspill occurs again and a new level comes in.

It is natural for the complexity to increase at each stage.

Complexes.

I am fairly confident that up to this point I am on the right lines. The rest of this paper is much more tentative and this final version is modified as a result of remarks in the discussion period. My thanks are due to all the critics and especially to those who are acknowledged in the text.

The other question I raised was the existence of Zeno-like arguments if the system is only enumerably infinite. My own starting point here is Alison's question: do the entities exist before labelling? I should have answered : yes; and so the labelling might have been differently done. The group of transformations is $u \rightarrow u'$ which preserves discrimination and is reversible, i.e. linear and non-singular operators. Since you will remember that my version of the hierarchy construction uses characteristic functions, which are therefore singular, this nicely uses up the rest of the algebra.

What then is the object left invariant under this group? Not just a single label or element but what I call, provisionally, a complex. It arises like this: at the first level I use as labels 1, 2, 12. The group of transformations swaps them round, so I can't point to any particular. To all the dcss generated by 1, 2 come elements at the next level and as 1, 2, 12 permute so do these elements. And as at the simplest level there was no choice about them, the same is true here too. All can be permuted. But at the next level certain operators arose, and others might have arisen so that some of a number of possibilities come up in the process. This state of affairs must survive under the group of transformations.

So here is the beginning of an invariant differentiation growing between the entities. What I do not at all understand at this moment is how, within this corpus of ideas, to make the distinction usually made between quantum mechanics (where this slow growth of differentiation between elements does not seem necessary) and macroscopic physics. Putting this difficulty on one side, continue the process and so increase the amount of differentiation. The result is what I am calling a complex. It consists of:

(i) a set of elements,

(ii) a set of functions for all their dcss,

(iii) a set of functions for the dcss of the functions in (ii),

... .

Now the important thing is that, contrary to the assumptions made in the CH, this does not stop. The Parker-Rhodes construction shows a limit for the maximum self-organisation. There is no limit if that constraint is not applied. (Is it here, perhaps, that a distinction between micro and macrophysics arises?)

You could code complexes by bit-strings if you like; but they are infinite bit-strings and so the set of them is non-enumerable. So, <u>if</u> one has a way to space from here, the Zeno-type problems will be neutralised. I do not have such a way yet but I am hopeful that a way exists on these lines:- the basic notion of the theory is that of a dcss. Any other set that arises has a unique discriminate closure. It seems possible to define a "discriminate topology" on the set of complexes in terms of a closure operation, in the manner of Kuratowski, with the closure being of course discriminate closure. But this is not quite straightforward. A closure operation is defined as having the property that the closures of the intersection and union of two sets are the intersection and union of their closures. Only the first half of this property holds for discriminate closure; the other has to be taken in the weaker form that the closure of the union of two sets is the closure of the union of their closures. So the actual setting up of a topology will be significantly different from the usual procedure and the exact nature of the difference still remains to be prised out. But if a clear topological picture emerges it will then be possible to define a metrical picture from it - as an artefact, as it were - which will satisfactorily deal with the construction of space.

That is my present position. But it may be that there are other ways forward. It would be possible to use topology in the reverse way (Lou Kauffmann). That is, instead of modifying point-set topology to absorb dcss, one could use the discrete system of elements (ignoring complexes for the present) as elements of an algebraic topology. Such an algebraic topology is normally constructed as a discrete picture of a continuous space; here it would be logically prior and the continuous space an optional additional form of representation. This path had its attractions in the distant past but I failed to get very far along it. Another attempt is probably overdue.

It is even possible that there is no problem (Tony Deakin, Mike Manthey, Pierre Noyes). The complexity of the top level gives a good approximation to the rational field; perhaps one should just not construct Zeno-like monsters (TD) or perhaps the three-dimensionality comes from Feller's theorem (PN). Or perhaps my problem is self-inflicted, from my insistence on a single hierarchy corresponding to an increase of information (Ted Bastin). If I did not insist on this I would be able to avoid some troubles by saying: sometimes the algebra corresponds to measurements of space and time, sometimes to measurements of properties of particles. Here my insistence stems from my deeply felt need to avoid anthropomorphic elements.

Some of this material is to appear in the autumn in Philosophica.

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on p.127, and also in ref.1, pp.141-2. I there attribute the failure of the attempt at constructing space to a dimensional trouble, but that was superficial.

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LOCUS DYNAMICS IN $\mathbb{T}^4$

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ABSTRACT

A general principle of orthogonal diphasic electronic scanning is described and used to introduce a concept of the partition of time into its diachronic (historical) and synchronic (contemporary) components. Simple applications to geometrical translations, rotations and reflections under a condition of rigid motion in the Euclidean plane are extended to demonstrate that time is inherently distributed as the dimension of length in real or virtual figures which exist in $\mathbf{R}^3$. It is then shown that the intersections of affine pairs of line elements allows the construction of temporal maps as a rigid $\mathbf{T}^4$ co-ordinate system. Modulation of a $\mathbf{T}^4$ grid by suitable signals will then permit the complete description of the locus dynamics of any system in an observer's defined frame of reference.

The methodology will ultimately facilitate accurate computation of the interactions of rigid bodies with ideally resilient substrates, and also the interactions of two or more non-rigid bodies, at the level of individual energy quanta and as relative functions in $\mathbf{T}^4$. Such studies will lead on to the identification of the role of evolutionary processes in the emergence of structure.

1 THE MATHEMATIC OF NATURE

By a convention, the datum line from which time on Planet Earth is measured is the prime meridian. The length of a day, as an agreed standard of time, was formerly defined as Greenwich Mean Time by means of a number of accurate chronometers.

In this paper, a different approach is adopted in order to allow the construction of temporal maps appropriate to a non-algorithmic mathematic expressed in modulo four arithmetic and in representation of the electromagnetic phenomenon of phase quadrature. That is to say, 'local' time at an arbitrary point on the earth's surface is referred to the noon zenith of the sun's light and the variations in the geometry of shadows cast by a notional gnomon illuminated by that light at various times of the day and year.

In Figure 1, the line labelled 'longitude' represents the change in the reciprocal vector of a shadow cast by a notional vertical gnomon located at the equator (latitude 0°) during the period 6 a.m. to 6 p.m. The dotted line labelled 'latitude' represents the maximum northerly excursion of a noon shadow for some point in the Northern Hemisphere (local longitude, 0°). In simple terms, the reciprocal vectors then form two parallelograms of forces as mirror images. The seasonal dynamics of these location-dependent parallograms will then depend on the maxima and minima which occur at the equinoxes and solstices, the quadrature points of the year.

In Fig. 2, a unit cube standing on a flat plane is specified as corresponding to a cell of the conceptual phase space of either thermodynamics or information theory. It is assumed to be illuminated by parallel rays of light from the sun; the edges of the arbitrary shadow shown are therefore also parallel. It can then be seen that, under a condition of projective geometry, the shape of the shadow can be resolved simply by application of Pythagoras's theorem in terms of the lengths, a and b, of the two sides adjacent to a right angle as ratios to the length, L, of the side of the unit cube.

Fig. 3 then shows that a unit cube located on a perfectly flat plane (stippled area) generates a virtual shadow system having the connectivity and configuration of a hypercube. This may be used to partition time into two components which are related to each other.

2 THE PARTITION OF TIME

The terms *diachronic* and *synchronic* were first used in linguistics by Saussure in order to distinguish between those changes in language which take place over a relatively long period of time, and those taking place at the present time, respectively. The corresponding temporal partition in the present instance is conceptually derived from the sequential combinatorics of quantized energy flows from the illuminated to the shadow area of a notional phase cube under a condition of relaxation. Consideration of Fig. 3 shows that the virtual hypercube will not only rise and fall in step with the limit cycle governed by the sun's longitudinal limit cycle, but will also rotate in accordance with the latitudinal limit cycle as well. In brief, energy radiated from secondary sources under Huygens' principle will flow along quantized gradients into shadow areas acting as sinks. The process will, over time, generate a 'mass/energy field' having the same 'shape' as the energy-accessible shadow area.

Fig. 4 represents the propagation of a quantized (i.e., bitwise) energy flow by means of a chain of simple electronic repeater amplifiers, each of which is a relaxation oscillator called a monostable. Propagation is initiated by injecting a fixed-amplitude electrical pulse of short duration at the input terminal of the datum monostable, M_0. After a short period of time, D, the pulse is re-transmitted over a wire link to the next circuit, M1. The inter-stage delay time, d, over the wire link is very short in comparison with D, and it should be obvious that the original pulse will be also be re-transmitted from M1 to the next in series, M2, after the same delay period. The total time for a pulse to pass completely through any one stage is then t, where t = (D + d), and the total time taken to pass through N monostables is N x t units of time.

Fig. 5 then shows a square array of cells, each of internal dimension D units of time, and separated from all of its neighbours by a 'membrane' of d units of time. Pulses are injected seriatim at the input terminals of rows s_4 to s_0 in representation of diachronic time, from which it is seen that they appear in temporal order t_4 to t_0 on the diagonal trace of the matrix. Fig. 6 is of a similar matrix operated in synchronic time in demonstration that synchronic and diachronic events are simultaneous only at the instant of sampling or sensing.

3 MIXING IN QUADRATURE

Figure 7 represents two cycles, C1 and C2, of a pure sine wave of constant frequency expressed in pulsed form. The unfilled circles represent nodes in the plane of the page, and the hatched arrows the corresponding antinodes of opposite senses (of either the electric (E) or magnetic (M) field vector). The symbols Q1 to Q4 represent the corresponding quadrature partitions in quarter-wavelengths. Energy is taken to be propagating from right to left in a system designated as being in external, or diachronic, time. It is further supposed that the information content of this signal may be abstracted without change by means of suitable electronic circuitry installed along any diagonal trace of the matrix (e.g., at dotted line). That is to say, the sampling circuitry supplies the energy necessary for sampling and merely repeats the signals passing through an external system to another system designated as internal and synchronic, all without perturbation of the external system.

Figure 8 represents the admixture of a diachronic signal (hatched arrows) with a synchronic signal (filled arrows) along the trace of a 4 x 4 quadrature matrix under the same condition. The relative phases of the signals are such that the antinodes of the synchronic signal occupy the spatiotemporal locations of the diachronic nodes. The result is a form of amplification in which the E and M field vectors add in quadrature, the power vector, (nominally half way between the paired vectors), then occupying a diagonal of the square location available (see 'cross section' in small diagram). The final signal is then represented as a mixture of both kinds of arrow and inspection shows that, in the example given, the correct phase sequence of rotations is maintained. A corresponding perturbation would be induced in a suitably resilient substrate in a plane adjacent to this mixed signal in a 'pure' energy system. That is to say, mixing two pulsed signals in quadrature will result in the generation of a linear signal equivalent to a wavelet in representation of available energy in a real system. The mixed signal is available for re-sampling in the example given and affords a means of automatic computation in phase quadrature.

In pulsed systems in which the phases are not exactly matched, mixing would either result in ultimate separation of the two signals or in the formation of crossovers leading to knot formation.

4 FIRST ORDER TEMPORAL MAPS

Fig. 9 shows the basic construction of a temporal map in which diachronic time is represented in right-to-left columns, and synchronic time in top-to-bottom rows. It is supposed that an electronic expression would consist of parallel planes of circuit boards, each organized in terms of modulo 4 x 4 (quadrature) matrices so as to allow nested operation. The diagram is thus equivalent to three layers coded as alternate (conjugate) upper and lower case letters A, B, C, D and a, b, c, d. Each of the small cells of the array is taken to contain a single active circuit as a unit of Gray (reflected) binary code (see bottom right hand corner).

The array is considered to be static in the sense that, when signals are injected into the parallel rows, the individual pulses, (equivalent to bits), are propagated through distances (i.e., numbers of cells) which are proportional to their recurrence frequencies. The condition necessary to achieve this is that the value of the inter-stage delay time, d, shall be insignificant in comparison with the frequency of the signal being propagated. This means that a complete symbol or pattern can be propagated through an array under a condition in which high frequencies initiate the beginning of recognition, and low frequencies the termination of a symbol. This condition also allows signals in parallel to overtake each other.

Such a configuration leads to a process of continued discrete differentation in almost unlimited parallel and involving little more than Pythagoras's theorem in the simple case. The four right-angled triangles are the only four conditions that need to be considered in that recognition of one pair of triangles is initiated by the leading vertical lines, and the other pair by the trailing vertical lines. All four hypotenuses represent rates of change in terms of ordered pair co-ordinates or map references (± *dN/dE*). Detection of irregular shapes (e.g., dark area) is by extension of the same principle.

The "dynamics" of the diagram can be imagined by reference to the small unfilled circles, representing "sources" and the filled circles, representing "sinks". In the example given, the various shapes will continue unchanged, travelling from right to left in diachronic time. Interconnection or interaction with a synchronic signal or signals will divert or repeat the various shapes to other arrays.

5 SECOND AND THIRD ORDER TEMPORAL MAPS

Fig. 11 is intended to represent a condition under which the regular temporal grid spacing of Fig. 9 is replaced by a "snapshot" of a grid made unsymmetrical by its dynamics. This may be achieved electronically, for example, by varying the inter-stage delay, d, while the grid is in operation. This is also the equivalent of a radio transmission in which the station tuning frequency, (i.e., carrier wave), is frequency modulated by the speech or music programme being broadcast. It is also equivalent to the "frequency-hopping" systems used for the security of military radio traffic in the field. Under this condition, the individual temporal cells will expand and contract in area in both vertical and horizontal directions, thus forming corresponding virtual rectangles. The thick rule lines in the diagram then represent affine pairs of hypotenuses in conjugate phases in process of being automatically directed from one point in the array to another in accordance with their temporal phases. The vertical and horizontal line elements are isochronous.

Figure 12 takes the process one stage further and represents the map grid as a dynamically ruled (quartic) surface of wavy lines. Coherent signals (dotted lines) are being propagated in both conjugate phases, represented as filled and unfilled circles. Some of the circles are represented as being either randomly "ON" or "OFF", or in the process of changing from one phase to another. Such signals would be the equivalent of electrical noise and would be available for diverting signals as variables in "general" rather than "specific" directions. This configuration may be taken as being equivalent to the mode of action of the animal central nervous system and also to involve the chemical activity of the brain as well as the electrical. In crude terms, the "spikes" or "action potentials" of the nervous system are equivalent to "words and phrases", the actual "sounds of speech" being at least partly carried at the level of the electron clouds of the biochemical substrates.

In summary, and given certain definable conditions, an extensive array of simple "active" circuit elements such as monostable repeater amplifiers is capable of maintaining phase coherence in complex populations of pulsed signals and of distributing those signals between two conjugate phases. The distribution of pulses is seen to be temporally four-dimensional, i.e., to be in $\mathbf{T}^4$ and not in $\mathbf{R}^3/T$.

6 SEQUENTIAL COMBINATORICS

The virtual hypercube connectivity of Fig. 3 is only one example of a universal mechanism, of which the general case depends on a principle of orthognomonic projection. Under such a condition, the edges of shadows will remain parallel and consistent with the concept of conventional orthographic projection. They will also be consistent with gnomonic projection under a condition in which the sun is the centre of projection, the earth's orbit governs the radius of projection, and the earth's surface is the plane into which projection takes place. At the quantum level, the curvature of the earth's surface will be insignificant in comparison with the radius of the orbit and the relationship between two points in projection is then that they are joined by a straight line which is also a geodesic.

Fig. 13 then represents the direct (p) and reciprocal (q) energy gradients set up on the earth's surface by energy received from the sun and expressed in terms of simple probabilities. That is to say, if p1 is the probability that an event will occur under direct irradiation, then the probability that it will recur in a reciprocal gradient at some later time is q1, where q1 = (1 - p1). This also means that, if event p1 does actually occur, then event p2 will not, since the outcome of p1 has been displaced in time such that it satisfies the condition of q2, the reciprocal of event p2. This confirms the exclusion principle in a way which is consistent with the superimposition of circadian rhythms on the seasonal cycle. The mechanism leads to the formation of phase locked loops, Fig. 13 being that of the trefoil knot as the simplest example.

Such a mechanism is holistically determinate under the condition of the simple Hamiltonian H(p,q), where p is the potential energy and q the kinetic energy of translation. This also means that quantum objects will be sorted in accordance with their relative masses and the energies supplied to them. For example, a light atom such as hydrogen will be moved further and faster under the influence of blue light than an oxygen atom under the influence of red light.

It must be assumed, therefore, that there are many differential mechanisms involved in the evolution of structure, and particularly in the evolution of life forms on Planet Earth. Fig. 14 represents the interaction of memory (stippled) with the "flow of consciousness".

7 LOCUS DYNAMICS

In conventional mathematics, a locus is a geometrical figure formed by all those points that satisfy an equation or function between or connecting a set of co-ordinates, or by a system of points, lines or surfaces which move in accordance with such a condition.

In the present instance, it has been shown that a process of computation is inherent in the energy gradients set up on the earth's surface by heat and light from the sun under the combined influence of the circadian and seasonal rhythms. It has further been shown that this process of computation can be executed as a single methodology based on systems of linear equations treated as affine pairs, the theoretical limit of resolution of this methodology being in quarter wavelengths of electromagnetic radiation of different frequencies.

It follows that any number system can be used in the manipulation of whole wavelengths on grounds of the availability of radix conversions. However, all number systems of the real world will then be modulo arithmetics in their relative and reciprocal expressions. Manipulations in phase quadrature (quarter wavelengths) must, however, be limited to modulo four arithmetic. The quadrature combinations then govern directions over time and generate the natural numbers as four perfect squares (e.g., p1/q1/p2/q2 in Fig. 13).

The linear relationship between space and time at the quantum level goes a long way to explaining how "the mind's eye" can equate map readings with real physical features on the earth's surface. It is, on this basis, supposed that the electrical activity of the central nervous system is also directed by energy gradients in what is usually called "the flow of consciousness" and that this flow interacts with what is usually called "long term memory". In Fig. 14, the vectors and their reflections represent a short-term "thought" flowing through a variable-density memory, the interactions between them being represented by crossings. Modulation is supposed to depend on the activation of a memory cell of lower energy (frequency), or by one of higher energy. On this basis, sensory stimuli will be continuously modulated so as ultimately to convey information to a motor cell. Although only a crude description of the activity of the central nervous system, this concept raises profound questions in both epistemology and experiential knowledge.

8 CONCLUSIONS

On the basis of a concept of the representation of observable events in terms only of time, it is difficult to avoid reaching a conclusion that the entire body of conventional mathematics, science and computing should be reviewed immediately.

The principal reason is that the origin and evolution of life on earth would appear to depend on what can only be called a "plan of generalized co-ordination". To all intents and purposes, the natural order is that everything arrives at the right place and at the right time. On the other hand, man-made methodology, and especially conventional mathematics, is the equivalent of not only trying to stop the world from revolving on its axis, but of trying to prevent it from orbiting the sun as well. In other words, the well-known "halting" problem of computation attributed to Turing does not exist in natural global systems, which are not only continuously supplied with energy, but also resonate until equilibrium is restored by continued averaging. On this basis, bringing a computation to a halt is only a necessity for those humans who wish to inspect a snapshot of a programme while it is running. This also means that the temporal order in which the concepts of conventional epistemology were historically assembled cannot lead to a completely objective or rational system of thought.

The "static", or asynchronous, approach has, unfortunately, been built into the educational system in "tablets of stone", a situation which will no doubt continue unless drastic action is taken. What is required is immediate replacement of such methods with teaching based on an appreciation of system dynamics expressed in terms of discrete mathematics. Fortunately, there is a plethora of games and puzzles of direct applicability to the problem.

The concept of holistic locus dynamics shows that all natural systems are temporally determinate since all observable events must be governed by the dynamic geometries of energy-driven sequential combinatorics. In other words, it is no longer valid to carry out scientific investigations on the basis of supposed mathematical laws which appear to correspond with observations. On the contrary, the interactions of any combination of systems or objects can only be accurately evaluated as ratios and in terms of locus dynamics at the quantum level of quarter wavelengths of electromagnetic radiation.

9 THE VORTEX CONJECTURE

If accepted, the concept of temporal maps will require a complete review of science and the practice of mathematics. As far as is known, for example, an approach based on the sequential combinatorics of phase quadrature has not been used in relation to the "big bang" theory of cosmology. The latter may, therefore, ultimately prove to be an artifact of mistaken mathematics.

If the need for a new paradigm of physics is postulated, it is conjectured that the ultimate expression of the mechanisms described above might well require a return to the concept of "vortices and smoke rings" of the Golden Age of Physics of the late 19th Century. For example, no really satisfactory explanation has been given of the ultimate fate of energy and matter drawn into a "black hole". It is all very well to say that black holes are so dense that not even light can escape. So where does the mass/energy go? If it "comes out on the other side", what and where is "the other side".

Fig. 15 is therefore intended to illustrate a conjecture that, because so much else appears to fit into a pattern of diphasic quadrature, then so might a black hole. The large arrows at locations 1 to 8 represent pairs of parallel signals which mix under a condition of enforced mutual orthogonality, the resultant signals then being reduced to quadrature form by computational ANDing (small arrows). It is then supposed that the parallel signals "fire" in rotational sequence such that they disappear into the black hole under a "squeezed" condition, only to re-appear on "the other side" as "cold dark matter" after sudden expansion, "the other side" being the conjugate phase. Under such a condition, the cold dark matter would not need to be symmetrically distributed since the "attractor" property of a black hole need not itself be symmetrical but could be allowed to fluctuate within its sustaining limit cycles.

At a conference held in November, 1991, at IBM's International Education Center at La Hulpe, Brussels, Professor Amiram Grinvald, of the Weizmann Institute of Science, Israel, showed that the electrical activity of the surface of the brain of experimental animals can be displayed in real time computer graphics using only eight false colours. This evidence is consistent with a concept that all wave motion can be represented in the octaphase of Figs 12 and 15.

