DISCRETE PHYSICS

AND

BEYOND

Proceedings of ANPA 10

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Preface

This volume of ANPA 10 Proceedings carries on the traditions of its predecessors but at the same time extends the field covered. The germ of ANPA was a small group interested in discrete physics and a particular construction called the "combinatorial hierarchy". The first three papers show the continuation of this interest, and since this interest has led to a number of foundational studies in mathematics and physics, the fourth and fifth papers are closely related to this approach. Somewhat the same may be said of the sixth but with the seventh we find an extension of the general ANPA way of independent thinking into the various human sciences and this is also the nature of the eight and ninth papers. Finally, the tenth and eleventh papers relate corresponding ideas with an emphasis on the computational side. The printed papers convey something, but only a part, of the excitement of the 1988 meeting of the Alternative Natural Philosophy Association in Cambridge. To the interested reader I can say only: if you feel that these papers are worthwhile, try to come to future annual ANPA meetings.

C.W. Kilmister

January 13, 1989

WHERE WE ARE*

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ABSTRACT

The conceptual consequences, cosmology and physical predictions stemming from the *combinatorial hierarchy* and a *discrete physics* model based on McGovern's *ordering operator calculus* are reviewed. We conclude that we have made a strong case for this approach to the foundations of physical science.

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1. INTRODUCTION

At ANPA 9[1], I was willing to admit that I believe we are engaged in trying to create a scientific revolution. Subsequent events have strengthened my conviction that this was not an idle statement, and rekindled some revolutionary ardor. Thanks to McGoveran's *ordering operator calculus*[2] we have not only been able to give a reasonably complete understanding of why a finite and discrete theory necessarily provides a common explanation for quantum mechanics and special relativity[3,4] but to make a start on meeting the three original tests of *general* relativity[5]. The clincher for me was McGoveran's calculation of the *second order* correction to the hierarchy exoskeleton (scale constant) value for electromagnetic interactions. This calculation[6,7] foreshadows a new era of quantitative predictive power for our theory, as I will discuss in my paper entitled *What Is To Be Done* at ANPA WEST 5[8]. The paper before you focuses on one task set for me by President Kilmister for ANPA 10, namely to provide an overview of what has *already* been accomplished. I discuss below progress which has been made in the conceptual foundations of our program, some of the cosmological implications, and the outline of our theory of elementary particle physics which is beginning to emerge. I conclude with "Homework problems" for ANPA 11.

2. CONCEPTUAL FOUNDATIONS

The conceptual foundations of our theory have been discussed in some detail by Gefwert[9] and McGoveran[2] in last year's proceedings. I emphasize that we follow the *modeling methodology* developed by McGoveran, which in the application at hand *starts* with the contemporary practice of physics as the problem to be modeled. In order to have a self-consistent formalism which can be related to this *epistemological framework*, it is necessary to develop a *representational framework*. As Bastin and Kilmister have emphasized, this framework must not use theory-laden language; we avoid this trap by insisting that the R-frame be strictly computable. To complete the modeling task we must establish rules of correspondence (a *procedural framework*) which connect the R-frame to the E-frame. In our application these will obviously include what is usually called "comparison with experiment", but will by no means be limited to this aspect of the problem. Many iterations of these steps in theory construction will have to be performed before we can decide that the theory is indeed successful, or will have to be modified or abandoned. Roughly speaking, this approach to physics is not very different in outline from the best contemporary practice; we have found it very useful to give more precision to this practice.

The place where we most obviously part company with contemporary practice is in the principles on which we base the construction of the R-frame, which in this application includes what McGoveran calls the *ordering operator calculus.* The five principles are strict finiteness, discreteness, finite computability, absolute non-uniqueness and strict constructability. We reject the continuum from the outset. We must state in advance how far we intend to count; if we find that we want to exceed this bound, all arguments must be re-examined. In the absence of further information, we must use equal prior probabilities for alternatives. We are necessarily context sensitive in our constructions and encounter indistinguishables in many of these contexts. This makes our theory richer than continuum theories. We must often consider the fact that many different "histories" could have led us to a particular point in a particular construction, and that in the absence of further information, we must assign equal weight to each of these. In return for this increased complexity some problems become much simpler for us. For those familiar with Kuhn's model for scientific revolutions, this should come as no surprise. Any new fundamental theory finds some problems easier to solve, and for other problems loses (sometimes for a long while) some of the explanatory power of the theory it is attempting to replace.

We now discuss several points where we believe we gain in explanatory power compared to conventional theories.

• 3+1 asymptotic space

In many conventional theories, the three dimensional structure of space and the sequential character of time are accepted as brute facts. Recently "string theories" give another argument. They accept both quantum mechanics and relativity, and start with a 26-dimensional structure whose uniqueness can be questioned. An argument in its favor is that, as the theory develops, this structure "compactifies" in such a way that only the usual 3+1 space-time is relevant at large distances compared to the elementary particle scale. For us, this 3+1 structure for events follows directly from McGoveran's Theorem, once our basic principles and rules of correspondence are understood. Assume dichotomous choice for any attribute we use to set up a metric and map onto D distinct sequences. We *synchronize* the sequential count by identifying a starting place in each sequence. Require that the labels which keep the sequences distinct cannot be used to distinguish the construction of one sequence from another (i.e. the construction is "homogeneous and isotropic"; labels specified only by discriminate independence are obvious candidates). Then McGoveran proves that the probability of being able to construct the n^{th} metric mark for all D sequences under these constraints is strictly bounded by $n^{-\frac{1}{2}(D-1)}$. Sum these probabilities up to some finite N and normalize. For $D = 2, 3$, there is a finite probability that the construction can continue to produce sequential homogeneous and isotropic metric marks in each dimension for any finite N, but for any larger number of dimensions this probability is strictly bounded by $1/N$; one can wait till hell freezes over for the next metric mark to occur in all $D \geq 4$ dimensions. Hence 3 homogeneous and isotropic dimensions must separate out once we count far enough (to 20 or so is enough for most practical purposes) using any universal ordering sequence that can be mapped onto the ordinal integers. We claim that this explanation gets to the heart of the matter for any theory such as physics that relies on finite counting.

• transport (exponentiation) operator

• combinatorial construction of π

One interesting development is that McGoveran has given, for finite N, the combinatorial definition of the exponential $e(N) = 1/N!$ as the ratio of all permutations to all complete permutations. This can be generalized to define a transport operator in terms of a finite "Taylor series" with a combinatorial definition of the coefficients. Since, in practice, any theory based on analytic functions has to expand them in finite Taylor series, and thus provide a combinatorial definition of the coefficients, we have explained to our satisfaction why physics is based on analytic functions and recovered for our theory the consequences of this. Another development is Mcgoveran's construction of finite coordinate patches with either square or radial symmetry. The ratio of perimeters gives one algorithm for computing $\pi(N)$ — in fact the Archimedean algorithm used in computer practice — and the ratio of the areas another. This reminds us that in our theory "π" is always, in principle, defined by a rational number depending on context and can be thought of as "empirical".

- limiting velocity
- supraluminal synchronization and correlation *without* supraluminal signaling

At a somewhat less fundamental level than the global "irreversibility of time" and the "3-dimensionality of space", all conventional theories take the existence of a limiting velocity as a "just so story". In contrast, we derive it from our fundamental principles. Attribute distance relative to some reference ensemble is defined as the number of computation steps which take the ensemble away from the reference ensemble minus the number toward - coincidence defined by local isomorphism *with respect to the attribute in question.* Therefore any attribute, reference ensemble and computational procedure define a "limiting velocity" or "computational band width" as the difference between these quantities divided by their sum (i.e. by the total number of steps or "computation time"). The Lorentz transformations follow in due course[2]. Further, since the transfer of causally effective ("physical") information requires the specification of *all* the attributes which go to specify a "physical object", the limiting velocity of physics has to be identified as the *minimum* of these limiting velocities. Hence we anticipate "supraluminal" correlation and synchronization without "supraluminal signalling", which in our view is the guts of the EPR situation[10].

• discrete events

Although our definition of "event" is very different from that used in second quantized field theory or S-matrix theory, we end up with our own discrete version of Feynman Diagrams, the CPT theorem, crossing and scattering theory. Whatever means we use to generate these diagrams and to obtain the combinatorial hierarchy, our theory as technically articulated depends on *bit strings* (ordered strings of zero's and one's or any two distinct symbols) which combine by *discrimination* (OREX, exclusive or,...): when the strings are the same $(aa)_n = (0)_n$ where the *null string* $(0)_n$ consists of n zeros; when they are discriminately independent $(ab)_n = (c)_n$, all non-null and c is distinct from a and from b. A 4-event is then defined by $(abcd)_n = (0)_n$, and by our rules of correspondence (the "counter paradigm") can be associated with the chain of happenings in the laboratory which lead to the "firing of a counter" or some conceptual equivalent. Our generation procedure leads to concatenated strings $(a)_{L+n} = (L_a)_L \| (A^a_x)_n$ where the first part is called the *label* and the second the *content*. The labels are of fixed length L and each is one of the $3 + 7 + 127 + 2^{127} - 1$ members of some representation of the 4-level *combinatorial hierarchy*[11,12]. Once the label strings are constructed, their *closure* under discrimination allows us to assign *invariant* attributes and parameters to them. The content strings can be any one of the possible 2^n strings of length n and grow in both length and number per label as the investigation proceeds.

• discrete Lorentz transformations (for event-based coordinates)

To go from this definition of event to event-based coordinates, we consider an event involving some label, and after the content strings have grown by some increment n, a second event involving the same label. Taking as our attribute the number of 1's in this incremental content string k_a and as our reference ensemble any string with $2k = n$, the attribute distance between these two events is $k_a - (n - k_a) = 2k_a - n$. Our rule of correspondence is to assign the invariant step length $\lambda_a = h/m_a c$ to the label a, and the time per step $\Delta t = \lambda_a/c$ where c is the limiting velocity. Then taking the first event as the origin, the distance between the two events $x = (2k_a - n)\lambda_a$ and the time $t = n\Delta t$ define the interval $s^2 = c^2t^2 - x^2 = 4k_a(n - k_a)\lambda_a^2 = (1 - \beta_a^2)n^2\lambda_a^2$ and the average velocity between the two events as $\beta_a c = (\frac{2k_a}{n} - 1)c$. Clearly the interval is invariant under the transformation $k' = \rho k, (n' - k') = \rho^{-1}(n - k)$ and the Lorentz transformations with $\gamma = \frac{1}{2}(\rho + \rho^{-1})$ in 1+1 space time or momentum-energy space follow immediately.

- relativistic Bohr-Sommerfeld quantization
- non-commutativity between position and velocity (for event-based coordinates)

Conventional theories have to take both the limiting signal velocity and the quantization of action as "given" because they have no way of deriving either concept. Granted this much, their second quantized free space theory can be formulated, but trouble starts once they try to embed the discrete, non-local events implied by quantum theory into the continuous space-time of special relativity. Because of the uncertainty principle this necessarily assigns an infinite amount of energy at each space-time point! Fifty years of struggling with this problem has produced, thanks to a generous input of practical information about elementary particles, a "non-Abelian gauge theory" which is finite, but which gives the universe at least 10^{120} times too much mass-energy; we return to this point when we discuss cosmology in the next section. For us, the reconciliation between quantum mechanics and relativity occurs at an appropriately fundamental level without invoking all this complicated technical apparatus.

We have seen above that our discrete principles require us to take discrete steps of finite length executed at the limiting velocity, achieving lower velocities, on the average, when some steps are toward and some away from the reference position. Because velocity can have a common significance in either space-time or energy-momentum space, we can use the invariance of the labels as the investigation proceeds either to assign an invariant step length h/mc or an invariant mass to each of the $2^{127} + 136$ distinct labels. Note that our definition of velocity, $\beta c = [\frac{2k}{n} - 1]c$, is invariant under the transformation $k' = Tk, n' = Tn$; for $T \geq 1$, T counts the number of positions "along the line" where events can (but need not) occur, or in the language of wave theory, where interference can take place. We could introduce the constant of action h by specifying the invariant mass and quantizing this periodic possibility using $E = h/T = h\nu$ rather than by taking the invariant length to be $\lambda = h/mc$. Clearly the two quantizations are equivalent and give us the *relativistic* deBroglie relations.

It is important to realize that this relativistic periodicity is defined even for a particle "at rest", i.e., with $n = 2k$. Since each step is executed at the limiting velocity, each step starting from rest changes the momentum by $\pm mc$, and we must take at least two steps in position and two steps in velocity in order to return to the rest position. This *zitterbewegung* associated with the rest energy mc^2 implies that even a "free particle" executes a periodic motion in phase space which encloses an area nh with n integral. Our theory automatically extends Bohr-Sommerfeld quantization to relativistic free particles. This fact underlies the success of our calculation of both the Sommerfeld formula for the fine structure of hydrogen and our correction to the lowest order hierarchy result $\hbar c/e^2 = 137$ discussed below. Further, since the changes in position and in velocity occur sequentially around this circuit, the determination of either becomes order-dependent and non-commutative. As is discussed in detail in Ref. 2, this non-commutativity between position and velocity is a necessary feature of any finite and discrete theory.

- conservation laws for Yukawa vertices and 4- events
- crossing symmetry

There was considerable discussion at ANPA 10 as to whether we in fact had proved the equivalent of vector conservation laws in 3-space for our "Yukawa vertices" and 4-events. This problem is only partly met in Ref. 4. Further analysis shows that we have precisely the conservation laws needed for 4-event crossing if one realizes that on mass shell 3-momentum conservation in a 4-event leaves only 9 degrees of freedom, and that the internal (in general off mass shell) velocity for the system connecting two incoming to two outgoing masses is defined by discrimination, (i.e. by the common string any pairwise decomposition of a 4-event that the definition allows). Consider the $a + b \to c + d$ channel $(ab)_n = (cd)_n$ and note that $|k_a - k_b| \leq k_{ab} \leq k_a + k_b$ and that $\beta_{ab} = \frac{2k_{ab}}{n} - 1 = \beta_{cd}$. The four external velocities and the four external masses taken together with *this* connecting velocity precisely specify all the momenta and angles for the conventional problem. Then the constraints implied by $(abcd) = (0)$ connect the $(ab) = (cd)$, $(ac) = (bd)$ and the $(ad) = (bc)$ channels in precisely the way crossing requires in the conventional theory. This will be spelled out in more detail some time during the coming year.

Of course *any* fundamental theory of MLT (i.e. mass-length-time) physics must compute everything else as physically dimensionless ratios once any three independent dimensional constants are fixed. Conventional theories take c and $\hbar$, and the structures implied by them for granted; we showed above that they are, for us, structural consequences of our basic principles. We share with other physicists the scale-invariant ~~*laboratory*~~ methods of relating c and $\hbar$ to arbitrary standards of mass, length, and time. Granting this much structure to conventional theories, conventional physicists still need some mass or dimensional coupling constant that has to be taken from experiment. Once again the existence of this unique constant — let alone a means of computing it within the theory — is not available to the theorist; this is not an obvious structural requirement of conventional practice. We not only obtain a first order estimate for the dimensionless $\hbar c/e^2 \simeq 137$, which would allow us to take as our third dimensional constant the quantized electric charge of elementary particles, but also the remarkable connection $\hbar c/Gm_p^2 = (M_{Planck}/m_p)^2 \simeq 2^{127} + 136$. This connection between the elementary particle, electromagnetic and gravitational scales naturally leads us to consider next the cosmological implications of the theory.

3. COSMOLOGICAL EXOSKELETON

- the equivalence principle
- electromagnetic and gravitational unification

Conventional cosmologies usually start from the general theory of relativity, which in turn starts from the postulate of the equivalence between gravitational and inertial mass and then explains gravitational effects as due to the space-time curvature introduced by the presence of mass-energy. Source-free electromagnetic fields of sufficient energy to trap themselves as standing waves held together by their own gravitational fields (geons) unify the two "classical" field theories in a conceptually satisfactory way, as was proved by Wheeler long ago. His theory depends only on G and c; it is *scale invariant.* Once a third dimensional constant (eg e^2 or h or any elementary particle mass) is introduced, the short distance behavior of the theory becomes ambiguous. The difficulty, of course, arises once again because of the continuum assumption that drags the theorist down to the natural cutoff length $\hbar/M_{Planck}c = [G\hbar/c^3]^{\frac{1}{2}}$ and below. The related problems of "quantum gravity" are a major field of contemporary theoretical physics research.

Once again the problem is much simpler for us. Because

$$\hbar c/Gm_p^2 = (M_{Planck}/m_p)^2 = 1.6937(10) \times 10^{38}$$

the hierarchy result,

$$2^{127} + 136 = 1.70147... \times 10^{38}$$

thanks to my interpretation[13] of Dyson's argument[14] implies that this is a first order calculation of the same number, we can take as our unit of mass either the proton mass or the Planck mass. Since ours is a *fundamental* theory, all masses must be computed in ratios to our unit of mass. We have no place in the theory for two different kinds of mass. Thus for us the "equivalence principle" is a deductive consequence and not a postulate. There is no need for us to "geometricize" gravity at this level of the discussion. Further, since the same hierarchy construction gives us both the electromagnetic and the gravitational couplings, the theory is "born unified", and the gravitational coupling affects everything, including electromagnetic quanta. We have here the starting point for a quantum theory of geons, a problem whose solution eluded Wheeler.

• the three traditional tests of general relativity

As will become clearer when we discuss the Bohr atom and the Sommerfeld formula in the next chapter, so far as non-relativistic "orbits" go, the Coulomb attraction and Newtonian gravitation can be described in the same way except for the scaling ratio we have already computed — i.e. $e^2/Gm_p^2 = [2^{127} + 136]/137$. Consequently, if we compare the energy of a photon emitted near the surface of the sun with the energy it delivers when absorbed near the orbit of the earth, we will find that it is "red shifted" by the observed amount, thanks to our relativistic kinematics. For a photon emitted by a star and subsequently traveling near the sun on its way to us, this "Newtonian" interaction produces only half the observed deflection of starlight. However, our theory gives us spin 1 traveling photons, and we believe (though have yet to demonstrate in detail) that it gives spin 2 gravitons as well as the Newtonian term. [If this assumption fails, our theory is in serious trouble.] For any spin 1 photon, only one of the two helicity states can interact with the spin 2 graviton, because it can do so only by flipping the spin of the photon one way; this provides us with the needed factor of two. For gravitons emitted and absorbed by macroscopic objects, the Newtonian term gives only one-sixth the observed precession of the perihelion of Mercury. In this case, all five possible orientations of the spin 2 gravitons with respect to the plane of the macroscopic orbit, and not just the two helicity states, are relevant, giving us the needed factor of six. This argument is discussed a little more carefully in Reference 5, and provides an approach to meeting the traditional tests of general relativity within our framework. We do not as yet know how to tackle the effects of strong gravitational fields in bulk matter (eg macroscopic "black holes") from first principles.

• event horizon

• zero-velocity frame for the cosmic background radiation

General relativistic cosmologies coupled with the recessional velocity interpretation of the Hubble red shift necessarily drag the theorist down into extreme densities at early times. Hopefully they also connect the cosmological problems of origin to elementary particle physics in a very exciting way. But once swallowed, the theorist finds himself in a cloud coo-coo land from which he can extract himself only by making heroic efforts. Our problems are again much simpler because we cannot even begin to talk about space and time— the content strings — until we have generated the hierarchy labels. Although the details will depend on just what generation scheme we adopt, it is clear that in order to start talking about particles, space and time, we will need to have on hand at least the 139 discriminately independent basis strings of fixed length from which the hierarchy can be constructed in due course. Then we can start forming the content label ensembles which describe velocities, and the 4-events which allow us to specify the conserved quantities and to talk about the baryon number and lepton number of the universe. Whenever and however the appropriate label string length is fixed, the content string length continues to grow. This content string length specifies the "event horizon" of an expanding (in fact, as Wheeler once noted, an "uncrunchable") universe. Once the strings have appreciable length the average velocity is zero because the most probable number of 1's is half the string length, This fact defines the unique "zero velocity frame" for the background radiation and everything else. Of course this is no more in conflict with "special relativity" than the brute fact of the experimental discovery of this frame is for conventional cosmologies.

• mass of the universe

We assume that our cosmology stems from a generation scheme of the *program universe*[15] type in which two arbitrarily selected strings either produce a non-null string by discrimination or increment all extant strings by a single bit arbitrarily chosen for each string. Once we have generated 139 discriminately independent (basis) strings and fixed the label length L, we will need at least 2^{127} discriminations involving about $[2^{127}]^2$ strings to close the hierarchy labels and fix quantum number conservation in the terms discussed in the next chapter — i.e. with the usual particulate interpretation. We conclude that there must be at least $[2^{127}]^2$ particles around when we first can start to talk about baryon number conservation in "space-time" in a way that relates (linearly ?) with the here and now universe in which we practice physics. We can now make a choice — on cosmological grounds — of our unit of mass. If this is the proton mass, the mass of the universe will be around $2^{254}m_p = 4.84 \times 10^{52} gm$, which as we will see shortly is about right according to standard interpretations of current observations. If we were to take the Planck mass as the unit, as has been suggested occasionally, we would be out by a factor of 10^{19}. So we settle on m_p, or something close to it, as the basic mass to which we will relate all others. The conventional wisdom is in much worse shape here than we are. Most of their model universes are buried under a pile of (BLEEP) that weighs 10^{125} times[16,17] too much for them to dig their way out from under it - except by the observation that we nevertheless exist, and that human ingenuity should be able to find an explanation. Current efforts to meet the problem usually involve an inflationary scenario which necessarily ends up with the critical density (i.e. just the right amount of matter to close the universe) and may have advantages in smoothing out early fluctuations, but we think it simpler not to get into the problem in the first place.

- fireball time
- critical density

Now that we have identified c, $\hbar$ and our unit of mass, the unit of time is fixed as $\hbar/m_pc^2$. Although it takes a minimum of 2^{127} discriminations to get all the labels, each label is picked arbitrarily; the sample space contains $[2^{127}]^2$ pairs. We conclude that it will take $[2^{127}]^2\hbar/m_pc^2 = 3.5$ million years. before we can talk about "space", "time" and "particles" in anything like an ordinary sense. Clearly the universe is still "optically thick" up to this time. Since the initial content strings are very short, they will have velocities which are substantial fractions of c, making the initial universe very hot. This heat will be further enhanced by the decays of higher generations of quarks and leptons once the receding event horizon provides them world enough and time to decay into. For our density (see below), 3.5 million years will also be about the right time for the transition from an optically thick to an optically thin universe to occur. This is our estimate of "fireball time" — i.e the time when the radiation breaks away from the matter.

Of course the "time" calculated above involves the usual fiction that we can reliably extrapolate the universal expansion now observed using the laws of physics established here and now back into the hot plasma that is implied before fireball time and indeed linearly all the way down to a point singularity. Clearly both space and time loose their usual significance long before the singularity is reached. In our model we must construct some substantial fraction of the labels before they have even a modest connection with their usual meanings. From fireball time on, when the universal expansion is matter dominated; the linear extrapolation (or retrodiction) is plausible. Our "fireball time" is consistent with a universe that is 1.5×10^{10} years old and a Hubble constant of 50 $km/[s \cdot Mparsec]$ if we extrapolate backward from the currently observed $2.7^o K$ background radiation. These assumptions fix the current radius of the event horizon as 5.92 $\times 10^{26}$ cm; with our mass of $[2^{127}]^2 m_p = 4.84 \times 10^{52}$ gm our model has a density of known particle types relative to the critical density ρ_c (i.e. the density needed to just "close" the universe) of $\Omega = \rho/\rho_c = 0.01175$. Here we have taken our figures from Faber[18] and hence under our assumptions take the critical density to be 4.75 $\times 10^{-30}$ gm. His limits for this number for visible matter are $0.005 \leq \Omega_{Vis} \leq 0.02$ and for baryonic matter are $0.04 \leq \Omega_{Bar} \leq 0.14$. The higher limits for baryonic matter depend on detailed arguments about the cosmo- and nucleo-genesis of light elements ("deuteronomy"). Until we have our own calculation for these processes, which will differ in significant ways from the standard ones, we take the observational ("visible matter") number as the one to compare with our model, and are pleased by the agreement achieved. We cannot really calculate the 10^9 photons per baryon implied by our numbers, and have taken it from observation, which is standard practice.

- dark matter

The prejudice of most cosmologists is that the universe should be closed, or "just closed". In fact the current fashion, as noted above, is to use an inflationary scenario which predicts $\Omega = 1$. I find an open universe much more satisfactory, particularly after reading Dyson's scientific eschatological analysis[19]. The observational "deficit" from the conventional perspective is now to be made up by "dark matter". Here they have a good observational case in that ten times as much of the mass of galaxies, as measured by Newtonian gravitation and the Doppler shift, is "dark" rather than electromagnetically visible. How much more there is depends, once again, on details of the cosmological model rather than on observation.

Here our theory makes a new prediction. Visible matter can only be understood by us in terms of the 137 labels for the first three levels of the hierarchy. But there are 3+7=10 labels that cannot be interpreted prior to the formation of the "background" of the 127 labels which make up level 3. Whatever they are, they must be electrically neutral and will occur, statistically, 12.7 times more frequently than the level 3 labels. They could form electromagnetically inert structures at any scale compatible with our finite scheme (quantum geons?). So our estimate of the ratio of the amount of "dark matter" left over from the "big bang" to the visible matter is 12.7; a better estimate will depend on what version of the early stages of *program universe* we use. To understand in more detail how we can expect to get dark matter out of our theory we must first understand how we get ordinary matter, which is discussed in the next chapter.

4. ELEMENTARY PARTICLE PHYSICS

- quantum numbers of the standard model for quarks and leptons

Our general derivation of conservation laws and crossing for 4-events applies to the labels and (because the labels close) can be used to define additive conserved quantum numbers. This is discussed in Reference 4; we omit several technical details here. Although the labels are constructed from the "bottom up" by generating the combinatorial hierarchy, the physical interpretation is most easily explained from the "top down".

• gravitation: $\hbar c/Gm_p^2 = 2^{127} + 136 = 1.70147... \times 10^{38}$ [$1.6937(10) \times 10^{38}$]

Level 4. Our method of construction[15] necessarily assigns to level 4 all labels which couple to the lower levels. The universal label is simply the anti-null string $(1)_L$ containing L ones, which obviously couples to everything and also takes a particle label into an antiparticle label. Clearly this can be identified with Newtonian gravitation, and indeed has the right coupling constant, since it occurs with probability $1/[2^{127} + 136] \simeq Gm_p^2/\hbar c$. For weak gravitational fields this will carry either a null or an antinull content string and hence define the gravitational "light cone". The next simplest strings will be the spin 2 gravitons, which also carry one of these two content strings and will be constructed from a lepton-antilepton (levels 1 and 2) and a quark-antiquark (level 3) pair, insuring that they also couple to everything.

• weak-electromagnetic unification:

$G_F m_p^2 = 1/(256^2\sqrt{2}) = 1.07896 \times 10^{-5}$ [$1.02684(2) \times 10^{-5}$]

$sin^2\theta_{Weak} = 0.25$ [0.0229(4)]

• quark-lepton generations

The charged weak bosons $W^{\pm}$ couple an electrically neutral neutrino to a charged lepton (electron or quark) in the same way for each generation. Using 16 concatenated strings of length 16 to represent 16 generations, they will occur with probability 256^{-2}. Because of a conventional difference between the way the Fermi and the Yukawa couplings are written, this corresponds to a Fermi coupling constant $G_F m_p^2 = 1/[256^2\sqrt{2}]$. For the neutral weak boson (Z_0) to also be pseudoscalar — the obvious first approximation — we need the weak angle (conventionally defined) to be $sin^2\theta_{Weak} = 0.25$ compared to the empirical value[20] of 0.229 ± 0.004. We have yet to carry out the mass ratio calculations for these particles. Electrons couple to the coulomb and spin 1 massless vector quanta (i.e. photons) within level 2 and to two flavors of quarks with 1/3 or 2/3 the same probability within level 3. Since the electromagnetic interaction crosses the first three levels, and the pattern repeats for higher generations, the lowest order calculation of the coupling is $e^2/\hbar c = 1/137$.

- color confinement — quark and gluon masses not directly observable
- $m_{u,d}(0) = \frac{1}{3}m_p$
- the generation structure

Level 3 contains two flavors of fermion-antifermion pairs (16 states) with three colors in an octet (8 states) making up the 128 -1 =127 distinct labels required for this level. The 1 is subtracted, as usual, because the null string is not allowed as a label. Since McGoveran's Theorem grants us only three asymptotic degrees of freedom, and hence (for the quantum numbers) only three exact (to order $1/[2^{127} + 136]$) conservation laws, we take these to be charge, baryon number and lepton number in order to correspond to experience. Then there is no way that colored quarks or their associated colored gluons can appear asymptotically. In other words "color confinement" is a necessary consequence of our theory. In the first generation, we can use three quarks to form fermion color singlets with no charge or one unit of charge (neutron and proton). Neglecting the internal (unexamined) energy, the quarks in these systems will have one third of a nucleon's mass. We anticipate that when neutrons and protons are probed at high energy the *effective* mass of the quarks and gluons will fall off (asymptotic freedom), but have not as yet proved that this happens. Mesons are quark-antiquark pairs in appropriate colorless color-anticolor combinations;, the usual connection to low energy nuclear physics is maintained.

Level 2 consists of electrons, positrons, massless spin 1 quanta (photons) and the coulomb interaction. *Level 1* contains the two chiral neutrinos responsible for parity non-conservation, but whether the associated quantum is a graviton or the Z_0 can only be determined by looking back up to level 4. As already noted, this pattern can repeat 16 times to form 16 generations. Necessarily the coupling from lower to higher generations will diminish dramatically with generation number because of the combinatorial explosion; we are not yet in a position to make this statement quantitative by calculating the Kobiyashi-Maskawa mixing angles.

• dark matter again

Now that we understand the coupling scheme in more detail, we can see that when the construction starts we will get labels corresponding to the first two generations 127/10 times as often as we get the third generation labels which first allow us to talk about electromagnetism and visible matter. Eventually some of these more complex labels will settle down into the pattern explained above, but initially will be coupled to each other only by pre-gravitation. This fact is our reason for believing that with more work we will have a model for "quantum geons" composed of neutrinos, gamma rays and gravitons with 10 identifiable quantum states, as we mentioned above when discussing cosmology. Whether this dark matter will nucleate correctly to form the dark matter of the galaxies is still conjectural.

• the hydrogen atom

A hydrogen atom consists of an electron and a proton whose mass ratio is discussed below. Since our first order scheme requires that only 1 in 137 of the events which bind this composite structure will be a coulomb event, the other interpretations of the labels average out in the first stage of the analysis; in other words

$$137N_B \textit{ steps} = 1 \textit{ coulomb event}$$

This means that we now have two frequencies (in dimensional units of $\mu c^2/h$), the *zitterbewegung* frequency corresponding to the system mass μ, which we take to be unity, and the coulomb frequency $1/137N_B$. Since these two motions are incoherent, the frequencies must be added in quadrature subject to the constraint on the energy E defining a bound state that in the rest system $E/\mu c^2 < 1$. One way to express this constraint is

$$(E/\mu c^2)^2[1 + (1/137N_B)^2] = 1$$

In the language of the ordering operator calculus, this is simply the normalization of the metric corresponding to the energy attribute under the appropriate constraint. If we take $e^2/\hbar c = 1/137$, this is just the relativistic Bohr formula[21].

• the Sommerfeld formula

In either the non-relativistic Bohr theory or the non-relativistic Schroedinger equation, the coulomb problem suffers from a degeneracy between the principle quantum number N_B and the orbital angular momentum quantum number ℓ, because the energy depends only on the principle quantum number, or, in the correspondence limit, on the semi-major axis of the ellipse. Thinking semi-classically, Bohr[21] and Sommerfeld saw that the relativistic mass increase, which is most important at perihelion in elliptical orbits, would break this degeneracy, and Sommerfeld[22] computed the effect. Dirac[23] arrived at the same formula in what appears to be a very different way, but one which also depends on lifting the degeneracy between two integers. For both Sommerfeld and Dirac the problem was, in a sense, easier than for us because in conventional theories irrational, transcendental, "empirical", ... numbers live in a different world than the finite integers. Their methodology allows these non-constructive entities to enter the argument at appropriate points. We must face a harder problem in our theory.

Let j be an integer, and let successive values of s differ by integers so that $s = n + s_0$. Although s_0 is rational, it lifts the degeneracy by being non-integral. If j and s_0 differed *only* by a rational fraction rescaling would restore the degeneracy. Hence the 137 coulomb rescaling from the combinatorial hierarchy exoskeleton, or any other single integral rescaling, is not enough to meet the problem posed. If we combine the two independent integer (except for s_0) counts by starting them off as close as we can while maintaining the distinction (i.e. "synchronize" the counting), we can require that s_0 be the value closest to j that s can have. This can happen in two distinct ways. There is no way in the problem posed that we can directly observe the "synchronization" of the two periods, and both possibilities correspond to "coulomb events". We can either assume that the synchronization corresponds to $137j\frac{steps}{(coulomb\ event)} + 137s_0^+\frac{steps}{(coulomb\ event)} = 1 + \epsilon$ or to $137j\frac{steps}{(coulomb\ event)} - 137s_0^-\frac{steps}{(coulomb\ event)} = 1 - \epsilon$ where ϵ is some rational fraction less than unity.

Here we must use care because these two equations have different meanings and cannot simply be interpreted as if they represented numerical quantities which can be combined by linear operations. As we saw in our derivation of the relativistic Bohr formula, independent frequencies must be combined in quadrature. We can form the specific product defining the squares: $137^2 j^2 - 137^2 s_0^2 = 1 - \epsilon^2$; ϵ still must be computed. Note that the two factors of this equation are the conditions on j and s_0 stated above. With j fixed, the two values of s_0 implied by this equation were called $s_0^{\pm}$ above. Since j is to be the norm to which we refer, we form $j^2 - s_0^2 = (1-\epsilon^2)/137^2 = a^2$. Taking $s = n + s_0$ as the appropriate number to define internal frequency for the bound state, we can follow our discussion above for the single frequency case and conclude that

$$(E/\mu c^2)^2[1 + a^2/(n + \sqrt{j^2 - a^2})^2] = 1$$

This is precisely the Sommerfeld formula, provided we can interpret a^2 as α^2 (to order a^3 or α^3) and know how to take the square root in our discrete theory.

- the fine structure constant: $\frac{1}{\alpha} = \frac{137}{1 - \frac{1}{30 \times 127}} = 137.0359674...[137.035963(15)]$

In order to understand how we can have two independent rational frequencies in our theory of this problem, we have to go back to where the 137 came from. In the absence of other information, the 3 + 7 + 127 labels have to be generated for each of the two labeled strings which are coupled by the two coulomb events that (minimally) allow a bound state to be specified. But, if the end result is to be distinct, the way this is done the first time must be distinct from the way it happens the second time. For both events to be coulomb, the second time through the first two levels must already have closed, so only 1 in 127 events would correspond to an indistinguishable repetition of the first process. Hence the population from which a coulomb bound state event is selected is reduced by 1 in 127 compared to statistical independence; this is standard statistics for sampling without replacement. But for two spin 1/2 particles (electron and proton) only 1 in 16 possibilities out of the spin, particle-antiparticle, dichotomies will also coincide; the null case cannot occur in our scheme, leaving only 1 in 15×127 cases to be excluded. We conclude that the expectation of the "second" event being degenerate with the "first" event is just $1/(15 \times 127)$, which defines $2\epsilon = \frac{1}{15\times127}$ as the interval around unity by which $137^2 s_0^2$ can differ from $137^2 j^2$. In the physical situation we are only interested in the portion that occurs within the period for j, namely $1-\epsilon$. Therefore the statistical estimate for the fraction of the number of the steps that are neither part of j or s_0 is $\epsilon = \frac{1}{2} \cdot \frac{1}{127\times15}$

This two factor analysis of the way ϵ relates to the normalization equation $j^2 - s_0^2 = a^2$ raises another subtle point. When experimentalists use the Sommerfeld formula and the fine structure spectrum of hydrogen to evaluate α, they fit their results to α^2 and then take the square root. In order for this to correspond to the calculation we have made, we must take $a^2 = (1-\epsilon)^2/137^2$, and we expect them to find that $\frac{1}{a} = \frac{137}{1-\frac{1}{30\times127}} = 137.0359674...$ in comparison to the accepted empirical value[24] 137.035963(15).

Looking ahead, it is important to realize that the Sommerfeld formula in fact only holds for the fixed center problem, and cannot be corrected for the case of two finite masses by using the non-relativistic system mass $\mu = \frac{m_1 m_2}{m_1+m_2}$. The formula to order e^4 is given :[25–27] by:

$$S_n = m_1^2 + m_2^2 + \frac{2m_1m_2}{[1 + Z^2\alpha^2/(n-\epsilon_j)^2]^{\frac{1}{2}}}$$

where

$$\epsilon_j = j + \frac{1}{2} - \sqrt{(j+\frac{1}{2})^2 - Z^2\alpha^2}$$

and j is the total angular momentum for the Dirac case, or the orbital angular momentum (ℓ) for the spinless (Klein-Gordon) case. This is to be compared with the invariant mass for two free particles with velocities β_1, β_2, which is

$$M^2 = m_1^2 + m_2^2 + 2m_1\gamma_1 m_2\gamma_2(1 - \beta_1\beta_2 cos\ \theta_{12})$$

where $\gamma_i = [1-\beta_i^2]^{-\frac{1}{2}}$ and θ_{12} is the angle between the two velocities. Note that the *zitterbewegung* of the two masses adds in quadrature, which we argue it should on general grounds. Hopefully by the time of ANPA 11, or with some luck by the time of ANPA WEST 5, someone will see how to extend David McGoveran's calculation to two finite masses; this critical step must be taken before we can go on to muonium and positronium.

