Objects in Discrete Physics

Proceedings of ANPA 11

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Preface

These proceedings contain papers contributed at, or based on, the 11th annual meeting of ANPA, the Alternative Natural Philosophy Association. 'Natural philosophy' is the last centuries' term for what we nowadays call 'natural science and mathematics', and it is ANPA's charter to provide a forum for scientifically well-founded discussion of fundamental issues in contemporary science. ANPA is 'alternative' because its members and supporters see great need for a fundamental re-thinking of the theories we have inherited.

As some readers of these proceedings already know, it was the work of Frederick Parker-Rhodes which led to the joint discovery by him and Ted Bastin of the physical implications of the combinatorial hierarchy (CH), the mathematical object that has formed the attractor of ANPA's endeavours from the its inception. Pierre Noyes, in his invited paper (as the FP-R Memorial Lecture) provides us with an extremely interesting reading of Frederick's physics and cosmology. The paper speaks for itself, and we thank you, Pierre!

Let me now approach the remainder of the CH papers somewhat elliptically. Clive Kilmister, dressing deceptively in traditional relativity-theoristic clothes, examines the 'clock paradox' in order to reveal to us an important fundamental point - that just as the resolution of the problem is not an empirical consequence - rather, an inherent one - for inertial frames, so should the case be for accelerated frames - but it is not, in traditional theory. Shucking then his sheep's togs, Clive closes by pointing out that the only recourse is to re-construct space-time, for which endeavour the CH-based finite and discrete theory is the only apparent candidate.

Taking the consequences of this conclusion (as it were), Pierre presents his annual attempts to weave a cloth of "real physics" out of the CH's discrete and combinatorial threads. As he himself writes, "On each occasion [ANPA 9,10 & 11] objections were raised by the audience which could not be successfully met while I was on my feet", and I am sure that all of us are looking forward to the next round! I should perhaps add, for those who missed it, that this session was (at least for me) the highpoint of ANPA 11. Speaking as the veteran of many thousands of lines of (eventually debugged) code, I think the process is converging however, since the objections seem to be increasingly revealing of actual physics. Stalking the back range of this work is David McGoveran, whose mathematics, besides putting a solid foundation under ANPA's discrete and finite program for physics, has also enlightened us. He concludes in the present paper that his "adding in quadrature" formulae, second order corrections to the pion mass and the value of the weak angle, and a new way of making contact with QED via Feynman's path integral formulation, strongly justify the entire discrete and combinatorial approach characteristic of ANPA.

The next two papers - if all concerned will forgive my analogy - represent the latest episodes of ANPA's very own soap opera, "As the Counter CHurns". As glib as the analogy however is the seriousness of the issue, namely "how does one count", if you really want to be formal about it. For those who might have missed earlier episodes, the mathematical foundations of 'counting' necessarily underlie any attempt to do combinatorics - and hence combinatorial physics. So this is serious business. Here, then, Clive comments on his previous construction, while Ted Bastin presents us with his version of same. Both Clive's mathematical confessions and Ted's early statement "Something from nothing: process takes place at the boundary between the known and the unknown" really got me thinking. I don't know what's happening to me ... I'm starting to enjoy counting formally!

The remaining papers fall into two groups. One group (the papers by Marcer and Deakin) I must refrain to comment on because I simply lack the background necessary to write anything at all intelligent. The remaining papers form - in my reading - a most interesting contrast to the CH-oriented papers above. In the latter, counting and combinatorics hold the centre stage. Nowhere however is the question asked, "what is it we are counting?", i.e., the CH is a most beautiful and subtle book keeper, but what are its records of? I do not herewith wish to exhume two earlier papers, both bearing the title "What do the bits of the CH really mean?" (one by Kilmister and one by yours truly). Rather, I wish merely to point out that the issue which seems to run through all these other papers is "what <u>is</u> an 'object'?". Almost as common is the closely related issue of the origin and mechanism of the hierarchy of objects we all see in the world around us. For better or worse, I can find no alternative to using my own paper as a mirror in which to reflect the others, and I apologise ahead of time if I have over- (or under-) interpreted their views.

All bearing eloquent testimony to the question not being idle (as surely the ghosts of Aristotle and Einstein (and P-R)) would agree, five papers - Manthey, Blizard, Clement, Pstruzina, and Gidney - tackle the issue of what constitutes an object. In addition, with the exception of Wayne Blizard, all take a solidly process-oriented view.

I apply the computational (multi-process) concept of 'synchronization', which implicitly births the notions of time, two-ness, and discreteness, to arrive at the conclusion that our notion of object is based on invariance in the time domain. Brian Clement's method of visual analogy - which akin to music should be seen rather than read - arrives at the same view: everything can be seen as waves. Louis Gidney concurs with me in the irreducible two-ness induced by the concept of an object: "Awareness (objects existing for a subject) is a particular form of the Relative Existence in all physical processes, which arises in complex systems when they interact holistically with their surroundings". Karel Pstruzina's 'endocepts' I find wholly compatible with my own ideas of both what constitutes a "cognitive" object and objects in general, because there is no difference.

Like all of the above, Keith Bowden too is interested in the issue of hierarchical organization. To his credit, I was able easily to grasp the thrust of Kron's Method of Tearing in spite of the mathematics' being beyond me (and what ANPA member can resist identifying with Kron?!). Wayne gets compound objects easily out of his axioms via composition, and I like his axiomatic approach to the problem (though we differ on other issues). In Brian's paper we see (literally) the hierarchical structure of objects. Louis wants, and Karel assumes, a hierarchic aggregational mechanism. The hierarchy (published elsewhere) formed by the cycles in my paper seems to me to fit nicely with the latter three.

Finally, Louis Gidney and Karel Pstruzina see, with me, a close connection between the concept of an object and that of consciousness. If we should come to understand the one, then surely we will understand the other.

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I would like to thank all of the authors for their stimulating contributions, and look forward to seeing them, and all the readers of these proceedings, at ANPA 12.

Mike Manthey

The Inevitable Universe
– Parker-Rhodes' Peculiar Mixture of Ontology and Physics*

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THE SECOND PARKER-RHODES MEMORIAL LECTURE

The unnamable is the eternally real.
Naming is the origin
of all particular things.

Tao Te Ching

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When asked to give a lecture on Parker–Rhodes' physics, I was somewhat non-plused. I almost replied "What physics?"—a point of view that Frederick expresses himself more than once in the book he was working on when he died.

But that would be unjust. Whatever his view, *I* assert that the discovery of the *Combinatorial Hierarchy* is one of the most important "discoveries"—or whatever you want to call it—in physics made in this century. His calculation of the proton–electron mass ratio is also a fantastic result that we are still trying to come to grips with. And his insight into early cosmology—what he called a "cold big bang" —which appeared in an early version of the *Theory of Indistinguishables*,[1] also had merit. His early universe is a lot closer to my own views now than I realized when I first encountered it. We will mention other insights as I go along.

But his views are so different from those of anyone I know or knew, that I have decided to let him speak for himself by reading passages from his manuscript *The Inevitable Universe*[2], or *TIU*, which was still unpublished at the time of his death, and add a few comments on them.

"PREFACE

"This book sets out the claim that one can infer, in strict logic, by a basically simple step–by–step argument, that whatever exists must exhibit a number of peculiarities, almost all of which are included in what is currently (or at least recently) believed by physicists about our world, both on the largest and on the smallest scale. Very little in the way of supplementary hypothesis is required, but the form and language of the argument is unusual, though fully rigorous at least up to the final interpretations. The unconventional features of the reasoning make the claim to rigour hard to assess—though I believe it well–founded—and they certainly explain, in part, why the thesis has not been set out before now.

"But, quite apart from the strangeness, opposition must be expected to arise from our culture's deep distaste for the notion of a successful *a priori* theory of this kind. The religious, averse to clarity, find it a blasphemous thought that mortal man might fathom the 'designs of the Creator'; over the fence, the scientists are equally reluctant to back hubris; and people at large find this idea, that mankind, willing to spend vast sums on threats of universal destruction, might also possess so majestic an insight into the nature of things too paradoxical to be thinkable.

"So where am I? I offer no majestic insights, certainly not 'the Truth'—though my work is nothing, if it is not a step towards the truth. And there's the rub—for, as Feyerabend has argued (F1), if ever we do reach consensus that some theory is uniquely 'true', we shall be into a new age of dogmatism. But won't there always be opposition, which if the theory isn't really true will prevail? and if it is true, why worry? Whether we would then be in for a dark night of dogma, or a blessed age of enlightenment, will no doubt be debated when the time comes. Meanwhile, the matter is hardly urgent; even the first step is yet to come.

"And it is a step across a void. Better men that me have 'stood on the shoulders of giants' (as Newton put it) to view the world, or found tools ready made by others; but I have had to stand on my own feet, and make my own (P). I have no refuge in a copious reference list, only some points of a corroborative nature can (honestly) be referenced. Nevertheless I have received welcome help and encouragement, mostly from colleagues in the Alternative Natural Philosophy Association, in conversation with Dr. Ted Bastin, Professor H. P. Noyes, Professor Clive Kilmister, Professor G. Schaefer, my son Adam Parker-Rhodes, and others. I may yet be condemned to the isolation of the would-be revolutionary; but I shall not live to see it."

.....

"LOGIC

"This book is addressed to a long neglected topic, namely the search for a coherent *a priori* theory to account for the fundamental nature of the physical world, and all that can be built on this foundation. The first author who seriously contemplated this task may have been Immanuel Kant—but he in the end concluded that it would be a logical impossibility, and I know of no less faint-hearted successor who returned to it. Success would imply that, in certain respects, the world we live in could not have been other than as we find it; I see nothing illogical in that. The human imagination has never been confined by mere fact, but our knowledge has always craved for an objective basis, a reliable givenness behind the manifold appearances out of which we have had to construct it."

.....

"METAPHYSICS

"My use of this term may well be thought a misnomer, since I by no means renounce interest in the empirical verification of any result that might follow from my 'metaphysical' arguments. But it is certainly not physics, in that neither observations nor experiment enter into what I shall do. If an adjective will solve anything, perhaps I might call it 'mathematical metaphysics'."

.....

"*Ontogeny*

"One may classify phenomena by origin, according to their dependence on four factors: creation, accident, necessity, contrivance. The last is, as far as we are concerned, the exclusive province of human beings which, being obviously non-fundamental, makes 'contrivance' necessarily dependent on the other terms. Whether either pure accident or pure necessity can account for anything is however a live issue.

"A phenomenon, even the existence of some object, is 'necessary' if it can be proved without relying on questionable assumptions that its non-existence is impossible. Thus rigorously defined, the category is normally understood to be an empty one. Nothing can exist because of mere reasoning. I propose to counter this opinion by numerous examples of 'necessary' existents; it is, for me, rather the case that accident and necessity are the *only* possible ontogenies to which truly fundamental phenomena can be assigned.

"Moreover, of these two, 'accidents' can only happen to things which already exist, so that necessity must be given precedence over accident. There is now no real opposition to the notion of accidental, that is to say, uncaused events—their non-existence would entail the prior existence of infinite information—but what, some might ask, of 'creation'?

"It is, of course, one of the canons of scientific method that divine intervention should not be admitted; but, precisely because it is 'canonical' it carries no weight with those who would reject it. In the present context, before I have justified my claim to establish necessary existence, creation must be kept, in the back of the mind at least, as the only remaining possibility. I am thus led to expect a picture of things in which necessarily existing particles rush about at random, impelled by presumably necessary laws, giving rise to all the basic phenomena of physics and chemistry, and where occasion offers of biology and, occasionally and after a very prolonged period of evolution, intelligence. Ultimately, necessity and chance must account for—or at least provide for—everything that ever happens anywhere. If not, the universe in incomprehensible."

.....

Thus Fredrick's view of the material part of the universe is remarkably similar to that of Epicurus and Lucretius. Later in the book he refers to particles that "swerve" as "clinons", after he has made an argument for their existence. Those which do nor swerve, he calls "aclinons". Relevant passages are:

"*Swerve*

"...it furnishes an observable basis for the spacetime metric, the need of which I have just argued."...

"Some particles do not swerve, except for a weak relativistic coupling to gravity; these I call 'aclinons'." ...

"...inertia must be *directly proportional* to gravitational mass..."

.....

These quotations are out of logical order, but seem appropriate in this summary to mention here. I now return to the main argument. Frederick develops his ontological point of view to a level where he feels he can make three basic claims, namely:

A: Something exists

B: Something whose definition incorporates no information exists

C: Given A alone, B is indubitably true.

Unfortunately his book still exists only as various partial manuscripts which some of the ANPA members received shortly before his death, and has yet to find a publisher. Fortunately, however, he developed a preliminary description of the the theory in an essay entitled "Agnosia" which I persuaded him to let me append to one of my papers[3] in a document that, together with the other appendices, is the closest we have to a *Proceedings of ANPA 7.* I mentioned it to him again after it had appeared, and after rereading it he remarked "It holds up pretty well, doesn't it?" or words to that effect. So I have his authority for urging you to reread it.

The argument in *Agnosia*, which he works over again in *TIU*, can be summarized as follows. He first makes what he calls in *TIU* his FIAT LUX assertion:

Something Exists

He then makes his second postulate:

This statement conveys NO INFORMATION

He claims that by considering the intersection between LOGIC and INFORMATION THEORY he can get out of the situation *within* logic that it is impossible to derive anything but tautologies from a postulate system. Thus his basic claim is that by considering two areas of thought together, he can arrive at a non-biological starting point for his theory. I repeat

THIS IS THE BASIC CLAIM

He then gives us a somewhat extended discussion of information theory, which I am not really competent to analyse, but ends up with the conclusion that it takes three bits to get the system started and that:

1 Bit is absorbed in FIAT LUX (Statement A)

1 Bit is absorbed in stating B

1 Bit left to start generating the cosmos

More bits ⇒ CREATOR

"The extra bits pertain to the Creator rather than to the Cosmos, and will not be looked at ever again in this work."

.....

"*Defining the Inchoative*

"The statement B of p. 13 is a recipe for a definition, requiring nothing but that it should incorporate no information. We may ask, for example, whether the definiend is divisible into parts, or an indivisible whole. If either is asserted so as to deny the other, information is imparted; therefore these apparent contradictions must somehow be reconciled. We can do this first at the purely verbal level, and then assign a meaning to what we have said which will realize this 'reconciliation'. If the outcome does not lead, as one might expect, straight into a contradiction, we can proceed to construct a general theory; if not, nothing will come of it.

"I therefore say that my definiend may be divided into parts, but only such as are each indistinguishable from the undivided whole; I have then to define 'indistinguishable' so as to save the prescription of 'no information' embodied in statement B. It is obvious that if each part is really indistinguishable from the whole, it too must be divisible again into as many parts as before—and so on forever. Likewise the whole must be one member of a set of indistinguishable parts of a superior whole; and so on again forever. It is clearly going to be a curious object we have to deal with.

"Next, we ask how many parts make one whole? If we state a number, any finite number, we impart information; an unspecified infinity does not do so (but an *unspecified finite* number contains information, by excluding both 1 and ∞). Thus, the statement B instructs me to define 'the Inchoative' as an infinite self-contained collection of indistinguishables."

Then he concludes that fragments of the inchoative exist, which is all he seems to need to develop his theory of indistinguishables. It was this passage in particular that led me to use a passage from the *Tao Te Ching* as the start of this paper. I have no idea whether or not Fredrick would have approved of making this connection.

"Identical, indistinguishable, distinct, are three parity–relations and cardinant, bipar, and indistinct are their three negatives, making six in all; I call the mathematics devised to embody them 'triparitous', in contradistinction to the conventional 'biparitous' maths which know only 'equal' and 'unequal'. Triparitous mathematics is called for whenever indistinguishables (called 'twins' when referring to their symbolic representations) are liable to occur. In particular, nearly all fragments of the Inchoative require this treatment. Because of their unimaginability, they exist only mathematically, having the ontology of conceivables. It is however possible in principle that some (perhaps all) of them might also have biparitous models or representations, which, under the right conditions, might promote them to imaginables. The question then is, what *are* the right conditions?"

As in *ToI*, Fredrick states these rules to be:

R:	If D is a triparitous domain, it is said to be 'rational' if and only if there is a set M such that
R1:	In M, every
R1.1:	element is expressed as a formula of a triparitous mathematic, using only symbols defined in D, and every
R1.2:	pair of elements which are images of elements cardinant (i.e. non-identical) in D are distinct in M, and every
R1.3:	relator
R1.3.1:	governs a relation having the same value over corresponding arguments whether in D or M, and
R1.3.2:	is an equivalence over all elements of M which image twin elements of D, whereas
R2:	in the domain D, every
R2.1:	element is represented by one but only one element of M, and every
R2.2:	parity–relation in D is determinable from the images of the corresponding arguments in M, and every
R2.3:	functor is represented by a functor defined in M in terms of functors already defined over D.

"At this point it is perhaps expedient to draw attention to the fact that I have now stated and defined *two* hypotheses, which I call the hypotheses of 'Agnosia' and of 'Rationality' hereinafter, namely:

Agnosia:	something exists
Rationality:	any domain, whose mathematical existence follow from the above, and is rational by the rules R, depicts an empirically observable feature, or perhaps more than one, of the real world.

That the hypothesis of Agnosia is the truth is as unlikely to be disputed as is that of Rationality to be conceded. I therefore devote, in effect, the rest of this book to an attempt (which I claim as successful) to justify the latter."

.....

As his first exemplar of a result that I trust we all agree gives some credence to his claim that there is pay dirt in the fragments of the inchoative he has succeded in naming, he cites what we now call the *combinatorial hierarchy*[4–6] , i.e. the four-term sequence $3, 10, 137, 2^{127} + 136$ which *terminates* because the last term is much greater than $(256)^2$. As he notes, this came to him before he had developed his *Theory of Indistinguishables* (*ToI*), so is out of logical order in terms of this book. Among other comments he remarks on this topic as follows.

"5.2 *The Initial Interpretation*

"This series has two peculiarities, which were noted simultaneously by Ted Bastin on first sight of the construction. First, that unlike a purely algebraically defined series, it terminates; and second, that the last two terms are 137, very close to $1/\alpha$ where α is the fine-structure constant, and $2^{127} + 136$ which at least of the right order for the corresponding constant for gravitation $1/\gamma$."

.....

Thus Fredrick's memory of his discovery is that he did *not* understand its physical significance at the time of the discovery, and that it was in fact Ted Bastin who took the ball and ran with it at that point. This is also the story I heard a few years later. To my mind Fredrick's assertion that he was *not* looking for a specific set of numbers adds great weight to the significance of the result. I suggest that we let him have this last word on the subject.

He then goes onto develop the theory of indistinguishables, using the following rules for sorting out the semantics of statemets in triparatous mathematics:

"The Rules of Concurrence

$=$	a b : a,b are identical	$\neq$	a b : a,b are cardiant (non–identical)
$\hat{-}$	a b : a,b are twins	$\hat{+}$	a b : a,b are bipar
$\perp$	a b : a,b are distinct	$\not\perp$	a b : a,b are indistinct

"These notations are used to characterize the relationship between two objects a,b as follows:

If a,b are two entities denoted by symbols which are
Indistinguishable and concurrent, then $= a \ b$

Indistinguishable but discurrent, then $\hat{+}$ a b

Distinct but concurrent, then (unless disproved) $\neq$ a b

Distinct and discurrent, then any relation may obtain"

.....

"I have claimed that the arguments to be deployed here carry the implication that there are certain triparitous constructions (existing in the mathematicians' sense, as 'fragments' of the Inchoative, definable—a further claim—as containing no information in its own specification) which are distinguished by carrying biparitous isomorphs along with them, and therefore imaginable and at least potentially observable;"

[In *ToI*, I insisted that Fredrick make explicit the implied postulate that anything which is observable has to be biparitous. For him this was to "obvious" to need stating. He agreed with me, but I see that recognizing the need for this postulate has slipped away from him againin this book. - HPN]

"and that these 'rational Sorts' do in fact correspond convincingly with some of the empirical facts, theoretical assumptions, and scale–ratios which underlie present–day physical theory.

"To support such a claim is to admit an element of *apriorism* which will be objectionable to many on philosophical grounds. There is a hint, if not more, here, of the 'synthetic *a priori*' which Kant claimed, controversially, as a non-empty class of propositions. Mine are perhaps more aptly termed 'analytic empirical'—another class (if it is not the same) which many have claimed to be empty.

"So there is likely to be a widespread hope that my reasoning can be demolished. It rests however on the concept of triparitous mathematics which has never, to my knowledge, been seriously developed before now. The slightness of its overall mathematical power is, I suppose, a sufficient explanation for this neglect. Indistinguishables are strictly non–lethal. They also defeat ordinary notation. ... But the notation I arrive at is still a ruly one, and still capable of rigour; though of course I may myself have failed to maintain it." .....

"So if you want to refute the 'inevitable universe' you can't any longer do so by *obiter dicta*—you must show either that my concurrence rules are irremediably wrong, or that they have been misapplied at some crucial point of the subsequent argument. In other words, you must 'beat me at my own game'."

.....

After having developed those parts of the theory of indistinguishables which are needed in the current context, Fredrick then goes on to develop his "physical interpretation" of the *rational Sorts* that this theory implies. His list of rational sorts is somewhat longer than in *ToI*, and is now asserted to be *complete.* His arguments for why and how they are identified as appearing in the world of experience are, in my view, bizare. None the less, I must admit that on looking over his catalogue of interpretations, I find that in my own development of the physics, I have often reached the same conclusions. Partly, this is because some of the interpretations came from me in the first place, although Fredrick in his usual casual fashion does not acknowledge this. But several of the identifications have only come to me recently, for reasons that are not consciously connected with my reading this book. Once again I have to pay tribute to Fredrick's intuitivegrasp of the physics in his scheme, and hope that others will take this list very seriously. For the notation, I refer you to *ToI.* Basically, the numbers are just the appropriate cardinals for what we would call discriminately closed subsets. After looking at the catalog, I make a brief effort to show how I think about the various things referred to.

"Catalogue of Interpretations

[ ]	The Big Bang, the universal expansion, the void
[1]	The first (proper) event
[1]	Time
[1]	The unique gravitational charge; degenerate form
$[1,1_{I_\phi}]$	Mass and Intertia
$[1,1_{I_{n\phi}}]$	Progress of linear motion
$[1,1,^n]$ etc.	The (n+1)the event in arbitrary ordering
[2]	Orientation coordinates
[2]	The two electromagnetic charge states
[2,1]	Polar coordinates for 3-space, motion on closed sum
[2,1]	Components of a meson
[2,3]	Components of a fermion
[2,1,3]	Descriptors of an aclinon
[3]	Cartesian coordinates for 3-space
[3]	The three colour-charge states
[3]	Degrees of freedom of physical dimension (M,L,T)
[3,1]	Spacetime coordinates
[3,1]	Mass length time and (irrationally) charge
[3,7]	Quark descriptors
[3,1,7]	Descriptors of a clinon
[3,1,127]	Reciprocal of fine-structure constant
[3,1,7,127]	Minimum unstable aggregate of electron-positron pairs
$[3,2,127,2^{127}-1]$	Reciprocal of gravitational coupling-constant
$[3,1,7,127,2^{127}-1]$	Minimum black hole
$[\infty]$	Atopic 'space'
$[\infty,1]$	Progress through atopic space.

[1]. In my bit-string model, this is the anti-null string and does indeed represent Newtonian gravitation.

[2,1]. Interpreting this as $(10) \oplus (01) = (11)$, this is also for me the basic Yukawa vertex connecting two particles to a meson.

[2.3]. Taking this to be level 2 of the hierarchy, this is indeed two fermions, two anti-fermions and three mesons in my scheme.

[2,1,3]. Taking this to be levels 1 and 2of the hierarch taken together, I have two chiral neutrinos, two chiral photons, 5 chiral gravitons and the Newtonian "action at a distance". These are the total number of massless "particles" or "aclinons" in

Fredrick's terminology. Once the full construction has been made, these can clump to form "quantum geons", which is my version of dark matter.

With regard to chirality, Fredrick makes the following profound remark, which I thoroughly endorse:

"If a chiral figures inhabits a space described by non–quantized

mensurands, its proportions could be progressively changed by arbitrarily small steps, ending in its becoming its antichire. This would show the chirality of both figures to be accidental, rather than an intrinsic property. All intrinsically chiral properties of material objects must therefore be quantized."

[3]. Taking this to be $(1100) \oplus (1010) \oplus (1001) = (1111)$, I agree with Fredrick that these three symmetric labels can represent 3 colors, 3 anticolors, black with (0000) colorless. The als are convenient lables for 3-space coordinates, and if one likes could be associated with mass, length and time or any three independent physically dimensional units.

[3,1]. I also identify this with space-time, but also with three absolutely conserved quantum numbers in space-time processes. Thus for me the association with charge is structural and not "irrational".

[3,7]. I agree with fredrick that the quark descriptors come in at level 3 of the hierarchy. Where we have an advantage is that thanks to McGoveran's theorem[7] we cannot go beyond three dimensions, and hence the colored quarks and gluons have to be "confined". Thus, for us [3,7,1] are the observable massive particles formed from quarks, or in Fredrick's terminlolgy "clinons".

[3,1,7,127]. This "minimum unstable collection of electron-positron pairs" is what Fredrick has sometimes called the "Noyes-Dyson argument", but does not bother to do so here. Similarly, [3,1,7,127,2^{127}-1] is my extension of the same argumnent to gravitation and does indeed define the minimum black hole.

[∞]. Here I quote Fredrick: "*Atopic Space*

"Somewhere where there is no geometry, but no lack of other terms of information. The suggestion is that the RD [∞] is well-adapted to represent such an unfamiliar 'space'. With no relevant metric, we can distinguish at most two points in any 'dimension'; but there is no pre-set limit on the number of different 'dimensions'. That describes for us a peninfinite Boolean lattice; the peninfinity we have already associated with [−∞], and the Boolean lattice property merely continues analytically my interpretation of [1], [2], and [3]."

"... I propose that the principle, that at least two elementary events are required to define any kind o geometry, may be helpful in producing a more explicit self-consistency in the account."

.....

Another novelty in this book is a completely different line of argument for his proton-electron mass ratio calculation in which the two particles are coupled by an inversion about the Compton radius or CR. The model, if one can call it that, is summarized by the following table.

	In the Proton	In the Electron
Colour state	Chromatic	White
Range	Confined within CR	Confined to beyond CR
Charge sign	Positive	Negative
distribution	by Fixed Ratios	by Random Values
Occupation	within CR	beyond CR
Vacation	beyond CR	within CR

As to the calculation itself, he makes the following comments:

"1: My speculations on the *nature of charge* give added support to the notion that the rational fragment [2,3] is peculiarly apt for representing the symmetries involved, being the smallest item in the catalogue which will serve—possibly the only one. This in turn fortifies the idea (much in need of fortification) of the 'inversion' relation between electron and proton."

[If I understand correctly what he refering to here, it is what I called $< 1/r > = 4/5$ in our presentation of his calculation[6].]

"2: The *distribution function* for the 'random' separations between vertices, whose form is perhaps mildly surprising, follows directly from the isotropy and homogeneity necessarily implied by the renunciation of 'geometry'—but compatible with many other models—and the tactic of rejecting all particular values. At least, this is compatible with the theory, and not with some others.

"3: The *degrees of freedom,* though they are only 3 (a sitting target for numerologists!) are surprisingly hard to justify expect by the argument from the [2,3] structure in Section 20.6; which structure they therefore reinforce somewhat."

[As already noted, our 3 degrees of freedom are a direct consequence of McGoveran's Theorem, and need no additional support.]

.....

I close this tribute to Fredrick's memory by several quotations which I found striking, and some of his concluding remarks:

"*Ontology versus Epistemology*

"A prevailing trend in the philosophy of science today is one that regards epistemology—how knowledge is obtained—as philosophically prior to ontology, the things we claim to know.",

.....

"For this philosophy, what I have claimed in this book to have done is impossible or absurd. ... I have always been curious to know why things are as they are, taking it for granted that they are as they seem; for how could our faculties have been granted by natural selection, if they had been systematically misrepresenting our environment? To the biologist the idea is absurd. Our natural senses are at least approximately veracious, and they can be indefinitely improved by technological aids.

"I therefore take ontology as prior to epistemology, and I claim to have shown that a small selection of 'what is' looks remarkably like what a seemingly rigorous theory says 'must be'." ...

.....

"*Objections to Inchoatism*

"Most scientists will not want to trouble themselves with any such maverick concoction as what I offer here. Only the young (and the young in heart) may be impressed. Many must die before the tide turns. The work may well be judged short on results, since so much that is high in current interest is left untouched by it. The hope (it is hardly a claim) that what is does explain might be sufficient *basis* on which to erect a more comprehensive and better balanced theory of physics in undeniably optimistic."

.....

"*The Parting of the Ways*

"It has been ably argued by Feyerabend (F1) that there ought always to be a plurality of theories and alternative views of the world for without this, as time and again in the past, men's minds will tend to think dogmatically and progress will slow down, even stop, because new thoughts have no foothold on the glacier of orthodoxy. In short, dogma is seen as the death of sciences.

"This is without doubt the most weighty objection to Inchoatism, because if it is once accepted it can hardly fail to seem irrefutable, and therefore it will be held dogmatically or not at all. If once the fundamental nature of things begins to appear transparent and objectively inevitable, the impulse of a curiosity no longer insatiable, but finally (it seems) fed, will cease from the world."

.....

"*The Inevitable Universe*

"But how much in Inchoatism is really new? In many ways it may seem like a relapse into classical ideas. Things figure in it more basically than processes; everything is perceived in its ground state, with minimal energy; unstable particles, consequently, are not seen at all. the blinding flash of the first millisecond seems more like a self-curing disease than a promising beginning. All this is very far from current preoccupations, more of an echo of former times.

"So how about the most conspicuous feature of the theory, its statement that there is no way the Universe could have been different from how it is, except in respect of historical accidents, themselves often repeatable in view of the immense size of it. There not only is not, but could not conceivably be, any alternative to compare it with. Logically necessary things undergo ceaseless random motions subject only—but decisively—to logically necessary laws, which allow the build-up of complexity even, in a few specially flavoured environments, to the appearance of life. Even more rarely, life may persist and evolve long enough to bring forth intelligence and creative imagination. Is that so?

"One could argue that to accept the possibility of successful apriorism is to trust human competence beyond plausibility; and that limitation of the results achieved limits this trust, while any looseness in the argument underminds all. But if one demonstrates, convincingly that is, that success is possible, however surprising, the objection fails. It's an all-or-none situation, and the reader is the judge. If you like it, pass it on; if not, spot the error or swallow the dose."

.....

"*The Great Coincidence*

"We find thus a mutual incompatibility between Inchoatism and any version of the anthropic principle. If the one is true, the other is mistaken."

.....

"*An Unhelpful Suspicion*

"Isn't it just a bit suspicious that the whole thing, thought up by a human mind, should explain the possibility of just such a mind?"

.....

"These however are mere speculations. In contrast, the opposition is predictable. They will say, truly enough, that it 'isn't physics'. They will say that it is impossible that any conclusions, let alone so many, should attend, legitimately, on so exiguous a premise (but absurdity does not imply falsity, else Ptolomy with his unmoving Earth would be with us yet). They will say that my Rational Sorts are such that they couldn't fail to have manifestations of one kind or another; but will they offer any viable alternatives? All this may turn out helpful propaganda for the Inchoatists; so the more intelligent opponents will say nothing and carry on. The blind eye weathers the storm."

.....

"It has, too, these twin attractions—for the atheist, that the Creator is all but exorcised from the foundations; and for the theist, that so inauspicious a start should be turned out so remarkably well for us self-styled 'intelligent beings'." ...

The Tao is like a well:
used but never used up.
It is like the eternal void:
filled with infinite possibilities.

Tao Te Ching

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Synchronization: The Mechanism of Conservation Laws

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Abstract

The computational mechanism of inter-process synchronization, which is fundamental to reasoning about concurrent computational phenomena, can be construed to provide a non-local mechanism for quantum number conservation laws. We argue informally that the framework so provided illuminates such quantum conumdrums as wave-particle duality and distant-correlation.

0. Introduction

While physicists' use of computers has traditionally been straightforward numerical computation, the view that computational mechanisms themselves can provide a basis for physical theory is slowly gaining popularity. An example is the use of cellular automata to model field phenomena. This particlar line of inquiry is characterized by the use of global synchronous time such that the cells in effect all change state simultaneously.

In this paper, we are concerned with a different computational arena, in which the processes which constitute the system change state asynchronously relative to each other. The result is non-determinism, one of whose roots being the mechanism considered in [1]. This state of affairs obtains continually in computer operating systems, computer networks, and highly parallel multi-computer computation engines.

The design and analysis of such asynchronous systems has been the focus of sustained inquiry within computer science for some twenty years. If one could name one single concept which has been revealed as being fundamental to all endeavors - both practical and formal - in the regime of concurrent processes, that concept would be synchronization. It lies literally at the core of our understanding of such systems, and gives both birth and profound meaning to such concepts as discreteness, time, and object.

Turning from computer science and synchronization to physics and conservation laws, we note in passing that the latter have typically been viewed as being, at bottom, profound empirical expressions of how Nature works. We, at least, are unaware of any inquiry into their mechanism. On the other hand, we are well aware of the failure of all prior attempts to provide a 'mechanical' model for quantum phenomena. As argued previously [1] however, we feel that computational - that is, information based - mechanisms may well succeed where other approaches have not. It is our hope that the present discussion of the intimate connection between synchronization and quantum conservation laws will inspire others to clothe the analogical bones herein presented. At the same time, we do not intend to give the impression that we have a complete answer to the physical issues in question. For example, our computational metaphor still lacks an energy (and hence, mass) concept. These flaws notwithstanding, we wish to describe what appears to be a new and promising way to view the concept of a quantum particle.

It is perhaps worth noting that although the message of this paper is couched in prose and proceeds at a generally intuitive level, the computational phenomena to which we refer have been treated formally by many authors [e.g. 2,3,4], in many different ways. Rarely, if ever, however, is any attempt made to relate these formal treatments to physical theory, the work of C.A. Petri [5,9] being a prominent exception; see also [6]. We also note that these formal treatments prefer, for technical reasons, less determinedly 'mechanistic' models than the one we use here (precisely because it is mechanism per se that is our focus). An example of the difference is that in our model, not only processes but also some actors (see below) possess state, which circumstance causes mathematical complications. In addition, some models [2,3] approximate true concurrency of events with non-deterministic sequential interleaving of same, a convenience which may not be physically appropriate. Most writers assume as well an *a priori* space-time in which events are embedded, an assumption this writer finds indefensible.

The structure of this paper is as follows. We first present the concept of synchronization and describe its basic properties. We next present the role of synchronization in the creation and maintenance of conceptual resources, and the connection of this to quantum conservation laws and wave-particle duality. Finally, we interpret the results of distant correlation experiments in the light of the framework we have established.

1. Synchronization

We will in the following use a low-level but otherwise generic form of what is known as a "message passing" model of computation. The basic elements of message passing models are *objects* or *actors* [7] which pass finite, internally ordered *messages* from one to the other over one-way perfect communication links of an otherwise unspecified character. Actors are active agents which receive messages, react to them in some particular fashion (e.g., as defined by some algorithm), and then send zero or more messages. If a given actor is *primitive* (as the synchronization actor we will focus on is), then its operation is considered to be *atomic* , in the sense that (a) its operation is instantaneous, and (b) it can only operate on one message at a time. Given some set of primitive actors, one can build 'composite' actors; the process concept defined below applies, via decomposition, to composite actors as well as primitive ones, a detail which we however will generally ignore. The reader is referred to [1] for further details.

The relationship between various actors in a computation can be visualized in the form of a directed graph where the nodes are actors and the directed arcs are communication links (see Figure 1).

```
0--->0--->0--->0--->0
```

Figure 1. A simple computation viewed as messages passed between actors.

The receipt of a message by an actor is called an *event* , and a *process* is defined as a totally ordered sequence of events such that, for any two consecutive events, the post-conditions of the first event logically imply the pre-conditions of the second event. It follows from the definitions that, associated with a given process, there will always be some single message (whose content will, generally, change as the event sequence proceeds). In fact, the *instantaneous state* of a process is defined to be the contents of its associated message together with that message's position between two given consecutive events. The actor net in Figure 1 would, with a single message in it, therefore illustrate a single process: its initiating event occurred at the left and its most recent event at the right. The process concept is seen to be inherently sequential.

If there were more than one message in the net, there would be multiple processes. The parallel activity which thus ensues is however of interest in the present context only to the extent that these multiple processes interact with each other. If they do not, then they are simply multiple, but individually purely sequential, processes. The reader is referred to [1] for a discussion of how this definition of process relates to that commonly used in physics (which is different from, but can be built out of, the above).

The events which constitute a particular process define a local time frame, where the most recent event is the local 'now'. Synchronization is, as we shall see, the mechanism by which we can relate the time frame of one process to that of another.

We enter the concurrent world when we are presented with two or more processes which interact with each other (see Figure 2).

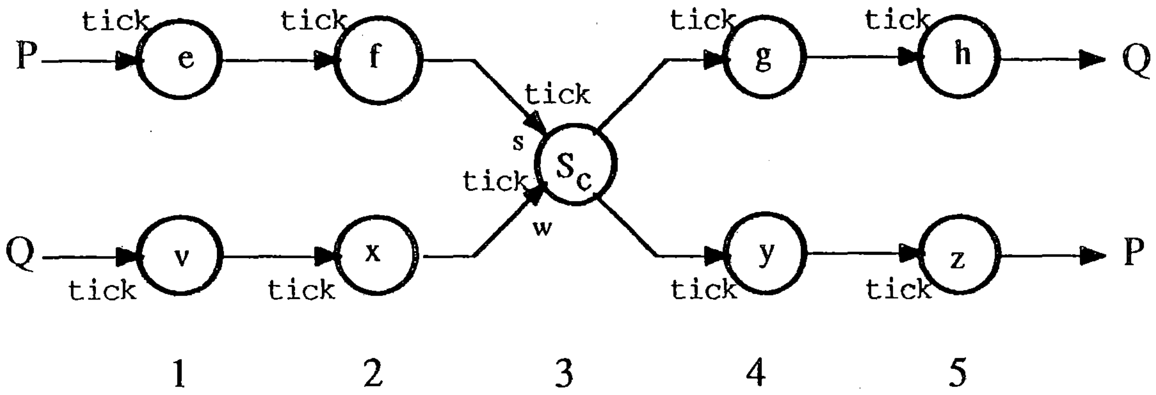


Figure 2. Two processes interacting over a synchronizer.

Figure 2 illustrates two processes, P and Q, which interact over a *synchronizer* actor S. We define a synchronizer's operation as follows:

1. There is a single internal bit of state which can take on one of two values, *open* (o) or *closed* (c), indicated with a subscipt to the S in the figure. Our synchronizer actor is a so-called binary semaphore; it is a standard exercise in the operating systems literature to show how, given one synchronization primitive, one can simulate another, at least in the sense of providing mutual exclusion and hence the basic before-after event relation, so there is no loss of generality in focusing on a particular primitive.

2. The little s on the synchronizer's in-leg indicates that any message entering there will Signal the synchronizer. Hence process P is a Signalling process. Such a Signal always sets the synchronizer's internal bit to 'open', irrespective of its current value. Signallers are never delayed except in the sense of the atomicity property mentioned earlier. The content of the message associated with P is unaffected by passing through a synchronizer.

3. The little w on the other in-leg of the synchronizer indicates that any message entering there will Wait. Hence process Q is a Waiting process. The further progress of a process arriving at the Wait in-leg of a synchronizer is dependent on the state of the synchronizer's internal bit. If the bit has the value 'open', then the process proceeds through the synchronizer, setting the bit to

'closed' on its way out. If on the other hand the bit currently has the value 'closed', then the process must "wait" until some signalling process Signals the synchronizer. The content of the message associated with Q is unaffected by passing through a synchronizer.

4. There is the possibility that more than one message/process is Waiting for the next Signal. Which of these is next through the synchronizer is a non-deterministic choice, reflecting the basic asynchrony expressed by the model. The same applies to Signalling processes, in that if multiple Signallers arrive at a given synchronizer, they pass through in arbitrary order

Here, process P enters from the above left (e), proceeds along and down to the synchronizer, which it Signals, and eventually exits at the bottom right (z). Process Q enters from the lower left (v), proceeds to Wait for P at the synchronizer, and will eventually exit at the upper right (h). P's local time ticks along, '1' (at actor e), '2' at f, '3' at S, '4' at y, and '5' at z. Q ticks along similarly. By making Q wait at S, the synchronizer establishes that ticks Q3, Q4, and Q5 occur *after* tick P3 (from this we cannot however infer that ticks Q1 and Q2 occurred *before* tick P3, since even though Q2 is before Q3, we cannot know Q2's relationship to P3: P2 could happen "before").

In a nutshell, *a synchronizer establishes a before-after relationship between two events in two different processes*. No more and no less. It is also important to understand that the integers 1,2,3, ... in the figure above are purely for labelling purposes, and do not in any way imply some sort of global time: rather, only the act of synchronization can establish time relationships between two processes.

Following [8], we say that two events are *concurrent* if one cannot establish which occured before the other. Q2 and P3 are an example, or Q4 and P4. If we have event *p* in process P and event *q* in process Q, and *p* is concurrent with *q* , then process P is concurrent with process Q.

Notice, as alluded to earlier, that the before-after relationship generated by synchronizers is restricted. For example, even with the synchronization, we cannot say whether P2 happened before or after Q2. Nor can we say whether event P4 happened before or after event Q4. P4 could conceivably happen a hundred years from now and Q4 and Q5 in the next few microseconds. All we know for sure is that, relative to process P, Q3, Q4 and Q5 happened after P3. Of course we still know what we knew before about events within the *same* process: P1 occurred before P2 occurred before P3 occurred before P4 occurred before P5, and similarly for process Q.

One could nevertheless complain that this isn't much of a 'synchronization', since it seems to give us so little. If we want more, we must put in more: there is no free lunch. If, for example, we wanted the events in P and Q to be totally ordered amongst each other, we must synchronize between each step of each process, in each 'direction'. The figure only shows a single sychronization, going from P to Q.

It is intuitively clear that synchronization is deeply involved with issues of relative time frames and hence relativity theory. However, besides that fact that further discussion in this direction would require some notion of geometry and space, this would also lead us away from the topic at hand.

Let us instead look at a simple computational application of synchronization. Figure 3 illustrates the interaction of two processes, a Reader and a Writer, over a shared primitive memory actor. The problem is to ensure that a Writer never updates the memory before the previously written value has been read by a Reader. We will assume for the sake of simplicity (and without loss of generality) that the memory is destructive read-out ("DRO") and contains no value initially (#), so that the Reader is initially waiting for the Writer.