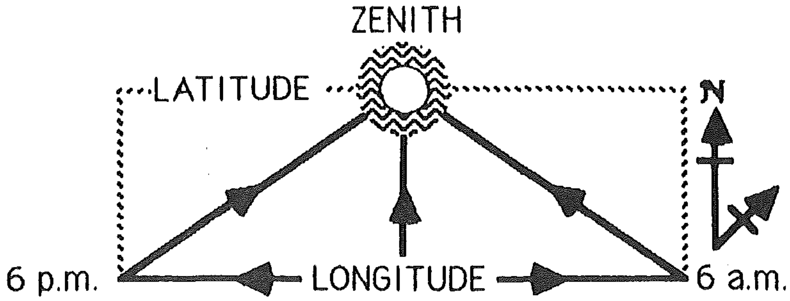

Fig. 1 Sunlight reciprocal vectors

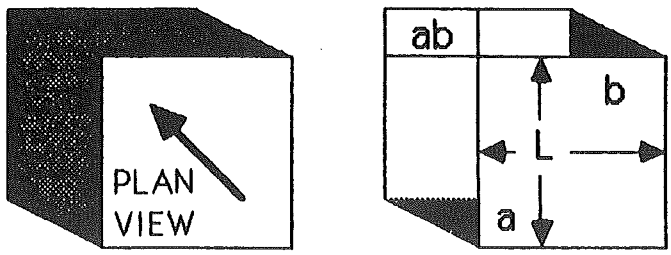

Fig. 2 Unit (phase space) cube and shadow

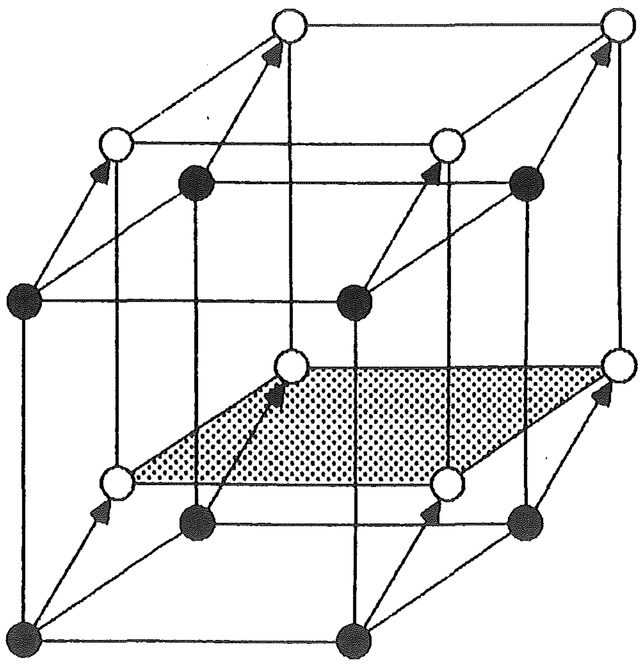

Fig. 3 Virtual hypercube connectivity

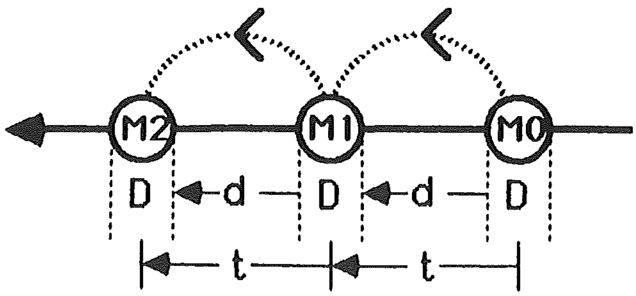

Fig. 4 Pulse propagation cascade

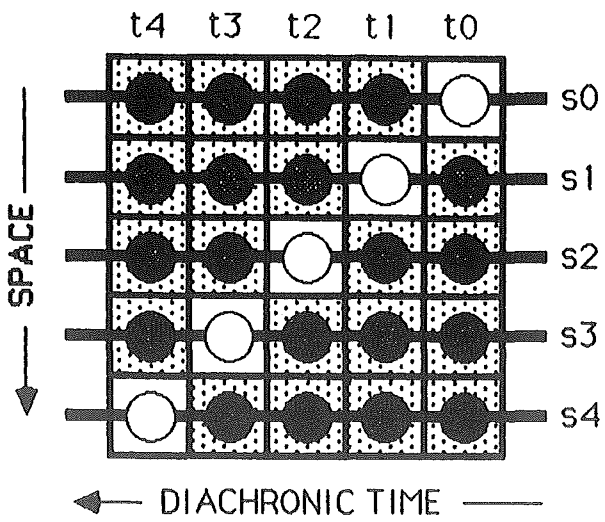

Fig. 5 Diachronic phase matrix

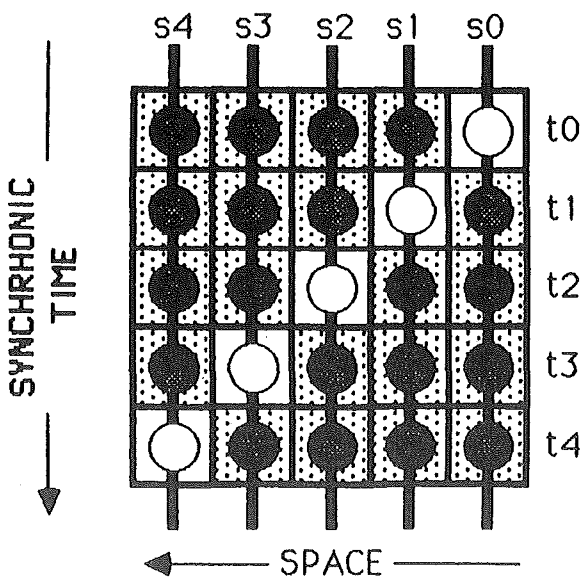

Fig. 6 Synchronic phase matrix

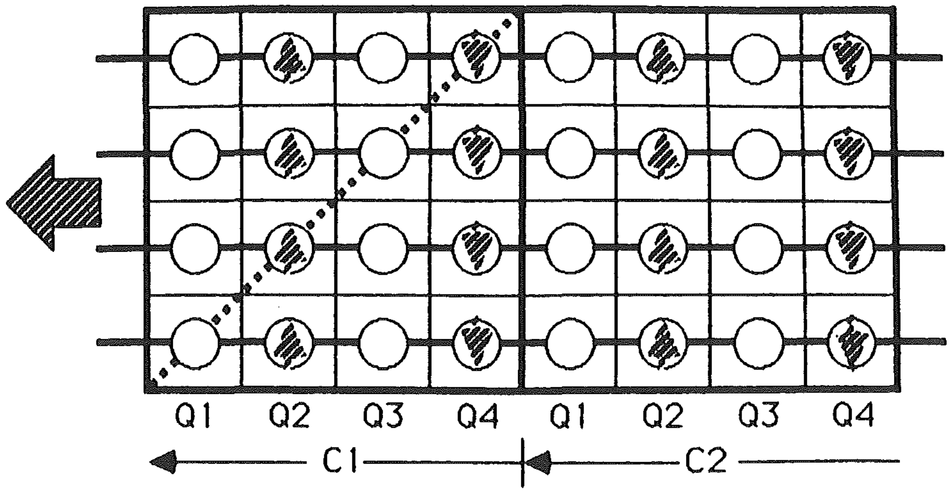

Fig. 7 Diachronic parallel phase quadrature

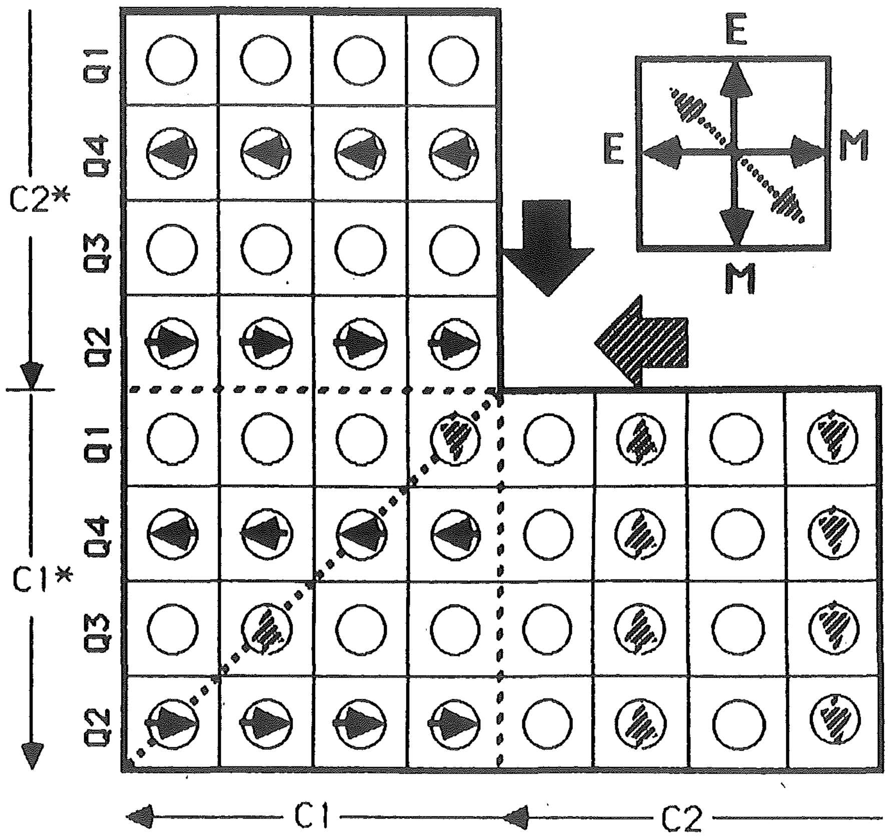

Fig. 8 Synchronic parallel phase quadrature mixing

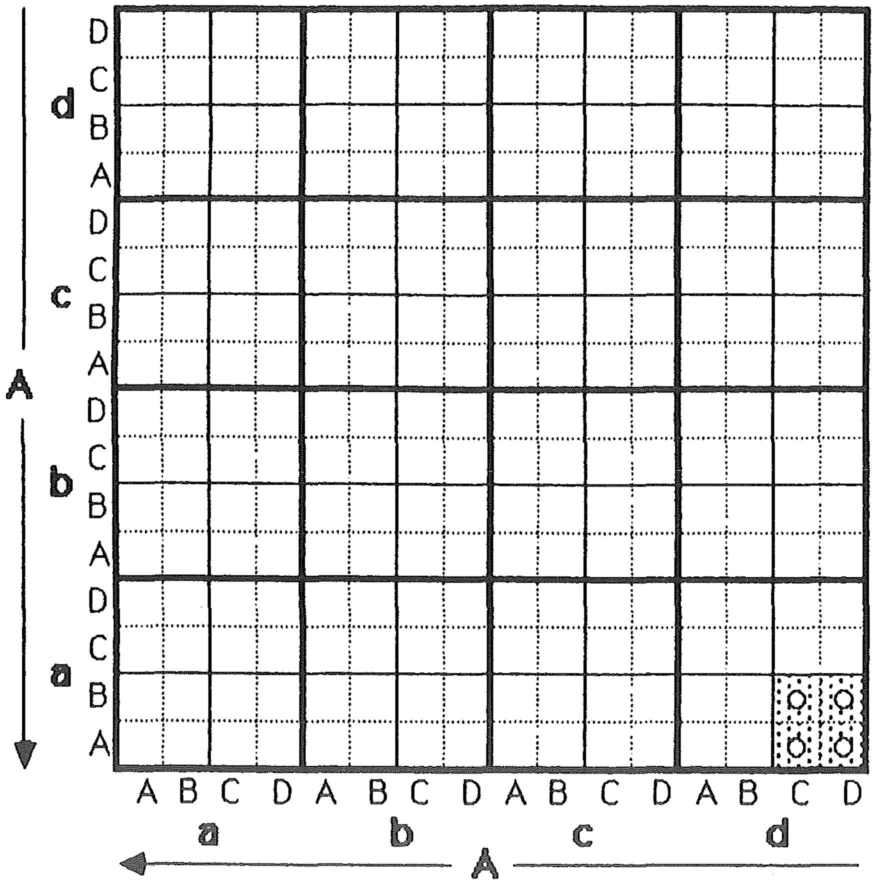

Figure 9 Static 1st order modulo-4 temporal map

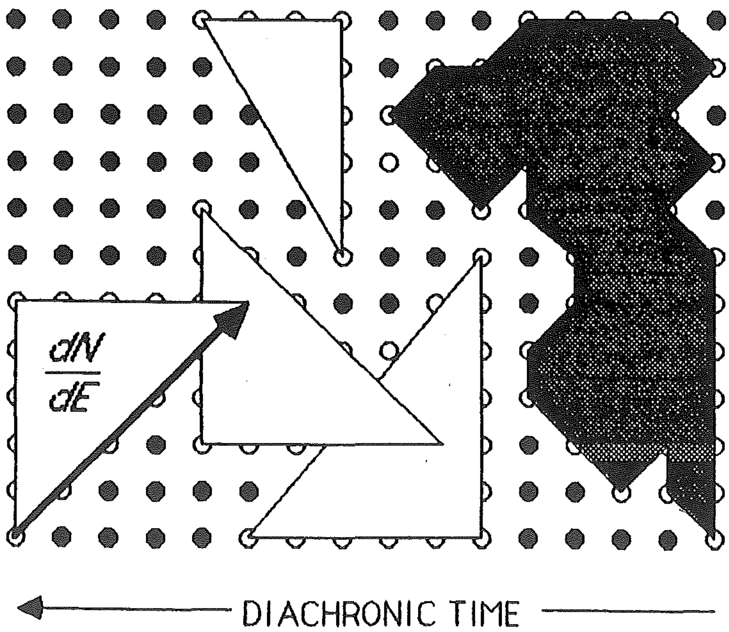

Fig. 10 Continued discrete differentiation

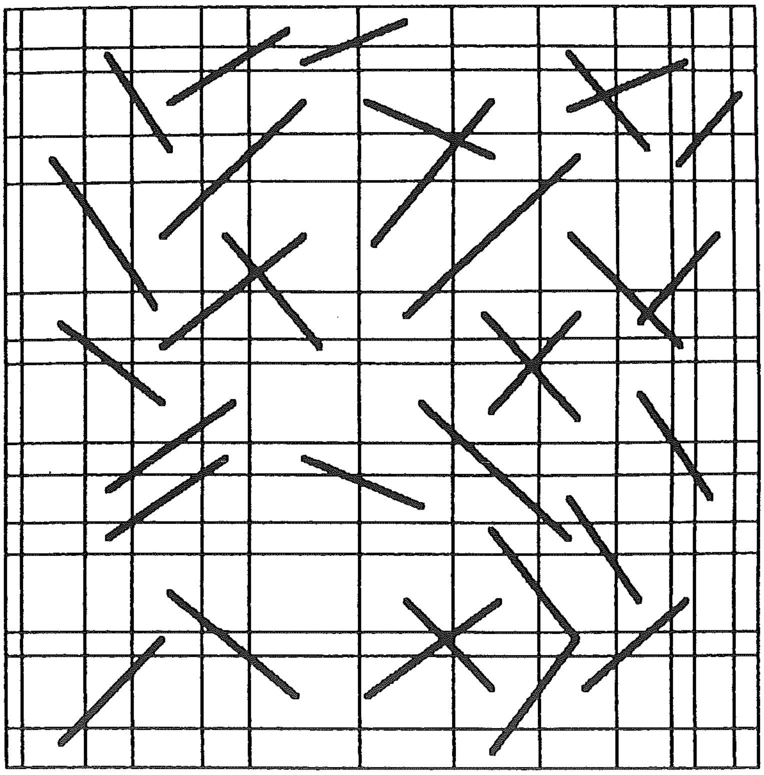

Fig. 11 Line segments in a 2nd order grid

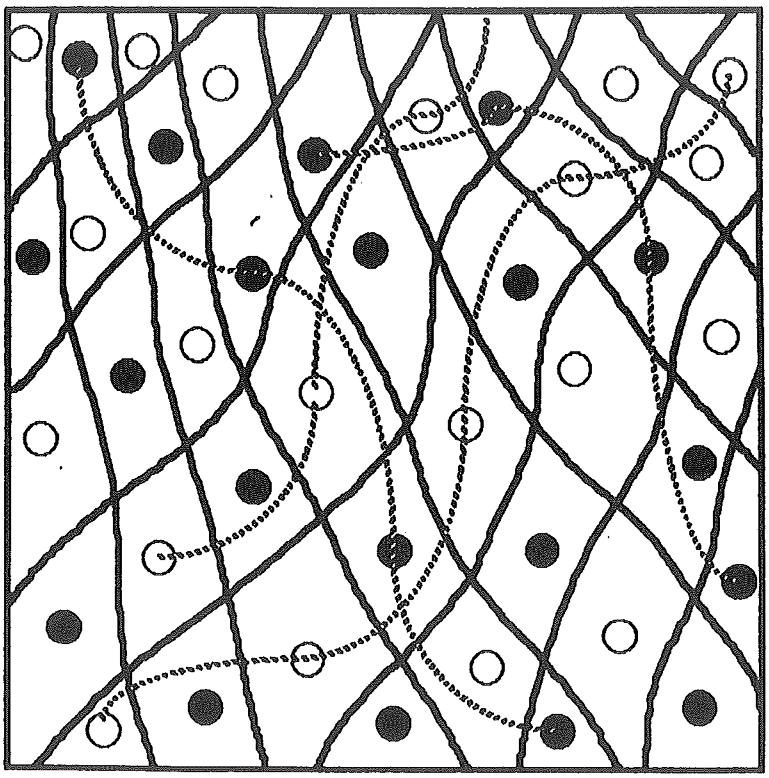

Fig. 12 Signal flow in a 3rd order memory grid

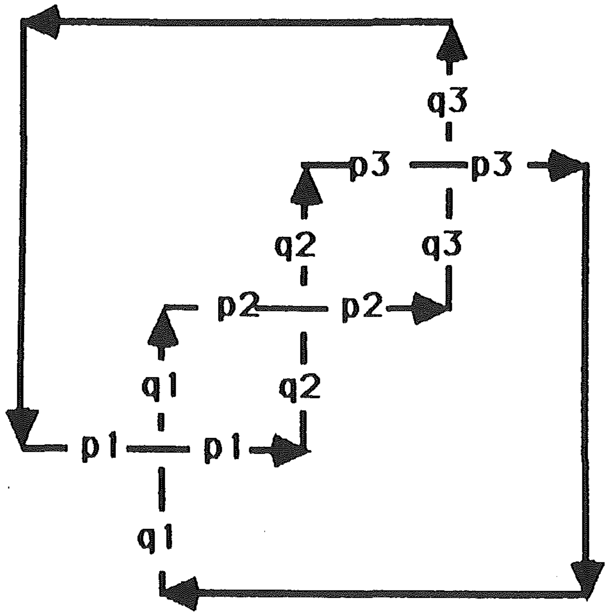

Fig. 13

DENSITY (FREQUENCY) INTERFEROMETRIC MAZE

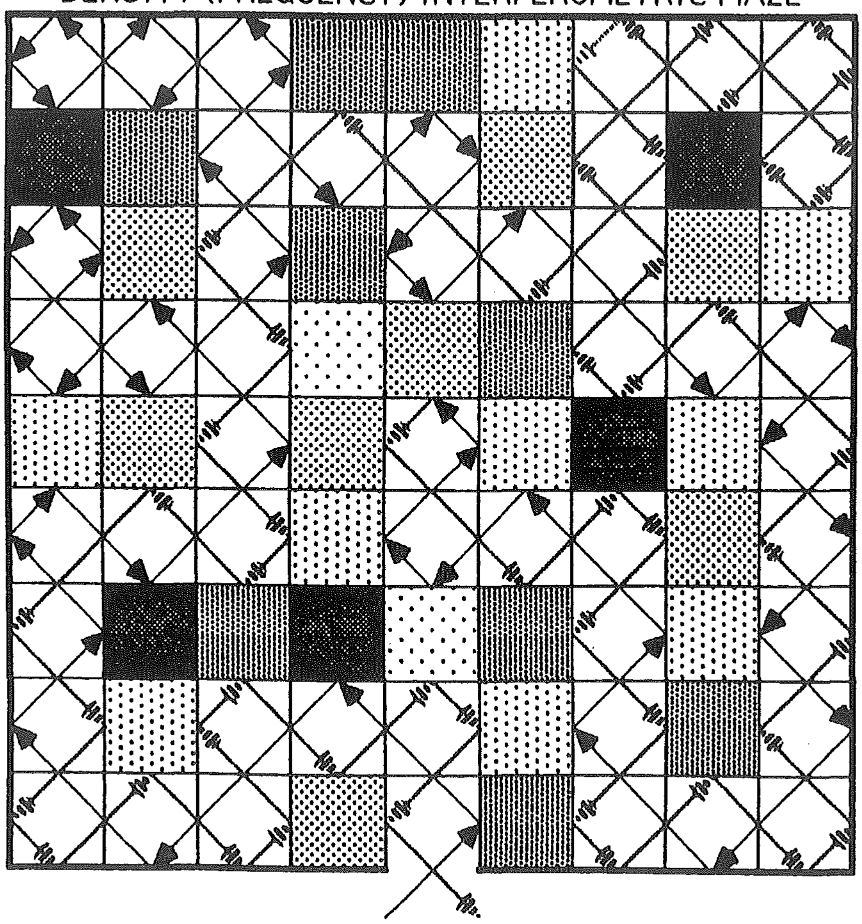

Fig. 14 SENSORY INPUT MOTOR OUTPUT

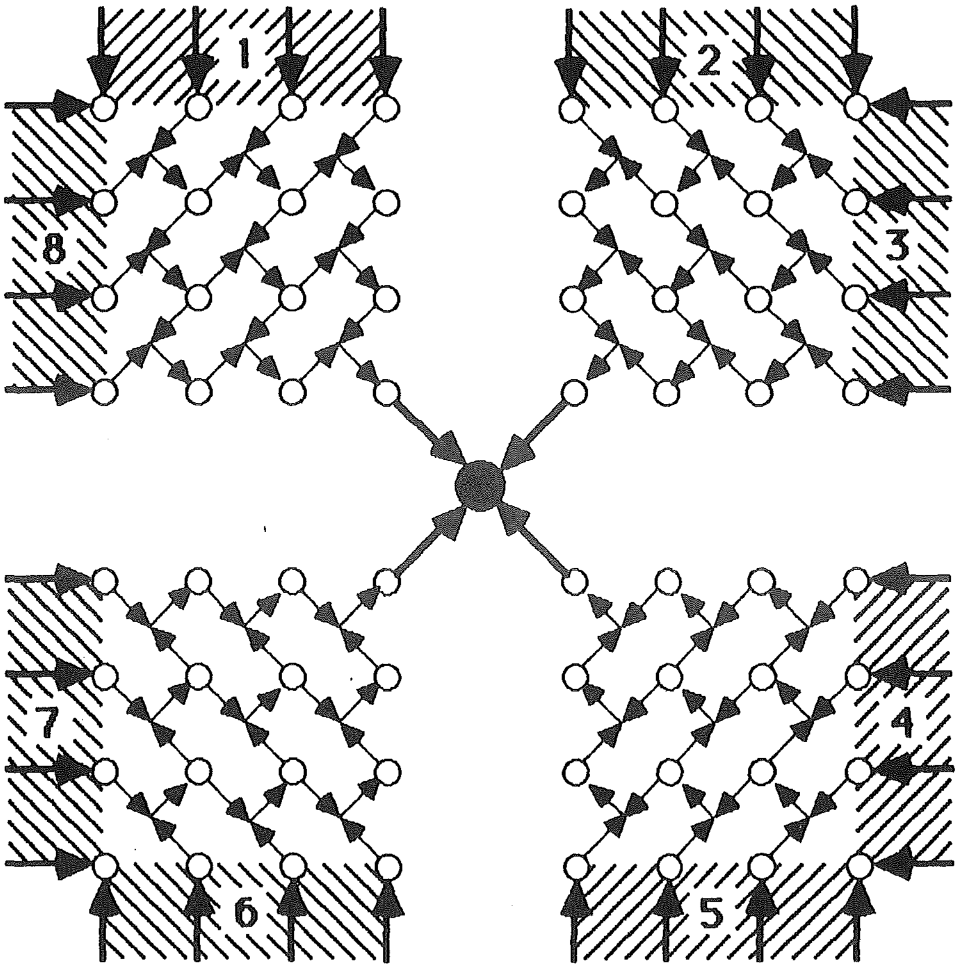

Fig. 15 Sequential rotary phase averaging in a vortex

Orthogonality and General Systems Theory

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Abstract

The evolution of the concept of Orthogonality from Mathematics into Computer Science is reviewed and extended into General Systems Theory. We look at applications to functional design and give a formal system theoretic definition of an Orthogonal System. The implications of orthogonality for design ***quality*** *are emphasised.*

1. Introduction

My research over the last few years has been largely devoted to the application of Huygens' Principle to system decomposition with particular reference, recently, to parallel computing. This paper, however, looks at a topic which has not been central to my work but which has influenced it strongly over that period. Orthogonality is a term which started its life in the world of Mathematics, and drifted, in a more general form, into the folklore of Computing. The paper aims to take it further, into that part of General Systems Theory that deals with design (if indeed it exists yet) and promote it as a general design principle.

Note that the use of the word "Orthogonal" in this paper is not strictly correct as we use the phrase in a context in which a Mathematician would say "Orthogonal and Complete". The full phrase however is too cumbersome to use continuously and in common parlance orthogonality often implies completeness so we will only use the full phrase for emphasis. We will use "Orthogonal but not Complete" where appropriate.

2. Orthogonality in Mathematics

The Oxford English Dictionary defines Orthogonality as "having to do with right angles". Specifically, two lines are orthogonal if they are at right angles to each other. Sets of Cartesian axes are orthogonal. In many applications a family of (non-intersecting) curves is given and it is required to find another family whose curves intersect each of the given curves at right angles. The curves of the two families are then said to be mutually orthogonal.

Legendre and Bessel polynomials enjoy a property which is called orthogonality and is of general importance in Engineering Mathematics. Two real valued functions defined on an interval are said to be orthogonal if the integral of their product over the period is zero. A set of such functions which have this property pairwise is called an orthogonal set. If, additionally, the integral of the square of each function over the period of definition is 1 then the set is called orthonormal. (Unless the functions are trivial then the interval of their squares over the interval is always non-zero.) An orthogonal set can always be normalised in the obvious way.

An orthogonal (actually orthonormal) matrix is a real matrix whose inverse is equal to its transpose, ie one for which

$$A'A = I,$$

the unit matrix. A clearly consists of a set of orthogonal unit vectors ie one for which

$$\begin{aligned} a_i'a_j &= 1 \text{ if } i=j \text{ and} \\ &= 0 \text{ if } i \neq j. \end{aligned}$$

The set is linearly independent and thus forms a basis for the system (a set of vectors that spans the space). A well known thereom states that orthogonality is a necessary and sufficient condition for linear independence. This property of some norm on unlike components of a system vanishing, and the same norm on like components existing (or in particular being unity in which case the system is often called orthonormal) is the signature of orthogonal systems.

Multiplication (of a vector) by an orthogonal matrix is called an orthogonal transformation. This is of particular importance because an orthogonal transformation leaves the inner product (x, Ax) and thus the norm invariant. In particular, in Euclidean space, an orthogonal transformation preserves the length of any vector and the angle between any two vectors, and is hence a rotation.

From these ideas we find that orthogonality breaks down into two related concepts. The x and y axes in the plane are said to be orthogonal. The x, y, z and t axes are orthogonal in the space-time continuum. First of all we have seen that the axes, being at right angles, are independent of one another. Secondly, in each of the two examples the axes are both necessary and sufficient to completely define the space. My brother, Alex Bowden (who at the time ran the R & D group of Prime Computers UK, at Harston Mill near Cambridge) has defined an Orthogonal (and Complete) System as **one whose components are, functionally, mutually exclusive and globally exhaustive**. That is, there are just enough of them to do their job. The term used by Time Series Analysts would be "parsimony". Physicists talk about the application of Occam's Razor (after William of Occam). Functional independence and parsimony are two most important properties of the components of orthogonal systems.

This is the sense in which Orthogonality is used in Computing and is the sense in which we wish to introduce it into General Systems Theory. It is considered to be a statement about the *quality* of a system about which more later. It refers to a system consisting of a set of independent or mutually exclusive (ie, different, unrelated) concepts. By a system we follow Mesarovic in calling the function of a system the system. We are not, unless stated, following Kron in thinking of a system as its physical manifestation. The set must span the system - it must be globally exhaustive. This is tautological if we are defining orthogonal systems. It is however an aim if we are thinking about system design.

Mutual exclusion, global exhaustion, parsimony and independence are subtly related concepts and it is necessary to be careful when using these terms. Clearly parsimony is related to necessity and global exhaustion to sufficiency and our definition can be viewed as being close to tautology. An orthogonal system is one containing both necessary and sufficient subsystems! It is the union of a minimal number of disjoint subsystems. If two subsystems are not both necessary to build the system then they are not mutually excusive (the reverse may not be true).

3. Orthogonality in Computing

Unfortunately although many people in the computer world know, use and have an understanding of the term "Orthogonal" in terms of software systems (particularly user interfaces), most have difficulty in explaining what they mean by it. (That of course is why it is a useful term, it conveys something which is difficult to describe otherwise.) In particular it is a very difficult term to define in general. I had been looking at this subject for many years before I prompted Alex to give me his definition. I have not come across a better one since. A typical attempt of a programmer to describe an orthogonal computer program is "one that provides a consistent, intuitive user interface. You can guess what to do." Thus we begin to see implications of quality. There is little in the literature which I have been able to find to provide a more definitive description. Thus one aim of this paper is an attempt to fill that gap.

Programming Languages

Algol68 was the first computer language designed with orthogonality in mind[6]. Its syntactical structures are independent and can be used arbitrarily in combination without fear of error. All other things being equal, an orthogonal computer language will allow the most concise form of expression (the lowest number of keystrokes or, at least, keywords.) It should allow any syntax to be used in any context provided this is meaningful and unambiguous. If ambiguity is possible then there is be a set of rules governing the meaning in each case (eg. for coercion.)

A simple example using a small number of syntactical structures should serve to illustrate this. For instance

if bool then x:=a[2] else x:=a[3] end

or something similar, will compile in most structured languages eg, Algol60, Pascal, Algol68. The following expression is more concise

x := **(if bool then a[2] else a[3] end)**

which will compile OK in Algol68 and Pascal but will produce a compilation error in Algol 60. Going a stage further

x := **a[if bool then 2 else 3 end],**

the most concise form of expression, will only compile in Algol68. The structures we have used here are, explicitly, assignment (:=), indexing ([...]) and the conditional, **if...then...else**.

This is a fairly simple example for Algol68. The language allows a very high degree of abstraction. The Algol68 compiler writer must do some fairly clever programming to ensure that all forms of expression will compile and do so correctly. However this involves no more than using a consistent (orthogonal) calling structure in the parsing procedure, whereas the Pascal and Algol60 compilers must actually check to see that the syntactical rules are not being disobeyed and print out appropriate error messages if they are, making them larger and slower. Computer systems are multilevel systems in which each level should aspire to orthogonality. The effect of interaction between these levels (operating system layers, levels of interpretation, hardware levels - RISC, microcode etc) on overall orthogonality would be a subject worthy of study.

Other languages make varying claims to Orthogonality. Occam, the parallel processing language for Inmos' Transputer, was named after the aforementioned William of Occam, but in its current incarnation makes little concession to its namesake. At the machine code level orthogonality was often invoked when comparing instruction sets such as those of the 6502 and Z80 microprocessors. The 6502 has more addressing modes than the Z80 but these cannot be used arbitrarily in any context and was thus claimed to be less orthogonal. At this point it is worth noting the reference to the degree of orthogonality of a system which will come up again later.

User Interfaces

Orthogonality is an important measure of quality in the study of user interfaces. In this context by user interfaces we are talking about operating system front ends, both the shell (or command line interpreter) and wimp (graphical) style interfaces or about application front ends such as those presented by programs such as graphic editors and word processors (and editors in general). Computer systems are becoming more orthogonal with time. Early bitmap editors offered features such as "elastic circles" and complex paintbrushes made from sprites. Later offerings allowed their simultaneous use, the sprite would be swept round in a circle creating a satisfying splurge. The latest version of WordPerfect however uses a different text editor for normal text and equations which is very unsatisfactory. Operating system shells (or command line interfaces) offer various features such as wildcards, variables, iteration lists etc, all of which should be independently implemented and simultaneously available. A major offender in this area is DOS5 which *still* does not have wildcard evaluation implemented in the command line interpreter but has the code individually and inconsistently embedded in each command which means that the effect of wildcards in a command line depends upon the command typed! It should be noted however that the reason that Unix initially appears to be so unfreindly is because of its high level of orthogonality. Apollo's now defunct Aegis shell is perhaps the most orthogonal command line interface yet developed without being unfriendly. We have high hopes for the Open Systems Foundation's OSF1.

In the field of HCI (Human Computer Interaction) researchers such as Thimbleby[8] have come up with sets of principles (Thimbleby calls them *gueps*, which appears to mean principles he's not sure about!) such as the "Principle of Least Surprise" which are aimed at good system design. A brief survey of these shows that most of these are manifestations of aspects of orthogonality. Two concepts which we have identified as possibly being independent of orthogonality are those of hierarchical structures and polymorphism. Von Blaauw's design principles for the IBM/360 architecture stated that generalisations (of systems) should be

Consistent
Orthogonal (by which he just meant having independent components)
Appropriate (necessary)
Parsimonious
Transparent (the implementation mechanism should be invisible)
Powerful (should do as much as possible)
Open ended (extensible)
Complete (sufficient)

We can see that all of von Blaauw's principles, with the possible exception of extensibility, are in fact subsumed by our definition of orthogonality! The exception is interesting. Most orthogonal software systems are extensible, ie extra functionality can be added and a mechanism to do so is provided. Another principle that might be added to the above list today is *seamlessness*, although this is clearly implied by orthogonality.

4. Product Design and Advantages of Orthogonality

My British Telecom Rapport answering machine is not orthogonal. It has a microphone for recording announcements. It has a loudspeaker for playing back messages. But you cannot use it for hands off telephone calls. This is bad design. (Telecom may call it vertical marketing but I call it either bad design or unethical.) Thus orthogonal design maximises use of component parts. I would like to see a module on orthogonality taught as a standard part of Engineering design.

In software terms my copy of WordPerfect has a different command set for the word processor and the equation editor. This makes it more difficult to learn than it would otherwise be and, further, uses up more RAM. The reason, of course, is that the Word Perfect Corporation tacked the equation editor on afterwards - it was probably a different product. This is sloppy and unprofessional. So orthogonal design makes systems easier to use, to learn and remember - they are *intuitive* - and uses less resources.

Similarly I was delighted when an early graphic art package offered rubber banding for drawing circles etc, and paintbrushes made from sprites and further allowed their simultaneous use. The sprite would be smeared around the route described by the circle. Early operating systems had different systems of wildcards in different contexts. DOS still does, wildcards work in some contexts and not others. There is no excuse for this in DOS5. It is easy to achieve - the wildcard evaluation must be put in the command line interpreter instead of each individual command (using up memory). Bill Gates must be perverse (but he's rich).

Using higher level and more structured, extensible languages for writing computer systems helps. It becomes more difficult to write nonorthogonal systems than consistent ones. It would be necessary to actually write a trap in for the special case. Orthogonal systems are easier to design, provided you take a high enough view of the original problem. There should only be one of everything. Their usage should not overlap.

Advantages of orthogonal systems

Easier to design (with a high level view)
Use minimal materials
Use minimal resources (eg. RAM)
Easy to use (learn and remember)
Intuitive (consistent)
Maximises use of components

5. A System Theoretic Formulation

Following Alex Bowden's definition the functions of the components of an orthogonal system form disjoint subsets of the task to be performed. Further, they form a partition of it in that they are also sufficient to perform that task, the overall system function. If the function of the ith component is f(S(i)) and the function of the overall system is f(S) then we can write

$S = \cup_i S(i)$ (a system is the union of its subsystems)

$f(S(i)) \cap f(S(j)) = 0$ where $\cap$ is the intersection operator and i#j

and $f(S) = \cup_i f(S(i))$, the union of all component functions.

Clearly $f(S(i)) \cap f(S(i)) = f(S(i)) \neq 0$ for nontrivial functions.

It can be seen that this is not quite good enough to describe the Bessel and Legendre polynomials mentioned above. A slightly more general version would replace the intersection by an arbitrary norm

$(f(i),f(j)) = 0$ and $(f(i),f(i)) \neq 0$.

Mesarovic[2] defines a General System as a mapping $S:X \rightarrow Y$ where X is an input set and Y is an output set or, more generally, as a subset $S \subset X \times Y$. As mentioned earlier S is clearly describing the functionality of a system rather than, for instance, its physical manifestation. (A general system may not, indeed, even have a physical manifestation). This is very convenient for us because this is precisely the aspect of a system that our definition of orthogonality constrains. Thus if we take Mesovaric's view of a system (a system *is* its function) then we can miss the word *functionally* out of our definition which becomes **an orthogonal system is one whose components are mutually exclusive and globally exhaustive.** We avoid the word subsystems (rather than components) just in case it has any implications that we have not thought of! A further implication of Mesarovic's definition is a simplification of the formulation in the last section. We may now write

$S = \cup_i S_i$ (a system is the union of it's subsystems) and

$S_i \cap S_j = 0,\ i \neq j$ (the subsystems are disjoint).

And $\forall i,\ S_i \cap S_i = S_i \neq 0$.

Or in general

$(S_i, S_j) = 0$ and $(S_i, S_i) \neq 0$.

One could describe this as a *proper* system.

As we have now mentioned subsystems we could look at the *decomposition* of S in Mesarovic's formulation. Each subsystem must be the Cartesian product of a subset of the input set and a subset of the output set, $S_i = X_i \times Y_i$ with $X_i \subset X$ and $Y_i \subset Y$, although the subsets of X and Y are not clearly disjoint.

Mesarovic's view of systems seems compatible with ours, although elsewhere we have suggested extending his work to include physical or spatially indexed systems as well as dynamic or temporally indexed ones. Indeed because of his preoccupation with the latter Mesarovic's work rapidly deteriorates into an abstraction of control systems theory, covering such aspects of a system as observability, controllability and optimality. We feel we have extended this work by formalising and emphasising orthogonality as an equally important concept from a different stable. Mesarovic's other major work in this area is of course that on hierarchical systems the importance of which we have stressed earlier.

6. Near Orthogonality

Many real systems are not orthogonal. In many cases this is due to sloppy design. In others it is due to a cost or technology constraint or apparant convenience. No programming languages are truly orthogonal. Algol68 has a number of structures that do essentially the same thing. A truly orthogonal control panel in an aeroplane may require the plane to travel at fifty times the speed of sound at some setting of the controls. My usual argument here is that if it looks as if a system would be better with a lower degree of orthogonality then not all the factors have been taken into account. If one stands back and looks at the overall problem one will find that one or more degrees of freedom relevant to the problem have been ignored. In our last example we could include a setting for maximum petrol consumption and the problem would be solved.

However other problems have a clearly near orthogonal solution. For instance, sticking to our aeronautical example, there is a branch of Control Theory called non-interacting control. This is a variation on classical control that considers a system with n inputs and n outputs like an aeroplane. Inputs include throttle, aerolons, rudder. Outputs include velocity, yaw, direction. Clearly it would be nice if certain controls only affected certain outputs, in practice this is difficult to achieve in conventional aeroplanes but modern jets use computer feedback in order to isolate the controls. Clearly this is an attempt to achieve a form of orthogonality, however even in the most expensive military equipment true independence is never quite achieved. The technique involved is called the Inverse Nyquist Array. At least for the control example this gives the beginnings of a measure of degree of orthogonality. The process involves minimising certain properties of the off diagonal elements of a particular array. In terms of our formulation we have the situation where the subsystems are no longer disjoint and we are in some sense trying to minimise the intersection $S_i \cap S_j$.

Finally, more speculatively, I was recently looking at a private (and I think unpublished) communication from Germano Resconi[7] in which he argues that the invariance in the gauge field in each level of a Unified Field Theory can be represented by a holographic transform from the previous level. It struck me that the breaking of gauge invariance is a deviation from orthogonality.

I wondered how far the hologram "analogy" can be taken? Consider the unusual form of system decomposition invoked by Kron's method of tearing[4]. This decomposes systems with a physical manifestation into a number of connected subsystems separated by a single connected "intersection network" with a dimensionality 1 lower than that of the system. The object is to produce a near independent set of subsystems where the interaction manifests itself through the intersection network. This deviation from orthogonality, according to Huygens' Principle, derives from a hologram like transformation.

7. Conclusions

Orthogonality arguments are also used in many area of the Physical Sciences, particularly when building models eg, in Physics and Engineering. As this paper is being presented to an audience consisting largely of Physicists we are not going to attempt to teach our grandmothers to suck eggs. It is sufficient perhaps to say that ontological Phycisists walk around with an Occam's razor in their back pockets. Many ontological arguments rely on parsimony of structure or expression. Similarly many arguments based on dimensionality are implicitly appealing to orthogonality. In a survey at the ANPA conference at which this paper was originally presented about half the audience admitted to knowing or using Orthogonality in the context in which we are presenting it.

Gabriel Kron, known for his work on system decomposition, used the words "orthogonal" or "complete" with respect to electrical networks. He used the words interchangeably. An electrical network is called an orthogonal (or complete) network when it is described by all its mesh and all its node equations. The outer product of the node and mesh transformation matrices is the unit matrix showing that the space of meshes is orthogonal to the space of nodes. The system is then, of course, overspecified and the equations are singular. It was these singular transformation matrices resulting from these machinations that got him into trouble with the tensor theorists.

In the course of our discussions on a number of occasions we have come across a system whose components were indistinguishable in type, eg axes in Euclidean space, subsystems in a torn physical system. Other systems have components which are not interchangeable in this way, eg the structures in our minisubset of Algol68 or the components of my answering machine. We refer to these as homogeneous and heterogeneous orthogonal systems respectively. We distinguish between systems indexed in time and in space by calling the latter dynamical systems and the former physical systems. Mesarovic and others dealt with the former, Kron dealt with the latter. Systems with both properties we call real systems. Computer networks are discrete digital real systems in this sense.

8. Acknowledgements

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An Alternative Natural *Philosophy*

A Critique of the Combinatorial Hierarchies Approach to Physics

by

N. V. Pope

ABSTRACT

What is the ANPA approach to physics, called the 'combinatorial hierarchies'? Is it just another mathematical gambling with numbers and axioms in the hope of hitting some numerological 'jackpot' which yields, as Stephen Hawking puts it, 'a complete understanding of the events around us, and of our own existence'?

This paper examines the unstated philosophical presuppositions on which such expectations are based and puts some questions which, hopefully, will challenge ANPA members into providing some clear philosophical answers.

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An Alternative Natural *Philosophy*

A Critique of the Combinatorial Hierarchies Approach to Physics

by N. V. Pope

In this talk I should like to avail myself, if I may, of the latitude offered in the description of ANPA's philosophical aims and objectives, for the consideration of alternative natural philosophies which may or may not be the same as that pursued by the Association under the title of the Combinatorial Hierarchies. I have studied that approach as closely as a non-mathematician may, and have to confess that I have so far failed to grasp its underlying philosophical intention.

But that's alright, because there are fellow members who, in similar fashion, have failed to grasp either the philosophy or the methodology of the Keele approach which I described in my last ANPA talk. Approaches to understanding in science-philosophy are still many and various. So, if you will bear with me I shall reiterate, more succinctly and convincingly this time, I hope, the point I tried to make last time, but which may have been obscured by a presentation which was too 'philosophical' and 'historical' for most practically-minded scientific and mathematical tastes.

Perhaps it will help, then, if I state exactly what my difficulties are in coming to grips with the 'Combinatorial Hierarchies' approach. As I understand it, and forgive me if I'm wrong – for nowhere have I seen it clearly stated – the aim of that approach is the same, in principle, as the one described by Victor Weisskopf in his speech marking the 25th anniversary of the founding of CERN. 'The object of research at CERN', said Weisskopf, 'is to study the ultimate constituents of matter and the ultimate forces of nature, the driving forces of all natural processes'.[1] As I see it, the Combinatorial approach seeks to do the same thing by the rationalistic process of combining abstracted numbers (basically, simple ones and noughts) according to various mathematical rules, with the aim of discovering combinations of these numbers, and hierarchies of these combinations, which will reflect – and perhaps even predict – those fundamental regularities in nature which are found, for instance, in scattering experiments carried out at places like CERN.

So, what's so difficult to understand about that? Well, in the first place, this approach, when you think about it, is extremely puzzling. Its underlying philosophical rationale is that once we discover the ultimately irreducible parts of things and the mechanisms by which these parts interact we will understand the whole of creation. In speaking to Weisskopf about this I discovered that he was so 'sold' on this way of thinking that he could scarcely conceive of any other.

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However, from the philosophical point of view – which Weisskopf, incidentally, would have no truck with – not surprisingly, as it turns out – there are some very serious problems with that reductionistic method. For instance, take these ultimate irreducibles, be they material particles, numbers or whatever. How do they associate? How do they combine? If everything, including minds, reduces to mindless material particles, of one kind or another, then what remains after analysis, to interrelate those particles? What can possibly combine them, since interrelations and combinations, as such, are not particles? For a relation or a combination to exist, something must comprehend not only each particle, singly, but also the two combined. And the same goes, of course, for pure numbers as for particles. The implicit assumption of reductionism, therefore, that when everything is reduced to mindless physical or mathematical simples, there somehow remain combinations and interrelations of those simples – that there is still, in the background, some all-embracing whole, or 'universe', which it makes sense to speak about – is a contradiction in terms.

There will be some, no doubt, who will say, 'Away with all this philosophical twaddle. Those relations are just there, what the hell!' But where? Are they confined to our own perceptions, as Hume supposed? Are they part of the formal structure of language, as Wittgenstein and others have suggested? Or are they 'in the mind of God', as Stephen Hawking presumes?[2] The question can, of course, simply be ducked and those who ask it and those who fail to answer it may doff their caps and go their separate ways, as Weisskopf and I did after our discussion at CERN. On the other hand, if people like me are to understand what is happening at ANPA – or at CERN, or SLAC or wherever – then those philosophical questions cannot be left to go begging.

I put it to you, then, that reductionism is a theory which is tenable only in connection with some highly questionable philosophical presuppositions which proponents of that theory, like Weisskopf, are understandably shy of producing for inspection. However, in order for me to understand what is being claimed by those who talk about the way these ultimate irreducibles 'combine' to form pairs, groups, a 'universe', or whatever, I need to be told something about these combinations. Do those ultimate irreducibles combine in themselves? To this, it is all too easy to reply, unthinkingly, 'Yes, of course they do!' But think about it – because this is crucial! How can they combine, when there is now nothing left but those particulars and since, by the definition of their natures, those particulars are incapable of comprehending themselves, let alone their plurality. I mean, with all awareness analysed out of existence, how can those elements interrelate? How can one and one of them ever make two? Are we, really, saying, with Hawking, that they combine in the mind of God? But how, in empirical science, can we presume to be privy to what God observes? In the mind, then, of man? But how can the mind of man be responsible for the way atoms and stars are formed? Analysed in that way, reductionism seems ludicrous. So, since I feel free to reject the unspoken assumption which underlies pure reductionism, that

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there are no relations, only relata – which I am sure no-one, knowing what he was saying, would seriously contend – I shall need to be told precisely how these relata are supposed to be related; that is, in what scheme of comprehension, man's, God's or whatever, that wholeness and interrelatedness of things which we call nature inheres, before I can form any opinion on what is being claimed. It was as an attempt to winnow-out and articulate what I saw as this 'missing philosophy' that I presented my 1991 paper.

Where, then, does all that wholeness and interrelatedness of things take place, without which there could be no science to speak of? The answer Wittgenstein gave, in his *Tractatus Logico Philosophicus*, was as follows. Like so many of his philosophical predecessors, Wittgenstein rejected the implicit assumption of people like Democritus, Weisskopf and Hawking, that it all takes place, somehow, absolutely, in the all-seeing eye of some ubiquitous deity. He also rejected the opposite logical alternative proposed by others like Hume and Kant, that these interrelations exist only in the mind of man. In his *Tractatus*, Wittgenstein put forward the view that the world reduces, in observation, to what he and Russell called 'logical atoms'. These were not the self-sufficient and dissociated *relata*, or atomic 'things in themselves', of Democritus, but elementary bits of logical interrelation between those phenomenal simples which Mach had already described as 'sense-data' (or 'instrument-data' as these later came to be regarded). The logical interrelations between these ultimate simples were thus of the sort we recognise in mathematics between numbers, and in logic generally as irreducible propositions of observational/instrumental language, such as 'Bright flash here', or 'This scale-reading there', and so on. This elementary logical relation of distinction and combination is, I gather, the basis of ANPA's 'combinatorial' method. Well, alright, in the ANPA approach, these original sets of 'logical atoms', or logical combinations of sense-elements, have become abstracted, so that they appear, as in the *Principia Mathematica* of Russell and Whitehead, as highly formalised mathematical equations; and the wider, more philosophical preoccupation of the Positivists with language in the round has largely been spurned. Nevertheless, and this is something which is all too easily forgotten in our mathematical dealings with numbers and their combinations, mathematics is not something which is written in the sky. It is a form of logic, which is never more, in the last analysis, than a special subdivision of language in the Wittgensteinian sense. So, thus far, one may identify the underlying philosophy of the Combinatorial Hierarchies approach to science as, basically, that of the *a priori* linguisticism which is associated with Wittgenstein and which became the method of the 1920s Vienna Circle.

However, if that is so, then the Combinatorial method fails in the same way that Positivism failed, as we shall see, and was rejected by its founder, Wittgenstein. So I think that if that method is to have any credibility outside the confines of ANPA, it must move on philosophically from its 1920s 'Logical Positivism' position towards something like the new position which I described

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in my last paper, which is based on the work of the later, or 'reformed' Wittgenstein. In other words, I do not think it is sufficient, in this day and age, merely to show how logical atoms – words, numbers, bits of behaviour or whatever – combine, however interestingly, in mathematical equations, in formal logic, in ordinary language or whatever, to reflect the regularities of nature. Somewhere it has to be made clear as to where – that is to say, in whose observations, man's, God's or whosever – these combinations are supposed to be taking place. In other words, we need to know what, precisely, is the epistemological status of those propositions and so on, in some identifiable philosophical approach which is not that of out-dated 'Logical Positivism'.

Where, then do those interrelations, those comprehensions, of systems of ultimate relata, take place? For instance, something which Logical Positivism was never able to cope with, and one of the main reasons for its rejection, was in failing to answer the question of how objects can interrelate at distances which are large on the scale of the so-called 'velocity of light'. If light travels in the way Einstein believed it did, then every atomic light-source is separated-off from every other by a time-interval $t = s/c$, where s is the distance between those sources and c is the 'velocity of light'. And since c is the velocity-limit on all causal interaction, then in what possible way can any of those sources interrelate? How can they combine? For things to combine they have to coexist in some holistic, comprehensive way, that is, simultaneously; and that condition of holistic simultaneity, or objective coexistence, is precisely what is banished from physics by Einsteinian relativity.

From those, then, who would say that these relations between simples are 'just there, what the hell!', if they will stay the course and not take refuge in philosophobia, we need some definite answer to the question of how they imagine one thing can possibly interrelate with another when those things are completely and inexorably isolated, or localised, by Einstein-separation – unless, of course, what we are talking about are not real events in real space and time but 'pure numbers', whatever these may be, and the way they are related in some Platonic world of 'Pure Mathematics'. Then, as an empiricist, I would want to know just how that ideal world of mathematics is supposed to exist and how it is supposed to connect with the real world of physical phenomena. This, I believe, is where people like myself who, while not expert in handling the actual mathematical processes of combination and so on, may nevertheless make useful contributions by helping to articulate the underlying philosophy, of which I have so far found no clear description in the ANPA literature.

As I see it, then, the philosophical stance which is required by the methodology of ANPA has to be the sort of post-Positivistic observationalism we find in Wittgenstein's later work and in Einstein's Special Theory of Relativity, in which the relations between those ultimate simples to which natural phenomena may be reduced, exist essentially in proper, language-articulated

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observation — that is to say, in some specified observational framework, or communicational system, of human observers and their non-human observational instruments. And by 'communication', here, I mean, as Wittgenstein did, all devices of language, verbal, instrumental and so on, including formal logic and mathematics. The difficulty of the reputed 'Einstein separation' due to the motion of light in space, which defeated the Positivists is then solved, I suggest, by recognising that Einstein was simply wrong in thinking of light as something physically travelling. To think of those ultimate simples, the quantum particles of illumination we call 'photons', as 'travelling in space' is, as I have demonstrated in my various papers, a category-mistake without which the theory of relativity can be expressed so simply and graphically as to be veritable child's play.[3]

What I am saying, in other words, is that what we call space, or 'the void', which separates things so inexorably, according to Einstein and reductionists generally, is not an underlying absolute precondition, or receptacle, for the conveyance of photons. It is the relativistic, observer-projected result of informational sequences of these observational quantum events. In other words, I am saying something which many of you may think preposterous but which is quite sensible when you think about it, that there can be no motion in a photon. The 'photon' is the ultimate quantum 'still' in a 'cinematographic' sequence of events which produces space and all that goes on in it, so that what we normally think of on the macro-phenomenal level as 'light in space' switches, at the quantum-sequential level of analysis, to become an observer-projected space in light.

That there is, as I say, no motion in a photon can be demonstrated by the ordinary time-dilation formula of Special Relativity,

$$t_{proper.} = t_{relative}\sqrt{[1 - (v^2/c^2)]} \qquad (1)$$

Here it is very clear that if v in that equation is put at c, the so-called 'velocity of light', then the proper-time t_{proper} of that light-signal is zero. And in consequence, of course, since t_{proper} is zero the proper distance, s_{proper} = c *times* t_{proper}, which we say the photon 'travels', is also zero. So how can something which, intrinsically, travels no distance and no time possibly be said to be 'travelling'? In what sense are its moments of emission and absorption to be regarded as separate in space? Since the so-called 'departure' and 'arrival' of a photon are indistinguishable, then to be emitted and absorbed, as we customarily describe it, is, for the photon, one and the same event.

So space and the 'velocity of light' do not exist at that photonic level of analysis, Q.E.D.! This has been a well-known, if not well-understood, consequence of Special Relativity for at least 66 years, since Gilbert Lewis's article to that effect appeared in *Nature*.[4] This removes the void which separates things

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so completely and inexorably according to Einstein and makes it a consequence of observational processions of these intrinsically spaceless and timeless quantum-events. That is, space ceases, as I say, to be a primordial receptacle for the conveyance of photons and becomes, instead, a kind of 'video-projection' out of information-processed observational sequences of the same.

Now, usually, someone or other, at this stage, gets the notion that what we are talking about, here, is some kind of solipsism, in which nothing exists outside and beyond one's own observations. This was certainly a problem for Positivism – another very good reason, indeed, for its failure. However, that was no problem for the sort of observationalist philosophy which followed, largely due, as I say, to the later Wittgenstein. Briefly, solipsism arises from thinking of the ultimate simples in any analysis of phenomena as 'sense-data', and of the relations between these as supplied exclusively by human perception. Since I cannot sense your sense-data and you cannot sense mine, then, obviously, the forms and relations I construct out of these entirely private 'sense-data' are inscrutable to anyone but myself. So how can I know that anyone else exists or that there is an outside world, other than in my own imagination? That is the sort of nonsense which reductionistic phenomenalism, or Positivism, generates.