• $m_p/m_e = \frac{137\pi}{\frac{3}{14}\left(1+\frac{2}{7}+\frac{4}{49}\right)\frac{4}{5}} = 1836.151497...$ [1836.152701(100)]

The m_p/m_e formula is due to Parker-Rhodes[28] Our theory differs from his. In the past we could only provide heuristic justification for the calculation. Now that we have a fully developed relativistic quantum mechanics, with discrete 3-momentum conservation, these past arguments become rigorous when we view the calculation as a calculation of the mass in the electron propagator — for us, a finite "self-energy". One puzzle was the extreme accuracy of the result, using 137 rather than the empirical value for $1/\alpha$. But now that we have found that the "empirical value" comes about in systems which lack spherical symmetry, or in combinatorial terms have two independent frequencies, and recognize that in the m_p/m_e calculation there is no way to define a second frequency, we have a rigorous justification for the formula as it stands. Numerically, we predict $m_p/m_e = 1836.151497...$ as compared with[24]: (old) 1836.15152(70) and (new) 1836.152701(100). We see that the proposed revision in the fundamental constants has moved the empirical value outside of our prediction by a presumably significant amount. For the m_p/m_e calculation the correction due to non-electromagnetic interactions could be large enough to affect our results.

• $m_\pi \leq 274 m_e$: $[m_{\pi^\pm} = 273.13 m_e,\ m_{\pi^0} = 264.10 m_e]$

The estimate of the pion mass was made long ago.[29] The model is due to our interpretation of Dyson's argument[14] that the maximum number of charged particle pairs which can be *counted* within their own Compton wavelength using electromagnetic interactions is 137. Taking these to be electron-positron pairs, we get the result. The argument in the past rested on the use of the Coulomb "potential". Now that we have a combinatorial calculation of the Bohr atom, we no longer need this extraneous element. If one looks at the *content* strings minimally needed to describe the possible states of the bound system, the saturation at 137 pairs emerges. As we can see from the Bohr atom calculation (eg by considering one electron or positron interacting with the average charge of the rest of the system), the first approximation for the binding energy is non-relativistic in that it neglects v^2/c^2 effects. Consequently the simplest estimate for the system mass, interpreted as the neutral pion mass, is just the sum of the masses, or 274 m_e, in agreement with experiment to better than ten electron masses. It will be interesting to calculate the α relativistic corrections (including the virtual electron-positron annihilation) and the neutral pion lifetime. Adding an electron-antineutrino pair to get the π^-, or a positron-neutrino pair to get the π^+, will be a good problem for sorting out our understanding of weak-electromagnetic unification.

5. CONCLUSIONS AND A LOOK FORWARD

By now the reader could grant that we have made a case for discrete physics as a fundamental theory. We have been led to many conceptual and numerical results that can only be obtained with difficulty, or not at all, by more conventional approaches. We believe the program will prove to be useful even if it ultimately fails. So far we have run into no insuperable barriers — frankly somewhat to my surprise. We have nailed down the quantum numbers in agreement with the standard model, and have computed reasonable values for the basic masses and coupling constants. Thanks to the high degree of overdetermination of elementary particle physics due to crossing and unitarity — Chew's bootstrap — we can expect to do about as well as conventional strong interaction theories. This means that when a difficulty *does* arise, it will suggest an area of phenomena that will deserve detailed experimental and theoretical examination.

Homework for ANPA 11

1.In the paper for ANPA 10, David McGoveran gave an argument for the $2 \times 15 = 30$ factor in the fine structure constant calculation as coming from $\binom{6}{2} + \binom{6}{4} = 30$ rather than the way it is computed here, using the states of two spin

1/2 particles. He is convinced that the fundamental combinatorial argument can be worked out without referring to spin. Of course the two arguments could turn out to be equivalent, or — in an interesting and perhaps testable sense — the fine structure "constant" will have different corrections for fermion-fermion, fermion-boson and boson-boson systems. A casual look at the relevant data does not rule this out. The reconciliation between the two points of view could lie in the multiple ways a fermion-antifermion pair can "define" a boson, and will deserve some careful work.

2.My revised abstract for the Conference at Imperial College on *Physical Interpretations of Relativity Theory*, (Ref.5) reads: "Starting from our discrete and finite version of relativistic quantum mechanics, we show that the first order estimates of $\hbar c/e^2 = 137$ and $\hbar c/Gm_p^2 = 2^{127} + 136 \simeq 1.7 \times 10^{38}$ derived from the *combinatorial hierarchy* allow us to solve the Rutherford scattering and hydrogen atom problems, and the corresponding gravitational problems as problems in probability. The three classical predictions of *general* relativity — red shift, bending of light, and precession of the perihelion of Mercury — follow when we include (as our theory requires) spin 1 propagating photons and spin 2 propagating gravitons. We predict that a *macroscopic* electromagnetic orbit would have 4 times the Sommerfeld precession for basically the same reason that Mercury has six times the Sommerfeld precession." Supply your own arguments for these conclusions.

3. Using your treatment of Rutherford scattering in problem 2, define the four quantum numbers $(m; \beta, \beta_{\|}, J_z)$ with J_z the angular momentum component in the $\beta_{\|}$ "direction" using labeled bit strings. Relate these to the basis states in the Pauli-Brodsky discretized version of second quantized field theory.

4. Calculate the fine structure spectrum of positronium and the first order line width correction due to singlet two-photon decay.

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Summary of **WHERE WE ARE**

General structural results

- 3+1 asymptotic space-time
- transport (exponentiation) operator
- combinatorial construction of π
- limiting velocity
- supraluminal synchronization and correlation *without* supraluminal signaling
- discrete events
- discrete Lorentz transformations (for event-based coordinates)
- relativistic Bohr-Sommerfeld quantization
- non-commutativity between position and velocity
- conservation laws for Yukawa vertices and 4- events
- crossing symmetry

Gravitation and Cosmology

- the equivalence principle
- electromagnetic and gravitational unification
- the three traditional tests of general relativity
- event horizon
- zero-velocity frame for the cosmic background radiation
- mass of the visible universe: $[2^{127}]^2 m_p = 4.84 \times 10^{52}\ gm$
- fireball time: $[2^{127}]^2 \hbar / m_p c^2 = 3.5$ million years
- critical density: of $\Omega_{Vis} = \rho/\rho_c = 0.01175$ $[0.005 \leq \Omega_{Vis} \leq 0.02]$
- dark matter= 12.7 times visible matter [10??]

Unified theory of elementary particles

- quantum numbers of the standard model for quarks and leptons
- gravitation: $\hbar c / G m_p^2 = 2^{127} + 136 = 1.70147... \times 10^{38}$ $[1.6937(10) \times 10^{38}]$
- weak-electromagnetic unification:
  $G_F m_p^2 = 1/[256^2 \sqrt{2} m_p^2] = 1.07896 \times 10^{-5} m_p^{-2}$ $[1.02684(2) \times 10^{-5}]$;
  $sin^2 \theta_{Weak} = 0.25$ $[0.0229(4)]$
- the quark-lepton generation structure
- generations weakly coupled with rapidly diminishing strength
- color confinement — quark and gluon masses not directly observable
- $m_{u,d}(0) = \frac{1}{3} m_p$
- the hydrogen atom: $(E/\mu c^2)^2 [1 + (1/137 N_B)^2] = 1$
- the Sommerfeld formula: $(E/\mu c^2)^2 [1 + a^2/(n + \sqrt{j^2 - a^2})^2] = 1$
- the fine structure constant: $\frac{1}{\alpha} = \frac{137}{1 - \frac{1}{30 \times 127}} = 137.0359674...$ $[137.035963(15)]$
- $m_p/m_e = \frac{137\pi}{\frac{3}{14}\left(1+\frac{2}{7}+\frac{4}{49}\right)\frac{4}{5}} = 1836.151497...$ $[1836.152701(100)]$
- $m_\pi \leq 274 m_e$: $[m_{\pi^\pm} = 273.13 m_e,\ m_{\pi^0} = 264.10 m_e]$

THE FINE STRUCTURE OF HYDROGEN *

REVISED and EXPANDED DRAFT
CIRCULATED FOR COMMENTS AND CORRECTIONS

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I. INTRODUCTION

The equivalent formulae of Sommerfeld and Dirac for the fine structure of hydrogen were crowning achievements of their day. The methods for achieving this result were apparently quite different and yet have recently been shown to be equivalent (The "Sommerfeld Puzzle" Revisited and Resolved by L. Biedenharn). In this paper we introduce a new and quite distinct derivation of the formula based upon the Combinatorial Hierarchy (Bastin, T., Studia Philosophica Gandensia, 4, 77, 1966) and the ordering operator calculus (McGoveran, D.O., Foundations of a Discrete Physics).

The Combinatorial Hierarchy led to the definition of the fine structure constant as 1/137 as a first order approximation. Our conceptual model of the hydrogen atom leads us to compute a second order correction factor for α. Estimates of the third order corrections (including $(1/137)^2$ and $(m_e/m_p)^2$) are beyond current experimental precision, so that the correctness of the second order approximation must be correct to about four decimal places <u>given</u> <u>our</u> <u>current</u> <u>conceptual</u> <u>model</u>. [Note that these statements apply to low-energy values of α. Further relativistic refinements are required for higher energy measurements (e.g., at the mass of the W.]

This theory is very different from that of quantum mechanics, not only in its conceptual but in its mathematical foundations. It leads to different results than do standard theories when they are extended beyond ordinary quantum mechanics to include relativity. Indeed, our theory is intrinsically relativistic and discrete, so that our model of what the Sommerfeld formula represents is different than the Sommerfeld semi-classical, the non-relativistic, or the Dirac models of the hydrogen atom.

II. PRELIMINARY CONCEPTS

<u>Terminology</u>

Throughout this paper, terms should be read as being mathematical rather than physical, unless explicitly given as physical. Thus, the terms event, cycle, period, and frequency do not have the familiar temporal and physical connotations.

Value and probability measures become indistinguishable in the ordering operator calculus. The probability measure is taken as counts of occurrences of some abstract event over an unnormalized (we do not mean unnormable) space - we only divide counts by the maximum cardinality N of the space when we want the relative probabilitistic frequency of some combinatoric attribute. So the space (e.g., 1-dimensional) is not on the [0,1] interval but on the [0,N] interval.

For our purposes, all objects of investigation will be treated as occurrences of what we call **combinatorial events** and the relationships between combinatorial events. When representing a given occurrence of some combinatorial event (using a bit string representation) we look for a bit string pattern (not substring) as output from the ordering operator (which could be the PU algorithm). If the combinatorial event recurs in a regular way, we say it is **periodic** and the number of bits generated between occurrences is the **cyclicity**. Then any generation of that many bits constitutes a **cycle**. These are mathematical, not physical terms. Whenever we use the term event, we shall mean a combinatorial event unless the term is otherwise modified.

A string of cyclicity n means that the string can be seen as the concatenation of similar substrings of length n. Under a physical interpretation (meaning the combinatorial event is interpreted as a physical event - see next paragraph), this cyclicity would be called a **period**. However, we are reluctant to use this term since it introduces connotations of physical time, which is not strictly implied by the mathematics. We will refer to a bit string representation of an event having a cyclicity as a **periodicity** for purposes of the discussion in sections IV and V.

By a **Program Universe event**, we shall mean a vertex in the sense defined by Noyes in the interpretation of Program Universe. It is not clear that all these vertices have a physically observable interpretation however, and so we distinguish between Program Universe events and physical events. Thus, a **physical event** as used here is a Program Universe event which has a physically observable interpretation. A physical event which participates in a Coulomb interaction will be referred to as a **Coulomb event**.

Ordering Operators

An **ordering operator** is a finite computational object which can be thought of as generating a **finite directed graph**, a mathematical object consisting of nodes and directed (i.e., ordered) arcs between nodes. Such a graph defines the ordering operator and vice-versa. Thus, the ordering operator calculus is process-oriented. When a node of the graph is connected by directed arcs leading either toward or from (but not both) two or more other nodes, a partial ordering is represented and the nodes are indistinguishable. Two nodes with a single directed arc between them are totally ordered. The possible walks of a graph generated by an ordering operator include all the possible "jumps" between indistinguishable nodes. One may think of such jumps as "virtual arcs", although no direction is implied since indistinguishables are not intrinsically ordered. Note that any node may be thought of as an abstract representation of a graph that has been previously constructed.

Ordering operators are quite general and powerful, but are always well-behaved. As a result, there is often no need to impose constraints of linearity on the mathematical objects constructed within the ordering operator calculus. Operators may act on graphs to generate an ordering of the indistinguishable nodes of the graph. Two or more operators may be thought of as operating concurrently on the same graph and operators may order two or more graphs to create more complex objects. Under a particular ordering operator, particular nodes of two otherwise distinct graphs may be taken as indistinguishable (e.g. when they are partially ordered under the given operator).

A sub-graph can be understood as representing a particular property or attribute of the graph, called a combinatorial attribute. It is often convenient to represent a walk of a particular graph by a binary sequence called a bit string. The representation of the walk is always in terms of some combinatorial attribute. One writes down a '1' if the attribute is encountered in the walk and a '0' if it is not after having traversed a number of nodes equivalent to the number of nodes required to traverse the sub-graph which represents the attribute.

If one decomposes the graph into several sub-graphs, these form a representation of the graph, so long as we know how to re-connect the sub-graphs. For a given attribute, the sub-graphs can be walked to generate bit strings representations. Given the context, the bit strings may be used to reconstruct the graph. It is often convenient then to think of ordering operators as generating bit strings instead of directed graphs. In this paper, we will use this concept of bit string generation when referring to the output of an ordering operator.

Probabilities

The ordering operator calculus treats all quantities as being combinatoric. This has the effect of (1) ensuring that all values and equations are algebraic, and more important (2) unifying the way in which both values (numbers) and probabilities are treated. The only difference is that values are typically given as integer counts (of the occurrences of some combinatoric attribute) and probabilities as that number normalized by the largest value then possible, i.e. the frequency interpretation of probabilities, so that the rational result lies between 0 and 1. If the probability space remains unnormalized, then there is no difference in the two spaces. Thus, in this paper, we will use the terms number, probability, and frequency interchangeably, context making it clear that normalization may be implied in order to obtain a probability or frequency from a number (count of occurrences).

Independence

There is an additional consideration. Ordinarily, there is the concept in probability that two events are either mutually independent or they are dependent. In the ordering operator calculus, due to the treatment of indistinguishables, there exists a smooth, though discrete progression from mutual independence to complete dependence. (Whether or not it is always appropriate to measure the degree of dependence via correlation coefficients is a matter of current investigation.)

This "spectrum" forms a multi-dimensional discrete and finite vector space which we have called a combinatorial attribute d-space. A d-set of completely independent combinatorial attributes forms a d-basis for the d-space. Two or more mutually independent attributes add as orthogonal vectors in the space.

It is also important to understand that the process notion of independence differs somewhat from the "static" definition. Because the size of the space is always changing in a process view, constraints and relationships come into play at various stages.

For example, although we might have a final relationship between x and y (whose sequences of values is counted by t), this relationship could not be said to be constructive until the space was large enough (say of cardinality N) to represent the computation of the relationship. Until the space has been generated to cardinality N, x and y are independent. Thereafter, they are dependent.

As a second example, and a simple extension of the first, suppose that x and y must be satisfy some relationship whenever t satisfies some condition. Then, it is not the case that x and y are correlated except when this condition on t is also satisfied. Between these pairs of values of x and y, no correlation can be stated.

An important point here: since this is a process system, independence can be transient. Clearly two periodic events are not independent if they are taken to have commensurate periods (just as the minor and major generators of an elliptic oscillator are not independent over time intervals larger than period of the system). However, they must be independent outside of this to 'lift the degeneracy' (i.e. - to get away from the equivalent of a circular orbit).

Objects

"An object is a conceptual carrier of [possibly complex] combinatorial attributes between [possibly complex] combinatorial events [which may correspond to physical events under a particular interpretation]."

Note that this definition reduces to the definition of particle given by Noyes (Noyes, H.P., and McGoveran, D.O., An Essay on Discrete Foundations for Physics, Physics Essays, 2, No. 1, (1989)) in presenting the interpretation of Program Universe:

"A particle is a conceptual carrier of conserved 3-momentum and quantum numbers between events."

Classical physics actually imposes many constraints on the nature of experiments as does quantum mechanics and relativity. A correct statement of a correspondence principle must take into account all the differences between these various domains. Between quantum mechanics and classical physics the primary differences have to do with how properties combine, which is only approximately correlated with object size. Part of the reason for this lack of correlation is due to the differences between how objects are viewed. In quantum mechanics, properties can be exchanged between objects and objects can be created and destroyed. In classical physics, object integrity is always maintain homogeneously - objects may be broken as in an elastic collision, but they do not change properties.

The classical notion of an object as having well-defined boundaries is contrary to normal experience. In addition, a similar problem exists with regard to what properties define an object. This problem has been analyzed in a manner different from that one used here by such researchers as Lofti Zadeh using "fuzzy logic" and, by application, "fuzzy pattern recognition". Those involved in other approaches pattern recognition (such as the clustering analysis methods of Nils Nilson) are closer to our approach and have long recognized the detailed problems.

While is it easy to see that one might argue that an upholstered chair is still a chair if the upholstery is removed, it is less easy to see that the question of chairness might or might not include the "warmth" of the object or its sentimental history or the airflow around the object. Indeed, the physical extent of an object into its environs is always in question. Anyone involved in the legal issues of property rights is well aware of this on-going and difficult debate. The point is that the classical physics notion of object does not meet the needs of everyday experience.

Part of the problem is due to the insistence that objects with well-defined boundaries and mix of properties PERSIST between observations. This is not, of course, altogether unreasonable. We are able to "intercept" the object along the classical trajectory and observe a confluence of roughly the same set of properties. But clearly this is not always the case: we can not always backtrack to a prior observation nor does an object evolve along all possible classical trajectories. We give reasons for this, but these are reasons which are valid only if one constrains experiment according to the classical physics world view. The notion of what constitutes a clean experiment not only serves to define the conditions, but to make them repeatable. And in so doing, the procedure eliminates numerous possible interactions with the environment, eliminates the complexities of impure objects and operations, and the complexities of what happens when the experimental prediction must be based on partial information.

In addition, the classical physics paradigm assumes that all objects are distinguishable. But this is no more true in the macroscopic domain than in the microscopic. Consider the "poor identification" of criminals based on eye-witness accounts. The lack of information, in principle available, which might distinguish the accused from the guilty leads to exchange of individuals as equals. A rose is a rose is a rose... The only difference between exchanging eggs in two omelets and electrons in two preparations is an ontological statement in the latter that the distinguishing information is, even in principle, not obtainable (see Bohm, Causality and Chance in Modern Physics for an extensive discussion of the lack of justification for or against this commitment).

The manner in which individuals decompose a complex object into supposedly less complex objects (and this is a major assumption) is heavily biased by culture. While it is certainly the case that decomposition of the environment is promoted along the lines of what can be easily manipulated. An object is perceived to be distinct if it can be separated physically from the rest of its environment, if it can be moved. But what can be and can not be moved is dependent on the technological devices available to the culture. And even if available, the perception will not be realized unless this is motivated by the needs of the culture.

If we give up the absolute character of the object (i.e. its Platonic reality) then we are left with the confluence of its properties. This gives us a characterization of an object at a particular time and in a particular context. We use pattern recognition techniques to correlate this confluence of properties with others. If there exists a strong correlation and a causal structure connecting the two confluences, we are ordinarily inclined to say that there is an object which carries the properties. (This is exactly what the magician depends upon to work his deception!) But this is a mistake. The interaction between and confluence of properties is an evolving aspect of systems. Treated in this way, the macroscopic world begins to look more compatible with the microscopic world.

There are other simplifications about the macroscopic world which classical physics promotes. For example, the notion that all classical operations commute. The notion that pure states do not exist. The notion that superposition is invalid. There are macroscopic counterexamples to each of these concepts.

A quantum mechanical interaction is more closely akin to a classical chemical reaction than to a classical object interaction: the properties may be exchanged between the reactants and products and apparent new properties may arise. However, the fundamental properties are always conserved. With a quantum mechanical interaction, many of the reaction parameters are only defined statistically, so that reactants and products are defined probabilistically.

Distance Functions and Independence

The adding of independent values arises from a straightforward consideration of (a) the requirements given in Foundations for a distance function and a norm, (b) the assumption of a locally flat space, and (c) the independence (which becomes orthogonality under a value measure) of the generations of the two probabilistic frequencies.

The ordering operator calculus defines an inner product as a recursive function n() which satisfies the following where a is any rational number and x is a d-vector on a vector d-space:

P8: $n(ax) = |a| * n(x)$ (1)

It is a distance function if it satisfies the following postulates:

P9: $n(x) >= 0$ and $n(x) = 0$ iff $x = 0$ (2)

P10: $n(x + y) <= n(x) + n(y)$ (3)

It is bilinear and symmetric (as it must be if the space is homogenous) if it also satisfies:

P11: $n(x + y)^2 + n(x - y)^2 = 2[n(x)]^2 + 2[n(y)]^2$ (4)

In terms of '0's and '1's, the distance function can be defined as

$$(I-D)/N \qquad (5)$$

where I is the number of increments, D is the number of decrements, and N is the number of possible increments and decrements. Note that the maximum value of I+D is not necessarily N in the event that indistinguishables are encountered. This is just a modified Hamming distance. The counts, I and D are, of course, not pure numbers - they depend upon the walk of the particular graph and also on the particular attribute whose occurrences are being counted. Thus, a count represents a particular sequence: it has a structure. The manner in which the count was obtained is important as to how we may manipulate the value.

Such a distance function is used to ensure that magnitudes are always used consistently. Clearly, the counts resulting from two partial walks of the same graph should add linearly - they are not independent if they must add to the length of the total walk. Independent walks, however, imply that no nodes may be walked twice. Adding squares implies that there exists a graph the walk of which could replace the two independent walks, and that there are decompositions of this graph which can represent each of the independent graphs. In other words, the replacement graph preserves the basis for the original graph. Then we must generalize the distance function given above to include d-vector resultants.

As in a Euclidean continuum, the resultant magnitude **c** of two orthogonal d-vectors in a locally flat (discrete but otherwise Euclidean) d-space of magnitude **a** and **b** is given by

$$a^2 + b^2 = c^2 \tag{6}$$

In a locally flat discretum however, the value of $\mathbf{c}^2$ may be computed, but the value of **c** as a square root may not exist. Instead, we look for the symmetric factors, so that the structure by which the count was obtained is taken into account. By symmetric factors we mean a pair of values **c'+e** and **c'-e** where both **c'** and **e** are rational and

$$c^2 = (c'+e)^2(c'-e)^2 = c'^2 - e^2 \tag{7}$$

Note that (7) assumes the commutativity of **c'** and **e**, but this can be proven from the construction if a and b are truly independent. In a non-flat space, this commutativity will fail. Since we deal only with algebraic quantities (i.e. with c either rational or irrational but not transcendental), there is always a solution set for (7). Thus independent quantities add via their squares if the space if locally flat (by local here we mean simply over the extent of the walk by which the quantities are obtained).

If the discretum is not locally flat, then there is a correction due to the "curvature". This fact will have a consequence when we discuss possible corrections to the results presented below.

Comparing New Theoretical Results to Old

The computational results of analysis of any particular experiment is dependent upon the interpretation according to a given theory. Of course, some of the analysis is dependent upon more obvious constraints given by the experimental situation. However, the determination of which numbers correspond to physical quantities is dependent upon the interpretation. For example, the relative importance of values of the factors of a polynomial versus the evaluation of the polynomial itself is key.

If the polynomial is incorrect (in that, like the wave equation, it does correspond to any structure in the system being modeled but is only a computational procedure), then the relationship of the factors to the evaluation of the polynomial is also incorrect.

Suppose that, in the correct theory, there are two factors - **(1-e)** and **(1+e)**. Since these factors are symmetrical, it is easy to confuse them and measure simply **+e** and **-e** so that only one value is perceived experimentally. It is then natural to assume that the polynomial is **K(1-e)(1-e)** rather than **K(1-e)(1+e)**, **K** being some third factor, if the extant theory ascribes only a simple structure (a value squared) to the polynomial. In order to compute the accepted experimental value, one must make this same mistake. However, the value of e is the theoretical value given by the new theory.

Within a methodology which provides for iterative improvement, it is considered to be a task of a new theory to explain how the theory it displaces computes the values it does, why the old theory computes correct answers when it does and why it computes incorrect answers when it does. It is important in all cases to understand why the two theories differ and why they agree. To the degree that this is possible, the theory be purposefully improved to provide a more encompassing explanation of the known facts.

Combinatorial Hierarchy

Many of the results of the ordering operator calculus depend on how the space is generated, that is, on what ordering operators are used to construct the space and on how they interrelate. For applications to physics, we have (since 1981) attempted to exploit the point of view that the ordering operators must also generate what is called the Combinatorial Hierarchy. The reasons for this selection have been discussed in other papers (e.g., McGoveran, D.O, Foundations of a Discrete Physics, in DISCRETE AND COMBINATORIAL PHYSICS, H.P.Noyes, ed.; ANPA WEST, 25 Buena Vista Way, Mill Valley, CA 94941).

Roughly speaking, one expects a hierarchy in physical processes which are decomposable into self-similar sub-systems. If a discrete, recursive process is an appropriate description of the elementary processes at work in the physical world, then clearly such a structure is called for in order to explain the rapid changes in degree of complexity as one changes scale.

Furthermore, at each level of the hierarchy, it must be possible to treat the elements as a complete and independent set of operators which operate on objects of less complexity and as a complete independent basis for representing more complex objects. If the representational basis is to be understood in a traditional (physical) sense, then independence (orthogonality) and linearity are appropriate properties. Simply put, these notions of completeness and independence mean that the hierarchy will contain its own means for abstract representation of all possible operations and all possible objects. The abstraction will be minimal and yet no conceptual power will be lost.

In addition, the algebraic structure for a sufficiently large numbers of elements (whether because the number of elements within a given level of the hierarchy is large or the hierarchy can be repeated) should be practically indistinguishable from a classical continuum structure. We are motivated to use a system which emphasizes the importance of discrimination (exclusive OR) over the field Z_2, since the ability to distinguish two objects is clearly of fundamental importance. That the Combinatorial Hierarchy satisfies this mathematical "correspondence principle" was pointed out to the author by Herbert Doughty, III (private correspondence).

The Combinatorial Hierarchy is generated from two recursively generated sequences. The first is governed by the recursion formula

$$n_{(i+1)} = 2^{n_i} - 1, \qquad (8)$$

(a formula familiar to those who have studied the Mersenne primes), and begins with the term $n = 2$ leading to the sequence 3, 7, 127, $2^{127} - 1$,... The cumulative cardinals of this series (ignoring the initial term) also form a series which has interpretive significance, namely 2, 3, 10, 137, ~1.7016... * 10e38 + 137,...

The second recursively generated sequence is governed by the formula

$$m_{(i+1)} = m_i^{\ 2} \qquad (9)$$

also beginning with the term **m** = **2** and leading to the sequence 2, 4, 16, 256, 65536,...

These two sequences have various justifications. Perhaps the clearest presentation has been given by Clive Kilmister (correspondence to H.P.Noyes, date Oct.16, 1978), paraphrased here as follows:

Definition: By a **combinatorial hierarchy** is meant a collection of levels related as follows:

a) the elements at level L are a basis of a vector space V/Z_2

b) the elements at level L+1 are non-singular (i.e. invertible) linear operators mapping $V/Z2$ into $V/Z2$

c) each element A at level L+1 are mapped to a subset S of the elements at level L by the correspondence: the proper eigenvalues of A [i.e., Av = v] are exactly the linear subspace generated by S.

d) each element at level L+1 is chosen independent, allowing the process to be repeated for level L+2, L+3, L+4, ...

Theorem 1: There exists a unique hierarchy (up to isomorphism) with more than 3-levels and it has the following successive numbers of elements:

$$2,\ 3,\ 7,\ 127,\ 2^{127} - 1$$

and terminates at Level 4 due to the fact that the operators have m^2 elements if the vectors are m-fold, and 2^n (required for V/Z_2 increases too fast.

One can think of the number of elements at each level of a combinatorial hierarchy as being the number of subsets of a set of n things. Given the definition of a hierarchy over the field Z_2, this generates the first sequence mentioned above. Over a finite field V, all operators which map V into V may be thought of as permutations in that they map elements of V into other elements of V and this map can be given a pairwise representation. Note then that is it possible to think of the number of independent operators as simply the number of permutations of n things taken two at a time (i.e. exchange of a and b corresponds to mapping a to b) or m^2 total at any level of the hierarchy, given m at the previous level. Clearly, this forms a complete set of independent operators - all other permutations of n things can be created by successive application of this set of permutation operators. This generates the second sequence mentioned above which governs a combinatorial hierarchy.

Mapping the first sequence onto the second and treating the second sequence as independent basis strings, one finds that the mapping is not uniquely possible beyond the fourth term or level. Thus the first sequence can not be a coordinate basis beyond level four of the hierarchy. The inter-relationship forms a **stop rule** which signals the end of global novel structure generation, although not necessarily the end of generation and redundant structure. In other words, if one codes the algorithm as a program, there will be a point at which no further novelty will result, although the program need not halt altogether. The cardinals of elements and operators at each level will be determined by the two sequences given above. As noted in Theorem 1, any two runs of the program will produce hierarchies which are isomorphic, even though the particular evolution may differ and the particular objects used to represent the elements may differ. For this reason, and since the details of evolution are generally not significant for us, we refer to **The Combinatorial Hierarchy.**

This way of thinking about the Combinatorial Hierarchy allows one to understand the conceptual nature of the increases in complexity. In a sense, the subsets of a set are objects created through a higher level of abstraction than is the set or its elements. Indeed, it would not be proper to say that the subset of a set S consisting of the element x is conceptually identical to the element x. To do so ignores the context of x.

Because the Combinatorial Hierarchy is defined over the field Z_2, it is now considered appropriate (and convenient) to think of the cardinals as counting binary strings, leading to the so-called "bit-string" representation, with the basic operations of discrimination (exclusive OR) and concatenation of arbitrary bits onto extant bit strings. This representation allows the hierarchy generation to be given in algorithmic terms, an exercise which, along with a particular interpretation of the physical meanings of the bit strings, forms the conceptual basis of the Program Universe (PU) interpretation and representation.

Labels

It is important for our purposes to understand the purpose of a basis representation such as is provided by the second sequence above. In particular, it provides a unique label or name for every distinguishable object to be represented in the space. This is true even for ordinary vector spaces: the basis representation allows one to uniquely represent a given point in the vector space as a unique n-tuple.

We were convinced long ago that the kind of labeling scheme suggested here is completely equivalent to Noyes' labeling scheme (interpretation of Program Universe), but noted that Noyes fails to make this relationship to the mapping space (i.e. the space of n^2 basis elements) explicit. Furthermore, Manthey does not provide a particular mechanism (code) within Program Universe (as written) for it. Indeed, this is one of the reasons for wanting to rewrite the program into "UP", being explicit in the constructive generation of Combinatorial Hierarchy everywhere rather than generating a structure which was "rich" enough to support the Combinatorial Hierarchy and simply showing that a model of the Combinatorial Hierarchy exists in the output of Program Universe.

We will refer to the generation of a bit string under various constraints (see the discussion on 3- and 4-events in Noyes, H.P. and McGoveran, D.O., An Essay on Discrete Foundations for Physics, SLAC-PUB-4528, pp. 35-43) as an event. This is simply a combinatorial event, provided no physical interpretation is implied. If the constraints are properly defined, the event can be interpreted as representing a physical event. Otherwise, it is simply an occurrence of a particular bit string having a computable probability of occurrence given the mechanism by which the background population of extant bit strings has been generated.

Keep in mind that there are two independent constructions in the Combinatorial Hierarchy (PU generated) space of strings which are distinct from the representation or mapping space. The first are given in a space which models the system, whereas the latter provide a common representation which appropriately labels the first strings. When strings generated by two (or more) independent constructions are mapped to the same label in the representation space, this does not mean they are identical but only that they are taken as representing the same information in the constructive context. It is incorrect to conclude that "if A maps to L and B maps to L, then A is the same as B." This allows the multiple runs of PU (or any other Combinatorial Hierarchy algorithm) generated strings to be truly independent even in the sense of interpretation and this must be so.

III. AN IMAGINATIVE PICTURE

The imaginative picture which we describe here is not meant to be accurate. Rather it is intended to motivate the reader conceptually. As we go, we will try to point out inaccuracies which might otherwise be misleading.

We begin by considering an experiment which we describe operationally as follows. The equipment consists of a digital sampling oscilloscope. Each count in a time period is based upon the occurrence of a particular digital pattern. There are two inputs. Each trace may be adjusted both horizontally and vertically. In addition, the magnification may be adjusted.

There are two ways in which a trace may be considered to be periodic. First, we may have a periodic structure in which the counts in each time period are periodic. In this way the amplitudes form a stable trace.

The second is an abstraction of the first to which we will appeal in the derivation which is to follow. Each trace occurs periodically. As a result, we can represent the entire trace as a pattern and set the input to detect this pattern. Then the trace consists solely of one or more points.

Each point on the trace represents what we call an event. Whether or not an event is elementary as in the first description or complex in the second does not matter. What is important is whether or not the events are periodic.

Suppose that we wish to compare two traces. Clearly, with the adjustments available, we can adjust the two traces so that the first points are coincident. However, there are not sufficient degrees of freedom to adjust the position of the beginning of the next period so that the corresponding initial points are guaranteed coincident as well. In particular, this will not be possible when either (1) the periods are relatively prime or (2) the minimal sampling period will not resolve the differences or (3) the resolution allowed by the adjustments is too course.

Suppose that we select one of the two inputs as the reference and adjust the second trace to be as nearly coincident with it as possible. If exact coincidence is not possible, then the best we can infer is that the first point of the second trace will derive from an event which occurs within one time increment from the corresponding point from the second trace, even if it is coincident on the trace. It will be convenient to think of the degree of coincidence between the two traces as the relative "phase" of the two traces. However, one must be careful not to let this classical language intrude on the analysis and remember the discrete nature of the system.

Now suppose that due to the experimental arrangement we are led to believe that the two traces derive from two independent aspects of the same source. Suppose further that the two inputs are known to be digitally produced; if we know exactly how they are generated, we can inquire as to the probability that the two inputs are distinguishable and under what circumstances the initial points of the beginning of each period of each trace remain distinct.

Again, one must not forget that the entirety is digital - the sources of the signals as well as the measurement apparatus and the adjustments it provides are digital and finite. Not only does this mean that the signals might not be perfectly synchronized in the measurement process, but also that the degree to which they are not synchronized is, in principle, statistically computable in a combinatorial fashion. In a sense, more information is available than would be the case if the signals were continuous - there are only a finite number of values by which the two signals may differ.

Suppose that the measurement apparatus with its intelligent inputs is connected to the source via a single channel - namely that we depend on computational pattern recognition to distinguish (i.e., separate or unmix) the two input signals. Suppose further that display consists of a single point per signal per period. Then pure synchronization of the two signals corresponds to indistinguishability. On the other hand, this "degeneracy is lifted" if subsequent periods beyond the first may not be synchronized on the display - they are thus shown to be distinct within the context of the experimental sources and the measurement apparatus. Given knowledge of the manner in which the two sources are generated and assuming perfect pattern recognition, we may compute the probability that the degeneracy is lifted in some way so that the display of the two traces differs by one or more divisions.

The difficulty with this picture is that it suggests that the problem of perfect synchronization can be solved by a measurement apparatus of higher resolution or other improvements. It also suggests that the two signals might be differentiated a priori. In our theory, neither of these is possible. The discreteness and the finiteness of the system is absolute and can not be circumvented. What follows will demonstrate the manner in which this situation explains a number of otherwise unexplainable phenomena. We will refer to the patterns being detected as bit strings and to their detection as bit string events.

IV. DERIVATION OF THE FINE STRUCTURE CONSTANT

Let us examine how the ordering operator calculus tells us to treat the problem of multiple bit strings events where the bit strings have some cyclicity n. We begin with a review of how a single periodic structure of bit string events can be used to derive the Bohr atom, modified from Noyes argument (McGoveran, D.O. and Noyes, H.P., On the Fine Structure of Hydrogen, SLAC-PUB-4730(rev. 3/15/89), submitted to Physical Review Letters).

The Bohr Atom

Part of a Program Universe bit string is interpreted as representing the quantum numbers generated by the Combinatorial Hierarchy and the remainder of the string of length n represents the increments, k 1's, and decrements, n - k 0's, in pre-space between events. Each step has an attribute velocity which we interpret to be the limiting velocity c and a length of h/mc; hence the attribute velocity between events is

$$\beta c = [(2k/n) - 1]c.$$

If we wish to model a "constant velocity" β, this restricts a string length of nN to have kN increments, defining the "deBroglie wavelength periodicity" N as the "positions" where events could (but need not) occur. Because the step-wise change in position h/mc implies a change in momentum mc, both of which can be reversed at the next step enclosing an area h in a discrete phase space (i.e., a discrete space having momentum and position as bases), or more generally enclosing a area nNh when we return to a cyclic starting point after nN steps, we have derived relativistic Bohr-Sommerfeld quantization from our model, including the zitterbewegung associated with the string mass specified by the system label.

We attribute the binding of m_e to m_p in the hydrogen atom to Coulomb events, that is, only to those events which involve one of the 137 labels at level 3, and hence occur with probability 1/137. (Noyes has phrased this as "on the average one in 137"; we will not introduce in this paper the complexity of assumptions which speaking of averages implies. Rather we assume that it is sufficient to be more deterministic for now so that the occurrence of a Coulomb event is strictly periodic in our sense. This does not mean that the strict periodicity is observable; remember that we have not introduced observable time in our construction.) The changes due to other events are indistinguishable in the absence of additional information. We can have any periodicity of the form 137j where j is any (positive) integer.

