Process and the Combinatorial Hierarchy

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Faruq Abdullah
26 August 1992

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COMBINATORIAL CALCULATIONS

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Historically I should begin, as in previous years, by saying something about the start of ANPA and about the Combinatorial Hierarchy which played an important part in its early development. This arose from a particular algebraic construction formulated by Frederick Parker-Rhodes in an attempt to describe what Ted Bastin was trying to do on an analogue computer. This construction was, as we now see, over the number field with two elements, 0 and 1, and a most important part was played in it by the addition operation in that field, +, in which $0 + 0 = 0$, $0 + 1 = 1 = 1 + 0$, $1 + 1 = 0$. This operation gives 1 or 0 according as the two elements are different or the same, and so became known in ANPA as DISCRIMINATION. The construction was in terms of vectors over the field, or as we would now say, bit-strings, and at the simplest level there were two linearly independent such vectors: $\begin{pmatrix}0\\1\end{pmatrix}$, $\begin{pmatrix}1\\0\end{pmatrix}$. Their sum, that is, the result of discriminating them, $\begin{pmatrix}1\\1\end{pmatrix}$ could be taken instead of one of them.

The next idea to enter was that of the discriminately closed subset, or dcss, that is, a set of vectors closed under the operation of discriminating any two DIFFERENT members. At the simplest level there were just three such dcss; two of these were sets with one member, one of the two quoted and the third contained both of them and therefore their sum as well. The basic notion in Frederick's construction was to replace a dcss by a linear operator (square matrix) having just the dcss as its set of eigenvectors. Then you can easily work out that at the simplest level the replacement was unambiguous. The three matrices had to be:

$\begin{bmatrix}1 & 0\\1 & 1\end{bmatrix}$, $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$, $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$. Now once this construction had been carried out these three square matrices can be coded as a new set of bit-strings, only now of length 4 and since they are linearly independent they give rise to 7 dcss for which some 4 x 4 matrices can be sought. At this next stage there is more arbitrariness because several matrices can have the same set of proper eigenvectors (eigenvectors whose eigenvalues are 1 and not 0). Frederick said, <u>choose</u> them to be linearly independent; this can be done so that, at the next stage, one has $2^7 - 1 = 127$ dcss and one has to find 127 16 x 16 matrices with the appropriate eigenvectors and themselves a linearly independent set. That needs some proof; but it can be done and so one reaches the last stage of $2^{127} -1 = 10^{38}$ (approx) dcss characterised by operators which are 256 x 256 matrices and so cannot possibly be linearly independent. After that the construction is forced to terminate. The cumulative totals of the number of operators 3, 10, 137, 10^{38}, terminating after the fourth, made it seem as if the construction must be something of importance in physics, and to some extent my work since then has just been understanding what this importance is.

This has been a slow business and as we meet each year to report progress there is a danger that the construction will become just an icon. To act against this danger, let me make a change this year by illustrating the fact that there are many unanswered mathematical (combinatorial) problems in the construction by showing you one. To do this more quickly, let us define a new notation, one which is actually a bonus from my foundational studies, but is here simply introduced as a convenient way of writing. It is to write for the vectors $\begin{bmatrix}1\\0\\0\end{bmatrix}$, $\begin{bmatrix}0\\1\\0\end{bmatrix}$, $\begin{bmatrix}0\\0\\1\end{bmatrix}$ simply 1, 2, 3 and so on and then $\begin{bmatrix}1\\1\\0\end{bmatrix}$ can be written as 1+2 and this in turn can be abbreviated to 12 in a calculation in which no numbers greater than 9 enter, (of course, with the ambiguity that 2 also denotes $\begin{pmatrix}0\\1\end{pmatrix}$ or $\begin{pmatrix}0\\1\\0\\0\end{pmatrix}$. Then a matrix operator can be written by writing its columns as vectors in a bracket separated by commas, and the whole of the first stage in the construction described above can be written as: 1, 2 generate the dcss [1], [2], [1, 2, 12] which are characterised by the operators (1, 12), (12, 2), (1, 2). (The value of the notation can be seen from the fact that (a,b)1 = a, (a, b)2 = b, (a, b)12 = ab and so on.) Now I can state the problem. There was no choice at the first stage but at the next level there was a choice and I said that Frederick's injunction was to choose the operators so that they were linearly independent. (You could tell when you had done this because they would then generate 127 dcss and not fewer.) If you actually work away to construct this level of the hierarchy, which you do by finding a set of operators and then checking that they are linearly independent, you usually find that they are. (I have always found this, in fact; but Pierre Noyes has not always been so lucky.) Why is this? One must expect the answer to be that of all possible choices, the linearly independent ones predominate; so that the answer is to calculate the probability that such an eventuality will ensue.

At this next level, rename the three basis vectors 1, 2,3. (If you are obsessed by bit-strings, vector-spaces, matrices, then you will say "take a new basis" but it is the same thing.) Then look at the 7 dcss and the possible forms of their operators:

Dcss	Operator
1	(1,a,b)
2	(c,2,d)
3	(e,f,3)
1,2,12	(1,2,g)
2,3,23	(h,2,3)

1,3,13	(1,i,3)
ALL	(1,2,3).

There are restrictions on the letters entering. Taking the first one, it is easy to see that a cannot be 1or2, b cannot be 1 or 3 and a+b cannot be 1 or 23. In all there are only 14 possible values for this matrix, and similarly for the other two of the first three. In much the same way, g cannot be 1,2,3 or 12 and so can have only one of three values. In all then, there are $2^3.3^3.7^3$ = 74088 sets of operators at this level, not a unique set as before. The question is then, how many of these sets give 127 dcss at the next stage?

One way of tackling this would be simply to compute, but it would be better to get another solution because the computation relies on the smallness of 74088 at this level, and the interest is really at the next stage where the numbers are bigger.

As soon as one starts trying to do this calculation one sees the relative uselessness of the linear algebra-vector space approach. It is not difficult to see the reason; most arguments there will depend on a determinant being non-zero, and when it is zero, a separate argument deals with the case. But at the level of 2 x 2 matrices how many are non-singular? Of the 16, only 6. It is true that the situation improves as the matrices get bigger, but more than than half are singular. So one has to tackle the problem differently.

Notice first that for any of the allowed values of g,h,i the four operators (1,2,g), (h,2,3), (1,i,3), (1,2,3) are linearly independent so that there will always be at least $2^4 - 1 = 15$ dcss. Is it possible that there should be exactly 15? In that case the four operators quoted must generate the other three operators having only one of the 1,2,3 as eigenvector (call these "singlet" operators). So one has to see what the four generate by discrimination and making a list soon shows that the only possible candidates for singlets are (1,i,g), (h,2,g) and (h,i,3). These will be the three singlet operators only if i, g satisfy also the constraints on a, b, h, g those on c, d and h, i those on e, f. Putting these conditions in easily gives just two possibilities: g,h,i = 13,12,23 or g,h,i = 23,13,12. Next one can determine the condition for producing two singlets in much the same way. The answer turns out to be that there are 9 ways of doing this. Similarly there are 12 ways of getting a single singlet and so 27 - 23 = 4 ways in which no singlet will be produced by discrimination. If no singlets are produced, then 127 dcss will certainly result. This will then happen in 4.14^3 ways. Again, if one singlet is produced this may be one of those listed (a HIT, say) and then only 63 dcss will result, but it may not (a MISS) in 13 out of the 14 ways and then 127 will result again. Here 127 is produced in $12.13.14^2$ ways. Continuing in this way one easily gets the results:

Number of dcss	Number of ways
127	67240
63	6642
31	204
15	2

So the probability of getting 127 dcss at the next level, so that the hierarchy construction can continue, is 67240/74088 = 0.9076.. It looks as if I was no more than reasonably lucky and Pierre was a bit unlucky.

It was not intended to exhibit an example of great physical importance but since the hierarchy is itself physically significant there must be some significance in any such example. One consequence of these calculations is the proof that the chance of any one element at the first three levels is no longer 1/137, as is often stated, but something less. For when there are only 63 dcss produced it will not always be the case that the element sought is actually one of these. The probability that it is there is, of course, 63/127. But to make up for that decrease, if it <u>is</u> present then the "equal probability" assumption says that the chance is not 1/137 but 1/(10 + 63) = 1/73. Similarly for the other cases of 31 and 15 dcss, so that the revised value for the probability of any one element is:

$$(1/137)[67240 + 6642X + 204Y + 2Z]/74088,$$

where

$$X = 63.137/(127.73) = 0.93096...$$

$$Y = 31.137/(127.41) = 0.81563...$$

$$Z = 15.137/(127.25) = 0.64724...$$

This probability turns out to be 1/(137.925..). Evidently any hope that the revised calculation would turn out to give the observed value of the fine-structure constant (1/(137.036..)) is extinguished. But two remarks may be in order. Firstly, this calculation opens the way for the theory to come up with probabilities which differ from the ratios of well-known whole numbers. Secondly, it shifts the problem of finding the true value of the fine-structure constant from finding the very small decrease of the probability from 1/137 to that of finding a much larger increase from the new value.

GRAMMARITHMS -
a dynamic number system

In the following paper I propose a NON-LINEAR number system which is not dependent on the INTEGER NUMBERS, nor the notion of VANISHING QUANTITIES, for its definition; the CONTINUUM arises spontaneously as a consequence of comparisons between GEOMETRIC entities, which in turn signify DIFFERENCES between DISCRETE STATES.

The symbolism is founded on W. R. Hamilton's conception of a "Science of Pure Time", and the corresponding notion of transitions between DISCRETE STATES by means of STEPS in TIME; the aim is to highlight the DYNAMIC aspect of the theory, to complement the "comparatively geometrical view" developed in the well-known QUATERNION calculus. The name GRAMMARITHM was suggested by Hamilton.

The STATES referred to can be taken to occur in an observable, discrete space (the STATE SPACE); the STEPS are taken to be FINITE and (generally) of DIFFERENT size and direction, but the PROCESS of TRANSFORMATION is left unspecified.

By considering the observed (or CONSTRUCTED) discrete states in the state space as SAMPLING or INTERPOLATION points in an ideal and continuous PHASE SPACE, the RIFS theory introduced by Barnsley et al. ensures the existence of a family of continuous trajectories through the points.

Such a trajectory would represent an idealized CONTINUOUS transformation, and would reflect the underlying DYNAMICS of the observed behaviour. The details of the dynamics would depend upon the trajectory chosen.

The STEPS in TIME can then be taken as INTERPOLATIONS or APPROXIMATIONS along the trajectory, and a method of adjusting automatically the LENGTH of each step to fit the LOCAL rate of change is proposed. By thus recursively adjusting the TIME SCALE to the COMPLEXITY of the graph, the behaviour of (quadratic) processes could be described in NORMAILIZED form.

The author hopes that this slightly different interpretation of the original theory of ORDER IN PROGRESSION might help bring about a closer understanding of the FORMAL EQUALITY of time and space.

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Q-TURTLE

The Quaternions are, basically, Turtle-Graphics[1] in 3 dimensions!

The interesting thing about Turtle-Graphics is that it offers a different way to look at geometry, namely, as depicting BECOMING, rather than the purely spatial relations of BEING. But to accomplish this in SPACE, it is necessary that the TIMING - the order of construction - be retained in its objects

Every quaternion can be seen as the quotient of two Q´s (one of which may be the base Q); this is the same as to say that we can describe the first "step" as that Q (Q1) which will distort the base vector of the chosen space in such a way as to constitute the desired first step. Q1 will thus be the GEOMETRIC RATIO between the base vector and the first "segment" of the graph. Then the next step can be described as that Q-transformation (or, ratio), Q2, which distorts the first segment into the desired second segment, and so on. The ORDER of the steps is essential, since quaternion multiplication is not commutative

But this notion of progressing from the PRESENT position to the NEXT by means of an EFFECTIVE STEP, is precisely how the Turtle operates.

DYNAMIC GEOMETRY

The descriptive Euclidean view of geometry is STATIC, CONTINUOUS AND ANALOG, whereas the constructive view is DYN AMIC, DISCRETE and DIGITAL.

In the Euclidean view, a circle CONSISTS of all the points that lie on the same distance (the radius) from a fixed point (the center).

But when you instruct the Turtle to MAKE a "circle" in the plane, you don´t use these concepts at all. Instead you tell the Turtle to turn "slightly" to one side, take a "small" step and repeat the process a suitable number of times (an "infinite" number of "infinitesimal" steps, in the limit). If this CONSTRUCTED circle has anything like a radius and a center, it remains to be proved: they certainly don´t occur in Turtle´s OPERATIONAL definition.

LOGO - the Turtle's natural habitat - is a highly recursive language and therefore well suited for recursively defined stuctures such as FRACTALS, and the introduction of hierarchical Turtle CLASSES provides for CONCURRENT processes operating in PARALLEL[2] - a tree sprouts leaves on all branches "simultaneously".

This permits the strict temporal sequentiality of recursion to be "fanned out" into layers of CONCURRENT processes operating in parallel, i.e., the substitution of SPACE for TIME.

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[1] See "LOGO-Turtle" on the ANPA diskette, or ObjectLOGO™ from Paradigm Software

[2] Note that here global side-effects of procedures correspond to non-local interaction!

Ordinals and quantities.

Nothing forces us to keep the difference, or step, between the VALUES of consecutive ORDINALS constant. In fact, for the treatment of ITERATIVE non-linear functions, it is rather inconvenient to denote SUCCESSIVE VALUES by INTEGER NUMBERS.

Logarithms provide an example of a non-linear value system for ordinal markers. It is a number system that can contain even exponential growth:

e^x is an explosion, but ln(e^x) is a line, and can be studied att leisure.

In a corresponding way, it seems possible to construct an ordinal number system which adapts to the dynamic behaviour of processes in order to reduce "irrequisite variety" and mask forth (put in canonical form) regularities that might otherwise go unnoticed.

In a dynamic non-linear environment we need to magnify vanishing phenomena as well as to contain exploding ones; we require a number system that adjusts the SCALE OF MEASUREMENT to the COMPLEXITY of the observed/described process.

In such a number system, the SUCCESSOR FUNCTION at any one point in the domain (progression) must depend on the surroundings of that point only, in the same sense that the regular onset of chaos depends only on the immediate (quadratic) surroundings of the summit point (Feigenbaums universality).

When plotting or sampling a function with a finite set of points, it seems reasonable to distribute the points densely for twisty regions of the graph, and sparser for relatively flat regions - the GRAININESS of approximation/measurement should reflect the RATE OF CHANGE in the process.

The center of curvature for a point on a graph is defined to make the corresponding circle CONGRUENT with the graph at that point, and the RELATIVE LENGTH of a step along the perimeter of a circle depends only on the ANGLE between the two radii that define, respectively, the beginning and end points of the arc.

By determining NEXT STEP in terms of the local circle of curvature and a fixed angular offset, the AMOUNT OF CHANGE between successive points can be kept constant. In this approach TIME is defined to make PROGRESS look simple, and the "power of resolution" stays fixed, no matter the (smooth) convolutions of the curve.

The curving line will thus, by construction, be sure to have only quadratic bends along its entire length, and if the orbit is supposed to exist in SPACE, rather than in the plane, then the tangents (that determine the location of the center of curvature) must be QUATERNIONS.

The method of construction is independent of any external coordinate system and uses only four characteristics, namely: WHERE (step number) in the PROGRESSION (F) we are, the PRESENT DIRECTION (F'), and the RELATIVE CHANGE (F").

And this way of describing a graph by its intrinsic features is of course also the way to tell Turtle how to draw the graph.

THE SUBJECTIVITY OF THE METRIC AND OF LATTICE STRUCTURES

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Abstract: It is shown how the space metric can be considered as a relative concept. This idea which has been discussed in the past is here expanded to show how arbitrary non-cartesian coordinate sets can be mapped to cartesian coordinate sets. Thus also a set of points equally spaced in a cartesian space can map to a non-uniform distribution of points in another space. Discrete structures in one space will remain discrete in another space but with a different number sequence assignment. Although the ensuing discussion will be carried out in terms of continuous spatial variables, it obviously would also apply to lattice point spatial geometies.

Also provided is a direct intuitive meaning for non-Euclidean geometries and general methods for the general solution of differential equations in arbitrary coordinates. This permits that non-local fluidic disturbances like rings or toroids be mapped to spherical ones which are local. Its application in GTR aids in the selection of a space metric for particular conditions.

Observers are required, called Coordinate System Inhabitants (CSIs) to whom an arbitrary orthogonal coordinate set appears to be literally Cartesian and to whom space in these coordinates is also literally Euclidean. This same coordinate set or space will, to an outside observer, appear to be distorted and non-Euclidean. It is the isomorphic mappings between these two observer's perceptions of a coordinate set which result in a number of useful physical and mathematical applications.

Such mappings are even shown to be possible under certain physical conditions between doubly and singly connected spaces. It is this which permits that non-local fluidic disturbances like rings or toroids be mapped to quiescent spherical ones which are local.

1. INTRODUCTION

We show here how space can be considered as a subjective concept which provides a direct intuitive interpretation of non-Euclidean metrics which aids in visualizing multidimensional curved spaces.

Such metrics are based on the historical discussions of these matters:

1. Gauss asked the question[1, 2]: "Can 2-dimensional intelligent beings, living on a 2-dimensional (spherical) surface, determine if their space is curved, and can they find the curvature by measurements made within the surface?" Although he showed that this is possible, this answer is based on tacit assumptions which will here be varied so that it is also not possible to know of or to measure such curvatures.

2. The replacement of the parallel axiom of Euclid by alternate axioms (see Ref. 1, Chapter 36) showed how the positive and negative curvature spaces that can be constructed result in non-Euclidean geometries which are at least self consistent.

3. The extension and generalization of these concepts by Reimann to the multidimensional case resulted in the well known relation for the line element[3]:

$$ds^2 = g_{ij}\, dx_i dx_j \qquad (1)$$

where the summation convention is used and g_{ij} is a symmetrical function (the metric) of the coordinates x_i and x_j. Although the 2-dimensional case considered by Gauss can be easily visualized and described, this not true for 3 or more dimensions, since the relation Eq. (1) is essentially local and explicitly non-global. Eq. (1), however, gives an algebraic relation which results at least in an unambiguous, algebraic, and self consistent (though local) understanding. Thus for the case of a 3-dimensional space with x_1, x_2, x_3, = x, y, z:

$$ds^2 = g_{ij}\, dx_i dx_j = dx^2 + dy^2 + dz^2 = \delta_{ij}\, dx_i dx_j \qquad (2)$$

which shows that the metric g_{ij} for this case is diagonal and unity. The metric is a tensor and retains its linear transformational properties if any other way of labeling the same space is used. Using the new coordinates α, β, γ, however, instead of x, y, z we get:

$$ds^2 = A\, d\alpha^2 + B\, d\beta^2 + C\, d\gamma^2 = M_{ij}\, dx_i dx_j \qquad (3)$$

where A, B, and C are the diagonal elements of M_{ij}, which is no longer unity although the line element still is equal to ds^2. These relations are based on the simple theory of coordinate transformations [4-6]. Such coordinate transformations and the unity of the metric will be shown to be of great importance for relative metrics. This because the thesis will be developed that *when one labels a space, one is making a subjective judgment*..

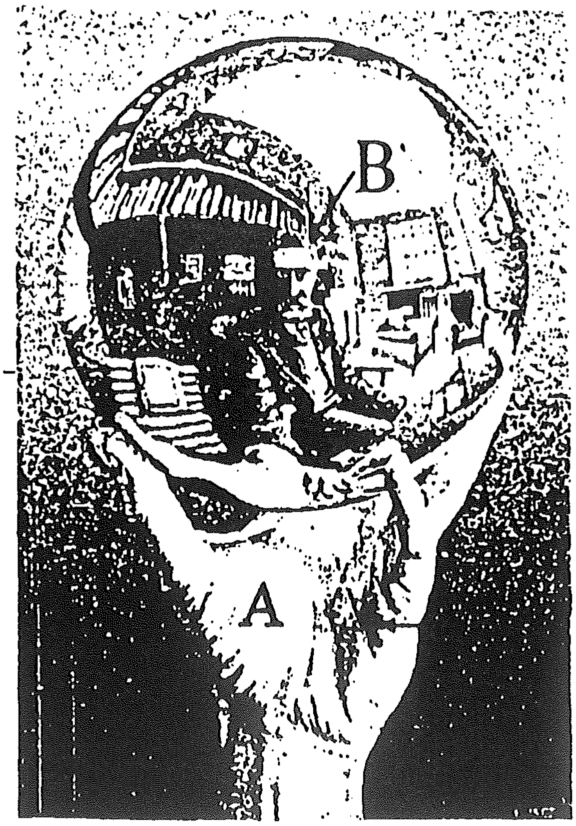
4. Helmholtz[7] maintained that there is no a priori specification that can be given to space, so that any designation for it (flat or curved) is equally conceivable. He concluded that this question can be solved in an empirical way by studying the characteristics of physical objects. In particular, the existence of a rigid body that can be rotated and displaced without a change in its dimensions is the reason for his conclusion that a Euclidean flat space is our common milieu. This practical conclusion is, in the light of modern physics (Special and General Relativity), not true, since there are implicit assumptions and explicit relationships like the Lorentz transformations concerned with variation of length in space and time of physical rest frames which have replaced previous naive perceptions of absolute rigidity of physical bodies. There is also a related and suggestive discussion of Poincare[8] on the relativistic considerations which enter into our comprehension of space that is similar to the Helmholtz discussion. Some of our initial developments of these matters have been presented[9, 10, 11].

It is because the Helmholtz[7] discussion gives an extremely clear example of our basis for relative metrics that it is given here:

"Let me first remind the reader that if all the linear dimensions of other bodies, and our own, at the same time were diminished or increased in like proportion, as for instance to half or double their size, we should with our means of space perception be utterly unaware of the change. This would also be the case if the distension or contraction were different in different directions, provided that our own bodies changed in the same manner.

Think of the image of the world in a convex mirror. The common silvered globes set up in gardens give the essential features, only distorted by some optical irregularities. A well made convex mirror of moderate aperture represents the objects in front of it as apparently solid and in fixed positions behind its surface. But the images of the distant horizon and of the sun in the sky lie behind the mirror at a limited distance equal to its focal length. Between these and the surface of the mirror are found the images of all the other objects before it, but the images are diminished and flattened in proportion to the distance of their objects from the mirror surface. The flattening, or decrease in the third dimension, is relatively greater than the decrease of the surface dimensions. Yet every straight line and every plane in the outer world is represented by a "straight" line or a "plane" in the image. The image of a man measuring with a rule a straight line from the mirror would contract more and more the farther he went, but with his shrunken rule the man in the image would count out the same number of centimeters as the real man. And in general, all geometrical measurements of lines and angles made with regularly varying images of real instruments would yield exactly the same results as in the outer world, all congruent bodies would coincide on being applied to one another in the mirror as in the outer world, all lines of sight in the outer world would be represented by "straight" lines of sight in the mirror. In short, I do not see how men in the mirror are to discover that their bodies are not rigid solids and their experiences not good examples of the correctness of Euclid's axioms. But if they could look out upon our world as we look into theirs, without overstepping the boundary, they must declare it to be a picture in a spherical mirror, and would speak of us just as we speak of them; and if two inhabitants of the different worlds could communicate with each other, neither, as far as I can see, would be able to convince the other that he had the true, and the other the distorted relations. Indeed I cannot see that such a question would have any meaning at all, so long as mechanical considerations are not mixed up with it."

The above is illustrated in the identical pair of pictures by Escher in Figure 1. In Figure 1a, Observer A is holding the reflecting sphere and sees in it the image of his own surroundings and himself. The image of his world in the sphere is labeled B. In Figure 1b the image observer B views the world of A. Neither observer believes his own surroundings are distorted but imputes distortions to the counterpart observer. It is in the above sense that space can be considered as a subjective concept. This is quite peculiar because each observer, A or B, considers his own surroundings as objectively Cartesian, Euclidean, etc., but imputes distortions to the image or counterpart world. Thus when each observer, A or B, labels his surroundings in what he considers an objective manner, it is an inherent act and not an arbitrary act. Thus labeling a space is not arbitrary, it is forced on each observer as the result of his own perceptions but it is subjective in the opinion of the counterpart observer.



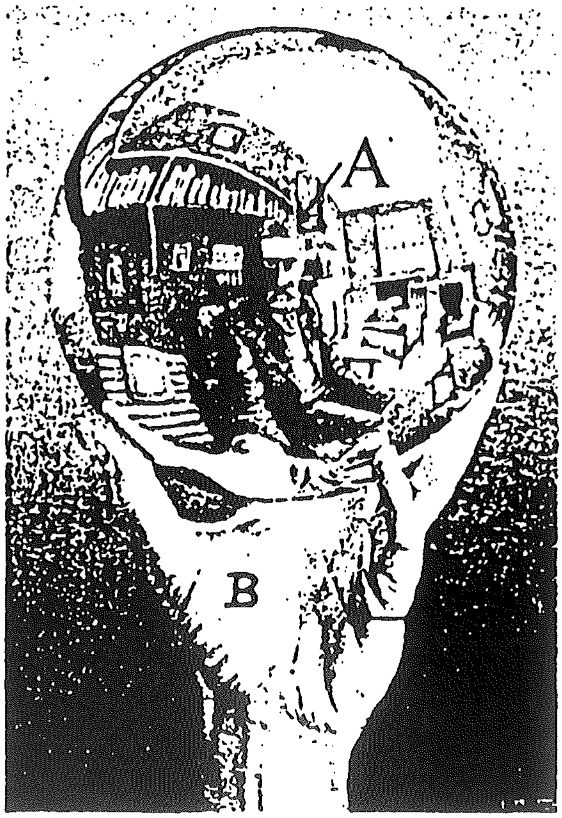


Figure 1. Escher Pictures Illustrating the Helmholtz Discussion & Relative Metrics.

This objective-subjective conflict of opinion is built into the situation and cannot be removed. The Helmholtz discussion appears to describe a fantasy; that is, the man in the spherical mirror is completely fictitious. Its connection with reality resides in the Builder-Ives views of the Lorentz transformations in that these transformations describe literal space and time distortions. Thus 2 living observers in two separate rest frames will have conflictual views as per the Helmholtz discussion.

Since it is the concept of rigid bodies which forced Helmholtz to the Euclidean flat space conclusion, then with respect to fluid dynamical entities like droplets, vortices, etc., the remarks of Helmholtz[7] and Poincare[8] take on a greater importance.

This idea will be illustrated for a 2-dimensional space, which will be discussed in detail below and in the Appendix. As shown in Figure 2 the plane can be labeled with x and y coordinates, which have the following significant, cartesian, euclidean properties:

A. EACH CARTESIAN COORDINATE AXIS EXTENDS WITH A UNIFORM SCALE TO $\pm\infty$ WITH RESPECT TO THE ORIGIN. THE COORDINATE AXES ARE ORTHOGONAL.

B. INCREMENTS OF LENGTH ARE INDEPENDENT OF POSITION AND ORIENTATION.

C. THERE IS A POINT ORIGIN AT WHICH ALL OF THE COORDINATES HAVE THE VALUE ZERO; THIS IS A SINGLY CONNECTED SPACE. THERE ARE POINTS ON EACH COORDINATE AXIS WITH THE VALUES ZERO AND ONE, WHICH DEFINE THE GAUGE OF THE DIMENSION.

(The above statements, will also apply to multi-dimensional spaces and will be called the 'A-C statements' when referred to in the rest of this paper).

The superposition of r, θ coordinates onto the plane of Figure 2 (in the form $re^{i\theta}$) would appear to be merely another way of designating points in the plane. Although this procedure appears to be unambiguous and objective, it can be shown to be subjective in terms of the following considerations. The properties of the x, y coordinates that are given in the A-C statements above and pictured in Figure 2 can be taken as statements made by an (x, y)-COORDINATE SYSTEM INHABITANT (or an (x, y)-CSI). For this (x, y)-CSI the use of the coordinates (r, θ) to label this plane (as is shown in Figure 2) is indeed merely another way of designating the plane. One notices, however, the (r, θ) metric tensor, M_{ij}, is no longer unitary and contains the elements of the coordinate transformations which mathematically relate the x, y to the r, θ coordinates.

One may now postulate an (r, θ)-CSI for whom the statements A-C above will refer NOT to the x, y coordinates but instead refer to the r, θ coordinates. To such an (r, θ)-CSI the same plane will appear as shown by the Cartesian axes r and θ of Figure 3. To this (r, θ)-CSI, Eq. (1) refers to r and θ so that x_i and x_j in Eq. (1) will be r and θ, and g_{ij} is a 2-dimensional unitary array. If this (r, θ)-CSI wishes to relabel his space with x and y, as shown in Figure 3, then he will treat them as r and θ were treated by the (x, y)-CSI in Figure 2. This is not to be understood as the mere switching of coordinate system designations. Thus, as in the Helmholtz discussion, each CSI will point to his native coordinates (to (x, y) or (r, θ) as the case may be) as the ones which are flat, Euclidean, Cartesian, etc.) and as the ones where the A-C statements above are literally true.

If one now has an arbitrary 3-dimensional coordinate system (α, β, γ) then an (α, β, γ)-CSI will always write:

$$ds^2 = d\alpha^2 + d\beta^2 + d\gamma^2 \qquad (4)$$

so that the metric tensor here is unitary for those coordinates. It is, therefore, not space itself which is described by the inhabitants, but rather the view of space based on the coordinate systems native to the inhabitants.

Returning to the 2-dimensional case, the (x, y)-CSI and the (r, θ)-CSI will each see their native coordinate system as Euclidean, Cartesian, etc., and will write a ds^2 using a unitary diagonal metric for their native coordinates. If each CSI wishes to use the counterpart's coordinate system to label space, then they will use simple coordinate system transformations for this purpose. This will result in a metric which is non-unitary. These non-unitary metrics will be interpreted by the native CSI as a simple coordinate transformation. The counterpart CSI, however, will interpret this metric in an entirely different way. The counterpart CSI will say that such a metric is simply wrong or meaningless, but since consistency can be demonstrated for such a metric, then that metric can be interpreted by the counterpart CSI as 'non-Euclidean".

This method can be extended to 3, 4, or more dimensional examples, as will be shown, and should also prove useful because:

1. The non-Euclidean metrics resulting from this method bear strong formal resemblances to many of the non-Euclidean metrics found in general relativity.
2. The method is obviously global, marking an extension in present concepts, and can be applied to n-dimensional spaces for n = 2, 3, 4, etc.
3. It is possible, for the 2-dimensional case of (x, y) and (r, θ) coordinates, to make rectilinear representations of rotational quantities like spinors and axial vectors like angular momentum.
4. The geometric concepts used for gravitational, electromagnetic, and other kinds of fields are difficult to visualize if the ponderable bodies with which they interact are rigid. If the bodies can instead be treated as fluid disturbances such as droplets, whorls, bubbles, etc., and if the fields are similar but non-quiescent entities, then fluid models of all these entities can be expressed in particular coordinate systems with certain geometric advantages. The motion of the fluids in such models can be modes of flow for particular coordinate systems which are parallel or perpendicular to surfaces of constant dimension value. The fluidic nature of the models will, of course, not be evident to the residents of those rest frames in which the observer is an inhabitant and thus conforms with the previous Helmholtz description. In this way such coordinate systems immediately acquire a physical meaning. They represent directions of fluid flow along or perpendicular to surfaces of constant dimensional value and the coordinate systems (which can be non-quiescent for moving bodies or fields) can themselves help to characterize the fluidic dynamical representations and their motions.

The examples to be treated make reference to the way in which such concepts can be applied and combined with the invariance of a super ds^2 or a super ds^4, see Section 3.

2. INITIAL CONSIDERATIONS

The above ideas form part of the basis for considering that our perceptions of space are subjective. Summarizing, five matters (a-e) can be treated with such concepts:

a. An intuitive or human meaning for non-euclidean metrics.

b. The literal interpretation of Lorentz transformations as space and time distortions perceptible to external observers but imperceptible to observers inside a rest frame.

c. The mapping of distorted to non distorted spaces (or metrics) and vice versa for the derivation of spatially and time dependent phenomena for particular spatial rest frames.

d. According to the above, the Reimann metric is no longer invariant and neither is space curvature nor any of the vector operations: gradient, the Laplacian, curl, divergence, D'Alembertian, etc.

e. Providing a technique for the general solution of fluidic flow and wave equation problems in arbitrary coordinates.

We comment on each of the items above:

1. (For a. above) Observers A and B each impute rest frame distortions to their opposites which can be put in the form of coordinate system transformations which each applies to the other, and the conclusion follows that when one labels a space one is making a subjective or relative judgment. Thus, non-euclidean metrics are judgments which a 'non-inhabitant' of a particular 'curved looking' space makes about that space. We need first to require Coordinate System Inhabitants (CSIs) which are defined as: *observers who specify literal physical Cartesian coordinate systems which appear to be non-Cartesian coordinate systems to outside observers.*

Thus all CSIs, are observers whose canonical judgement of their own space is Euclidean and to which the usual Cartesian designation is the simplest and clearest expression. Their judgments of other 'curved' spaces appear objective to them but not to the inhabitants of those other spaces.

2. (For b. above) One finds a useful application in the affine rest frame distortions where the usual Lorentz transformations are taken as literal for external observers of a rest frame while the internal observers consider their space to to be cartesian and invariant.

3. (For c. above) Detailed mappings will be given for the 2-dimensional case whereby a plane is usually designated both with Cartesian coordinates (x, y) and with Polar coordinates (r, θ) superimposed in the form $r\,e^{i\theta}$, as in Figure 2. This is the way the (x, y)-CSI sees the plane. On the other hand, the (r, θ)-CSI sees the r, θ coordinates as literally Cartesian as in Figure 3 or 7 and sees the x, y coordinates as literally Polar coordinates. These two points of view can be connected mathematically by a sequence of linked isomorphic mappings of the initial r, θ coordinates as seen by the (x, y)-CSI of Figure 2 to that of the r, θ coordinates as seen by the (r, θ)-CSI of Figures 3 or 7. One proceeds from Figure 2 to Figure 4 via an r-gauge exponential mapping and then from Figure 4 to Figure 5 via a logarithmic mapping, and then from Figure 5 to Figure 6 via a 3-dimensional stereographic mapping. The unfolding of this last mapping onto a plane yields the final point of view of the (r, θ)-CSI as shown in Fig. 3 or 7 which has the

obviously Cartesian form. Each mapping in the sequence results in a set of direct coordinate transformations. The mapping sequence is diagramed in Figure 8, where

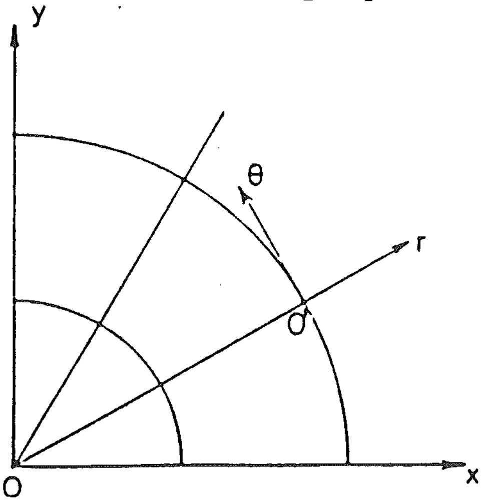


Figure 2. The (x, y)-CSI View of the 2-Dimensional Plane.

The A: (x_0, y_0) space; $(r_0 e^{i\theta})$ for the same (x, y)-CSI.

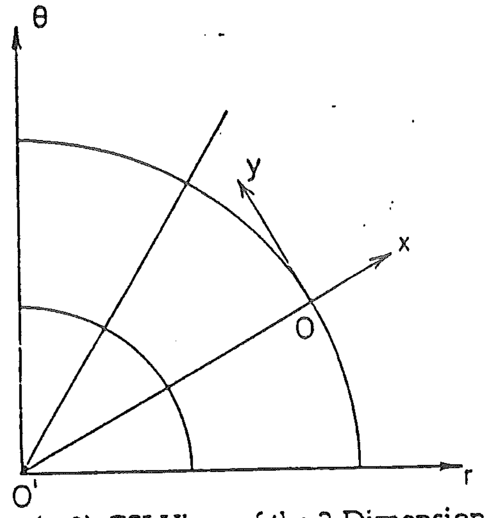


Figure 3. The (r, θ)-CSI View of the 2-Dimensional Plane.

The D: (r_3, θ_3) space; $(x_3, e^{i\theta_3})$ for the same (r, θ)-CSI.

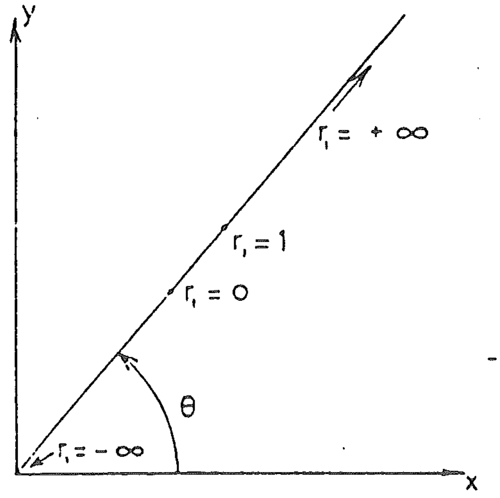


Figure 4. B Space. B: (r_1, θ_1); (x_0, y_0) is superimposed (r-gauge mapping from A).

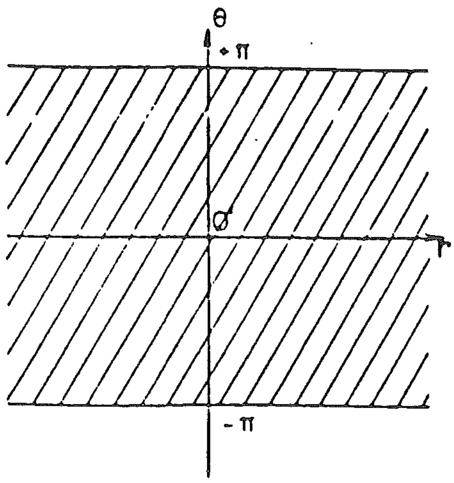


Figure 5. C Space. C: (r_2, θ_2) space; (log mapping from B).

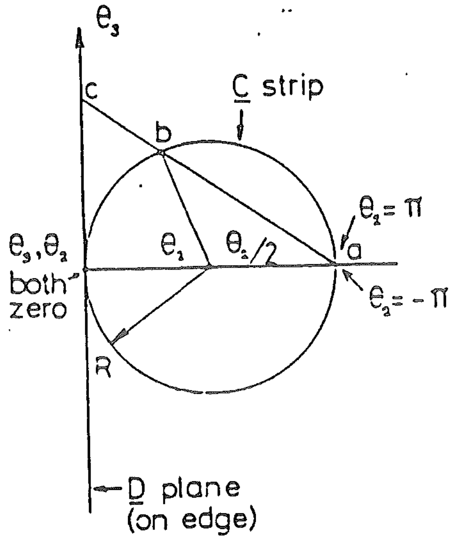


Figure 6. D: (r_3, θ_3) Space; (two-dimensional stereographic mapping from C).

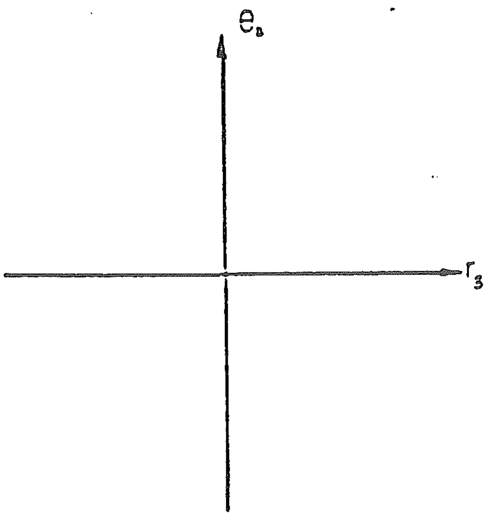


Figure 7. D: (r_3, θ_3) Space (The unrolled plane view, identical with Figure 3).

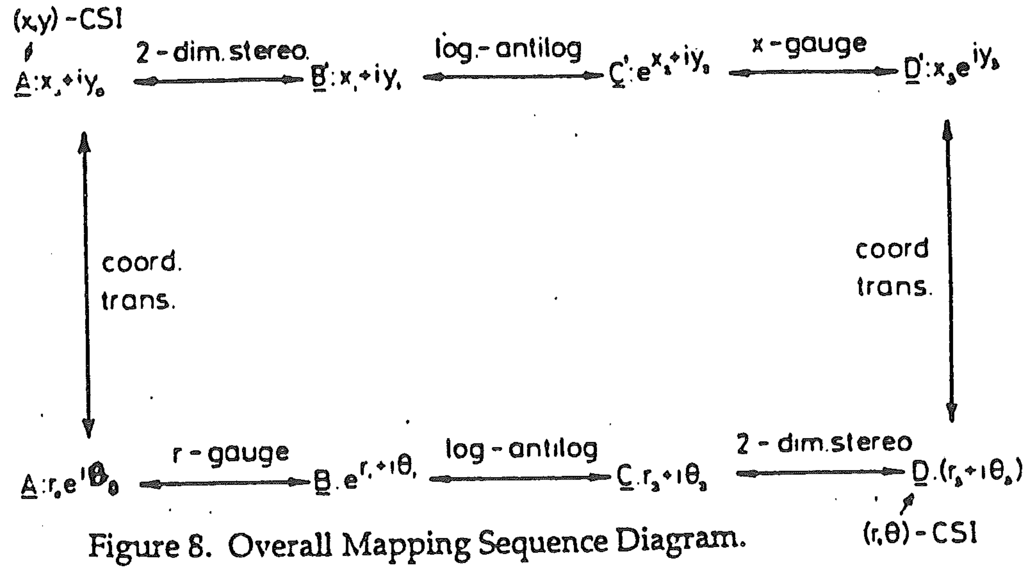


Figure 8. Overall Mapping Sequence Diagram.

the observers (and their spatial perceptions) are labelled A, B, C, and D, respectively. A, gives the (x, y)-CSI perceptions and D is the (r, θ)-CSI spatial perception; B and C correspond to intermediate stages of the A to D mapping. The mapping back from the (r, θ)-CSI point of view to the (x, y)-CSI point of view is also shown and is along the path D to C' to B' to A. These mappings have been given in greater detail[9, 10, 11] and summarized in Appendix A.

4. (For d. above) Since the metric is no longer invariant, one can, for example, derive rigorously rotational and irrotational flow fields and solve many other space curvature problems expressed in other coordinate systems.

5. (For e. above) Since this method maps arbitrary coordinate spaces to the Cartesian ones, one may start with the Cartesian coordinate system which can correspond to parallel lines lines of fluid flow and then map these to arbitrary coordinate systems which simultaneously will convert the lines of parallel flow to flow lines following the arbitrary dimensional designations. Thus, for example, in mapping to cylindrical or spherical coordinates one may start with the general and well known solutions to various wave equations in Cartesian coordinates and map them to the general solutions in cylindrical or spherical coordinates. This, however, is merely practice and serves to confirm the technique, since such solutions are already well known. The general solutions, however, for wave equations in toroidal and many other coordinate systems are not known and this method give promise of providing such solutions.

The physical meaning for these techniques resides in the 2-fluid plenum introduced elsewhere[12,13]. Such fluids in particular fluidic particle models have particular 3-vector fluid flows and 1-vector charge densities in 4-space. Coordinate sets which follow the charge density and flow distributions are then natural schemes for expressing these fluidic entities.