However, if you think of the ultimate phenomenal simples, not as 'sense-data' (whatever those may be) but as natural action-quanta, which are not peculiar to that instrument of observation we call our bodies but which are the currency of all instrumental communication whatsoever, animal, vegetable or mineral, then the physical 'self' loses its uniqueness as a centre of reference. The relations between those simples, relations of distinction and comparison, distance, time and so on, are then no longer uniquely those supplied by human sensibility and language but are manifest in things and processes throughout. Thus, 'observation' becomes decentralised, spread into the community and the environment at large by communicational transforms of the sort described by Ayer and exemplified by Lorentz, In such a world of common currency of simples and their relations, in definite and inter-communicable instrument-frames, what question can there be of 'solipsism? With what sort of 'outside' could such an all-embracing 'inside' possibly be compared?

What, you may ask, has all this got to do with the Combinatorial Hierarchies? I'm not exactly sure. All I know is that in order to be able to speak about 'combinations' of entities, assuming that these entities are anything like those ultimate irreducibles we call quanta and which, as I hope I have shown, we cannot sensibly think of in the 'photonic' sense as 'travelling', we have to be able to state where, that is, in what schema of comprehension, those combinations are supposed to be taking place. I have disposed, I think, of the notion, born of Logical Positivism, that these combinations are confined to any subjective, or solipsistic schema and that although they do not occur in any absolute space and time of the sort envisaged by physicists from Democritus to

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Newton, they are nevertheless objective 'public property' insofar as they may be truly discerned and discussed.

However, if these combinatorial 'atoms' are not quanta – if, as someone described it to me, they are just the ultimately simple ones and noughts of pure mathematics – then I need to know how these ultimately simple ones and noughts of purely idealistic combinatorial theory are supposed to relate to those other ultimate simples which are physical photons and suchlike, especially since, unlike those simple, ideal, ones and noughts, the photons are many and various in terms of quantities like energy and frequency. The difficulty, for me, is that no-one, it seems, ever defines what these simple ones and noughts of combinatorial theory are or how they relate to the quanta of nature. So I am left wondering, are they photons and other quanta of that kind, or are they pure abstractions of language, undefined except in opposition to one another? In that case, as I say, how are these number-sequences supposed to relate to reality other than in some purely incidental way? What more understanding of nature would we hope to achieve by that method than monkeys would achieve in fortuitously reproducing a Shakespeare sonnet by strumming randomly on typewriters?

Rationalists, of course, as opposed to empiricists, characteristically believe – as Russell and the early Wittgenstein believed – that beyond human experience and language there is some Platonic realm of super-existence consisting of sets of logical and mathematical axioms, theorems and principles which, as soon as they are discovered, will enable us automatically to reconstruct the whole of nature in such a way as to make our actual experiences of it unnecessary. To an empiricist this is complete nonsense, since for him, mathematics, no matter how impressive or 'elegant' it may be, is simply a branch of logic, which is never more than a branch of language and is therefore answerable, philosophically, to methods of linguistic analysis. Language is thus eminently open ended. The world, that is to say, is essentially capricious, indeterminate and stochastic, in a word, informational. This gives us freedom of communication, implying real responsibility for action and therefore real morality, as exemplified in human society and humane theology. The Rationalistic dream of a world of pure 'clockwork', or of automatic mathematical combinations of abstracted simples, is therefore as inimical to empiricistic science and philosophy as it is intrinsically nonsensical.

The trouble, of course, is that in the ideal world of pure mathematics there does appear to be an 'absolute' space and time – or space-time in the case of General Relativity – which we can view in an overall, God-like way which has no parallel in the real world. Yet where can this ideal exist other than in our minds or in our conventions of language? It is all too easy for us to extrapolate from this ideal to a 'universe' which we find nowhere in perception but which we may presume to comprehend by exalted methods of pure reason. Thus

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we have modern cosmologists and cosmogonists like Hawking, David Deutsch, Frank Tippler, Chris Isham, *et al*,[6] who preach a mathematical gospel about 'The Universe' and its contents in the same way that the priests of yore used to preach to us - often in Latin - about 'Heaven' and its communion of saints and angels.

In order to avoid that kind of mysticism, mathematicians, in my view, must remain mindful of what their ultimate symbols and so on mean in connection with ordinary usage and must structure their theories in the real, relativistic space and time of natural phenomena rather than in some idealised and universalised 'God's-eye-view' of space and time. I would say that nothing is more nonsensical than to seek to describe, as some theoreticians do, a time in which time itself begins, evolves and ends. No matter how mathematically 'elegant' a theory may be, couched in such terms, it can never amount to anything more than gobbledygook, and any benefit it might turn out to have in intriguing us or in advancing our scientific understanding can be, at best, no more than fortuitous.

The only sensible way, then, in which I can think of these ultimate simples and combinations, of combinatorial theory, if it is any help, is in the Aristotelian rather than in the Platonic way, as *in rebus* not *ante res* — that is, in things not beyond things. This means that the ultimate simples and relations of combinatorial theory are to be identified with those of direct and instrumental perception. Otherwise, I confess, I cannot even imagine what we are talking about.

Now, lest there are some who think that logical or philosophical refutations are not concerned with empirical evidence but 'only with words', let me add something about reductionism. Any such idea, that the world as a whole is no more than a sum of absolutely self-sufficient parts, be they numbers, bits of matter, 'sense-data' or whatever, and that whatever goes on is determined either by automatic mechanisms or equally automatic logical and mathematical combinations of those parts, is easily refuted by straight observation. If the world were no more than an epiphenomenon, a specious collection of such absolute units, as reductionism entails, then no change in the interrelations among those units could possibly affect them in any material way — since, according to that hypothesis, relations as such wouldn't count. So, no motion of one atom relatively to another would be able to affect their material character in physical processes since, as a relation, motion could have no material influence. The atoms of material processes would therefore remain what they were, no matter what. This, however, is plainly refuted by the fact that in relativistic physics, the very mass of an atom, as a participant in physical processes, changes with relative motion. Indeed, neither the mass nor any other property of that atom can be described in isolation, that is, absolutely, but only in relation to other atoms. So it follows that the atom (or whatever

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happens to be the current 'fundamental particle') cannot be absolute, other than in some completely isolated and indescribable, God's-eye-view way.

The same goes, of course, for the 'photon'. If this be taken as the ultimate quantum of optical phenomena, then the frequency, $\nu = e/h$, of such an event, associated with an observed object, could not be affected by motion of that object since, again, motion is no more than a relation between the object and the observer, which, *ex hypothesi*, cannot affect those relata in any way. In that case, the distributions of photons in atomic line-spectra could not be influenced by the relative motion of the observed object. This is plainly refuted by the fact that the frequencies of photons are so affected. As we all know, they are relativistically Doppler-shifted as a result of motion. This could never happen if they were like the pixel-events on a television-screen, which occur in the way they do, independently of the motions of the screen-objects.

The reductionist hypothesis, then, that the world we see before us is unreal and accidental and that reality resides uniquely in absolute particulars of some kind or another, is as untenable as to suppose that the letters of the alphabet have some 'absolute' pronunciation, no matter what words they appear in, or that the coins in our pockets retain some 'absolute' value which is not affected by variations in the exchange-rate. It was the radical 'flipover' in thinking, from this sort of reductionism towards relationism or relativism, which made Wittgenstein claim, among his Vienna Circle followers, to be the 'first non-Wittgensteinian'.

In my view, then, in any analysis of phenomena, any question as to which is either ontologically or epistemologically prior, the whole or the part, the relations or the relata, is as nonsensical as the argument about the hen and the egg. Can there be an inside before there is an outside, or an up before a down? How, then, can there be atomic parts with no comprehended wholes? As Wittgenstein came to realise — taking his cue perhaps from the *Gestalt* psychologists, Ehrenfels and Wertheimer — every analysis, scientific, philosophical or whatever, is essentially holistic; that is to say, it begins and ends in experience as a whole, and the microscopic parts of that experience take their character from the whole as much as the whole does from those parts. Does ANPA follow suit or does it remain bound to the philosophically outmoded reductionistic assumptions of the *Tractatus*? I, for one, would very much like to know.

To wind up, then, here is how I would summarise the Keele position. Suppose that in classical physics, which is essentially reductionistic, you wanted to paint a picture of an atom, how would you represent it? By a dot, say, on a canvas? But without comparing that representation with the atom itself, how could it ever be judged to be a good or a bad likeness? So in classical physics, which is essentially reductionistic, man's ideas of nature and nature itself end

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up on opposite sides of an unbridgeable epistemological gulf. In that case, to suppose, as classical physicists did, that the relation between physics and nature was one of representation, was optimistic, to say the least.

On a relationist theory, on the other hand, there is no such trouble with representation. The basic assumption of the Keele thesis, for instance, is that the sum of all the various ways in which we interrelate things, which is called language, is not the whole but only a small part of that system of natural relations which the Greeks called *logos*. Nevertheless, those relations between things (relata) that are expressed in properly formulated human language – in physics and mathematics, for instance – are essentially the same as those we find in nature. Thus, we try to identify, in words or symbols, the natural hierarchies of relata and their interrelations; and when we have done that to a reasonable level of satisfaction we may justifiably feel that to that extent we have truly represented natural reality. If we have not, then our anticipations of natural events and processes will sooner or later prove false and will require readjustments of the sort which could never be called for in that way if our ideas were as severed from physical reality as classical physics supposed. So the relation between nature and that conception of it towards which physics evolves is one of identity – as one might declare 'that's him' on being presented with an identikit picture of a criminal, even though there may be nothing in the picture which is anything like those bits and pieces of the criminal to which a reductionist thesis might reduce him.

The conceptual 'flip-over', then, which the Keele approach requires is from classical reductionism towards relationism. In the old way, perception comes to us retail, by devious and unfathomable causal processes, beginning in hidden atomic 'mechanisms' and ending, somehow, in mental 'ideas'. In the new way we deal with nature wholesale and direct. This aims at completing the work of Einstein who, to the annoyance of his philosophical mentor, Mach, laced his relativism with classical precepts which have no place in true relationist philosophy. The prime example of this, as I have shown, is Einstein's conception of light as the ultimate interrelating agency which takes seconds, centuries and, in most cases, millennia, to do its job of connecting-up and combining things. It is the uncritical inclusion of this relic of reductionism, or atomistic de-combination, in what is called the 'Combinatorial' approach which, I confess, makes that approach very difficult for me to understand.

ADDENDUM

Now I had intended to finish right there, but then I thought it might be better not to end on such a critical and destructive note. My more positive suggestion, then, is that the true aim of research, at places like CERN and SLAC, should not be to seek, vainly, behind the old Cartesian, 'veil of perception' for 'hidden mechanisms' consisting of Weisskopf's 'ultimate constituents of matter

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and ultimate driving forces of nature', but to study directly the functionings of the things and parts of things that stand right there before us. That is to say, our purpose in ANPA, as I see it, should be to classify and divide physical phenomena, so as to discover, by means both experimental and logical, how many different divisions and permutations there may be of their various measure-categories. Thus, what we should expect to find, at the ultimate ground-floor level of such an analysis, is not some hard and self-sufficient 'object' of any sort, far less some abstract and self-sufficient, one or nought, but some limit at which reductions in the dimensions length, mass and time, all converge. Such a convergence-factor, if that's what we may call it, is Planck's constant, the product of whose internal parameters, in arbitrary units of kilograms, metres and seconds, is h. This, as I say, is not an object in any ordinary sense, but a kind of ground-terminal for branching hierarchies of dimensional interrelations, where the natural units of length are defined in terms of those of mass and time, and where, circularly, the natural units of mass and time are defined in terms of those of length and so on. So, 'No-one has yet succeeded in splitting h', as Weisskopf assured me. But indeed, how could they when to squeeze h in one way is to expand it in another? Yet within h — at its divergent basement levels, as it were — there are contained the remaining two of what he, Weisskopf, has dubbed 'the three spectroscopies'. The first of the three is, of course, the ordinary, 'above-ground', Balmer-Rydberg photon-spectrum. So the remaining two, the sub-h spectra, let's call them, are those of the mass-energy, force, spin, and so on — depending on how you cut it — of the products of those bashings which h receives on the anvils of our 'atom-smashing' laboratories.

My suggestion, then, for whatever it is worth, is that instead of chucking numbers — or protons — around, *ad lib*, hoping to hit some numerological jackpot, we should be studying the actual hierarchical combinations and internal parameters of h and $\hbar$, corresponding to Weisskopf's 'three spectroscopies'. To deal with all the various mathematical interrelations and sub-relations between these natural units is beyond my own range of expertise, yet I feel that I may have gone some little way towards it in my work with Anthony Osborne, at Keele. This work is based on something Clive Kilmister wrote in *New Scientist*, nearly thirty years ago,[6] in which he described how the mass of a light-transmitter decreases — how, as he says, the transmission of information is the transmission of energy, which is equivalent to mass. At Imperial College, on Monday, September 14th, Anthony and I will show how, as part of the radical, Keele approach to theoretical physics, we advance this idea by deducing the Balmer-Rydberg formula for the photon-spectrum directly from the formulæ of Special Relativity — that is, purely dimensionally and numerically, without the unnecessary ritual of talking about light or electromagnetic displacement currents propagated a void. We also demonstrate how, from this spectrum-formula, in the same, purely numerical and dimensional way, there may be deduced sub-h — or, rather, sub-bar-h — measures equivalent to the electron, proton and neutron. We can also demonstrate variations in G, due to involutions and convolutions of

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angular momentum, which unify gravitational force and Coulomb force. Is that or is that not a contribution to the 'Combinatorial Hierarchies'? Will somebody please tell me?

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[2] Hawking, S.: *A Brief History of Time*, Bantam Press, London, 1988:

> 'If we do discover a complete theory, it should in time be understandable in broad principle by everyone, not just a few scientists. Then we shall all, philosophers, scientists, and just ordinary people, be able to take part in the discussion of the question of why it is that we and the universe exist. If we find the answer to that, it would be the ultimate triumph of human reason – for then we would know the mind of God.'

[3] Pope, N. V.: 'Relativity is Kids' Stuff', *School Science Review*, June, 1989, **70** (253), p.86.

[4] Lewis, G. N.: 'Light Waves and Light Corpuscles', *Nature* No. 2937 **117**, February 13, 1926.

[5] BBC.TV, 'Antenna', Circa July, 1992: 'Time-Travel', Deutsch, D., *et al*

[6] Kilmister, C. W.: 'Gravitation After Einstein', *New Scientist*, No.346, July 1963, pp. 34-36.

ADDENDUM 2

In the discussion which followed, a further 'slot' was allocated for supplementary explanation of the 'Keele approach'. This short talk was delivered *extempore*, with the use of a hastily prepared blackboard diagram depicting a linguistic analogy between the bogus question' 'Have you stopped beating your wife?' and the equally bogus Cartesian question 'Is reality Mind or Matter?' In further discussion, the various elements of this diagram, sequenced on the blackboard by various rubbings-out and replacings of marks, were rendered by a succession of OHP transparencies designed to be more or less self-explanatory. The following are a tidied-up and slightly edited version of those diagrams.

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BUM QUESTION:

'Have you stopped beating your wife?'

This is a misleading question. But a society in which such a question becomes institutionalised develops a language in which there is no answer other than a simple yes or no.

YES-PARTY

NO-PARTY

BUM ANSWERS TO
A BUM QUESTION

LINGUISTIC ANALYSIS

'Linguistic' philosophers refuse to answer the bum question. Seeing it as misleading they seek to revise it and to refine out – by 'analytic' methods of argument similar to those used in psycho-analysis – all the various institutionalised vestiges of the false dichotomy.

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BUM QUESTION:

'Is reality Mind or Matter?'

Descartes posed this misleading question, which became institutionalised in the scientific and philosophical traditions of our society. Our language has developed around it in such a way that it can only be answered in terms of 'Either Mind or Matter'.

MATTER-PARTY

NEWTON

FARADAY

MAXWELL

PLANCK

EINSTEIN

et al.

MIND-PARTY

BERKELEY

HUME

KANT

MACH

EARLY WITTGENSTEIN

et al.

BUM ANSWERS TO A BUM QUESTION

MOORE

AUSTIN

RYLE

LATER WITTGENSTEIN

et al.

LINGUISTIC ANALYSIS

'Linguistic' philosophers refuse to answer the bum question. Seeing it as misleading they seek to revise it and to refine out from our society – by 'analytic' methods of argument similar to those used in psycho-analysis – all the various institutionalised vestiges of that Cartesian schismatism.

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LINGUISTIC ANALYSIS

Following Wittgenstein's philosophical 'sea change', this analytical approach fell into bad odour, especially with scientists. This was because 'linguistic' philosophers saw themselves as custodians and defenders of 'Ordinary Language', protecting it from encroachments by the 'scientism' of the Cartesian dichotomy. This 'anti-scientistic' philosophy, sometimes called 'Naive Realism', took as its standard the language ordinary people use in describing what they see, touch, hear, and so on. In seeking to enforce this 'proper usage', these philosophers were sometimes referred to by their critics as 'the language-police'.

This 'new philosophy' became fashionable among arts-orientated students. Getting rid of Descartes' 'bum question' relieved them, they believed, of any need to take account of science in the pursuit of philosophy.

Wittgenstein's dictum, 'Language is alright as it is,' thus led to a form of impressionistic, arts-based 'philosophy' in which all references to science or scientific ideas were 'rubbished'. The wave of enthusiasm for that sort of sophistry has now all but completely subsided on our university campuses.

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THE KEELE APPROACH

At Swansea and Bangor, and now at Keele, a development of linguistic philosophy, called Normal Realism, rejects this sophistic reaction to science. Normal Realism accepts that changes in language due to science are inevitable. It sees no necessity for hostility between ordinary and properly formulated scientific language and stresses the need for the two to evolve together.

To this end, Normal Realism applies methods of linguistic analysis with the aim of refining out of scientific language those vestiges of the Cartesian schismatism which maintain chronic confusion. Chief among these is the language of 'the velocity of light'. Removing the Cartesian dichotomy makes redundant this idea of light as a space-travelling intermediary between matter and perception.

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A PRIME EXAMPLE OF HOW LINGUISTIC ANALYSIS WORKS IN PHYSICS

What replaces 'light in space' when the Cartesian schism is removed? The following is an example of how, in conjunction with science, ordinary-language usage can be mobilised to answer that question.

What is space? In ordinary language it is a vacuum, or void where, by literal definition, there is no intervening or mediating object. We therefore see things in space immediately — that is, literally, without mediation. Gilbert Lewis confirmed this in 1926 by demonstrating that in space, optical interaction is an immediate and proper-time-instantaneous quantum 'touching' between object and observer.

So the idea of light 'travelling in space' between object and observer is a redundant relic of the Cartesian false dichotomy. What we see in unmediated perception are the objects themselves — or, at least, those parts of them with which, at any time, we are in direct quantum contact. At the quantum level, therefore, every particle is potentially the immediate neighbour of every other, as in the random access memory (RAM) of a computer.

Because sequences, by definition, take time, then in any sequential interaction — for example, a two-way, or 'reflected' signal — the quantum transfer takes time (as we have seen) in the way that people like Fizeau and Michelson ascertained. It is the way in which these sequences are observationally processed that distance (distance-time) is projected.

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QUANTUM POTENTIAL

This immediate random access potential (RAP) of quantum particles is what we at Keele call quantum potential (see also our article in the current *Physics Essays* journal).

Because it is proper-time-instantaneous it solves the 'action-at-a-distance' (EPR) problem created by so-called 'Einstein separation' and is supported by the results of experiments carried out by Alain Aspect, Carrol Alley, *et al.*

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Chaos, Biology, and Physics

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I'll use this opportunity of the ANPA presidential address to try and relate my work to the combinatorial hierarchy, the main interest of ANPA. The title refers very generally to the three subjects to be discussed. In this paper, concepts from chaos theory will be shown to apply to both physical and biological systems. I have always been interested in what I call a "compactness of description" concerning the functioning of systems in nature. Although I am a biologist and specialize in living systems, I view the universe as a single self-organized evolving system with cosmological, prebiotic, and biological evolution being successive stages of a single evolutionary process. Although my papers are written in what a computer scientist would call a high level language when compared to the formal writings of many ANPA members, the "compactness of description" I attempt to achieve could also be called parsimony. In his *An Essay on Discrete Foundations for Physics* , (1) Noyes states that parsimony is one of the principles that presumably all practitioners of laboratory physics would agree is desired in a successfull theory. Parsimony isn't restricted to the practice of physics. It should be a desired attribute of any scientific theory, and be agreed on by practitioners of science in general. An early book on chaos theory in non-linear systems said that the goal of the research was to "capture the essentials in a nutshell" (2). I am looking for the conceptual description behind what can also be a mathematical description. I try to formulate principles which can hopefully be given mathematical description. The idea of hierarchy falls nicely out of this view since reductionism run rampant, which much current science is, is actually non-compactness of description. Thus, not all practitioners of laboratory science apply the principle of parsimony. However, while many scientists are content to continue to collect facts which are all variations on a theme, the true discoverers know when it is time to shift to a new level of description. I have always applied this principle by asking whether new facts at a certain level will actually contribute new information or a higher level of description is now warranted by the collection of facts. The hierarchy construction is perhaps the most general embodiment of this principle since a switch occurs to a new level when there is no more information to be gained by the process at the current level.

I also find similarities in my approach to the modeling methodology of McGoveran (3). In this system the R or representational framework is the formal mathematical description, the E frame is the epistemological or knowledge based frame, and the P or procedural frame describes the mapping between them. When I said above that I look for the conceptual description behind the mathematical description I mean that I try to express the E frame description which corresponds via a particular P frame mapping to a particular R frame description. In my previous papers in the ANPA proceedings I try and relate my E frame principles to the R frame work of other ANPA members using a type of P frame mapping which has not yet been properly expressed mathematically. A final point of contact between my work and the modeling methodology is in the recursive process which is used to refine the modelling. I too continually compare my E frame descriptions to R frame work and try to construct more precise and accurate P frame mappings between them.

I focussed my attention on self-organization and chaos theory because they revealed some common principles of organization in diverse systems. Here was a domain of universal behavior which included the universal routes to chaos and the ubiquitous 1/f or flicker noise that was seen in so many different systems. Self-organization was related to phase transitions, another domain of universality in nature. Central to these principles was a new type of geometry known as fractal geometry (4). This was a new geometry of nature that described complex and irregular objects through a specification of the simple algorithms that generated them. Simple fractals sets are self-similar objects with a linear relation between the entire object and its parts. The strange attractors in chaotic dynamical systems are fractal attractors. The structure of systems at phase transitions is self-similar or fractal, and universality in chaos is due to the fractal structure of a phase transition in a mathematical space.

I was particularly intrigued by a non-linear fractal object known as the Mandelbrot set (M set) which came out of work on quadratic mappings of the complex plane carried out in the early 1900s by Fatou and Julia. This object can be continually magnified until the computer runs out of power revealing a world of organic appearing forms. It was the resemblance of these forms to real biological systems that presented a great mystery that I decided to study. I approached it as a problem of determining the reason why these organic forms reason appear in purely mathematical mappings. I have discussed the answer in a number of previous ANPA papers (5, 6). The details needed to justify the following remarks can be found in those papers.

The forms seen in the Mandelbrot set result from an intersection of number theory, topology, dynamic stability, and the theory of computation. The forms in the M set are those that are structurally stable for this mapping. Structural stability is used as it is in catastrophe theory. This is where the topology enters and the forms are the result of dynamic stability principles. This stability is related to the concept of self-organized criticality since the stable forms are those for which the forces involved in the mapping are balanced. The forms are organized around the topological singularities of the quadratic mapping. There is a taxonomy to the forms which exhibit an increasing complexity due to a topological bifurcation series. A quote by Feigenbaum from the preface of a recent book is relevant here: "many of you are aware of the existence of a certain object called the Mandelbrot set. Virtually none of you are aware that that it's ubiquitous existence in those sufficiently smooth contexts in which it appears is the consequence of universality in the transition to chaos" (7). Number theory enters in the pattern of topological events. The positions of stability are organized around rational numbers. This is determined by the number theoretic behavior of the quadratic mapping. It also brings in the theory of computability since the quadratic mapping is an example of a halting problem. The decision of whether a point is inside or outside of the M set depends on whether a computation halts or not. This in turn depends upon the number of iterations. A striking confirmation of the relevance of these ideas can be seen in a quote from the same recent book referred to above: "It has been described as the most complex and possibly the most beautiful object ever seen in mathematics. It's most fascinating characteristic has only just recently been discovered: namely that it can be interpreted as an illustrated encyclopedia of an infinite number of algorithms" (7). The border of the M set at any number of iterations relates to the border of decidability for the halting of this particular computation. It also illustrates the notion of the edge of chaos since any orbits that will still escape from the set have periods which exceed the number of iterations tested.

The border of the M set also demonstrates the notion of minimal connectivity at a phase transition since the Julia sets defined by C values at the border of the M set are those which have multiple basins of attraction which are minimally connected. Any C value which is outside the M set defines a Julia set which is in separate pieces. The border marks a phase transition in the connectivity of Julia sets.

In dynamical systems the transition point to chaos or the "edge of chaos" defines the maximum complexity in the organization of a particular system. This maximum is related to algorithmic complexity. At the border of the M set, there is no shorter procedure for determining whether or not a computation will halt than to run the computation. You cannot tell in advance whether it will halt using a shorter algorithm. For some algorithms, it is possible to characterize their activity on the scale of order to chaos. The most efficient algorithms are those whose activity is at the transition point to chaos. An efficient algorithm is one whose sub-operations work coherently together. This is another example of the "economy of description" which I used above as an example of the efficency of hierarchical description. The "edge of chaos" as a state of organization is one which has the most information or complexity without any redundancy (8). This is an excellent example of the parsimony principle. The edge of chaos is the state of minimal yet complete connectivity or coherence in a system. The edge of chaos is related to the idea of the exoskeleton produced by the combinatorial hierarchy.

I believed that this collection of mathematical principles underlying the forms in the M set were so general that they should not just apply to the realm of quadratic mappings. What seemed most exciting to me was the possibility that these same principles were responsible for the similar forms seen in actual biological systems. I developed the idea that this same set of principles was also responsible for the morphology of structures in nature. I called this the origin of pattern and form and related actual biological systems to the structures in the M set using the concept of self-mappings (5, 6). A self mapping is the mapping of a system onto itself. In the self-mapping of the complex plane, at each time step each point is mapped by a function to another point in the complex plane. When the trajectories generated from each point of this self-mapping of the complex plane are classiifed by where they go, the patterns seen in the Mandelbrot set occur. Cellular automata simulations are rule based self-mappings which are carried out pixel by pixel. They generate dynamic patterns that are somewhat reminiscent of a simulated ecosystem. One popular cellular automata is called the game of life. A real living system can also be seen as the self-mapping of an organism, where at each instant, each configuration is mapped onto another configuration of the system. This involves higher level processes because complex systems are organized in a hierarchy and the rules or functions that determine the evolution of configurations is not carried out only for each of the smallest units in a system. There could also be lower level mappings occurring on a point by point basis. If biological systems are considered abstractly then we can think of some physical process as a self-mapping which generates aggregates which further self-organize in hierarchical systems where the self-mappings take place via rules which operate under higher assemblies. The self-mappings still occur at the elementary unit level as well. Something analogous occurs in the cellular automata game of life where the neighbor rules occur at the pixel level but the initial conditions can be engineered so that logical operations occur at higher levels of organization. A universal Turing machine can be simulated using the cellular automata game of life which is itself a simulation on a computer.

The laws of Newtonian physics can be modeled as mappings by using difference equations instead of differential equations (9). Quantum mechanics can also be seen as a mapping since it is a prescription which maps initial states into final states. The combinatorial hierarchy is a recursive process which generates a collection of information strings (10). It too can be seen as a self-mapping through the notion of a program universe. The program universe developed to generate the combinatorial hierarchy, is a self-limiting, self-mapping which generates new elements of the domain being mapped as the mapping proceeds. Given this general description of self-mappings allows the generation of forms in actual biological systems to be compared with the generation of forms in the Mandelbrot set. This is shown in rows 6 and 8 of Table 1. Any self-mapping of an actual system should follow similar computational and mathematical principles. A striking illustration of this concept is that the principles of structural stability as described by catastrophe theory control the topological aspects of form in real biological systems and in the Mandelbrot set. I discuss this in detail in some of my earlier papers (5, 6).

In my approach, I define structure as the sets left invariant by specific global or self-mappings of particular spaces or systems (11). The nature of these structures is determined by the principles of topology, number theory, computation, and dynamical stability which together account for the origin of pattern and form. I will now give some examples showing how my principles for the origin of pattern and form has proven to be a very powerful concept for simply describing the essential features of complicated phenomena.

The first example is in the domain of structural stability where catastrophe theory describes biological processes in terms of the singularity structure or envelope of a mapping. This singularity structure is what I have shown to be the basis of the structures in the M set. There is some additional; theoretical evidence by others illustrating the importance of the singularity structure of a mapping for the form of actual biological systems (12).

The second example involves the harmonics of natural form which follows the rational number organization which I have described as the number theory portion of the origin of pattern and form. This pattern of organization involves the golden section and fibonacci series and has been shown to underly the structure of many biological organisms (14). In addition, a harmonic bifurcation sequence appears to characterize the synthesis of specific developmentally regulated RNA and protein molecules in developing embryos (13).

The third example is the new field of controlling chaos (15). This is the newest and currently the most exciting branch of chaos research. The central idea is to keep the system in the appropriate portions of the phase space where the parameters produce periodic orbits. In earlier papers I have shown that all of the structure in the M set is organized around rational numbers. That is because rational numbers produce periodic orbits in the self-mapping which generates the M set. All of the structure is organized around a skeleton of periodic orbits. In fact periodic orbits have recently been called the skeleton of chaos and even quantum chaos. This brings us to the fifth example which is quantum mechanics. Even a mapping in a continuum generates structures which are quantized because they are organized around periodic orbits which in turn reflect organization around rational numbers. The same harmonics of rational numbers seen in the Bohr atom is seen in the organization of forms in the M set and in the harmonics of actual biological systems.

Other quantum mechanical examples include the harmonic structure of windings in the fractional quantum hall effect and the Yang-Baxter relation in string theory which involves the harmonic structure of topological windings. The Ahharonov-Bohm effect shows that when the topology of a mapping is changed, so is the dynamics in a manner which reflects non-local topology rather than local causality. This is the well known quantum non-locality.

The relation of quantum mechanics to the origin of pattern and form is quite complex and was the subject of my paper at ANPA 13 (16). The reader should be familiar with that work before attempting to fully understand the contents of this paper. The most relevant observation for the combinatorial hierarchy is that fractal quantum spacetime is not pre-existing but is generated by the measurement process. One can think of this as a particular kind of self-mapping where spacetime events and particle trajectories are defined by measurement. The uncertainty principle and the de Broglie relation are the geometric constants of the mapping which define the relationship among the structures seen on different scales. The combinatorial hierarchy is also a self-generating process. It is in fact an iterative self-generating process which produces a set that has fractal properties.

In previous papers I have defined the cosmology of the M set as the scale dependent structures which define a small copy of the M set yet are outside of it (5, 6, 16). A copy of the M set encodes the same lifelike forms as the large M set which, as I have shown previously, contains temporal information. This same temporal information can be used to describe the evolution of the "lifeforms" encoded by the M set as universe and can also be used to describe the evolution of the M set itself as the universe. These computer graphic observations allows an interesting definition of spacetime in the M set program universe which as I have previously shown has interesting fractal peoperties. I have also previously shown how the computer graphic data can be related to Feynman graphs and to other aspects of fractal quantum spacetime. The structure of these spacetime points and their relation to Feynman graphs is shown in Figure 1.

The fractal dimension of quantum mechanical paths is one at large distances and approaches two at small distances. This is true for each dimension of space and for time as well. This is also the fractal dimension of Brownian motion and the fractal dimension of the boundary of the M set. The boundary of the M set can have a fractal dimension of 2 with many different types of structures. This a function of bifurcation depth and is related to the observation that there is a hierarchy of scattering processes related to Feynman graphs which occur as a function of scale in fractal quantum spacetime and the boundary of the M set. In brownian motion the trajectories are made of straight segments with simple bends. As bifurcation depth increases the simple bends are replaced by more complex networks of lines and bends. There is a field theory of pattern identification which shows differences in classification to reflect differences in topology (17). The topology manifests through changes in the singularity structure or envelope. As I have described previously, information is recursively involded into the singularity structures of a mapping as the logical depth or topological bifurcation depth increases. The increase of complexity of quantum mechanical systems is a matter of history but one which reflects an increase in logical or topological depth.

I have been trying to illustrate the idea described in Table 1 that all structure can be defined as invariant sets from global mappings. For the case of the global quadratic mapping of the complex plane that generates the M set, the structure of this invariant set is remarkable. The set is essentially made up entirely of small copies of itself. Furthermore, this object can be viewed as both the most elementary particle in the universe and the entire universe itself. . As shown in rows 6 and 8 of Table 1, the relationship between the universe and the and particle in the M set is analogous to the relationship between an organism and it's genome in biology.

After making this observation I asked whether there was any object in nature which, like the M set, was an invariant set from a global mapping and was also both the largest and the smallest entity. I concluded as shown in row 5 of Table 1 that the superstring was potentially an analogous object. I discuss this in detail in the proceedings of ANPA 13 and refer the reader to that paper (16). At first it was not apparent that superstring theory describes an object made up of copies of itself, but the theory of third quantization allows one to describe a scale transformation between the universe and the most elementary structure. This seems to be an example of the universe recoding itself. Work on inflation has also uncovered evidence for a self-reproducing universe (18).

If the universe encodes itself, the code is in the nested families of periodic orbits which underly complex systems. Because of the relation between cosmology and biology depicted in Table 1, a reflection of this organization should be found in the genome of an organism. Evidence for long range fractal correlations in genomes has recently been reported (19).

The notion of loop space has recently become popular as a model for quantum gravity (20). Although the relation to string theories is not yet clear, from the viewpoint in this paper both strings and the loops of loop space are examples of the field lines which I have shown to underly the structure of the M set. These field lines are the mathematical equivalents of the field lines in electrostatics and conformal mappings are important in both electrostatics and the mathematics of the M set. Strings in string theory including superstrings are known to be related to topological flux lines. This is also true for the loops of loop space since according to Rovelli, "guage theories originated from Faraday's idea of the description of electric and magnetic force in terms of loops that fill the space: the loops of the loop representation are precisely the quantum version of Faraday's force lines which historically gave birth to guage theories" (21). This is yet one more example of where my principles of pattern and form are applicable to systems seemingly far removed from the quadratic mapping of the complex plane where they were discovered.

As Table 1 tries to show, the notion of invariant sets from global mappings allows my principles of pattern and form to be compared to other processes. In the case of the quadratic mapping of the complex plane, the topology is defined by the two dimensionality, and the single hump in the class of all curves defined by this mapping. There are aspects of both one and two dimensionality in the mapping. Fractal dimension shows that the concept of dimensionality is subtle which helps to explain the difficulty people have defining it in the combinatorial hierarchy. The metric structure of the mapping comes from the ordering of points in the complex plane. The quadratic mapping of the complex plane, even has discrimination since the iteration number defines a maximum counting number which determines the resolution with which trajectories can be discriminated. I am suggesting that mappings can be defined in more general spaces where the ordering of points and perhaps even the topology are not well defined. In the most general case one must describe the self-organization of the topology and the numerical ordering of points. This is what I believe the combinatorial hierarchy work is trying to do. I site McGoveran's *Foundations of a Discrete Physics* specifically (22) because I believe the problem of ordering points, the definition of dimensionality, and McGoveran's theorem are particularly relevant to this more general type of self-organization process. This still leaves open the problem of the process that carries out the actual mapping. The patterns in the M set are visualized by a particular type of decoding of the raw information strings of the complex plane. This decoding is what turns the ordering of points in the complex plane into patterns. In the combinatorial hierarchy, the decoding is accomplished by the second series involving the matrices. The relation of these processes to actual processes that occur in nature is not clear. If our world is an invariant set from a global mapping then the dynamics must be generated by a process, and the combinatorial hierarchy must al least be isomorphic to the actual process that carries out this construction of the universe. The fractal quantum spacetime approach strongly suggests that this process is quantum mechanical measurement. If the combinatorial hierarchy describes the inevitable result of a world ordered by discrimination it would seem to be related to quantum mechanical measurement which defines an elementary physical process of distinction.

I have recently been struck by the idea that the peculiar structure I found in the Mandelbrot set where the object is made up entirely of copies of itself, bears an intriguing relation to how Parker-Rhodes defines the inchoative without using information (23). To begin, I do a gedankin experiment and ask what type of world a sentient observer would find himself in if his world were the program universe of the M set. For example, we pretend that an organic form in the M set were free to move and study the world. We pick an observer of an extremely small scale, one visible at a magnification of 10^{20}. If we set the observer loose on the real axis, he would observe a fractal lattice of spacetime structures and if he attempted to define an elementary object he may become aware of the M set as an underlying invariant. If the absolute scale of the M set is outside of his powers of observation, there will be only a network of objects made of smaller copies of itself. In his unpublished manuscript *The Inevitable Universe*, Parker-Rhodes defines the inchoative as "a peninfinite self-contained collection of indistinguishables". He points out that "this definition fails to specify whether it is divisible into parts or indivisible by asserting both. It avoids assigning any numbers to the parts and denies whether they are distinct or identical". The M set is made up of an indeterminate number of parts each made up of an indeterminate number of parts because these numbers are scale dependent and iteration dependent. If one of our hypothetical observers were unaware of the large M set because it is outside of their powers of observation, then our observer may conclude that one of the small M sets is the entire universe when it is actually only a part of the universe. As I discussed above, and in a previous paper (16), this problem is related to the problem of self-reproducing universes in quantum cosmology. Can a baby universe be considered part of the universe which gives rise to it? Is the idea of a baby universe hidden behind the event horizon of a mini black hole, related to the problem of bit strings which can't be classified continuing to be generated by program universe? Parker-Rhodes points out that "any member of a self-contained sort has members indistinguishable from itself as well as belonging to a whole indistinguishable from itself". One can define conditions of observation where the relation between parts and wholes in the M set is poorly defined. Perhaps one example would be attempting to describe the structure of the M set without the benefit of the computer graphics. In this case, one is describing a structure based on a nested series of unstable periodic orbits, where the parts can be made up of as many parts as the whole. The actual number of parts in any segment is determined by iteration number, which is analogous to the maximum counting number in McGoveran's theory. I am not suggesting that the M set is any type of model for the inchoative. I am merely suggesting that there may be a relation which is worthy of further exploration. The M set is clearly both conceivable and imaginable and therefore it is in a different logical category than the inchoative which is conceivable yet unimaginable. Julia sets were originally described using formal mathematical descriptions, and their intricate and organic beauty was in no way apparent from this description. Before it was visualized using computer graphics, the Mandelbrot set was only a formally defined object, conceivable to some mathematicians but its intricate appearance was unimaginable based on that description.