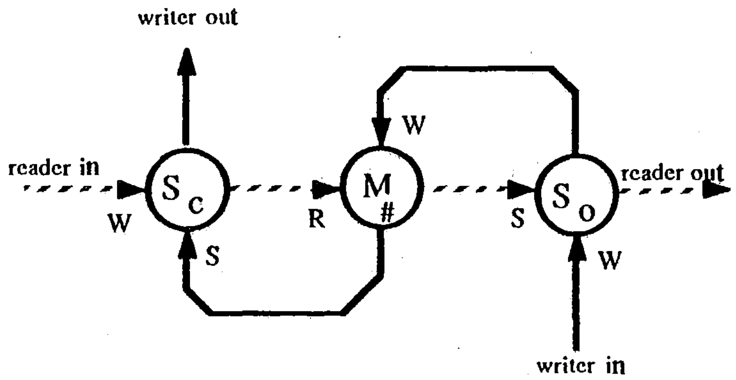


Figure 3. Two processes communicating over a shared, sychronized memory.

The Writer, arriving at the right-hand synchronizer, finds it 'open', so it goes through immediately, setting it to 'closed'. Both synchronizers are "now" closed, and the Writer is alone "inside" in what is called a "critical section". The Writer writes a value into the memory and thereafter Signals the left-hand synchronizer, setting it to 'open' and exiting the critical section. Since the right-hand synchronizer is closed, any subsequently arriving Writers will have to "wait" until a Reader process has emptied the memory again.

The focus of activity now shifts to the left-hand synchronizer, which is now the only one that is 'open'. A Reader may already have arrived, in which case it was "waiting", and hence can now pass through the synchronizer, setting it to 'closed' in so doing. Now the Reader, all alone in the critical section and both synchronizers being 'closed', proceeds to read (and hence empty) the memory, and thereafter Signals the right-hand synchronizer. Our little system is now back in its initial state. Its activity will oscillate between Writing and Reading forever, and each process mutually excludes the other in its activity.

A useful way to think about such synchronization patterns is found in mine railroads, where to ensure mutual exclusion on a particular section of track, a bucket with a stick is placed at its entrance. The driver of a train wishing to use the section of track removes the stick, replacing it in the bucket when he has finished. Hence the stick represents a permission to use the section of track, and a train may not proceed without one.

In the case of synchronizers, we imagine that an 'open' synchronizer contains a virtual such stick. A Waiter, in the process of going through an open synchronizer, takes this stick with it, which action implicitly closes the synchronizer. Similarly, to Signal means to place a stick in a synchronizer, causing it to be 'open'. It is crucial however to understand that unlike the mine-train sticks, synchronizer sticks are virtual - they exist implicitly in the position of the process in the network, and not as "real" state bits in the message. Sticks are, in themselves, naked 'instants' of time.

The Reader/Writer example is typical of the use to which synchronization is put in computer systems. Sometimes one speaks, as we have here, of using synchronization to establish and maintain critical sections, and other times (as we will henceforth) one speaks, equivalently, of using synchronization to establish and maintain mutually exclusive use of some "resource" among a set a processes. The resource in our example is the permission to enter the critial section. Hence sticks and resources are intimately related; we will return to this in Section 3.

Notice that what we have done in the example of Figure 2 is to use synchronization to create a new, larger "atom" of action. What goes on in a critical section is, by definition, inaccessible to any process outside it. In the most profound sense, synchronization gives birth to the very notion of atomicity, and hence of discreteness. Hence, it would be quite surprising if synchronization had nothing at all to do with the discreteness which Nature exhibits at the quantum level. We will return to this aspect in section 3.

It is important to realize that the (necessarily discrete, viz. the stick counting) resources established via synchronization are fundamentally conceptual in nature, that is, that the implied mutual exclusion is a logical necessity. Consider for example a piece of chalk. It is logically inconsistent with the notion of writing on a blackboard with a piece of chalk to talk of two people using the same piece of chalk at the same time. Mutual exclusion on the use of the chalk is necessary if we are to understand how such things behave. Use of a printer on a multi-user computer is similar - it is inconsistent with our notion of how such a printer should function to allow two users to write on the printer simultaneously, since this would hopelessly interleave their output. The Pauli exclusion principle is, we believe, of a similar nature: it is logically inconsistent with the empirically demonstrated discreteness of electron states that they should behave any other way.

2. Observing Synchronization

Let us now turn to the issue of "observing" synchronization. We use the term "observe" to mean that there exists some set of processes (the 'system'), of which the observer process is one, which interact via synchronization. The states of these processes, the observer process not least, constitute the only available information regarding the state of the system, i.e., we do not permit god-like omniscience.

The basic experimental set-up for observing synchronization can be described by the diagram in Figure 4.

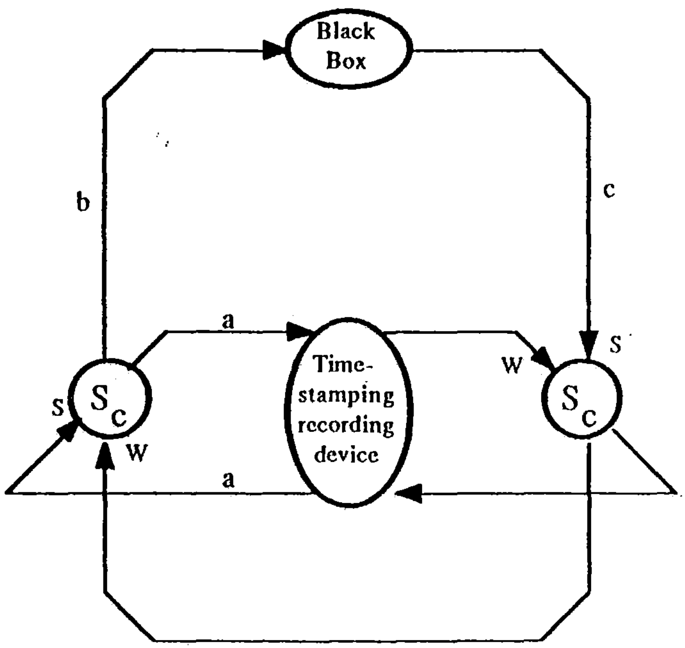


Figure 4. The basic set-up for observing synchronization.

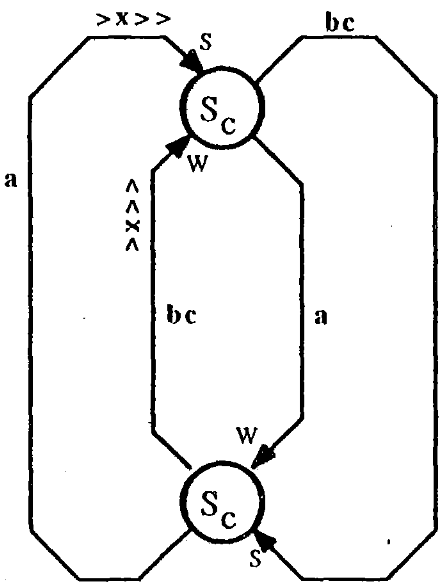
Process **a**, representing the experimenter, releases at the lefthhand synchronizer a probe process **b**, which interacts with the black box. As process **b** proceeds on its way, process **a** records the (local) time at which it did this using the "time stamping device" in the middle, and then proceeds to Wait at the righthand synchronizer until the black box reacts (presuming it does). We will assume for the sake of the example that the 'time' between the release of process **b** and the time-stamping of same is insignificant compared the 'time' the black box takes to react, although as we have seen in the preceding discussion, this in itself is a sticky issue. In the last analysis, however, the time-stamp can only pair events.

Process **c** represents the (presumed) reaction of the black box to the probe, and Signals process **a** at the righthand synchronizer, whereafter process **a** records the 'time' of this event and can now release a new probe. The same assumption of the insignificance of delay applies here too.

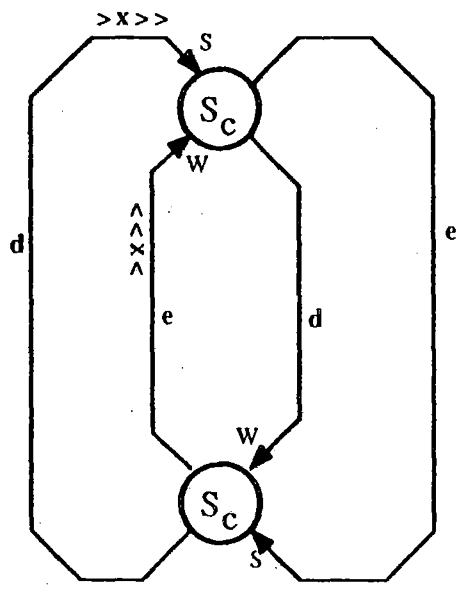
Notice that processes **b** and **c** can be combined into a single extended process **bc**, which can loop just as process **a** does. After some number of probes and reactions, the 'times' recorded by the time-stamping device can examined to see what time-wise relationship - if any - exists between the left- and righthand synchronization events. Recall that since we are here only examining synchronizational interactions, the content of message/process **bc** is unimportant - only relative timing is of interest.

Let us assume now that processes **a** and **bc** continue to 'loop' around as described, and that the black box reacts in some regular way with the probe. Even without the time-stamping device (which merely produces a permanent record), we could correctly say that process **a** *resonates* with the black box system, i.e., process **a** and the black box are two systems which resonate with each other via a "medium", process **bc**, which "transports" a "force" or "field" whose influence we see in process **a**'s behavior.

Let us now look a little more closely at an archetypal interaction between our observer (process **a**), from which the time-stamping device has been deleted, and a minimal black box (see Figure 5).



The Observing System



The Black Box System

Figure 5. Detail of the (uncoupled) observer and black box systems.

Initially, as shown in the figure, we have two non-interacting systems, the observer on the left, and the black box on the right. The synchronizers are all initially closed and messages ">x>>" are located as shown. Given this initialization, both systems will oscillate in splendid isolation, their constituent processes each trading a single stick back and forth internally. Although the two systems are shown as being identical in structure, this of course need not be so.

Now we interconnect the two systems (Figure 6).

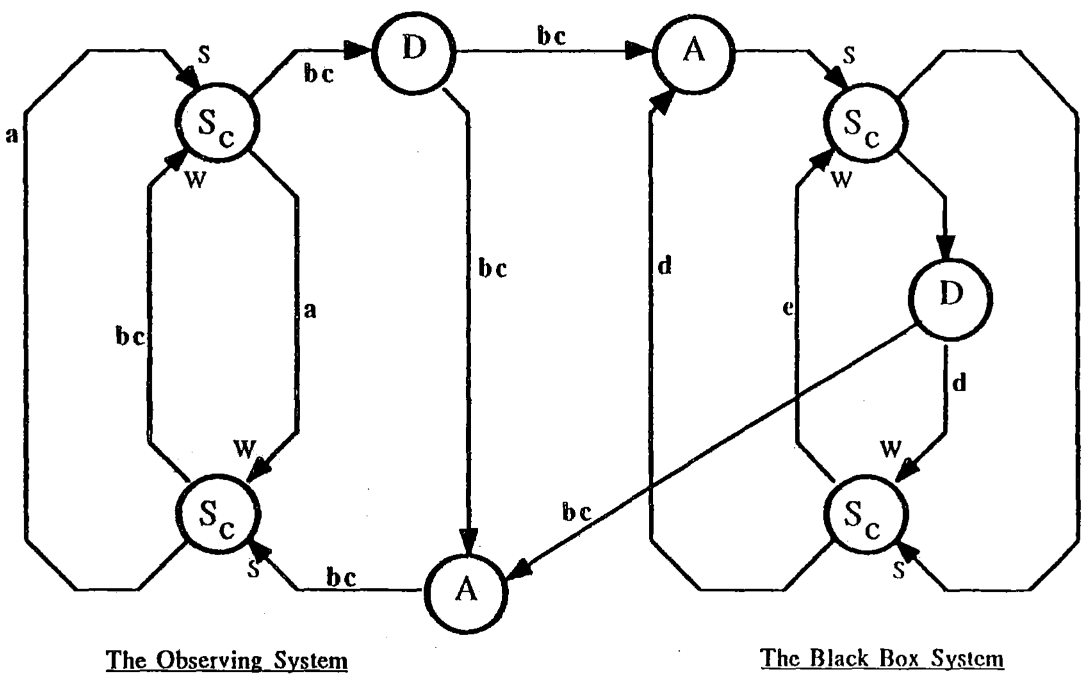


Figure 6. The observer and black box systems connected together.

The top middle decider ("D") in Figure 6 makes a decision which we control (by means not shown) to switch our probe process **bc** into the black box system when we so choose. The other decider - in the black box - expresses the fact that the black box behaves differently when **bc** is present from when it is not. Process **bc** always returns back to whence it came, whereas process **d** always remains in the black box. How it is that the black box does this is not important, and this decider merely reflects the fact that it behaves like this. The two arbiters ("A") are necessary simply in order to make the net a legal actor net and have no direct bearing on the situation.

As noted earlier, as long as the top switch-decider simply sends **bc** back into the observer system, both systems will simply oscillate. What will happen when we instead send **bc** in to probe the black box system? If it should occur that **bc** gets to the top (right) synchronizer before **d** does, then it will deposit a stick and return back to the observer system. Presuming that this happens continually, what will be the effect of this action on the behavior of the two systems?

Let us look at process **e**'s behavior first. Due to the appearance of **bc** at more-or-less periodic intervals, there are now roughly twice as many sticks potentially available to **e** as there were previously. If it had been the case that process **e** had frequently had to Wait at the top right synchronizer for **d**, this would now occur less frequently. The oscillations of the black box will then almost certainly be different in this case - **e** will circulate much "faster". On the other hand, if **e** almost never had to wait for **d** (**d** was "faster" than **e**), then little change in **e**'s behavior would result, since the stick from **bc** would be "overwritten" by **d**'s.

Now what about the observer system - what effect does **bc**'s continual probing have on it? The story is similar to the one we just heard. **bc** is almost certainly going to be "slower" in its Signalling of **a** because of its (**bc**'s) detour through the black box. This will almost certainly mean that **a** will circulate more "slowly", and hence the oscillation of the observer system will in general be affected as well.

We could look at some more examples, but we have enough here to draw our central conclusions. First and foremost, the effects of persistent synchronizational interaction will be fundamentally oscillatory in nature - a correlation of events in the time domain. This obtains because we are investigating a time-based (or rather, a time-creating!) phenomenon. Notice especially that this kind of behavior does not appear in memory-based interactions (cf. [1]). Vice versa, the kind of non-deterministic behavior we find there does not appear here: the two kinds of interaction - communication and synchronization - are fundamentally different.

The second conclusion we can draw is that both parties to the interaction are affected - observer and observee. The effects have been lumped under the term 'resonance', but it could as well have been called interference. The exact character of the resonance depends naturally, as we have seen, on the internal 'timing' of the two respective systems. In the end, we must turn to Nature - empirical observation - for a unit of physical time, and then pray to the god of isotropy that all hydrogen atoms are the same.

The third conclusion is that while we were engaged in observing, we in fact created a new and larger system - an observer + black box system. This compound system itself exhibits a characteristic compound oscillatory behavior, exactly what we would expect when we couple two individually oscillating systems together.

We need not necessarily observe periodic behavior when observing synchronization - only when there are sticks being handed off on a regular basis. The number of sticks circulating also has an effect. It is obvious that if there is a large number of sticks circulating in each net, we wouldn't see much effect from adding (or consuming) one more. Rather, it is when there is a small number that the effect really stands out (shades of Bohr's correspondence principle, to which, incidentally, we do not entirely subscribe for reasons lying outside the scope of this paper).

The essential point is that, since synchronization is the means by which time itself is promulgated, observations of synchronization of necessity lie in the time domain. It follows that any observation involving persistent synchronizational interaction with the same target object will necessarily be oscillatory in character.

3. Conservation

The resources of which we have been speaking are what are called "non-consumable" resources, in that such resources are acquired, used, and then released for re-use by other processes. A typical computer operating system contains many such resources - memory and disk blocks, printers, tape drives, table slots, etc. Analogous to the single stick in our example, a system with three tape drives would have three sticks, and the statement that there are always (barring failure, etc.) three tape drives in the system is equivalent to saying that there are always three sticks. Sometimes the operating has them, and sometimes the user processes have them, but there are always three of them.

We return now to the alternating Writer/Reader example of section 1. In this example, there is a single stick, which is initially in the right-hand synchronizer. A Writer picks the stick up there and places it in the left-hand synchronizer, and a Reader picks up the stick there and moves it back to the right-hand synchronizer. The thing to notice here is that there is always exactly one stick, which one-ness is intimately related to the mutual exclusion property of the construction as a whole.

This invariant state of affairs can be expressed in equational form, a so-called *resource invariant* :

$$N = N_0 + \sum \text{releases} - \sum \text{acquisitions}$$

where N_0 is the initial amount of the given resource and N is the number currently available to requesting processes. In the case of the alternating Writers/Readers, $N_0=1$, and in the case of the computer system with three tape drives, $N_0=3$. In the latter case, we could say that the system cum users obeys "the law of conservation of tape drives". Moreover, this is not at all meant facetiously, but rather to motivate the point that *synchronization is the mechanism by which such laws are created and maintained.*

Not only is synchronization, in addition, the mechanism by which time is propagated between processes, it is furthermore, as mentioned earlier, the mechanism by which conceptual atoms or objects are created. Indeed, one can argue that it is the very property of conservation that supplies the continuing existence through time that we require of anything which is to be viewed as an object. Quantum mechanical particles, which are known and identified solely by their quantum numbers, are a particularly naked example of such 'conceptual' objects. Viewing then the world of quantum numbers - integers or half-integers which count up and down in units of one - we find their analogy to computational resources compelling. Eddington's definition of a particle as the carrier of conserved quantum numbers, translated into the computational metaphor, becomes "a particle is a set of processes whose interactions conserve various sticks". This is what we mean when we say that synchronization is the mechanism of (quantum) conservation laws.

Recall now that besides being a mechanism for conservation laws, synchronization is also the mechanism by which instants of time are, so to speak, transferred from one process to another. Viewed from this perspective, the (e.g., two) processes involved in a stick-conserving relationship will alternately pass a stick to each other. Hence, the character of the object so created will inevitably be oscillatory. We advance therefore the idea that in this we find a basis for wave-particle duality: if we interact with the stick-conserving entity as a whole, we see that whole; if on the other hand we interact with its constituent processes, we see these processes' oscillatory relationship with each other. We believe that the fecund profundity of the synchronization concept is sufficient for taking this idea seriously in spite of the (presumably temporary) absence of an associated uncertainty principle, which is clearly hiding in the woodwork.

4. Non-Locality

Consider now some system in which the constituent processes interact via synchronization. We described earlier the inevitably oscillatory character that observation of such an object system will have, and just saw the connection between this and wave-particle duality. In this section, however, we are interested in how to observe a single synchronization.

We will think about this in the following way. Suppose that, referring to Figure 2, the Waiting process Q, after its synchronizastion with P, interacts with an observer process O (not shown). We pose the question: how can we know, via O, if Q in fact actually did wait for P, i.e., was the synchronizer 'open' when Q arrived or not?

First off, we can eliminate the possibility of looking (via O) at Q's current state as reflected by Q's message content, for this is totally unaffected by passing through a synchronizer. As explained earlier, the only difference between a process' state before and after passing through a synchronizer is just exactly its position in the network and no more.

Second, we can discard the possibility of appealing to some sort of global clock, because O and Q would have to synchronize with this clock in order to establish a timing relationship. This, in the last analysis, puts us back where we began.

Since any observation of synchronization is ultimately in the time domain, and time is both irretrievably relative and mediated only via synchronization, we are ineluctably driven to conclude that the only way to tell whether Q was delayed or not is to look at the effect a delay has on all the processes constituting the system in question. This effect will be revealed in the fact that various events (subsequent to the synchronization) might occur in a different order: the system will, in general, behave differently depending on the order in which the various processes arrive at "later" interaction points and the events which thus ensue.

Hence, ultimately, we can only know of Q's past in terms of correlations between the interactions of the processes constituting the system. If Q was in fact not delayed, one set of correlations will obtain, whereas if Q did in fact have to wait, a different set of correlations will obtain. In other words, we can only know about a synchronization in terms of the system as a whole. In other words, even though synchronization occurs locally, we can only see its effects non-locally. In other words, *synchronizational interactions will reveal themselves in terms of non-local, correlatively related effects.*

We are clearly on the doorstep of a discussion of distant correlation effects, but we lack one last piece before we can put the puzzle together: how can we square the empirical facts with the absolute ceiling on the speed of light, and hence on the speed at which an effect can propagate? In fact, the answer to this question is already in hand: since synchronizational interactions do not transfer information (viz. the message content is unchanged), and the limitations regarding lightspeed refer only to information bearing signals, we see that synchronizational effects are free to be supra-luminal.

If we now view the EPR experiment in the framework we have established, we see that there is very little left to explain. Bohr argued that the two anti-correlated particles, though distant from each other, still constitute a "bound" system. We agree, and add to this that it is precisely the conservation law which binds them. Furthermore, such conservation laws are the stuff from which our very concept of a coherent object is constructed. Indeed, were it not for the conservation law binding the two particles, we would be unable to construct the empirical correlation at all.

We have, in addition, argued that the mechanism underlying such conservation laws is synchronizational in nature. Hence any synchronizational interaction with a stick-conserving system, not least a physically distributed one, can avail itself of supra-luminal speeds and will in general reveal itself in terms of correlations. If we now model the two anti-correlated EPR particles as stick-conserving subsystems of a common stick-conserving super-system, then each subsystem is free to evolve on its own, whilst they exchange, say, one stick between them to form the super-system. The "breaking in" of the empirical observation will discover that the super-system's stick is randomly in the one place and not the other.

Thus, with neither paradox nor magic, is the EPR rabbit pulled out the computational hat.

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THE LOGIC OF SOLID OBJECTS*

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This paper is a discussion of ideas and issues introduced in [1]. We begin with a story that is said to have taken place in late 1987 or early 1988. It is most probably true, but it has not been verified. The London Symphony Orchestra (abbreviated by LSO) was booked to perform several concerts in Israel. An Israeli official was visiting London several months before the scheduled concerts. He was astonished to discover that the LSO was scheduled to perform in London on exactly the same dates as the Israeli concerts. Fearing a mistake had been made, the official enquired at the LSO administration offices. He was told that no mistake had been made – the LSO was to perform in London and Tel Aviv on exactly the same dates. The official asked how this was possible. He was told that certain key members of the LSO would be with each of the two orchestras. Other musicians would be hired to bring both orchestras up to full complement. When this information was transmitted back to Israel by the official, the Israeli concerts were cancelled by the Israeli government on the grounds that the *real* LSO could not be in two distinct places at

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exactly the same time. This is an accepted principle of law: if an individual is at location x at time t, then that same individual is most certainly not at location $y \neq x$ at time t (otherwise known as "having an alibi"). The Israeli decision was upheld in the courts. The entity LSO could perform at at most one location at any given time.

The formal theory outlined below does not attend to symphony orchestras or suspected criminals. The LSO story does illustrate, however, one of the main principles (that of Axiom II) underlying the theory of solid objects. We now define the formal theory OST (Objects, Space and Time).

The axioms of OST are intended to formalize the motion of *small solid* objects on a discrete location grid with reference to a discrete time sequence "tick, tick, tick, ...". By *small* we mean that objects occupy a single location point. By *solid* we mean that no two objects occupy the same location point at the same time. The theory OST uses three sorts of variable symbols: A, B, C, ... denote *objects*; x, y, z, ... denote *location* points; and r, s, t, ... denote *time* values. The intended interpretation of the ternary predicate A_x^t is "object A is at location x at time t"; of the binary predicate N(x,y) is "x is a neighbour of y"; and of the time term t' is "the tick immediately after t". The logical axioms of OST are those of the first-order predicate calculus with equality. The non-logical axioms of OST are –

I	$\forall A \, \forall t \, \exists x \, A_x^t$
II	$(A_x^t \wedge A_y^t) \rightarrow x = y$
III	$(A_x^t \wedge B_x^t) \rightarrow A = B$
IV	$(A_x^s \wedge A_x^t) \rightarrow s = t$

V	(i)	$\forall x \forall y\, (N(x,y) \rightarrow x \neq y)$
	(ii)	$\forall x \forall y\, (N(x,y) \rightarrow N(y,x))$
	(iii)	$\forall x \exists y \exists z\, (N(x,y) \wedge N(x,z) \wedge y \neq z)$
VI	(i)	$\forall t\, (t \neq t')$
	(ii)	$\forall t\, (t' \neq 0)$
	(iii)	$\forall t\, (t \neq 0 \rightarrow \exists s\, t = s')$
	(iv)	$\forall s \forall t\, (s' = t' \rightarrow s = t)$
VII		$(A_x^t \wedge A_y^{t'}) \rightarrow N(x,y)$
VIII		$(A_x^t \wedge B_y^t \wedge N(x,y)) \rightarrow \sim (A_y^{t'} \wedge B_x^{t'})$

For Axioms II, III, IV, VII and VIII, we mean the universal closures of the formulae stated. The formal details of the theory OST are given in Section 2 of [1].

Axiom I states that objects are always located: for every object A and every time t, there exists a location x at which A is located at time t. In other words, there are no unlocated objects.

Axiom II states that if object A is located at x and located at y at time t, then x and y are the same location. In other words, an object cannot be at two different locations at the same time. For now, we are dealing with *small* objects – objects which always occupy exactly (at most) a single location. However, as we shall see, this principle generalizes to larger objects as well.

The LSO is not a small solid object. However, the story nicely illustrates the principle idea of Axiom II. The space occupied by a solid object is "connected" – there cannot be a piece of object A 'here' and a piece of object A 'there'. We expect solid objects to be *whole* and *local* in this sense. This is not the case for pieces of shattered pottery which no longer constitute a solid object. The entity West Germany (not a solid object either) violates this principle – West Germany includes West Berlin which is located inside East Germany.

Axiom III states that if object A is at x at time t and object B is at x at time t, then A and B are the same object. In other words, distinct objects cannot occupy the same place at the same time. The theory OST, therefore, deals with *solid* objects.

In general, every solid object with extension occupies space *exclusively*. No other solid object is able to co-inhabit the space. For many animals (including *Homo sapiens*), this exclusivity of space occupied by their bodies is extended outward to a territorial exclusion zone in which other animals are prevented (or discouraged) from entering. This same exclusivity is fundamental to Archimedes' Principle. The amount of water displaced by a solid object which is totally immersed in it, exactly equals the volume of that solid object, no more and certainly no less.

In Chapters 5, 6 and 7 of [3], Richard Sorabji surveys various classical answers to the question – Can two bodies be in the same place at the same time? The usual answer ('No') rests on the idea "... that [solid] bodies cannot interpenetrate, in other words, that they cannot be, and their parts cannot be, in exactly the same place" ([3], p. 61). This idea was not accepted by all the Greeks. For example, the Presocratic philosopher Anaxagoras held that there is a portion of everything in everything. Sorabji concludes, "So either in a stronger, or in a weaker sense, Anaxagoras does seem to have put stuffs in the same place. He must even allow that stuffs can *move* into the same place ..." ([3], p. 65). Aristotle, on the other hand, strongly opposed the idea of bodies in the same place. In fact, he used the idea of bodies in the same place in *reductio ad absurdum* arguments against other people's views. In short, Aristotle held that any answer to the above question other than a firm 'No' was an absurdity ([3], pp. 72–73). He asked, if two bodies were in exactly the same place, would there still be *two*? ([3], p. 75).

Sorabji defends the Stoic response to Aristotle. In Stoic metaphysics and chemistry, there are many instances of bodies interpenetrating – that is, two bodies being in the same place. For a lively and lucid discussion of the Stoic versus Aristotelian debate on this point, see Chapter 6, pp. 79–105 in [3]. Some Neoplatonists like Plotinus sided with Aristotle, while others allowed interpenetration as exceptions. The issue of bodies being in the same place was also important for Christian theology (for details, see [3], pp. 120–122). Sorabji concludes his discussion with a quote from Milton's *Paradise Lost* –

> Obstacle find none
> Of membrane, joint or limb, exclusive bars.
> Easier than air with air, if spirits embrace
> Total they mix.

and the rather remarkable observation, "What Milton tells us is that angels, unlike humans, make love by total interpenetration ..." ([3], p. 122).

There has also been a good deal of discussion recently as to whether or not certain elementary particles (like bosons) occupy the same place at the same time (for references, see [3], p. 73). However, such a question may, in fact, be meaningless for such particles. David Sanford has argued that there is nothing wrong, in principle, with the idea of solid objects occupying the same place at the same time. For a summary of Sanford's thought experiments (with discussion and references), the interested reader should consult [3], pp. 74–76.

Lawrence Lombard defines *events* as changes in objects, where an object is said to change if and only if it has a property at one time and lacks it at another. He states "... when an object changes by having and then lacking a property belonging to a given quality space, the object comes to have another property from that same space." ([2], p. ix). Lombard finds that events are like physical objects in that they are occupiers of places at times. However, events are unlike physical objects in that distinct events may occupy the same place at the same time (a situation called *multiple occupancy*). For further details of Lombard's theory of events, see [2].

Axiom IV is controversial. It has exactly the same form as Axioms II and III. It states that if object A is at location x at time s and at time t, then s and t are the same time. In other words, an object cannot be at a location at distinct times. This is a property of solid objects which we call *non-returnability*: objects do not return to the same location at different times. This certainly would seem to run counter to everyday experience. My coffee mug returns to the same location on my desk every morning. Yet "the same location on my desk" is a relative location. My desk is *not* in the same location (relative to the sun) every morning, and the sun itself is *not* in the same location relative to the centre of the galaxy.

If for "object" we read "system" and for "location" we read "state", then the statement "no system is ever in the same state twice" seems less difficult to accept. Nevertheless, why rule out objects being in the same place at different times? The reason is two fold:

(i) Axiom IV looks good on the page. It's syntax is symmetric with respect to Axioms II and III. Formal symmetry is a powerful argument in science and mathematics.

(ii) Axiom IV has interesting consequences –

1. there are no objects at rest – everything is in motion, and
2. all motion is open or expansive motion – the path of an object cannot intersect itself.

In short, Axiom IV forces motion and defines allowable types of motion.

The three Axioms V determine the binary predicate N where N(x,y) is intended to read "x and y are distinct neighbouring locations". The four Axioms VI set up a linear sequence of time points where t′ is intended to read "the tick immediately after t".

We do not want our objects to leap across vast distances in a single time interval. Axiom VII requires that if object A is to move from location x to location y in a certain time period, then it must locate at neighbouring locations between x and y during that time period. Axiom VII is in the same spirit as Axiom II – objects are *local* and move only to neighbouring locations. Axiom VII states that if object A is at location x at time t and at location y at time t′, then x and y are neighbours. Thus this property of motion of objects could be called *discrete continuity*.

Axiom VIII is necessary in OST in order not to violate the spirit of Axiom III. Axiom VIII states that if objects A and B are located at neighbouring points x and y respectively at time t, then they cannot switch locations at time t′. If A is at y at t′ and B is at x at t′, then the objects A and B must "pass through each other" at some time between t and t′. The idea that objects can pass through each other certainly violates the idea that objects cannot be located at the same place simultaneously.

Axioms II, III, VII and VIII fit nicely with the observations of Cornell University psychologist Dr. E.S. Spelke [4], who states "... everyone seems to believe intuitively that objects move as cohesive and bounded bodies on continuous paths through unoccupied space. We believe these things so strongly that we hesitate to consider something a physical object if its parts are scattered around a room, if it appears in different places without ever moving between them, or if other solid bodies can pass through it." Dr. Spelke's research suggests that even young infants (at about five months) appear to endow objects with four abstract properties: *cohesion* (the movement of an object preserves its integrity: objects move as wholes), *boundedness* (the movement of an object preserves its distinctness: objects do not blend into other objects), *substance* (objects move through unoccupied places: objects do not pass through one another) and *spatio temporal continuity* (objects move along connected paths: an object does not move from one place to another without tracing a continuous path between them). Dr. Spelke concludes, "Learning may enrich our conceptions of objects, but learning does not appear to overturn the conception with which we begin." For further details, see [4].

One can introduce the notion of *splitting* of objects. At time t, object C is at location x. At time t′, the object C ceases to exist, and splits into distinct objects A and B at neighbouring points y and z. At time t, A and B do not exist. They "come into being" at time t′. Thus splitting allows objects to scatter to distinct locations without contradicting Axiom II. Similarly, if objects A and B are at distinct location points y and z at time t, they can *fuse* to create a new object C at location x at time t′. Location x must be a neighbour of both y and z. Fusion allows distinct objects to join together at the same location without contradicting Axiom III. Thus splitting and fusion allow situations to hold that are prohibited by Axioms II and III while maintaining the consistency of OST. One can show that splitting and fusion are duals: each splitting is a fusion with the direction of time reversed, and each fusion is a splitting with the direction of time reversed. For the details of introducing splitting and fusion into OST, see Section 3.1 of [1].

Large objects (objects which occupy many location points at a given time) can be introduced into OST with a few minor changes. We introduce a fourth sort of variable symbol: X, Y, Z, ... called *location set* variable symbols and a binary predicate symbol $\in$ such that the intended interpretation of the atomic formula $x \in Y$ is "the location point x is an element of the location set Y." For an object

A at time t, let A^t_X stand for $\forall x(x \in X \leftrightarrow A^t_x)$. Axiom I now implies that there is always a non-empty location set at which objects are located.

Axiom II must be revised to be the formal equivalent of "an object occupies at most one location set at any one time *and* location sets are pathwise-connected". By "pathwise-connected" we mean that for any two points in a location set there must be a path (a subset of the location set) in the location set that connects them. In other words, location sets do not occur as disjoint pieces.

Axioms III, V and VI remain unchanged. The obvious reformulation of Axiom IV is the universal closure of

$$(A^s_X \wedge A^t_X) \rightarrow s = t.$$

In other words, large objects are never "at rest" and can never "return" to *exactly* the same location set. The reader who felt uncomfortable with the original Axiom IV may find its reformulation for large objects more acceptable: large objects may return to *virtually* the same location set, but never to *exactly* the same location set.

If we define NN(X,Y) (read "X and Y are neighbours") to mean X and Y are distinct and every location point belonging exclusively to one has a neighbour belonging to the other, then Axiom VII can be replaced by the universal closure of

$$(A^t_X \wedge A^{t'}_Y) \rightarrow NN(X,Y).$$

One must also revise Axiom VIII to disallow "switching" of large objects; that is, distinct neighbouring large objects cannot switch location sets between some time t and its immediate successor t′. Additional formal detail is given in Section 3.2 of [1].

With large objects at hand, one has the possibility that the location sets of objects may change their size, or change their shape over time. To remain as general as possible, objects in a revised theory OST could be classified as follows:

1. *solid* – objects with constant size and shape
2. *scaled* – objects with constant shape but variable size
3. *fluid* – objects with constant size but variable shape
4. *organic* – objects with variable size and shape.

Under these definitions, a small object which remains small over time is a solid object.

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ANPA HAS MUCH TO CELEBRATE

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To paraphrase Ehrlich[1], with the appearance of J.H. Conway's "On Numbers and Games"[2], ANPA has much to celebrate.

It is Conway's important discovery that the familiar Dedekind cut and Von Neumann's ordinal constructions are part of a more general construction which leads to the proper class of numbers No, the real closed ordered field which describes all numbers great and small [1,2]. Conway shows in particular that No for which Knuth[3] coined the term surreal numbers, is a complete binary tree of height On, On being the class of all ordinal numbers and that No has a natural canonical power series structure[2]. Conway also showed that there is a characteristic 2 analogue of No, a field he calls On_2, which shares all its properties and is in fact the simplest way of turning the class of all ordinal numbers in a field. On_2 concerns Conway's construction of the surreal numbers where the distinction between L and R (and so between + and -) is abolished; it has the simplest canonical power series structure.

Kilmister in his Brouwerian Foundation[5] has shown that the Combinatorial Hierarchy concerns computation over On_2, which following Conway[2], Alling in his "Foundations of analysis over the Surreal Number Fields"[4] shows is therefore optimal hyperconvergent, i.e. in the final analysis essentially finite and polynomially complex. Ehrlich[6] also shows that such computation concerns the optimal universal model for a theory T over the language L of the sets (0,1) where computation is essentially a model of input-output. Since models concern the semantics of formal theories, the Combinatorial Hierarchy therefore concerns an optimal universal representation in terms of (0,1) of the semantics of T which Clement, Coveney and Marcer[7] and Kauffman[8] have shown concerns the processes of wavefield physics, and as a representation of a universal model of relational systems[6], these semantics will describe not only the relations among the structures of wavefield physics, but the properties of the domains within which such structures exist, or can come to exist, and the behaviour of the structures in the domains.

In particular Conway succeeded in showing that although No has many field automorphisms, it has a unique birthorder preserving automorphism, where the height of an element in No is called its birthorder, and since On_2 shares this property with No, it can be postulated following Kauffman[8] that such field automorphisms on On_2 will exactly explain the reversible and the irreversible properties of time, appropriate to a domain where Lorentz invariance is an inherent property ie. a 3+1 space-time, and characterizes the field.

The Combinatorial Hierarchy therefore models by means of the sets (0,1) the relations among the structures, their geometries, and the spaces within which they exist, the unique optimal birthorder process appropriate to the universal model of wavefield physics, where such universal models generate all possible representations of the theory they represent.

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DISCRETE PHYSICS & THE LORENTZ TRANSFORMATION

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INTRODUCTION

As past President of ANPA I would like to begin by extending a welcome to both new and old faces, and, before we get to business, my thanks to Faruq Abdullah, without whom there would have been no organisation at all. The committee are also very grateful to Professor Michael Redhead for his good offices in making this room available. Before I begin the main part of my paper I ought to say a little, for newcomers, about the corner of ANPA activity (originally the whole of ANPA!) in which I am operating. This is discrete physics. Years ago Bastin and I were involved in an algebraic theory that would be useful in explaining the curious logical statuses of elementary particles and of dimensionless constants, say, for example of the charge e on an electron and the ratio $\hbar/e^2 = \frac{1}{\alpha}$, the number whose value is near to 137. Parker-Rhodes found an algebraic construction, the meaning of which was unclear, but which gave rise to the numbers 3, 10, 137 and 10^{38}. Much of my activity since then has been putting a meaning into this, and Ted Bastin will say more of this in his paper.

SPECIAL RELATIVITY

Quite a few years ago in one of these meetings Irving Stein attempted a derivation of the Lorentz transformation by a combinatorial argument, using statistical formulae. I want to recant my opposition at the time, not so much to say that Stein's derivation was right, because I still have many doubts about it, but to say that I was wrong in not seeing that some such derivation was in fact needed. I will explain how this need comes about. It is easiest to do this in a personal account of how my ideas have developed.

There are two ways of setting up special relativity, (a) using both rulers and clocks, (b) using clocks alone. Einstein's original 1905 paper is of type (a), but nowadays one usually scoffs at arguments of type (a) because no realistic theory of rigid bodies exists in special relativity and so one abstracts from Einstein's argument a slightly different (b) version. In this version one uses Einstein's rule for assigning a time to distant events for observers at rest in coordinate-systems "in which Newtonian mechanics holds", what Bridgman calls "spreading time through space"; that is, one sends a light-signal at time t_1 and receives its reflected mate back at time t_2, and assigns time t and distance x to the reflection event, where $t = \frac{1}{2}(t_1 + t_2)$, $x = \frac{1}{2}(t_2 - t_1)$. (Here such units have been chosen for space measurements that the speed of light has unit value.) It is most important to realise that this rule produces an ontological change from the classical picture of Newtonian time. Instead of one place functions from events to numbers, $E \rightarrow t = f(E)$, the world contains two place functions $(E,O) \rightarrow t = f(E,O)$, where O denotes an observer.

Of course, $t_2 = t + x$, $t_1 = t - x$ and so, if two such observers are involved (fig. 1) it is clear from a mere scaling argument that

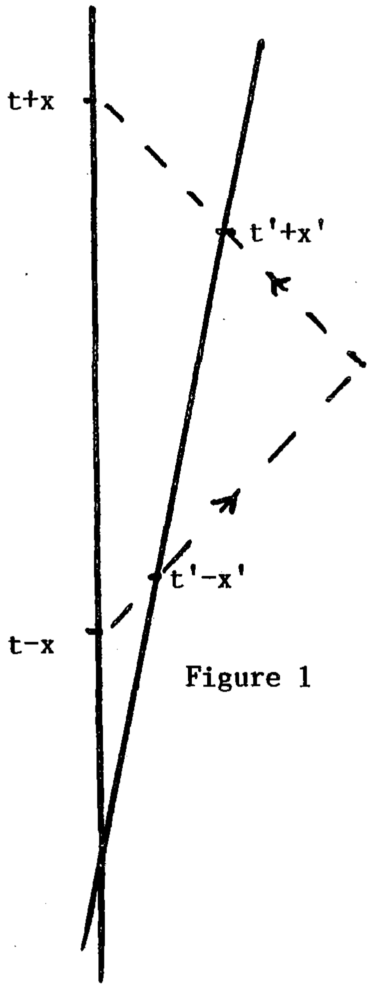


Figure 1

(if only the two observers synchronise watches at zero when they are coincident)

$$t' - x' = k(t - x),$$

where k is some constant depending on the velocity of separation. Then assuming that all inertial observers are completely equivalent, it follows that also

$$t + x = k(t' + x')$$

with the same constant k. From these two equations the Lorentz transformations follow at once and, by differentiating the equations it also follows that

$$dt'^2 - dx'^2 = dt^2 - dx^2$$

and if we call this invariant, ds^2, the square of the proper time, then $dx' = 0$ gives $ds = dt'$, so that it follows as a necessary part of the theory that uniformly moving clocks measure proper time.