'The Relativity of the Metric' via all the above considerations has an exact mathematical meaning. It refers to the fact that any CSI will write a unitary diagonal metric for that CSI's native coordinates and a non-unitary one for other coordinate systems. In the above example, the (x, y)-CSI will write a unitary diagonal 2x2 matrix for the (x, y) coordinates. This CSI will also write a diagonal nonunitary matrix for the (r, θ) coordinates which are merely algebraic transformations for x and y. The (r, θ)-CSI, on the other hand, will write a unitary diagonal matrix for the (r, θ) coordinates and will write a non-unitary matrix for the (x, y) coordinates which are also algebraic transformations of his cartesian appearing (r, θ) coordinates. They will, of course, disagree with each other on the designation of the space but can at least consider the counterpart designations as non-euclidean and at least consistent. This idea has a general and fundamental importance. It is introduced and used here for the mapping of the Hertzian dipole kidney shaped toroidal half wavelength field distributions to quiescent spherical field distribution and thus finally to the canonical particle (see final sections).

3. MATHEMATICS OF RELATIVE METRICS

It appears to be generally untrue that arbitrary dimensional designations can conform to the A-C statements given previously. Thus angular dimensions are either discontinuous or periodic and radial dimensions have only positive values. Nevertheless, conformance with respect to A-C statements can be established by modifying by means of a method of sequential isomorphic mappings, those meanings for the coordinates which appear irreconcilable with the A-C statements. The new meanings for the coordinate designations would then apply. Before doing this, the method of simple coordinate transformations is first summarized[4-6].

Suppose the set of Cartesian coordinates (x, y, z) and another coordinate set (α, β, γ) label a 3-space. The relationships between these coordinate systems are:

$$x = x(\alpha, \beta, \gamma) \qquad y = y(\alpha, \beta, \gamma) \qquad z = z(\alpha, \beta, \gamma) \tag{5}$$

and the surfaces α, β, γ = a constant, form an orthogonal (assumption) system. The differential element of length is defined by:

$$ds^2 = dx^2 + dy^2 + dz^2 \tag{6}$$

and the following functions may be constructed with the usual conditions on continuity and differentiability:

$$\begin{aligned} 1/U &= [(\partial x/\partial\alpha)^2 + (\partial y/\partial\alpha)^2 + (\partial z/\partial\alpha)^2]^{1/2} \\ 1/V &= [(\partial x/\partial\beta)^2 + (\partial y/\partial\beta)^2 + (\partial z/\partial\beta)^2]^{1/2} \\ 1/W &= [(\partial x/\partial\gamma)^2 + (\partial y/\partial\gamma)^2 + (\partial z/\partial\gamma)^2]^{1/2} \end{aligned} \tag{7}$$

so that ds^2 may be expressed as:

$$ds^2 = (d\alpha/U)^2 + (d\beta/V)^2 + (d\gamma/W)^2 \tag{8}$$

On the basis of the previous discussion, the above equations are merely those that would be written by an (x, y, z)-CSI. One may now write the following set of equations for an (α, β, γ)-CSI: (the equation numbers will be the same as above, but primed, to facilitate comparisons):

$$\alpha = \alpha(x, y, z), \quad \beta = \beta(x, y, z), \quad \gamma = \gamma(x, y, z) \tag{5'}$$

and the surfaces x, y, z = a constant, form an orthogonal system. The differential element of length will, however, be called ds':

$$ds'^{\,2} = d\alpha^2 + d\beta^2 + d\gamma^2 \tag{6'}$$

and the following functions can be constructed:

$$\begin{aligned} 1/U' &= [(\partial\alpha/\partial x)^2 + (\partial\beta/\partial x)^2 + (\partial\gamma/\partial x)^2]^{1/2} \\ 1/V' &= [(\partial\alpha/\partial y)^2 + (\partial\beta/\partial y)^2 + (\partial\gamma/\partial y)^2]^{1/2} \\ 1/W' &= [(\partial\alpha/\partial z)^2 + (\partial\beta/\partial z)^2 + (\partial\gamma/\partial z)^2]^{1/2} \end{aligned} \tag{7'}$$

so that $ds'^{\,2}$ may be expressed as:

$$ds'^{\,2} = (dx/U')^2 + (dy/V')^2 + (dz/W')^2 \tag{8'}$$

Both coordinate systems have been restricted to the requirements of Eqs. (5) and (5') and this carries the usual conditions on the Jacobians of the transformations; the transformations are bilaterally, explicitly, and finitely resolvable. Now if the coordinates (α, β, γ) are merely another set of Cartesian coordinates which are displaced or rotated with respect to the (x, y, z) set, then the (α, β, γ) set will immediately satisfy the A-C statements and all of the above equations can be immediately applied. This corresponds to the case of a linear transformation linking (α, β, γ) and (x, y, z).

However, in the case of radial and angular coordinates like r and θ, the A-C statements are not obviously true. Whereas for (x, y, z):

$$-\infty \leq x \leq +\infty, \quad -\infty \leq y \leq +\infty, \quad -\infty \leq z \leq +\infty \tag{9}$$

for the r and θ one must write:

$$0 \leq r \leq +\infty, \quad -\pi \leq \theta \leq +\pi \tag{10}$$

where the principal values have been used for θ. However, performing the mapping:

$$r \rightarrow r', \qquad \theta \rightarrow \theta' \tag{11}$$

such that:

$$-\infty \leq r' \leq +\infty \quad \text{and} \quad -\infty \leq \theta' \leq +\infty \tag{12}$$

holds, then the r' and θ' coordinates will satisfy the A-C statements and the primed variables above can be used. Such mappings, therefore, will be necessary for coordinate systems having radial and angular dimensional designations.

Returning now to the case of the (x, y, z)-CSI and the (α, β, γ)-CSI, we have, for an (x, y, z)-CSI :

$$ds^2 = dx^2 + dy^2 + dz^2 = (d\alpha/U)^2 + (d\beta/V)^2 + (d\gamma/W)^2 \tag{13}$$

or in terms of the metric:

$$ds^2 = g_{ij}\, dx_i dx_j \tag{14}$$

where $x_i, x_j = x, y, z$ and g_{ij} is δ_{ij} is unitary. Also we have:

$$ds^2 = M_{ij}\, du_i du_j \tag{15}$$

where $u_i, u_j = \alpha, \beta, \gamma$ and the term M_{ij} is non-unitary.

For an (α, β, γ)-CSI, however,

$$ds'^2 = d\alpha^2 + d\beta^2 + d\gamma^2 = (dx/U')^2 + (dy/V')^2 + (dz/W')^2 \tag{16}$$

using Eqs. (6') and (8'), or in terms of the metric:

$$ds'^2 = \delta_{ij} du_i du_j \tag{17}$$

where $u_i, u_j = \alpha, \beta, \gamma$ and we also have:

$$ds'^2 = N_{ij} dx_i dx_j \tag{18}$$

where $x_i, x_j = x, y, z$ and N_{ij} is non-unitary.

In words, an (x, y, z)-CSI will give a unitary diagonal metric for dx, dy, dz as in Eq. (14) and will assert that the metric seen in Eq. (15) is the same as (14), but expressed in the α, β, γ coordinates. An (α, β, γ)-CSI, however, will give a unitary diagonal metric for $d\alpha$, $d\beta$, $d\gamma$ as in Eq. (17) and will assert that the metric seen in (18) is the same as (17), but expressed in x, y, z coordinates. Each CSI should be able to derive the equations of the counterpart CSI by imagining a counterpart observer and making use of the A-C statements for the counterpart coordinates. In the canonical case, each CSI will state that the counterpart equations are either meaningless or are a non-Euclidean (and at least consistent) way of labeling space. Each CSI will note that: (1) The respective ds^2 and ds'^2 are not equal, (2) The non-unitary metric for the counterpart CSI bears a formal resemblance to a non-Euclidean metric (his own non-unitary metric is a simple coordinate transformation, in his opinion).

Both CSIs will adhere to the truth of the A-C statements for their native coordinates. It is in this sense that the metric can be considered as a relative concept. This implies that the non-Euclidean metric can be traced to the presence of another coordinate system with its own or native CSI.

Now suppose that each of the coordinates (α, β, γ) do not satisfy the A-C statements; then the sequence of mappings can be made:

$$(\alpha, \beta, \gamma) \rightarrow (\alpha', \beta, \gamma) \rightarrow (\alpha', \beta', \gamma) \rightarrow (\alpha', \beta', \gamma') \tag{19}$$

such that $(\alpha', \beta', \gamma')$ will finally satisfy the A-C statements. These mappings must be independent in order to preserve the orthogonal nature of the dimensions, so that, in any order:

$$\alpha \rightarrow \alpha', \qquad \beta \rightarrow \beta', \qquad \gamma \rightarrow \gamma' \tag{20}$$

The mappings of Eqs. (19) and (20) can be affine, projective, contact, logarithmic, exponential, etc., transformations. Although these are meant to be isomorphic transformations, mappings between singly and doubly connected spaces might also be performed, as follows. Since these are meant to describe physical situations and fluidic models, the inevitable singularities of these latter transformations can be kept track of, and imputed, to a part of the physical model under consideration where its effect is physically trivial. Thus mappings which violate the topological integrity may be performed if the singularities are thrown in projection to infinity or are put into a finite region where they map to vanishingly small volumes of fluid flow and hence to vanishingly small amounts of the energy needed to set up the fluid model.

Since the values of ds^2 and ds'^2 are no longer equal, they are no longer equal and invariant. It is necessary to examine how an invariance principle may still be constructed which includes this metrical relativity. Two comments from Reimann's famous essay are relevant[3]:

1. "Thirdly, one might instead of taking the length of the lines to be independent of position and direction, assume also an independence of their length and direction from position. According to this conception, changes or differences of position are complex magnitudes expressible in three independent units". Here Reimann notes that both length and orientation together can be invariants for the 3-dimensional and absolute spatial considerations which he uses.

2. "The next case in simplicity (to $ds = ([\Sigma(dx)^2]^{1/2})$ includes those manifolds in which the line elements may be expressed as the fourth root of a quartic differential expression."

On the basis of the first remark: If the two CSIs are characterized as an (x_i)-CSI and a (u_i)-CSI in accordance with Eqs. (14)-(18), where $i = 1, 2, 3, ...n$, where n is the number of dimensions, the line elements for each CSI can be identified with an x or u subscript, thus for the (x_i)-CSI:

$$ds_x^2 = \delta_{ij}\, dx_i dx_j = M_{ij}\, du_i du_j \qquad (21)$$

where M_{ij} is from Eq. (15).

For the (u_i)-CSI:

$$ds_u^2 = \delta_{ij} du_i du_j = N_{ij} dx_i dx_j \qquad (22)$$

where N_{ij} is the same as in Eq. (18). One may construct an invariance principle from the first Reimann remark above as the sum of the 2 above expressions:

$$dS^2 = ds_x^2 + ds_u^2 \qquad (23)$$

Using this together with Eqs. (21) and (22), one gets:

$$[\, \delta_{ij} + N_{ij}\,] dx_i dx_j = [\, \delta_{ij} + M_{ij}\,] du_i du_j \qquad (24)$$

Using Eqs. (21) and (22) again:

$$ds_x^2 + N_{ij}\, dx_i dx_j = ds_u^2 + M_{ij}\, du_i du_j \qquad (25)$$

which gives a relation between ds_x^2 and ds_u^2. These expressions are not immediately useful, although it can be shown that ds_x^2 is based on elements of length, while ds_u^2 (for the 2-dimensional cases previously considered) is based on the sum of the squares of two elements, one of which is displacement and the other is orientation, (for the 2-dimensional case) thus;

$$ds_u^2 = dr^2 + d\theta^2 \qquad \text{(See below)}$$

as in Eq. (23) and somewhat according to the first remark of Reimann above. It is to be noted that the θ in the above expression should be a quantity that is in length units, which will be accomplished (in the 2-dimensional example to follow) during the mappings from the original r, θ designations.

On the basis of the second remark of Reimann, one may also construct a super dS, defined as:

$$dS^4 = ds_x^2 \, ds_u^2 \qquad (26)$$

so that, using Eqs. (21) and (22):

$$dS^4 = [\,\delta_{ij}\, dx_i dx_j\,]\,[\,\delta_{kl}\, du_k du_l] = [\,M_{ij}\, du_i du_j\,]\,[\,N_{kl} dx_k dx_l\,] \qquad (27)$$

with the result that:

$$M_{ij} N_{kl} = \delta_{ilkl} \qquad (28)$$

The equations (26) - (28) appear to be more useful. The invariance of dS^4 gives, via Eq. (28), a relation between the M and N matrices, so that one may be derived from the other. This will not be immediately useful until both sets of coordinates have become similar; that is, only after both sets simultaneously satisfy the A-C statements. This means, for example, that the origin of the (r, θ) coordinates in Figure 1, and the angles at which discontinuity occurs ($\theta = \pm \pi$) must be mapped in general to infinitely distant regions for the (r, θ)-CSI. This is demonstrated in the details of the 2-dimensional mapping example given in the Appendix.

4. TOROIDAL & MULTIDIMENSIONAL MAPPINGS

The mapping of photex field configurations (half wavelength dipole field configurations) have been discussed[13] where it was suggested that each such: toroidal entity, or 'smoke ring' or photex could be projected to finite length right circular cylinders. A further sequence of mappings of the cylinder was also suggested whereby the axis of the cylinder is shrunk to a point while its surface is made subject to the condition that its distance from that point is deformed into a fixed distance. This makes the cylinder and its contents into a sphere (See Figure 4 [13]). The poles of this sphere are singularities of the mapping of the photex toroid to the cylinder. In a physical sense these singularities are negligible since the amount of energy of the fluids at each face of the cylinder ends have vanishingly small energies. The final sphere can then be shrunk to the canonical point particle to give a particle representation for the photex toroid. All this must occur in a 4-space since the initial mappings of the expending Hertzian toroidal field configuration must first have its radial dimension mapped to a continuously increasing radial space dimension which results in quiescent models for the toroids, cylinders, and spheres. This results in a dual view of these photexi. They are indeed continuously expanding entities as shown in the Hertzian dipole wave pictures which is a physical representation applicable to physical rest frames. Their representation as quiescent spheres or particles is a representation of a (non-physical) electromagnetic rest frame[13]. They do provide, however, a way of considering such discontinuous entities as non-local in physical rest frames and as local in conceptual electromagnetic rest frames. We note here that from the viewpoint of the electromagnetic rest frame, ordinary quiescent matter will appear as continuously deforming rapidly shrinking objects.

This paper is meant to introduce the subject of relative metrics and it is evident that:
1. The method outlined here is readily applicable to multidimensional spaces since the mappings of each of the dimensions are independent of each other as in Eqs. (19) and (20).
2. Proofs of the Helmholtz vortex theorems may be simply inferred by mapping rectilinear fluid motion in an (x, y) space to rotation in an (r, θ) space. The Gauss and Stokes theorems may also, therefroe, be mapped to the rotational case.
3. The quantum mechanical rectilinear operators may be mapped to spinor quantities.
4. Axial vectors can be shown to be derivable directly from these considerations.
5. The non-linear generation of fluidic entities might be dealt with by means of the growth of coordinate sets which can be broken off at far field conditions.
6. There are many kinds of canonical general solutions of various wave equations in Cartesian coordinates. These consist usually of the orthogonal set of sinusoidal Fourier functions. Mapping of these original equations to other coordinate sets should simultaneously map their general solutions consisting of the sinusoidal Fourier functions to general solutions consisting of transformed Fourier functions. This points to the conclusion that orthogonal functions (like Bessel, Hankel, etc.) are derivable from such spatial mappings. If all orthogonal functions map to each other using the appropriate space, then their properties can be more easily clarified and systemetized. A single arbitrary function in one space, however, will have a different appearence in each space to which it is mapped, although a knowledge of the counterpart CSI relations will permit construction of the original function.

5. FINAL REMARKS

This compressed discussion merely introduces a set of linked concepts for the reconciliation of the many points of contention in STR/QM. It proceeds from the basis of the Builder-Lorentz version of STR which provides operational covariance. Thus, the rest frame distortions as evident in the Lorentz transformations, are considered to be literal but unmeasurable inside the rest frames. A continuous dual charged fluid for vacuum space is introduced from which fluidic and operationally covariant models for the fundamental particles are constructed. The electron droplet model provides a fluid model and mechanism for the generation of em waves as vortex shedding behaviour of the accelerated or decelerated electron droplet. This results also in the provision of a realistic model for the hidden variable of QM (the shed vortex which has been named the photex with the energy which comes directly and only from h, Planck's Constant[13]). The relativity of the metric is necessary to provide a special reference frame for em waves wherein such waves can be particles. All these ideas will remain merely speculative suggestions unless and until experiments suggested[12,13] by these concepts are confirmed in the laboratory.

The tenets of STR/QM have been replaced with axioms based on the above to provide literal pictures of physical reality. Of course, these new axioms which have been introduced are indeed axiomatic and no explanation exists for them. It fits, however, with the idea that it can become part of an ongoing sequence of deeper theories in the future. If these ideas are ultimately shown to be useful, they would, however, still demonstrate the importance and significance of the philosophy of STR/QM which, I believe, consists of providing useful new methods for tentatively leapfrogging defects in determinate approaches until more inclusive determinate approaches are proved useful and this has also been presented[14].

Final Note: It should be emphasized that the introductory exposition is considered the most important part of this paper with its reference to Helmholtz and Poincare. Here the subjectivity of space is presented, which if measurements derived from these considerations are confirmed, then this idea will mark an extension of differential metrics to global ones and provides a counterexample to the status of geometric thinking as given by Kant, by Reimann, and by Gauss.

APPENDIX
RELATIVE METRICS FOR THE 2-DIMENSIONAL (x, y) vs (r, θ) COORDINATE SYSTEMS & SPACES

The clearest procedure for the construction of the 2-dimensional (r, θ)-CSI conception of space is to perform a sequence of mappings of the (r, θ) coordinates starting from Figure 2, such that the final (r, θ) mappings result in Figures 3 or 7. When this has been done, the A-C statements will fully apply to the meaning of (r, θ) via the mappings as given in Eqs. (19) and (20). Such mappings are necessary because the (r, θ) coordinates as initially defined by the (x, y)-CSI do not satisfy the A-C statements. Although the coordinate designations do not change during the mappings; gauge, orientation or even more complicated transformations can be made, in the opinion of the appropriate CSI. After the mappings, when the A-C statements are satisfied, the final physical size of the CSI will also need to be considered, in order to make sure that the CSI size is still finite in finite regions of the spaces and conforms to the A-C statements.

The examination of this case depends on the clear differentiation of 3 different CSIs:

1. The (x, y)-CSI to whom the (x, y) and the (r, θ) coordinates are as shown in Figure 2.
2. The (r, θ)-CSI to whom the (r, θ) coordinates are as shown in Figure 3.
3. The super-CSI: this corresponds to we who are examining both cases and taking the viewpoints of each of the above CSIs at various times during the analysis.

In order to follow the mappings, (x, y) and (r, θ) will be given numbered subscripts for each of the mappings steps. In addition, capital letter designations will refer to the illustration of a particular mapping and sometimes will designate that space, e.g., The A space as shown in Figure 2. Since it is the (r, θ) which will be mapped, their designations will be carried along, thus:

Situation A: (r_0, θ_0) and (x_0, y_0) See Figure 2
Situation B: (r_1, θ_1) See Figure 4 (29)
Situation C: (r_2, θ_2) See Figure 5
Situation D: (r_3, θ_3) See Figure 6 and then Figure 7.

This should not obscure the point that for a CSI the coordinates are unique, so that no matter how we, as super CSIs, describe a space in any situation, the (x, y)-CSI or the (r, θ)-CSI will always call his native coordinates (x, y) or (r, θ), respectively. In Figure 2 we have the space A with the coordinate designations:

$$A: (x_0 + iy_0) = A: r_0\, e^{i\theta_0} \tag{30}$$

This is because (x, y)-CSI will describe the transformation between coordinates as:

$$x = r_0 \cos \theta_0, \qquad y = r_0 \sin \theta_0 \tag{31}$$

for the transformation $(r_0, \theta_0) \rightarrow (x_0, y_0)$, and

$$r = [(x_0)^2 + (y_0)^2]^{1/2}, \qquad \theta_0 = \tan^{-1} (y_0/x_0) \tag{32}$$

for the transformation $(x_0, y_0) \rightarrow (r_0, \theta_0)$. This is also displayed in the mapping sequence diagram, Figure 8, as the coordinate transformations on the left hand side. The set (x_0, y_0) in the upper left hand corner thus maps to the set (r_0, θ_0) in the lower left hand corner via the 2-headed arrow marked, "coordinate transformation". In the above 2 equations the range for the (x, y) and (r, θ) sets are:

$$-\infty \leq x_0 \leq +\infty, \qquad -\infty \leq y_0 \leq +\infty \tag{33}$$

and

$$0 \leq r_0 \leq +\infty, \qquad -\pi \leq \theta_0 \leq +\pi. \tag{34}$$

Neither expression in Eq. (34) satisfies the A-C statements. As super CSIs, we wish to first perform a mapping of the point $r_0\, e^{i\theta_0}$ of Eq. (30), to change the range of r. This will be called an "r-gauge" mapping (exponentiation mapping of r from A to B :

$$\text{A: } r_0\, e^{i\theta_0} \xleftrightarrow{\text{r-gauge}} \text{B: } e^{r_1 + i\theta_1} \tag{35}$$

(The mappings will define the correct units).

For the arrow to the right the relations are:

$$\theta_1 = \theta_0, \qquad r_1 = \log r_0 \tag{36}$$

For the arrow to the left the relations are:

$$\theta_0 = \theta_1, \qquad r_0 = e^{r_1} \tag{37}$$

This now changes the range in r at B given in (34) so that

$$-\infty \leq r_1 \leq +\infty.$$

Notice the simplicity of the final form of (35). Figure 4 shows that this mapping has converted the origin of Figure 4 into the point at negative infinity for the r variable (r_1). The origin of r_1 corresponds to the $r_0 = 1$ point of Figure 4.

The next mapping carries the (r_1, θ_1) point to the right in the mapping sequence diagram, Figure 8, to the viewpoint C. This mapping is called the "log" mapping (log mapping from B to C, and antilog mapping from C to B):

$$\text{B: } e^{r_1 + i\theta_1} \xleftrightarrow{\text{log-antilog}} \text{C: } r_2 + i\theta_2$$

For the mapping to the right the relations are:

$$r_2 = e^{r_1} \cos \theta_1, \qquad \theta_2 = e^{r_1} \sin \theta_1 \tag{38}$$

For the arrow to the left the relations are:

$$r_2 = (1/2) \log (r_2{}^2 + \theta_2{}^2), \qquad \theta_1 = \text{Tan}^{-1} (\theta_1/r_2) \tag{39}$$

since $r_1 = r_2$ and $r_1 + i\theta_1 = \log (r_2 + i\theta_2)$. This mapping results in the C space of Figure 5, which in view of the restriction on θ to principal values, is confined to the strip between

$-\pi < \theta \leq +\pi$ and $-\infty \leq r \leq +\infty$. The purpose of this mapping is to orthogonalize the θ and r dimensions. This is a global condition which is necessary to make (r, θ) satisfy the A-C statements.

At this point 2 choices are possible for the θ mappings. If the restriction to principal values is abandoned, then we, the super-CSIs, have arrived at the r, θ situation which satisfies the A-C statements, since the ranges for both r and θ become $\pm\infty$. This quite general condition on θ is useful in the physical example of a vortex, which is considered in the body of this paper. It corresponds to the unfolding(unwinding) of the multileaved Reimannian surface, only one leaf of which is displayed in Figures 2, 4, and 5.. Additional periodic and non-periodic conditions can be set on these leaves.

The second choice is to retain the principal values of θ, thus:

$$-\pi < (\theta_0, \theta_1, \theta_2) \leq +\pi$$

and now perform an additional mapping that will change the range of the limits to

$\pm\infty$ for θ_3. Here a projective stereographic mapping can be made by visualizing the strip of Figure 5 taken into 3-dimensional space, rolled into a circular cylinder such that the $-\pi$ and $+\pi$ edges touch each other, and then making this cylinder tangent to an infinite plane D. This plane has the Cartesian axes (r_3, θ_3) and the line of tangency is $\theta_2 = 0$ on the cylinder and $\theta_3 = 0$ on the plane. This mapping, shown on edge in Figure 6, also converts θ to a distance measure and shows the centerline of projection at a, going through every point of the strip b, to all points of the plane, c. The mappings are:

$$\text{C: } (r_2 + i\theta_2) \xleftrightarrow{\text{2-dim. stereo}} \text{D: } (r_3 + i\theta_3)$$

For the arrow to the right the relations are:

$$r_3 = r_2, \qquad \theta_3 = 2\tan(\theta_2/2) \tag{40}$$

$$r_2 = r_3, \qquad \theta_2 = 2\tan^{-1}(\theta_3/2) \tag{41}$$

It is here where θ acquires the connotation of distance and the range $\pm\infty$. Even the values of θ_0, θ_1, θ_2 could have been previously considered as distances with the length ranges $\pm\pi$ are based on a strip cylinder radius of one as shown in Figure 5. This is so in Eqs. (40) and (41) for θ_2, θ_3.

The final mapping of the (r, θ) to the point $(r_3 + i\theta_3)$ of the D of the mapping diagram also brings (r, θ) into conformance with the A-C statements and to the points of space D. As shown in the mapping diagram, as super CSIs, we have proceeded from the point $(x_0 + iy_0)$ in the upper left hand corner in a counterclockwise direction to the lower right hand corner to the point $(r_3 + i\theta_3)$. The end points start from the (x, y)-CSI and terminate at the $(r_3 + i\theta_3)$-CSI.

One notes now that the forms $(x_0 + iy_0)$ and $(r_3 + i\theta_3)$ are identical. Each form is what each CSI will use to identify his native coordinates. Figure 8 also shows the reverse path for the mappings back from the (r, θ)-CSI to the (x, y)-CSI viewpoints. We may, therefore, continue in a similar counterclockwise fashion around the mapping diagram. Thus those mappings will continue as follows:

$\text{D: } r_3 + i\theta_3 \longleftrightarrow \text{D: } x_3 e^{iy_3}$ (coord. transformation)

$\text{D: } x_3 e^{iy_3} \longleftrightarrow \text{C': } e^{x_2 + iy_2}$ (inverse x-gauge trans.)

$\text{C': } e^{x_2 + iy_2} \longleftrightarrow \text{B': } (x_1 + iy_1)$ (antilog trans.)

$\text{B': } (x_1 + iy_1) \longleftrightarrow \text{A: } (x_0 + iy_0)$ (inverse 2-dim. stereo trans.)

The complete symmetry of this situation is apparent to us as super CSIs. This means that as super CSIs we have no right to consider the upper left hand corner designations as any more valid than the lower right hand corner designations of Figure 8 (or Figure 2 as any more valid than Figure 3). The conclusion is that we may treat (r, θ) starting from the $(r_3 + i\theta_3)$ point in an identical fashion to the way that we treated (x, y) when we started from the point $(x_0 + iy_0)$.

This completes the description of the mapping diagram, but the sequence of mappings between the upper left and lower right hand corners could have been simplified to:

$$A: (x + iy) \longleftrightarrow D: (r + i\theta). \qquad (42)$$

Although this could have been done directly, the intermediate substitutions were necessary in order to keep track of the singularities, infinite points, etc., so as to ensure complete conformance with the A-C statements. One more matter needs to be discussed: the size of the observer (the CSIs) must be considered in the mappings. The above mappings, which are for a flat 2-dimensional space, bear on a tacit assumption of great importance, here and in the Gaussian consideration of 2 dimensional spherical surfaces. This concerns the range of a dimension which is allotted to the observer, e.g., the volume of the observer (or CSI) as seen by the counterpart CSI. Such a consideration is silly when considering an (x, y)-CSI in Figure 2 or a (r, θ)-CSI in Figure 3, since any fixed size can be permitted by either CSI. However, if the (x, y)-CSI is to give a correct description of the relative metrics, he must impute to the (r, θ)-CSI an infinitely small size at the origin O of Figure 2. Likewise, the (r, θ)-CSI will make the same statement about the (x, y)-CSI for the origin of Figure 3. In the mapping from C to D the stereographic projection will require that a fixed small area approaching the region of $\pm \pi$ for θ, will in the final Figure 7 cause the range in θ to increase without bound. This would be evident to the (x, y)-CSI but not to the (r, θ)-CSI; but such variations are to be expected and perceived by the super-CSIs.

These considerations bear on the Gauss argument mentioned at the beginning of this paper, because it is evident that Gauss made no such assumptions like the A-C statements about his observers. He assumed instead that his 2-dimensional beings had a fixed, finite size which did not vary as they moved about the spherical surface. If he had made allowance for metrical relativity, then he would have had to project the surface of the sphere unto a plane, using the well-known stereographic sphere to tangent plane projection and then his observers would have varied radically in size as they moved from the point of tangency to the pole opposite that point.

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A NEW CONCEPTION OF THE PHYSICAL WORLD (2)

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Read to ANPA 13. Cambridge University. September 13th 1991.

ABSTRACT: The substance theory derived in "A New Conception of the Physical World (1)" is developed by demonstrating its ability to shed light on a wide range of problems in fundamental physics.

To deserve the name, an explanation must take an analytico-synthetic form. Ideally, an exhaustive explanation of human experience must first succeed in analysing this experience into irreducible, self-explanatory elements, and then go on to show how, via a logically implicative process, all the entities of the empirical world arise as particular syntheses of these. The so-called scientific world-view does, in fact, recognisably approach this genuine explanatory form. Via an analysis of the commonsense world, in which atomic theory plays the central role, it has arrived both at certain ultimate particles - the quarks, leptons, or whatever - and at a few basic laws of motion determining the modes of interaction of these particles. It then seeks to exhibit all things as dynamic syntheses of these particles, emerging and enduring solely through the operation of these laws. But, for all that, as an explanation of human experience it is a near-total failure.

It can offer no reason why its ultimate particles should exist, and the little it has to say about their nature is both nebulous and self-contradictory. Worse, it is compelled to postulate, in addition, a wholly unintelligible medium, space, within which these intrinsically unchanging particles can exist and interact. It cannot tell us why particle motions should be governed by the particular laws it postulates: the connection is quite arbitrary. Nor has it succeeded in discovering any logical connection unifying these laws themselves. And if it has enjoyed a strictly limited success in deriving the entities of the physical world out of its laws and particles, this, most emphatically, cannot be said of its attempts to account for biological and - a fortiori - human organisms and behaviour. It has produced wholly convincing evidence that living organisms have emerged by natural processes out of the so-called inanimate world, but has just as surely demonstrated to the informed and impartial mind that its postulated laws and particles radically fail to account for them. And this explanatory incapacity is more than confirmed at the human level, where only intelligences rendered prematurely senile by prolonged exposure to bad philosophy could seriously entertain the notion that the limitless abundance, range, complexity, and intensity of our inner life could conceivably be accounted for in terms of motions of nonsensical particles through unintelligible space. Finally, there is the huge, age-old, and ever-growing body of evidence - occult, parapsychological, mystical - which, unambiguously implying the existence of a supersensible realm, falsifies scientific materialism so absolutely as to leave its protagonists without defence - apart, of course, from silence, barefaced denial, feeble temporising, and feebler ridicule.

If, then, the form of orthodox scientific explanation is essentially true, but its content wildly untrue, where and how did it go astray? Certainly, its basic untruth lies in its conception of the ultimate entities of the physical world: of minute, undifferentiatedly enduring bodies interacting within a spatial medium. With such ultimate building-blocks as these, all attempt to achieve a rational account of the world is doomed from the outset. But these ultimate elements have been arrived at via an exhaustive analysis of the everyday world, so that it is within this analysis that we must look for the fatal errors. And only a very modest level of neuropsychological and epistemological sophistication is needed - but all too seldom attained - in order to locate, expose, correct, and transcend them.

The naive realist, and that effectively includes the man of science, is rightly convinced that he directly perceives an objectively existing, external, world, but falsely predicates of the entities of this world attributes that belong, not to them, but only to his own perceptions of them. In a word, he confuses the object of his perception with his perception of the object: mistakes for object what is, in reality, subject responding to object. He perceives the world, but he does not experience it; he experiences only himself. Hence, the true datum ab initio of philosophical inquiry is very definitely <u>not</u> a world of enduring bodies of various qualities and properties interacting within three-dimensional space, but only an ever-varying flux of affectively toned sensations (including, of course, somaesthetic) and memories, out of which, for sound biological reasons, we have gradually - in both the ontogenetic and phylogenetic sense - constructed this world. When, with this truly psychologically primitive content as our raw material, we perform an exhaustive logical analysis - such as I attempted in my contribution to ANPA 12 - we arrive at ultimate elements very different from those of scientific orthodoxy, and from which the logical derivation of a rationally coherent world-conception, incorporating without strain the whole of human knowledge and experience, becomes a perfectly feasible enterprise.

This true analysis discloses the existence of no undifferentiatedly enduring particles, nor any spatial medium through which anything moves. Instead, it discovers only qualification sequences, where every sequence (and all possibilities exist) is a logically generated succession of qualified simples, each of which is either an absolutely simple One, or its Negation, differently qualified. Qualification may be loosely defined as 'the change in something when the existence of another is taken into account', and the absolute One as 'self-realisation'. Schematically represented there is only:

FIGURE 1

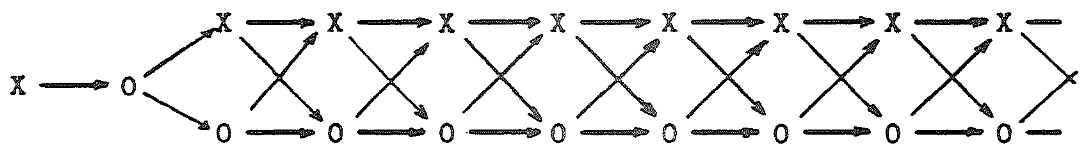
This process, generating 2^{n-1} qualification sequences by the nth instant, is what we know empirically as the temporal process. Last year, using empirical data, we calculated that one instant = 1.880×10^{-23}s.: that is, there occur 5.319×10^{22} consecutive simple events in one second. All complex entities are <u>natural groups</u> of qualified simples: simples which belong together through their very natures as particular parts of the total generative process. They are thus collectively something more than an arbitrarily chosen number of simples. And every such group is self-selected from the totality by virtue of its sole possession of some particular attribute.

Our Cosmos must be such a natural group which, by virtue of its size and complexity, might better be termed a system. In order to bring to light the nature of this system we are obliged to work from two directions: attempting to integrate the rational evidence of our ontological schema with the empirical evidence provided by physics.

THE TEMPORAL PROCESS

X THE ABSOLUTE

0 NULLITY



INSTANT 1 2 3 4 5 6 7 8 9

2^{n-1} QUALIFICATION SEQUENCES GENERATED IN n INSTANTS

FIGURE 1

(In this talk we shall not be proceeding to the biological level.) At ANPA 12 we made a tentative start on the construction of this model, and intend today to go - no less tentatively - somewhat further into detail. But first, for the benefit of those who have not read my first paper, I will restate one or two fundamental points.

To the extent that sequence numbers remain constant, the principle of selection, whatever its nature, must, in effect, be choosing between an X and a 0 at every instant on every sequence. Our physical knowledge leads us to postulate that, at every instant, every sequence is participating in selecting the continuation of every other; but that, as between sequence and sequence, the relative strength of this selective effect (which we identify with force) is precisely graded. Gradation of selective effect <u>is</u> the ontological essence of distance. Moreover, we assert that the time which elapses before this selective influence takes effect is directly proportional to the distance. So that we have force $\propto \frac{1}{\text{distance}^2}$ (squared, because, then, the product of number of sequences and distance away is constant for all distances) and distance $\propto$ time. In the first case the constant of proportionality has the dimensions of a charge squared, and will, in fact, be equal to e^2, where $e = 4.803 \times 10^{-10}$ e.s.u.; in the second it has the dimensions of a velocity, and is, of course, equal to c, where $c = 2.998 \times 10^{10}$ cm.s^{-1}. All cosmic sequences we conceive to be of the form n(2X,0) or n(X,20), where n may conceivably be of any integral value at any time on any sequence. The former we shall identify with positrons and the latter with electrons, though it could equally be the other way about. Hence, the assumption by n of different values implies that the frequencies of consecutive X/0 alternation of the electrons and positrons are changing, though the X/0 ratio (the charge) remains always at 2. And it is precisely these changes of frequency which are being brought about by the interselective influence, which is thus wholly electrical in nature: like (X - X and 0 - 0) charges repelling, and unlike (X - 0) attracting. So that the selective influence (force) is exerting a synchronous dual effect: It changes the X/0 frequency of the sequences even as it changes the rate of change of their distances (magnitudes of mutual selective strengths) apart. There will be a precise correlation between these two effects: $\frac{\text{rate of change of distance } (v)}{\text{frequency } (f)}$ = constant. This constant has the dimensions of a distance, and is equal to the absolute minimum distance (d), which we derived at ANPA 12, and which = 5.636×10^{-13} cm., the classical diameter of the electron.

FIGURE 2

There is also a certain proviso to be registered respecting the meaning of 'frequency'. The frequency, f, I am referring to above is what I term the location frequency, and this need not be equal to the frequency of alternation, f_A. It may take, according to circumstances, any one of three values: f_A, $\frac{n}{n+1} f_A$, or $\frac{n}{n+2} \times f_A$, where n is the duration, or period (in instants) of one X/O alternation, and when n = 3, it also takes the values $\frac{n}{n-2} \times f_A$ and $\frac{n}{n-1} \times f_A$. The following diagrams should make this clear.

FIGURE 3

Before proceeding further with this model I should like to quote a few highly relevant sentences from the late Guy Burniston Brown's brilliant pioneering work "Retarded Action-at-a-distance" (Pub. 1982). On p.73 he writes: "If wave-mechanics had been thought of as a force-frequency theory, a genuine physical basis might have been found for the explanation of the phenomena, so that it would not be just "a dodge and a very good dodge too" (as Eddington called it). We might then consider more carefully the idea that all force is a series of very high frequency impulses - of attraction or repulsion.

The author has made a very tentative indication of the way in which this hypothesis could be used to give a physical explanation of electron 'diffraction'..." Here, p.137, are his opening sentences: "The hypothesis of action-at-a-distance is acutely concerned with the evidence for the wave-like behaviour of matter. It is well-known that in the days of the mathematical theory of wave-mechanics, developed with the aid of analogy, an attempt was made to give the wave-function ψ a physical meaning, and that this has failed, many writers falling back on what is merely a verbal subterfuge by saying that matter has a 'dual character'.

1 unit of time (instant) = 1.880×10^{-23} s.

1 unit of distance = 5.636×10^{-13} cm.

Maximum frequency (F) = 5.319×10^{22} Hz.

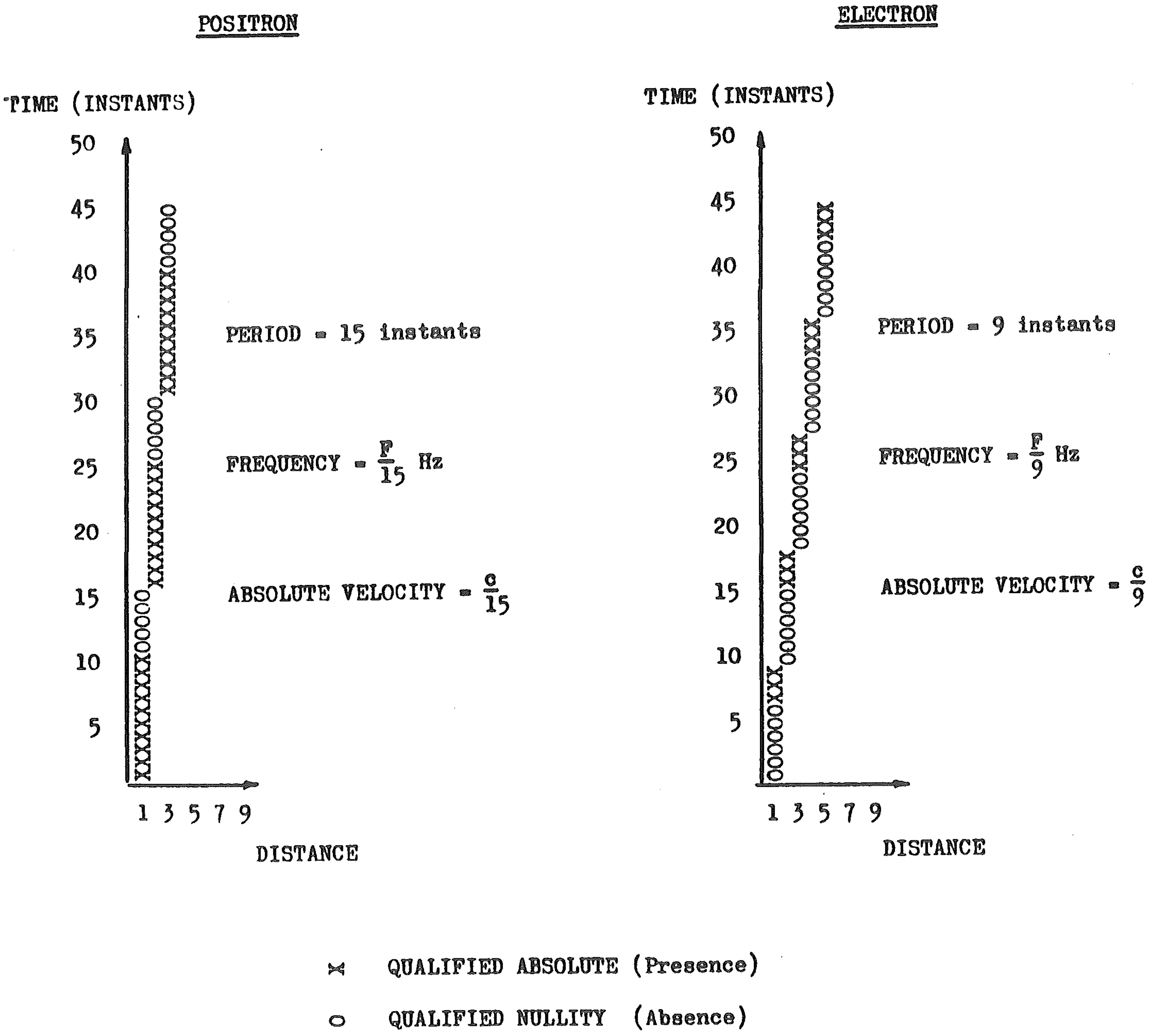


FIGURE 2

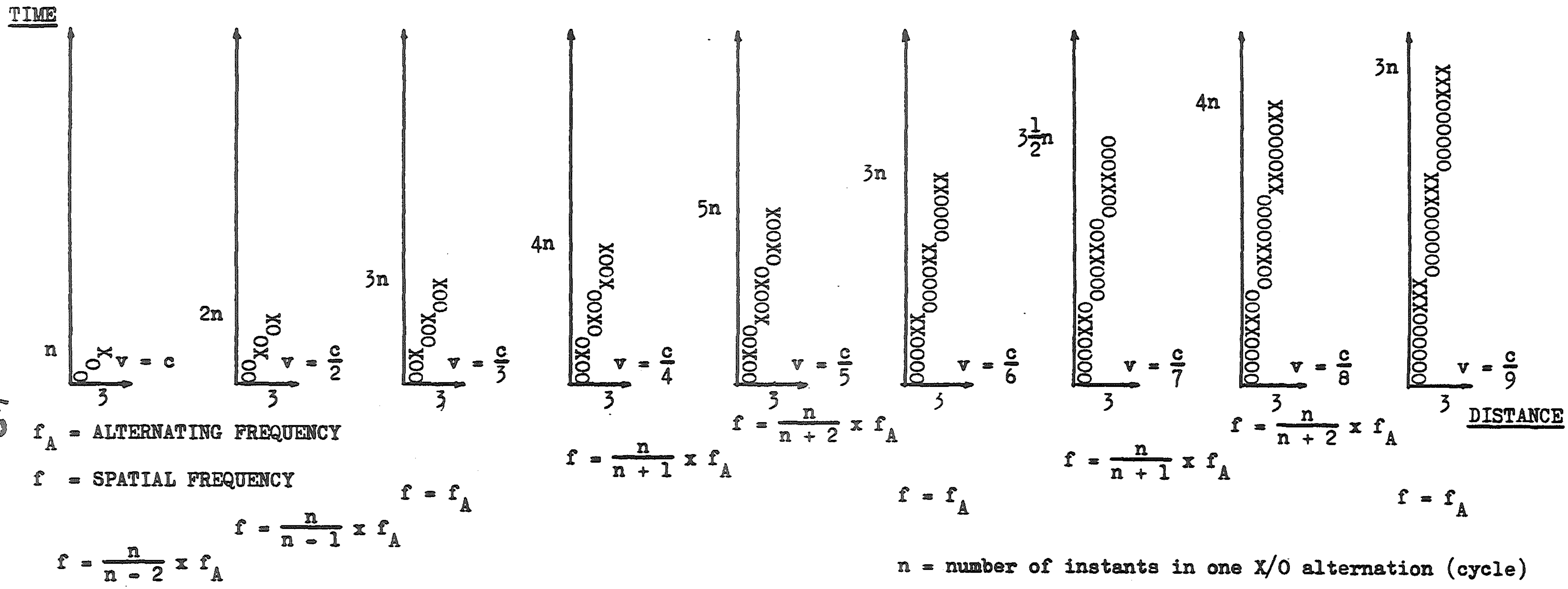

TIME
DISTANCE
n
2n
3n
4n
5n
3n
3½n
4n
3n
v = c
v = c/2
v = c/3
v = c/4
v = c/5
v = c/6
v = c/7
v = c/8
v = c/9
3
f_A = ALTERNATING FREQUENCY
f = SPATIAL FREQUENCY
f = n/(n - 2) x f_A
f = n/(n - 1) x f_A
f = f_A
f = n/(n + 1) x f_A
f = n/(n + 2) x f_A
f = f_A
f = n/(n + 1) x f_A
f = n/(n + 2) x f_A
f = f_A
n = number of instants in one X/O alternation (cycle)

FIGURE 3

On the theory of retarded action-at-a-distance and no ether, the only way to deal with the phenomenon of electron diffraction is to suppose that interaction is intermittent; that is to say, the force between two particles is not continuous but periodic." He takes the de Broglie frequency, $\frac{mvc}{h}$, as his frequency of intermittence and goes on to produce some initially very fruitful results. Nowhere does Burniston Brown put forward a matter theory, but he does make one statement (p.73) which shows which way his thoughts are tending: "It [that is, action-at-a-distance] is a retarded interparticle force that does not require 'waves'; and being purely mutual between two particles, any 'quantum' effects are due to the individual structure of the particles, and not to any 'quantisation' of 'radiation'."