Perhaps the best way to phrase it is that the M set may carry structural relations to the inchoative as do the rational fragments derived from it. There is certainly some type of conceptual relationship since the M set is a result of necessary mathematical and logical aspects of global mappings, and the universe according to Parker-Rhodes is a reflection of the inchoative. He states that the combinatorial hierarchy mirrors the inchoative and as I have shown in Table 1, the combinatorial hierarchy is an example of a global mapping. There should be a relation between the invariant sets derived from a global mapping and the rational fragments derived from the inchoative by pure logic. The global mapping would have to be general and not start with spacetime or a well ordered set of points. Fractal quantum spacetime defined by measurement is one example, and the combinatorial hierarchy and program universe viewed as actual process is another. Hopefully, table 1 is a step towards comparing these various approaches.

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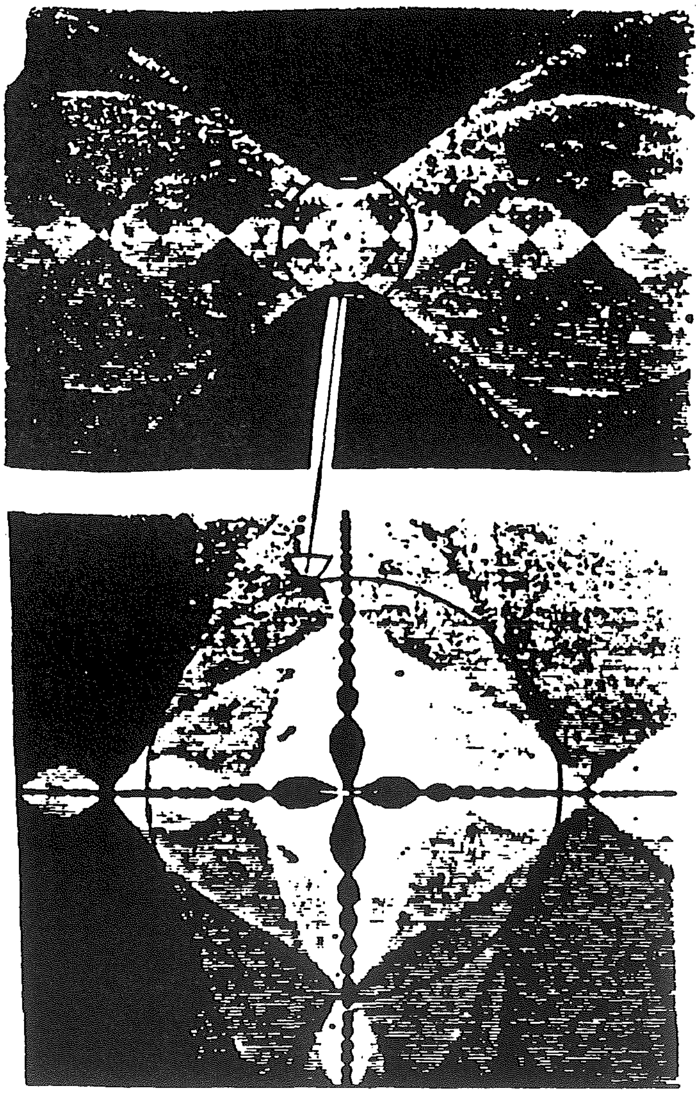
LEGENDS TO FIGURES

Figure 1. Fractal Spacetime points in the Mandelbrot set. Stage 1 in the upper left shows a magnification of spacetime points along the real axis at the tip of the Mandelbrot set. Stage two shows a magnification of the spacetime points along the axis orthogonal tto the real axis. For a detailed explanation of this bifurcation sequence see (4, 5). Stages three and five show magnifications of the spacetime points but not the axes for stages three and five.

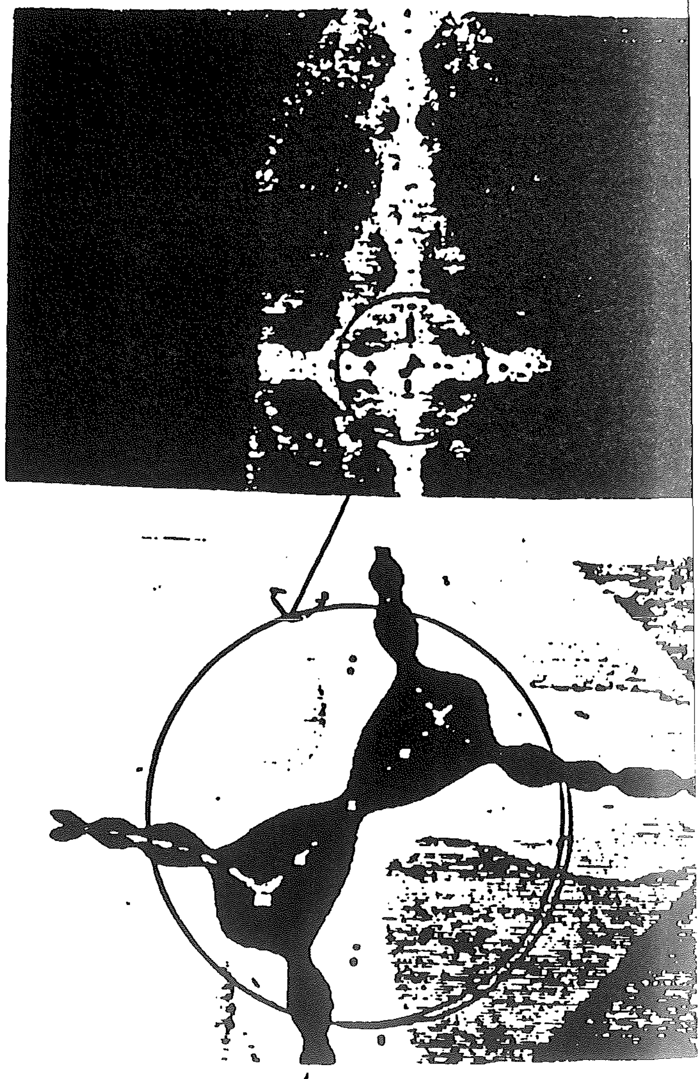
Table 1

INVARIANT SETS FROM GLOBAL MAPPINGS

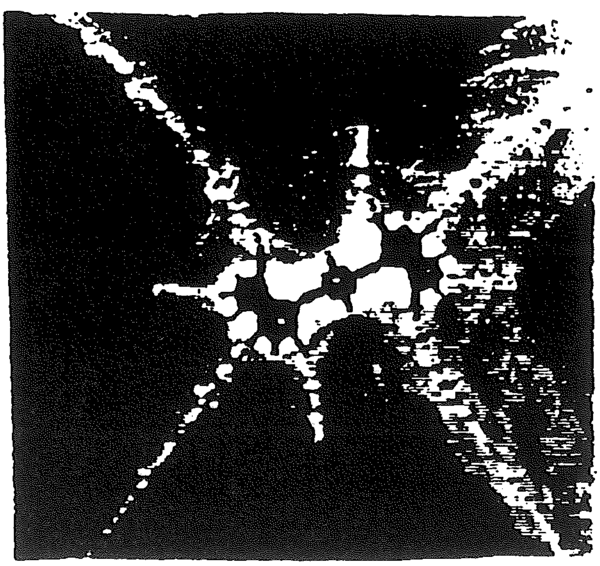
System	Space	Global Mapping	Ordering of Points	Invariant Set	Scaling Symmetry
Quantum Mechanics	3+1 D	Quantum Measurement	Given by 3+1 D	Quantum Space Time	Self-Similar
Multiple Reduction Copier	2 D	Settings on Copier	Implicit in Transformation	Fractal Set	Self-Similar or Quasi Self-Similar
Combinatorial Hierarchy	3+1 D	Power Set Construction	Self-Generation of Bit Strings	Universe or Program Universe	Quasi Self-Similar or Multifractal
Self-Mapping Fractals	Complex Plane or Riemann Sphere	Geometric Inversion or Squaring	Fixed by Nature of Space	Poincaré and Klein Group; Schrodinger Equation; Julia and Mandelbrot Sets	Quasi Self-Similar
Superstring Theory	Riemann Sphere or Teichmuller Space	Conformal Transforma-tion	Fixed by Nature of Space	Superstring	Universe and Baby Universe or Super-string
Biological Development	Egg or Womb	Gene Expression Mechan-ism	Positional Information	Organism	Organism and Genome
Genetic Code	Code or Sequence Space	Associated Gene Expression Mechanisms	Hamming Measure of Sequences	Triplet Code or Set of Computable Triplet Code	Quasi Self-Similar or Multifractal
Mandelbrot Forms	Parameter Plane	Decoding of Squaring Transformation by Escape Time Algorithm	Fixed by Nature of Space	Mandelbrot Set with Harmonic Code	Mandelbrot Set and Baby Mandelbrot
Julia Sets	Dynamic Plane	Decoding of Squaring Transformation by Escape Time Algorithm	Fixed by Nature of Space	Julia Sets with Harmonic Code	Julia Sets and Baby Julia Sets



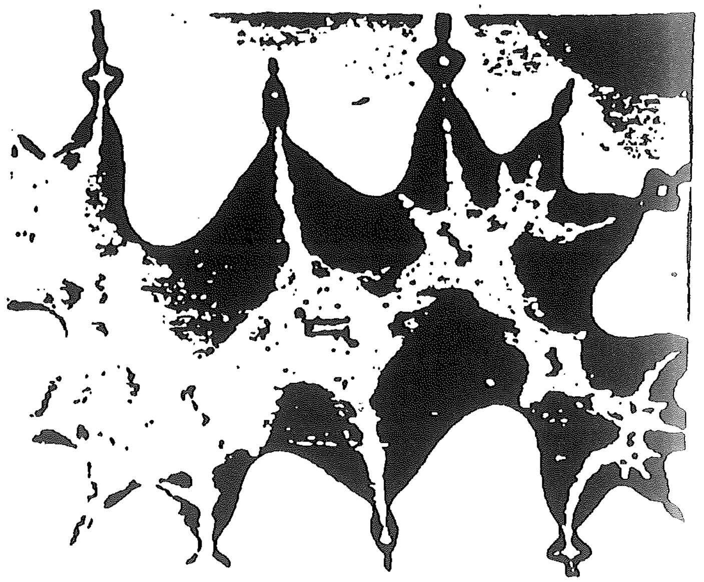
stage 1



stage 2



stage 3



stage 5

Figure I

A Test for Classical Psychospinors †,‡

by

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Abstract

The present paper supplements Oshins' ANPA West 8 paper, "The Search for Classical Psychospinors," by including a specific experimental test proposed by Oshins during the post-session. After a short summary of Oshins' work on *classical* and *quantum* spinor representations in psychology, a brief overview of the history of certain hand motions found in some martial and meditative arts is discused. Shepard's experimental demonstration of the phenomena of mental rotation of internal imagery is described. Georgopoulos' rotating, neuronal "population vector" realization for Shepard's cognitive model is briefly described. Hamilton's realization of SU(2,C) as "turns" is discussed. Georgopoulos' neurophysiological approach to Shepard's cognitive mental rotation model is adapted to Oshins' proposal for *classical* psychospinor representations for self-referential motion. Oshins' "population turn" hypothesis for neurophysiological correlates of *classical* psychospinors is made explicit. Extensive end notes provide technical elaborations upon the content of the paper.

Section I: Introduction

In 1976 I proposed using *spinor* representations of the Pauli rotation-reflection algebra in psychology (Oshins & McGoveran, 1980; Oshins 1982, 1984a,c,d). This was done in order to reconcile controversy in the psychological literature over schizophrenia, treated as a logical phenomena (Oshins and McGoveran, 1980; Oshins 1982b/1983 rev., 1984a, 1987b, 1989a,b,c(to appear), 1992b, 1994 (to appear); Hilgard, 1989), and as a formal alternative to Brown's (1973) *Laws of Form* approach to self-referential paradoxes (Oshins and McGoveran, 1980; Oshins 1990, 1991a,b, 1993a.).

In Oshins and McGoveran (1980), we also proposed using spinor representations in order to realize Shepard's mental rotations as unitary transformations in Hilbert space[1]. In particular, I was interested in how and why the brain would code the natural "orientation-entanglement relation" of the human arms in

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‡ This paper is dedicated with deep gratitude to John Weakland, my friend and mentor for the past dozen and a half years.

terms of quaternians and spinors: If one picks up an object, such as an alarm clock, and rotates it 360^0 with respect to some reference frame, such as oneself; it winds up where it started. In contrast, if one holds ones hand with the palm up, as if holding a cup of tea on the palm, and rotates it, while keeping the palm up so as not to spill the tea, one finds it necessary to turn the palm around the rest of oneself *twice* (ie. 720^0), once below the elbow and once above it, before one winds-up where one is/was.[2,3,4,5] This is illustrated thus (adapted from Bernstein & Phillips, 1981):

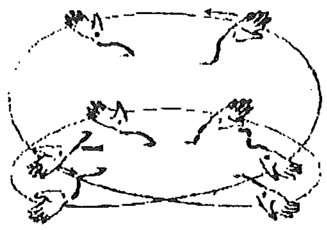


Figure 1: Double Covering of Palms found in Martial Arts Pa Kua Chang and Pencak Silat and in the Philippine folk dance Binasuan ("wine dance").

I have proposed two psychological interpretations for spinors:

(1) as a solid object/impenetrable region of space (Biedenharn & Louck. 1981, Ch. 2, pp. 7-26; Ch. 4, pp. 180-203; and Ch. 7, § 7.10, Sect. b., pp. 528-532 ; Oshins, (Oshins 1983c, 1984a,c, 1985, 1986a/87e, 1987b, 1988a, 1993b), instead of rigid objects (Shepard & Chipman, 1970; Shepard & Metzler, 1971; Shepard & Cooper, 1976; Shepard, 1978, 1979, 1984; Carleton & Shepard, 1990a,b). Such *classical spinors* are even-dimensional, *continuous realizations* of the simply-connected, double covering group [SU(2,C)] of the odd-dimensional rotation group [SO(3,R)][6]; and

(2) as quantum dicotomies/bits with ambiguity, obeying Dirac's "principle of linear superposition" of *rays* in projective Hilbert space. This latter lead to my "synaptic spanning" model for "*quantum* parallel processing,"[7] presented as a quantum alternative to McCulloch-Pitts' "synaptic summation" model (McCulloch & Pitts, 1947) and to Pribram's hologram/"synaptic superposition" model (Pribram, 1971). *Quantum spinors* are *dicotomic* [2-valued] realizations [eg. "right/left", "up/down"] of the same symmetry group [SU(2,C)], such that the dicotomy itself can be realized as collineating with any continuously parameterized direction --- the vector associated with the dicotomy can have any direction in 3-space. (Oshins 1984a,c, 1985, 1986a/87e, 1987b. [ft.nt.10 of note 10 and note 14], 1988a, 1993b; Oshins, et. al., 1992/89).

As a result of citing a preprint by Yuri Orlov (Oshins & McGoveran, 1980), I was asked to serve as scientific spokesperson for Orlov's related work on "doubt states" and "the wave logic of consciousness" (Orlov, 1981, 1982; Oshins, 1983a), smuggled from the Soviet prison camp (Greenbaum, 1986; Oshins, 1983c, 1984c, 1986b, 1987b). Although Orlov explicitely insisted that his wave logic was not of a quantum nature (Oshins, 1991, ft.nt.6 and references therein) --- ie. "Our hypothesis is that the experience of doubt is not of a quantum mechanical nature...." (Orlov, 1981, p. 88) --- it was close enough that I became convinced that if I could show a basis for using quantum physics in psychology, I could provide a tool for physicists in fighting for Orlov's freedom from the prison camp. Since Professor Sidney Drell had urged me to find an empirical basis for my ideas, if I hoped to get the attention of physicists, and since I had already provided a basis to believe in spinor representations in the brain, I turned my attention to possible magnetic effects, ultimately proposing several ideas for experimental inquiries using a SQUID (Superconducting Quantum Interference Device) (Oshins, 1984a,d, 1985, 1987b; Oshins, et. al., 1992/89; Aharanov & Susskind 1967; Bernstein & Phillips 1981).[8]

In 1989, Steve Zins and Myles Hayes independently drew my attention to Geogopoulos's population vector approach to measuring neurophysiological corrolates to Shepard's mental rotations (to be described below). It occurred to me that I might be able to adapt Geogopoulos's technology in my search for *classical* spinor states of the brain. Thus, the origin of this paper.

SECTION II: HISTORY OF PA KUA PALMS (Jou, 1980; Lu-t'ang, 1983; Miller, D. (ed.). (bi-monthly); Painter, 1981; Smith, 1967; Veith, 1949; Wilhelm, 1967; Ying-arng 1973)

It is said that in the "legendary period" in China the first legendary emperor Fu Hsi divined on the back of a tortoise an archetypical coding for the patterns of nature. Specificly, the story goes that he interpreted cracks on the tortoise as triplets of solid and broken lines. The solid lines stood for yang (masculine, light, active, etc.) and the broken lines stood for yin (feminine, dark, receptive, etc.)[9]. The eight possible triplets of combinations of yin and yang formed the pa kua (8 trigrams). Pairs of trigrams (the inner and outer trigrams) formed the hexigram structure of the *I ching* which is a Chinese "classic" of wisdom and destiny.

The third legendary emperor Huang Ti was the "father of Chinese medicine." He proposed a system of points (tsubo) and channels (merideans) on the human body through which the various "energies" (chi) are supposed to flow. He originated the tradition of accupuncture and the theory of 5 elements (wu hsing). The theory of 5 elements was similar to the childhood game of paper-rock-scissors (hand-fist-fingers). It was supposed to represent the laws of accupunture through so called creative and destructive cycles[10] In Huang Ti's codification scheme, all of the so-called yin (zang) meridians were on the inside/front of the body and all of the yang (fu) meridians were on the outside/back of the body.[11]

It is said that in the early 1800's a eunich named Tung Hua-chuan came to prominence in the palace of the Ching Emperor. While serving at court, Tung was supposedly carrying his trays in such an agile and skillful manner that he was discovered to be a master of an unknown martial art called pa kua chang (eight trigram palm). Pa kua chang is a so-called "internal" (mind/image/will) art, based upon cultivating one's *intention* and having much in common with such moving meditations as t'ai chi chuan (t'ai chi boxing). One exercise done in this art involves doing a double-covering with both hands coordinated.

In addition, there are two palm positions out of the so-called "8 mother palms" that are essentially the same in form except for the intention of the change. Specificly, when the hand is held as if embracing someone with the palm facing inward toward the body it is called pao chang (embracing palm). When the hand is held in the essentially the same manner except having the intention on the outside of the hand as if going to strike someone with the outside/back surface, it is called liao chang (warding-off palm). So we see that what we are essentially coding is the intential of the hand with respect to the rest of oneself.

Furthermore, there is a series of meditative motions that consist of walking a circle in different poses with the hands in different relative relationships (the so-called 8 "inner palms"). For example, consider the following picture of Sun Lu-Tang, the Pa Kua master and founder of Sun style T'ai Chi [See also pictures, pp. 85-95 in Oshins (1987b)]:

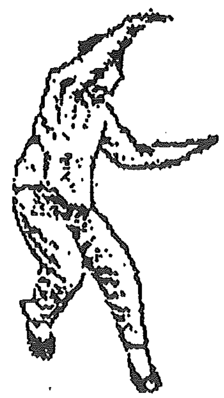


Figure 2: Sun Lu-Tang performing an "inner palm" meditative posture from Pa Kua Chang.

Let us imagine Sun Lu-t'ang's body as being somewhat like a bat or a spreaad out pelt, the palms determining a pair of planes tangent to a right circular cone: As proven in the end notes, a parameterization for a *classical* spinor can be made in terms of the loci of intersection of two planes rotating around the cone (Mercer, 1963; Gaposhkin, personal communications; Zins, personal communications)[12]:

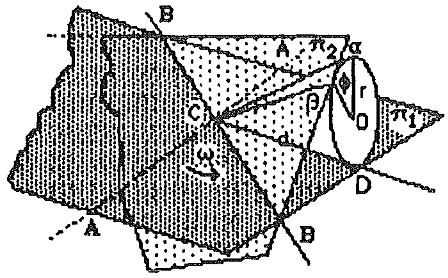


Figure 3. Inner palms as planes circumnavigating a cone as a spinor parameterization.

In the next section I will briefly describe Shepard's pioneering work demonstrating the phenomena of mental rotation of internal imagery and Georgopoulos' extraordinary finding of a cortical "population vector," which is supposed to reflect a monkey's collective, neuronal activity in the motor cortex, corresponding to a cognitive (mental) rotation which is necessary for the monkey to perform a physical task. In the following section I will describe briefly the mathematics of "turns," which provides a realization of the quaternian algebra and thus its spinorial parameterization, suggest that one might adapt Georgopoulos' technology to search for *classical psychospinor*, and provide an explicit test which would demonstrate or reject such an hypothesis.

SECTION III: SHEPARD'S MENTAL ROTATIONS AND GEORGOPOULOS' POPULATION VECTOR (Shepard, 1978, 1979, 1984; Shepard & Cooper, 1976; Shepard & Chipman, 1970; Shepard & Metzler, 1971; Appenzeller, 1989; Georgopoulos, Schwartz, & Kettner, 1986; Geogopoulos, et. al., 1989; Oshins, 1984a,c,d, 1985, 1986a/87e, 1987b, 1988a, 1992a, 1993b; Oshins & McGoveran, 1980; Oshins, et. al., 1989)

Shepard's Mental Rotations[13],[14]:

Shepard has shown that in mentally comparing differentially oriented, asymmetrical geometric objects, the time required to accurately discriminate whether or not a second object is a mirror image or an equivalent object is linearly proportional to the relative angular orientation, thereby, indicating that rotations of mental representations take place during these comparisons. For example (adapted from Shepard & Metzler), subjects were asked to compare the top block set with the bottom block set in each of the three groups depicted below:

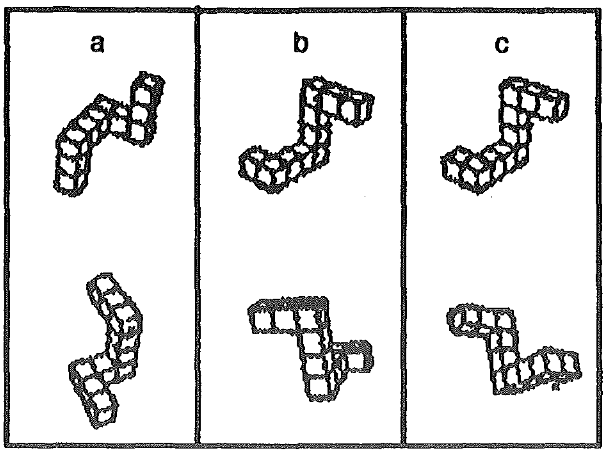


Figure 4 Shepard & Metzler's Mental Rotation Experiment: For figures 4a and 4b, a rotation is all that is necessary to aline the objects so that they coincide. For figure 4c this is impossible since one of the pair is the mirror image of the other. The time measured to check for sameness or inversion is linearly proportional to the angle necessary for the rotations.

The reader will be convinced that the first two sets are identical other than in orientation but the last set would require an inversion of one of the pair in order for them to have to be alined. Shepard and his colleagues showed that the amount of time to compare was linearly proportional to the angle through which one would have to rotate (within a plane in 3-space) in order to aline the objects. This frequency was not very sensitive to whether the rotation lay within the picture plane or out of the picture plane, thereby indicating a 3-dimensional aproximate isotropy to the mental rotation. With some practice, the reader will also be able to see the objects apparently rotate in the mental image during the task of making the comparison

Georgopoulos' Population Vector[15]:

Georgopoulos and his colleagues showed that they could define a representation of collective activity in the motor cortex of a monkey called a population vector that has a directionality which strongly corrolatable with the direction of motion of a monkey's arm during a conditioned task. Specificly, he found that (1) each neuron has a preferred direction for which its firing rate increased the most; (2) during one movement, neurons with many different preferred direc tions showed changes in activity; (3) during one movement, cells tuned for directions close to the direction of movement showed largest increases in firing; and (4) cells tuned to different directions than the direction of movement showed smaller changes such that the frequency of discharge of a particular neuron during movement varied linearly with the cosine of the angle between the neuron's preferred direction and the direction of motion. The population vector is a sum weighed vectorial contributions of individual cells, where one weighs the preferred direction of the individual cells with cells difference in activity level from some offset level[16].

Having found a such directionally tuned, collective neuronal representation (the population vector), Georgopolous and colleagues used it to "probe" the cognitive task of a monkey making a mental rotation in order to perform a motor task.. After a preparatory signal, the monkey was trained to move a handle in the direction of a dim light, but if the light was bright, the task was to move the handle in a direction 90 ^{0} counter-clockwise from the dim direction (ie. perpendicular to it). This was all done in a plane havin 8 equally spaced, randomized pairs of directions Georgopoulos found that the population vector developed before the actual movement and that when the task involved changing the direction of motion, the population vector (representing the collective neuronal activity) rotated uniformly with an approximately constant angular velocity! Thereby, he succeeded in in finding "the first direct visualization of a cognitive process in the brain."

SECTION IV: OSHINS' POPULATION TURN HYPOTHESIS FOR CLASSICAL PSYCHOSPINORS[17] (Oshins, 1992a, 1993b):

WHAT ARE TURNS[18]

Turns provide a way to represent rotations in 3-space: (1) if you let your thumb (of say your right hand) point in the direction of the axis of rotation, with the other fingers curling around an arc in the direction of rotation; and (2) if you imagine this directed arc to lie on the surface of a (unit) sphere along a great circle and allow its length in degrees (from tale to head) to be 1/2 the angle of rotation, you will observe that you can parameterize all 3-dimensional rotations in terms of these 1/2-angle directed arcs of great circles, or *turns*. [I will not discuss the reason for the 1/2 here. It is in Professor Biedenharn's chapter and devolves around the fact that all translations can be made out of pairs of reflections in parallel planes and all rotations can be made out of pairs of reflections in intersection planes]

One combines turns to realize the rotation *product* by means of *addition* of turns, i.e.. one lines up the 1/2-directed arcs head to tail (modulo great circle transport) in a manner that is *similar* to adding vectors heat to tail (modulo parallel transport in order to align them). The result of *adding* two turns gives the turn that corresponds to what would be the result of *multiplying* the two rotations. The fundamental difference between adding turns as opposed to adding vectors is that the order of addition of the two turns in general makes a difference as is the case for rotations (i.e.. $T_2 + T_1 \neq T_1 + T_2$ when $R_2 \circ R_1 \neq R_1 \circ R_2$) whereas the result of adding two vectors is order independent (i.e.. $V_1 + V_2 = V_2 + V_1$).

OSHINS' POPULATION TURN HYPOTHESIS (Oshins, 1992a, 1993b):

I am predicting that the collective activity pattern which Georgopoulos refers to as a population vector will actually transform as a "population turn" or "population psychospinor". Specifically that: (1) the activity currents will add as turns do to realize the product of the rotation, and (2) that the order of the addition will be relevant as is the case for 3-dimensional rotations. This representation only works for the (more fundamental) 1/2-integral representations (i.e.. spinors/turns/quaternions) but also lets one build the vector and tensor representations. The converse does not hold.

I think that such addition would impart an extraordinary adaptive advantage over multiplication since addition is so much simpler. It would seem to be a natural way for activity to combine and a reasonable, possible evolution of Georgopoulos' efforts. On the other hand it might also not work since it would require the addition to be order dependent. Then again, this property of "noncommutivity" in itself might be valuable in some way.[19]

SECTION V: CONCLUSION

In this paper I have reviewed some pioneering work by Shepard and by Georgopoulos in the mental imagery and neurophysiology of mental rotations, respectively. Reasons have been put forth to suggest that there may be more "fundamental," *classical* spinor brain states that could be measured using Georgopoulos' technology. Furthermore, an experimental hypothesis --- that the brain uses spinor representations, not vector representation, in realizing mental rotations --- has been proposed which is capable of demonstrating (accepting or rejecting) the hypothesis. Since all translations and rotations can be generated by pairs of spinorial reflection through such half angles, in parallel planes or intersecting planes, respectively, the described hypothesis may reveal the method by which humans (and other animals) code sensory perception of Euclidean motions

In addition there is reason to believe that efforts to corrolate flows of cross-sensory modalities, such as found in the motor cortex by Georgopoulos and as predicted in the vision cortex by Carlton, would be very valuable. Similar corrolates with flow of magnetic interference that is anticipated to be found using SQUID helmets would also be possible and would open the door for a nonintrusive ability to monitor brain activity in search for cognitive capacities and capabilities[20]. Such would be invaluable from a policy consideration. Specificly, one might be able to monitor an individual to determine if the individual had the capacity to have consciousness --- perhaps through negation permitting synchronizations (Oshins, 1989a,b; Hilgard, 1989) --- and thereby choice. This could obviate much of the philisophical bantering about issues such as "criminal insanity" and "date rape." If one can not form the necessary concepts, then one can not be held *responsible* for the consequent actions.

A final comment about the search for psychospinors and the quaternionic arm: Should we have neurophysiological corrolates to the spinorial motion of arms it may, indeed, lead to something very important —at the level of DNA in psychology, ie. the building blocks of internal representations of experience. Further investigation should be into the possibility of coupling the two arms through the waist.[21] This would give $SU(2,C)_L \otimes SU(2,C)_R$. This is close to a representation of the Dirac time-space algebra and thus related to the group SL(2,C) of Finkelstein's relativistic quantum logic. There may well be a developmental coordination facility that is responsible for the synchronizations (or co-channel) necessary for negation and to compact the linear group into the unitary group of ordinary quantum logic (Oshins, 1984d; 1989a,b and references therein; Hilgard, 1989)

END NOTES

1 Specificly, we propsed "the more fundamental, simply connected, covering group SU(2,C) instead of the traditional orthogonal rotation group O(3,R)", specificly, to represent your work on mental rotations (Shepard & Chipman, 1970; Shepard & Metzler, 1971; Shepard & Cooper, 1976; Shepard, 1978, 1979, 1984) as unitary transformations in Hilbert space (Oshins & McGoveran, 1980, footnote 8; Oshins, et. al., 1984).

Some of the correspondences between orthogonal rotation operators $\mathbb{R}$ and their simply-connected unitary covering group elements $\mathbb{U}$ are shown below.

$$[\mathbf{r} \rightarrow \mathbf{r}' = \mathbb{R}_u \mathbf{r}] \longleftrightarrow [\mathbf{r}\cdot\sigma_r \rightarrow \mathbf{r}'\cdot\sigma = (\mathbb{R}_u \mathbf{r})\cdot\sigma = \mathbf{r}\cdot(\mathbb{U}_R \sigma \mathbb{U}_R^{-1})]$$

$\mathbf{r}$ = position vector , $\mathbb{R}_u$ = orthogonal rotation ,

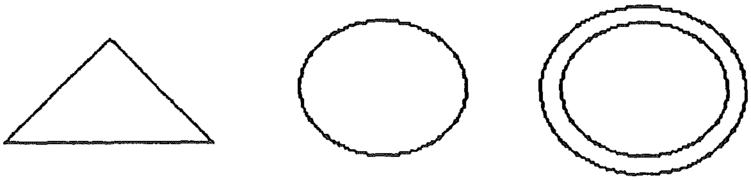
σ = Pauli matrix embodyment of spinors , $\mathbb{U}_R$ = unitary rotation ,

$\mathbf{r}\cdot\sigma$ = Cartan matrix representation of vector ,

$$\pm\, \mathbb{U}_R \xleftrightarrow{\;2 \rightarrow 1\;} \mathbb{R}_u , \quad [2\rightarrow 1 \text{ homomorphism}].$$

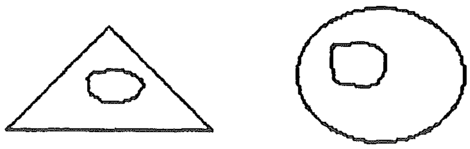
The difference between the covering group and the traditional rotation group lies in the global topology, specificly that the covering group is "simply-connected" whereas the traditional rotation group is "doubly-connect" --- an issue of "topological homotopy."

A decade ago, L. Chen (1982) appears to have demonstrated that the perceptual system identifies geometric objects according to their topological homotopy class! If the effects persist, it is a magnificent discovery. Basicly, the type of experiments that he performed had test subjects compare the similarity vs. difference between the following topological structures:



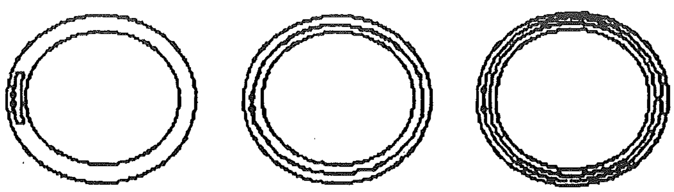
A B C

Chen found that test subjects identified the triangle A with the circle B due to them having the same homotoy group, which means that closed paths or loops with the same (arbitrary) base point can be continuously deformed into each other. In this case the fundamental or Poincaré group is the trivial group. This means that all paths are *contractable* and the space is *simply-connected* (like the quaternian covering group of the orthogonal rotation group), and is illustrated below for A and B:



A B

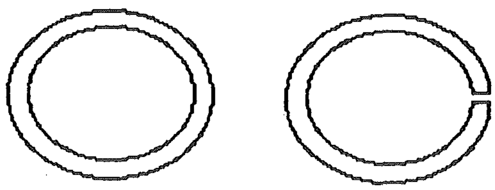
On the other hand, for the ring C, one has more than one possible paths that cannot be shrunk into eachother. Indeed, for the ring (annulus) the homotopy/Poincaré group is isomorphic to the group of additive intergers $\mathbb{Z}$ (likewise, for example, the n-dimensional Torus is isomorphic to the direct sum of n copies of the group of integers). Consider:



etc.

C' C'' C'''

In 1984, Steve Zins and I independently suggested the obvious --- looking for differences when employing the following two:



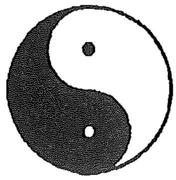
B B'

2 This "double-covering" property of the rotation group was first drawn to my attention by Claude Anderson in 1966. (Mathematicians and physicists refer to this as Dirac's "belt trick" (eg, Kauffman, 1991) I first came to understand the perceptual imprt of this from Kenneth Cohen in 1980 in his class on Pa Kua Chang at the Academy of Taoist Healing Arts.(Oshins 1983c, 1984a,c, 1985, 1986a/87e, 1987b, 1988a; Painter, B ,1981). (Painter refers to this exercise as "white serpant serves tea." I have been told by several martial artists that this is an incorrect name. Pa Kua champion Zhang Xian Ming, has told me that the form

is called "pang (or ping) wan chung," translated by Pa Kua Sifu Adam Hsu as "leveling hand motion." It refers to ancient practices of leveling the ground with horizontal hand motions when preparing to plant seeds.

This motion is found in other meditative and martial arts, such as the in the Philippine folk dance Binasuan ("wine dance") (Bernstein & Phillips, 1981, p. 122; Oshins, op. cit.; Kauffman, op. cit.) and Pencak Silat (Crista Hansen, personal communication, circa 1987).

3 If one does the palm up "double covering", with both palms in a symmetric manner with respect to reflection down the medial plane, then one finds that one circumnavigates with the palms a closed ball above the elbows and then with the back of the palms a closed ball under the elbows. In the ancient *Su Wên (Conversations with the Yellow Emperor) (Veith, 1949)*, which is where acupuncture originates, one learns that the ancient Chinese codified the body such that *all yin organs* are on the front side of the body, viz. inside palm, and *all the yang organs* are on the back side of the body, like a turtle, viz. outside palm/back of hand. My claim, and original idea, has been that this is circumnavigating a T'ai Chi (Yin/ Yang) symbol! More recently (Oshins, 1993b) I have suggested that this proximate technique can be used to realize Wing Chun kung-fu's bong sau/tan sau" movement out of the Kauffman/Oshins "quaternionic arm" discussed and referenced below in end note 5.



End Figure 1: T'ai Chi Symbol: If one performs the double-palm covering symmetricly --- considering the insides of the palms to be Yin and the outsides of the palms to be Yang --- one circumscribes a T'ai Chi symbol. If one approximately does the same movement, assymetricly, with the palms making a *tiny circle*, one effectively is doing Wing Chun's "bong sau/tan sau" movement (Oshins, 1984a, 1986a/87e, 1988a, 1992, 1993b; Oshins, et. al., 1989)

I believe that this may be a way to get mind to code the relative relationship of part of oneself with respect to the rest of oneself (self-referential motion) and can explain the concepts of being "centered"/ "one"/"integrated"/"extended"/"whole" etc. which one strives for in meditation. To this latter idea I am indebted to L.C. Biedenharn for introducing me to finite size/impenetrable region of space spinors (March 10, 1982 letter from L.C. Biedenharn to E. Oshins). Such *classical* spinor representations necessitate an intrinsic 3-dimensionality [ie. a *solid object*/ impenetrable region of space as opposed to a penetrable *rigid object* (Biedenharn & Louck. 1981, Ch. 2, pp. 7-26; Ch. 4, pp. 180-203; and Ch. 7, § 7.10, Sect. b., pp. 528-532 ; Oshins, op. cit.)

4 One can also do this by fixing the hand frame. One finds that one must walk around oneself *twice* in order to wind up where one began! (Oshins, 1985, 1986a/87e, 1988a). This discovery of mine has been incorrectly credited to Scott Kim on p. 331 of Martin Gardner's recent edition of *The Ambidextrous Universe* (March 18, 1991 letter from Scott Kim to Martin Gardner).