So long as j is the only periodicity, we can say that

137j steps = 1 Coulomb event

or

137j steps/Coulomb event = 1 (10)

Thus, (10) is a units conversion factor or, in our terminology, a rule of correspondence. Now characterize the system by some attribute which we identify with the energy. The value of the total system rest energy in terms of the system mass u is just μc^2 (where $\mu = (m_e m_p)/(m_e + m_p)$. We meet the bound state requirement that the attribute associated with the energy E be less than that associated with the system rest energy. Since the internal frequency 1/137j is generated independently form the zitterbewegung frequency which specifies the mass scale, we can express the system rest energy in terms of a portion which is due to 1/137j proportional to that part which independent of this frequency (which we will call E). There are two ways in which these two portions may combine (to represent probabilities in the frequency sense) combinatorially, namely

$$E(1 + 1/137j) = \mu c^2 \text{ and } E(1 - 1/137j) = \mu c^2$$

These behave as two factors in the observed system, combining by multiplication, yielding

$$(E/\mu c^2)^2\ [1+(1/137j)^2] = 1 \qquad (11)$$

Identifying $e^2/\hbar c$ as 1/137 to first order as implied by our interpretation of the Combinatorial Hierarchy, (11) is just the relativistic Bohr formula (Bohr, N., Phil.Mag, 332-335, Feb. 1915).

The Bohr-Sommerfeld Atom

In a system of multiple combinatoric constructions, the constraints are always sufficient to define the possible combinatorial paths leading to a particular generation of an attribute instance or combinatorial event (or particular confluence of them). In the present case, the event is either a Coulomb event or confluence of the multiple periods.

The constraints are just a combination of theoretical and operationally justifiable polynomial conditions. These must be understandable in terms of the rules of correspondence and also consistent with the algebraic operations which standard theory uses to compute the standard values of any structure constants of the system under investigation.

Now combine a second distinguishable period **s** with the first **j**, considering two independent periodic bit strings, which we represent by a count of the number of respective cycles as j and **s**. Let **j** have a cyclicity j_0 of one and therefore be of integral period. This just means that we represent an entire period with one bit. We will refer to this integral period requirement later as constraint **C(1)**. Let **s** similarly be of integral periodicity. Now we explore how to combine the period counts for these periodic bit strings. Note that, in the system so far constructed, there is no absolute scale. If the cyclicities of j and **s** were identical, they would be indistinguishable and no question of when **s** begins with respect to j is meaningful. However, it has not been assumed that the periodic bit strings need have the same cyclicity, so that if only the number of periods are counted, it is possible for the beginning (or end) of a period of j not to match the beginning (or end) of a period of **s** regardless of how a cycle of **s** is represented.

The best that can be done is to represent **s** in such a way that the magnitude of each period is integral and can be mapped locally to a single period of j. We must then take into account the relative origins of the two representations. Furthermore, we must also consider the possibility that the particular period of **s** actually occurs after multiple periods of $\mathbf{s_0}$. If the lowest of these s_0 is given in the same representation as that of j, then s_0 is not necessarily an integer, because the lowest period of j need not correspond to a minimum value for s. (Physically, this is similar to two waves which are slightly out of phase.) However, without further constraints being imposed the difference between successive values of **s** (due to counting lowest periods by twos, threes, fours, etc.) must still be integral, i.e.

$$s_n = n + s_0. \tag{12}$$

We refer to this as constraint **C(2)**.

The combinatorial paths are related to Feynman paths (Feynman and Hibbs, Quantum Mechanics and Path Integrals), which represent information (contributions to the kernel). Each path represents a distinct factor for a polynomial constraint on the , all of which are required to produce the 'observed' constraining condition (in this case, the relationship between j^2 and s^2; since j and **s** are independently constructed. As a result, whereas information is summed, the independent probabilities represented by combinatorial paths are multiplied (Nature, January, 1989).

There is not a non-integer starting point here for s_0 at all but a mapping between to independent integer counts that results in a non-integer representation of one of j or s.

From the ordering operator calculus, distances are only comparable if they add according to a distance function. Given the conditions which a distance function must satisfy, and assuming a locally flat space, independent quantities add according to the familiar Pythagorean formula (i.e., in quadrature). This fact has special meaning within the ordering operator calculus; it is the squared values of independent quantities that are related to each other in an observable manner. Only completely dependent quantities add linearly.

Since j and s are independently constructed, they must add according to a single norm or distance function if they are to be treated as comparable, i.e. they add in quadrature. We can think of constructing this relationship by considering the two discretely generated paths by which j and s can combine. The use of j or s has no independent meaning so long as only one periodicity occurs in the system under consideration. If, however, both j and s occur concurrently and it were possible to state that both j and s simultaneously begin and end synchronously and are still distinguishable, then we could give (11) as in (13) below.

$$137j \frac{\text{steps}}{\text{Coulomb event}} + 137s_0 \frac{\text{steps}}{\text{Coulomb event}} = 1 \qquad (13)$$

There is no way in the system as constructed that j and s could be distinguished under these conditions. Rather, we must assume that some asynchrony in the periods of j and s (i.e., a non-zero "relative phase") occurs and that this leads to two possible conditions depending on whether s begins before or after j in whatever common representation we choose for comparisons of the two.

$$137j \frac{\text{steps}}{\text{Coulomb event}} + 137s_0 \frac{\text{steps}}{\text{Coulomb event}} = 1 + e \qquad (14)$$

and

$$137j \frac{\text{steps}}{\text{Coulomb event}} - 137s_0 \frac{\text{steps}}{\text{Coulomb event}} = 1 - e \qquad (15)$$

From this, and ignoring the physical units for now, we have:

$$137j + 137s = b_+, \quad \text{or} \quad j + s = (b_+/137) \qquad (16)$$

or

$$137j - 137s = b_- \quad \text{or} \quad j - s = (b_-/137) \qquad (17)$$

Note the subscripts on b, representing two distinct values. We either add or subtract values of 137s depending on whether the relative beginning of s with respect to j is positive or negative value in order to "synchronize" the periodicities. The b_- and b_+ are rational values which are less than or greater than 1 by some small amount e in accordance with our understanding of using symmetric factors as discussed above:

$$b_+ = 1 + e \tag{18}$$

and

$$b_- = 1 - e. \tag{19}$$

In a continuum theory, the values of **b** are not differentiated. Instead, a symmetric average is used which for us is just the assumption of a unique root rather than the existence of multiple polynomial factors, and this assumption is not valid in a discrete theory. Remember, however, that in the discretum both factors must be used to obtain the "squared" value. The usual value is then just the square root of the product of $(b_-)(b_+)$, or

$$(b_-/137)(b_+/137) = (j + s_0)(j - s_0) \tag{20}$$

$$= j^2 - s_0^2 . \tag{21}$$

Let this last quantity be represented by a^2 :

$$a^2 = j^2 - s_0^2. \tag{22}$$

Thus, from (18), (19), (20), and (22),

$$a^2 = (1 - e^2)/137^2 \tag{23}$$

Note that a^2 is a <u>theoretical</u> quantity, whereas the multiple values of 137b are in principle observable. Thus we must be careful when comparisons to empirical values are made below.

Solving for s_0 in terms of the above equation, and putting in the constraint **C(2)** that higher values must differ from lower values by integers **n**, and that s_0 must be positive, we have that

$$s_n = n + \sqrt{j^2 - a^2} \tag{24}$$

although we must remember not to solve for the square root in the usual fashion if we are to compare our results with experiment.

<u>Computing the Probability that j and s Coincide</u>

We must now compute the probability that j and **s** are mapped to the same label in the representation space. To do so, we impose further constraints on j and **s** by noting that all events must be mapped to a single basis representation, as discussed in the subsection entitled <u>Labels</u> above. This space is constructed within the Combinatorial Hierarchy in the usual manner.

We can understand the quantity **a** as an event probability corresponding to an event **A** generated by a global ordering operator which generates the entire structure under consideration. Note again that these are probabilities in the frequency interpretation sense of probabilities. Then **a** is given by a combinatorial analysis of the relative frequency of a particular attribute instance (an event) within the given construction (corresponding in probability parlance to the sample population of possible events). We can now demand that additional constraints on the sampling be met as well, based on knowledge of the construction of the population. Each of the two events having probability counts **j** and **s** are derived by sampling from the same population.

For us, the population which defines **a** is defined via some <u>ordering operator which generates the combinatorial hierarchy.</u> **Footnote:** In the presentation given at ANPA 10, I attempted to justify the number of labels required differently. That means of computing the probability that **j** and **s** coincide in the representation came after the method used in the current version of the paper. It was motivated by (1) a misunderstanding of a physical constraint on the allowed values of k for valid velocities in Noyes' interpretation of Program Universe, and (2) a desire to make the mathematical and conceptual presentation less difficult to follow.

In order to represent the 127 basis strings at level 3 of the Combinatorial Hierarchy, the canonical basis strings must be of length 7. Just prior to the completion of level 3 via the generation of s, no more than 6 bits are required. (No relationship to total angular momentum is implied here by the use of the symbol L). For lengths L_a of a in terms of bit counts and L_{s0} of s_0, $L_a = L_{s0} + 1$.

The combinatorial ways of generating only labels which have even numbers of 1 bits, and eliminating those which under discrimination map any string either to the null string or to itself, the number of labels available for s_0 is 30. These can be divided into two sets of 15, each the dual of the other set.

There appears to be some connection between the two methods of computation. Under the requirement for constant velocity, the partitions of a representation string of length n = 6 = pN must have k = qN 1's for integer p, q, N < n. Except for N = 1, this condition reduces to the condition that k is even. How this relates to the representation is not clear, although I suspect the situation is more than coincidence. Also, if one uses Feynman's method of computing the amplitude of a discrete one-dimensional path integral (Feynman, R., and Hibbs, A., Quantum Mechanics and Path Integrals, problem 2-6, pp.34-36, c.1965), one sees that this constraint is equivalent to demanding that the amplitude is positive. This likely has some Program Universe interpretation in terms of spin.

This operator could be specified by the Program Universe algorithm, for example. In particular, let the population consist of the 137 basis strings defined at the first three levels of the hierarchy. Let a be the probability that a Program Universe event completes a set of 137 basis strings. In Noyes interpretation via the Noyes argument, this means that 1 out of every 137 vertices is a Coulomb event.

Now if the event S_0 is to be independent of **A**, it can not have a string representation that is greater than or equal to that of **A** (i.e. it must have shorter period in the terminology of the interpretation). For simplicity, we will use the canonical bit strings basis representation, in which the bit strings which are used to represent (i.e., label) the generated bit strings are just their binary enumeration (see Footnote below).

We must generate a population of 127 basis strings in order to represent level 3 of the Combinatorial Hierarchy since, under the interpretation, we require level 3 to have the Coulomb event probability (as in the Bohr atom model discussed above). These bit strings do not represent the occurrences (counts) of the steps and Coulomb events; they serve as labels for them. To combine the two independent constructions for j and s, they must have a common representation in this space of labels subject to the constraints of independence, namely, they should not require the same label at the same point in the generation.

The next constraint C(3) on the values of a, s and j is given by understanding what counts as a string that participates in a physical event.

If one considers the number of discriminately independent strings required to generate level three (again excluding the trivial initial strings of 0 and 1), we have 137. Indeed one may consider these strings as being discriminately independent with respect to the strings 0 and 1 of the initial level as a basis. Until the full basis is generated however, we must use as the basis the sixteen strings at the previous level.

Any elementary Program Universe event corresponding to level 3 of the Combinatorial Hierarchy occurs once in every 137 strings. For a particular representation, such an event maps to a specific basis string at level 2. This mapping can be understood as providing a distinguishable label for the string. We can not then include the ten discriminately independent strings from levels 1 and 2 as these are already represented by the basis strings in the mapping. Thus, one maps 127 discriminately independent strings at level 3 (as used for j) to the 16 basis strings (labels) available at level 2. However, the null string does not meet the requirements of the interpretation, reducing the number of available labels to 15.

Any other periodicity generated in the same period as j must then map to one of 15 basis strings if it is to have a distinguishable representation and must be one of the 127 strings that can be generated at level 3. There are then 127*15 possible distinguishable representations that differ for any given event **s**. Thus an indistinguishable string representation will occur one out of 127*15 times. Due to our construction, we do not know if s_0 completes its cycle later than or earlier than **j**. This corresponds to twice as many possible ways in which the 127 strings can be labeled, so that we have 127*30 possible ways j and s can jointly occur.

Now comes the final constraint **C(4)**, the independence of the events **S** and **A**. Thus, given a level 3 Combinatorial Hierarchy event **A**, it is decomposable into independent sequences of events j and s subject to the constraint that j and s are indistinguishable with a probability of

$$e = \frac{1}{30 * 127} \qquad (25)$$

The event which completes $\mathbf{s_0}$ may occur anywhere within an interval **2e** around the Coulomb event for **j**. The expectation of this event is just **1/(127*30)** according to (25) above. However, as noted in the subsection on <u>Comparing New Theoretical Results to Old</u>, we must take care in comparing our theoretical results to experimental values. Note that the value given by (25) can either subtract from or add to the probability of a Coulomb event. These correspond to two different combinatorial paths by which the independently generated sequences of events may close (the "relative phase" may be either positive or negative). In the particular physical situation, we are only interested in the portion that occurs within the period for j, namely 1-e. The reason for this is strictly operational. If the event which complete the period of s is later than that which completes the period of j, it will be counted operationally with the next period of **j**. Therefore the empirical value which corresponds to the probability that all $\mathbf{s_0}$ vertices are generated distinguishably within one period of j is:

$$1 - \{ \frac{1}{(127*30)} \} \qquad (26)$$

The difference $\mathbf{d^2}$ between $\mathbf{j^2}$ and $\mathbf{s^2}$ is then just computed as the square of this "root":

$$d^2 = (1-e)^2 \text{ or } 1 - 2e + e^2 . \qquad (27)$$

It is now legitimate to work only with the single factor since we claim that d^2 corresponds to the square of α as the actual empirical value). Substituting, we obtain the following for **d**:

$$= \quad 1 - \frac{1}{127*30} = \frac{3809}{3810} \qquad (28)$$

Performing the substitution into **(1/137)**α for **d** we obtain

$$\frac{\alpha}{(137)} \simeq \frac{3809}{3810} \qquad (29)$$

Multiplying through we obtain the value
1/137.0359674... **(30)**

in comparison to the accepted empirical value of

$\alpha =$ 1/137.035964(15).

recently replaced by

1/137.0359895(61) at $Q^2 = m_e$

(M.Aguiar-Benitz, et.al., Particle Properties Data Booklet, April 1988, p.2, p.4.)

V. THE FINE STRUCTURE OF THE HYDROGEN ATOM ENERGY LEVELS

Now that we have the relationship between **s**, **j**, and **a**, we can consider a quantity **H'** interpreted as the value(s) of the "Hamiltonian" energy attribute **H** expressed in dynamical variables at the **137j** value of the system. We seek to obtain a formula which governs the values of **H** in this system of two periods. According to our previous derivation of the relativistic energy (Foundations of a Discrete Physics), we represent this energy in units of the "mass" for the bit string defining j as

$$\mu c^2 \qquad (31)$$

where μ is a bit string invariant which transforms as mass under the Lorentz transformations, and **c** is the minimum limiting velocity for combinatorial system. The independent amount of additional energy due to the shift of **s** relative to **j** for a period can be given then as a fraction of this energy by

$$(a/s_n)(H') \qquad (32)$$

One may wish to think of this quantity as a correction term to the quantity H' due to the fact that two independent periodic bit strings serve to define the system, s being the second such string. These independent values may then be either added or subtracted to yield the total system energy over one period, subject only to the constraint C(5) that the resultant may not exceed the total energy μc^2. This gives two factors:

$$1 - (a/s_n)\ (H'/\mu c^2) = H'/\mu c^2 \qquad (33)$$

and

$$1 + (a/s_n)\ (H'/\mu c^2) = H'/\mu c^2 \qquad (34)$$

These factors (33) and (34) then multiply just as the factors for a multiplied above. We then obtain the (elliptical) equation

$$(H'^2/\mu^2 c^4) + (a^2/s_n^2)\ (H'^2/\mu^2 c^4) = 1 \qquad (35)$$

Note that both (35) and (22) - rearranged with j as the dependent variable - constitute expressions of legitimate distance functions as demanded by the definitions given above.

On rearrangement of the terms of (35) and substitution of the equation for s_n (22), we have the desired Sommerfeld result for the fine structure of hydrogen:

$$H'/\mu c^2 = \left[1 + \frac{a^2}{\{n + \sqrt{(j^2 - a^2)}\}^2} \right]^{-(1/2)} \qquad (36)$$

Footnote: As Kilmister has pointed out, an inequality is appropriate for the general form of the Hamiltonian energy (Not only due to the physical bound state requirement that the energy be less than or equal to the system energy, but also due to the mathematical conditions for adding combining independent combinatorial quantities.) This is the case if one allows for other than a flat space with totally independent generators of the two portions of the energy attribute. As soon as the degree of independence and curvature is known, the inequality goes away and a corresponding correction is asserted.

WHAT DO THE BITS IN THE COMBINATORIAL HIERARCHY MEAN?

The 1988 Presidential address by C W Kilmister

I started off this meeting by pleading for consideration of what we meant by the clever calculations that we carried out rather than further elaborations, and I want to carry out my own precepts. My promise in the Proceedings to ANPA 9 was to tackle the problem of what the elements of the Hierarchy were, physically; this was in answer to Mike Manthey's paper in the same proceedings which showed what they were not. However, I found that I could not make much headway with that until I first got clear what part the elements really played in the system, and that is what I shall be mainly concerned with here.

There have been so many discussions of how the Combinatorial Hierarchy (CH in what follows) is concerned with a FINITE PROCESS theory that I shall take this for granted. But the near unanimity about this has partly been achieved by not probing too much into what the process is. I begin by stating baldly that I shall consider a process which is modelling the developing quality of a scientific investigation. But I would like to qualify that at once to prevent misunderstanding, because it has to be understood in a particular context. It is now over thirty-five years since Ted Bastin and I put forward some ideas on physics which still seem to be important. One of our contentions then was that, just as we would not wish to distinguish thoughts sharply from their (spoken or written) expression, so in physics we want to consider a physical theory as made up thoughts, sentences and manipulations with bits of the physical world. The theory does not exist in our minds, as it were, independent of ~~of~~ bits of the physical world. And so also we would not wish to draw a hard line between the physical world and our knowledge of it. That is the context in which I talk about a scientific investigation and I hope it rescues me from an appearance of subjectivity. My intention, then, is to capture in a discrete model this continually developing quality; something which is well-recognised in orthodox views, but as something accidental, not as here to be included as part of the system. As a result of my investigation, I shall derive six results, most of which have figured as assumptions in other discussions of the CH. They are: (i) The importance of the discrimination operation; (ii) the need for vectors over Z_2; (iii) the hierarchy structure; (iv) the importance of discriminately closed subsets; (v) the appearance of linear operators in the hierarchy and, finally, a new proof that the CH actually exists.

Let us think first about this developing aspect which gives rise to a change in the nature of the investigation. There must be some sense (which we need to make clear below) in which the state of affairs is complete or closed and this completeness provides the need for the change of direction. The system then changes to a different level; it must be self-organising in the accepted sense that it changes its functional structure qualitatively without specific interference from outside. So, taking account of the assumed finiteness, the system consists of a series of finite closed levels; I call this being HIERARCHICAL.

In the course of the investigation, one of the ways in which a change takes place is by the arising of new entities. When such a new element of reality comes along, we must, in order to have a handle on it in the articulation of the theory, give it some designation or LABEL. It is only by this means that a mathematical theory will be possible; and this labelling is not to be seen as an arbitrary or trivial matter. Rather, it is the essential core of the whole representation. Convenient labels will be the well-known signs, 1,2,3... but how these are to be used is something about which I shall have more to say.

The next question is, when we label the new entity, are we quite sure that it really is a new one or might it be another copy of an old one? What can we mean by this? There must be some sense in which we count two objects as equivalent and then we'll give them the same label. By saying this, we are importing something very like Leibniz's "identity of indiscernibles", a notion likely to be very congenial to elementary particle physicists who consider all the time objects with a limited set of attributes, but with this difference: Leibniz wanted to consider an object as defined by its bundle of attributes but got into difficulties because of contradictions about whether spatial and temporal position were to be counted amongst the attributes. We are going to pursue the same line as Leibniz but as space and time do not yet exist in the system no such contradiction can arise. (Of course, one does not get something for nothing; in due course we shall have the obligation of constructing space and time.) Of course, when an object enters the theory there is a sense in which it must differ from any other object (for example, in the point at which it was introduced) so we are making a decision about whether it is none the less convenient to consider it as the same. Decisions like this must naturally survive new developments in the theory. The situation is a little like the Kripke models for intuitionistic logic where the notion of "forcing" is introduced on a partially ordered set, with the requirement that once a proposition is forced it remains forced at every later node of the set. But there is a difference here because you will recall that the development of the theory is intended to represent the progress of a particular investigation; so that an eventual contradiction is allowed and will correspond to the point at which the investigation is finished and another one is going to begin. We shall use the term STABLE for a system which satisfies the requirement that propositions about the system at one stage remain true when new elements are added.

A device which automatically incorporates the stability requirement without making an appeal (which would be illegitimate from our point of view) to a fixed "given" set of already generated elements is to use the idea of a function, that is, a rule which gives a value for each value of the argument. Don't import any modern ideas of domain or range here; there is simply a rule, and the rule is then constrained to be such that the values of the function f for elements equivalent to already generated elements lie in a fixed set Z which is disjoint from the set of values for truly new elements. This may not be the only way to proceed; but it does encapsulate the basic notion introduced by Bastin in 1952. Before I discuss the possible form of the function f, there are two further assumptions that I want to make clear. The first one, which I distinguish as being UNIVERSAL is that this method is to be applicable to any of the sets of elements under consideration. This assumption can be read in two ways; either it puts obligations on the functions, or it restricts the family of sets we are to consider. It will be in the second way that it will be used below. Secondly, and this assumption is not of the same fundamental importance as the others but merely a matter of convenience, which it turns out can be fulfilled without loss of generality, I shall assume that the functions are DIVERSE by which I mean that the value of fx is always different from x. The origin of this assumption can be seen in the idea of a computing technique, which was at one time a useful way of generating these ideas; for then $fx \neq x$ means that there is a sure indication that f has indeed operated. But without this motivation, it proves to be still useful to restrict oneself to diverse functions.

Now f will not be unique; leaving aside the possibility of having different "signal sets" Z, two f's will do exactly the same job even though they may have totally different sets of values outside Z. We can call two f's that do just the same job EQUIVALENT. Now think of the sort of f which will serve for comparison with a single element. Such a simple comparison will not occur very much, but it will sometimes occur and the limitations in this case do much to determine the general form that any f will have. The limitations on f arise from the fact that it represents an equivalence relation and a little work on the three well-known characteristics of an equivalence relation easily shows that the form of f for a single element u, f_u say, is always equivalent to a special f, f^*_u, for which the set Z has only a single element and we introduce for that element an ideal one, denoted by 0, and such that if we write

$$f^*_u = u + x,$$

then the operation + is commutative and associative and further

$$x + x = 0$$

for all x. This is my result (i), the importance of the discrimination operation, for that is what + is.

It will save time in the exposition if I come clean about what is to happen next and at the same time go back a little way in the recent history of my discussions. In the past we have had two notations for discrimination systems, neither of which quite fits. Under the influence of Conway's lovely book "On Numbers and Games" I have preferred to regard the labels 1,2,3.. as elements of a finite field in the same way as Conway, and then discrimination was simply his addition ("Nym-addition"). This notation was completely isomorphic to the original vector-space over Z_2 notation which entered ANPA discussions with Frederick Parker-Rhodes' first construction of a hierarchy. There is no mystery about the isomorphism; the connexion is simply that, for instance, 11 is written in the scale of two as 1 + 2 + 8 = 1011 and this is matched with the vector $[1,1,0,1]^t$. The reason that the vector space notation does not quite fit is that a definite dimension has to be given to the vector space, from outside as it were, whereas in practice one does not want to draw any distinction between [1,0], [1,0,0], [1,0,0,0] and so on. This same trouble shows up in the finite field notation in a more subtle way. Firstly, we shall want to identify the element 2 in the field (1,2,3) with the 2 in (1,2,3,4,5,6,7). In fact, Conway seems to take this identification for granted, but it would be better if the notation made it explicit. Secondly, the labels in the finite field notation do not represent the arising of new elements in the most convenient way. Thus, we can give the label 1 to the first element, and 2 to the second, but then 3 is the element which arises from the discriminating of 1 and 2 and the next new one from outside has to be called 4. This second difficulty is not insuperable, but merely a matter of convenience; the first one is more serious. Be all that as it may, it is already clear in these imperfect notations that my result (ii) has been made explicit.

The new notation which I shall now introduce does not have these disadvantages; it is not just a matter of notation, though. That is just how it presents itself; what is really at issue here is the nature of the elements in the system which is exactly described by the notation constructed for them, and this nature is just what my paper is about. It will save time to introduce the notation baldly and then explain it afterwards. An element is now a finite sequence of integers,

$$r = (r_1, r_2, \ldots r_k),\ r_1 < r_2 < r_3 < \cdots < r_k,$$

and is called a STRING (of integers, implied). In ANPA Language, string usually means a bit-string; this use is not exactly the same, but sufficiently alike for no confusion to arise. Any finite set of integers, S say, can be made into a string by the operation $S \rightarrow \text{str } S$ defined by:

(a) Delete every pair of equal integers in S,

str:

(b) then re-arrange the integers in ascending order.

For example, if S = [1,3,3,3,2,2,5,5,5,5], then str S = (1,3,5). In the initial stages of the system, when we shall find that none of the integers exceeds nine, it is convenient to omit the commas and to write str S = 135. The order of r, ord r, is defined as the last integer, r_k and the norm of r, N(r), is the number of integers in r. So, in the example,

ord str S = 5, N(str S) = 3.

We can then define the sum or DISCRIMINATION of two strings by

$$r + s = \text{str}(r \cup s).$$

As a matter of convention, if, say, S = [1,1], then str S is the empty string, which we denote by 0. The connexion with the earlier notations is simple; a string is just an equivalence class of vectors over Z_2, the integers labelling the rows of the vector which are occupied by 1; so that str S in the example stands for [1,0,1,0,1], or [1,0,1,0,1,0], or [1,0,1,0,1,0,0] and so on. We can order the strings in the following way: first arrange by the order, then amongst those of each order remove the largest integer and repeat the process. So a list of strings in order is:

1,2,12,3,13,23,123,4,14,24,124,34,...

and since the corresponding finite field notation for these elements is

1,2,3,4,5,6,7,...

it is clear that the ordering defined is the same as the one defined by treating the finite field labels as ordinals.

All of that follows by asking about the comparison with a single element, but now we have to extend the system to compare putative new element with a developing set $S = (u_1, u_2, ..)$. It will not be possible to do this by "testing each u_i in turn", because then at each stage we should have to ask "Have I used this u_i or not" and in testing that we set off on an infinite regress. We can avoid this by using again the device of a function $f(S,x)$ which (taking for granted the initial stages of the previous discussion) vanishes if and only if one of the $u_i + x$ vanishes. The previous symmetry between u and x has now gone, so a more convenient form is to write $f(S, x) = F_S(x)$ and to think of F_S as a characteristic function for the set S. We can define an addition operation between two characteristic functions, F, G say, by the obvious rule:

$$F(x) + G(x) = (F + G)(x)$$

for all x in play up to the point at which the definition is being applied. Then it is easy to see that this operation is just like the original discrimination operation, so that the set of characteristic functions is itself a discrimination system. This fact is important in showing that the system is hierarchical and giving rise to the hierarchy structure.

The idea of F vanishing if and only if one of the arguments vanishes puts one in mind of a product operation in a system without null divisors. And in fact it is possible to define a product version of a characteristic function equivalent to your F; I call this the PRODUCT CONSTRUCTION. Conway carries out one version of this construction and shows that it leads to successive closed sets of sizes 3, 15, 255, 65535, ... Every so often you have to introduce a new label to extend the table; each of these elements is of norm unity. Let us call such elements BASIC. These basic elements used to extend the system are the elements of the set F:

$$F = (f_0, f_1, \ldots) = (2,3,5,9,\ldots)$$

where $f_r = r^* + 2$ (writing as usual $r^* = 2^r - 1$). But Conway also gives an alternative which is easier to carry out; this is defined by the two rules for basic elements:

If r,s are basic, and $r < s$ and $s \in F$, then r.s is the basic element $r + s - 1$ and $s^2 = (s - 1, s)$.

The results for all other products are to be got by appropriate use of the commutative, associative and distributive laws and it is not too hard to show that the resultant system does satisfy all those laws and has no divisors of zero. So that is the product construction; what we want from it is not so much the details of the product as the numbers of the successive closed sets, for these numbers prove to be the same for other constructions as well.

Now I want to show that the use of the product construction in a process theory has the disadvantage of not fulfilling the four conditions of being simultaneously hierarchical, stable, diverse and universal. Then I shall show that a modification of it to ensure the fulfilment of the conditions takes us to something which is very near to the original Parker-Rhodes system. If you start, as an ANPA audience would, from that construction, this must seem a roundabout way of proceeding. I want to emphasise instead the extreme naturalness of the present approach, so that it is a surprise and a disappointment that it does not quite do what we hope of it. I could just do this by giving a counter-example but it is more instructive to see how the difficulty arises. I do this by formulating a way of using the techniques, which I call the

INITIAL SCHEME, in the form of a detailed example. Nothing can happen until some element has arisen, and when this has happened there is no loss of generality in calling this by the least label, 1. The function F is now defined by F(x) = x + 1. Further elements, if they do not make F zero (in which case they are merely repetitions of x = 1) will eventually give a non-zero value. Call such an element by the least unused label, 2, and then, since F(2) = 1 + 2 = 12, it follows that the combined operation of generating a new element and testing it has produced two new elements, 2 and 12. So the next F is not a quadratic, but a cubic:

$$F(x) = (x + 1)(x + 2)(x + 12)$$

and I leave it as an exercise to you to prove the quite useless fact that this is just $x^3 + 1$. The set [1,2,12] is closed under + and so arbitrary discriminations will produce nothing new. Finally a new element will arise for which F is non-zero, and we can call it by the next label, 3. By another simple exercise for the reader, F(3) = (1234), so again we have two new elements added and the new F is a quintic. This is the point at which the initial scheme runs into trouble. If arbitrary discriminations take place, now or later, they may give rise, internally as it were, to elements that have not been mentioned, but these new elements are not the ones which we envisaged as arising as part of the investigation. When the next one does arise as part of the investigation, we give it the next label, 13; a third exercise for you is to show that F(13)= 24. That is fine, so long as 13 has not been produced by a discrimination, say of 1 with 3, and so long as such a discrimination never occurs in the future. But if it does so occur, we do not expect that the element produced by the discrimination will be the same as the 13 which we have introduced here.

This trouble only arises at the second step; why is this? When we had only introduced 1 and 2 and produced 12 by discrimination, the set [1,2,12] is closed under further discriminations of two different elements, that is, it is a discriminately closed subset (dcss). This shows the way out of the difficulty; we impose the restriction that we only ask the question of whether a new element belongs to, not the set of elements that just happen to have arisen up to now, but to their DISCRIMINATE CLOSURE. The construction is now universal so long as we understand the requirement as restricted to the dcss. This is the result (iv), the importance of dcss.

Even with this restriction there is a further blemish in the product construction, which I describe as failing to be COMPACT. In the above example it will still be the case, even when we confine attention to dcss, that the cubic F will be $x^3 + 1$ and so F(3) = 1234. This value for F not only takes one up to the next dcss, D[1,2,3] = [1,2,12,3,13,23,123], as would be expected, but requires the use of a further element, 4, outside that. I view this as an unnecessary extension, so without physical content. Fortunately, so long as we confine our attention to dcss, there is an alternative to the product construction, which I will call the ARRAY CONSTRUCTION. This also introduces a considerable mathematical simplification. It is based on the result:

> If and only if S is a discriminately closed subset, one of equivalence class of characteristic functions for S, F say, satisfies:(a) F is compact,
>
> (b) $F(u + v) = Fu + Fv$ for any two different labels u, v.(Remember that a label is a non-zero element of the algebra.)

You can see that I am very near to the result (v) on the appearance of linear operators in the hierarchy, but I am not quite there for these characteristic functions are not quite linear, since the linearity condition is stated only for different values of u, v. They cannot really be linear for then we should have

$$F(0) = F(u + u) = F(u) + F(u) = 0$$

although 0 does not belong to any of our dcss. So we change the definitions a little by saying, for any operator F, that if F(u) = 0 and u ≠ 0, then u is a PROPER zero of F and if F(0) = 0 then 0 is an improper zero. Then call F a pseudo-characteristic function of a dcss S, or for short, a SIGNAL FOR S, if and only if S is the set of proper zeros of F. The proof of the previous result equally proves

> If and only if S is a dcss, there is a linear compact F which is a signal for S.

In the vector space notation, when a fixed dimension was assumed for the vector space, such linear operators may be written as square matrices. With the same restriction here, there is a corresponding reduction to ARRAYS. An array, R, is a set of k elements,

$$R = [r^1, r^2, r^3, \ldots r^k],$$

where none of the k elements is of order greater than k. Such an array is defined as a linear operator by defining its operation on basic elements by

$$R(s) = (r^s) \quad \text{if } s \leqslant k$$

the value for other elements being got by the linearity.

Now that serves to reconcile the present notation with the square matrices, but only under the restrictive condition of fixed dimension. Our operators are not arrays since this restriction does not hold.

I should emphasise another point about an array. The r_k are not restricted to being labels, that is, one or more of them may be zero and indeed the zero array [0,0,...0] is allowed. But because we shall later be concerned with a multi-level structure in which the step to the next level involves questions of linear independence the appearance of the zero array gives rise to some inconvenience. In the original construction of Parker-Rhodes this is neatly avoided by taking as a signal for S, not F where

$$u \in S \rightarrow Fu = 0 \quad \& \quad u \neq 0,$$

but G where

$$u \in S \rightarrow Gu = u \quad \& \quad u \neq 0.$$

Since the signals are linear, the connexion between the two is simply that F = G + I, where I is the identity operator. We shall ultimately go over to the more convenient Parker-Rhodes version, but to avoid a break in the argument we will continue with the original signals for the present, knowing that we can at any stage convert the results easily.

Although the operators F,G are not arrays, it turns out that an operator can correspond to an array in a way that I shall now make clear. I begin by giving an algorithm which will define F over the set of successively larger dcss, D_1 = [1], D_2 = D[1,2] = [1,2,12], and so on. It is clear that

$$D_{r+1} = D_r \cup L_{r+1}, \quad L_r = [r] \cup [r+D_{r-1}]$$

At each stage in the algorithm there are two sets under consideration (as well as the D_r for which the values are being defined). Call these the TARGET set, T_r, and the set of VALUES V_r. At the r th stage values are to be assigned to elements of D_r, i.e. L_r since those for D_{r-1} will already have been assigned. The procedure is then:

1. Does L_r contain a zero, z, of F? (i.e. a member of S).

IF NOT:

2a. Put $Fr = \min u\ [u \in T_r]$.

3a. Put $V_{r+1} = D[V_r \cup [u]]$.

4a. Put $T_{r+1} = (T_r \cup D_r) - V_{r+1}$.

5a. Proceed to stage r+1.

IF SO:

2b. Put Fz = 0.

3b. Put $V_{r=1} = V_r$.

4b. Put $T_{r+1} = (T_r \cup D_r) - V_{r+1}$.

5b. Proceed to stage r+1.

At stage 1, we set $L_1 = D_1 = [1]$, $V_1 = \phi$, $T_1 = S$.

Let me illustrate this by constructing the operator for the dcss $[23, 24, 34] = S = T_1$.

$L_1 = [1]$; no zero, so F1 = 23, $V_2 = [23]$, $T_2 = [1,24,34]$.

$L_2 = [2,12]$; no zero, so F2 = 1, $V_3 = [1,23,123]$, $T_3 = [2,12,24,34]$.

$L_3 = [3,13,23,123]$ with zero 23, so put F23 = 0, $V_4 = V_3$,

$T_4 = [2,12,3,13,24,34]$.

$L_4 = [4,14,24,..]$, F24 = 0, $V_5 = V_3$, $T_5 = D_4 - V_3 = [2,...]$

$L_5 = [5,...]$; no zero, F5 = 2,$V_6 = D_3$, $T_6 = D_5 - D_3 = L_5 \quad L_4$

$L_6 = [6,...]$, F6 = 4, $V_7 = D_4$, $T_7 = L_6 \quad L_5$.

$L_7 = [7,..]$, F7 = 5, and so on.

It is clear that once the members of S have been dealt with, so that there are no more members and so no zeros of F, an infinite "tail" develops which is quite useless. Of course, if a different algorithm were used, the details of the tail would in general be different, but a tail would result just the same. And if the algorithm is fixed, then once the first four stages have been performed, there is no need to write down the rest, as it can be reconstructed as needed. So we can TRUNCATE operators, and consider just their initial segments. It is only necessary to include the segment with all the members of S (all the zeros of F); the rest may be truncated. Another way of putting this is to notice that the length of the segment is the maximum order of elements of S, and so we can always truncate so that the truncated form of F is an array. This really concludes the explanation of the occurrence of linear operators (matrices) in the hierarchy, the fifth of the results I promised at the beginning.