<u>Energy and frequency.</u> Kinetic energy $= \frac{1}{2} mv^2$, so that, for an electron (assuming constant mass) K.E. $\propto v^2$. But, in our model, absolute velocity of electron (v) $\propto$ electron frequency (f), so that K.E. of electron $\propto f^2$, or $E = Kf^2$. This constant is easily evaluated from our data, and can be expressed as various combinations of fundamental constants; for example: $\frac{\alpha\hbar}{F}$, or $\frac{1/2mc^2}{F^2}$, or $\frac{e^2}{F^2 d}$, where F is our maximum frequency of 5.319×10^{22}Hz. The last is the most physically illuminating, since it is expressed entirely in concepts basic to our model: charge, frequency, and distance. This constant $K = 1.447 \times 10^{-52}$g.cm^2, and has the dimensions of a moment of inertia. With this relation in mind we turn to the atom, where the basic quantitative connection between energy and frequency is provided by the formula $\Delta E = h\nu$. In what follows we are, incidentally ignoring the misleading notion of potential energy, and identifying energy entirely with kinetic energy. In any case, since, for orbiting electrons, total energy is numerically equal to kinetic energy, magnitudes are unaffected. Since $E = \frac{\alpha\hbar}{F} \times f^2$, then $\nu = \frac{\Delta E}{h} = \frac{E_i - E_j}{h} = \frac{\alpha}{2\pi F}(f_i^2 - f_j^2)$, where f_i and f_j are the sequence frequency of the electron orbiting in the ith and jth levels respectively. Moreover, there is a simple formula connecting sequence frequencies (f_i) and orbital frequencies (ν_i). It can easily be shown that $\nu = \frac{i\nu_i - j\nu_j}{2}$, giving $\nu_i = \frac{\alpha}{\pi F i} \times f_i^2$.

<u>The fine structure constant and the Bohr diameter.</u> Before we leave the subject

of atomic levels, there is another interesting relationship worth noting. $\frac{v_j}{v_i} = \sqrt{\frac{r_i}{r_j}}$. Now, imagine this relationship holding while the atom shrinks to the size of an electron. The diameter of this shrunken atom will be the electron diameter of 5.636×10^{-13} cm. = our minimum distance (d); and we have made the basic postulation that a unit charge attracted from infinity to another of opposite sign will have an absolute velocity of c at this unit distance. Therefore, $\frac{v_o}{c} = \sqrt{\frac{d}{2a_o}}$. But $\frac{v_o}{c} = \alpha$. Therefore, $\alpha = \sqrt{\frac{\text{unit distance}}{\text{Bohr diameter}}}$, or Bohr diameter = $\frac{\text{unit distance}}{\alpha^2}$. A fundamental unsolved mystery about the atom is: why should an electron hold off from an attracting proton (but not the equally charged positron) when it is still the Bohr radius away from it? Surely the answer must lie somewhere in the structure of the proton. Is a clue afforded by the circumstance that the so-called 'strong force' operating in atomic nuclei, and, therefore, presumably, within the proton, is $1/\alpha$ x the coulombic force which operates between proton and electron?

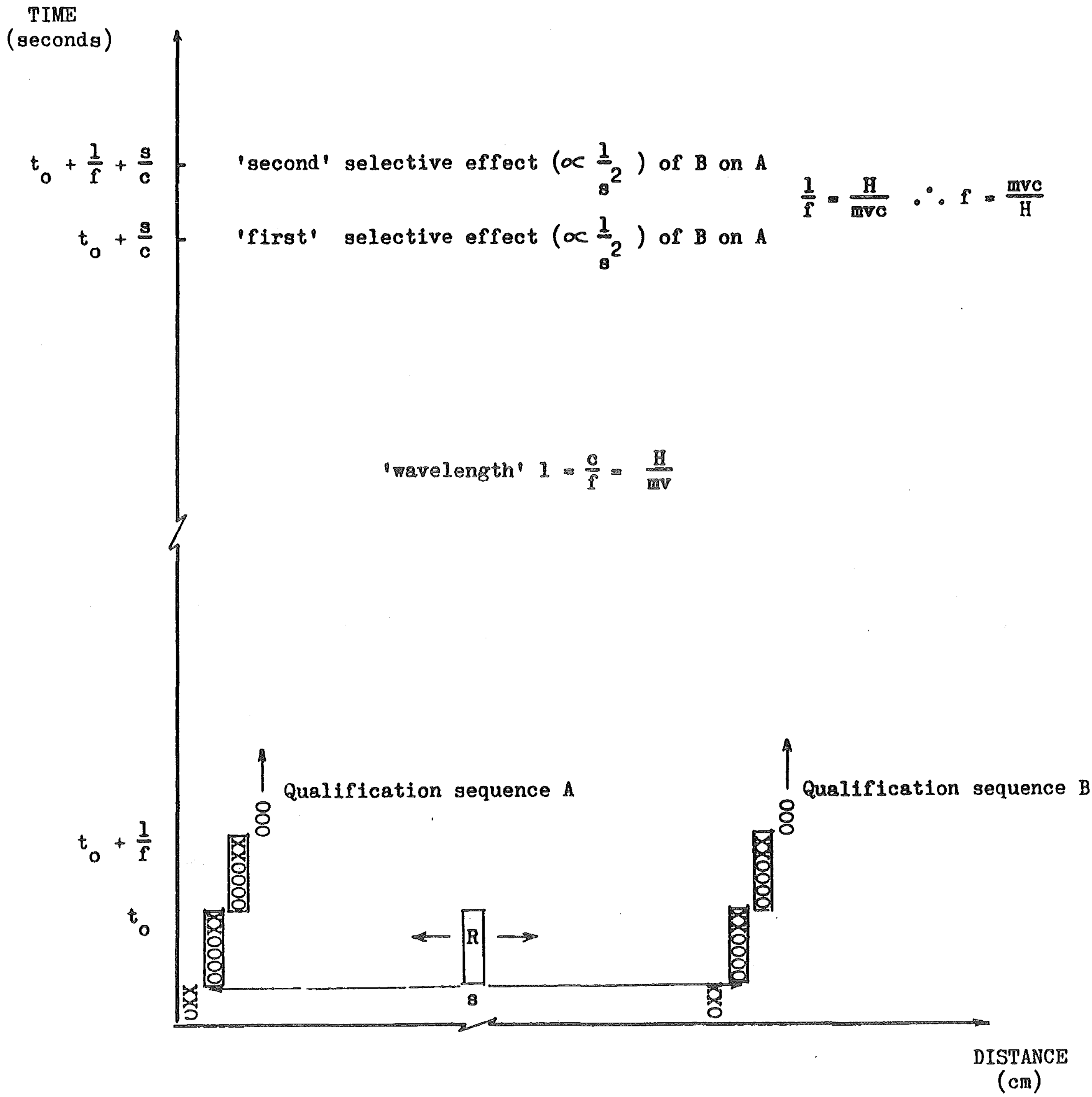
The unit of action. Action has the dimensions of mass x velocity x length. The accepted basic unit of action $h = 2\pi m v_o a_o$. Yet h enters essentially into formulae for electron waves and frequencies. It seems illogical that a unit defined from the atom should be essential for a quantitative description of the electron, or, what really amounts to the same thing, that a fundamental unit should be defined by means of a combination of non-fundamental units. We postulate a smaller unit of action, H, as the ultimate unit. Since action = mass x velocity x length, we would expect this ultimate unit to be the product of the fundamental units of these dimensions, or $H = m \times c \times d$. This works out at $H = \frac{\alpha}{\pi} \times h$, or 1.539×10^{-29} erg-s. Moreover, Energy = Action x Frequency, and it is easily shown that $mc^2 = H \times F$, or $H = \frac{mc^2}{F} = \frac{\text{twice maximum kinetic energy}}{\text{maximum frequency}} = \frac{2e^2}{Fd} = 2KF$.

The de Broglie wavelength. Louis de Broglie postulated that a wavelength λ is associated with an electron such that $\lambda = \frac{h}{mv}$, where v is the velocity of the electron. If this wave, as seems logical, is travelling at the speed of light, then $\nu = \frac{mvc}{h}$. It is readily shown that our electronic frequency, f, which, as we shall shortly see, is also the frequency of interaction, $= \frac{\pi}{\alpha} \times \nu$. But $H = \frac{\alpha}{\pi} \times h$. Therefore $fH = \nu h$. Hence, $fH = mvc$, and thus for our frequency of interaction, $f = \frac{mvc}{H}$. Since the 'wavelength' of these pulses of interaction (that is, the distance between successive pulses) $l = \frac{c}{f}$, then we immediately obtain $l = \frac{H}{mv}$. So that when the substitution is made of $H = \frac{\alpha}{\pi} \times h$ for h, the de Broglie wavelength and frequency arise naturally in this conception.

Radiation as pulses of interaction. We envisage the n simples contained in each period - the time for which the sequence is in one spatial location - as constituting one unit, or pulse of interaction. At a time $\frac{s}{c}$ seconds subsequent to t_o, the instant of completion of the pulse, these n simples interact synchronously on a one-one basis with their n exact contemporaries on all sequences s cm. distant. Distance apart of successive pulses will thus be c x period, or $\frac{c}{f}$. Therefore, 'wavelength' $l = \frac{c}{f} = \frac{cd}{v} = \frac{H}{mv}$, and frequency of interaction $= f = \frac{mvc}{H}$. The interaction will exert its selective effect $\frac{s}{c}$ seconds subsequent to t_o on the sequence containing the pulse. That is, we conceive interaction as operating on an outgoing principle, in contrast to the incoming radiation of the mechanistic model.

FIGURE 4

Effect of increasing velocity on force and acceleration. As the velocity of an electron increases from $\frac{c}{n+1}$ to $\frac{c}{n}$, acceleration $= \frac{\Delta v}{\Delta t} = \frac{\frac{c}{n} - \frac{c}{n+1}}{\frac{n}{F}} = \frac{cF}{n^2(n+1)}$, where n is the number of instants in one period, and $cF = 1.595 \times 10^{33} cm.s^{-2}$. So that, assuming constant mass, Force $\propto$ acceleration $\propto \frac{1}{n^2(n+1)}$. Therefore, an accelerating force which is below the value needed to produce an acceleration of $\frac{cF}{n^2(n+1)}$ will have no effect at all. Hence, as n decreases, this threshold value increases. But, of course, when this threshold value is exceeded, the greater the acceleration that will be produced. Thus, for a given force, as velocity increases the fewer the sequences it will act upon, but the greater the acceleration it will produce on those upon which it is able to act. This makes for a quantitatively complex situation; and certainly the derivation of a satisfactory formula for the motions of fast particles in a given electric field has so far proved elusive.



R = repulsion

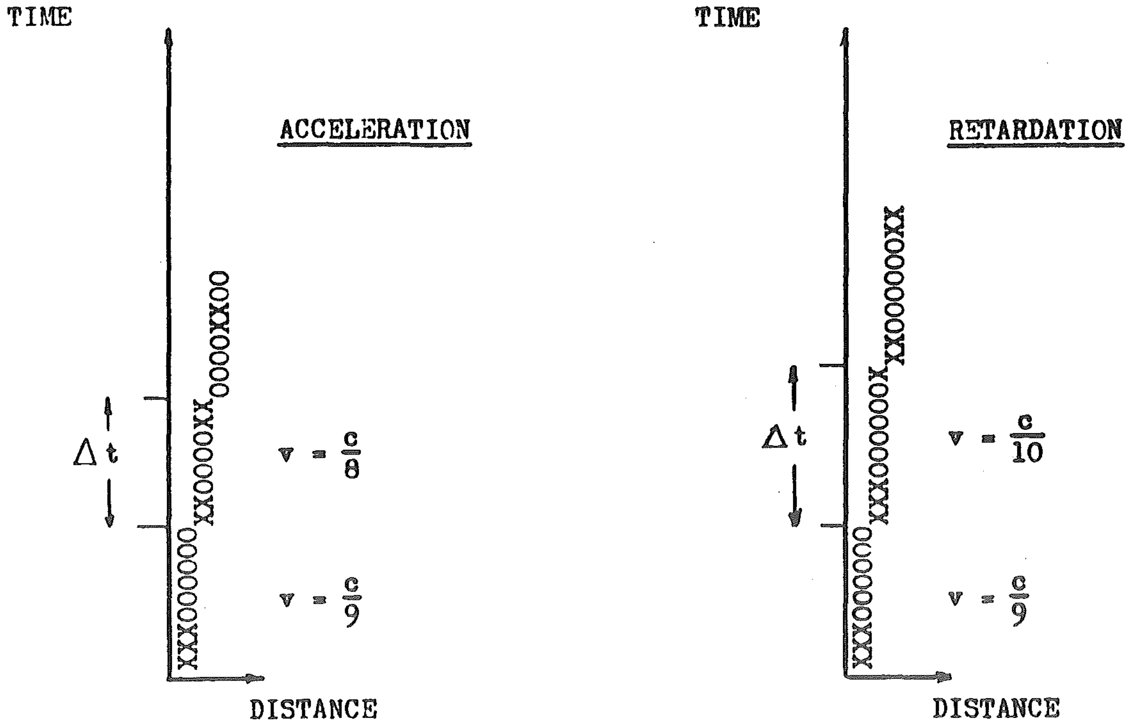
FIGURE 4

J.P. Wesley (Selected Topics in Advanced Fundamental Physics (1990), p. 268) has this to say on the subject: "Apart from the 5 meagre and uncertain data points provided by Bertozzi [1964] there is no experimental evidence available showing how the mass or kinetic energy might actually vary as a function of the time-of-flight velocity of a fast particle. There is, thus, essentially a complete lack of pertinent experimental information presently available that might reveal how fast particles with mass and charge might behave. In the absence of requisite experimental facts it is impossible to construct any meaningful fundamental theories in electrodynamics or in mechanics. Fundamental physics is today at an experimental bottle neck."

FIGURE 5

Relation between mass and charge. At ANPA 12 we obtained our minimum distance $d = 5.636 \times 10^{-13}$ cm., the classical diameter of the electron, by the following method: Acceleration $\propto \frac{1}{s^2}$, or $a = v\frac{dv}{ds} = \frac{k}{s^2}$. By integrating, and setting $v = 0$ when $s = \infty$ and $v = c$ when $s = d$, we found $k = \frac{c^2 d}{2}$, so that acceleration, $a = \frac{c^2 d}{2s^2}$. Then, using data from the Bohr atom, we went on to determine d. But we are also claiming that accelerations are produced solely by forces of attraction and repulsion due to interaction between positive and negative charges. Now, a single charged sequence has less effect than one-hand clapping; an interaction between two charged sequences is necessary to produce acceleration - that is, to constitute a force. Hence, we have Force $\propto \frac{e \times e}{s^2}$, and, because, in the c.g.s. system the constant of proportionality is made equal to unity, Force $= \frac{e^2}{s^2}$. Taking these two equations, acceleration $= \frac{c^2 d}{2s^2}$ and force $= \frac{e^2}{s^2}$ and dividing the second by the first, we obtain $\frac{\text{Force}}{\text{Acceleration}} = \frac{2e^2}{c^2 d}$. This ratio has the dimensions of a charge. We give it a special name; we call it mass, and, so far as the single sequence is concerned, this is all that mass is. Evaluating this ratio from $\frac{2e^2}{c^2 d}$ gives mass $m = 9.108 \times 10^{-28}$.

But, living in, and investigating, a universe of events, as we do, let us suppose that we had made our concept of charge, one of interaction: that is, thought of e^2 (I, say) rather than e as charge. In which case $\frac{e^2}{s^2} = \frac{I}{s^2}$ would have, like $\frac{c^2 d}{2s^2}$, the dimensions of an acceleration, and when the two equations were divided we would



$$a = \frac{\Delta v}{\Delta t} = \frac{\frac{c}{8} - \frac{c}{9}}{\frac{8}{F}}$$

$$= \frac{Fc}{8^2 \times 9}$$

$$= \frac{1.595 \times 10^{33}}{576}$$

$$= 2.769 \times 10^{30} \text{cm.s}^{-2}$$

$$a = \frac{\Delta v}{\Delta t} = \frac{\frac{c}{10} - \frac{c}{9}}{\frac{10}{F}}$$

$$= \frac{-Fc}{10^2 \times 9}$$

$$= \frac{-1.595 \times 10^{33}}{900}$$

$$= -1.772 \times 10^{30} \text{cm.s}^{-2}$$

FIGURE 5

obtain $1 = \frac{2I}{c^2 d}$, or $I = \frac{c^2 d}{2} = 2.553 \times 10^8$ units of charge. Hence, one could conceive of a physics in which mass and force did not appear. From a philosophical standpoint this is perfectly acceptable. As Burniston Brown is at great pains to point out, when Newton, for mathematical reasons, introduced the concept 'mass', he was very careful to define it as a mere measure number - a number which he took to be ultimately grounded on the number of ultimate particles composing a body - and not to conceive it as some mysterious attribute that a body possesses. To puzzle over what is meant by the mass of an ultimate 'particle' is to fall victim to the same kind of gross category error that we alluded to last year regarding the duration of an instant event. With regard to force, as Burniston Brown asserted, "In the physical world causes are forces." Now, as I pointed out in my ANPA 12 paper, apart from the actual generation of the qualification sequences, all causes are just different instances of association of the generated simples. And we have seen that the physical world is as it is because the particular sequences which compose it are <u>associated</u> by conforming to a single principle of mutual selection. This, we identified with force. But this 'force' is <u>expressed</u> as the actual composition of the sequences, as their X/O frequencies changing in coordinated fashion; so that these coordinated changes do not manifest force at all. They manifest only charge, time, distance, frequency, period, velocity, acceleration, etc. Of course, commonsense man conceives the interactions between bodies in terms of pushes and pulls: in other words, in terms of his own feelings as a causal agent in <u>his</u> interactions with bodies. But this sense of effort arises through mnemic causation, which, as I pointed out in ANPA 12, is a mode of causation other than that of mutual selective influence, and often in conflict with it, arising within the physical world, ultimately because of the cumulative nature of the temporal process, and which gives rise, via the biological, to the human world. Thus, in our commonsense conceptions of the interactions of bodies we anthropomorphise the physical world. And, of course, this formed the substance of Hume's argument in his celebrated analysis of causation, whose paradigmatic mode, true child of the Enlightenment that he was, he took to be transmission of momentum by impact. The essence of this analysis was the true claim that all we actually experience when perceiving such body body impacts is a succession of sense impressions.

Finally, before leaving this topic, let us return to the formula $m = \frac{2e^2}{c^2d}$. We obtained this formula by postulating, among other things, that a sequence attracted from infinity by another sequence of equal and opposite charge has acquired the maximum velocity c at the time it is separated from the attracting sequence by the minimum distance d. The kinetic energy of this sequence will then be $\int_d^\infty \frac{e^2}{s^2}\,ds = \left[\frac{-e^2}{s}\right]_d^\infty = \frac{e^2}{d}$. But, from the above formula $\frac{e^2}{d} = 1/2\ mc^2$. Hence, this maximum kinetic energy can, so far as numerical calculations are concerned, be expressed in either of these two forms. But it is $\frac{e^2}{d}$, not $1/2\ mc^2$, containing, as this does, the dummy symbol m, which is physically significant. In other words, $1/2\ mc^2$ is basic to modern physics, not in its own right, but only because it is numerically equal to $\frac{e^2}{d}$. Historically, the science of mechanics, because it lies closer to the phenomenal surface, preceded the more fundamental science of electricity. Hence, we tend to conceive electrical concepts in terms of mechanical. But, in terms of ontological priority, it should be the other way round.

Variation of force with phase, absolute speed and relative velocity.

FIGURES 6, 7, 8.

All these modifications of the basic inverse square force (coulombic) of attraction and repulsion between positrons and electrons due to motion (and, probably, phase) are what phenomenal science terms 'magnetic force'. As Burniston Brown says, " ... magnetic force is only the force due to interacting moving electric charges ... " (Retarded Action-at-a-Distance, p.47).

The fundamental importance of phase. We have already noted how change of phase between qualification sequences can radically affect their interaction. The accompanying tables should make it clear that the effect of phase displacement, certainly at absolute speeds greater than c/10, is both very great and very complex.

TABLE 1

The effects of phase displacement bear strongly upon such questions as the composition

O	R	O
O	R	O
X	R	X

O	R	O
O	A	X
X	A	O

1 —— PHASE —— 2

FIGURE 6

A = Attraction

R = Repulsion

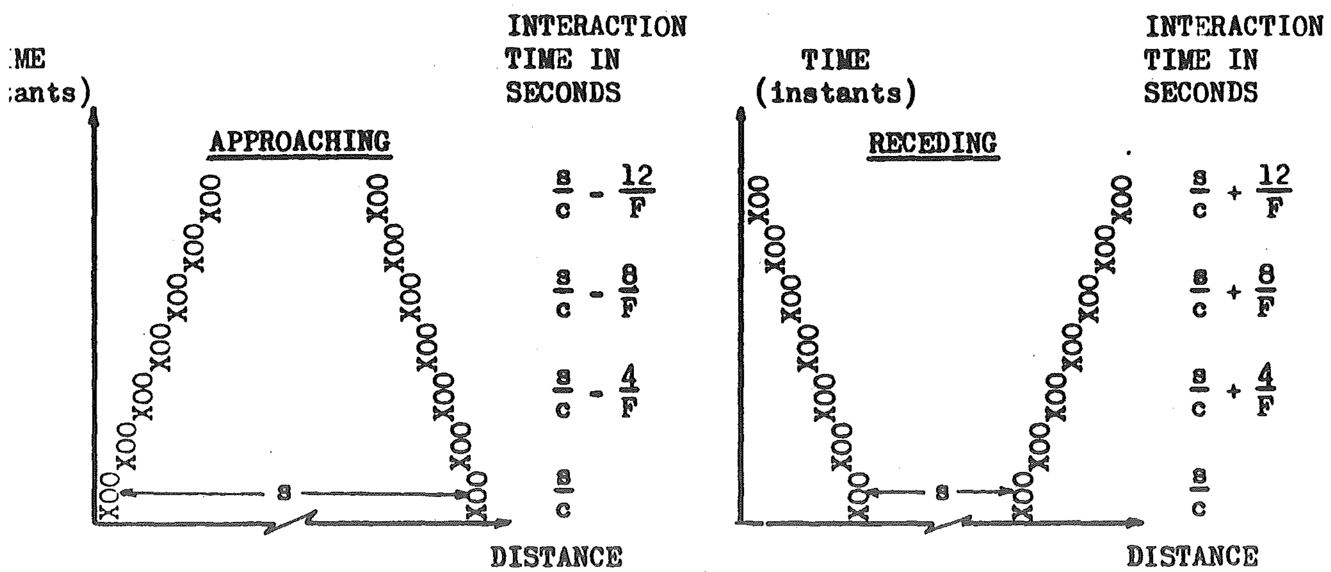
O	R	O
O	R	O
X	R	X
O	R	O
O	R	O
X	R	X
$\frac{c}{3}$		$\frac{c}{3}$

O	R	O
O	R	O
X	A	O
O	R	O
O	A	X
X	R	X
$\frac{c}{3}$		$\frac{c}{6}$

ABSOLUTE SPEED

FIGURE 7

RELATIVE VELOCITY



NUMBER OF INTERACTION PULSES AFTER N CYCLES:

	In instants (n instants = 1 period)	In frequencies (f) (F = maximum frequency)
CONSTANT	N	N
APPROACHING	$N \times \frac{n}{n - 2}$	$N \times \frac{F}{F - 2f}$
RECEDING	$N \times \frac{n}{n + 2}$	$N \times \frac{F}{F + 2f}$

FIGURE 8

TWO ELECTRONS OR TWO POSITRONS
(For an electron and a positron attractions and repulsions will be interchanged)

ABSOLUTE SPEED	PHASE DIFFERENCE				
	0	1	2	3	4
c $\frac{c}{2}$ $\boxed{\frac{c}{3}}$ $\frac{c}{4}$ $\frac{c}{5}$ AND c $\frac{c}{2}$ $\boxed{\frac{c}{3}}$ $\frac{c}{4}$ $\frac{c}{5}$	0A 3R $(\frac{1}{3})$	2A 1R $(\frac{2}{3})$	IDENTICAL WITH 1		
c $\frac{c}{2}$ $\boxed{\frac{c}{3}}$ $\frac{c}{4}$ $\frac{c}{5}$ AND $\boxed{\frac{c}{6}}$ $\frac{c}{7}$ $\frac{c}{8}$	2A 4R $(\frac{1}{3})$	4A 2R $(\frac{1}{3})$	2A 4R $(\frac{1}{3})$		
c $\frac{c}{2}$ $\boxed{\frac{c}{3}}$ $\frac{c}{4}$ $\frac{c}{5}$ AND $\boxed{\frac{c}{9}}$ $\frac{c}{10}$ $\frac{c}{11}$	4A 5R $(\frac{1}{3})$	4A 5R $(\frac{1}{3})$	4A 5R $(\frac{1}{3})$		
$\boxed{\frac{c}{6}}$ $\frac{c}{7}$ $\frac{c}{8}$ AND $\boxed{\frac{c}{6}}$ $\frac{c}{7}$ $\frac{c}{8}$	0A 6R $(\frac{1}{6})$	2A 4R $(\frac{1}{3})$	4A 2R $(\frac{1}{3})$	4A 2R $(\frac{1}{6})$	
$\boxed{\frac{c}{6}}$ $\frac{c}{7}$ $\frac{c}{8}$ AND $\boxed{\frac{c}{9}}$ $\frac{c}{10}$ $\frac{c}{11}$	8A 10R $(\frac{1}{3})$	8A 10R $(\frac{1}{3})$	8A 10R $(\frac{1}{3})$		
$\boxed{\frac{c}{9}}$ $\frac{c}{10}$ $\frac{c}{11}$ AND $\boxed{\frac{c}{9}}$ $\frac{c}{10}$ $\frac{c}{11}$	0A 9R $(\frac{1}{9})$	2A 7R $(\frac{2}{9})$	4A 5R $(\frac{2}{9})$	6A 3R $(\frac{2}{9})$	6A 3R $(\frac{2}{9})$
	1st	2nd	3rd	1st	2nd
	PHASE				

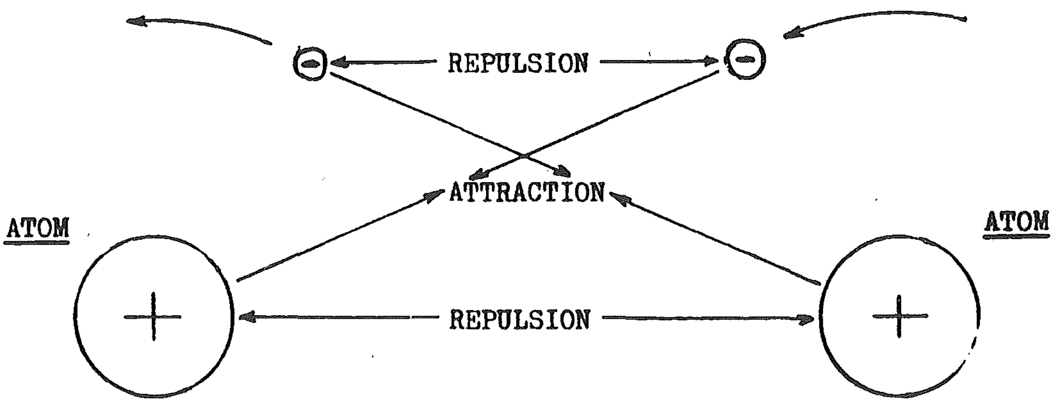
TABLE 1

of the atomic nucleus, annihilation and pair production, proton electron repulsion, the exclusion principle, the proliferation of sub-atomic particles, and doubtless much else. The main points to note are (i) that, at certain phase displacements at certain absolute speeds, like sequences can attract and unlike repel; (ii)that, at any speed, when all phase displacements are taken into account, repulsion between like sequences always outweighs attraction, and, similarly, overall attraction always outweighs repulsion between unlike charges; (iii) that there is no gradual change in force with phase displacement at these high speeds: it is often large, and sometimes a reversal; (iv) that equal numbers of electrons are, and permanently remain in, one of three possible phases, with positrons similarly fixed in these <u>same</u> three phases. There is thus universaal phase lock.

<u>Gravitation as residual electrical attraction.</u> We have seen that different phases, absolute speeds, and relative velocities affect the magnitude of the force operating between two sequences at any given distance apart. And although atoms are, on the whole, electrically neutral, their equal numbers of ultimate positive and negative elements are far from being in balance as far as these three attributes are concerned. Hence, one would not expect attractions and repulsions between two atoms to cancel each other out exactly. We find, in fact, that there is a <u>very</u> small overall attraction of the order of 10^{-37} x the coulombic force, and which, like that force, obeys the inverse square law. This residual electromagnetic force is gravitation.

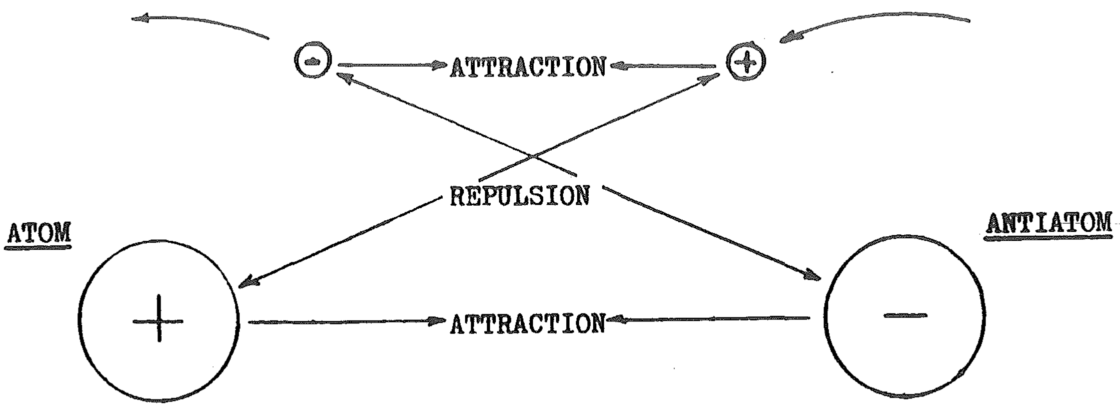
<u>Antigravitation and cosmology.</u> We cannot think of any reason why the different atomic roles played by positive and negative constituents should not be reversed - why there should not be a negative charge at the centre of orbiting positrons.
A cosmological theory developed by the two Swedish physicists, Oskar Klein and Hannes Alfven contends that the cosmos is not just symmetrical as regards numbers of ultimate charges, but also, broadly speaking, in the spatial arrangement of this charge: that there are regions of the cosmos occupied by antimatter even as our neighbourhood is by matter, and in which, of course, matter is as rare as antimatter is here. It is

a very attractive theory, making - so I would claim - a far greater intellectual appeal than the big-bang. Its weakest point has always seemed to me the lack of an adequate mechanism for separating matter and antimatter. But the gravitational theory I have just touched on puts this mechanism in our hands. If two neutral atoms exert an overall minute mutual attraction, then, when these attractions and repulsions are exactly interchanged, as they will be for an atom and an antiatom (Figures 9&10), there must be an overall minute mutual repulsion - antigravitation, in fact. Under the action of this mutual repulsion, hydrogen and antihydrogen forming in the same region, would gradually separate out.



MINUTE OVERALL ATTRACTION

FIGURE 9



MINUTE OVERALL REPULSION

FIGURE 10

THE SEQUENTIAL PARADIGM

(A Constructionist Analysis of Relativistic Action-at-a-Distance)*

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ABSTRACT

It is suggested that nothing less than a radical revision of physics-language is necessary to solve those 'action-at-a-distance' paradoxes which have become the bane of modern physics. The source of these paradoxes, it is argued, is the customary description of 'the speed of light *in vacuo*', which is shown, in the context of relativity, to be both unempirical and self-contradictory.

As the editor of a philosophical journal, I once visited CERN, in Geneva (around 1975), where I had a discussion with the late John Bell. He told me he had once been an Arts man like me, but had since 'crossed the floor' from Arts into science. 'This', he said, is 'where it's at! There are huge problems, here,' he said, 'which need to be addressed, problems of a *philosophical* nature rather than of run-of-the-mill science. So, where the hell are the philosophers?' he complained. 'What are they playing at? They should be here, in droves, helping us!'

Some time afterwards, with this sentiment of Bell's stewing away on my mental back-burner, I read Ted Bastin's 1977 article in *The Encyclopædia of Ignorance* [1] which caused me to move my stew-pot from the back of the stove and put it right on the fast ring. What gave me this increased sense of urgency was the following passage from Bastin's article. 'I shall argue,'

* Also circulated at the International Conference in Memory of John Bell: 'Bell's Theorem and the Foundations of Modern Physics', Cesena, Italy, 7th -10th October, 1991, by kind permission of ANPA.

he wrote,

> 'that there are two paradigms discernible in this current situation which are incompatible. One, which I shall call "the classical paradigm", is so familiar in its application that it practically constitutes physicists' thinking as we have it at the moment. The other - which I shall call "the sequential paradigm" - had been forced into existence by our experimental knowledge but has no background of thinking out of which it naturally emerges. The latter is the one we need in our present situation; the former is the one which alone we can think with.'

If this is not a central philosophical problem, I thought, then I don't know what is! Thus galvanised, and taking this passage of Bastin's as the articulation of the problem to be addressed, I mobilised myself to answer Bell's call for philosophical reinforcements. This meant that, like Bell, I had to 'cross the floor' from Arts into Science; but, unlike him, I did not seek formal Science qualifications. I decided instead to serve Science as a <u>philosopher</u> - as what John Locke called a 'philosophical underlabourer to science', tidying-up the conceptual matters and leaving the technical details to those properly qualified to deal with them. In this ancillary role I have shown, I hope, in a recent paper,[2] that this new and well-termed [3] 'sequential paradigm' of Bastin's is not entirely without philosophical backing. As I have analysed it, there is a tradition of philosophy, starting with Locke and Berkeley, leading via Hume and Kant to the ideas of Ernst Mach and the post-positivistic school of Ayer and the later Wittgenstein, which all but anticipates that new, experimentally obligatory, way of thinking.

However, my purpose here is not to elaborate on the history of that movement. It is to show that, as always, whenever some revolutionary paradigm-revision takes place, there is a crucial, fairly simple conception upon which that revolution hinges. In the case of the Copernican revolution, for example, that hinge-conception was the shift from thinking of our earth as the centre of the planetary system towards thinking of it as just one of the planets orbiting the sun. That was neither a highly technical nor a highly mathematical thing to achieve. It could have been done by almost anyone - except, of course, for the sheer social inertia of the prevailing paradigm. In the same way, all we need, in order to facilitate the paradigm shift Bastin talks about, is to rid ourselves of the problematic conception of the 'speed of light *in vacuo*' and replace it with some more suitable description of the facts.

However, that 'speed of light' concept is as central to our ideas of the world and our relation to it as the earth was to pre-Copernican cosmology, so the very suggestion that there may be no such thing as the 'speed of light in space' seems to us not just absurd but outrageously absurd. Let us see why.

The 'classical paradigm', Bastin writes,

> '... has its basis in the idea of a continuous background of space and time, which are imagined as perfectly smooth, perfectly homogeneous, infinitely divisible continua, mathematically modelled by the continuum of all real numbers as it was finally formalised by Dedekind and Cantor between 1870 and 1880. Physical entities are located within these continua of space and time. The basic entities are particles – idealised as single points whose position in space changes continuously and smoothly with time – and fields – distributed through space, and at each point of space having a certain intensity which again varies smoothly both with time and with changes in the special point at which the field is considered. ... Indeed, the classical paradigm has become for the physicist a mathematical elaboration of common sense and the automatic vehicle of his thought.'

Now if, during the heyday of this classical way of thinking, some physics-student had asked his professor where the *observer* or describer of this paradigm state of affairs was located, he would have created instant puzzlement. What is thus described, he would have been told, is simply the *universe*, which is everywhere, all at once. The describer is incidental. He is some tiny and insignificant part of that universe, along with other observers and co-describers. They all co-exist, in the same present moment, whether known to one another or not. So what does it matter who describes it, or from where?

Okay, says our awkward student, so how do we *know* that the 'universe' exists in that unified and simultaneous way? The standard reply would have been that this knowledge is gained from information received by our sense-organs and other instruments via mediating agencies like light and sound. Then, surely, says our awkward student, if each of us is no more than a tiny, insignificant part of that universe, observing it in that way, then how can we presume to refer to it as though it were something we knew in its entirety – as a *universe*, that is? That is one of the great unanswered and unanswerable mysteries of the classical paradigm. How can we, who are inescapably finite, presume to speak as though we comprehended the infinite?

A small number of philosophers, however, have discerned that this postulated 'universe' of the classical paradigm was never more than an institutionalised and peculiarly dogmatic projection of human imagination. What science *ought* to have said was, 'We are finite beings who have *some* knowledge, though never complete knowledge, of things around us; knowledge which may, nevertheless, be continually revised and enhanced by exploring those things and their interrelations with the best means available.' Nothing is gained – and undoubtedly much is lost – by *prescribing*, in the manner of the classical paradigm, the nature of what is to be discovered – which has the same sort of absurdity about it as the song in which Columbus tells the King of Spain he is setting out to discover the United States!

So it is scarcely surprising that the shores on which science has now washed up are nothing like those of the classical prescription. The classical paradigm arbitrarily conceived 'the universe' as a stretching-to-infinity of matter separated by void, but it is plain that physical bodies distanced in that way continue to influence one another. They orbit one another and behave in ways which are consistent with the laws of overall conservation of energy, angular momentum and so on; but how can they do that with a *void* – literally a *nothing* – between them? So, what physics gave with its one hand it had to take back with the other. It gave us a real, self-extended void, and then filled that void with all sorts of 'fields' and 'æthers' in order to reinstate physical continuity. And so began that process of conceptual proliferation which makes theoretical physics the tangled web of 'force', 'field' and 'action-at-a-distance' concepts which it is today.

So, to the question of how we can know about the existence of distant objects, the answer classical physics gave was that those objects – or processes within them – somehow 'disturb the æther', which disturbances are conveyed to the observer in ways similar to those in which sound travels in air or water. And, of course, when the constant ratio, *c*, of distance-units to time-units was discovered, what was more natural than that it should be interpreted as 'the speed of light' in the æther – or '*in vacuo*' as opposed to its speed in refracting media? Another logical adjunct to the æther theory, accordingly, explaining *how* these waves were conveyed was the electrodynamics of Faraday, Maxwell and Hertz, with its mediatory, 'bumpety-bump'-type explanation of light-propagation in the form of electromagnetic waves (then called 'æther-waves') at the characteristic speed *c*.

The crisis for the classical paradigm began, of course, when it was realised that the existence of this universal medium of interconnection – call it 'æther', 'field', or whatever – should be detectable by motion with regard to it, in the way that the motion of a ship, becalmed, can be detected by any eccentricity in the propagation of rings with respect to a pole poked into the water. But we now know, of course, thanks to the famous experiments of Michelson and Morley, that in fact, no motion of the observer with respect to any such medium can be detected, that we are as badly-off measuring motion with respect to an æther as measuring motion with respect to a pure void.

So, as it began to be realised at the time, something was terribly wrong with that standard way of thinking. The most illustrious device for averting the necessary radical upheaval was, as we all know, Einstein's theory of relativity. But as the notoriously abstruse character of that theory attests, it has over-interpreted the relevant facts. What nature seems to have been trying to tell us, in its plainest terms, was that those dimensions of distance and duration which characterise physical phenomena are not the dimensions of our fancied 'voids', 'fields', 'æthers' or whatever, but *dimensions of those informational systems we call observations*. What nature was saying, in effect, was, 'Look, those dimensions which you see in things are *relative* to you, the observer, in the way that colours and the feel of things are relative. All I do, objectively, is to supply the *information*, in ultimate quantum bits.. What you do with that information – how you process it, interpret it, project it, report it to one another, write it up and so on, is a *relative* matter, to be decided in communication between you and your fellow observers.'

However, a straight radical switch from the classical approach to physics into that relativistic, or observer-centred mode would have been too traumatic for the prevailing paradigm. Nevertheless, that the dimensions of distance and time are relative had become an inescapable consequence of experiment and was bound to have been established sooner or later by one theory or another.[4] Meanwhile, for sociological reasons, some kind of fudging was necessary. So in Einsteinian relativity the old absolute background of 'the universe' tacitly remained, preserving the comforting illusion that nothing had really changed. That is why 'relativity', since the advent of Einstein's theory, has come to mean something peculiarly – not to say abstrusely – different from what is ordinarily understood by that term. The true fact which had emerged, defining what Bastin calls the 'sequential' alternative, was that *all* separations between events, both those we call distance and those we call time, are, uniformly, *observational* and are all, uniformly, *time* separations, all measured in the same arbitrary units of seconds – or in metres divided by the constant *c*, which is the same thing. Time thus ceases to be the absolute, classical, one-dimensional sequence of events and becomes, instead, an intercommunicational network of otherwise completely independent, *relative*, *four*-dimensional systems.