5 Kauffman and I have also adapted this double covering property in terms of the "quaternionic arm" (Oshins, op. cit.; Oshins, et. al., 1989; Kauffman, 1991; Hart & Kauffman, 1991): Hold the right arm directly in front and perpendicular to the body with the palm up. The right thumb will point to the right. Rotate the right arm by 180^0 so that the palm faces down and the thumb points to the left. This realizes i ($180^0/2$). Next, rotate the arm 180^0 with the palm continuing to face down, around a verticle axis, until the fingers point towards you and the thumb points to the right. This realizes j ($180^0/2$). Finally, rotate the arm 180^0 with the thumb continuing to face right, until the fingers once again point forward. Here we have two choices: If the path is such that the fingers point up during the rotation, one finds that the arm returns to its original position, This realizes -k ($-180^0/2$) since if the path is such that the fingers point down during the rotation, one finds that the arm does not return to its original position, but to the same position that would

obtain by a rotation of 360^0 around a vertical axis with the palm up during the entire process. This latter choice realizes k ($180^0/2$) such that ijk = -1 ($-360^0/2$).

6 SO(3,R) = SU(2,C)/{+1,-1}.

7 The name "synaptic spanning" comes from the fact that the "span", or the space generated by the states or rays, is the quantum correspondent of logical "or" in the non-distributive, quantum lattice.

8 For the reader's convenience I provide a brief listing of some of these experimental ideas from Oshins (1984a):

(a) Self-Referential Motion: The coherent, chiral superposition of right- and left-handed, information signals and the identification of half-angle representations, when coding the natural "orientation-entanglement relationship" of the human arms, would indicate that the mind uses spinor realizations of mental rotations, which necessitates an intrinsic 3-dimensionality to the coded physical space (solid objects), and simply-connected, global group structures. I have predicted psychological applications of what are know in the physical literature as "Aharanov-Susskind-Bernstein Effects," such as a reversal of spinorial, brain current activity as a consequence of relative self-rotation by 2π.

(b) Symmetry in Nei Chia (Noi Kung): In a manner similar to Bernstein's resolution of translational human movements into Fourier components, I hope to attempt a decomposition of motion of a Chinese "internal" (image-based) martial art (such as Pa Kua Chang or T'ai Chi chuan) into normal modes having rotational symmetries, such as (spinor) spherical harmonics or Bessel functions in order to demonstrate integrated movement and an adaptive economy for certain natural motions.

(c) Projective Ray Representations: The determination of intensity conserving "ray representations," when adding an information state to itself, would provide a quantum alternative to the "hologram hypothesis", a natural interpretation of spatial frequency as "the generator of translations", and an enticing interpretation of "psychic force" and "psychic energy" (viz. Freudian "cathexis") through Heisenberg's equation.

(d) Consciousness and Negatives, Metrics, and Compactness: The capacity for inter-hemispheric synchronization could be responsible for and correlate with a negative function and a metric in cognitive space. (I have proposed a model in which conscious processes would realize a unitary restriction of the 2-dim. special linear group through synchronizing the preparation and the determination of information states, based upon Finkelstein's "relativistic logic". The compact, metric preserving, covering group of the 3-dimensional rotation group, which I have related to Shepard's work, would result.)

(e) Natural Fourier Transforms: The brain might perform a natural Fourier transformation through global phase relations that have a symmetric structure corresponding to the square root of an inversion in a projective unitary gauge space.

(f) Phase-Locked Hallucinations: It may be possible to correlate phase-locked neuronal firing patterns between brain centers during the hallucination of synaesthesic events (eg. seeing a color upon hearing a sound), and of those involving the syncretic fusion of linguistic and sensory modes (eg. perceiving a foul odor when experiencing a "rotten person").

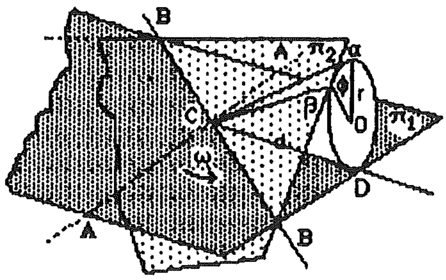
Regarding (8d), I point out that the relation relation between the Lorentz group SO(3,1;R) and it's simply-connected covering group SL(2;C) is similar to the relation between the orthogonal rotation group and its covering group from end note 6 above, ie. SO(3,1;R) = SL(2,C)/{+1,-1}. Here, we discover though that whereas the unitary, Lorentz group only has ∞ - dimensional representations, the linear, covering group has finite representations. But since they are not unitary, they do not conserve probability as is ordinarily assumed in standard quantum interpretations. Finkelstein's negationless quantum logic and my negationless quantum psychology both employ the special linear group.

9 The interplay of the yin and the yang formed the T'ai Chi symbol of end note 3, above. Actually, this notion of patterns of change (from yin to yang and from yang to yin) is supposed to have come from the observation that as as the sun passed a mountain the part that was light turned dark and the part that was dark (shaddowed) turned light (Veith, 1949; Wilhelm, 1967).

10 The creative cycle can be identified with n -> n+1 mod (5) and the destructive cycle can be identified with n -> n+2 mod (5) (Oshins, 1984d).

11 See end note 3 above.

12



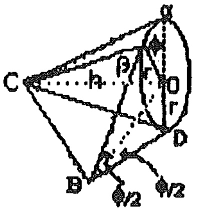
$$\omega = \tan^{-1}\left(\frac{d}{r}\tan \phi/2\right)$$

,

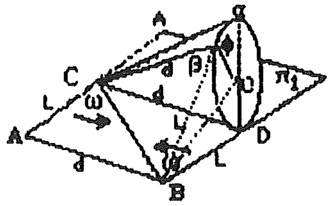
End Figure 2: Inner palms as planes circumnavigating a cone as a spinor parameterization.

Let two planes π_1 and π_2 lie along the side of a right circular cone such that the lines of tangency of the planes with the cone meet the base of the cone at the points α and D, respectively. The two planes intersect each other along the line AA. As plane π_2 rotates around the cone, without slippage, the line of tangency to the cone changes its point of intersection with the base of the cone to point β. Call the new line of intersection of the two planes BB. The base radius Oα of the cone travels an angle φ in the base of the cone when the radius Oα travels to the radius Oβ, ie ∠αOβ = φ. Correspondingly, the line of intersection of the two planes π_1 and π_2 traces and angle ω in the plane π_1, ie ∠ ACB = ω.

Let h be the height of the right circular cone and r to be the radius of the right circular cone.



End Fig. 2A: Symbol defining picture of: (a) height *h* of cone; (b) similar right triangles Δ OBβ and Δ OBD which lie in the same plane as the base of the cone.



End Fig. 2B: Symbol defining picture of: (a) slant height *d* of cone; (b) angle ω measuring the change in the line of intersection of the two planes π_1 and π_2; & (c) side *L* of Δ OBβ and Δ OBD.

Thus, ∠ DBO = ∠ βBO. Since radius O is perpendicular to plane π_1 and radius Oβ is perpendicular to plane π_2, the sum of ∠ DOβ + ∠ DBβ = ∠ DObβ+ (∠ DBO + ∠ βBO) = π. Likewise, ∠ DOβ + ∠aOβ = ∠ DOβ + φ = π. Therefore, ∠ DBO = ∠ βBO = φ / 2.

Let h be the height, r be the radius, and d be the slant height of the right circular cone. Thus $d^2 = h^2 + r^2$. It follows that the intersection of the two planes provides a 1/2-angle or spinor parameterization:

$$\tan\omega = \frac{d}{L} \qquad \tan(\phi/2) = \frac{r}{L} \qquad \tan\omega = \frac{d}{r}\tan\phi/2 \qquad \omega = \tan^{-1}\left(\frac{d}{r}\tan\phi/2\right)$$

If I consider my *self* to be situated at the locus of intersection of my two palms than *I* should realize such a spinor representation!

13 Eloise Carlton (personal communication, circa 1983; Carlton, 1988; Carlton & Shepard, 1990a,b; Pribram & Carlton, 1986) has put forth a related, yet fundamentally different, approach toward mental imagery and neurophysiclogical states in visual perception. If I understand her correctly, she has represented cortical activity patterns at a given period of time in terms of the Hilbert space of the electromagnetic field, having a functional form identifiable with Gabor elementary functions (minimum uncertainty packets/Gaussian envelope of plane waves). She then attempts to characterize mental rotations (and other apparent motion) in terms of paths of the unitary action of the Euclidean group acting on this function space. It is hypothesized that this action would correspond to a geodesic path in the group space. The cortical activity paths would thereby be determined by the internal representation path.

This extraordinarily beautiful approach differs from a quantum approach in that the electromagnetic field can *not* carry the unitary representations of a quantum theory as has sometimes been asserted (Pribram & Carlton, 1986; Oshins, 1990, 1991a, & in prep.). Although this model will not work as a quantum model, it may well be a valid and correct classical model.

Furthermore, I do not accept Carlton and Shepard's belief that classical physics is somehow inadequate to represent any motion of a rigid object which can be identified with a path in phase space based upon an alleged difference between what they refer to as "kinetic physics" and "kinematic geometry". It seems to me that they are unnecessarily *defining away* the possibility of "psychic forces" of some sort by insisting that to a physicist an image must follow force-free rectilinear motion. This is interesting since it is standard to go from Galilean invariance (of which the Euclidean group is a subgroup) to Newton's laws to action principles (by imposing covariance) to geometric orbits (via the "action metric"). (Wheeler, 1965, especially the section "geometrical properties of dynamical orbits" in chapter IV, Transformation-theoretic analysis of Newton's equations). Put simply, I believe that there should exist close to an isomorphism between the group geodesic approach of Carlton and Shepard and standard, classical, physical modeling.

In addition, Carlton and Shepard seem to believe that physicist treat the Euclidean group as if the direct product applied to the translation and rotation groups instead of the semi-direct product. This is not the case. In my own approach I identified the Lie algebras of the "generators" of these groups with Schwingers type I and type II variables, ie. with momentum and angular momentum. (Schwinger, 1970) That the Euclidean group employs the semi-direct product follows from the fact that the momentum is a vector operator and thereby does not commute with the angular momentum.

Finally, although there are mathematical reasons that make using the covering group of the Euclidean group desireable, I see nothing in the phenomena which they describe that would require using the fundamental spinor representations instead of the ordinary vector representations. But then again, I may be wrong or lacking in understanding of their approach.

In any case, it would be interesting, and possibly very important, to try to identify the flow in the visual cortex suggested by Carlton's representation of Shepard's model for mental rotations, and more general imagery, with the flow in the motor cortex predicted by Georgopoulos' population vector approach. A significant aspect of any such inquiry is that it is difficult (or impossible) to define temporal "zones of *simultaneity*" for different sensory domains of the brain because they have different transduction processes (Ruhnau and Pöppel, 1991). The ability to temporally coordinate such different sensory modalities may be developmentally sensitive and significant. (Oshins & McGoveran, 1980, conclusion) It may indeed shed light on the issue of negation and synchronization between preparation and determination of information states which I proposed based upon Finkelstein's relativistic quantum logic (Oshins, 1989a,b; Hilgard, 1989)

14 I am grateful to Roger Shepard and Eloise Carlton for providing me with access to their work before publication, for trying to help me to understand their approaches, and for jcomments helpful to my own di-

rection. I am also indebted to Pierre Noyes for his direct help and for his relentless efforts to help us to communicate and understand our varying appraoches.

15 I would especially like to thank David Adelson and Mike Williams for helping me to understand Georgopoulos' experiment.

16 This end note sumarizes Georgopolous' computation of a population vector as described in the text:

Georgopolous' Population Vector Computation: (3-dim. direction; 282 neurons (41 of which are non-directional) ∴ 224 fit model).

1. consider a single directionally tuned neuron:

It is observed that single neurons are *broadly tuned*, ie. the firing activity changes substantially with movement in *any direction*.

$\vec{M}$ = *movement vector* of unit length defined by directional cosines.

$d(\vec{M})$ = *frequency of discharge* of a *particular neuron* during motion inthe direction of the movement vector $\vec{M}$.

2. Neuronal ensembles:

$\vec{C}$ = *preferred movement vector* for which individual cells activity is highest

Θ_{cm} = ∠ between direction of preferred movement vector $\vec{C}$ and actual movement vector $\vec{M}$

b is a coefficient that varies from neuron to neuron

$$d(\vec{M}) = b + k \cos \Theta_{cm}$$

Single neurons are broadly tuned, ie. the activity changes substantially with movement in any direction. Thus the movement is not coded by individual cells that respond to movement in only one direction. Instead, we assume that movement is coded by means of a neuronal ensemble.

Three assumptions:

(1) Each cell indexed by i makes a vectorial contribution along a preferred direction $\vec{C}_i$.

(2) The magnitude of the vectorial contribution of of the i th cell, ie. the length (amount) of the vectorial contribution, w_i, is a function of the movement direction $w_i = w_i(\vec{M})$. The vectorial contribution $w_i(\vec{M})$ to the ensemble vector is a measure of the change in the i th cells activity level from an offset level b_i which represents a baseline activity for the i th cell. Specificly,

$$w_i(\vec{M}) = d_i(\vec{M}) - b_i$$

where $d_i(\vec{M})$ is the discharge frequency of i th cell for movement in the $\vec{M}$ direction.

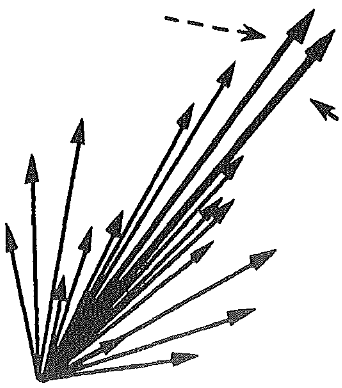
Assumptions 1 & 2 imply that the weighed vectorial contribution of the i th cell is:

$$\vec{N}_i(\vec{M}) = w_i(\vec{M})\,\vec{C}_i$$

$\vec{N}_i(\vec{M})$ points in the preferred direction $\vec{C}_i$ of the i th cell, if $w_i(\vec{M}) > 0$, and points in the opposite direction, if $w_i(\vec{M}) < 0$.

(3) The ***Neuronal Population Vector*** $\vec{P}_i(\vec{M})$ --- which represents a *sharply-tuned, collective directionality* of the 224 broadly- and directionaly-tuned, individual neurons --- corresponding to movement in direction $\vec{M}$ results from a *weighed vectorial sum* of the contribution vectors $\vec{N}_i(\vec{M})$ of the individual neuronal cells (See End Figure 3, below):

$$\boxed{\vec{P}_i(\vec{M}) = \sum_{i=1}^{224} \vec{N}_i(\vec{M})}$$



End Figure 3: Georgopolous' narrow-band, directional Population Vector (collective activity pattern of 224 wide-band, directional, motor neurons in monkey) alines up with Movement Vector representing direction of motion: A depiction of the clustering of "jets" of individual neuronal activity corresponding to movement in a direction $\vec{M}$. The weighed vectorial sum of contributions of these individual neurons results in a collective population vector $\vec{P}_i(\vec{M})$ that approximates the movement direction. The movement direction $\vec{M}$ is usually found to lie within a a 95% confidence interval of a cone around the collective neuronal population vector $\vec{P}_i(\vec{M})$ .. (Adapted from Georgopoulos, Schwartz, & Kettner, 1986, Neuronal Population Coding of Movement Direction, *Science* , 26 September 1986, *233*: 1416-1419.)

The movement direction $\vec{M}$ is usually found to lie within a a 95% confidence interval of a cone around the collective neuronal population vector $\vec{P}_i(\vec{M})$. It appears from preliminary investigation that the results are insensitive to the origin of motion. (Ibid., ft. nt. 15) [This might indicate that the neuronal population vector is coded in "homogeneous coordinates" and may have a simple projective representation.] It is also found that the direction of the population vector $\vec{P}_i(\vec{M})$ for a particular movement vector $\vec{M}$ was basicly insensitive to recalculation using a random sample of 224 neurons drawn with replacement from the original population of neurons or when the number of contributing cells was randomly reduced (Ibid., ft. nts. 11 & 12). Thereby, the neuronal population vector $\vec{P}_i(\vec{M})$ appears to be a robust of the direction of movement $\vec{M}$.

In the experiments involving the monkey performing a cognitive task of mental rotation , the following additional observations are pertinent:

(A) All of the experiments have taken place in only one plane of motion --- the frontal plane to the monkey (October 31, 1991 letter from Georgopoulos to Oshins). Thus, there is no indication yet that the activity patterns will properly simulate the 3-dimensional rotation group. Nevertheless, it is quite likely that the rotations have approximately a 3-dimensional symmetry as is approximately true in the original mental rotation studies by Shepard and colleagues.

(B) Effects of *coupling both hands* as suggested by Oshins (examples in this context are: Oshins' Aug. 28, 1989 memo to W.A. Little, Oshins' Dec. 31, 1990 letter to L.C. Biedenharn, and Oshins' July 8, 1991 letter to A.P. Georgopoulos) have not yet been attempted (October 31, 1991 letter from Georgopoulos to Oshins).

17 I am grateful to Larry Biedenharn, Peter Gaposhkin, and Lou Kauffman for helping me to understand some of the relevent mathematics I have also benefited greatly from conversations with David Adelson, Fred Young, and Steve Zins on these issues.

18 This end note sumarizes some of the properties of Hamilton's Turns (largely adapted from Biedenharn and Louck (1981):

Definition: A *turn* T_{AB} = $\underset{\rightarrow}{AB}$ is "half the directed arc of the rotation parametrized as an ordered pair of points on the surface of a unit sphere, modulo great circle transport." [The under-arrow $\underset{\rightarrow}{AB}$ is used to reflect the fact that the turn is ordered and directed but is *not a vector* (see Biedenharn and Louck, 1981, Ch. 4, "The theory of turns adapted from Hamilton") which should be clear if only because addition of turns is noncommutative.] The normal $\hat{n}$ to the great circle of the directed arc points in the direction of the axis of rotation using a right handed rule to determine the sense of the rotation. Figures 4A and 4B below illustrate the geometric realization of a turn and the noncommutivity of the addition of turns.

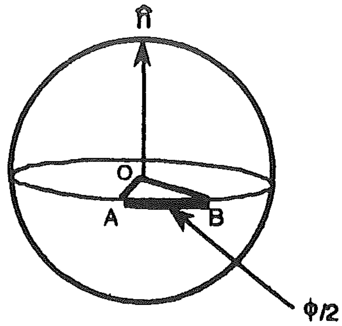


Fig. 4A: Geometry of *turn* T_{AB} = $\underset{\rightarrow}{AB}$: (a) directed arc $\underset{\rightarrow}{AB}$ of length $\phi/2$ realizes a rotation by ϕ ; (b) normal $\hat{n}$ points in the direction of the axis of rotation .

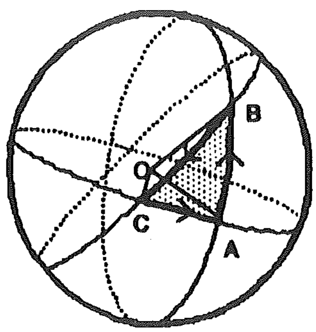


Fig. 4B: Addition of turns $\underset{\rightarrow}{AB} + \underset{\rightarrow}{BC}$: Turns add head to tail like vectors but modulo great circle transport. Thus, turns $\underset{\rightarrow}{AB} + \underset{\rightarrow}{BC}$ = turn $\underset{\rightarrow}{CA}$ as described in the text.

Properties of Turns (Biedenharn & Louck, 1981, Sect. 4.2)::

1. Two turns are *equivalent* if they can be superposed by displacing either one along the great circle containing it, ie. "modulo great circle transport". [This corresponds to *parallel transport equivalence* for the addition of *vectors*].

2. Two turns defin ed by any pair of diametrically opposite points are equivalent. Let T_π denotes this equivalence class. Since T_π *commutes* with all turns, it represents a *scalar turn.*

3. Turns form a *group* under addition: In analogy to vectors, the *sum of two turns* is defined by addition head to tail modulo great circle transport. This addition is *noncommutative*, but *associative.* The turn of zero length T_0 ($\equiv T_{2k\pi}$, k=0, 1, 2...) realizes the *identity* operation (It has *no*length, direction $\hat{n}$, nor sense). The *inverse* to a turn T^{-1} ($T^{-1}{}_{AB}$) = - T (+ T_{BA}), ie. the turn with the same great circle and lenth, but opposite sense. This group of *Hamilton's turns* is *isomorphic* to SU(2.C) as 3-parameter objects giving a "finite size spinor" as "impenetrable object" in 3-space instead of a 2-parameter object such as Cartan's "point spinor." (Biedenharn and Louck, 1981; Sects. 2.3-2.4 & 2.7, note 2; and pp. 186 & 191)

4. Turns have two involutions:

(A) *inverse turn* T^{-1}, ie. $(T^{-1})^{-1} = -(-T) = T$.

(B) *non-identity, scalar turn* T_π associated with a pair of diametrical points --- having length π, but *no* direction $\hat{n}$, nor sense --- provides a second involution defined as $T^c \equiv T + T_\pi = T_\pi + T$, ie. $(T^c)^c = (T + T_\pi) + T_\pi = T + T_{2\pi} = T + T_0 = T$.

(C) If one identifies the two equivalence classes of scalar turns $T_0 \equiv T_\pi$, one effectively factors the group SU(2,C) into the proper, orthogonal group SO(3,R), ie. $Z_2 = \{T_0, T_\pi\}$ and SU(2,C) / Z_2 = SO(3,R).

5. Turns of length $\pi/2$ ($T_{\pi/2}$) have special properties:

(A) Denote a generic turn of length $\pi/2$ by E, ie. E $\equiv T_{\pi/2}$. It follows that $E^c = E^{-1} = E$ or equivalently that $E + E = T_\pi$.

(B) Theorem (Biedenharn and Louck, 1981, p. 187): An arbitrary turn can always be decomposed (non-uniquely) into a sum of two turns of length $\pi/2$, ie. $\forall$ T, $\exists$ E and E′ such that T = E′ + E. (Proof: Trivially, let E′ = $T_{\pi/2}$ from the tail of T to the intersection of the normal to its plane with the unit sphere and let E = $T_{\pi/2}$ from this point to the head of T.)

(C) Theorem (Biedenharn and Louck, 1981, pp.189-190): If $\hat{e}$ and $\hat{n}$ denote unit vectors from origin perpendicular to planes defined by turns E and T where E is an arbitrary turn of length $\pi/2$ and T is an arbitrary turn, then $\hat{e} \cdot \hat{n} = \cos \| T\text{-}E \| / \sin \| T \|$, where $\| T \|$ denotes the length of turn T.

(D) Hamilton's Triplet of "Right Versors" (Ibid.; Hamilton, W.R. (1866/1959): Hamilton's triplet of right versors can be put into a 1-1 correspondence with his quaternians or "quotients of vectors".

19 Some additional information that would be useful: (1) It would be useful to know the relationship between the left hands activity flows and the activity flows due to the right hand, symmetrically and anti-symmetrically, both combined and independently. (2) What might be the relationship between your population vector and spatial frequency. [Spatial frequency data probably relates to translational degrees of freedom (as opposed to rotational) and may be able to be accommodated with the population vector approach as a (the?) way to realize Euclidean motions (translations and rotations)] (cf also Carlton and Shepard); (3) the population vector uses "homogeneous coordinates" which means that the cell activities all add using the same origin. Your cover photo from the original population vector article (Science, 9/26/86) seems to indicate that the population vector is isotropic, i.e.. for a similar movement in two different directions the corresponding activities reflect only the movement directions and not an intrinsic directionality in the population vector space. It would be important to know if this is so since state vectors are homogeneous and (4) There is some data in the psychological literature about unitary vs. analyzable representations of data, where the former uses a Euclidean metric (i.e.. which squares the distances, then adds them, then takes the resultant square root) and the latter uses a city-block metric (i.e.. which adds the absolute value of the components). Perhaps one might fortuitously use a different metric structure in computing the population vector collective

activity? If one wanted to add the activities of both hands, would one add them all up or would one find separate population vectors for each hand and then add them up? These would not be equivalent procedures.

20 It would also be interesting to see if one could corrolate Georgopolous' population vector with the anticipated magnetic field interference in the brain under the act of mental rotation. Professor W. Fairbank told me in 1984 that there is a plan to make a helmet coupling 30 or so SQUIDs that might be able to find such correlations (See also, "Thinking Cap: Superconducting SQUIDs peer into minds --- and hearts," *SCIENTIFIC AMERICAN*, March 1991, p. 112).

21 There is a very famous Chen style t'ai chi chuan exercise called chan-ssu chin (silk-reeling exercise) in which the waste is used in a rotational fashion as the leader and coordinator of the arm motions (Jou, 1980)

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MNEMIC CAUSATION

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ABSTRACT:

Papers read at ANPAs 12 and 13 outlined a new conception of the physical world. The chief aim of the present paper is to show how life emerges as a logically necessary consequence of a physical world so conceived. Centrally, this entails defining mnemic causation, and relating it, both qualitatively and quantitatively, to physical force. The latter part of this paper examines briefly the part played by mnemic causation in a wide-ranging succession of ontological contexts.

Fatally handicapped by its near-blindness to noumenal/phenomenal distinctions, orthodoxy succeeds only in standing the physical world on its head. This rationally inverted world of undifferentiatedly enduring particles in motion within a spatial medium bequeaths us two sets of problems - the origin and nature of, and relations between, the physical fundamentals, and the causal and substantial connections between these fundamentals and life, mind, and spirit - equally insoluble within this theoretical framework. In two previous talks (published in the ANPA 12 and ANPA 13 Proceedings) I outlined a noumenal conception of the physical world, free as far as possible from all phenomenal contamination. This theory, so I would claim, stands the physical world the right way up, and so is able to provide solutions to the first set of problems. My aim in this third talk is to outline solutions to the second set. The key to all these is the mnemic causation of my title. Since mnemic causation follows logically from my conception of the physical world, it is is impossible to understand its nature without prior familiarity with this conception.

The attribute of the physical process as conceived in my theory which most profoundly distances it from all materialistic conceptions is that it is cumulative: all its past states being preserved intact, so that all its changes are, at bottom, additions. In the orthodox theory the dynamic configuration of particles in space that constitutes a body is, in essence, temporary. Sooner or later it breaks up under the impact of physical forces, and therefore ceases to exist. And under the action of these same forces its constituent particles enter into all manner of other essentially ephemeral arrangements with other particles. All that is permanent are the ultimate particles, the laws of nature, and space and time - whatever these may be. In the conception I am advancing nothing like this occurs. There is only the instant by instant redefinition of The Absolute and Nullity against the already existing, ever growing, body of past instant qualifications. What, then, happens to any present as, overlain instant by instant by further qualifications, it recedes ever further into the past? The short answer is: nothing. _It_ remains intrinsically unchanged; what changes is the universe growing round it. But, of course, in ceasing to be part of the present an entity changes in one supremely important respect: it ceases to be a change. The universe is always the universe at a certain time, and what it is then is all that is changing at that time: how, in terms of new qualifications and new associations, it is distinguished from what it was at the immediately preceding time. But though they are no longer associative changes, the associations remain. How could it be otherwise? The ultimate units, the simples, are merely added to. They, in themselves, remain unchanged. But the particular associations of the simples as the universe unfolds are no less part of the universe than the simples so associated: it is in these arrangements that the simples actually exist. And though these associations cannot change internally, they can enter, through association by similarity (sympathetic association), as intrinsically unchanging units into any number of further associations. In fact, since every part of the universe has something in common with every other, every past entity, simple or complex, will enter into some degree of sympathetic association with every future entity to form a basic, comprehensive, association within which all closer, more specific, associations exist.

But in thus sympathetically associating with its latest qualifications - what I term the physical present - does the past play any part in actually determining the course of the Cosmos? For sympathetic association to occur there must first be something to associate, and 'something' in this conception is either a simple or an association of simples. Where, then, will associations arise locally within the Cosmos? Wherever forces result in regular changes of period (or frequency): that is, wherever n is changing regularly - either on a single sequence, or, much more frequently and importantly, on a number of sequences collectively. But since velocity and acceleration vary precisely with frequency and change of frequency respectively, wherever, phenomenally, we find regular motion, there too, noumenally, must be experiential synthesis. In other words, regular motion is the phenomenal index of sequential, or rhythmic pattern. Such rhythmic patterns can be thought of as concrete universals. In the inanimate world we find such regularity in the rotations, revolutions, and oscillations of sub-atomic particles, atoms, molecules, crystals, and certain plasmas. And in the animate world, in the metabolic processes of cells, and in the ordered activities of systems of cells, most especially in the firing patterns of nervous systems - in their resting states, but more complexly in perceptual regularities and behavioural patterns.

All syntheses are processes, and therefore require time to manifest. As a new instance of a particular kind of cosmic frequency pattern occurs in the present, it associates seriatim with its past instances. Now , let us suppose that this present instance is interrupted in its manifestation on some one of its constituent sequences through the action of disruptive physical forces issuing from the Cosmos at large. What will happen? Let the sequence pattern be labelled schematically A-B-C-D-E-F, where each letter stands for one X/O period, and let it have reached D in this particular instance before the disruptive forces would selectively impose some alien period, K, say. But in sympathetic association with present instance A-B-C-D- there exists physically past A-B-C-D-. And since the complete pattern A-B-C-D-E-F already exists in this physically past, but sympathetically present, instance, D is contiguously associated with E. Now, we must not forget that all sequences exist, so that both continuations, K and E, exist in the universal physical present: it is solely a question of what frequency (or period - we may think indifferently of either) is the cosmic continuation.

Certainly K, as the collectively determined selection of the physically present cosmic sequences - we will call it the intersequential frequency - is being selected in by the whole of the cosmic physical present. But this physical present does not constitute the whole Cosmos. It does, of course, in the orthodox conception, but in our conception all past cosmic sequences continue to exist, including those that composed at various times and places the particular rhythmic pattern under consideration; and these, through sympathetic association, exist also in the present.

Now, clearly, the natures of physically past sequences have been just as much determined by the Cosmos as have those of physically present sequences. And sympathetic association, which makes them part of the cosmic present, is a universal and fundamental type of association. So that this sympathetically associated past exerts no less a cosmic selective influence than the cosmic physical present: it is still the Cosmos self-selecting its sequences, but via a different associative route. Ultimately, this comes about because types of synthesis arise within the Cosmos whose mode of organisation is fundamentally different from that of the Cosmos itself. The Cosmos is in no sense a giant organism! Although these modes of organisation - organisms - arise as parts of the Cosmos, because they are constructed on a different principle of association from the Cosmos itself, they possess a measure of autonomy. Hence we may say that the Cosmos exerts two selective influences: one proceeding from its own mode of organisation qua Cosmos, and the other via these other modes of organisation which grow up naturally within it. In that case, it might be asked: Why are not both the frequencies, K and E above, selected in? Were that to be the case neither would be a cosmic continuation in the sense of being selected in by the Cosmos as it actually is, but only by one aspect of the Cosmos artificially severed from the other. No, every <u>cosmic</u> sequence must be selected in as an expression of the Cosmos' dual selective nature. Just how, then, do these two influences, operating through two different routes, combine to select a resultant frequency? But before we investigate this, a small point of nomenclature: we have called the intersequential influence, resulting as it does in change of frequency, necessarily accompanied by acceleration, a force. But since the influence exerted by the past produces the like effects, it too must be classified as a force. We call it mnemic force, or, since all forces are causes, mnemic causation.

There are two basic factors, inherent in the situation, which determine the magnitude of this resultant; the first of which makes the mode of combination of these two forces quite different from that of two intersequential (physical) forces. This is that if both the intersequential and the mnemic force select the same frequency this frequency is selected. Obviously, since then there is no conflict. Since, if one of these forces were altogether absent the same frequency would be selected, it is obvious that each is merely duplicating the other. The second factor is that, if both forces were absent, the sequence frequency, and therefore the absolute speed, would remain constant. Science knows this as inertia. So that if either influence selected in zero change it would be otiose, since were it altogether absent the consequence would be no different. It is as if the situation incorporates a third influence: inertia; and that this always selects in the present, or existing, frequency. The presence of these two factors already, therefore, determines the resultant in a number of cases. In the accompanying table and those that follow, since all sequences are of the form (an, bn) or (bn, an), where a and b are cosmic constants, it is easiest to denote all sequences by their n-value, which I will call the period root.

I denotes intersequentially selected sequence; M denotes mnemically selected sequence; X denotes existing sequence; R denotes resultant sequence:

I/M	n+m	n-m	n+m	n-m	n
M/I	n+m	n-m	n	n	n
X	n	n	n	n	n
R	n+m	n-m	n+m	n-m	n

Also, of course, intersequential and mnemic force are interchangeable in the sense that, for any pair of selections, the resultant would be the same irrespective of which value was intersequential and which mnemic.

We are left, then, with the following four cases, with $m \gg p$:

I/M	n+m	n-m	n+m	n-m
M/I	n+p	n-p	n-p	n+p
X	n	n	n	n
R	?	?	?	?

In the first two cases the selective changes are in the same sense, but of different magnitudes. Let us consider the first. Now, we already know the following:

I/M	n+m	n+m
M/I	n+m	n
X	n	n
R	n+m	n+m

Since the two extreme values of the lesser change (p) give an identical resultant, why should any intermediate value yield anything different? It, too, will give a resultant of n+m. This clearly arises from the basic circumstance of each force duplicating the other when both select the same value. Here, we have partial duplication; that is, to the extent that the two forces are the same, each is merely duplicating the other. But since the greater change (m) must equal cp, where c is greater than unity, the lesser change is wholly comprehended by the greater, and so is wholly duplicated. For the second case, the acceleration, we merely substitute n-m for n+m and n-p for n+p in the above argument, obtaining a resultant of n-m.

We come finally to the second pair of cases. Here, the selective changes are in opposing senses, one an increase, the other a decrease. Neither force duplicates any part of the other, so the preceding argument will not apply. The two cases may be considered together. The resultant change will not be the algebraic sum of the two changes, since then the answer will come out differently according as one works with periods or frequencies (basically because $\frac{1}{m} - \frac{1}{p} \neq \frac{1}{m-p}$) - a state of affairs which is clearly absurd. In any case, one would expect the resultant change to take into account the existing frequency; a constant algebraic sum would really constitute a change of different magnitude for every value of n. Clearly, we must think in terms of ratios - just, in fact, as we do for charge. We would then expect that if one force increased the existing frequency in the ratio r:1 (doubled it, say), and the other decreased it in the same ratio (that is, halved it), then the frequency would be unchanged. This yields for the first case, $\frac{R}{n} = \frac{n+m}{n} \times \frac{n-p}{n}$ or

$R = \frac{(n+m)(n-p)}{n}$; and for the second, $R = \frac{(n-m)(n+p)}{n}$. All this is more simply expressed by $R = \frac{I \times M}{X}$, or, as I prefer to write: $R = \frac{MI}{X}$.

This formula will not, in general, yield an integral value for R. But, as there are no fractions of instants, only integral departures from the existing sequence are possible. Either the resultant achieves a unit change of n or it does not. Let us take an example. Let X be the sequence (10,5), I the sequence (4,2), and M the sequence (12,6). Then, working with period roots, which is easiest: $R = \frac{MI}{X}$; therefore $R = \frac{6x2}{5} = 2.4$. Hence n has decreased from 5 to 3, and the resultant is the sequence (6,3). This formula obviously also holds true when either selection equals zero: that is, when I or M equals X. It can even be made to yield the correct resultant in the case of two selections in the same sense provided duplications are taken suitably into account. But since we already know that the resultant is always the greater of the two selected changes this is a trivial exercise.

In view of the fact that an increase or decrease of n by unity implies an acceleration or retardation of the order of $\frac{10^{32}}{n^3}$ cm s^{-2}, we should expect the overwhelming majority of accelerations and retardations to occur in unit steps. For this majority the accompanying simple table of changes ($\Delta n = \pm 1$) holds true:

I/M	-1	-1	-1	0	0	0	+1	+1	+1
M/I	-1	0	+1	-1	0	+1	-1	0	+1
R	-1	-1	0	-1	0	+1	0	+1	+1

It will be realised that mnemic causation has no directional implications; it will speed up or slow down a sequence but cannot directly alter its direction. It can achieve this indirectly, of course, since the change in absolute speed that it produces will affect the magnitude and direction of the physical forces acting on the sequence, and this may well give rise to directional change. Hence, where direction is biologically important, mnemic forces must act within such physical situations as ensure that their intervention produces a change in the right direction.

A good example is provided by the neurons. If, in the laboratory, the axon of a neuron is stimulated at a point somewhere along its length, the nerve impulse will travel both ways. But in the living organism the neurons are so connected that the impulse is always initiated on the cell body, and therefore always travels from the cell body, never towards it. Also, we must realise that since, in a rhythmic frequency pattern, the sequences mnemically change their speeds, such change must affect the physical forces acting between them. So that a viable pattern must be taking these changes in physical force, which no doubt are producing changes in direction, into account. But because mnemic forces derive directly from past physical forces one would, in any case, expect this.

Without mnemic causation, that is, with physical forces alone acting, the Cosmos would be no more than an ordered, but meaningless, interplay of attractive and repulsive forces. It is mnemic causation, which, by modifying these forces on the basis of past experience, imparts a creative dimension to the Cosmos, building into, and upon, it experiences of ever greater complexity. By way of introduction to this creative process, I will devote the remainder of this paper to indicating the bearing of mnemic causation upon a few themes of basic empirical interest.

Mnemic Causation and the Principle of the Conservation of Energy:

It is a settled dogma of orthodoxy that the principle of the conservation of energy necessarily precludes the existence of non-physical forces, among which, by orthodox conceptions, mnemic causation would rank. But this dogma can find no firm support on either rational or empirical grounds. It is no more than a "hypothesis of impotence" long overdue for refutation. It is not, of course, rationally entailed in the fundamental nature of things, but is a mere analytic consequence of Newton's Laws of Motion. On the assumption that these alone are operative in nature, the principle logically follows. But we are rejecting this assumption; as we have just seen, from the very nature of our physical fundamentals a force from the past is also acting in the physical present. Hence, in our conception, the principle of the conservation of energy, as it is ordinarily understood, does not hold good.

Empirically, the principle rests on no more substantial grounds. Mnemic causation acts principally on chemical bonds. But mechanistic dogma takes for granted that the magnitudes of the bond dissociation energies are precisely what they would be were only conventional physical forces at work. But this is no more than a blatant assumption for which there is no evidence whatsoever. To justify this assumption the bond dissociation energies empirically obtained would have to equal those calculated from first physical principles, using the Schrödinger equation. But even in calculating the ionisation energies of atoms some form of approximation has to be employed, because of the cross-terms arising from repulsions between electrons. When we come to even simple molecules, the vibrations of the nuclei, the rotation of the whole molecule, etc., render the complete wave equation far too complex to solve. Radical simplifications (e.g. the Born-Oppenheimer approximation) have to be introduced - with one eye, it might be added, on the empirically determined value. As for complex organic molecules, with molecular weights of hundreds of thousands, the very notion of calculating bond dissociation energies from first principles is ludicrous. In most cases these cannot even be obtained empirically with any degree of accuracy; since they have to be determined in free solution, and it is universally conceded that values so found will frequently differ substantially from the true values obtaining within the complexly structured context of the living cell. We know, of course, that energy output from an organism is approximately equal to energy input; but the magnitude of the energy we are considering is very small - well within the margin of experimental error. An apt analogy would be the energy expended by a driver on pedals and steering wheel, compared with that obtained from the combustion of petrol vapour.