So far all I have been doing is to refine the apparatus for the expression of the hierarchy construction. Now I turn to this strange construction, from which all this ANPA work has developed. It really needs another lecture to explore the why and wherefore of it so I shall content myself with noting how this more refined apparatus provides a more intuitive proof that the hierarchy defined by Frederick Parker-Rhodes does exist. We can translate the original construction into our terms very easily. It all depends on the result noted above, that the set of characteristic functions itself forms a discrimination system; this is easily extended to show also that the set of truncated signals for a discrimination system itself forms a discrimination system. The Parker-Rhodes construction begins with 2 elements, which give rise to 3 dcss. The 3 signals for these are linearly independent and constitute the next level, giving rise in their turn to 7 dcss. It is possible to choose their signals linearly independent and so the next level has $2^7 - 1 = 127$ signals for the corresponding dcss. These give rise to 10^{38} dcss; the original construction envisaged all this in the vector-space picture, so that the original two elements were 1,2 in our notation. The linear arrays for them were 4-dimensional; so that the 7 arrays at the next level were 4^2 = 16-dimensional, the 127 at the next were 256-dimensional and the requirement that those at the next level should be in a 256^2-dimensional space, although there is need for 10^{38} of them, prevents their being chosen to be linearly independent and so the construction stops. Little is changed by our new apparatus, except that there is no convincing reason to impose a particular dimension on the arrays beforehand, so that the distinction for us at the last stage is not that further progress is impossible, but that the last step involves a huge increase in the size of the arrays, whereas the earlier stages did not.

But what was a puzzle for a time in the earlier discussion of the construction was the proof that it was actually possible to find the required 7, 127 arrays which would fulfil all the requirements (including being a linearly independent set). Seven out of 16 was not too surprising, but 127 out of 256 seems to be living slightly dangerously. In the event, Pierre Noyes showed there were 127 operators by finding them, but it was tedious to verify that his set was indeed linearly independent. I gave a quite different proof that there were 127 such operators, but this was only an existence proof and did nothing to tell one what they were. The key question is how big the dimension of the vector space of signals has to be to accomodate all the constraints. Suppose there are r elements and so $r^* = 2^r - 1$ dcss; then it is obvious that there is no hope for linear independence if the order of the arrays is n where n^2 is less than r*, since n^2 is the dimension of the vector space of the arrays. It is rather surprising that this is also enough:

THEOREM Given r elememts. so r* dcss. Their r* signals have truncated forms which are arrays of order n if and only if $r^* \leq n^2$.

The following proof is not the most elegant, but it seems to me most instructive because it shows why the result is true. (Since the particular application to r = 7, r* = 127 gives n = 12 as enough, rather than n = 16, there is some need to see why this can be true.) I begin for the case r = 2 and use the algorithm above to construct the signals for the dcss [1],[2],[1,2,12] which I denote for shortness as 1,2,12 respectively. The result can be tabulated by giving the value of the signal for 1,2 respectively:

	1	2
1	0	1
2	2	0
12	0	0

For the case r = 3 we can proceed similarly and a first shot gives the following table:

	1	2	3
1	0	1	2
2	2	0	1
12	0	0	1
3	3	1	0
13	0	1	0
23	2	0	0
123	0	0	0

Each line of this table is good by itself, but the signals are not linearly independent as can be seen by adding together the ones for 2, 12, 23. A general procedure for dealing with this situation is to go back to the last signal causing trouble and try to modify it so as to cure the trouble. Here it is the signal for 23. The 2 needs to be changed but of the six other values it might have 1 is excluded by diversity as is 12 or 13, 3 is excluded by linear independence as is 23 and 123 is excluded by diversity. Accordingly one moves back to the next row contributing to the trouble, which is that for 2. Here it is easy to see that 2 cannot be replaced by 1, 12, 3 but 13 will serve to meet all the requirements, A similar piece of work serves to do the case r = 4, so showing that the signals for the 4* = 15 dcss can be chosen in 16 dimensions, which is surprising. At the next stage, however, the 5* = 31 signals need 6^2 = 36 dimensions; and it is much easier to satisfy the linear independence since the table we are to fill in now has six columns, not five. The extra column can be used to help over the linear independence. At each later stage of the theorem these extra columns are present in increasing numbers. Thus for the application to r = 7 the number of columns is 12 not 7 and this gives great freedom to rectify any troubles.

So where do I stand on my original question about the physical meaning of the elements of the hierarchy? I have time only for a sketch. Each element is an element of reality, to quote Einstein's phrase and, just as Ted Bastin and I held in 1952, it is not that some are more physical than others, but some are at a simpler stage. The elements at one level, if I am not mistaken, are not elementary particles - this is a point of difference from "Program Universe" - but a simple version of such a particle <u>is</u> provided (and this is the hint given by the hierarchy construction) by a set of elements at the four different levels. So we are really dealing with a graded algebra. But how do these graded elements correspond to particles? The answer must lie - and this sets out some of my programme for the coming year - in conservation laws, both of quantities like momentum and also of quantum numbers. How do all these fit into the system? I hope to be able to say something more about that when I come to ANPA 11; I hope to see you all then.

SETS, SORTS, MULTISETS AND REALITY

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INTRODUCTORY REMARKS

Sir Arthur Eddington said, "Proof is the idol before which the mathematician tortures himself". What Eddington failed to say, however, is that the mathematician *enjoys* such torture. Alas, there will be no tortuous proofs given in this paper.

The title - SETS, SORTS, MULTISETS AND REALITY - is blatantly misleading. The *primary* topic is multiset theory, with a little bit of set theory, sort theory and reality mixed in for good measure. Set theory is described briefly because it is the backdrop against which the other theories are discussed. I am sorry to say that my renewed efforts to understand sort theory (as presented in [3]) have been unsuccessful. I find the mathematical formalism extremely difficult, and I have not as yet made the necessary effort to understand Chapter IV of [3] in which the axioms and definitions of sort theory are given. The little I do understand, I will discuss in a moment. I have even less to say about reality which should come as a surprise to no one. Reality *is* a difficult concept. I believe the human mind has a natural tendency to view reality in terms of multisets of objects. I will also speculate about the possible structure of the real universe in terms of the structure of the formal universe of multisets, multisets which contain both positive and negative numbers of elements. Details will be given shortly.

The subject matter of this paper lies outside of classical mainstream mathematics. Multiset theory falls within the area of non-classical logics and set theories which is only a small part of mathematical logic as a whole. Mathematical logic itself is not held in the highest regard by many (or most) mathematicians. The mathematics surveyed in this paper is unconventional, and is as yet, outside the mathematics tradition. From certain points of view, it might even be seen as controversial mathematics. Whatever it is, I hope you will agree that it is at least challenging, beautiful and exciting, and perhaps even important for future mathematics and natural philosophy.

SET THEORY

We begin by thinking about the formal assertion

$$x \in y \ .$$

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The intended interpretation of this formula is "y is a set of elements, one of which is x". We can write this as

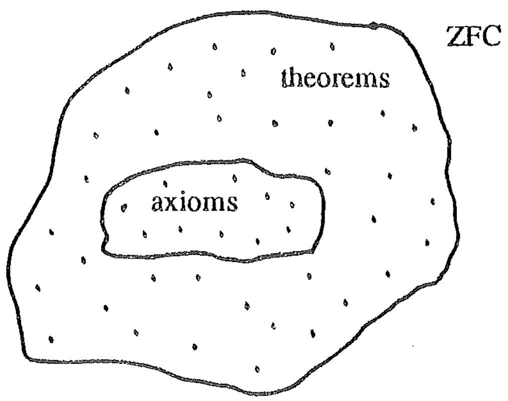
$$y = \{..., x, ... \}$$

where curly brackets are used to enclose a list of elements in y. The order of elements is ignored.

Where did this stylized Greek epsilon come from? It was introduced by the Italian mathematician Guiseppe Peano in 1889 and it is based on a Greek word meaning "is". The rationale behind this notation is as follows: if y is the collection of all blue objects, then to assert "x is blue" is equivalent to asserting "x belongs to y". Hence, $x \in y$.

The concept of set membership, as in $x \in y$, has proved to be a very general and powerful concept (not without its difficulties). In English, the word "set" possesses vast generality. In fact, it has the longest definition of any word appearing in the Oxford English Dictionary. Formal set theory has evolved over the last 100 years from (as early as Bolzano) R. Dedekind, G. Cantor (generally acknowledged as the father of modern set theory), E. Zermelo, A. Fraenkel, J. von Neumann and many others. It has been built up using only the atomic formula $x \in y$ and classical first-order logic. The result is a formal theory of sets called ZFC (Zermelo-Fraenkel set theory). The C in ZFC does not refer to Cantor, but to Choice - the classical axiom of choice. Most (if not all) of modern mathematics can be formalized withing ZFC. In fact, it is difficult to conceive of modern mathematics without the use of the epsilon $\in$. ZFC is generally acknowledged as the foundation of modern mathematics, although this is a debatable point. Category theory, for one, has challenged this claim recently.

What exactly is ZFC? ZFC is a collection of formal sentences that make assertions about sets. The formal sentences are constructed (in a well-formed manner, the details of which we do not discuss) using only the variable symbols x, y, z, ..., the symbol $\in$, and the logical symbols $\sim$, $\wedge$, $\vee$, $\rightarrow$, $<\!\!->$, the quantifiers $\exists$ and $\forall$, and brackets), (where necessary. ZFC is, therefore, just a vast (infinite, in fact) collection of sentences that say things (that are "provable") about sets.



Certain of these sentences in ZFC are special and are called *axioms*. These are in ZFC because they say things about sets which are "self-evident" or "desireable" from the point of view of the set theorist.

For an excellent discussion of why the axioms of ZFC are accepted as axioms, and a discussion of the historical birth of the axioms of ZFC, see P. Maddy, Believing the Axioms I, *Journal of Symbolic Logic*, June 88. In fact, they came into being not to secure the foundations of mathematics and avoid paradoxes, but rather to list the precise assumptions needed to prove a particular theorem. The axioms are the *seeds* of the theory (there are infinitely many axioms) and the mechanisms of growth are logical rules (substitution and modus ponens) by which all sentences in ZFC (called *theorems*) can be generated. A sentence is in ZFC if it has a *proof* in ZFC (a finite sequence of formulae which ends with the sentence to be proved). Examples of axioms of ZFC are:

$\forall x \, \forall y \, (\forall z \, (z \in x <-> z \in y) \rightarrow x = y)$ and

$\exists y \, \forall x \, {\sim} x \in y$.

When formulae get too long and complicated, one economizes by using *definitions*. For example, instead of writing the formula $\forall z \, (z \in x \rightarrow z \in y)$ one simply uses $x \subseteq y$ instead. A theorem of ZFC is

$$\forall x \, \forall y \, \forall z \, ((x \subseteq y \wedge y \subseteq z) \rightarrow x \subseteq z) \ .$$

This would be very cumbersome to write if the defined symbol $\subseteq$ was not used.

We will not discuss problems of paradoxes, consistency and completeness which have plagued set theory over the years. Nevertheless, most mathematicians think set theoretically, and set theory can still claim to be the traditional (if not the best) candidate for the foundation of mathematics. Given the power and generality of set theory, it comes as something of a surprise to find out that certain very simple collections of objects cannot be *directly* written down in the language of set theory: collections like

$H, H, T, H, T, T, H, \ldots$

x, y, y, x, y

3, 2, 2, 7, 7, 3, 2 .

These are simple patterns or collections of objects. What they have in common is that elements repeat (occur in the collection more than once). The problem is that in classical set theory, the elements of sets must be *distinct* - elements of sets cannot repeat (or repeated elements are ignored).

The axiom of ZFC written down earlier,

$$\forall x \, \forall y \, (\forall z \, (z \in x <\!\!-\!\!> z \in y) \rightarrow x = y) \ ,$$

forces the equality (the identity) of, for example, the "two" sets $\{1,2\}$ and $\{2,1,2,2,1\}$. Why? Because an element of one is always an element of the other, and, therefore, by the axiom $\{1,2\} = \{2,1,2,2,1\}$. Therefore, in classical set theory, repeated elements in sets collapse to distinct elements.

There are *indirect* means of talking about repeated elements in ZFC but these can be quite clumsy. For example, one can employ functions, cartesian products, sequences or families of sets. One could write the multiset $[x, y, x, x, y]$ as the function $\{<x, 3>, <y, 2>\}$ or as the sequence $<x, y, x, x, y>$. In families of sets, one gives the same member of the family different indexes if one wishes to count it more than once. Also, to obtain two copies of a set A, one constructs $A \times \{x\}$ and $A \times \{y\}$ where $x \neq y$. There are problems with all of these approaches. One would like to be able to say formally "there are 3 copies of x and 2 copies of y" in a simple and direct manner.

MULTISET THEORY

Recently, *multisets* (sets with repeated elements) have become popular in theoretical computer science, but they have also been used in mathematics and philosophy. As early as 1888, R. Dedekind defined a concept equivalent to that of multiset. Since antiquity, numbers have been represented as multisets of units.

Multiset theory is not the only new set theory that has appeared recently. There has been a great variety and a wide range of alternative or non-classical set theories, including fuzzy set theory, L-valued set theory, Boolean and Heyting-valued set theory, intuitionistic and constructive set theory, set-valued sets, semisets, Church's set theory with a universal set, paraconsistent set theory, Parker-Rhodes' Sort Theory (which we discuss later), quantum sets, rough sets, et cetera, et cetera.

Even classical logic itself (within which ZFC is formulated) has been abandoned by some in favour of intuitionistic logic, many - valued logic, triparitous logic, paraconsistent logic, fuzzy logic, quantum logic, modal and temporal logic, relevance logic, and more recently (1984) something called linear logic.

To modify ZFC to meet the needs of multisets (and other non-classical set theories) there are two basic strategies:

1. change the logic that underlies ZFC,

   or

2. change the language and axioms of ZFC while maintaining the underlying logic.

We choose the second of these two strategies.

We develop (in [1]) a formal theory called MST (multiset theory) that is like ZFC but that can handle multisets in a simple and direct manner. Consider again the formula

$$x \in y \quad .$$

We would like to be able to say more than just "x belongs to y" or "x does not belong to y". We want to *qualify* the epsilon of membership. To do this, we introduce a measure of the *degree* or *extent* of membership of x in y. We replace the binary predicate $\in (x,y)$ by a ternary predicate $\in (x,y,n)$ where the variable n describes the *extent* to which x belongs to y. We write the formula $\in (x,y,n)$ more suggestively as

$$x \in^n y \ .$$

The intended interpretation of the formula $x \in^n y$ is "x belongs to y with degree (or extent) of membership n". For multisets, we read $x \in^n y$ simply as "x belongs to y exactly n times".

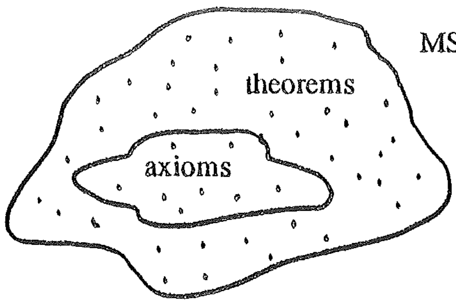
Depending upon what values the variable n is allowed to take, one gets a variety of different variations of set theory:

0, 1	classical ZFC set theory where	$x \in^1 y$ is $x \in y$ $x \in^0 y$ is $x \notin y$.
$[0,1] \subseteq \mathbb{R}$	Zadeh's fuzzy set theory where	$x \in^p y$ is $\mu_y(x) = p$
lattice L	L-fuzzy sets	
complete boolean-algebra $\mathbb{B}$	Boolean valued set theory	
complete Heyting-algebra $\mathbb{H}$	Heyting-valued set theory	
natural numbers $\mathbb{N}$	Multiset theory MST	

the integers $\mathbb{Z}$	"Shadow Theory" MSTZ
other algebraic structures, semi-rings, fields etc. etc.	for example,Eilenberg's *K*-sets or field-valued set theory.

Anyone who has seen the axioms for a vector space (or a Hilbert space) will notice that there are two kinds of variable symbols: u, v, w, ... denote *vectors* and α, β, γ, ... denote *scalars*. The axioms for a field are assumed for the scalars, and other axioms for vector addition, scalar multiplication, ... et cetera are given. Classical set theory ZFC uses only one kind of variable symbol: x, y, z, ... denote *sets*. Multiset theory MST, on the other hand, is more like the theory of vector spaces. It requires two kinds of variable symbols: x, y, z, ... denote *multisets* and k, l, m, n, ... denote *multiplicities* (the number of times elements belong to multisets). The multiplicity variables satisfy axioms for the natural numbers, and both kinds of variables satisfy other axioms that are similar to, but more general than, their corresponding axioms in ZFC.

Just like ZFC, the formal theory MST is a large collection of formal sentences that make assertions about multisets.



As before, the sentences are constructed by certain rules, and theorems are derived from axioms by classical substitution and modus ponens.

Some axioms of MST are

$$\forall x \, \forall y \, (\forall z \, \forall n \, (z \, \varepsilon^n \, x \leftrightarrow z \, \varepsilon^n \, y) \rightarrow x = y)$$

$$\forall x \, \forall y \, \forall n \, \forall m \, ((x \, \varepsilon^n \, y \wedge x \, \varepsilon^m \, y) \rightarrow n = m)$$

$$\exists y \, \forall x \, \forall n \, \sim x \, \varepsilon^n \, y \quad .$$

Again, *definitions* are used as shorthand, so that instead of writing out

$$\forall z \, \forall n \, (z \, \varepsilon^n \, x \rightarrow \exists m \, (n \leq m \wedge z \, \varepsilon^m \, y)) \ ,$$

we simply write $x \subseteq y$. We can prove the theorem

$$\forall x \, \forall y \, ((x \subseteq y \wedge y \subseteq x) \rightarrow x = y)$$

which is a formal sentence in MST.

In MST, we can define *sets*:

Set (y) stands for the longer formula

$$y = \phi \vee \forall x \, \forall n \, (x \, \varepsilon^n \, y \rightarrow n = 1) \; .$$

We can also define *hereditary sets*:

HSet (y) stands for the longer formula

$$\text{Set}(y) \wedge \forall x \, (x \, \varepsilon \, y \rightarrow \text{HSet}(x))$$

where $x \, \varepsilon \, y$ is defined to mean

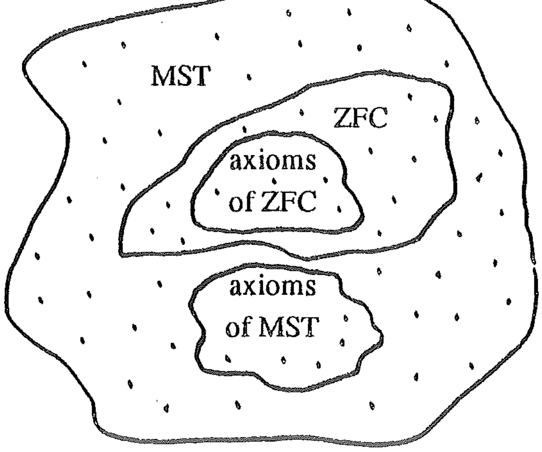
$$\exists n \, (n > 0 \wedge x \, \varepsilon^n \, y) \; .$$

Explanation: The elements of multisets are themselves multisets, which in turn may have elements themselves, which are also multisets, et cetera, et cetera. To say that a multiset is a *hereditary set* is to say that

1. it is a set
2. all of its elements are sets
3. all of its elements of elements are sets . . .

This process ends after some finite number of steps. *Hereditary set* means every multiset involved in the construction of the set is a set (everything in the "transitive closure" is a set).

It is the hereditary sets in MST that are the exact analogs of classical sets in ZFC. If one collects together all the sentences of MST that make assertions about hereditary sets (and only hereditary sets), then one obtains an exact copy of ZFC.



Therefore, the axioms of ZFC are now theorems of MST (basically, they are just the axioms of MST restricted exclusively to hereditary sets). We, therefore, say that ZFC is *contained* in MST, or MST is a *generalization* of ZFC, or ZFC is a *special case* of MST.

In a formal mathematical theory, the most devastating thing that can happen is to discover a *contradiction*. Contradiction is the cardinal mortal sin in mathematics. The reason is that once a contradiction is present, *everything* becomes proveable - and the whole idea of "proof" and "truth" breaks down. *Consistency*, that is, freedom from contradiction, is the highest virtue. When one develops a new theory, one would like to show that contradictions are no more likely in the new theory than in the prevailing theory. We do not know if ZFC is consistent. We suspect that it is. No one has yet found a contradiction in ZFC. But someone could uncover such a contradiction tomorrow. For a new theory like MST what one does is to prove *relative consistency*; that is, one proves that if ZFC is consistent, then MST is consistent (in other words, if there is a contradiction in MST, then there is also a contradiction in ZFC). One does this by constructing a "model" of MST in ZFC. The details of this construction and the proof that it is a model of MST are given in [1]. The new theory MST is such that something is gained (the ability to talk about a much wider variety of collections of elements) and nothing is lost (it contains ZFC and it is relatively consistent).

SORT THEORY

Some confusion may arise from the use of the equality symbol $=$ in the theory MST. The *equality* of multisets is not intended to mean the *identity* of multisets. Perhaps a different symbol like $\doteq$ or $\sim$ should be used instead of $=$ in MST. What we have in mind in some general equivalence relation (that is not identity) - a binary relation $\sim$ that is relfexive, symmetric and transitive. The logical axioms of MST include the axioms for the first-order predicate calculus with equality. Therefore, the usual axioms of equality hold including the substitution of equals:

$$(\phi(x) \wedge x = y) \rightarrow \phi(y) \ .$$

To assert $x = y$ in MST is not to assert that x and y are identical (the same thing), but rather that x and y are very much alike - in fact, x and y are as much alike as is possible without being the same thing. To assert $x = y$ is to assert that x and y are indistinguishable - there is no way to distinguish between x and y - they are exactly alike in all respects except number. The multiset $[x,x] = [x]_2$ contains *two* objects. Yet we still assert $x = x$ without meaning that there is only one x. To assert $x = y$ is to assert the *Parker-Rhodes Principle of Indistinguishability* ([3] p.7); namely,

(i) if C is a class containing x, then the result of removing x and replacing it by y is again the class C

and

(ii) if C is a class containing x, then the result of inserting the element y into C is a new class C' different from C with one more element than C.

The concept of *identity* is tied to the concept of *singularity*; that is, *sameness* implies *oneness*. What we have in mind in multisets is a *plurality* of objects without a *diversity* of objects. The multiset $[x,x]$ has cardinality *two* even though there is nothing which distinguishes the x's.

Therefore, repeated elements in multisets satisfy the Parker-Rhodes criteria for indistinguishables: they behave as identicals when elements of different classes, but they behave as a plurality when they are elements of the same class (they each contribute to the cardinality of the class). Parker-Rhodes insisted that repeated elements in multisets should be called *indiscernibles* because they can be labelled (in his view, indistinguishables cannot be labelled). The theory of sorts developed in [3] is a radical departure from classical mathematics. It is "... a novel mathematical system parallel to, but largely unlike, Set Theory." The fundamental difference is that sort theory is *triparitous* in nature, whereas set theory (and multiset theory) is *biparitous* in nature.

Given objects a and b, in what relationship can they stand to each other? In classical biparitous mathematics, there are only two such relationships:

(i) a and b are *identical* (they are the same object, there are not two objects, just one) in which case we write $a = b$

(ii) a and b are *different* (they are not the same object, there are two distinct objects) in which case we write $a \neq b$

In this case, each relationship is the negation of the other: non-identity is difference and non-difference is identity.

In triparitous mathematics (invented, I believe, by A.F. Parker-Rhodes) there are three such relationships. To (i) and (ii) is added a third possibility:

(iii) a and b are *indistinguishable* (they are twins, they are a plurality of two but with no distinguishing characteristics) in which case one writes $a \hat{-} b$ ([3], p.5, also Ramsey [5] pp. 180-183)

In this case, the negation of one relationship is the disjunction of the other two, so that, for example, non-identity is the same as difference or indistinguishability, and non-difference is identity or indistinguishability. Parker-Rhodes calls non-indistinguishable (or simply, distinguishable) objects *bipar* since they are either identical or different (as is the case classically).

Parker-Rhodes felt that to every multiset there corresponded a sort with the same structure, and to every sort, a multiset with the same structure. He felt that there was a greater degree of *constructiveness* on the multiset side, but he also felt that one simply could not do multiset theory using only biparitous mathematics. The first axiom of Sort Theory ([3] Axiom 4.1, p. 57) "... sets this theory off sharply from Set Theory". It states (I think?) that

"If x is a class and y is a class
and $x \in y$ and $y \in x$, then $x = y$".

In other words, classes which are mutual members are identical. But it is noted that the converse is not the case, "... identical classes are not necessarily mutual members".

A *sort* is defined ([3] pp. 66-67) to be a class of which

"... it is *not claimed* that *every* pair of members is bipar,
i.e. either identical or distinct."

It is added that "... it may be claimed that *some* pairs are so, and it may even be the case that *all* are provided this has not been proved or asserted." Also ([3] p.68), members of a Sort, if any, must be Sorts. This is a further difference from Set Theory (with urelements) in which Sets may have non-empty members which are non-sets. Also

"... there is nothing which does not have members..." ([3] p. 57) and "... the search for ultimate members of a Sort leads to an infinite recursion, which is the motive of defining elementary Sorts as being their own members" ([3] p.68)

These statements confirm that the epsilon of class membership in Sort Theory is not well-founded. However, as interesting as this theory appears, I have not been able to "get my head around it" yet. I can, therefore, say nothing more about the Sort Theory in [3].

NEGATIVE MEMBERSHIP

I now wish to discuss what I consider to be the most exciting extension of multiset theory. In the years 1970-72, when I was a graduate student in Canada, I read several popular accounts of the positron, antiparticle annihilation, et cetera. One such book was called, I believe, *Worlds and Anti-Worlds*. In fact, I *misread* these popular accounts because I came to believe that the result of a particle-antiparticle annihilation was, literally, nothing! It was only much later that I realized that energy is omitted from this interaction. This does not matter, however. I had in my mind the idea that if I held the "stuff of reality" in one hand and the "anti-stuff of reality" in my other hand, I could bring my hands together, and the result would be nothingness. I thought about what this would mean in set theory. Is it possible, for any set A^+ of elements, to define an anti-set A^- of "elements" such that the result of "joining" A^+ and A^- together (the result of such a "union") would be $\varnothing$, the empty set. This idea I wrote down in 1971 as follows:

$$A^+ \uplus A^- = \varnothing \ .$$

I knew some set theory so I had a reasonably good idea about the nature of the set A^+ and the empty set $\varnothing$. I spent a great deal of time thinking about the possible nature of the anti-set A^-: Was it unique? What did its elements look like? How did they belong to A^-? What was it about the elements themselves or the nature of their membership in A^- that could cause them to "undo" the set-theoretic structure of A^+? I also thought a great deal about the "generalized union" operation $\uplus$. How could such a union "erase" the presence of elements (their very existence) in A^+? I thought about these ideas for two years and got absolutely nowhere with them. I then spent the next 10 years doing other things. In 1982, I began thinking again about sets and anti-sets.

At this time, I started using the notation

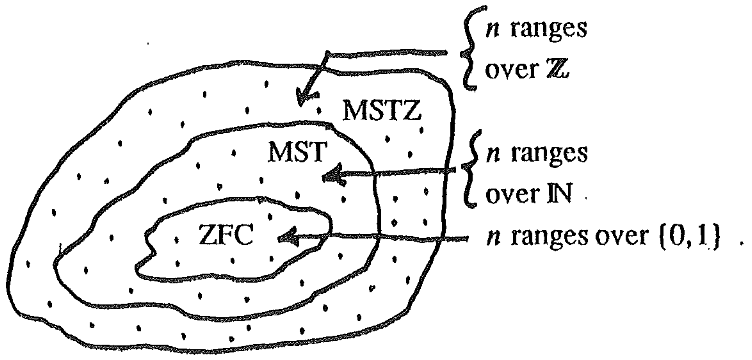
$$x \in^n y$$

in which I raised the set-theoretic membership epsilon to the "power" (or the "order") n. It was the use of this notation that lead to the further development of the theory. I could see that if n was either 0 or 1 then the result is just $x \notin y$ and $x \in y$, respectively. I had heard about fuzzy set theory, so I could also see that if n took values in the real unit interval $[0,1] \subseteq \mathbb{R}$ then $x \in^n y$ expressed the fuzzy membership of x in y. The idea of going "beyond one" then suggested itself. Hence, for $n = 2, 3, 4, \ldots$ the "set" y must contain more than one copy of x. Hence, writing $x \in^4 y$ suggested that among the elements of y there occurred, somewhere in y, four copies of x. This was my first notion of the concept of a multiset. A friend of mine, at the University of Victoria, suggested I look at H. Weyl's *Philosophy of Mathematics and the Natural Sciences* in which there was a discussion of "aggregates". Weyl's "aggregates" were collections of objects in which certain objects occurred more than once. Weyl applied "aggregates" to problems in the natural sciences. I then went to Oxford in 1983 and eventually learned about "multisets" through a reference given to Robin Gandy by T.J. Smiley. This opened up the literature on multisets, and I learned that no formal theory of multisets existed. Many people were using multisets (under various names) and there were many different notations and conventions. There seemed to be a real need for a formal theory of multisets:

> "... the lack of adequate terminology and notation for this common concept [multiset] has been a definite handicap to the development of mathematics" Knuth (1981)

To this was added the observation that the lack of an adequate theory of multisets had also impeded the development of logic and philosophy. Meyer and McRobbie (1982). The time seemed ripe for a formal theory of multisets that was based as much as possible on classical ZFC set theory. Thus, MST was born. In MST, the multiplicity variable n in the atomic formula $x \in^n y$ ranges over the natural numbers $\mathbb{N} = \{0, 1, 2, 3, ...\}$. It was a natural next step to consider a theory MSTZ in which n ranged over all integers (both positive and negative) $\mathbb{Z} = \{0, 1, -1, 2, -2, ...\}$. The type of negative membership we have in mind can be illustrated as follows: begin with the multiset $[x,x] = [x]_2$ which contains exactly two copies of x. Remove an x. The result is the multiset $[x]_1 = \{x\}$. Remove an x. The result is the empty multiset $\varnothing$. Continue this process. Remove an x. The result is the multiset $[x]_{-1}$ which contains -1 copies of x. Remove an x. The result is the multiset $[x]_{-2}$. Et cetera. This process of going "below the empty set" is conceptually difficult for some. It may be helpful to think in terms of deficit accounting. On reflection, the conceptual difficulties are exactly the same as the conceptual difficulties that were voiced with the introduction of the negative numbers. Despite such conceptual difficulties, the formal use of collections of objects which contain negatively many elements has been of interest, and has found application, in the literature (H. Whitney (1931), R. Rado (1975), T. Hailperin (1976, 1986), W. Reisig (1985) and R. Feynman (1987)). Detailed references are available on request.

The formal theory MSTZ (developed in [2]) is much like the theory MST. It is a collection of formal sentences (theorems) that make assertions about multisets in which elements belong any integral (positive or negative) number of times.



The *cardinality* of a multiset (the sum of the multiplicities of its elements) is a measure of the plurality of elements which it contains (the number of elements which it contains with repeated elements counted severally). A typical multiset in MSTZ might look like

$$A = [x,\ y,\ z]_{-1,\ 4,\ -2} \quad .$$

There are -1 copies of x, 4 copies of y and -2 copies of z. Its cardinality is 1 (the integer sum of -1, 4 and -2). The elements x, y and z are themselves multisets with elements of their own (which may belong with positive or negative multiplicity). Et cetera. This process always stops after a finite number of steps. The multiplicities are (finite) integers, but the number of distinct elements may be infinite. For example, the multiset

$$[1, 2, 3, 4, \ldots]_{1, -1, 2, -2, \ldots}$$

is a respectable multiset of MSTZ.

Therefore, in MSTZ it is necessary to define both positive and negative cardinal numbers (finite and infinite).

Within the theory MSTZ, the dream of 1971 about sets and anti-sets has been realized. One can show that in MSTZ, for every multiset A, there exists a unique multiset A^- (called the *shadow* of A) such that

$$A \uplus A^- = \varnothing$$

where $\uplus$ is the "additive union" of multisets (the multiplicities of common elements are added and those of non-common elements are left unchanged). The elements of A^- are exactly the elements of A with multiplicities of opposite sign. So, with our example above,

$$A^- = [x, y, z]_{1, -4, 2} \;\; .$$

The "strangeness" of A^- is not with respect to its elements (they are no different than those of A), but with respect to the "dual" nature of their membership in A^-. Adding corresponding multiplicities we get

$$[x, y, z]_{-1, 4, -2} \uplus [x, y, z]_{1, -4, 2} = \varnothing$$

where zero multiplicity is the same as non-membership. The definition of $x \subseteq y$ of MST

$$\forall z \, \forall n \, (z \in^n x \rightarrow \exists m \, (n \leq m \wedge z \in^m y))$$

works equally well in MSTZ. Only non-zero multiplicities are counted. Therefore, there are no proper (non-empty) msubsets of $\varnothing$.

Examples

$$[x]_{-10} \subseteq [x]_{-6}$$

$$[x]_{-1} \subseteq \{x\} \subseteq [x,y]_{5, -8}$$

$$[x, y, z]_{1, -7, -2} \subseteq [x, y, z]_{2, -2, 7}$$

One can show that the defined msubset relation $\subseteq$ behaves classically; that is, it is relfexive, anti-symmetric, and transitive. In MSTZ, even a singleton set like $\{x\}$ has infinitely many msubsets:

$$\{x\},\ \emptyset,\ [x]_{-1},\ [x]_{-2},\ \ldots\quad .$$

Simple algebra of multisets in MSTZ:

binary $\cup$ - take maximum of non-zero multiplicities

binary $\cap$ - take only minimum of two non-zero multiplicities

binary $\uplus$ - add the respective multiplicities.

$$[x,\ y]_{-1,\,2} \cup [x,\ z]_{3,\,-1} = [x,\ y,\ z]_{3,\,2,\,-1}$$

$$[x,\ y]_{-1,\,2} \cap [x,\ z]_{3,\,-1} = [x]_{-1}$$

$$[x,\ y]_{-1,\,2} \uplus [x,\ z]_{3,\,-1} = [x,\ y,\ z]_{2,\,2,\,-1}\ .$$

In MST, $\bigcap x \subseteq \bigcup x \subseteq \biguplus x$.

In MSTZ, $\bigcap x \not\subseteq \bigcup x$, $\bigcap x \not\subseteq \biguplus x$ and $\bigcup x \not\subseteq \biguplus x$.

In MSTZ, one can prove that

1. to every y, there exists a unique *root set* y^* such that $\forall x\ (x \in y \text{ <-> } x \in^1 y^*)$ (in MST $y^* \subseteq y$, but not in MSTZ) ([2] p. 22)

2. to every y, there exists a unique *shadow mset* y^- such that $\forall x\ \forall n\ (x \in^n y \text{ <-> } x \in^{-n} y^-)$ so $y \uplus y^- = \emptyset$ and $(y^-)^- = y$. ([2] p. 23)

3. one can define, thereby, *unrestricted complementation*: for any msets x andy y, define $x - y$ by $x \uplus y^-$. Therefore, $\{x\}^- = ([x]_1)^- = [x]_{-1}$ is simply the result of removing a single copy of x from the empty set; that is, $\emptyset - \{x\} = \emptyset \uplus \{x\}^- = \emptyset \uplus [x]_{-1} = [x]_{-1}$. ([2] p. 25)

4. to every y, there exists a unique *hereditary shadow* mset $y^{=}$ such that $\forall x \, \forall n \, (x \in^{n} y <\!\!-\!\!> x^{=} \in^{-n} y^{=})$ where $y^{=}$ is the result of taking the shadow of y, the shadow of elements of y, the shadow of elements of elements of y, ... through all msets used in the construction of y. Also $(y^{=})^{=} = y$. ([2] pp 27-28)

The hereditary shadow operation $^{=}$ is useful in MSTZ to define *negative cardinal numbers* needed for the cardinality of certain multisets in MSTZ (like $[x, y]_{3,-7}$). For any classical cardinal number λ of MSTZ (a hereditary set in ZFC $\subseteq$ MST $\subseteq$ MSTZ) we define its *negative* to be simply $\lambda^{=}$. For example, the finite cardinal $\hat{3}$ is the von Neumann (hereditary) set

$$\{\varnothing, \{\varnothing\}, \{\varnothing, \{\varnothing\}\}\} = \hat{3}$$

containing *three* elements.

The finite cardinal $-\hat{3} = (\hat{3})^{=}$ is

$$[\varnothing, [\varnothing]_{-1}, [\varnothing, [\varnothing]_{-1}]_{-1,-1}]_{-1,-1,-1}$$

since $\varnothing^{=} = \varnothing^{-} = \varnothing$.

The theories discussed thusfar (ZFC, MST and MSTZ) are collections of formal sentences that make (proveable) assertions about sets and multisets. We now turn our attention to the universes about which they make assertions. For ZFC, the *cumulative hierarchy* of classical sets is denote by V and it is constructed from the ground up (from the empty set) using the power set operation.

The Principle of the Power Set Operation:

> Any level of the hierarchy is a collection of sets. Any subset of that collection is a candidate for membership in the next higher level of the hierarchy.

For example, if the level contains just two elements like $\{x, y\}$, then the next level is $\{\varnothing, \{x\}, \{y\}, \{x, y\}\}$ which contains exactly the subsets of $\{x, y\}$.

The entire set-theoretic universe of ZFC is constructed from the empty set $\varnothing$ using the power set.

$$V_0 = \varnothing$$

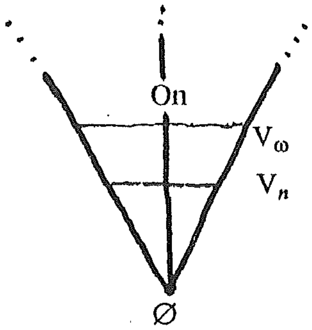
$$V_1 = \{\varnothing\}$$

$$V_2 = \{\varnothing, \{\varnothing\}\}$$

$$V_3 = \{\varnothing, \{\varnothing\}, \{\{\varnothing\}\}, \{\varnothing, \{\varnothing\}\}\}$$

$V_4 = \mathbb{P}(V_3)$ ...

We note that $\hat{0} \in V_1$, $\hat{1} \in V_2$, $\hat{2} \in V_3$, $\hat{3} \in V_4$ and, in general, the *ordinal* $\alpha \in V_{\alpha+1}$. It is usually pictured as



A great deal (if not all of mathematics) takes place

within the class $V = \bigcup_{\alpha \in On} V_\alpha$.