As interpreted by Einstein and Minkowski, however, these relative time-systems were incorporated within another kind of 'universe', a geometrodynamical super-æther, or super field-continuum with absolute dimensions, not of space and time but of *space-time*, of which all the 'relative' spaces and times were regarded as variously orientated three-dimensional slices. This deterministic interpretation of 'relativity' was made even more abstruse by the fact that the axioms of the theory were couched in terms of the classical 'speed of light'. But this 'speed of light' was now a very strange 'speed' because although it was a *finite* speed it was also an *unreachable* speed (for anything except light) and, if this were not sufficiently confusing, it was the *same* speed for all *differently* moving observers.

Meanwhile, the quantum theory, as Bastin says, has produced a formalism which has strikingly different implications from those of both the classical paradigm and the 'relativism' which has been added to it. This should, ideally, lead to a new 'purely sequential' way of thinking, which takes full and proper account of quantum discreteness, contiguity and indeterminacy. A sequence of discrete, indeterminately occurring events has a certain probability or improbability; and improbability (*pace* Shannon [5]) is a measure of *information*. In classical continuistic/deterministic physics where, by definition, there was no inherent improbability, then neither could there be any inherent information. And because observation is essentially informational, this makes a paradox even of the fact that we observe anything at all.

The same applies, of course, to the neo-classical 'space-time continuum' of Einstein and Minkowski. By contrast, in the sequential paradigm, the information inherent in the improbability of directly observed and discrete quantum occurrences is what constitutes physical *phenomena*. Phenomena are, thus, not the dissipated remnants of 'light-rays' propagated in the depths of space, as in classical physics, but are the immediate and pristine sources of information from which our knowledge of the world is constructed. The information they contain and which we directly observe forms the 'bottom line' of any truly empirical science.

From the viewpoint of the sequential paradigm, then, those quantum elements of light which we call 'photons' *do not travel*. They simply *occur*, in observation, with no discernible end nor beginning of the sort we imagine they have in classical space and time. In short, in the classical paradigm, photons travel. In the sequential paradigm they don't. And that, I would say, is the central difference between the two paradigms.

In any case, the sheer contradictoriness of the classical 'travelling' conception of light in Einsteinian relativity is easily demonstrated. As every first-year student of relativity knows, the proper time of a photon travelling at the so-called 'speed of light', c, is zero, as is shown here:

$$t_{Proper} = t_{Relative}\sqrt{(1 - (c^2/c^2))} = 0$$

It also follows from the length-contraction formula that the photon's distance-scale shrinks to zero. So there is *no intrinsic motion of light-quanta*. And that is the contradiction in Einsteinian relativism. Our textbooks tell us that light 'travels at speed *c*', but at that so-called 'speed' the photons are intrinsically stationary. Photons are the fastest things there are, we are told, yet in themselves they register no distance and no time. What can be more self-contradictory than that? The plain and simple fact is that photons are purely *sequential*. They do not 'travel'. They are the ultimate quantum 'stills' in a motion which is entirely 'cinematographic'. A full and proper recognition of that entirely successive and unmediated, *informational*, character of observational events defines, I would say, what Bastin calls the sequential paradigm.

Now philosophers like Wittgenstein have shown how vitally our thinking-processes depend on the correct or incorrect use of language. I propose, then, that the required 'Copernican shift' from the classical to the sequential paradigm may be facilitated by revising the 'speed of light' language along these new, purely photon-sequential or 'cinematographic' lines. First let us take the word 'speed'. 'Speed' is defined as the quotient of distance by time. However, as a consequence of relativity, all unmediated optical distances are *times* in the ratio of arbitrary units *c*, so the 'speed of light' is the constant ratio of two observational *time* measures which have customarily (but unnecessarily) been measured in different units. Further, in relativity this constant applies not only to optical interactions but to *all* physical interactions whatsoever, so any specific reference to 'light', in respect of that fundamental constant becomes redundant.

However, insofar as that constant *c* retains the essential observational features of a 'speed', in the way signals (as opposed to 'photons') are 'cinematographically' propagated in observational space with time, we cannot simply erase that 'speed' interpretation from our vocabulary – the very suggestion of which, as Bill Honig pointed out to me at the conference, would be 'immoral'. So I suggest that for all these various reasons the constant time-by-time quotient which we customarily call the 'speed of light' may be more appropriately referred to as the *speed of time* – since, in every logical and factual sense of experimental physics, that is plainly what it is. There is then no more reason to think of *c* as a 'propagation *in vacuo*' in observing a sequence of distant events than in experiencing a sequence of birthdays.

In this purely 'sequential' interpretation, 'light' ceases to be a space-travelling intermediary between ourselves and physical objects and reverts to its original commonsense meaning of *what we see*, a three-dimensional *Gestalt*, or matrix of discrete quantum elements which are the unanalysable instants, or 'pixel-events', of the phenomenological 'video-display', with the same sort of informational matrix-structure as the sets of noughts and ones that we deal with in modern computer-technology. From this information, that is, from the four-dimensional patterns of distribution of these quantum elements in observation, the observer – the information-processor, so to speak – 'holographically' *projects* those dimensions of distance and time which are characteristic of physical phenomena.

At Keele University, my colleague, Anthony Osborne and I, in a cross-disciplinary Maths/Philosophy project,[6] are working on the philosophical and mathematical implications of modifying, in the manner suggested, that old and compulsive idea of 'light-speed' and its associated 'God's-eye-view' of the space in which that 'travelling' takes place. Our approach is this. We see that physicists have no quarrel, basically, with the standard laws of conservation, of energy, angular momentum and so on, for systems of particles in general. But due to the bogus idea of the 'causal limit' set by the so-called 'speed of light' it has become impossible to apply those laws other than 'locally', that is, to systems of particles on levels like, say, those of ordinary engineering. For systems of particles whose separations are astronomical, no such spontaneous overall balance, it is supposed, can be sustained because the distances are too great for any reciprocal influence limited by the 'speed of light' to pass between them. However, it is plain that this balance *is* maintained, not only locally but also non-locally – you only have to look at the ordered whirling of the galaxies to convince you of that! Besides, Bell's condition of the mathematical inequality involving measurable quantities, which has to hold if the 'locality' principle is true, has been shown to be violated,[7] demonstrating very clearly that physical laws like those of conservation apply <u>non</u>-locally as well as locally. So, in order to maintain that simultaneous balance, something 'spooky', it is supposed, must be passing between those particles 'telepathically' — at speeds in excess of light.

That this is absurd is shown by our relativistic equations, according to which anything acting at such a 'superluminal' speed would have to get there, by its own reckoning, less than instantly, faster than time, in effect, as in the nonsense-jingle:

> There was a young fellow named Bright
> Whose speed was much greater than light.
> He set out one day,
> In a relative way
> And returned on the previous night.

However, this nonsensical notion of a 'superluminal' influence is redundant, because, as we have seen, *the instantaneous connection required by the laws of conservation is assured by the (one-way) zero proper-time of photonic interaction*. This, moreover, is the only instantaneity that can be both invariantly and empirically defined. In any such photonic instant, the object and the observer are literally touching (as G. N. Lewis has described it [8]) so that each, simultaneously, feels the same quantum jolt. In fact, in sequential terms, they are the same event — the same quantum 'still' in the 'cinematographic reel'.

How, then, does a two-way (e.g. a reflected) light-signal which is instantaneous in both directions take time to travel, as in Michelson's famous toothed-wheel experiment? The answer is clear when we remind ourselves that in measuring the motion of an object over some observational distance there is not just one time by which that motion may be measured but at least <u>two</u>: one is the time registered by the observatory clock and the other is the time registered by the object itself (minus, in both cases, for conventional reasons, the distance-divided by c). I shall refer to these as the *relative time* and the *proper time*, respectively.

The photon, then, has two 'speeds'. Its *relative* speed is the observational distance s divided by the relative time s/c, which is, of course, c, and its intrinsic speed over that same observational distance is, as we have seen, $s/0$, which is infinite. In proper-time terms, the sum of the times of the outgoing and reflected photons is zero, whilst in relative-time terms the total time is $2s/c$, which is the same *as if* the signal travelled from source to mirror and back at the 'speed c, as experiment confirms. This can be seen very clearly in the Pythagorean Cone-Model of quantum relativity which Osborne and I have developed at Keele.[4] [6]

Now, we must not underestimate the sheer intellectual difficulty of getting rid, even for hypothetical purposes, of the classical 'speed of light *in vacuo*' and replacing it with a consistent conception of all physical processes, including light-propagation, as pure successions of observational quanta. The number of man-hours required to come fully to grips with this prototype theory and develop it consistently to a stage at which it can fairly compete with the prevailing paradigm is undoubtedly more than most hard-working experimental physicists can afford without some blue-chip assurance that in the end they will not have wasted their time. How would they convince their sponsors – especially if these are industrialists – and especially in the present economic climate – that manhours spent in this way would be 'cost-effective'?

I do not know where any such 'blue chip assurance' is to be obtained, especially in the present situation, where the choice of theoretical approach has become so much of a gamble. All I can say is that the mathematical consequences of relativity are, demonstrably, much easier to derive without 'light-velocity' than with it.[9] Also, having dedicated some effort to exploring, in depth, the philosophical implications of this new way of thinking, my bet is, for whatever it's worth, that if the paradigm shift Bastin talks about can be managed – and I believe it will be as soon as the old 'speed of light' language is replaced by the more conceptually appropriate *speed of time* – then we may see a new sunrise on our understanding in which the present, seemingly-insoluble paradoxes of relativity and quantum-physics will simply disperse like morning mist.

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THE GENERATION OF THE COMBINATORIAL HIERARCHY I: CH IN THE CONTEXT OF THE EVOLUTIONARY SYSTEMS FRAMEWORK (ESF).

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1. Introduction.

One persistent problem within the Combinatorial Hierarchy is, paradoxically, its generation. So far, what we have is more a formal *description* of what happens when we go from one level to the next, than a formal *explanation*.

In this paper we intend to draw insight from ESF (evolutionary systems framework) in the hope that aspects of the problem of the generation of the hierarchy will become clearer.

2. Genesis of the Evolutionary Systems Framework.

ESF was developed with the intention of dealing with the question, within general systems theory, of the ever-present lack of unifying principles among qualitatively different entities.

Imagine for one moment what would happen in the universe of infinite sets in mathematics if we erased the concept of cardinality while maintaining (in a platonistic vein) all the sets available before erasing it? How could we relate, without theory, all the qualitatively different sets, and not take the risk of relating elements belonging to sets of different cardinality? The work simply could not be done unless we went to pre-Cantorian mathematics. And even there we would need an ordering principle that would distinguish between numbers of different cardinality. The reason for the existence of these ordering principles in mathematics is that knowledge is constructed of the ability to relate and make distinctions; if knowledge in mathematics is increased, it is so relative to its ability to relate and make distinctions.

A real world was already there before human discourses. The hypothetical situation of sets of different cardinality without knowledge of them would be an analog of the actual situation of the world of the real. We are in a world about which we know very little. A typical scene in any part of our world is a number of objects, different in quality, interrelating constantly and everywhere. What ordering principle -or set of principles- will allow us to negotiate this thick web of different qualities, while maintaining a certain sense of orientation and purpose?

What we have now is pockets of knowledge of various kinds but, critically, with dramatic conceptual isolation between them. Consider then that:

(1) There is no match between our explanatory power and the enormous range of qualitatively different manifestations of reality.

(2) This may be compared to a field where there is a balance between the world of interacting objects and the world of theoretical constructs enabling their understanding and representation -in other words, where growth in the number and complexity of objects corresponds to growth in explanatory power and theoretical synthesis.

(3) This comparison helps us to measure, in a more tangible way, the immense difference between the situation in a field of coherent knowledge like mathematics as opposed to fields dealing with the real world.

The problem we need to deal with is a major and complex one: the *lack of ordering principles*. By that we mean rules (i) allowing distinctions among qualitatively different objects, and (ii) permitting the ordering of such qualitative differences.

In attempting to outline broad principles, we need, first of all, to define their context of applicability, and this itself is not a light issue. The proposed hypothesis is that, for the world of real objects, the whole physical universe, U, is the smallest context in which an ordering principle regulating qualitative differences among entities can be defined (in their coming into being as well as in their interrelations). Considering that differences in quality are the outcome of evolution, any ordering principle has to cover the full range of phenomena to which that concept applies. The "ordering" ranges in scale from the system, U, to the smallest manifestation of systemic individuality; and, in terms of evolutionary time, from the Big Bang to the very instant in which I am preparing this text and beyond.

3. ESF assumptions.

The main assumption is that the universe, if it is to be considered in the variety, complexity and, at the same time, coherence known to us, can only be understood and explained if defined as a *constructive universe*. By "constructive" I mean that variety and complexity can only be obtained by processes of manipulation and combination, operated on and by the initial, given, U. These are understood to be processes increasing the levels of differentiation as well as resulting in the accumulation of structure; those two, together with their linking factor, will be represented as subject to law.

Before going further, I wish to explicitly justify the *necessity* of having to deal, from the outset, with the *whole of U* if we want to be able later on to represent or explain any specific (evolutionary) system. The reasoning is as follows:

i) In a constructive U, the ways of constructing will be either first principles or deducible from them. Therefore, an attempt to explain any particular constructible system (e.g., an evolutionary system: ES), has to pass through the unveiling of general (universal) principles.

ii) General principles, although necessary, are not sufficient. We also need ways to reflect the unique specificity that each constructive system has. We would be facing an insoluble problem if there were no way of linking general principles with the particular, which is the stuff of actuality, and so needs representation.

iii) The link between the general and the specific is brought about by a chain of cycles: the end of one cycle being the ground from which the next cycle will emerge. In other words, apart from the initial U, there is no constructive or evolutionary system that does not have its roots in a previous, less specified, ES.

iv) Evolution of and within U, takes the form of a constructible hierarchy of hierarchies called a "metahierarchy" (Alvarez de Lorenzana and Ward, 1987). The metahierarchy is an evolutionary chain the links of which are sets of combinatorial expansion cycles (in ANPA's jargon: CH or combinatorial hierarchy).

v) Only by aiming at the most general -and deepest- principles of constructibility, will we be able to understand and explain the specificities of individual ESs. Keeping in mind the dichotomous nature of constructibility within evolution (undifferentiated vs differentiated), I have taken the view that our problem requires three basic principles upon which construction can be defined and take place: two principles of systemic unfolding (one dealing with the undifferentiated, the other with the differentiated), plus a third dealing with the way in which their conjunction will take place. These elements form the framework of ESF.

4. Undifferentiation and indistinguishability.

Undifferentiation is lack of distinctions and an *undifferentiated system* is a system that has no distinctions within itself. In the language of thermodynamics it could be said that an "undifferentiated system" is in a state of minimum entropy or maximum potential. In the language of mathematics such a state could be formally represented by a collection of indistinguishables or a *sort*.

In a previous paper (Alvarez de Lorenzana, 1991) it was mentioned that I came to the concept of undifferentiation well before I knew anything about the combinatorial hierarchy or indistinguishables. The idea behind the concept of undifferentiation is to be able to separate the *initial conditions*, on which the system has no bearing, and anything that is systemically implemented or constructed. Those "initial conditions" affect the system as a whole, while any systemic implementation is always *localized* to some degree. It is in this particular sense that it could be said that "initial conditions" define a *collection of indistinguishable components* while, on the other hand, any systemic activity presupposes some ordering relation.

This is an essential point because its consequences are so far-reaching. We only have to remember the difficulties faced in Artificial Intelligence when trying to program the most menial tasks and the dreaded *combinatorial explosion* in which it ends so frequently. The combinatorial explosion can be dealt with to a certain extent by how we define the system. The system is the functional domain and that means that the fewer interacting parts there are in the system, the smaller of an exponential we will have to carry for the number of possible outcomes. So, most of our efforts will have to go in the direction of keeping the functional domain as small as possible for as long as possible. The combinatorial explosion will always come for any situation that is sufficiently close to reality; all that can be done is to delay it. The cut-off in the CH is one example of a combinatorial explosion. But, also, the CH is one of the most (perhaps the most of its kind) efficient structure in the quest for slow growth of the functional domain.

I want to give a few pointers as to what an undifferentiated (evolutionary) system is and how and why indistinguishables are a useful formal tool for its representation. But before we do that some minimal background on ES is necessary.

First of all it is helpful to set out the general goal explicitly. We want to work with the *smallest functional domain* possible at all times. This is a must for *any* finite system that is not a closed system. In our quest we have to be aware of the fact that functional domains are affected by any relation that it is not a *unary* relation. "Unary" relations are relations of a component with itself, i.e., they are *properties* of the components and if they are carried by every component then, those properties, will affect all components and will be representational of the system as a whole. So, we want to define the system *only* in terms of unary relations in order to harness the combinatorial explosion, and that excludes any *ordering* relations which are, by definition, binary, ternary, etc.

In essence that is where the validation for undifferentiation and, *a fortiori*, indistinguishables lies. At the same time it is important to realize that this limitation in the formalization of ESs is, in fact, reflecting the very nature of the real phenomena that is being represented. That is, the problem is in no way confined to formalization: in finite systems there are finite resources; some of those resources are invested in *mapping the environment*, i.e., there is a need for the system, in order to be adaptive, to implement an internal representation of the environment within the system, which in turn has a cost in terms of system resources. One important condition for any ES to be effectively viable is to maintain at a minimum, while modelling the environment, the cost ratio of the pair resources-adaptation.

The undifferentiated components are the basic systemic *core* with which we start. There is a particular quote, where "scalar values" are defined, that I also find useful for defining undifferentiated system components:

"Scalar values represent 'the smallest semantic unit of data', in the sense that they are *atomic* -they have no internal structure (i.e., they are nondecomposable) so far as the model is concerned. Note carefully, however, that having no internal structure so far as the model is concerned is not the same thing as having no internal structure at all" (Date, 1990).

Date goes on to say:

"...a city name certainly does have an internal structure (it consists of a sequence of letters): however, if we decompose such a city name into its constituent letters, then *we lose meaning*. It is only if the letters all appear together, in the right sequence, that the meaning becomes apparent" (ibid.).

Some years ago I wrote:

"A core system is a system whose parts have a different ontological status than the system itself. That is, in order to deal with a part independent of the system we have to cross an ontological boundary below which the systemic phenomena of interest do not appear" (Alvarez de Lorenzana and Ward, 1987).

It is this boundary ("semantical" or "ontological") that is crucial as a guide-post for determining where undifferentiation *and* indistinguishability have to be situated. Having made that point clear, the next step is to determine which are the "properties" to be defined.

Because ESF is meant to be as general as possible, the properties to be chosen have to go along with that intent of general applicability. Those properties are (Alvarez de Lorenzana and Ward, 1987):

(1) *Minimal length interval* (L^*). This is the minimal length that can be resolved by the system. Anything smaller than this length cannot be registered (measured) by the system and therefore has no systemic meaning or representation and no systemic response.

(2) *Minimal time interval* (T^*). This is the minimal time interval within which any perturbation can be registered by the system. Any interval less than T^* has no systemic meaning or representation and therefore no systemic response.

(3) *System's cycle* (c). This is the combination of L^* and T^* to form a ratio to be called *cycle*:

$$c = L^* / T^* \qquad [1].$$

The cycle determines an *absolute velocity* for the system. It is a parameter that applies to all systemic activity and therefore characterizes the activity of all components in the system, whether interacting among themselves or with the environment. The cycle of an ES describes the maximum velocity at which systemic actions are defined for that ES.

(4) *Total receptive capacity* (h). This is the total amount of environmental information immediately available to an ES given the number and characteristics of its information channel(s).

(5) *System's potential* (ρ). This property represents the intrinsic potential for development of the ES. It refers to a state of highest density of the system in question or, conversely, to a state of lowest system's entropy (a good analogy in physics would be the Compton wavelength, where maximum confinement and maximum energy density of a particle coexist). The importance of ρ in the characterization of any ES is that it allows the establishment of an ordering for all ESs. Through that ordering we will be able to design and represent proper relationships between ESs of different degrees of complexity: this is, precisely, one major goal for ESF, as it was stated at the beginning of the paper.

It is time to go to the main theme of this section, that of undifferentiation and indistinguishability. For any given ES, its state of undifferentiation refers to a plurality of properties of its initial components. In other words, at the point of the system's undifferentiation can be distinguished, neither L^w or T^w, nor can its atomic components. This means that at that point there is a *threefold indistinguishability*: in space, in time and in terms of the atomic components. Quoting from a previous paper:

"...we should be remainded that there are three kinds of indistinguishability involved: component-wise, spatial and temporal. They are different and indispensable; they are not opposed nor does one reflect indistingushability better than the others; they are complementary.

"Component indistinguishability takes the form of **sameness** of system objects; spatial indistinguishability takes the form of **parallelism**; temporal indistinguishability takes the form of **concurrency**" (Alvarez de Lorenzana, 1991).

In the light of these comments and as a working hypothesis we could consider *multilayered neural networks* as a possible model for ESs. Such a working hypothesis in no way pretends to be a limitation or even a preference; rather, it is an instantiation, subject to scrutiny and discussion.

Having said that, there is still the fact that neural networks are becoming more germaine to physics as time goes by. The first connection to the field came through what is now called Hopfield networks. Hopfield, himself a physicist, wrote a paper in 1982: "Neural networks and physical systems with emergent collective computational abilities" (Hopfield, 1982) and since then the links with physics haave increased in number and variety.

5. Neural networks and Indistinguishability.

i) Linear separability.

The formal model of a neuron can be attributed to McCulloch and Pitts (McCulloch and Pitts, 1943).

An n-input neuron is modelled by a *linear threshold function*:

$$F: \{0,1\}^n \rightarrow \{0,1\} \qquad [2]$$

From all the inputs there is a *weighted sum* of the inputs to be defined by

$$S_t = \sum_{i=0}^{n-1} w_i x_i \geq \mu \qquad [3]$$

[S_t:weighted sum;w_i:individual weigh;x_i:inputs; μ:threshold]

For a neuron to fire, the weighted sum must reach or exceed the threshold. Excitatory inputs increase the sum; inhibitory inputs decrease the sum. It is easy to see that this scheme provides a way to digitized information so the computation and transmission of data can take place very simply.

Within the ANPA approach we can think of the "n" inputs as a bit-string of zeros and ones of length n. The input patterns are then classified in the usual dichotomy of ones (excitatory) and zeros (inhibitory). Moreover, the set of inputs is said to be *linearly separable* because the dichotomy is obtained by a linear threshold function. It can also be stated that a set A is linearly separable when defined by n+1 parameters: n input coefficients (w_i) plus the threshold μ.

ii) The perceptron convergence theorem.

Neural networks became interesting because of their potential for learning (i.e., the ability to isolate and recognize patterns). There is a theorem, proven by Rosenblatt (Rosenblatt, 1962) which says that: given a neural network and any linearly separable set of inputs, the network will be able to discriminate all the patterns contained in the data set, in a finite number of trials, irrespectively of the initial values of the neuron's parameters.

In the context of the CH the theorem means that, *given any level of the hierarchy and the discrimination operation, a base of independent vectors will be found for that level in a finite number of trials* (ticks, according to PU).

iii) The XOR (discrimination) operation, multilayer neural networks and 3+1 dimensions.

One interesting element about neural networks is that the XOR operation *cannot be defined* in a two-dimensional network which were the kind initially used by Rosenblatt. Multilayer (multidimensional) neural networks, on the other hand, allow the XOR operation to be defined. In fact, 3+1 dimensional neural networks are the ones which have to be used in order to define the XOR operation (any higher dimension can always be decomposed and redefined in terms of 3+1 dimensions without losing any information).

At this point I want to make two brief suggestions in relation to ANPA's work:

(a) The 3+1 dimensionality comes very naturally into multilayer neural networks. For one, we have been defining and using the XOR operation, within the CH, as if it did not have any implication or prerequisite in the body of the theory. Moreover, the XOR operation, to my knowledge, has never been related or linked to dimensionality and, far less, to indistinguishability.

(b) It seems to me pertinent to take the issue of indistinguishability more seriouly into the CH approach, particularly if we take into consideration that neural networks *are* indistinguishables. I also pointed out in ANPAWest 7 some concerns coming from information theory (which is now becoming more relevant to theoretical physics, in particular cosmology) about the fact that we are in the presence of a finite universe within which there are no energy cost-free activities of any kind, including communication between its parts, and that such a constraint can be tamed by the use of indistinguishables (see Bremermann, 1982; Landauer, 1987)

To end this section and in order to bring neural networks closer to the CH, it might be interesting to note that the input coefficients define a binary address and that the threshold defines the size of a region to which the neuron responds; in other words, the threshold defines a *distance* in the same way that the coupling constants of the CH do. So, in fact, thresholds and coupling constants are the same (at least up to isomorphism).

All the models of neural networks that I have seen so far *do not* consider a hierarchical structure encompassing different scales (from local to global). In that sense, the CH structure deals, from the outset, with a major issue in a very self-contained and coherent way.

6. General Principles of ESF.

(A) Principle of Combinatorial Expansion (PCE).

Recall that in ESF we always start with an *undifferentiated* system.

From a formal representation point of view this amounts to saying that we start with a *singleton* of some cardinality N>1. In other words, we have a collection of objects which lack an ordering relation between them. We have to remember that undifferentiation is a *threefold indistinguishability*: in space, in time and component-wise.

From a system's point of view, undifferentiation means a confinement (up to indistinguishability) in space and time of the system's atomic (i.e., identical) components: in other words, the system is, in terms of its dynamics, reduced to one locus (e.g., the analogy given for physics was the Compton wavelength).

So, PCE deals with the unfolding of undifferentiated systems. The mathematical expression will have to convey the construction of ordering relations among the collection of indistinguishable components in space and time. Correspondingly, the systemic expression will have to convey the implementation of space-time scales where qualitatively different interactions take place: beginning with the most intense and short ranged to the weakest and far reaching ones.

The mathematical expression takes the form of the CH structure. The systemic expression takes the form of development or developmental processes by means of learning (through discrimination, measurement, pattern recognition or whatever other ways) and scaling.

(B) Principle of Generative Condensation (PGC).

The CH has a cut-off rule that is manifested in the PCE by the termination of developmental processes in an asymptotic limit. Such a limit is tied to the ES in question, i.e., to a kind of linear optimization that is specifically bounded.

Progress at that particular point is still feasible: but in order to invoke the kind of change necessary (to be used), we have to go beyond optimization, as that would not do any more. The "asymptotic limit" situation requires changes in the *blueprint* of the system. And changes in the "blueprint" amount, in fact, to a *different* system altogether. Recently I related this particular situation to Gödel's undecidability theorem whose quote will be useful again here if we equate undecidability with asymptotic limit:

"In 1931 Gödel proved that in any proposed axiomatic theory of mathematics there are true sentences (theorems) that cannot be derived (proved) from the axioms. The corollary to this is that the consistency of any formalization can only be proved in a more powerful formalization. What does a "more powerful formalization" mean? It means that in order to overcome undecidability the axiomatic framework should be enriched with new axioms (or axiom). In Godel's words:

'...even disregarding the intrinsic necessity of some new axiom, and even in case it has no intrinsic necessity at all, a probable decision about its truth is possible also in another way, namely, inductively by studying its 'success.' Success here means fruitfulness in consequences, in particular in 'verifiable' consequences, i.e., consequences demonstrable without the new axiom, whose proofs with the help of the new axiom, however, are considerably simpler and easier to discover, and make it possible to contract into one proof many different proofs. [...] There might exist axioms so abundant in their verifiable consequences, shedding so much light upon a whole field, and yielding such powerful methods for solving problems (and even solving them constructively, as far as that is possible) that, no matter whether or not they are intrinsically necessary, they would have to be accepted at least in the same sense as any well-established physical theory'." (Chaitin, 1982).

In terms of specific ESs, the outcome at this point is nothing less than the *invention* of a new technology (perhaps based on experiences and dead ends of previous ones), the *punctuated equilibrium* of biological systems, new socio-political organizations, etc.

The basic idea of this principle is that components of the last level of the combinatorially expanded system (the one that has reached the asypmtotic limit) will become an enriched environment from which new atomic components will be defining undifferentiated systems, with new properties and new possibilities for combinatorial expansion.

(C) Principle of Conservation of Information (PCI).

The conjunction of the two previously mentioned principles brings about a third and last one. Cycles of development alternate with evolutionary changes and, moreover, evolutionary jumps are preceeded by differentiation; i.e., differentiation (which is the outcome of discrimination) brings about systemic development which, in turn, provides the necessary information to "design" evolutionarily new systems.

In a finite Universe, increasing complexity can only be obtained if an equivalent number of constraints is put in place; in other words, the amount of information implied by any ES should be constant. Otherwise miracles would be the main source of evolution (Alvarez de Lorenzana and Ward, 1985).

This principle, in itself reasonable, is difficult to measure in practice at present. Nevertheless, a partial confirmation of it came in 1988 through the work of J.P. van Bendegem (van Bendegem, 1988). Within the frame of a finite mathematics, he develops what he calls "quasi notational systems." Such systems have increasing complexity within a fixed operational space (or memory, if one wishes to trade the size of a sheet of paper for the size of a computer memory). In that situation there is a need for trading free memory vs amount of structure to be carried over to the next (quasi notational) system.

7. The Generation of the Combinatorial Hierarchy.

In attempting to generate the CH the most difficult - and still unresolved- issue is how to define the ground base for that generation. I will colloquially describe what I forsee as a viable option to initate the generation of CH.

It could probably be said that there is a consensus as to where we start:
"Interactions take place between a corpus of construted things and an unknown background which only manifests itself through the order in which things can be constructed." (Bastin, 1991)

"*Something from nothing*. Process takes place at the boundary between the known and the unknown" (Bastin, 1989).

These two quotes present an initial dichotomy from which the hierarchical construction stems or unfolds: on the one side there is the "unknown background" or, simply, the "unknown", and on the other side there is the "corpus of constructed things" or "known."

The relation between the two could be partially established from the definition that is given of the first entity: "...unknown background which only manifests itself through the order in which things can be constructed."

There is a precedence relation between those two entities. In fact, this could be a sufficiently abstract definition of an environment-system relationship as it was presented at the begining of this paper for ESs (evolutionary systems).If we call the "unknown" ***environment*** and the "known" ***system***, then the precedence relation goes from the environment to the system ($f: E \rightarrow ES$), in the sense that the former "manifests itself through the order in which (the latter) can be constructed." But if that is the case, then we cannot qualify that relationship as "something from nothing" because the "order in which things can be constructed" is a subtle but nevertheless substantial handout. On the other hand there is reason to talk about "something from nothing" because we get an ordering for the system from void spatio-temporal loci. That void loci we are going to call it vacuum.

In this paper it is assumed as a working hypothesis that the vacuum has a spatio-temporal ordering: that of the continuum, the reason being that, in this case, the equivalent of the *quantum of action* that would give us a kind of Heisenberg's set of scales, happens to be infinitely small. If this supposition turns out to be correct, then it could be stated that the complete absence of matter (vacuum) brings about the continuum by default. Our system, on the other hand, having a discrete and finite quantum of action h, will have to redefine (or model, or encode) such a continuum in a discrete and finite way. But because initially our system is confined to almost a point (what in cosmology is called Planck's mass and Planck's density), there is no ordering at all: it is an undifferentiated system that is composed of elements which we call indistinguishables.

So, initially (to bring in the big bang scenario), we start with a collection of components which are indistinguishable due to the fact that they are spatially and temporally confined to a very small locus (could it be said that the universe is confined to within its Compton wavelength?). The undifferentiated collection we call *system* and the ordered spatio-temporal vacuum we call *environment*, both being, respectively, Bastin's "known" and "unknown."

There is a boundary between environment and system. There is a process that takes place at the boundary. The nature of the boundary I will live to the physicits. What about the nature of the process? As implied by Bastin's comments, what takes place is a transfer of information from the environment to the system. Being void of matter-energy, no information can be drawn from the vacuum other than continuum spatio-temporal ordering. At the same time, the system, being discrete and finite, will only be able to encode any "continuum spatio-temporal ordering" in terms of its discrete and finite attributes. The spatio-temporal ordering can only be induced into the undifferentiated system as a *many-to-one* relationship.

The "ordering" to be induced is done by means of the operation called *discrimination* (Δ). This operation can only be defined among a collection of elements that are equivalent in some sense; in other words, Δ establishes differences within similar elements.

The "similar" part of Δ refers to *length interval*, or *dimension* on a bit string of some (relational) space. The important issue here is that the similarity condition for the definition of Δ, only allows interaction (dynamics) between elements or components which are within the range of the "length interval" or "dimension." This forces the interacting parts (in this case our "environment" and "system") to construct a hierarchy of levels of interactions, each of them covering a certain range of "length intervals" or "dimensions."

In the case of our CH we will have levels of 3, 10, 137, $10^{38}+137$ bits respectively, and that is where the dynamics take place within the system and between the system and the environment. In each level there is a certain number of independent categories to choose from (3, 7, 127, $2^{127}-1$) and the amount of information conveyed by any one category or linear combination of them will obviously depend on the number of categories that exist at that particular level.

As was mentioned before, if we assume parallel-concurrent systems as the architecture of undifferentiation, then it turns out that we need 3+1 dimensions in order to define Δ . That is, the operation that plays such an important role in the combinatorial physics being put forward by Bastin and Kilmister, cannot be defined in less than 3+1 dimensions as was demonstrated long ago. The reason for this necessity being that *linear separability* in neural-networks requires multilayered elements, otherwise, discrimination (or XOR) cannot be defined. Seen from our previous categorization point of view, it means that the minimal systemic condition for categorization of inputs is 3+1 dimensions. So, our minimal systemic level of discrimination requires three bits of information.

The formal mechanism we want to use in order to convey the internalization (construction) of *spatio-temporal ordering* from the environment (or vacuum) into the system, will have to reflect the many-to-one relationship characteristic of such a process.

8. The CH as a mathematical structure.

We have had for some years what could be called the *standard model* of the CH, as it was developed by Clive Kilmister: "Where did ANPA come from and what is the Combinatorial Hierarchy?" (Kilmister, 1991).

In 1988-89 Herb Doughty presented two papers (one at ANPA 10 and the other at ANPA West 5), where he introduced what he called *Finite Double Fields*. In his own words:

"A double field brings together in a single structure two algebraic fields in such a way that the multiplication of the first field is the addition of the second field. In all but the smallest finite double field, the first field is of two power order, and the second is the field of integers modulo a prime which is one less than that power of two. Any prime which is one less than a power of two is called a Mersenne prime.

"...a remarkable sequence of numbers long conjectured to contain only primes. The sequence begins with the primes 2, 2^2-1+3, $2^3-1=7$, $2^7-1=127$. The conjecture that all of the numbers in this sequence are prime became interesting in 1876 when Eduard Lucas proved that the next number in the sequence

$2^{127}1=170,141,183,460,469,231,731,687,303,715,884,105,727$ is prime" (Daughty, 1988).

Some time after becoming aware of "finite double fields" and, in particular, the fact that the addition operation in one field was isomorphic to the multiplication operation of the other field, the idea of a *transform*, as a link between levels, came to me. I mentioned the plausibility of a transform within the CH structure to Herb Doughty and David McGoveran (at ANPAWest 6, 1990); each of them answered me, separately, that it might be possible. It is a well-known fact that transforms are one of the tools most frequently used in physics, engineering, and many other fields. Moreover, transforms, because they have been widely used, do carry a considerable amount of specific and *well-interpreted* cases. In other words, if we could bring into the CH structure the "transform" instrument, it could only help ANPA in connecting the combinatorial approach to main stream physics.

I explored the classic literature on transforms. From there I went to look into *discrete* transforms (in particular Discrete Fourier Transforms). Later I went into what is called Number Theoretic Transforms and, eventually, found the Mersenne transform. [**Definition:** A Mersenne transform is analogous to a discrete Fourier transform, defined in the ring of integers with multiplication and addition modulo *a* Mersenne number. The inverse transform is similarly defined.] So, from Doughty's finite fields to the Mersenne transform.

Now, suppose we interpret as a *transform* the relationship between the addition-multiplication operations of the two fields of the "double field" structure. Can we build the CH? Most certainly not. With the transform we can switch from one field to the other, but such a mechanism does not take us any further *up* in the CH due to the fact that if we take the transform of a transform we go back to the initial field. So, iteration on the transform between the two fields of the double field structure take us nowhere beyond the next level (above, below) in the building of CH.

We need to address, specifically, the somewhat difficult issue of how to generate a level from a previous one; in other words, to go from a *description* of the generation (which -it seems to me- is all we have up to now) to an *explanation* of such a critical process.

The approach that I am proposing is based on a paper (Park and Komo, 1989) where the following theorem could be proven:

Given any level L_n of the CH ($n<4$), sequences for the basis of the L_{n+1} level can be found. That is, each of the elements of the sequence for L_{n+1} is a vector expressed as a linear combination of elements of L_n, times a set of basis elements for L_{n+1} over L_n. The autocorrelation of those sequences increases substatially if they are defined in L_{n+1}.

The paper is interesting in two respects. One, it shows that from a given field, a new one can be generated which fits the CH's characterization of consecutive levels, and therefore allowing proof of the generation of the CH structure. Two, it also gives a criterion for the pertinence of such a generation: the new (generated) elements have a *better autocorrelation* within the structure.

We have to have in mind that the context within which the CH generation takes place is a many-to-one mapping of the environment into the system, and that due to the difference in size (E>>ES), the modelling is done by stages in order to improve or redefine the model. In other words, there is a need to increase the length of the strings (here is the answer to why?) and the question is *how*.

The autocorrelation criteria is the answer to the "how." It seems to me not only plausible but interesting. In the generation of the CH we are contemplating phenomena that involve cyclicity, changes in scale and, at the same time, invariances. Autocorrelation should be relevant to all those factors.

Neural networks were mentioned in a previous section of this paper. In relation to that I want to bring attention to the following statements (Bruck and Blaum, 1989): "To perform maximum likelihood decoding in a linear block error-correcting code it is shown to be equivalent to finding a global maximum of energy function of a certain neural network." Moreover: "Given a linear block code, a neural network can be constructed in such a way that every local maximum of the energy function corresponds to a codeword and every codeword corresponds to a local minimum."

In the light of these two quotations it can be said that neural networks have a rich algebraic structure, to the point of allowing an equivalence between the two formalisms. In other words, I would like to suggest, at this point, an isomorphism between those two approaches. Moreover, it is my intention to formally prove such an isomorphism; i.e., that the CH can be stated as a codification (via a linear block error-correcting code) of an environment by a finite system (E>>ES) made up of MLT-indistinguishables. And given the equivalence stated in the before mentioned paper (Bruck and Blaum, 1989) between that particular type of coding and neural networks, it should be possible to assert their equivalence to a neural network scenario.

To finish with this particular issue, the isomorphic link between indistinguishables and neural networks does have some consequences, if only for modelling purposes.

Another element to be brought into this picture is Bender's approach to the discretization of Riemannian geometry (Bender, 1975). He encapsulates his thoughts in the following *continuum-discontinuum theorem*:

To every algebraic form describing the geometry of a continuum (in single symbol coordinates x^1), there corresponds a relation in integers (a Diophantine form) and conversely"

Bender's quantization is based on a discretization of space which itself is based on a many-to-one mapping from the continuum to the integers. Such mapping is defined so as to select from the continuum: "...certain discrete sets of integers which characterize a relevant *discontinuum* " (Bender, op.cit., p.11).

I want to see Bender's algebraic mechanism for the discretization of the space continuum as isomorphic to expressing any n-dimensional vector space by means of its corresponding *basis*, i.e., a set of n-linearly independent vectors, with which the whole vector space can be represented.

In the particular case of Z_2, Bender's proposition for discretization would take us to the CH scenario, whereby any possible value of any level of CH could be considered as suitably *compressed* in the corresponding set of linearly independent elements of that level.

Moreover, Bender even makes a connection between the diophantine form of its discretization and the Mersenne primes: the same numbers that came about in Doughty's papers on "double fields."

The last element that should be considered in this formal endeavor that I am proposing, has already been mentioned although indirectly. I am referring to the important (although subtle and difficult to convey) relationship environment-system as was defined by Bastin. How do we bring order, from the environment into the system, in this "something-from-nothing" way?

This is the point at which homomorphic deconvolution (based on the maximum likelihood deconvolution, MLD) comes into the formalization attempt.

The essence of MLD, very simply stated, is to define a given system from the output data (which is the only information available). In other words, the available data is the result of the convolution of the input (environment) signal with the system's transform of that signal (i.e., the filtering of the environmental input) by the system. It is my conviction that this is the formal equivalent of Bastin's request (recall: "an unknown background which only manifests itself through the order in which things can be constructed"). Homomorphic deconvolution deals, specifically, with the many-to-one and onto mappings, which is the case for the problem under scrutiny.

9. Work in progress.

Because the formal reasoning has not yet been fully implemented, I will present the line of thinking that supports my ongoing attempt.

I would like to think that the information gathered so far gives what I call *partial but sufficient evidence*. That is, isolated evidence on all the aspects of the problem in question. Although dispersed, the evidence is formally compatible and, therefore, "linkable" to all other partial evidence.

This means that there is, in principle, sufficient ground for a complete and comprehensible body of theory based on those dispersed but proven facts.

I. The dichotomy Environment-Evolutionary System.

1. We start with a dichotomous scenario: Environment (E) and Evolutionary System (ES).

2. A primordial dichotomy could be assumed:

-The (quantum) vacuum as the primordial E.

-The physical universe as the primordial ES.

3. We define development of an ES as *self-resemblence scaling* (Sahal, 1981).

4. The formal representation of development is given by the CH structure.

5. For any given "dichotomous scenario" (including the "primordial") a pre-condition has to be met:

The ES in question starts always at the point of indistinguishability.

6. Given any pair (E, ES), it is necessary that E>>ES, for "development" to take place.

7. Dynamics between any pair (E, ES), can be represented as $f:E \rightarrow ES$.

8. The functional relation represented by 'f' is: many-to-one and onto.

9. Colloquially it could be said that: f maps complex input patterns to simple output actions.

10. Moreover, the complex-to-simple or many-to-one mapping process allows the ES to identify equivalent classes of events and, a fortiori, to construct simple models of E.

11. From the ESF point of view, these are the minimum necessary conditions for the implementation of development within any ES.

II. The generation of the CH.

12. In order to have development (self-resemblence scaling) within any given ES, more conditions have to be met. Those conditions refer to the internal structure of the ES; more specifically, the part of the structure that has to do with the mapping process of E by ES.

13. We have assumed that ES excerts a *filtering action* on E by means of a many-to-one mapping of the inputs.

14. The outcome of that process is a simplified model of E by ES.

15. That model is, nevertheless, an enriched structure compared to the initial filter within ES.

16. In order for the scaling process to continue, the filtering structure has to be enriched.

17. In order for the scaling to be enriched according to self-resemblence, the output of the first filtering process has to become the *new* filter.

18. The "new" filter is made out of a collection of different combinations of previous (initial) filters.

19. The output of the new filter will be, itself, subject to the same process of clustering, giving, in the end, a *newer* and *richer* filter, etc.

20. Only if this buil-up of internal structure, within the ES, takes place, can we say that development is occurring and that the CH is being fully generated.

21. This process of enriching the ES filtering device induces a parallel process of refinement in the modelling of E by ES, which brings about the scaling aspect of development.

22. On the other hand, the self-resembling aspect (which could be self-similar in the mathematical sense, as in fractals), comes from the fact that the generation of each filter (what we call "a level of the hierarchy") is a recursion of the immediately previous filter (equivalently, level).

