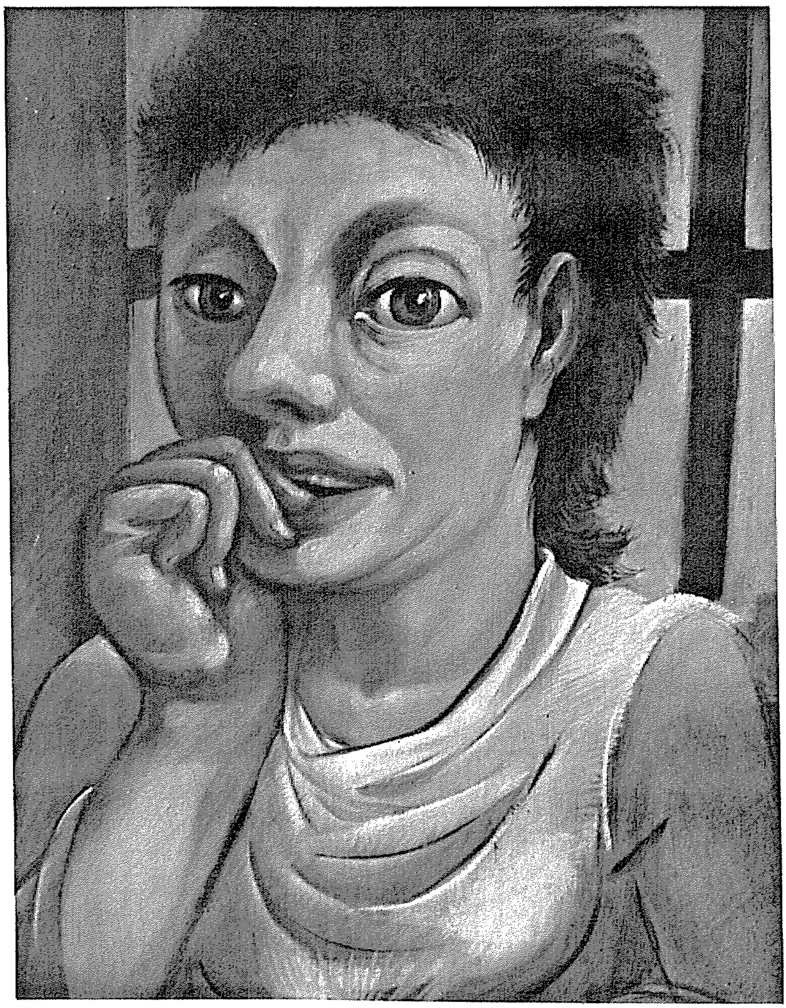
# ANPA WEST

## Journal of the Western Chapter of the Alternative Natural Philosophy Association



Volume Two, Number Three - Fall 1991

# ANPA WEST 8

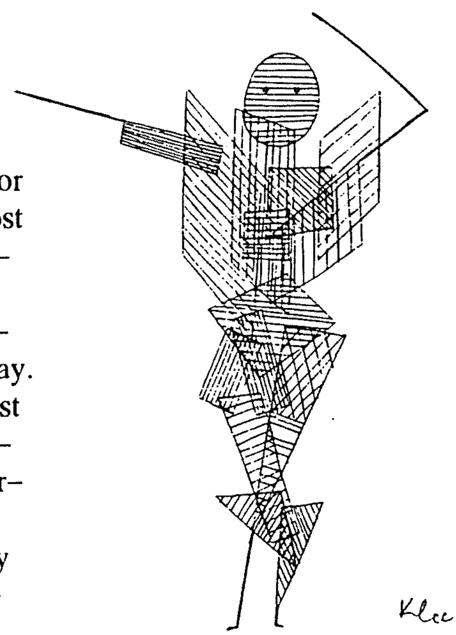
## EIGHTH ANNUAL MEETING

### of the Western Regional Chapter of the Alternative Natural Philosophy Association

### 15–17 February 1992
### Cordura Hall, Stanford University

Preregistration, $10; registration at the meeting, $15. If you wish to preregister and/or present a paper, send your registration fee and an abstract of your paper to:

Fred Young
128 Lyell St.
Los Altos, CA 94022
(415) 949-4728




Papers selected by the local committee for oral presentation will be scheduled for at most 40 minutes presentation followed by 20 minutes discussion. Papers not scheduled for presentation during the first two days (Saturday and Sunday) will be discussed on Monday. Any papers in camera ready format of at most 20 sheets (8 1/2 " by 11") – preferably less – which are given to the Secretary before Saturday noon, Feb. 15, will appear in the INSTANT PROCEEDINGS the next day. Only those who pay for these proceedings on Saturday will receive them; cost will be what it takes to produce them over-night divided by the number of reprints.

ANPA WEST – Volume Two, Number Three – Fall 1991
Editorial Office: Tom Etter, 409 Leland Avenue, Palo Alto, CA 94306

## From the editor.

For those of you who have been wondering what happened to your interactive ANPA West journal, we're not there yet. INRAC, the language in which the new journal was to be written, was created for an earlier generation of computers and needed some upgrading. We thought this would be a snap, but it wasn't. It's been like trying to remodel an antique house that has obsolete plumbing, dangerous wiring, hidden dry rot, irregular dimensions, and no plans – we finally decided it's a lot easier to just start over. The new INRAC will be much better, but the delay is frustrating, most of all to those of us who are eager to use it. Give us six more months ...

In the meantime, things have been happening at ANPA. In this issue there are two articles presenting some new thoughts on Spencer–Brown's Laws of Form in relation to quantum mechanics, plus several short progress reports. Also, please note the announcement on the previous page of the ANPA West 8 meeting coming up on Feb. 15.

# IN THIS ISSUE

# The Primary Algebra of Spencer-Brown is Non-Boolean

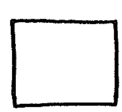
by Louis H. Kauffman

The purpose of this note is to provide a complement to Tom Etter's discussion about pre-logic in relation to Laws of Form [E]. I will show here, that the primary algebra of Laws of Form is not a Boolean algebra, and how we come to interpret it as Boolean. The primary algebra becomes Boolean when we adopt a fixed viewpoint in relation to it. The primary algebra is a precise structure that underlies the Boolean point of view.

One motivation for this essay is the investigation of what is common to the development of formal systems. By taking the very simplest systems of symbols, and submitting them to creative scrutiny we can begin to see universal patterns. Of course, the very act of taking a discrete starting point (using written symbols rather than sound and gesture) biases the endeavor. I suggest that it is precisely this bias that makes the Boolean or near-Boolean patterns appear near the beginning. In another essay, I will discuss how patterns related to quantum logic, quantum theory and special relativity also appear near the beginning of the development of formal language. These are the pervasive patterns of our thought, speech and experience.

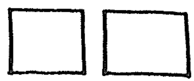
At the end of this note I discuss the principle of idemposition (common boundaries cancel) in relation to both pre-logic and Laws of Form. There is a common theme, with the realm of formal idemposition providing a background prior to and more primitive than the primary arithmetic that underlies Laws of Form.

Now to work. Recall first the primary arithmetic ([S] and [K]). It is an arithmetic generated by distinctions made in (or of) a given space. A convenient representation is the plane of writing, with a rectangle representing one distinction. (Spencer-Brown uses an abbreviated rectangle, with subtle differences of language. In the spirit of the present essay these subtle differences are quite significant. Nevertheless, the use of the rectangle is easy to grasp as a first pass through this domain.)

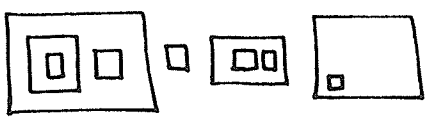


Rectangles can be placed inside one another

or next to one another

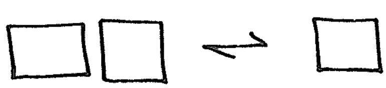


The primary arithmetic is generated by all such drawings of non-intersecting rectangles

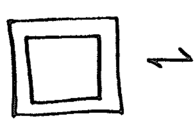


and two rules of operation

1.(Calling)

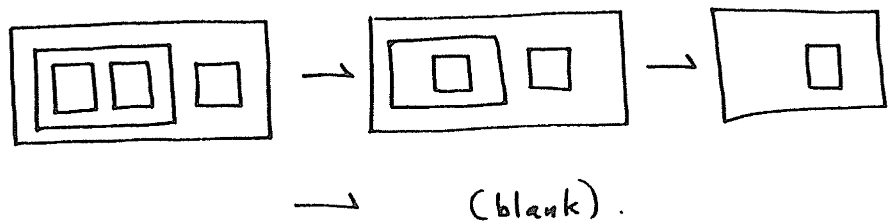


2.(Crossing)



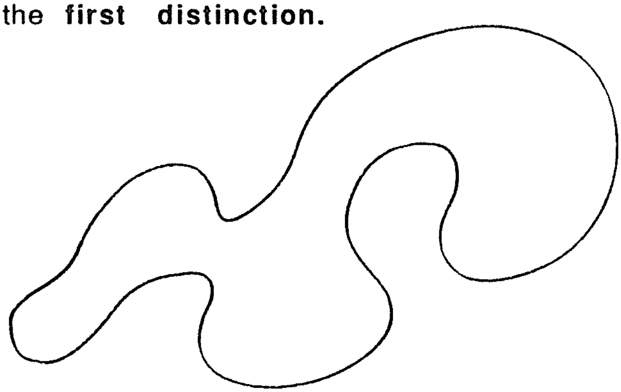
In applying calling, two empty adjacent rectangles can be replaced by a single rectangle, or a single empty rectangle can be replaced by two adjacent empty rectangles. In crossing, two nested rectangles (with no other occupants of the nest) are erased, or two nested rectangles are produced in a formerly blank space.

Any expression in rectangles can be reduced by applications of calling and crossing to either an empty space or to a single rectangle. For example

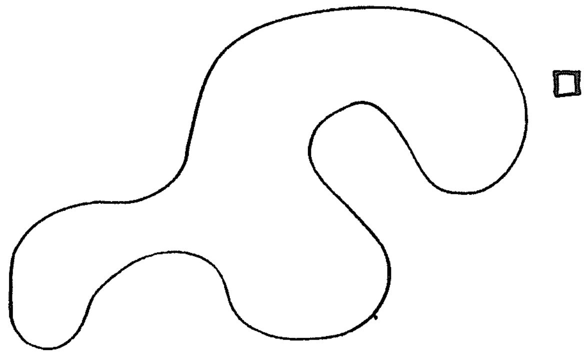



It is an interesting to prove that there is no sequence of calls and crosses taking the empty space to a space with one rectangle. Try this as a first exercise in Laws of Form!

Some terminology: I shall refer to empty space as **the void**, to a space containing one rectangle as a **marked space**. The expressions in this arithmetic of rectangles are regarded as referring to the sides of a distinction that is initially given and called the **first distinction.**



The two sides of the first distinction are distinguished, and so we can choose a mark of distinction (the rectangle) to distinguish them, by marking one side of the first distinction with the mark of distinction.



The side with the mark is the **marked side.** The side without the mark is the **unmarked side**. Crossing refers to the act of changing sides, while calling refers to the act of affirming a given side.

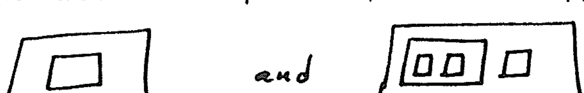
An expression in rectangles is said to be marked if it reduces to the marked state, while it is said to be unmarked if it reduces to the unmarked state. Any expression is either marked or it is unmarked. The **value** of an expression is this designation. Thus the mark itself has the value (i.e. *is* ) **marked**, while the nest of two rectangles  is unmarked.