In a very similar way, the entire multiset universe of MST can be constructed from the empty set ∅ ([1] p. 44).

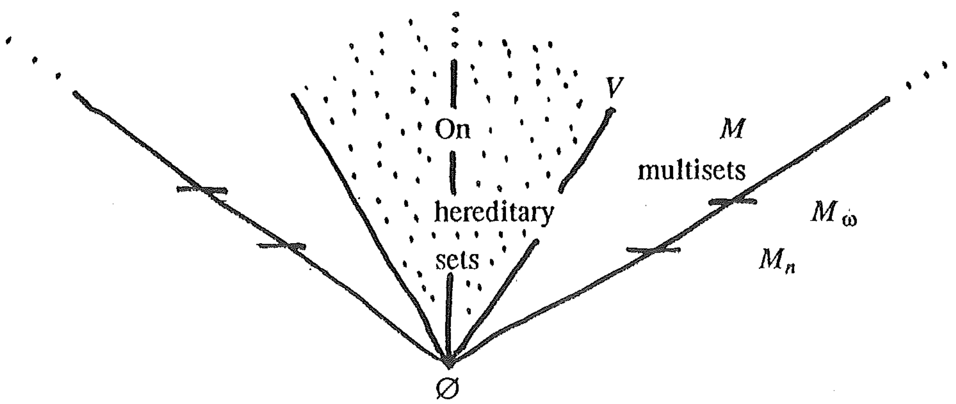
$$M_0 = \varnothing$$

$$M_1 = \{\varnothing\}$$

$$M_2 = \{\varnothing, \{\varnothing\}, [\varnothing]_2, [\varnothing]_3, \ldots\}$$

$$M_3 = \{x \mid x^* \in \mathbb{P}(M_2)\} \text{ where } \{\varnothing, \{\varnothing\}\} \text{ is in } M_3 .$$

$$M_4 = \ldots\ldots\ldots\ldots\ldots\ldots$$



Again, the MSTZ universe can be constructed in a cumulative hierarchy ([2] p. 26):

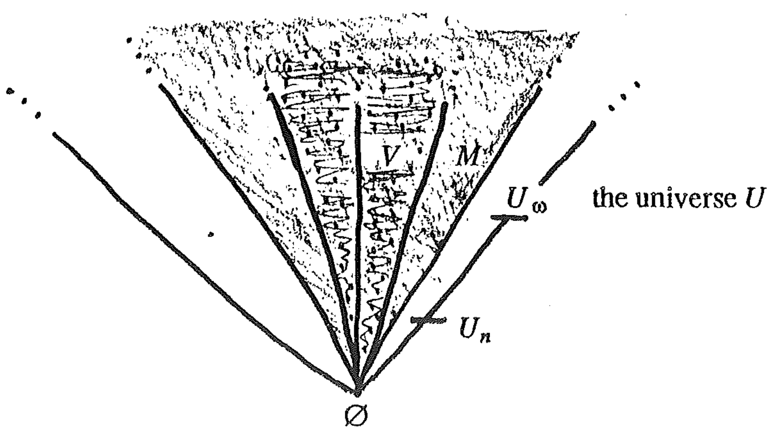
$$U_0 = \varnothing$$

$$U_1 = \{\varnothing\}$$

$$U_2 = \{\varnothing,\ \{\varnothing\},\ [\varnothing]_{-1},\ [\varnothing]_2,\ [\varnothing]_{-2},\ \ldots\}$$

$$U_3 = \{\varnothing,\ \ldots\ [\varnothing]_n\ \ldots,\ \ldots\ [\{\varnothing\}]_n\ \ldots,\ [\varnothing,\ \{\varnothing\}]_{n,m}\ \ldots$$

$$\ldots\ldots\ [[\varnothing]_n]_m\ \ldots\ldots \text{ et cetera.}$$



In U is $V^=$ and $M^=$, as well as all mixed msets with both positive and negative multiplicities.

CONCLUDING REMARKS AND REALITY

Even if Leibniz was right in saying that there are no two things that are exactly alike (everything is unique, there are no elementary particles) which exist in nature, he did admit such entities in abstract theory. Indeed, the human mind tends to disregard distinguishing characteristics of real objects. Therefore, instead of *sets* of objects (in which every element is different) the human mind has a natural tendency to abstract away certain distinguishing characteristics and manupulate *multisets* of objects (in which some elements occur more than once).

For example, let us accept the fact that everyone in this room is unique; that is, the collection of people in this room is a *set* of (say 20) distinct elements. It is more often than not useful to think of collections of people as a "population sample" with elements distinguished only by certain criteria. Each such sample by sex, by religion, by age, by profession, by height, by weight, et cetera, of the people in this room, gives rise to a different multiset of cardinality 20. For example, $[M,\ F]_{17,3}$ or $[\text{Phy, Phil, Math}, \ldots]_{8,5,3,\ldots}$ or $[C,\ J,\ H,\ M,\ B,\ A,\ O]_{10,2,3,2,1,1,1}$. In many respects, this is how science deals with the "stuff" of reality: substances (multisets of compounds), compounds (multisets of molecules), molecules (multisets of elements), elements (multisets of particles), particles (multisets of quarks), ...

In my enthusiasm for negative membership, I do not wish to claim too much. Yet in the very simple notation for the multiset

$$[x, y, z]_{-2,3,-8}$$

(or any other multiset in the universe U) there is hidden many levels of complexity. The elements x, y and z may be any multisets in U, and *their* elements in turn. By "travelling down" the epsilon chain only a finite number of levels (well-foundedness) great depth and complexity can be expressed.

In my boldest moments, I think that the formal structure of U (which is no more than an abstract conceptual pattern) and the inter-relationships between objects in U, say something important about the actual structure of *reality* (the stuff of existence, the sum total of what there is, all that is the case). Such things are difficult to put into words, as difficult as to describe the mind itself. What I am unable to describe has to do with the balance of surplus-deficit systems, symmetry, duality, yin and yang, the substantive versus the shadow existence of things, the presence and absence of things that underpins all of reality. What I am unable to describe has to do with "reaching into" the empty set and pulling out a multiset x and knowing that by so doing its shadow x^- also comes into being to maintain balance. Pulling out a y and seeing its y^-. Combining x and y in various ways and seeing these ways reflected in the looking glass that is x^- and y^-. What we observe, sense, interact with, are bits and pieces, waves and patterns, distortions and tips of icebergs. What I am unable to describe has to do with the conscious and unconscious universe, the substance and shadow of things, that which lingers unknown in the darkest depths of reality.

I am convinced that the acceptance of multisets and negative membership by mainstream set theorists would lead the way to something of a revolution in mathematics, and thereby, in all of the sciences that use mathematics as their formal language. With a new language comes the ability to express new thoughts, a liberation of the intellect. Notation often anticipates ideas. With such a new language at hand, the mathematician, the scientist, the natural philosopher, would not be forced into viewing indistinguishable objects and negative multiplicity as extraordinary or pathological phenomena.

I feel it appropriate to end with the words of Damaris Parker-Rhodes:

In my end is my beginning,
and the way out is the way in.
([4] p. 177)

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FINITE DOUBLE FIELDS

An Idea Related to the Combinatorial Hierarchy

December 1988 version

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The August 1988 version was read at ANPA 10 by David McGoveran

0. Motivation. A *double field* brings together in a single structure two algebraic fields in such a way that the multiplication of the first field is the addition of the second field.

In 1968 when I first thought of double fields, I was naively trying to think about discrete physics.

Since having read many popular works on cosmology in the early 1950's including several by Eddington, I have attempted on my own to look for mathematical clues to the cosmological ratios and the nature of space and time.

In 1956-7, while a Caltech freshman, I got to take Chemistry from Pauling, Math from Bohnenblust and a non-credit course called Physics X, in which about 20 diverse undergraduates got to take turns asking the late Richard Feynman any question on any topic. What an inspiration!

Soon, at Ohio State, I got to take many inspiring courses from fine mathematicians including Bumcrot's Projective Geometry and Zassenhaus's Experimental Number Theory.

I heard a rumor that the set of bases for the vector space of real numbers over the field of rational numbers had an uncountable infinity of elements, none of which could be pointed to with any finite amount of information. This started an active interest in the question of what kind of mathematical existence the physical universe has. Ever since, it has been my guess that the observable universe is a finite and discrete mathematical structure. But *which* finite discrete structure?

Soon, I learned that every projective plane which can be embedded in a three dimensional space is Desarguesian, and can be coordinatized with the elements of a division ring. Wedderburn's theorem says that every finite division ring has commutative multiplication. A commutative division ring is called by algebraists a *field*.

Ever since learning these things, I have felt that the observable universe is probably embedded in some many dimensional space over some kind of finite field. But which kind of finite field?

The impression that many people have had that space is continuous is easy to account for if its coordinates are from the field Z_p of residue classes modulo some large prime p. The abstract structure Z_p can occur in many ways. In one of them, a continuous space containing the rational numbers is divided into cells of length P with subcells of length 1. If the prime were large enough, we could not look inside the subcells, so I doubt that we could *observe* any difference between this situation, and any other situation in which the Z_p structure occurred. In particular, the Z_p structure formed by the second and third operations in a double field of equivalence classes of polynomials over Z_2 would look just like it.

One large scale difference between being in a space containing Z_p and being in a space containing the rationals is that if Z_p is involved, the matrix in which space time is embedded would *necessarily* wrap around in each dimension like a torus and be devoid of singularities. (Doesn't this remind you of Hawking?)

Another difference is that over Z_p, every function has a formal derivative. In fact, every function is a polynomial of degree at most $p-1$. The space of all functions over Z_p is a finite dimensional vector space. It has exactly p^p points.

Furthermore, over Z_p, there are pairs of pxp-matrices, A and B, such that $AB - BA = I$. Wouldn't it be nice to not need infinite dimensional Hilbert spaces.

If for some prime p, Z_p is involved, which prime? In what relevant way would one prime be more special than another?

On the other hand, properties of the events and particles of quantum mechanics seem to involve bit strings and an exclusive-or operation. The only finite fields with this operation are those with a number of elements which is a power of two. Of all finite fields, these are the least like Z_p. Is there any natural way that both could be involved?

If p is any prime and k is any positive integer, there is a unique (up to isomorphism) finite field with p^k elements and these are the only finite fields. The prime p is called the characteristic of the field with p^k elements. Since the subfields of a field are of the same characteristic as the field itself, neither Z_p for any odd prime p, nor any field of characteristic 2 can contain the other *as a subfield.*

There is a catch! To be a subfield, it has to be a subset which is a field under the same addition, and the same multiplication, Actually two fields of different characteristic cannot have either the same addition or the same multiplication. It is however possible for the addition of one to be the multiplication of the other. For this to happen the prime p must be one less than a power of two, or else itself be two.

This basically is how I first came up with the idea of double fields in 1968. Immediately, I began telling my friends about double fields. Interestingly, most of the mathematicians I talked with liked the idea whether or not it had applications to physics, but none of the active research physicists that I talked with were interested because they all felt that space was continuous.

For many years, although I continued, in my spare time, to pursue the matter (along with many others), I felt that my naive ideas on the nature of the universe were in the nature of a long shot.

In 1986 at the International Congress of Mathematicians in Berkeley, I gave a brief talk introducing finite double fields, and pointing out their relevance to a very old problem in number theory.

In this paper we will look at the finite double fields. It would be interesting to know whether or not there are infinite double fields. *If there are not,* then the results of this paper together with the Loenheim-Skolem theorem imply that there are only finitely many Mersenne primes, settling an ancient Greek conjecture about perfect numbers.

However, it seems likely that there are infinitely many Mersenne primes. In fact, there is a particular infinite sequence of numbers which is conjectured to contain only primes. Beginning with *2*, for each number i in the sequence, let its successor be $2^i - 1$. The sequence begins with the primes *2, 3, 7,* and *127.*

The conjecture that all of the numbers in the sequence are prime became interesting in 1876 when Eduard Lucas proved that the next number in the sequence $2^{127}-1 = 170,141,183,460,469,231,731,687,303,715,884,105,727$ is prime.

With the double fields associated with the primes in this sequence, there are some auxiliary conjectures which may contribute to an induction proof that all of the numbers in the sequence are prime. This would settle the Greek conjecture in the other direction.

Finally, on 5 March 1988, at a meeting of the Northern California Section of the MAA, I first heard of ANPA, The Alternative Natural Philosophy Association. For 10 years now, ANPA people, mainly in England and California have been developing a finite discrete model of the universe for which double fields seem directly relevant.

In particular, the ANPA model is built upon the fields of 2-power order involved with the primes in the sequence above, which they know as *the combinatorial hierarchy.* Recently, with this model, David McGoveran, Pierre Noyes, and their colleagues have enjoyed great success in coherently blending relativity and quantum mechanics, and in deriving fundamental ratios that have not been predictable with any other model.

By the way, if all of the numbers in the hierarchy sequence are prime, which I now doubt, then ironically, a continuous model based upon the combinatorial hierarchy would be a possibility.

1. Sizes of finite double fields. We begin with the following definition.

DEFINITION 1. *A double field is a set D with three operations delta, addition, and multiplication, having identity elements* ∞, *0, and 1 respectively, such that D under delta and addition, and D without* ∞ *under addition and multiplication are both fields.*

Given a finite double field D with n elements; D under delta, D without ∞ under addition, and D with neither ∞ nor 0 under multiplication are of course Abelian groups having n, n-1, and n-2 elements respectively. The middle group, having n-1 elements, is the multiplicative group of the first field and hence is cyclic. Since it is cyclic and is the additive group of the second field, it must be of prime order. So, n-1 is prime. n is also the cardinality of a finite field, so it must be either a prime or a power of a prime. Since either n or n-1 must be even, we see that n-1 must be either 2 or a prime which is one less than a power of 2, that is to say a Mersenne prime.

Since Z_3 under multiplication is isomorphic to Z_2 under addition, there is a double field with 3 elements. If $p = 2^i-1$ is a Mersenne prime, then Z_p is a field. Since the multiplication of the field with 2^i elements is cyclic, it is isomorphic to the addition of Z_p. So, there is a double file with 2^i elements. This proves theorem 1.

THEOREM 1. *There are double fields with n elements if and only if n-1 is either 2 or a Mersenne prime.*

2. Multiplication by ∞. The one part of the operation tables of a double field which is not defined explicitly by the operation tables of the two participating fields is multiplication by ∞. However, it is easy to see that the only definition of multiplication by ∞ which is consistent with one being a multiplicative identity and multiplication distributing over addition is:

DEFINITION 2. *For all a including zero,* $a \times \infty = \infty = \infty \times a$.

Definition 2 leaves intact commutativity and associativity of multiplication, and distributivity of multiplication over addition. One is still a multiplicative identity. But ∞ like 0 has no multiplicative inverse, and ∞ has replaced 0 as the multiplicative annihilator.

3. From Polynomials to Double Fields. Let $p = 2^i - 1$ be a Mersenne prime, and let ϕ be an irreducible polynomial of degree i over Z_2. Let D_ϕ under delta and addition be the field of polynomial residue classes modulo ϕ. This is the standard Galois field with 2^i elements. Only the names of the operations and their identity elements have been changed. Note that ∞ is the equivalence class containing the polynomial 0, while 0 is the equivalence class containing the polynomial 1 which is x^0.

The addition group, being polynomial multiplication modulo ϕ, is the cyclic group with p elements, so each of its elements except for 0 is of order p. Consider the p monomials x^0 through x^{p-1}. Since each of them except x^0 is of order p, no two of them fall into the same equivalence class. So every equivalence class except ∞ contains a unique monomial of degree 0 through p-1. We will use the exponent of this unique monomial of least degree as a name for the equivalence class containing that monomial. These names are the integers modulo p. And ∞ is the corresponding name for the one element of D_ϕ which contains the polynomial 0, and no monomials. These names will be referred to as the numeric names of the equivalence classes.

Each equivalence class also contains a unique polynomial of least degree. We will call the coefficient string of this unique polynomial of least degree the polynomial name of the equivalence class. For each equivalence class, its polynomial name tells its role in the D_ϕ under delta and addition, and its numeric name tells its role in the field D_ϕ without ∞ under addition and multiplication.

Since $x^i \times x^j = x^{i+j}$, and the order of x is p, one way to perform addition (which is polynomial multiplication modulo ϕ) is to add the numeric names of the equivalence classes modulo p. Similarly, to perform multiplication (the third operation on D_ϕ), we just multiply the numeric names of the equivalence classes modulo p.

monomial of least degree	numeric name	polynomial name	polynomial of least degree
	∞	000	0
1	0	001	1
x	1	010	x
x^2	2	100	x^2
x^3	3	011	$x+1$
x^4	4	110	x^2+x
x^5	5	111	x^2+x+1
x^6	6	101	$x^2+1.$

Δ	∞ 000	0 001	1 010	2 100	3 011	4 110	5 111	6 101
∞ 000	∞ 000	0 001	1 010	2 100	3 011	4 110	5 111	6 101
0 001	0 001	∞ 000	3 011	6 101	1 010	5 111	4 110	2 100
1 010	1 010	3 011	∞ 000	4 110	0 001	2 100	6 101	5 111
2 100	2 100	6 101	4 110	∞ 000	5 111	1 010	3 011	0 001
3 011	3 011	1 010	0 001	5 111	∞ 000	6 101	2 100	4 110
4 110	4 110	5 111	2 100	1 010	6 101	∞ 000	0 001	3 011
5 111	5 111	4 110	6 101	3 011	2 100	0 001	∞ 000	1 010
6 101	6 101	2 100	5 111	0 001	4 110	3 011	1 010	∞ 000

+	∞ 000	0 001	1 010	2 100	3 011	4 110	5 111	6 101
∞ 000	∞ 000	∞ 000	∞ 000	∞ 000	∞ 000	∞ 000	∞ 000	∞ 000
0 001	∞ 000	0 001	1 010	2 100	3 011	4 110	5 111	6 101
1 010	∞ 000	1 010	2 100	3 011	4 110	5 111	6 101	0 001
2 100	∞ 000	2 100	3 011	4 110	5 111	6 101	0 001	1 010
3 011	∞ 000	3 011	4 110	5 111	6 101	0 001	1 010	2 100
4 110	∞ 000	4 110	5 111	6 101	0 001	1 010	2 100	3 011
5 111	∞ 000	5 111	6 101	0 001	1 010	2 100	3 011	4 110
6 101	∞ 000	6 101	0 001	1 010	2 100	3 011	4 110	5 111

×	∞ 000	0 001	1 010	2 100	3 011	4 110	5 111	6 101
∞ 000	∞ 000	∞ 000	∞ 000	∞ 000	∞ 000	∞ 000	∞ 000	∞ 000
0 001	∞ 000	0 001	0 001	0 001	0 001	0 001	0 001	0 001
1 010	∞ 000	0 001	1 010	2 100	3 011	4 110	5 111	6 101
2 100	∞ 000	0 001	2 100	4 110	6 101	1 010	3 011	5 111
3 011	∞ 000	0 001	3 011	6 101	2 100	5 111	1 010	4 110
4 110	∞ 000	0 001	4 110	1 010	5 111	2 100	6 101	3 011
5 111	∞ 000	0 001	5 111	3 011	1 010	6 101	4 110	2 100
6 101	∞ 000	0 001	6 101	5 111	4 110	3 011	2 100	1 010

FIGURE 1.
The double field of equivalence classes of polynomials over Z_2 modulo $x^3 + x + 1$

4. From Double Fields to Polynomials. Given any double field, D, with 2^i elements as specified by its three operation tables, we first assign each element its numeric name, by adding ones using the addition table. Next, using the delta table with the numeric names, we define a function C from the polynomials over Z_2 onto D:

$$C\left(\sum a_j x^j\right) = C\left(\sum_{(j/a_j = 1)} x^j\right) = \Delta_{(j/a_j = 1)} mod\,(j,\, 2^i - 1)$$

Obviously C maps polynomial addition onto delta, In particular, the polynomial zero being the empty sum is mapped onto the delta identity ∞, as is the difference of any two polynomials which are mapped onto the same element of D. Among the nonzero polynomials mapped onto ∞ there is a unique one of least degree. Let us call it ϕ. Since the equivalence class mapped onto ∞ is closed under linear combinations, every element of the class is a multiple of ϕ. The function C partitions the polynomials over Z_2 into equivalence classes, with each class mapped onto one element of D. It is easy to see that the equivalence relation is simply congruence modulo ϕ. Since there are 2^i equivalence classes, the degree of ϕ is i. If ϕ were factorable over Z_2 then its equivalence classes would not even form a field. So it is irreducible.

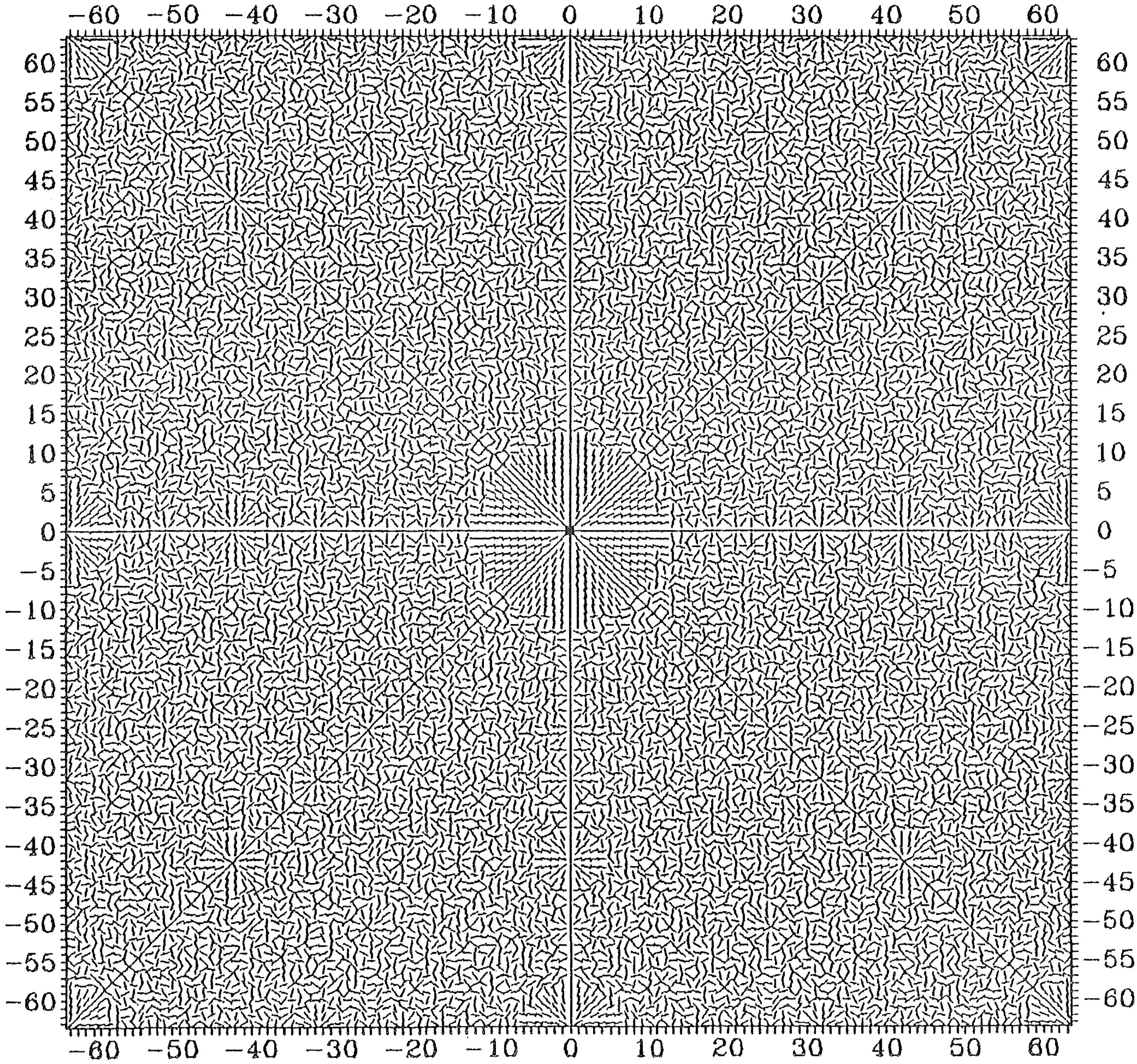
5. Summary. The results of the last two sections are summarized in theorem 2. A Mersenne index is an integer, i, such that 2^i-1 is prime.

THEOREM 2. *There is a one to one correspondence between even double fields and irreducible polynomials of Mersenne index degree over* Z_2. *In Particular, every even double field is isomorphic to a double field of equivalence classes of polynomials in one variable over* Z_2 *modulo a* ***unique*** *irreducible polynomial of Mersenne index degree.*

It is well known that if i is a Mersenne index then the number of irreducible polynomials of degree i in one variable over Z_2 is $(2^i-2) / i$. This then is also the number of double fields with 2^i elements. The question of whether there are infinitely many finite double fields is equivalent to the famous unsolved question of whether there are infinitely many Mersenne primes.

Since there are double fields, how about triple fields and beyond? Each finite n-tuple field would require the existence of two n-1 tuple fields with cardinalities differing by one. Since the first two operations of the *only* odd double field are isomorphic to the last two operations of the smallest even double field, there is a unique finite triple field and nothing beyond.

6. Coming up. At the 28 January 1989 ANPA West meeting at Stanford, I plan to pose a few concrete questions concerning ways in which geometry over the prime fields of the hierarchy may be relevant to combinatorial physics. To stimulate your curiosity, I have included here an illustration prepared for that talk.



Lines Through the Origin in the Plane
Over the Field of Integers Modulo 127

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Introduction.

The essential mystery of quantum mechanics is nowhere better revealed than in the so-called two slit electron diffraction experiment. Electrons are shot one at a time at a barrier in which there are two slits or holes; those passing through are detected by a photographic plate on the other side. We observe that there is a certain spot S on this plate which the electrons never hit. Then the experimental situation is changed by closing one of the holes in the barrier, and we observe that some electrons *do* hit the spot S! How can this be?

Two explanations have been proposed. The first, called the hidden variable theory, is closest to common sense; it goes something like this. Obviously the electron either goes through hole A or through hole B; suppose it goes through A. Now when B is closed the electron can hit S, but when B is open it can't, which means that something must go through B to deflect it. It's hard to find anything wrong with this reasoning, but in fact its seemingly innocuous conclusion runs into serious, and many would say fatal, trouble due to a theorem by Bell to the effect that if such "guiding" entities exist, they transmit their influences faster than light, in conflict with the theory of relativity.

The second and more radical explanation, known as the Copenhagen interpretation, says that the electron has no properties at all, including location, except in relation to some experimental situation. Since the electron literally has no *trajectory* apart from how it is measured, it is meaningless to ask where it would have gone if the other hole had been open, and the puzzle about what causes it to avoid a certain spot can't even be stated. The Copenhagen interpretation was invented by people whose driving passion was the new physics, and who needed an argument against those who would have them search for "mechanisms" where their best instincts told them none were to be found. I believe that their best instincts here were right. However, their so-called interpretation was a rather perfunctory afterthought to their physics, and is far from a complete account of the meaning of the quantum. Indeed if the Copenhagen interpretation is right, there is a much bigger story here than quantum mechanics, which is just the tip of the iceberg.

The quantum puzzles are usually regarded as being about the nature of quantum reality: we ask what kind of things might electrons be that behave so oddly when the hole is closed. I suggest that this is the wrong question. We should be asking instead about the nature of quantum change. What sort of change in the experimental situation could lead to such an odd change in the behavior of the experimental objects?

The word "change" is being used here in a broader sense than usual. The relativistic fusion of time with space, though it is popularly understood to imply the spatialization of time, can better be taken to imply the "temporalization" of space, as Einstein himself agreed (in a discussion with Emile Meyerson). Some hint of this is found in our commonplace concept of change of viewpoint. I see an arrangement of objects on the table. I change my viewpoint by going to the other side of the table, and there is a change in the arrangement. But exactly this same change can be said to "occur" with me remaining in my chair, provided you are also in your chair on the opposite side; we then speak of the "change" between your and my viewpoints. One describes such spatial "changes" in just the same way one describes temporal changes, in terms of <u>transformations</u> on a <u>state space.</u> Fifty years before relativity, Hamilton recognized the deep connection in physics between these two kinds of change when he discovered that his so-called canonical transformations, which were invented to describe changes of viewpoint on a classical mechanical system, also describe its changes with time according to Newton's laws.

In speaking of change in a quantum context, I shall mean change in this larger sense. Going from one to another quantum observable is like changing viewpoints, although the viewer can no longer be considered a passive spectator as he can in classical physics. One sometimes hears that the conceptual problems in quantum mechanics stem from the fact that the viewer unavoidably disturbs what he sees. In fact they go deeper than that; they involve the breakdown of a belief which is almost axiomatic in modern science, which I shall call <u>logical atomism</u> (This is not to be confused with what Bertrand Russell called logical atomism, which was a misnomer). Let's take a brief look at several kinds of atomism. For the Greeks, and for modern science before the twentieth century, atoms were material objects:

<u>Material atomism.</u> The <u>atoms</u> of an object are its smallest material parts. The <u>state</u> of an object is an arrangement of its atoms, and <u>change</u> is their rearrangement.

We now know that the so-called material atoms and their smaller parts share the mutability of objects in the everyday world, so material atomism is dead. But from its ashes has arisen a more abstract kind of atomism which has proved to be more durable, in part because it is a powerful way to describe the lawfulness of change. A law of change is a rule that applies not only to the change of actual states but to the change of all possible states; material atomism is itself such a rule. Now a law of change can be interpreted as saying that change rearranges the collection of all possible states according to certain constraints. Even if atoms come and go, the collection of the object's possible states goes on forever, provided we construe "possible" in a wide enough sense. Thus it would seem that by regarding the object as "composed" of its possible states, we obtain by fiat a new and irrefutable kind of atomism:

Logical atomism. The logical atoms of an object are its possible states. An object is defined in terms of its so-called state space (or phase space), which is an arrangement of its logical atoms. Change, as before, is the rearrangement of atoms, with the laws of change being conservation laws that describe unchanging aspects of the arrangement.

An object's logical atoms constitute a range of mutually exclusive possibilities or cases. Logical atomism says that the situation before and after a change always share such a range of cases. It is just this seemingly irrefutable truism that is challenged by quantum theory! In the electron diffraction experiment, the case range after the hole is closed is in a deep sense incompatible with the case range with the hole open; no larger range can encompass them both. Postulating logical atomism for quantum states leads to logical contradictions.

The problem of incompatible case ranges in quantum theory was explored in detail by von Neumann who came up with the idea of constructing a "system" of such ranges constituting an alternative non-Boolean "logic" which he called quantum logic. This is a very seductive idea, especially since there are some remarkable theorems showing that quantum logic, despite its austere structure (it is based on the two relations "P implies Q" and "P contradicts Q"), implicitly contains the whole structure of Hilbert space, and implies the Born probability rules too! (see ch. 1). Unfortunately, quantum logic hasn't done much to clarify the foundations of quantum theory. I think the trouble is that it tries too hard to hang onto the forms of logical atomism. It fosters the illusion that Hilbert space is some new kind of state space; in fact Hilbert space is not one state space but many, and these many are related to each other by an essentially new kind of change, a kind that *cannot* be construed as the rearrangement of states. The real challenge is not to modernize logical atomism so as to make it look tenable, but rather to get beyond it so we can understand this new kind of change.

Does the failure of logical atomism in quantum theory mean we must give up on atomism altogether? It may come to that eventually. But atomism has at least one more trick up its sleeve, and, for quantum change, this one really works. Let's slightly reformulate logical atomism so that the atoms are not the states themselves but certain sentences about the states, namely those of the form "The object is in state S". A change is now a rearrangement of these sentences. But note that it is also a rearrangement of their Boolean compounds, which are sentences like "The object is in state S or the object is in state S'" and "The object is not in state S". Let's now take a bigger step and redefine the atoms of the object to be *all* of these sentences, both simple and compound; call this sentential atomism. Just as material atomism is a conservation principle in logical atomism, so logical atomism is in turn a conservation principle in sentential atomism:

Logic: A set of sentences structured by the Boolean algebra of their logical connectives (see Ch. 3). We can think of logic as generated by NOT and OR.

Sentential atomism. The atoms are sentences about the object. The arrangement of atoms includes their logic. Change, as usual, is their rearrangement.

Principle of conservation of logic. Change rearranges sentences in a way that preserves their logic. In particular, it preserves the status of those simple state sentences which are, to use the logician's technical term, the atoms of the Boolean algebra.

Logical atomism, which is is tacitly assumed by every branch of empirical science except quantum mechanics, can be defined as sentential atomism with conservation of logic. We saw that logical atomism can't handle quantum change. But sentential atomism is a wider category. The present paper is an account of a kind of sentential atomism that actually does handle quantum change.

That such an atomism exists is, I believe, the most radical message of quantum mechanics; it means that quantum change *explicitly* violates the principle of conservation of logic. Quantum change re-immerses its object in a strange proto-world, a "world" lacking contradictions or consequences, where even the contrast between truth and falsehood is missing, its place taken, as we shall see, by a more primitive symmetrical dichotomy between truth and "anti-truth". This new atomism, which I shall call pre-logical, is extremely simple and easy to define abstractly:

Pre-logic. Like logic with OR replaced by XOR (exclusive OR). That is, a pre-logic is a collection of sentences structured by that part of Boolean algebra definable in terms of NOT and XOR.

Pre-logical atomism. Sentential atomism with conservation of pre-logic.

Pre-logic has had an odd history. Scientific development seems to like the flashback. A new field will start out with a few dramatic discoveries from what should be its advanced phase before it quite recognizes its proper subject matter and enters its elementary phase. In the late 1950's several English physicists and mathematicians became discontent with standard quantum mechanics as a basis for particle physics and began exploring a new approach which they loosely termed combinatorial physics. They felt that a good theory should be able predict the dimensionless scale constants, which orthodox theory still hasn't done, and soon came up with a rather esoteric mathematical construction which yields these numbers with surprising accuracy. Over the years others have joined their enterprise, and this original construction has turned into a more full-bodied theory, now known as bit string physics, which predicts quite a number of other things about particles. Very little of this will concern us here - it's much too advanced. What does concern us is that the basic operations of bit string physics implicitly make use of pre-logic.

What the bit strings of bit string physics seem to signify is what logicians call extension. Now it conveniently turns out that the concept of extension provides just the handle we need to get a mathematical grip on sentential atomism. We shall see in chapters 3 and 4 that even without pre-logic, the theory of extension has a good deal to say about quantum theory, revealing that certain aspects of quantum structure are built into the very notation of modern mathematics. We shall be studying extension in the context of the predicate calculus, which is modern mathematical notation in its most general form.

What is extension? The extension of a quality is usually defined as the set of all things possessing that quality, e.g. the extension of red is the set of all red things. This definition is too open-ended to yield a manageable concept. Does the set of red things include red things in dreams? In fairy tales? This kind of question seldom arises in mathematics, since the mathematical context normally supplies a well-defined reference set of which the extension is a subset; the extension of even is a subset of the *integers*, for instance. We shall follow mathematics here in always regarding an extension as a subset of some well-defined reference set.

Suppose we make a list of the members of a reference set. We can then represent the extension of a quality with respect to that reference set by checking off those entries in our list of which the quality is true. We can do this by writing "1" for true and "0" for false, thus representing the extension of the quality by a bit string.

Extension vector of a quality: A bit string whose places correspond to the members of a reference set, and whose 1's mark those members having the quality in question.

It's better not to think of extension as belonging to the quality itself but to a sentence that predicates it; thus instead of speaking of the extension of red, we'll speak of the extension of "It's red" or "x is red". A sentence with several variables doesn't predicate a quality but a *relation,* and we can define its extension in terms of a multi-dimensional two-valued array of which the sentence's variables are the indices. This is what we do when we draw a graph. Take the sentence "y = f(x)". A graph of this sentence is a two-dimensional array of black and white points displaying the functional relationship between its indices x and y as a black line. The graph shows the sentence's extension, which is to say, the black points mark just those joint values of x and y for which "y = f(x)" is true. If we think of black and white as 1 and 0, the graph becomes what we shall call an

Extension tensor of a sentence: A bit array whose indices are the sentence's free variables, where 1 means that the sentence is true, 0 that it is false. More generally, an integer array defined for some of the sentence's free variables whose entries are favorable case counts (see ch. 3), a case being defined as a joint value of all the sentence's free variables.

Extension tensors are very useful in analyzing the structure of compound sentences. We'll see (ch. 3) that all compound sentences are reducible to a special kind of compound sentence called a formula in which the only compounding operator is AND and in which no variable occurs more than twice. A variable which occurs once in a formula is called live, one which occurs twice is called a dummy. A problem in algebra, such as finding the solution or solutions to a set of simultaneous equations, can always be expressed as the problem of finding the extension tensor of the live variables of some formula (ch. 3). We now come to an important theorem:

Fundamental theorem of extension. The extension tensor of the live variables of a formula is the tensor product of the extension tensors of its components, with dummy variables contracted.

The fundamental theorem says you can theoretically solve any (finite) problem in algebra by calculating a tensor product; whether this could be useful in practice, I don't know. Our interest in this theorem is of a very different kind, having to do with the dummy variables rather than the live ones. As we shall see, dummy variables are closely connected with states and transformations.

Basic quantum theory can be formulated in terms of the so-called von Neumann density matrices, which represent quantum states, together with their transformation matrices (see ch. 1). There are two key rules here:

1). The diagonal elements of the density matrix represent measured probabilities.

2). The general rule for change of state is $S' = TST^{-1}$, where S is the old state, S' the new, and T is the transformation matrix.