There are two reasons why the energy contribution from mnemic causation should be very small in comparison with the physical energy with which it is associated.

Firstly, because it works with, not against, the grain of physical force. Obviously: since mnemic causation is, at bottom, only past physical force operating, through sympathetic association, in the physical present. The constructive processes which produce living organisms are preponderantly endergonic. Energy for these is <u>not</u> provided by mnemic causation, but by coupled exergonic processes. The role of mnemic causation is not the initiation of processes effectively running counter to physical force, but rather of subtle control over, and regulation of, those physical forces making for organic order. Whenever physical forces tend to stray from their constructive metabolic pathways, the effect of mnemic causation is always towards bringing them back on course. It may be contended that enzymes, by greatly speeding up reaction times along these pathways already achieve this. But mnemic causation is continually operative no less within enzymes than without. But for this instant by instant guidance exerted by mnemic causation in the service of order, metabolic processes would rapidly peter out in chaos. Secondly, this precise, regulative role of mnemic causation will sometimes require it to accelerate 'particles', and sometimes to retard them. So that as the algebraic sum of these additions and subtractions, its total energy contribution will be numerically less - conceivably much less - than either. Thus, the magnitude of the <u>effect</u> of mnemic causation does not bear a simple relation to the size of its overall energy contribution, whether positive or negative.

<u>Mnemic Causation and Hierarchical Order</u>:

Living organisms are psychophysical systems, mnemic causation being the effect of the organism's past experience - the psyche in the broadest sense - on the cosmic processes which, thus modified, constitute the system's physical present. In response to environmental changes, these organic physical processes themselves constantly change in order to maintain the organism as a biologically viable synthesis. The elements of such organic changes, which are, of course, themselves complex processes, evince a marked regularity and repetitiveness. If we consider any small region of the organism, then the qualification sequences of which this is ultimately composed will constitute, at different times, the ground of different higher order processes of varying complexity, themselves going to make up the organism as a behavioural synthesis at any particular moment. Now, it is this overall synthesis, noumenally a unified experience, which is actually occurring; its constituent processes occurring only as parts of this whole, inclusive process. Hence, what is occurring in any region of the organism will change according to whatever overall, multi-channelled process it happens, at that particular time, to be part of. Thus, in any region, a certain repetitive process may commence or cease at different times relative to associated processes in other regions, or, depending on what is happening elsewhere, take any of a number of variant forms. The physical situation of near-equipotentiality may remain effectively unchanged at all times; but mnemic causation, the effect of the past instances of the particular overall process acting in the present, will bias this physical equipotentiality now this way, now that, to give rise to the different constituent processes. The organism, considered as a framework of energetic possibilities, may be likened to a road system of innumerable multiple junctions, and the biological processes composing it, to the traffic passing through this system. The traffic must pass along the roads which the energetically favourable constraints lay down for it, but which routes it proceeds by at any junction depend not upon any attribute of the roads, but only upon itself.

Mnemic Causation and Negative Affect:

It is in the causal duality of the Cosmos, as we have described it, that suffering has its ultimate root. Living organisms are systems of coordinated processes. But the different states which such a system can assume vary enormously with respect to the orderliness of their activities. Courses of connected activity spread through the system, but this, in itself, says nothing as to the degree of regularity with which they do so. Normally, for two basic reasons, this regularity is of a very high order. Firstly, because energetically efficient activities are invariably orderly activities, and such efficiency is essential for survival. And, secondly, because mnemic causation, as far as possible for a psyche of given dispositional limitations within a particular situation, will always select the most orderly continuation. As the expression of a psyche, essentially an emergent manifestation of a self-realisatory Absolute, it will always maximise that psyche's self-realisation. In experiential terms, the unfailing tendency of mnemic causation is to maximise positive affect. This high level of order, which defines the norm for healthy organisms, is experienced as physical well-being. But, in two basic ways, organisms often fall markedly short of this norm. If the organism becomes old, or sick, or injured, that stream of proprioceptive information which makes so large a contribution to regular, ongoing activity, becomes, with lowered efficiency, much less regular. Also, in the healthy organism, there have evolved through natural selection sense receptors which dispatch to, or trigger within, the central region of smooth, harmonious activity, bursts of markedly anomalous events whenever the organism commences some injurious activity. Since the psyche's nature is to maximise order to the best of its ability, mnemic causation initiates behaviour which withdraws the organism from the harmful, disruptive stimulus. In both these ways, anomalous events replace normal, harmonious ones, and hence contradict - usually effectively - the mnemically selected continuation. These inharmonious, anomalous, contradictory events, as inappropriate substitutes for orderly, collectively constitute <u>dis</u>order, and, as such, are experienced as negative affect; the severity of which depending on the degree of disorder. It must be emphasised, however, that this disorder relates to the derogation of the organism as a harmonious system of experience, and not to its impairment as a biologically efficient entity, although, for the reasons indicated, the two will be broadly commensurate.

<u>Mnemic Causation in the 'Inanimate' World:</u>

The biological world has emerged as a consequence of the modification of physical force by mnemic. But what of the infra-biological world? What part does mnemic causation play here? Since mnemic force operates wherever there exist repetitions of patterned unities, it is certainly active in the world of sub-atomic 'particles', atoms, molecules, crystals, and plasmas. But, is it effectual, in the sense of significantly changing the Cosmos? That is, does it do something significantly more than duplicate the action of physical force? The difficulty here lies in not knowing just how much in the way of patterned order can be achieved by physical forces acting alone: a difficulty compounded by the fact that, at this level of minimal individuality, if mnemic causation has substantially changed the world, it will have done this so uniformly and so universally as to render conceptual separation from physical force impossible in our present state of knowledge.

Within what kinds of ordered, repetitive situation should we, then, expect mnemic force to be effectual? To be effectual - that is, to dominate physical force sufficiently to significantly change the cosmic contents - it must reconcile the two opposites of constancy and change. Without constancy there can be no repetition, and hence no mnemic causation. But if this constancy is internal, that is, if the unity is essentially that of monotony, then such slight changes as may occur will be overridden by physical forces, and so rendered ineffectual. Such reconciliation is most obviously embodied in cyclical chemical change. Here, repetitions of chains of short, discrete, abrupt, emphatic accelerations will override the great majority of the disruptive physical forces they will encounter within the sheltered situations which, by the manipulation of these same physical forces, they have created for themselves. With such disruptive physical forces largely rendered nugatory, mnemic force is free to choose constructively among the alternative chemical reactions that present themselves in situations of near-equipotentiality.

One type of mnemically favourable situation on an infra-chemical level occurs where, owing to the delocalisation of orbiting electrons, certain compounds resonate between different forms. But the most likely locale for the effectual action of mnemic causation at the infra-chemical level is the atomic nucleus. The unstructured, liquid drop, model is almost certainly false. Far more probably, the constituent protons and neutrons collectively form a precise, cyclical, dynamic structure. And since these nucleons are, themselves, composed of negative and positive electrons (our single qualification sequences) interacting in precisely structured dynamic patterns, here, one might legitimately infer, is a type of situation where mnemic causation could sustain a stability beyond the scope of physical forces acting alone. If this is indeed the case, and if the number, shape, size, location, and orientation of the electronic orbitals are determined by the precise conformation of this structured nucleus, then, since chemical properties depend on these orbital attributes, it follows that mnemic causation plays an essential role in the emergence of orderly chemical reactions, as such.

Mnemic Causation and the Evolution of Experience:

Mnemic causation is the creator and sustainer of organic order. And outside such order it does not operate. Thus, in our schematic example, were not E contiguously associated within the rhythmic pattern A-B-C-D-E-F, it would not be mnemically selected as an alternative to the physically selected, organically disruptive K. Moreover, were not the past instance of rhythmic pattern sympathetically associated, the A-B-C-D- contiguously selecting the E would not even be in the present to select it. Clearly, therefore, mnemic causation is an influence for preserving the existence of organic order in opposition to the dissipative tendencies of physical force, which, as the expression of a different type of order - cosmic - is indifferent to it. This indifferent cosmic order does, in fact, produce the basic physical situations from which mnemic causation first effectually operates, but in no way serves, in itself, to sustain the far greater order which is raised upon such foundations by mnemic causation. If it does so serve, it is only through mnemic causation's steering it towards that end: an end which, without such channelling, it would destroy. It is mnemic causation which, without the involvement of any nonsensical in-built orthogenetic nisus, brings about that emergence of organic order on ever more complex levels of experiential organisation, we know as biological evolution.

Two fundamental parameters interact to achieve this. One is simply the basic fact of the auturgic maintenance, by mnemic causation, of experiential organisations of an order immeasurably more complex than any that could arise from the sole operation of physical forces. The other is that, as living organisms change the physical environment they inhabit, they open up new ecological niches: that is, they render possible new ways of life - new ways of thriving vis-a-vis this increasingly biological environment. But some of these hitherto non-existent ecological niches created by a more complex ecosystem require more experientially complex organisms to successfully exploit them. And such biologically successful exploitation, by its very advent, adds to the complexity of the ecosystem, thereby creating some yet more complex ecological niches which only yet more experientially complex organisms can successfully exploit. Thus complexity feeds upon itself.

The evolution of experience is only very subsidiarily a matter of increasing elaboration of the sensory system and musculature. Principally, it consists of ever fuller assimilation of that other 'external' world - the inexhaustible reservoir of past experience. Mere similarity between past and present, however indispensable as a foundation, will not, of itself, take the organism very far in this direction. What is required is active access to the past: selective access directed by mnemic causation issuing from the organism's awareness of its situation as a feeling, striving entity vis-a-vis the world. Through this very activity, the substance of the organism's world becomes increasingly that of memory based imagination; perceptions given ever-deepening significance by, and responses organised more and more with respect to, this transperceptual context.

But the efficient sustainment of this context, in all its constantly changing relevance, cannot be achieved without the evolution of massive physiological underpinning. The key anatomical development is, of course, that of an extensive, near-equipotential neocortex. As a comparatively detached region of highly regular activity, this provides a cerebral part-alternative to the visceral body image, distracting much less, by virtue of this extreme regularity, from associated past experience. But, more than this, past experiences evoked by the perceptual situation can mnemically activate the cortex in very different ways, producing widely varying degrees and distributions of neuronal excitation, which, themselves, become associated with widely divergent, mnemically induced, proprioceptions and vestigial kinaesthetic sensations. In this way, mnemic causation, both involuntary and voluntary, can bring about the association of particular states of the nervous system with particular kinds of psychical content, where these are discriminated not just on intrinsic grounds, but also in respect of what we might term their functional position in relation to the organism's present purposes. In thus passing from memory to memory not just passively by the 'free' associations dictated by their intrinsic similarity and contiguity, but actively, by making such associations depend also on mnemic control of the nervous system, the organism is able to harness the past in service of the present so efficiently as to make possible ways of life of an altogether unprecedented order of experiential complexity.

This whole mental development has its roots in the prior evolutionary process of the mnemic construction of an 'objective' world of solid bodies interacting in three-dimensional space, upon a nucleus of visual sensations. Through mnemic activation of both the neocortex and the motor system, this nucleus is integrated into a complex fabric of interneural excitations, proprioceptions, and kinaesthetic sensations born of visual, tactile, cutaneous, kinaesthetic, proprioceptive, and interneural memories: the whole thus constituting an elaborately integrated groundwork of memory and perception. From the manipulation of perceived objects in perceptual space to the manipulation of imaginary objects in perceptual space is but a step, and but another to the manipulation of imaginary objects in imaginary space - and that is to have reached the threshold of thought and, with it, the human condition.

Mnemic Causation and Metachoric Experience:

All cosmic sequences have been selected either by physical force acting alone, or else jointly by physical and mnemic force in the way we have outlined. The latter sequences are those belonging to a psychophysical organism. The fact that, when physical forces alone are acting, sequences are selected into the Cosmos but not into a psyche, naturally raises the question: Are there sequences which are selected into a psyche but not into the Cosmos? In considering this question we must never lose sight of two essential points. The first is that all sequences exist - no less for not being selected into the Cosmos. The second is that the spatially grounded structure of the Cosmos demands that all its sequences begin their separate existence by branching off the same parent stem. Apart from that, cosmic sequences, qua cosmic sequences, do not bifurcate, making their continuation at every instant a case of straight either/or. But no such basic constraint binds mnemic selection. Certainly, it must always act, with whatever degree of effectiveness, on all the cosmic sequences collectively constituting the physical component of the psychophysical organism; but this does not necessarily preclude its selecting into the psyche, so long as they are the sympathetically associated expression of its present state, additional, non-cosmic - or, as I prefer to call them, metacosmic - sequences, bifurcating from the cosmic. Such metacosmic selection - at least, on a large scale - will not be common, since biological considerations require that, in general, the psyche is fully preoccupied with its physical situation. On the imaginative, human level it tends to occur when the neocortex becomes abnormally free from influx of exteroceptive and proprioceptive information; a state of affairs which allows mnemic force effectively to override physical over large areas of the cortex. These metachoric states can be deliberately induced by what we may summarily label shamanic and yogic techniques, or else they arise involuntarily through cerebral malfunction, shock, sickness, accident, etc. This process of mnemic selection of metacosmic sequences gives rise to such types of experience as OBEs, NDEs, apparitions, and lucid dreaming. What, one wonders, is the fate of metacosmic sequences in a TNDE - a too-near-death experience?

MIND, HIERARCHY AND COSMOS

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ABSTRACT

Using a wavefield model of the brain under development called the Huygens' Machine, a mathematical specification for mind is derived. This defines consciousness as mind/brain interaction. Such interactions are the quantum measurements between the brain a classical sensory prepared macroscopic apparatus, and the dynamic quantum vacuum a non classical medium. This Descartian model of mind points to the fundamental spectral theorem of Hilbert and Von Neumann, as defining mind - a quantum computational universal attractor. A mechanism that will be shown to be at the heart of the Combinatorial Hierarchy and thus can be postulated to be the computer constructor universal chaotic attractor central to cosmological evolution that the Hierarchy models in terms of labelled bit strings.

Introduction

Any debate on the nature of consciousness must be largely sterile without a hard scientific model of the human brain, or of biological brains in general.

Such a model of the brain called the Huygens' Machine is under development. B.E.P. Clement et al, in press. It is based on a mathematical formalization of a generalization of Huygens' Principle of secondary sources describing the propagation of any wavefront or signal in a non-linear medium. M. Jessel, 1954. This mathematical formalization is by means of General System Logical Theory (GSLT) developed by Germano Resconi and Maurice Jessel (Jessel, M. and Resconi, G., 1986). This formalism employs commutative diagrams like that given below. This for example, specifies how the signals propagated in a medium change the mappings of any particular form of physical behaviour OP or wavefield F(O) in order to control the machine or how the machine may be controlled in order to compute a particular form of physical behaviour.

1)

$$\begin{array}{cccccc} F & = & F(O) & \xrightarrow{\quad OP \quad} & S_{or} & = \; OPF \\ T & & \downarrow & OP & \downarrow & \\ & & F(t) = TF(O) & \xrightarrow{\qquad\quad} & S_{or}^{s} & = \; (OP,T)F(O) \end{array}$$

where OPF = S_{or} is the particular physical behaviour that connects the sources S_{or} to the field F; F(t) = TF(O) describes the means by which the wavefield propagation takes place, and S_{or}^{s} = (OP,T)F(O) are the secondary sources of the wavefield where (OP,T) = OPT - TOP is the Lie product or commutator and T is a suitable weighting operator. Huygens' principle (Jessel, M., 1973) says that the perturbation of the field F that passes through a surface S containing a wave source S_{or} is identical to the perturbation that is obtained by removing the source and substituting it by an appropriate system of secondary sources S_{or}^{s} distributed on the surface S. Thus for the diagram above to be true, T must be chosen to be a Heaviside-like operator so that TS_{or} = O except on S. Such diagrams specifying the nature of signals and signal pathways therefore determine the morphology and dynamics of the Huygens' Machine, where for example a set of secondary sources S_{or}^{s} perfectly simulate a source S_{or}. That is Huygens' Machine information processing employs actual physical simulations of processes and not their mathematical representations in the form of programs.

The signal states, signal flows or signal synchrony such a machine (a non-linear medium) would require to carry out analogue and quantum mechanical computation, have been researched and found to be in broad general agreement with the known dynamics and morphology of biological brains. This will be described later. In this machine model the physical processes constituting the nature of information, perception, cognition, and consciousness are all well defined.

Consciousness

In particular consciousness is mind/brain interaction, the means of machine control when the Huygens' Machine becomes a classical wavefield apparatus prepared with sensory data or brain, able to compute quantum mechanically by interaction with the dynamic quantum vacuum - a non-classical medium. This is in exact analogy with the means by which a classical prepared apparatus makes generalised quantum measurements in high energy particle physics. It follows Descartes original conception of brain and mind as two fundamentally different substances. The first the brain - a classical macroscopic apparatus or machine: the second the interaction with the dynamic quantum vacuum, a mean zero energy field or sea from which quantum appear and disappear in accordance with the Heisenberg uncertainly relation. Such vacuum perturbations play the role R of mind, a ghost in the machine since the dynamic quantum vacuum is itself not directly observable. R is completely specified in the Huygens' machine by means of the fundamental spectral theorem of Hilbert and Von Neumann represented in the form of the following commutative diagram, B.E.P. Clement et al, in press.

Diagram II

$$\begin{array}{ccc} f & \xrightarrow{\quad R \quad} & R(f) \\ F \big\downarrow & & \big\downarrow W \\ F(f) & \xrightarrow{\quad R \quad} & R(F(f)) = W(R(f)) \end{array} \qquad - 1)$$

R are the rotations which are the simplest way of obtaining the self-adjoint operators F for any vector f of the Hilbert space, given a multiplicative weighting operator W, or vice versa. Such vacuum perturbations therefore clearly endow mind with all the potentialities of a universal machine, although a non-classical one able to perfectly simulate any algorithm or physical process (classical or quantum) as has been shown by D. Deutsch, 1985 by different means. (See below)

Such a commutative diagram constitutes the basic logical unit or module in General System Logical Theory (GSLT) by means of which Huygens' Principle of secondary sources is mathematically formalised and by which the experimentally validated but little known entropic processes of active control and active absorption of wavefields known as Field Reshaping Theory or Holochory employed by Huygens' Machines are described. For example

$$\begin{array}{ccc} x & \xrightarrow{OP} & y \\ T_1 \downarrow & & \downarrow T_2 \\ T_1x & \xrightarrow{OP} & T_2y \end{array} \qquad 2)$$

concerns the operators OP, T_1, T_2, x and y. It says that $OPT_1 = T_2OP$ or $T_2 = OP^{-1}T_1OP$ meaning the processes or operators T_1 and T_2 are equivalent i.e. that any property of T_1 is under these conditions that of T_2. Such similarity transforms for example describe the basic control structure for the sensory apparatus of the Huygens' Machine where a wavefield inside the machine is used to perfectly simulate a corresponding wavefield arriving at a sensory apparatus. Perfect simulation is thus assured because any property T_2 inside the machine is equivalent to the property T_1 at the sensors. And Diagram 1) is of course such a diagram as

2) where F and W are equivalent. Thus 1) specifying mind guarantees that any such perfect simulation as is required, will be possible with a Huygens' Machine which is able to compute quantum mechanically as described above.

Having therefore defined consciousness, let us return to the corresponding definitions of information, perception and cognition which can all be experimentally demonstrated here and in the laboratory.

Information, perception and cognition

Take visual perception, remembering that in the case of light, that when electromagnetic radiation in the visible spectrum v_s impinges on a 3 dimensional object o_i, an interference pattern $P(o_i,v_s)$ can be produced. This pattern provides the definition of information therefore in the new paradigm. Furthermore from this pattern $P(o_i,v_s)$, the 3 dimensional object image $o_i'(v_s)$ can with suitable physical apparatus be recovered, and subject to suitable conditions o_i and $o_i'(v_s)$ can be made totally coincident. Such a wave condition is called phase conjugate (Pepper, David M., 1985) and can be demonstrated with respect to our own perception. Snap your fingers at some point about your head and observe the direction, amplitude and location of the acoustic object image your brain creates. It is located outside your head at the precise point at which your fingers snapped; such a phase conjugate condition applies to any kind of physical wavefield. Focus your eyes on a close object and reach out and touch it. Your fingers tell you that the tactile object image (a real image) the brain creates is located exactly in every particular where your eyes tell you the visual object image (a virtual image) the brain also creates is to be found. Now tap the object with your finger and the acoustic object image of the tap comes from the object too. Thus the process of converting an interference pattern $P(o_i,v_s)$ to an object image $o_i'(v_s)$ coincident with the object o_i defines in this paradigm the process of perception. Furthermore the Huygens' Machine model of this paradigm says that such definitions of information and perception equally apply to moving objects and their corresponding spacio-temporal object images.** Suppose the object o_i is a ball in flight that is then successfully caught - what have our brains done? They have learnt through visual and tactile perceptual experiment how to transform such a dynamic 3+1 dimensional inertial object image of the ball, so that it comes to rest in our hands as a gravitational object image i.e. a weight supported by our muscles. That is to say the same fundamental perceptual process applies to visual, tactile, acoustic, inertial, gravitation wavefields etc. and it can be inferred that the corresponding patterns $P(o_i,v_s)$ for any experienced 3+1 dimensional object o_i and wavefield v_s can be stored within the brain and used in the future for the purposes of cognition. Thus if brains have the ability to store such patterns, as the above experiment tells us they do, (and subpatterns for example those in the visual spectrum restricted by a filter to say blue or red) and have the ability to compare such stored patterns with newly arriving ones from our senses, then such a comparison will not only enable the brain to

** simply the application of relativity for example the Lorentz invariance between visual and retinal cortices that is essential for survival recognise the objects o_i defining the process of cognition but will by means of a suitable system of filters enable the brain to taxonomize all objects according to their properties (i.e. to differentiate a blue or red object from other objects by colour). This can be done from the stored experiential patterns or subpatterns P without any need of calling or labelling a subpattern blue or red. Such comparisons allow an object seen from one perspective to be identified with the same object seen from a totally different perspective in a single operation (with the possible exception of a small set of perspectives that are pathological or illusions) and since such stored patterns are of spacio-temporal object images, memories of the past are quite literally potentially memories of the future i.e. predictions and actions. Such an analogue machine would therefore be in principle incomparably faster than any analogous digital system for say visual perception, which must operate by reconstructing objects from a knowledge base of all their parts or "bits" to achieve a corresponding successful comparison rather than simply establishing the knowledge base after the fact as the analogue machine can. That is to say the analogue machine works top down, while the analogous digital system must work bottom up. Thus analogue machines using holographic processing to implement computation rather than Boolean operations define a new paradigm in which the nature of information, and the processes of perception, cognition and learning are all well defined. This is in contrast to both digital computation and neural nets where as yet computer science has no agreed upon corresponding definitions. For example (Feynman, R., 1986) information in the form of 0s and 1s used in digital systems follows from the usual classical analysis of Boolean operations which assumes there to be two standard voltages representing the local 0 and 1. These are not used in brains because information stored in this form cannot efficiently reflect the 4 dimensional spacio-temporal nature of reality as the patterns P can, and which brains must do if they are to survive.

How therefore are Huygens' Machines organised?
This novel machine model based on Jessel's generalization of Huygens' Principle of secondary sources, mathematically formalised by means of GSLT developed by Resconi and Jessel, employs Field Reshaping Theory or Holochory of which holography and holophony are special cases. Field reshaping Theory has over the last twenty years revolutionized acoustical wavefield research where Jessel first applied his discoveries , M. Jessel, 1988. But it applies to any form of wavefield such as for example biochemical wavefields and aerodynamic flows as demonstrated by Resconi who has used it to describe biochemical reactions and the flight of aerofoils, H.E. Fatmi et al, 1990.

It leads to an abstract model of machines and a control theory described formally in terms of Heaviside-like or all or nothing operators (-directly applicable to neural firing) and in terms of Lie commutators, algebras and groups. This is in direct contrast to the Boolean algebras which describe digital machinery. It is a formalism used to describe general dynamical systems in both classical and quantum physics, and one capable of dealing with exponential complexity through abstract Lie transformations. The product of such transforms are dynamical invariants - the very essence of perception and cognition. Thus it brings fundamentally different and advantageous formal physical modelling techniques to bear on brain organization, dynamics and the processes of cognition. Such architecture or organisation is modular, an entirely natural consequence of the model, and has been shown to correspond with

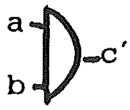
a) an entropic hole or Maxwell demon operating in a neural membrane, figure 1
b) the neuron itself, as a Lie group germ, figure 2
c) a cortex, figure 3 and,
d) with the left/right hemispheres, and their joining the corpus callosum found in advanced brains. figure 4

(a) is the analogue computational gate or hole in a surface which opens and closes to admit an ion, particle or molecule.

(b) consists of a surface of such computational gates controlled by potentials generated by an arborization corresponding to a dendritic tree above the surface. It constitutes a threshold or firing mechanism represented by a Heaviside or all or nothing operator, that is, in this model the neuron is a local computing surface.

(c) consists of a surface of such neurons as (b) controlled by potentials produced by an arborization above the surface consisting also of such neurons (b); that is a cortex or global computing surface, and

(d) the logical primitives essential to digital computing machines (Feynman, R. 1986) are conventionally, AND

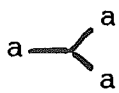


a	b	c′
0	0	0
0	1	0
1	0	0
1	1	1

NOT

a	a′
0	1
1	0

FANOUT



and

EXCHANGE

In the model of Huygens' Machine able to carry out quantum mechanical computation allowing it to perform reversible Turing computation (Deutsch, D. 1985) AND and NOT or the single element NAND are replaced by the logical primitive XOR (and its inverse) such reversibility requires as already shown.

FAN OUT is the arborization referred to above, represented by a Jacobi identity, or higher form, i.e. a commutator tree, as below

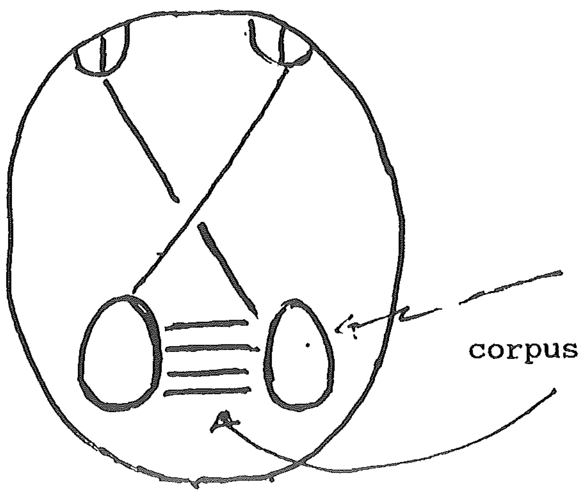
$$(((x,y),z),w)+(((y,x),w),z)+(((z,w),x),y)+(((w,z),y),x)) = 0 \quad 4)$$

where (a,b) = ab - ba is the Lie produce or commutator, and x,yx,z,w etc are operators by means of which a simulation taking place in the Huygens' Machine is described. Such higher forms (4) as Hoffman (Hoffman, W.C., 1966 and 1989) has pointed out correspond to a "tree structure" in the Lukasiewicz's theory of parentheses.

Lie products (a,b) are the fundamental units of description of the Huygens' machine at both the local and global levels of its information processing morphology ensuring that i) the global computing surface is a complete map and ii) that each and every Lie product or germ is potentially covariant and calculable.

EXCHANGE described through such a Lie product can therefore be postulated as the only remaining representation for the Huygens' morphology or signal flow at the level of global computing surface or cortex in the quantum mechanical machine. Such a postulate requires that this cortex be divided into two corresponding to switchstates 0 and 1, and joined one to one by unit wires reversible computing requires (Fredkin, E. and Toffoli, T, 1982) to provide a canonical labelling essential to the global computational process. This is indeed the case, such brains postulated to compute quantum mechanically i.e. the human brain has a left and a right hemisphere joined by axons (as its unit wires) called the corpus callosum. Indeed such an EXCHANGE morphology or signal pathways are seen with respect to the visual system, (figure 4) and similarly with respect to the global acoustic and self inertial (balance) perception i.e. ears; global olfactory perception i.e. nose; global tactile manipulatory perception (arms and hands) and global perambulatory perception i.e. legs.

<u>FIGURE 4</u>



Furthermore the signal pathways appropriate to the phase conjugate condition (Pepper, David M., 1985) are also present. Such phase conjugation generally achieved by what is called four wave mixing, explains why the signals from the left and right halves of each eye go to different hemispheres.

Why should brains evolve so as to compute quantum mechanically? In the Huygens' Machine, machine organization concerns both the local states of neurons or local computing surfaces on the one hand, and the global states of the machine or global computing surface on the other. Survival by learning will dictate that while the brain or intelligent machine process in parallel the multitudinous complexity of local states and local state changes - each of greatly varying significance to that survival - that a mind or "I" i.e. a self must develop that focuses on the most significant current global state i.e. W in (1) and switch attention when required from that state to the next. Further more such a global state in the Huygens' Machine will for example in the case of visual perception be a holographic dynamic picture in spacio-temporal dimensions giving a global visual stream of conscious perception like that of which we are all aware. The Huygens' Machine model points to four principal benefits once this is achieved in terms of an enhanced morphology and dynamics, B.E.P. Clement et al, in press:

a) the potentiality to simulate or communicate information digitally or symbolically, in accordance with a canonical labelling schema, i.e. a natural language,
b) a further substantial increase in computing power such that the particles on the local computing surfaces may function

as 'bits' in the digital sense or even as universal quantum mechanical machines,

c) the ability to switch instantaneously from one massively parallel computation to another i.e. to change from one W to another in (1) by means of a rotation R in the global quantum mechanical state space of the machine, should survival require it. And this is indeed observed in human brains, and

d) to behave as non-classical machine through interaction with the quantum vacuum using signal synchrony not as in c) inside the brain itself, but outside in the sense that nonlocality provides a link with other brains or with the universe as a whole. That is there is a sense in which mind on this model provides an awareness of the whole universe and its ongoing evolution and of which creativity in the intellectual sense is simply part of an on going creation in which mind is privileged to participate. The present not space is the final frontier, and one that each of us can locally change for good or ill. We cannot however change the global character of that on going creation as this concerns the whole Universe, although mind can undoubtedly be the agent for such global change. We can of course oppose global change by means of local change, but this ultimately will be swept aside by what the Chinese called the Tao. In this model too, the dynamic quantum vacuum can be identified with another Chinese concept Chi.

Consciousness and Cosmogenesis

The commutative diagram (1) expressing the fundamental spectral theorem of Hilbert and Von Neumann is open to diverse physical interpretations or experimentally validable hypotheses.

The first, already given concerns mind or the capability of the Huygens' Machine as a classical prepared wavefield apparatus to compute quantum mechanically through its interaction with the dynamic quantum vacuum.

Others are that it provides a specification for

a) a universal model of quantum mechanical computation defining all the allowed relations or mappings. This is confirmed by the work of D. Deutsch, 1986, who has shown that if the rotations R are confined to the state space of a bit $Z_2 = (0,1)$ i.e. to the Hilbert space of a particle with spin states, spin up and spin down, then such rotations can perfectly simulate any physical process or algorithm. Deutsch also shows universal quantum mechanical computation so defined, contains universal reversible Turing computation as a submodel.

b) what nanotechnology describes as a "quantum dot", E. Coroccan, 1990, an example of which is single electron working as a computational element dimensionally confined to 2, 1 or zero dimensions so that it operates entirely as (1) specifies in its configuration space,

c) the dynamic quantum vacuum or its interactions, P. Marcer, 1992, in press,

d) the Creation or Big Bang, P. Marcer, D. Dubois, 1992, where the Universe at its inception exists solely within its configuration space, the Hilbert space. This would say that at its inception, the Universe was a 'quantum dot' without space or time, or perhaps more accurately that space-time is a property of Universe, and not the continuum within which the Universe is often or generally imagined to exist. In this case (1) can be considered as specification for the Unified Field.

Evidence in favour of the last hypothesis d) concerns

i) the fact that (1) constitutes an example of a universal relational model of a theory as defined by Philip Ehrlich 1989 i.e. all the allowed relations or mappings of the theory in this case quantum mechanics. Ehrlich has shown that any such model of a theory, has a unique birthorder field automorphism or birthordering. Such a birthordering appropriate to the Second Law of Thermodynamics in a theory such as quantum mechanics where all the dynamical processes are reversible would therefore provide a model of the Cosmogenesis and one that remarkably is unique. It would define the natural or birthorder by which the measurable Universe would be seen to emerge from the original postulated quantum dot without space or time, ie.. the Tao.

ii) M. Berry, 1988, has hypothesised that there exists a self-adjoint Hamiltonian operator the eigenvalues of which are the imaginary parts E_n of the non trivial zeros of the Riemann Zeta function obtained by quantizing some still-unknown dynamical system without time-reversal symmetry whose phase space trajectories are chaotic. The proof would follow from the truth of the Riemann Hypothesis. But from (1), the only process without time-reversal symmetry is its unique birthorder field automorphism, and so we can hypothesize that Berry's Hamiltonian is that of the Cosmogenesis. That this is likely to be the case follows from (1). Choose the vectors f so that they constitute a normalised linearly independent orthogonal conjugate basis for the Hilbert space i.e. e_a a = 1,2,3....... where $e_a . e^b = \delta^b{}_a$ is the Kronecker delta $\delta^b{}_a = 1$ when a = b and 0 otherwise.

The self-adjoint Hamiltonian operator required must be the one that corresponds to a realization of the whole basis e_a. But since F the self-adjoint operator in this case, and W its corresponding multiplicative weighting operator have by the diagram equivalent properties, W too must exhibit a corresponding linearly independent basis in multiplicative form. Such a basis is that exhibited by the Riemann Zeta function where the primes p play the role of the basis, and are indeed known to be linearly independent (i.e. form such a basis for the integers). Such a linear independence of the primes equivalent to the unique factorization theorem of the numbers on behalf of W, can therefore be expected to correspond to a unique Hamiltonian on behalf of F especially if the rotations R are confined as D. Deutsch, 1985, requires to those of the state space of a bit Z_2, i.e. the Hilbert space of a particle with spin states, spin up and spin down, eigenvalue 1/2, providing by the Pauli exclusion principle, a unique canonical labelling any form of computation must have. This all points strongly to both F and W in this case having the same eigenvalues (to within a multiplicative constant) which are indeed the imaginary part E_m of the zeros of the Riemann Zeta function, which the Riemann Hypothesis postulates all lie on the line x = 1/2 or the eigenvalue of R. It points to a cosmogenesis which in terms of its wave properties, is deterministic in the same sense that the Schrodinger wave equation is deterministic, and in which chaos, chaotic trajectories and attractors play the major role in determining that evolution. This would be expected from Huygens' Principle describing a non-linear analogue form of computation of which the essential signature is chaos. It points away from the Big Bang, Big Crunch or periodic attractor model of cosmology currently in vogue, towards a model or evolution without finite limit towards an asymptotic attractor; the Great Attractor.

iii) The above importance of a linearly independent basis e_a points to S. Amari's, 1991, Dualistic Geometry of the manifold (of higher order neurons) as providing a solution to the problem of the dichotomy between quantum mechanics and general relativity.

Amari's dualistic geometry of the manifold concerns geodesic behaviour by means of two coupled Riemannian metrics such that these are dually flat manifolds each having two different criteria of linearity or flatness coupled one to the other so that the metric tensor of the first g^{ab} is the inverse of the other g_{ab} implying the existence of mutually dual or conjugate bases in their tangent spaces, where

$$g_{ab} = e_a . e_b \quad \text{and} \quad g^{ab} = e^{xa} . e^{xb}$$

and e_a are those referred to above. Amari then demonstrates the Pythagorean relation between the dual orthogonal geodesics at a point Q, demonstrating that such behaviours do indeed take place in the Hilbert space in exact correspondence with (1). In Amari's dualistic Geometry e_a are the tangent vectors along the coordinate curve θ^a at a point θ in the first of the dually coupled co-ordinate systems such that it is the inner product $g_{ab}(\theta)$ that defines the metric, and similarly in the second dual co-ordinate system z along the co-ordinate curves z_a the corresponding inner product $g^{ab}(z)$ defines the second metric, where

$e_a . e^{xb} = \delta^b{}_a$ is the Kronecker delta.

The Pythagorean relationships between the dual orthogonal geodesic at a point Q is then $D(P,Q) + D(Q,R) = D(P,R)$ for points P, Q and R in the manifold, where $D(P,Q)$ is the divergence between P and Q from P to Q as defined by the measure

$$D(P,Q) = \psi(\theta_P) + \phi(z_Q) - \sum \theta^a{}_P . z^Q{}_a$$

and ψ and ϕ are the two convex potential functions with which two coordinate systems are connected by the Legendre transformation

$$\theta^a = \partial/\partial z_a \phi(z) \quad \text{and} \quad z_b = \partial/\partial \theta^b \psi(\theta)$$

so that

$$\psi(\theta) + \phi(z) - \Sigma \theta^a z_a = 0$$

The coordinate system θ is regarded as a linear coordinate system with an e affine or e covariant derivative in the manifold on which the e (for exponential connection) flatness is based, and similarly for z which has a m (for mixture connection) flatness to use Amari's nomenclature. Amari shows that θ and z coordinates are unique to within affine transformations and therefore, for a constant matrix A^k_a and its inverse A^a_k

$$\theta^k = \sum A^k_a \, \theta^a \quad \text{and} \quad z_k = \Sigma A^a_k z$$

are also a dual pair of the primal affine and dual affine coordinate systems, and the same dualistic structure holds in these coordinate systems. Such inverse dual affine systems are typical of Lie transformations that (1) would lead us to expect.