If two expressions X and Y have the same value, we shall write

X=Y.

Thus

It is important to understand that this notion of equality refers only to the value of the expressions, not to their appearance. Thus




are distinct expressions of the void.

The primary algebra arises at the level of the description of value. For example here is a first result.

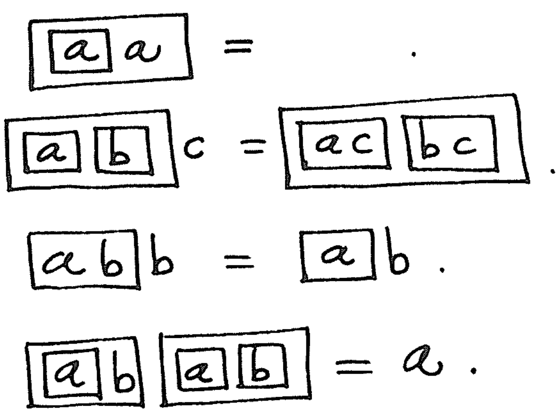
**Theorem.** For any expression A, AA=A.
**Proof.** Either A is marked or A is unmarked. If A is marked, then AA= □□ = □ =A. If A is unmarked, then AA= =A. Thus, AA=A in every case. QED

Proofs of algebraic identities are rather easy at this stage. Here is another one:

 = A

I leave the proof to the reader. We have made a real transition into algebra here. If A is an expression in rectangles, then so is the result of drawing a box around A. Thus "boxing" becomes a new operation in the algebra.

You can then check many algebraic identities, by seeing that they are true about the arithmetic:



This algebra is called the **primary algebra** of Laws of Form.

The primary algebra is so close to being a Boolean algebra that it requires some fine tuning to make the distinction.

Recall one version of Boolean algebra: A Boolean algebra B is a set with two binary operations + and x, one unary operation ' ( a' is the result of applying the unary operation to a.), and two distinguished elements 0 and 1 such that 0'=1 and 1'=0. The operations + and x are commutative and associative. The binary operation has order two: a''=a for any a. Other properties hold such as

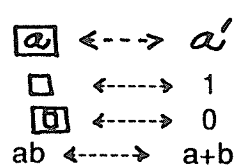
0. a+b = (a'+b')' for any a,b in B.

1. a+a' =1 for any a in B.

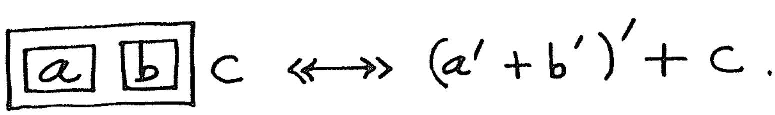
2. (a+b)xc = (axc +bxc) for any a,b,c in B.

These properties, are not minimal (e.g. you can deduce a''=a from the other properties given above). They do suffice as a definition of a Boolean algebra.

We can translate primary algebra into Boolean algebra by the dictionary:

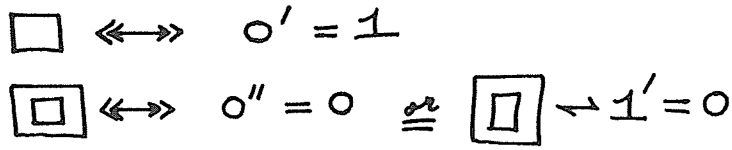



where the expressions on the left are in the primary algebra, and the expressions on the right are in Boolean language. We also have to introduce parentheses in the Boolean format, where they were not needed in the primary format. For example




This need for extra parentheses arises from the fact that the rectangle itself acts as a parenthesis in the primary algebra.

Another difference concerns the translation of the unmarked state. It might seem that there is an ambiguity since any blank can be regarded as two blanks or even infinitely many blanks! Should we include these extra blanks as copies of 0 in the translation to Boolean algbra?! Obviously not, but this means that we have to decide that an entirely blank space will translate as 0, and otherwise we do not do it! For example




This matter of the void is very important. In the primary algebra the notation is set up so that it is quite natural to have empty spaces in the notation. Many rectangles have empty insides, and the unmarked plane is itself an expression of the void. The primary algebra speaks about the patterns of calling and crossing in relation to a first distinction. These patterns are imaged in a language of distinctions (the rectangles) that resonates with the original idea of inside/outside for a first distinction.

Translation to Boolean algebra involves fixing of notation for the unmarked state, and the introduction of other distinctions (the parenthesizations) that come in the wake of insisting on a specific name for the void.

Thus I say that the primary algebra is not a Boolean algebra. Boolean algebra is a interpretive form for the primary algebra, but the primary algebra is not Boolean. In a Boolean algebra there are more distinctions, and in particular the unary operator is distinct from a value in the algebra. In the primary algebra the unary operator acting on the void is identical to the marked state, and this is directly present in the notation.

The primary algebra is to Boolean algebra as the smile of the Cheshire cat is to the cat.

**Exclusive Or**

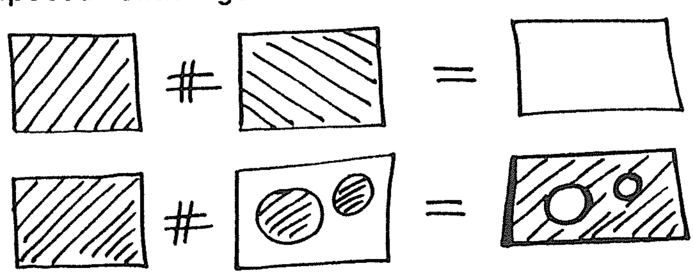
As the pre-logic of Tom Etter is based on the structures generated by exclusive or, it is worthwhile looking at the formation of exclusive or in the primary algebra. Let A#B denote the exclusive or of A and B. We have AB as the usual OR in the primary algebra, and can write

Whence exclusive or (EXOR) is given by the formula

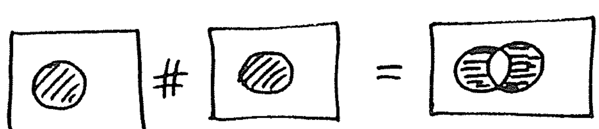
In these terms the operation of crossing is given by

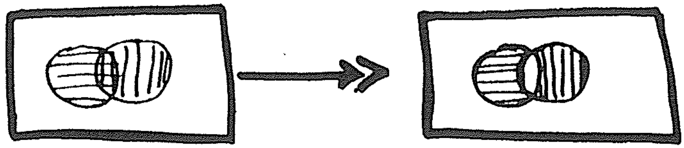
Thus EXOR generates the operation of complementation.

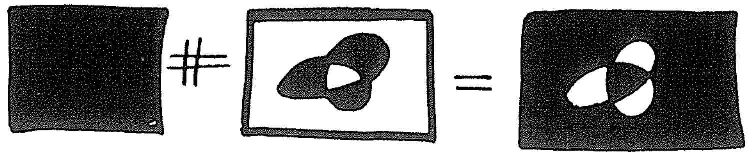
In fact, it is worth looking briefly at the Venn diagrammatic interpretation of EXOR. If a distinction is indicated by a shaded area in the Venn diagram, then we adopt the rule that **superimposed shadings cancel.**



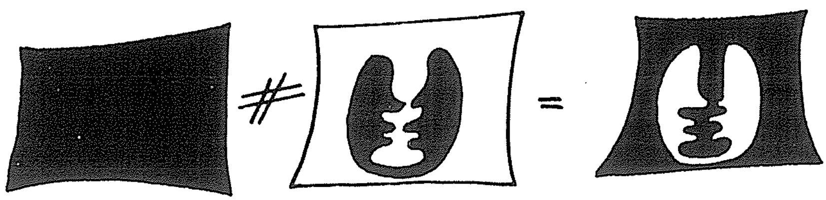
The result of superimposing two Venn diagrams is then their EXOR:







And of course the act of superimposing "■" with A reverses the colors, producing the complement! (Here the mark is regarded as the shading of the entire universe.)



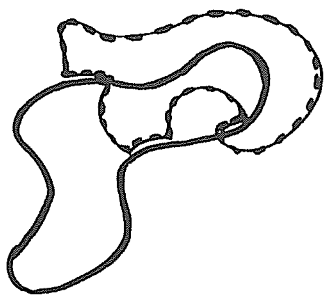
It is clear from Venn diagrammatics that the operation EXOR is really closer to the original spirit of Laws of Form than the later Boolean rules that appear at the level of the primary algebra. Thus we could say (as Tom has suggested) that Laws of Form is generated, or passes through a pre-logical region that is not yet Boolean. It is not yet Boolean in an even simpler sense than the non-Boolean nature of the primary algebra. This very early non-Boolean world is susceptible to a rich field of interpretations. This interpretive structure deserves to be compared with quantum logic.

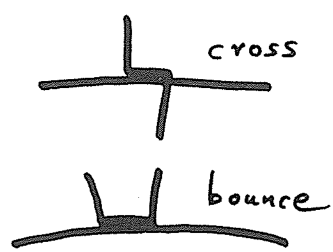
I will not discuss quantum logic here, but it should be emphasized that we are discussing the translation and unification of different languages, where the most condensed language is subject to a multiplicity of interpretations in the more sophisticated complex languages that surround it. Properties of such translations are very similar to the properties of translation from worlds of quantum mechanical events to the worlds of macroscopic observation.

Oshins [O] has suggested that quantum logic is a model for processes of learning and experiencing that we undergo in growing up, and that the internal structure of human thought follows patterns of quantum mechanics. We do not acquire the ability to consciously create formal systems until rather late. This points to the fact that we are creating a drastic oversimplification when we, as adults, attempt to write what appear to be very simple formal systems. These systems are inevitably pervaded by our entire learning experience, just as a single word of English needs a knowledge of the whole English language to be properly understood.

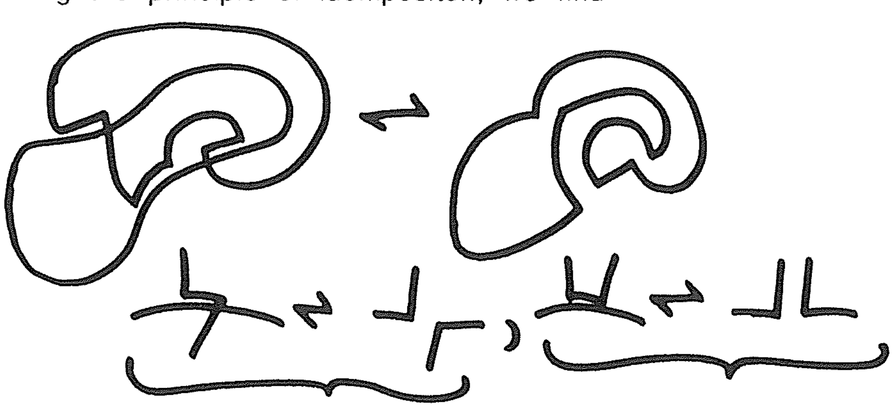
**Idemposition**

The principle of idemposition is due to Max Aintree [A]. It states that **common boundaries cancel.** In order to illustrate the principle we need some common boundaries. Therefore consider interacting plane curves that share a bit of boundary either in the form of a crossing, or in the form of a bounce (see below).