Now according to Pascal's original definition, probability is the number of favorable cases divided by the total number of cases. Recall that the extension tensor gives favorable case counts, which are thus proportional to probability. Suppose we are given a formula containing the dummy variable x (x occurs twice). Define the state S of x as the extension matrix on the two places in the formula where x occurs, pretending that these places are occupied by different variables (ch. 4). It can be shown that diagonal of this matrix is the extension vector of x, thus satisfying rule 1 for quantum states. Suppose the formula is divided into two parts P and Q linked by the dummy variables x and x', which means that both x and x' are live (occur once) in both P and Q, and there are no other such variables. Let S be the state of x and S' be the state of x'; then the extension matrix T of P in x and x' will be called a transformation of S into S'. If T is non-singular, it follows from the fundamental theorem that $S' = TST^{-1}$, which is rule 2 for quantum transformations. Thus we see that two of the most basic rules of quantum theory, which are generally regarded as peculiar to physics, actually have to do with the compounding of mathematical notation.

Where does this leave quantum theory? To get to quantum theory proper, as given by Hilbert space or quantum logic, two things must be added to the above account. First, the states must be of a special kind which are unchanged by flipping them around their diagonals, and second, the entries in the extension tensors must be able to take negative valuex (actually, there needs to be a further slight specialization through symmetry to get complex amplitudes, but this is a minor detail at this stage). Now the first addition is innocuous enough, since such symmetry is a feature of perfectly ordinary formulae. The second, however, sounds completely insane. What could it possibly mean for a sentence to be true for a *negative* number of joint values of its variables?

Obviously, to let case counts go negative is no mere technical adjustment, no mere tweaking of the parameters. Try as one may, and I do urge the reader to try, there is no way to understand a negative case count in terms of the concepts we use in thinking about ordinary objects. Is there any way to avoid them? That, in essence, is what hidden variable theories are all about, and it explains their appeal. On the other hand, the great appeal of negative case counts, apart from avoiding non-locality, is that they explain quantum phenomena far more simply and directly than the hidden variable theories. Consider the two hole anomaly, where opening hole B prevents the electrons going through A from hitting the spot. Hidden variable theory must account for this by some elaborate Rube Goldberg mechanism for which there is not a hint of direct evidence. With negative cases counts, on the other hand, one has the much more obvious explanation that opening hole B introduces a number of new ways in which the electron can get to the spot; it just so happens that this is a negative number, cancelling the positive number of ways it can get there through A. Even if this is logical gibberish, you've got to admit it's simple.

The essential message of the present paper is that negative case counts are inextricably linked with pre-logical change. This is not the place to state the argument rigorously - that's what the whole paper is about - but I'll try to hit some of the high spots in a way that will keep us oriented.

First of all, in going from logic to pre-logic there is a useful rule of thumb which is sort of like the rule of thumb for "quantizing" a set of classical equations. This rule is implicit in the above definition of pre-logic; here it is more explicitly:

Rule of thumb. To find the pre-logical form which corresponds to a given logical form, state the latter in terms of OR and NOT and then replace all the OR's by XOR's.

Let's see how this applies to extension, which we can think of in the broadest sense as having to do with regions, parts, wholes, separation and merging. The extension tensor represents one kind of extension - call it case extension - where the regions are possibilities, their separation means that they contradict each other, and regions are merged into larger regions by OR. There is however another and more familiar kind of extension, that of facts, where separation means that the facts in question are uncorrelated items of information, and where the merging operator is AND.

Now knowledge rests on our ability to isolate, to separate out, knowable parts of the world. What happens to that ability when we apply our rule of thumb? With case extension, very little, since OR and XOR operate identically on separate regions. But with the extension of facts, it's a very different story. According to our rule, the pre-logical equivalent which we must substitute for AND is IF AND ONLY IF! This substitution utterly destroys our ability to isolate facts; "It's raining in New York AND the 49ers won in L.A." turns into "It's raining in New York IF AND ONLY IF the 49ers won in L.A.", for instance. In a pre-logical world, there is non-locality with a vengeance; not only is everything connected, everything is totally squashed together.

This might sound like new age truth to some people, but I regard it as a reductio-ad-absurdum. To put it bluntly, there *is* no pre-logical world. Pre-logical facts aren't all merged into one, because there aren't any such facts; pre-logic is prior to truth and falsehood. It's not a different kind of reality but, as I mentioned earlier, a different kind of change. More exactly, it's that about ordinary reality which persists through pre-logical change. Once we have understood this clearly, we can indulge in a bit of make-believe:

Every Boolean framework of sentences has its pre-logic as an aspect of its Boolean algebra. Now let's pretend that some of the sentences in this framework are "facts" in a "pre-world", meaning that they merge not by AND but by IF AND ONLY IF (which we'll now abbreviate to IFF). Is there a formal way to separate these "facts" from each other analogous to the way we separate facts in a logical world? In the ordinary world, two situations are separate if the number of their joint cases is the product of the numbers of their individual cases. This is the key mathematical fact behind the fundamental theorem of extension; it's what makes extension tensors work. Now it turns out that we can retain this key fact in pre-logic, and hence also the fundamental theorem, if we replace falsehood by a different kind of "truth-value" called anti-truth.

The case count of a sentence is the number of cases for which the sentence is true; cases for which it is false don't enter into the count. Cases for which it is anti-true, however, do enter into the count - negatively.

Favorable case: Adds 1 to the count.

Unfavorable case: Doesn't affect the count.

Anti-favorable case: Subtracts 1 from the count.

The pre-logical extension tensor of a sentence is an array of 1's and -1's, or more generally, any array of integers; these integers are called amplitudes. Now if we define formulae as before except with IFF replacing AND, we have exactly the same fundamental theorem of extension. As far as the tensors go, the only way you know you are in pre-logic is that you run into negative case counts. But as we saw above above, this is just what is needed in the theory of extension tensors to get quantum mechanics.

This in effect ends the story that will be told here. It is of course only the barest beginning of the theory of pre-logic. The next major project is to show how to describe the classical world, with its full Boolean logic, within the mathematical framework of pre-logical extension tensors, and then to analyze quantum measurement as a classical event in relation to pre-logical change. The general idea here is straightforward enough. First of all, the composition structure of sentences can be defined in terms of that of their extension tensors, so it's just a matter of getting the numbers (amplitudes) right. To get a classical world, one needs two things: there must be 0's or near 0's representing falsehood which are relatively stable under transformation, and the other amplitudes must be positive. Both of these are essentially large-number effects involving pre-logical formulae having large numbers of "hidden" dummy variables. The first requirement is related to an abstract form of the second law of thermodynamics, while the second, curiously enough, is related to quantum theory, since the self-adjointness of pure quantum states, which can be seen to have a statistical origin, guarantees that classical probabilities are never negative.

I think of the present work as being in the broad tradition of the Copenhagen school. It strongly supports Bohr's contention that quantum mechanics requires a classical context. It goes further than that to assert that reality is classical; there is no "quantum reality". But it also reminds us that reality isn't the whole story. There is also change. The deepest lesson of quantum mechanics, as revealed through pre-logic, is that reality is far more malleable than we had ever imagined.

Biomagnetics and Organic Germanium: two holistic therapies in need of application and explanation

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Introduction

My aim is to introduce the potentialities of two therapies I have come across in recent years. As scientific adviser to a small British mental health charity I had been exploring new therapies which may alleviate the immense suffering of the mentally ill - suffering which extends well beyond the patient to include the despair of close relatives and friends.

In this day and age when nationalism and religious ideologies still have a firm hold on the imagination of man it is pertinent to note that when a person is labelled as a schizophrenic he or she is not called an Indian schizophrenic or an English schizophrenic - mental illness cuts across all such psychological barriers that man has erected.

My own interest in these matters was driven by personal circumstances involving a desire to end the suffering experienced by my wife following the birth of my first daughter seventeen years ago with subsequent deterioration such that my second daughter, now eleven, was born in a psychiatric hospital.

Twelve psychiatric admissions have shown me the limited but significant value of psychotropic drugs. The drugs often control but rarely effect any deep healing. The search has extended my observational horizons beyond those in which I was trained (physics) to areas which include amongst others, clinical ecology (generalised allergies to the environment) and magnetic sensitivities involved in biological organisms and their orientation behaviour. For example the homing ability of pigeons remains a mystery although a magnetic sense is implicated.

Biomagnetic therapy

Biomagnetic therapy operates on the eight extra meridians of acupuncture (1,2) but instead of needles employs magnets the size of a small disc shaped pill and field strength approximately 400 G (0.04 T). The therapy was first developed by a Japanese acupuncturist Dr. Osamu Itoh some 15 years ago. In 1977, at the 5th World Congress on acupuncture in Tokyo, the technique was demonstrated and explained to a British acupuncturist, Dr. Terrence Williams who was impressed by the simplicity and power of the method. He returned to his busy practice in the UK and soon began to use the method while at the same time researching the technique including the incorporation of magnets in the place of needles. By 1983 he realised the technique had revolutionary potential since many diseases hereto considered incurable were being cured. He realised that the technique must be made more widely available and in February 1984 a teaching body was formed, the British Biomagnetic Association (BBA). Dr. Williams personal history on the development of biomagnetics has been documented (3). The BBA has produced an introductory information sheet, a small booklet and a number of newsletters with case histories. The information sheet and booklet are titled "Biomagnetic Therapy - the science of the future". Clearly if this is really the case then the explanation given in the information sheet (included here) must be partial since it is couched in terms of the science of today. The remarkable case histories which follow merit, in my view, serious attention by ANPA members to this exciting area.

Biomagnetic therapy - some case histories

Over the past year I have compiled eight case histories of regular and long term treatment with biomagnetic therapy. These case histories have been documented with a report both from the practioner and the patient and are available in full. Two practioners were involved. Below a summary of each case history is reproduced according to the main presenting complaint, but first I quote the words of the young lady with a severe brain haemorrhage. The reason for this quote is that there are, in my view, in her words, some generic features characteristic of the treatment-namely a restoration of peace and tranquility with prolonged treatment concomitant with improved physical symptoms.

> "My experience of the treatment has been absolutely amazing. At first I did not experience the true benefit of the treatment, but as the treatment became more intense I experienced a sense of true serenity and peace which I had not experienced in years, it also encouraged my body to be more relaxed, hence my mind is now much more serene, but it is not just a physical serenity, it goes much further than that.
>
> I've now found a deep peace of mind I have not experienced in years. It has also improved my memory"

Case Histories

1. Avril - female, age 30.

Presenting complaint and history:	Severe disorientation following a cerebro-vascular haemorrhage in 1978 for which a brain shunt was inserted. Memory loss, frequent headaches, spasms in left leg which was always cold. Suicide attempted on several occasions.

Treatment began August 1987 - once a week, by October 1987 mood improved - feeling more energetic, memory improved - could come to clinic by public transport herself. By April 1988 free of headaches, walking easier, by November 1988 - No cramps, memory continuing to improve - feels she is still making progress - her quoted comments on the treatment were made at this time.

2. Joan - female, age 41.

Presenting complaint and history:	Sciatica which developed over a period of 18 months following a horse riding accident.

Treatment in 1986. The first treatment cured her sciatica following a marked shortening of her leg on the application of the magnets. Two more treatments cleared residual pain in the lower back and pain in left knee.

Patients Comments	".... this method of treatment is by far more dignified and complete than any which I have seen in orthodox medicine in which I have been involved for the last 20 years".

3. Jacky - female, age 30

Presenting complaint and history: Ulcerative colitis for 15 years. Main complaint for which treatment was required - chronic backache and left leg ache (sciatica).

Treatment began April 1987. After two treatments leg much improved - pain only occasional. After sixth treatment much better but still getting cramps in bed. Bowel motion less frequent and more formed.

Patients Comments: Sciatic pain disappeared after several treatments as did morning back aches which had existed since late teenage years. Colitis symptoms improved.

4. Adrian - male, age 36.

Presenting complaint and history: Ankylosing spondylitis for 20 years. Continual use of pain killers. Tripping or catching toe on a pavement shook spine causing severe pain and agony. Difficulty in getting up from a lying position, colds and infections could result in complete incapacitation.

Treatment began Autumn 1985 initially once per week for 6 weeks then gradually to once every three to four weeks. By three months marked improvement in all symptoms. By June 1986 - no stiffness in back, neck and hips, generally free of pain. Use of pain killers almost nill. Generally well - occasionally gets a painful joint but only single joint pathology and not multiple as before.

Patients comments: "Having exhausted traditional medicine in my search for a treatment for my Ankylosing Spondylitis(AS), I have relied entirely on Dr. O'Leary's biomagnetic treatment to relieve unpleasant symptoms over the last few years ..... . As soon as I feel the symptoms developing I have a course of biomagnetic treatment. The treatment always stops the symptoms getting worse and usually they go away entirely until the next 'attack' many months later. My perception of the effectiveness of treatment is that sometimes the 'attack' can be stoped and cleared up in just one session, sometimes two and never more than about three. During the treatment I am not aware of any unusual sensations but invariably I will become relaxed and 'centred'. Usually the symptoms will have gone by the end of the session, or have gone by the next morning. Over the last year I have tried maintenance treatment, once per month, rather than wait for the symptoms to show themselves and then have treatment. So far I have had no serious or lasting recurrence of the AS symptoms.

Being a dedicated sceptic, I only seek biomagnetic treatment because it works - I do not pretend to understand how it works".

5. Madeleine - Female, age 38

Presenting Complaint and history:	Premenstrual tension(PMT) increasing in severity over the past 5 to 7 years. Had tried hormone treatment without success.

Treatment began July 1987 - weekly. After 2nd treatment noticed improvement - mood better but still very tired. After three months all PMT symptoms disappeared.

Patients Comment: "I began suffering from premenstrual syndrome after the birth of my second child in 1982. I came to dread the week before my period as during this time I felt tired and depressed .... . After only two treatments with Dr. O'Leary I began to feel more vibrant and energetic than I had felt since the birth of my first child .... . I now feel a new kind of confidence. I don't worry about my health in the way I used to because I can recognise when my body is out of balance and I am confident that a few visits to Dr. O'Leary can restore the balance and my body's ability to deal with the stresses that undermine one's health.

6. Ilva - female, age 64.

Presenting complaint and history:	General arthritis (osteo) of 30 years standing with complications.

Treatment began March 1987 with patient sitting in chair due to severe pain. By 5th treatment patient sufficiently pain free to lie flat on table for treatment. Continual remarkable improvement over the next few months with increased mobility - able to take and enjoy long walks. Patient said she did not know who she was now - since she had been in severe pain for 30 years. Progress continued over the next 6-12 months. Patient now returns intermittently for a "booster".

Patients Comments "..... The benefits from biomagnetics have been absolutely great. No medications taken to date since my first treatment. Within the first half hour, the pain on the neck first went. After two treatments I was able to go on a business trip to Paris and walk almost without pain all day at an exhibition - something I had not been able to do for a very long time. Prior to the biomagnetic therapy as well as having pains in my back, I also had severe electric shocks at the back of my head - this would often happen in the middle of the night. I had pains all over my body that with the heart condition would wake me up screaming in agony. I never slept more than a couple of hours without being woken by the pains. Walking any distance without pain was out of the question - I was often reduced to tears. Now I can sleep 6-7 hours without waking up, walk a mile without pain, do my shopping without help and more important without pain or tears".

7. William - male, age 61.

Presenting complaint and history:	Multiple Sclerosis for 8 years with frozen right shoulder, little use of right arm and gross weakness of right leg but still ambulatory with walking stick. Cramps in both arm and leg. Extreme urgency of maturation, marked constipitation.

Treatment began November 1987 - weekly. First treatment caused severe headache (lasted 6 hours) but then urinary flow easier, cramps reduced, feet warmer, improved libido. By fourth treatment cramps gone, constipation improving. After 25th treatment expressed a feeling of inner calm.

Patients Comments: "... I feel very "bubbly" now as I used to. My libido is much more in evidence now. If it wasn't for the fact that I have nearly lost the use of my right side I would be perfect".

8. Andrew - male, age 27

Presenting Complaint and history	Cerebral Palsy resulting from a Whooping Cough Immunisation which he received at the age of six months resulting in incoherent speech, bi-lateral spasticity making sitting in a wheelchair and handling in bed difficult. Developed diabetes 10 years ago - insulin dependent. Irritable bowel syndrome and myopia.

Treatment began April 1987 twice weekly for 7 weeks then weekly until July 1987 (when patient moved away). After first treatment patients immediate reaction was of feeling relaxed yet energised - no spasms in legs for 5 hours, slept particularly well. After second treatment developed an abscess on the left parietal area which supurted pus for three days. At same time his eyesight improved - could watch TV without glasses, speech became clearer, spasms decreased by 50% and sleep became deeper. After 4th treatment irritable bowl condition had resolved and given place to normal formed bowel movements. After 7th treatment could now stand momentarily and had new weaker glasses. By 13th treatment diabetes had become very stable - not subject to the swings that had occurred before commencing treatment. Patient much more positive and outward going - accepted on a residential course for handicapped people to do a course in computer science.

Patients Comments: (recorded into a tape recorder) "At first I felt a bit dubious, but eventually felt I could cope with it quite well. I was afraid of increased diarrhoea - but I got used to the treatment and it was a good idea. By the records it was really good. I wish everybody to know about it - and I hope it continues as I have benefited from this quite a lot. My whole frame of mind is relaxed. I am not a tense person any more, before I was really tense. Without the treatment I wouldn't have coped because my course at the moment is really outgoing. It is really good and I would recommend it to anybody.

Organic Germanium

Organic germanium is a trace mineral with unusual properties since it is not stored anywhere in the body and is completely eliminated within a few days following ingestion. The late Dr. Kazuhiko Asai of Japan spent some 30 years of his life in researching germanium - first inorganic germanium extraction from coal and later on production of a synthetic organic germanium compound named Ge 132. This water soluble compound was first synthesised in his laboratory in November 1967 following years of unsuccessful attempts at producing an organic compound from inorganic origins. Indeed the 132 in Ge132 refers to the 132nd compound attempted by Asai and his staff in the coal research institute Asai had established following the second world war and with the aim of rebuilding the Japanese industries. Dr. Asai passion for producing an organic germanium compound was driven by his early measurements of the presence of germanium not only in coal but in all living plant materials particularly those tradionally valued for their healing properties. He suspected that organic germanium maybe of great significance to life. The first test of the Ge 132 was naturally on himself and within a few weeks his rheumatic afflication was reversed.

In 1969 the Asai Germanium Clinic was established. In 1978 a nationwide organisation was established in Japan to foster research and applications. Over the last decade research has extended beyond Japan to Europe and U.S.A. In Germany another organic compound has been sythesised. From the early days reports of the use of organic germanium for a range of diseases began to emerge. Dr. Asai's own story is told in two english translations (4,5) and more recently a paperback has appeared (6), updating the scene.

Asai considers both mechanisms of action and clinical applications. In mechanisms the oxygen enriching dehydrogenation function is the key to Germanium's beneficial action according to Asai. Other significant features are its immune enhancing properties (including reports on gamma interferon production) and its protection of the endorphins. The clinical studies cover cancer where large dosages of the order of grams per day are used to arthritis and mental illness where lower dosage have been used. In cancer a number of trials have been reported on and a very significant feature is germanium's remarkable pain relieving properties. In chronic illnesses the length of treatment can extend over months and even years.

At present there is no forum within the U.K. for studying organic germanium. My knowledge has come from reading the literature and through personal contacts with those supplying and using organic germanium. In U.K. Jan de Vries who runs a large alternative natural health clinic in Scotland, has the greatest experience of the use of organic germanium. He had the good fortune to work with Asai before his death.

Dr. Asai himself treated many thousands of patients with his organic germanium compound. He showed with animal studies that Ge 132 was virtually non toxic and preferred to call germanium a "health giving substance" capable of "preventing and reversing diseases in both adults and children". Asai recognised that the explanation of germanium's action may not lie within established science in its present stage of development - so clearly there is possibly much to explore here.

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Two interesting recent books which study electromagnetism and man and look beyond current ideas are "The Body Electric" by Robert O. Becker and Gary Seldon (Marrow, N.Y. 1985) and "Electromagnetic Man" by Cyril W. Smith and Simon Best (Dent, 1989).

Note: Since publication of these proceedings an article by Duncan Campbell (New Statesman, 8th September 1989) has appeared claiming germanium to be toxic. The article is misleading since it confuses GeO_2 (germanium dioxide) with Ge132. GeO_2 is known to be toxic whereas, to date, Ge132 appears to be safe. Clearly this points to the need for a forum for the *serious* study of organic germanium.

Faruq Abdullah
12th September, 1989.

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The British Biomagnetic Association

INTRODUCTION TO BIOMAGNETIC THERAPY .....

"THE SCIENCE OF THE FUTURE"

Healing with magnets is a very old science; Healing with Acupuncture is also very old and very widely used in the treatment of all types of diseases.

Biomagnetic Therapy takes both these healing systems and brings them into line with modern understanding, terminology and technology. Very exciting developments are taking place in our understanding of the effects of magnetic fields on the organs and systems of the human body, particularly in the case of conditions which were thought to be incurable and some of this is beginning to be reported in the media.

There are different types of magnetic therapy being practised but Biomagnetic Therapy is based upon two main factors (1) the tried and tested science of Acupuncture and (2) the recognition of the necessity to take into consideration the body's own magnetic force and the correct channeling of these energies to ensure a safe and effective method of healing.

The technique of Biomagnetic Therapy was developed some nine years ago by Dr. Osamu Itoh, a doctor of Acupuncture and a Member of The British Acupuncture Association. Dr. Terence Williams, also a doctor of Acupuncture, introduced Biomagnetic Therapy into his Clinic and found that it could achieve better, safer and longer lasting results than the use of needles. He also discovered that certain diseases which would not respond to any other type of treatment, were being cured. This was in 1976 and since that time a great deal of research and tests have been carried out.

One of the most important discoveries made was that the majority of illnesses - whether they were organic or functional - will produce an irregularity in the body's structural alignment. Having made this discovery it was then necessary to find out why this should be so and why Biomagnetic Therapy was so effective.

To understand how **Biomagnetic Therapy** works, we must first of all explain how the human body must function in order to maintain health and well-being.

The most important part of the body is the Brain and within the Brain there is a command centre which is responsible for distributing energy and nourishment to all the cells. This centre is able to control and maintain the precise conditions needed to cope with the many disturbing and adverse influences which affect the human body. We know that no disease or discomfort will materialise until this control is lost.

The most common cause of the Brain's failure to control the balance of energies required to keep the body in perfect function is Stress.

Stress can be any stimulus that creates a change in the chemistry of the body and to which one or more parts of the body must respond if the change is to be normalised. For instance, if you run to catch a bus or a train, your muscles need oxygen and nourishing blood to remove the acids which build up in the tissues.

Your heart and lungs accelerate and presently the acids are removed and normality returns. That was not stress because the body responded correctly. If this were to continue to the point at which your body is unable to neutralise the situation, then the results could be potentially fatal and heart failure could result from such a situation as described. What is stress varies from one person to another; an exciting challenge for an athlete may be a disaster for an untrained person.

Stress can be any stimulus physical, mental or emotional, prolonged beyond the point to which the body can adapt.

The Brain is continually having to adapt to changes in our internal and external environments. This it does magnificently but there may come a time when it is exhausted and unable to command the correct responses. It's ability to maintain the body in an energised and stabilised condition has been over-ridden by a build up of stress which has resulted in a congestion of the flow of energy. Since this energy can no longer flow, other systems and organs become depleted and can not function normally.

It is widely accepted that the Thalamus (part of the Brain) is the "Command Centre" and controls the energies and nourishment required to regulate the Endocrine system (Glands, hormones); we now know that an imbalance here can adversely affect the Muscular system. As muscles are always involved where there is a distortion of the Skeletal system, i.e. slipped discs, twisted pelvis, arthritic hip or knee, you can begin to understand the effect each organ or system has upon the others and that it is not only the part of the body which is giving you pain or discomfort that is involved but, to some extent, all of the functions of the body are affected.

As stated earlier, most illnesses will produce an irregularity in the structural alignment of the body and it is for this reason that Practitioners of Biomagnetic Therapy will first of all examine their patients for any difference in the length of the legs, the length of the arms and any other structural abnormalities.

The Biomagnetic Practitioner will then, by the application of Biomags, immediately correct any structural distortions and, in so doing, will release the congestion and restore to the Brain the ability to normalise the body's energy flow to all the systems and organs. Healing will then begin to take place naturally and simply.

In some instances, patients will experience an immediate and noticeable benefit from their first treatment, many from the second and third treatments but the badly depleted and chronic conditions may take longer to replace the necessary energy and to stabilise it. The body's natural healing processes require considerable time to rebuild damaged tissue.

Patients are often surprised at the simplicity of the procedure. They go to their Practitioner with many symptoms and expect these symptoms to be individually evaluated. However, Biomagnetic Therapists are concerned with removing the cause of these symptoms as quickly and effectively as possible.

To show the difference between treating the symptom and removing the cause, let us take an example of a situation with which most people have at some time had experience.

You discover that one wall in your house is developing cracks; you could, for a time, make the cracks disappear by covering them with wallpaper. However, as you have not dealt with the cause of the cracks in the wall, you realise that it is only a temporary measure and that it will eventually widen and the problem increase. When this happens you will have to treat the cause, by which time much greater effort and expense may be required to repair the damage. Most conventional therapy is like papering over the cracks - it very effectively masks the symptoms for a time - but they will return and in all probability be much worse.

Practitioners find that the main difficulty patients have in understanding this treatment is its simplicity. It is usually very quick and may not last more than 10 - 15 minutes. Extending the time, rather than improving the results may well have the opposite effect. Most times the patient feels nothing other than a great sense of relaxation and many patients fall asleep during treatment.

The human body is very sensitive to magnetism and the treatment (which is cumulative) often works for days balancing all the body's systems - heart, lungs, endocrine, circulation, digestion, nerves, muscles, vertebrae, etc., plus, of course, the mental and emotional spheres.

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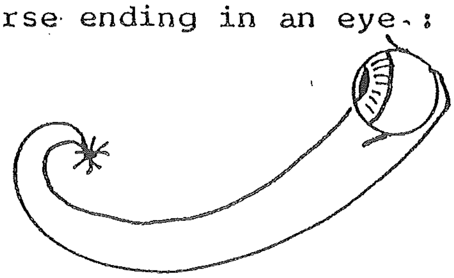
Does physics require a philosophy of reality ? Since Copenhagen it has tended to be noncommittal about this, but the use of terms such as "alternative natural philosophy" and "pre-physics" seems to imply something rather more definite.

Copenhagen did not set out to rubbish philosophy - what it stated was the factual observation that our knowledge of reality can be expressed only in algorithms, with which it is very hard to disagree - in fact "alternative natural phi losophy" probably should read "alternative-algorithm physics". All models, including extreme virtualist or mentalist models like those of Nagarjuna or Plato, are inevitably confronted with two entities, O and ~O. The problem about algorithms lies in determining the relative contributions made by each of these. We also have to accept the constraint imposed by the "anthropic universe", that in a non-anthropic world there would be no algorithms. Wheeler used to draw a diagram showing the emergence from a singularity of a widening universe ending in an eye :



a philosophy of reality depends on our ability to speculate on what would be there when the eye is shut. The word "reality " is tainted semantically with "apprehensible", though not if we go along with Wittgenstein - reality is everything which is the case.

The importance of this for the actual project of doing physics is that our basic prephysics model takes a line through O and ~O : our knowledge of "the real" consists of a pile of math describing the behaviour of entities, such as particles, which we actually postulate as observers (they are not "things" in the Democritean sense) and which we study in the hope of refining the algorithms until they approach a complete description of all observables. There is presumably no limit to this approximation, and it is all we are about to get.

Socrates made himself unpopular by asking apparently naive questions. If he belonged to ANPA he would quite possibly ask "What is physics studying ?" Prior to quantum mechanics this would have been a reasonably simple question. Now, in the light of Copenhagen, all of us, whether pursuing discrete physics or some other model, would probably say "We are looking for the most comprehensive algorithm, containing the fewest difficulties, to connect observed phenomena." Neuropsychology has possibly a rather easier reply if asked the same question. It is studying what is meant by observation and algorithm-construction, what it is that observes phenomena, and how the observing-constructing system ("Mind") is connected with phenomena which are not-Mind.

It is risky to guess what the next Socratic question might be, but it could well be "Are not these the same project ?" The initial justification for this particular come-back is that the two exercises are inseparable - all valid physical models either include observation or take it as given. They have to, because there is no physics without it (though we can, and 19th century physics which worked on an objective Lebenswelt did, take it out of the finished model : "reality is like that"). But there is a more fundamental and speculative reason for the question - although neuropsychology still tends to a Helmholzian consensus (Mind is the name we give to an activity of a biomechanical system, brain) the best safety shot, if we are at all aware of the problems which this raises in physics, is to say that Mind is a programme - token functionalism - with the rider that although all minds with which we are familiar are transduced by brains, if a thinkable system could enable the programme to be run by silicon chips, ectoplasm, or, as Dewdney recently suggested in Scientific American, the Aphraphulian computer made of pieces of string, then computers, ghosts, and catscradles could be said to have minds, or at least to be indistinguishable by Turing's criteria from mind-possessing systems.

Now it could ~~equally~~ be argued that "token physicalism" applies equally to the knowable universe : the substrate of physics consists of coded information - in other words, a programme. This is the justification of the second pseudo-Socratic question.

One can clarify this by analogy. Consider a videotape of _Hamlet_. This is a prosaic object which contains a digitally-encoded sequence of 1's and 0's, represented by magnetized areas on oxide tape, laser-readable pulses, etc., etc. It can also be played, given suitable equipment, and will then generate a sequence of pictures and sounds. If these are observed by a (human) brain, and, accordingly by Mind of the type which generates physics and looks for algorithms, the result is a Shakespeare play. Given this condition, what is the physicist studying ?

The Shakespeare play is a Lebenswelt - roughly the shadow sequence on the wall of Plato's Gedanken-cave = and obviously we start by analysing this. More detailed observation, showing it to be made up of lines and frames, leads to the inference that it is virtual and dependent on a finite sequence of 0's and 1's which store it as information. How precisely this programme is encoded might or might not be ascertainable, but we can establish that what we are seeing depends on a "Hamlet reality" R_H of which the phenomenal reality R_P on the screen is an "explicate". R_P is unfolded sequentially (it exists in a virtual 3 + 1 space on the screen which would simulate a real 3 + 1 space if our screen were holographic) while R_H exists somewhere en=bloc, as the whole text of this writing exists en-bloc until we read it sequentially : the videotape itself is, for the purposes of R_P, outside time and outside the display on the screen. Now the 0's and 1's are physically present on the tape-representation, but unless we have some way of inferring this, our universe of observation (a) is extradimensional with respect to R_P and (b) consists of information. Any algorithm describing it applies not to the tape but to a programme - token functionalism applied not to "Mind" but to the substrate of observation. Hence the point of the second Socratic question. The thing is made harder if we apply the metaphor to what we study in physics, because both the tape-machine and our observing minds are held to be specified by what is on the tape and should therefore lie either in R_H or in R_P - in fact, if we apply this to most models in inexplicit pre-physics they alternate dizzily between the two, or produce a recursive system (the "hyperloop") with a no-exit loop between observer and observed as parts of _alles was der Fall ist_. Spencer Brown propounded this, but has ha[illegible] cor[illegible]ing [illegible]ers.

It is luckily not necessary to work out exactly what physics is about before doing any physics, but it is quite important to speculate about any algorithm what exactly it is an algorithm _for_ ; subjacent reality, phenomena, or our cerebral method of ordering experience. Since the last of these is adapted to middle order phenomena, and also determines them (it imposes things like sequence and causality which may not apply at extremes of the sensible or inferrable world, in subatomic or cosmological theory) it may be invincibly unsuited to going beyond R_P : R_H is going in some respects to be counterintuitive, while R_P introduces ~~matter~~ material - the dramatic significance of _Hamlet_, which depends on human preconceptions and preoccupations. The algorithm obviously leaves these out, but even so we have to deal with Mach's view that nature "simply is : ~~causation~~ cause and effect... belong but to the abstractions we make in ordering information".

One view is that we have to ignore this kind of rumination, soldier on, and see where that goes - in the expectation that the domain of a satisfactory algorithm (one which fits all the known results and predicts others) will become clear as we develop it. But now that we talk about pre-physics, a more sophisticated idea than the wait-and-see implicit in metaphysics, it would at least be valuable to hypothesize what that domain is. In neuropsychology, which up to now has proceeded on naive dualist or reductionist assumptions under the impression that one or other, most recently the reductionist, is "scientific" and all others naive or unjustified, one has to create testable hypotheses. It is, after all, the nature of the phenomena we call Mind which we are supposed to be studying, as in physics we study such phenomena as matter or energy which are appearances to Mind, and Mind is what we are using to look for algorithms. There might be algorithms without an observer, at an abstract level, but they would be rather like the Sleeping Beauty, who only gets kissed awake in an anthropic universe. If we have an obligatory imaging system interposed between what is and whatever is analyzing it, we need to use the same system to analyze itself, so far as that is possible, and reverse any transformations which it makes. We only get _Hamlet_ if the tape is played to an audience.

One of the plusses for discrete-physics algorithms is that they are better targetted than others : hidden parameter or many-worlds models, while mathematicallysustainable, put a different game cartridge in the machine : discrete models, if they check out, would interface with any mindlike system and are Turing compatible. They describe what is on the tape, the information which carries R_H onto R_P : few others are philosophically pin-pointable to the same extent. Their relationship to alternative models, such as those of Bohm or of Everett could in fact be superpositional rather than alternative, like the double-take between wave- and particle-mechanics which the discreet model resolves - there may be observations which make it necessary to live with a kind of polytheism in physics without contradicting the basic simplicity of having a single, monotheistic model. Our tradition prefers either/or to both/and situations, but it could be that this is our problem (it cannot be Nature's). This could be disheartening for the discrete-theory modellers, but it does nothing in fact to devalue their model. There could be more on the tape than our transducer habitually transduces.

Neurophilosophers do need to explain to mathematical physicists what they are on about. Their problems do not arise from difficulties over the observer phenomenon, and whether participation alters <u>events</u> : it is a priori, and concerns the definition of "event" itself. In physics we have a system of which the observer and his instruments are part. If we started with a totality ॐ, we have either to make a primal division of ॐ into O the observer and $\sim O$, the observed, which is what the unspoken pre-physics of most algorithm-formation implies, or we need to devise our algorithm in a recursive mode : two parts or domains within ॐ are operating reciprocally on eachother. An algorithm for $\sim O$ as O sees it is only one limb of this process. For most practical purposes in physics that would not matter - a good algorithm would describe or predict all observed phenomena: taking this view is called "avoiding metaphysics". Unfortunately it means that we have to settle for a reality consisting of concepts without providing us with a way of discriminating between classes of concept: we feel intuitively that there has to be a difference in kind between the concept "electron" and the concept "the Nonconformist Conscience", but uninspected conceptualism does not provide one. This, like most sophisms, is a demonstrative overstatement. But supposing we say "The phenomena we conceptualize as particles are analogous to the gamepieces in a video game.

They do not behave like "things", and we have some difficulty at this non-middle-order level in stating exactly what a "thing" is - for all we know it is an eigenstate belonging to a particular eigenvalue, or a coincidence of several eigenstates belonging to several eigenvalues, as Dirac says. Never mind, when approximately these eigenvalues turn up, we see a blip on the screen which exhibits measurable behaviors and effects, and by playing the video game according to the rules we have ascertained we can manipulate it predictably, as we can Pac Man. Pac Man is not a thing, and neither is this, but we can still play the game. Pac Man and this particle entity differ from mere concepts - we can see Pac Man (he is a display) and our instruments can "see" this entity, while we can often see its en-masse effects.

I think that would be quite acceptable for physics if we operate it in vacuo, but it causes severe problems for neuro-psychologists and hence for an algorithm of completeness. After all, the Pac Man reality is obviously $\backsim$O, like R_H, even though it takes our imaging system to make a cartoon-figure out of an array of nonlocal nonsimultaneous light pulses : we are outside the box. But in the video-game called physics one part of the operation - that to which phenomena appear, the mind of O - has to be included in the system. Without O there may be reality but there are no phenomena. This argument was academic in the hands of, say, Berkeley, but it is not academic in physics: not, that is, if we do them from a pre-physics which includes Helmholzian mind, i.e. an activity reducible to the interaction of brain cells or silicon chips, because these are made up of the virtual entities, coinciding eigenstates, probability waves, or what have you, into which the substrate of physics dissolves - they too are virtual figures on the video screen, which makes both viewing the screen and analysing what appears there a recursive activity of the game pieces. Pac Man is playing Pac Man. When Pac Man views R_H, he sees R_P, but he is himself being specified by information in R_H.

What we have here is a loop in comprehension as regards our definition of $\backsim$O. It does not lead precipitously into psycho-physical dualism as Leibniz understood it, and has little to do with classical idealist-realist debate in philosophy (this time we are talking about actual observations, not pre-theory). It does award a Nobel Prize to Descartes, because we are back with the idea that the only ungainsayable entity is observerhood. We are on the verge of assuming pure mentalism exactly as 19th century physics assumed pure mechanism and real objects, though neither simplification is likely to be right. It is difficult to accept a superpositional both/and answer empathically, but it might be possible to devise one mathematically (it would write the observed paradox into the algorithm without necessarily explaining it - "this formula describes what happens"). We have seen the start of this project, which involves analysing both the O and the ∨O limbs of the operation, inserted implicitly into e.g. many worlds models, and rather more explicitly into implicate-explicate models. What mind-studiers would like to ask particle-studiers is how, if at all, these considerations affect the discrete physics model. Does it address Spencer Brown's point that the physicist "himself consists of the very particulars he describes", and if so, how ?

There are some indications how one might go about giving an answer. The discrete model, based on strings, is computer-compatible, which means that it can address a Turing-defined mind. I would have some reservations on the Turing definition very similar to those which make most psychologists reject classical behaviourism : as a matter of fact, if we compare neuropsychology with contemporary physics, they have a mirror-image relationship to their substrates. Physics is trying to formalise counterintuitive features of quantum reality, but takes a "hard" model of Mind. Neuropsychology is becoming aware of counterintuitive, or more correctly intuitive but unconventional, aspects of Mind, but is still largely physics-illiterate and based on the "hard" pre-quantum view of Nature generally. Where physics has ERP and von Neumann's regression, neuropsychology has to deal with paradoxes like the Feigl autocerebroscope. Haldane was probably right, when we put the two disciplines together, in surmising that Nature is not only odder than we think but odder than we can think : this is the challenge to mathematisation.