23. It could be said, in a somewhat conjectural vein, that any ES, in the course of its development, changes its morphological characteristics so as to maintain invariant its systemic identity (functional properties).

24. In an even more conjectural vein, it could be said that the CH is a formal instantiation of allometric growth (as in biology) and that the distribution of its components follows Pareto's law.

25. The *internalization* of the mapping of E by ES is proven by the mere fact that evolution and evolutionary processes exist. We don't want to go any further than this at this point.

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On the Measurement of π*

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1. Introduction

1.1 GALILEO'S MEASUREMENT OF π

The idea for this paper came to me from Stillman Drake's discussion[1] of the actual *historical* route by which Galileo arrived at his "times squared law" for free fall. What Drake shows is that Galileo found, *by measurement*, that if the time t_ℓ it takes for a pendulum of a specific length ℓ to swing to the vertical through a small arc is 942 units, then the time t_d it takes a body to fall from rest through a distance equal to that length ($d = \ell$) is 850 units. Although Galileo had no way of knowing this, we now believe that this ratio must be given by

$$\frac{t_\ell}{t_d} = \frac{\pi}{2\sqrt{2}} = 1.1107...$$

"anywhere that bodies fall and pendulums oscillate".[2] Consequently, we can now assert that Galileo's measurement of 942/850 = 1.108 to four places was the first kinematical measurement of π. His measurement agrees with the currently predicted value to considerably better than 1 percent accuracy.

Drake's discovery of "Galileo's constant", which he symbolizes by $\sqrt{g} = \pi/2\sqrt{2}$, is the result of a lifetime dedicated to painstaking research into the question of *when* and *how* and *to what accuracy* Galileo, in historical fact, arrived at his results. I do not have space here to do justice to his arguments. Drake spent many hours in "hands-on" examination of what remain of Galileo's working papers. His conclusions are supported by watermarks on the paper, when Galileo had arthritis, what it was prudent to destroy before the Inquisition seized his records, what was a fragment preserved by *literally* "cut and paste" from lost manuscripts, ... The methodology and many important conclusions were reported some time ago.[3] Some important conclusions rely on recent measurements with reconstructed apparatus. Equally important investigations of the records of Galileo's telescopic observations are relevant in demonstrating once again that the founder of *experimental* physics was a superb observer. I hope those of you who have not yet had the pleasure of reading this scientific "detective story" will be motivated to take a look at it.

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1.2 CONTEXTS FOR THE MEASUREMENT OF π

I call Galileo's measurement *kinematical* because it involves the measurement of time in conjunction with length. The first *geometrical* measurement of π is lost in the mists of prehistory. The Bible quotes a value of 3, which sometimes makes trouble for fundamentalists, and even state legislators who are influenced by them.† The success of Euclidean Geometry established the presumption that π for perimeters, areas, and volumes is the same. So far as I know, David McGoveran was the first to suggest[4] that in Discrete Physics these three numbers are conceptually distinct and subject to empirical measurement; his argument is still implicitly geometrical. On the other side, once the concept of *mass* is introduced, the meaning of π changes again. I call measurements of π that involve length, time and mass *dynamical.* Following Einstein, Wheeler's geometrodynamics relates mass to the curvature of space, and freezes the universe into a static 4-space. I think of his theory as "geometrostatics" in contrast to our constructive, context-dependent approach which necessarily introduces multiple connectivities. These connections cannot be "flattened out" for the same reason that parallel processing computers of sufficient complexity cannot be reduced to a single Turing machine. Penrose[5] seems to be unaware of this latter fact.

Drake's analysis suggested to me that, just as our finite and discrete theory has a natural unit for velocity, it *also* has a natural unit for *acceleration.* This further suggests that, once physicists get used to the idea of accepting π as a context-dependent *empirical* number on the same footing as c, it will become easier to convince them that coupling constants and mass ratios can be computed from general structural requirements. This paper is a first effort in that direction.

† Toward the end of the nineteenth century my father, when he was Professor of Chemistry at Rose Polytechnic Institute, once testified against legislation that had been proposed in the Indiana state legislature which would have required π to be exactly equal to 3 throughout the State!

2. Kinematical Units

2.1 Some Remarks on Dimensional Analysis

Dimensional analysis can start with the observation that, historically, the unit in which any physical quantity is measured is arbitrary. The units are chosen initially for convenience in measurement, and only as theory develops are comparisons made. Today these comparisons are customarily carried out in terms of theory-laden "fundamental constants". Physicists are used to the scale-invariant Newtonian system of units based on mass, length and time. But this, too, is arbitrary. In his excellent book on metrology, Petley notes that in different branches of physics and engineering more than three units may be useful and are in fact employed. In addition to mass, length and time, electrical engineers are accustomed to use charge, or some equivalent, as an independent dimensional concept with an independent unit. Petley finds that up to seven dimensional units[6] may be employed in standard contexts.

A fact that is often ignored in dimensional analysis is that measurement of zero or infinity is *impossible*. One way to build this fact into the methodology is to base measurement on *ratios* of finite quantities. Prior to Galileo, such ratios were always taken between quantities of the same logical type. In their rigorous mathematics, both Galileo and Newton used the Eudoxian theory of proportions, drawn from the paradigm of length ratios in Euclidean geometry. For instance, in the measurements mentioned in the first chapter, Galileo took the ratio between two times. It was only later in his work that he took the critical step of taking the ratio of a length to a time, and allowed this velocity to pass through all values starting from zero, or diminishing to and increasing from zero. Newton, following Galileo, allowed velocities to pass through zero without changing their direction. This is one way to extend the Euclidean concept of a *point* to space-time. Historically these continuously varying quantities which can include zero led to the conflict over "infinitesimals". Operationally, finite measurements in classical physics remained restricted to the comparison of finite ratios.

In a scale invariant theory based on the calculus, there was no conceptual difficulty in using continuous quantities represented empirically by measured finite ratios, once the calculus itself was given acceptable mathematical rigor. The situation in special relativity is usually represented as specifying a maximum or limiting velocity, but this is not the only way to talk about it. In a conventional relativistic wave theory in a dispersive medium, or for relativistic deBroglie waves, c is the geometric mean between the phase and the group velocity: $c^2 = v_{ph}v_{gp}$. One way of looking at the EPR "paradox" is to note that *causal* information transfer (forward light cone) is limited by the group velocity, while space-like *correlations* involve a supraluminal phase velocity. These distant coherent effects are no puzzle in the classical electromagnetic wave theory for dispersive media, and need be no puzzle in relativistic quantum mechanics if one accepts the deBroglie wave dispersion theory as a brute fact. In a sense, the absence of a material model for deBroglie wave dispersion need be no more puzzling than the absence of a material model for the electromagnetic ether. One is represented by a universal constant with dimensions L/T and the other by a universal constant with dimensions ML^2/T. Operationally, one can cut the Gordian knot there if one wants to.

The situation changes once there is a maximum or a minimum quantity in the theory, in addition to some convenient reference value which may or may not have deeper theoretical significance. In papers presented at this conference, both Constable and Reed have exploited this fact in different but related ways. I have thought a lot about how their different approaches work, and this manuscript has profited from these considerations. Prior to the development of quantum theory, there was no reason to believe that physics required the insertion of invariant maximum or minimum quantities. In our RQM theory [Relativistic Quantum Mechanics = RQM = Reconstruction of Quantum Mechanics] scale invariance is broken by the invariant length h/mc, rotational invariance is broken by the smallest quantized unit of angular momentum $\frac{1}{2}\hbar = \frac{h}{4\pi}$ and the mass scale is set by the largest coherent mass $[\frac{hc}{G}]^{\frac{1}{2}}$. Quantization based on h/mc being the length unit looks "dynamical" in that it involves a mass parameter, but careful operational analysis[7] at the kinematic level reveals that in practice the theory depends only on h/m, and mass ratios relative to any convenient reference mass, until gravitational phenomena are discussed. So long as this reference mass parameter is *arbitrary* we can still discuss measurement in terms of length and time units, and c as the geometric mean between two unknown upper and lower bounds. We will follow this approach before breaking scale invariance.

2.2 VELOCITY-ACCELERATION UNITS

We are accustomed in theoretical physics to use arbitrary units of length L and of time T. Theoretical expressions assume that quantities identified with "length" and "time" always use the same units. Otherwise no consistent way to compare theoretical predictions with experiment would be possible. Once the limiting velocity c appears, the same assumption applies. Using c in the theoretical expressions carries the implicit assumption that when a *numerical* value is required for comparison with experiment c will be given an appropriate numerical value in those units. The units themselves remain disconnected and arbitrary. Special relativity has given unique significance to the limiting velocity c which goes far beyond its connection to the Maxwell Equations and the "speed of light". For theoretical physicists it became customary to use "$c = 1$" in theoretical discussions. This is often confusing to the uninitiated, though considerably less dangerous than the "theorist's approximation" $\pi^2 \approx 10$ for order of magnitude calculations.

The route currently taken in SI units is to use the definition $c = 299\ 792\ 458$ *meter sec*$^{-1}$. This creates an unusual metrological situation, which is mentioned in Petley's book. Neither this convention nor $c = 1$ is quite general enough for my current purpose. Noting that Drake has introduced a pure number $\mathbf{g} = \pi^2/8$ to specify a connection between length and the square of a time — the square of "Galileo's constant" — we can also specify a pure number $\mathbf{c}$, which we can call "Einstein's constant". Since these are pure numbers, *theoretical equations* — which as mathematical expressions are themselves pure numbers — can contain $\mathbf{c}$ and $\mathbf{g}$ as *arbitrary* constants. Particular numerical choices, such as $\mathbf{c} = 1, \mathbf{g} = \pi^2/8$, simplify some expressions at the cost of complicating others. The choice is different from, but just as arbitrary (until the structure of the theory is taken into account) as the choice of the length of the king's foot, the weight of his head, and the time it takes to fall to the ground in the Place de la Republique as units of length, mass, and time. My proposal is to take the basic equations for velocity- acceleration units to be

$$\frac{L_c}{T_c} = c\mathbf{c};\ \frac{L_g}{T_g^2} = g\mathbf{g};\ T_{c,g} = \frac{c\mathbf{c}}{g\mathbf{g}};\ L_{c,g} = \frac{[c\mathbf{c}]^2}{g\mathbf{g}} \tag{2.1}$$

where $\mathbf{c}, \mathbf{g}$ are *pure numbers* picked for theoretical convenience, and c, g are physical parameters in units of L/T and L/T^2 respectively. As Drake has shown, practical experiments can be designed to *test* a specific theoretical value for $\mathbf{g}$ independent of the system of units L, T. We hope the systems of units presented below will show how this notation can be usefully employed to make new connections between theoretical ideas and physical parameters.

SI Velocity-Acceleration Units

The SI system uses $L = meter$; $T = sec$ for length and time. After long discussion, it has now picked

$$c = 299\ 792\ 458\ meter\ sec^{-1} \tag{2.2}$$

as a *convention* which encapsules a host of empirical information. Of course, it is still possible to question, empirically, whether the "velocity of light" is indeed the same in different empirical situations. Only the metrological *language* relating metrology to laboratory practice has to be changed.

What is also much more obviously conventional is to define a "standard gravity" by the relation[8]

$$g = 9.806\ 65\ meter\ sec^{-2} \tag{2.3}$$

We can take this convention as defining a standard unit for acceleration. Take $\mathbf{g} = 1 = \mathbf{c}$. Then the units of time and length in this system, re-expressed in SI units, become

$$T_{c,g} = \frac{299\ 792\ 458}{9.806\ 65} sec = 30\ 507\ 323...\ sec \approx 0.966\ 719\ years$$

$$L_{c,g} \approx 0.966\ 719\ light\ years \tag{2.4}$$

In my verbal presentation at ANPA 13, I remarked that "This makes human interstellar travel within our galaxy feasible with current technology." Clive gently suggested that this remark needs elaboration. The argument assumes that (unless or until "anti-gravity" becomes a technological possibility, rendering g irrelevant) any interstellar drive will have to accelerate humans at something like g or less, making c/g the appropriate time scale. In fact, if one uses internal rocket power which delivers one g to the initial mass of the ship, and continues to deliver one g relative to the galactic frame, the remnant of the ship could reach almost anywhere in a ship-time close to c/g. But the acceleration *inside the ship* would squash the passengers flat well before that time was approached. However, using an *external* drive delivering g to the passenger compartment (interstellar hydrogen ram-jet, or the like), the ship could cross the galaxy in 20 years or so with the passengers experiencing only normal gravity. The conceptual design of ram-jets fueled by interstellar hydrogen has been discussed in terms of current technology; hence my remark.

A design of interstellar ships capable of reaching a few percent of the velocity of light is closer to current realization. Dyson has presented two designs, using deuterium bombs for the Orion-type propulsion system. The radiating design would move a community of 20,000 people at one parsec per century, and the ablating design would move a community of 2,000 people at 10 parsecs per century (1 parsec $\approx$ 3.3 light years). Since we have evolved under g, and are limited by that heritage, I find it amusing to note that if c/g were ten times smaller, interstellar travel would already be an interesting engineering topic; if it were ten times larger, most engineering schemes would almost inevitably have to wait for radically new technologies to be invented or discovered. Those who believe in the "anthropic principle", of whom I am not one, will undoubtedly take off from this fact, if they have not done so already.

Combined Galilean and Einsteinian Units

In modern notation, Galileo's law for falling bodies can be written

$$d = \frac{1}{2} g t_d^2 \tag{2.5}$$

where we use t_d rather than "t" to remind us of the experiment from which it comes. The related result for the time t_ℓ which it take a pendulum of length ℓ to swing through a small arc to the vertical, which came from Newton's dynamics, can be written

$$t_\ell = \frac{\pi}{2}[\frac{\ell}{g}]^{\frac{1}{2}} \tag{2.6}$$

Consequently, when $d = \ell$, $\frac{t_\ell}{t_d} = \frac{\pi}{2\sqrt{2}}$ as already asserted. In order to utilize this combined collection of theoretical, experimental and historical facts, Drake proposes a system of "Galilean Units" based on Galileo's constant which can be defined, in the notation already established, by taking

$$g_G = \pi^2/8 \tag{2.7}$$

which is *locally* valid in any region where g is some arbitrary acceleration which is constant over the relevant region within experimental error. This definition can obviously be extended to many more regions than the environments specified by Drake as "anywhere that bodies fall and pendulums oscillate".

In analogy to our definition of Galilean units, we define "Einsteinian units" by taking

$$c_E = 1 \tag{2.8}$$

and c some conventional or empirical value based on laboratory experience. We combine these two conventions to obtain "Galileo-Einstein" units

$$T_{GE} = \frac{2^3}{\pi^2}[\frac{c}{g}]; \; L_{GE} = \frac{2^3}{\pi^2}[\frac{c^2}{g}] \tag{2.9}$$

Since $8/\pi^2$ differs from unity by about 25 percent, the time unit is again close to a year, which makes the length unit close to a light year (by the same factor).

Centripetal Acceleration-Radius Units

Newton, starting from Galileo's parabolic law for projectile motion and the observation that the acceleration measured by Galileo is always directed toward the center of the earth, arrived at the conclusion that a projectile launched above the atmosphere parallel to the surface with a velocity $V_\oplus = \sqrt{gR_\oplus}$ would continue to move in a circle of radius $R_\oplus$ around the center of the earth with this constant velocity. This is obvious from the symmetry and the geometry of the situation once one accepts Galileo's "vector" addition of velocity and acceleration. Newton went on to draw *dynamical* conclusions from this kinematical calculation, but we need not follow his chain of thought.

Galileo, starting from laboratory measurements of space and time intervals determined what we now believe to be the dimensionless constant $\pi^2/8$ to reasonable accuracy. His methodology allowed him to do this using *arbitrary* units of length and time thanks to the Eudoxian theory of proportions. Newton's *theory* for g allowed him to connect local velocity and acceleration measurements to a "non-local" distance $R_\oplus$. I call this distance *non-local* because it can only be inferred from laboratory measurement and an *astronomical* theory based on Euclidean geometry, parallax,...

The situation just described has three natural length and time parameters, the radius r, the circumference (length of the trajectory back to the starting point) $2\pi r$, and the time to return to the starting point (period) T. It also has the locally measurable acceleration g with which we are already familiar. We can (following Newton), define a centripetal acceleration equal to g and a corresponding circular velocity, $v_{gr}^2 = gr$. One way to define the time unit is to take $L_{gr} = r$, $T_{gr} = \frac{2\pi r}{v_{gr}}$. Compared to the Galilean Units defined by Drake, we find that

$$\frac{L_{gr}}{T_{gr}^2} = 4\pi^2 g; \ \frac{L_G}{T_G^2} = \frac{8g}{\pi^2} \tag{2.10}$$

Since the length of the orbit used in the definition of v relies on the geometric "π for perimeters", and is not *locally* specified as noted above, comparison of these two kinematical measurements of π can be thought of as a test of whether the "plane of the orbit" is *flat*.

We can extend our analysis from satellites in circular orbit to planetary motion, using the velocity and distance at perihelion and the semi-major axis of the ellipse, as we discuss below. Drake tries to do this in a way that, he believes, gives him "non-Keplerian" results discussed in the last chapter of his book. In the light of the analysis which follows, we believe this claim should be treated with caution.

Velocity-Radius Units; coupling constants

Since the essential parameter in gr units is a velocity, once we introduce Einsteinian units, the situation is described by a *single* dimensionless parameter

$$\beta_{gr}^2 = \frac{gr}{c^2} \tag{2.11}$$

This allows us to relate this classical analysis directly to bit-string quantum mechanics.[9] For any rational fraction velocity $\beta = \frac{u-w}{u+w}$ any bit-string with n_0 0's and n_1 1's for which $n_1 = Nu$ and $n_0 = Nw$ will serve as a model. For any step-length interpret N as the number of times the dimensionless periodic boundary condition $\lambda = 1/\beta$ is repeated. If the bit-string is used to model a circular orbit where $\lambda = 2\pi r$, the periodicity represents the probability of an interaction which, on the average, delivers just enough centripetal acceleration to maintain the circular velocity β. For any rational fraction velocity (in units of c), the probability of an interaction occurring compared to λ steps in straight line motion at that velocity specifies a dimensionless coupling constant

$$f^2 = \beta = 1/\lambda \tag{2.12}$$

The dividing line between "weak" and "strong" interactions defined by $f^2 = 1$ is just the interaction which will produce $v = c$ for a radius $r = \lambda/2\pi$. In this way our analysis achieves universal significance, independent of the unit of length. We still have an unknown parameter f^2 characterizing specific systems.

2.3 General Kinematical Units

Once one goes beyond the conceptual fusion between space and time symbolized by $\mathbf{c} = 1$ and allows time as an *independent* component of measured (and perceived?) experience, one can conceive of kinematical theories which have dimensionless constants other than $\mathbf{c} = 1$ for the linear relation between length and time and some dimensionless value for $\mathbf{g}$ relating length to the square of a time. The earliest such system is contemporary with Galileo, and can be ascribed to Kepler — in the apochrophal sense in which Drake defines units based on "Galileo's constant". It specifies an arbitrary constant which relates L^2/T to a *Keplerian* system of units based on his Second Law. In the spirit of our previous discussion we could, in any well specified observational context, pick a dimensionless constant $\mathbf{K_2}$ called "Kepler's Constant" and combine it with Einstein's constant. Or we could start from Kepler's Third Law, or from some definition appropriate to quantum mechanics, or ... Once we have done this, the assumption that there are only two fundamental kinematical constants will have non-trivial consequences.

Units based on Kepler's Second and Third Laws

Kepler discovered (using Tycho's data) that for the planets the line from the sun to the planet sweeps over equal areas in equal times. This defines the dimensional combination, $\frac{L^2}{T}$, rather than the linear ratio and the $\frac{L}{T^2}$ ratio we have so far considered. If we were geometrically motivated, we could introduce a unit of area based on Kepler's First Law (elliptical orbits with the sun at one focus) by using the area of the ellipse (πab, with a the semi-major and b the semi-minor axis). This would give us a kinematic way to measure π for areas and compare it with π for perimeters. Rather than take this route, I use recent work on foundations with Pat Suppes and Acacio DeBarros, parts of which will be reported elsewhere.[10,11] We can now construct finite and discrete Lorentz transformations from three integers (see Appendix).

Consider a circular orbit of radius r and a minimal step-length Δr. Any minimal step between two points on the orbit which (which keeps the radius constant) specifies an isosceles triangle with base Δr and sides r. The square of the area of the triangle [using the general formula for sides a, b, c that $16A^2 = (a+b+c)(a+b-c)(b+c-a)(c+a-b)$] and units of Δr^2 is

$$[\frac{\Delta A}{\Delta r^2}]^2 = [r/\Delta r - \frac{1}{2}][r/\Delta r + \frac{1}{2}] \tag{2.13}$$

Since the time increment $\Delta t = \Delta r/v$ in units of the period $T = 2\pi r/v$ is $\frac{1}{2\pi}[\frac{\Delta r}{r}]$ we can define the half-integer $j = \frac{r}{\Delta r}$ and the integer $\ell = j - \frac{1}{2}$ with the consequence that in these units

$$[\frac{\Delta A}{\Delta t}]^2 = \ell(\ell+1)(\frac{1}{2\pi})^2 = (j^2 - \frac{1}{4})(\frac{1}{2\pi})^2 \tag{2.14}$$

We conclude that the natural unit in which to express Kepler's second law is $1/2\pi$ for ℓ and $1/4\pi$ for j. We take $1/4\pi$ to be the minimal kinematic unit for angular momentum per unit mass. In the past we have quantized bit-string physics using the invariant step length $\lambda_0 = h/mc$. Consequently, if we know the mass scale, the minimal unit for angular momentum is $\frac{1}{2}\hbar$ This gives us an alternative, but consistent, route to quantization. Note that since we computed the area, we get only the *orbital* angular momentum with maximum projection $\pm\ell\hbar$ on some reference direction. To identify the spin contribution we can derive the Dirac equation in this framework, which we have done elsewhere.[12] Once again the π which comes in is the relation between linear and circular measures of periodicity.

Since h/m has dimensions of L^2/T, we can make our constant for Kepler's second law consistent with relativistic quantum mechanics simply by taking

$$\frac{L_{K_2}^2}{T_{K_2}} = K_2^{RQM} = \frac{1}{2\pi}\frac{h}{m} \tag{2.15}$$

or with Galileo-Einstein units by taking

$$K_2^{GE} = \frac{L_{GE}^2}{T_{GE}} = \frac{2^3}{\pi^2}\frac{c^3}{g} \tag{2.16}$$

Once we have Kepler's First Law that the orbits are elliptical rather than circular, with the sun at one focus, and we generalize our version of velocity-radius units to perihelion velocity and distance, Kepler's Third Law is simply a consequence of (kinematic) dimensional analysis:

$$K_3^{GE} = \frac{L_{GE}^2}{T_{GE}^3} = \frac{\pi^2}{2^3}cg \tag{2.17}$$

Thus, once one accepts my way of combining Galileo's and Einstein's kinematics, Kepler's Third Law — although dependent on reference to well defined geometries — is only a consequence of a choice of units which are easily related to the earlier definitions. Drake's final chapter entitled "Galilean Units Today" applies Galilean units to accepted astronomical data (relative to Mercury). At least from my point of view his numerical results are no surprise, and to call them "non-Keplerian" becomes more of a semantic than a physics issue.

Kinematical RQM Units

Although m appears in my quantum version of Kepler's second law, it remains arbitrary, as indeed it must in any kinematic system. Nevertheless, the essential parameter h/m can be given kinematic significance relative to some sufficiently stable reference particle, as I have already mentioned in earlier work[7]. Take $c = 1$, define velocity by a counter telescope, and measure the double slit interference pattern in arbitrary units of length. This defines the relativistic deBroglie wavelength for that particle as a function of velocity. Lengths characteristic of other particles can be determined in the same way. Length ratios are now operationally specified. For any choice of reference particle, the constant h/m_0 can be given universal significance using Kepler's second law, as we saw above. Mass ratios remain empirical. The choice of m_0 remains arbitrary.

3. Dynamical Units: Gravitation

In conventional parlance, as used for example by Drake, kinematics involves a *description* of motion while dynamics involves a causal, and in principle calculable, *explanation* based on the concept of *force*. Mach's *Science of Mechanics* tries to banish the concept of "force" from the subject because of its residual anthropomorphic connotations. His treatment remains Newtonian, however, in that his mass ratios are based on Newton's Third Law. In recent years, I have become increasingly aware of the desirability of separating kinematics from dynamics in the discussion of relativistic quantum mechanics. As noted above, I have replaced Newton's Third Law by operationally defined deBroglie wavelengths which specify the equivalent of mass ratios. Then I must *derive* relativistic three-momentum conservation. This is done in the Appendix. It remains to give mass an absolute significance.

Thanks to the work we have done above, this is straightforward. We have already defined the minimum unit of angular momentum in terms of h/m and the shortest possible length as that which gives orbital velocity c. Taking this minimum length as $\hbar/Mc = r$ and $gr/c^2 = 1 = GM/rc^2 = GM^2/\hbar c$ we have that the limiting mass for a *coherent* elementary system is $M = [\frac{\hbar c}{G}]^{\frac{1}{2}}$, which should come as no surprise. To get this result related to the proton mass takes a little more work.*

Zurek and Thorne[13] have shown that the number of bits of information lost in forming a rotating, charged black hole is equal to the area of the event horizon in Planck areas, i.e. the *Beckenstein number*[14]. Wheeler[15] has suggested that this could be a significant clue in the search through the foundations of physics for links between information theory and quantum mechanics. If one accepts the conservation of baryon number, as attested by the *experimentally* unchallenged stability of the proton, one can argue that the proton is a stable, charged, rotating black hole with baryon number +1, charge $+e$, angular momentum $\frac{1}{2}\hbar$ and Beckenstein number $N = \hbar c/Gm_p c^2 \simeq 1.7 \times 10^{38}$.

Consider an assemblage of N proton-antiproton pairs with all quantum numbers zero which contains an additional proton; this system has baryon number +1, charge e, and angular momentum $\frac{1}{2}\hbar$. Suppose the average distance between each pair is $\hbar/m_p c$. Then the gravitostatic energy E is

$$E = \frac{NGm_p^2}{\hbar/m_p c} = N\frac{Gm_p^2}{\hbar c}(m_p c^2)$$

which is equal to the proton rest energy when $N = \hbar c/Gm_p^2$. This is analagous to the bound $N_e = 137 \simeq \hbar c/e^2$ on the number of charged particle-antiparticle pairs established by Dyson[16] when he showed that the renormalized QED perturbation series in α is not uniformly convergent. No particulate constituent of the gravitational system we envisage can escape; the escape velocity exceeds c. Yet proton-antiproton pairs can annihilate to produce Hawking radiation[17], which is not, necessarily, bound to the system. The predictable endpoint of this system, granted baryon number conservation, charge conservation and quantized angular momentum conservation is a system with mass and conserved quantum numbers indistinguishable from those of the proton. Since this system started from

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⋆ An earlier version of the next two paragraphs — SLAC-PUB-5588 — was rejected by *Phys. Rev. Letters* because it was too novel to be published as a "Comment".

$N = \hbar c/Gm_p^2$ indistinguishable pairs, the number of bits of information *lost* in this way can reasonably be called "the Beckenstein number of the proton". Of course, any particulate mass can be gravitationally stabilized in this way, if it cannot decay to lighter particles. That the proton is the lightest (indeed, the only known) stable baryon makes the identification unique.

4. Appendix: Integer Lorentz Transformations

4.1 Basic Algebra

Given three positive-*definite*, finite integers n_i, n_j, n_k with the three indices i, j, k finite, distinct, cyclic, positive-*definite* integers, i.e.

$$n_i, n_j, n_k, i, j, k, \in 1, 2, 3, ..., N;\ N\ fixed;\ i \neq j \neq k \neq i\ cyclic \tag{4.1}$$

we can define

$$t_{ij} := n_i + n_j;\ t_{ij}\beta_{ij} := n_i - n_j := x_{ij} \tag{4.2}$$

$$\tau_{ij}^2 := t_{ij}^2 - x_{ij}^2 = 4n_in_j = t_{ij}^2(1 - \beta_{ij}^2) := t_{ij}^2\gamma_{ij}^2 \tag{4.3}$$

with the consequences that

$$t_{ij}\beta_{ij} + t_{jk}\beta_{jk} + t_{ki}\beta_{ki} = 0 \tag{4.4}$$

and

$$-\beta_{ij} = \frac{\beta_{jk} + \beta_{ki}}{1 + \beta_{jk}\beta_{ki}} \tag{4.5}$$

Further, since

$$|t_{ij} - t_{jk}| \leq t_{ki} \leq t_{ij} + t_{ki} \tag{4.6}$$

we can define

$$\odot(t_{ij}, t_{jk}; t_{ki}) = \odot(t_{jk}, t_{ij}; t_{ki}) := \frac{1}{2}[t_{ij}^2 + t_{jk}^2 - t_{ki}^2] \tag{4.7}$$

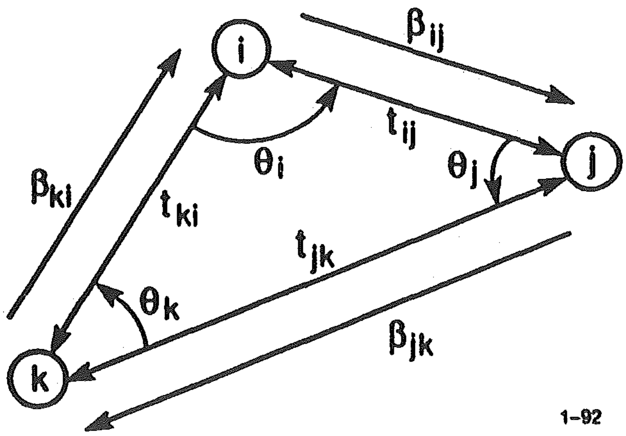
and draw a triangle (see Figure 1) with sides t_{ij}, t_{jk}, t_{ki} and angles

$$cos\ \theta_k := \frac{\odot(t_{ij}, t_{jk}; t_{ki})}{t_{ij}t_{jk}} = \frac{t_{ij}^2 + t_{jk}^2 - t_{ki}^2}{t_{ij}t_{jk}} \tag{4.8}$$

Any one side can be interpreted as a combined rotation and boost taking the position and velocity of one event to another event with respect to a third event, as we will now show.

4.2 The Three-Counter Paradigm; [The RQM Triangle]

Figure 1. Kinematical interpretation of the three integers n_i, n_j, n_k.



The figure can be thought of as three counters with associated clocks — synchronized using the Einstein convention— which keep a record of the time of arrival or departure of a signal, and whether it was a particle or indistinguishable locally from a gamma-ray. Since we are using units with c=1, the *distances* between the counters i, j, k are simply t_{ij}, t_{jk}, t_{ki}. If we launch a signal with velocity β_{ij} from counter i toward counter j and simultaneously launch a signal with $-\beta_{ki}$ from i toward k which, on arrival at k, triggers a signal from k to j with velocity $-\beta_{jk}$, the signals from i to j and from k to j will arrive simultaneously at j. This explains why, if we pay proper attention to signs, we obtain the usual Lorentz velocity addition law independent of how far away counter k is from the ij path.

Note also that our cyclic convention can be used to define a direction out of the plane of the triangle whose sign reverses either if we change our convention from cyclic to anti-cyclic or if we interchange two of the indices. Clearly this is the "parity" transformation P. In contrast to classical relativistic kinematics, our finite assumption forces us to consider transformations which do *not* conserve parity. Further if we reverse *all* velocities — which corresponds to time reversal T — this discrete transformation produces the same result as the (cyclic ↔ anti-cyclic) parity operation. Consequently the physical paradigm we use to interpret the formalism automatically guarantees that at this stage the theory is invariant under P^2, T^2, PT *and* TP. Full CPT invariance will have to wait until we define conserved quantum numbers analagous to and including electric charge. However, if we include forward or backward "motion in time" in order to define a conserved difference between the number of particles and the number of antiparticles, or left-right motion in a single direction to conserve helicity, we can immediately invoke these conservation laws to construct finite and discrete solutions to the Dirac equation in 1+1 dimensions[12].

Although, thanks to the velocity addition law derived from the usual clock synchronization convention, the paradigm obviously has an Lorentz-invariant significance, we have yet to establish formal Lorentz invariance.

4.3 Boosts; definition of c, λ_0

Although the last section interpreted the algebra of Section 4.1 as describing three synchronized counters fixed in the laboratory, it is also interpretable more abstractly as describing coordinate transformations. Consider first the connection between counter i and counter j. Note first that the separation $t = t_{ij} = n_j + n_k$ and the velocity $\beta_{ij} = (n_i - n_j)/t_{ij} := x_{ij}/t_{ij} = x/t$ only involve the two integers n_i and n_j. If we take as our referent the vanishing of these two integers, symbolized by $(0,0)$, for the 1+1 space-time integer coordinate (x,t), the square of the invariant interval between the two events at i and j is $t^2 - x^2 = 4n_in_j$ independent of the value of n_k or of the position of the counter k. If we take the counter k as the referent for both $(x',t') = (n_j - n_k, n_j + n_k)$ and for $(x",t") = (n_k - n_i, n_k + n_i)$, with invariant intervals $\tau^2_{ki;j} = 4n_jn_k$ and $\tau^2_{jk;i} = 4n_kn_i$ respectively, we see that

$$\tau^2_{jk} = [n_i/n_j]\tau^2_{ki} := \rho\tau^2_{ki} \qquad (4.9)$$

Note that this connection between these two invariant intervals is again independent of n_k and hence of the arbitrary reference system represented by counter k. Clearly $\beta = \frac{\rho-1}{\rho+1}$ is simply the boost along the $i-j$ direction which brings the event at i and the event at j to a coordinate system in which the two events are at rest. Once this is understood, the Lorentz transformation taking (x',t') to (x'',t'') is easy to work out.

Our clock synchronization convention and resulting derivation of the Lorentz transformation establishes the fact that c has the customary physical significance. Note that the unit of length is arbitrary. If we take $\lambda_0 = h/m_0c$, this corresponds to our earlier quantization assumption. h/m and mass ratios measured by a double slit plus collimators follow.

4.4 ROTATIONS; DEFINITION OF $\hbar$

For rotations, instead of an invariant interval, we need to preserve an invariant length. Recall that the square of the area of the triangle is given by

$$16A^2 = (t_{ij} + t_{jk} + t_{ki})(t_{ij} + t_{jk} - t_{ki})(t_{ij} - t_{jk} + t_{ki})(-t_{ij} + t_{jk} + t_{ki})$$

$$= 16(n_i + n_j + n_k)n_i n_j n_k \tag{4.10}$$

Take

$$t_{jk} = r = t_{ki}; \ \Delta r = t_{ij} \tag{4.11}$$

Require that equal areas be swept out in equal times. Then, if the minimum step *for rotations* (including straight line periodicities) is $\hbar/mc = \Delta r$, the area swept out by this minimal step is, in these units,

$$[(mcr/\hbar)/(mc\Delta r/\hbar)]^2 = (j - \frac{1}{2})(j + \frac{1}{2}) = \ell(\ell + 1) \tag{4.12}$$

where we have defined $j = r/\Delta r$ in order to bring out the formal similarity between this quantal version of Kepler's second law and the usual quantization of angular momentum for particles with spin $\frac{1}{2}$. The details will be presented elsewhere.

Note that this route defines $\hbar$ independent of c. Then, relative to any stable mass, scale invariance is broken.

4.5 GENERAL LORENTZ TRANSFORMATIONS IN A PLANE

Since we can now boost to a rest system, rotate, and boost to the final system, the basic problem of the Lorentz invariance of our theory has been solved. Given two arbitrary integers n_i, n_j representing events at $(0,0)$ and (x,t) connected by the velocity $\beta = x/t = p/E$ we can obviously always find a third event relative to which, in the rest system, the two distances satisfy Eq. 4.11. Taking $\Delta r = \frac{\hbar}{mc}$ gives us the unique quantum number j either for a free particle (impact parameter) or for a circular orbit. We already have the invariant interval $\tau^2 = 4n_i n_j$ giving us two of the free particle quantum numbers. The third comes from using an integer 3-space coordinate system. The reference mass remains arbitrary until we introduce gravitation, which we can do via the combinatorial hierarchy. The self consistency between the *linear* step-length $\lambda = h/mc$, the unit for angular momentum of $\frac{1}{2}\hbar$ derived from Kepler's second law, and the deBroglie relation $p = h/\lambda = \hbar k$ in fully invariant form is what convinces me that, *finally*, I have the correct elementary starting point for relativistic quantum mechanics. Working out the details will take a book, which I am writing.[18]

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The Physical Foundations of Computer Science and Conscious Machines

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Summary. The eludication of brain function as analogue computation using the recent paradigm of active wave control, is set out as the agenda for the solution to many of the outstanding problems of Computer Science. From this physical processes defining the nature of information, learning, perception, cognition, consciousness and mind follow.

The Physical Foundations of Computer Science, the remit of the BCS Cybernetics Machine Group, offers an understanding of the factors and principles governing computational efficiency. These not only provide an explanation of the workings of the transistor and the success of the various integrated silicon technologies that have followed it, [1,2] but must similarly elucidate the structure and function of brains. We believe that a sufficient basic understanding of the physical foundations exists that for example (see below) the largely empirical practice of designing high performance concurrent computer architectures can be replaced by a foundational approach, and that this approach will demonstrate an increasing convergence with the computing architectures and dynamics of biological systems ie brains.

An empirical step on this convergence has been the emergence of the discipline of neutral sets: [3] however as Francis Crick [4] has pointed out, while these nets are conceptually closer to the computational behaviour of biological brains, they are still unrealistic in important aspects. This again illustrates the necessity of an approach to the understanding of computation from a physical rather than a mathematical foundational viewpoint.

The Evidence.

Natural neutral systems, analogue devices, are superbly efficient simulators and modellers of reality. The most efficient digital circuits consume about 10^{-9} joule per operation, while neurons expend only 10^{-16} joule [1,5,6]. This is because in digital systems data and computational operations must be converted to binary code [7]. It is a process that in silicon requires 10^4 digital voltage changes per operation, whereas analogue devices require only one step for the same operation. This is a decrease in power consumption and number of operations of about 10^4.

Nor is analogue computation simply a matter of optimizing the number of computational operations. There is its capacity to represent information in context and to adapt. Analogue circuits respond to the difference in signal amplitude rather than absolute amplitude and this context of the signal may be some more complex set of signals or pattern including those that constitute learning [5,6]. This response may also be to the phase of the signals, as for example has been demonstrated to occur in biological systems by W Heiligenberg [8] in the case of gymnotiform fish, and as is essential if 50% of the information encoded in signals is not to be lost. This then would allow a fundamentally different processing paradigm by exploiting the processes of physical law. For example take visual processing.

When electromagnetic radiation in the visible spectrum v_s impinges on a 3 dimensional object o_i, an interference pattern $P(o_i,v_s)$ results. From this pattern $P(o_i v_s)$, a 3 dimensional object image $o'_i(v_s)$ can be recovered using for example a phase conjugate mirror so that o_i and $o'_i(v_s)$ coincide [9,10]. Thus a machine with such a mechanism can be defined as perceiving o_i(utilizing the complete information within the spectrum v_s available to it) by means of $o'_i(v_s)$ (as we do!)°°Such a mechanism provides awareness of objects o_i and their exact locations, but not the cognition of what they are. This however could be learnt by the machine if it were provided with the facility to store the patterns P_i and subpatterns, (for example those in v_s restricted by a filter to say blue

°° see inset page 4

or red as a means by which to taxonomize all objects o_i in v_s) and the ability to compare them. Such a comparison would then allow the machine to recognise the object o_i again or to differentiate a blue object o_i from other objects. Furthermore such a comparison process allows an object seen from one perspective to be identified from any other in a single operation (with the possible exception of a small set of perspectives that are pathological or 'illusions'). Such a analogue machine would therefore be in principle incomparably faster than any analogous digital system for visual perception, which must operate by 'building' objects from a knowledge base of all their parts to achieve each successful comparison rather than simply establishing a knowledge base of all the patterns it has experienced or perceived after the fact as the analogue machine can. That is to say the analogue machine works top down while the analogous digital system must work bottom up.

The above evidence shows that analogue computation using natural laws to implement mathematical functions rather than Boolean operations, can define a distinctly different processing paradigm in which to understand the nature of information, [11,12] perception, learning and cognition, about which is yet computer science has no agreed upon definitions, but about which biological neurons through evolution and adaption, demonstrate the capacity to process correctly and efficiently. Information in the form of 0s and 1s as used in digital systems follows from the usual classical analysis of Boolean operations which assumes there to be two standard voltages representing the local 1 and 0. It is not used in brains for reasons of computational efficiency, and because information stored in this form cannot easily reflect the dynamic 4 dimensional spatio-temporal nature of reality as for example the patterns P_i above can. [13,14]

Real time control and safety-critical behaviour.

Once again, as implied above the evidence points in favour of biological and analogue systems. In real-time control it is essential to iterate to the control problem solution or near problem solution as rapidly as possible, but as Hopfield and Tank confirm, [15] in highly complex non-linear calculations corresponding to real systems, only analogue elements capture this computational ability. Two state devices once again do not. Equivalently in safety-critical systems ,if the system canbe shown to operate safely between extremes A and B ,a control calculation must return the control to the interval. But it is known that digital computational control systems do not guarantee the continuous behaviour that this criterion requires as analogue systems can. Can it therefore be mere coincidence that biological brains compute analogically to achieve real time safety-critical behaviour as evolutionary survival indicates to be the case.

AS WE DO !

Demonstrate phase conjugancy with respect to your own perception.

Snap your fingers at some point about your head and observe the direction,intensity and location of the acoustic image the brain creates .It is located outside your head at the precise point at which your fingers snapped. Remembering phase conjugate perception as defined applies to any kind of wavefield,focus your eyes on a close object, and reach out and touch it. Your fingers tell you that the tactile object image thebrain creates,is located exactly in every particular where your eyes tell you the visual object image the brain creates,is to be found. Now tap the object and the acoustic object image the brain creates comes from there too. All object images are outside your head.
Suppose theobject is a ball in flight successfully caught. Our brains have learnt how to transform the dynamic(ie 4 dimensional) inertial object image of the ball to rest in our own reference frame by means of our hands,And once the ball is at rest, it is maintained at rest by means its gravitational object image or weight. The latter concerns the basic reference frame to which our brains habitually refer. We must ther -efore conclude that Mach's Equivalence Principle,and special and general relativity are fundamental to the way our brains simulate our models of the world, and that all the processes simulated match the phase conjugancy of natural law determining independently the same propert -ies of the objects in reality.

The Foundational Approach.

There is already a considerable body of research by means of which to understand how such analogue architectures might be constructed. Empirically very large integrated silicon analogue circuits have been constructed which for example further mimic the actual neural architectures of the retina. [5,6] And theoretically there is Gabor's universal, non-linear filter, predictor and simulator which optimizes itself by a learning process [16] recently generalized by means of the new computing principle of Fatmi and Resconi, [17] and the related abstract computing architectures based on Hugens' Principle of secondary sources. [18,19,20] All these it was recently shown by Bowden [21] contribute to the explanation of how any problem may be solved in principle by the computing architectures ånalogue and digital, [22] based on Kron's method of tearing. [23] This methodology decomposes the original problem to be solved into a set of independent subproblems, and a problem on the intersection network by means of which the subproblems overlap. It permits its solution by means of a hierarchical architecture of processors computing concurrently, and for which in principle an optimum architecture can be found. [24] It says that the nature of wave propagation itself is the principle underlying such hierarchical architecture; that there is essentially no difference between the transmission and the storage of information would seem to confirm this. [25] It infers once again that the basic nature of information concerns wave interference patterns as first elucidated by Gabor as holography. Other approaches which fit well into this picture are (a) that of Slechta of quantifying the atomic and molecular properties of materials by means of the Self Consistent Continued Fraction (SCCF) method which conceives of brains as generalized lasers; [26] (b) that of Clement of constructing a universal electronic circuit element [27] for geodesic computation [14,28] that physical principle [29] and the optimum Bowden infers, problem-solving architectures of Kron require, [24] and (c) the extensive 25 year mathematical investigations of Hoffman [30] - these are grounded in homology (analogy) theory (on which Kron's methodology is also based) and have resulted in a description of the visual cortex as a contact bundle [31] in good agreement with the neurophysiological evidence.