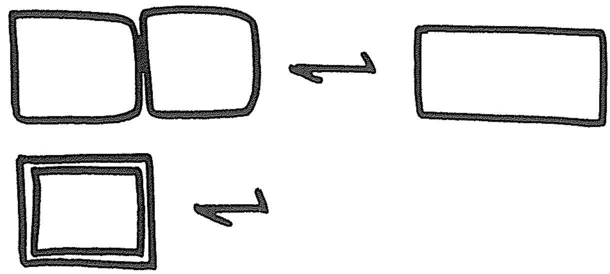





Following the principle of idempositon, we find



In particular, we see that the principle generates instances of both calling and crossing:



By allowing a primitive underlying idea of local action (at the boundary), the principle of idemposition generates patterns for the arithmetic of Laws of Form. The arithmetic of Laws of Form is obtained by freezing and selecting two particular forms of idemposition (calling and crossing), and allowing them to act at a distance. The result is the illusion of absolute clarity of distinction.

### Idemposition and Exclusive Or
Idemposition is a form of EXOR, since it partakes precisely of the pattern of cancellation of common colors. The colors are transposed to the boundary.

### Conclusion
In conclusion, I invite the reader to reconsider the structure of any domain of discourse in regard to its primary distinctions. There are extraordinary realms just beneath the surface of our thought and speech.

## References


[A] Aintree, Maxwell. (private communication.)

[E] Etter, T. *The Laws of Form are Non Boolean!*
*(Subsequent to this writing, Etter's draft article has been greatly revised and retitled. See page 19. - Editor's Note)*

[K] Kauffman, L. H. *Schrodinger's Cat and the Cheshire Cat - Quantum Mechanics and Laws of Form.* ANPA West Journal, Vol 2, No. 2, Fall 1990.

[O] Oshins, E. (1987) *Quantum psychology notes. vol 1: A Personal Construct Notebook.* (privately distributed by the author).

[S] Spenser-Brown, G.. **Laws of Form**. The Julian Press Inc., New York, (1972).

## Addendum

The primary algebra is a formal algebra or *expression algebra.* That is, I am considering the elements of the algebra to be the different expressions composed of boxes and symbols, modulo only the background equivalences obtained by shifting the boxes and symbols in the plane without crossing any boundaries.

We have adopted the equals sign to indicate that expressions have the same vaue, even though they may be distinct.

All of this can be formalized by using the notion of an equivalence relation, and pointing out that at the beginning we have two equivalence relations, one generated by the simple equivalence of expressions, and the other generated by value. The curious thing is that we do reach an end to this process of articulation of equivalence relations, and this end has to do with notational convention.

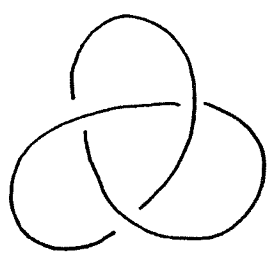
An equivalence relation is defined upon a given set of elements.

In order to articulate an equivalence relation, we must specify the set under consideration. The set of rectangles drawn notationally in the plane is specified only notationally, *unless* we wish to regard the plane as a mathematical plane , and speak of rectangles via coordinates. If the rectangles are simply (!) notation, then we know the rules for their specification only through an understanding of the conventions. Since we are concerned with providing a non-circular foundation to this matthematics, we are constrained to rest upon convention and notation sooner or later.

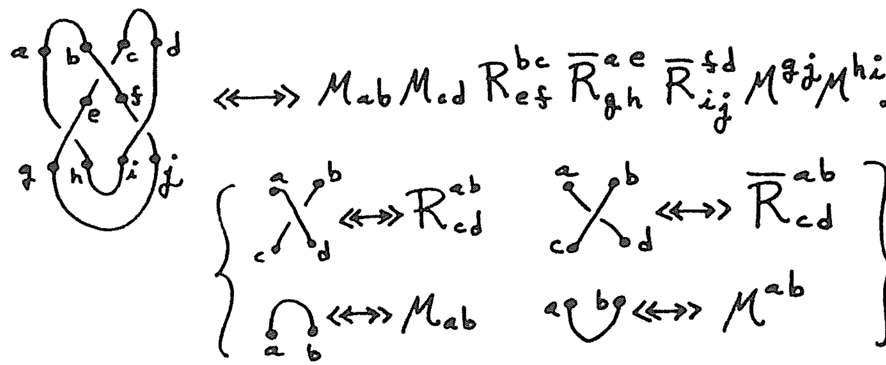
This matter of the notation is usually taken for granted because we either use very standard notation such as letters and signs from printed English, or we very explicitly state the conventions for using new language. Thus the usual notion of a formal system consists in giving the elements of the language, the rules for combining these elements, and rules of operation and transformation of the resulting expressions. This is the domain of modern logic.

Rather than saying that the primary algebra is non-Boolean, we could have said that it is a formal domain (expression algebra) with a Boolean interpretation. Because the now-taken-to-be-standard form of Boolean algebra (with its given distinction between unary operator and values in the algebra) is distinct from the primary algebra, we can see this distinction as meaningful. In Boole's day, the distinction would have been less clear.

Expression algebras occur in other areas of mathematics. A good example is the algebra of combinatorial group theory where the groups are defined by generators and relations. Here the group itself is taken to be the quotient under the equivalence relation generated by the group axioms. The empty word occus in this context, but can always be represented by the symbol 1, standing for its equivalence class. Another example is the diagrammatic theory of knots and links. Here the expressions are the diagrams themselves



These diagrams are notation in the system. Since this is topology, the rules of transformation are motivated by topological and geometrical concerns. In recent years (See [K].) there has been sucess in relating knot theory to statistical mechanics and quantum field theory. A significant part of this relationship derives from the idea of superimposing an abstract tensor diagram on the link diagram, and thereby making a direct connection with techniques in statistical mechanics. The tensor diagrams express formal amplitudes in quantum mechanics, and these amplittudes can be used to calculate topological invariants of the knots and links.



I think that we are just beginning to become conscious of the power and creativity involved in the use and evolution of diagrammatic formal systems.

[K] L.H. Kauffman. **Knots and Physics**. World Scientific Pub. (1991)

# ARE THE LAWS OF FORM NON-BOOLEAN?

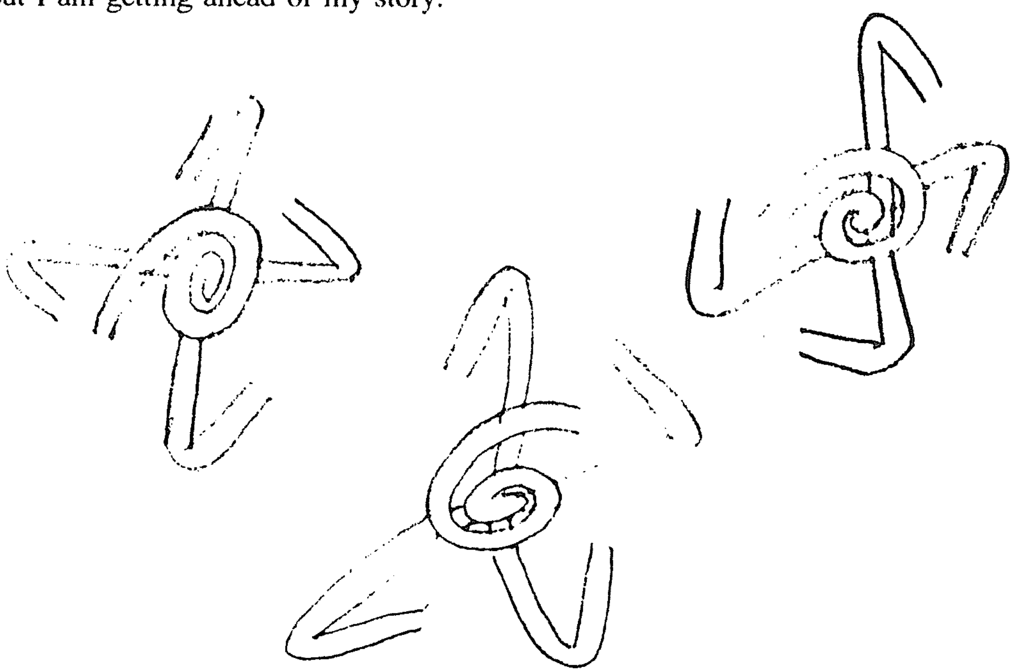
by Tom Etter

In my first attempt at this article I plunged recklessly ahead into the thicket where Spencer-Brown had the courage to precede me. After reading Lou Kauffman's response, I reformulated my thoughts. Kauffman's excellent exposition of Spenser-Brown basics precedes this article. If you haven't read this essay, please do so before proceeding further. And pay close attention! It's simple, but subtle.

As you have now learned from Kauffman, in the Spenser-Brown arithmetical there are two binary operations on squares, calling and crossing, represented by juxtaposition and enclosure respectively. When we go from arithmetic to algebra, we write the calling operator by juxtaposing variables, for instance XY. But how do we write the binary crossing operator in terms of variables? We would have to enclose one variable in another, a neat trick if you can do it! Alas, you can't; in Spenser-Brown algebra, crossing is only a unary operation, like negation. I hadn't realized this when I wrote my original paper, in which the "primary algebra" had a binary crossing operator, which does make sense in a linear notation, as we shall soon see.

If the essence of Laws of Form is the enclosure notation for crossing then I would have to agree with Eddie Oshins [O] that it is hopelessly Boolean and therefore not very relevant to quantum mechanics. My linear notation, which leads to a non-Boolean logic very similar to quantum logic, would then have to be called something else. Indeed, what I am going to present here is in some respect very different from the Laws of Form, since my primary algebra takes crossing as its only primitive – calling comes in later as a defined concept. However, I'm not willing to resign from the Laws of Form club quite yet, since it was Spenser–Brown's mysterious law of crossing, "To recross is not to have crossed", that got me started in my new, if perhaps heretical, direction. The only way I could see to make literal sense of this seemingly paradoxical law was to use homogeneous variables (see below), which is what revealed the quantum–like stage that logic must go through in its evolution from pure negation to Boolean classicism.

But I am getting ahead of my story.



# PART I. QUANTUM LOGIC.

Since I'll be presenting a variant of what von Neumann called quantum logic, let's first look at the original. There are many detailed and rigorous accounts of quantum logic in print, so my account here will be broad and informal.

Quantum logic isn't logic in the ordinary sense of the word; it's not about how to reason correctly, but about the role of words like AND, OR and NOT in what we say about what we observe.

Let's start not with quantum phenomena but with ordinary objects. Take a chessboard, for instance. Looking at a chessboard, we say things like "There's a bishop on square 3,5.", "Both queens are still on the board", "No queen has been taken yet" etc. Notice that the last two statements express quite different ideas, but nevertheless give the same *information* about the state of the board, taking that word in Shannon's sense as a narrowing of the range of possibilities. Such statements will be called *equivalent,* and we'll say that equivalent statements express the same *proposition.* The operation of AND, OR and NOT on these propositions is what will be called the *logic* of the chessboard.

A statement conveys an idea. A proposition is the choice made by an idea at a place. A logic is the joining, disjoining and negating of choices at a place.

Since propositions are equivalence classes of statements, it's important to keep in mind Kauffman's distinction between the two basic kinds of equivalence, as presented in the addendum of [K]. To quote: ".. at the beginning we have two equivalence relations, one generated by the simple equivalence of expressions, and the other generated by value." I shall use different symbols for these two relations: The equal sign "=" will stand for what Kauffman calls background equivalence, while value or foreground equivalence will be shown by a double equal sign, "==".