THE CLOCK PARADOX

So much for what has become a popular way of setting up special relativity. In terms of this I was considering (as all workers in the subject, unless they have very strong wills, are forced to do from time to time by an army of cranks) the clock paradox. One twin stays at home while the other accelerates away in a space-ship and then returns to find his brother older than he is. About 35 years ago

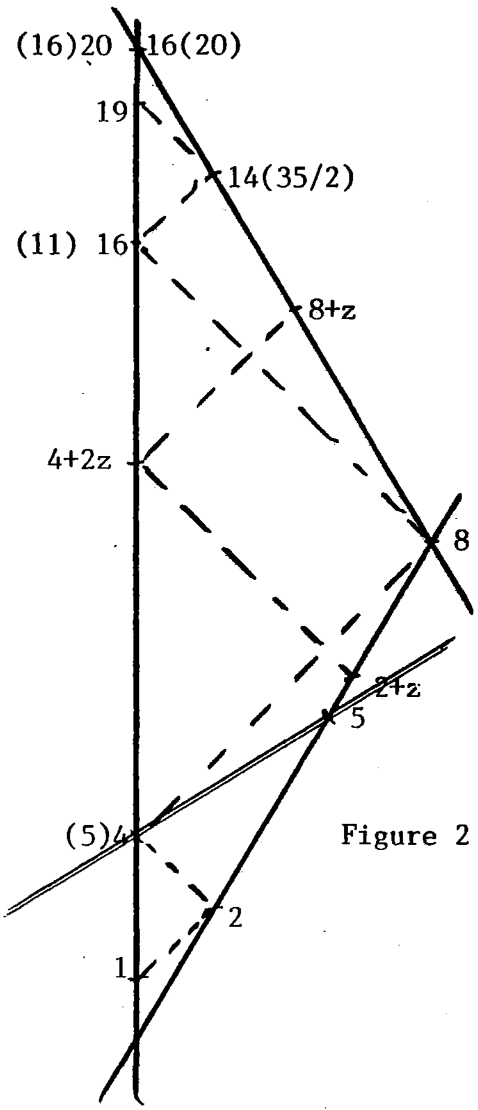


Figure 2

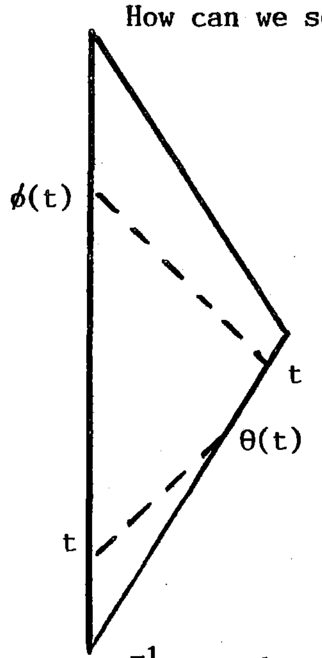
I propounded a simplified version in which the accelerations were replaced by using a pair of travelling observers to replace the one moving twin, they synchronising watches at the point where they pass. Figure 2 shows a particular version of this in which, to get rid of mere algebraic compexity, I have taken the special value of 2 for k, which it is easy to see corresponds to a speed of separation of 3/5. Numbers out of brackets are measured times, and those in brackets are assigned by the other observer. Since 16 is indeed less than 20 the result is explained but that is not quite the end of the story. What becomes of Dingle's "each twin says the other's clock goes slow"? There are two separate confusions in Dingle's statement. Talking of clocks "going slow" suggests that the theory makes a statement about the behaviour of clocks - a kind of universal malfunctioning analogous to the specific malfunctioning that a pendulum clock would undergo in the experiment. This is wrong; the theory simply states that there is not so much time elapsed for the moving twin and that is why his clock measures less.

The second confusion is more complex. A way of seeing it is to put some more detail on the bare statement that there is no symmetry between the twins, since one is at rest in an inertial frame all the time, whilst the other is not (either he is accelerated, or in the other version he is two observers). [In the discussion after the paper, Michael Redhead drew my attention to one way of doing this. It is in terms of planes of simultaneity; those for the stay-at-home twin would be horizontal lines in the diagram. For the out-going twin they would be a family of parallels like the double line in fig. 2 which has been constructed by joining the point to which time 5 is assigned to that at which 5 is measured. These lines slope upwards in the lower part of the diagram and so, by symmetry, downwards in the upper part. Between t = 4.8/5 = 32/5 and t = 20 - (32/5) = 68/5 there are no points on the line of the stay-at-home twin simultaneous with those of the pair, though they are simultaneous with events on the outgoing observer after he has relinquished his role at t' = 8 or with events on the incoming one before he has taken over at t' = 8. The sudden, discontinuous change of observer is mirrored by a jump from the upward sloping family of lines to the downward one. During either the upward sloping regime, or the downward one, the moving twin can say that the stay-at-home has less time elapsing than himself, or, in the convenient jargon, the stay-at-home's clock goes slow. But this is cancelled out in overall effect by the sudden change. This analysis answers Dingle's point very clearly, but it is more appropriate to the consideration of the moving twin as two separate observers. It is possible to give a different analysis based on a "let's pretend" view of the two moving observers as a single one.]

Now of course he is then an accelerated observer and the acceleration is an infinite one. This simply leads to some improper functions entering; this need not frighten us for in a physical situation the sharp corner can be smoothed out. The improper functions are just a simpler way of calculating than one which goes into the detail of the smoothing out. But he is an accelerated observer, none the less; Einstein gave his rule for spreading time through space for an observer at rest in an inertial frame. Can this observer use the same rule? Einstein took this point up in 1907 when he asked "Is it conceivable that the Principle of Relativity also holds for frames that are accelerated relative to each other?" By the Principle, he means the assumption that natural laws are independent of the state of motion of the reference frame. He goes on to discuss the special case of a constant rectilinear acceleration of one frame and relates this to a homogeneous gravitational field. It is well-known that this was an intermediate stage for Einstein on the way to his formulation of general relativity eight years on. But let us investigate the generalisation of Einstein's remark, not in the way he later came to, but, more modestly, to the case where the acceleration is not constant in time. And the extreme case of this is exactly that thrown up in the clock paradox. So let us first use the rule to set up a coordinate system, and then see what gravitational field we have created by our behaviour.

Returning to fig.2, a signal sent from the moving twin between $t' = 2$ and $t' = 8$ will be reflected & return to the twin when he is on the return journey; so now the arithmetic becomes a little more complicated. If the signal is sent at $2 + z$, for z between 0 and 6, it reaches the stay-at-home at $4 + 2z$, which is at a time-difference $20 - (4 + 2z) = 16 - 2z$ from the reunion. The reflected signal therefore reaches the homecoming twin at $\frac{1}{2}(16 - 2z)$ from the reunion, that is, at time $16 - (8 - z) = 8 + z$. Einstein's rule gives for the reflection event a time (assigned) $5 + z$ and a distance 3, the distance being independent of z! Since the measured time of the reflection event is $4 + 2z$, the moving twin completes his characterisation of the others behaviour by saying that, between the two periods in which his clock goes slow there is one in which it goes twice as fast. During this period, he sees the stay-at-home as stationary; but the fast running of his clock is no contradiction to special relativity since there the similar running of clocks at rest refers to those at rest in inertial frames.

How can we square this with Einstein's introduction of a gravitational field? Referring to fig.3 it is easy to express the functions θ, ϕ in terms of the Heaviside unit function H defined by



$$H(x) = 0,\ x \leqslant 0,\ H(x) = 1,\ x > 0.$$

In fact, we get

$$\theta(t) = 2t + (6 - 3t/2)H(t - 4),$$

$$\phi(t) = 2t + (12 - 3t/2)H(t - 8).$$

The inverse functions are similarly found as

$$\theta^{-1}(x) = \tfrac{1}{2}x + (3x/2 - 12)H(x - 8),$$

$$\phi^{-1}(x) = \tfrac{1}{2}x + (3x/2 - 24)H(x - 16).$$

Then the generalisation of fig. 1 gives at once

$$t' - x' = \theta(t - x),\quad t' + x' = \phi^{-1}(t + x)$$

which can be written out explicitly. The metric in the traveller's coordinate system is $ds^2 = A(dt'^2 - dx'^2)$ where the conformal factor

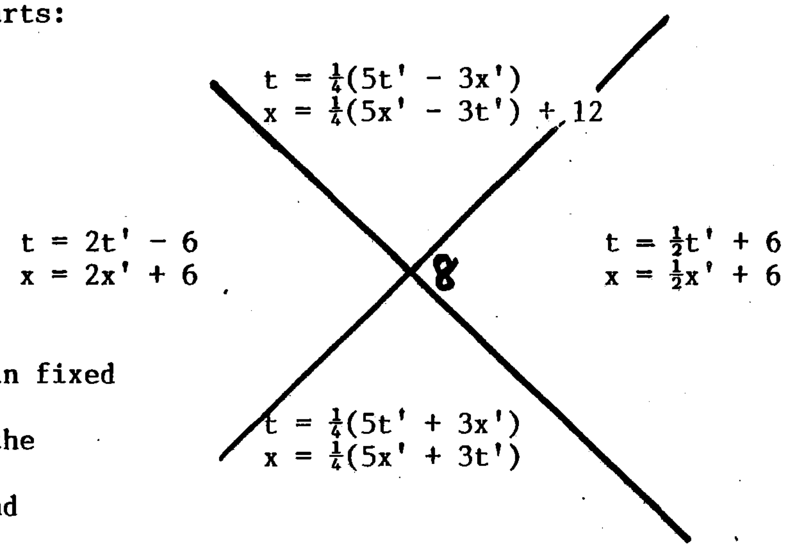
$$A = 1 + 3H(t' - x' - 8) - 3H(t' + x' - 8) - 9P/4,$$

where P is written for the product of the two H functions. (Thus

$$P = H(\max[t' - x' - 8,\ t' + x' - 8]).\ )$$

Because of the Heaviside functions in the metric there are non-zero values of the affine connexion (indeed Dirac delta functions) on the surface $(t' - 8)^2 = x'^2$, the light-cone of the change-over event. It is only on this surface that there is any (apparent) gravitational field (apparent because it is of course reducible, for the Riemann tensor must vanish everywhere).

The light-cone divides the t-x plane into four regions and it is straightforward to break down the nonlinear coordinate transformations into their linear parts:



Now dx/dt will remain fixed in crossing any of the light-cone lines, and so it is easy to see that the effect of crossing any of the bounding lines in the direction of increasing time is to compound (by the Einstein velocity rule) a velocity 3/5 (from left to right) with that of the particle. In particular, the gravitational field has no effect on a ray of light. The path of the stay-at-home twin is a freely-falling one in this field. It is because of his initial speed of -3/5 that he is seen as at rest in the central part of the trip. The gravitational field has the form of an imploding-exploding wave centred on the change-over point.

THE REAL PROBLEM

I have allowed myself to be led up a side turning in going into such detail about the clock paradox; my excuse is that the work does not seem to have been done before. But now it is time to turn more seriously to the question of accelerated observers. Evidently they can use Einstein's rule to set up a radar coordinate system, but the process may have some funny answers and in some cases (but not this one) the coordinates set up will not cover the whole of the original space. So the question arises of the physical meaning of setting up such a coordinate system. This was exactly the problem neatly by-passed by Einstein (after some years of agonising) when he went over to the full-scale general relativity theory in which the coordinates are, from the first, stated to have no physical meaning. Inertial observers are characterised by the two properties (a) that they are all equivalent, (b) that clocks measure proper time. For accelerated ones Milne (for instance) was so keen on keeping (a) that he was prepared to give (b) away and to re-calibrate his clocks. General relativity, on the other hand, assumes (b) and is content to reject (a) (or, more precisely, to take in such a generalised form that it is rejected for our purposes). Both of these strategies are answering different questions from mine. I want to know whether accelerated clocks measure proper time. It seems to me unsatisfactory if this is answered by an empirical appeal of the form: it is assumed that ds is measured in general relativity and this fact is used in the experimental checks on the theory, so the success of these checks is evidence in favour of ds being measured even when clocks are accelerated.

In the first place, the empirical evidence is at present rather thin. But this may well be remedied; my real objection is that the corresponding result for inertial observers is not an empirical one but an immediate consequence of the theory. The corresponding fact should have the same logical status for the accelerated case.

My conclusion, then, is that this can be put right only by going back to the original construction of space and time, and it so happens that the only theory in the market to do this is the discrete one with which we are working. Nor is this just an accident. For the problem is to show how separate dimensions, three of space and one of time, emerge from the process of getting knowledge about the external world - four dimensions emerging in some way from one process. This increase of dimension is very difficult to imagine in a continuum theory; in the discrete theory there is no such difficulty because dimension is not intrinsic. The sort of devices with which people show that the rationals are enumerable are that sort of dimension-changing device. It is true that the result of such a treatment will be to give a discrete version of special relativity, and yet the discreteness seems to play no important part, but this does not matter. To return to Irving Stein - I am not committing myself to supporting his particular construction, but I do see some such construction from a discrete picture as necessary.

NOTE

In the discussion it was suggested to me that both Robb and Minkowski might be of help; I do not know, but my impression is that something more basic is needed.

BIT-STRING SCATTERING THEORY*

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ABSTRACT

We construct discrete space-time coordinates separated by the Lorentz-invariant intervals h/mc in space and h/mc^2 in time using *discrimination* (XOR) between pairs of independently generated bit-strings; we prove that if this space is homogeneous and isotropic, it can have only 1, 2 or 3 *spacial* dimensions once we have related time to a global ordering operator. On this space we construct exact combinatorial expressions for free particle *wave functions* taking proper account of the interference between indistinguishable alternative paths created by the construction. Because the end-points of the paths are fixed, they specify *completed* processes; our wave functions are "born collapsed". A convenient way to represent this model is in terms of complex amplitudes whose squares give the probability for a particular set of observable processes to be completed. For distances much greater than h/mc and times much greater than h/mc^2 our wave functions can be approximated by solutions of the free particle Dirac and Klein-Gordon equations. Using an eight-counter paradigm we relate this construction to scattering experiments involving four distinguishable particles, and indicate how this can be used to calculate electromagnetic and weak scattering processes. We derive a non-perturbative formula relating relativistic bound and resonant state energies to mass ratios and coupling constants, equivalent to our earlier derivation of the Bohr relativistic formula for hydrogen. Using the Fermi-Yang model of the pion as a relativistic bound state containing a nucleon-antinucleon pair, we find that $(G^2_{\pi N})^2 = (2m_N/m_\pi)^2 - 1$.

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1. INTRODUCTION

At ANPA 9, 10 and again at ANPA 11 I attempted to explain how to go from discrimination between bit-strings to a relativistic quantum scattering theory. On each occasion objections were raised by the audience which could not be successfully met while I was on my feet. Part of the problem at ANPA 11 was that I made an incorrect connection between McGoveran's Theorem (FDP Section 3.4, pp 30-34)[1]:

Theorem 13. The upper bound on the global d-dimensionality of a d-space of cardinality N with a discrete, finite and homogeneous metric is 3 for sufficiently large N.

and the definition of "event" in my scattering theory. This theorem establishes our right to claim that we have *explained* the 3+1 structure of "space-time" in our finite and discrete context. The theorem was not shaken. However, I admit that my attempt at ANPA 11 to tie it directly to the scattering theory was flawed. At ANPA 11 I distributed a few sheets with the same title as this paper and a heading "SLAC-PUB-formulae". These are now only of historical interest; the flaw occurs at Eq. 1.8, p.3 in that manuscript. A correct derivation based on the construction provided in this paper is given at the end of Sec. 2.4 below.

My intent at ANPA 11 was to present "a systematic discussion of the kinematics and dynamics of the bit-string scattering theory which has been developing within the framework of discrete physics." Subsequently I have come to realize my attempt failed for a deeper reason than the technical flaw noted in the first paragraph. The problem was that I was trying to use a discretized random walk in space-time as the basic paradigm without paying sufficient attention to the broader context within which the "random walk" is constructed. The random walk model restricted to causal space-time trajectories captures much of the essence of relativistic quantum mechanics, as Stein taught us long ago[2–4] , but does not lead to specific characteristics of the Schroedinger equation without an additional quantum postulate[5]. Karmanov[6] has suggested that a "Stein-like" random walk in

Feynman's sum over paths using a *fixed* imaginary step length of $i\ h/mc$ in space and $i\ h/mc^2$ in time can still lead to the Dirac equation in 1+1 dimensions for $x >> h/mc$ *and* $t >> h/mc^2$. Work by Karmanov, McGoveran, HPN and Stein[7] has not only provided a rigorous proof of this contention but led to what I believe to be a resolution of the problems raised by my earlier attempts to construct a bit-string scattering theory.

It is characteristic of the *ordering operator calculus* that when there are paths with interfering alternatives due to the sharing of indistinguishable possibilities that we obtain *at least* two alternative types of path; further, we do not have sufficient information to choose between them without extending the context. However, in our construction we find that the sum of the squares of the number of alternative paths in each type provides the correct normalization condition for our calculated probabilities. This is a specific example of the "adding in quadrature" which David McGoveran discusses in his contribution to this conference[8]. It is then a matter of convenience to introduce the complex amplitudes of quantum mechanics; no mysticism is involved. That is, we can go from our construction of rational, real probabilities for real "interfering alternatives", to the complex interfering alternatives which Feynman finds characteristic of quantum mechanics[9].

Once we have seen how these alternative paths are generated, anchored to the space-time trajectories and restricted by them, but not confined to the lattice of space-time points in the way a "classical" random walk would require, my earlier work on scattering theory falls into place. I hope that with this paper available prior to ANPA 12, I will for once be able to confront at least a fraction of the (for me) more exciting applications which are now hull up and moving rapidly toward us.

2. QUANTUM WAVE FUNCTIONS AS A CONTINUUM APPROXIMATION

2.1. BIT-STRINGS

We specify a ***bit-string***

$$\mathbf{X}(S) = (...,b_i^x,.....)_S \tag{2.1}$$

by its S ordered elements

$$b_i^x \in 0,1; \quad i \in 1,2,....S; \quad 0,1,...,S \in \textit{ ordinal integers} \tag{2.2}$$

and its norm by

$$|\mathbf{X}(S)| = \Sigma_{i=1}^S b_i^x = X \tag{2.3}$$

Define the ***null string*** by $\mathbf{0}(S)$, $b_i^0 = 0$ for all i and the ***anti-null*** string by $\mathbf{1}(S)$, $b_i^1 = 1$ for all i. Define ***discrimination*** (XOR) by

$$\mathbf{X} \oplus \mathbf{Y} = (...,b_i^{xy},...)_S = \mathbf{Y} \oplus \mathbf{X}; \ b_i^{xy} = (b_i^x - b_i^y)^2 \tag{2.4}$$

from which it follows that

$$\mathbf{A} \oplus \mathbf{A} = \mathbf{0}; \ \mathbf{A} \oplus \mathbf{0} = \mathbf{A} \tag{2.5}$$

We will also find it useful to define

$$\bar{\mathbf{A}} = \mathbf{A} \oplus \mathbf{1}; \ \textit{hence } \mathbf{A} \oplus \bar{\mathbf{A}} \oplus \mathbf{1} = \mathbf{0} \tag{2.6}$$

2.2. ONE DIMENSIONAL AMPLITUDES

Consider two independently generated strings $\mathbf{A}(S), \mathbf{B}(S)$ restricted by $|\mathbf{A} \oplus \mathbf{B}|=n$ and $A - B = c$. We call these the *boundary conditions.* We now construct two substrings $\mathbf{a}(n), \mathbf{b}(n)$ by the following recursive algorithm starting from $i, j = 0$ and ending at $i = S,\ j = n$.

$$i := i + 1$$

$$if\ b_i^A = 1\ and\ b_i^B = 0\ then\ j := j + 1\ and\ b_j^a := 1\ and\ b_j^b := 0$$

$$if\ b_i^A = 0\ and\ b_i^B = 1\ then\ j := j + 1\ and\ b_j^a := 0\ and\ b_j^b := 1$$

$$if\ (b_i^A - b_i^B)^2 = 0\ then\ j,\ b_j^a\ and\ b_j^b\ do\ not\ change$$

Once we have made this construction,

$$\mathbf{a}(n) \oplus \mathbf{b}(n) \oplus \mathbb{1}(n) = \mathbb{0}(n) \tag{2.7}$$

and we can interpret the string $\mathbf{a}$ as representing a "random walk" in which a "1" represents a step forward and a "0" represents a step backward, as in the Stein paradigm. Define

$$a_j = \Sigma_{k=1}^{j} b_k^a;\ b_j = \Sigma_{k=1}^{j} b_k^b \tag{2.8}$$

We call the "points" $(a_j - b_j, j)$ connecting $(0,0)$ to (c,n) a *trajectory*; the new ordering parameter j then represents "causal" time order along the trajectory. Note that $a + b = n$ and $a - b = A - B = c$ for any trajectory because of our boundary conditions.

We can also define a *path* in the larger space s_i, A_i, B_i where

$$s_i = \Sigma_{k=1}^{i} s_k = \Sigma_{k=1}^{i} b_k^A b_k^B \tag{2.9}$$

$$A_i = \Sigma_{k=1}^{i} b_k^A (b_k^A - b_k^B)^2 + s_k; \; B_i = \Sigma_{k=1}^{i} b_k^B (b_k^A - b_k^B)^2 + s_k$$

Note that by construction $A_i - B_i = a_j - b_j$ and hence A_i, B_i is tied to the same trajectory in the $(a_j - b_j, j)$ plane; it acquires a third "orthogonal" coordinate due to those cases when both A_i and B_i are incremented by 1. Note also that there is no way from our boundary conditions or from the trajectory to tell those cases from those where i advances but neither A_i nor B_i nor s_i is incremented. All we know is that $s_{AB} = \Sigma_{k=1}^{S} b_k^A b_k^B$, lies in the range $0 \leq s_{AB} \leq S - n$. It is these *indistinguishable* paths which create the interfering alternatives in our model.

We now ask how many paths characterized by the unknown parameter $s = 0, 1, 2, ..., S - n$ satisfy our boundary conditions. By construction each path is tied to the n points which compose a trajectory, and can be chosen in n^s ways. Note that we have broken the causal connection between path and trajectory. Of the total number of ways of choosing a path characterized by s from the $S!/(S-s)!$ possibilities, only $S!/s!(S-s)!$ are distinct. Consequently, the probability of having a path characterized by s is

$$\frac{S!/s!(S-s)!}{S!/(S-s)!} = \frac{1}{s!} \tag{2.10}$$

Thus the total number of paths is

$$P(n; S) = \Sigma_{s=0}^{S-n} \frac{n^s}{s!} = \Sigma_{s=0}^{S-n} p_s(n) \equiv exp_{S-n}(n) \tag{2.11}$$

where $exp_{S-n}(n)$ is the *finite exponential.* This is a general result for the *transport operator* referring to *attribute distance* as has been proved by McGoveran in FDP, Theorems 36-40, pp 55-58.

2.3. ADDING IN QUADRATURE

Although Eq. 2.11 specifies the total number of paths, given S, n, it conceals a four-fold ambiguity arising from the construction. However the sequence of paths is generated, the order adopted in the sum implies a recursive generation of the terms $p_s(n) = n^s/s!$ given by

$$p_{s+1}(n) = np_s(n)/(s+1); \; p_0(n) = 1 \tag{2.12}$$

The first ambiguity is the fact that we do not know whether $S - n$ is even or odd outside of the uninteresting case $S = n$ when paths and trajectories coincide; hence we do not know whether the sum terminates in an even or an odd term, which affects the statistical calculation in an interesting way. The second ambiguity arises because, however s is ordered, we do not know how many cases arise because *both* A_i and B_i are incremented, or *neither*. To include this dichotomy we split the even and odd sequences themselves into two sequences corresponding to these alternatives which we call 11 and 00, giving four recursion relations:

$$p^{e,11}_{s+4}(n) = \frac{n^4}{(s+4)(s+3)(s+2)(s+1)} p^{e,11}_s(n); \; p^{e,11}_0(n) = 1$$

$$p^{o,11}_{s+4}(n) = \frac{n^4}{(s+4)(s+3)(s+2)(s+1)} p^{o,11}_s(n); \; p^{o,11}_1(n) = n$$

$$p^{e,00}_{s+4}(n) = \frac{n^4}{(s+4)(s+3)(s+2)(s+1)} p^{e,00}_s(n); \; p^{e,00}_2(n) = n^2\frac{1}{2}$$

$$p^{o,00}_{s+4}(n) = \frac{n^4}{(s+4)(s+3)(s+2)(s+1)} p^{o,00}_s(n); \; p^{o,00}_3(n) = n^3\frac{1}{6} \tag{2.13}$$

At some point which depends on whether (a) $S - n$ is even or odd and/or $2s_{AB}$ is greater or less than $S - n$, this four-fold ordering of the terms in the sum over s has to stop, and may or may not leave some terms unaccounted for. Calling the contribution of these terms, to the sum ΔP, we find that our construction allows us to decompose the sum over paths as follows:

$$P(n;S) = \Sigma_{s=0}^{S-n}[p_s^{e,11} + p_s^{o,11} + p_s^{e,00} + p_s^{o,00}] + \Delta P \tag{2.14}$$

We are now in the general situation discussed by McGoveran in this conference[8] where we know that for any specific generation of the paths which meets our boundary we could compute the decomposition, but because of interfering indistinguishable alternatives the actual probability calculation eludes traditional approaches. Consider first a situation with two alternatives, with P_1 and P_2 paths characterized by each separately. Since the total number of paths is P, the elementary treatment takes $P_1 + P_2 = P$, but this cannot represent the situation when they are *independently* generated and hence define a joint probability space with $P_1 P_2$ elements. In order to satisfy both constraints, we form $P_1^2 + P_2^2 = P^2 - 2P_1P_2 \equiv R_{12}^2$, which is identically satisfied if the two are not independent. If, due to indistinguishable paths which we do not know how to assign to either P_1 or $P - 2$, we have indeed made the two independent in the sense that the product P_1P_2 is no longer constrained other than by the inequality $2P_1P_2 < P^2$, we can adopt $R_1 2^2$ as the measure of the square of the number of paths in this new space. Taking the product $2P_1P_2 = f^2P^2$ where f is some rational fraction less than unity, we thus arrive at the general result

$$P_1^2 + P_2^2 = R_{12}^2 = P^2(1-f)(1+f) \tag{2.15}$$

which has been derived by McGoveran[8] by considering case counts including indistinguishables.

Returning to the case at hand, and considering even and odd paths, this restricts P_e and P_o to the circle defined by

$$P_e^2 + P_o^2 = R^2 \tag{2.16}$$

independent of how P is partitioned between P_e and P_o. We can now *define*

$$\psi_{S-n} = P_e + iP_o \tag{2.17}$$

with the normalization condition

$$\psi^*\psi = R^2 \tag{2.18}$$

Clearly we can divide ψ by R to get the normalization condition $\psi^*\psi = 1$ when we are modeling the case that a single system traverses the trajectory with certainty.

Our next step is to exploit the remaining ambiguity arising from the bisection of the interval between 0 and $S - n$ due to the indistinguishable parameter s_{AB}. We have already taken the basic step in Eq. 2.14 by splitting the sum into four rather than two independent collections of paths. Now that we have recognized that the amplitudes — whose square gives a quantity which can be normed to form a probability —, can be complex, we have no conceptual barrier to forming real combinations which can be negative as well as positive. The obvious choice is to form those which lead to the finite sines and cosines, i.e. by subtracting the two components of the odd or even series from each other:

$$R\ cos_{S-n}(n) = R\ \Sigma_{k=0}^{\frac{1}{2}(S-n)}(-1)^k \frac{n^{2k}}{(2k)!} = \Sigma_{s=0}^{(S-n)}[p_s^{e,11} - p_s^{e,00}] \tag{2.19}$$

$$R\ sin_{S-n}(n) = R\ \Sigma_{k=1}^{\frac{1}{2}(S-n)}(-1)^{k+1} \frac{n^{2k-1}}{(2k-1)!} = \Sigma_{s=1}^{(S-n)}[p_s^{o,11} - p_s^{o,00}] \tag{2.20}$$

The two constructions can now be combined by taking the normalized wave function to be

$$\psi_{S-n}(n) = exp_{S-n}(in) = \Sigma_{s=0}^{\frac{1}{4}(S-n)} \frac{(in)^s}{s!} \tag{2.21}$$

Thus, by taking proper account of the interference between *independently generated* paths which share *indistinguishable* elements, we claim to have *derived* Feynman's prescription for quantum mechanics as a "sum over paths" with imaginary steps. In other words, we have shown that by using the *ordering operator calculus* to count the paths we can construct a completely finite version of "wave functions" with finite step lengths and real probabilities. The i has been introduced simply for mathematical convenience and carries no deeper significance. The *significant* aspect of the system that eventually leads to observable quantum interference is the fact that our construction of space-time from bit-strings includes interfering alternatives due to paths which share *indistinguishable* elements.

2.4. CONSTRUCTION OF SPACE-TIME COORDINATES

1+1 dimensions

In any universe of bit strings of length S, all quadruples such that

$$\mathbf{A} \oplus \mathbf{B} \oplus \mathbf{C} \oplus \mathbf{D} = \mathbf{0} \tag{2.22}$$

are called *events.* Note that this implies that

$$\mathbf{A} \oplus \mathbf{B} = \mathbf{C} \oplus \mathbf{D}; \ \mathbf{A} \oplus \mathbf{C} = \mathbf{B} \oplus \mathbf{D}; \ \mathbf{A} \oplus \mathbf{D} = \mathbf{B} \oplus \mathbf{C} \tag{2.23}$$

$$\mathbf{A} = \mathbf{B} \oplus \mathbf{C} \oplus \mathbf{D}; \ \mathbf{B} = \mathbf{C} \oplus \mathbf{D} \oplus \mathbf{A}; \ \mathbf{C} = \mathbf{D} \oplus \mathbf{A} \oplus \mathbf{B}; \ \mathbf{D} = \mathbf{A} \oplus \mathbf{B} \oplus \mathbf{C} \tag{2.24}$$

Consider an event defined by four independently generated strings $\mathbf{F}, \mathbf{B}, \mathbf{R}, \mathbf{L}$ whose norms are F, B, R, L; all must be less than or equal to $|\mathbf{1}| = S$. For the moment we need only define a fifth integer n by

$$|\mathbf{F} \oplus \mathbf{B}| = n = |\mathbf{R} \oplus \mathbf{L}| \tag{2.25}$$

We will return below to the additional flexibility provided by the remaining two equalities in Eq. 2.17. Our intent is to construct a discrete square coordinate mesh (z_i, t_j) with $(2n+1)^2$ points within which we can model piecewise continuous ordered trajectories (z_k, t_k) which connect the "endpoint" $(0,0)$ to some "endpoint" (z,t) lying on the boundary of the square

$$t = \pm n, \ -n \leq z \leq n; \ z = \pm n, \ -n \leq t \leq n \tag{2.26}$$

The order parameter $0 \leq k \leq n$ traverses any space-time point along the trajectory only once; in addition we require that

$$z_{k+1} - z_k = \pm 1; \ t_{k+1} - t_k = \pm 1; \ (four\ choices) \tag{2.27}$$

The description is *static* in the sense that it can be read either from 0 to n or from n to 0 and still describe the *same* trajectory. Note that in contrast to previous discussions, (a) we consider space-like as well as time-like trajectories, and (b) that the length of the strings $S \geq n$ is not specified; it is some finite integer named in advance of the construction. Note further that since we specify both endpoints, we are describing a *completed* process. The "wave functions" we will eventually construct on this mesh will be "born collapsed". All our results will belong to the "fixed past"; whether we should or should not use our theory to *predict* the future, either in a deterministic or a statistically deterministic sense, is a separate issue we will not discuss in this paper. We have picked our boundary conditions $(0,0) - - - (z,t)$ in the process of specifying the problem.

Any space-time point (z_k, t_k) not on the axes $(z_k, 0), (0, t_k)$ lies in one of the four quadrants $(+,+), (-,+), (+,-), (-,-)$. We fix the sign convention in terms of our parameters R, L, F, B by the rule

$$(+,+) \leftrightarrow R > L, \ F > B; \ (-,+) \leftrightarrow R < L, \ F > B$$

$$(+,-) \leftrightarrow R > L, \ F < B; \ (-,-) \leftrightarrow R < L, \ F < B \tag{2.28}$$

The interiors of the light cones are specified in the usual way: Time-like $(t^2 > z^2)$: Forward $t > |z|$, Backward $t < -|z|$; Space-like $(z^2 > t^2)$: Right $z > |t|$, Left $z <$

$-|t|$. We can now define our bounding endpoints in terms of our basic parameters, and four new parameters r, l, f, b by

$$Time-like, Forward \leftrightarrow z = R - L = r - l; t = n = r + l$$

$$Time-like, Backward \leftrightarrow z = R - L = r - l; t = -n$$

$$Space-like, Right \leftrightarrow z = n = f + b; t = F - B = f - b$$

$$Space-like, Left \leftrightarrow z = -n; t = F - B = f - b; \tag{2.29}$$

The advantage of introducing the new parameters r, l, f, b is that they make it easy to define what will become Lorentz invariants. Explicitly

$$t^2 - z^2 = \tau^2 = 4rl = n^2(1 - \beta^2) \ with \ \beta = \frac{2r}{n} - 1$$

$$z^2 - t^2 = -\tau^2 = 4fb = n^2(1 - \omega^2) \ with \ \omega = \frac{2f}{n} - 1 \tag{2.30}$$

Since the sign of τ^2 specifies the light cone type and hence whether we use β or ω, and the sign of one or the other specifies the appropriate octant (up to an overall sign ambiguity which can only be resolved by reference to the laboratory situation which is being modeled), we can start from z, t and calculate n and β or n and ω unambiguously; from these we get r, l, f, b. This leaves one unknown parameter $s = S - n$ which we will use to characterize all possible trajectories which fit our boundary conditions by requiring that

$$R = r + s; \ L = l + s; \ F = f + s; \ B = b + s \tag{2.31}$$

As we have shown many times[10] it is easy to give meaning to the concept of Lorentz invariance in our discrete context. Defining $r' = \rho r$, $l' = \rho^{-1} l$, τ^2

is obviously invariant, and if we define $\gamma_\rho = \frac{1}{2}(\rho + \rho^{-1})$, $\beta_\rho^2 = 1 - \frac{1}{\gamma_\rho^2}$ we have immediately that

$$z' = \gamma_\rho(z + \beta_\rho t); \; t' = \gamma_\rho(t + \beta_\rho z) \tag{2.32}$$

Although our original derivation applied only in the forward light cone, the current context allows us to extend it to the square space-time mesh we constructed above. Then we find that n is simply the space-like or time-like interval in some rest system, and the coordinate-system dependent quantities β, ω can be ignored until we start making ***physical*** use of the model for discussing laboratory experiments.

In order to relate this model to the ***single*** bit-string "random walk" we have used in the past, we construct two substrings $\mathbf{r}(n), \mathbf{l}(n)$ by the same recursive algorithm starting from $i, j = 0$ and ending at $i = S, \; j = n$ which led to Eq. 2.7 above. Once we have made this construction,

$$\mathbf{r}(n) \oplus \mathbf{l}(n) \oplus \mathbf{1}(n) = \mathbf{0}(n) \tag{2.33}$$

and we can interpret the string $\mathbf{r}$ as representing a "random walk" in which a "1" represents a step to the right and a "0" represents a step to the left, as in the Stein paradigm; the new ordering parameter j now represents *causal* time order along the trajectory when $F - B > 0$.

<u>3+1 dimensions</u>

Clearly, until we make physical application of the formalism, the distinction between space and time in this construction is only suggested by our choice of symbols (z, t); the Minkowski symmetries are maintained. That is, *any* set of labels R,L,F,B; r,l,f,b; n, z, t which maintain the connections defined above model the same situation. An appropriate interchange of the symbols 0, 1 which maintains both the dichotomy and the asymmetry [i.e. $\mathbf{1} \oplus \mathbf{1} = \mathbf{0} = \mathbf{0} \oplus \mathbf{0}$] used above makes the whole scheme "label invariant" within the *combinatorial hierarchy*[11] labeling scheme constructed by Kilmister; his latest version was presented at this conference[12].

To distinguish space from time in the model, we include additional *spacial* dimensions which we require to be *homogeneous* and *isotropic* in the sense that none of the symmetry properties depend on the choice of the labels $x, y, z, \ldots$. One of the great conceptual advantages of our constructive approach is that McGoveran has *proved* that in our theory the extension from 1+1 space-time to 2+1 and 3+1 has to stop there. This is *McGoveran's Theorem* (FDP Section 3.4, pp 30-34) which we already quoted in the first paragraph of the introduction. To see how this applies in our context, fix the F, B pair as defining the universal ordering parameter j for causal space-time events, and try to construct not only the z coordinate from the R,L pair as above but three additional independently generated pairs $W_+, W_-; X_+, X_-, Y_+, Y_-$ to construct the coordinates $w = W_+ - W_-, x = X_+ - X_-, y = Y_+ - Y_-$, and for consistency in the notation replace L,R by Z_-, Z_+ with z $= Z_+ - Z_-$.

Following the same procedure as above, we generate four substrings $\mathrm{w}_+(n)$, $\mathrm{x}_+(n)$, $\mathrm{y}_+(n)$, $\mathrm{z}_+(n)$. Since these four strings are *independent* by hypothesis, they cannot discriminate to the null string, so we need a definition of *event* appropriate to this situation. We take this to be those values of j for which all four strings have accumulated the same number of "1" 's, i.e.

$$\Sigma_{k=1}^{j} b_k^{w+} = \Sigma_{k=1}^{j} b_k^{x+} = \Sigma_{k=1}^{j} b_k^{y+} = \Sigma_{k=1}^{j} b_k^{z+} \tag{2.34}$$

The extension to D rather than 4 spacial dimensions is obvious. This reduces the probability of events occurring after j space-time steps in D dimensions to the probability of obtaining the same number of "1" 's in D independent Bernoulli sequences after j trials,

$$p(j) = \frac{1}{2^{jD}} \Sigma_{k=0}^{j} \binom{j}{k}^{D} < j^{-\frac{D-1}{2}} \tag{2.35}$$

Clearly this definition of events defines a "homogeneous and isotropic" d-space, but the probability of being able to *continue* to find events for large values of j

vanishes for $D > 3$. Consequently we need only consider three spacial dimensions. Thus, provided we have some clear way to label independent bit strings, we can extend our construction of 1+1 space-time to 3+1 space-time, but no further. We will return to the calculation of scattering probabilities in this space in the next chapter.

2.5. DISCRETE FREE-PARTICLE WAVE FUNCTIONS

Time dependence of the Schroedinger wave function

In the past we have used the macroscopic space-time interval between two counter firings as our basic means of connecting our theory to the laboratory (the "counter paradigm"). We have now extended our model to include left-right motion in space *and* "forward-backward motion in time" using a single global ordering parameter. The Lorentz invariance of the system allows us, in principle, to talk about one *or* the other by going to the appropriate "rest system". To relate this to the time *Zitterbewegung* of an isolated system of rest energy mc^2, which has period $T = h/mc^2 = 1/\nu$ or angular frequency $\omega = mc^2/\hbar$, we require a new variant of our "counter paradigm" which touches an empirical reflection of this basic aspect of the theory. Fortunately Feynman has supplied us the clue by allowing us to think of a particle moving "backward in time" as an antiparticle moving forward in time.

This line of thought suggests that, just as we use counter telescopes to define "monochromatic velocities" which are the starting point for making measurements that exhibit deBroglie wave interference effects, we think about external devices before and after the counter firings [which in a rest system would be $t_0 = n_0(h/mc^2)$ seconds apart] that tell us whether a particle or an antiparticle enters or leaves the system. This is easy for charged particles, since all we need do is to put a region of magnetic field between the two counters in the entrance and exit counter telescopes, and see which way the macroscopic space-time trajectory bends. In this way, we find that particles of opposite charge bend in opposite directions, and that on the macroscopic scale a single particle has the same charge in the entrance and exit telescopes. Although we cannot reverse time flow, we can reverse velocities, and find that for all the time intervals between the four counter firings kept the same, particles of opposite charge *and* opposite velocity follow identical trajectories, although they are obviously traversed in the opposite sense. Thus the same experimental setup does allow us to distinguish "forward" from "backward" while passing along the *same* macroscopic trajectory, and we accept both this fact and the conservation of charge (or more generally number of particles minus the number of antiparticle with all other quantum numbers reversed) as part of our *rules of correspondence* connecting our constructed model to laboratory experience.

With this experimental preliminary out of the way, we can now immediately apply our general "adding in quadrature" result to construct free particle wave functions. For time-like intervals, we need simply take the period to be $h/mc^2 = h/E$ in the rest system, and find that *any* isolated system with rest energy $mc^2 = E$ has a combinatorial wave function which can be approximated by a solution of the equation

$$\pm i\hbar\partial\psi/\partial t = E\psi \tag{2.36}$$

We emphasize that our solutions are derived only when restricted by the space-time boundary conditions which represent *completed* processes. For us it would be a *serious error* to try to interpret this or any other Schroedinger-type equation as describing the *causal* evolution of a complex amplitude.

The Klein-Gordon Equation

We have seen that this time evolution can be transformed from the rest system with $\tau^2 = n_0^2(h/mc)^2$ to an arbitrary system with $\tau^2 = c^2t^2 - z^2$ in which the velocity between the endpoints of the trajectory is $\beta = z/ct$. Consequently we have already constructed the discrete solutions which can be approximated by continuum solutions of the equation

$$\partial^2\psi/\partial z^2 - \partial^2\psi/c^2\partial t^2 = (mc/\hbar)^2\psi \tag{2.37}$$

Extension to 3+1 dimensions is immediate.

The Dirac Equation[7]

The Dirac case differs from the Klein-Gordon case because a step to the left or to the right can have either left or right helicity, and spin-conservation adds a second conservation law to the particle-antiparticle conservation implied by our boundary conditions and reflected in our use of complex amplitudes. Consequently, in addition to the two independent time sequences $t_\pm(s)$ we must have two independent space sequences $z_\pm(s)$ ordered by the *same* global ordering parameter i and characterized by the *same* path parameter s. We can take over the same space-time boundary condition used above with $Z_+ = R$ the steps to the right and $Z_- = L$ the steps to the left, and use a imaginary step length for the $\pm c$ *Zitterbewegung*, but the wave function now has two initial states α depending on whether the initial step (or helicity) is positive or negative, and two final states β. If $\Phi_{\beta\alpha}(B)$ are the number of *trajectories* with B bends, the extension of our prescription derived above, which is equivalent to Feynman's[13,14] except that our step length is kept *fixed* at $i\ h/mc$ (ih/mc^2), amounts to calculating[7]

$$K_{\beta\alpha}(b,t_b;a,t_a) = \Sigma_{B\geq 0}\Phi_{\beta\alpha}(B)(i)^B \tag{2.38}$$

As we have shown elsewhere[7], the *exact* combinatorial result is

$$K_{-+} = (i)\Sigma_s(-)^s\frac{r^s}{s!}\frac{l^s}{s!} = (i)\Sigma_s(-)^s(\frac{\tau}{2})^{2s}\frac{1}{(s!)^2} \to iJ_0(\tau) \tag{2.39}$$

where we have used the fact that

$$4rl = [(r+l)^2 - (r-l)^2] = [c^2(t_b-t_a)^2 - (b-a)^2](\frac{mc}{h})^2 = \tau^2(\frac{mc}{h})^2 \tag{2.40}$$

is the square of the invariant interval. Applying the same reasoning to calculate the other three components, our final result is

$$K(z,t;0,0) = \frac{1}{2}\begin{pmatrix} -\frac{(ct+z)}{\tau})J_1(\tau) & iJ_0(\tau) \\ iJ_0(\tau) & -\frac{(ct-z)}{\tau}J_1(\tau) \end{pmatrix} \tag{2.41}$$

which for our boundary conditions is the solution of the Dirac equation

$$-i\sigma_z\partial\psi/\partial z - m\sigma_x\psi = i\partial\psi/\partial t \tag{2.42}$$

where $\hbar = 1 = c$, σ_x and σ_z are Pauli spin matrices and ψ has two components. Again, extension to 3+1 dimensions appears to be immediate.