Quantum Mechanical Computation.

The above theoretical work is further supported by the fact that computation based on an understanding of wave phenomena and Huygens' Principle allows a natural extension into the microscopic domain [32] to Deutsch's postulated processes of universal quantum mechanical computation. [33,34] This would imply, both Feynman [35,36] and Deutsch have shown, an interface between analogue computation and its digital counterpart, shedding light on the undoubtedly most important feature that distinguishes human from other biological brains, symbol processing. But where symbols as labels, are stored as the interference patterns of objects like 0 and 1 which have nothing to do with the magnitudes of 0 and 1 respectively except where the symbols are subject to an environment or context in which the operations of arithmetic apply. For example in the case of electric currents where the conservation of charge dictates that they will add and subtract. More generally however, one may only assume with respect to 0s and 1s that the operations equivalent to exclusive OR or its converse apply, ie there is a physical operation of difference or similarity of the symbols, which follows from the empirical evidence that one can distinguish between them. [37,38] Since these rules are closely concerned with Sierpinski triangles, this implies that macroscopic physical behaviour must be closely concerned with the canonical behaviour of chaos [39] or equally with the surreal arithmetic of On_2 which is Conway's simplest surreal number field. [38,40] Both are known to map onto the integers and thus mathematically concern computational behaviours that correspond to the set of all the Turing computable functions C(T). The former by means of Zurek's information metric [41,42] which utilizes algorithmic complexity [43] as a thermodynamic measure shown to be equivalent to entropy, and the latter by means of the Combinatorial Hierarchy, [44,45] a model of discrete quantum physics giving results equivalent to the standard model of quantum physics but providing other predictions too. [46]

Furthermore the features of safety criticality and real time control captured by analogue systems are also features of (a) interval topology and hyperconvergence of analysis over the surreal number fields [47] and (b) of simultations using automata employing canonical chaos. [48,39]

However a most important implication of quantum computational machines, is a physical model of consciousness which takes such machines beyond classical paradigm. [49,50,51]

Conscious Machines

A conscious machine is defined as an analogue computational architecture as already envisaged above ie suitably prepared classical apparatus as in elementary particle physics, capable of carrying out quantum computations or generalised measurements, where consciousness is the machine's interaction with the quantum vacuum. [52] Exactly analogous interactions are the processes producing the Lamb shift of atomic spectra and the Casimir effect in capacitors. [52] These are well defined and experimentally validated phenomena even though the quantum vacuum itself is unobservable ie a zero energy (or information?) field. But they are simply second order corrections to the spectral levels and the capacitance respectively, while in the conscious machine such interactions are primary and run the machine by stealing energy from the quantum vacuum and restoring it in due time as allowed by the Heisenburg uncertainty relation. [32] And a sufficient condition for this is probably the phase conjugacy of the zero energy fields themselves such that the new definitions of nature of information, perception and cognition as proposed, continue to hold even for these zero energy wavefields. [53] Under these specific conditions, consciousness or mind/brain interaction running the machine constitutes a true ghost in the machine which as well as providing the analogue machine with the capabilities of symbol processing, also endows it by means of quantum vacuum with properties quite unlike those of any classical machine [54] as is evidentially the case in human brains. This further suggests that phase conjugate symmetry is a naturally occuring physical condition in nature appropriate to all kinds of waves and fields. This for example would allow the mass of an object o_i to be defined as its 4 dimensional gravitational object image from which both Mach's equivalence principle and

Eintein's General Relativity [55] are implied; ie the 4D 'perception' of an object gravitationally is its mass. It is a symmetry that resolves the dichotomy between general relativity and quantum mechanics at the level of the quantum vacuum and a matter therefore of some implication for both cosomology and consciousness, as proposed here.

For example quantum phase conjunction in brains computing quantum mechanically constitutes by the definition of perception given above, conscious awareness. This strongly suggests that the quantum vacuum is the source of for example both the cosmic creation [52] and human creativity [56]. These are emergent properties of the same fundamental core process evolution that we call the birth of the universe and the birth of an entirely novel idea, concept or theory. which often takes place in the process of learning as the well known Eureka phenomenon.

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Distinguishing Indistinguishables

David McGoveran

The third A.F. Parker-Rhodes Memorial Lecture

I. Introduction

When Clive Kilmister invited me to give this lecture, I thought about it for several days before accepting. In addition to being uncertain of my ability to attend this year, I was also undecided at to what I might talk about that would honor Parker-Rhodes while at the same time being a topic to which I might make some small contribution (or at least, not to babble too incoherently).

The title of this talk is some evidence of my decision to proceed; this will almost certainly not be a distinguished lecture, but it should not be indistinguishable from other lectures either. I apologize in advance if it should turn out that printed copies of the talk are not available, but I will nonetheless endeavor to keep this conceptual and philosophical so that pen and paper are not needed.

Parker-Rhodes gave great importance to the concept of indistinguishables. He found the treatment in standard mathematics to be superficial and full of assumptions. This situation caused his own work on the topic to require discussions which were subtle and hard to follow. This situation carried over into the formalism as well. So I feel that my interpretation of his work and to the extent to which I will indulge myself here may be in error, but is at least an honest attempt. This leads to a serious warning however: listen carefully and skeptically to quotations from The Theory of Indistinguishables lest you follow me too well and down the wrong path.

I do not intend to explain the Parker-Rhodes theory; that is a subject appropriate for a year long graduate course in foundations of mathematics. Instead the approach will be to introduce the key ideas behind Parker-Rhodes indistinguishables and then to tell a lot of stuff about other kinds of indistinguishables. In this way perhaps one can learn to distinguish these various sorts of indistinguishables (pun intended) when they are encountered.

II. Parker-Rhodes Indistinguishables

From a foundational point-of-view, Parker-Rhodes' indistinguishables are interesting in another way. As will be discussed shortly, indistinguishables are usually defined in terms of the properties of classes of indistinguishables.

However, Parker-Rhodes saw indistinguishables as fundamental and saw them as underlying the physical plane (as distinct from the human and organic planes). He did not think of these as being reducible one to the other, although he clearly thought of them as layered one above the other. He therefore defined two types of indistinguishables: primary and secondary. A primary indistinguishable is a fortiori strictly unobservable. A secondary indistinguishable is an 'inscrutable' meaning that perhaps they only hide their differences from the observer and could be distinguished if the observer only knew how. We will return to this distinction below.

Parker-Rhodes felt that there were two key ideas behind his indistinguishables: first, how 'identicals', 'indistinguishables', and 'unequals' might be defined, and second, the notion of a triparitous relation based on this definition. The definition of classes of indistinguishables was defined so that whether entities were the same, different, or indistinguishable depended on how they contributed to the cardinalities of the classes to which they belonged. This idea property allowed Parker-Rhodes to define classes of indistinguishables with observable properties, but in which individual indistinguishables were not observable. Part of this notion is contained in the 'triparitous' parity-relations. Thus comparing indistinguishables led not only to the results 'identical' and 'distinct', but to 'twins', 'non-identical', 'bipar', and 'indistinct'.

This same idea of the non-uniqueness of negation is found in other areas of mathematics and logic: many-valued logics, non-distributive lattices, intuitionism, etc. With Parker-Rhodes however, it formed the basis for a different kind of mathematics; he found that his notation could not be interpreted uniquely without knowledge of the context, but managed to keep his sense of humor:

"...my notation will not be 'context-free'... This introduces a serious complication into the theory -- for it is a feature of all normal mathematics that it is couched in a context-free notation... One can't hope that this will help to popularize the theory."

Parker-Rhodes was well aware of the burden that this context-sensitivity placed both on himself and on his readers:

"Difficulties of exposition make themselves felt from the first, and it is not difficult to understand why the idea of indistinguishables has been so long neglected. Quite apart from the philosophical problems...these difficulties include not only, as is inevitable, a revised and more complex axiom-schema to replace the familiar rules about the substitutability of equals, but also a whole preliminary section of analysis which can ordinarily be passed over in silence. This concerns the semantics of mathematical notations."

When it came to doing something with indistinguishables, Parker-Rhodes defined two operations: correlation and predication. Correlation consists of identifying classes of entities by matching their respective cardinalities with known physical concepts. Predication consists of predicating numerical values to the class based on the correlation; for Parker-Rhodes this involved a much more complicated process.

It seems that Parker-Rhodes was also aware of the literature in regard to indistinguishables and found it wanting:

"It is therefore at first sight surprising that there exists no branch of mathematics, in which a third parity-relation, besides equality and inequality, is admitted..."

"The concept of what I here call 'indistinguishability' is not unknown in logic, albeit much neglected. It is mentioned, for example, by F. P. Ramsey ... who criticizes Whitehead and Russell... for defining 'identity' in such a way as to make inditinguishables identical."

As we shall see, Parker-Rhodes ideas are distinguishable from other definitions of indistinguishability.

III. CLASSICAL INDISTINGUISHABILITY

The notion of indistisnguishability is precluded from having a fundamental role in set-theoretic mathematics. The definition of a set forces each member to be distinct. This at once places the discussion of indistinguishables on a level different from that proposed by Parker-Rhodes.

There are a variety of ways in which one normally encounters indistinguishability in classical mathematics and logic. Perhaps the most common idea is that of an equivalence class; a class of entities are said to be equivalent under a particular relationship and thus form a class. This notion is easy to work with and is found in set theoretic discussions of indistinguishability. In this case, the indistinguishability is not fundamental; it is an abstraction based on ignoring the properties which make the members of the class distinct.

A variation on this kind of indistinguishability is found in statistics. Boltzmann statistics aries because the 'boxes' are equivalent under the distribution function. Variations occur in Bose-Einstein and Fermi-Dirac statistics where it is the equivalence of certain properties of the entities that are questioned rather than an equivalence under the counting operation.

IV. MULTIISETS

A number of attempts have been made to remove the restriction of member distinctiveness from set theory; these attempts are usually called multiset theories. Among the efforts along this line have been Zohar Manna's work on 'bags' and Wayne Blizard's work on an axiomatic multiset theory. Both Manna, et.al., and Biz ard follow the usual practice of defining the properties of the collection rather than the properties of the atoms of the collection. Manna defines the ways in which 'bags' can be combined -- a kind of combinatorics for classes of indistinguihables. Bliz ard follows the practice of defining the inference rules for multisets: this leads to rules, for example, for substitution of indistinguishables in place of the rules of substitution for equals.

While these approaches are highly satisfying from the formal point-of-view, I suspect they would leave Parker-Rhodes with the feeling that the cart was before the horse; if one thinks of indistinguishables as fundamental, it is disconcerting to be able to define them only after 'distinct' elements are defined.

V. RELATION THEORY

An approach which was pointed out to me by Pat Suppes involves defining a relation between atoms which is slightly different from an equivalence relation. In particular, this relation meets the usual requirements for an equivalence relation (symmetry, transitivity, and reflexivity) except for transitivity.

In some sense this is more satisfying that the classical and multiset ideas; it does deal directly with the atoms. But it like the other approaches, it leaves one without any understanding of how one arrives at the decision that two atoms are distinct or equivalent under some relationship.

VI. ORDERING OPERATOR CALCULUS

Without engaging in too much discussion, I would like to set forth some of the ideas underlying one more concept of indistinguishability. This is one I know more about since I have made it up as I went along. It's my own idea.

Like Parker-Rhodes, I think of indistinguishability as fundamental and consider the possible relationships between indistinguishables to be key. Unlike him, I place a stronger importance on process. Parker-Rhodes referred to the kind of indistinguishables found in the ordering operator calculus as inscutables and did not deal with them. In the ordering operator calculus, the 'reason' for an entity being an inscrutable is well-defind. It is based on the concept of computability: if there exists a decision procedure which can be completely represented within the system under consideration and which serves to distinguish two entities, then those entities are distinguishable. Otherwise, they are indistinguishable. For two entities to be identical, every property must be held in common and a deciion procedure must exist to identify the equivalence of each such property.

Two ideas are at work here. Like Parker-Rhodes theory, the ordering operator calculus is context-sensitive. Unlike Parker-Rhodes, I have insisted that syntax and semantics are essentially the same. In particualar, properties arise from the context in which an entity is embedded -- from the structure of the relationships which an entity has with other entities. This means that entities do not generally occur in isolation unless they are without properties. It follows immediately that all entities in isolation are indistinguishable; this is reminiscent of Parker-Rhodes Inchoative Plane.

However, the ordering operator calculus takes the idea a step further; if there is no decision procedure which would distinguish two entities then these can be treated as though they are in isolation. On the one hand, this means that we can not determine whether or not we are 'at the bottom of the heap.' One the other hand, it means that it doesn't matter; any 'bottom' will do.

Ordering operator calculus indistinguishability is context sensitive with a vengeance. It is not a context-sensitivity of notation but is manifest; it can not be removed except by considering an extremely restricted context.

These ideas remove the distinction between inscrutables and indistinguihables, between correlation and predication, between syntax and semantics. They place the definition of indistinguishability at the level of the atom rather than just the collection by refusing to entertain a reductionist doctrine in any form. Parker-Rhodes and I never had a chance to engage in detailed discussion about these differences in our theories, but we did agree on the intent. I can only hope that he would have approved of the 'correlations' and 'predications' which have been achieved in the interim. His influence on the work has been strong, even if subtle. One thing is certain: distinguishing indistinguishables takes great care -- and an effective procedure.

WHAT IS THE MEANING OF THE COMBINATORIAL HIERARCHY?

Clive W. Kilmister

An active discussion took place in which different ANPA members put their own points of view. This is a very short sketch, which no doubt does justice to none of the contributions, but may show their diversity.

MIKE HORNER saw the hierarchy as a step towards a general theory of information, in a sense that would include decoding in biology and at the other end of the spectrum Jacques' theory of organisations.

FRED YOUNG placed the importance in the self-organising character of the hierarchy, and its parsimony of assumptions.

PIERRE NOYES agreed with this but emphasised the way in which the construction involves the memory of comparisons as well as the self-organising property which leads to the cut-off.

PETER MARCER agreed with Mike Horner that there was a sense in which the construction "explains everything" but he wanted to point out as well the significance of discrimination being to do with the heat engine model of reversible computation (Jablonski 1990)

JUAN ALVEREZ DE LORENZANA opened the discussion by pointing out the value of the hierarchy in socio-economic systems. There is a natural connexion with development, where there is need for a neutral hierarchy and the CH provides a decent formal model.

MIKE MANTHEY saw the importance of the hierarchy in terms of its book-keeping potential - a value-free structure devoid of content (and to be compared with sychronisation in computing).

GÖRAN ENGSTRÖM agreed about the book-keeping aspect but for him the important thing is ordinals rather than integers. So that, while "not believing in" the CH he felt that we should forge a common language to market ideas.

LOUIS GIDNEY saw the importance to lie in the escape that the CH offered from the grip of cartesian dualism

DAVID MCGOVERAN reverted to discrimination. It is a common enough operation in many fields, but once it is to be represented it leads inexorably to the CH.

Fractal Geometry and Quantum Mechanics

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Introduction

Einstein's goal of a complete geometric description for physics has never been fulfilled. To develop the theory of general relativity Einstein utilized non-Euclidean geometry. This generalization of geometry proved to be incompatible with quantum mechanics, the other great cornerstone of 20th century physics. In this paper I will argue that the further generalization of geometry to fractal geometry provides a conceptual basis for the geometrization of physics. By applying fractal geometry, a plausible unification of general relativity and quantum mechanics is indicated. However, I will also argue that this unification requires the discrete constructive approach taken by ANPA (1), and suggested by Cherbit rather than the continuum approach of Nottale. Based on these conclusions I propose a conceptual geometric basis for superstring theory that reveals the unknown starting point that Witten and others are searching for.

Data compiled by Richardson indicated that the length of a coastline or border between two countries is a function of the unit of length used in the measurement. This was one of the observations that led Mandelbrot to the concept of fractal geometry (2). In the measurement of a continuous curve the length diverges as the unit of measurement shrinks. To understand this, we utilize a mathematical model of a coastline known as the Koch curve. The construction of the Koch curve is shown in Figure 1. In this construction we start with the real interval from 0 to 1. This is called the initiator. An operation called a generator is carried out recursively. In this case the operation is to replace the interval with the 4 part peaked line segment shown in the figure. This is repeated on each of the four segments. A limiting object is produced which resembles a coastline. This curve is described by the following equation:

$$L' = l'(\Delta x')^{D-1} = \left(\frac{4}{3}l\right)\left(\frac{1}{3}\Delta x\right)^{D-1} = L = l(\Delta x)^{D-1}.$$

At each stage, a segment is replaced by 4 segments, each being 1/3 the length of the segment being replaced. The above formula shows the divergence of length as the unit of measure decreases. This considers the curve to be a 1 dimensional object. The exponent can be replaced with a number that keeps the length finite. This number is called Hausdorff dimension. For the Koch curve, the Hausdorff dimension is In4/In3=1.268. The Koch curve is a 1.268 dimensional object. It fills space in a manner that is between a line and a plane. It has a fractional dimension.

The following equation shows how the notion of space filling and dimension is generalized to non-integer numbers:

$$0 < \lim_{\varepsilon \to 0} \{N(\varepsilon)(\varepsilon\pi^{1/2})^{D}/\Gamma(1 + D/2)\} < \infty$$

The Fractal Dimension of Quantum Mechanical Paths

The first researchers to realize that quantum mechanical paths can be associated with a Hausdorff dimension were Abbott and Wise in 1981 (3). The quantum mechanical path is defined by considering a series of measurements with spatial resolution Δx and temporal resolution Δt. The path is obtained by connecting the centers of the regions where the particle is known to lie at each measurement. Because of the Heisenberg uncertainty principle, the path becomes increasingly erratic as the particle is localized more precisely. The particle path length will be: $\langle l \rangle = N \langle \Delta l \rangle$

In the case of 0 average momentum, the uncertainty principle indicates that:

$$\langle \Delta l \rangle \propto \hbar \Delta t / m \Delta x.$$

These paths can be characterized in the same way that the Koch curve was characterized using the expression:

$$\langle L \rangle = \langle l \rangle (\Delta x)^{D-1}$$

The Hausdorff dimension turns out to be 2. A more elaborate formula can be derived for the wave packet which also gives a Hausdorff dimension of 2. When the distances being resolved are much larger than the particle's wavelength, the path is a classical curve with a Hausdorff dimension of 1. When the distances resolved are small compared to the particle's wavelength, the path is space filling with a Hausdorff dimension of 2. In the transition region, the Hausdorff dimension changes rapidly as a function of spatial resolution. A similar analysis was done for a particle in a harmonic oscillator potential and also gave a Hausdorff dimension of 2 (4).

The relativistic case was first addressed in 1983 by Ord (5). He realized that pair creation would bring about a transition in the fractal dimension of time at the Planck time h/E for a particle. Unaware of this work Cannata and Ferrari proposed incorrectly that a particle path undergoes a second transition to a fractal dimension of 1 at the Compton wavelength (6). Also unaware of this work, Nottale proposed that the particle undergoes a transition to a fractal dimension of 3 at the Compton wavelength (7).

A very detailed investigation of the relativistic case was published by Nottale in 1989 (8). Although still unaware of the work of Ord, he confirmed the transition of the fractal dimension of time to 2 at the length c*tau where tau=h/E.

<u>Fractal Quantum Spacetime</u>

The main purpose of this paper is to show how the fractal nature of quantum mechanical paths relates to the combinatorial hierarchy and to my previous work on the Mandelbrot set. The first to suggest that the fractal nature of quantum paths could provide a fundamental conceptual insight into relativistic quantum mechanics was Ord (5). He was the first to suggest that the characteristics of fractal geometry would help to forge a geometric link between relativity and quantum mechanics.

Significant development of this approach occurs in the Nottale article of 1989 (8). He sets out to show that the new concept of fractal geometry could provide a unified basis for general relativity and quantum mechanics which the non-Euclidean geometries could not. The curvature of space in general relativity is a universal geometric property of space which determines geodesics. The uncertainty principle and the De Broglie relation are also universal characteristics of the quantum spacetime. They cannot be given a conventional geometric interpretation, but can be given a fractal geometric interpretation.

As described above, the original derivation of Abbott and Wise of the Hausdorff dimension of a quantum mechanical path provides a fractal geometric interpretation of the relationship between position and momentum. The De Broglie relation and the uncertainty principle are the structural constants describing the operation performed recursively during the construction of a fractal set. They are similar to the p and q which describe the scale factors and generator for construction of the Koch curve. In the case of a quantum mechamical path they describe a filling in or interpolation of the path by increasingly refined measurements.

At each step in the process of constructing a final fractal set there is set of finite resolution. These form a nested sequence of sets. The concept of spacetime in such a constructive approach must include the notion of a zoom spacetime. This has been pointed out by Nottale (8). At any finite resolution the coordinate axes have thickness. The quantum path at any level of resolution is one of the nested nets we have described. When the resolution is significantly larger than the particle's De Broglie wavelength the Hausdorff dimension of the path is 1 and the trajectory is classical.

The fact that the Hausdorff dimension of the coordinates of a particle path at high spatial resolution is 2 focuses further study on a particular class of fractal sets: space filling curves of Hausdorff dimension 2. In the literature of fractal geometry, a well known set of curves of fractal dimension 2 are the paths of Brownian motion (2). On finer and finer scales, these come closer and closer to filling space. A well known fractal curve of Hausdorff dimension 2 which is constructed from a simple generator is the Peano-Moore curve. Because quantum trajectories are not restricted to a plane, the best model is the construction of a self-avoiding fractal curve of Hausdorff dimension 2. This curve would be some sort of a wrinkled surface (8).

Nottale proposes that fractal quantum spacetime is a self-avoiding continuum of topological dimension 3+1. To develop this continuum approach, he utilizes an intrinsic definition of a fractal curve which describes the angle between successive segments along the fractal curve at any given level of resolution. This brings in rotation and allows an analysis of spin in quantum mechanics. Spin is attributed to increasingly rapid rotation between segments of the curve at increasingly fine resolutions, corresponding to successive stages in the fractal construction. An interesting fact that emerges is that spin can only be a finite quantity if the Hausdorff dimension of quantum particle paths is 2.

Intrinsic description of fractal curves also allows Nottale to define a fractal derivative on the curve which is a summing of segments over a finite section of the curve. To describe the final fractal curve he utilizes non-standard analysis which is well suited for fractal curves (9). He also shows that quantum paths can be described by a construction involving fractal surfaces.

Because we are taking a computational rather than continuum approach, our discussion diverges from Nottale's at this point. However, there are two features of Nottale's approach that are important to this paper. The first is the notion that quantum spacetime is comprised of all geodesics which can be drawn through the points of each of the nested sets in the fractal sequence at any level of resolution. This originates from the observation that in the construction of an object such as the Koch curve, only the points at each stage are part of the final curve and the segments can be replaced by curves. The type of fractal set needed for quantum spacetime is different from an ordinary fractal set in that it is not a preexisting collection of curves or sets. **Quantum space time is constructed via the measurement process.**

The second important feature involves the fractal derivative. This depends on resolution, and is a d-1 dimensionali cut or slice through a d dimensional fractal set. This is like taking a hypersurface in relativity. It is a projection effect. Information about the d dimensional object is always lost in the measurement process.

The Construction of Spacetime

In the approach followed here, the presence of all possible geodesics between spacetime points is due to the fact that quantum space time is not a pre-existing curve, set, or object. The aspect of fractal spacetime which is analogous to the points in the stages of construction of the koch curve are the discrete events. To Nottale, these events are connected by all possible continuous geodesics. In the combinatorial hierarchy the overall global connection of events is described in a radically different manner. Nottale's interpretation specifies a continuum yet at any finite resolution there are gaps between events. Even if they are connected by continuous geodesics quantum space time has only events and extended possible connecting links. There is no evidence for a continuum.

Cherbit, on the other hand, has emphasized the importance of discreteness in the construction of fractal quantum spacetime (10). He has shown that a maximum velocity for light and a minimum spacetime resolution follows from this approach.

In pursuing the analogies between the construction of fractal sets and quantum spacetime we find that the combinatorial hierarchy approach is presented in the language of an overall construction of quantum spacetime (1, 10). It is in fact the simplest possible construction based on the distinction and representaion of information. The initiator is the elementary distinction, and the generator is the recursive formation of the power set. This particular construction has a natural cutoff as do members of an interesting class of fractal constructions which are non-linear.

In this paper we will discuss members of this class known as self-mapping fractals. The constructions we have discussed so far require both an initiator and a generator for their specification. Non-linear fractals are defined as the sets left invariant by the self-mapping of a space or manifold onto itself. The simplest transformations of this type are the geometric inversions. Example of inversions which are of physical significance are the Poincaré limit sets. These are the limit sets for the recursive self mapping of a hyperbolic manifold. These transformations have been shown by Tomashitz to model the ground state of the Schrodinger wave function on curved complex spacetime (12). Tomashitz states that this is the first time a quantum mechanical ground state has been derived from Newtonian mechanics in a rigorous non semi-classical way. The critical element is the global behavior of the highly unstable bounded trajectories. The ground state energy is the folding degree of the infinitely pleated envelope of the bounded trajectories. The relation between the energy bounded trajectories and the Hausdorff dimension is given by the following formula:

E_0 = ground st. to energy

$$E_0 = \frac{\hbar^2}{R^2 m}\delta_H\left(1 - \frac{\delta_H}{2}\right)$$

The domain of the trajectories is completely given by their geometry. Gutzwiller has derived the energy levels of the hydrogen atom as a fractal set using purely classical quantities (13).

This set of unstable bounded trajectories can be compared to the bounded unstable trajectories for quadratic mappings, the next elementary class of self-mapping fractals. This is where the subject of this paper links up with my previous research. The sets of unstable bounded trajectories for the quadratic mapping in the complex plane are known as Julia sets and the associated parameter space map is the Mandelbrot set (M set).

In previous work I showed that computer graphic observations of the Mandelbrot set provided a representation of spacetime in a program universe (14, 15). This was shown in the construction of small copies of the M set. The border is constructed by interpolation of structures mathematically related to field lines in the conformal mapping. The construction is started with 1 dimensional linear elements containing points and at each interpolation stage there is a bifurcation. As the boundary of the M set is approximated more closely, the Hausdorff dimension approaches 2. The Hausdorff domension of the actual border has recently been proven to be 2 (16). Thus, as a fractal set there is a relation between the boundary of the Mandelbrot set and quantum spacetime.

One interesting possible relation which will not be further discussed in this paper is the link of the M set boundary to Gödel's theorem and the halting problem. As I showed in the ANPA 12 proceedings, the complexity of the boundary shows the highly non-linear relation between sequence and halting for a particular computation. The pattern of halting sequences among the ordered points in the complex plane cannot be completely predicted in advance. A link to quantum mechanics is suggested by the hierarchy derivation based on distinguishing information strings.

Spacetime Points in a Fractal Program Universe

In previous work I showed that although it is represented as a static "snapshot", the M set exhibits a temporal dimension (14, 15). This is seen in the evolutionary sequences I have described in previous papers. Underlying the evolutionary sequences is a series of topological bifurcations. As I explained above successive bifurcations yield a fractal dimension which comes closer and closer to 2. At each step in the sequence there is a fractal structure making up each spacetime point. The idea that what appears to be a spacetime point at a given level of resolution actually has a complex fractal structure was also suggested by Nottale.

In Figure 2, I show the increasingly complex structure of the spacetime points at several steps in the bifurcation sequence. The Hausdorff dimension of the connecting links between the spacetime points on the field lines or filaments, starts at 1 and progressively increases approaching two as a limit. If fractal curves of Hausdorff dimension 2 are self-avoiding they can only exist in 3 dimensions. A well known model in fractal geometry is the Peano-Moore space filling curve. This curve is self-avoiding and can come arbitrarily close to but not fill space in 2 dimensions. The quantum mechanical case must involve a 3 dimensional analog of this curve. This implicates a particular type of fractal curve as a model for the quantum mechanical case.

As described above, the De Broglie length and Planck time for a particle are structural constants of the fractal spacetime related to the p and q parameters of the Koch curve construction. Nottale has shown that there is a relation between stages in a fractal construction and Feynman graphs of increasing order. He showed how classical time can be 1 dimensional while relativistic quantum time can flow backwards. The backward flow of time is due to the emission of antiparticles from emitted photons in the self-energy calculations. I previously made a similar proposal about the relation between stages in a fractal construction and Feynman graphs. This was based on computer graphic observations of the M set. The basis of this conclusion can be seen in the magni*f*ications in stages 1 and 2 of Figure 2. What looks like a 1 dimensional coordinate at low resolution is seen to have higher dimensionality at higher resolution.

The complex structures can be considered to be increasingly complex scattering diagrams. These diagrams correspond to spacetime points at successive levels in the fractal construction. These increasingly complex scattering diagrams would be described by graphs of increasing order. Nottale has shown that the final fractal which he describes using non-standard analysis has an equal number of segments moving forward and backward in time providing a model for the reversibility of time in quantum mechanics.

The diagrams in Figure 2 occur in parameter space maps and are therefore pictures of configuration space interactions. At a given level of resolution, each scattering diagram has a central element through which all configuration space interactions occur. Detailed examination indicates that it is a small copy of the M set at the center of each diagram. In fact, the field lines which construct the border of a copy of the M set are constructed entirely of small copies of the M set and connecting segments. The points inside the M set copies do not halt, and are part of the final fractal set for the mapping. The only points on the connecting segments which do not eventually halt are also copies of M. The boundary of the M set as the set of non-halting points for the quadratic computation is made up entirely of small copies of itself.

This is a remarkable observation. The incredibly complex biological like forms near the boundary of the M set are made up entirely of copies of the M set connected by combinatorially organized filaments. I previously described how the biology of the M set can be observed along the border, whereas M set cosmology, the construction of the M set, can be observed on the real axis (14, 15). Even the spacetime point analogs which resemble scattering diagrams are made up of copies of the M set and connecting links. The set left invariant by the non-linear quadratic self-mapping is made of elementary entities which are small copies of the particular program universe called the M set with it's diverse collection of mathematical "lifeforms". This object encodes at it's boundary all of the possible structures in the program universe of the quadratic mapping.

Singularity Envelopes and Superstring Theory

The question which immediately occurs is whether there is an analog of this remarkable structure in physics. Is there in nature an object which encodes all possible structures? I propose that the concept of such an object as an elementary particle has been discovered theoretically and is the subject of superstring theory. In the remainder of this paper I will describe the evidence for this conclusion.

A popular description of the concept of the superstring explains that all of the myriad particles found in nature can be considered to be the vibrations of a string. This is much like music where the notes can all be considered modes of a vibrating string. The basic particles correspond to the notes of the string, and the laws of nature correspond to the harmonies or relations among the notes. As I have explained in previous papers, the structures near the border of the M set are all determined by the collection of periodic orbits, which are in turn organized around a system of positional information involving rational numbers. This pattern has been confirmed, and it has been stated that periodic orbits are the skeleton of chaos.

It has been pointed out by Witten that superstring theory is a 21st century theory that dropped into the 20th century (17). When compared to a theory like general relativity which began with a conceptual principle, superstring theory has been evolving backwards (18). It began with the observation that the Euler Beta formula described S matrix scattering theory (19). A principle of duality was discovered that explain aspects of crossing in S matrix theory. It was originally proposed as a theory of hadrons but the success of quantum chromodynamics and problems with the dual model relegated it to obscurity. The theory was ressurected as a unified theory when it was found that it contained a spin 2 particle with the properties of a graviton and it yielded a finite quantum gravity.

Later work showed that the duality principle involved an extended object or string (20). The backwards evolution has not been completed, the conceptual principles have not yet been discovered. According to Witten, discovery of the the geometric conceptual basis is the goal of physics for the next 50 years. General relativity began with a geometric principle that led to a field theory and it has not yet been successfully quantized.

In the remainder of this paper I will show that a combination of my work on the origin of pattern and form, and the constuctive computational approach of the combinatorial hierarchy explains why a quest for the elementary entity in nature turned up a superstring. I will also argue that the superstring is a very different type of object than most researchers suppose.

In my explanation of the origin of pattern and form I proposed that the organization of systems and structures in nature are attributed to a combination number theory-as a pattern of harmonics organized around rational numbers, topology-as it applies to universality in chaos and structural stability, and Godel's theorem as represented in the halting problem.

The program universe I investigated involved quadratic mappings in the complex plane. Clearly this cannot serve as a model for nature because it differs in a number of significant ways from the type of program universe which has been shown to be able to model nature. The major difference is that the quadratic mappings assume a pre-existing system in which the result of computations can be displayed. In addition, there is a spatially fixed system of ordering based on the location of points in the complex plane. The program universes used to model nature have a more general notion of order. Furthermore, the program universes which model the combinatorial hierarchy can be considered self-generating systems.

The Conceptual Geometry of the Superstring

I will list some major aspects of superstring theory and show that the conclusions I reached concerning the quadratic mappings in the complex plane provide a conceptual basis for superstring theory when generalized to mappings of more abstract spaces.

(1) The mathematics of superstring theory is very closely related to the mathematics of self-mapping fractals. In both cases, structurally stable sets are being described which remain invariant under the self-mapping of a space onto itself. The topology of structural stability is one of the three components in my explanation for the origin of pattern and form. Both fractal geometry and superstring theory describe holomorphic mappings of the Riemann sphere. Superstring theory also involves self-mappings of Riemann surfaces of higher genus. Self-mappings of the Teichmuller spaces of superstring theory give rise to bounded sets which are analogous to the Mandelbrot set (21).

The motion of a superstring produces a 2 dimensional world sheet. There is a close relationship between this object, and phase transitions in statistical mechanics (22). Phase transitions are where the concept of self-similar scaling first entered the physics literature. In fact, a novel type of phase transition known as self-organized criticality has been proposed as the general mechanism responsible for the widespread occurrence of fractal objects in nature (23). This theory has recently been applied to the guage hierarchy problem (24)

Conformal mappings are used in superstring theory, statistical mechanics, and mappings of the Riemann sphere in fractal geometry (22,25). A Hausdorff or fractal dimension of 2 applies to the superstring world sheet, the boundary of the Mandelbrot set, and the paths of quantum mechanics on the order of the De Broglie wavelength. These fractal sets are related to what I call the edge of chaos. This is the edge of distinction which involves the Godel's theorem part of my explanation for the origin of pattern and form.

(2) Superstring theory originated from studies of the S-matrix for hadron scattering. A principle of duality was discovered that involved a relationship between S and T channel crossings. The theory evolved backwards to the discovery that what was involved in duality was the concept of an extended object in the crossing channel. Although this theory was dropped for hadron interactions because of the success of the quark model, it was later shown to give a finite theory for all particle interactions. It quickly emerged as the leading candidate for a unified theory of all interactions.

An interesting fact that is never mentioned in the literature of superstring theory is that Chew's bootstrap theory for hadron interactions was based on a combination of the principles of unitarity, analyticity, and crossing which would be expected to hold in any fundamental theory. Chew proposed that nature was a bootstrap of consistency principles that all fit together (26). The modern theory of superstrings into which the dual model evolved, is closer to Chew's original bootstrap concept than the original dual model of hadron scattering.

The concept of basing a theory on a set of "self-evident" consistency principles has been followed to it's logical conclusion by the ANPA work, particularly as it is expressed in the 5 principles of McGoveran's Foundations of a Discrete Physics (27).

In my work, the suggested conceptual relationship between the boundary of the Mandelbrot set and the superstring was based on finding this object in the center of what passes for scattering diagrams in the complex quadratic program universes I have analyzed. The relation of the scattering diagrams to the spacetime points in the Nottale quantum fractal spactime was described above.

(3) In the fractal approach to quantum spacetime, at any stage of the construction, which corresponds to the "zoom" spacetime at any finite level of resolution there are extended objects. These correspond to the possible paths connecting any two events. In this conception, the "arena" in which interactions occur is a nested sequence of sets approaching a final fractal object. This final fractal object has been called the exoskeleton on which fractal measures lie (28). The combinatorial hierarchy construction also produces an exoskeleton of nature. It is interesting that in the Belousov-Zhabotinski reaction, the prototype self-organizing chemical reaction, the coherence is organized around topological filaments which are an actual physical example of this exoskeleton (29).

(4) There are finite and discrete versions of superstring theory which utilize p-adic mathematics (30). This mathematics is the basis of a recent concept called ultramatricity which is used to describe fractal scaling in systems as diverse as neural nets and evolutionary trees (31). The objects of p-adic superstring theory are known to have interesting fractal properties.

(5) In superstring theory, although it is not known by this name, the principle of absolute non-uniqueness is being applied in case counting of the categories of paths in any given topology. Calculations are guided by determination of the categories of paths on a manifold and the structurally stable sets left invariant by global mappings.

(6) In superstring theory, a relation known as the Yang-Baxter relation underlies much of the phenomena (32). The Yang-Baxter relation originated in non-linear dynamics in soliton theory (33). These non-dissipating waves are related to stationary waves. This can be understood by considering the braiding or integer winding of trajectories on toroidal phase spaces in dynamical systems theory. Solitons appear in equations which are integrable. This integrability is due to the integer windings of trajectories and ratios

between them described by rational numbers. To get solvable models in superstring theory, rational conformal field theories are used.

This aspect of superstring theory is related to the number theory portion of my explanation for the origin of pattern and form. I showed how all of the forms in the Mandelbrot set are organized around rational numbers. As I discussed above, this also underlies the structure of chaotic systems. In fact, the idea of periodic orbits as the skeleton behind chaos (34) is also related to the notion of the exoskelton in fractal mappings and the combinatorial hierarchy.

(7) In dynamical systems theory, The D+1 dimensional phase space is often studied using a D dimensional Poincare section (35). In previous work I have described how the Mandelbrot set is a static picture or slice through a higher dimensional phase space which is needed to represent the dynamics (13, 14). If the dynamics could be visualized then at each step, each point is mapped to another. The carrying out of this mapping involves events outside the 2 dimensional complex plane.

Witten describes how Atiyah said that it is necessary to get out of "flatland" to understand superstring theory (36). This involves the D and D+1 dimensions of a section and it's associated higher dimensional phase space. The role of knot theory, braid groups and the Yang-Baxter relation in superstring theory comes form this general dynamical behavior of mappings.

(8) One of the most promising non-perturbative approaches to superstring theory invoves catastrophe theory and associated renormalization group flows. It is the bifurcation sequences in the singularity envelope of the Mandelbrot set that determines the inceasing complexity of forms.

(9) One of the major problems in the conformal mapping approach to superstring theory is the large number of theories which are possible. In quadratic mappings of the complex plane, any quadratic equation can be congugated to the simplest quadratic equation by a coordinate shift. This may provide a clue to the meaning of equaivalence classes of conformal mappings in string theory.

(10) The outstanding problem in string theory is the lack of a geometric concept. The current quest involves the local and global symmetries of general relativity and Yang-Mills theory. Conceptually, this involves the underlying principles of global mappings and localized or bounded unstable trajectories in self-mapping fractals which I have used to model biology (14, 15) and Tomaschitz has used to model quantum mechanics (12).

(11) In the attempt to develop a field theory of strings, the mapping of the string onto itself and the interactions of strings are used. In my previous work only the self-mappings of a manifold is described. I have pointed out that my program universe is artificially simple and doesn't include interactions between objects. The combinatorial hierarchy approach includes both self-mappings and interactions.

(12) The construction of scattering vertices in string field theory is closely related to the construction of three and 4 fold scattering vertices in the combinatorial hierarchy work.

(13) In superstring theory, the concept of the topological invariants of a manifold is very important. In my work on pattern and form, I showed how topological bifurcations are of key importance. Structures are organized around the topological invariants corresponding to a particular stage in the construction.

(14) There are a number of mathematical miracles in superstring theory which involve monster groups (37) and the efficient packing of spheres in higher dimensional spaces (38). This may be related to optimzation in my work on the edge of chaos (13, 14), and parsimony in the ANPA work (1). In addition, it has been suggested that aspects of superstring theory including the p-adic version argue that number theory is the ultimate physical theory (39). All of these concepts concern the inevitability of mathematical relations in physical theory and can be related to Parker-Rhodes inevitable Universe.

Self-Reproducing Universes

In previous work I have shown that all of the structures near the boundary of the Mandelbrot set are organized around the envelope or singularity network of the quadratic mapping (14,15). If any of the spacetime points in these networks is magnified sufficiently, it is seen that the network is comprised of small copies of the Mandelbrot set connected by filaments that follow a combinatorial pathway. This is essentially a network of open and closed string-like entities. These structures are the elementary entities in this two dimensional program universe in the complex plane. As I described above, this observation led me to the idea that the analog of this phenomenon in nature is the superstring as a carrier of all possible orbits. I propose that the superstring is analogous to the envelope of unstable trajectories at the boundary of the Mandelbrot set.

It has been pointed out that a second quantized or field theory of strings, is equivalent to a third quantized theory of quantum fields (40). Third quantization is used in quantum gravity to describe a superposition of individual universes. Introduction of multiple connectivity via wormholes gives rise to the concept of baby universes. This picture provides insight into the vanishing of the cosmological constant and may help to explain the values of the coupling constants. In the field theory of strings, the coupling constants appear at the second quantization level. The superstring in string field theory is analogous to a baby universe in third quantization.

In quadratic mappings of the complex plane, the Mandelbrot set contains copies of itself. The Mandelbrot set can be considered as a type of program universe. In the sequential construction of the Mandelbrot set by topological bifurcation which I have reported previously, there is eventually a recoding of the starting object. As described above, I have related these small copies to superstring theory. However on the quadratic plane, they are not just the elementary unit or particle from which all else is constructed. They are small copies of the entire universe or baby universes. The universe and the elementary particle are related to one another by a scale transformation. I propose that the superstring is also related to the universe by a scale transformation, and that recoding of the M set by itself is an example of a self-reproducing program universe. Self-reproducing universes have been introduced into cosmology by Linde as a natural consequence of his work on inflation (41).

Since an analog of self-reproducing universes can be seen in the program universe of quadratic mappings of the complex plane, it is likely that these phenomena can be investigated at the level of informational coding. The earlier example of a self-reproducing informational universe was the self-reproducing cellular automata of von Neumann. Cellular automata evolve discretely in time through a rule depending only on local neighborhood. The original model was very complex and had 29 states per cell or pixel with a 5 cell neighborhood (42). Most of the states are internal, since the simulation itself is 2 dimensional. These internal dimensions may provide a model for hidden or compactified dimensions in superstring theory. In cellular automata, much simpler self-reproducing systems have been developed (42). This suggests that it may be possible to formulate a much simpler model for nature than superstring theory. In the bit string model of discrete physics investigated by ANPA, there is a relation between the interactions of bit string scattering, and the rational periodic orbits which organize the envelope or boundary of the M set. Since the latter is related to the superstring, I propose that the combinatorial hierarchy model is an example of a unified theory of nature that is much simpler than superstring theory.

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Legends to Figures

Figure 1. Construction of the Koch snowflake curve. This Figure shows the first six stages in the recursive construction of the Koch snowflake curve. The initiator is the straight line shown in the bottom of the Figure. The generator replaces each segment with a four segment peak a shown in the Figure.