The essential difference between the two is this: With background equivalence, more commonly known as equality, the equivalence classes themselves are the objects of our attention. Often we don't even notice the individuality of their members, whose role is to present, or to represent, some single thing they all share. Foreground equivalence, on the other hand, is a *relation* between things whose differences are important to our subject matter. Take, for instance, the relation of topological equivalence among Kauffman's knot diagrams; on the one hand, we must remain aware of the differences between diagrams in making connections to other branches of mathematics like tensor theory, while on the other hand, it's their sameness that concerns us when it comes to grasping the concept of knot.

The notation "==" is a reminder that we are dealing with two things that are one. We'll write it's negation as "=\=".

Background/foreground here is of course a shifting boundary. Kauffman gives us good examples of the forward progress of this boundary in the direction of greater abstraction. Sometimes we need to move backward, though, to pull "=" apart into "==" and look separately at several things that were perhaps too hastily merged into one. Such is very much the case with propositions in quantum mechanics.

Let's stay with the chessboard a bit longer. Suppose the white queen is off the board. There are many ways to say this: "The white queen is off the board", "I don't see a white queen", "I'm sorry to say that your queen is gone" "If you look, you'll realize that your queen isn't there", "Horowitz is spelled with a 'w', and you've lost your queen" etc. Here we have various pieces of information about the queen, the board, seeing, realizing, possibly looking, and Horowitz. But in the context of the states of the chessboard, and assuming we see correctly, all these statements are true of exactly the same set of states. This means, according to our definition, that we must regard them all as expressing the same proposition.

We allow ourselves these casually interchangeable forms of expression in everyday life secure in our faith that the facts are indeed the facts, and will not be altered by how we look at them. But suppose we literally looked daggers at them! In the case of the chessboard, our daggers would send the pieces flying in all directions, and the differences among what is, what is seen, what would be been seen or seen again etc. could not be so casually ignored.

Because we have a long direct experience of the states of everyday objects like the chessboard, it seems natural when we reflect on measurement and its logic to start out with states. But we have no such direct experience of quantum states. If, as all the evidence suggests, quantum states would unavoidably be altered by measurement, then it becomes problematical to speak about quantum states at all. But without states, what becomes of propositions? If a proposition is the choice made by an idea, what does the idea choose? Without states, what is information? Quantum mechanics forces us to take such questions seriously.

As empiricists our safest course is not to postulate quantum states but to start to start, that is, with quantum *observation* and define quantum propositions and states in terms of observation as best we can. Here's how this is done in von Neumann's Hilbert space approach.

Let the notation A.S mean a statement S about the outcome of a quantum measurement A.

Von Neumann Equivalence. Given measurements A and B, A.S == B.T will be defined as meaning that if we were to measure A and then measure B, S would be true if and only if T were true.

Quantum Proposition: An equivalence class under von Neumann equivalence.

Notice how far we have come from our first encounter with the chessboard, where we abstracted propositions from statements about the actual board. "If we were to measure A and B" is a variation of "Once upon a time we measured A and B". In the name of empiricism, we have erected the edifice of physical fact upon the ground of fiction! Even our equivalence itself is a fiction: "Once upon a time we measured A and then measured B, and because A came out S, B came out T, and vice versa". Of course our fictions are designed to be not only plausible but instructive, which is why we call them science rather than science fiction.

Von Neumann equivalence does in fact live up to its name as an equivalence relation. Here's what this means for observation:

Reflexive law. If we measure AA, i.e. if we measure A and then immediately measure A again, both measurements always yield the same result; this implies that for any S,
A.S == A.S.

Symmetric law. It doesn't matter whether we measure AB or BA in determining equivalences, i.e. if A.S == B.T then B.T == A.S.

Transitive law. The indirect equivalences of the form A.S == C.T that can be inferred by measuring AB and BC are also the direct equivalences revealed by measuring AC.

Quantum logic is the logic of quantum propositions. More exactly, it is the set of all quantum propositions about a given quantum object as structured by the Boolean operators AND, OR and NOT. The singular term "quantum logic" has become well established, but it carries a misleading connotation of there being one logic where it is more accurate to say that there are many. The quantum propositions fall into many classes, each of which is a Boolean logic; let's call them Boolean *frames*. These various Boolean frames overlap, and the pattern of their overlaps is what defines the global structure called quantum logic. Von Neumann tried to extend AND and OR into non-Boolean global operators, but this proved to be a mistake. AND and OR are essentially Boolean concepts. It's just that in the quantum realm there is a *different* AND and OR in every frame, just as there is a different RIGHT and LEFT in every spatial frame.

There are two important things to know about quantum logic: first, it's not Boolean, and second, it's the core of quantum mechanics as a theory. Let's take them in order.

Think of Boolean frames as flat pieces of paper, and quantum logic as a paper house made by gluing the frames together, where the glue is von Neumann equivalence. The question arises whether it's possible to flatten such a house without tearing it. For quantum logic, this means interpreting all of its frames as sub-algebras of a single Boolean algebra without severing any of the von Neumann equivalence bonds between frames, and the answer is no. The word sub-algebra here can be interpreted in several ways, so let me use another:

Factor. A subset of a Boolean algebra B is called a *factor* of B if it is itself a Boolean algebra under the same operators.

To say that quantum logic is non-Boolean is to say that all of its frames cannot be factors of a single frame, consistent with the global (interframe) von Neumann equivalence relation. This is an important point for hidden variable theory, which tries to interpret quantum mechanics within a single frame. Granted that a such an interpretation requires tearing or separating some frames, might this be done in an orderly way, as when we flatten the Earth into a Mercator projection? The answer would seem to be no, since it turns out that there are very simple sub-structures of quantum logic that can't be flattened (see appendix) and these are distributed throughout the whole structure, which can thus only be flattened by ripping it to shreds. Any way you cut it, hidden variables are a mess.

A brief aside about non-locality: A flat Earth has at least one bad non-locality, since when you sail off the East edge you instantly appear at the West edge. The so-called non-localities of Bell's theorem come from our unconsciously regarding quantum logic as flat, which leads us to substitute some fantasied causal mechanism for the equivalence bonds we have inadvertently severed by flattening it.

Now we come to the second point. I regard the core of quantum mechanics as the Born probability rule together with unitary dynamics. There is a remarkable theorem by Gleason that in essence deduces the Born rule from the structure of quantum logic alone. What this theorem shows is that if we assign a probability distribution to every frame, and if these distributions are consistent between frames in the sense that equivalent statements always have the same probability, then they must satisfy Born's rule for some state in a uniquely defined Hilbert space. It's then as easy step to show that Schrodinger's equation, in its most general form, is equivalent to quantum logic rotating at a steady rate with the passage of time. We've seen that quantum logic isn't flat; now we see that it's round!

In conclusion, I would like to give a capsule summary of the mathematical structure of quantum logic that will tie it to the quantum-like logic of Laws of Form.

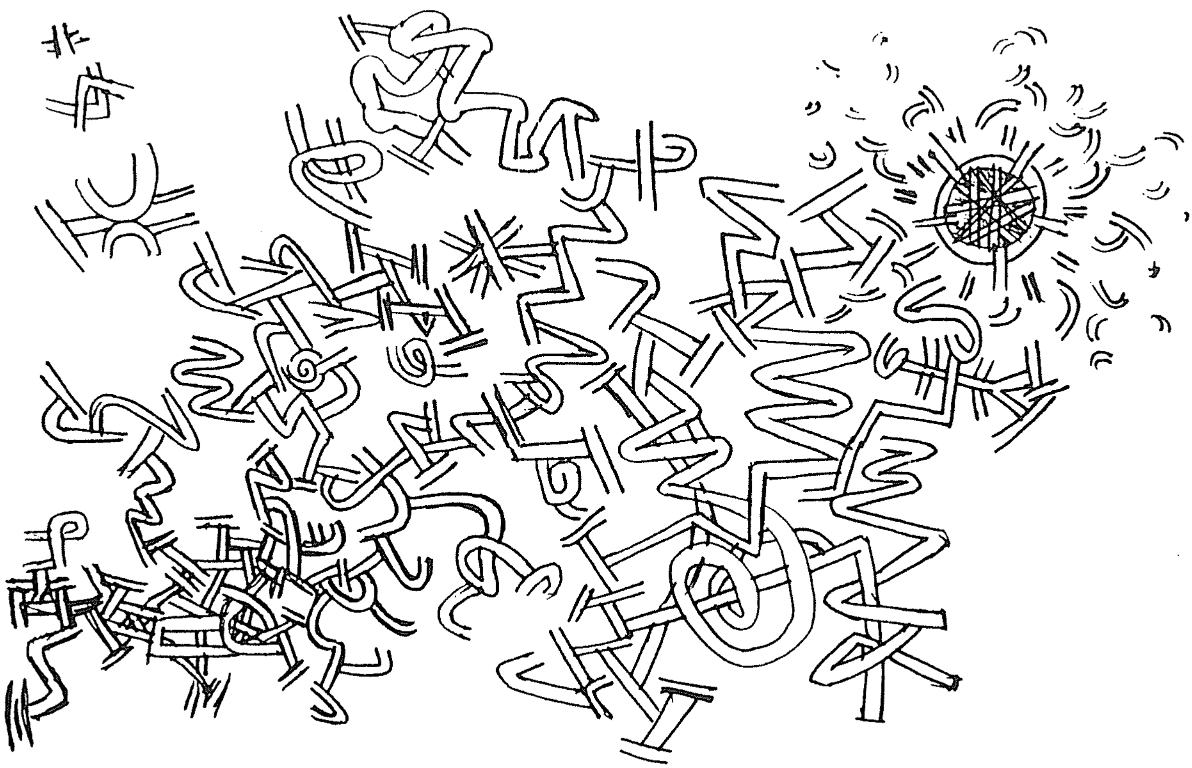
Let's confine our attention to those frames that are not proper factors of other frames; such a frame will be called *complete*. A complete frame is sometimes referred to as a complete set of commuting observables. Define an *atom* of a frame to be a proposition which is as informative as possible. For instance, in the logic of a numeric state variable x, "x=3" would be an atom, since nothing else you could say about x would add any more information. An atom of chessboard logic is a complete specification of the state of the chessboard. An atom of a quantum frame is a pure quantum state.

Clearly every proposition in a logic is a disjunction of its atoms. If we arrange the atoms in a linear order, then we can represent every proposition by a bit string, where the 1 atoms are in the disjunction, the 0 atoms out. Let's write these bit strings as the diagonals of matrices whose off-diagonal elements are all 0's. Note that the conjunction P&Q then becomes the matrix product PQ, and negation is the complement 1−P, (1 is the identity matrix) with the 0's and 1's in the diagonal reversed. Thus our diagonal bit string notation enables us to represent the logic of the frame within matrix algebra. This is true of any Boolean algebra, not just quantum frames.

Suppose a proposition is common to two frames. In general, its matrices in the two frames will be different. How are these two matrices related? Now we come to the heart of the matter.