Defining Social Systems in terms of their process dynamics.

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Introduction.

The outcome of a process dynamics view in socio-economic phenomena is to bring Social Sciences into the realm of developmental theories. The developmental theories are in great need of dealing fully and in a formal way with the always difficult question of structural change. The combinatorial hierarchy (CH) is a formal scheme which provides such a setting (see Bastin et al, 1979); but only if we are able to define social systems in a way that allows us to use such formal language.

The paper pretends to briefly develop the following points:

1. The modelling relation: Natural vs Formal Systems.
2. Developmental view of Social Systems.
3. Some pre-requisites for a meaningful formalism.
4. Limitations of the CH-formalism in Social Sciences.

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1. The modelling relation: Natural Systems vs Formal Systems.

Any attempt to understand the external world presupposes that the sequence of events that we are able to perceive is not totally arbitrary. Quite to the contrary we register some regularities and, therefore, seek some kind of relationship between those events.

The relationship between events in the external world is called **causality**. We could not do science without an underlying belief in a certain causal order which, on the other hand, is already attested to by the basic fact of the viability of our own existence.

The task of theory and theoretical science is to translate the prescribed causal order into a formal language (such as Mathematics) from which can be drawn a representation of the causally related phenomena.

There is no causality involved in a formal system, only (logical) implication among propositions. So, when we speak of theory we are aiming at the possible correspondence between a certain class of events in the external world and a certain class of propositions in a formal language.

That «certain class of events», when represented in a formal system, determines a **model** of a particular natural system; and when a «certain class of propositions» is related to a natural system, we say of the natural system that is a **realization** of such (formal) model.

A key issue in establishing a good formal representation of a natural system is to be able to relate causal order (in the natural system) to rules of inference (in the formal system).

Having an adequate formal representation of our natural system allows us to work out possible scenarios in a well defined and consistent manner and then translate the end results (predictions), formally obtained, into specific rules and instances or our natural system (see Fig. 1).

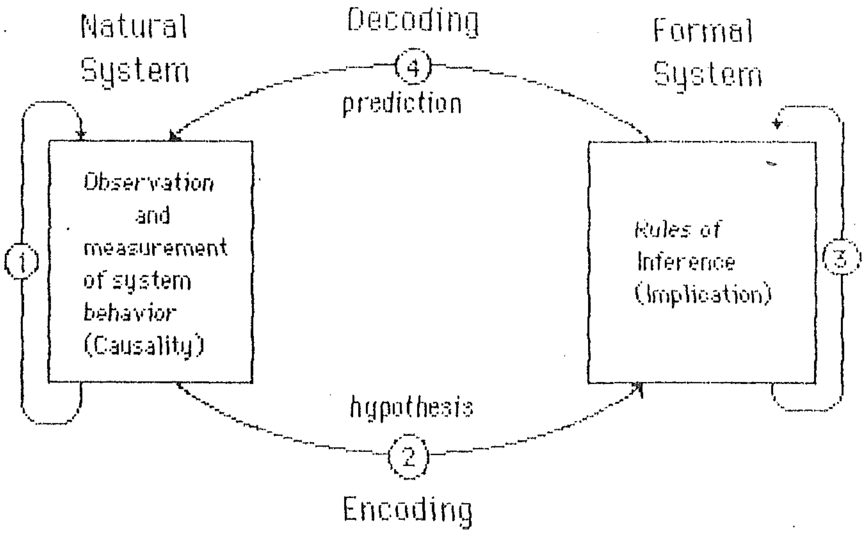

Fig. 1: Arrow 1= arrows 2+3+4. The modelling relation NS<->FS is established when the diagram commutes.

2. Developmental views in Social Systems.

Imagine for a moment the following scenario:
We have a two-dimensional spatial map of a particular geographic zone in which the flora of that region is plotted. We are faced with the task of determining the kind of ecological dynamics that is present between the different species of flora. There is a mountain in that zone but for some reason (unknown to us) the lines that determine the altitude are missing and we assume the same altitude for the whole region.

Any attempt to causally relate the different flora species would run into trouble. There is a scale factor that is missing without which there is no proper way of representing -and far less explaining- the plant distribution as it appears on our map.

We have had a somewhat similar situation in Social Sciences for quite sometime. Not with the different scales in altitude but with the different **scales of development**. A good indicator of this is the long tradition of isolation between Sociology and History as a manifestation of the separation between the micro and the macro phenomena. But, was not that also the case in Physics? Since when have elementary particle physicists and cosmologists been talking to each other? Only when it became apparent that they were talking about the same things; and that only happened when they took into account the developmental character of the universe as a relevant aspect of its own nature.

In Social Sciences it is also becoming more and more evident that in order to understand social systems we have to take into consideration their developmental nature.

Development is inextricably linked to spatio-temporal multiscaling and multiscaling to (multi) couplings. But that is not enough; we also need to account for the process or processes that brought those scales into being, and account for them in a coherent and essential way, that is, without calling for some causality external to the system.

If this is the case we are then forced to take a fairly strong constructive stand when dealing with multi-scaled systems. Here is where a major stumbling block lies. Just as in the case of the formal language (read Mathematics), the right tools have not been available in Social

Sciences.

3. Some pre-requisites for a useful formalism in Social Science.

It is my intention to simulate (as soon as the right tools are available) different models of societies. Given that intention, it seems feasable and particularly useful to choose David McGoveran's **Ordering Operator Calculus or OOC** (see his paper on «Foundations...»). His five basic principles are seemingly sound formal counterparts to any developmental natural system and, in particular, in our case, social systems. My views on how to define social systems were expressed well before I knew anything about the OOC (see Alvarez de Lorenzana and Ward, 1985; 1987) and still can be accomodated -to that calculus- without restrictions or distortions. I defined developmental systems (there to be called *evolutionary systems*) in terms of «global properties»; and system's development as the constructed expansion through -and in interaction with- the surrounding environment by means, only, of the relentless use of the systemic properties. It just happens that my «global properties» are what in the OOC are called *attributes*. That should be enough commentary on this issue.

Apart from the appropiateness -in principle- of OOC as a good formalizing tool, I want to make some comments on two aspects of the theoretical scheme.

A. Defining a model by coding a particular natural system into our formal language:

(i) **Constructibility**. The view that I have taken in relation to social systems is that they are constructive in nature (see previous reference).

The social system *builds* its own systemic (i.e., relevant) environment through contrast (discrimination). It is therefore appropriate to have as their formal representatives ordering operators.

The mathematical objects on which ordering operators act (i.e., attributes), have as their natural counterparts the so called «global properties» (see reference above).

The **ordering process** at the formal level, corresponds to the **developmetal process** at the natural system level.

(ii) **Indistinguishability**. This characteristic is an important gauge for all the components of our systems. It implies that:

-below that level there are no components anymore, but only basic component properties or canonical variables (remember: the *systemness* of the human body brakes down at the cell level no matter how complex and multicellular a chosen part of our body might be). That is what the mentioned «global properties» should be thought of as.

-above that level there is systemic ordering and differentiation.

The great importance of establishing this boundary relies on the fact that by doing so we can distinguish between the two kinds of causality that are implied here. At the formal level this is manifested by having to deal with two sets of inference rules. Something that, fortunately enough, the

OOC (as well as Parker Rhodes' *Theory of Indistinguishables*) can afford because it is **context dependent**.

Another issue that arises at the formal level in relation to the necessity of starting with indistinguishable components when defining social systems, is of a paradoxical nature. Indeed, Barber's Paradox, so well known in logic, appears in Social Sciences. In this instance there is a confusion between an individual as an indistinguishable component of a social system and an individual as a social subject having a role or a rank in the hierarchical structure of that particular social system. The outcome of the confusion is an excessive emphasis on the psychological and behavioral aspects of the social construction of reality which greatly distorts facts.

B. Decoding the results of our formal model into our natural system:

The collection of indistinguishables translates as a particular undifferentiated natural system. Here too we have to be able to make certain distinctions.

(i) We have to distinguish between two kinds of indistinguishables:

-physical indistinguishable components of a given system, such as human beings in a social system.

-systemic indistinguishable units, such as basic elementary units of production and reproduction of a social system.

The distinction and relationship between the two kinds of indistinguishables is of a deep nature and great importance for Social Sciences, as it deals with one of the most involved issues in the theory of history and probably in many of the branches of sociology.

It is «deep» because it characterizes the evolutionary status of the social systems that we want to deal with.

It is «important» because any interaction or relationship between a particular social system and its environment (or between it and another social system) has, as the domain of possible invariant transformations, the overlapping subset between indistinguishables of each part involved (in that interaction or relationship). And the overlap has to be defined in terms of both kinds of indistinguishables. It should not be difficult to guess that those two «kinds» of indistinguishables are Parker Rhodes' *identicals* and *twins*.

(ii) The CH can only be an abstract representation of a particular model of our natural system. In fact, it probably is the simplest representation possible; labeling each systemic instance with a bare characteristic function (or string of them). In order for our system to be representable in those terms we have to conveniently define it, so that, indeed, a mere set of characteristic functions, plus the operations defined on it, can conveniently represent a potentially very complex system. Here is where some equivalent of an appropiate set of canonical variables for a social system becomes important.

4. Limitations of the CH-formalism in Social Sciences.

Being able to define a social system in terms of a small number of variables allow us to use McGoveran's OOC. From there we get a CH structure that represents the systemic unfolding through increasing systemic spatio-temporal scales, which is what is called **systemic development**.

In social systems «systemic development» has a horizon which, to my knowledge, tends to be asymptotic. Interestingly enough, new systems come into being on the periphery of the old systems. I am talking about structural periphery, i.e., what is known as L_4 in the CH jargon.

It could be pointed out that I might be invoking some metastatement of an ontological nature. That the statement is ontological I agree; I can only present -at this point- instances that occur in social reality. But facts do not necessarily disqualify themselves at the formal (epistemological) or theoretical level; to the contrary, sometimes they force us to look into the matter more carefully.

Even in what could be called developmental physics we should realize that from the combinatorial hierarchy we can only go as far as the production of light nuclei. In order to account for heavier nuclei we have to go into star formation and that, in itself, is equivalent to invoking some constructuve mechanism other than the one established in the CH-formalism.

To that mechanism I have given the name **generative condensation**. The basic idea of this principle is that components of the last level of the combinatorially expanded system (the one that establishes the limit stage, i.e., L_4) will condense and become a new enriched environment from which new (more informed) systems of indistinguishables, with new values for their global porperties (or canonical variables) and new possibilities for combinatorial expansion will eventually come into being. Thus, this principle generates a hierarchy springing up from within the final level of a combinatorial hierarchy, much as the irrational numbers could be said to emerge from within nested sequences of rationals.

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A NOVEL DERIVATION OF THE COMBINATORIAL HIERARCHY BY MEANS OF LIE COMPUTABILITY

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ABSTRACT

It is shown that since in the theory of Lie computability that natural systems modelled by not necessarily commutative operators determining non stationary, i.e. far from equilibrium dynamic behaviour, will naturally give rise to stationary behaviour, i.e. to processes that correspond to stationary values modelled by operators that commute, that such stationary processes can be represented in {0,1} i.e. Z_2, and will be specified by the Brouwerian Foundation of the Hierarchy as previously defined.

The Hierarchy therefore models the natural evolutionary irreversible development of the natural world for which the standard quantum theory allows the computation of eigenstates and eigenvalues.

INTRODUCTION

The Combinatorial Hierarchy is a growing universe of labelled bit strings generated by a simple recursive but non-terminating algorithm. It's development by a central caucus of members of the Alternative Natural Philosophy Association has continually reinforced the belief that the class of labelled bit strings which is generate on the field Z_2 constitute a coherent model of a discrete quantum physics based on minimal assumptions. In this discrete quantum physics, the families of labels at different levels of the Hierarchy have been found to be in exact correspondence with the families of elementary particles as these are currently understood from experimentally validated high energy particle physics (and through, it must be said the more orthodox quantum mechanical models on which such experimental validation has depended) (2,14,15,16).

Some uniquely important features of the Hierarchy, by which it differs from the more orthodox models of quantum physics, concern of course its discreteness, and

a) the addresses of the labelled bit strings which stochastically model R^4 Lorentz invariant space-time for all classes of entities with the same label and therefore,

b) that the Hierarchy via such addresses, manufactures an actual fabric for space-time, and the evolutionary development of the labels within that fabric. This is in complete contrast to orthodox quantum mechanics where an R^4 space-time continuum is assumed, and we might infer already exists. Whereas in the Hierarchy it is coming into existence continually.

c) the Hierarchy naturally organizes itself into four levels which are

characterized by an increasing complexity to which actual model generated values of
$3, 10, 137 \simeq hc/e^2$ and $2^{127} + 136 \simeq 1.7\times10^{38} \simeq hc/Gm_p^{\,2} = (M_{Plank}/m_p)^2$
are assigned where h is Plank's constant, c the velocity of light, e the charge on the electron, G is the gravitational constant and m_p and M_{Plank} and the proton and Plank masses, respectively.

This leads me to postulate that the Hierarchy and the orthodox models of elementary particle physics are not necessarily alternatives, but are essentially complementary, because the Hierarchy is a model of becoming characterized by irreversibility and a non terminating evolution, while the orthodox models are of being, characterized by reversibility. This is an attempt to establish a basis for this postulate, so that the algorithm may be considered as the program for a self defining, self organizing quantum computation with a mapping onto Z_2.

THE ELEMENTS OF LIE COMPUTABILITY AND METHODS

Based on the concept that it is physical processes, which determine the ultimate capabilities of machines, the quantum theory of computation is a new revolution in computer theory made explicit by Deutsch in 1985 in a paper entitled "Quantum Theory, the Church - Turing Principle and the Universal Quantum Computer"(3). The essential problem identical to that which faced Turing (7) and others in 1937, is how to design and then physically construct the universal machines corresponding to the new theoretical model. Is there as exists in digital control theory, a universal gate corresponding to the NAND gate, and a control theory corresponding to that based on Boolean algebras? And what physical form and function will such new universal gates have?

A proposal has been formulated for a theory of computability and machines (1) using control structures defined in terms of Lie algebras, which are the experimentally validated formal systems suitable for the modelling of dynamic behaviour in both classical and quantum physics (9). The proposal uses a notation but with different lettering that is suggested by Bertrand Russell's Introduction to Mathematical Philosophy, page 54, by which Resconi and Jessel (10) have devised General System Logical Theory, GSLT, which is an extension of the General System Theory of Mesarovic and Takahara (11).

It has been demonstrated that GSLT concerns an extended theory of computability in the sense of Deutsch's Church-Turing Principle that,

"Every finite physical process can be simulated by a universal computing machine".

so that computability is defined as the modelling of a natural system by a formal system, and where in this case the formal systems are Lie algebras, or locally continuous transformations, rather than the Boolean algebras appropriate to Turing's theory (7).

GSLT, or Lie computability, is therefore a model of a universal computer based on Lie algebras, where the recursive control structures are specified in terms of Lie products, as formalized in the commutative diagram,

$$\begin{array}{ccc} F(0) & \xrightarrow{\;OP\;} & S_{or} \\ \downarrow T & & \downarrow B \\ F(t)=TF(0) & \xrightarrow{\;OP\;} & S^{s}_{or}=[OP,T]F(0) \end{array}$$

where F(t) = TF(0) defines a recursive structure by means of the operator T and the diagram formalizes Huygens' principle that

> "The perturbation of the field F that goes out through a surface S that contains a wave source S_{or}, is identical to the perturbation that can be obtained by cutting off the source and substituting it by appropriate secondary sources S^S_{or} distributed on the surface S".

and where in the case of a closed surface

$$S^S_t = \int_S [OP,T]F(0)\, dS = S_{or} = \text{constant}.$$

The Lie product [a,b] = ab - ba or in the quantum theory the generalized Poisson bracket (9) may therefore be interpreted as representing in the theory, the universal gating device, analogous to the NAND gate of digital control theory, which takes the form of the secondary Huygens' sources of some physical field or virtual dynamic 'hologram' on a general surface S. What will be called a holochor from the Greek "holos" meaning whole, and "choros" meaning field. Lie computability therefore defines the generalized processes of which holography and holophony are special cases.

This new approach has been chosen because it is directly interpretable in terms of the neurophysiological evidence using the universal gate of Lie computability, the generalized analogue of the NAND gate of digital control theory, as the model for the neuron (6).

The equivalance between Deutsch's quantum theory of computation and that of Lie computability and machines, follows from the fact that the Schrodinger theory on which Deutsch's paper is based, is a necessary consequence of Heisenberg's commutation rules. (H.Weyl, The Theory of Groups, and Quantum Mechanics, Dover, page 280). And it is seen from Dirac, The Principles of Quantum Mechanics, Oxford University Press, chapter IV, 3rd edition, 1947, that the commutation rules determining the dynamical behaviour in both classical and quantum physics, correspond to those formal systems which may be specified in terms of Lie products, or generalized Poisson brackets, and therefore Lie algebras.

The conceptual basis of Lie computability is therefore the same as quantum computation and quantum mechanics, that although the uncertainty principle limits what it is possible to know about pairs of observables, a and b, such as position and velocity, these may be represented jointly by a wave, the evolution of which is deterministic with time. Uncertainty only enters when it is perceived as necessary by an observer to measure one of the pair of observables, say a. Then the uncertainty principle says that if a is measured with a high resolution, this will mean that the other member of the pair b, can only be known to a low resolution. The propagation of waves is determined by the Huygens' principle of secondary sources, and the Feynman principle of least action confirms the validity of the Huygens' principle with respect to the class of all special relativistic quantum mechanical models. The basis for Lie computability is the generalization of Huygens' Principle to include non-stationary behaviour determined in terms of secondary sources of the wavefields, experimentally validated in acoustics by Jessel and others (4).

The necessary control of such far from equilibrium behaviour may be achieved by physical means, see below, using the Huygens' secondary sources of the fields. Lie computability therefore explicitly allows the description of dissipative and open dynamical systems as well as the non dissipative and closed dynamical systems to which the standard quantum theory is usually confined (17). In this way Lie systems or machines follow the traditional classification of dynamical systems, corresponding to limit points, limit cycles, and chaotic ("strange") attractors found in the studies of non linear ordinary differential equations, and finally a fourth category, universality as suspected by S. Wolfram, (5) (preface to Physica 10D, 1984 devoted to cellular automata as alternative models of physical systems, i.e. universality with respect to Turing computability and Von Neumann constructability).

The theory shows that control structures based on Lie algebras, contain those based on Boolean algebras as a stationary subclass, in the same way that Deutsch's quantum theory of computation contains Turing's theory as a submodel. The new theory breathes fire into quantum computability by demonstrating that it should be possible to design Lie machines, which only when subsequently physically constructed, will execute the prescribed behaviours beyond the capabilities of digital machines that Deutsch's theory infers is possible. It is in this sense that Deutsch's Church-Turing Principle

> "Every finite physical process can be simulated by a universal computing machine".

may violate the existing Church-Turing Hypothesis,

> "Every 'function which would naturally be regarded as computable' can be computed by the universal Turing machine".

From the above diagram it is possible to define a Lie machine through the generalized Huygens' process by means of a reference operator OP which specifies the physical processes or laws that must be satisfied, for instance

$$OPe = \frac{\partial^2 e}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 e}{\partial t^2}$$

specifies a medium in which the variables e are waves. Thus it is possible to write the diagram above in the following way

$$\begin{array}{ccc} e & \xrightarrow{\quad OP \quad} & S_{or} \\ T\downarrow & & \downarrow B \\ Te & \xrightarrow{\quad OP \quad} & BS_{or} = TS_{or} + [OP,T]e \end{array} \qquad (1$$

so that $T(\ .\,/e) \xrightarrow{OP} B(e\,|\,S_{or})$

where T acts on e

B acts on S_{or} and depends on the field e

and B can be used so as to determine the change in the sources in the field $e(x)$ such that when these new sources are known in a natural system, they will create via the operator OP^{-1}, the required transformation.

Hence, although in many cases it may be impossible to determine OP^{-1} by mathematical means, the universal computer specified in the diagram can realize the prescribed behaviour by physical means; that is, it is necessary to construct the machine on the basis of the theory and when energy is supplied the machine itself will solve the problem, and execute the prescribed behaviour.

Further the generalization of the Huygens' Principle based on Lie algebras has been demonstrated to be a novel formulation of the variational principle of least action or more generally of stationary process, and hence corresponds to computational paths which optimize the computational and physical resources required to achieve the prescribed behaviour.

Many physical phenomena such as wave propagation, the thermodynamic dissipative process, and the demon of Maxwell's thermodynamic paradox, which may be conceptualized as the ultimate thermodynamic limitation to machines, can be shown to be universal gates or computers in this new abstract model of universal computation which following Deutsch contains Turing's model as a submodel.

That is, it demonstrates a universal control theory for which the existing universal digital control theory based on Boolean algebras is a stationary subclass.

THE NOVEL DERIVATION OF THE COMBINATORIAL HIERARCHY

What is of interest are not those mappings from one Boolean algebraic structure to another which are the province of digital or Turing computational models, but in line with the definition of Lie computability the recursive mappings from the more general Lie algebraic structures onto the Boolean algebraic structures, which can be represented in Z_2. This class of mappings is of great importance, because Lie computability describes physical systems and so this class of mappings onto Z_2 concerns representations of those physical systems with closure

$$S_t^s = \text{constant}$$

which might be interpreted as the Universe, i.e. all that exists.

This class of mappings, transformations or processes is of course, related to those digital models which simulate locally continuous processes to some degree of approximation but is now, we may suppose much larger than the mappings of one Boolean algebraic structure onto another.

Let us therefore examine this class of mappings for those which have representations on $\{0,1\}$

The starting point is the set of relations governing commutation relations and which define the Lie algebras of some not necessarily associative algebra U, i.e.,

$[u,u] = 0$ or $u^2 = 0$

$[u,c] = 0$ where c is a number, which may be considered a a special case of a dynamical variable.

$$[u,v] + [v,u] = 0$$

$$[u_1 + u_2,v] = [u_1,v] + [u_2,v] \qquad (2$$

$$[u, v_1 + v_2] = [u,v_1] + [u,v_2]$$

$$[u_1u_2,v] = [u_1,v]u_2 + u_1[u_2,v] \qquad (2 \text{ (Cont'd)}$$

$$[u,v_1v_2] = [u,v_1]v_2 + v_1[u,v_2]$$

$$[u,[v,w]] + [v,[w,u]] + [w,[u,v]] = 0 \quad \text{The Jacobi indentity}$$

But regarding uv as equal to uXv as defining the class of all operations X such that the above relations apply, and a subclass of X are the operations X' such that X' corresponds to the rules

$$\begin{aligned} F(a,a) &= 0 \\ F(a,b) &= F(b,a) \\ F(a,F(b,c)) &= F(F(a,b),c) \end{aligned} \qquad (3$$

so in correspondence with the above, X' is the subclass which is associative, commutative, and discriminative (in the ANPA sense), since the earlier relations are not necessarily associative, or commutative.

But one interpretation of 3) is as logical connective functions, where the zero 0 must be taken as equivalent to the truth value false. The above equations 3) therefore determine the class of logical behaviours derived from the more general algebraic behaviours 2), where negation by failure holds since $F(a,a) = 0$ in 3) and negation plays a special role in 2) since $[u,v] = uv - vu$. That is to say, 3) determines logical systems where a proposition A is only equal to $\neg\neg A$ when A is $\neg B$ and it is provably the case that B is $\neg A$ inferring that metareasoning is required, and where the logical connectives commute and so there is backward as well as forward reasoning. It therefore appears that 3) defines a suitable theory that can be found within intuitionistic higher order logic by means of what Dana Scott calls modest sets (12). Modest sets are often called partial equivalence relations meaning that equivalence is only defined to hold on some subset of $N = \{0,1,2, \ldots\}$ the set of natural numbers.

that is,

a) there is a binary relation on N, $=_p \subseteq N \times N$ such that

b) $n =_p m$ always implies $m =_p n$ and

c) $n =_p m$ and $m =_p k$ always implies $n =_p k$

so that the modest set is the structure $P = \langle N, =_p \rangle$

that is what is modest about these sets is their countability.

However, for the moment, we are not directly interested in the logical structures that result from 3), but in those representations in Z_2 that conform to 3) but arise from 2) and therefore from the n dimensional space V_n. That is that represent the mapping process

$$G \longrightarrow V_n \ / \ Z_2 \qquad (4$$

But 3) and 4) are just the basis for a completely recursive generation of the Combinatorial Hierarchy as demonstrated by Clive Kilmister, and called the Brouwerian Foundation (2).

It is of interest to note that in the derivation Kilmister follows

i) a methodology used by Conway (13) by which the number system for ordinary arithmetic may be generated via the generalized Dedekind cut $\{|\}$ as a subset of more general entities called games. This methodology yields a single operation for G of the form that if L are left and R the right disjoint subsets of a set S of previously constructed elements then $\{L|R\}$ is adjoined to S. This has the great advantage that it is completely recursive (as the model of Lie computability requires) γ in the strong sense that no starting point is needed and the first element generated is $\{\emptyset|\emptyset\}$ where $\emptyset$ denotes the empty set. This element plays a special role and is denoted by 0 or zero, (as is again required by the equations 2) and 3)). From the arguments above therefore it is reasonable to postulate that since 0 must in the model indicates a physical state or process, but 0 means false, this must mean a state that is unattainable, i.e. the absolute zero of temperature.

ii) that 3) follow as a result of defining an equivalence relation D on S using G to specify what Kilmister calls discrimination, and represented in numerical form by elements of what Conway calls the curious field on On_2 which is the "simplest" way of turning the class of all ordinal numbers into a field, and using addition + over this field so that F(A,B) = A + B.

iii) the labelling, address structure of the Hierarchy then emerges, from: a proof by Conway that if S' is a closed discrimination system that there is an isomorphism between (S',+) and (V_n,+') where V_n is the n dimensional vector space over the field Z_2 with +' denoting vector space addition, and the imposition of an economy process and the definition of discriminate closure, so that information can be represented in a more economical way at a higher level. This results in hierarchical completion, which occurs so as to maximize the information carrying capacity of the structure and defines bounds on the amount of information that can be dealt with. This economy process would evidently correspond to the variational principle of stationary process that is a feature of the Huygens' processes of Lie computability, as would the concept that Conway uses of "simplest".

The Combinatorial Hierarchy may therefore be said to be the discrete elements, the representation on the field Z_2, recursively generated that are in exact correspondence with locally continuous representations generated from Lie computability from the same boundary condition, the empty set. The physical interpretation of the Lie model says that this surface on which this exact correspondence takes place together with the appropriate representation of the Hierarchy at any point in its evolution from this boundary condition, constitutes the holochor or dynamic holochoric image of that boundary condition, and that these discrete elements must be considered as resulting from the generalized diffraction effects of the Huygens' process that Lie computability defines, as arising from the boundary conditions, i.e. that there is no 'observable' universe at the beginning of the evolution, i.e. no representation on Z_2, and the Universe is closed, i.e. is all that exists. We may say therefore, following Gabor, that if each discrete element pinpoints the exact location and properties of a secondary source of the holochoric image, that this carries all the information conveyed by the whole wave except that its sharpness or uncertainty of detail may be worsened. And the Hierarchy tells us, that stochastically these secondary sources will always be confined to an R^4 that is Lorentz invariant.

That is, at the beginning of the evolution when the representation on Z_2 is null Ø, all the information is carried by the wave in non stationary form, and that some of this information is gradually transferred into discrete form as structural elements in the Lorentz invariant R^4.

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CONNECTIVITIES, COMBINATORICS AND TOPOLOGIES

OF

BINARY EVENTS

In his book, *The Tao of Physics*, Fridtjof Capra quotes a Zen Mu-koan which poses the riddle: "You can make the sound of two hands clapping. Now what is the sound of one hand?". This indicates a deep, intuitive understanding, on the part of early Oriental philosophers, that all events of the observable Universe are primarily binary events. If an object or system never interacts with another object or system, it will presumably continue on its journey in space-time until it reaches "infinity". Since communication of information is a form of interaction, we would never have direct knowledge of its existence. By the same token, if events of the observable Universe are binary, then the foundations of mathematics should also be binary.

Now, the fund of human knowledge is dependent on two principal factors. The first is an inherent ability to recognize a pattern, and the second is the ability to compare a pattern recognized in the "present" with one which existed in the "past". If it is accepted that an adequate working definition of "pattern" is "that which is repeated", then it soon becomes self-evident that patterns may be classified as spatial (structural, or static within a local frame of reference), temporal (dynamic) or spatiotemporal (complex, i.e., involving some combination of mass, energy and information).

On this basis, memory involves temporal connectivity or <u>phase</u>.

This presents an immediate difficulty since, if quantum physics holds that energy is only available in "small packets", it follows that phase should be similarly quantized. This seems to be a startling error of omission from theoretical work, although the (Oxford Concise) dictionary definition of the word in everyday usage, which is "stage of change or development", is not explicit with regard to any limitation on the permissible extent of sub-division of the process of change. Clearly, only macroscopic, and not quantum, changes of phase are normally taken into account.

Insofar as memory is a temporal sequence, the question of quantization of phase is easily resolved. Consider a simply-connected maze, i.e., one in which all the barriers are connected to an outer, continuous barrier. Suppose it to have a single entrance door, and that its pathways are mapped as a matrix of unit squares or cells, one of which is a desired "goal state". In mythology, Theseus penetrated the labyrinth of Minos and, having slain the minotaur, regained the outside world by winding up a thread, (thoughtfully provided by Ariadne, later deserted), which had been laid along the path from the entrance. A simple modification of this technique would be to tie on an extra thread at every point where the path branches in order to investigate every blind alley,- a "total-solution" network. Replacing thread with stiff wire, and adding suitable directional labels, would allow a tree structure to be withdrawn via the entrance and re-assembled in the open where it could be analysed at leisure. (In passing, this approach is essentially a general problem-solving procedure, long sought by the artificial intelligence community.)

Now suppose the human problem-solver to be replaced by a cube, (preferably a gaming die), having a good sliding fit between the barriers on either side of the pathways of unit width. Suppose it to be rolled over, stepwise on its sides, from the entrance to the goal state. This would, of course, involve orthogonal changes of direction whenever branching or blind alleys are encountered. The successful route could then be recorded as a sequence of numbers, (or dots representing quantum objects), representing the sides as they come into contact with the path, a sophistication of a child's chalk marks.

However, a difficulty arises if an additional requirement is made that a particular side must be in contact with the ground when the die reaches the goal. Such a problem could, of course, be solved by trial and error, as in the natural world. Another way is to cheat, by correctly orientating the die in the goal cell and then rolling it backwards to the entrance, a method similar to reductionism. Yet another way would be to use a large number of cubes, or to provide the maze with mirrors and flood it with light. Whichever approach is adopted, it is not only a question of phase but of orientation as well. In the case of a simply-connected maze, there would be only one possible phase-and-orientation state for the initial, or "starting", position. In the case of a large, multiply-connected maze, there would exist a number of possible pathways and, with still greater complexity, a number of possible initial states. At the limit, all possible starting states would be equivalent, with nothing to choose between them. Nevertheless, all maze problems are potentially soluble by extension of the Trémaux algorithm, which can be shown to be related to interferometry and Hamilton circuits.

Examination of the possible cube-rolling sequences reveals an interesting fact; the temporal sequence of only two of the identifying numbers in a complete rotation of four is reversed when the direction of rolling is reversed. This produces different rolling sequences if a sequence of four moves is not completed, but the cube rolled left or right at some intermediate stage. In terms of rotation, this is equivalent to phase reversal of only half the cube. A somewhat similar result is obtained if a die rolling along a level plane is stopped by a vertical wall one unit high and is then subject to appropriate optimum forces to "climb" the wall, then continue rolling along a plane parallel to the first and separated from it by one unit. The die suffers a 180° phase lag (half a cycle) in such a manœuvre.

This is of no consequence in the case of the movement of a single die. It is, however, of the utmost significance in a hypothetical collision between two dice, since they will exhibit a behaviour of phase relative to each other. What is more, it will not, in an absolute sense, be possible to determine what this relationship is without specific information being available on their initial phase states, (corresponding to different entrance points of a maze). Usable information might, however, be available from previous states and, in this respect, it should be noted that the practice of communications engineering affords means of resolution. In this discipline, a distinction is usually drawn between <u>differential phase</u>, in which a designer is concerned only with changes from 0 to 1 or vice versa, and <u>reference phase</u>, in which one dynamic component of the system is taken as conceptually fixed so that all other possible phases of the system may be referred to it.

Complete definition of the initial phase state allows construction of a "memory-thread" bit string for any dynamic situation as a finite state machine or automaton, and for comparison with the "real world" for modelling purposes. The theoretical procedure is first to imagine a die, impelled by some arbitrary force, to be set in motion in free space in which there is also a population of dice, each die having a quantum-spinning local Cartesian coordinate system relative to the observer's die, the latter initially orientated, as reference phase, by definition or convention of the observer. The numbered sides of all the dice are then equated to the six components of phase space applicable at the times and locations of collision, and the observer's die is allowed to grow into a bit string in which all incremental additions to memory are expressed as "resultant phase bits" of direction or momentum. For the purposes of simple illustration, the resultants need only be probabilities corresponding to two sets of numbers from 1 to 6. It will be recalled that the dots on opposite faces of a die add up to seven, a number which appears several times in the Parker-Rhodes expression for the proton/electron mass ratio. For complex situations, relative energy levels would be substituted for simple probabilities.

Clearly, it would not be practical for a die to trail a memory thread like Theseus; it would need to be accomodated inside the die itself. This also implies an internal gyroscope mechanism, to enable the die to record "which way up" it is. (For this purpose, a self-organizing structure based on a special 27-cube hypercube will be described later. In such a structure, the central cube has a special property in that it has no face on the external surface of the outer cube; the central cube can also be assigned the special number fourteen.) Under these conditions, the observer's die may be said to have an internal environment, and its chance encounters with other dice to constitute the effects of an external environment. Growth of the bit string would, of course, be governed by the energy relationships of discrete collisional processes subject, in addition, to the principle of least action. This indicates that "following the path of least resistance" would have been as important as competitive "survival of the fittest" processes in the evolution of living organisms. In purely physical systems, an overall dimensionality of twelve is implied; one or two of these would need to be expressed as permanent structures for the purpose of memory. In general terms, this is then a first approximation to 10- or 11-, and to 26-dimensional, superstring theories of physics.

Obviously, such systems would not be amenable to conventional computation since, for example, the definition of a set, or the range and domain of a function, would need to be amended after each addition of a "phase bit". Both the internal environment (lumen), and its response to the external environment (numen), would change after each increment. Such structures and environments could, however, readily be directly modelled using suitably adapted electronics, principally simple relaxation oscillator circuits. In these, two active elements (e.g., transistors) are cross-coupled in such a way that their fundamental frequency of oscillation, and the symmetry of the waveforms generated, are governed by the time constants of several (usually resistive and capacitative) components. The outputs used in commerical apparatus are normally two square waves in antiphase, (often reduced to short-duration pulses). Waveforms of a nominally triangular form are usually available at active element terminals (e.g., transistor bases) and may also provide a "trigger" function. Taken together, however, the various waveforms may be regarded as capable of separating the odd and even harmonics of a sine wave in a similar manner to a Fresnel lens and thus, in principle, capable of defining any specific function by switching and Fourier synthesis. Additional features of relaxation oscillators are that they possess both integrating and differentiating circuits, and that the internal (crossover) circuit is a first-order, or "bang-bang" controller with a phase lag of up to 90°, while the externally-connected components may constitute a second-order controller delivering a signal into a reactive load and thus having a response governed by "damping" effects and resulting in a phase lead response of up to 180° or more.

Circuits of this kind are usually "hard-wired" in commercial equipment, e.g., digital computers, so as to give a reliable, fully-defined, and usually "programmable", system response. Circuitry which was both variable and switchable was common in the days of "analogue" technology and, by analogy with biochemical systems, a concept of "separable" time constant connectivities is now re-introduced, together with a new form of binary arithmetic. Full advantage can then be taken of the capabilities noted above.

For this, an amended symbol is proposed (Fig. 1) and definition of the circuit function, performance and connectivity of any relaxation oscillator is then easily presented in tabular form. Using a simplified symbol for purposes of illustration allows demonstration of the principle of the ring oscillator (Fig. 2), in which two contra-rotating pulse trains are present in inner and outer circuits. It is also possible to connect a large, virtually unlimited, number of oscillators so that they bear a direct relationship to both mathematical and biological structures (Fig. 3). In brief, these circuits then correspond to algebraic rings and fields which, under constant-frequency conditions, would be governed only by simple modulo arithmetics (symbol, M). The "inner" and "outer" products may be equated, in broad terms, to chemical and electrical activities, respectively, found in living organisms. The principal advantage of these configurations is that phase relationships may easily be represented as unit vectors within unit cellular arrays. In this sense, the integer one is taken as the identity of the group of natural numbers and as the scale invariant. Another advantage is that they could be made to operate automatically under conditions of a residue theorem, and to yield "phase re-setting", either with cessation or change (modulation) of frequency, or with cessation or change delayed until after expiry of a "time-out" period similar, for instance, to the overwintering period of the seed of a plant or of a butterfly chrysalis. Configurations of this kind are directly comparable with concepts of crystal form, solitons, and with knot and lattice theory. As consequence, adequate explanations and applications of fractals and cellular automata may be formulated. In addition, they may be directly related to ray optics, and to processes of diffusion, shear and convection, leading to an ultimate configuration as a large-scale, generalized flow net formed of hypercubes in phase alternation and applicable to thermodynamics and fluid dynamics. From this, it has been concluded that the principle of least action governs the phenomena of superconductivity, the operation of the genetic code, and the neuronal action of the central nervous system of animals. It has further been concluded that the fundamental mechanism is the same as that of the optical phase conjugate hologram and that the origin of life was due to phase quadrature effects induced at the molecular level by the influence of energy incident on the surface of Planet Earth. The maze may still be used as a model of living cells on the basis that equilibrium between internal and external environments can be maintained without violating any principle of thermodynamics. In this case, a biological semi-permeable membrane is considered to be equivalent to the outer boundary (membrane) of a multiply-connected maze having many entrances and exits, each controlled by oscillations of the boundary itself as an expression of the balance between anabolism and catabolism. In terms of conventional mathematics, a living cell may be considered to possess a central, single node or singularity. Kirchhoff's law may then be applied to processes of electrochemical equilibrium, at least under cell maintenance, rather than growth, conditions.