Figure 2. Fractal Spcetime points in the Mandelbrot set. Stage 1 in the upper left shows a magnification of spacetime points along the real axis at the tip of the Mandelbrot set. Stage two shows a magnification of the spacetime points along the axis orthogonal to the real axis. For a detailed explanation of this bifurcation sequence see (14, 15). Stages three and five show magnifications of the spacetime points but not the axes for stages three and five.

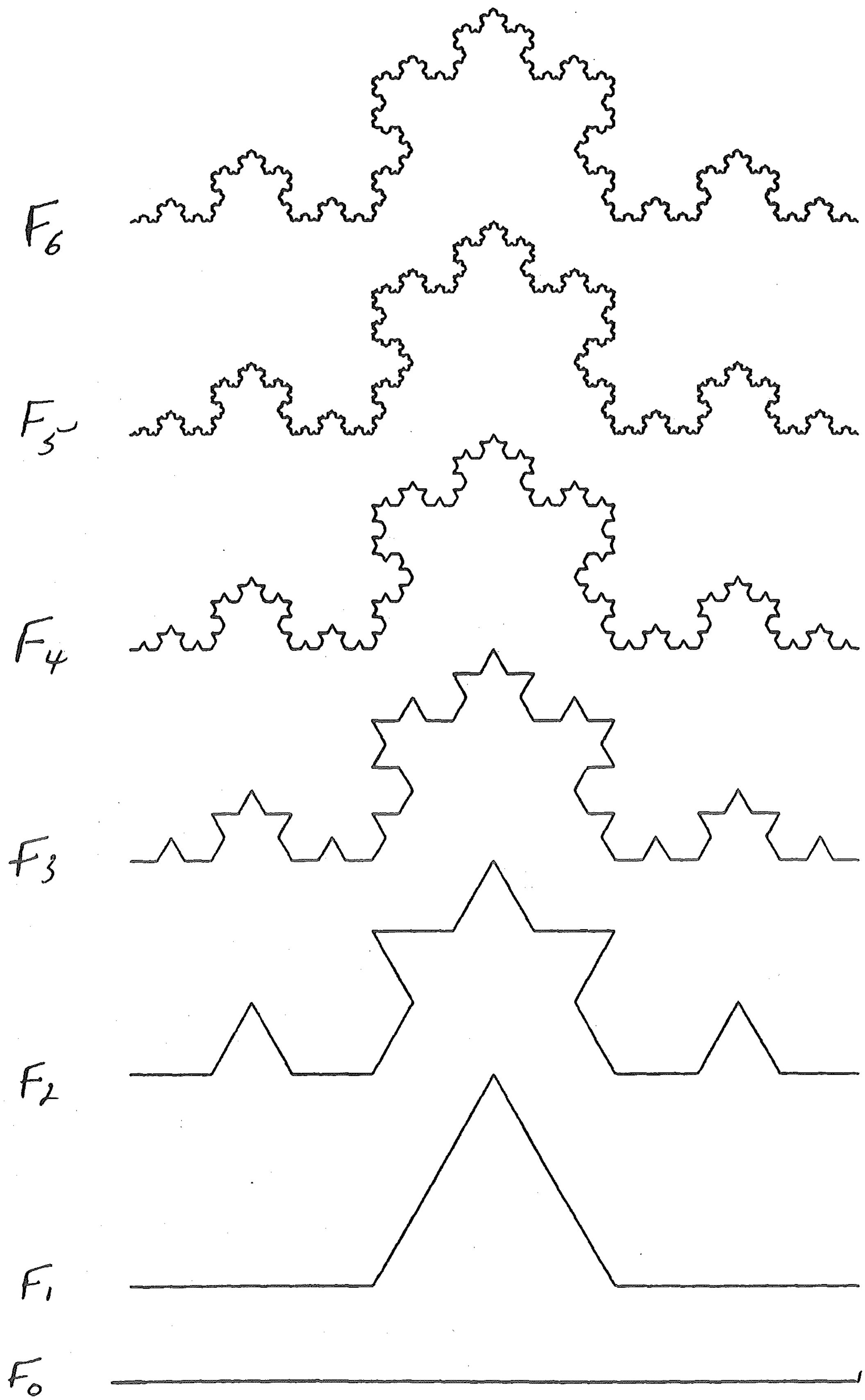

Figure 1

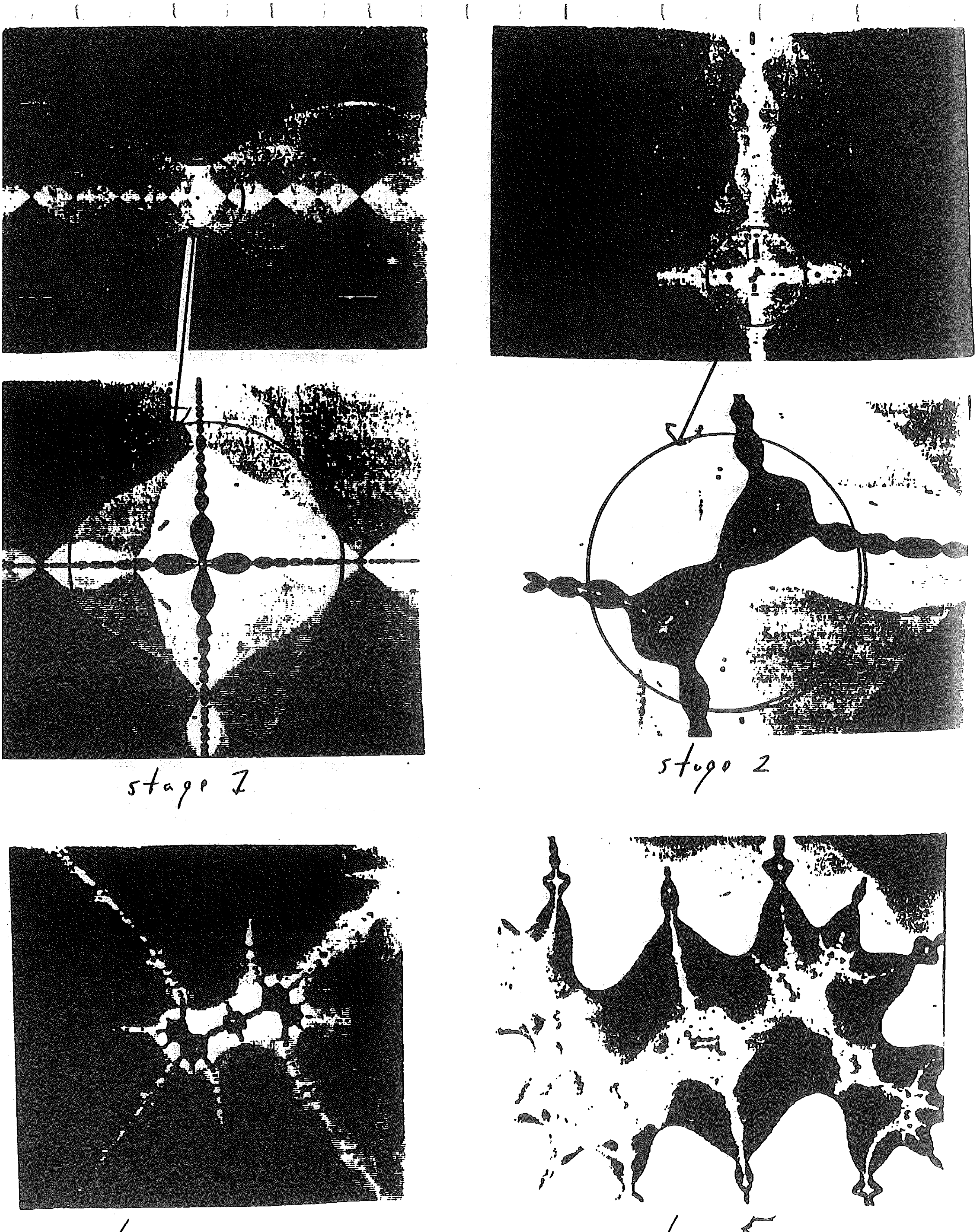

Figure 2

BACKWARD PROGRESS IS STILL PROGRESS

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Hamlet: For yourself, sir, shall grow as old as I am, if like a crab you could go backward.
Polonius [aside]: Though this be madness, yet there is method in't.

I wonder if a suitable motto for ANPA could be culled from that quotation? My general programme for research, as has been well described this year by Ted Bastin, is to reformulate Frederick's construction, or something near to it, in terms of PROCESS and to do so without any anthropomorphic overtones. My present position, very like Ted's, is that of trying to make progress in the interpretation.

Let me begin by recalling to you what is set out in ANPA 11. The process is one in which entities which have not yet been labelled or named have then to be labelled. I contend that I do not, in describing this labelling process, have to decide on the question raised by Alison Watson, of whether the entities already exist before coming into the process or not. I do not wish to draw a sharp distinction between epistemology and ontology. Once an entity has been labelled it is called an element. Its label then takes part in the mathematics (if you want to make a distinction between mathematics and physics). Next one takes into account that one does want to talk about indistinguishables (e.g. electrons). Each new entity has to be tested to see whether it is indistinguishable from one already labelled - and if so to give it the same label - or not. So the process must include something which I would now express (using here a new symbol, *, for which I would thank David McGoveran though he might well say that his introduction of it was essentially different) as: If a is some already labelled element, then

$$(a, *) \longrightarrow [\text{a signal of indistinguishability}]$$
$$\searrow [\text{no such signal}]$$

where the symbol * denotes an entity about which (at the time of writing the symbol in the description of the process) nothing whatever is known.

Then, continuing this modified description of ANPA 11 (so that it becomes rather what I <u>wished</u> I had said) I took Z as the set of signals of indistinguishability, so that

$[(a, *) \longrightarrow x, \quad x \in Z]$ if and only if [For * write a].

I argued that Z must be a fixed given set, so that there was no loss of generality in taking it to have one element (meaning by this that the process would be just the same if Z had only one element). To the further question: How is it possible for the process, having determined that $(a, *) \longrightarrow x$, to determine whether $x \in Z$? (For it looks as if one then has to go on to the further question, if s is in Z, then does $(s, x) \longrightarrow$ [element of Z]? and so on in an infinite regress.) I answered that the process must get round that difficulty in this way:- The (single) element of Z is not a label, so that if $(a, *) \longrightarrow$ [element of Z], this situation is at once ascertained. Then I called the element of Z 0, and used 1,2,3.. as an alphabet for labels.

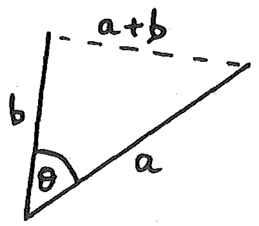
Out of that beginning I can get, by mere elaboration, discrimination, the extension to comparison with a whole set of already determined elements, bit-strings, the need for eigenvectors, the hierarchy construction, the numbers 3, 10, 137, 10^{38} and all that. I will not elaborate that here except to repeat one thing. Frederick's construction of the hierarchy involves CHOOSING a linearly independent set of operators. If I am to escape the anthropomorphic mathematician, I cannot imagine that to be incorporated in the process. Rather, in the process operators are constructed. They may be linearly independent in one run, and not in another. If enough runs happen, we make the "ergodic hypothesis" that all possibilities are attained. The actual Frederick construction simply gives a kind of boundary inside which the partial hierarchies given by the process reside.

Suppose all that done to everyone's satisfaction. We are now face to face with the problem raised by Ted Bastin in this meeting: to construct physical space. I have not solved this. Instead, I want to make four disjoint sets of remarks. It is possible that one or more of these sets may ultimately lead to a solution, or none of them. We shall see.

I. ANGLES. Let me start by clearing out of the way some misleading remarks I made in previous years. I then argued like this: to a bit-string a there corresponds its Hamming norm, N(a) (the number of 1's in the string). And the fact that this is a norm, that

$$N(a + b) \leqslant N(a) + N(b),$$

(note the mathematical pun, for + on the left-hand side means discrimination, on the right-hand side addition of numbers) characterises the discrimination operation, in which the number of 1's can decrease but that of 0's can only increase. But because it is a norm, this means that I can use it to define an angle in a diagram like this :



This is only a diagram; but it suggests a possible programme - to start with the angles so defined and construct space from them. This programme, as far as I can see, fails and the reason for this is interesting - if enough elements "meet at a point" the angles between them are all defined, but they cannot be embedded in a three-dimensional space.

II. NEED FOR TWO POINTS OF VIEW. I remind you of Ted's list of things that we do not yet know:

1. We do not know how to describe two things separated in space.

2. We do not know what dimensions are (or what the dimensional

structure of a physical space means).

3. We do not know how to construct quantum numbers.

4. Not merely can we not use classical space in discussing the quantum events; we cannot use it in relativity either, without providing a combinatorial explanation.

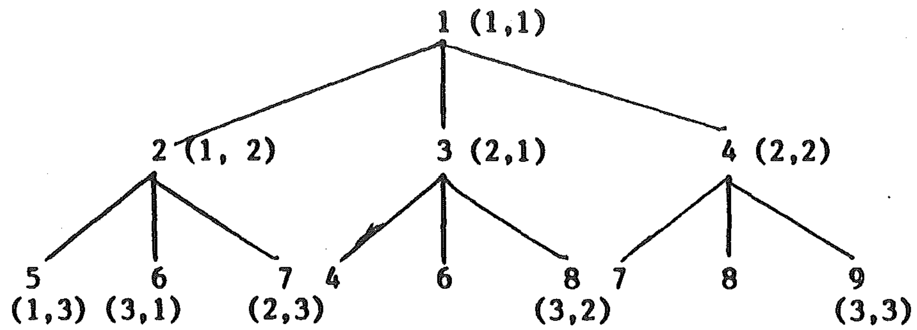
I rephrase the last: in so far as we have a physical picture, it is seeing the world, in Leibniz's phrase, from one point of view. Why is this? Ted suggested to me during the year that the trouble lay with my Z, in particular with my assumption that Z had only one element. My first reaction was negative: "the process works the same...". But, in desperation, let me explore some more. I do it by the scientist's traditional method of knowing an answer (more or less) and trying to work back to it.

I imagine two processes. How can that be? Really there is only one but in some way that I do not yet see either it splits, or it can be split, into two. The only possible difference in point of view will be that

$$(a, *) \longrightarrow 0, \text{ but } (a, *)' \not\longrightarrow 0'$$

or vice versa. So it begins to look as if we can get away with three elements for Z when there are two processes (and $2^n - 1$ when there are n). One is (0, 0); then (0, *), (*, 0). But of course these last two, involving the new symbol I mentioned earlier, are really enumerably many. Thus Z will not be a finite set any more, but since it must be given and fixed it must be recursive. That could be elaborated easily enough; but how will it look when the process goes on? Recall first the old "one-process" case: the first element is 1 and then when a different one comes it is called 2 and in the process of testing the compound label 1+2 (or 12 for shortness) is generated by the process. Next a new one is 3 and in the course of testing that 13, 23, 123 will have come up. (That is not what really happens. How could it?

For when the process starts testing, this procedure would need it to test whether what it is testing against is one that has been used before or not... and so another infinite regress. No, at the third stage above, the process must be one of considering the whole dcss [1,2,12] and so it needs an operator A such that A1 = A2 = A12 = 0 and no other elements exist which A annihilates. So if the process finishes by finding A(*) = 0 it has to go back into the cycle to test the dcss of A until it comes up with the one for which A annihilates it and it has one member. That member then gives the label to be written for *.) It is quite hard to see how to extend this to the 2-process case. If we continue the cheat above, the first step is the same, so the label is (1, 1) which we can call 1 in the new version. Next, however, there are three possibilities for A(*), viz. (0, *), (*, 0), (*, *), and the labels will then be (1, 2), (2, 1), (2, 2), which we may rename 2, 3, 4. At the next stage there is another three-way split, so we have a tree developing:



But if I give myself this finished tree and look at one branch, say 1, 3, 8, ... how can I disentangle the two points of view? And where did I go wrong in my earlier analysis that suggested a single member for Z?The discrepancy does not lie in the infinite character of Z because the fact that it is determined by a recursive rule means that the infinity is harmless. Rather it lies in the assumption that the information in the signal is so minimal, so there is a lesson here - the abstraction has left out the concept of information.

III REFERENCE. Let us go back a bit, and notice that the problem (as it was for Frederick) lies in indistinguishables. I claim to have a theory of labelling, or I could also have said of NAMES. We are back with what the philosophers call SINGULAR TERMS or REFERRING EXPRESSIONS. Can we get any help by seeing what has been said before? Not perhaps by worrying about the equivalence of the morning star and the evening star, about the present king of France or about the round square. But twenty years ago A N Prior said "By a name logicians generally understand an expression that we use to indicate which individual we are talking about when we make a statement." Disregarding the anthropomorphism which we could easily rewrite, we see that we cannot dodge "which individual" if we have indistinguishables, so something needs to be worked out afresh.

We are really going back to the problems brought into philosophy by Russell ("On Denoting") building on ideas of Frege, that is, the problem of "denoting phrases" which have no meaning in themselves but there is an analysis which provides a meaning for any proposition containing them. But if you turn to Frege or Russell for help, you find the analysis very much couched in terms of thought or belief and that is just too anthropomorphic to be got rid of. But all need not be lost, as Gareth Evans has shown (Evans, 1982). He analyses the notion of "a person perceiving something" in terms of a new primitive "being in an informational state with content X", so carrying out the analysis entirely in terms of an information transfer system. How then does he deal with "seeing the world from two points of view"? He notes that first we must make sense of "two informational states embodying the same piece of information" and then he retreats into a comfortable realist position of saying that this will happen when they originate from the same informational event. But perhaps we can help ourselves out at this point by means of the notion of process in the hierarchy. I have not yet been able to think enough about this.

IV. CLIFFORD ALGEBRAS. We have to build bridges. We need to look over the wall (or perhaps this year I should write Wall) and see, amongst those tilling the unregenerate continuum vineyard, to whom it is worth reaching out. For a long time I have studiously ignored the school who seemed to me to have re-discovered Clifford algebras just as Ted and I had given them up (Bastin & Kilmister 1952) but I do commend a recent book to you (Hestenes & Weingartshofer 1991) although it is a conference proceedings. For one thing, this book contains a treatment of the Dirac equation free (in a non-trivial way) of $\sqrt{-1}$, and that, if it is correct, would make the appearance of $\sqrt{-1}$ in quantum mechanics a non-problem.

But if we are going to build any bridges in that direction, do not let us do it by any irrational crackpot method of identifications. We should ask, rather, what our algebra, say in the form expressed by bit-strings, is actually doing for us. The answer is that it is expressing dichotomous choices. If we write down one variable, x, is can be only 0 or 1; we do not have to say any more, it is all built in. Contrast this with a quantum mechanics approach. They prefer (and why not) to work with 1 and -1; no real difference there. They also work in terms of operators and states. For a dichotomous choice there must be two eigenstates so, if the operator is $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the equation for the eigenvalues:

$$(a - k)(d - k) - bc = k^2 - (a + d)k + ad - bc = 0$$

must have roots 1 and -1, giving $a + d = 0$ and $ad - bc = -1$. There is a little subtlety in getting the general solution of these conditions, but ignoring that for the moment it is easy to see the possible cases: $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$.

These three, together with the unit 2 x 2 matrix, form a basis for all 2 x 2 matrices, and of course we have here the Pauli spin matrices or the quaternion units (up to factors of i). This is all for one dichotomous choice; for more a direct product construction will serve, and so our bit-strings of length n are to be compared with the direct product of n quaternion algebras, or to the Clifford algebra generated by 2n anticommuting units.

Perhaps next year I shall be able to say which of these four lines is more promising.

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The Neuron, a computational model utilizing quantum devices.

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A model is given by which biological neurons may compute using quantum devices and dimensional confinement. Connections with quantum mechanical computation, neural nets and standard computation employing non-procedural programming using matrices and decision tables are made.
Microneurophysiological studies could both validate the model and furnish design criteria and morphology for equivalent man-made systems, so that nature's nanotechnology will guide the Feynman machinists and Quantum mechanics of tomorrow.

The Neuron, a computational model utilizing quantum devices.

Man has now pixelated a surface with individual Xenon atoms so as to spell out the letters IBM[1,2]. It is not therefore difficult to envisage a control morphology (later described) appropriate to a similar surface where such individual atoms under that control are made to act like 0,1 switches[1,3], and where such individual atoms are admitted to that surface subject to that same control by means of an entropic hole in the surface membrane[4]. Thus under the control morphology envisaged the spin states 0,1 of atom i admitted to the surface may be denoted by σ_i giving a Hamiltonian contribution on the surface of

$$H_i = \sum_j J_{ij} \sigma_i \sigma_j \qquad (1)$$

where J_{ij} measures the 'exchange' interaction between the pair of spins σ_i and σ_j. This specifies a behaviour corresponding to the Sherrington-Kirkpatrick neural net model of a 'cortical' map on the surface[5]. But Deutsch[6,7] has shown that each such spin constituting a 0,1 switch or bit, has a universal quantum mechanical state space capable of perfectly simulating any physical process or algorithm. That is to say subject to a suitably enhanced control morphology each such atom i with spin σ_i will act as a quantum computational dot[8] working by dimensional confinement[6,7] which the control morphology,[4] the surface and the entropic hole provide.

Under these conditions the network of atoms Hamiltonian H_1 where $H_1 = \sum_i H_i$ constitutes a physical realization equivalent to a system for non-prodedural programming using matices and decision tables. That is to say the two dimensional array of atoms (i,j) on the surface realize a matrix showing the data elements available from the input record in one dimension i to the output record in the other j. Most of the intersections of such a 'data-data' matrix will be empty (ie the entropic holes are closed) - implying the input data element which crosses an output data element at that intersection does not affect its value. Where, however the intersection is not empty ie an atom is present , this atom functions as Deutsch has shown as the algorithm or physical means by which the output value is computed. That is to say the set of atoms (i,j) is the means by which to identify a set of data elements such that every unique occurance of the set ' triggers' one complete output cycle, as indeed happens in biological neurons. But H_1 a spin glass behaviour, is also important Sourlas[9] has shown because it constitutes an error correcting mechanism, which is able to saturate Shannon's well known cost performance bounds for the transmission of information. That is to say such a matrix of atoms acting as 0,1 switches will subject to learning as a neural net be both a reliable and optimal means of computation.

The Control Morphology.

In a biological neuron, the control morphology determining the above input/output data, consists of dendritic trees[10] which develop as the result of learning experiences. In this model these apply potentials V_k representing the input or output data to each of the atoms (i,j) by means of their charges Q_k where k ranges over i and j, producing a Coulomb energy behaviour H_2

where $$H_2 = \sum_k Q_k V_k \quad (2)$$

This corresponds to another well validated neural net model employed by Leon Cooper[11] and his associates at the Nestor

Corporation, and so is demonstrably an effective learning mechanism for such input/output data. The actual mechanism appropriate to the model described here by which each dendrite tree is structured in order to generate the required potential V_k has been described elsewhere.[4] Conversely since as Crick[12] has pointed out, while neural nets are conceptually closer to the computational behaviour of biological brains, they are still unrealistic in important aspects, the use of 1) and 2) to satisfactorily represent net behaviour and learning, would be explained by the model presented here. Furthermore the model here indicates through its various components, that a single neuron contains within itself, the design for a whole brain as a computational network of synapses[4] which it is currently believed that the man-made neural nets represented by 1) and 2) exactly mimic. It might therefore be that a single neuron is a blueprint for a brain but certainly if such neural net mechanisms operate within the individual neuron on the neural surface in the form of a matrix of atoms utilising dimensional confinement provided by entropic holes and constrained by the particular structure of the particular neuron's dendritic trees, this is an explanation of why brains of even a few neurons are so computational powerful. Each neuron on this model is computationally equivalent to a supercomputer.

What would be the implication that within the control morphology of an individual neuron, that the matrix of atoms on the neural surface function as a neural net, and subsequently that with a suitable enhancement of that control morphology (that provided by the brain?[4]) that the matrix of atoms function like a network of quantum mechanical computers[7] with each atom functioning as a universal machine? To carry out quantum mechanical computations by means of transformation in the state space of a single bit or atom with spin, it will be necessary to modify the local vacuum properties by pumping photons [13](of V_k) so that the atom k or suitable matrix of atoms, behave like a laser. Of course it may be that the charges on the neural surfaces are due to ions without spin in the quantum mechanical sense. Nevertheless the hole-occupancy of the neural membrane may be cast into an occupation number representation with similar consequences. In either case, experimental validation of the dielectric properties of actual neural membranes, and estimations of the potentials on the surface could serve to confirm the likelihood of the model proposed.

This will, I believe be the case, and there will be very substantial pay-offs from paying close attention to the technological accomplishments of nature.

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MAXWELL'S DEMON AND THE FOUR-COLOUR MAP THEOREM

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ABSTRACT

It has previously been asserted (1) that:

(i) Traditional mathematics is a tautology in that its true foundation is electromagnetism, and

(ii) There exists a Mathematic of Nature which is expressed as electromagnetic phase quadrature.

If these assertions are accepted, then it follows that the Mathematic of Nature may be expressed in terms of a simple modulo four arithmetic applicable to phase quadrature.

The well-known four-colour map problem is therefore re-examined, first in terms of a simple geometry in the Euclidean plane, then in terms of a modulo four projective geometry in explication of Maxwell's demon.

A method for finding the dynamic geodesic pathways in NP-complete problems is suggested, and diphasic systems are postulated in which very large numbers are accommodated as binary ladder exponents representing geometrical place values.

1. INTRODUCTION

In their book, *The Mathematical Experience*, Davis and Hersh (2) ascribe the four-colour map problem to a conjecture made by Francis Guthrie in 1852 that any political map drawn on a flat surface, (or on the surface of a sphere), can be tinted without using more than four colours. The conditions sufficient and necessary are that all individual domains may be of any shape, and that those that share a border in common may not be of the same colour. The conjecture is believed to have been based on the accumulated experience of cartographers. Many attempts at formal mathematical resolution of the problem have been reported, including one based on computer analysis by Appel and Haken in 1976, also discussed by the above authors.

2. MINIMUM MENSURABLE LENGTH

The solution proposed in this paper relies on an axiom that a boundary is only definable in terms of a minimal linear mensurable distance, (symbol, $\underline{l}$), in the dimension of length, which then allows conventional grid or matrix mapping as a tiling in the Euclidean plane (Fig. 1a). It can then be seen that while the central minimal tile of area $\underline{l}^2$ shown in Fig. 1b can be surrounded by four tiles of differing colours W, X, Y and Z, it cannot be coloured with any of these other than by sub-division. Sub-division by colouring on either side of a diagonal is admissible in a geometrical sense since the diagonal of a square is $\sqrt{2}$ greater than the length of a side. This process is therefore applied to the four corner tiles in Fig. 1c, the process then being repeated (under rotations) for the four tiles vertically and horizontally adjacent to the central tile. The last can then be self-coloured using any one of four colours not included in the set of all adjacent colours. The same consideration applies to a central bi-coloured tile (indicated by inclusion of both diagonals in Fig. 1d).

If exchange of bicoloured tiles is allowed, then an alternating pattern of self-coloured tiles having sides $\sqrt{2}\underline{l}$ x $\sqrt{2}\underline{l}$ can be constructed (Fig. 1e). Now boundaries can, by definition, only be connected at the corners of a minimal square. If this is not physically possible, then it is certainly possible for squares having sides $\sqrt{2}\underline{l}$ in length, or for higher multiples of $\underline{l}$. The concept of $\underline{l}$ therefore explains the diphasic (odd and even) nature of connectivity systems equivalent to face centred cubic (FCC) crystal symmetry (Fig. 1f).

3. THE PHYSICAL BASIS OF LENGTH

Colour is a measure of length in that the wavelength of electromagnetic (EM) radiation is defined as the velocity of light, c, divided by a frequency, f, (the number of oscillations in one unit of time). It follows that $\underline{l}$ will not be constant, but will vary according to the frequency of light used to make a measurement. Fig. 2a shows a hypothetical extreme case in which a minimal area is defined in terms of four colours which are widely separated in the visible spectrum. The result is an arbitrary quadrilateral (stippled) rather than a square. However, according to Von Aubel's theorem (3), lines joining the centres of squares erected on opposite sides of a quadrilateral are equal in length and mutually orthogonal. This implies that a mixture of EM frequencies constitutes a self-mapping, i.e., self-organizing, system of discrete trajectories definable within finite Cartesian co-ordinate systems as reference frames.

The differentiating properties of colour taken as a source of information indicate that the mechanism of Maxwell's Demon is immanent in Nature, as illustrated by consideration of Fig. 2b, which is a simplified version of Gabor's *perpetuum mobile* of the second kind. The mechanism originally postulated (4) was the detection of a "molecule" in the upper half of a container by means of a system located in the lower half which circulates light in both halves. Detection actuated movement of a piston into a chamber, after which the piston was moved by the molecule, the system being sealed off by a sliding mirror. A natural version of Maxwell's demon would require only that the piston and mirror mechanism be replaced by EM energy of appropriate kind in order to perform the same functions. In the present instance, Gabor's hypothetical apparatus has been drawn so as to emphasise the analogy with bicoloured (diphasic) tiles.

Two forms of EM energy would be necessary to establish a demon, one to act as an "automatic sliding door" capable of distinguishing between high and low expressions of energy, the other to act as the "high" energy that becomes enclosed. This implies a feedback mechanism, for which only the rotational and phase quadrature properties of EM radiation need be invoked. In the simple case, $\underline{l}$ is defined as one quarter wavelength of EM radiation of nominally constant frequency in order to avoid the complication of chromatic aberration (Fig. 2a). The perimeter of an iterated minimal area is thus $(\underline{l}_1+\underline{l}_2+\underline{l}_3+\underline{l}_4)$ modulo 4, equivalent to W, X, Y and Z.

4. THE TRAVELLING SALESMAN PROBLEM

The travelling salesman problem (5) is taken as an example of the operation of Maxwell's demon by feedback in phase quadrature.

In Fig. 3, EM energy of constant frequency generated by a transmitter/receiver (TR) is radiated to a receiver/transmitter (RT). Its minimum detectable <u>amplitude</u> is $\underline{1}$, thus making the main grid spacing the equivalent of 41, or one cycle of oscillation.

One cycle of oscillation is sub-divisible into four equal parts which may be represented in discrete form as nodes and antinodes of amplitude (0, +1$\underline{1}$, 0, −1$\underline{1}$), or as angles (0°, 90°, 180°, 270°) or simply as ordinary numbers (0, 1, 2, 3), as in the diagram. As numbers, they may be regarded as "even" (0, 2) and "odd" (1, 3) pairs. Cancellation in antiphase (180°) takes place when such signal pairs are reflected onto themselves as shown in the plane of the page. However, doubling addition takes place instead when the signals differ in phase by a further rotation at 90° to the plane of the page, i.e., 270° quadrature.

Now, the feedback pathways to right and left may not be of exactly the same lengths. This may result in loss of phase quadrature coherence on the outward journey from TR to RT, and on the return journey from RT to TR. All of these differences can be accounted for by considering the rotational nature of the signals in terms of the fourth roots of unity. This is accomplished by taking one signal as reference phase, and the other as differential (or relative) phase. The complete table of relationships is then as given in Table 1.

[The case in which two signals are not orthogonal to each other, (i.e., not in exact quadrature), appears to be capable of explaining a learning function in animal behaviour (6) given by the expression:

$$P[n] = 1 - (1-P[1]) \times (1 - \partial)^{(n-1)},$$

where 1 is the value of the first correct response, P is the probability of an animal responding correctly on trial n, and ∂ governs the slope of the learning curve (corresponding to an angle between two EM signals). A similar curve can be derived for the rate of increase of current passing through an inductance and the rate of rise of a charge on a capacitor.]

A simple combinatorial-geometrical approach to the travelling salesman problem is based on the following assumptions:

1. Journeys between towns are reducible to line segments.

2. Both ends of line segments are defined as points.

3. Points are defined as ordered pairs (Eastings and Northings).

4. The sequence of towns, and the town from which a journey is started, are arithmetically irrelevant to the shortest possible overall journey, even though all combinations of distances may be tabulated (Fig. 4a). The actual journey is a closed loop.

In Fig. 4b, each horizontal arrow pointing to the right on the central diagonal represents a line segment between a pair of towns. Each has a corresponding arrow crossing it, also representing a line segment between another pair. The connections between the towns are taken at random. All towns are represented at one end or the other of an arrow. It will then be seen that the co-ordinates (ordered pairs) of all the towns in the system can be cancelled by their reciprocals simply by subtracting the sum of the distances represented by all the vertical arrows from the sum represented by all the horizontal arrows. This represents a partition of the overall distance into approximately equal outward and return halves.

The residual (diagonal) distances for the outward and return portions of the journey totalled separately are noted, and one of the horizontal arrows exchanged for one of the vertical arrows. The distances of the outward and return journeys are again noted and the exhange retained if the overall distance is less than that obtained in the previous trial. The process is repeated with all other horizontal and vertical pairs until the lowest values are found. Provided a proper record is kept, the individual journey segments can then be assembled into their correct order by inspection. This approach corresponds to certain concepts of circulant matrices (ref. 2, p. 176), lattice theory (7), and an algorithm due to J. H. Conway (8), one example of which is a game or tournament in which player W beats player X, X beats Y, Y beats Z, and Z beats W, a situation which is often the basis of "sucker bets". These probability paradoxes can be stated in terms of "heads" and "tails". That is to say, they are already in a form of binary arithmetic and therefore have a decodable information content.

5. NONTRANSITIVE PARADOXES

J. H. Conway's algorithm uses what he calls "leading numbers", which allow calculation of the probability of a player X being able to select a quadruplet sequence, (e.g., THTH), which is more likely to occur in a continuous finite sequence before one selected by player W (e.g., HTHH). This is based on "waiting" times, i.e., the number of throws needed, on average, for a particular n-tuplet sequence of T's and H's to occur; these are given as 20 and 18 for THTH and HTHH, respectively. The paradox is that an event which is less frequent in the long run is likely to occur before one which is more frequent, which can be related to the properties of an "average waiting time". Quadruplets are a particular case in that anomalies occur with six pairs of combinations, a situation also met with left-handed and right-handed dice. Probability distributions can be constructed in matrix form (ref. 8, p. 64), outlined in Fig. 5a.

The specific procedure involves writing H-T sequences as X over X, W over W, X over W and W over X for sequences of n-tuplets. Martin Gardner gives the examples shown in Fig 5b. These are based on a reducing combinatoric, i.e., a '1' is written over the top if the sequence of the leading 7 symbols (from left to right) in the lower row corresponds with the sequence which follows the first symbol in the upper row. The symbol '0' is then written over the top if the sequence of the first 6 symbols in the lower row does not correspond with the sequence that follows the second symbol in the upper row. And so on for all four rows, for which the results can be converted to base ten numbers to yield a ratio of probabilities that X will beat W.

The travelling salesman problem is clearly of the same form except that the ratio of X to W to be sought would be a minimum value, and ideally 1:1. In more general terms, the form of the "multi-trefoil" knot of Fig. 4b is clearly a sophisticated averaging mechanism in that both horizontal and vertical arrows are alternately in the forward and return halves of the overall journey, thus making a modulo four system. It is also apparent that the overall knot mechanism is a reduced Fourier integral involving only hypotenuse elements of sin and cos terms. This is also to say that the underlying principle of Fig. 5a can be extended to represent any function.

[N.B. The term "arrow" has been used in preference to "vector" so that the latter can be reserved to multiple wavelength systems.]

6. AUTOMATIC PATTERN GENERATION

Nontransitive paradoxes applied to non-polynomial (e.g., travelling salesman) problems in mathematics not only provide an explanation for Maxwell's demon, but also suggest the feasibility of developing a method of synthetic mathematics for their generation.

The expression:

$$\sum_{n=1}^{\infty} 2^{-n} = 1$$

can be modified to:

$$\sum_{n=Q_f}^{\infty} 2^{-n} = 4$$

where Q_f are the fourth roots of unity (+1, +i, -1, -i) in representation of the phase quadrature operators (in continued fraction/probability form) of an oscillation of a fixed frequency, f.

A structure generated by a mixture of frequencies can then be calculated using the following modification of Shannon's expression for negentropy, H:

$$H = -k \sum_{i=1}^{n} f_i \log f_i$$

where f_i is an average frequency (equivalent to a probability) and n is the number of frequencies for which the quadrature characteristics, Q_f, are to be calculated in modulo four form.

The bilinear operator w = (z-1)/z+1) can then be applied so as to represent self-organizing systems. The end product of such a procedure would be the generation of structure in the form of standing waves, together with a mechanism for the capture of energy in the form of travelling waves, both sets being complementary to each other and relative to the observer's frame of reference. Fig. 6 is the (octahedral) phase quadrature projective geometry of the interaction of two waves of the same frequency and amplitude, a problem of the kind often encountered in the field of fluid dynamics.

7. DIPHASIC HOLOGRAPHIC MEMORY

Fig. 7a is of a diphasic (chequerboard) system with arrow alternations overlaid as a phase conjugate hologram (5). The symbol B represents a binary system, and the symbol C its conjugate. The symbols Au and Ad represent notional augend and addend, respectively, as an indication of an addition process carried out on the trace of a matrix in the complex plane. The augend and addend could, in principle, be on separate limbs of a twisted structure, as for the configuration of the genetic code. The same configuration, (as in Fig. 5b), would be appropriate to the cerebral hemispheres and the cross-connections would then be some explanation of the functions of the inter-hemisphere central commissures. This is also to say that the nervous impulses in the two halves of the brain might very well be in antiphase so that only difference signals are "processed" (9).

The principal feature of Fig. 5a is, however, that memory elements could be added in unlimited parallel, and as circulatory ring circuits, at each and every 2 x 2 matrix. The diagonal elements of such a configuration would then be place values as ladder exponents, thus explaining the enormous memory capacity of the animal central nervous system. The overall configuration would also be that of FCC crystal symmetry such that all 2 x 2 matrix elements would be capable of continuously communicating with each other by means of a diphasic "nearest neighbour" principle. The necessary connectivity for continuous exchange of information in circulant matrices would be that of truncated octahedra in contact through their square faces.

The configuration is also highly relevant to finite element analysis in that rectangular lattices with cross-bracing struts (Fig. 7b) could, in principle, be dealt with along the same lines (10). This is also to say that "solid" structures would be expected to deform in different ways according to their internal structures, and also in accordance with the direction and force of applied loads. The principal feature of a solid is that it can resist shear forces and, in this respect, it would be expected that a three-dimensional phase-conjugate lattice circuit would be able to act as a "carrier" of electronic representations of injected energy so as to provide a completely accurate model of interactions down to the level of quanta (11). It is therefore postulated that the electron clouds of atoms and molecules act as information carriers in physical systems and that they are therefore subject to a law of conservation of phase.

8. THE FOUR-TIER HIERARCHY

It must be supposed that the expression of physical laws in real systems involves extremely large numbers. It must also be supposed that phase, and particularly phase quadrature, is of paramount importance. For example, 18 ml of water in a tablespoon contains the Avogadro number (6.3×10^{23}) of molecules "at rest" in an inertial reference frame (the spoon). However, merely rotating the spoon through 90° results in the water being spilled, thus converting "order" to "chaos". The converse process, that of converting chaos to order, must presumably involve a very large number (Avogadro+) of orthogonal (phase quadrature) transitions. The question is how such transitions are initiated.

Fig. 8a postulates a series of binary transitions radiating from a single, notionally static, object. The reason for supposing this to be the case is that the contact rotation of one object about another of the same size leads ultimately to the FCC phase structure. That is to say, alternating layers are able to rotate in opposite directions, unlike mechanical gear mechanisms, which require "idlers". Structures similar to the "radiating FCC phases" of Fig. 7a have, indeed, been observed in the crystallization of amorphous silica (12).

It is therefore postulated that the four-tier hierarchy is an expression of extremely large numbers as geometrical ladder exponents. That is to say, series such as:

$$(((((2)^{2})^{2})^{2})^{2})^{2} \ldots\ldots$$

This is illustrated in Fig. 8b, where the 2's are ladder exponents on the diagonals of a system of nested squares and represent the present observable stage of expansion of the series. In other words, the four-tier hierarchy is probably a memory state, similar in structure to the computer language, LISP.

Fig. 8b also gives a representation of a Feynman diagram as a thick rule zig-zag line connecting two "large-number" energy states. This appears to be appropriate to reduced states of matter or energy as minimal energy diagonals in "free" space, as for the travelling salesman problem. This approach appears to offer the prospect of massive simplifications in theoretical physics, particularly as Fermat's classification of prime numbers as sums of (4k + 1) or (4k+ 3) squares in which the only modulo 4 squares are 0 and 1 (ref. 7).

9. SUMMARY

The concept of a modulo four arithmetic appropriate to the description of the combinatorial geometry of phase quadrature corresponds with Berry's geometrical phase (13) and can easily be explained to a lay public by expressing it in monetary terms.

In this approach, the first step would be the notional conversion of ordinary decimal currency to a new, modulo-4, currency. The unit of lowest value in a "binary" coinage might be called a "point" and would have the same value as a contemporary penny (in the United Kingdom). The next higher value coin could be called a "line", and would have a value of four points (pence). Above that would be a "square" (sixteen points or four lines), and above that again a "cube" (of 64 points, or 4 squares, or 16 lines). Casting and posting would be by divide-by-four, with carry and remainder, over four columns. Such values would also constitute a hypercube "direct-in-binary" system not requiring further "processing" in a machine of appropriate design.

A similar procedure could be adopted for all units of the real world currently expressed in decimals, e.g., kilometres, kilogrammes, etc. There would then be direct numerical equivalence between monetary units and physical units for everyday uses (i.e., ignoring "chromatic aberration"). This would allow direct calculation of environmental costs in parallel with production costs as normally reckoned in management accounting procedures. The justification is that of the discipline of dimensional analysis (14), in which all measures can be expressed as force [F], mass [M], length [L] and time [T]. They are related to each other by the expression:

$$ML/FT^2 = 1.$$

Environmental costs are divided into two categories. The first are "external" costs, which includes items such as contributions to global warming, and the second are "user" costs, which is a measure of damage to more local resources. All such costs are additional to the traditional costs of labour, raw materials, etc., and can only be properly assessed within a holistic system.

Quite apart from this, the advantages of using geometrical modulo-4 combinatorics for any form of computation should be obvious.