Equivalence law. There exists a Hilbert space H such that every proposition matrix represents an ortho-projection operator on H, and two such matrices are equivalent if and only if they represent the same projection. (footnote 1)

This equivalence law effectively determines the mathematical structure of quantum logic, which can be succinctly defined as the set of all ortho-projections on a Hilbert space together with the operators PQ and 1-P. In general, the product PQ is not a projection, so PQ as an operator in quantum logic is only defined for certain pairs of propositions. The criterion for PQ to be a projection is that PQ = QP (footnote 2). Whenever PQ is defined it represents conjunction in some Boolean frame, while 1-P always represents negation. Thus a Boolean frame is a set of ortho-projections closed under PQ and 1-P; a complete frame is a Boolean frame that is not a proper subset of any other Boolean frame, or to put it another way, it's a maximal set of commuting ortho-projections.



# PART 2. PRIMARY THEORY.

## I. The Primary Arithmetic.

Let's now turn to our linear variant of Laws of Form. Unlike Spenser–Brown's Laws of Form, our primary algebra is based on crossing alone; calling enters the picture as a defined symbol. Like Spenser–Brown, we start with a single mark symbol; ours is "0". An expression will be any string of zeroes, including the null string. We'll use all three equal signs, "=", "==" and ":=" to make statements about expressions. There is one axiom:

The Law of Cancellation. 00 == the null string

Definition of Arithmetical Equivalence. Two expressions are called *equivalent* if each can be derived from the other by applying the law of cancellation. Since every expression reduces to either 0 or the null string, there are just two equivalence classes or *values.*

Let's now introduce some new symbols by definition. A definition is a decision to let a new symbol express what is already expressed by an old one, so a defined term is not only equivalent to what it replaces, but *equal* to it. We shall use the computer symbol ":=" to declare this equality.

Definition of the Unit. 1 := 00

Here are some examples of equality and equivalence: 00 = 1, 10 = 000, 010 = 0000, 000 == 0, 010 == 00, 111 == 1. The multiplication table for 0 and 1 is a mixture of equalities and equivalences:

| | |
|---|---|
| 00 = 1 | 01 == 0 |
| 10 == 0 | 11 == 1 |

We'll call this the *primary arithmetic.* If we interpret 1 as truth and 0 as falsehood, it's the truth table of the Boolean operator IFF. Let's now define a new symbol "⊕" having the truth table of EXOR (exclusive OR), the Boolean dual of IFF.

Definition of EXOR. For any expressions X and Y, $X \oplus Y := 0XY$

This definition is ambiguous if there is more than one "$\oplus$"; for instance, is $X \oplus Y \oplus Z$ equal to 0X0YZ or to 00XYZ? We'll resolve this ambiguity by assuming that the left scope of $\oplus$ is always the beginning of the expression, so the second is correct: $X \oplus Y \oplus Z$ = 00XYZ. Of course, $X \oplus Y \oplus Z$ is equivalent to both 0X0YZ and 00XYZ.

What follows will be intuitively clearer if we work mostly with EXOR rather than IFF. Here is the multiplication (truth) table of EXOR, as one can quickly confirm from its definition.

| | |
|---|---|
| $0 \oplus 0 == 0$ | $0 \oplus 1 == 1$ |
| $1 \oplus 0 == 1$ | $1 \oplus 1 == 0$ |

## II. The Primary Algebra.

Let's now push arithmetical equivalence into the background. This means that arithmetical expressions are now only pointers to their values; we no longer care *how* they point, just *where*. Our two values will now be denoted by their shortest non-null expressions, 0 and 1. We will of course now write "=" where above we wrote "==".

Expressions in 0 and 1 are constants; algebra begins with variables. By an *expression* will now be meant a string consisting of 0's, 1's $\oplus$'s and variables. We'll use small letters for variables, capitals for unspecified expressions. We must keep in mind that these capital letters are not themselves expressions as we are now using that term; rather, they belong to our informal *metadomain* of language *about* expressions.

Algebraic equivalence is defined by the:

Law of Algebra. For any expressions X and Y, X == Y if and only if any substitution of constants for all of the variables in X and Y makes them into equal constant expressions.

This rule plus the primary arithmetic lead to the following two theorems:

Cancellation Theorem. XX == 1.

Proof: Any substitution of constants for the variables in X will give both occurrences of X the same value. Thus XX will reduce to either 00 or 11, which both reduce to 1.

Reordering Theorem. XY == YX. (proof similar)

The familiar method of truth tables is a reliable way to use the law of algebra to prove simple equivalences. For instance, to prove the reordering theorem we might write:

```
X   Y ¦  XY == YX ?
-----¦-----------
0   0 ¦  1 ==  1
0   1 ¦  0 ==  0
1   0 ¦  0 ==  0
1   1 ¦  1 ==  1
-----¦-----------
      ¦    yes!
```

With the above two theorems plus the rules of arithmetic we can reduce any expression to an expression in which no variable occurs more than once, the variables occur in alphabetical order, the first character is either 0 or 1, and there are no other 0's or 1's. Call such an expression *canonical*. It's easy to see that no two canonical expressions are equivalent. If we are working with a total of n variables, there are $2^{(n+1)}$ canonical expressions, and thus $2^{(n+1)}$ values.

EXOR is defined in the primary algebra just as it is in the primary arithmetic, i.e. $X \oplus Y$ := 0XY. Note however, that now it operates on $2^{(n+1)}$ values instead of just 2. These become the elements of the EXOR group, whose structure is characterized by fact that every element is its own inverse. 0 is the EXOR group identity.

The value algebra has a further structure arising from the fact that a particular non-zero element has been singled out as the constant 1. As far as the group structure is concerned, the choice of this element is completely arbitrary. Singling out 1 is equivalent to introducing a unary operation of negation:

Definition of Negation. $\sim X := 1 \oplus X$. Note that $\sim X = 0X$, so the mark symbol 0 as a multiplier is synonymous with negation, as in Spenser-Brown theory. Note also that $1 == X \oplus \sim X$.

Pre-Logic. The term pre-logic, as I have used it elsewhere [E], is defined as a value algebra given by Boolean EXOR and negation, or equivalently, a group whose every element is its own inverse and in which there is singled out a particular non-0 element which we call 1.

Pre-logic is that part of Boolean algebra generated by EXOR and NOT, or by EXOR and IFF, or by IFF and 0. It can be shown that a binary Boolean operator is definable in pre-logic if and only if there are even number of 1's in its truth table. Thus pre-logic doesn't include AND, OR, NAND, NOR, or material implication, which shows that it's a much weaker structure than Boolean algebra.

The elements of a pre-logic are formally indistinguishable from each other, except for 0 and 1. We've seen that pre-logic emerges when we add variability to crossing, and we've also seen how our canonical expressions are in natural 1-1 correspondence with the elements of pre-logic. These canonical expressions are quite distinguishable however, and not just by arbitrary notational features; they fall into different formal classes according to how many variables they contain.

Something here is at odds with the spirit of Spenser-Brown's law "To re-cross is not to have crossed". The expression xzyw, for instance, suggests a series, a history, of four possible re-crossings. It is not equivalent to any expression with fewer than four variables, which says that its fourness is more than just an accident of representation. And yet Laws of Form is trying to explore a realm prior to history, prior even to the possibility of fixing and tagging things so as to be able to count them. Our notation, which is the source of the forms we are studying, should reflect this better.

There is a way to make it do so. Let's make a list of all non-constant canonical expressions except for those consisting of a single variable. Now let's assign, as a definition, a new variable to each of these expressions in the list; for instance, $n := xy$. This doesn't add any new expressions, since we now have $n = xy$. But notice that it generates two new notational equivalences, $x == yn$ and $y == nx$. The three expressions x, y and n are now formally on an equal footing. It turns out that this is true of all variables, which are now *homogeneous.* That is, if we take the defining equations as equivalences, we can exchange any two variables and obtain a new set of true equivalences of exactly the same form. Taking single variables as the canonical non-constant expressions ends the artificial distinction between simple and compound values.

## III. Primary Space.

As mentioned above, the calling operator, written "&", will enter the picture as a defined symbol. The idea here is to create the conceptual apparatus of calling in a way that retains the symmetries of the primary crossing algebra. By so doing, we find that when we reach Boolean algebra, the particular Boolean algebra we reach is one frame among many. And here is the punchline: This frame is related to the others almost exactly as it is in quantum mechanics! The differences are that the scalar field of the common vector space is now binary rather than complex, negation takes the place of the inner product, and most important, this vector space is not pulled out of a hat but is already there in the Boolean algebra of the classical frame!

We'll now push primary algebraic equivalence into the background along with primary arithmetical equivalence. This means we'll write $x \oplus y = y \oplus x$, $x \oplus x = 1$ etc., reserving "==" for new equivalences involving &.

With 0 as the mark sign, the arithmetic multiplication table for calling is that of Boolean AND. This table is what defines &; more exactly, it defines four new synonyms for 0 and 1:

Definition of & for constants:

$0\&0 := 0$ $\qquad$ $0\&1 := 0$

$1\&0 := 0$ $\qquad$ $1\&1 := 1$

where it is to be understood that the 0's and 1's on the left can always be replaced by equal expressions.

When we combine these definitions with the arithmetic of ⊕, truth tables reveal that & and ⊕ satisfy the distributive law $C\&(D \oplus E) = (C\&D) \oplus (C\&E)$, where C, D and E are of course constants. Interpreting ⊕ as addition and & as multiplication, we get the binary field, with 0 and 1 as its elements. Note that we must now use parentheses to specify the scope of the two operators; this adds two more symbols to our growing notation.

What happens if we try to extend & to expressions with variables? If one of the arguments is constant, we can evaluate the resultant immediately by using the multiplication table for & plus the law of algebra: given any substitution of constants for variables in an expression X, we have 0&X = X&0 = 0 and 1&X = X&1 = X. In effect, what we have done here is to make calling into a pair of unary operators 0& and 1&. Since we originally made Spenser-Brown's unary crossing operator into a binary operator, at this stage we could use Spenser-Brown notation with crossing and calling reversed! Taking 0 as mark, 0& would mean enclosure by a square. Since 1 = 00, the Spenser-Brown symbol for 1& would be double enclosure. Since 1& is the identity operator, double enclosures would cancel, just like two squares juxtaposed.

Unary calling operators like 1& and 0& will be very important for our project. Since binary calling is conjunction, we'll speak of a unary calling operator as a *junction.*

The junctions 1& and 0& satisfy both a left and a right distributive law, i.e. C&(X⊕Y) = (C&X)⊕(C&Y) and (C⊕D)&X = (C&X)⊕(D&X), where C and D are of course constants (the reader may want to check this by writing out the truth tables). This means that we can regard our primary algebra as a *vector space* over the binary field of 0 and 1, taking & as scalar multiplication.

Primary algebra is a bit odd as a vector space, since the scalar 1 is also a vector. Recall that ~X = 1⊕X. It follows that 1 = X⊕~X. Identifying 1 with a vector is thus equivalent to introducing a negation operator on the vectors. At this stage, the abstract structure of primary algebra is that of a binary vector space with negation; we'll call this structure *primary space.*

Abstract Definition of Primary Space: A vector space over the binary field together with an operator ~ with the property that there exists an element U such that for all X,
~X = U+X

Note that such a negation operator is not possible in any other vector space, since with any other scalar field we wouldn't have ~~X = X. Negation in primary space resembles orthogonality in Hilbert space, though it is a weaker concept; indeed, when you introduce a full orthogonality relation into a primary space it becomes a Boolean algebra.

When you choose a basis in primary space, every vector becomes a bit string. There is an important analogue in primary space to the concept of ortho-normal basis in Hilbert space:

Proper Basis: A basis in which the constant 1 is represented by the bit string of all 1's. (Note that 0 is the bit string of all 0's in any basis).