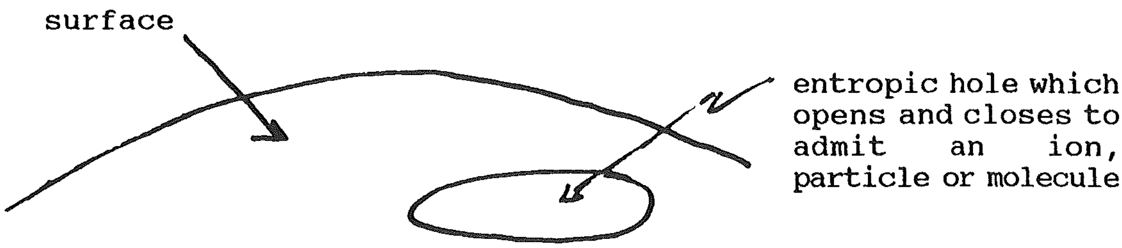
Amari's proof therefore furnishes a mathematical demonstration that geodesic behaviour appropriate to General Relativity can indeed be realised with the dynamics of the Hilbert space by means of such dually coupled Riemannian metrics. And a well known example of this is known that of the Klein-Kaluza equation, where the dual metrics are those of General Relativity and classical electromagnetism.

Conclusion

The Combinatorial Hierarchy - a canonically labelled bit string model of discrete quantum physics based on the EXOR operation and equivalence classes as shown by Clive Kilmister in his Brouwerian Foundation, can therefore employing model checking as an alternative to theorem proving, be identifed with the cosmogenesis modelled as Ehrlich's unique birthordering of (1) in the language of the sets $Z_2 = (o,1)$ and with Berry's self-adjoint Hamiltonian operator whose eigenvalues are the imaginary parts of the zeros of the Riemann Zeta function.

FIGURE 1

Huygens' Computational Gate or Maxwell Demon
(Coveney, P.V. et al, 1991)



Categorical diagram representing its working

$$\begin{array}{lccc} & & OP & \\ \text{entropy} & S(O) & \longrightarrow & o^s \\ T_1 & \downarrow & & \downarrow \quad T_2 \\ & & OP & \\ & S(t) = T_1 S(O) & \longrightarrow & T_2 o^s \end{array}$$

entropy production (can create or destroy states in a molecular or ionic field)

$T_2 o^s = S(O)\, \partial T_1/\partial t + J^s . \nabla T_1$

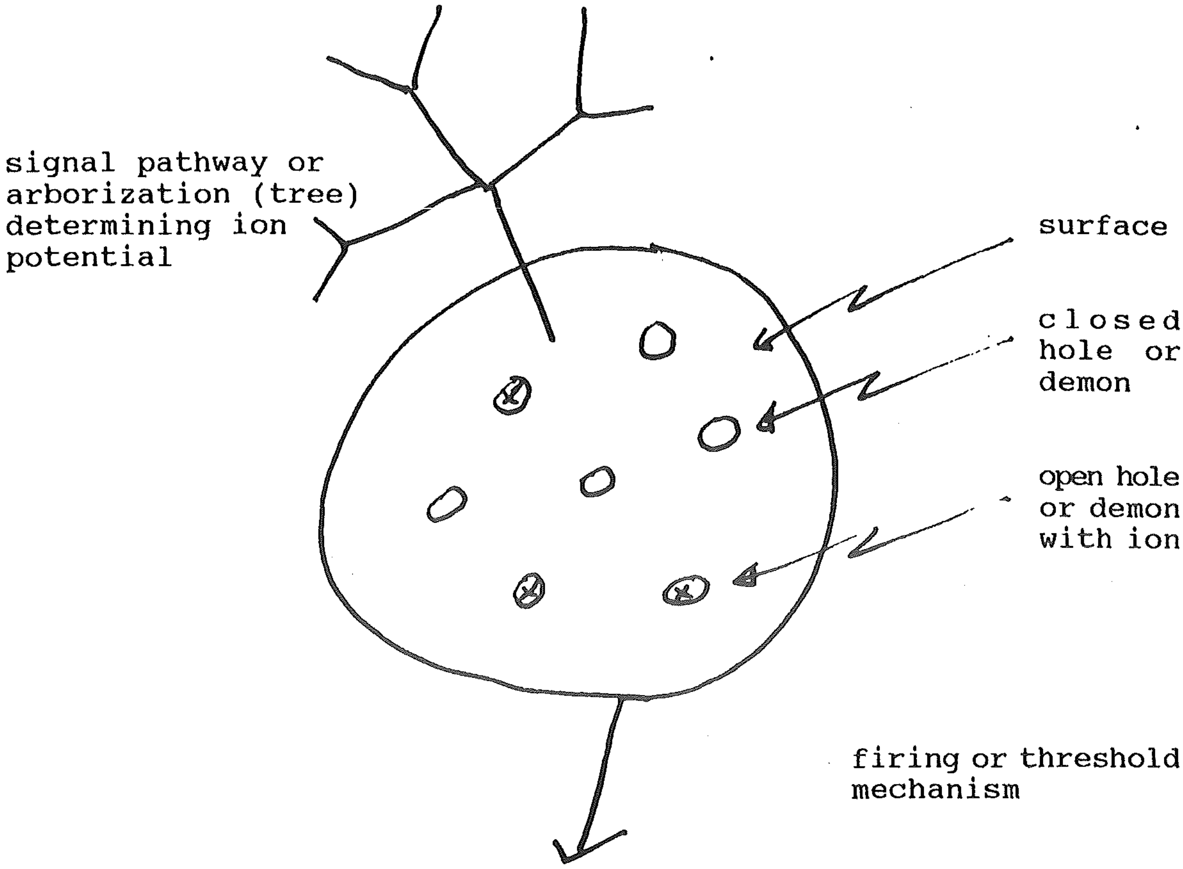
because the entropy in an isolated system is governed by the continuity equation

$\partial S/\partial t + \nabla . J^s = o^s = OPS(O)$

where $J^s = S\underline{v}$ the flux of the entropy over the surface of the hole.

FIGURE 2

Model of local computing surface or neuron
(Clement, B.E.P. et al, in press)



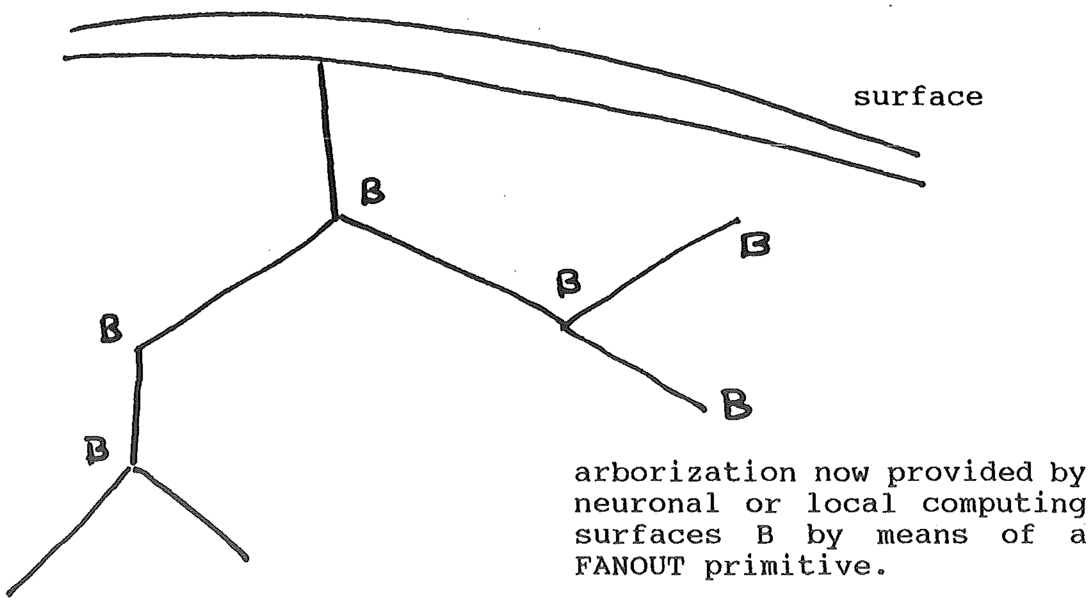
in actual neurons - arborization is provided by dendrites and firing or threshold mechanism by axon and synapse.
In Huygens' Machine the first is described in terms of a Jacobi relation or higher form as below

$$(((x,y),z),w)+(((y,x),w),z)+(((z,w),x),y)+(((w,z),y),x) = 0$$

in terms of Lukasiewicz's theory of parentheses, and the second is described in terms of a Heaviside-like or all or nothing operator.

FIGURE 3

Global computing surface or Cortex
(Clement, B.E.P. et al, in press)



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THE FRACTAL MACHINE: THE WHOLENESS OF THE MEMORY CHAOS

by Daniel M. DUBOIS

INTRODUCTION

This paper gives theoretical arguments in favour of the wholeness of chaos in the brain memory as in the fractal machine. We conjecture that the fundamental origin of the memory dynamics is based on the propagation of phases in the electrobiochemical oscillations in the neural membranes. The interferences of the oscillations are well determined by a composition on their phases, like in holography. This leads to a non-local emergence of phases space-time patterns, like in quantum mechanics. We conjecture that the essential dynamics of the memory and cognition is localized in the oscillations of neural membranes, the amplitudes of which could be lower than the firing threshold of neurons. The role of the firing threshold could be the selection of informations from these oscillations by an adequate variation of the threshold value.

THE BRAIN AS THE FRACTAL MACHINE

Let us consider the neural nodes in the neural frame of the brain as oscillators with frequencies pw, p=1,2,3,... and one common phase f, given by a trigonometric function: X=A*SIN(pwt +f). In taking a discrete time t=0,1,2,3,...in the unity of the fundamental period T=2pi/w, with a unit amplitude A=1, which is the constant amplitude of the pulses, the successive values of X at the discrete times t will have always the same value: X=SIN(f) and will seem static by a stroboscopic effect. If we look at the neural frame for the values of all the neurons with the common phase f=-pi/2, all the neurons will be at their inhibitory state X(t)=-1, t=0,1,2,3,...

If we perturb a neuron by a phase shift x, at time 0, we will have X(1,0)=SIN(x+f)=+1 and X(i,0)=SIN(f)=-1, i=2,3,4,... for f=-pi/2 and x=+/-mpi where m=1,3,5,...is an odd integer number, where the index i is the number of the neurons in the frame. It means that the frame gains one bit (binary digit) of information just in advancing or delaying the phase of one neuron of half its period T/2. This corresponds to a frequency doubling, because the frame is formed of two groups of oscillators with half a period phase difference.

In a one dimension neural frame, each neural node, with one input and one output, will be connected to its backward neuron and forward neuron . Let us consider 16 neural nodes at 16 discrete times t=0,1,2,..,16. At the initial conditions, the first neuron X(1,0)=SIN(pi-pi/2)=+1 and all the others X(n,0)=SIN(-pi/2)=-1, n=2,3,...,16. Let us take the composition, i. e. the local rule at each node,

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$X(n,t+1)=-X(n,t)*X(n-1,t)$ where n=16,15,14,..,1 is the path, i. e. the way to apply the rule, from the right to the left. We obtain, in considering $X(0,t)=-1$ as boundary condition :

n =	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
t=0	+1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
t=1	+1	+1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
t=2	+1	-1	+1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
t=3	+1	+1	+1	+1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
t=4	+1	-1	-1	-1	+1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
t=5	+1	+1	-1	-1	+1	+1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
t=6	+1	-1	+1	-1	+1	-1	+1	-1	-1	-1	-1	-1	-1	-1	-1	-1
t=7	+1	+1	+1	+1	+1	+1	+1	+1	-1	-1	-1	-1	-1	-1	-1	-1
t=8	+1	-1	-1	-1	-1	-1	-1	-1	+1	-1	-1	-1	-1	-1	-1	-1
t=9	+1	+1	-1	-1	-1	-1	-1	-1	+1	+1	-1	-1	-1	-1	-1	-1
t=10	+1	-1	+1	-1	-1	-1	-1	-1	+1	-1	+1	-1	-1	-1	-1	-1
t=11	+1	+1	+1	+1	-1	-1	-1	-1	+1	+1	+1	+1	-1	-1	-1	-1
t=12	+1	-1	-1	-1	+1	-1	-1	-1	+1	-1	-1	-1	+1	-1	-1	-1
t=13	+1	+1	-1	-1	+1	+1	-1	-1	+1	+1	-1	-1	+1	+1	-1	-1
t=14	+1	-1	+1	-1	+1	-1	+1	-1	+1	-1	+1	-1	+1	-1	+1	-1
t=15	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1
t=16	+1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

Notice that the last sequence is identical to the first one.This result is very general (D. Dubois, 1992): we can take any initial sequences of 16 bits and we obtain the same sequences at time t=16. In fact, at times t=2**k, k=1,2,3,4,..., we obtain the first 2**k bits of the initial sequences. The same result is obtained between the intervals of times : for example, between times t=8 and time t=16, the first 4 bits at time t=8 are obtained at time 12 and the first 8 bits at time t=8 are given at time t=16. This is a fundamental property of fractals to be self-similar. The pattern of these activated (+1) and inhibited (-1) spatio-temporal neural nodes is the Sierpinski mat designed by nested triangles. This neural pattern is invertible: in taking the same initial and boundary conditions, we can propagate the composition (D. Dubois, 1992)

$$X(n,t+1)=-X(n-1,t+1)*X(n,t), \quad n=1,2,3,..,16 \text{ for } t=1,2,3,..,16$$

with a path from the left to the right at each times. We obtain the mirror image of the preceding fractal structure. This composition computes the value of the nth neuron $X(n,t+1)$ at time t+1 in function of the value of the (n-1)th neuron $X(n-1,t+1)$ at the same time t+1. This is what is called HYPERINCURSIVITY (D. Dubois and G. Resconi, 1992). In fact, we make the hypothesis that the propagation duration of the composition along a path is a fraction of the period T of oscillations of neural nodes. Experimentally, it is well-known that the speed of signals along the axons is very rapid in comparison with the period of neural membranes oscillations. Moreover, it is well-known that the phase propagation velocity has no limit, e. g. the photons phase propagation velocity can be upper than the light speed c. Let us look at the first step of hyperincursive iterations given by the above composition:

n =	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
t=0	+1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
t=1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1	+1

This first sequence is equal to the 16th sequence of the fractal structure given above and the second sequence is equal to the 15th sequence of the same above fractal structure. In one time step, the whole neural frame is at the same phase as the first stimulated neuron. It is the characteristics of a phase transition, like in physics.

An analogy between this neural dynamics and the spin theory can be made in considering the activated neuron +1 similar to an electron with spin +1/2 and the inhibited neuron -1 to an electron with spin -1/2 : recall that the spin can be assimilated to a rotation frequency of a particule. A spin frame should be placed in a oscillating magnetic field for perfecting the analogy. The theory of the analogy to magnetic field in an iron material is given elsewhere (D. Dubois and G. Resconi, 1992, p. 131). The parallelism between fractal networks dynamics and quantum mechanics was suggested by D. Dubois (1990, 1992). An analogy between neural frame and elementary particles theory could be made for the CPT (Charge, Parity, Time) invariance . General important properties of symmetries of hyperincursive processes are given in D. Dubois and G. Resconi (1992).

The composition laws given in the above basic model are propagated along particular paths in the neural frame are based on the coupling of two oscillators by their products, which correspond to the addition of their respective phases which means that the backward neural nodes drive the forward neural nodes by either advancing or delaying their phases and propagate them through the whole neural frame.

We can write the composition for the phases and amplitudes as follows:

f(n+1)	+ f(n)	=> f(n+1)	-X(n+1)	*X(n)=>	X(n+1)
0	0	0	-1	-1	-1
0	1	1	-1	+1	+1
1	0	1	+1	-1	+1
1	1	0	+1	+1	-1

with the phases given in +/-m*pi units, m=1,3,5,...where m is an odd integer in knowing that the addition or substraction of two odd phases give an even phase equal to 2*pi=0 for periodic functions like the SIN one. We recognize the well-known exclusive OR (XOR) Boolean rule truth table proposed before (D. Dubois, 1990).

If we look at the amplitudes signs (+1 corresponding to activated neurons and -1 to inhibited ones), we obtain the above table which gives rise to the same XOR rule. The multiplication of the signals can be simply computed by the addition of the phases. Notice that if we take into account any absolute values for the amplitudes, the XOR rule is still true for the signs (+ or -) of the amplitudes : it is related to the membrane ions potential. So, the XOR rule is related to the parity, the phase, the charge and the time invariance. Physically, we called this propagation of the XOR rule, the "propagation of the difference" (D. Dubois,1990): it means that any neuron reacts if and only if it detect any difference between two inputs, even if it is between a backward neuron and itself. In mathematics and computer science, there are many ways to formulate this propagation of the "difference operator". The modulo function, MOD N, where N=2 in the binary case, permits to obtain the above XOR table: for example f(n+1)=(f(n+1)+f(n))MOD2. In Pascal programming, we can write: f(n+1):=f(n+1)< >f(n) where f is defined as a Boolean variable.

The time-dependent one-dimensional space frame, defined above, can be transformed into a two-dimensional space frame in considering the time steps as space steps in using the following composition: x(i,j)=(x(i-1,j)+x(i,j-1)) MOD N where N=2, i=0,1,2,3,...16 is the column index and j=0,1,2,3,...,16 is the line index of the two-dimensional frame. The initial conditions being x(1,1)=1 and all x(i,j)=0, we propagate the composition following the path from i=1 to 16 for each j=1 to 16.

A generalized Sierpinski triangles mat is obtained by the simulation of the phase propagation in a two-dimensional neural frame with the above composition with N=16, the 16 colors being the 16 phase values x(i,j) at each node (i,j) of the frame (D. Dubois, 1992).

WHOLENESS OF THE CHAOS FRACTAL MEMORY

The hyperincursive simulations of a fractal memory are given in D. Dubois (1992) where order/chaos transitions and the multiplication of the initial image through the whole frame are well-seen, the basic process consisting in ondulatory interferences like in holography and quantum mechanics. All the neurons with the same phase are synchronized. The composition is based on the following rule : x(i,j)=(x(i-1,j)+x(i,j-1)+x(i,j)) MOD N where x is the phase,i the line index,j the column index of a neural node, and N is an integer number which gives the total number of phase shifts defining a complete oscillation. Each successive phase of neurons x(i,j) is computed by taking the sum MODULO N of its phase x(i,j) with the phases of its west neuron x(i-1,j) and north neuron x(i,j-1), where the mathematical modulo is written MOD in computer programmes. It is well-known that the synapses cumulate their input signals and discharge periodically. The amplitude of the synaptic potential is X(i,j)=SIN(x(i,j)*2*pi/N-pi/2) with the stroboscopic time T (the period of the oscillations). The composition is propagated with a path from the left to the right, line by line, i. e. for each line (j=1 to J), we compute successively the composition for each column (i=1 to I). The initial state of the memory is given by an image on a black background. The black color represents the phase 0, which corresponds to inhibited neurons. After 1 iteration on each node, the phase pattern looks chaotic. Once at the end of the frame, we begin again and again the iterations with the same composition and path. We will give only the fundamental process of the memory dynamics. After 8 successive paths, an ordered pattern begins to appear. After 16 paths, the initial image appears in a big number of spots in different colors (phases). After 32 paths, the initial image is distributed by multiplication through the whole frame. At the 64th path, a lower number of repetitive initial images is observed, which are the result of the merging of images of the preceding figure. Two superposed images give a change of phase, signature of an interference effect of ondulatory type. At the 65th path, i. e. only one more path than the preceding figure, the pattern is drastically changed into a non-recognized image. This pattern is locally chaotic and is globally similar to the chaos after 1 path. In fact, the structure presents a local phase order for values of the number of paths equal to 2**n, n=1,2,3,4,..., i. e. after 2,4,8,16,32,64,... oscillations of the neural frame. The chaotic patterns happen for odd numbers of paths between two ordered states. So, we could predict that the structure will be again in an ordered state after 64+16=80 paths, if the process is fractal like the Sierpinski pattern we obtained in the one-dimensional case. Indeed, at the 80th path, a similar figure like the 16th path is obtained. After 113 paths, the pattern presents ordered chaotic heaps, the global shape of which being similar to the initial image. Each heap contents approximatively 1/20th of the total number of neural nodes of the frame. If any part, even important, of the frame is destroyed, the resting frame can reconstitute the initial image. It is an experimentally proved property of the brain memory. At the 128th path, the frame is at its second cycle of 64 successive paths. The initial image is now distributed 10 times through the whole frame without superposition on each other. This frame can be taken as an initial condition for a new series of iterations, even with half a frame. After one supplementary path, gives again a global chaotic pattern. The order/chaos transitions repeate endless, and the pattern comes back to the initial pattern at the 256th path.

When the pattern is formed of separate heaps, we can continue the simulations after fragmentation of the frame in any spot containing 1 heap, without destroy the image given in the initial conditions. We saw that after 1 path, the pattern was chaotic: if we remove the spot where the initial image was given as input, the simulations show that it is impossible to rediscover the initial image by the successive paths.

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AN ELECTROMAGNETIC METHOD OF HEALING

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September 1993

Contents

1.0 The History of An Invention Born of Necessity

This paper reports on the invention of Steven Walpole, an electronics engineer, who turned his attention towards the development of an electromagnetic method of healing when he was told that he would have to "live with" the severe migraines that he developed following a car accident in 1979. Born in 1958 Walpole developed a passion for electronics at the age of ten and a year later began suffering from migraines - typically once or twice a month with each lasting from one to three days. Ten years later the severe migraines which followed his car accident were diagnosed by the Princess Margaret Migraine Clinic, London and classified untreatable. Walpole was not prepared to accept defeat and the last thirteen years have involved the development of several stages of his invention once he found he could cure his own migraines with electromagnetic methods. Even more difficult has been his attempts to market his invention to others suffering from migraines. Indications are now emerging that the method can also be used for cases other than migraine. Walpole has produced an account of his story in a personal publication. "The Development of Electromagnetic Medicine" by Stephen J Walpole available from Energy Medicine Development Ltd, 17 Owen Road, Diss, Norfolk, IP22 3ER, England. The rest of this paper presents a brief study made by the author or Steven Walpole's invention first reported in the 1991 ANPA Newsletter.

2. Principles and Inventions

2.1 Principle

A. Measurement (EEG)

- Measure EEG signal
- Fourier transform and display power spectrum (0.5 to 30 Hz in 0.1 Hz steps).
- look for "missing" frequencies (in each 0.1 Hz interval - "missing" defined as below normal level).

B. Feedback (Magnetic Field Generator)

Inject back into body a magnetic pulse train at the missing frequency using a magnetic field generator, worn on any part of the body.

2.2 Inventions

(i) Mark 1 (1985) - "Cigarette box" shaped generator (Number used 200)
Measurement - 3 electrode EEG system
Feedback - CMOS oscillator feeding a pulse train into "bolt" shaped coil.
Four fixed frequencies 3.3, 4.4, 5.8, 9.7 Hz one of which is selected.

(ii) Mark 2 (1986) - "Disc" shaped generator (Number used 350)
Measurement - 3 electrode EEG system
Feedback - Developed Mark 1 into surface mount technology with a pancake shaped coil underneath.

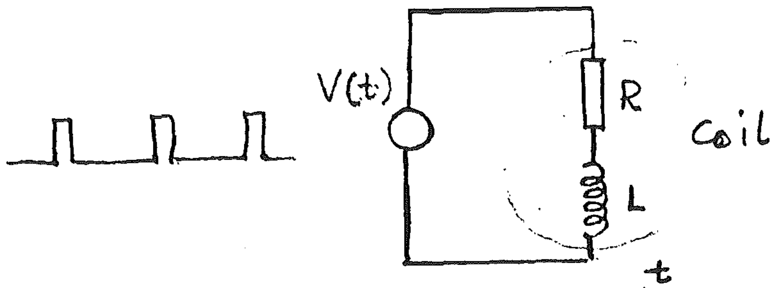
(iii) Mark 3 (1989) - "Delta key" watch generator (number used 450)
Measurement - 3 electrode EEG then 2 electrode (Cz 0z) EEG with DFT processing.
Feedback - Microcomputer controlled digital watch with doughnut coil. One fixed frequency (1.15 Hz) plus one adjustable frequency according to "missing" frequency found in patient.

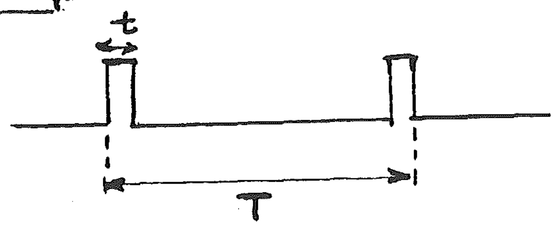
(iv) Mark 4 (1991) - "Novagen" generator (Number used 1500)
Measurement - 2 electrode EEG then later novel crystal/LED based EEG
Feedback - programmable chip for selecting up to <u>four</u> "missing" frequencies and sequentially injecting pulse trains at each frequency (12 seconds duration for each frequency). coil - doughnut shaped. Fixed 1.15Hz replaced by random variation around this frequency - present all the time.

2.3 The Magnetic Field Generators

2.3.1 Pulse Trains

Equivalent coil model





Input voltage pulse train

If Missing frequency, f then T = 1/f

pulse width for Mark 1 and 2 designs t = 0.25ms (fixed)
for Mark 3 and 4 t = 0.125 ms (fixed).

The pulse width was empirically tested for (with patients). For example t=0.125ms was arrived at in say a patient with 10 Hz missing frequency (for which T = 100ms) by trying t = 50 ms (50/50 mark space ratio) and reducing this in steps until the optimum, according to patient response, was found. The lowest value of t tested for was 0.03 ms. The final optimum value (0.125ms) was obtained by testing on a few sensitive people - one of whom was Julie Wenn.

2.3.2 Coils

the number of turns for each coil design was also arrived at by empirical testing.

(i) bolt (Mark 1) - 600 turns of 0.2mm Cu wire (tested range 200 to 1000 turns).
Typical R = 12.6Ω, L = 3.8mH ∴ time constant τ = L/R = 0.3ms

(ii) pancake (Mark 2) and Mark3 2000 turns of 0.0675mm Cu wire (tested range 1000 to 2500).
R=280Ω L≈26mH ∴ τ = 0.1 ms

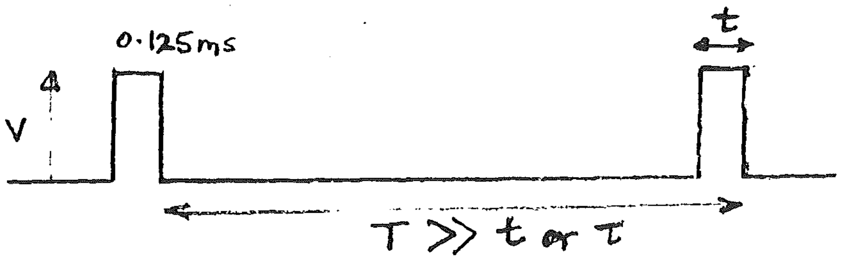
(iii) Doughnut (Mark 4) 300 turns of 0.16mm Cu wire (tested range ?)
typical R = 21.4Ω, L = 8.7mH ∴ τ = 0.4 ms.

2.3.3 Shape of Injected Magnetic field pulses

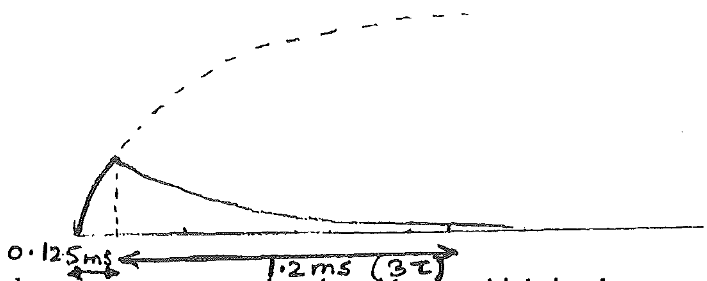
The time constant τ of the coils modifies the current through the coil (a first order equivalent circuit) and since this is responsible for the magnetic field - the magnetic pulses are accordingly shaped.

The highest clinically significant missing frequencies are thought to be of the order of 15Hz ($\therefore$ T $\simeq$ 60 ms) and since the coil time constants are typically less than 1 ms the magnetic field will die away between pulses. So typically we will have (taking τ = 0.4 ms and t = 0.125ms).

driving voltage



current (and therefore mag field) through coil for single pulse.



It should be noted that the pulse length turns out to be about 1 ms which is close to a typical neuron depolarisation pulse time.

2.3.4 Value of the injected magnetic field

This is calculated for the present design (Mark 4 - NOVAGEN). The device is fed from a 3 volt battery and the coil has a resistance of about 20 Ω so the maximum current is 3/20 A = 0.15A. Because the time constant $\tau << t$ the actual peak current will be (with τ = 0.4ms).

$$I = I_0(1 - e^{-t/\tau})$$

$$= 0.15\,(1 - e^{-0.125/0.4})$$

$$= 0.04\text{A}$$

A simulation of the NOVAGEN coil was done using a Finite element (FE) package. Assuming that the NOVAGEN is placed at a distance of 1 cm from a body (to allow for thickness of clothing etc) the field pattern generated is displayed in the attached figure 1 (assumed 1 A through coil). The value of the field is fairly constant under the plane of the device (figure 2) and will have a value of

$$\frac{0.04}{1} x\ 0.005T$$

$$= 20\ x\ 10^{-5}\ T$$

$$= 200\mu T$$

Given that 1G (gauss) = 100 μT this is 2G or 4 times the earth's field strength (0.5G).

2.3.5 Conclusions on Magnetic Field Generators

Similar simulations performed with the Mark 1 invention, which had a 9V battery driving it, shows that the field strength magnitude is larger in that device by about a factor of 3. Interestingly the empirically derived coil designs each have approximately the same time constant and this to a large extent determines the shape of the magnetic pulse, so the tentative conclusion must be that the injected field strength is not as important as the shape of the pulse injected into the body. Steven Walpole claims that the field generators are effective as long as they are within about 2cm of the body. At 2cm from the body the field generated by the NOVAGEN will be about 1G, so we may tentatively conclude that the pulsed magnetic fields must be around (greater than?) the earth's field strength.

3.0 CONCLUSIONS

The conclusion that seems to be emerging:- namely that Steven Walpoles invention provides a pulsed magnetic field around the earth's field in strength and a shape that is close to an individual neuron pulse is interesting. A recent letter in LANCET* (attached here) indicates that there is a human sensitivity to weak magnetic fields. This work shows that the EEG power spectrum in 50% of healthy volunteers can be altered by imposed 60 Hz CW magnetic field activity (applied for 2s on and 8s off). The strength of the 60Hz applied field which seems to alter the EEG is around the earths field strengths in magnitude. The letter proposes that a single neuron or a perineural cell is involved in a "Transduction mechanism" that is not within conscious perception.

There has not been time, in this study, to explore, adequately, Steven Walpole's crystal/LED headset. That is left for the future.

Finally it should be noted that the method has been applied to, more than 2000 patients with a current success rate of over 50%, some patients have responded spectacularly.

(*) THE LANCET VOL. 338 DEC. 14 1991 pp. 1521-1522

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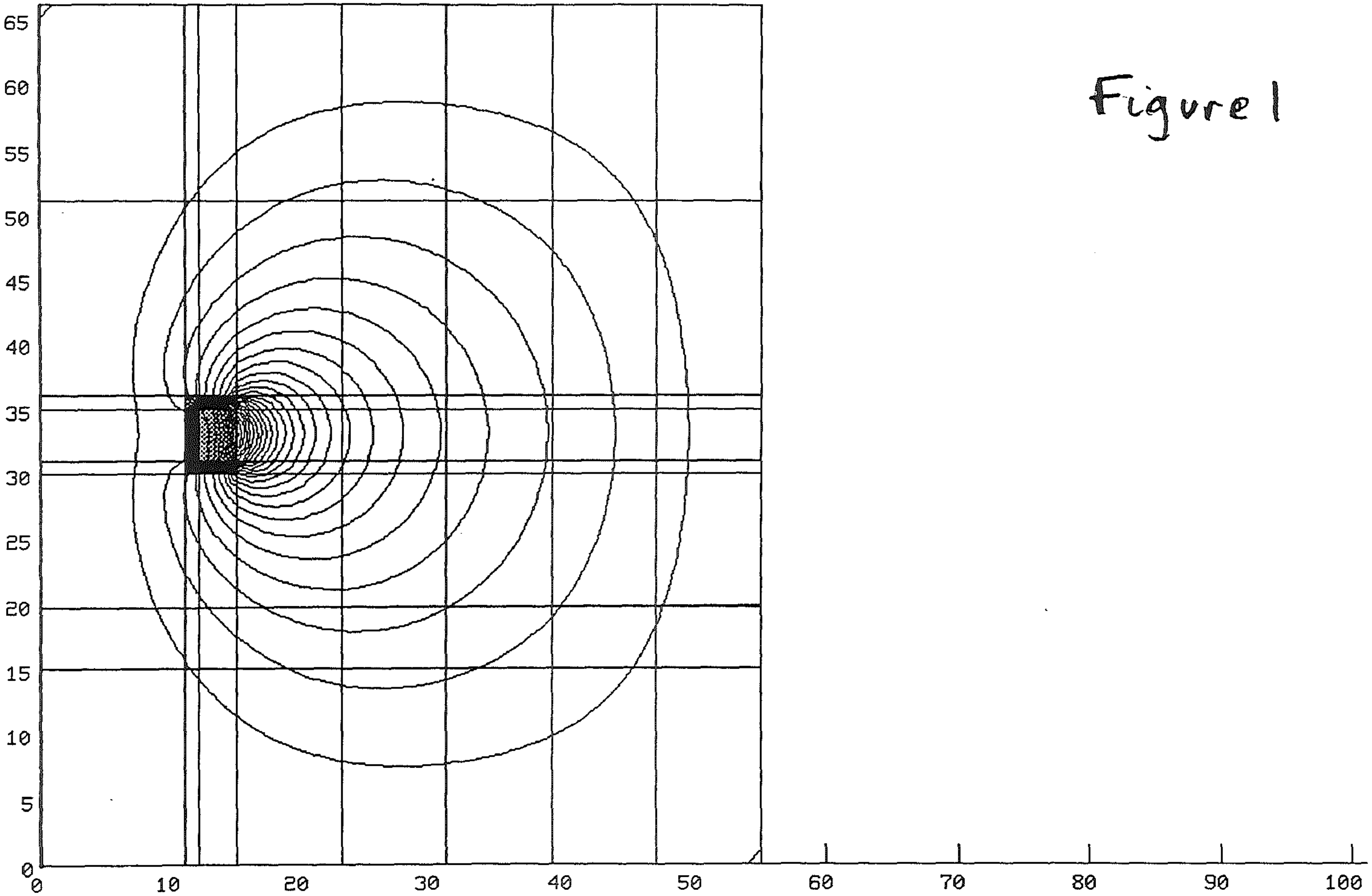
Figure 1



ELEM=LINE SYMM=AXI SOLN=RA FIEL=MAGN
Static Solution Mesh 7260 Elements 56 Regions

PE2D > COIL1 MUR=2000 POT CONTOURS N=20 | Thu Aug 27 16:35:08 1992

shelltool - /bin/csh

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iesun6% at now +1 minute
iesun6% screendump -e | psraster -i>x11
at> <EOT>
job 2851 at Thu Aug 27 16:33:00 1992
iesun6%
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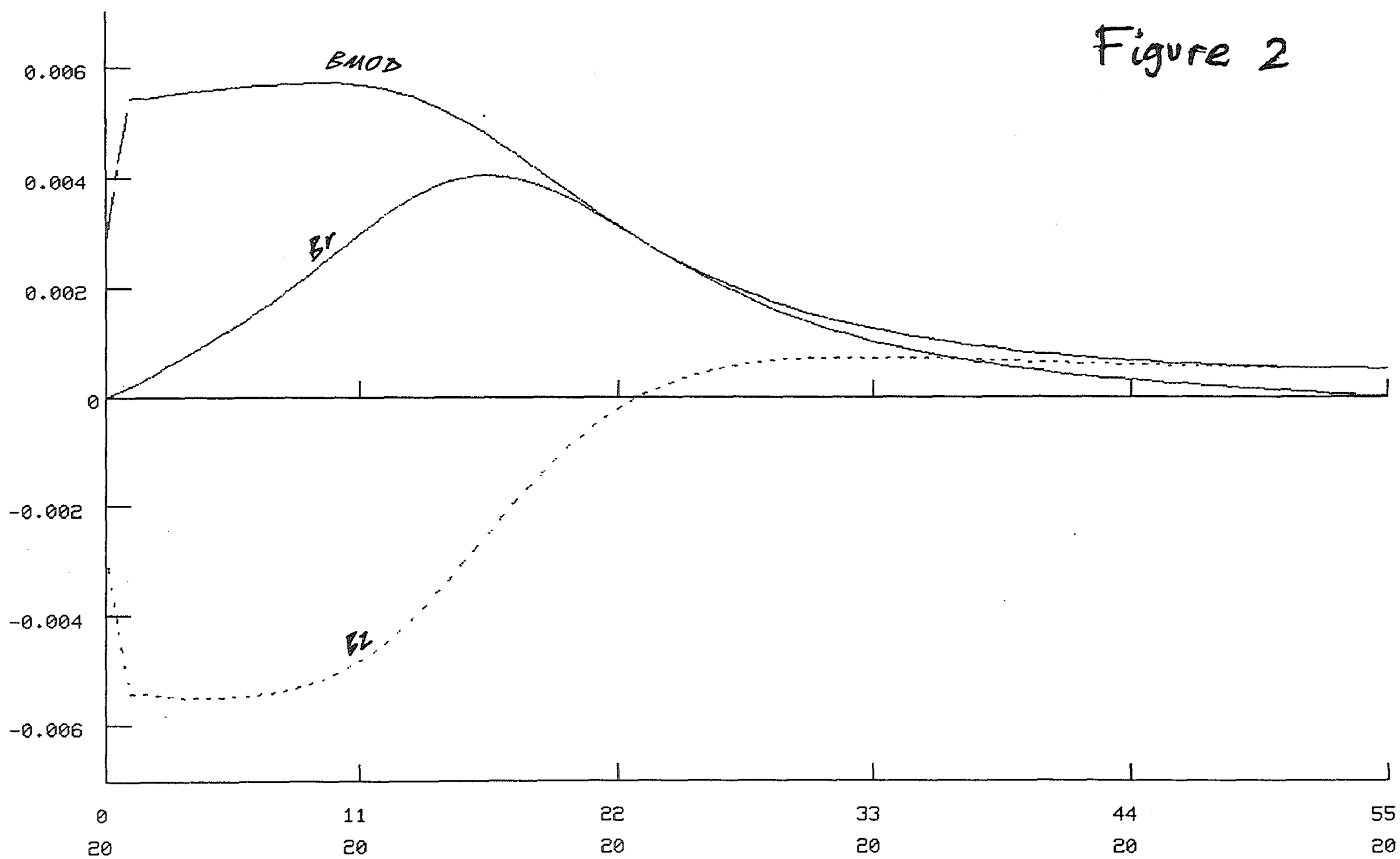
27/Aug/92 17:15:25 Page 1 : LINE X1=0,X2=55,Y1=20,Y2=20,AUTO=NO,VMIN=-0.007,VMAX=0.007,NP=100,COMP=BR,LTYP=3

Figure 2



Values of BR

Values of BZ

Values of BMOD

shelltool - /bin/csh

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iesun6% at now +1 minute
at> screendump x
at> <EOT>
job 2851 at Thu Aug 27 17:19:00 1992
iesun6%
iesun6% screendump -e | psraster -i>x22
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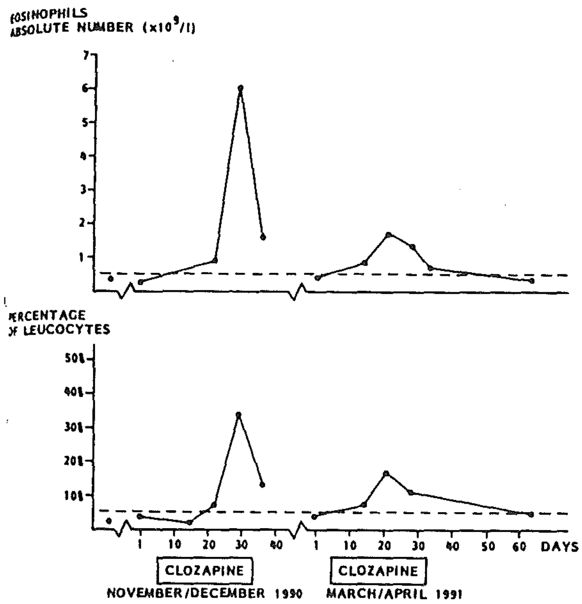
PE2D > COIL1 MUR=2000 B=f(r) z=20 mm | Thu Aug 27 17:20:00 1992



Eosinophil counts and percentage of leucocytes in patient with schizophrenia before and during challenge and rechallenge with clozapine.

osinophilia were noted. After a further increase in the following week to 18·1 × 10^9/l leucocytes, of which 34% were eosinophils, clozapine was discontinued and laboratory values rapidly became normal. The patient did not have any symptoms during this period. Other drugs were continued. The patient did not have collagen disease, leukaemia, Hodgkin's disease, allergic illnesses, or symptoms suggestive of hypereosinophilic syndromes, such as Loeffler's syndrome or endocarditis. Parasitic infections, however, were not excluded. In February, 1990, a severe relapse of refractory schizophrenia necessitated reintroduction of clozapine. Four days before starting treatment with clozapine 200 mg daily, leucocytes and differential count were normal (figure), but after ten days leucocytes and eosinophils were again abnormal and remained raised during the subsequent eleven days of treatment. Clozapine was then discontinued and all abnormalities rapidly disappeared. Again there were no symptoms. At this time parasitic infections were excluded.