<u>Momentum-space equations</u>

A major conceptual advantage arising from our finite and discrete approach to relativistic quantum mechanics using end-point boundary conditions is that we obtain the momentum-space wave function without additional effort. We have already seen that for the interval specified $z = (r-l)(h/mc)$ and $mc^2t/h = (r+l) = n$; consequently the velocity in units of the limiting velocity c is $\beta = z/ct = \frac{2r}{n} - 1$ Since we have already established our discrete version of Lorentz invariance for the equations, we must use the implied definition of energy $E = \gamma mc^2$ and momentum $p_z = \gamma\beta mc = \beta E/c$. This gives us immediately the Klein-Gordon equation in "momentum space"

$$(p_z^2 + m^2)\phi(p_z) = E\phi(p_z) \tag{2.43}$$

where (and from now on) $\hbar = 1 = c$. Another way to see this is to recognize that our energy (or momentum) conservation law, allows us to treat the left-right *Zitterbewegung* in z as a one-dimensional problem analagous to our treatment of forward and backward movement in time. Thus we can immediately conclude that $\psi_{p_z}(z) = e^{ip_z z}$. Since the space-motion and the time-motion are generated independently in our model, we can multiply the two independent amplitudes to obtain

$$\psi(z,t) \rightarrow e^{\pm i(p_z z \pm Et)} \tag{2.44}$$

and hence provide an alternative derivation of the Klein-Gordon equation which is completely equivalent to our treatment above. Clearly, this route applied to two amplitudes which conserve helicity at the end points along the same lines will yield the 1+1 Dirac equation in momentum space, and make extension to 3+1 dimensions even easier to accomplish. For instance, instead of p_x, p_y we can use $p_\perp, j$ and the wave function $e^{i(p_\perp r_\perp + j\phi)}$ where the boundary condition on ϕ is periodic with period 2π or 4π depending on whether j is integer or half-integer.

3. A BRIEF LOOK AT INTERACTIONS

3.1. The 8-counter Paradigm for 4-events

So far we have considered only single particle wave functions modeled by pairs of bit-strings of length S from which we constructed coordinate substrings $\mathbf{c}(n)$, $c \in x, y, z, t$ subject to the constraint

$$\mathbf{c}_+(n) \oplus \mathbf{c}_-(n) \oplus \mathbf{1}(n) = \mathbf{0}(n) \tag{3.1}$$

Now label four sets of coordinate strings by strings of length 4 and some discriminately independent choice of three of them, eg $(1010) = \mathbf{a}, (1001) = \mathbf{b}, (1111) = \mathbf{1} = \mathbf{q}_0$. Clearly these model the seven strings and discriminately closed subsets of the second level of the *combinatorial hierarchy*[10,11,12]. Such a choice provides a convenient way to model two types of particle, their antiparticles and three quanta. This description carries with it two additive conservation laws[10] and Feynman diagrams corresponding to the processes

$$\mathbf{a} \oplus \bar{\mathbf{a}} = \mathbf{q}_0 = \mathbf{b} \oplus \bar{\mathbf{b}}$$

$$\mathbf{a} \oplus \bar{\mathbf{b}} = \mathbf{q}_+ = \bar{\mathbf{a}} \oplus \mathbf{b} \tag{3.2}$$

$$\mathbf{a} \oplus \mathbf{b} = \mathbf{q}_- = \bar{\mathbf{a}} \oplus \bar{\mathbf{b}}$$

We now put this together as an 8-counter paradigm in figure 1. If we now construct the *content stings* corresponding to each of these labels, following the recursive algorithm leading to 2.24, but now in 3+1 dimensions, considering all the pairs in Eq. 2.16 and all the triples in 2.17, we will find that we have the full *crossing-symmetric* kinematics for all 2-2 elastic and anelastic scatterings and all 3-body decays or bound states. We leave these technical details to another publication.

To get the (free particle) Bohr-Sommerfeld quantization for finite velocity, note that our definition of velocity (*and* coherence length) necessarily implies a periodicity N_λ specified by ω or by $\beta = \frac{2k}{n} - 1 = \frac{2N_\lambda k_\beta}{N_\lambda n_\beta} - 1$ where k_β, n_β have no common factor. If we are considering boundary conditions defined by constant (average) velocity between a coherent string of possible time-like events or constant average separation along a string of possible space-like separated events — which is the usual condition for free particle quantum mechanical interference — this means that two events can be causally connected or spacially correlated only when they are some integral number N_λ of (relativistic) deBroglie wavelengths apart. Thus, in addition to the *Zitterbewegung*, we have a grosser quantized "random walk" with the positions where events can either occur (in time-like sequence) and/or are correlated in their space-like separations specified by their mass and some velocity parameter $0 < \beta < 1$ in terms of the deBroglie wavelength. In contrast to the unobservable *Zitterbewegung*, this structure can extend over macroscopic dimensions for low velocities and leads to our relativistic Bohr-Sommerfeld quantization[1,10]. The space-time, and momentum-energy conservation laws come from the deBroglie periodicities in the velocities, since events can occur only when all particles have moved an integral number of deBroglie wavelengths along their trajectories. We will spell this out in detail elsewhere.

3.2. Scattering cross section, resonances and bound states

Our eight counter paradigm with constant velocities for labeled trajectories connecting all pairs of counters defines a scattering volume measured by deBroglie wavelengths with a linear dimension of at least one wavelength perpendicular to some beam direction. This allows us to define a cross sectional area $4\pi(\hbar/p)^2 = 4\pi/k^2$ which can be taken out of the beam and scattered in any direction. If the total scattered intensity is a^*a compared to the beam with no scattering, this corresponds to an isotropic average differential cross section

$$P = N^2(1 + a^*a) = 1; \; d\sigma/d\Omega = t^2/k^2$$

where the complex amplitude $t = a/\sqrt{1 + a^*a}$ represents the interfering alternatives between scattering and not scattering and arises from the same type of "adding in quadrature" analysis we developed above.

For a situation where the probability of scattering is small, and is indeed isotropic (i.e. the probable size of the scatterer is much smaller than the deBroglie wavelength defined by the beam velocity and mass), we can model this by an "interaction" with coupling constant f^2 to which t is proportional. This is the actual situation for the universal Fermi interaction between distinct spin-1/2 particles one of which is a massless neutrino. Since there are four particle types, and each has two spin states and is a particle or antiparticle there are 16×16 possible initial states and similarly final states. Thus the probability of any one specific type of scattering occurring is $1/(256)^2$ which is our calculation of the Fermi constant. Put in appropriate units this calculation is good to 7%, and as McGoveran reported at this meeting, a correction similar to that computed for the fine structure constant gives four figure accuracy in comparison with experiment.

Of course recognizing the coulomb interaction parameter$e^2/\hbar c$ implied by the combinatorial hierarchy as 1/137 was one of the earliest successes of the program. At first sight the "short-range" approach used above for scattering due to the Fermi interaction looks hopeless when applied to the 'infinite range" coulomb interaction. Actually the fact that there is no range makes the coulomb case independent of h (for non-identical particles), which is why Rutherford was able to find a classical solution. However, once one introduces finite angular momentum conservation and recognizes that the infinite cross section which the classical (or non-relativistic quantum mechanical) theory predicts in the forward direction has to be modified to take account of finite angular resolution, a simple treatment becomes possible. It fits neatly with the McGoveran[15] calculation of the Bohr[16] -Sommerfeld[17] -Dirac[18] -Biedenharn[19] problem, and will be presented elsewhere.

One can unify these treatments by considering a system whose free constituents (eg two masses m_1, m_2) have invariant four-momentum squared $s = (m_1 + m_2)^2$ and one bound ($s < s_0$) or resonant ($s > s_0$) state at s_0 due to some interaction energy $f^2\mu$. This state decays once the resonance is formed with a lifetime $1/\Gamma$ where Γ is the width of the resonance. The two invariant attributes $(s - s_0)$ and Γ^2 must be added in quadrature by the now familiar argument to give

$$(f^2\mu)^4 = (s - s_0)^2 + \Gamma^4 \tag{3.3}$$

This our derivation of the relativistic Breit-Wigner probability unitarity condition for a single resonance, familiar in S-matrix theory.

Note that for $\Gamma = 0$ this formula describes bound rather than scattering states. To emphasize its importance I call it "A *handy-dandy non-perturbative formula* for bound states and resonances". It allows a uniform treatment of strong and weak interactions, and might allow us to abolish the *infrared slavery* of QCD[20]. In particular, it allows us make the following calculation. If we follow Fermi and Yang's suggestion[21] that the pion is a bound state of a nucleon-antinucleon pair, $s_0 = m_\pi^2$, $(m_1 + m_2) = 2M_N$ and

$$(G^2 m_\pi)^2 = (2m_N)^2 - m_\pi^2 \simeq (14m_\pi)^2 \tag{3.4}$$

which is, so far as I know, the first *prediction* of the pion-nucleon coupling constant.

But this just scratches the surface of what now becomes possible for both strong (QCD), electromagnetic-weak (unified) and gravitational interactions. I wish I had time to sketch some of these applications, but must close here. The summary table give an inkling of what lies ahead.

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FIGURE CAPTIONS

1) The 8-counter paradigm: four pairs of counters on a sphere of diameter D which are uniquely sensitive to particles of type a, b, c, d and may be traversed in either direction. For explicitness, the single counters of each type carry the index i or f for initial or final firing. The same apparatus can record may different types of 4-events. Suppose that by using devices exterior to the counters we can identify particle types $W, X, Y.Z$, W_i^a means that a particle of type W fires counter a as one of the initial firings, etc. Then $(W_i^a; W_f^a)$ calibrates the counters and, together with similar calibrations for all pairs defines what we mean by "no scattering". We consider all cases when four counters fire: (1) $(W_i^a, X_i^b; W_f^b, X_f^a)$ elastic scattering, illustrated in the figure. (2) $(W_i^a, X_i^b; Y_f^c, Z_f^d)$ anelastic scattering. (3) $(W_i^a; X_f^b; Y_f^c, Z_f^d)$ 3-decay. (4) $(W_i^a, X_i^b, Y_i^c; Z_f^d)$ coalescence. (5) $(W_f^a, X_f^b; Y_f^c, Z_f^d)$ 4-decay (requires a source inside the scattering volume).

Summary of **WHERE WE ARE** in January, 1990

General structural results

- 3+1 asymptotic space-time
- combinatorial free particle Dirac wave functions
- supraluminal synchronization and correlation *without* supraluminal signaling
- discrete Lorentz transformations for event-based coordinates
- relativistic Bohr-Sommerfeld quantization
- non-commutativity between position and velocity
- conservation laws for Yukawa vertices and 4- events
- crossing symmetry, CPT, spin and statistics

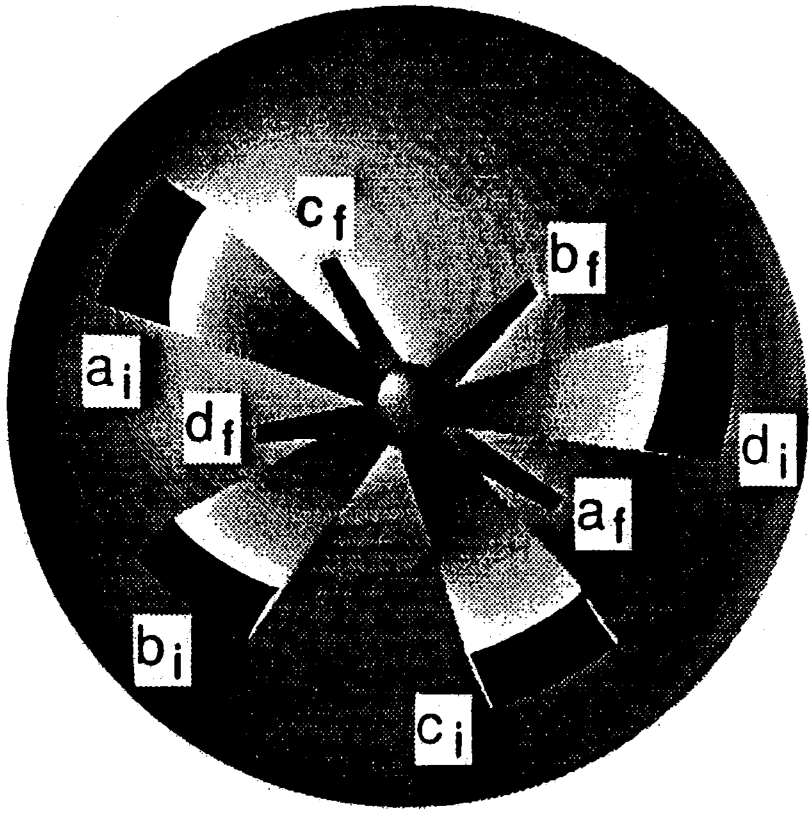
Gravitation and Cosmology

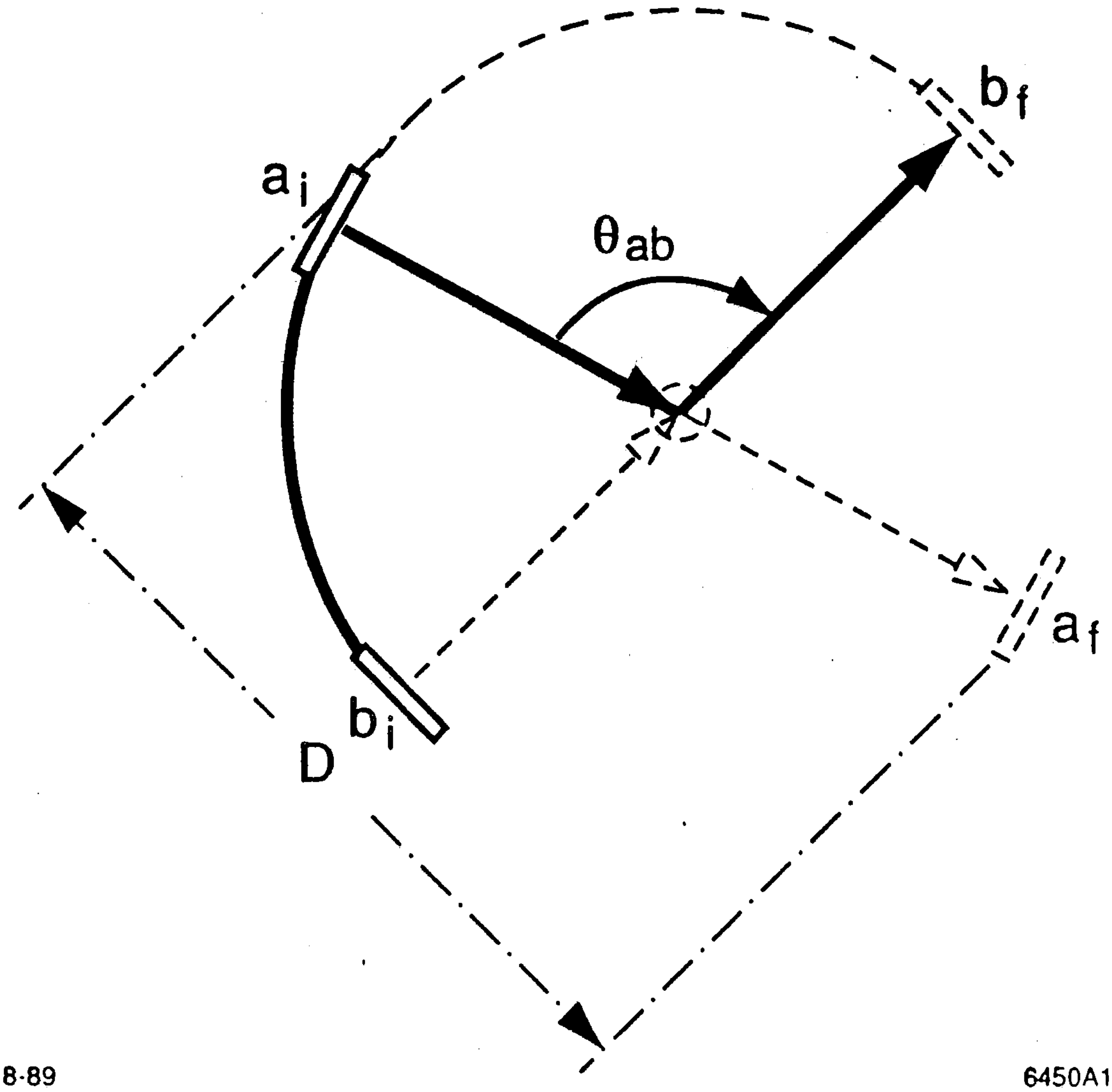
- the equivalence principle
- electromagnetic and gravitational unification
- the three traditional tests of general relativity
- event horizon
- zero-velocity frame for the cosmic background radiation
- mass of the visible universe: $(2^{127})^2 m_p = 4.84 \times 10^{52}\ gm$
- fireball time: $(2^{127})^2 \hbar / m_p c^2 = 3.5$ million years
- critical density: of $\Omega_{Vis} = \rho/\rho_c = 0.01175$ $[0.005 \leq \Omega_{Vis} \leq 0.02]$
- dark matter = 12.7 times visible matter [10??]
- baryons per photon $= 1/256^4 = 2.328... \times 10^{-10}$ $[2 \times 10^{-10}?]$

Unified theory of elementary particles

- quantum numbers of the standard model for quarks and leptons
with confined quarks and exactly 3 weakly coupled generations
- gravitation: $\hbar c/Gm_p^2 = 2^{127} + 136 = 1.70147...[1 - \frac{1}{3\cdot 7\cdot 10}] \times 10^{38}$ $=1.6934... \times 10^{38}$ $[1.6937(10) \times 10^{38}]$
- weak-electromagnetic unification:
  $G_F m_p^2/\hbar c = (1 - \frac{1}{3\cdot 7})/256^2\sqrt{2} = 1.02\ 758... \times 10^{-5}$ $[1.02\ 684(2) \times 10^{-5}]$;
  $sin^2\theta_{Weak} = 0.25(1 - \frac{1}{3\cdot 7})^2 = 0.2267...$ $[0.229(4)]$
  $M_W^2 = \pi\alpha/\sqrt{2}G_F sin^2\theta_W = (37.3\ Gev/c^2 sin\ \theta_W)^2$; $M_Z cos\ \theta_W = M_W$
- the hydrogen atom: $(E/\mu c^2)^2[1 + (1/137N_B)^2] = 1$
- the Sommerfeld formula: $(E/\mu c^2)^2[1 + a^2/(n + \sqrt{j^2 - a^2})^2] = 1$
- the fine structure constant: $\frac{1}{\alpha} = \frac{137}{1 - \frac{1}{30\times 127}} = 137.0359\ 674...$$[137.0359\ 895(61)]$
- $m_p/m_e = \frac{137\pi}{\frac{3}{14}\left(1+\frac{2}{7}+\frac{4}{49}\right)\frac{4}{5}} = 1836.15\ 1497...$ $[1836.15\ 2701(37)]$
- $m_\pi^\pm/m_e = 275[1 - \frac{2}{2\cdot 3\cdot 7\cdot 7}] = 273.1292...$ $[273.12\ 63(76)]$
- $m_{\pi^0}/m_e = 274[1 - \frac{3}{2\cdot 3\cdot 7\cdot 2}] = 264.2\ 1428..$ $[264.1\ 160(76)]$
- $(G_{\pi N}^2 m_{\pi^0})^2 = (2m_p)^2 - m_{\pi^0}^2 = (13.86811 m_{\pi^0})^2$

  [( )] = empirical value (error) or range





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1. Introduction

This catchy (and ambiguous) title occurred to me when I realized (1) I would not be able to attend the ANPA 11, (2) I did not know how much time I could devote to writing a paper for presentation in my absence, and (3) I could not be certain of which advances could be discussed nor how mature they might be. Having thus set your expectations I proceed.

It is my desire to convey the directions which my research into the foundations of discrete physics has taken since ANPA 10. Under the circumstance of little time, these are largely conceptual directions although some results are so obvious to me that they may be convincing to others as well. The topics which I wish to outline are three:

1. A new presentation of the "adding in quadrature" formula (which appears in my paper "The Fine Structure of Hydrogen").
2. An explanation of how to apply the same second order correction methods for other computations, especially the mass of the pion and the value of the weak angle.
3. A new way of making contact with QED via Feynman's Path Integral formulation.

2. Adding In Quadrature: Why?

Over the last year, the one aspect of my computation of the second order correction to the fine structure constant which was most difficult to explain was what Noyes has referred to as adding in quadrature. As I have pointed out, adding in quadrature is actually a consequence of having what I think of as simultaneous and independent "paths" in the system. In Feynman's terminology these are alternative "paths". My own, largely non–verbal, conception of the systems involved has finally found expression in terms of a certain complex system and the frequency with which certain alternatives will be manifested.

The terminology I shall use is that of mathematical probability (see Uspensky). This is important to remember since the meanings of the terms in the physical sciences can be quite different. So as not to burden the reader with having to look up the definitions of terms in the references, I will define a few key ones here.

By an *event* I mean a well–defined, abstract arrangement or class of arrangements. The event (arrangement) may manifest in either space or time or both. An arrangement is identified by its properties or an equivalence class of properties. In fact, a manifestation of an event may be purely abstract. It may be defined only in some mathematical context and so be not physical at all. I will use the term *statistical event* when I mean this kind of event, since this differs from physical events.

By an *occurrence* of a statistical event I mean a particular manifestation of that statistical event. Each particular way (one detailed arrangement among the arrangements of an equivalence class of arrangements) in which the statistical event can be manifested is called a *case*. For example, the results of throwing a die is an occurrence and each of the possible results (a face with 1, 2, 3, 4, 5, or 6 points) is a case. The number of ways in which a case can occur is called a *case count*—these are the number of favorable cases for a particular outcome.

The confluence of one or more statistical events will be said to form a *system*. Exactly how these multiple statistical events are arranged in the system also defines a statistical event, albeit at a higher level of complexity. I will refer to the partitions of the system, each consisting of a component statistical event as the *sub–systems.*

The system under investigation consists of two or more statistical events. For convenience, I will speak of only two. These two events can occur in various ways, called the cases for each event. A particular set of possible cases provides a *representation* of the sub–systems; any given representation may or may not be orthogonal and complete in the sense that it allows us to distinguish (a) each of the cases and (b) the sub–systems.

We can think then of a particular manifestation of a statistical event as sampling from a population consisting of p copies of all the possible cases. The populations (in particular, the individual cases) for two or more statistical events are not necessarily distinct. The information available about the system is (1) the number of (not necessarily distinct) members of each population, (2) the relative frequency with which each case occurs, and (3) the representation system.

For example, we might know that there are 2^q cases and q^2 possible labels to represent those cases for each sub–system. The problem is then as follows: Is there a way of representing the system in terms of the 2^q cases with the q^2 labels (*i.e.* a map) of each sub–system such that (1) the maximum amount of information about an event in the total system is obtained and (2) any known constraints regarding relative frequencies, cases, or populations are respected?

The answer is sometimes yes, given information about the structure of the system. Following a constructive point–of–view, we are required to construct the statistics about the system from the cases for each of the statistical events.

Consider two statistical events A and B. Let these two statistical events jointly have a total of m cases (which we write as $(a) + (b)$) subject to two conditions: (1) The statistical events A and B are *statistically equivalent* in the sense that there is a mapping for which pairwise mappings of cases have the same case count. (2) They are also *independent*, however, in the sense that the (a) and (b) cases are distinguishable, *i.e.* assignable to A and B respectively, except for n cases. These

n cases must be assigned to either A or B arbitrarily.* For example if $n = 1$, we do not know whether A consists of either $(\frac{m}{2} - 1)$ or $(\frac{m}{2} + 1)$ distinguishable cases. Thus, whenever one of the n indistinguishable cases occur, we must recognize that a statistical event of both types (*i.e.* A and B) has manifested. So in this sense, n is a measure of the degree to which A and B are not independent. Note that the number of cases *must* be non–integral for odd numbers of cases. However, this is not a problem if, as here, we have assumed that A and B are statistically equivalent in terms of the number of case counts (condition (1) in the preceeding paragraph). If A and B are not statistically equivalent, then we do not simply divide m by 2 in the formula.

The necessity of assigning the n cases to A or B is a problem if we assume the independence of A and B. When we count cases, there is an implied assumption that we know what we are counting cases of ... that the cases are classifiable into those which should be counted for A and those which should be counted for B. We have no place for indistinguishable cases, and so must find a way to assign the n cases to either A or B. Since we know m and we know that A and B are identical except for the n cases, we know that if we assign n to either A or B, then $(\frac{m}{2} - n) + (\frac{m}{2} + n) = m$, and then the value of (a) is $(\frac{m}{2} - n)$ and that of (b) is $(\frac{m}{2} + n)$ or vice–versa.

As an example, consider a special die. The faces on this die are either green or blue with the exception of n faces. These n faces are turquoise. An omniscient observer can distinguish green and blue from turquoise. Our real observer is not even aware that the color turquoise exists and so always sees either green or blue, even when the face is turquoise. In this example, I assume that there is no particular bias toward either green or blue and so the real observer says a turquoise face is either green or blue with equal probability. This assumption of equal a

* Note that n is integer unless A and B are periodically repeating statistical events. Then it is possible that x cases out of every y repetitions of A and/or B are indistinguishable. Then we may take $\frac{x}{y}$ as the expectation value of n. If A and B are not strictly repeating, the relationship must be analyzed more carefully and the expectation value of n must be computed accordingly.

priori probabilities is the only reasonable one when there is no information to the contrary.

Suppose we have one such die so that there are 6 faces total with one turquoise face on the die, three of green (or blue) and two blue (or green). A green face is a favorable case for the statistical event A and a blue face is a favorable case for the statistical event B. Our real observer assumes that the die contain no turquoise faces and in fact that green and blue are equally distributed. Whenever our real observer throws a die, if the turquoise face comes up, it can be identified as either green or blue. If it is identified as blue, this simply re-establishes the statistical equivalence of A and B with $(a) = (b) = \frac{m}{2}$. A and B are treated then as completely independent. Note that this is equivalent to there being no turquoise faces at all. Otherwise one obtains $(\frac{m}{2} + 1)$ for (a) and $(\frac{m}{2} - 1)$ for (b).

We can made the situation more interesting, if less physical, by using two dice, one of which has a turquoise face and both of which are otherwise either totally green or totally blue. In this system, the turquoise face couples A and B. This couples the two dice when the judgment call is "bad"—*e.g.* identifying as green a turquoise face on an otherwise blue face die. When the judgment call is "good", each of the two die always contributes to either statistical event A or statistical event B (but never to both) and so they are independent in the context of A and B.

If there is no way of telling whether or not the turquoise face is on the "green die" or the "blue die", this situation corresponds to the kind of system described in the computation of the fine structure constant.* In the physical experiment, we are always looking for data regarding a coupled system (*i.e.* an "orbit" composed of two oscillations, one corresponding to the major axis and the other to the minor axis of an ellipse. By definition, the judgment call is always "bad". When the call is "good", we obtain a system that is partitionable into two statistically equivalent

* In the ordering operator calculus, the attribute distance between the two turquoise faces can be zero so that they are truly indistinguishable. This kind of coupling is pre-supposed in the analysis of the fine structure constant.

and completely separable sub–systems. We would call the results *noise* in the experimental system and throw out the data. Such data comes from the case where the "orbit" is circular.

To continue, for event A, the probability of one of the (a) cases manifesting is then $\frac{1}{(m/2)} * (\frac{m}{2} - n)$ and similarly for event B. We write these as (A') and (B'), where (A) would be the true probability of an A case were we able to deal with the partial independence of A and B. Under the assignment of n to either A or B, the events are once again independent. The compound probability (AB) of simultaneous occurrence of cases belonging to A and B is then

$$(A')(B') = \left[\frac{1}{(m/2)}\right]^2 \left(\frac{m}{2} - n\right) \left(\frac{m}{2} + n\right)$$

Given such a compound probability, under the (real observer's) assumption of complete identity and independence of A and B, it is natural to then compute the $(A) = (B)$ as

$$(A)^2 = [(A') * (B')]$$

This, on expansion and rearrangement give the "adding in quadrature" formula"

$$\left(\frac{m}{2}\right)^2 (A)^2 + (n)^2 = \left(\frac{m}{2}\right)^2 .$$

However, the physical way in which (A') and (B') are measured in a physical system may determine whether or not both (A') and (B') contribute. For the fine structure constant measurement, it is only (A'), corresponding to $(\frac{m}{2} - n)$ that contributes to the measured compound probability:

$$(A) = [(A') * (A')]^{1/2}$$

In the case of the computing the fine structure constant, we have $m = 4{*}127{*}15$ and $n = 1$. The factor of four comes from two independent events having $127 * 15$

cases each, any pair of which may be ordered as A then B or B then A, *i.e.* in two more ways.[†] Each repeated occurrence of the compound event A and B is a mutually exclusive case and it takes 137 such cases to have a Coulomb event. The total probability for a set of mutually exclusive and independent cases is then just

$$137 * \alpha = \left[\frac{1}{(m/2)}\right] (A')$$

But (A') is just $(\frac{m}{2} - 1) = (2 * 15 * 127 - 1)$, so that

$$\alpha = \left(\frac{1}{137}\right) \left[1 - \left(\frac{1}{2 * 15 * 127}\right)\right]$$

The two statistical factors contribute to the combined event A AND B. By multiplying the (almost) independent probabilities, one obtains a formula for "adding in quadrature" on rearrangement of the terms. The formula is, of course, different if there are more than two almost independent events.

To obtain the general case, one must consider more than two statistical events (of number k) and all the various possibilities for judging the indistinguishable cases as being favorable to one or more of these statistical events. All possible assignments of the n indistinguishables must be taken into account. Whenever n is a multiple of k, there is the possibility of an equi–probable assignment to all k statistical events. Otherwise one must obtain terms in the relative frequencies which look like binomial coefficients. I hope to have an opportunity to spell out the general case in the near future.

† Note that in the general case of p multiple events the factor of two is replaced by the number of permutations of p events.

3. Other Second Order Corrections

The currently accepted empirical value of the fine structure constant is 1/(137.035963(15)). According to the combinatorial hierarchy and Program Universe, the calculated value to first order would be 1/137. Note that this is the value obtained if (A') is $\frac{m}{2}$—*i.e.* if $n = 0$ and events A and B are in fact identical. When the structure of A and B, and the possibility that they are not independent ($n = 1$), are taken into account, this yields a second order correction. The value obtained is given by

$$\left(\frac{1}{137}\right)\left(1 - \frac{1}{2 * 15 * 127}\right) = \frac{1}{(137.0359674)}$$

in close agreement with the empirical value.

By following an argument directly analogous to that presented in computing the fine structure constant, one can obtain corrections to the weak or Weinberg angle and to the Fermi coupling. This should not be surprising since there is a relationship between the "weak" structure constant and the fine structure, as introduced by Glashow:

$$g_W = \frac{e}{\sin \theta_W}$$

Thus, given that the definition of the fine structure constant is $e^2/\hbar c$ and weak structure constant analogously by $g_W^2/\hbar c$, we have the ratio of the fine structure constant to the weak structure constant:

$$\frac{a}{a_W} = \sin^2 \theta_W$$

The weak coupling g_W and the Fermi coupling G_F are related by

$$G_F = 2^{1/2} g_W^2 r_W^2$$

where r_W is the range of the weak force.

Just as the Coulomb event depends on 127 cases of level 3 of the combinatorial hierarchy and the 16 - 1 possible labels to represent them, the weak event depends on the 7 cases of level 2 and the 4 - 1 possible labels to represent them. This gives the number of cases as $3 * 7$ analogous to $15 * 127$. However, the measurement of the weak event is not in the context of a bound system like that of the hydrogen atom, but rather is understood in the context of a decay process. There is no reference frame in which to distinguish event A from event B. As a result, the cases for each of two events are not orderable: the factor of two that occurs in $\frac{m}{2}$ for the Coulomb event does not occur for the weak event. Thus $\frac{m}{2}$ is $3 * 7$ and the correction term (A') is $(1 - \frac{1}{3*7})$.

Fermi Coupling Constant

The empirical value of the Fermi coupling (as given in terms of the proton mass and factoring out the square root of two and the proton mass squared that would otherwise appear) appears as

$$G_F * (2)^{1/2} = 1.02684(2) * 10^{-5} .$$

To first order, this value is calculated from the combinatorics of Program Universe as

$$\frac{1}{(256 * 256)} = 1.07896 * 10^{-5}$$

The second order correction gives

$$(1.07896 * 10^{-5}) \left(1 - \frac{1}{3 * 7}\right) = 1.0275808 * 10^{-5}$$

again in close agreement with the empirical value.

The Weak Angle

The currently accepted weak or Weinberg angle squared empirical value is

$$\sin^2\theta = 0.229(4)$$

Again, according to Program Universe the ratio of weak to Coulomb events is 2:1 so that the calculated first order value of the weak angle is 1/2 and the square of the weak angle is then

0.25.

The second order correction applied to the weak angle (not to the square) and then squaring gives

$$\left(0.5 * \left[1 - \frac{1}{3*7}\right]\right)^2 = 0.2267573$$

again in good agreement with the empirical value.

Note that we have no "running constants" in our theory nor do we have perturbative approximations. Ours are true corrections due to well–defined, finite system effects. We do anticipate a proper correction factor correlated with the energy of the system.

4. Some Further Speculations

Having had reasonably good success with computing these physical values, I am inclined to make a few conjectures under the assumption that similar corrections would work for other physical values. Three have been presented by Program Universe to–date: the charged pion/electron mass ratio, the neutral pion/electron mass ratio, and the gravitational structure constant.

Charged Pion/Electron Mass Ratio

The empirical value of the charged pion/electron mass ratio is

273.13

The pion is represented in Program Universe as 137 electron positron pairs plus either an electron—antineutrino or a positron–neutrino pair, suggesting a mass ratio of

275.

First note that this is a bound system of two sets of 137, *i.e.* electron type events and positron type events. Thus ordering is important and the factor of 2 discounted in the correction of the weak structure can not be discounted here. Suppose that the charged pion electron/positron pairs are weakly coupled ($3 * 7$ cases) via the 7 labels required to represent level 2. Furthermore, suppose that there are two cases (via exchange of weak labels) which can not be distinguished as belonged to either the electron or the positron set (*i.e.* $n = 2$) for each pair.

The second order correction is then

$$275 * \left[1 - \frac{2}{2 * 3 * 7 * 7}\right] = 273.12925$$

in excellent agreement with the empirical value.

Neutral Pion/Electron Mass Ratio

The empirical value of the neutral pion/electron mass ratio is

264.10.

Program Universe suggests that the pion consists of 137 electron/positron pairs, so that the first order computed value is

274.

Suppose the neutral pion is a more complicated system. If there are three indistinguishable cases instead of two, and only manifesting half the time, and that it does not couple back to the 7 labels in level 2 as the charged pion does, the second order correction is*

$$274 * \left[1 - \frac{3}{2*2*3*7}\right] = 264.21428$$

Gravitational Structure Constant

The empirical value of the gravitational analogue to the fine structure constant is

$$\hbar c/Gm^2 = 1.6937(10) * 10^{38}$$

where m is the proton mass.

The first order value computed by Program Universe is

$$(2^{127} - 1 + 137) = 1.70147 * 10^{38} \ .$$

Suppose that there is a coupling between a combinatorial hierarchy level 2 weak event with $3 * 7$ cases and a compound level 1 - level 2 event with $3 + 7 = 10$ cases. These then couple to give $3 * 7 * 10$ possible cases. If one of these cases is indistinguishable ($n = 1$) and order is unimportant, one obtains for a second order correction

$$(1.70147 * 10^{38}) \left[1 - \frac{1}{3*7*10}\right] = 1.6933675 * 10^{38}$$

again in good agreement with the empirical value.

⋆ Compare this with Noyes recent argument in "Bit String Scattering Theory" – SLAC PUB 5085, January 16, 1990 – from estimation of the pion-nucleon coupling constant.

5. Path Integrals and Event Networks

A short while ago Karmanov suggested (correspondence to Noyes) that there was a close correspondence between the random walk approach to quantum mechanics of Stein and the path integral approach of Feynman. He pointed to Problem 2–6 worked out by Feynman and Hibbs which is a discrete one–dimensional path integral yielding the appropriate amplitude and kernel for a relativistic free particle moving in one dimension, equivalent to the Dirac equation. He later referenced Jacobson and Schulman.

At about the same time, I was working on the problem of interconnecting Noyes bit string events to provide a global causal structure. It has long been my belief (coming from a general relativistic or geometrodynamic point–of–view) that 4–space events are the observables which should define the space–time causal structure, rather than assuming a space–time causal structure in which observable events occur. Since Program Universe provides a mechanism for generating "events", only some of which have physical significance, we can generate a collection of such physically interpretable events. The question then arises "how shall these events be seen as interconnected?".

A clue to this is provided in FDP in the discussion on persistent objects. The idea is that a physical system is identifiable over its evolution if its properties are conserved. To cast this another way, we say that a set of events describes a single system or object if all the events in the collection have the same set of properties. At the quantum level, these properties are specified by the conservation laws and the quantum numbers. If the system is understood to be evolving in time, then there must be an ordering to the events—we must be able to state a criterion by which we say that one event is later than another. These ideas are explored a bit further in "The Fine Structure of Hydrogen".

This suggests an event network, defined by the combinatorics of the bit–string events themselves. Nodes in the network denote events. Arcs denote connectivity between events. The arcs are directed. The events which can be interconnected are constrained by the conservation laws—not because of any intrinsic factor—but because without them no property persists and events can not be ordered into a causal structure.

The event network is characterized by ordering in terms of the generating operator (call this Tick) and by units of "action". The first is NOT physical time, but is probably related to it. In particular, the complexity of the network increases with Tick. Given the event network, a key problem is how to translate from this "Tick–action" space into a space–time causal representation.

This problem has been only partially solved to date*. In the event network, two connected events may be Tick ordered. However, an increment in the action can not occur without a Tick. On the other hand a Tick can occur without an increment in the action. Events which are connected in this way are indistinguishable in the event network except by their connectivity.

In order to research Karmanov's comments, I read Feynman and Hibbs "Quantum Mechanics and Path Integrals". It quickly became clear that path integrals are formed over event networks in a continuum space–time representation. Here the difficulty is the opposite of mine; "how does one constrain the numbers of possible events so that the expression for the probability amplitude does not diverge?" This appears as an infinity of possible paths, related to the others by a phase factor. In the convergent cases, the phase serves to diminish the significance of the contribution of most of the paths. It does this in two ways: (1) by there being a nearby path that cancels out the contribution of the path under consideration, (2) by reducing the contribution to the amplitude directly so that for paths at infinity the contribution is zero.

Consider a discrete version of the Feynman path integral. Each point along the paths has a particular action as is clear from the Lagrangian form of the action integral which Feynman uses to define the amplitude. Clearly it is possible to

⋆ September, 1989

translate a Feynman path integral formulation into an event network in Tick–action space. Of course, this is a backwards approach from my perspective, but it does show that the event network in Tick–action space can be used to solve problems that are solvable by Feynman path integrals.

The classical version of the event network for a free particle leads immediately to the correct expression for the dynamics. Furthermore, the classical event network inherently obeys the principle of least action.

This deserves some explanation. A particle or object is said to follow a *continuum classical path or trajectory* if, given two space–time points on the path, there is always a third point "between" them such that the particle will be found at the third point. In other words, any other path would not preserve the unique identity of the "particle". There always exists a path through the network which consists of the least action and this path also has the fewest indistinguishables. Of course, our network is discrete. The definition is suitably modified so that the "classical path" is not infinitely divisible.

A similar definition is important in understanding what is meant by *interacting particles* in an event network. Suppose that two particles are identifiable at entry to the network and also at the end (exit) of the network. If it is possible to define a classical path for each of the particles on the event network, then they are *non-interacting*. Otherwise, they are interacting. Note that for the portion of the network where two particles interact, they lose their identity—they are not separable in terms of the space–time causal structure. This definition is consistent with our definition of particle (or object) as a conceptual carrier of properties between events.

This definition is also consistent with Feynman's definition of interacting particles for the path integral formulation of quantum mechanics and QED. However, there is an important difference between the classical network and the (relativistic) quantum network. In particular, the allowed paths which preserve the properties of the particle may go backwards in time. In addition, indistinguishable events contribute to the amplitude, whereas in the classical network they do not. The reason for these differences is straightforward: there need not be a unique path in the event network which serves to propagate all the properties of the particle—it may be propagated by multiple paths. This does not imply that multiple particles will always interact in the quantum network.

The last conceptual point to be made is that the event network is easily modified to take "potentials" into account. This modification effectively results in changing the density in space–time of the observable events. This is affected in the translation to a space–time causal representation of the network or a path on the network.

In solving any particular problem, the key is to develop the appropriate event network probability amplitude. First the network is constrained by the number of nodes that are possible. This is done combinatorially from the bit string representation of the properties involved in the system. The network is to be treated as a complex 4–event or 3–event. What takes place within the event is unknown from the outset, but can be solved according to the conditions on the input and output. These limit the possible kinds of nodes that can be generated from Program Universe. In particular, due to the conservation laws, directly observable new properties can not arise within the event complex.

At this point I am working to define how one computes the number of paths within the network if the number of nodes that can be generated under the constraints is n. The following is known straight–away:

1. The number of possible arcs in graphs consisting of n–nodes is $\frac{n^2}{2}$.
2. The number of directed graphs is twice that number.

We must then determine:

3. The number of directed graphs which are connected—*i.e.* there are no isolated subgraphs and every node either initiates or terminates a directed arc?
4. Of these directed and connected graphs, how many have at least one directed

arc initiated at each of the "entry nodes" and at least one directed arc terminating at each of the "exit nodes"? Each of these graphs represents a possible configuration for the interior of the event complex.

5. For each of these graphs, how many ways are there to walk the graph such that the walk begins on the "entry nodes" for the network and terminates on the "exit nodes"? Each such walk corresponds to another way of generating an n–node graph representing the interior of the event complex, *i.e.* an ordering operator.

6. How many of these walks are indistinguishable from another such walk under some permutation of the node labels? This is essential to computing the amplitude: we need to know both how many unique walks there are and how these are weighted by the permutation of node labels.

I have not had the time to pursue these questions. However, I am fairly certain they are solved problems in graph theory. I am also fairly certain the answers can be cast in terms of combinations and permutations. Once they are solved and cast in this form, it is a simple matter to express the formula in terms of the discrete transport operator as defined in FDP and to draw the correspondence to the Feynman amplitude (which is expressed as a phase "transport"⋆).

I am confident that the event network formulation in "Tick–action" space is richer than the path integral formulation and that it provides a linkage between the bit string events of Program Universe and an accepted formulation of QM and QED. It comes to us fully relativistic and finite—there are no divergences nor need for renormalization.

⋆ See FDP for a derivation of the transport operator.

6. Conclusion

In my absence from these pursuits, I invite others to contribute to these computational endeavors in order to speed us on.

7. Postscript

Since this paper was written, these ideas have been applied to the finite and discrete construction of the $1+1$ Dirac equation in such a way that the finite step length is preserved. The method appears to apply to the $2+1$ and $3+1$ problem as well.