To put it in a nutshell, primary space is Spenser–Brown's primary algebra with the truth tables for calling and crossing reversed.

## IV. Junctions, Projections and Units.

We still haven't finished with the definition of &. So far we have managed to evaluate X&Y using the rules of primary algebra alone. But it turns out that if both X and Y contain variables, this is no longer possible. That is, there are many ways to interpret & as an operator on primary space without contradicting primary algebra. Instead of choosing a single &, our project now is to explore this whole class of &'s and see how they relate to each other. We'll do this in two stages. In the present section we'll study an extended class of unary operators of the kind I called junctions, which turn out to be projections on primary space. The theory of junctions seems to be where the real action is, and since this is mostly new material I must regrettably be somewhat more technical in my presentation here than I have been so far. In the next we'll look more briefly at & as a binary operator.

We have defined the junctions 1& and 0& in primary algebra. Can we define junctions of the form P& in primary algebra where P contains variables? If we interpret this question as asking whether P&X can be evaluated within primary algebra the way 1&X and 0&X can, the answer is no. P&X, as it stands, has a new value. If we are to interpret the form P& as an operator mapping primary space onto itself, this form must be coupled with a new equivalence relation that reduces these new values to old ones.

Although primary algebra can't evaluate P&X, it does have some important things to say about it. Using truth tables we can immediately prove the following three laws:

Linearity. $P\&((C\&X) \oplus (D\&Y)) = C\&(P\&X) \oplus D\&(P\&Y)$ where C and D are constants.

Idempotency. $P\&(P\&X)) = P\&X$.

Unity. $P\&1 = P$.

The first identity says that P& is a linear operator on primary space. The second says that it's a *projection* operator. The third we have already encountered as scalar multiplication.

Suppose we started out with any projection operator J(X). The law of unity could then be used as a *definition* of P, i.e. P := J(1). We could then go on to define the form P&X along with a new equivalence relation == by the condition P&X == J(X). Would this make P& into a junction?

Not necessarily. Suppose that 1 is in the null space of J, i.e. J(1) == 0, and that J is not the null projection, i.e. there is some X such that J(X) =\= 0. Then if we should define P&X := J(X), we would have P&1 == 0, so by the law of unity, P == 0, and J(X) == P&X == 0&X. But primary algebra says that 0&X = 0, so J(X) == 0, contrary to assumption. Since we want the & of J to agree with the & of primary algebra, we cannot allow 1 to be in J's null space.

Junction Defined: A *junction* is either the null projection or a non-null projection J such that J(1) =\= 0.

Junctions, as projections, bring into primary algebra the beginnings of the concept of *place*. It's not far-fetched to speak of the range of a junction J as HERE and the null space of J as elsewhere. J as a mapping tells us what's HERE: things HERE are mapped onto themselves, while things elsewhere are mapped onto 0. More generally, things that are only partly HERE are mapped onto the parts of themselves that are all HERE. If we think of P := J(1) as what is expressed by J, then J, in its treatment of other expressions, is like an egocentric talker who can only hear from others what he has already said himself.

Spenser-Brown's crossing operator moves us from one place to another across a physical line. The operator that generates our present primary algebra was called crossing, but this is something of a misnomer, since neither its notation nor our early defined concepts gave us anywhere to cross from or to. But now, given a junction, we have both; we can now speak of what's in and what's out. Does our term "crossing" live up to its promise? Does crossing reverse inside and outside?

By crossing I now of course mean the unary operator of crossing, i.e. negation. But at this point we actually have two concepts of negation: ~P:= 1⊕P for expressions P, and ~J := 1⊕J for projections J (footnote 3). Let's first look at the latter, which is complementation (1 here means the identity operator). Recall from Part I that the complement of a projection J is the projection whose range and null space are those of J reversed. Complementation thus does reverse inside and outside, HERE and elsewhere. There is still a problem, though, since the complement of a junction need not be a junction! We need another condition on junctions:

Propriety. A *proper* junction is defined as a junction whose complement is a junction. I.e.

a junction J is *proper* if J is either the identity projection 1& or else J(1) =\= 1.

If we think of 1 as everything, then propriety says that not everything is HERE. The definition of junction say that there must be something HERE, so if elsewhere can be exchanged with HERE there must be something there too. The pure void can hardly be considered a proper place to go to – it's just not done!

Diagonal Matrix Theorem. A projection on primary space is a proper junction if and only if it has a diagonal matrix in a proper basis. (Recall that a proper basis is a basis in which the bit string for 1 is all 1's.)

Proof: Choose a basis {Bi} in which a proper junction J has a diagonal matrix. {Bi} divides into two parts, {Ri} and {Ni} where {Ri} spans the range RJ of J and {Ni} spans the null space NJ. J will be diagonal for any {Ri} and {Ni} that span RJ and NJ, and since the non–constant vectors are homogeneous, we can always pick an {Ri} and {Ni} such that the component bit strings of 1 in RJ and NJ are all 1's, which makes {Bi} proper. This shows that if a junction is proper it is diagonal in a proper basis. The converse is immediate, since if {Ri} and {Ni} are all 1's, then neither component of 1 can be 0.

I don't know whether improper junctions will ever turn out to be worth studying in depth, but for now we'll stick with proper junctions and drop the "proper" in speaking of them. The diagonal matrix theorem makes it possible to see clearly how our two forms of negation are related. First, one more definition:

Unit of a Junction. The expression J(1) will be called the *unit* of junction J. 1 is the unit of the identity operator, and it will sometimes make for greater clarity to speak of 1 as the unit of the primary algebra.

Crossing Theorem. The unit of the complement of a junction is the negation of the unit of that junction. I.e., if J is a junction, ~J its complement, then ~J(1) == ~(J(1)).

Proof: If we choose a proper basis in which J is diagonal, then the unit of J will be all 1's in {Ri}, all 0's in {Ni}. But the unit of ~J will be all 0's in {Ri} and all 1's in {Ni}, making it the negation of the unit of J.

Every junction has an expression as its unit. By the principle of homogeneity, every expression is the unit of some junction. But now the question arises: given some expression P, is there more than one junction J of which it is the unit? In case P is a constant, the answer is no, as we have seen. If there are only two non–constant expressions, then it's easy to see that there are only two non-constant junctions, so the answer is also no. For all larger primary spaces the answer is yes. We'll now classify the various J's that go with a particular P. First we'll look at the *forms* of the J's of P, then at the duplicates of each form.

Form, as I am now using the term, is what remains the same under isomorphism. The isomorphisms of primary algebra are the non-singular (i.e. 1-1) linear transformations of primary space that preserve 1. If a junction J' results from J by such an isomorphism, we'll say that J and J' have the same form:

Junction Forms. J is called *similar* to J' if J' == $TJT^{-1}$ for some isomorphism T. An equivalence class of similar junctions will be called a *junction form.*

Any isomorphism can be given by a 1-1 mapping between two proper bases. From this it follows that two junctions have the same form if and only if their ranges and null spaces have the same dimensions (in the finite-dimensional case, it's enough to assume that their ranges have the same dimension). Since any division of a proper basis into two subsets defines a junction, there are junctions of every dimension. The principle of homogeneity says that there is an isomorphism reversing any two non-constant expressions, so every non-constant expression is the unit of a junction of every form except 1& and 0&. If J(1) == P, and J' is similar to P, then J'(1) == P if and only if T(P) == P for some T that maps J onto J' (if $TJT^{-1}(1)$ == P, then, since T(1) == 1, $TJT^{-1}(1)$ == TJ(1) == T(P) == P).

In brief: Among non-constant expressions, every P is the unit of a J of every form except 1& and 0&, and of all copies of that J that can be reached by a T having P as an eigenvector.

After this rather detailed exposition, let me conclude with a few less formal remarks. The concept of junction, which extends von Neumann's deep insight in quantum mechanics about the connection between propositions and projections, may well be the central concept in the new more general science that is starting to emerge out of work on quantum foundations. If we think of a junction as what defines HERE, then junctions are the building blocks of a very broad conception of space that encompasses not only physical space but logic and matter and much more still to be explored. Though we have been treating primary algebra as something that is simply given, if we reflect on what gives us this particular primary algebra HERE, we see that it must be a junction. The universe of discourse is the range of a junction. How about the universe itself? WHICH universe? Why, this one HERE.

## V. Boolean Logic.

Let's now make & into a binary operator. It might seem that we have already done this, since P and X in the form P&X can be any two expressions. If for every expression X we choose a particular one of its junctions J, then we can indeed define a binary operator b(X,Y) := J(Y). But is it AND? Must all these junctions add up to conjunction? Would it be consistent with primary algebra to write b(X,Y) as X&Y?

As one can quickly verify with truth tables, any binary operator X&Y that satisfies our multiplication table for 0 and 1, and also the law of algebra, must satisfy the following two laws:

Associative Law. X&(Y&Z) = (X&Y)&Z.

Commutative Law. X&Y = Y&X.

These two laws are of course meaningless for junctions taken singly, which is why we didn't encounter them sooner.

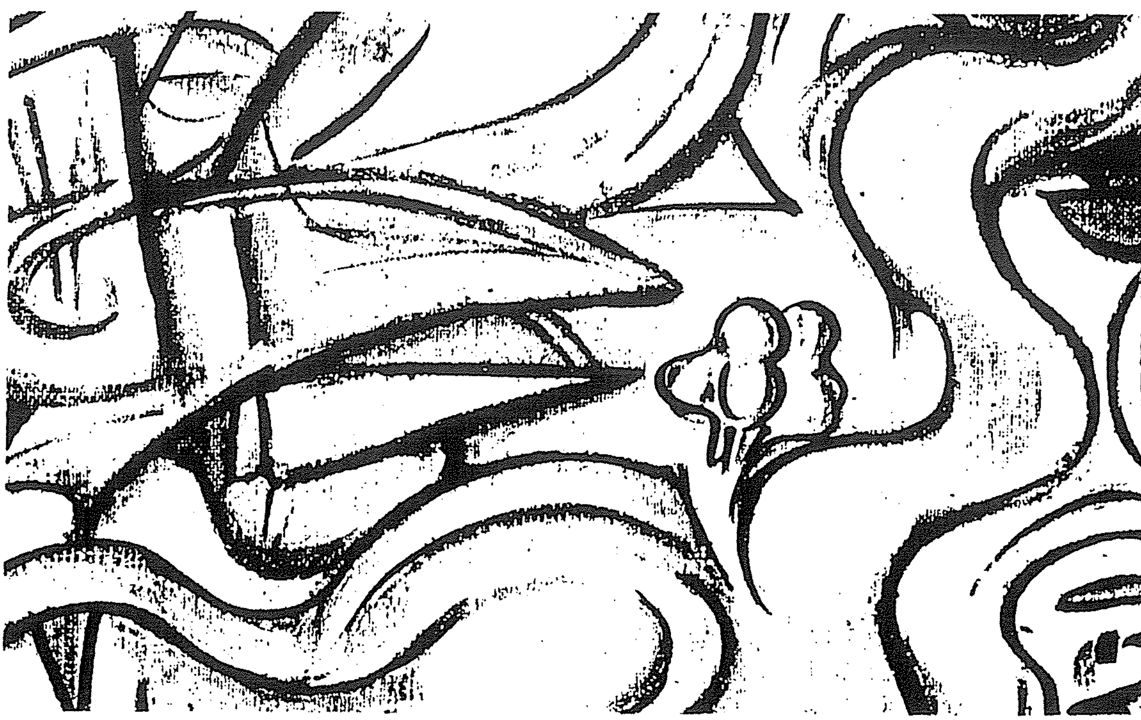
Applied to our binary operator b(X,Y), the associative law says that the (functional) product JK of junction J of X and junction K of Y is the junction of b(X,Y). This means that KJ is the junction of b(Y,X), so the commutative law tells us that J and K are commuting projections. More generally, it tells us that if we choose a junction for every expression, and if these junctions cohere into a single binary operator of conjunction, they must all commute. Most pairs of junctions don't commute, so the answer to our question above is no.