The foregoing indicates a need for a new kind of mathematics for the purpose of four-dimensional, or spatiotemporal, computation. What follows is a preliminary description of the primary geometry of such a system, followed by an outline of its associated temporal element. Formal definitions, leading to proposals for the design and construction of a "direct synthetic mathematics machine", are in course of preparation.

Fig. 4 is of a 27-cube hypercube made up of three layers of nine cubes indexed such that the central cube is assigned the number 14. Each cube is regarded as representing a potential dynamic location of a quantum object, e.g., a molecule, atom, or electron, subject to the influence of quantum objects of an external environment. Cube 14 may thus be considered as being "protected" by a process of continuous optimization to equilibrium. As a "floating" centre, node or singularity of stability, all that enters cube 14 must be balanced by all that leaves. In this case, however, "all" is to be taken as meaning any combination of mass, energy and information, rather than simply mass or energy, as in the usual definition of a node. In other words, cube 14 is a "quantum" of phaseless, or steady-state, space which may be both zero and infinitely dense over time in terms of conventional mathematics. Fig. 5 gives all possible internal optimization vectors, by orthogonal resolution onto Cartesian coordinates, for three dimensions of space. The reduced (resultant) form of these vectors is the usual (tesseract) hypercube as given in Fig. 6. Under these conditions, the surface cubes, 26 in number, are free to re-arrange themselves, leaving cube 14 undisturbed, rather like Rubik's cube.

The next stage in the process is to imagine that the 27-cube contains a (face-centred) inscribed octahedron dual, equivalent to the central spinning mirrors of a three-dimensional interferometer illuminated by temporal sequences of light of different colours. The octahedron contains an inscribed cube as dual, this containing an octahedral dual, and so by continued regression down to the minimum measurable distance, presumed to be two Planck lengths. This mechanism may then be regarded as a "recursive quantum gyroscope", able to maintain a memory trace of past spatiotemporal states as a nested sequence of phase shifts between concentric layers. In other words, the total memory structure may be envisaged as being like an onion in which there is a maze between each of the skins. Solution of a maze then depends on concentric re-positioning of layers actively sliding over another until the correct "phase fit" is achieved.

The final stage in the process is the application of Shannon's measure of information (negentropy) as the (sigma) sum of a lengthy, but specific, series of log frequency terms in parallel, rather than as a serial addition of a number of log probability terms.

The mechanism of parallel switching from one state to another may be envisaged by extension of the principle of a computer keyboard character-coding matrix in a single dimension to coding in four dimensions. That is to say, in **R4**, or ordered quadruplets, (or permutations of four from six), of the natural numbers. Fig. 7 represents a conventional computer keyboard with switches capable of connecting, when depressed, a set of wires arranged in rows and columns; that between row 3 and column 1 is shown depressed such that the de-coding circuitry (not shown) propagates the appropriate binary code for the character "A". Extension to spatiotemporal computation would involve addition of three more key-switches connecting the row and column wires at mid-points of each of the cells, together with two further switches connected to adjacent planes of wires. In addition, the switches would carry signals of differing frequencies instead of having the usual "on-off" action.

The overall action of such an arrangement is then as depicted in Fig. 8, in which R1/R2 carry odd harmonics, and the mid-point connections carry even harmonics (modulo arithmetics M1 to M4). The effect of signals of frequencies M1 to M4 propagated for sufficient lengths of time would then be to carry them to other rows and columns distant from R1/R2 and C1/C2 in a manner analogous to a variable-focus Fresnel lens. This then constitutes the temporal element and leads on to a general consideration of quantized phase.

In Fig. 9 are given the usual topological representations of a torus, a Klein bottle and a projective plane. By inspection, interconversions are conceptually effected by simple phase changes. Conversion of a Klein bottle to a torus involves only phase reversal of side a; that for conversion to a projective plane involves reflection in the plane and phase reversal of side b.

A regular pattern emerges in a more complex arrangement employing orthogonal signals propagated along crossed parallel pairs of wires in the directions indicated by arrows in Fig. 10. This indicates the manner in which a system may trap energy and thus become self-organizing. Although a Klein bottle cannot be constructed as a real "rubber sheet" structure because of self-intersection of the plane, there appears to be no reason why it could not exist as a virtual structure in discrete mathematics, or as a real energy system. In Fig. 11, suppose that energy is able to enter the open bell-mouth of a Klein bottle and, in doing so, alters the phase relationship of that system. This gives two possible derivatives. One is the torus, leading to chained structures by repetition, and the other is the projective plane, which has the form of an energy pump (Fig. 12). With the addition of binary division processes as quantized energy supply, it is then easy to imagine the origin of self-energized systems capable of setting up cellular automata and fractal structures.

Modified, or "phase-quantized", knot theory provides a simple approach for investigations of this kind. Suppose the simple trefoil knot of Fig. 13 is made of string having a unit square cross-section and that it is "mapped" onto an orthogonal matrix board using pins (Fig. 14). It is then evident that it has several layers, better appreciated by conversion to a three-dimensional wire frame structure (as used in computer-aided design programmes) (Fig. 15). This can be reduced to minimum volume by reduction of the lengths of paired "sides" until no voids remain. For a dynamic system able to coil like a worm, this would then be a means of generating fractals in accordance with the principle of least action. Clearly, only simple further modification is required in order to investigate the seven known forms of crystals.

When re-drawn as a system of cubes, Fig. 10 is seen to be both a self-correcting process consistent with the concept of a phase conjugate hologram, and also a path of least resistance, consistent with the phenomenon of superconductivity. Fig. 16 illustrates the two orientations necessary for superconduction, together with the relationship between the phase-alternation error correcting Gray code and the Hamming distance. Gray code is a form of binary arithmetic in which only a single bit is altered in passing from one bit pattern to the next pattern in a sequence. It is thus "simply-connected". While this sequence does not correspond with the conventional sequence of denary numbers, it can be shown that words of specific lengths represent reflections. Indeed, Gray code is also called reflected binary. It is easily demonstrated that Gray code is entirely appropriate for computation based on the concept of the phase conjugate hologram and that, in this form, it is very much more efficient than conventional binary arithmetic. In addition to the former having zero redundancy, the latter is multiply-connected since progression through the sequence of bit patterns involves simultaneous changes of several bits at regular intervals, e.g., when passing from denary 15 (1111) to denary 16 (10000) there are 5 switching operations. In other words, reflected binary computation is able accurately to select information and totally to reject the effect of noise, as would be expected of an interferometric process.

A general conclusion to be drawn from the above is that the neuronal action of the human central nervous system already relies on four-dimensional holographic spatiotemporal computation based on Gray code and that human capabilities are currently limited by a social organization and culture almost completely restricted to computation in the two dimensions of sheets of paper.

Fortunately, it can readily be demonstrated that synthetic spatiotemporal computation is conceptually much simpler, and much more powerful, than traditional mathematics. Mathematical philosophies based on such abstract concepts as lines possessing the property of length but without the necessary accompanying physical property of width are not simply not valid in the real world. For example, if a network is taken to be the spatial complement of a tiling pattern, the lines which define the network cannot be of zero width since it would then not be possible to distinguish between individual tiles. In order to distinguish the new concepts from those of existing discrete mathematics, it is proposed that the term "quantum" mathematics be used for the new form.

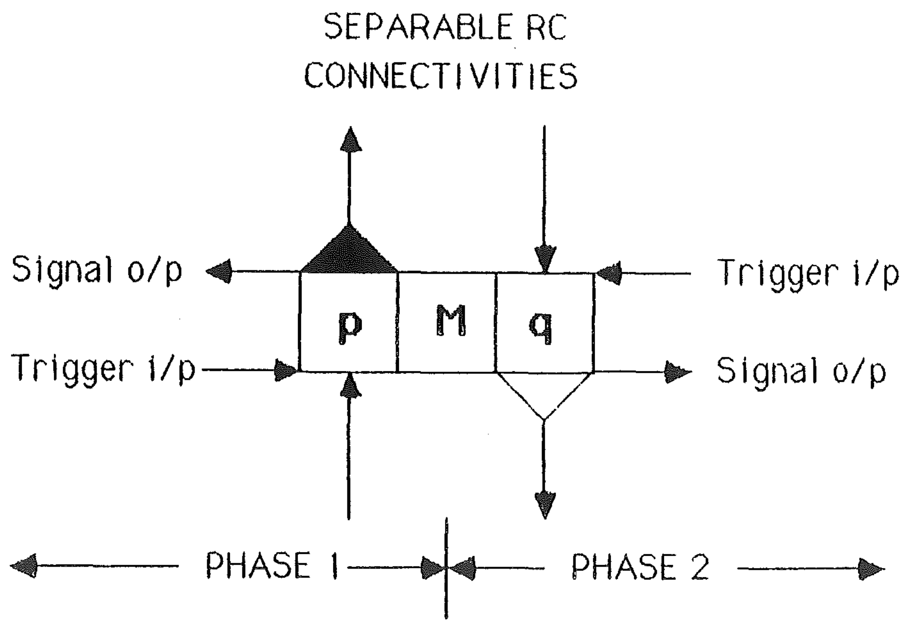

Fig. 1 Proposed separable connectivity symbol for generalized relaxation oscillator (p phase active, q phase inactive)

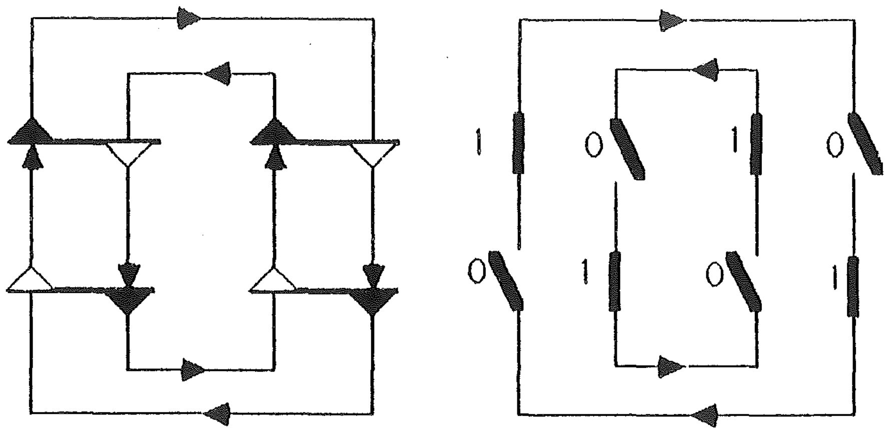

Fig. 2 Simple ring oscillator (As switch mode)

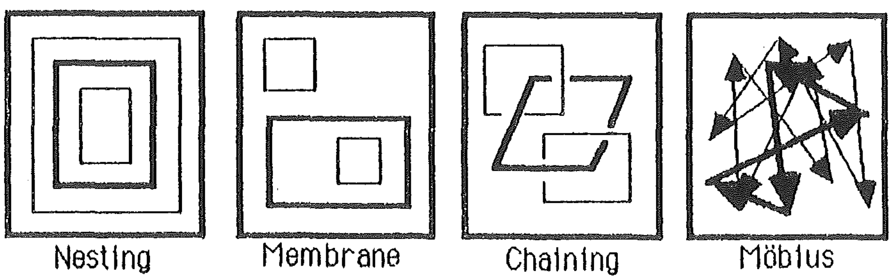

Fig. 3 Complex ring oscillators

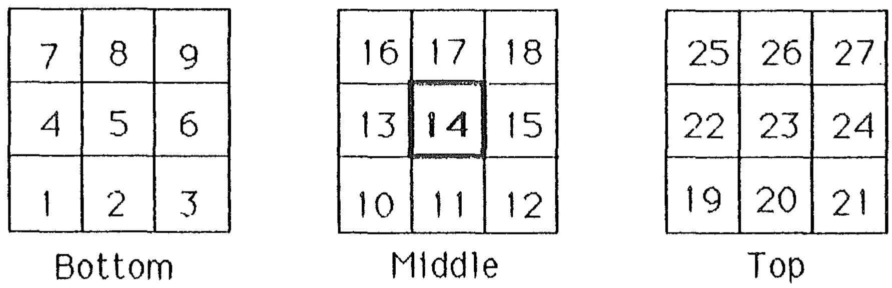

Fig. 4 Three-layer 27-cube hypercube numbering

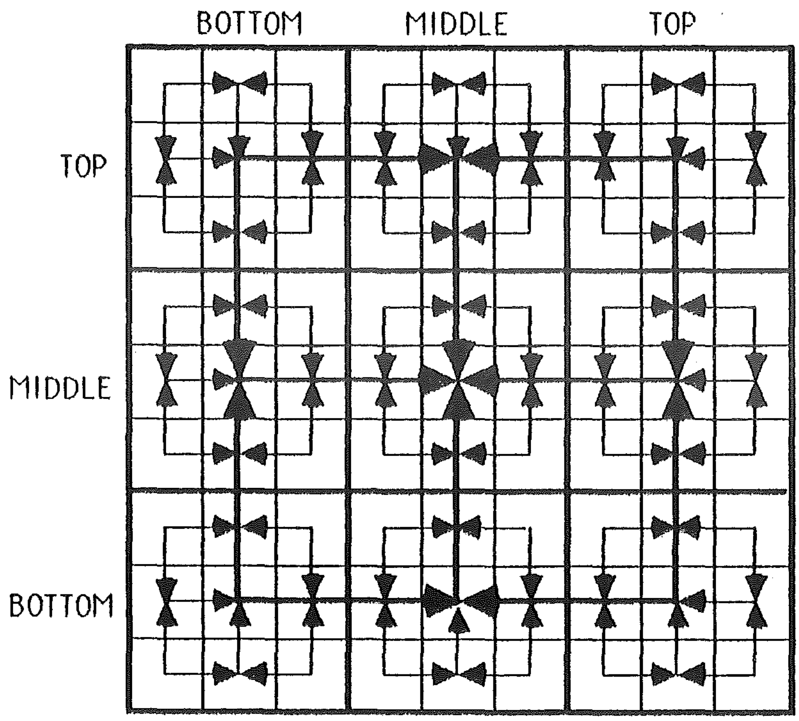

Fig. 5 Hypercube optimization vectors

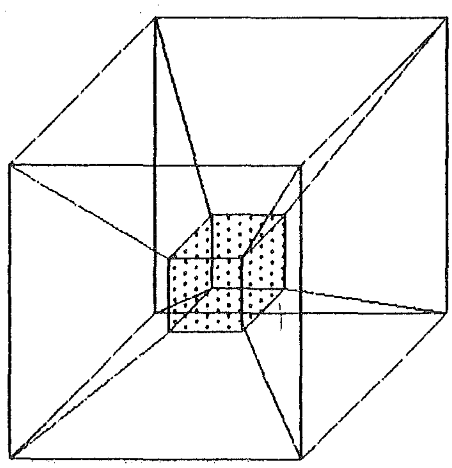

Fig. 6 Conventional hypercube (tesseract)

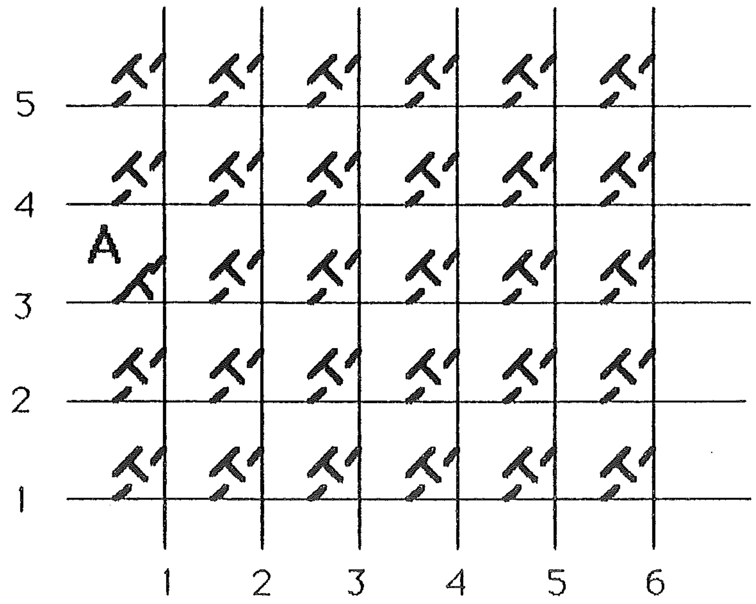

Fig. 7 Computer keyboard matrix

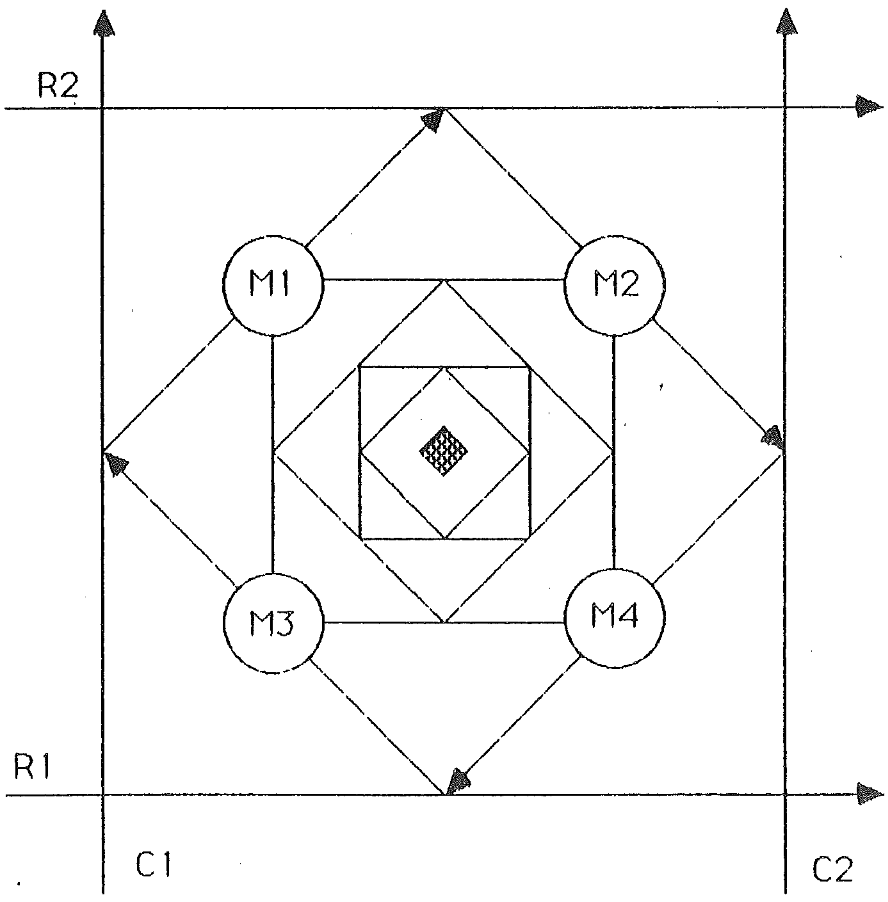

Fig. 8 Recursive cellular automaton

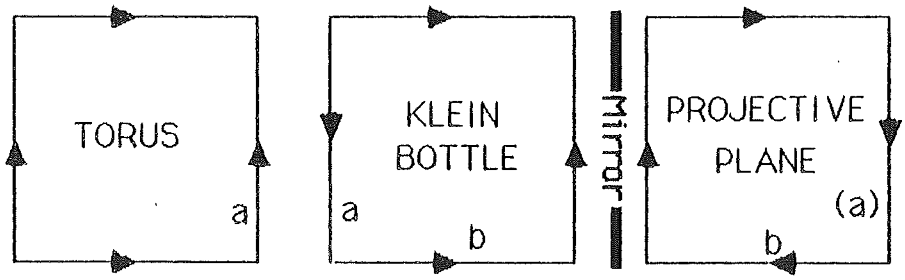

Fig. 9 Topological phase relationships

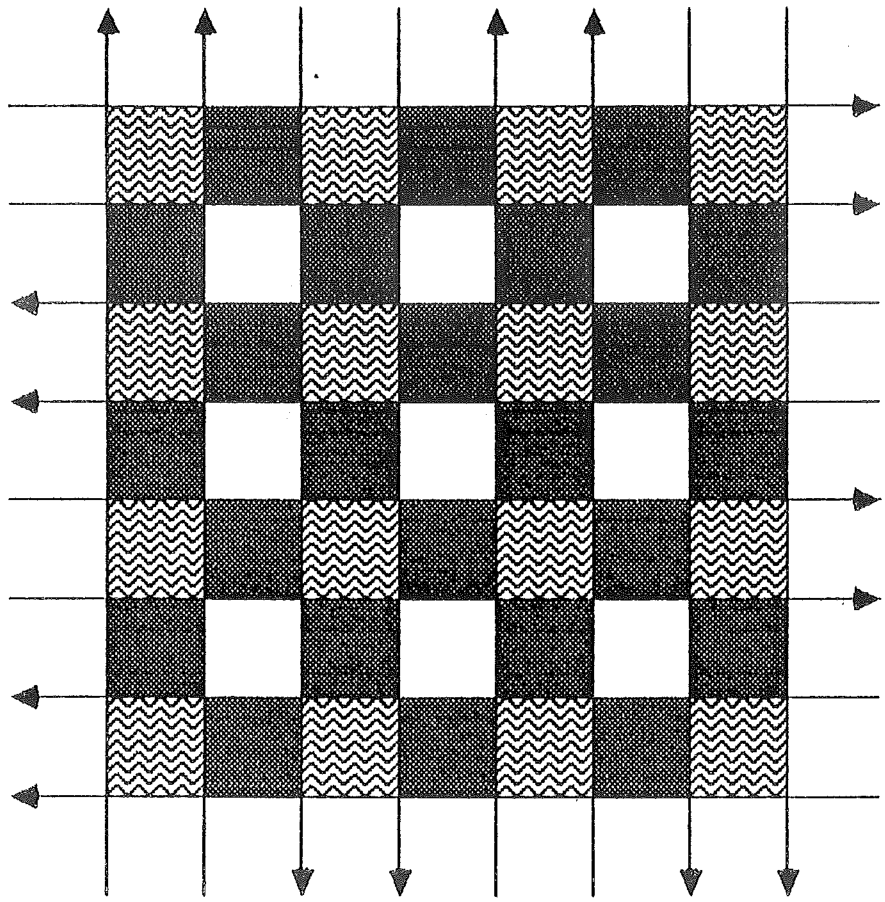

Fig. 10 Four-dimensional spatiotemporal computation phase relationships

Key

Torus — Klein bottle — Projective plane

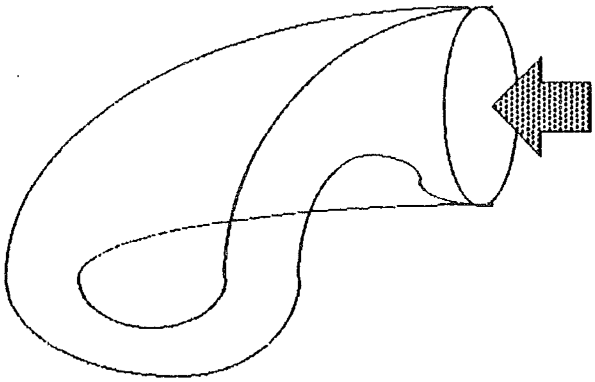

Fig. 11 Klein bottle as energy trap

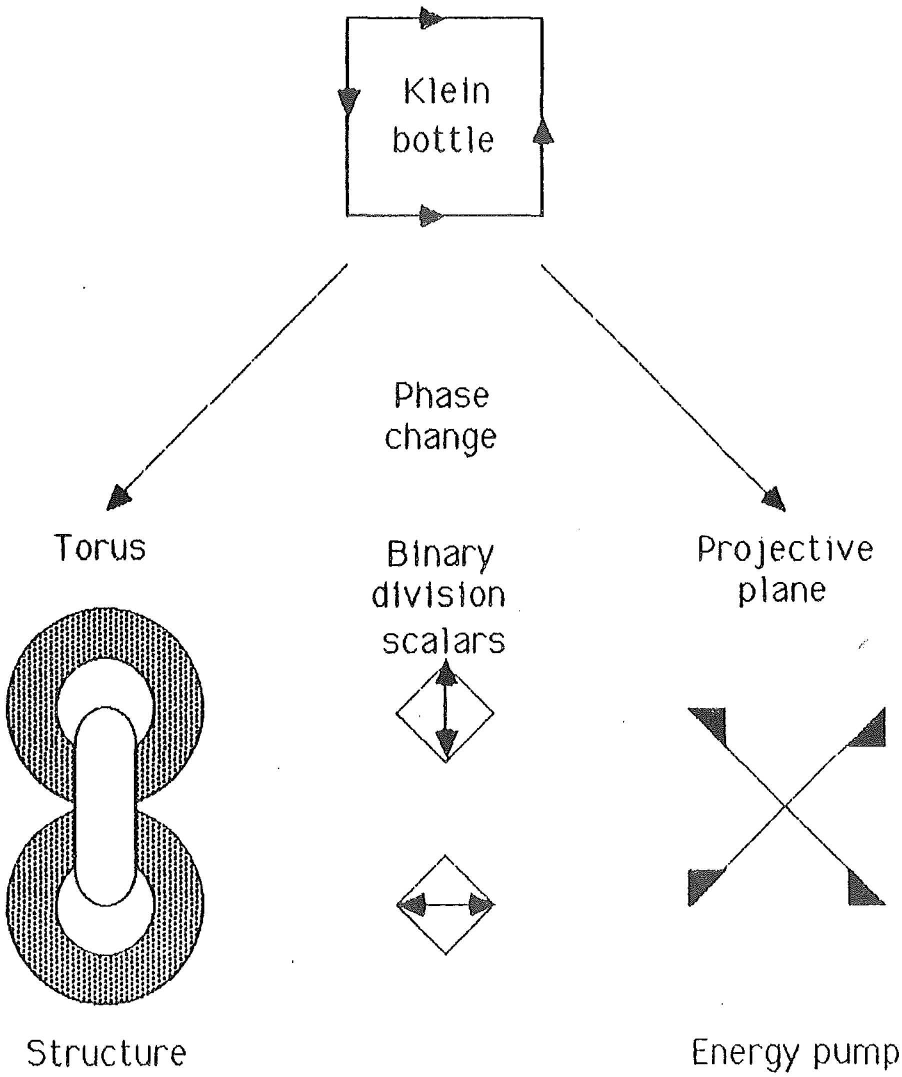

Fig. 12 The relationship between energy, structure, time and information

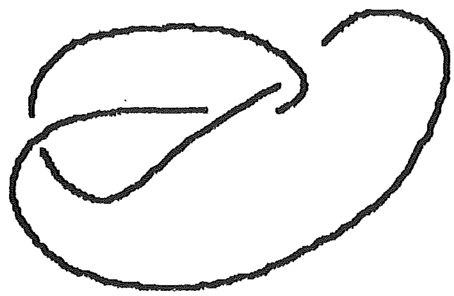

Fig. 13 Simple trefoil knot

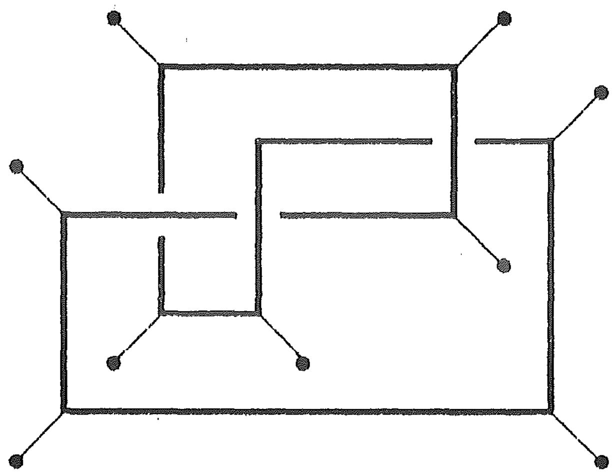

Fig. 14 Trefoil knot as matrix

Separation at A and connections at B-B and C-C yields two interlinked tori.

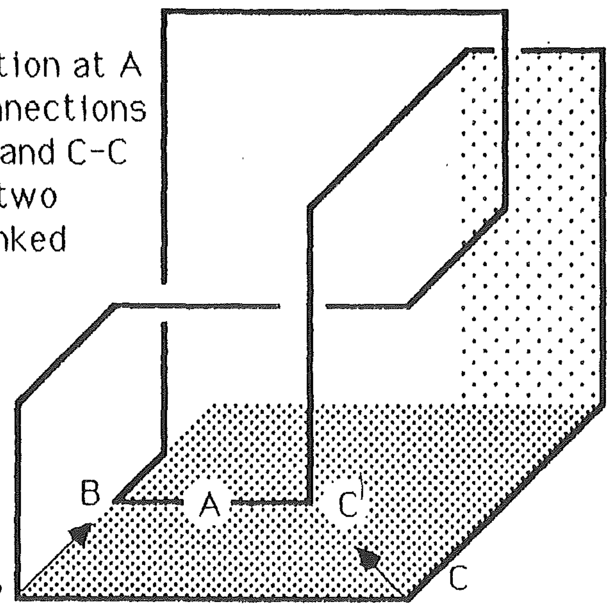

Fig. 15 Trefoil knot as an array of cubes

Phase alternation, superconductivity and the principle of least action

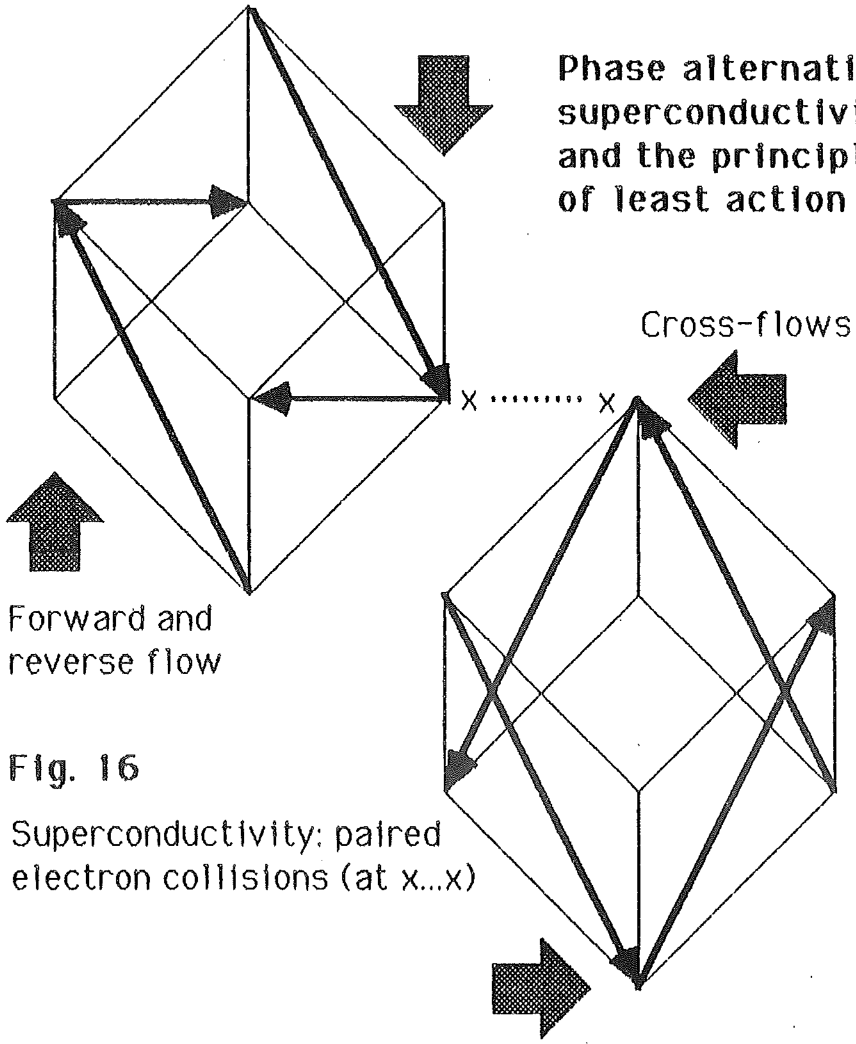

Fig. 16

Superconductivity: paired electron collisions (at x...x)

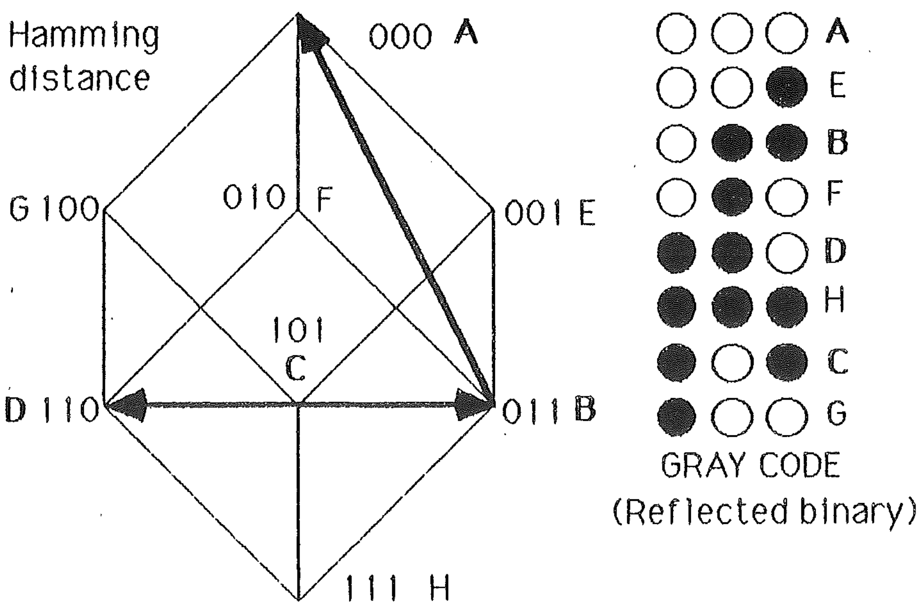

Fig. 17 Gray code and Hamming distance

Alternative Natural Philosophy Association

Statement of Purpose

1. The primary purpose of the Association is to consider coherent models based on a minimal number of assumptions to bring together major areas of thought and experience within a natural philosophy alternative to the prevailing scientific attitude. The combinatorial hierarchy, as such a model, will form an initial focus of our discussions.

2. This purpose will be pursued by research, publications and any other appropriate means including the foundation of subsidiary organizations and the support of individuals and groups with the same objective.

3. The Association will remain open to new ideas and modes of action, however suggested, which might serve the primary purpose.

4. The Association will seek ways to use its knowledge and facilities for the benefit of humanity and will try to prevent such knowledge and facilities being used to the detriment of humanity.

Organization

1. The Executive Council is the governing body of the Association. The Coordinator, Secretary and Treasurer serve at the pleasure of the Executive Council, in consultation with the Membership. Decisions of the Executive Council are subject to majority vote of the Council as a whole.

2. Members of the Association are (a) members of the Executive Council and (b) others nominated by the members and approved by the Executive Council.

3. Vice-Presidential candidates are nominated by the Membership and the Executive Council during the first year of the President's term of office. Any nomination must be accompanied by a statement from the nominee that he will serve a full term if elected. If more than one nominee exists, selection will be made by mail ballot to the Membership decided by plurality of votes. The vice-President is elected biennially to serve for one year concurrently with the then current President, and the two following years as President. The vice-President becomes a member of the Executive Council on election, and retiring Presidents remain on the Council for five years. Presidents must serve for one full term (1 year as vice-President and 2 years as President) and cannot run for re-election until three years after their initial term has elapsed.

4. The President is the official representative of the Association in external affairs, and has the responsibility for calling meetings of the Membership, at least annually, for the determination of overall policy.

5. The Treasurer is the responsible financial officer of the Association for the receipt and disbursement of funds and shall maintain appropriate records of Association activities, membership, mailing lists, etc.

6. The Secretary is responsible for keeping minutes of the Membership and Executive Council meetings, production of a newsletter to keep members of the Association informed of its activities, and such other duties as may be assigned.

7. President, Secretary and Treasurer will not be paid for their services but may, as appropriate, receive funds for travel expenses, secretarial help, etc.

8. The Coordinator may be paid an appropriate salary for his services, funds permitting. These services will include the organization of meetings and the editing of the proceedings of such meetings for publication, coordination of and participation in the research activities of the Association, preparation when appropriate of research reports and publication of such reports, and such other duties as may be assigned.

9. The Executive Council has selected an independent Advisory Board. It may adopt its own rules for operation and replacement of members. The Executive Council may nominate candidates to the Board. Any member of the Board, or the Board collectively may make recommendations to the Executive Council, or directly to the Membership. Action taken on such recommendations must be promptly reported by the Executive Council to the Board in writing.

10. Dues are currently £10.00 ($15.00 US) per annum.

Executive Council: Dr. Faruq Abdullah, Dr.John Amson, Dr. Ted Basin, Mr. Anthony M. Deakin, Prof. Clive W. Kilmister, Mr. David McGoveran, Dr. Michael J. Manthey, Prof. H. Pierre Noyes.

President (1987-89): Clive W. Kilmister, 11 Vanbrugh Hill, Blackheath, London, SE3 7UE, England.

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