In a letter dated November 1, 1989, Karmanov has written that the finite step length does not survive when computing the $1+1$ Dirac (or $2+1$ or $3+1$) from a Stein random walk model (along the lines of Feynman and also Jacobsen and Schulman). However, it turns out that the derivation of the amplitude assumes that the number of trajectories is given by the binomial coefficient and this was then approximated as (number of right [or left] turns) to the k power, divided by k factorial for large N. This approximation is wrong if the step length is fixed. I have pointed out (November 28, 1989) that the number of trajectories should be given by my expression for the "total attribute distance" (derived in FDP) since in our model all "turns" are indistinguishable, there being no a *priori* coordinate system to distinguish them. This situation is similar to the more familiar case of "spin flips". The total attribute distance is just the needed formula arrived at by Karmanov earlier by an approximation in the large N limit, but is now precise and independent of N. The Dirac equation follows and the finite step length survives.

8. References

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Towards a synthesis

Contribution to ANPA Proceedings - December 1989

Ted Bastin

This contribution is in three parts which contain separate material and are introduced separately.

PART I **Process mathematics - the argument**

This workpaper is an attempt to present the argument which runs through the refounding of the hierarchy mathematics which Clive Kilmister has been doing for three years or so very briefly in a sequence of numbered steps. I am guided by two needs: firstly, to explain the motivation for the whole construction, and this can only be effectively commented on as the ideas develop; and secondly, to explain what I call the necessity of the steps. By this I mean allowing the reader to see that the course taken was the only one seriously open, so that he does not have the cumulatively destructive feeling that what he is being asked to contemplate is arbitrary. Both these needs tend to be suppressed by the mathematician's method of exposition, which has different criteria.

What I am talking about is an information theory based on process. The argument does not depend any longer on its specific application to physics. My summary does not necessarily represent Clive's views, though we have worked closely together.

* * * *

1. **Something from nothing.** Process takes place at the boundary

between the known and the unknown. Putative entities from the unknown are tested to see if they should be added to the stock of known ones, or whether they have already been acquired.

2. Labels. A number of entities come to be given a single label. This label stands for an element. In particular it can be used to recall the element.

3.Characteristic Functions. To allot a label to an element u a decision has to be made whether u is new or has been acquired before. We imagine the a variety of functions which we have to narrow down by noticing that some are equivalent to eachother in that they do the same job. The characteristic function F operates on an element u, forming F(u) which is a label.

3. Unique element, discrimination. Elements and labels are recognizable only through their operation since they are represented by the same symbols. There has to be a limit to this freedom, and hence there has to be an element -call it 0 -which cannot be a label, and is therefore unique. The only thing there is to identify it with is the result of forming F(u) in the cases when u has been had before. The testing process is called discrimination.

4.Discrimination. This is between an already labelled element and one that has not yet been labelled. Either it is different, in which case you label it, or else it is not, and you don't. However in the second case the act of discriminating has produced something which can't be an element because nothing has come in from the outside, and so must be a lebel. This is not yet the label of anything, though our further discussion will show it will become such in due course, because of the way the mathematics has to use all the labels in order. This therefore constitutes a difference between elements and labels: elements must have labels, but the converse is not true.

5. First theorem. Unambiguous labelling, and therefore well-defined process, are possible if we proceed by construction of successively larger discriminately closed subsets (dcs's). It is easily appreciated if one does the construction that if dcs's are not completed before new elements are added then the labelling is ambiguous. It is important to notice that at this stage we can only test the new element against single elements. The proof uses Conway's device of always selecting the smallest used as the formal justification, but it is also well-known that no other choice provides a non-trivial hierarchy.

PART II SCATTERING Ted Bastin

October 1987

Michael Horner asked me what my present aim was for the present stage of the combinatorial physics project, and I said that it was to be able to give a lecture, or a course of lectures given time, on the particles and scattering. he asked how long I thought it would be before that were possible, and I said two years. That wasn't two years ago yet, but as a result of a week spent with Pierre here I now think I could do what was proposed, and am sending round this short note on the way I imagine the essential bits to fit together to provide a position from which I could start and from which a variety of different investigations would radiate like the spokes of a wheel, providing the separate lectures.

I am being quite personal about this. I am describing what MY mind will accept as a way of thinking which does not separate into only arbitrarily connected parts. On the one hand I do not say that other minds may not encompass other unities, nor, on the other, that other people ought to be able to take over mine.

Three main strands of thought appear in my attempt at a synthesis, and each has emerged as a consequence of a major work. There is Pierre's theory of the coupling constants which for the present purpose I regard as the way in to the understanding of the quantum numbers specifying the particles (in a way which I think is not so far from his own perception). There is David' work on the second order corrections to the coupling constants which result from the need to bring into association unrelated hierarchy constructions, and which bring in aspects of his operator calculus. And there is Clive's deepening of the foundations of the hierarchy algebra which shows that the hierarchy originates in the need to label elements in a constructive theory, and in the course of which demonstration a lot is discovered about the use of labels. They are not crude stick-ons any more which require paste and paper or computer technology or coloured chalk

THE STARTING POINT

What has freed thinking up for me was a remark by David in the paper which he sent to this year's conference. It now seems likely that I have read more than David meant into the remark, though it may be that I have seen its significance in a new way. the remark occurs in "Advances in the foundations", read at the recent conference, is on page 6, line 4, and reads "However, the measurent of the weak event is not in the context of a bound system like that of the hydrogen atom. There is no reference frame in which to distinguish event A from event B. As a result, the cases for each of two events are not orderable: the factor of two that occurs in m/2 for the coulomb event does not occur for the weak event. Thus m/2 is 3x7 and the correction term (A') is (1-1/3x7). Pierre thinks that the 'ordering' which stimulated my thinking, referred to the fact that the last place in the label could be either inside or outside the bounded interval with equal probability, and that a factor 1/2 would enter the frequency.

The electric effect it had on me was different. I saw this as the point at which one could first speak of non-bound <u>and therefore separated</u> systems because one had for the first time a combinatorial way of giving meaning to non-boundness and spatial separation at the same time and as equivalent concepts. I mentioned this to David amongst other points in a letter and he said "Bound and unbound refer to the physical situation being modeled. By a bound system I mean one which does not admit of decomposition into two separable physical space-time coordinate systems. By starting with events, there is no guarantee that the notion of two or more objects each with a local space-time frame is possible in a coherent fashion." This remark enshrines a large part of David's position on the mixed initial situation which is characteristic of the quantum world, which I shall have to talk about more, but it still seems to me to need to import from elsewhere some meaning to the notion of separabilty which can apply sometimes. I could compress all my beefing about program label consistent with the other constraints operating at that point of the construction. No claim is made that this is the only way to construct; it is only that this way acheives the desired result and therefore is equivalent to anything else that might be 'really' going on in the world so that no harm can come by using it.

6. Second theorem. This uses the labelling structure of theorem I to complete the specification of the characteristic functions. The argument is similar to that of theorem I but now the new element is compared with the existing subsets of elements already acquired. It is shown that this program can be carried through if the characteristic functions satisfy a linearity condition that the function of a sum of two elements is the same as the sum of the functions of each separately. The characteristic function of 'its own' dcs is zero, but it leaves all the other dcs's unchanged. This used to be how we justified the use of linear algebra, but now it appears as a sort of bonus coming from the requirement that the labelling should not be ambiguous.

7. Arrays. (essentially matrices without size restriction) are the simplest way to represent the characteristic functions as they have been determined by theorems I and II. The rows and columns of the array denote respectively what the characteristic function operates on, and the result of the operation. The matrices in the familiar algebra are obtained by cutting off the arrays at a particular dimension.

8.Levels. The reason for restricting the generation of arrays to produce the levels familiar from the old algebra goes back to the principles concerning labels from which we began. During the construction, labels and elements are used indistinguishably. The familiar principle of making stops to the construction at which all the existing dcs's are used as the basis for a new phase of construction, thus generating levels, is explained because it is the only way of getting the use of labels and elements distinct. In fact they are only distinct at the completion of each level.

The recursive structure of levels is determined completely once a first level is fixed, which gives a unique character to that level, so that additional arguments are needed to justify just that one choice. (Namely of three elements excluding the zero, and three dcs's.) The type of symmetry in that level has been universe into the one objection to this assumption.

Now, things have moved a long way, and Pierre worked continuously when he was here to acheive for my benefit a minimal scattering theory, making it possible for me to get my personal clarification. His remaining sticking point was that he had to have a multiplicity of bit strings for each label,and given that, was prepared to be flexible about the physical identification of program universe. The multiplicity of strings is needed to define a single physical variable which it is appropriate to call velocity, and which carries the notion of physical separation. He agrees in any case that program universe is only one of a class of possible models. He was of the opinion that Clive's recent work is restricted to a single construction of the hierarchy and that he has gone back on the indefinitely continued generating operation which stimulated program universe. This account is certainly not the whole story, and I shall hope to show that the true situation is much more interesting. David is probably much more firmly committed to defining a coordinate system independently and prior to the discussion of scattering, but discussion of that would take too long for here.

Starting the way I do, I am now free to take a (spatially) separated system as one in which the label of one of the constituents is incomplete. (David would call this a lack of specification, and it is from this that he gets his characteristically quantum-mechanical probability with 'adding in quadrature'.) Then the main principles fit in pretty much as Pierre and David have developed them:-

I **Coupling constants** These characterize the different scatterings which exist, and the most important have been successfully identified with frequencies or normalized probabilities on the supposition that there are finite numbers of possibilities allowed by, and calculable in, the hierarchy. The most important thing to see at this stage is that these numbers must exist in the nature of the case, and be settled before any theory can be constructed, though in current theory this will have to be empirical.(I have not dealt with the correction terms yet).

II **Particles** These are names given to different arrangements of going beyond 10^{39}). Pierre calls the primordial number which gives the individuality to a scattering process a velocity, for reasons which will appear in the next paragraphs.

VI **Basic properties of labels** Clive's theory of the origin of labels has two important consequences for my present discussion. The first is that it is only when we are at a level change (i.e. with an exactly full level) that we can speak unambiguously of levels as opposed to elements. I have already made use of this principle in connexion with bound and scattering states. The second is that the bit-string representation is something which we arrive at only as the end-point of a long mathematical argument, and there is a funny twist at the last step which Clive was inclined to see only as a curiosity. The final matrix form of his characteristic function has an infinite 'tail' along a diagonal or parallel to the diagonal. However this is determined by the matrix and can be ignored since it gives nothing new, in all cases except that of the null matrix. This case is different, since in it there is no matrix to determine anything. Switching to picturesque language now, I say that we have to see the matrix operator as mounted on a string of indefinite length and content. Moreover, since one string is as good as any other, we can discriminate these strings and start setting up labels all over again if we wish and get the Hamming numbers characteristic of the velocities. The main freedom to play about with is, to put it picturesquely again, that it is a statistical matter how two strings which have freedom to slide about on strings of zeroes of indefinite length, overlap, if they overlap at all. And needless to say this is where we break from Clive's hierarchy construction to David's. This is my sketch of what Clive's theory provides for the program universe enterprise, and to my mind it is most of what the universe programmers ought to want.

VII **The photon** I shall follow program universe in referring to the strings of variable length as the 'content strings'. Then the simplest configuration for velocity is strings to be either all zero or all ones, and there are only two velocities, which we call +c and -c. Other strings are naturally associated with mass, and this is the first appearance of that term. Angular momentum does not have to be defined except as a verbal convenience to answer the question what does a massive particle in a bound state possess? Pierre has an account of the difference between head-on scattering and scattering when there is a moment about the centre of gravity, couched, of course in counts and angles. I do not go into the question of exchange, but that cannot be long delayed.

VIII **Events** There is one more element in Pierre's list of things which he has consistently insisted on, and that is his wanting a fourth label string to complete the set of three required to specify a scattering. I think this completion defines what he calls an event (and Pierre is clear that event is a less basic concept than velocity). It has been explained in terms of conservation, since it is in a way the constant sum under discrimination of the other three; though 'conservation' seems to depend for its force very largely on continuum ideas which are inappropriate. Again it seems to mix up the treatment of labels and content strings to require closure of the label set. Pierre speaks of a 'happy accident' that the label strings have the same number as the content strings to specify an event, but he is not at all happy with that. Another bit of the jigsaw puzzle is the succession of time nodes (the ticks of program universe) which may be related to the formation of events as a basic property of a process model. This brings me back to a basic principle arising out of Clive's work, discussion of which I left hanging in the air earlier. This was the fact that label and element are only distinct concepts when a level is complete, and I conjecture that the necessity for the completion which Pierre requires may be needed to provide the former kind. I had thought of this kind of thing only in connexion with the highest level, but perhaps one has to have it throughout.

My vision is getting a bit rarified, I shall stop.

PART III

The construction of arrays -practice in the mathematical detail

zeroes and ones in labels of the length required by the hierarchy algebra to specify the coupling constants. Places in the label are called quantum numbers. Conventionally one imagines travelling objects, and this convention is entailed in the use of the word 'scattering'. Allocation of quantum numbers is done phenomenologically using the coupling constant values. The neutrino is the name for the 'particle' with only two states; the leptons have four -two spin 1\2 and two charge. The appearance of mass at the lepton level is connected with the representation of the photon, and we haven't got there quite.

III Multiplicity Pierre's theory of scattering and coupling constants really only makes sense with the possibility of independent hierarchy constructions, but this only becomes clear with David's account of the hydrogen atom and his introduction of unrelated 'asynchronous' constructions for the bound state with the resulting successful calculation of the fine-structure correction. Also subsequent extension of the basic idea by both of them to other bound states.

IV Counting and angles We must associate the counting of frequencies of scattering with the multiplicity of constructions, and McGoveran's theorem shows that any measure attached to probabilistic phenomena will show a recursive threefold structure (the hierarchy being itself an example) and we take this as justification for associating the counts of numbers of constructions with angles (I would say 'labelling them with angles' if that term were not already bespoke).

V 'Velocity' Pierre requires the counting defined in the last paragraph to be represented in bit strings. I have been held up for long by this requirement appearing by analogy with classical space-time physics, but now am free of this superstition and agree that there is an overriding abstract case for it. By 'represented in bit strings' I mean that we must be able to infer a primordial mechanics using the discrimination operation, to explain the scattering as due to the interaction of an indefinite number of individual strings. (The number may have to be less than 10^{80}, but we have to be able to be 'indefinite' to the extent of

(This is a workpaper prepared by Ted Bastin from a transcript of a special purpose lecture by Clive Kilmister. It covers essentially theorem 2 of PART II.)

This workpaper is one of several whose aim is to make the mathematics of the combinatorial hierarchy, as the core of a process view of information theory, available to the non-mathematician. The basic position behind the workpapers is that there is great benefit to be had by exhibiting the principles and motivations of the mathematics in an extremely simple form so that the progression of the thinking is apparent. The usual objective of the mathematician is different, and may actually be effectively in conflict with this position. On the other hand, the non-mathematician will probably feel that his understanding of the principles is 'up in the air' in an uncomfortable way if he does not know what it would be like actually to do the mathematics. Therefore I make an example of the second of the two main pieces of deduction in Kilmister's recent work in which he has provided new foundations for the hierarchy mathematics (which, incidentally, enable it to be seen separately from the application to physics, and therefore as a putative part of information theory). The aim has been to put in all the guiding comments which go with a verbal presentation, and which the mathematician would regard as something which the student should outgrow, since it is part of mathematical (as distinct from the logical or philosophical) expertize to acheive a minimal statement.

I (Clive speaking) am going to show that whatever the characteristic function for a dcs may in fact have been, one can always replace it by one which has two properties which will enable us to use the techniques of linear algebra. (We used to jump in at the deep end with the assumption that these were available for use, and give some sort of justification by use of quasi-physical arguments.) The two conditions are:

(1) what I shall call compact. This condition has been used already and means that when we label a new entity we always choose the smallest possible label that is allowed by the

labelling that has already taken place. It is the constructive device that was used by Conway, as already quoted.

(2) a form of <u>linearity</u>. The function of the sum of two elements is the sum of the functions of those elements:

$$F(u+v) = F(u) + F(v)$$

This is the usual definition, but we change it slightly to hold only when u and v are different.

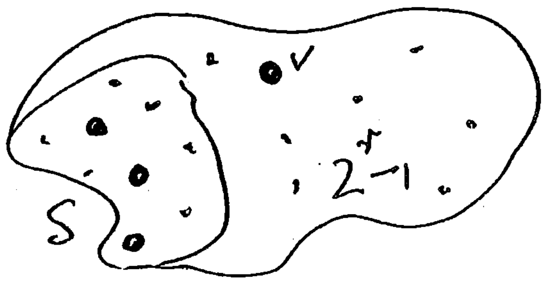
I now give an algorithm which will actually construct the F which is the characteristic function for a dcs which satisfies the above conditions. The proof of the theorem that thereis one is simply the algorithm which generates it. We want to specify the function F, and that simply means having to give the value of the F(u) for all values of u. Our aim will be to put this in the form of a rule, since there is a potentially infinite number of them, but at the beginning we shall have to take the different u's separately.

1. Firstly: F is supposed to be the characteristic function of S, and this means that if u is in S then F(u) is zero. That is the definition of the characteristic function -that it is to vanish for the elements in S. Hence F(u) = 0 if u is in S. Moreover all we know about 0 is that it is the same unique element as we had before. It is also the case that F(u) mustn't be zero unless u is in S. It must vanish for the whole of S and not for anything else, and that is the way in which it points to S. We shall have to make sure in the course of the construction that no extra zeroes turn up. F has to be constructed in such a way that it can't vanish anywhere outside S.

2. The next stage is to look outside the dcs. the elements outside have an order which was established in the first deductive part of the of the work and which established the dcss (I use 'dcss' as the plural of 'dcs') as arising naturally out of the labelling which defined this order. So now you pick the first element which you haven't dealt with already: you have dealt with the whole of the dcs S so you pick the first element outside it -the smallest- using the labels. Let v be the first or smallest. So we write F(v) = u_1 (and we must remember that F(v) mustn't be equal to v). v is outside S, and so if I pick for F(v) an element of S then it will certainly be different

from v itself, and if I pick the first element of S then I am certainly doing as well as I possibly can about the compactness, because S is certainly involved, and I am here choosing the first element of S.

So far we haven't mentioned linearity, and we now push in the linearity requirement for the elements which we have considered so far. If I take any element of S -say u - then F(u+v) has got to be F(u) + F(v). But now because u is an element of S, F(u) is 0, so 0 + F(v) = 0 + u_1, so that the whole lot must come to u_1. That means that we have assigned the values of F for v and the whole set of elements consisting of any element of s plus v. Now that is itself the next larger dcs containing S, because it consist of S plus one element outside, together with the result of discriminating that with all the elements of S. So we have F for the next larger dcs containing S. (S must have a size which is 1 less than a power of 2 because it is a dcs. For example suppose |S|, the size of S, is 7 because S is a dcs generated by 3 elements and has the form 2^r-1. Then v is one more element, and then the 7 elements of S added to v make another 7, giving 15 in all. This is then the next dcs. This was not our original way of developing a structure containing dcss, but I have been thinking this way for some time: you could speak of the structure as a nest of dcss. In the diagram the elements of S are in the smaller shape, and the element v is in the larger shape outside. Now S has size 2^r-1 where r is the number of generators. So we can take the r generators (which are blacker than the rest) together with v itself. Then in this case those three generators together with v produce a larger dcs including v and others. if now you ask how many elements does this larger one have? the answer is that it has the whole of S itself; which is 2^r-1, and then it has v in it which is an extra element; and then it has all the elements which are got by taking v with one of the elements of S, which is another 2^r-1:



$$(2^r-1) + 1 + (2^r-1) = 2x2^r - 1 = 2^{r+1}-1.$$

Of course the final sum is the size of the dcs with r+1 generators; these being the r generators of S together with v itself, which has got added on as an extra generator. This last bit is a sort of aside to the main proof.

I take an element outside the starting dcs which is 1. Let it be the element 2. That is the v in the theorem. Then, as well as 2, I add to 2 the elements of the original dcs which is only the single 1, and then I have the element which I call 12, or 1and2. (Notice that as we are now speaking of labels, how we write the association of 1 and 2 is still only a matter of convenience). Now taking these three together is the next largest dcs and this has 3 elements and the two generators 1 and 2. Then I look outside this at another element, and we choose the next one which is the element 3. Notice again that we have the elements available to choose, and are able to make the choice: it is not a matter of attaching the label as it was in the first of part the construction (which does not appear in this workpaper). Now I have also the elements 13 (or 1and3), 23 (or 2and3), and 123 (or 1and2and3). I add on those four elements to the three which I have got already, and that gives me a dcs with 7 elements.

Well, so the process goes on, and the next element to put in would be 4, and the result of discriminating that with all these 7 that we have got already would be to give another 7. There would be eight elements here, and those added to the 7 you've got already will give 15. It is evident that this process will in due course include every element which you have generated. See the table below. The outcome is that if I have specified F for these successively larger dcss, you will have specified it completely.

labels	dcss
(1)	{1}
(2, 12)	{3}
(3, 13, 23, 123)	{7}
(4, 8 new elements)	{15}
	{31}
	–

Now I introduce the first element which is outside this. Call it v. I define f(v) to be the first element of S which I call u_1, then I know that this will specify F not only for S, which I know to be 0 but also on a larger dcs which consists of v together with v plus elements of S. Call the enlarged dcs S_1. We know that S_1 contains S, so now we have got F specified all over this dcs S_1, and the next stage is to choose the smallest element which we haven't got to yet. That is to say it is outside S_1. Call it v*. (We notice again that the mathematician looks at every element and is able to do that because of our proof of the equivalence of this construction with anything that may be going on).

To give the value of F for v* we come back to the requirements of compactness and linearity. The former requires us to take the smallest element of S_1, except that u, which was used earlier on as a value must be avoided otherwise we should get spurious zeroes of F which we don't want to be included in the set. So we must take F(v*) as the smallest element of S_1 which we have not used already. Call it u_2 (not u_1). Now we have to plug in the linearity. The only problem that has to be settled is the value of the F of v^*+u_1 where u_1 is any element of S_1, and we want to make sure that that is F(v*) + F(u). We can do this by simply defining the values of F(v*) + u to be what they should be, that is F(v*), which is u_2, together with the value of F(u) where u is an element of S_1. There are two kinds of elements of S_1 now; some of them are already -were already- in S, and for them F(u) would be 0, and others which were not in the original set S but were in S_1. For these, F(u) would be u_1. Hence:

F(v* + u) where u is any element in S_1

= F(v*) + F(u)

$$= u_2 + \begin{cases} 0 \text{ if } u \text{ is in } S \\ \quad\text{or} \\ u_1 \text{ if } u \text{ is in } S_1 \text{ but not in } S \end{cases}$$

So this will give you the values of F for the following elements:- firstly those we have already done in S_1 which included S itself, and now we have done it for v* and for the elements which consist of v* + any element of S_1. So we have done it for the next larger dcs generated by the process described earlier. So now F is specified for S_2, the next larger dcs, namely (S_1, v*, v*+S_1). And so on.

Summary of the continuing process. You choose the smallest element which you haven't dealt with by this process already; you assign the value of F for that element as the smallest element of S_2 which you haven't used yet. This ensures the compactness. Then you assure the linearity by defining F for the sum of the new element and all the old ones in such a way that the linearity is satisfied.

Detailed example

Take S to be the dcs which consists of the element 1, the element 23 (2+3), and the element 123. These three are indeed a dcs becauseif you take the first and discriminate you get the third, and so on in any order. I could take the example where the elements were 1, 2, and 1+2, but the characteristic function would be too simple to be instructive. The rule is as follows:

(1) The F for any element in S has got to be 0. So F for any element in S is 0, and F(1) = F(23) = F(123) = 0, and only these may be 0.

(2) then one looks at the other elements which are not in s, and this answers the question how do you choose the next element. One has got all the elements on the level ordered, so one just looks along them and removes the ones that have been dealt with already. The next element is 2, and we have to specify F(2) because we haven't had 2 yet. The rule says take the first element of S as the value of F(2). So F(2) = 1. Now consider linearity. You have it for S, now you want it for the elements of S taken with 2.

(3) F(1+2) = F(1) + F(2) = 1, (because of linearity and because F(1) = 0.

F(3) = 1 [similarly, and F{(2+3) + 2} = F(2+3) + F(2) = 0 + 1 = 1]

F(1+3) = 1 [similarly F(1+3) = F(1) + F(3) = 0 + 1 = 1]

This specifies the f's for the dcs (1, 2, 3, 2+3, 3+1, 1+2, 1+2+3).

(4) Now we carry on applying the rule and pick the first element which is outside this dcs which is 4. So you take F(4) and you pick for this the smallest element of this S_1 which you haven't used already (starting over again). You have used 1; so you have to use 2. So F(4) = 2, and the linearity requires

F(1+4) = F(1) = 2

F(2+4) = 1 + 2

F(3+4) = 1 + 2

. . .

F(1+2+3+4) = 2 (F's for 1,2,3,4 and all elements generated from them).

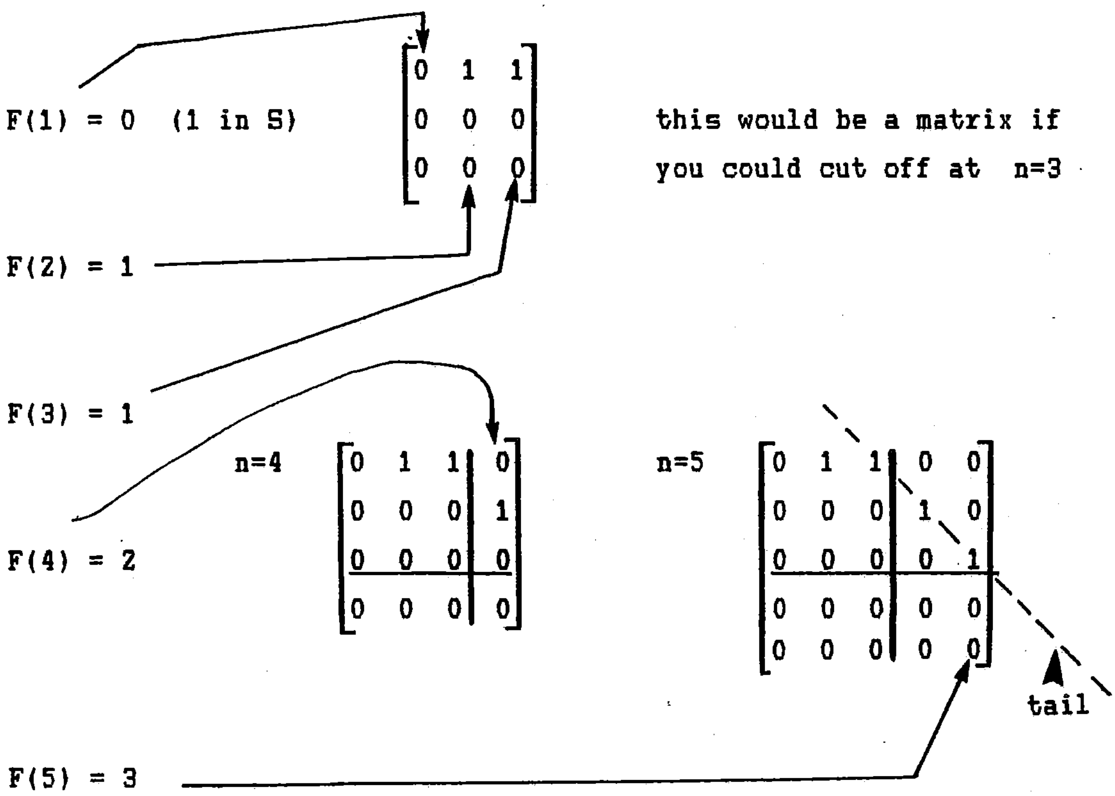
(5) The next outside is 5, and we find we have had already 1, 2, 1+2. 3 has not appeared as a value of a function because F(2) is 1, and F(4) is 2, and linearity will do no more than add the 1 and the 2. So F(5) is 3. Then you put in linearity to get the values of F for all those elements which consist of 5 plus smaller elements.

A surprising consequence of my development leading up to the matrix operators and linear algebra which had been obscured before is that one does not get precisely to the matrices because tha arrays have what I call an infinite tail which can be ignored in all cases except in one vital case. I am not concerned with the consequences at present but only with the nuts and bolts.

Once you have got up to 4 you see a simple rule emerging since there really isn't much to do except put in the linearity to give the values for a whole dcs. F(5) has to be the next one up, and F(6) mhas to be the one after 3 which is 4, and so on. ASfter 4 the later values are an infinite tail, and in this particular case F(n) is always n-2. It depends on the structure at the top end how many you have used up.

To construct matrices given the F's we use the rules:-
5 is a shorthand notation for the vector 0, 0, 0, 0, 1. 3 is the same for 0, 0, 1, 0, 0. 2+3 is 0, 1, 1, 0, 0, and so on. Let us first look at the situation as though it were

appropriateto cut these vectors off and make a 3x3 matrix out of 3-vectors as we do when we want to talk of strings of 0's and 1's. The table shows the F's and their relation to the matrices.



. . . .

The 1's in the tails of the two new matrices does indicate something about the F's which we have constructed, but we have essentially got all the information because the rulewill tell you how to construct the later matrices. However there are important exceptional cases. If the original matrix inside the dotted line consisted entirely of zeroes (the null matrix) then it does not convey the necessary information about the F's because we shouldnT be able to construct the infinite tail from it. In particular you can't tell where the vector stopped and you started adding on zeroes up to where the original vector was. Parker-Rhodes' trick was to add on the identity matrix along the diagonal so as to eliminate what he realized was the difficult case of the null matrix, but there is no justification for this procedure in my treatment, and it indeed obscures something which is likely to turn out to be very important.

ANPA, THE CH, PU & THE UK

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INTRODUCTION

Pierre Noyes's paper in ANPA 10 called "Where we are" and his scattering theory in this year's conference, with his "What is to be done", gives a good idea of the progress of the ANPA Bolsheviks across the Atlantic. By contrast, this is intended to be a thoroughly revisionist document showing the (limited) Menshevik progress in the UK over the past year. I shall begin by commenting on my paper in the ANPA 10 Proceedings; I am glad that I do not find too much that must be changed, though if I were writing this year the emphases would be different. Ted Bastin has dealt with a better version of what I tried to do then in his paper. I confine myself to some detailed changes. Then, in the last part of the paper, I turn to what was promised at the end of last year's.

THE MATHEMATICIAN'S CLOVEN HOOF

I am content with the first part of the ANPA 10 paper but half way down p.56 alarm bells start ringing because of the arbitrary way in which the idea of a function is introduced. The process theory must work by itself; there is no scope for the mathematician to insert his foot in the door from outside. My training makes it hard for me to guard against this danger; I gratefully acknowledge the insistence of Ted, and also of Alison Watson, in improving my performance here. The distinction I was blurring last year was this: we are asking the question "Is this element a new one or another copy of an old one?" and so the system must have an operation, call it "testing" which, applied to a putative new element, gives a signal of OLD or NEW. If it is a new one, it has to be given a new label; if it is old, some further investigation is needed (looking at subsets of the elements already in play) to determine which old one it is another copy of and so which label to give it. But in either case, labels will be given, and when this has been done we can work with the labels. We are then doing mathematics; and it is obvious that we can then write the testing operation in terms of $f_S(a) = b$, where S is the set of already labelled elements, a is the putative new one and b is a signal label with some property P (where P(b) means that a is old and $\sim P(b)$ means that a is new). But, in an important way, all this is devoid of interest because it can be written down only AFTER the labels have been given. This way of writing things introduces a mathematical function f_S though it does not define it, for there are many different f's allowed by the restriction stated above. Any of these might be the way in which the system is "actually working" but we cannot tell which one because the system functions in the same way with any of them. It is now fair for the mathematician to enter and to choose to analyse these processes in terms of particular forms of f_S which suit him, so long as these forms are amongst those allowed by what has been said.

The guide to finding a characteristic function for S is the method briefly sketched in ANPA 10 for doing the same when S has only one element. In that case, since $f_u(v) = 0$ represents an equivalence relation between u and v, it must be the case that (i) $f_u(u) = 0$, (ii) If $f_u(v) = 0$, then $f_v(u) = 0$ and (iii) If $f_u(v) = 0$ and $f_v(w) = 0$, then $f_u(w) = 0$. Amongst the various f's which do the same job there are some which satisfy these conditions because they actually satisfy the stronger versions: (ii) $f_u(v) = f_v(u)$, (iii) If $f_u(v) = f_u(w)$, then $v = w$. (This fact is not obvious, but a proof of it is not difficult, using "Conway's trick".)

In looking for f_S the condition (ii) is no longer relevant, (i) becomes (i) $f_S(u) = 0$ if and only if u is in S, and the analogue of (iii) is (iii) If $f_S(v) = 0$ and $f_S(w) = 0$ then either $v = w$ or $f_S(v + w) = 0$. In fact, this is just the statement that S is a dcss. It is useful to rewrite this slightly as

(iii)' If $f_S(v) = 0$ and $f_S(w) = 0$ and $v \neq w$, then $f_S(v + w) = 0$.

Now, as in the simpler case, there will be amongst the f's some which satisfy (i) and the following stronger form of (iii)':

(iii)" If $f_S(v) = f_S(w)$ and $v \neq w$, then $f_S(v + w) = 0$.

This is shown by a recursive construction (as it must be since we have to provide for new u's as they come in). This consists essentially of defining f(u) as u proceeds upwards to be the least element in play which will satisfy the constraints. This construction is then the one described in ANPA 10 on p.67, but the difference in emphasis is this: the construction has been shown here as arising naturally, and it is then a matter of proof that the operator constructed is a linear one, whereas last year I was still playing the part of the furtive mathematician, bringing in the concept of linearity from nowhere, and then showing how to make f

This shows that the judgment above that "all this is devoid of interest", though true enough at the level of the operation of the system, has to be taken with a pinch of salt when one goes on to analyse all the ways in which the system might work.

LINEARITY

I do not want to alter the paper in ANPA 10 as far as the part immediately following the introduction of functions is concerned, but a similar point comes up on p.64 when the mathematician seems to be insinuating the notion of linearity without much rationale. (I may say in passing that there is too much emphasis in that paper on the product construction. Since it is the aim of that part of the paper to show that the product construction is not a satisfactory one, the construction is not worth such attention to detail. But the details are correct.) Once the importance of dcss (discriminately closed subsets) has been recognised, I would prefer to proceed as follows:

First, let us introduce some shorthand notations. If U is any set of elements, then $S \equiv D(U)$ denotes the discriminate closure of U, that is, the set that results from adjoining to U the results of arbitrary discriminations between different elements of U and between the elements so produced. If U has exactly r independent elements, then $S = D(U)$ has $r^* = 2^r - 1$. If U is specified by its elements, $U = [u_1, u_2, \ldots u_r]$, drop the brackets in D(U) to write $S \equiv D(u_1, \ldots u_r)$. Similarly, if another element u is added to U and the discriminate closure $D([u] \cup S)$ is formed, write D(u,S) for it; finally, if $U = [1,2,3\ldots r]$, write D_r for D(U).

conform.

The recursive construction goes like this:

1. If u is in S, define f(u) = 0, so defining u on the dcss S.

2. If v is the least element outside S, define f(v) = u_1 where u_1 is the least element of S.

3. For every u in S it will be the case that f(v + u) =u_1 by using the condition (iii)" replacing v, w by v, v + u; for then (iiii)' will show that each u for which f(v + u) = u_1 is a member of S. This defines f on the dcss S_1 = D(v,S).

4. If v_1 is the least element outside S_1 , define f(v_1) = u_2, f(v_1 + u) = u_2 for any u in S, where u_2 is the least element of S_1 apart from u_1.

5. To define f(v_1 + w) for any w in S_1 but not in S note that (iii)' prohibits u_1 and u_2 as values and so (Conway's trick again) define f(v_1 + w) as the least value apart from them, which is in fact u_1 + u_2. This defines f on the dcss S_2 = D(v_1, S_1) and the possible non-zero values of f are the dcss D(u_1, u_2).

[NB: Observe the way in which linearity enters here through the use of Conway's trick to satisfy (iii)".]

6. The general step of the recursion is then: If f has been defined for the dcss S_r with valuesin D(u_1, u_2, ...u_r) and v_r is the least element outside S_r, define f(v_r) = $u_{r'}$, f(v_r + x) = $u_{r'}$ + f(x) where $u_{r'}$ is the least element of S_r outside D(u_1, u_2, ... u_r), x is any element of S_r and (as a matter of notation) r' is the successor of r. This defines f for $S_{r'}$ = D(v_r,S_r).

So much for the detailed changes that I see as needed in last year's paper.

CONSERVATION LAWS

Last year my hostage to fortune was to predict that the way ahead towards physically known particles was to investigate how conservationlaws fitted into the graded algebra (whose elements are quartets of elements of the hierarchy, one at each of the four levels). Here is my rather limited progress.

We are concerned here with discriminations, which we see as the germ of a particle scattering vertex. It is at such a vertex that we want to study conservation. There are two related versions of the "conservation story" in the orthodox approach. In the first, modelled on electric charge, numbers are attached to the three elements entering the discrimination and conservation means that the sum of two of these numbers equals the third.("As much goes in as comes out.") In the second, modelled on momentum, numbers are attached to the three elements and conservation means that the sum of any two of these is less than the third. The numbers can then be used to construct a "triangle of momenta" and so angles and this second version can be subsumed under the form of the first by saying "As much goes in as comes out, vectorially". This link between the two versions is easy in the orthodox approach because it takes place in a fixed space-time arena. Such a link has to be in the future for us, when space and time have been constructed. But this link also provides us with a clue; it suggests that it will be through the definition of angles in this way that a space-time arena will be constructed.

A few words of withdrawal of optimistic hopes are in order about the second type. I have argued that the Hamming value of a bit-string is essentially the only norm (that is, the only way of associating a number with a bit-string which satisfies the triangle inequality for the three elements at any discrimination and such that zero is associated only with the null string). This is true enough if you do not limit the meaning of the elastic word "essentially" too much but of course if one defines the norm N(a) of a bit-string $a = (a_1, a_2, \ldots a_n)$ by

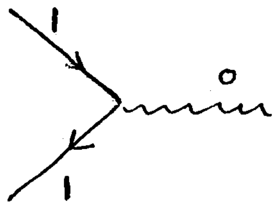
$$N(a) = \sum c_r a_r, \quad \text{where all } c_r > 0$$

or, equally, if a is the row $a = (p,q,r\ldots s)$, then

$$N(a) = c_p + c_q + c_r + \ldots c_s,$$

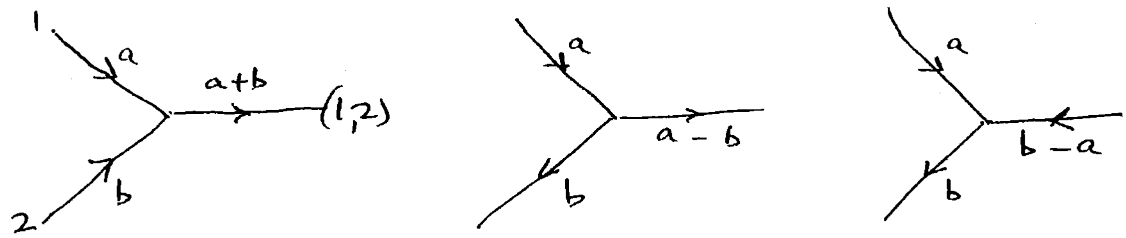
then N is also a norm, because the Hamming norm (for which $c_r = 1$ for all r) is one. For rows, which we have agreed are the right form for the elements of the system, the length is not fixed beforehand and so the c_r must be given by some rule (possibly a recursive one). Different rules will give rise to different sets of angles. So we must bear in mind that the Hamming norm may not be the only one with which we are concerned, but it is likely to be the most important.

Let us turn to my present sticking-point, the need to talk (in making connection with the orthodox approach) about "in" and "out". I can make the difficulties clear by looking at some simple cases. First consider the lowest case possible, below the lowest level of the hierarchy, where bit-strings of length 1 are discriminated and so there is just one vertex, $1 + 1 = 0$ (except for the trivial case of $0 + 0 = 0$ which will recur at every level). If we assume as part of the idea of conservation that the element 1 has sufficient individuality that the same number must be associated with it in both cases, then, since we also have that zero is to be associated with the nul string, the "outs and ins" must be as summarised on the following figure:



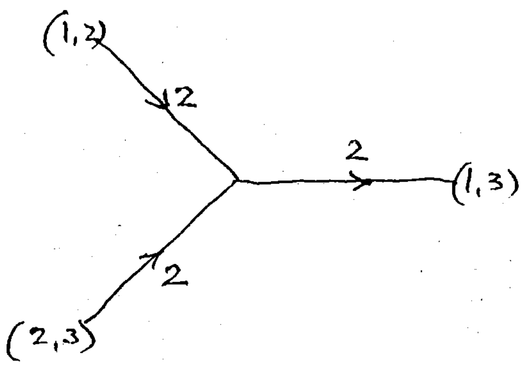
remembering here and later that all arrows are to be understood relatively (they could all reversed without changing anything). I should also warn that these are diagrams; the angles between the lines are of no significance.

Turning to rows of higher order, say of order r, there are then $r^* = 2^r - 1$ elements and so r* vertices of the type just considered, about which no more needs to be said. The number of vertices not involving a zero is evidently $r^*(r^* - 1)/6$. If r = 2, the lowest level of the hierarchy, there is just one such vertex, 1 + 2 = (1,2). What of conservation at this vertex? This is a formal matter only. If we associate a, b, c with 1, 2, (1,2), then either c = a + b or c = a − b (or b − a) with appropriate arrows:



If we fix on a and b, the fact that we have different possible associations for the remaining element shows that the theory has not yet reached the stage when an element can be unambiguously associated with a known physical particle. If you expected that, as I did, then this is a difficulty; but it has to be accepted and so may yet turn into an opportunity.

Something new turns up when r = 3. There are now $3^*(3^* - 1)/6$ = 7 vertices. Three of these are of type 1 + 2 = (1,2), three of type 1 + (2,3) = (1,2,3) and there is no more to be said about these than as above. In fact, we can summarise the discussion for these vertices by saying that the Hamming value will serve as the conserved quantity at all six, and that using it defines a system of arrows. It is quite otherwise with the seventh one:



If one sticks with the Hamming norm, then one is forced to go over to the second type of conservation here, introduce angles and say that two elements of 2 each go in at angles of 120° to the direction of the outward going 2 and to each other. (NOT, let me emphasise, in a particular three-dimensional space, or indeed a two-dimensional one; these are angles from which such a space is later to be constructed.)

It is because of the choice of the Hamming norm that the seventh vertex is the troublesome one, but the Hamming norm is just serving as an indicator here. In fact there can be no assigning of numbers to the seven elements which is conserved at all seven vertices. For suppose there were and to avoid ambiguities in the discussion suppose that at each vertex all three arrows are drawn inwards, so that some changes of sign in the assigned numbers have been made. The conservation conditions are then seven linear equations in the seven numbers assigned. A few moments investigation shows that there is no non-zero solution of these equations over the rationals. (One can show that the determinant does not vanish or one can simply try to solve.) Thus as soon as one gets above the bottom level of the hierarchy angles have to be brought in to save the day on conservation.