Mathematically the situation is really quite simple. In order to have a binary AND, and with it a Boolean logic, we must choose among the J's of the P's so as to make all the J's commute. There are many ways to do this, a different way for each proper basis {Bi}. For a particular {Bi}, the J of a P is the junction having a diagonal matrix whose diagonal is the bit string of P. The resulting logic is what in Part 1 was called a complete frame. In Hilbert space the complete frames are the maximal sets of commuting ortho-projections. In primary theory the complete frames are the maximal sets of commuting junctions.

In Part 1 we saw that we can identify propositions about the state of an object with projections. In the everyday world, and in classical science, all such projections commute, and indeed it is impossible even to say what vector space they project onto, since their matrices are only 0's and 1's. As we "progress" to quantum mechanics, though, we find that there are non-commuting pairs of projections, in this case on Hilbert space, whose failure to commute corresponds to the fact that as items of information they cannot be conjoined.

In Part 2 we seem to be moving over the same ground from the opposite direction. First we emerge from pre–logical darkness into the faint dawn of bare discrimination. We find ourselves HERE, not elsewhere. THIS separates from that. The world faintly emerges, but only as a projection. But then we find we can *cross* from HERE to there and back, and lo! there are TWO projections, two worlds, an inner and an outer, us and them, self and other.

Zooming quickly up to the exalted perch of the contemporary mathematician, we see that these worlds are but two among a myriad. But almost no two of this myriad can communicate. Like Leibnitz' monads, their windows are mostly closed; no information can be exchanged with the others. Closed, that is, except within those communities where it doesn't matter which is first, i.e. where all members *commute*. In finding our own thoughts within such a community, we arrive at the logic of the everyday world.



# CODA. DECONSTRUCTION.

My exposition of primary theory has been more or less in the style of what I call Erector Set science: you start with simple parts and assemble them into a progressively bigger and better construct that you offer, subject to empirical test, as a portrait or "model" of the world. This is how it's done these days. But in the present case the style is somewhat at odds with the message, which is that science today is basically in need of a radical *deconstruction.*

Our situation resembles that of our ancestors who learned that the world is not flat. In a flat world, up and down are absolute, belonging to space itself and giving to every material object its place in the order of higher and lower. But in our actual world, if you travel too far afield you can no longer assemble your various observations about what is higher and lower into a single hierarchy. The facts concerning up and down simply don't cohere; even though you speak truly, you can't keep saying this AND this AND this etc. and get a progressively bigger and better construction. As long as you remain in one place all is well, but the truth doesn't seem to be portable! As Yeats put it "The center cannot hold. Mere anarchy is loosed upon the world".

Such is the time of deconstruction. Save what you can of the parts, but prepare to assemble them in new ways. And don't be in too much of a hurry to reconstruct. The first requirement is to explore, to take note of what is changing during your enlarged travels, and of what stays the same. And then of course to make maps – not maps as static pictures, but maps as records and stories of your explorations, both real and imagined. Let the new order emerge, don't try to force it.

Today with quantum mechanics the truth seems to have lost its portability when you travel from one to another of what Bohr called "experimental arrangements" on a quantum object. You can no longer assemble your observations from such various viewpoints into a single piece of knowledge about the state of the object; again, the facts simply don't cohere. But now the trouble doesn't seem to be with the relativity of ideas like up and down, but with coherence itself. The reason you can't say this AND this AND this etc. is not that this, this and this etc. are relative, but that AND itself is relative! Conjunction has fallen apart; what remains is a plurality of incompatible junctions.

I think we are in for a period of exploration that will produce surprises like nothing we have ever seen before. I hope the kind of theorizing offered in this paper will prove useful to the new mapmakers.

## APPENDIX. A small house that can't be flattened.

The concept of flattening a set of frames is actually a bit more complicated than was indicated in the main text.

House. Define a *house* H as a set of quantum proposition, i.e. ortho–projections on Hilbert space, such that if P and Q are in H, then 1–P is in H and if PQ = QP then PQ is in H. Given any set S of projections, we will speak of the house *generated* by S as the smallest house containing S. It's clear that every house is a union of overlapping frames each of which is a maximal subset of commuting projections.

Implies. Given propositions P and Q, define P<Q, which we read as "P implies Q", or "All P's are Q's", to mean P&Q = P. For quantum propositions this of course means PQ = P.

Flattening. Given a house H and Boolean algebra B, define a *flattening* of H into B as a 1–1 mapping f(P) of H into B such that

F1. f(1–P ) = ~f(P)

F2. If PQ = QP then f(PQ) = f(P)&f(Q)

F3. If f(P)<f(Q) then P<Q.

F1 and F2 say that every frame of H is mapped onto a factor of B, which is how I described a flattening in the main text. There is a strong empirical argument for F3. Note first of all that if P<Q is false then there will be some repetitive experimental arrangement in which not all P's will be Q's; this follows from Born's probability rule interpreted as governing observable relative frequencies. Now if B is to be a Boolean model of the situation, then certainly it must exhibit this experimentally confirmable fact, i.e. not all f(P)'s can be f(Q)'s. If they are, i.e. if f(P)<f(Q), then P<Q, which is F3.

We shall now prove the following

Theorem: Given any three orthogonal states, there are nine propositions definable in terms of these three states that generate a house for which there is no function f that satisfies F1 and F2 without violating F3. Such a violation can be quantified in terms of observable relative frequencies, and amounts to a deviation of more than 8% from the value predicted by quantum mechanics.

Proof:

Let b, b' and c be orthogonal states, i.e. (ortho) projections onto orthogonal 1–dimensional subspaces. Let's also represent these states by unit vectors, to which we'll give the same names b, b' and c. Define four more state vectors as follows (k here is 1 over the square root of 2):

| | |
|---|---|
| $p := k(c+b)$ | $p' := k(c+b')$ |
| $q := k(c-b)$ | $q' := k(c-b')$ |

We are now only concerned with ortho–projections, which we can represent by those subspaces that are their ranges. Their logic is then represented by the following subspace relations: ~P is the ortho–complement of P, P&Q the is set intersection of P and Q, and P OR Q is defined as ~(~P&~Q), which is the subspace sum of P and Q, and P–Q is defined as P&(~Q). We shall only consider these operators to be defined if PQ = QP, which means that they describe Boolean relations, i.e. relations among propositions within a single Boolean frame.

Since b, b' and c are orthogonal and thus commuting, we can define the 2–dimensional propositions B and B' as follows:

B := (b OR c)

B' := (c OR b')

Note that p and q are orthogonal as are p' and q'. This means that we can write B = (p OR q) and B' = (p' OR q').

| | | | | |
|---|---|---|---|---|
| | p | | p' | |
| b | B | c | B' | b' |
| | q | | q' | |

Define U := (B OR B') = (b OR c OR b'). Define P as the 2–dimensional proposition which, as a subspace, is spanned by p and p', and Q as that spanned by q and q':

p P p'

b B c B' b' = U

q Q q'

Since p and p' are linear combinations of b, c and b', $P<U$, i.e. $PU = UP = P$. Similarly $Q<U$. This means that $U-P$ and $U-Q$ are well-defined, and they are 1-dimensional since P and Q are 2-dimensional and U is 3-dimensional.

Now let's consider the house H generated by b, c, b', p, p', q, q', P and Q. We've met three of its other members: B, B' and U, and these are the only other members we need be concerned with. Suppose we try to flatten H with a function f, and we get as far as satisfying F1 and F2. Let's see what this entails:

Since B = (p OR q) we have f(B) = (f(p) OR f(q)), and similarly, f(B') = ((f(p') OR f(q')). Since U = (B OR B'), we have f(U) = (f(B) OR f(B')) = ( f(p) OR f(q) OR f(p') OR f(q')).

f(p')

f(p) f(B')

f(U) f(B) f(q') =

f(q)

Since $p<P$ and $p'<P$, by F2 we have $f(p)<f(P)$ and $f(p')<f(P)$, and similarly for q and q'. Thus $f(U) < (f(P) \text{ OR } f(Q))$. But we know that $P<U$ and $Q<U$, so $(f(P) \text{ OR } f(Q)) < f(U)$, which means that f(U) = (f(P) OR f(Q)). From this we conclude that $f(U-P) < f(Q)$.

Now if F3 were true, we would have $U-P < Q$. Is this in fact the case? The easiest way to find out is to build a simple three dimensional model from which one can read off the relevant trigonometry. At the bottom of the page are two cutouts for making such a model; the triangles represent the diagonal planes of a pyramid which shows the angles among p, p', q, q', b, and b', the edge lines, and c, the common center line. P is the plane of the pyramid face containing the edges p and p', Q the same for edges q and q'. U is physical 3–space.

U–P is a line perpendicular to P. If, as F3 requires, that line is contained in Q, i.e. if U–P < Q, then Q would have to be perpendicular to P. A quick look at the pyramid shows that this is not true; a calculation shows that the actual angle between P and Q is about 73 degrees. Thus f cannot satisfy F3; the house H cannot be flattened.

By Born's rule, the 73 degree angle means that about 8% of U–P's are not Q's. But all of f(U–P)'s are f(Q)'s. Thus any model of H based on a mapping f that identifies all of its frames with factors of a single frame must lead to observational errors of at least 8% in relative frequencies. Any mapping of H into a Boolean frame that is consistent with the observed relative frequencies must sever some of its inter–frame bonds, i.e. some of its von Neumann equivalences. Since there is a different house H for any three orthogonal states, any Boolean model of quantum logic must tear it to shreds.

## *Footnotes.*

*(1) A Hilbert space is a vector space over the complex field with an inner product. A projection P is a linear operator such that PP = P. P is specified when we specify two subspaces: the range of P, and its null space, the latter being the set of all vectors v such that P(v) = 0. A projection is called an ortho–projection if its null space is orthogonal to its range. The null space of an ortho–projection is the ortho–complement of its range, so that an ortho–projection can be specified by specifying its range alone.*

*(2) Proof: Suppose PQ is a projection. Then PQ is self–adjoint, i.e. PQ = (PQ)* = Q*P* = QP, so P and Q commute. But if PQ = QP, then (PQ)(PQ) = (PP)(QQ) = PQ, so PQ is a projection.*

*(3). Recall that in Part I we saw that the complement operator ~J can be written as 1–J, where 1 is the identity operator. Since subtraction and addition are the same operator in the binary field, this becomes ~J := 1⊕J for primary space, which reveals the interesting parallel between quantum negation for projections and primary negation for expressions.)*

## *References.*

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# CONSTRUCTION OF THE DIRAC EQUATION

Pierre Noyes

A bit-string construction of all solutions of the free particle Dirac equation connecting two space-time events has recently been presented and subjected to searching criticism and discussion in Pat Suppes' seminar (Philosophy 242a). When it comes to comparison with experiment, all of relativistic quantum scattering theory, and second quantized relativistic field theory, has to be reduced to algebraic expressions which can be computed from these single particle "solutions" once the particle number "space" has been given appropriate finite and discrete extensions. That we can construct this same basis from bit-string physics should help convince some establishment physicists that we are modeling the same physics that they are.