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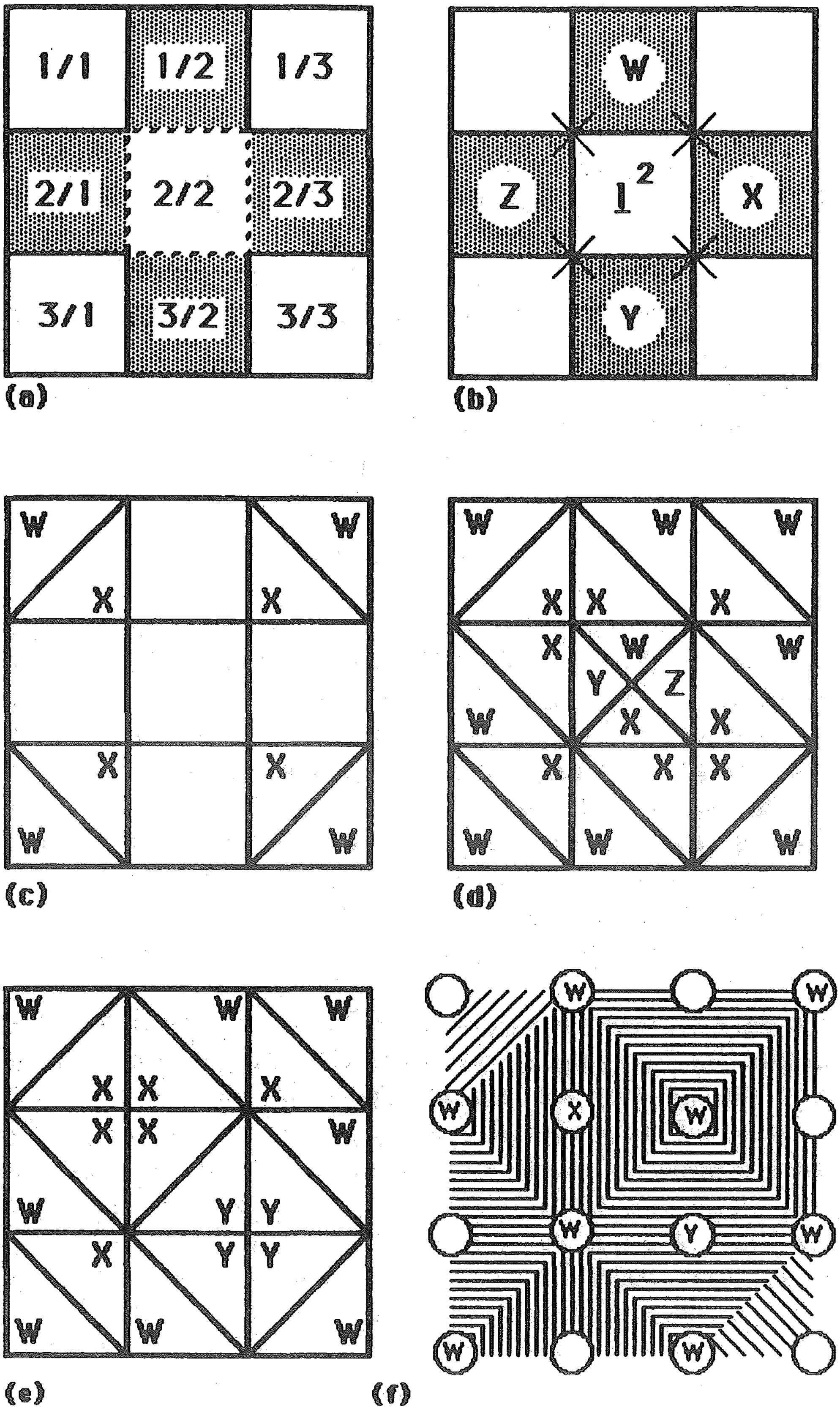


Figure 1

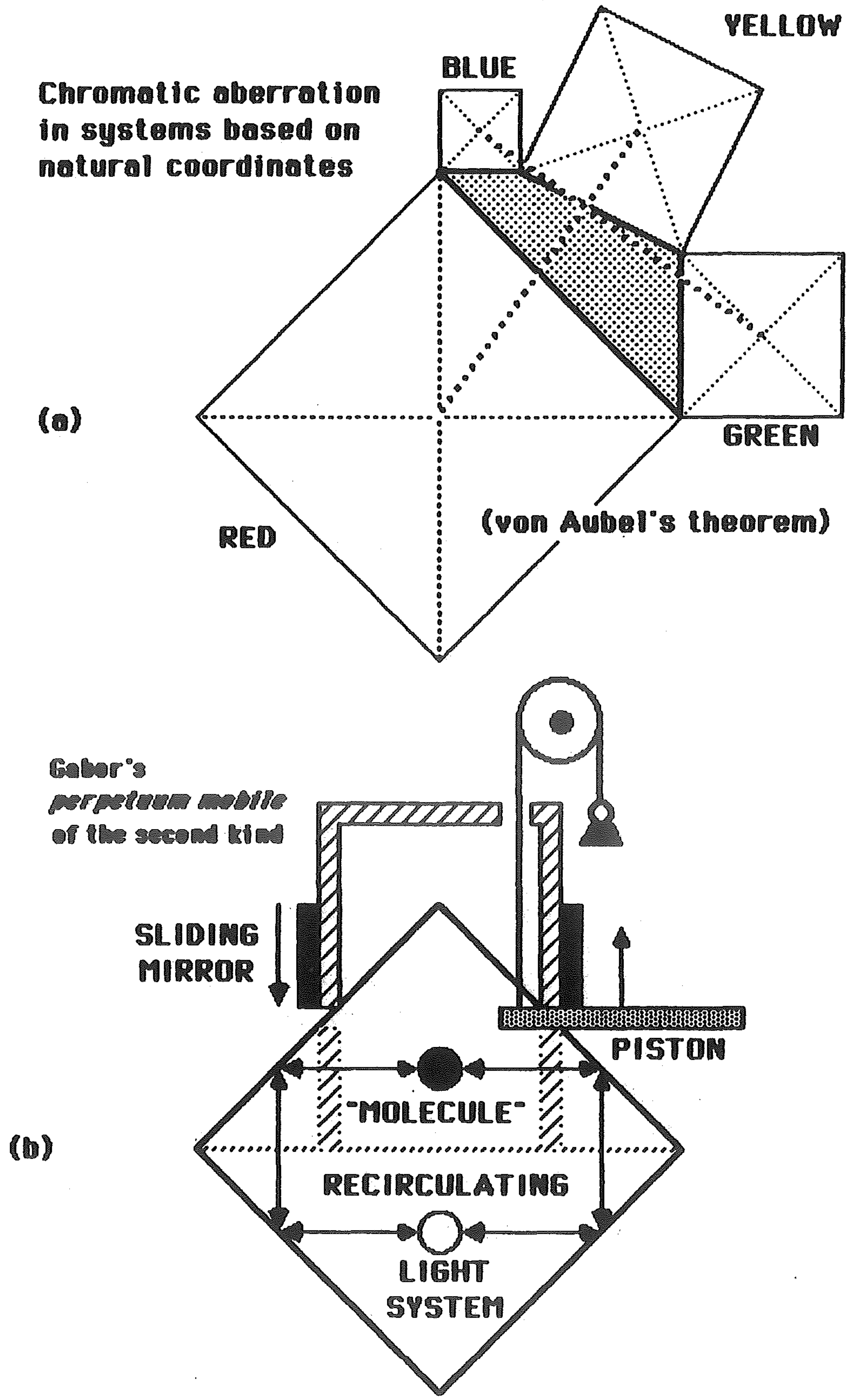

Chromatic aberration in systems based on natural coordinates
BLUE
YELLOW
GREEN
RED
(von Aubel's theorem)
(a)
Gabor's perpetuum mobile of the second kind
SLIDING MIRROR
PISTON
"MOLECULE"
RECIRCULATING
LIGHT SYSTEM
(b)

Figure 2

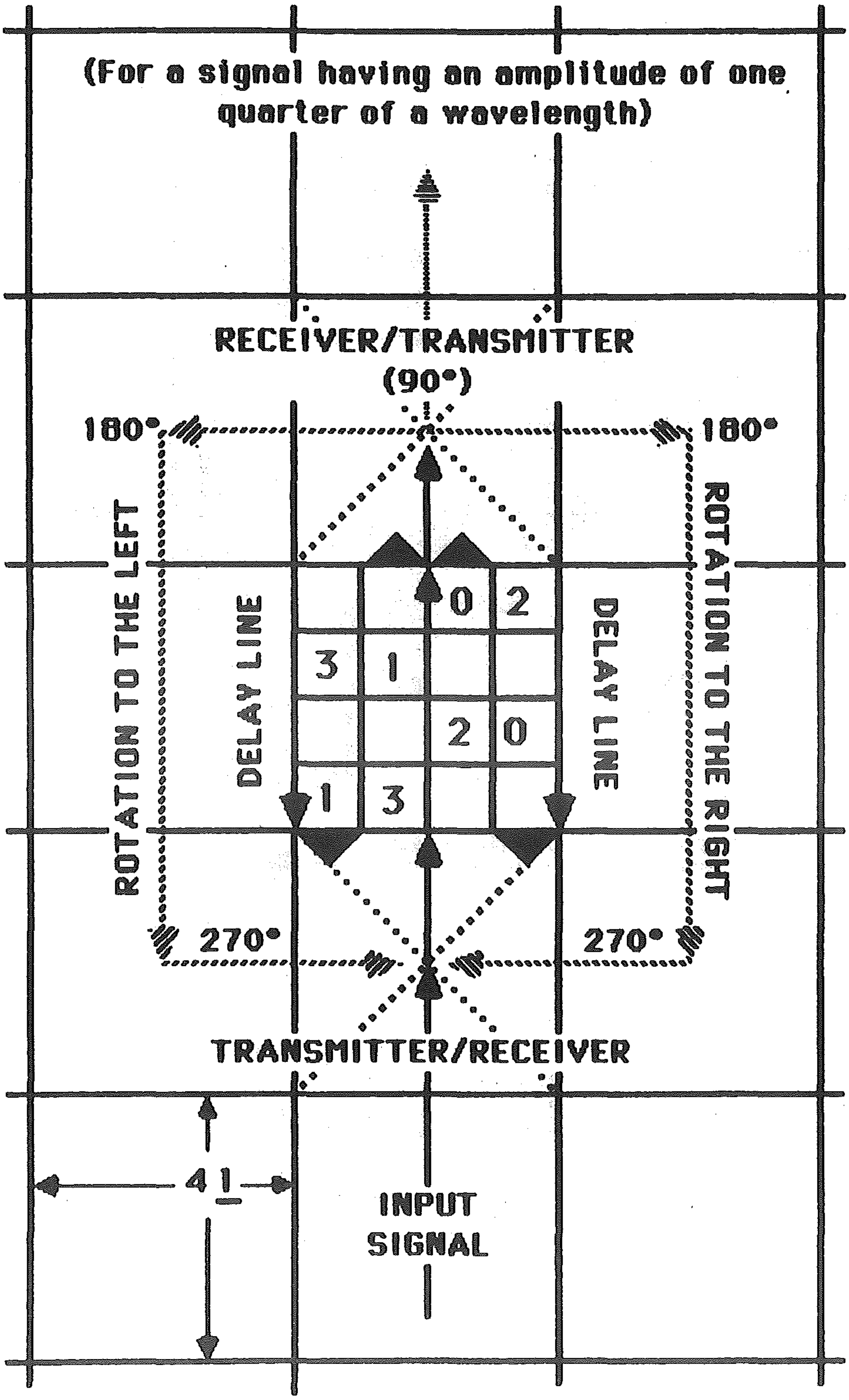
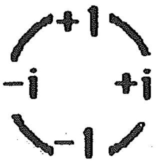

(For a signal having an amplitude of one quarter of a wavelength)
RECEIVER/TRANSMITTER
(90°)
180°
180°
ROTATION TO THE LEFT
ROTATION TO THE RIGHT
DELAY LINE
DELAY LINE
0
2
3
1
2
0
1
3
270°
270°
TRANSMITTER/RECEIVER
4 1
INPUT
SIGNAL

Figure 3

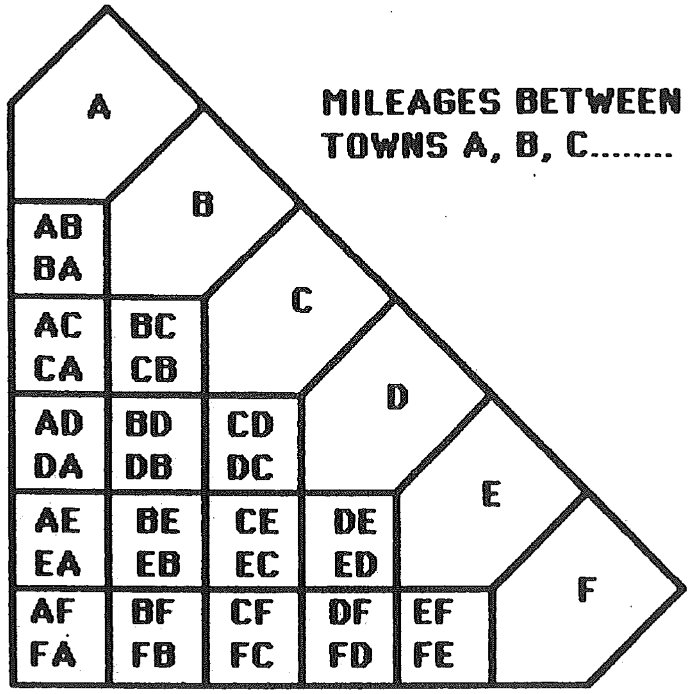
ADDITION IN PHASE QUADRATURE



		REFERENCE PHASE (STARTING AT +1)			
		+1	+i	-1	-i
DIFFERENTIAL PHASE	+1	+2	(1+i)	0	(1- i)
	-i	(1-i)	0	-(1+i)	-2i
	-1	0	(i-1)	-2	-(1+i)
	+i	(1+i)	+2i	(i-1)	0
	-i	(1-i)	0	-(1+i)	-2i
	-1	0	(i-1)	-2	-(1+i)
	+i	(1+i)	+2i	(i-1)	0
	+1	+2	(1+i)	0	(1-i)
	-1	0	(i-1)	-2	-(1+i)
	+i	(1+i)	+2i	(i-1)	0
	+1	+2	(1+i)	0	(1-i)
	-i	(1-i)	0	-(1+i)	-2i
	+i	(1+i)	+2i	(i-1)	0
	+1	+2	(1+i)	0	(1-i)
	-i	(1-i)	0	-(1+i)	-2i
	-1	0	(i-1)	-2	-(1+i)

Table 1

(a)


MILEAGES BETWEEN TOWNS A, B, C........
A
B
C
D
E
F
AB BA
AC CA
BC CB
AD DA
BD DB
CD DC
AE EA
BE EB
CE EC
DE ED
AF FA
BF FB
CF FC
DF FD
EF FE

(b)

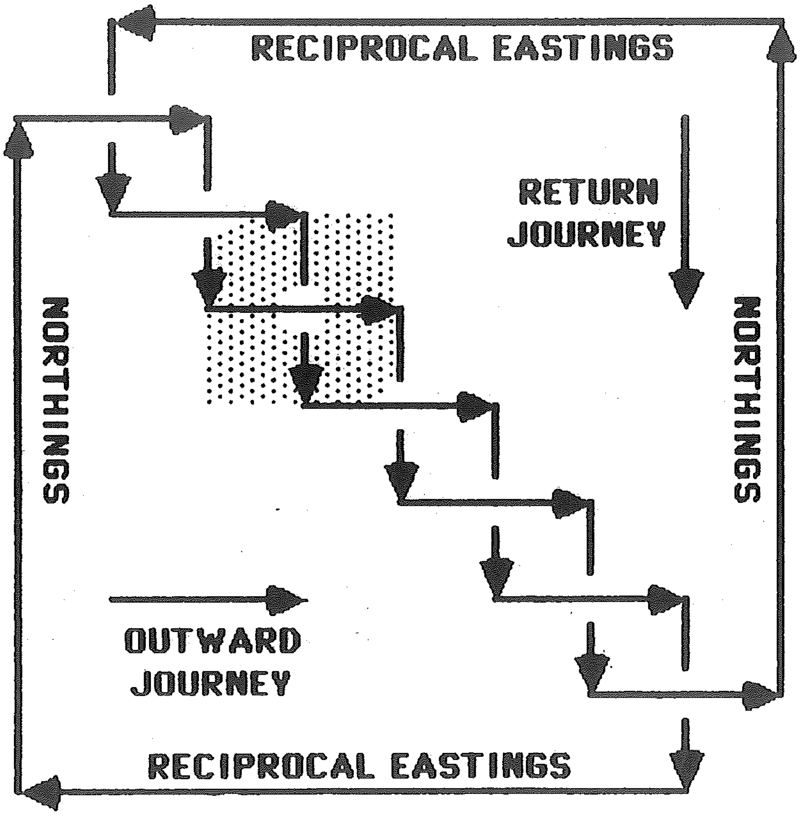
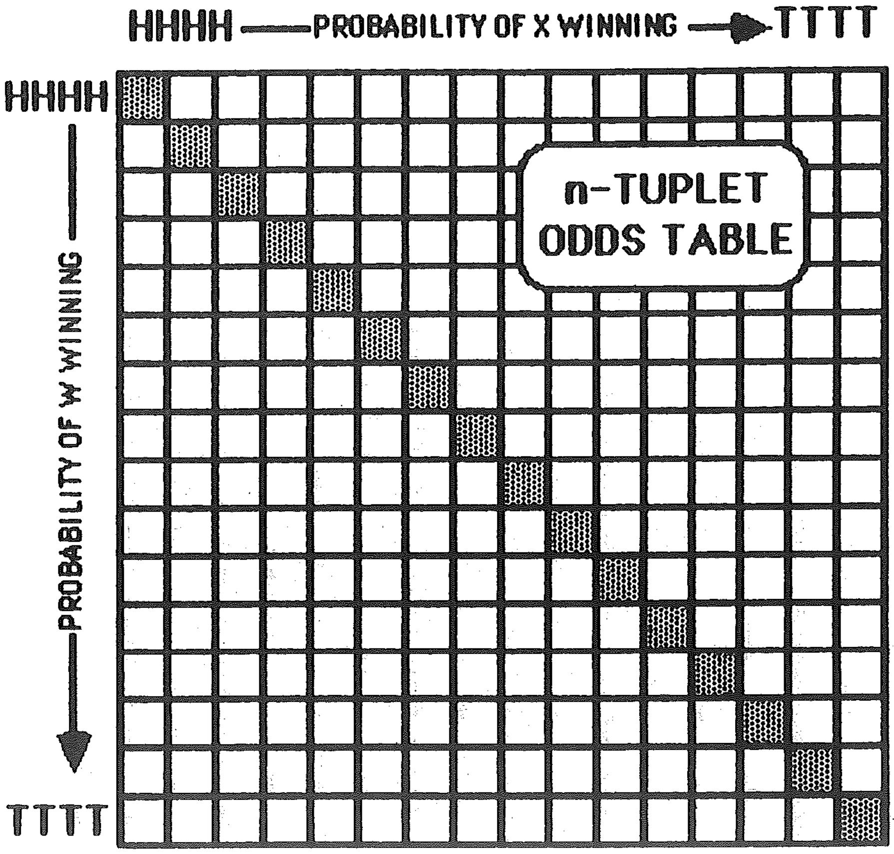

RECIPROCAL EASTINGS
RETURN JOURNEY
NORTHINGS
NORTHINGS
OUTWARD JOURNEY
RECIPROCAL EASTINGS

Figure 4



(a)

1000100=68 W=HHTHHHT W=HHTHHHT	0000001=1 W=HHTHHHT X=THHTHHH
1000000=64 X=THHTHHH X=THHTHHH	10100011=35 X=THHTHHH W=HHTHHHT

(b)

WW - WX :	XX - XW
68 - 1 :	64 - 35
67 :	29

J. H. Conway's algorithm for calculating the odds of X's n-tuplet beating W's n-tuplet.

[From Martin Gardner's
Time travel and other mathematical bewilderments
W. H. Freeman and Company, ISBN 0-7167-1925-8]

Figure 5

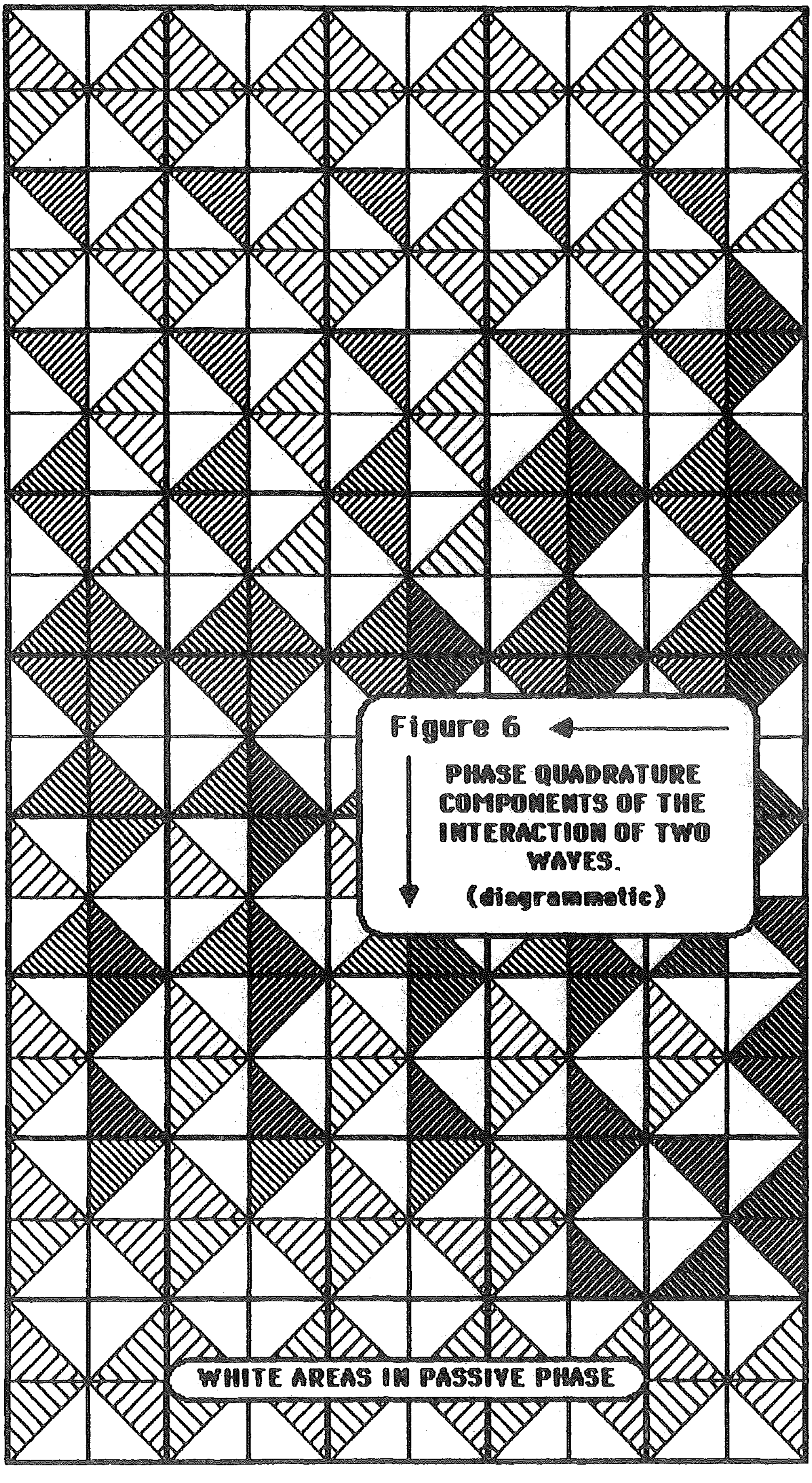


Figure 6
PHASE QUADRATURE COMPONENTS OF THE INTERACTION OF TWO WAVES.
(diagrammatic)
WHITE AREAS IN PASSIVE PHASE

(a)

(b)

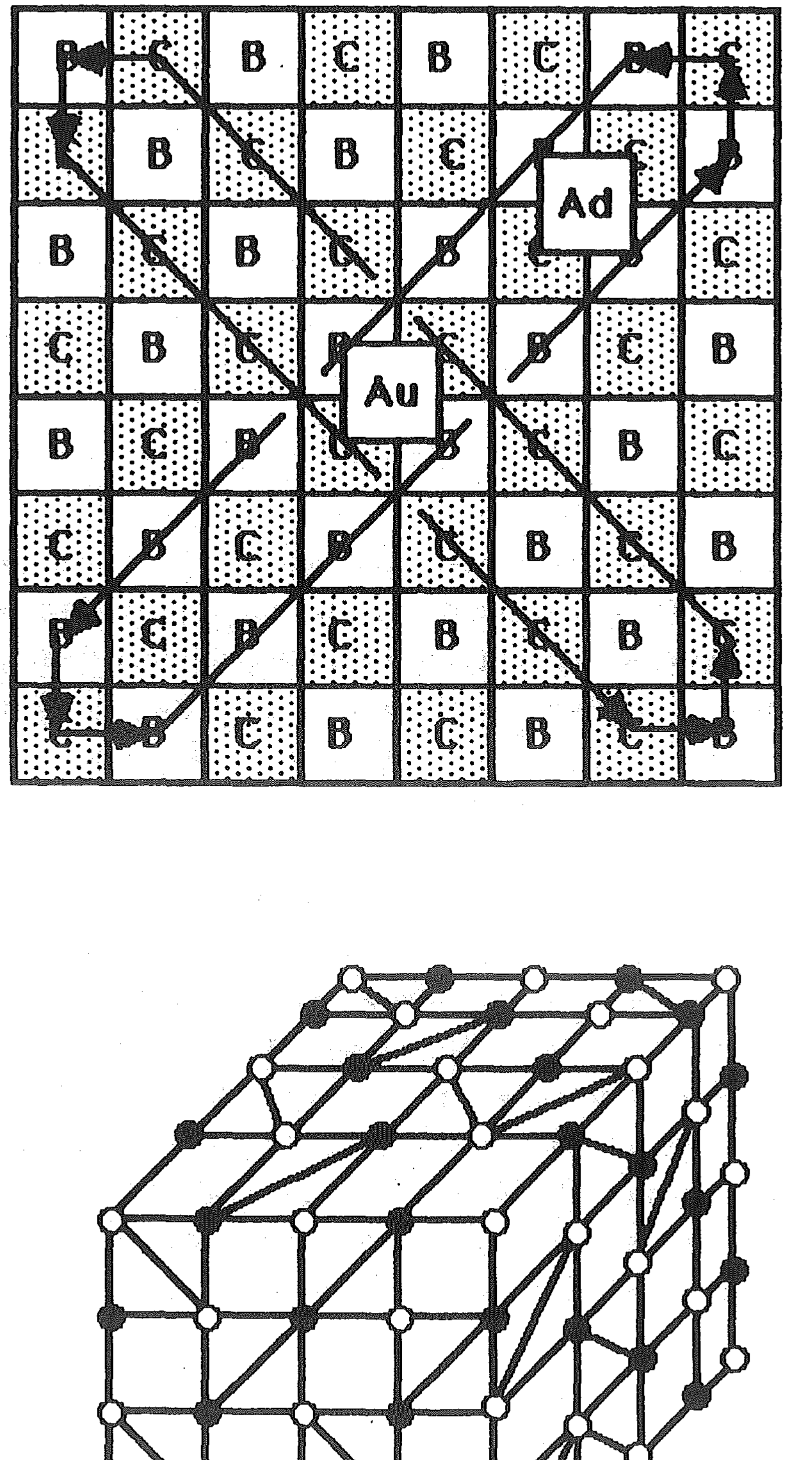


Figure 7

(a)

(b)

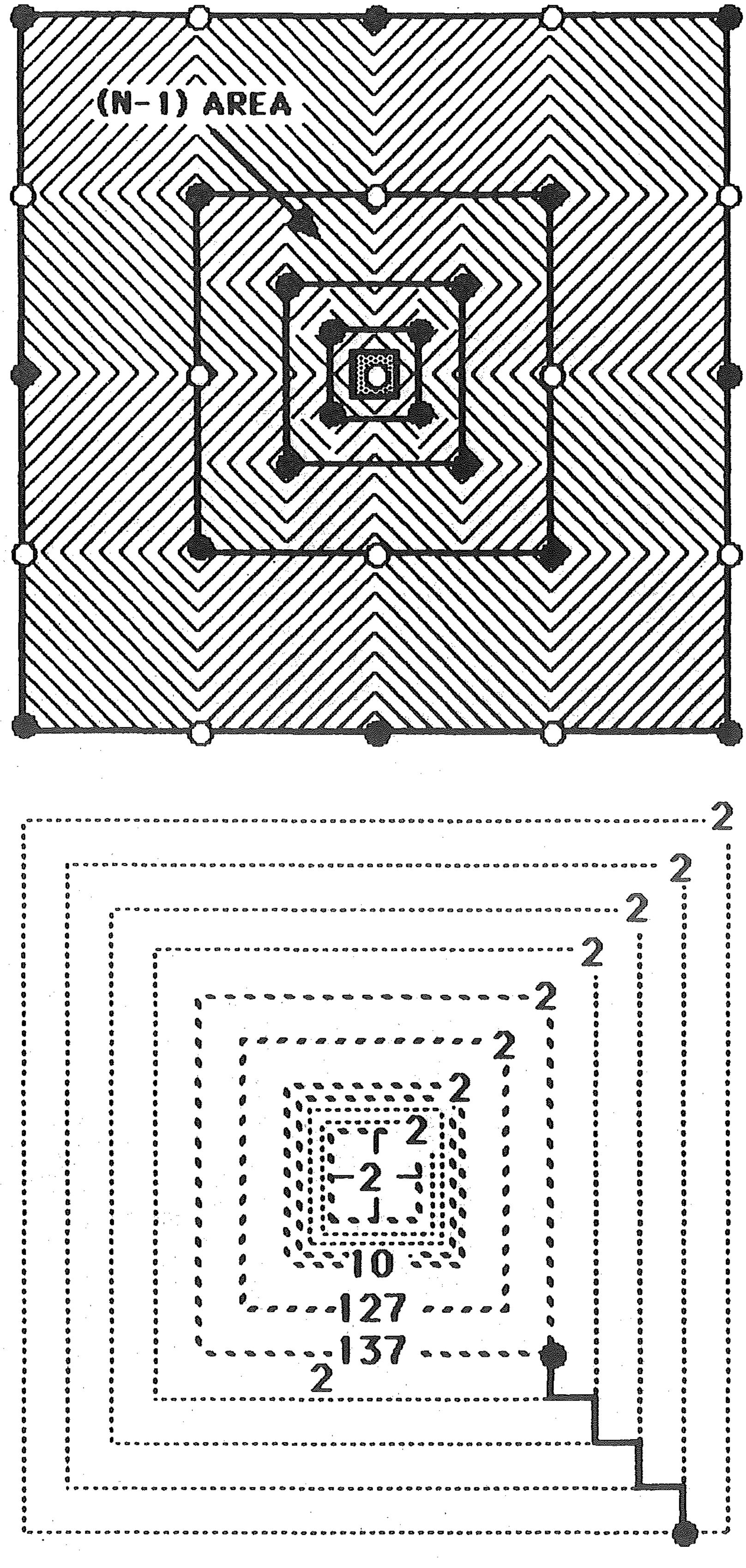


Figure 8

The maximum and minimum values of physical variables

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Abstract

Relying upon Planck's Law, the Law of Gravity, the fact that 'c' (the speed of light) is the maximum speed that can be observed, and the assumption that the beginning of the universe was a unique event, it is argued that physical variables possess both maximum and minimum values. It is also argued that, in all cases examined, the ratio between any pair of such values depends upon the dimensionless number c^5To^2/Gh, where 'To' is the age of the universe, 'G' is the Gravitational Constant and 'h' is Planck's Constant.

It is deduced from these findings that the universe began as a relatively small body characterised by the Planck Mass, Length and Time. At the periphery of such a body mass energy is equal to gravitational energy, thus permitting matter to be created spontaneously. It is proposed that such a process is the means whereby the universe grew to its present mass and size.

The maximum and minumum values of physical variables, and the relevance of such values to the creation of the universe.

Geoffrey Constable

We have become accustomed to the fact that 'c' (the speed of light) is the maximum speed that can be observed. Furthermore, we consider other variables in terms of minimum 'quanta' in order to explain the results of experiments. The main thrust of this paper is that all physical variables are characterised by maximum values - similar in concept to the maximum value ascribed to the speed of light - and are also characterised by minimum values - similar in concept to the mimimum value of angular momentum ascribed to the spin of the electron. Thus it is argued that the whole of nature is both constrained and incremental.

The arguments that support these claims rely upon the following framework:

a that we can accept Planck's Law and the Law of Gravity;

b that the speed of light is the maximum speed that can be observed; and

c that the beginning of the universe was a unique event, i.e. that it is reasonable to assume that the universe commenced at a particular place and time.

It follows from (c) above that the age of the universe is the maximum observable period of time. We shall refer to it as To. Observing a period longer than To would imply either that the universe existed before its creation (which is absurd) or that the future is already in existence (which common experience tells us is not so).

It seems obvious from this deduction that the maximum distance in the universe is c.To. For example, a photon emitted at the creation of the universe would have travelled such a distance and would, therefore, now be at the periphery of the universe. We shall refer to this distance as Ro, the radius of the universe.

As To is the maximum period of time in our universe, it might be guessed that the minimum frequency would be 1/To. This, however, cannot be so. If the universe rotates at the equivalent angular velocity $2\pi/To$, the maximum tangential velocity at the periphery would be $2\pi.c$, which is not possible. Instead, the maximum possible tangential velocity is 'c' which suggests that the minimum angular velocity is 1/To and that the miminum frequency is $1/2\pi To$.

This value for minimum frequency can be inserted in Planck's Law, from which it appears that the minimum amount of energy in the universe is

$h/2\pi To$ (h being Planck's Constant), ...(1)

which may, for convenience, be written as $\hbar/To$.

Note: such a finding was disclosed in 1950 by Whitrow, using different arguments - see Ref 1.

This energy is small - approximately 10^{-44} cgs units - and does not relate obviously to any known particle. However, a photon of such energy would have a wavelength similar in length to Ro. Such a photon would resemble a simple standing wave and would thus have an energy flow that is stationary with respect to the universe. This leads to the proposal (see ref 1) that the rest energy of this photon should be described by the expression $\hbar/To$ and that its rest mass is given by $\hbar/c^2.To$ - the minimum mass that is possible given (1) above. Unfortunately such a rest mass is too small to detect (using current techniques) by examining any variation of 'c' with frequency. (Ref 2)

This concept of minimum energy (and, hence, minimum mass) enables maximum and minimum values to be calculated for many other variables. Each maximum value will be denoted by a suffix 'x' and each minimum value by a suffix 'n'. In each calculation, care has to be taken to devise a model that is consistent in all respects with the framework given above and which describes an experiment that is theoretically feasible.

For example, we can imagine that a particle of minimum mass, M_n, is located at the periphery of the universe. It is reasonable to assume that the gravitational energy of such a particle is equal to the minimum energy, E_n, referred to above. If the mass of the universe is M_x, the gravitational energy of the particle is given by the expression

$$G.M_x.M_n \times 1/c.To.$$

Putting this equal to the minimum energy and substituting $\hbar/c^2.To$ for M_n reveals that

$$M_x = c^3.To/G \text{ (where G is the gravitational constant) (2).}$$

Note: this finding was announced by Milne in 1936 using different arguments - see refs 3 and 4.

It seems reasonable that the mass of the universe calculated in this way should be regarded as the maximum mass that is possible. Proposing that a mass exists that is larger than that of the universe would be a contradiction of terms. As one would expect, this mass is large - some 10^{55} grammes. Somewhat surprisingly, however, it increases linearly with time. Thus the universe is not only becoming larger (as the radius c.To expands), it is also becoming more massive.

This prediction concerning the mass of the universe is of the same order as eqivalent predictions from other sources. For example, there seems to be agreement among many scientists that the universe consists of some 10^{79} protons (ref 5) which, taking into account Avogadro's number etc, indicates a mass similar to that given above. More specifically, Einstein is credited with the calculation that the mass of the universe is $\pi Ro.c^2./2G$ which, when c.To is substituted for Ro, is in fair agreement with the prediction given in this paper.

A mimimum value for angular momentum can be predicted by imagining two particles of mimimum mass separated by a distance Ro and rotating with an angular velocity 1/To about their midpoint. The resulting angular momentum of such a system is given by the expression

$$2.\hbar/c^2.To \times (c.To/2)^2 \times 1/To = \hbar/2 \quad ....(3)$$

It is now generally accepted that the spin of the electron - also $\hbar/2$ - is the smallest quantum of angular momentum, which agrees with the prediction above.

Thus this theory provides predictions concerning the mass of the universe and electron spin that agree reasonably well with other findings. It may, therefore, be applied elsewhere with some confidence. Table 1 summarises some of the max/min values for various physical variables that can be predicted by this means.

Note: the concept of minimum and maximum values for such variables is not new. Whitrow outlined such an idea (Ref 1) but limited it to minimum and maximum values of photon-energy, the maximum value (for a photon) corresponding to the mass-energy of a proton.

Minimum length - see Table 1 - can be calculated by locating a minimum mass very close (i.e. minimum distance) to the maximum mass in order to yield maximum energy due to gravitational attraction. This model leads to the equation

$$c^5.To/G \text{ (maximum energy)} = h/c^2.To \times c^3.To/G \times G/L_n$$

$$\text{whence } L_n \text{ (minimum length)} = G.\hbar/c^4.To \quad ....(4)$$

Minimum acceleration can be derived by considering the acceleration due to gravity experienced by a body at the periphery of the universe. The acceleration of such a body is given by the expression

$$G.c^3.To/G \times 1/c^2.To^2.$$

This reduces to

$$c/To, \quad ....(5)$$

a value that is (interestingly) numerically but not dimensionally similar to that of 'G'. It is conceivable that a means will one day be developed for measuring accelerations of this order so that the quantisation of linear acceleration in this region can be investigated (ref 6).

The ratio between any pair of max/min values appears to depend upon the large dimensionless number

$$c^5.To^2/G\hbar \quad ....(6)$$

This number is, therefore, significant. It is also enormous (10^{124}). Since it represents the largest quantity of something (eg the mass of the universe) divided by the smallest quantity of that same something (e.g. the smallest mass that can exist), it must represent the maximum number of sub-divisions (in this case the maximum number of particles) that it is possible to have. In other words, since it is not possible to have a larger number that has any physical meaning (except by waiting for time to elapse), for all practical purposes this particular number can be taken as representing infinity.

Note: it has been suggested by Professor Kilmister that a proof of this observation may exist. The four constants concerned are the only ones that are likely to be relevant in this context. Straightforward dimensional analysis reveals that they must be combined as shown (or be collectively raised to some power) if a dimensionless number is to result.

As will be noted from Table 1, many (but not all) of the max/min values are time-dependent. Most time-dependent max values increase with time (mostly in a linear manner but sometimes as the square of To) while most time-dependent min values decrease with time (mostly in a reciprocal manner but sometimes as the reciprocal square of To).

Consequently, by going backwards in time, there must come a point where the max and min values of a variable coincide. We may take the max and min values of mass as an example. At such coincidence

$$c^3.To/G = \hbar/c^2.To.$$

(We get the same result irrespective of the pair of values that is chosen).

Solving for To yields the relationship

$$Tpl^2 = G.\hbar/c^5. \quad \text{.........(7)}$$

The symbol Tpl denotes that this time period is the same as the 'Planck Time' - the period of time that is obtained by solving the three equations that describe the speed of light, Planck's Law and the Law of Gravity to yield absolute values for mass, length and time.

Substituting the value for Tpl into the expression for minimum mass gives a value

$$Mpl^2 = c.\hbar/G \quad ....(8)$$

where the symbol Mpl denotes that this mass is identical to the Planck Mass.

Substituting the value for Tpl into the expression for maximum length (c.To) gives a value

$$Lpl^2 = G.\hbar/c^3 \quad ....(9).$$

where the symbol Lpl denotes that this length is identical to the Planck Length.

Thus, as we go backwards in time starting from the present, it is possible to chart a continual decline in the mass (c^3.To/G) of the universe; continual, that is, until we encounter the point where the maximum mass and the minimum mass are identical. The age of the universe at this point is Tpl, or approximately 10^{-45} seconds. It seems reasonable, therefore, to regard this point as being the beginning of the universe as we know it.

This early body - we shall refer to it as the Planck Universe - has some interesting features. Its mass energy (Mpl x c^2) is similar to its gravitational energy (G.Mpl^2/Lpl). Consequently, such a body might be able to appear randomly and spontaneously, with no net energy change to the system.

The gravitational energy of a particle of mass 'm' located at the periphery of the Planck Universe is given by the expression

$$- m \times c^3.To/G \times G/c.To. = m.c^2.$$

Thus the gravitational energy of a particle at this point is again equal to its mass energy. Consequently, new matter could appear at this periphery with no net change to the energy of the system, a finding that would permit the Planck Universe to create new matter spontaneously and thereby to expand. Furthermore, this ability to create new matter without inputting energy remains, even though the size and mass of the universe continues to grow (note that the gravitational potential at the periphery does not depend upon To and depends only upon 'c' - a constant).

As observed above, the mass energy of the Planck Universe is $Mpl.c^2$. If the Planck universe carried an electric charge Epl, the energy associated with this charge would be Epl^2/Lpl. However, 137 is the maximum number of charged particles (ref 7) that can be packed into a region determined by the relevant Compton Wavelength (Lpl), which suggests that in an energetic body such as the Planck Universe a charged particle would be capable of occupying any of the relevant states. In other words, we may guess that in this regard there would be 137 degrees of freedom and that the energy equation should be written

$$137.Epl^2/Lpl = Mpl.c^2.$$

$$\text{or} \qquad Epl^2 = \hbar.c/137 \qquad ....(10)$$

This is none other than the expression for the Fine Structure Constant. It is well known that the charge for Epl thus determined corresponds closely to that of the electron. This charge is not time-dependent and, therefore, cannot increase or decrease with time. This suggests that the charge of the electron is fundamental and is, indeed, the minimum charge possible.

Turning to the present universe, we can imagine a system that consists of two minimum charges separated by the maximum distance Ro. The minimum energy is $\hbar$/To (see eqn 1 above) and the electrostatic energy would appear to be E_n^2/c.To. However, noting that the number of degrees of freedom associated with the wave function for a pair of charges is 137 (ref 8), it seems reasonable to write the energy balance as

$$137.E_n^2/c.To = \hbar/To.$$

This is the same as the result given in (10) above.

The size of the maximum charge possible (E_x) can be obtained by setting E_x^2/c.To equal to the maximum energy possible - c^5.To/h.G. This gives

$$E_x^2 \quad = \quad c^6.To^2/137.G \quad ....(11)$$

Interestingly, according to this analysis, the ratio E_x/E_n is the square root of the large dimensionless number given in (6) above.

To summarise, that the universe is expanding radially and at the speed of light arises from the relationship Ro = c.To. Despite commencing as a particle of mass aproximately 10 microgrammes (Mpl), it is believed that it could have attained its current enormous mass through the continuous creation of matter at its surface. However, there appears to be no need to invent a means whereby charge can be created spontaneously and the initial charge of the Planck Universe (if any) is likely to be the same as the overall charge of the present universe.

Finally, it is noted that the Schwarzschild Radius defines the critical radius to which matter must be compressed in order to form a Black Hole. This radius is:

$2MG/c^2$, where M is the mass of the body in question.

We know (eqn 2) that the mass of the universe throughout time is given by the expression

$c^3.To/G.$

Substituting this value into the expression for the Schwarzschild (crictical) radius yields

$2c.To$, ...(12)

or twice the actual radius of the universe. This implies that the matter of the Planck Universe was compressed within a radius that is smaller than the Schwarzschild Radius. Consequently, this universe was a small 'Primordial' Black Hole, which grew and grew as time elapsed. Moreover, the expression (12) shows that the universe at present is still a Black Hole, though a very large one.

Although some of the numbers that are derived from the findings of this paper are larger than those that have arisen so far from considerations of the Combinatorial Hierarchy, the conclusion that the physical world is incremental in nature is common to both approaches. Perhaps it will prove possible at some point in the future to derive numbers as large as 10^{124} from the Combinatorial Hierarchy and thereby to cast more light on the processes described above.

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Table 1 Maximum and minimum values of common physical variables.

Variable	Maximum Value	Minimum Value
Time	To	G.ħ/c .To
Length	c.To	G.ħ/c .To
Mass	c .To/G	ħ/c .To
Energy	c .To/G	ħ/To
Speed	c	G.ħ/c .To .
Acceleration	c .To/G.ħ	c/To
Momentum	c .To/G	ħ/c.To
Angular momentum	c .To /G	ħ
Force	c ./G	ħ/c.To .
(Charge)	c .To/137.G	c.ħ/137