Nor should one be misled by the fact that there is only one vertex causing trouble here. If one takes the Hamming norm as an indicator again and considers rows of length n, a simple calculation shows that the number of vertices not forcing angles to be introduced is $\frac{1}{2}(3^n + 1) - 2^n$ (so for $n = 3$, $14 - 8 = 6$) out of a total of $n^*(n^* - 1)/6$ and so the ratio of these, though 6/7 for $n = 3$, is 966/2667 for $n = 7$, the next level of the hierarchy. For large n the ratio is roughly $(3/4)^n$ and so tends to zero; and so for $n = 127$ this ratio is about 1.35×10^{-17} – almost all vertices force angles to be introduced.

To conclude; there is still a great deal to be done. We have to be more explicit about how these considerations fit into the idea of a graded algebra, where the general form of a particle is

$$u = u_3 \oplus u_7 \oplus u_{127} \oplus u_N$$

(and N has been written for the large number), and so a vertex is really a quartet of vertices in which the lowest dimensional one has no angles, but simply conserved quantum numbers, the highest has almost all vertices equipped with angles, and there are still the two intermediate ones to deal with. I am quite at sea over such elementary matters as, how can the different sets of angles produced at different levels be reconciled? But ultimately the big questions are about whether the angles produced can be fitted together to form a space, and whether that space will be three-dimensional.

On General Physical Systems Theories

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"'But in your tractate,' I said, 'that's exactly what you stipulate reality is: a two source hologram.' 'But intellectually thinking it is one thing,' Fat said, 'and finding out it's true is another!'"

Philip K Dick, Valis, Bantam 1981.

Abstract

This paper describes the history of the theoretical background behind Kron's Systems Theory. Traditionally both Kron's network theory and his Method of Tearing, or system decomposition, were justified by appeal to the elements of algebraic topology, or homology theory. It is argued that Kron's work and Jessel and Resconi's General System Logical Theory are both part of a larger general theory of (macroscopic) physical systems, based on Huygens' Principle. We suggest that, seen from this point of view, the success of Kron's Method is due to holographic transformations similar to that described by Bohm as the holomovement.

* * *

Despite widespread lack of recognition Gabriel Kron (1901-1968) was responsible for the ideas behind much of the Engineering Analysis we use today, in particular electric circuit theory (and, by analogy, other sorts of network, such as thermal and fluid flow) and rotating electrical machine theory. He also developed a General Systems Theory of physical systems, based on his circuit theory, with dual pairs of covariant and contravariant (or "across" and "through") variables; and the efficient method of analysing physical systems by splitting them up into a number of unconnected subsystems, solving each one separately, and then recombining them into an (exact) overall solution. The subsystems are connected only to a common boundary layer, or "intersection network". The analysis procedes by calculating the solution at the intersection and then assuming that the solution for any subsystem depends only upon the local boundary conditions and the intersection solution at the subsystem boundary.

Kron's methods were criticised for lack of a theoretical background, despite the fact that they worked. He used non- conventional "tensors-in-the-large" in his circuit theory (actually just matrices with covariant and contravariant indices) by analogy with field theory tensors (in-the-small); and strange singular connection tensors. He was also fond of unsubstantiated "generalisation postulates". The controversy raged until a mathematician called J Paul Roth [2] showed how Kron's circuit theory was based on algebraic topology or Homology Theory (Appendix III). This is the global (in-the-large) theory of connected systems, as opposed to differential topology, which only looks at local (in-the-small) phenomena. Note that the word "homology" means, essentially, analogy. Mathematically Roth's approach was rather unconventional. Looking at the natural (twisted!) isomorphism between the homology sequence of the electrical network (viewed as a multidimensional polyhedron, or complex) and it's dual cohomology sequence, Roth derived Kron's circuit equations (Appendix II). The homology sequence of a complex consists of a series of boundary operators each of which annihilates the previous one (and is thus called an exact sequence). For an electric circuit this exhibits Kirchhoff's laws. Similarly de Rham cohomology includes an exact sequence of differential operators (for example, curl grad = 0, div curl = 0). Roth's approach was validated by Steenrod, one of the founders of homology theory. Later, a Japanese mathematician, S Amari showed that the Method of Tearing could be derived from homology theory (Reference [3]). Homology theory works at a very high level of abstraction, proving the great generality of Kron's approach.

Many other workers contributed to both theory and practice. Jack Lynn [4] at Liverpool University, amongst others, looked at fields as differential forms, a special kind of homology theory. In the seventies his colleague, and my doctoral supervisor, Harry Nicholson [5], at Sheffield, approached the subject using Scattering Theory, utilising the Redheffer star product, and included optimal control systems in the list of systems analysed. My thesis [6] surveyed the work that had been done to this point (1982) and looked in detail at the use of Roth's isomorphism diagram. The networks now covered included electrical, magnetic, thermal, economic, mechanical, fluid flow and control systems, in both their discrete and continuous time forms (see Fig (1)). Each of these is described by a dual pair of variables (eg, voltage and current, prices and commodity flows) whose product form a utility function (electrical energy, money) to be maximised or minimised (or conserved). Fields and waves described by the theory include electromagnetic, sound and mechanical stress, fluid flow, thermal, economic, and gravitational. These complexes (networks) and differential forms (fields) are described algebraically by Singular Homology Theory and de Rham Cohomology Theory respectively. An isomorphism between these (and thus between network theory and field theory) is given by the de Rham Theorem.

Fig(1) Classification of Networks

NETWORK	COVARIANT	CONTRAVARIANT	QUOTIENT	PRODUCT
electrical	voltage	current	impedance	power
magnetic	mmf	magnetic flux	reluctance	energy
thermal	temperature	heat flow	1/conductivity	power
mechanical	stress	strain	stiffness	density
	force	acceleration	mass	power rate
rotational	torque	twist	stiffness	energy
fluidic	pressure	flow rate	resistance	power
economic	prices	commodity flow	distance	money
control systems	costate	state	weighting matrix	disutility
general	potential	flow	impedance	metric

On the practical front the Method of Tearing was widely used on small and medium scale systems until the advent of modern computers. Large scale systems suffered from the problem of storage of the subsystem matrix inverses even with the advent of cheap memory. With recent high speed computers it was found that indirect, iterative (or "brute force") methods would converge in a reasonable time, thus Kron's method lost favour. Indirect methods do, however, suffer from the problem that if a boundary condition is changed, the entire problem has to be resolved; Kron's method has the advantage that not only can boundary conditions but also entire subsystems can be modified without resolving the rest of the system. Further, indirect methods are relatively difficult to parallelise whereas Kron's method is naturally parallel. The author's implementation of Kron's method on a Transputer pipeline is discussed in Ref [7]. I have also recently derived a finite recursive, hierarchical version of Kron's algorithm which should relieve the problem of storing the subsystem inverses.

Kron thought of his theory as a natural philosophy, rather than just a method of analysis. This leads to a view of the Universe as consisting, in an important sense, entirely of boundaries. In Alison Watson's "The Birth of Structure" [8] she considers the property of being a boundary to be basic to the fabric of the universe. "Objects become objects only by virtue of having recognisable boundaries... The state of affairs would then appear as a sequence of derivations: time as derived from spatial relations, and space as derived from relations between and within the boundaries of objects. The necessity of form, as delineated by boundaries, can then be recognised as a prior condition for the arising of space... A boundary, then, taken in the Cartesian sense, is not now the edge of something, but rather that by which we distinguish somethings. The emphasis has shifted from the something to the distinguishing act." Recursive tearing leads to a picture of the world as a sort of infinite, hierarchical foam. Bubbles within bubbles. Everything consists, in this sense, of the set of all its possible hierarchical dissections.

As a theory of physical systems an approach based on homology has a number of disadvantages. First of all homology theory is a very difficult branch of maths. Also it is axiomatic. General Homology Theory, of which Singular and de Rham are special cases, is based on the seven Eilenberg-Steenrod axioms. (Actually all theories based on the axioms are considered to be identical (isomorphic). Other different theories can be generated by dropping or modifying one or more of the axioms.) Whether a theory based on the ES axioms CAN be sensibly called a physical theory is something that needs investigating. We are interested in making an approximation to macroscopic reality just to the right of GST in the spectrum between General Systems Theory and Physics.

I was therefore intrigued when Tony Durham, the journalist, and Peter Marcer of the BCS pointed out to me that the work of Jessel and Resconi called General Systems Logical Theory (GSLT) [9] has much in common with Kron's, yet is based on Huygens' Principle, an eminently physical principle. Further investigation quickly revealed that Kron's Method is based on Huygens' Principle. Kron's intersection network is a Huygens' surface, a hologram-like entity that stores information about its contents. HP in the form that Jessel and Resconi present it (Appendix IV)

$$sF=OP^{-1}([OP,s]F+sS)=OP^{-1}(OPsOP^{-1})S$$

is the continuous version of Kron's equations for a linear discrete system

$$x_i = Z_i^{-1} C_i (Y- +_{j=1}^{n} C_j' Z_j^{-1} C_j)^{-1} (+_{j=1}^{n} C_j' Z_j^{-1} y_j)$$

(Note the importance of the Lie bracket is that it can be seen to vanish everywhere but on the boundary.) This is a satisfying result. It is however interesting that Kron's work and that of Jessel and Resconi are so different as they are both GSTs based on Huygens' Principle. This is because Kron was interested in analysis (in the Engineering sense) whereas Jessel's interest is in Holography, wavefield physics (he was a student of Louis de Broglie) and field reshaping. These are different problems, thus the two GSTs are just parts of a larger whole; the correspondence between the two equations above constitutes the common intersection.

A good approach to making a General Physical Systems Theory would seem to be to start with the work of Mesarovic [10] et al, who defined a General System as a subset S of the Cartesian Product UxY of an input set U and an output set Y. The mapping S:U->Y is equivalent to OPF=S. These sets could be indexed over a real set T representing time. Similarly U and Y could be indexed over the integers for a discrete system, or over polynomials in s or z representing Laplace and z transforms. The difference beween our systems and those of Mesarovic is that they possess a topology. Now extend the index set to allow any connected set for networks or vector space for fields. The sets can be graded (dimensionally indexed) and allow boundary operators. This Homological Systems Theory is the subject of future investigation. The scope of the theory must include analysis and field reshaping as well as traditional subjects such as observability and optimal control.

In terms of the latter Huygens' Principle looks very like being Bellman's Principle of Optimality. It seems to be the crucial principle behind the solution of all boundary value problems. We propose that Bellman's Principle of Optimality or Invariant Embedding and hence Pontryagin's Maximum Principle are special cases of what we will now call the Kron-Huygens' Principle, applied to one dimensional systems. This is very satisfactory from both the GSLT and the homological point of view as both theories have previously tried to incorporate Optimal Control Theory. Bellman's Principle states that "if u* is an optimal control and if x*(t) is the optimal state trajectory corresponding to u* on the interval [t0,t1], then the restriction of u* to any subinterval [s0,s1] of [t0,t1] is an optimal control for the initial pair (x*(s0),s0) and the target set (x*(s1),s1)" or more simply stated "portions of an optimal trajectory are optimal". This Principle holds for any dynamic system dx/dt=f(x,u) and any cost function minJ(x,u). We claim that the pair (x*(s0),x*(s1)) constitutes the Huygens' surface or intersection network.

In a similar way GSLT can be used to describe the process of holography, the subject from which the theory evolved. A hologram is the interference pattern recorded on a photographic plate exposed simultaneously to coherent laser light reflected from an object and incident directly from the original source. The hologram itself is (part of) a Huygen's surface which when illuminated by laser light will reproduce an apparent three dimensional image of the original object. David Bohm [11] considers "the relationship between the transformation E in the illuminated structure and concomitant changes in the hologram which follow these transformations. In the illuminated structure, E can be characterised as a point-to-point correspondence in which any similar locality is transformed as a similar locality. The corresponding change in the hologram is described by E'=ME/M. This is not a correspondence of points in the hologram to each other in which the property of locality of such sets of points would be preserved. Rather, each region is changed in such a way that depends on all other such regions. Nevertheless, the change E' in the hologram evidently determines the change E in the structure that can be seen when the hologram is illuminated with laser light."

The atomic unit, or Elementary Logical System, of GSLT is a commutative diagram of a similarity transform. Jessel and Resconi consider successive transformations represented by a sequence of these diagrams. This is precisely the sort of hierarchical sequence of transformations that Bohm calls an ordering or an enfolding. The ELS can be used to display Kron's Tearing transformation or the holographic process in a natural way. In the Kronian Universe complete information about any particular subsystem is held in its surface, that is in the interface between subsystems. From GSLT we can see that this information is effectively held in holographic form. We don't actually have to tear the system up for this to be true, it is always true everywhere. According to David Bohm, the corollary is also true: complete information about the entire universe is enfolded into every region. Bohm calls this the Implicate Order of the Universe. "...this enfoldment and unfoldment takes place not only in the movement of the electromagnetic field but also in that of other fields, such as the electronic, protonic, sound waves etc. There is already a whole host of such fields that are known, and any number of additional ones, as yet unknown, that may be discovered later. Moreover, the movement is only approximated by the classical concept of fields... more accurately, these fields obey quantum mechanical laws... (which) may only be abstractions from still more general laws, of which only some outlines are now vaguely to be seen. "...we call this totality by the name holomovement." [11]

Finally, a number of questions come to mind regarding Huygens' Principle (1678). Firstly there is the problem of diffraction which was covered in a modification by Fresnel (1815). Will Kron fail for diffraction? Then to what (imaginary) physical systems does Huygens apply? Obviously they have to be connected. The Principle of Local Causes comes to mind. If causes aren't local then surely HP won't work. Kirchhoff showed that Huygens' Principle (for the wave equation) can be derived from the wave equation. What about the diffusion equation? What are the restrictions on OP in the equation OPF=S? In what ways does the Principle of Local Causes or HP limit the kinds of fields we may experience? How does Huygens' Principle look in terms of homology theory?

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Appendix I Kron's Method of Tearing

Over the period from the nineteen twenties to his death in 1968 Gabriel Kron developed a physical philosophy based on his early work in electrical machine theory and electric circuit theory and analysis. This philosophy saw physical systems as being "modellable" as generalised electrical networks which could then be analysed by the Method of Tearing or Diakoptics as he called it. For instance a suspension bridge could be stress analysed as a mechanical network with both inductive and capacitive effects. Continuous systems can be approximated by finite element-like networks which approach exactness as the mesh becomes finer. A physical system here is one whose variables can be classified into contra- and covariant (or "through" and "across") variables such as current and voltage. Their product would be a measure of utility such as energy.

The Method of Tearing is a form of physical analysis by decomposition with the special property that when a system is 'torn up' it must be done by actually removing a layer of components, the intersection network, at the interface between adjacent subsystems. The analysis then procedes by calculating the solution at the intersection and then assuming that the solution for any subsystem only depends upon the local boundary conditions and the intersection solution at the subsystem boundary.

For example for a one dimensional system we could physically decompose into four subsystems like this

```
X1:i:X2:i:X3:i:X4
```

where the X's represent the subsystems and the i's the intersection network. Note that there are four subsystems but only one intersection network. This is made more clear in the two dimensional case where it can be seen that the intersection is connected

```
X:i:X:i:X:i:X
..i...i...i..
iiiiiiiiiiiii
..i...i...i..
X:i:X:i:X:i:X
```

For efficient analysis the size of the intersection network should be minimal, that is a single component layer in a network or an n-1 dimensional surface in an n dimensional system.

For a 1-dimensional linear system the system matrix looks like this

$$\begin{bmatrix} Z1 & & & & C1 \\ & Z2 & & & C2 \\ & & Z3 & & C3 \\ & & & Z4 & C4 \\ C1' & C2' & C3' & C4' & Y \end{bmatrix}$$

where the Zi are the subsystem matrices, Y is the intersection network matrix and the Ci are the connection matrices giving the topology of the interconnections between the subsystems and the intersection. As none of the subsystems have any common boundary the rest of the matrix is null. This can be considered to be no more than a particular reordering of the system matrix induced by the tearing operation.

So we can write the system equations

$$Zx + Cy = b \qquad (1)$$

$$C'x + Yy = c$$

```
where Z is the block diagonal system matrix
      Y is the intersection network system matrix
      C' is the partitioned connection matrix [C1':C2':C3':C4']
      x is the partitioned vector of subsystem solutions
      y is the intersection network solution vector
      b  is  the  partitioned  vector  of   subsystem   boundary
conditions (voltage and current sources)
      c is the vector of intersection network boundary conditions
```

The last equation is the system equation of the intersection network which itself is adjacent to all the subsystems, hence the appearance of Ci' in every term. Note that for a two dimensional system the subsystem matrices Zi are themselves of the form

$$\begin{bmatrix} Z & C \\ C' & Y \end{bmatrix}$$

where Z is block diagonal. This recursive structure extends naturally to multidimensional systems.

Rearranging equations (1) gives the basic equations of diakoptics for a linear system

$$y = (Y - C'Z^{-1}C)^{-1}(c - C'Z^{-1}b) \qquad (3)$$

which gives the intersection vector and

$$x = Z^{-1}(b - Cy) \qquad (4)$$

which gives the subsystem solutions. Note that the equation for y involves only the inversion of a matrix of the order of Y, which can usually be made quite small, and the inversion of Z, which is block diagonal and thus involves only the inversion of (all) the Zi. Equation (3) is often referred to as a projection of the boundary conditions onto the intersection network. The equation for the n subsystem solutions naturally splits into n subsystem equations

$$x_i = Z_i^{-1}(b_i - C_i y) \qquad (5)$$

which as stated depend only on the intersection vector and the local boundary conditions. The intersection vector is given by

$$y = \left(Y - \sum_{j=1}^{n} C_j' Z_j^{-1} C_j\right)^{-1} \left(c - \sum_{k=1}^{n} C_k' Z_k^{-1} b_k\right) \qquad (6)$$

Note that although all of the vectors and matrices given above are assumed to be real, the theory works for any suitable field eg, complex numbers or polynomials. In particular solution of the equations where the field is polynomials in the Laplace transform s, allows a dynamic analysis, a wave theory rather than a field theory. The development of polymorphic programming languages capable of dealing with algorithms of this type could be important in future implementions of this type of algorithm.

Appendix II Kron's Electric circuit Theory

An electrical circuit consists of a one dimensional network through whose branches flow currents and across whose branches exist potentials or voltages responsible for the flows (or vice versa). The flows and potentials are subject to conservation laws (known as Kirchoff's laws) at nodes and around meshes. The branches consist of impedances governing the dynamic relationships (Ohm's law) between the local branch voltages and currents. Typically these impedances are either linear (resistance), first order integration (capacitance) or first order differentiation (inductance). Also distributed around the network are a set of voltage and current generators or sources responsible for initiating and/or sustaining the dynamic evolution of the system variables, voltages and currents. The system equations can be written and solved in two dual forms

$V=ZJ$ or		$J=YV$ or	
$E+e=Z(I+i)$	Ohm	$I+i=Y(E+e)$	(1)
$C'e=0$	Kirchoff	$A'i=0$	(2)
$i=Ci'$		$e=Ae'$	(3)
$E'=C'E$		$I'=A'I$	

where i is the vector of b branch currents
  and I of corresponding generators,
    i' of m mesh currents (not vector transpose),
    e of b branch voltages,
    E of corresponding generators,
    e' of n node-pair voltages
  and Z is the (diagonal) matrix of branch impedances,
    Y is its reciprocal of branch admittances

and C is a bxm matrix of incidence numbers from the set {1,-1,0} depending whether the ith oriented branch is {positively, negatively, not} incident to the jth oriented mesh. In a similar way the A matrix relates mesh and node variables.

It can easily be seen that A'C=0 and that the following diagram, essentially due to Roth, commutes (gives the same answer whichever way you go round a loop)

```
                        C         A'
         0  ->   i'    ->   J   ->    I'    ->  0
                 ^          ^         ^
          /(C'ZC)|         Y|         |A'YA
                 |          |         |
         0  <-   E'    <-   V   <-    e'    <-  0
                        C'        A
               meshes   branches  nodes
dimension        2          1       0
```

Remember that the mappings are into and out of spaces J,V,i' etc and not between variables. The rows of the diagram are known as short exact sequences (because A'C=dd=0 and C'A=bb=0). The upper sequence relates contravariant variables, ie currents of successively lower dimension via boundary operators d2=C and d1=A'. The lower sequence relates covariant variables, ie voltages of increasing dimension via coboundary operators b. The vertical mappings (impedances) are isomorphisms. Multiplying (1) through by (2) and substituting (3) gives the branch currents and voltages in terms of the system sources

$$i=C(C'ZC)^{-1}C'(E-ZI) \qquad e=A(A'YA)^{-1}A'(I-YE) \qquad (4)$$

A great deal of system structure is shown in this diagram which is not apparant from the equations. This is can be seen in my own version of Roth's diagram, Fig (2), which is a sequence of mappings between Venn diagrams in which the internal structure of the spaces is shown explicitly and the mappings can be read as between variables. Note how the solutions (4) can be read off the diagram. Application of the method of tearing to an electrical network decomposes (by reordering) the matrix inversions in equations (4) into the form shown for a 1-dimensional linear system in Appendix I.

This form of Roth's diagram is actually an isomorphism between the chain and cochain complexes of an electrical network. Now this only works because the so called homology groups of an electrical network vanish (that is, because all loops in the network are boundaries of meshes, ie there are no holes) and does not work in general eg, for a torn network. To get round this we must, like Roth, invoke homology theory. For an electrical network Roth's diagram is essentially the same whether we use complexes or homology.

Appendix III Homology Theory

Diakoptics has been demonstrated in the linear case, but Kron claimed it to be a general process not restricted to a particular class of system (or at least valid for a very wide class of systems) and indeed made this assumption in many of his analyses. For the lack of an underlying mathematical background his methods were criticised, but they seemed to work. The unstated diakoptical proposition, that "The solution for any subsystem depends only upon the local boundary conditions and the intersection solution at the subsystem boundary", we will call Kron's Principle. Recent implementations of finite element packages, particularly on parallel processors such as the Transputer make such an assumption for field problems in continuous systems. The technique is known in the genre as "substructuring". Its justification by the genre is not known to the author. Solution is generally by iteration, although the author has recently implemented a direct solution on a Transputer array using Kron's method.

In 1955 the mathematician J Paul Roth showed how Kron's electric circuit analysis was related to Homology Theory or Algebraic Topology. Homology Theory describes a multidimensional space as a "chain complex" with sequences of boundary operators, d, in which consecutive pairs of operators annihilate,

$$\xrightarrow{d} C_{r+1} \xrightarrow{d} C_{r} \xrightarrow{d} C_{r-1} \xrightarrow{d}$$

the arrows give mappings between "chains" Cr of successively lower spatial dimensions with dd=0. This is equivalent for an electric network to saying that the boundary of a boundary is zero, that is Kirchhoff's voltage law (C'A=0). Roth showed how, by comparing the homology (voltage) and the dual cohomology (current) sequences of an electric circuit, the solution equations could be read off the resulting commutative diagram, thus introducing the concepts of impedance and energy minimisation into homology for the first time. He also showed how the mapping between the two sequences (impedance) includes a "twisted isomorphism". This isomorphism is the central vertical mapping in my version of Roth's diagram as shown in Fig (2). It can be seen that the kernal of the following mapping at each end of the central isomorphism maps, not to the kernal at the other end, as might be expected, but to the cokernal.

The surface of a given n-dimensional object (chain) is called its boundary. An object whose boundary vanishes is called a cycle. In general the image of a given boundary operator is contained in the kernel of the next, all boundaries are cycles but all cycles are not necessarily boundaries. The equivalence class of n- dimensional cycles mod boundaries is called the n-dimensional homology group. The homology groups of a simply-connected 1- dimensional network such as an electrical network are trivial (zero) - although still interesting, as Roth showed - because all the cycles are boundaries, but for a subnetwork of a torn network this is no longer true. For higher dimensional objects (that is continuous systems) homology essentially tells us how many holes something has in it.

Kron's theory appears to apply equally well to continuous systems and to networks. There is a mathematical justification of this in the de Rham Thereom. The most general representations of fields we deal with in continuous space are called differential forms. They are the objects under integral signs and can be thought of as a generalisation of tensors to n-dimensional space. In the calculus of differential forms the traditional differential operators div, curl and grad in three dimensions are replaced by just one boundary operator d and one coboundary operator b in many dimensions, along with the Hodge star operator * which maps between dual variables eg, d=*b*. The space of differential forms along with their coboundary operators forms a "cochain complex". The homology groups of this cochain complex form the de Rham Cohomology Theory. The de Rham Thereom gives an isomorphism between the de Rham Cohomology and the Singular Homology Theories. Thus any principle in one theory has counterpart in the other. There is an implication here that has relevance to the work of ANPA, that it is impossible to tell whether we inhabit a discrete or a continuous universe from our vantage point, except perhaps if a discrete theory can be found that gives predictions unobtainable from a continuous one (or vice versa!)

Modern Homology Theory is an axiomatic system. The seven Eilenberg-Steenrod axioms define a Homology Theory on a "Sufficient Category", ie. a suitable algebraic structure. Both the classical singular and de Rham theories described above, along with a number of others, satisfy the axioms. A homology theory is considered to be a functor from the axioms into the algebraic structure concerned. Thus as all the theories thus far mentioned are isomorphic and constitute the same functor, they are considered to be the same homology theory. (The de Rham Thereom is a proof of this; the proof procedes by invoking yet another theory called sheaf cohomology.) Other theories such as K-theory and cobordism drop one or more of the axioms and thus are truly different theories.

Homology Theory condenses information about a topological space X, or (in a similar way) about other mathematical objects, into a family of Abelian groups

$$H_0(X), \ldots, H_n(X), \ldots,$$

(where the suffices correspond to spatial dimensionality) and about continuous mappings f:X->Y into a family of group homomorphisms

$$f_n : H_n(X) \to H_n(Y).$$

Thus an algebraic picture of the topological space is given by its homology groups and their homomorphisms, from which properties of the space can often be found. One means of finding the homology groups of a topological space is by calculating complexes.

Let the free Abelian group generated by a collection of n- dimensional 'singular' simplices (simple polyhedra) in X be Cn, then the boundary of each simplex consists of a collection of n-1 dimensional simplices and we can write the homomorphism

$$d : C_n \to C_{n-1}$$

where d assigns to each simplex S the alternating sum dS of its boundary simplices. Each element of the group is given by a linear sum over the simplices which constitutes its basis. The coefficients of this sum determine the voltages across (or currents flowing in) the branches of a network. Thus if the system is dynamic the coefficients may be polynomials (in the Laplace transform). This defines a "simplicial complex" C

$$0 \leftarrow C_0 \overset{d}{\leftarrow} C_1 \overset{d}{\leftarrow} C_2 \overset{d}{\leftarrow} C_3 \overset{d}{\leftarrow} \dots$$

in which dd=0 so that the image dC(n+1) of d is a SUBGROUP of the kernal Zn of d:Cn->C(n-1). The nth homology group of the complex C and hence of the space X is the quotient group (equivalence class)

$$H_n(C) = Z_n / dC_{n+1}.$$

In the formal theory the homology groups are defined as members of a "long exact sequence", that is a sequence of groups and homomorphisms such that the image of one homomorphism is EQUAL to the kernal of the next. It can be shown that for chain complexes the homology groups are unique and as shown above. Thus in our arbitrary (eg maybe torn) network it no longer matters that all loops (cycles) are not meshes, as Roth's diagram drawn using homology sequences rather than complexes is exact. There is a connection between smooth manifolds and networks at this stage of the argument. In any given space there is a limit to the complexity of the "triangularisation" C of X necessary to fully describe the space by Hn(C). For instance imagine covering the surface of a torus with a collection of nonoverlapping triangles. When do we have enough triangles to represent the topological properties of the torus?

Given any group G there is a corresponding complex whose homology is appropriate to the group. Each type of algebraic system has homomorphisms of appropriate type associated with it, and under composition of homomorphisms these systems constitute a category. Now, if A and B are abelian groups, the set Hom(B,A) of all group homomorphisms f:B->A is also an abelian group. For B fixed, it is a covariant functor on the category of all abelian groups A, each homomorphism a:A->A' induces the map a*:Hom(B,A)->Hom(B,A') which carries each f into its composite af with f thus a*:f->af. For A fixed Hom is contravariant. Each b:B'->B induces the map b*:f->fb or b*:Hom(B,A)->Hom(B',A) in the opposite direction.

Thus Hom(?,A) applied to a complex

$$0 \leftarrow C_0 \xleftarrow{d} C_1 \xleftarrow{d} C_2 \xleftarrow{d} C_3 \xleftarrow{d} \dots$$

induces a (cochain) complex

$$0 \to Hom(C_0,A) \xrightarrow{d^*} Hom(C_1,A) \xrightarrow{d^*} Hom(C_2,A) \xrightarrow{d^*} \dots$$

with the arrows in the opposite direction. So if a(i) is a function in Hom(C(i),A) then a(i)d:C(i+1)->A is a function in Hom(C(i+1),A) and the adjoint d*:a(i)->a(i+1) where d* means postmultiply by d or in the matrix case where the contravariant variables are treated as row vectors, the conjugate transpose of d. Here the factor group ker(d*)/im(d*) is the nth cohomology group Hn(C,A) of C with coefficients A.

In an electrical theory A is a function representing energy. If the covariant variables are voltages then Hom(?,A) contains functions of the form "multiply by an electric current". Thus a cochain complex is a complex of current flows. Note the historical anomoly in terminology in that a cochain complex, and hence cohomology, is generated by a contravariant functor. Further associating this with the contravariant variable set (currents rather than voltages) is an arbitrary convention that we use here. (The same arbitrariness is found when we try to assign cause and effect to the two dual variable sets.) By analogy with the electrical case we can use complexes to model mechanical forces and displacements (A is mechanical energy) or even economic prices and commodity flows (A is money).

The homology of a complex C determines its cohomology (up to a group extension), provided each Cq is a free abelian group. We don't need to invoke duality to define a cochain complex; the formal definition is as follows. A cochain complex C* consists of a sequence of K-modules and homomorphisms

$$\dots \to C^{q-1} \to C^{q} \to C^{q+1} \to \dots$$

defined for all integers q such that at each stage the image of a given homomorphism is CONTAINED in the kernel of the next. The homomorphism

$$d^* = b = b^q : C^q \to C^{q+1}$$

is called the qth coboundary operator. bb=0 (Kirchoff's current law, A'C=0) but the sequence is not long exact in general. Only the homology sequences are exact. The kernel Zq(C*) of bq is the module of qth degree cocycles of the cochain complex C*, and the image Bq(C*) of b is the module of qth degree coboundaries. The qth cohomology module Hq(C*) is the quotient module

$$H^q(C^*) = Z^q(C^*)/B^q(C^*) = ker(d^q)/im(d^{q-1}).$$

It can be seen that what Roth did in the construction of his diagram was to explicitly demonstrate the isomorphism between the homology and cohomology sequences of an electric circuit modelled as a chain complex.

We now describe briefly how Shun-ichi Amari [4] used these concepts to lay down a mathematical justification for Kron's method based on Roth's ideas. We dissect a 2-dimensional complex (electrical network) X into two parts X0 and X1 where X1 is closed, that is for any chain in X1 all its faces are also in X1. Conversely X0 can be seen to be open, any mesh (branch) having at least a branch (node) in X0 must itself be in X0. Neither X0 nor

X1 need be connected but for diakoptics X0 (the intersection network) will be connected and of minimal size to split X1 up into a number of disconnected closed subsystems. (In Onodera's dual codiakoptics X1 is the intersection network and dissects X0.)

Now define the four projection and injection operations

$$p_j : C \to Cj \text{ and } i_j : Cj \to C \quad j=0,1$$

where Cj is a chain in Xj. Amari states that "the following two relations are easily proved from the facts that X=X0+X1 (all simplices must either be in X0 or X1 but not both) and each Xi is the complement of the other.

$$i_0 p_0 + i_1 p_1 = 1 \text{ and } p_j i_k = d_{jk}$$

where d_{jk} is Kronecker's delta.

Thereom. No informations are lost (sic) by operating i0p0+i1p1 on a chain of X, that is, by projecting a chain of X into X1 and X0 respectively and then gathering them by the injections from both X0 and X1. This theorem shows the validity of using the dissection processes or diakoptics."[3]

Now define the boundary and coboundary operators

$$d_j = p_j d i_j \text{ and } b_j = p_j b i_j$$

on Cj. Since X1 is closed di1C1 is always on X1 only so that

$$p_0 d i_1 = 0 \text{ and } p_1 b i_0 = 0.$$

But X0 is not closed therefore di0C0 may have an X1 part so we define

$$p_1 d i_0 = d_{10} : C0^r \to C1^{r-1}$$

$$p_0 b i_1 = b_{01} : C1^r \to C0^{r+1}$$

Amari shows that if X is an electrical network with the following two restrictions (1) there are no mutual couplings (inductance) between X0 and X1 (2) all current sources I1 lie in the X1 part only and all voltage sources E0 lie in the X0 part only, then the equations of diakoptics (and codiakoptics) can be written

$$\begin{bmatrix} d_1 y_1 b_1 & d_{10} d_0 \\ b_{01} b_1 & b_0 z_0 d_0 \end{bmatrix} \begin{bmatrix} e_1 \\ i_0 \end{bmatrix} = \begin{bmatrix} I_1 \\ E_0 \end{bmatrix}$$

where $e_1 = C1_0$ and $i_0 = C0^2$

are unknown vectors of node voltages and mesh currents

and $I_1 = d_1 C1^1$ and $E_0 = b_0 C0_1$

are vectors (strictly linear sums over the basis simplices) of impressed currents and voltages respectively. The equations can be written

$$\begin{bmatrix} Y & C \\ -C' & Z \end{bmatrix} \begin{bmatrix} e \\ i \end{bmatrix} = \begin{bmatrix} I \\ E \end{bmatrix}$$

which is essentially the form given in the previous section.

Appendix IV General System Logical Theory

Recently Maurice Jessel in France and Germano Resconi in Italy have developed a General System Logical Theory (GSLT), based on Jessel's Theory of Secondary Sources and Resconi's Logical Systems Theory. This is claimed to be a generalisation of the General Systems theory of Mesarovic et al. Jessel's original work in Holophony (three dimensional sound placement) had many practical applications such as the development of Active Noise Control. The theory of secondary sources also applies to Holography; the combined field is called Holochory.

The starting point is Huygens' Principle which can be stated "the perturbation that goes out (or in) through a closed surface S that contains (excludes) a wave or field source is identical to the perturbation that can be obtained by cutting off the source and replacing it by appropriate sources distributed on the surface S".

Consider the wave equation

$$OP\ F = Sor$$

where OP is a differential operator such as the Laplacian, but possibly with a time derivative, F is a field distribution in space and Sor is a set of (original) sources, eg charge. It should be noted that this is the continuous (distributed) version of the basic system equation that diakoptics tries to solve. Kron's aim is a fast solution of the equation

$$F = OP^{-1}\ Sor$$

Jessel and Resconi are more interested in changing the form of the field F by modifying the sources. They consider the identity

$$OP(sF) = sOPF - [OP,s]F$$

where the last term is the Lie bracket (OPs-sOP)F. The scalar space function s is a field modifier. For instance it can be used to define a Huygens' surface S by setting its value to 1 outside the surface and 0 inside the surface. Its value on the surface is undefined but in general may be complex. As s is a constant scalar everywhere else the Lie bracket vanishes everywhere but on the surface. The first term sOPF = sSor simply defines any sources that are not inside the surface. Thus the Lie bracket gives the secondary sources required to reproduce that element of the field not produced by any sources external to the Huygens' surface and we write

$$Ssor = [OP,s]F$$

and $$OP(sF) = s\ Sor + Ssor.$$

If the sources are all inside the surface then the first term vanishes because either s or Sor is always zero. This is Huygens' Principle.

Jessel and Resconi claim that a change in the field F due to the modifier s induces a corresponding "anticausal" change s' in the sources. They show this on the commutative diagram

```
          OP
     sF   ->  sSor+Ssor

    s ↑         ↑ s'=s+[OP,s]OP

     F    ->  Sor
          OP
```

and $s' = OP\,s\,OP^{-1} = s+[OP,s]OP^{-1}$.

They call a diagram of this form an ELS or Elementary Logical System. s and s' are said to be similar when there exists a 1:1 relation OP such that the diagram commutes. For instance if OP is a matrix it must not be singular. The domain and codomain of s (and s') must be of the same type. Bertrand Russell [11] saw similarity as a very basic concept and noted that "when two relations are similar, they share all properties that do not depend on the actual terms in their fields... Even statements involving the actual terms of the field of a relation, though they may not be true as they stand when applied to a similar relation, will always be capable of translation into statements that are analogous." Thus similarity is not unlike isomorphism which in turn is a special case of exactness.

A Logical System is a sequence of such ELS as shown below

```
                  OP2    OP1
     Fn ... F3 ->  F2 ->  F1

    sn ↑   s3 ↑   s2 ↑   s1 ↑

     Fn ... F3 ->  F2 ->  F1
                  OP2    OP1
```

Thus s1 is similar to s2 and hence s3 and so on. Structures like this also crop up when we investigate multilevel hierarchical tearing. Note that if OP(i) is a boundary operator then the horizontal sequence is a chain complex (and the diagram is a chain mapping). Note also that this is precisely the sort of hierarchical sequence of transformations that David Bohm calls an ordering or enfolding.

For example consider the electrostatic equation

$$\mathrm{div}\ D = \rho$$

where the charges, rho are contained within a surface defined by s then the secondary sources to reproduce the field outside the surface are given by

$$\begin{aligned} Ssor &= [\mathrm{div}, s]D \\ &= \mathrm{div}(sD) - s\,\mathrm{div}\,D \\ &= D.\mathrm{div}(s) \end{aligned}$$

and $sD = \mathrm{div}^{-1}\ (\mathrm{div}^{-1}\ \rho).\mathrm{div}(s)$.

or

$$\begin{array}{ccc} sD & \xrightarrow{div} & D.div(s) \\ s\uparrow & & \uparrow div^{-1}(.).div(s) \\ D & \xrightarrow[div]{} & rho \end{array}$$

which gives the field in the region outside the surface in terms of secondary sources on the surface which in their turn are calculated from the original sources inside the surface. Note how similar this is to Kron's procedure. We claim here that the intersection network of Kron's Method of Tearing is in fact a Huygens' surface. The ELS for Kron's Method looks like this

$$\begin{array}{ccc} x & \xrightarrow{Z} & b-Cy \\ [I\ 0]\uparrow & & \uparrow [I\ 0]-C(C'Z^{-1}C-Y)^{-1}[C'Z^{-1}\ -I] \\ \begin{bmatrix} x \\ y \end{bmatrix} & \xrightarrow[\begin{bmatrix} Z & C \\ C' & Y \end{bmatrix}]{} & \begin{bmatrix} b \\ c \end{bmatrix} \end{array}$$

If it is required only to solve the ith system then s will take the special form

[0 ... 0 I 0 ... 0]

where the I is in the ith position. The intersection vector is a holographic field containing all the information from the boundary conditions. The holographic form is easier to visualise if a dynamic system is considered.