Our construction starts from the firing of two counters. We first consider all possible "trajectories" consisting of left and right space-time steps connecting the two firings. The step length is the Compton wavelength ($h/mc$); each step is executed at the speed of light. Feynman introduced a similar model some time ago. He used *imaginary* step lengths proportional to $h/mc$. He had to let his step-lengths approach zero in order to reproduce the oscillating Bessel function series which are the computable representations of the solutions of the Dirac equation. We obtain these series directly — keeping our *real* steps *fixed* in length. We succeed in doing this because our bit-string theory allows us to use the two physical ideas of "background" and "conservation laws" in a way that was not available to Feynman.

Our theory allows events to occur only at intervals which are an integral number of deBroglie wavelengths apart; since the theory also requires all masses to be commensurable, this result insures (relativistic) 3-momentum conservation. Enlarging our "free particle" problem to include global 3-momentum conservation, we can (for purposes of the current construction lump) the massive laboratory background in which the counters are embedded into a single external coordinate with large mass, undergoing the same type of zig-zag motion at a much higher frequency. A simple bit-string model of this situation is provided by comparing two independently generated stings. When one contains a 1 where the other contains a 0 the particle moves to the right and in the converse situation to the left. When a sequence of 1's changes to a sequence of zeros the particle zigs, and in the converse situation it zags. When both strings contain the same symbol (two 1's or two 0's), the local space and time coordinates represented in our discrete version of the Dirac equation do not change. The the second class of cases corresponds to the many possible situations in which something is going on in the background which does not affect our single particle in any significant way. At any such point along its space-time trajectory, the particle neither changes its local position by $h/mc$ nor its local time by $h/mc^2$. In the count of possible cases in a "path space" which includes this background degree of freedom we have many more possibilities than Feynman considered. Yet each of our trajectories can be put into one-to-one correspondence with each of his. Another way of putting it is that we are using a configuration space in which the background dimension(s) are orthogonal to the particle motion, but must be included in the case count. In this way, we arrive at the case count originally derived by David McGoveran from much more general considerations. This allows us to get the pieces of the series solution to the Dirac equation directly while keeping the step length fixed. Our bit-string comparison also allows us to derive the relativistic velocity addition law and establish our discrete version of Lorentz invariance connecting 3 or more counter firings.

The second physical idea we require comes from a still subtler aspect of our discrete theory, which was not fully developed in the seminars. So far as the space-time structure goes, the position-velocity discreteness gives us the usual commutation relations for position, momentum, and angular momentum. We find that space-time ("content") strings with an odd number of 1's represent fermions (half integral angular momentum in units of $\hbar$, which includes the Dirac case) and two of them combine to give a boson (integral angular momentum). But our theory also requires each content string to have a "label" string which represents the quantum numbers of a particle, and when the 0's and 1's are interchanged represents the quantum numbers of the corresponding anti-particle. Putting these together gives us the invariance under particle-antiparticle interchange ("charge conjugation"), left-right reversal ("parity") velocity reversal ("time reversal") which is respected in the laboratory (CPT invariance). Conventional theory derives this most easily from the Dirac equation, and has trouble with more general derivations. For our bit-string theory it is "obvious" from the start; I have called it "Amson Invariance".

Since we are talking about a spinor,we must insure spin conservation. We start by taking account of the spin direction relative to the direction of motion, i.e. the "helicity", both for the overall motion and for the individual steps. We then find that spin conservation requires us to take the difference between the number of cases with an even and an odd number of bends along the trajectory. This gives us the minus sign that Feynman's sleight-of-hand put in by using $(i)^2 = -1$. For two space-like connected events, we invoke Feynman's idea that a particle moving backward in time is the same as an anti-particle moving forward in time. [Of course this was suggested to Feynman by CPT invariance.] Since (for the one-particle equation) the number of particles minus the number of anti-particles must be +1 at the start and end of any observed class of trajectories, we again have to take a *difference* between two classes of calculated trajectories.

In short, our construction works because (a) we include a crude model for the "background" against which the particle is observed and (b) we impose a conservation law.

# A Comment on the Combinatorial Hierarchy

by David McGoveran

*(This note was read before the Light Hearted Philosophy Group, Stanford University, Nov. 6, 1991.)*

I am continually amazed by the abysmal state of modern science. For too long the state of affairs expressed so clearly in the Bohr/Einstein debate (see footnote 1) has been accepted without further questioning; so much so that I sadly confess to viewing most so-called progress in science as little more than so much engineering, data collection, and curve-fitting. But then I am ill-equipped to make such judgements public.

In contrast to this appalling situation, I find the point of view expressed by Kilmister and Bastin refreshing; not only because it attacks foundational issues directly and its practitioners are undaunted by the tremendous difficulties involved, but also because it does so in a manner that I see extending the very essence of relativistic thinking while preserving what I consider to be most valid in Bohr's thinking.

On the one hand, the importance of statistical aspects of nature is reified in the combinatorial aspects of Bastin's and Kilmister's work. At the same time, there is room in the theory for a deterministic view – one which I believe Einstein would have found encouraging even though it is not as "mechanistic" or "materialistic" as he might have preferred. In addition, the whole work hinges on the idea that no meaningful contact can be made with nature except by starting from the dimensionless numbers. All other contact is too theory-laden. This *is* relativity applied to the entirety of empirical science and not just to "reference frames" and "transformation laws". It insists that no predisposition, whether particular coordinate frame or particular units of measure, be treated as intrinsic. The combinatorial hierarchy numbers identify certain important dimensionless constants, and, when coupled with the requirements that the entirety of the generation of these numbers meet certain not-too-hard-to-accept criteria (see footnote 2), they become more compelling in terms of the observed evidence of intrinsic structure in the universe. There is something important here. And it is mathematically unique!

What remains in question is the interpretation, the putting on (or in) of flesh. But this is part of the non-invariant (i.e. the non-intrinsic) aspect of any theory; the tools chosen for the task and the language chosen for describing the task's results can vary in power and context. One must not confuse these with the subject of the task itself. My own efforts have been to emphasize how very much they bring to the task... and thereby to expose the importance of what is truly intrinsic. In psychology this might be called a mathematics of framing. But I have little fear of any success. We hold too many principles to be self-evident; and hold dearly to their being different for each of us.

Footnote 1. If I had to decide, I would say that I fall more on the side of Einstein. I do not believe that the Providential Authorities play dice nor that the Old One is malicious; on the other hand I would not be surprised if man is perverse in his ability to understand nor that the creator has a sense of humor and finds amusement in man's insistence on looking only under the street lights. Einstein felt strongly about the continuum and about the existence of a particular causal structure; I find these to be "artifactual". Bohr insisted on an intrinsic statistical structure; I find this attitude defeatist and unimaginative while greatly admiring his abilities to tease so much prediction out of such a view. For me, the value of the statistics is beyond dispute but the reason for them given by Bohr is nonsense.

Footnote 2. For example, expressibility in a vector space, hierarchically organized, automatic termination, etc. -- I have described these in more detail in *Justifying the Combinatorial Hierarchy,* ANPA 12 Proceedings.


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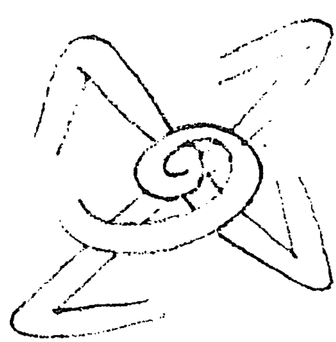

# *PRESIDENT'S MESSAGE*

*By Fred Young*

*Whenever I attend an ANPA meeting at either Stanford or Cambridge I am struck by how many different ways there are of approaching the core set of problems which interest ANPA members.*

*ANPA meetings have always involved a deep fundamental interdisciplinary approach to science, focusing on the overall organization of the universe and our knowledge about it. Now ANPA meetings are no longer the only places that interdisciplinary scientific ideas are discussed. The appearance of new subjects like chaos and fractal geometry has focussed renewed interest on the idea that common functions can be observed in diverse systems. The physics of information and complex systems is now joining conventional particle physics in the quest for fundamental theories. My own research is in the area of complex systems and it seems to be converging on a common basis of understanding with the main ANPA research interests. At ANPA 13 there was a general discussion on the meaning of the combinatorial hierarchy that was continued in the U.S. by myself, Pierre Noyes, David McGoveran and Tom Etter. These discussions helped me to clarify some ideas of how to facilitate intercommunication among ANPA members using very different methods and approaches.*

*I find that it is useful to break things down into self-organization, self-reproduction and self-generation. Self-organization immediately suggests the field initiated by Prigogine which,because of the modern computer, joined with the subjects of chaos and fractal geometry to become non-linear science. Part of this subject involves cellular automata, which gave us the game of life and introduces self-reproduction. This subject goes back a long way in computer science to von Neumann's original self-reproducing cellular* *automata. Self-reproducing and self-organizing systems occur within some already developed system of computation and information transfer.*

*The third category of self-generation deals with the primary organization of information processing which gives rise to both observed reality and observers. Self-organization and self-reproduction occur within pre-existing contexts, and utilize units with pre-existing qualities. This pre-existence is unsatisfactory in a fundamental theory, and studies of self-generation try to understand the primary processes responsible for a universe in which other processes of organization can occur. The study of self-generation is probably the major focus of ANPA members. The combinatorial hierarchy, program universe, McGoveran's foundation for a discrete physics, and Etter's pre-logic all involve studies of a self-generating universe.*

*My own studies indicate that the subjects of self-organization and self-reproduction become highly relevant to fundamental physics when studied using computer simulation. To make the story complete I would have to be able to bring in self-generation, a research task for the future.*

*The computational approach which ANPA has taken since its beginning is becoming increasingly widespread in physics and other sciences. Combining particle physics with information and complex systems as ANPA has done for years is a promising direction for scientific unification. This will become increasingly apparent to other researchers as the idea spreads that information plays a fundamental role in physics. The three "selfs," self-generation, self-organization and self-reproduction, are convenient categories for discussing interdisciplinary subjects and they are likely to be found together in any fundamental theory. Some popular approaches to scientific unification such as superstring theory seem to neglect self-generation and may therefore not be completely fundamental.* ●

## ALTERNATIVE NATURAL PHILOSOPHY ASSOCIATION

### Statement of Purpose

1. *The primary purpose of the Association is to consider coherent models based on minimal number of assumptions to bring together major areas of thought and experience within a natural philosophy alternative to the prevailing scientific attitude. The combinatorial hierarchy, as such a model, will form an initial focus of our discussion.*

2. *This purpose will be pursued by research, conferences, publications and any other appropriate means including the foundation of subsidiary organizations and the support of individuals and groups with the same objective.*

3. *The Association will remain open to new ideas and modes of action, however suggested, which might serve the primary purpose.*

4. *The Association will seek ways to use its knowledge and facilities for the benefit of humanity and will try to prevent such knowledge and facilities being used to the detriment of humanity.*

## ILLUSTRATIONS

All incidental illustrations, unless otherwise noted, are by Suzanne Bristol

Cover: Suzanne Bristol, *The ANPA Question*, 1991
p. 2: Paul Klee, *The Music Teacher*.
p. 51: Suzanne Bristol, *The Gift* (detail), 1989.