A causal relation between leucocytosis and eosinophilia and the use of clozapine is highly probable in this patient. There was a clear temporal relation between adverse effects and course (challenge, "dechallenge", rechallenge), and other causes were ruled out or regarded as unlikely in view of the course. Although blood eosinophilia has been noted in some studies,[3,4] we are not aware of any published case reports. Moreover, these effects are not mentioned in most data sheets (although blood count monitoring is obligatory) even though eosinophilia occurs in 5–10% of those taking clozapine.[3,4] Medical practitioners prescribing clozapine should be aware of this effect, especially since it may accompany other white blood cell disorders.

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Fluoxetine doses

SIR,—It would be unfortunate if the conclusions drawn by Dr Baldwin (Sept 28, p 828) were to dissuade clinicians from increasing the dose of fluoxetine in patients not responding at the recommended "standard" dose of 20 mg. Whilst his comments are entirely reasonable and probably valid for most patients with depression treated in general practice or outpatient clinics, patients with more severe depression encountered in specialist units may respond selectively at a higher dose.

Whilst it is true that dose escalation from 20 to 60 mg in patients not responding at 20 mg did not produce significant further improvement overall,[1] some features of depression such as cognitive disturbance significantly improved with the higher dose. There is a built-in handicap when assessing changes in Hamilton rating scale for depression data in patients treated with fluoxetine. Appetite and weight changes contribute to the total score, patients with a diminished appetite and weight loss due to fluoxetine (more likely at higher doses) appearing more depressed in terms of the total. This makes interpretation of this study difficult. Had the comparison been between 20 and 40 mg in non-responders, the results might have been different. A fixed dose study[2] showed some evidence of a curvilinear response to fluoxetine with 53% responding at 20 mg, 61% at 40 mg, and 48% at 60 mg, all significantly superior to placebo (27%). Several studies[3,4] suggest the need, in some patients with more severe depression, for a dose of 40 mg to achieve remission and to prevent early relapse.

Whilst such higher doses might be accompanied by an increase in the rate of serotonin-related side-effects and hence compromise compliance in an outpatient setting, they should remain part of a treatment plan for inpatient units.

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Human sensitivity to weak magnetic fields

SIR,—Exposure to electromagnetic fields (EMF) may increase the risk for cancer.[1-3] This implies the existence of a mechanism for detecting the EMF and transducing it into a biological signal. We have been testing the hypothesis that the EMF is a stressor which causes signals in the central nervous system that subserve detection and response.[4] We have found evidence for these signals in rabbits[5] and describe here our first study in man.

After they had given their informed consent fourteen healthy volunteers were exposed to magnetic fields produced by Helmholtz coils.[6] The field strengths were similar to those produced near household electrical appliances. The saggital plane was perpendicular to the coil axis; the head and upper chest were within a field region that was uniform to within 5% of its nominal value. The average background 60 Hz magnetic field was less than 0·1 mG. All measuring equipment was located remotely from the room that contained the coils and the volunteer.

We measured P(f)—the power in the electroencephalogram (EEG) in μV^2 at frequency f in Hz averaged over 2 s as determined by Fourier transformation of the EEG voltage signal—with the field on and off. The magnetic field was presented for 2 s, with a mean period between stimuli of 8 s (range 5–11 s, varied randomly). The volunteer did not know when the magnetic field was on, and there were no visual or auditory cues. The effect of the EMF was assessed by comparing the EEG recorded during application of the field with that recorded during the 2 s immediately preceding the application. About 60 trials were done, and the first 50 artifact-free ones were

EEG FREQUENCIES AFFECTED BY EXPOSURE TO MAGNETIC FIELDS

Volunteer	Magnetic field (G)	EEG frequency (Hz) affected
1 (31, F)	0·25	None
2 (37, M)	0·25	O—2·5, 17
3 (30, M)	0·25	P—11, 12
4 (23, M)	0·5	O—5·5, 7
5 (22, F)	0·5	C—1·5, 3, 3·5, 10, 10·5, 13
6 (47, M)	0·5	None
7 (23, M)	0·5	C—10·5, 15
8 (18, M)	0·5	None
9 (25, M)	0·5	None
10 (28, M)	0·25	None
11 (30, M)	0·5	P—11, 12
12 (36, M)	0·5	None
13 (35, F)	0·5	None
14 (30, M)	0·25	C—6, 10 12; O—1, 1·5

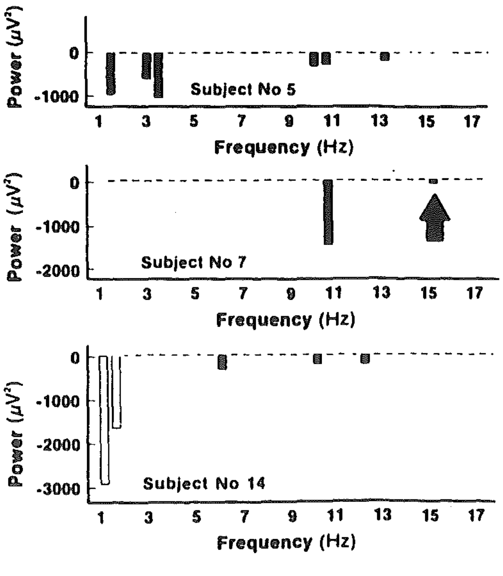
C, P, O are central, parietal, and occipital electrodes, respectively.

used in the subsequent analysis. Sham exposure was used as a control.

The EEG was recorded from the central, parietal, and occipital electrodes (10–20 system) filtered to pass 0·3–35 Hz, and the signal was then divided and simultaneously recorded on an electroencephalograph and sampled at 200 Hz. The coefficients at 1–18·5 Hz in increments of 0·5 Hz were obtained from the Fourier transform and analysed by Wilcoxon signed-rank test.[7] The criterion for concluding that a volunteer had detected the magnetic field was that the field produced at least two bilateral successes (difference between corresponding exposed and control epochs significant at $p < 0·05$) in at least one pair of electrodes, provided those changes were in the same direction. The probability that an effect might be due to chance was $p < 0·02$ (binomial distribution).

Half the volunteers responded to the EMF by significant changes in EEG (table); no significant effects were observed during sham stimulation. P(f) for the frequencies significantly affected in three volunteers is shown in the figure. In all cases, less power was observed during the stimulus epochs compared with the control epochs.

Since the field-on and field-off epochs lasted only 2 s the locus of field transduction was probably in the nervous system—either a neuron or a perineural cell. Classic somatosensory pathways involve spike potentials and conscious perception of the stimulus,



Change in EEG power (P) from 3 volunteers at frequencies significantly affected by the magnetic field.

$P = P_B - P_O$, where P_B and P_O are the mean power values recorded during the field and control epochs, respectively. Data from C (■) or O (□) electrodes.

but information may be added at non-spiking regions of a neuron via voltage-gated channels whose cumulative effect is encoded by a subsequent spike.[8] If the EMF produced an afferent signal consisting of such a modification of spontaneous neural activity, that could explain our observation of altered brain electrical activity in the absence of conscious perception.

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Neopterin and psoriasis

SIR,—Dr de Rie and co-workers (Nov 9, p 1208) question our finding (Sept 21, p 759) that neopterin concentrations correlate with psoriasis area and severity index (PASI). We found such a correlation during follow-up but did not have the no-treatment data described by de Rie et al. However, only one of our patients had had increased serum and urine neopterin levels before treatment with cyclosporin, and there was no close association between pretreatment neopterin levels and PASI, which accord with de Rie's data. During our follow-up, neopterin concentrations rarely exceeded the upper limit of normal but increases and decreases in PASI were paralleled by neopterin concentrations in all patients. We still think that endogenous interferon-γ is sufficient to induce production of neopterin by monocytes in psoriatic lesions. However, the changes in serum and urine neopterin concentrations may be less striking than those seen in other clinical situations like virus infection,[1] simply because the molecule may not sufficiently penetrate from the periphery into the bloodstream.

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New clinical phenotype of branched-chain acyl-CoA oxidation defect

SIR,—Clinical manifestations of multiple acyl-CoA dehydrogenase deficiency due to electron transfer flavoprotein (ETF) or ETF ubiquinone:oxidoreductase deficiency can vary considerably from a neonatal form, with multiple malformations, severe acidosis, hypoglycaemia, and hyperammonaemia, to milder, later-onset phenotypes.[1] Urinary excretion of glutaric acid, ethylmalonic acid, C_6–C_{10} dicarboxylic acids, and several acylglycines is often indicative of an affected patient.[2] In some cases, riboflavine treatment may be effective.[3] We report here three unrelated Italian children presenting with a novel clinical phenotype, characteristic neuroradiological features, abnormal organicaciduria and biochemical evidence for branched-chain acyl-CoA oxidation defect.

The patients presented with neonatal hypotonia followed by severe progressive pyramidal dysfunction with spastic diplegia,

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DISCRETE ANTI-GRAVITY*

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ABSTRACT

Discrete physics, because it replaces time evolution generated by the energy operator with a global bit-string generator (*program universe*) and replaces "fields" with the relativistic Wheeler-Feynman "action at a distance", allows the consistent formulation of the concept of signed gravitational charge for massive particles. The resulting prediction made by this version of the theory is that free anti-particles near the surface of the earth will "fall" up with the same acceleration that the corresponding particles fall down. So far as we can see, no current experimental information is in conflict with this prediction of our theory. The *experiment crusis* will be one of the anti-proton or anti-hydrogen experiments at CERN. Our prediction should be much easier to test than the small effects which those experiments are currently designed to detect or bound.

This is a slightly revised version of the paper presented at
ANPA WEST 7, Cordura Hall, Stanford, Feb. 16-18, 1991

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* Work supported by the Department of Energy, contract DE-AC03-76SF00515.

Although this century has witnessed two basic revolutionary developments in physics, usually referred to as relativity and quantum mechanics, reconciliation between these two radical departures from conventional thinking has been hard to achieve. By now most particle physics theorists believe that non-abelian gauge theories — a very narrow sector of the overlap between quantum mechanics and *special* relativity— remove the infinities that more naive approaches invariably produce. Nevertheless a distinguished physicist and philosopher[1] can still say that he is uninterested in comparing *relativistic* quantum mechanics with experiment because it does not have a rigorous mathematical basis. It is widely believed that the problem of achieving a quantum theory of gravitation, which most physicists would think of as the reconciliation between quantum mechanics and *general* relativity, has yet to be achieved. For example, in his lectures at Schladming in March, 1991 C.J.Isham gave a survey of some of these difficulties and asserted that he finds a basic incompatibility between the foundations of quantum theory and general relativity.

The situation is significantly different with regard to the compatibility between the basic phenomena on which belief in relativistic particle kinematics and relativistic quantum particle mechanics rests. These phenomena can be formalized and reconciled if one is willing to adopt a novel approach to the problem. We model particle physics and physical cosmology by constructing "space", "time", and "particles" using a finite and discrete set of concurrent computer operations rather than trying to embed discrete quantum events in a pre-existing continuum. One can characterize the theory by a modern version of the older materialist slogan, — "Chance, events and the void suffice." This work started back in the 1950's with research by Bastin and Kilmister and in collaboration with Amson, Pask and Parker-Rhodes led them to the discovery of the *combinatorial hierarchy*[2] in 1961. A preliminary connection to particle physics[3] was presented in 1979. However, it was not until 1987 that HPN was prepared to claim[4] that this research program had indeed led to a reconciliation between quantum mechanics and (special) relativity.

Discrete Physics, or "bit-string physics" as it is sometimes called, draws on computer science and constructive mathematics for many of its basic ideas.[5] That there were obvious connections to gravitation has been known since 1961 when Bastin pointed out (Ref. 2) that the last two terms in Parker-Rhodes' 4-level terminating combinatorial hierarchy (3, 10, 137, $2^{127} + 136$) are suggestively close to the dimensionless scale constants $\hbar c/e^2 \approx 137$, $\hbar c/Gm_p^2 \approx 1.7 \times 10^{38}$. The connections were made still closer once the construction of relativistic quantum mechanics and physical cosmology had been sketched out[6] and the theory had been shown to imply the three classical predictions of *general* relativity[7]. More recently it has become possible to discuss the relationship between our approach to the gravitational problem[8] and work by Wheeler[9] based on the entropy of charged, rotating black holes.

Although *discrete physics* has had considerable success in calculating masses and coupling constants that are already known (see *Predictions*, following references), many of which cannot be calculated by conventional theories, so far a numerical prediction prior to experimental measurement has not been possible. We came close to being ready to predict that the width of the Z^0 would limit the number of types of neutrinos to the three used in the current version of the standard model for quarks and leptons. After the SLAC measurements started to come in, we realized that there is only room for three generations at the level of $1/256^4$ coupling, a statement that could not be made within the standard framework.

The encounter between the authors of this paper at PIRT II (the conference where Ref. 8 was presented) has opened up a radically different way in which our approach could be tested. In a paper prepared for PIRT II, shown to HPN but not formally presented, and in private discussions, SS gave strong arguments for the discreteness of physical processes at the Planck length (which is one way of looking at discrete physics), and for abandoning the equivalence principle. The testable conclusion was that, as stated in our abstract, the anti-proton[10,11] and anti-hydrogen[12–15] experiments now being prepared or proposed at CERN should show that anti-protons and anti-hydrogen "fall" up with the same acceleration that protons and hydrogen fall down. This should be much easier to demonstrate than the small departures from conventional theory that these experiments are currently designed to detect or bound. One purpose of this paper is to encourage the experimenters to try a preliminary run before all the refinements needed for high precision are in place.

As we discuss in greater detail in FDP and DP, we view the task of theoretical physics as an application of a general *modeling methodology* in which we start with a rough idea of the phenomena we wish to model (in our instance laboratory physics and observational cosmology *as practiced*), construct a representation which can be given logical and mathematical precision, and then introduce rules by which this quantitative structure can be compared with laboratory phenomena already available or carried out to test the expected consequences. We are prepared to repeat this cycle — or variants of it — many times before we achieve a satisfactory model. In the spirit of Bridgman, we try to make our basic *rules of correspondence* between the mathematical structure and actual laboratory practice as direct and simple as possible.

We start by using the *counter paradigm* (see DP) to connect the SI units of length and time to the corresponding length and time intervals in our bit-string model. Consider two counters a distance L apart which fire sequentially with a time interval T between the two firings. We model these two events by two independently generated[16] *bit-strings*, which when compared by *discrimination* (similar to the XOR operation of computer practice) produce a string with $r + \ell$ symbols, r of them being "1" 's and ℓ of them being "0" 's. Our rule of correspondence is that the distance interval between the counters is given by $L = (r - \ell)(h/mc)$ and the time interval between the firings is $T = (r + \ell)(h/mc^2)$. We are now under the obligation to give separate operational meaning to the symbols c, h and m. Implicit in our paradigm is the assumption that the uncertainties in distance and time measurements are much greater than h/mc and h/mc^2 respectively. This is currently true for *direct* measurements of the type described in the paradigm.

We define "velocity" by $V := \beta c := L/T$. The symbol "c" is referred to as the *limiting velocity*. It represents the empirical generalization that no experiment where proper care was exercised in the elimination of background has given sequential counter firings for which this limit for L/T was exceeded. This experience is now codified in SI units by defining "c" as the *integer* $c := 299\ 792\ 458\ ms^{-1}$.

To obtain h/m, we prepare a beam of particles of constant velocity βc (with $\beta << 1$) incident on a pair of slits a distance d apart and measure the spacing s between the interference maxima in a counter array perpendicular to the beam line from the slits to the array at a distance D behind them. Then $h/m := \beta c(sd/2D)$. The invariance of this number for a given type of particle beam over a large range of velocities and in various geometries summarizes current experience. If we can prepare different types of particle beams with the same velocity incident on the same geometrical arrangement used to measure h/m, we can define and measure mass ratios by $m_1/m_2 := s_2/s_1$. In most of particle physics, it is convenient to use either the proton mass m_p or the electron mass m_e as the reference mass, particularly since there is no empirical evidence that either is unstable. Intercomparison is achieved by an overall fit to all data considered relevant, with the current result[17] $m_p/m_e = 1836.152\ 701(37)$.

Our specification of mass ratios is not a conventional one. An alternative that is available to us is to allow two constant velocity particle beams $[V_1, V_2]$ with relative angle θ to cross each other and scatter into relative angle ϕ with velocities $[V_1', V_2']$. Within experimental uncertainties, initial and final velocities lie in a plane. Further, for any reference direction in that plane such that $\theta_1 - \theta_2 = \theta$ and $\phi_1 - \phi_2 = \phi$, we find that given sufficient and sufficiently precise data we can always determine two masses (relative to some arbitrary, finite reference mass) such that $m_1 V_1 cos\ \theta_1 + m_2 V_2 cos\ \theta_2 = m_1 V_1' cos\ \phi_1 + m_2 V_2' cos\ \phi_2$. All these statements are, of course, subject to appropriate qualifications about experimental uncertainties, and the allowed range of the parameters. They are equivalent to Mach's definition of mass ratios starting from Newton's Third Law. We cannot accept his starting point because it is *scale invariant*, while our fundamental paradigm breaks scale invariance by invoking a unit of length (time) which is h/mc (h/mc^2). We abandon Mach's definition in favor of the definition provided above in terms of particulate quantum interference. We have argued elsewhere that we can derive Mach's specification of mass ratios from our discrete model[18]. This conclusion follows in our model because events involving constant velocity particles can occur only at "points" separated by an integral number of wavelengths $\lambda = h/\beta mc$.

The careful reader will note that our definition of mass ratios imposed the "non-relativistic" restriction $|\beta| << 1$. So far as we know, there are no *interference* experiments that distinguish the non-relativistic deBroglie wavelength $h/\beta cm$ from the relativistic deBroglie wavelength $h/\gamma\beta cm$, with $\gamma^2\beta^2 = \gamma^2 - 1$. David Fryberger, Pat Suppes and HPN are investigating whether current technology might allow this statement to be revised. The modified definition of mass ratios is obvious and immediate. The experimental decision between the relativistic and non-relativistic alternatives could provide an *experiment crusis* separating alternative relativistic and non-relativistic quantum mechanical models.

We have taken care to spell out what we mean by "mass ratios" in our theory because the fusion of the concept of "inertial mass" with "gravitational mass"— the "equivalence principle" — was Einstein's starting point in constructing the general theory of relativity. From his point of view, the interesting part is yet to come. For us, the concept of "mass" stops with what he (and Newton) would call "inertial mass", — mass ratios measured by conservation of momentum in collisions, and in our quantized theory by deBroglie wave interference. We have discussed elsewhere[19] how our bit-string theory can accept the macroscopic "field" concept of classical physics as a continuum approximation to our discrete theory. Here we take a more radical stance by bringing to the fore aspects of discrete physics that suggest a fundamental conceptual break with continuum physics and allow us to abandon both the concept of "energy" and the "equivalence principle" at the same time. Whether or not our prediction proves to be correct, we believe that the issue we raise of the incompatibility between the $\mathcal{CPT}$ invariance of the theory and the equivalence principle deserves careful investigation in any framework that the reader accepts for his own work.

Our discussion of "gravity" follows from our understanding of electromagnetic interaction in our model, and our successful calculation of both the fine-structure spectrum of the hydrogen atom and the *value* of the fine-structure constant[20]. The tentative interpretation of the third combinatorial hierarchy constant $137 \approx \hbar c/e^2$ as the number of events which provide the "background" for each "Coulomb event" that keeps the atom bound is reinforced by our derivation of the *relativistic* Bohr formula[21] from this starting point. Including a second degree of freedom leads to the Sommerfeld formula[22] and a combinatorial correction to the fine structure constant which brings it close to the accepted empirical value. In conventional renormalized QED, the first calculation amounts to calculating the binding energy in the Coulomb gauge, and the second to including the spin-dependent corrections of order 1/137. This suggests that in our theory the particulate states of two particles bound gravitationally will have the Bohr spectrum with coupling constant $\frac{Gm_p^2}{\hbar c}(\frac{m_1 m_2}{m_p^2}) \approx \frac{m_1 m_2}{m_p^2}/1.7 \times 10^{38}$ replacing 1/137. Such particulate bound states have yet to be observed, except for aggregates of matter so large as to overcome the very small coupling constant and to make the quantum levels unobservable. That quantum mechanics nevertheless applies to gravitation was demonstrated by quantum interference effects involving *single* neutrons near the earth. From our point of view, "spin-dependent" corrections can be expected to be smaller by 1.7 parts in 10^{38}, which enormously simplifies our analysis of the anti-proton experiment.

Whether or not the force between two particles is attractive or repulsive is most simply established by whether or not they form a bound state. This was the starting point for the Bohr atom, which assumed that — as was known for macroscopic charged objects — elementary particles of opposite charge would attract each other, and of the same charge repel each other. In scattering states either attractive or repulsive electric forces for particles with positive energy lead (classically) to hyperbolic orbits, the only difference being which focus of the hyperbola the reference particle occupies. This difference is not directly observable at the atomic level. However, the short range nuclear force, which has to be attractive in order for nuclei to form, can interfere with the coulomb force in the scattering of like charges (eg. proton-proton scattering) and hence confirm the assumption that the electric force between these two like charges is indeed repulsive. No known phenomenon would lead us to question the assumption that like electric charges repel and unlike charges attract at the particulate level.

The situation for gravitation differs in that no elementary particle states which are bound gravitationally have been observed, or can be expected to be observed with currently available techniques. Similarly, quantum interference effects between gravitational scattering and known interactions are may orders of magnitude below current detection threshold. That neutrons are attracted by the earth was shown using external reactor beams shortly after World War II, and beautiful cold neutron interference experiments show that this force also has the expected coherent quantum mechanical effects. But to our knowledge there is no *direct* experimental evidence that either anti-neutrons or anti-protons are attracted to rather than repelled by the earth. In this sense the CERN anti-proton and anti-hydrogen gravity experiments offer a unique and clear window through which to look at a basic phenomenon that is otherwise inaccessible.

Since we still lack this experimental information, we next ask what theory would lead us to expect. This is a very complicated question in conventional relativistic quantum field theories because as already noted there is currently no consensus as to how to formulate a theory of "quantum gravity". In contrast, discrete physics already contains the connection between the proton mass, the Planck mass and Newton's gravitational constant $[\hbar c/Gm_p^2 = (M_{Planck}/m_p)^2 \approx 1.7 \times 10^{38}]$ as a *prediction* of the theory. Further, we have argued above that for particulate experiments only the Newtonian term will be significant. Few physicists would argue with the proposition that like electric charges attract, unlike electric charges repel, and that either two particles or two antiparticles would attract each other gravitationally. What we need is a theoretical argument at to whether a particle would either attract or repel an anti-particle gravitationally. To make the argument, we must first explain how the Coulomb attraction and repulsion arise in discrete physics.

As we have already noted, the hyperbolic scattering trajectories (Rutherford scattering) produced by Coulomb attraction and repulsion are not distinguishable in quantum scattering experiments without further information. However, for a particle-antiparticle pair, there is an additional contribution to the scattering in a *relativistic* quantum theory due to the pair coalescing to make an "off energy shell" or "virtual" photon. This Bhabha term interferes with the Rutherford scattering and is readily observed at high energy, confirming directly the attractive force between particle and antiparticle. However, when particle is changed to antiparticle ("crossing") this term becomes simply one of the two "coulomb exchange terms" which appear in the (repulsive) Coulomb scattering between two identical particles. That this virtual photon is still characterized by zero "rest mass" is the starting point for the "renormalization group equation", — a subject we will approach from the discrete physics point of view in subsequent research. Discrete physics contains all these standard results.

The "crossing symmetry" which is invoked here comes from the $\mathcal{CPT}$ invariance of the theory. In our bit-string model the choice between which of the two dichotomous symbols in the bit-string we call "0" and which "1" is simply a choice between one representation of the combinatorial hierarchy and a distinct dual representation. This property in our context is called Amson invariance, and is discussed on pp 7-10 in a recent technical note[23]. Briefly, we have to interpret our model in such a way that when we interchange "0" 's and "1" 's in a string (the "bar" operation) the fixed ("label") part of the string that contains discrete quantum numbers such as charge has to reverse their sign as well as reversing velocities and reflecting spacial coordinates. Since, other than magnitude and the distinction that like particles attract each other rather than repel, the gravitational interaction in the Newtonian approximation is indistinguishable from the electromagnetic interaction in the Coulomb approximation, we interpret "crossing" or $\mathcal{CPT}$ invariance to require a particle and an anti-particle to *repel* each other gravitationally. *This is our* *prediction.* What remains for us to do is to show that if we accept this prediction, there are no currently observable consequences other than the dramatic prediction of what we expect in the CERN anti-proton and anti-hydrogen experiments.

The easiest question to dispose of is whether or not we expect we expect gravitation to break $\mathcal{CPT}$ invariance in a way that can be observed using current technology. The answer is that it does, at least globally, if we accept the conventional interpretation of cosmological data as showing that the matter of the universe consists primarily of protons, nuclei and electrons rather than anti-protons, anti-nuclei and positrons, and that there are around 2×10^{10} photons per baryon. This small trace of matter is well predicted, to a first approximation, by our theory, as we argue in Ref. 7. The point that is less clear is whether our model for the approximately 12.7 times as prevalent "dark matter" as composed of gravitons, photons, neutrinos and anti-neutrinos gravitationally bound will indeed act gravitationally as matter rather than anti-matter, as it must if it is to explain the observed linear radial doppler shift dependence of the light from galaxies. If neutrinos and anti-neutrinos repel, as we are required to assume for consistency, then our "dark matter" will contain the same trace of matter relative to photons that we have already estimated for electrons and nucleons. So far, this does not seem to cause us any difficulty.

The conventional treatment of gravitation in special relativity starts from the mass-energy equivalence $E = mc^2$ and treats this energy, whatever its cause, as a source of gravitational field. This gives the red shift of light emitted by the sun correctly, but fails by a factor of 2 to explain the displacement of stellar positions near the sun observed during a solar eclipse, and fails by a factor of 6 to explain the observed precession of the perihelion of Mercury. As we argue in Ref. 7, all that is needed to explain these two effects is the spin 1 character of traveling photons and the spin 2 character of traveling gravitons. The full paraphernalia of the Einstein theory is, from our point of view, overkill and should — if possible — be dispensed with by invoking Occam's Razor. The problem we face is whether electromagnetic and gravitational radiation are attracted by matter in our theory.

Since massless radiation cannot carry gravitational charge, we would expect both types of radiation to be attracted by either matter or anti-matter in the same way. Then our explanation of the classical tests of general relativity *and* explanation of "dark matter" as consisting of stable "quantum geons" would still survive. Lacking a full theory of quantum gravity formulated along the line we propose for deriving the classical Maxwell theory as a continuum approximation, we cannot be sure. But we can make a few qualitative arguments.

The basic difficulty in comparing our theory to a theory of gravitation where energy rather than matter is the "source" of the gravitational "field" is that field energy does not appear in our theory. Massive particles have energy and momentum connected in the usual way. They have velocities in the reference frame at rest with respect to the $2.7^o K$ cosmic background radiation which can be modeled by rational fractions lying between -1 and +1, specifically by the difference between the number of "0" 's and "1" 's in a bit string divided by the sum of those numbers. But "massless quanta", apart from their helicity quantum numbers, are modeled simply by the null or the anti-null string and contain no possibility of defining their "energy", "momentum" or "wavelength" other than by context. To indirectly infer these "classical" parameters we *must* model both the "source" and the "sink" of the radiation by the change in velocity of at least two massive, charged particles. In other words we have no choice but to adopt the Wheeler-Feynman "action at a distance" point of view. It is still possible to discuss "photon-photon scattering". This process is studied at SLAC in precisely the way that this description requires, that is by measuring the change in velocity of the source and target particles and any additional charged particles emitted in the process. The photon-photon process itself does not depend on whether the charges and currents which emit and absorb the "photons" are positive or negative. If the treatment of gravitational radiation can be carried through along the same lines, which appears to be possible — but difficult— then the photon-graviton interaction will be independent of whether the source of the gravitational radiation is particles or antiparticles, and all our earlier results will survive.

We conclude that it is possible, and perhaps even likely that the bit-string model of discrete physics can indeed be shown to *predict* that anti-protons will "fall" up near the surface of the earth with the same acceleration that protons and hydrogen fall down.

That the correct starting point for a theory of anti-gravity is the denial of the equivalence principle was originally suggested by SS. That discrete physics might provide a convenient theoretical framework for such a theory was suggested by HPN. We are indebted to M.C.Duffy for bringing us together at the second conference on *Physical Interpretations of Relativity Theory*, and for the stimulating intellectual environment provided by individuals at that conference willing to question established scientific dogma in a systematic way.

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Predictions made by Discrete Physics: March, 1991

For background see papers by H.P.Noyes and D.O.McGoveran: "An Essay on Discrete Physics", *Physics Essays*, **2**, 17-100 (1989) and SLAC-PUB-4528; "Foundations for a Discrete Physics", SLAC-PUB-4526; and "Discrete Gravity", *Physical Interpretations of Relativity Theory, II*, M.C.Duffy, ed., Imperial College, London, 1990, pp 196-201 and SLAC-PUB-5218.

EMPIRICAL INPUT

$c, \hbar$ and m_p as understood in the "Review of Particle Properties", Particle Data Group, *Physics Letters*, **B 239**, 12 April 1990. Numbers are quoted in the format [( )] = empirical value (error) or range.

[a][$G^2_{\pi N}$ = 13.3(3) from R.A.Arndt *et.al.*, . *Phys. Rev. Lett.*, **65**, 157 (1990). F.Sammarruca and R.Machleit (*BAPS*, **36**, No. 4 (1991)) note most modern models for the nuclear force use the strong empirical ρ coupling and therefore require $G^2_{\pi N} > 13.9$; the smaller vector-meson-dominance-model value for ρ is compatible with the Arndt value.]

COUPLING CONSTANTS

Coupling Constant	Calculated	Observed
$G^{-1}\frac{\hbar c}{m_p^2}$	$[2^{127}+136]\times[1-\frac{1}{3\cdot 7\cdot 10}] = 1.693\ 37\ldots\times 10^{38}$	$[1.69358(21)\times 10^{38}]$
$G_F m_p^2/\hbar c$	$[256^2\sqrt{2}]^{-1}\times[1-\frac{1}{3\cdot 7}] = 1.02\ 758\ldots\times 10^{-5}$	$[1.02\ 682(2)\times 10^{-5}]$
$sin^2\theta_{Weak}$	$0.25[1-\frac{1}{3\cdot 7}]^2 = 0.2267\ldots$	[0.2259(46)]
$\alpha^{-1}(m_e)$	$137\times[1-\frac{1}{30\times 127}]^{-1} = 137.0359\ 674\ldots$	[137.0359 895(61)]
$\alpha_s(m_\pi^2)$	$\frac{1}{7} = \frac{m_\pi}{m_N}$	[? ?]
$G^2_{\pi N}$	$[(\frac{2M_N}{m_\pi})^2 - 1]^{\frac{1}{2}} = [195]^{\frac{1}{2}} = 13.96..$	[a]$[13, 3(3), > 13.9?]$

MASS RATIOS

Mass ratio	Calculated	Observed
$\left[\frac{M_{Planck}}{m_{proton}}\right]^2 = \frac{\hbar c}{Gm_p^2}$	$[2^{127}+136] = 1.70147\times 10^{38}$	Proton mass is gravitationally generated
m_p/m_e	$\frac{137\pi}{\frac{3}{14}\left(1+\frac{2}{7}+\frac{4}{49}\right)\frac{4}{5}} = 1836.15\ 1497\ldots$	[1836.15 2701(37)]
$m_\pi^\pm/m_e$	$275[1-\frac{2}{2\cdot 3\cdot 7\cdot 7}] = 273.12\ 92\ldots$	[273.12 67(4)]
m_{π^0}/m_e	$274[1-\frac{3}{2\cdot 3\cdot 7\cdot 2}] = 264.2\ 143\ldots$	[264.1 373(6)]
m_μ/m_e	$3\cdot 7\cdot 10 = 210$	[206.768 26(13)]

General structural results

- 3+1 asymptotic space-time
- combinatorial free particle Dirac wave functions
- supraluminal synchronization and correlation *without* supraluminal signaling
- discrete Lorentz transformations for event-based coordinates
- relativistic Bohr-Sommerfeld quantization
- non-commutativity between position and velocity
- conservation laws for Yukawa vertices and 4- events
- crossing symmetry, CPT, spin and statistics
- Fields replaced Wheeler-Feynman ' 'action at a distance"

Gravitation and Cosmology

- consistent formulation of gravitational charge
- electromagnetic and gravitational unification
- the three traditional tests of general relativity
- event horizon
- zero-velocity frame for the cosmic background radiation
- mass of the visible universe: $(2^{127})^2 m_p = 4.84 \times 10^{52}\ gm$
- fireball time: $(2^{127})\hbar/m_p c^2 = 3.5$ million years
- critical density: of $\Omega_{Vis} = \rho/\rho_c = 0.01175$ $[0.005 \leq \Omega_{Vis} \leq 0.02]$
- dark matter = 12.7 times visible matter [10??]
- baryons per photon $= 1/256^4 = 2.328\ldots \times 10^{-10}$ $[2 \times 10^{-10}?]$

Unified theory of elementary particles

- quantum numbers of the standard model for quarks and leptons with confined quarks and exactly 3 weakly coupled generations
- gravitation: $\hbar c/Gm_p^2 = [2^{127} + 136] \times [1 - \frac{1}{3\cdot 7\cdot 10}] =$
  $1.70147\ldots[1 - \frac{1}{3\cdot 7\cdot 10}] \times 10^{38}$ $=1.693\ 37\ldots \times 10^{38}$ $[1.693\ 58(21) \times 10^{38}]$
- weak-electromagnetic unification:
  $G_F m_p^2/\hbar c = (1 - \frac{1}{3\cdot 7})/256^2\sqrt{2} = 1.02\ 758\ldots \times 10^{-5}$ $[1.02\ 684(2) \times 10^{-5}]$;
  $sin^2\theta_{Weak} = 0.25(1 - \frac{1}{3\cdot 7})^2 = 0.2267\ldots$ $[0.2259(46)]$
  $M_W^2 = \pi\alpha/\sqrt{2}G_F sin^2\theta_W = (37.3\ Gev/c^2 sin\ \theta_W)^2$; $M_Z cos\ \theta_W = M_W$
- the hydrogen atom: $(E/\mu c^2)^2[1 + (1/137N_B)^2] = 1$
- the Sommerfeld formula: $(E/\mu c^2)^2[1 + a^2/(n + \sqrt{j^2 - a^2})^2] = 1$
- the fine structure constant: $\frac{1}{\alpha} = \frac{137}{1 - \frac{1}{30\times 127}} = 137.0359\ 674\ldots$ $[137.0359\ 895(61)]$
- $m_p/m_e = \frac{137\pi}{\frac{3}{14}\left(1+\frac{2}{7}+\frac{4}{49}\right)\frac{4}{5}} = 1836.15\ 1497\ldots$ $[1836.15\ 2701(37)]$
- $m_\pi^\pm/m_e = 275[1 - \frac{2}{2\cdot 3\cdot 7\cdot 7}]$ $=273.1292\ldots$ $[273.12\ 67(4)]$
- $m_{\pi^0}/m_e = 274[1 - \frac{3}{2\cdot 3\cdot 7\cdot 2}] = 264.2\ 1428..$ $[264.1\ 373(6)]$
- $\alpha_s(m_\pi^2) = \frac{1}{7}$
- $(G_{\pi N}^2 m_{\pi^0})^2 = (2m_p)^2 - m_{\pi^0}^2 = (13.868..\ m_{\pi^0})^2$