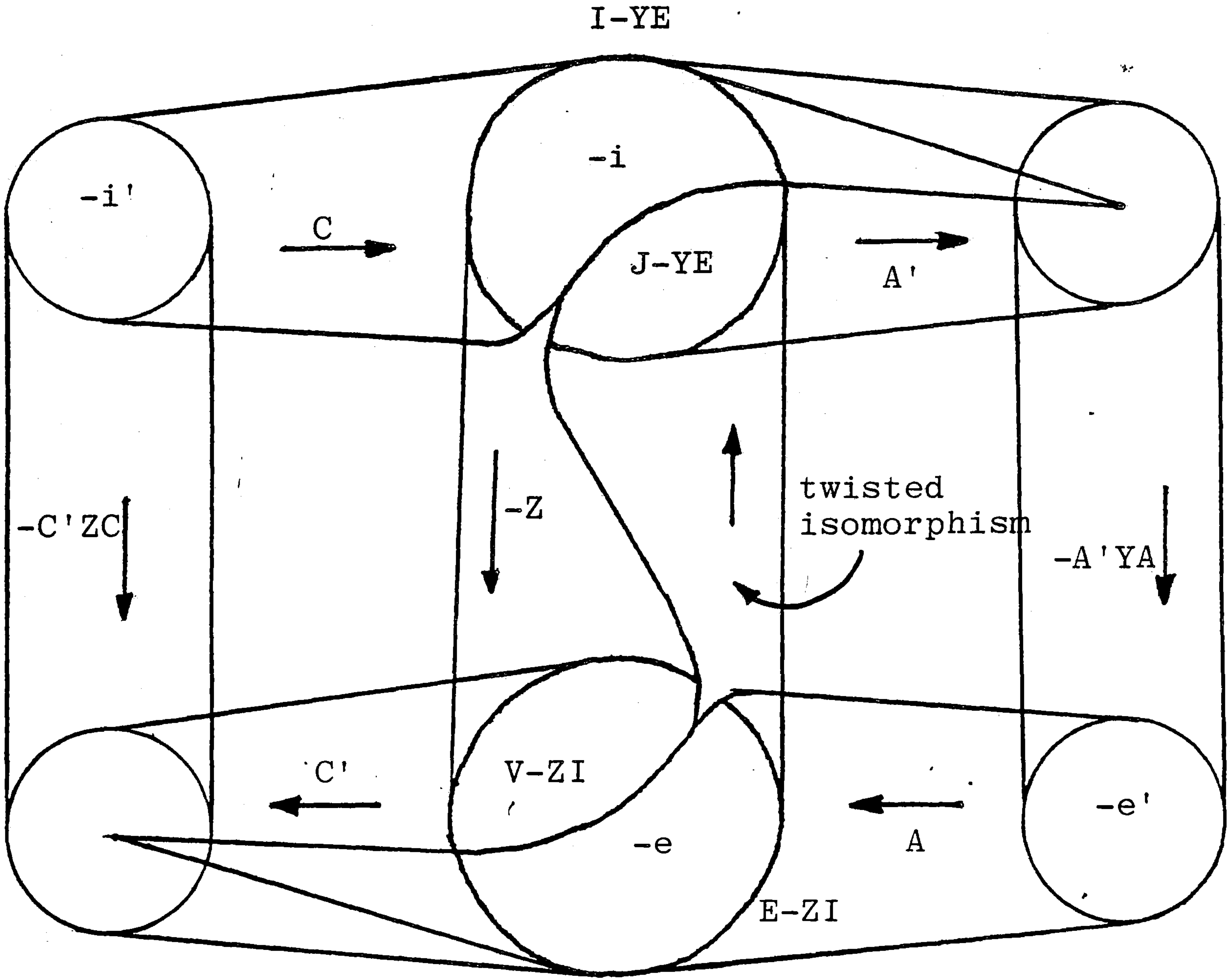
We will now compare GSLT with Kron's method. Consider the Huygens' surface described above enclosing a set of primary sources which can be modelled by secondary sources on the surface. Now reverse the situation. Consider an observer inside a Huygens' surface observing a set of primary sources outside the surface. HP still says that the set can be replaced by a set of secondary sources on the surface. Now consider an arbitrary n- dimensional boundary value problem (maybe with holes) with all the sources on the surface. Tear the system into two subsystems (1) entirely within the surface of the system and thus nowhere touching the boundary and (2) entirely surrounding the first system and thus adjacent to the entire boundary. Now an observer within subsystem (1) is exactly in the situation he was in before. The boundary sources can be entirely replaced by a set of secondary sources on the intersection network and the solution in subsystem (1) given by the equation

$$sF = OP^{-1} (OP s OP^{-1}) S.$$

s=1 inside the Huygens' surface and s=0 everywhere outside. For a discretised system putting b1=0 (subsystem (1) has no primary sources) and c=0 (the intersection network has no primary sources) Kron's equations give

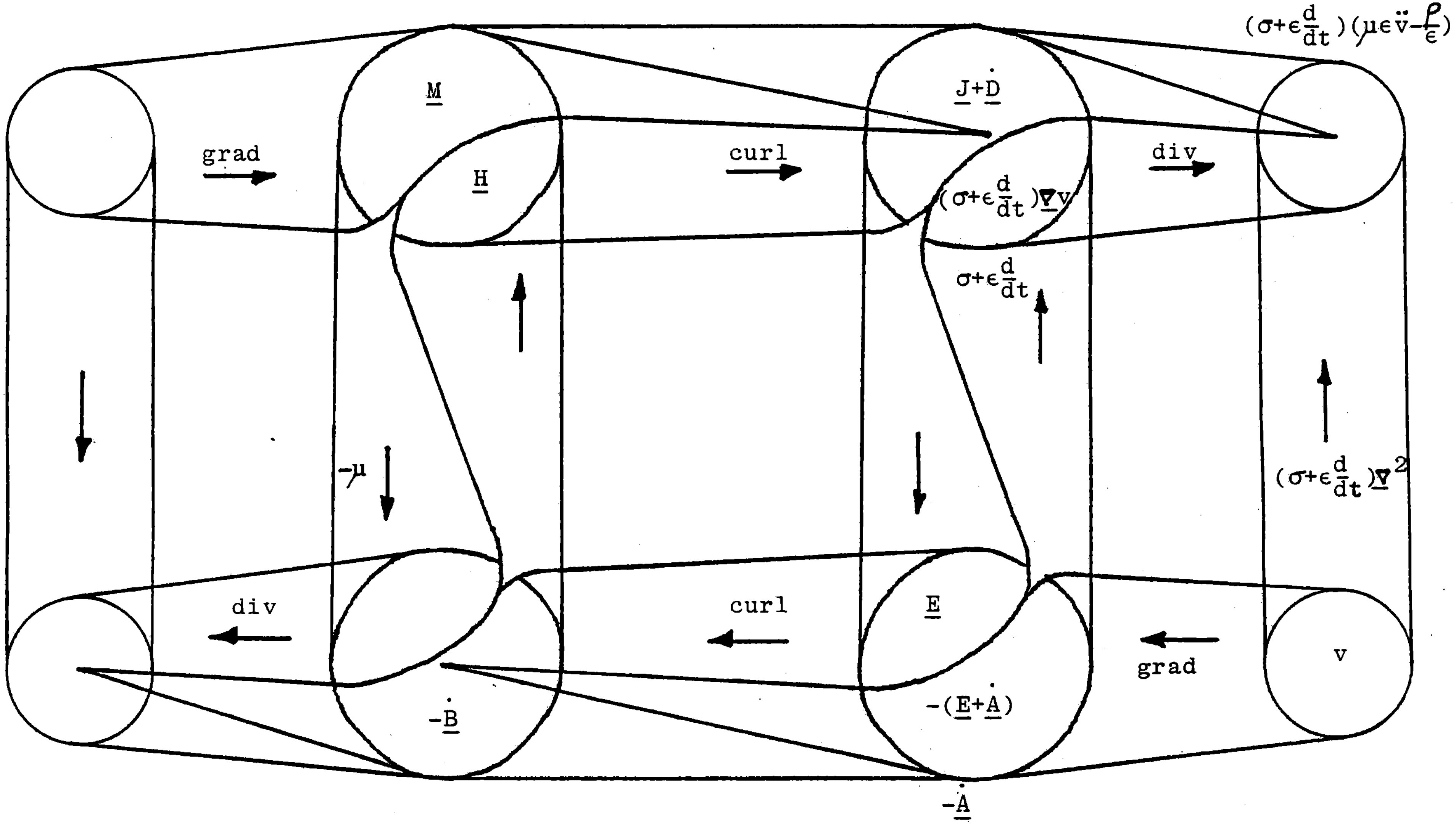
$$x_1 = Z_1^{-1} C_1 \left(\sum_{j=1}^{n} C_j' Z_j^{-1} C_j - Y \right)^{-1} C_2' Z_2^{-1} b_2 .$$

The correspondence between the two equations is clear. S is b2. The last OP is Z2, ie OP on system 2. C2' does the same job as s, ie restricts the solution to the inner region. The brackets (notice the inner structure absent from GSLT) in Kron's equation have the same effect as the middle OP in Jessel's equation, ie to calculate the secondary sources producing the restricted field, and the final OP (Z1) calculates the solution for the inner region. It can be seen that this correspondence extends naturally to multidimensional systems torn into many subsystems. If we take Amari's approach and take X0 to be connected and minimal and splitting system (1) up into a number of unconnected closed subsystems X1, then X0 must be the union of system (2) and the intersection network. Therefore for X0 to be minimal system (2) must be vanishingly small and in fact consists of a thin layer just inside the boundary. Thus what we called Kron's Principle, earlier, is Gabor's version of Huygens' Principle applied to a discrete system. This can be seen to be a consequence of de Rham's Thereom. There is much scope for further work investigating the correspondence between these two, discrete and continuous, algebras.



Fig(2) Roth's diagram for the electrical network problem

Fig (3) Extended Roth's diagram for Maxwell's equations



GEODESIC COMPUTATION

ABSTRACT

A previous conjecture that a process of computation is immanent in the Universe, and that it depends entirely on the principle of least action, is further developed against a new background concept that uniqueness is a necessary condition for existence. It is first asserted that distance, as illustrated by a procedure for solution of the shortest (static) network problem, implies a first-approximation state of latent minimal energy when applied to populations of discrete objects under dynamic conditions in the simple case; extension to a system of contiguous triangles leads to a concept of minimal energy networks under a condition of forced phase alternation producing strings, ribbons and braids. A system of minimal-energy topology and geometry, (including knot theory), is outlined, leading to initial definitions of line segment generators, minimal surfaces and volumes, and a minimal code indicating the existence of a dynamic symmetry common to the chemical elements, the genetic code, and the functions of the animal central nervous system. A general extension to the four-tier hierarchy is suggested.

1. COMPLEMENTARITY OF FIELDS AND OBJECTS

In theoretical physics, contemporary wisdom holds that there are four fundamental forces, that these forces generate fields, and that objects move under the influence of such fields. Where a lay view on such matters exists, it is usually that fields are qualitatively homogeneous, since they cannot be distinguished by the senses; they are all taken as hidden and mysterious. On the other hand, the lay view of objects is that they are directly observable in the real world, and readily distinguished by their heterogeneity.

Now, Einstein's famous equation $E = Mc^2$ indicates that energy (= fields) and mass (= objects) are related, and that this relationship is mediated by the velocity of light in some way. It therefore seems appropriate to seek the origin of heterogeneity in the properties of *fields*, rather than in the diversity found among objects and, if possible, to link it with the concept, generally accepted among scientists, of the degradation of energy.

It is therefore hypothesized that:

1. Mass has its origin in fields.

2. The heterogeneity of matter has its origin in the heterogeneity of fields.

3. The specific expression of heterogeneity in matter, (e.g., the chemical elements), and, therefore, among material objects, is a consequence of the evolution of minimal-energy pathways among fields consistent with the degradation of energy (= thermodynamics) and the principle of least action (= nuclear physics).

4. Fields and objects are complementary to one another.

5. The relationship between fields and objects is expressible in terms of information theory. This implies that a process of computation is immanent in the Universe.

6. That the proper measure of universal information is not the multiply connected system of (0,1) binary arithmetic but the simply connected Gray, (or reflected), binary code (1).

2. UNIQUENESS AS NECESSARY CONDITION FOR EXISTENCE

Electrons are known to exhibit both wave-like and particle-like properties. In this respect, they could be said to represent a stable transition state between that of a field and that of matter. It is also known that a simple collision between an electron and a positron results in mutual annihilation and the release of electromagnetic (EM) energy in another form. If both these "particles" are really only "bound" energy, such events become readily explicable on the basis of their coming into exact antiphase.

The converse of this situation would be that of "particles" which are in *precisely the same phase.* But this would result in the appearance of giant fields by unlimited doubling, a situation at variance with experience. By analogy, all snowflakes have the same hexagonal form, but all within a limited range of sizes. Then again, in spite of the "macroscopic" hexagonal symmetry, no two snow flakes with exactly the same microscopic form had been observed until very recently. Similarly, computer modelling has indicated that the odds against two snowflakes being exactly alike are extremely high. In effect, snowflakes epitomize the fundamental combined paradox of the self-similarity of fractals and the unexpected appearance of chaos in apparently stable dynamic systems.

This paradox could, however, be resolved by taking account of the effects of quantized phase (2) in those cases where multiple phase-locked loops come into existence. Under such a condition, variety on the microscopic scale could be attributed to the existence of multiple phase-locked loops nested inside other, larger, phase-locked loops governing the macroscopic form.

A concept of quantized and nested phase-locked loops is therefore invoked as an overall mechanism by which fields and objects ultimately settle into minimal-energy configurations under a process often referred to as *annealing* (i.e., degradation of energy over time). It is then necessary to invoke a concept of *uniqueness as necessity for existence* in order to explain why this minimising process does not go to completion, whatever state that may be. A conjecture concerning an ultimate state of the Universe can, in fact, be made and, for this reason, the above phrase is widened to *aperiodicity as necessity for existence over cosmological time.* In summary, bulk matter is both oscillatory and aperiodic.

3. FIRST APPROXIMATION MINIMAL ENERGY NETS

The dictionary definition of geodesic is *the least distance between two points.* That is to say, a straight line in Euclidean geometry or a great circle arc in non-Euclidean. In what follows, **least distance** will be taken to imply **least energy expenditure.**

A simple geometrical method exists (3) for finding, (by construction in plane geometry in the simple case), the shortest network connecting three points. This is expressed as a point internal to the triangle formed by the three points as vertices and is known as Steiner's point. A property of this point is that line segments drawn from it to the vertices form a system of three 120° angles.

If taken three at a time, systems of points distributed at random could, therefore, be regarded as a network of contiguous triangles, each with a notional Steiner point. The network would then be endowed with an additional virtual minimal energy network to which the static points, (representing objects possessing potential energy), would move if they were suddenly to acquire translational energy in the absence of constraints. But this says nothing of the temporal or spatial order in which the objects would move.

A system of triangles in the Euclidean plane is, however, highly unstable. A single triangle is able to resist forces acting in its plane tending to extend or compress any of its sides but is weak in relation to torsional forces propagated through the plane. This is confirmed by the fact that the sum of the internal angles of a Euclidean triangle is 180°, the same as that for the twist in a Möbius band.

It is, however, hypothesized that a discrete object having a velocity higher than the average for the population of objects forming its local environment would reach its nearest Steiner point before other objects having lower velocities and greater distances to travel. Objects would, therefore, tend to form chains in decreasing order of their velocity-inverse distance products. In the absence of collisions, the trajectories of such objects would become interwoven.

Systems consisting only of identical objects, e.g., monatomic gas atoms, would re-disperse under conditions of collision. Chemical reaction would, however, be possible in heterogeneous systems, confirming the significance of heterogeneity.

It is then supposed that, in a heterogeneous sytem, those particles having the highest velocity (= energy, = temperature, = frequency, = probability) constitute phase one of a biphasic system. This may be regarded as a Hamiltonian partition function such that:

$$H \text{ (total Hamiltonian)} = H_0 + V$$

where H_0 is the unperturbed Hamiltonian, V is a "noise" factor, and H_0 and V are interchanged in alternate phases. (All is flux!)

Clearly, determining the combinatorics of a large collection of objects sorted in decreasing order of their velocities would be a daunting mathematical task, even as a first step in studying the possible structures which might later arise. It is, therefore, necessary to resort to modelling and, for this, consideration of a system of soap bubbles confined between two glass plates, also bounded peripherally, is instructive. The individual bubbles in real systems lose their spherical shape on compression and take up polygonal form over a limited range of sizes; the resulting network is stable in the absence of disturbances: However, the whole system adjusts itself rapidly and automatically if air is gently withdrawn from one of the bubbles. Clearly, the information required to effect this change is propagated throughout the entire system; it does not need "programming". Under the condition of partition by alternation hypothesized, this information is coded in simply connected binary (Gray code).

The individual bubbles in such a system may also be considered to be domains connected by a network of virtual minimal energy pathways, i.e., Hamiltonian pathways. This is confirmed by the technique for defining configuration space obstacles (4), in which the sides of, say, a triangle and a quadrilateral are drawn as sequences of vectors "walked" in opposite directions, all re-drawn from a common origin, then re-constructed into a new polygon by taking the vectors in a combined sequence, (clockwise or anticlockwise), and using them to construct a new polygon as the configuration space obstacle; its sequence of vectors is largely drawn alternately from the old sets.

It is therefore supposed that a population of discrete objects tends towards a minimal energy configuration over time, i.e., it "crystallizes." Face centred cubic is said to be the "densest" crystal symmetry (5); formal proof appears to be straightforward.

4. SOAP BUBBLE DYNAMICS AND BACON'S BILATERAL CIPHER

A soap bubble expands and contracts so that its external and internal air pressures are continuously equilibrated. At the simplest level, this involves the exchange of a single quantum of energy, equivalent also to the exchange of a single bit of information This can be expressed as the bilinear function $E = (n-1)/(n+1)$, where E is the energy exchange and n is a partition function of soap molecules. The direction of exchange is initially governed by a convention so that, for example, $(n-1)$ applies to the inner surface of the soap film on rearrangment of a molecule to the outer surface, i.e., $(n+1)$.

Now, the process of exchange of a single bit of information between the inner and outer surfaces of a soap bubble is equivalent to the binary symmetric channel (Fig. 1a) and can further be represented in terms of Bacon's bilateral cipher (6, Fig.1b). In this, single cleartext characters written round the outside of a circle are encoded as a sequence of binary characters written round the inside; groups of four are given in the example. This can be also be written in spiral form (Fig. 1c), when it becomes apparent that it can be used to define the cells of a braid made of a single thread; the sequence of ciphertext groups can be shown to be the same as a special case of Gray, or reflected, binary code (7), and to be similar to the construction of one of Napier's "bones" (8). Since Gray code can be related to Hamiltonian pathways which visit each of a number of points "once and only once", it follows that the overall mechanism is also that of minimal energy.

The groups-of-four sequences can also be related to a *minimal* set of three elements labelled A, B and C, together with a *labelled* empty set, E. In the present instance, however, the empty set is replicated as necessary in all subsets of the power set so as to maintain the same number of labelled elements in each as in the original set. Under this condition, ordinary numbers base ten are generated by a process of omission of labelled elements. This gives eight sets with a total of thirty-two elements which can be related to the spatiotemporal encoding of a single cycle of EM radiation.

When all thirty-two elements are laid out as a 4 column x 8 row matrix, it is then seen that a continuous loop can be formed by sliding alternate columns in oppposite directions. This generates obligate pairs of cells, as for the nucleotide bases of the genetic code. The code can also be modelled as spirals of 45/90/45° prisms.

5. SPATIOTEMPORAL ENCODING OF QUANTIZED PHASE

The left hand matrix of Fig. 2a represents a tube of unit square section, four units deep, when opened flat. The straight line represents a single, spiral, continuous incident EM wave, i.e., a "ray" or soliton, propagated as IR along the central axis. Conceptually, the field vectors, (E & M), of IR contact only the midpoints of the top, right, bottom and left sides of the cells of the assembled tube at times T0, T1, T2 and T3 corresponding to spatial coordinates S0, S1, S2 and S3 under a condition of diffraction. The diagrams below the matrix represent the field vectors for a clockwise rotation from the point of view of an observer looking into the tube, the far end of which is closed and from which the soliton is reflected. The right hand matrix is the reflected ray, (RR). (Visualizing the matrix cells as rotating makes IR appear rectilinear.) The overall matrix corresponds with the sixteen ways in which EM radiation can interact with matter.

Now, the E vector is unaffected under a condition of reflection; the M vector is delayed by180°. Reflection therefore creates a virtual image in the normal way. However, M cannot continue its normal journey in space and is turned back into the tube. In doing so, it effectively suffers a 90° loss of "space". RR therefore effectively lags IR by 270°, i.e., IR and RR come into phase quadrature. Under a condition of propagation in free space, (i.e., without local inductive or capacitative loads), the energy vectors (bisectors of E and M) will then add, as in the bottom diagram. Assuming the condition applies equally to anti-clockwise rotation, it is then seen that the computational EXOR operator can be generated among a population of many solitons.

This assessment is of significance in terms of relativity since IR and RR are not in the same reference frame: RR effectively has its origin in a secondary source, even though it lags IR by 270°.

The significance in the present instance is that the extent of phase quadrature addition is dependent on the length of an unobstructed axial pathway. An appropriate model is that of diffraction/reflection in a crystal of sodium chloride. If diffraction takes place on, say, sodium atoms, then omission of axial chlorine atoms allows reflections equivalent to a series of ordinary numbers. Sodium chloride crystallizes in the face centred cubic symmetry. (Inclusion of the EXOR function allows automatic solution of the "travelling salesman" problem as a system of phase-locked loops.)

6. GENERATION OF LINE SEGMENTS AND RIBBONS

Inspection of the "pure", (i.e., bisecting), energy vectors of IR and RR combined in quadrature in Fig. 2b shows that they form a rotating ribbon twisted to an apparent zero, (when viewed from the side), at the half-wavelength node, (between T1 and T2); the "edges" of the ribbon are, however, drawn from different signals.

The input/output end of the ribbon, (at To,So), can be conceptually "cut free" and joined; the structure so formed can then be shown to be equivalent to a simple trefoil knot (Fig. 3a). The ribbon may be regarded as being generated by a triple oscillator, i.e., a ring circuit in which the phases of two oscillators are inverted in sequence by the action of the third oscillator (Fig. 3b). This then affords the concept of a photon as being a linear entity generated as the antiphase reflection of an oscillation and which is capable of travelling unchanged through a medium such that subsequent interaction with matter causes a second antiphase change. This is then in accordance with the requirements of information theory equivalent, for example, to the transfer of a signal from a radio transmitter to a receiver. The double 180° phase shifts ensure that the phase of the received signal is the same as that of the transmitted signal for whole wavelength pathlengths and explains the presence of a reflective layer in the retina of the eyes of animals. The same mechanism is found in rotary offset printing, and in the cooperativity of enzymes in both anabolic and catabolic pathways in biological systems (Fig. 3c).

The above is then consistent with knot theory. Kauffman (9), for example, describes the principle of the triple oscillator (p. 108), its expression in ribbon form (p. 84), and demonstrates the presence of a Möbius band in the simplices of an octahedron (p. 154); he also relates knot theory to Spencer-Brown's calculus of indications (10).

A trefoil-connected system in which each oscillator has two (e.g., ON/OFF) phases generates a total of six phase states (11), the same number as the sides of a cube. Moreover, a trefoil knot has three binary crossover points which constantly alternate between "OVER" and "UNDER" when traced by a "wave packet", a condition which Kauffman (p. 149) calls a "tyrant" and notes as being "an extraordinarily good communicator". Finally, knots may be represented as systems of mirrors or 45/90/45° prisms, configured as a "butterfly" or a "bivalve mollusc", for example.

7. PHASE STATES OF KNOTS, RIBBONS AND BRAIDS

Knot theory does not usually take account of the length of the thread or string from which the closed loop is formed. Clearly, a condition of minimal energy would require the string to be of minimal length and, in turn, that this would be related to phase alternation in some way. Phase alternation and a minimal energy configuration of a trefoil knot initially of unspecified, but non-minimal, length can, indeed, be demonstrated by simple geometry.

In Fig. 4a, it can be seen that a trefoil knot can be configured so that a notional "wave packet" always passes round the two loops of the circuit in the same direction; both passes have the same, (covariant), phase. In Fig. 4b, it passes round the two loops in opposite directions; the two passes are in (contravariant) antiphase. It is conjectured, therefore, that this may be sufficient to explain the strong and weak nuclear forces, i.e., the configurations are equivalent to gluons and vector bosons, respectively. It is further conjectured that this can be explained by the bilinear function $E = (n-1)/(n+1)$, since numerator and denominator are both even when n is odd, and odd when n is even (obligate pairs). This condition then corresponds to the odd and even harmonics of a sine wave and, therefore, to Fourier generation of square and triangular waves constituting the octahedral/cubic dual of geometry and face centred cubic symmetry of crystals. That is to say, n can be a frequency. To reduce either configuration to minimal length would, of course, entail removing portions of the string equivalent to one or two units (a minimum of half a wavelength) at a time. This corresponds with the previous half-wavelength diffraction/reflection concept of number but does not exclude induction of a phase change, (i.e., a "tyrant"), at any node.

The bilinear function is also the locus of points on a hyperbola in the complex plane projected onto a line in the Euclidean plane. It can, therefore, be used in the definition of number systems which are relatively prime (modulo m) or which are reciprocal (modulo m). Modulo arithmetics are, of course, equivalent to the outputs of oscillators. Taken together, and with other minimal-energy geometries not given in this paper, such conditions would appear to offer the prospect of quantized phase states which are mutually exclusive and, therefore, which are aperiodic relative to each other. This would confirm the hypotheses of Sheldrake and Lovelock (12,13).

8. MINIMAL VOLUMES, THE COUPE du ROI AND SPINNING OSCILLATORS

Matveev and Fomenko (14) have recently demonstrated a minimal topological volume. Their diagram shows nine four-way crossover points connected in a particular configuration.

Their configuration is, however, easily converted into a system of three mutually-orthogonal circles, each of which uses two of the crossovers and two branches of each of the remaining three four-way crossover points; the remaining two of each of the latter can then be connected in various sequences. The picture that emerges for a system of fields is then that of a sphere with six crossover points at maximum separation, configured as phase-locked loops, and each driven by one "wave packet" (=line segment generator); several relative phase relationships are possible.

Now the Hamiltonian path for six points exhibiting maximum separation on the surface of a sphere is equivalent to the edge of the Coupe du Roi (15, Fig. 3d). A simple sequence of cuts and folds reduces this to the equivalent of a quadrant of a circle six layers thick. This can be further reduced, by congruences, to a trefoil knot containing three oscillators, one in each "virtual ring".

It follows, from de Moivre's theorem, that the outputs of these dipolar oscillators can be represented as points equally spaced on each of their respective constituent circles. That is to say, much like the well-known Yin-Yang symbol. It also follows, from the previous arguments, that incessant output of a spinning oscillator will be in Gray code sequences. (N.B. Dipolar = ON/OFF or IN-PHASE/ANTIPHASE, giving six phase states for any triple oscillator.)

If the structure of an object has its origin in such a mechanism, it follows that it can be expressed as a modification of the Galilean coordinates, (x, y, z, ict), to (|w|, x, y, z), where |w| is a fixed number of oscillations, (of known amplitude, frequency and phase), of a (triple) line segment generator, these oscillations being partitioned in a phase-determined manner between the three Cartesian coordinates. If the **orientation and spin** of the object so formed is then **re-defined** as a reference, (i.e., initial), phase state, the result of any subsequent collision with another object similarly defined could, in principle at least, be determined exactly. It would be a spatiotemporal reflection of a minimal energy state of its environment.

9. SYSTEMS OF POINT CHARGES

A formula given (16) for the potential energy of a totally connected system of point charges is:

$$E = 1/2 \sum_{i=1}^{n} Q_i V_i$$

where E is the potential energy, Q is the value of a point charge and V is the effect over network distances of all other charges on Q. The formula is derived by conceptually withdrawing charges from the system one at a time under a condition of zero velocity and acceleration. But $Q_i V_i$ already defines the initial potential energy, and this is reduced to half by the derivation, leaving the fate of the other half unstated or unavailable. It is hypothesized, therefore, that seriatim withdrawal of charges from such a system would result in dynamic rearrangements of the remaining charges towards minimal energy networks. Overall, this is a binary partition of potential energy, half as dissipation to "infinity" and half as "mass" (= gravity).

Bearing in mind the previous arguments concerning minimal codings as obligate pairs, a diagram based on this partition and including all four (n, n+1, n+2, n+3) families of radioactive elements can be constructed (Fig. 5). This also corresponds to representation of the properties of a chemical element as an Argand diagram, implying similarity to EM radiation. On this basis, an alpha particle could be regarded as an inner unit of stability based on obligate pairs of neutrons and protons. By assuming that alpha particles are emitted in alternate phases, Fig. 5 can readily be converted to the same double helix structure as the genetic code (DNA) and to a Cockcroft and Walton voltage doubler or, better still, to a double-sided Marx voltage multiplier (17). It is therefore supposed that the structure and stability of the elements has its origin in the same phase conjugate hologram structure previously proposed for both DNA and the animal central nervous system (18). The manifestation of this structure in the generality of objects can then be accounted for by the four spatial configurations found at an intersection of two EM waves at a single node as basis for the formation of closed, i.e., phase-locked, loops. The properties of such loops would be governed by frequencies in descending order, (as expression of the degradation of energy), these frequencies also being relatively prime (modulo m) and reciprocal (modulo m) as mechanisms making them relatively unique (aperiodic).

10. HYPERBOLIC SYSTEMS

Systems of constant-frequency, constant relative phase and static relative location radio transmitters have been in use as navigational aids for half a century or more. In their early versions, (e.g., GEE, Loran), ordinary maps were overprinted with families of hyperbolas and used in conjunction with cathode ray tube displays for the purpose of providing navigational "fixes" for aircraft; frequency modulated systems are used for the determination of altitude.

It should now be evident that, if mass is derived from oscillatory and spinning energy systems, then atoms should emit radio signals, albeit weaker and more complex those of man-made origin. The following is an example of what then emerges:

The usual formula for a rectangular hyperbola referred to its asymptotes as axes is:

$$xy = a^2$$

If x and y are reciprocal, (modulo m), then, for a single soliton, this may be re-written as:

$$x/y = a^2$$

Since nothing is changed by alteration of the symbols, this may be re-written as:

$$E/M = c^2, \qquad \text{or } E = Mc^2$$

On the basis of the previous demonstrations, this equality could be re-stated as a polynomial:

$$M = 1/c^2 \sum_{s=1}^{n} Q_s V_s$$

where M is mass, c is the velocity of light, s is the number of solitons of EM energy, Q_s is the charge distribution of a soliton (i.e., including phase), and V_s is the effect of all other solitons on Q. This is of the same form as Shannon's measure of negentropy of information.

11. SUMMARY

All of the foregoing implies a single universal (EM) force and is essentially represented in Fig. 6. That is to say:

(i) A minimum energy oscillatory partitioning mechanism.

(ii) A minimal set theory derived from the properties of EM radiation and expressed as a minimal binary code.

(iii) Minimal surfaces, leading to minimal volumes, which are amenable to description in simple geometrical terms, but which lead to extreme combinatorial complexity in the absence of a minimal energy constraint.

(iv) Quantized phase effects requiring discrete, one-to-one mathematics in order to avoid manifestations of chaos.

(v) EM phase-locked loops leading to the appearance of structure.

(vi) Dependence on hyperbolic transformations, including rotations under a bilinear operator.

(vii) Relatively prime and reciprocal modulo number systems as expression of a uniqueness principle.

(viii) Forced phase alternation as necessity for omnidirectional propagation of information by radiating oscillators (19).

It is concluded that:

1. Natural phenomena are encoded and propagated as reflections embedded in binary Gray code fields.

2. That other number systems, (including conventional binary code and denary numbers), are unsuitable for scientific work since direct decipherment of spatiotemporal Gray code sequences requires a unique spatiotemporal procedure (20).

3. The principle of Napier's abacus (21) would be a suitable basis for a spatiotemporal (geodesic) computer.

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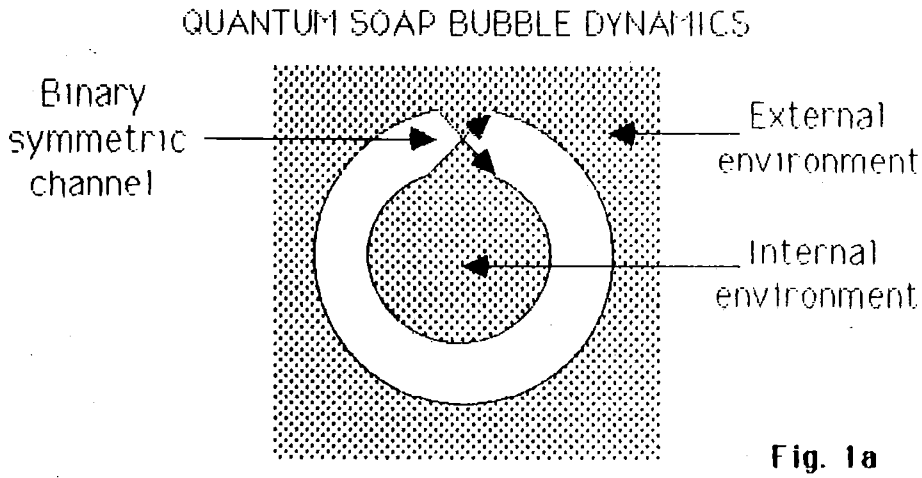


$E = (n-1)/(n+1)$

Fig. 1a

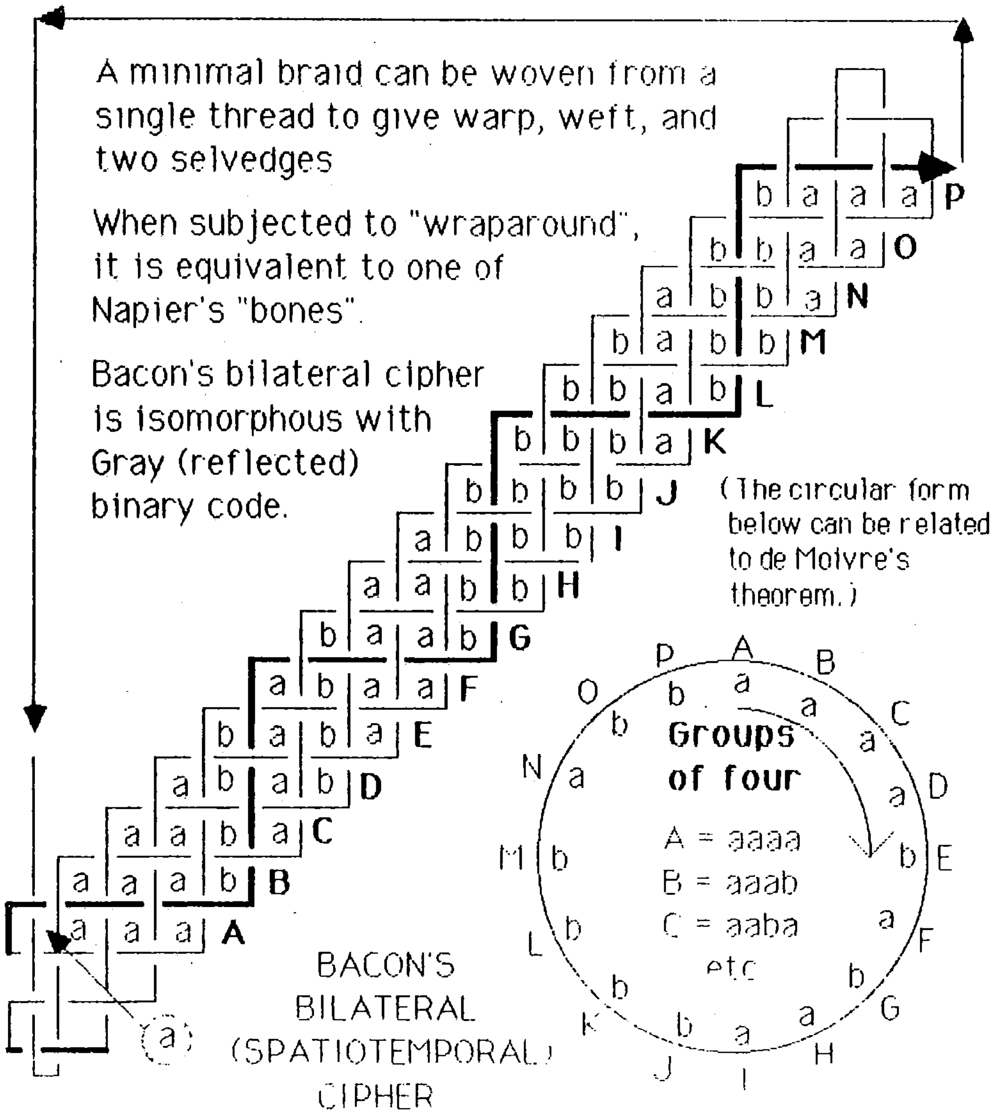
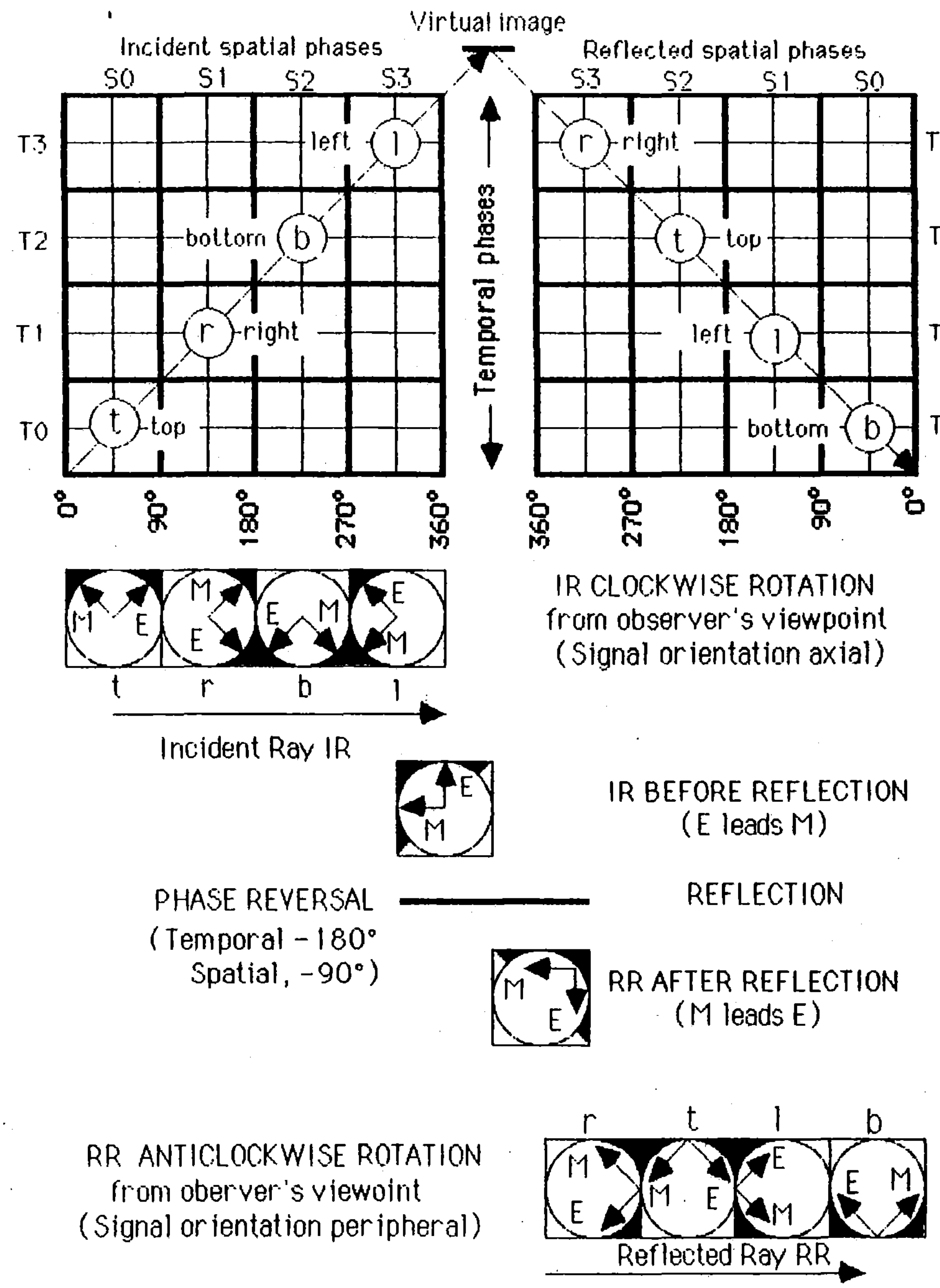


Fig. 1c

Fig. 1b



(Energy is located in the black areas)

Fig. 2a

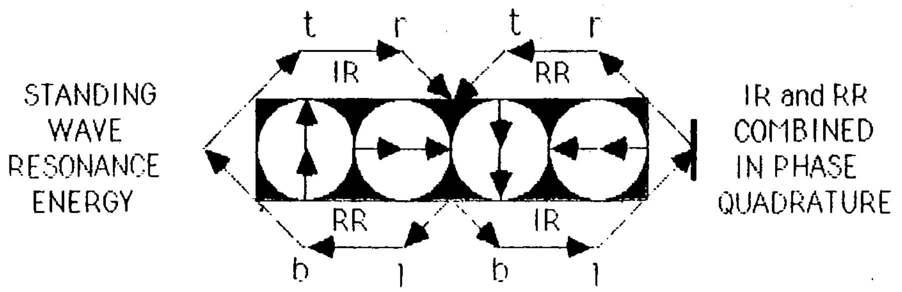


Fig. 2b

Trefoil knot (3 crossings)

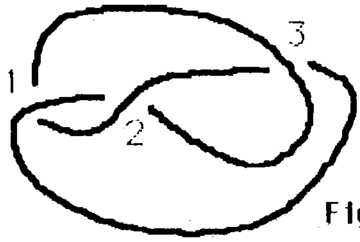


Fig. 3a

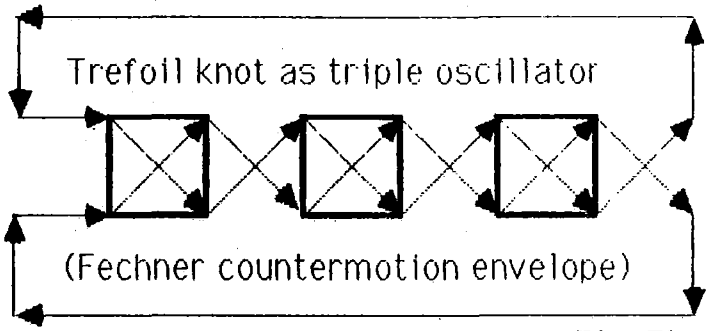
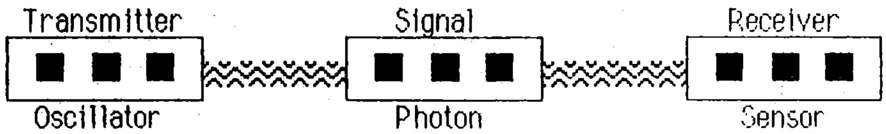
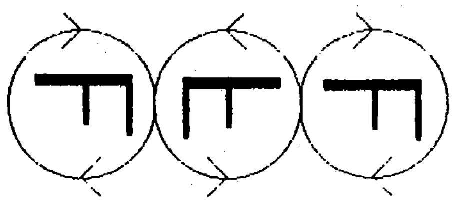


Fig. 3b



Offset rotary press



Two 180° phase shifts

Ink Offset Print

Fig. 3c

The edge of the Coupe du Roi is a Hamiltonian pathway in phase quadrature (3 x 90°).

Quadrant folds:

C,1,2 onto C,1,6
C,2,3 onto C,3,4
C,5,4 onto C,3,4
C,5,6 onto C,1,6 and
C,1,6 onto C,3,4 (6 layers)
yields a trefoil knot by congruence

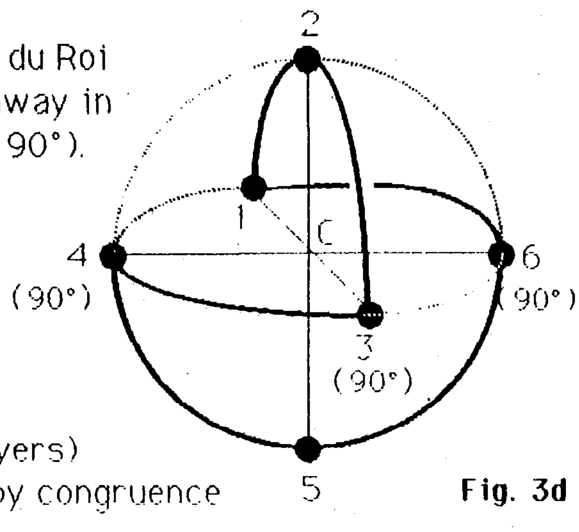


Fig. 3d

TREFOIL KNOT CONFIGURATIONS

(Over in thick rule, under in thin)

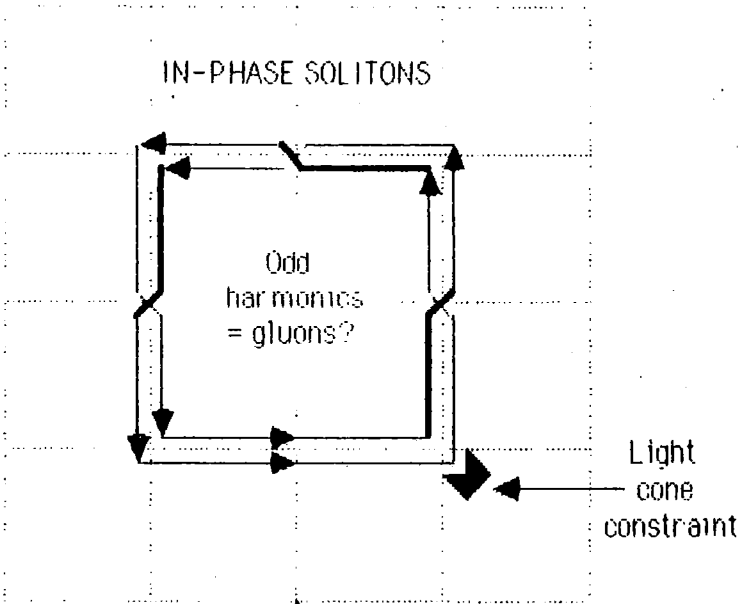


Fig. 4a

$$E = (n-1)/(n+1)$$

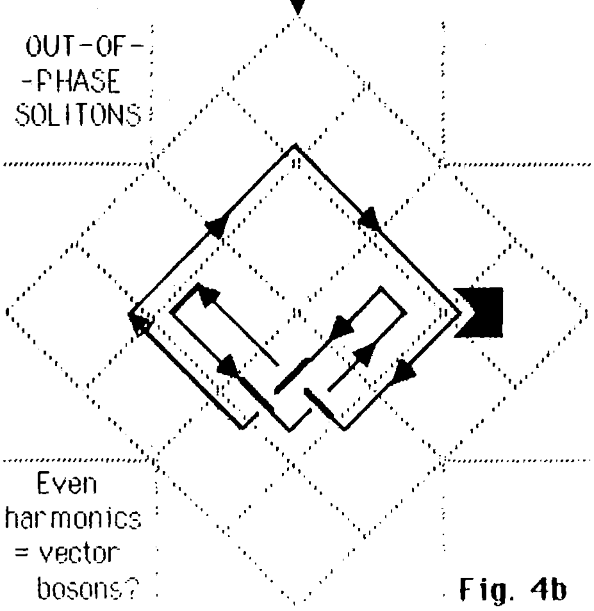


Fig. 4b

When n is odd, E is even and vice versa

(Both circuits maintain over , under , over , under)

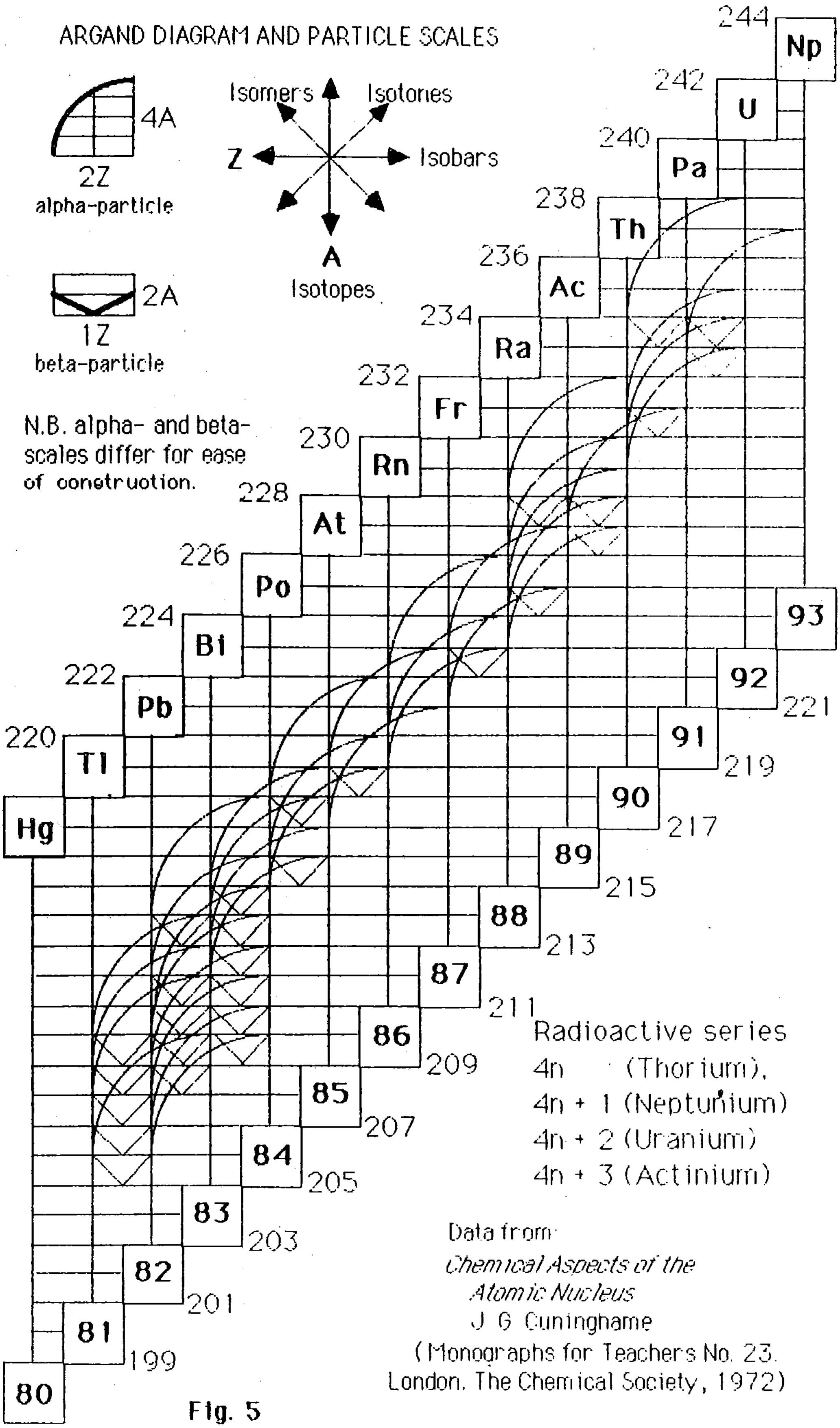
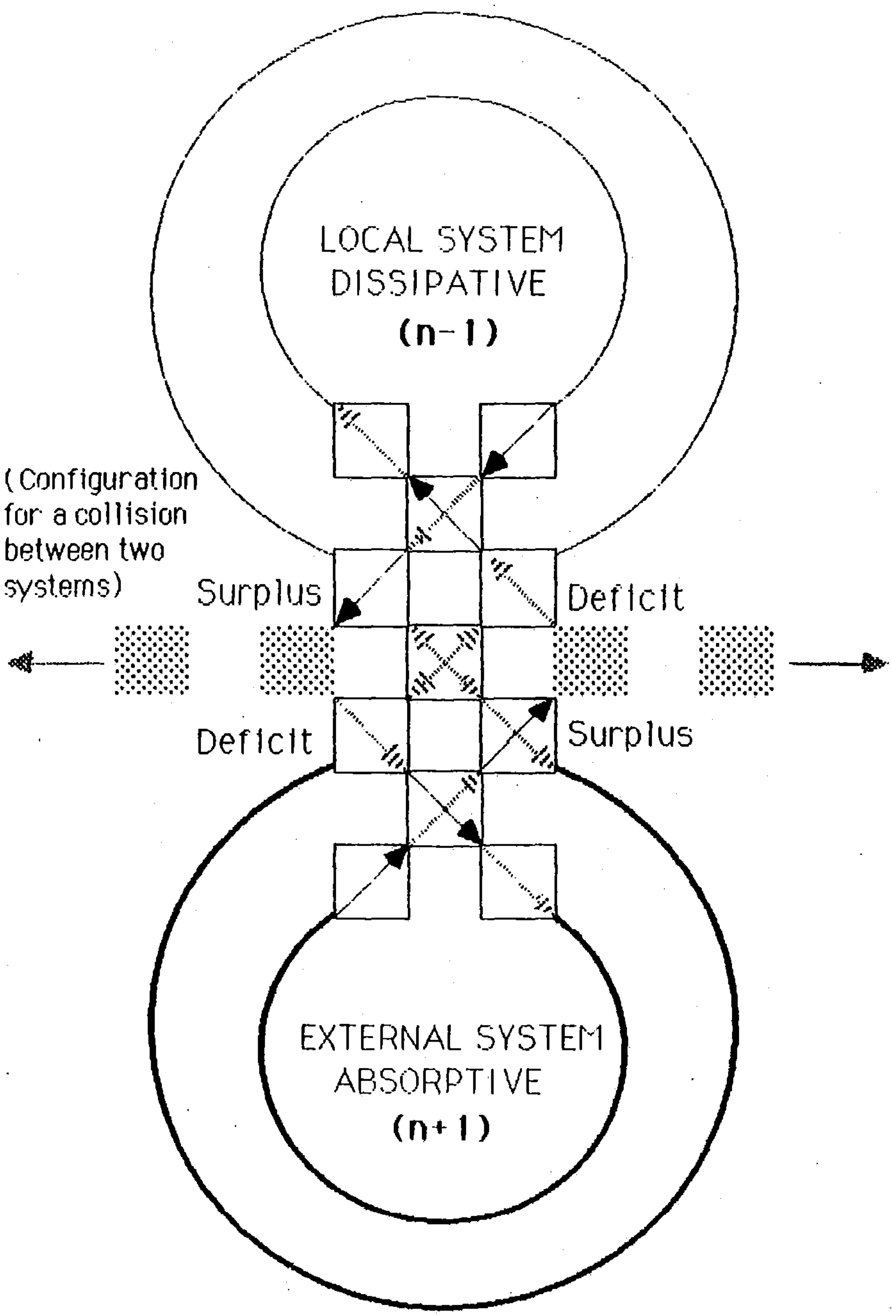

ARGAND DIAGRAM AND PARTICLE SCALES
4A
2Z
alpha-particle
2A
1Z
beta-particle
Isomers
Isotones
Z
Isobars
A
Isotopes
N.B. alpha- and beta-scales differ for ease of onstruotion.
244 Np
242 U
240 Pa
238 Th
236 Ac
234 Ra
232 Fr
230 Rn
228 At
226 Po
224 Bi
222 Pb
220 Tl
Hg
93
92 221
91 219
90 217
89 215
88 213
87 211
86 209
85 207
84 205
83 203
82 201
81 199
80
Radioactive series
4n (Thorium),
4n + 1 (Neptunium)
4n + 2 (Uranium)
4n + 3 (Actinium)
Data from:
Chemical Aspects of the Atomic Nucleus
J G Cuninghame
(Monographs for Teachers No. 23.
London. The Chemical Society, 1972)

Fig. 5

QUANTUM PARTITION AS THE DISCRETE HYPERBOLIC BILINEAR FUNCTION:

E = (n−1)/(n+1)

(Generalised four-tier hierarchy)



A Lorenz attractor is formed by two mutually-orthogonal systems. A local system may also be nested inside an external system.

Fig. 6

A MODEL FOR AN EVOLUTIONARY EPISTEMOLOGY BASED ON THE AXIOMS FOR UNIVERSALLY EXTENDING CONTINUA

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The deletion of the Archimedean condition from the novel categorial axiomatization of the Universally extending Continua[1], yields that for J.H. Conway's surreal numbers[2], which describes all numbers both great and small. Similarly eliminating the distinction between left and right (and + and -)[2] yields an axiomatization for a universal model of optimal computation $C(On_2)$, i.e. computation in the minimal number of computational steps through the progressively more complex concepts of sums, products, inverses, algebraic, transcendental, and iconic extensions[3]. This corresponds to the various classes of process suitable for modelling classical, relativistic and quantum mechanical wavefield physics[3]. Uniquely in this system, the birthorder process for this surreal field provides the only means for absolute comparison or measurement since all other processes are essentially reversible and relativistic[3]. This is as the Second Law of Thermodynamics would require and it says that all measurement is necessarily therefore dissipative; it is in agreement with de Broglie's proof that the principle of least action and the Second Law are equivalent. The Combinatorial Hierarchy[4] is therefore by Kilmister's Brouwerian foundation[5], the corresponding model of evolutionary complexity against this norm, i.e. the mapping from $C(On_2)$ onto the birth-ordering n , n = 0,1,2, ... The axiomatization for $C(On_2)$ is free from the concept of probability yet defines probabilistic computation, i.e. each computational process describes the computational complexity of the mechanism or algorithm needed to overcome the progressively more complex form of problem to be solved. Thus quantum computation is classed as transcendentally probabilistic or complex. Probability can therefore be explained combinatorially or computationally without any reference to an axiomatization for randomness, as various models of chaos.

The axioms for the Universally Extending Continua therefore encapsulate those for continuous and discrete mathematics, universal computation, wavefield physics and probability, and hence are, I believe, a suitable model for an evolutionary epistemology appropriate to defining any form of evolutionary complexity, structure or knowledge, to solve any problem. In particular it is demonstrable that $C(On_2)$ may define the genetic code appropriate to biological adaptive complexity[3].

McGoveran[6] has shown by means of arguments grounded on probability that asymptotic dimensionality (appropriate to the Combinatorial Hierarchy) is of a 3+1 space-time. It can therefore be inferred that the 3+1 space-time continuum is the unique dynamic asymptotic basin of attraction or homeostasis appropriate to the Universally Extending Continua as its more general dynamical processes of homeorhesis, and that there is a corresponding geometric probability associated with the space-time continuum as its model of 'chaos'. The known inability to prove the topological equivalence of 4 dimensional manifolds shows that geometric probability is ultimately incommensurable as a mathematical continuum requires. It can be inferred however, that since in this model the 3+1 space-time continuum is a unique dynamic asymptotic basin of attraction that such geometric probability as corresponds to it, is physically bounded (defining the Universe as we know it). This is again in agreement with $C(On_2)$ where the models of chaos are progressively sums, products, inverses, algebraic, transcendental and iconic extensions, with the iconic extension symbolizing the boundedness of optimal computation as a physical process. This suggests that the iconic form of chaos will be directly observable and manifests itself as dissipativity, which it is known can be overcome in a bounded domain by free energy[7]. A definition of geometric probability by Stoka[8] and its use by Fatmi and Resconi, suggests that a suitable formal dynamical model for the Universally Extending Continua is provided in their paper "A New Computing Principle" where they generalize by means of Lie algebras the novel mathematical model for computation advanced by Gabor based on his understanding of the physical nature of holography. See also reference 10.

Similarly applying Gabor's principles of holography set out in his paper with Goss on interference microscopy[11], to the models of chaos inferred from $C(On_2)$ for the Continua, one sees that one arrives at that appropriate to Hilbert space as $C(On_2)$ requires to be the case. The principles of the interference microscope require illumination of an object by coherent reference waves only a quarter of a wave apart to produce firstly the hologram of the illuminated object, and then secondly its reconstructed real image free from the conjugate image which usually accompanies it. Applying this condition to the probability waves modelled by $C(On_2)$ which are coherent since they originate from a single unique state corresponding to the empty set (where 3+1 space-time has no meaning) gives as a result, the non-dissipative residue of the interference of such waves, the model of linear superposition of quantum mechanics, which concerns the set of mutually orthogonal dimensions. It therefore appears that discretization or quanta appear out of the Universally Extending Continua as a consequence of the constructive interference or resonance of the probability waves which might be thought of as the computational complexity of the vacuum. Such constructive interference might also be considered as a global phenomenon for which phase conjugation[12] provides the experimentally validated confirmation, and so is the fundamental process symbolized by the iconic processes of $C(On_2)$.

* * *

The recent discovery of a possible great attractor for the whole of the space-time continuum, is therefore appropriate to and explained by this model particularly since the Combinatorial Hierarchy as a model of evolutionary computational complexity has also been shown to be a model of discrete quantum physics appropriate to the experimentally validated families of elementary particles[4], for the initial levels of evolutionary complexity where it models only fixed point dynamical solutions or structures.

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The Nature of our Thinking

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I claim that sciences stand before a new type of rationality which consists of a new attitude towards the knowledge of truth. I think we can distinguish three approaches to the truth in our former cognition of reality (1). The first approach was dominant in greek philosophy and it is the coherent conception of truth, which also underlies Socrates's dialectic. The second approach emerged at the Renaissance and the essence of this conception of truth is correspondence between reality and our cognition of reality. The third approach abdicates from the requirement of absolute truth. Knowledge of truth is every cognition which brings us achievements by which we can change reality of the world or can exploit our knowledge. We use these three approaches to the truth in our everyday scientific work.

But I think we are at the beginning of a new conception of truth. In it truth is conceived neither coherently nor correspondingly nor practically but truth is a construction from our ideas. Truth is not hidden in the reality of the world and sciences do not uncover it. Scientists must construct truth but do not do so arbitrarily. Scientists must define the conditions which are necessary for the validity of truth and this is not easy. Science must overcome one obstacle when we accept such a conception of truth; it is the limited abilities of scientists. Therefore the next step in the field of scientific research must be the steps to better cognition, to more productive thinking.

If we want to describe the process of thinking we must primarily analyse our consciousness. We observe only a manifestation of thinking in speech or in culture or in science in the same way that we observe, for example, a table or chair. But behind these actions is something more essential; it is the structure of thinking, a content of thinking, a process of thinking and a criteria of thinking. In the same way a table hides the structure of reality, structure of atoms. But we have other possibilities which can be very profitable; these are the investigation by means of peculiar states of consciousness under the conditions of hypnosis, during sleep, or when we look into schizophrenia, de'ja' vu effect etc.

What is my assumption?

In my opinion, when we are under the conditions of hypnosis, our thinking is blocked. Thinking is not connected with conscious processes and is replaced by the rapport of the hypnotiser. Depending on the intensity of hypnosis, various layers or sections of thinking are blocked. In sleep it is similar. And with the help of this methodological base we can consider the structure of both; consciousness and thinking.

First of all it is possible to analyse perception, which is a substructure of consciousness. Man under the conditions of hypnosis perceives outer stimuli, but his precepts are modified by the hypnotiser's rapport, for example in the perception of pain, or the perception of deceptive appearance of persons or actions, and in the case of automatic writing. The precepts depend on both; the senses to the brain and the brain evaluates them and stores them in the memory and so on. But under the conditions of hypnosis the appreciation of the stimuli, the keeping of the stimuli in the mind and so on, are changed. And we can see that not all the precepts are changed, but only the part of the precepts which belongs to thinking. And I should like to claim that thinking evaluates all the stimuli.

I can give an example. If we prick a person with a needle when he is under the conditions of hypnosis, then he will know about it, he will have 'sensory pain', but he will not feel 'suffering pain'. He has an impression that it is somebody else who has been pricked. And moreover, if this man gets the order from the hypnotiser to write automatically about his feelings, he will write simultaneously: 'It hurts, why do you hurt me?'

Using the example of the perceptions of pain under the conditions of hypnosis we can demonstrate also the fact that thinking includes the mechanics of retention and forgetting. This is because through the hypnotiser's rapport man has the ability to remember the pain or he can forget about it - forget about the stimuli. This is also the basis for the assertion that only part of the memory (thus not all the memory) is controlled by thinking.

In the same way, we can say that thinking comprises the criteria of the evaluation. And in the same way we can construct a structure of thinking which is only a substructure of consciousness. And when we analyse sleep, particularly the REM phase of sleep, we can discover the same structure of thinking. Thinking is not ousted from consciousness by the rapport in sleep, but thinking does not control and does not organise the process of consciousness. Thinking is switched off and consciousness works with the contents of thinking, processes of thinking and criteria of thinking as if chaotically, randomly.

Our thinking is a special mirroring of objective reality. We can cognize the reality of the world in different ways but every time the reality appears to us in the external stimuli form. All sensations we elaborate gradually at the several stages. At the first stage we accept the outer stimuli by our sensory organs and marshal them. These processes are nowadays meticulously analysed by cognitive science and neuroscience. We know for example which groups of neurons work when we recognise horizontal or vertical lines of a picture, which group of neurons participate in distinguishing a face, how a picture is reduced to pieces and after this how these are synthesised in the visual cortex and so on. It is similar in the process of hearing, though there are differences, especially between how we perceive the sounds and how we perceive the utterance.

There are differences above all in the process of perception as age changes. The crucial point is, in all probability, puberty when the thalamus as a filter of sensations gets under the control of the cerebral cortex. After puberty perception is not prior to the mind, but on the contrary our thinking chooses among the outer stimuli. A 'child thinks as it perceives; on the contrary an adult perceives as he thinks'; this, Vygotsky's thesis, is a summary of sophisticated processes which happen in reality in our brain (2). The brain processes differ if a child or adult holds, for example, a book and perceives it. A normal adult has an abstract model of a book before his perception. This abstract model of a book arises from habituation. It is by such a process that we carry out abstraction from very often repeated perception. By this abstraction we can distinguish both; what is, among our precepts, invariable and essential and what general properties or features belong to the class of things. Similarly we have an abstract model of the various things and I label them 'endocepts'. It is Arieti's term and I take over this one (3). K.H. Pribram uses the term 'neuron's model'; W. Penfield prefers the term 'pattern'; D.H. Hubel speaks about 'coding's processes'; B. Russell calls it 'scheme'; and J. Fodor uses the term 'prototype'.

Though endocepts are closely related to awareness of percepts there are differences between awareness of our percepts and all the sensations that our brain has recorded. We are consciously aware of only a limited amount of these percepts at any moment. Endoception is the opposite of perception. It is an inner recall of our life through the world. The endocept is also a storage of all contents of thinking that we carry out and by which we scrutinise the world. But an endocept could not be identified with subconscious structures as displayed by a Freudian-type analysis. Rather it comprises large systems of past experience, images which do not currently release actions, are not easy to express in words but are felt as dispositions to thinking.

If one adult perceives for example a book, he recalls the endocept a book and his thinking confronts the endocept with the actual sensations. We can distinguish a thing only when we compare with endocepts, with memory of previous percepts. Here is the steady ground for categorising the evidence which is firmly rooted in isomorphy confirmation of percepts by means of our endocepts. I am in agreement with J. Fodor who wrote: 'I really have no idea how cow perception works, but let's follow the fashions and suppose, for purposes of discussion, that we can use some sort of prototype-plus-similarity-metric. That is, the perceptual recognition of cows is effected by some mechanism which provides solutions for computational problems of the form: how similar - how 'close' - is the distal stimulus to a prototypical cow? My point is that's the way it's done, then cow perception might be mediated by much the same mechanisms that operate in a large variety of other perceptual domains as well - in fact, in any domain that is organized around prototypes. This is because we can image a quite general computational system which, given specification of prototype and similarity metric for an arbitrary domain of percepts, will then compute the relevant distance relations in that domain. It seems plausible, that is to say, that procedures for estimating the distance between an input and perceptual prototype should have pretty much the same computational structure wherever they are encountered. (4)

We conjecture or we make a hypothesis 'what it is' at every moment of our perception. And it is thinking which confirms or refutes our conjecture. Thinking makes a zig-zag course between endocept and the actual sensations and confronts these actual sensations from the criterion of novelty. It means our thinking traces if something new is in our percepts. If it is not, if percepts comport with our endocept for example of a book we do not really perceive a book but we only confirm our inner endocept by the sensations which are isomorphy. It would be very troublesome for our brain to reflect on a book for a long period when we have read it. Therefore our brain chooses the strategy of confirmation of our endocept of a book. It is perhaps the tendency to a stable condition of the neuron model, as Hopfield supposes, that is a response to excitement by outer stimuli. G. Mandler labelled this process the judgment of familiarity and he claimed that either it is no conscious effort as an immediate response to the events or we must search in our long-term memory whether it is an old or new occurrence (5).

Thinking is not in this sense any organ in the brain, it is only a way by which we cognize reality of the world, here on the level of perception.

It is very similar in our quotidian life. We are living in a relatively stable environment and it would be very difficult for us to reflect the whole of our environment at every moment. But in the case that the world is some way that we do not expect it to be, of course, we perceive the world too. We instantly give it attention if something new is in our actual perception. For example, a book is damaged, or there are misprints in a book and so on. In this case our thinking reflects differences between endocept and actual perception and we either complete, if it is possible, our endocept with novelty, or we reconstruct it, or we must form a new one. These processes depend on the quantity and the consequence of the inclusive novelty in the actual sensations. Sometimes new information has only a virtual character, when new stimuli are small or they are not frequent. We shift aside such information into the periphery of our consciousness and they make a latent agent. Similarly turbulent informations are not able to create a new endocept because they bring too many new stimuli, they are confused and call up a chaos. Such information we push aside to the periphery as well. These are two bounds and between them is a possibility for creation of a new endocept from the percepts.

The persistent shuttle movement between endocept and actual sensations which is carried out by thinking also evaluates the new stimuli from the vantage point of 'pleasure and pain'. Perhaps this is a point when our emotions begin. In other words the new stimuli need to be evaluated for us to know whether to seek or avoid them. This process probably gives birth to endorphines influencing in the first place the limbic system in our brain. So the nervous system determines our behaviour. And it is also a further filter which sensations must run through. Thinking is here still in the level which, in the tradition of German philosophy, is labelled as 'das Verstand'. Thinking at this point works with the material object and therefore it is observable thinking. I. Kant defined this type of thinking as thinking which is mediated by the sensory organs. Because of the differences between our perceiving of sounds or pictures and the contents of sentences I can not accept that Kantian definition.

But thinking can work otherwise as well as with concepts. In such processes thinking does not work with sensations, percepts and images but with ideas. The latter are most frequently externalized as words and represent a content of thinking. Therefore man can work with ideas as with things in a practical life. Man can combine or compose ideas and so he can create new ideas,. Thinking is here on its own field and can evolve concepts; it is a level which is called, in the tradition of German philosophy, 'die Vernunft'. Hegel has asserted that concept is evolved on the level of 'Vernunft' by the means of negation. A concept involves its defining only as the negation of other meanings. We can determine a saltiness as that which is not bitter, is not sweet, is not sour, because we can not immediately tell what is a saltiness. Hegel has distinguished three types of negation - 'leere negation', 'bestimmte negation' and 'absolute negation'. The latter has been typical for evolving the concept in the shape of a triad. Such dialectic advancement of concepts is still very profitable.

The inference of propositions by the means of logic has been very often used in philosophical theories too, but in my opinion logic is a limited means for the purpose of construction of new ideas. Thinking is a more abundant means for evolving the concepts which can obtain new achievements outside of the rule of logic as well.

It is apparent that the rudiment of all inference is identity and difference that means, old Hegel's negation. But everytime we come to this deep ground we remain at the darkness of the depth and our thinking can be paralysed. Unravelling of ontological rudiments can explain to us 'how the world is' but very often imperfection is the motive power of the development of ideas. I claim the creation of new ideas can be described also by the means of generators and inhibitors of thinking which can act in both ways - discursively and divergently.

Discursive (rationally organised) thinking is goal-directed and sequential. The goal is attained by a specific, well-defined process of individual operations. In contrast, divergent thinking is only partially goal-directed. Typically, it proceeds in diverse directions and does so almost unrestrainedly because its goal is only vaguely defined and wide, one where the alternative solutions of the thinking process can be said to compete or one which is devised so as to produce alternative answers. Discursive thinking is conscious, volitional, whereas divergent thinking is spontaneous and open ended. Discursive thinking is largely concerned with apprehending the objective reality, with reflecting the objectively existing processes (cognitive thinking). Its guiding principle is accuracy. If its aim is seen as the construction of a new whole, then the construction assumes the form of synthesis of the individual apprehended parts. Divergent thinking, in contrast, goes beyond the facts by virtue of its spontaneity, vaguely outlined combinations are among its probable end-products (constructive thinking).

Both these polarities are generated or inhibited by a number of factors which affect the thought processes. Without reflecting its generators and inhibitors thinking has a limited scope. Generators and inhibitors act in both ways - discursively and divergently.

The functioning of generators and inhibitors is interconnected. The inhibitor, by blocking certain thought operations, makes room for others to be released. Thus every inhibition is simultaneously generations and vice versa. The power of generators and inhibitors varies. Beside the extreme cases of total generation or total blockage, there exist partial arresting or slowing- down or even disintegration of thinking resulting in tentativeness or uncertainty. Generators and inhibitors act differently on different personality types. The functioning of generators and inhibitors is frequently subconscious. This may lead to the false conviction that thinking can not operate differently from the way it actually does. This in turn gives rise to the fallacy claiming that there is no need to analyse thinking or make it more accurate.

More knowledge of the thought processes and of logical reasoning without their employment in real thinking is useless. The use or rejection of great number of facts and logical procedures depends on correct inhibition or generation. Therefore a correct analysis and employment of both has the same importance as the amount of findings. Mere knowledge without ability of correct application is dead.

The place for purposeful combination is germane to thinking is at this point. By means of generators and inhibitors we can combine ideas and construct new ones.

Well-known generators and inhibitors are:
- dominance;
- association;
- action;
- the first information;
- analogy;
- metaphor. (6)

When we are at the level of Vernunft our thinking operates with ideas and either by means of negation or logic or generators and inhibitors of thinking we construct more abundant endocepts.

Our endocepts are shaped by thinking on both levels: by the habituation on the level of Verstand as well as by the combination of concepts on the level of Vernunft.

Therefore we can speak about the constructive function of thinking in this case. I suppose it is very important for scientific work because we must form, or we must construct something in our head before we uncover it in the reality of the world. We must know what we look for. Without mental construction which precedes our experimental work we cannot distinguish the facts and define them into a system.

However, I should like to claim that a very important and often neglected stage in which a new endocept is informed is the rhombencephalic phase of sleep. This phase of sleep which is very often labelled as REM sleep is a very interesting stage of our consciousness. During the night it appears four or five times and there are serious reasons for assuming that it is a stage when our brain processes elaborate and evaluate all stimuli collected from the previous day.

During REM-sleep we not only dream but are dreamt. We are near to meditation. REM-sleep is germane to memory processes because 'if a list of nonsense words is learned, and memory is tested eight hours or twenty-four hours later, more of the list will be remembered after 24 hours, given an intervening period of sleep, than after 8 hours without sleep. It seems that memory traces are strengthened during sleep, maybe especially by paradoxical (REM) sleep; and since they presumably depend upon the durable molecules of brain protein, this can be understood.'(7) Activity of our consciousness during REM-sleep resembles a vigil, the brain processes are correlated by rapid eye movement (REM) which gives evidence about processes of thinking; men have an erection, women have a clitoral engagement; during this stage of sleep uterine contractions set in too, and our brain slips into a natural processes in which the present-day information becomes involved with our endoceptive structure of thinking and so on. K.H. Pribram describes this process in this way. 'When we examine EEG recordings we can see that after we go back home and put up for the night, the records from the whole day march past but in reverse sequence. ...a new examining of the stimuli sets off from a more advantageous point' (8).

The endocept is every morning either more abundant in the last day's experience or steady, when we were experiencing nothing new. Our consciousness immediately reflects human action and objective reality of the world but there is also a different part of our consciousness which has arisen in the course of long-time reflection and observation of human action. This way by which our consciousness works has been shaped by the history of social relations and therefore it is the outcome not only of an ontogenetic process (memory, experience are only component parts), but a phylogenetic process as well.

However, man does not always employ his capability of incorporating into his thought process actual and simultaneously also endoceptive thought structures. In everyday life, endoceptive thinking is forced back into deeper structures by every new fact comprised in actual thinking. In this way thinking gets focused on reality. This reality-oriented state of thinking makes it impossible for man to break through the barrier of the algorithmizing thought processes. As long as actual thinking does not draw upon endoceptive thinking, it stays at the level of observative thinking. It is dominated by the factual character of the objective reality, it is focused on it and is distracted by reflecting details. There are many examples revealing the difficulty of defying observative thinking both in real life and man-made puns and puzzles.

Only after breaking through the barrier of observativeness can we mobilise the above treated historically and socially evolved structures of thinking. 'Our brains are at their most efficient when allowed to switch from phases of intense concentration to ones in which we exert no conscious control at all.' (9) In succeeding in doing this we do not claim that every problem will be solved, but merely that we have managed to put to use more of our own thinking equipment and are nearer our self-realisation.

Very often there emerges a process leading directly to the solution of the given problem. This is termed intuition and consists in our suddenly seeing through reality and conceiving a new, creative approach to the studied problem. Research among 230 biologists showed that with 83% of them the solution of important problems took on the form of an unexpected 'burst' of intuition.

There are several methods of unblocking actual thinking and bringing into play endoceptively formed structures and I claim that the most profitable is the rhombencephalic phase of sleep (REM sleep) which still permits the preservation of the integrity of thinking. We must learn to maintain our conscious between vigilance and dream in the state which reflects our processes of thinking but still retains a single theme, and in which, if necessary, we can still by an act of will come back to the starting point of our thought. It is not easy to achieve. The difficulty lies in the fact that the integrity of thinking may in this phase be disturbed. Behind the shut eyes there run products of our imagination, thinking is diverted to remote associations and free imagination enters in. Quite a few scientific and practical problems are known to have been solved thanks to the reflection of thinking in the rhombencephalic phase of sleep. I.D. Mendeleyev's periodic system of elements and A. Kekulke's discovery of the benzene ring are two well-known examples.

However, if we will grip this special but natural stage of our consciousness then we can usually expand the capacity of our thinking. Scientific work is nowadays very demanding and we can see that this sort of knowledge touches the limit of our mental faculties. Therefore, we must mobilize all the possibilities that we have including REM-sleep. But not only that. It is, in my opinion, part of the continuation of the evolution of nature by way of human thinking.

Also, therefore, this phase of our life deserves further and meticulous investigation. There are also further methods of unblocking our actual thinking which can burst into intuition. There is a method which is connected with the use of mind-influencing drugs; or of working under enormous stress; or rhythmical and stereo-typed work when only the exploratory reflex is switched on and thinking moves along the endoceptive path. Arousing of the endoceptive structure of our thinking can lead us also to some specific exercises for thought. There are many tasks in the field of investigation of the nature of our thinking.

More knowledge of the thought processes and of the logical reasoning without their employment in real thinking is useless. The use or rejection of great number of facts and logical procedures depends on correct inhibition or generation. Therefore, a correct analysis and employment of both has the same importance as the amount of findings. Mere knowledge without ability of correct application is dead. I am aware entirely that there does not exist any universal solving method. When we try to learn to control our thinking in all dimensions we will not obtain advice on 'how to win every game of chess', but we can coach ourselves as a 'better chess player'.

Thinking can by means of generators and inhibitors work up both, outer and inner stimuli. Here thinking does not relate to a material object or to objective images but to abstract ideas. Results of such processes of thinking we usually express by language and so the new ideas stand outside of us like other objects or stimuli. We accept them very often because they have emerged from our contents of thinking and were evolved by our operations of thinking. But it does not mean that they are right and that they will be an enduring constituent part of our personality. For verification of our new ideas we have to put them to the test of further questioning and man has to pass by means of

REM sleep persistent adoption of these new ideas, which, after that period of sleep, will be affiliated to our endocepts.

This is also a basic difference between human and artificial intelligence. Man creates his personality every day during the sleep as a synthesis of his endocepts with novelties which are automatically evaluated from axiological, ethical and other criteria. Man is every day in the morning rather different. Artificial intelligence has no endocepts and does not sleep. Of course, there are other differences especially in the distribution of energy flow or in principle how formation is confined, but by the functioning of our mind we belong to nature.

Much of our thinking we control and much will remain spontaneous. It is very important what we live through. Our past underlies our future perceptions, thinking and behaviour. Only by persistent integration of our past and actual being do we create ourselves, we share our abilities, we form our personality. It is an unceasing process of self-creation in both nature and social environment and this is the way by which we are natural and social beings.

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GENERAL RELATIVITY AND ELECTROMAGNETISM DERIVE FROM QUANTUM MECHANICS?

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ABSTRACT

Evidence is accumulating that both GR and EM can be derived from QM as a single unified theory. The bases for the theory are

- the usual apparatus of self adjoint operators acting in an Hilbert space (and associated theorems);
- the usual apparatus of states, represented by normed vectors in the Hilbert space, which evolve with a continuous scalar time;
- axiom(1)- observables are bounded;
- axiom(2)- the system is isolated;
- axiom(3)- the observables of classical mechanics, when subject to a sucession of measurements, are *maximally deterministic* (MD) in the quantum mechanical sense;
- axiom(4)- the laws of physics should be expressable in a form independent of coordinate systems;
- the emphatic rejection of the Correspondence Principle as an aid to theory.

A unique evolution operator, together with equations of motion, can be derived from quantum algebra alone. This, from its form, appears to represent a particle subject to symmetric tensor, vector and (invariant) scalar fields acting in a space-time of n dimensions; the fields can be proved to have the tensor character claimed; also the divergences of the tensor and vector fields can be shown to vanish. From the manner of derivation the particle appears to have 'low energy'; however the energy may be 'low' only on a cosmological scale.

When the vector and scalar functions are set to zero the equations of motion have the form of the geodesic equations for a Riemannian n-space; the interval is proportional to the QM time and the metric tensor is proportional to the tensor field of the evolution operator; it is necessary to assume that Plank's constant is 'small' in order to arrive at this result.

The theory does not yet determine the signature. However, when the vector and scalar fields are non-zero, n=4 and the metric tensor is Galilean (zero gravity), the evolution operator has a recognisable form:- that of the

Hamiltonian of a relativistic particle moving in an EM field. This interpretation is correct, in detail, only if the fields can be regarded as the EM vector and scalar potentials; which leads us to consider fields.

Suppose that a bounded observable function of the coordinates is MD; see axioms (1) and (3). Then the MD property can be expressed by requiring that a certain integral, over the Reimannian n-space, shall be stationary. The resulting PDE involves the observable, the QM probability density and the metric tensor. The QM probability density cancels through if we take the 'classical particle' to be at its maximum; the result is a PDE for the observable as a function of the classical particle coordinates.

The components of the vector and scalar fields (considered as MD observables) turn out to satisfy the generalised d'Alembert equation; this supports the notion that they are EM potentials. The elements of the metric tensor, given the same treatment, satisfy equations which, in the weak field (low energy) case, approximate the Einstein law of gravity for empty space. If we demand that the coordinates are also MD we only slightly restrict the choice of coordinates. The interval (considered as a function of one of its end points) turns out always to be MD.

In the Newtonian approximation the Einstein field equations reduce to a single PDE for the gravitational potential:- Laplace equation in 3 (spacelike) dimensions. If, we pretend that we do not know the dimensionality (but that we do know the signature), we have Laplace equation in n-1 dimensions. Now evaluate the total energy for any plain circular orbit; this involves the spherically symmetric solutions of the PDE. We find, by studying the expression, that the orbit is bound and stable only if n=3 or 4.

Despite these successes many problems remain to be solved before the theory can be regarded as coherent. For example, the equations obtained for the gravitational field are approximate and are not tensor equations (see axiom (4)); is this because the form of the evolution operator is approximate or is it because we have picked the wrong quantities to be MD? Similarly d'Alembert's equation is a tensor equation only if the dependent variable is an invariant; the elements of the vector field are not (by definition) invariants; so, again, have we picked the wrong quantities? There is a strong argument (on physical grounds- see axiom(4)) to suggest that we should pick, as MD observables, only invariant functions of the tensor, vector and scalar fields; but, by what criteria should we pick them?

My thanks are due, once again, to Prof. Clive Kilmister for his patient review of my mathematical rambles, for his encouragement and for much illuminating discourse.

On Designing a New Ontology for Natural Philosophy

Summary of an informal talk given at ANPA11 (September1989) by

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Physics has needed a new philosophical foundation since quantum effects were discovered at the turn of the century. One is proposed which, as a bonus, clarifies how consciousness arises from physical processes. It resembles Leibniz's Monadology but using modern insights, it is less contrived.

It involves a radical paradigm-shift. To make sense of it, several topics, each vulnerable to dispute taken separately, must be juggled like simultaneous equations. It is presented as a "design experiment".

We abandon identification of Reality with Being, and identify it with Activity. Existence is relativised, seen as deriving from activity, and as the "germ" within physical processes from which arises awareness (the Relative Existence of an object *for* a conscious subject). The following summary gives an overview:

1.) It is a category-mistake to say that events or acts *exist*, because they are not objects. They are time-like. The proper predicate is *occur*.

2.) The Real "primary stuff" of the world is assumed to be a flux of quantum acts. So we should use *occur* to predicate them, not *exist*.

3.) Spatial extension and material particles are produced by acts, within which particles have only a Relative (and transitory) Existence *for* each other but no Absolute Objective Existence (that concept is dropped).

4.) Awareness (objects *existing for* a subject) is a particular form of the Relative Existence in all physical processes, which arises in complex systems when they interact holistically with their surroundings.

From (3) to (4), from the bare, undifferentiated, transitory existence of a particle for another within the simplest quantum act, to the richness of human consciousness is a long road. But using these ontological ideas, it is a more promising road than the one starting from molecules conceived as existing "classically".

To travel it we need, in addition to the idea that Relative Existence arises within acts, an entropic principle according to which, if interaction becomes constant (or constantly repetitive), Relative Existence fades, such that inside a system, the separate "experiences" of parts merge into a single unified "experience".

The present aim is not to describe every hierarchical detail, but only to remove conceptual blockages and to offer a principle for, the evolution of consciousness from bare Relative "Existence-for", by the merging of simpler systems of activity.

5.) The "ego-centric predicament" can be avoided if we assume human consciousness, unlike simple awareness, is a product of interaction between people rather than a property of individual brains. Then the world must come to exist for us during infancy as a shared experience, public from the start.

6.) Thus our primary notion of existence lies in a habit of perceiving things while simultaneously imagining how they "exist for" others, including the viewpoint of an impartial objective observer. This reveals the origin of the concept of Absolute Objective Existence as an instance of "existence-for-others". Nothing else in experience (itself an activity) suggests this concept or the need for it.

7.) Traditional atomism was "bottom-up" (building the universe from simpler parts). Perhaps in rejecting it we should proceed "top-down" from appearances. This would side-step the implausible "merging" of disembodied acts. In a world *as given* they are "merged" already. The proposed scheme then has the role of predicting that experiments should reveal a bottomless world of discrete, act-like processes.

Alternative Natural Philosophy Association

Statement of Purpose

1. The primary purpose of the Association is to consider coherent models based on a minimal number of assumptions to bring together major areas of thought and experience within a natural philosophy alternative to the prevailing scientific attitude. The combinatorial hierarchy, as such a model, will form an initial focus of our discussions.

2. This purpose will be pursued by research, publications and any other appropriate means including the foundation of subsidiary organizations and the support of individuals and groups with the same objective.

3. The Association will remain open to new ideas and modes of action, however suggested, which might serve the primary purpose.

4. The Association will seek ways to use its knowledge and facilities for the benefit of humanity and will try to prevent such knowledge and facilities being used to the detriment of humanity.

Organization

1. The Executive Council is the governing body of the Association. The Coordinator, Secretary and Treasurer serve at the pleasure of the Executive Council, in consultation with the Membership. Decisions of the Executive Council are subject to majority vote of the Council as a whole.

2. Members of the Association are (a) members of the Executive Council and (b) others nominated by the members and approved by the Executive Council.

3. Vice-Presidential candidates are nominated by the Membership and the Executive Council during the first year of the President's term of office. Any nomination must be accompanied by a statement from the nominee that he will serve a full term if elected. If more than one nominee exists, selection will be made by mail ballot to the Membership decided by plurality of votes. The Vice-President is elected biennially to serve for one year concurrently with the then current President, and the two following years as President. The Vice-President becomes a member of the Executive Council on election, and retiring Presidents remain on the Council for five years. Presidents must serve for one full term (1 year as Vice-President and 2 years as President) and cannot run for re-election until three years after their initial term has elapsed.

4. The President is the official representative of the Association in external affairs, and has the responsibility for calling meetings of the Membership, at least annually, for the determination of overall policy.

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6. The Secretary is responsible for keeping minutes of the Membership and Executive Council meetings, production of a newsletter to keep members of the Association informed of its activities, and such other duties as may be assigned.

7. President, Secretary and Treasurer will not be paid for their services but may, as appropriate, receive funds for travel expenses, secretarial help, etc.

8. The Coordinator may be paid an appropriate salary for his services, funds permitting. These services will include the organization of meetings and the editing of the proceedings of such meetings for publication, coordination of and participation in the research activities of the Association, preparation when appropriate of research reports and publication of such reports, and such other duties as may be assigned.

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